Valuation MiniProject 5 - Design and Valuation of an Up-and-Out Call Option on the S&P 500: Pricing, Risk Analysis, and Hedging Strategy

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An **up-and-out** call option is a type of barrier option, which is a financial derivative with a payoff contingent on the underlying asset reaching or failing to reach a specific price level during the option's life. A few primary purposes being **Cost Reduction** since up-and-out calls are typically **cheaper** than a plain vanilla call option because of its **barrier feature.** The option is also useful for traders or hedgers who expect the price of the underlying asset to increase but not exceed the **barrier level**, as a result provides **Speculative/Hedging positions.**

We then proceed to choose our parameters, Risk free rate is assumed to be as 1 yr treasury rate on a continuous basis as 4.15% and the dividend yield as SPY's annual yield equivalent to 1.32%., **Sigma**: Average of the implied volatility calls.

Our approach is:

- We start with **simulating stock price paths (**via **Monte Carlo** and **GBM**): We use the GBM model to simulate 10,000 paths of the stock price over time. GBM assumes the stock price changes continuously over time with drift (trend) and randomness (volatility).
- We then **calculate the option payoff**. **For each simulated path**, if the stock price at any point exceeds the barrier($500), the payoff is $0 (option knocked out). Else, the payoff is (**max(S[t] – K,0**), S[t] being the price at maturity, and K=300
- **Discounting** payoffs: The option's value today is the **PV** of the expected payoff. Discount each payoff back to the **present value** via the risk-free rate.
- **Computing Mean and Variance**: We calculate the mean and the variance. Mean of the discounted payoffs is the estimated price of the up-and-out option. Variance gives us a sense of some price uncertainty.
- Calculating the **Trade's PnL:** We hedge the option by holding N shares of the underlying stock. For each simulated path, we compute the PnL of the hedge by comparing the hedging portfolio's value to the option payoff.
- **Minimizing the PnL** variance via Solver: We then use the Excel's solver to find the optimal N shares that minimizes the variance of the hedging PnL, ensuring hedging strategy is stable.

This resulted in average option value = $ 4.76 and the error range was within 2 standard deviations that ensured accuracy.

Hedging Strategy: *For each short position in a call option, the trader should hold **0.6848 shares** in a long position.* The portfolio has a positive expected return of 6%, but the high variance (3.9221) indicates significant return fluctuations. Negative skewness (-1.3203) suggests a tendency for extreme negative returns, while the kurtosis (2.5712) indicates that returns have heavy tails. The 95% and 99% VaR and CVaR values highlight considerable potential losses, with up to 38% and 74% possible under extreme scenarios.

Quantitative Risk: **Vega Risk**: Unexpected volatility increases can raise option values, leading to losses, especially for short up-and-out options. **Mitigation**: Monitor implied volatility and use volatility-neutral hedging strategies. **Delta Risk**: Large price movements near the knockout price can cause significant P&L swings due to imperfect hedging. **Mitigation**: Implement dynamic hedging adjustments in volatile regions. **Downside Risk (Skewness -1.32)**: Negative skewness creates significant downside risk, especially near the knockout threshold. **Mitigation**: Use protective puts to cap extreme losses.