

Simplified CDO Analysis

Group Members

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Task 1: Build a simulation model of the aggregate quarterly cash flows of the portfolio, as a function of the quarterly cash flows.

Step 1

- Generated random variables z for 10,000 cases
- Moment matched these random variables by demeaning and dividing by standard deviation of their respective columns. We observe that after moment matching the mean is close to 0.01 and standard deviation is close to 1.
- We fetched 10 z values for 10 bonds on case by case basis depending on the case number mentioned in the Red Box.

Standard independent z values										
Case	Bonds									
1	1	2	3	4	5	6	7	8	9	10
	-0.22	0.75	-0.41	-1.46	-1.32	-0.56	2.26	-1.40	1.27	-0.50
Average	0.01	-0.01	0.00	-0.01	-0.01	-0.01	0.00	-0.01	0.00	0.01
Std	0.98	1.01	1.00	0.99	1.01	1.00	0.99	1.00	0.99	1.00
Random Z numbers (10,000 cases)										
1	-0.22	0.75	-0.41	-1.46	-1.32	-0.56	2.26	-1.40	1.27	-0.50
2	-0.49	-0.73	-0.75	-0.83	1.34	-1.89	0.51	-0.46	0.22	-1.98
3	-0.42	-1.82	1.27	0.98	-0.95	-0.27	0.05	-0.21	0.06	-0.20

Step 2: Transform uncorrelated z values into correlated and find the default dates

- Determined the Target Matrix with the correlation as 20%
- Formulate the Cholesky matrix for this target matrix and cross verify by matrix multiplying the Cholesky matrix with its own transpose to get back the target matrix

[illegible]

- | | | | | | |
|---------------|---------------|--------------|-------------|---------------|--------------|
| Annual PD % | 4% | | | | |
| Quarterly PD% | 1% | | | | |
| Default Dates | | | | | |
| Bond | Independent z | Correlated Z | Probability | [- LN(1-u)] | Default Date |
| 1 | -0.22 | -0.22 | 0.41 | 0.54 | 53 |
| 2 | 0.75 | 0.69 | 0.75 | 1.40 | 140 |
| 3 | -0.41 | -0.31 | 0.38 | 0.47 | 47 |
| 4 | -1.46 | -1.38 | 0.08 | 0.09 | 8 |
| 5 | -1.32 | -1.41 | 0.08 | 0.08 | 8 |
| 6 | -0.56 | -0.82 | 0.21 | 0.23 | 22 |
| 7 | 2.26 | 1.78 | 0.96 | 3.28 | 328 |
| 8 | -1.40 | -1.46 | 0.07 | 0.08 | 7 |
| 9 | 1.27 | 0.92 | 0.82 | 1.72 | 171 |
| 10 | -0.50 | -0.64 | 0.26 | 0.30 | 30 |

[illegible]

Step 1: Assign scheduled cash flows for Tranches A and B, assign the remaining cash flows to Residual class

Tranche	Face Value	Coupon %	PI at midlife rate
Total	100	1.50	100.00
Class A	75	2%	100.00
Class B	25	1.50	100.00

Step 2: Based on these Scheduled Cash flows , we calculated the Simulated Cash Flows.

[illegible]

Step 3: Ran a What-If-Analysis to execute the steps 1-2 for 10,000 cases and retrieve the present values of cash flows for every class case-by case basis.

What-if Table					
				Default Indicators	
Case	Class A	Class B	Residual	A	B
1	20.97	11.46	71.32	0	0
1	20.97	11.46	71.32	0	0
2	20.97	11.46	64.72	0	0
3	20.97	11.46	78.59	0	0
4	20.97	11.46	86.04	0	0
5	20.97	11.46	65.16	0	0
6	20.97	11.46	86.13	0	0
7	20.97	11.46	85.78	0	0
8	20.97	11.46	85.08	0	0
9	20.97	11.46	72.89	0	0
10	20.97	11.46	77.53	0	0

Step 4: Found mean and maximum error for these 10,000 cases of present values for Class A, B and Residual Class as well as counts of Defaults for Class A and Class B.

				Default Indicators	
	Class A	Class B	Residual	A	B
Mean	20.97	11.46	79.25	0	0
Max Error	3.0E-14	2.2E-14	0.23		

Step 5: Found the CDO market value, cost of risk and eventually the value of classes

Covariance Matrix			w'market row sums	Mean	Risk Charge	Value
2.30132E-24	-1.6923E-24	2.3077E-24	2.91672E-24	20.97	5.06906E-25	20.97
-1.6923E-24	1.24446E-24	-1.6997E-24	-2.14755E-24	11.46	-3.73229E-25	11.46
2.3077E-24	-1.6997E-24	136.1078435	136.1078435	79.25	23.65461307	55.59

Bonds Market Value	\$ 88.03
PV Expected cash flows @ Risk Free rate	\$ 111.68
PV Risk charge	\$ 23.65
Grand sum	136.11
Cost of risk	\$ 0.17

Task 3: Finding out highest notional value of Tranche A that will support credit rating of Moody's Aa for Class A.

Step 1: Explicitly laid out the generated cash flows for 10,000 simulations in Task 1-2

Non Defaultable	
Face value (in \$ MM)	10
Principal	4
Coupon Price	0.05
Defaultable	
Principal	6
Coupon Price	0.05

Case	Realized Cash Flows for All Classes																			
	Time (in Quarters)																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1.5	1.5	1.5	1.5	1.5	1.5	1.41	1.23	1.23	1.23	1.23	1.23	1.23	1.23	1.23	1.23	1.23	1.23	1.23	83.23
2	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.23	1.23	1.23	1.23	1.14	1.14	77.14
3	1.5	1.5	1.41	1.41	1.41	1.41	1.41	1.41	1.41	1.41	1.41	1.41	1.41	1.41	1.41	1.41	1.41	1.32	1.32	89.32
4	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	95.41
5	1.5	1.41	1.41	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.23	1.23	1.23	1.23	1.23	1.23	77.14
6	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	95.41
7	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.41	1.41	1.41	95.41
8	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.41	1.41	1.41	1.41	1.41	1.41	1.41	1.41	1.41	1.41	1.41	95.41
9	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.41	1.41	1.41	1.41	1.41	1.41	1.41	1.32	1.32	1.23	1.23	83.23
10	1.5	1.41	1.41	1.41	1.41	1.41	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.32	89.32

Step 2: Compared the generated cash flows to the scheduled cash values. Then used solver to find the maximum notional amount for Class with probability default constraint according to the Moody's Aa Requirement (0.1%)

Face Value	52.52
Number of defaults	13
Probability of default in a 5 year period	0.0013
Probability of no-default in a 5 year period	0.9987
Annual default probability	0.02601%

Solver Parameters

Set Objective:

To: ☒ Max ☐ Min ☐ Value Of:

By Changing Variable Cells:

Subject to the Constraints:

Add

Task 4: Assume that tranche A is priced to yield 50 bppa over the risk-free rate. Using your new notional value for Tranche A in Task 3, keeping the Class B issuance at \$10MM in face value, and the discount rate for B as 4% per year over treasuries, answer the following questions

- 1. What is the probability of default for Class B and the expected loss given default?**

Class B statistics	
No of Defaults	184
P(default in 5 years)	0.0184
P(no defaults in 5 years)	0.9816
Annual default prob	0.00371
Total B cash flows expected per case	12
Total B cash flows actual for 10,000 cases	118927.212
Loss due to defaults	1073
Loss per default	5.830
Loss given default	48.59%

- 2. Value of Class A, B and Residual (equity) [Quarterly]**

Value of class A at 1.5%p.a (Increase of 50bppa on Risk Free Rate = 1% + (50)*(0.01%))	\$ 53.77
Value of Class B at 5%p.a.	\$ 9.48
Value of residuals	\$ 22.40

- 3. ROE for the bank**

ROE per quarter	5.215%
ROE per annum	20.859%

Task 5: Discuss the reasons why the bank might have done this structure

1. Risk Distribution and Reduced Balance Sheet Exposure

The bank, by structuring the CDO into tranches (Class A, Class B, and the residual equity), the bank effectively distributes credit risk among a broad range of investors. High-risk portions, such as the residual equity and lower-rated tranches, are transferred to investors seeking higher returns. This reduces the bank's exposure to default risk on speculative-grade bonds and limits the concentration of risky assets on its balance sheet, contributing to greater financial stability.

2. Income from Structuring and Origination Fees

The bank benefits from significant revenue streams by structuring and issuing CDOs. These upfront fees, collected from investors, provide an immediate source of income, independent of the performance of the underlying debt. By acting as a structuring agent, the bank builds relationships with institutional investors, which could lead to additional business opportunities and recurring income through future deals. This fee income also diversifies the bank's revenue sources beyond traditional lending activities.

3. Capital Optimization and Enhanced ROE

By converting debt into marketable securities and selling them, the bank frees up capital that would otherwise be tied up in risky loans. This allows the bank to continuously originate new loans and reinvest in more profitable ventures, increasing capital efficiency. As a result, the bank can generate a higher Return on Equity (ROE) without being restricted by regulatory capital reserve requirements, maximizing profitability while maintaining financial flexibility.

4. Improved Investor Targeting Through Tranche Customization

The creation of multiple tranches within the CDO allows the bank to cater to a wide range of investor risk appetites. The senior tranches (e.g., Class A) are designed for conservative investors seeking stable returns, while lower tranches appeal to those willing to take on more risk for higher yields. This flexibility increases the marketability of the CDO, as it aligns the bank's product offering with investor preferences, leading to better demand for the CDO tranches and more favorable pricing for the bank.

5. Regulatory and Credit Risk Management

By securitizing and selling the underlying bonds, the bank can achieve capital relief, as regulatory requirements typically impose higher capital charges for speculative-grade assets. Offloading these assets into a CDO reduces the bank's need to hold capital against them, allowing it to manage its balance sheet more efficiently. Additionally, the tranching structure provides credit enhancement for the senior tranches, increasing the likelihood of higher credit ratings. This mitigates overall credit risk exposure and satisfies regulatory requirements while maintaining profitability.