Miniproject 2: Bootstrapping the U.S. Yield Curve: From Bonds to ZCBs: Methodology for the Project

FRE 6103: Valuation

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The objective of this project is to construct a Zero-Coupon Bond (ZCB) yield curve using U.S. government bond data. The yield curve represents the relationship between bond yields and their respective maturities. To achieve this the following metrics were calculated:

- ***Last Coupon date*** using the formula =COUPPCD (Date of Modelling, Maturity Date, coupon payment frequency)
- ***Next Coupon date*** using the formula =COUPNCD Date of Modelling, Maturity Date, coupon payment frequency)
- **Days since last coupon:** Date of Modelling minus Last Coupon date
- **Days to Next Coupon:** Next Coupon date minus Date of Modelling
- **Accrued Interest (in percentage) and Accrued Interest (in dollar)** terms was calculated by applying the following:

A = P x r (d / t)/2	A = Accrued interest P = Face value r = interest rate of Treasury bond d = # of days since last coupon payment t = # of days in current coupon period

- **Invoice Price** = (Bid Price + Ask Price)/2 + Accrued Interest (in Dollars)
- **Time to Maturity = (Maturity Date – Date of Modelling) / 365**

The primary equation used to fit the yield curve for different bonds is as follows:

$$y(t) = a_1 + a_2 * t + a_3 * \ln(t) + a_4 * e^{(a_5 * t)} + a_6 * t^{0.5} + a_7 * t^2$$

This equation models the **yield *y(t)*** as a function of **time to maturity *t***. Each term in the equation contributes to the shape and flexibility of the yield curve. The rationale behind using this equation is as follows:

- **a_1**: A constant term that provides the base level of the yield, ensuring the curve does not cross unreasonable yield levels for very short maturities.
- **$a_2 * t$:** A linear term that captures a direct relationship between the time to maturity and the yield. As t increases, yields typically increase to compensate for the time value of money.
- **$a_3 * \ln(t)$:** A logarithmic term that adds flexibility at the shorter end of the maturity spectrum, as bond yields often change more rapidly for bonds with shorter maturities.
- **$a_4 * e^{(a_5 * t)}$**: An exponential term that captures the long-term behavior of bond yields, which may rise or fall exponentially with increasing maturity.

- **$a_6 * t^{0.5}$:** A square root term that introduces smooth, non-linear growth, allowing for mid-term adjustments in the yield curve.
- **$a_7 * t^2$**: A quadratic term that ensures the curve can adjust its curvature as needed for long-term maturities, allowing for steepening or flattening of the yield curve.

To fit this equation to the bond data, we employed **least squares minimization**, which seeks to minimize the sum of squared errors (SSE) between the observed bond prices and the prices predicted by the model.

The **sum of squared errors (SSE)** for my model was **138.2615**, indicating a good fit between the modeled and actual bond yields.

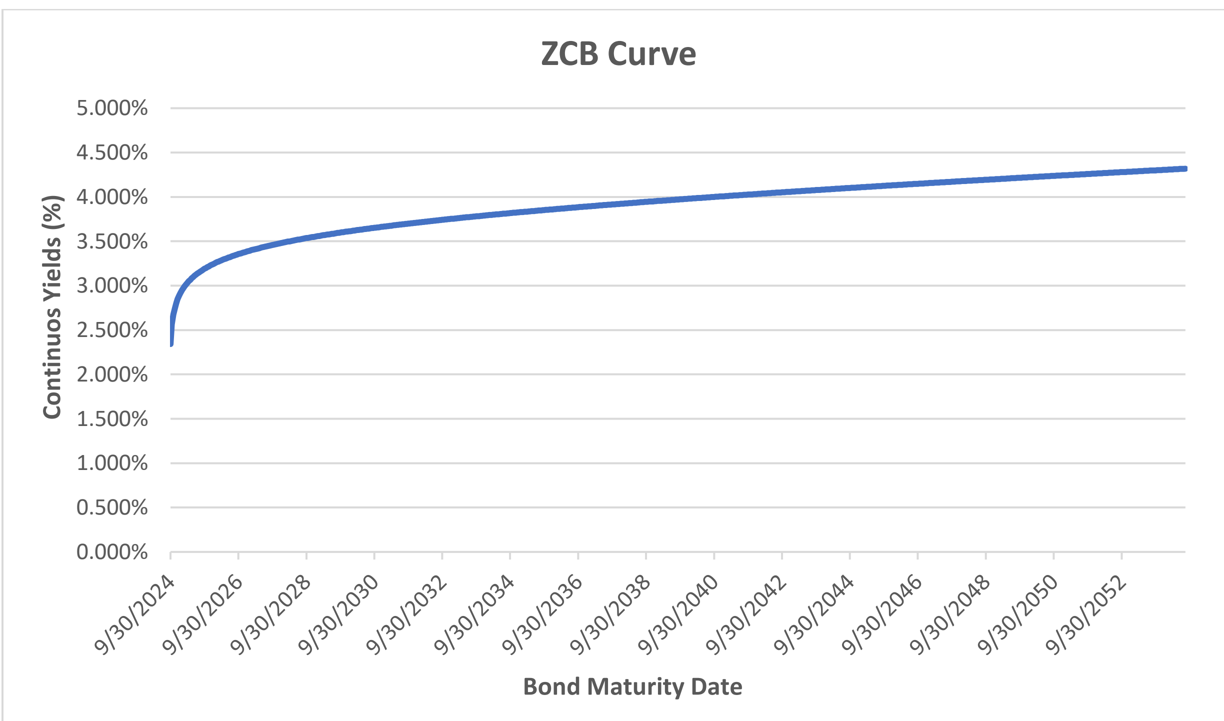
The specific parameter values I obtained from this fitting process were:

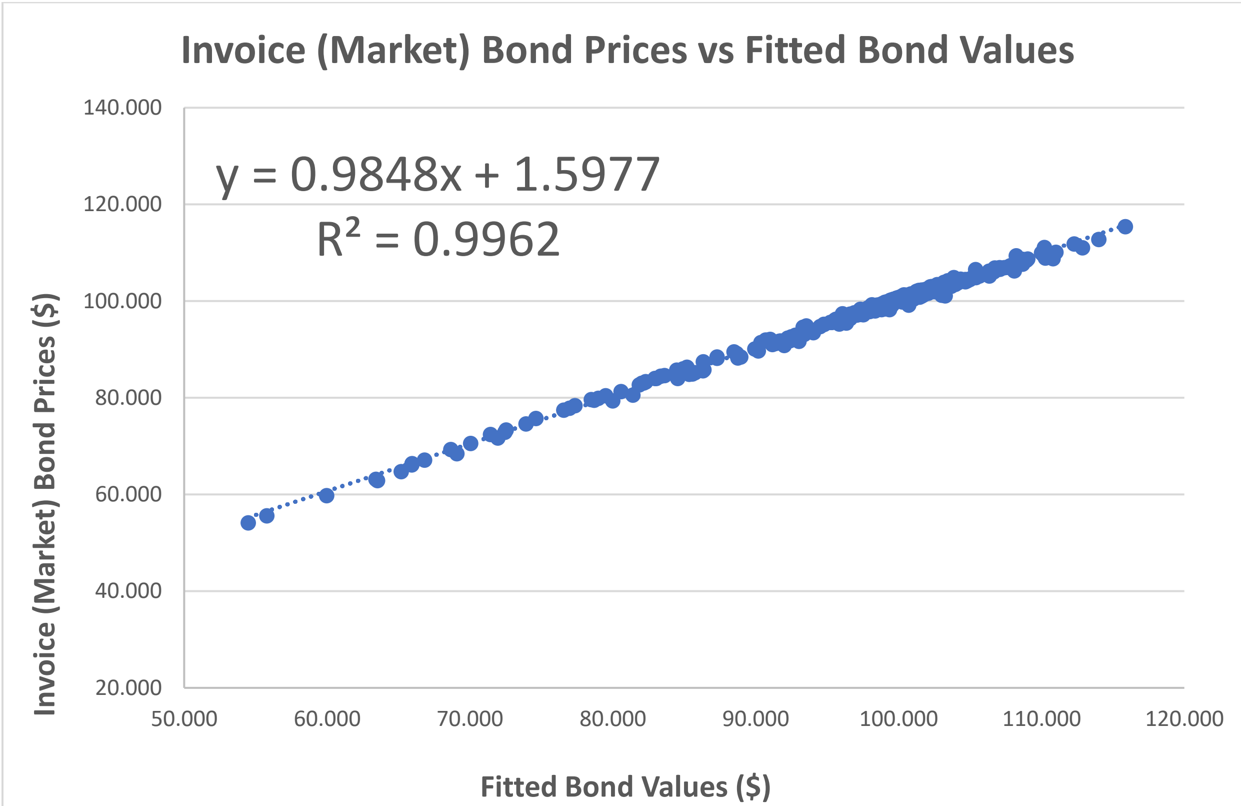
Parameter	Value
a1	0.020377
a2	0.000111
a3	0.002238
a4	0.011329
a5	0.001376
a6	0.000013
a7	0.000000

SSE	138.2615

These parameters provided a well-fitted yield curve that reflects the market behavior for U.S. government bonds.

RESULTS




Invoice (Market) Bond Prices vs Fitted Bond Values
y = 0.9848x + 1.5977
R² = 0.9962
Invoice (Market) Bond Prices ($)
140.000
120.000
100.000
80.000
60.000
40.000
20.000
50.000
60.000
70.000
80.000
90.000
100.000
110.000
120.000
Fitted Bond Values ($)