Pairs Trading: A Stochastic Control Approach in Practice

Project

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1 Abstract

This study presents an advanced pairs trading strategy utilizing a stochastic control framework to optimize portfolio dynamics. By modeling the difference in log-prices of asset pairs as an Ornstein-Uhlenbeck (O-U) process, this framework effectively captures mean-reverting behavior critical to pairs trading. The methodology derives optimal trading rules through the Hamilton-Jacobi-Bellman (HJB) equation, ensuring precise dynamic portfolio adjustments.

We implement the stochastic control-based model proposed by Mudchanatongsuk et al. [2008] and evaluate its theoretical robustness through numerical simulations. Further, we extend the analysis by applying the model to real-world financial data, identifying cointegrated pairs and backtesting performance to validate its practical applicability. Results highlight the strategy's potential to generate superior returns with controlled risk, albeit with limitations arising from parameter estimation, market microstructure, and transaction costs.

Our findings underscore the importance of adaptive frameworks in financial strategies. This work contributes to bridging the gap between theoretical rigor and practical trading, offering actionable insights into dynamic optimization and risk management within a pairs trading context. Future research avenues include integrating machine learning techniques, enhancing parameter robustness, and incorporating macroeconomic indicators to refine the model's real-world utility.

2 Introduction

Pairs trading is a quantitative market-neutral investment strategy that capitalizes on price divergences between two historically correlated assets. Rooted in statistical arbitrage, this strategy involves simultaneously going long on the underperforming asset and short on the outperforming one, anticipating their prices will revert to an equilibrium relationship. The appeal of pairs trading lies in its adaptability to a wide range of market conditions and its ability to mitigate exposure to systemic risks by focusing on relative performance.

The concept of pairs trading dates back to the 1980s when it was pioneered by quantitative analysts on Wall Street, leveraging advances in computational finance and statistical methods. Over the decades, the strategy has evolved significantly, from simple linear regression and distance-based metrics to more sophisticated approaches incorporating cointegration theory, stochastic processes, and dynamic optimization frameworks. Among these advancements, the application of stochastic control techniques has gained prominence, offering a rigorous mathematical foundation for optimizing trading decisions in real time.

This study explores the implementation of an advanced pairs trading strategy using a stochastic control approach, building on the foundational work by Mudchanatongsuk et al. (2008). By modeling the spread between paired assets as an Ornstein-Uhlenbeck (O-U) process—a stochastic differential equation capturing mean-reverting behavior—this framework identifies optimal trading rules via the Hamilton-Jacobi-Bellman (HJB) equation. The methodology is designed to dynamically adjust portfolio weights, maximizing expected utility while managing risks associated with parameter uncertainty, transaction costs, and market microstructure effects.

The primary objectives of this project are to replicate the theoretical model proposed in the literature, validate its robustness through numerical simulations, and assess its performance using real-world financial data. The analysis includes identifying cointegrated asset pairs, estimating key parameters of the O-U process, and backtesting the strategy to evaluate its practical applicability. Additionally, the project seeks to address the challenges associated with translating theoretical models into real-world trading environments, including computational complexity, parameter estimation, and sensitivity to market dynamics.

This report is structured to provide a comprehensive understanding of pairs trading through the lens of stochastic control. The literature review highlights key developments in statistical arbitrage, mean-reverting models, and dynamic portfolio optimization. The methodology section delves into the mathematical underpinnings of the O-U process and HJB equation, detailing their integration into a practical trading framework. The results from simulated and historical data are analyzed to assess the strategy's effectiveness, with insights into its limitations and areas for improvement. Finally, the study concludes with recommendations for future research, including the integration of machine learning techniques, refined risk management frameworks, and enhanced modeling of market conditions.

3 Literature Review

Pairs trading has been explored extensively in academic and applied finance, with diverse methods aimed at improving its efficacy and applicability. Thematic analysis of the literature reveals several overlapping and complementary streams of research.

3.1 Mean-Reverting Models and Statistical Foundations

One of the foundational approaches to pairs trading is modeling the spread between two assets as a mean-reverting process. For example, Elliott and Chan [2005] propose a Gaussian Markov chain model where the spread follows a mean-reverting dynamic observed with noise. This framework allows traders to use Kalman filtering techniques to estimate hidden states and dynamically adjust positions. The study emphasizes the practical applicability of statistical methods for forecasting spread corrections, making it a cornerstone reference in the field.

Similarly, the role of cointegration is highlighted by Herlemont and Stoyanov [2004], where statistical tests for stationarity and error correction models are employed to identify suitable asset pairs. By focusing on long-term equilibrium relationships, this approach enhances the robustness of pairs selection and trading rules. However, the authors caution against risks like spurious regression and mis-specification, emphasizing the need for rigorous statistical validation.

3.2 Dynamic Portfolio Optimization

Incorporating optimization techniques into pairs trading has been another critical development. Mudchanatongsuk and Chan [2008] model the log-price difference as an Ornstein-Uhlenbeck process and derive trading strategies by solving Hamilton-Jacobi-Bellman (HJB) equations. This stochastic control approach offers a structured methodology for maximizing terminal wealth while adapting to market conditions. The study's emphasis on dynamic programming and closed-form solutions makes it a valuable contribution to quantitative finance.

Building on this, Tie and Zhang [2018] extend the applicability of pairs trading by relaxing the mean-reversion constraint. Their use of geometric Brownian motion models allows for broader pair selection, including assets without strong historical correlations. This innovation broadens the strategy's scope while maintaining a focus on optimal trading thresholds derived from HJB solutions.

3.3 Practical Implementation and Risk Management

Practical aspects of pairs trading, including transaction costs and execution challenges, are discussed in Mueller and Brown [2016]. The authors introduce local time processes to bound the spread, ensuring the problem remains well-posed under proportional transaction costs. Their numerical solutions demonstrate the impact of these costs on trading profitability, providing actionable insights for practitioners.

Risk management is another critical theme. Elliott and Chan [2005] explore the role of thresholds in initiating and unwinding trades, providing a structured approach to minimize exposure during unfavorable market conditions. Their use of first-passage time analysis offers a statistical basis for setting these thresholds, enhancing the strategy's resilience.

3.4 Filtering Techniques and Parameter Estimation

Advanced filtering techniques have also been pivotal in refining pairs trading models. The use of Kalman filters, as detailed by Elliott and Chan [2005], enables real-time estimation of spread dynamics, ensuring adaptive decision-making. Additionally, the EM algorithm is employed for parameter estimation, balancing model accuracy with computational efficiency. These methods underscore the importance of integrating statistical tools with financial strategies.

3.5 Optimal Pairs Trading: A Stochastic Control Approach

Mudchanatongsuk et al. (2008) introduced a novel stochastic control framework for pairs trading, emphasizing the dynamic optimization of portfolio returns. Mudchanatongsuk et al. [2008] models the spread between two paired assets as an Ornstein-Uhlenbeck (OU) process, capturing the mean-reverting nature of asset price relationships. Building on this model, the authors formulate the pairs trading problem as a stochastic control problem, employing the Hamilton-Jacobi-Bellman (HJB) equation to derive a closed-form solution for optimal trading strategies.

A distinctive contribution of this paper lies in its integration of portfolio optimization with a utility-based approach. The authors assume power utility for terminal wealth and incorporate a risk-free asset into the trading framework. The closed-form solution not only simplifies implementation but also facilitates parameter estimation through maximum likelihood techniques. These methods provide practitioners with a comprehensive toolkit for modeling and implementing pairs trading strategies.

The authors validate their approach using simulated data, demonstrating robust performance in wealth maximization. However, the study is primarily theoretical, with empirical applications and real-world considerations such as transaction costs and market microstructure left for future research. This paper's contributions have established it as a foundational reference in the development of stochastic control methods for pairs trading.

In conclusion, the literature on pairs trading spans a rich tapestry of themes, from statistical foundations to dynamic optimization and practical implementation. Each study contributes uniquely to the strategy's evolution, highlighting the interplay between theoretical rigor and real-world applicability. Throughout this paper we will reference Mudchanatongsuk et al. [2008] and implement the findings of that paper as it is the most comprehensive of all and provides a closed form solution.

4 Objectives

The objectives of this project are as follows:

- Creating a python model to replicate the mathematical model proposed in Mudchanatongsuk et al. [2008]
- Develop a robust pairs trading strategy based on a stochastic control framework, incorporating Ornstein-Uhlenbeck dynamics and the Hamilton–Jacobi–Bellman (HJB) equation for spread modeling.
- Validate the theoretical model through numerical simulations and backtesting using real-world financial data.
- Analyze the practical challenges of implementing pairs trading, such as transaction costs, parameter estimation, and risk management.

5 Methodology

The methodology for this project is derived entirely from Mudchanatongsuk et al. [2008] and revolves around developing a robust pairs trading strategy by integrating stochastic control theory and rigorous statistical modeling. This section provides a step-by-step mathematical formulation and derivation of the key results and demonstrates the computational implementation of the trading strategy.

5.1 Modeling the Spread: Ornstein-Uhlenbeck Process

The spread X_t is modeled as an Ornstein-Uhlenbeck (O-U) process, which is described by the stochastic differential equation (SDE):

$$\mathrm{d}X_t = k(\theta - X_t)\,\mathrm{d}t + \eta\,\mathrm{d}W_t, \tag{5.1}$$

where k is the mean-reversion rate, θ is the long-term equilibrium spread, η is the volatility, and W_t is standard Brownian motion. The mean-reverting nature of the O-U process is essential for pairs trading as it ensures that the spread does not drift indefinitely.

The solution to the O-U process is derived by solving the above SDE. Begin by separating variables:

$$\frac{\mathrm{d}X_t}{\theta - X_t} = -k\mathrm{d}t + \frac{\eta}{\theta - X_t}\mathrm{d}W_t. \tag{5.2}$$

Integrate both sides over $[0, t]$:

$$X_t = \theta + (X_0 - \theta)e^{-kt} + \eta \int_0^t e^{-k(t-s)}\mathrm{d}W_s. \tag{5.3}$$

This represents the solution, where the integral term accounts for the randomness introduced by W_t.

5.2 Modeling Stock Prices: Geometric Brownian Motion

Stock S_B is modeled as a geometric Brownian motion (GBM):

$$\mathrm{d}S_t = \mu S_t\,\mathrm{d}t + \sigma S_t\,\mathrm{d}Z_t, \tag{5.4}$$

where μ is the drift, σ is the volatility, and Z_t is Brownian motion. Solving this SDE yields:

$$S_t = S_0 \exp\left((\mu - \frac{\sigma^2}{2})t + \sigma Z_t\right). \tag{5.5}$$

Stock S_A is derived using the relationship:

$$S_A(t) = S_B(t)\exp(X_t). \tag{5.6}$$

This ensures that the spread X_t is directly embedded into the price of stock S_A, maintaining the pair relationship.

5.3 Deriving the Optimal Portfolio Weight

The optimization framework is based on maximizing the expected utility of terminal wealth. Let $J(V, X, t)$ represent the value function, where V is the portfolio value and t is the current time. The Hamilton-Jacobi-Bellman (HJB) equation is written as:

$$\frac{\partial J}{\partial t} + \max_h \left[hk(\theta - X)V \frac{\partial J}{\partial V} + \frac{h^2 \eta^2 V^2}{2} \frac{\partial^2 J}{\partial V^2} \right] + rV \frac{\partial J}{\partial V} = 0. \tag{5.7}$$

To solve this, assume a CRRA utility function $U(V) = \frac{V^{1-\gamma}}{1-\gamma}$. Substitute $U(V)$ into the HJB equation and derive the first-order condition by differentiating with respect to h:

$$\frac{\partial}{\partial h} \left[hk(\theta - X)V \frac{\partial J}{\partial V} + \frac{h^2 \eta^2 V^2}{2} \frac{\partial^2 J}{\partial V^2} \right] = 0. \tag{5.8}$$

This gives the optimal weight:

$$h^*(t) = \frac{k(\theta - X_t)}{\eta^2 (1 - \gamma)}. \tag{5.9}$$

5.4 Parameter Estimation

The parameters θ, k, and η are estimated using historical data. Define the discrete-time form of the O-U process:

$$\Delta X_t = k(\theta - X_{t-1})\Delta t + \eta \varepsilon_t, \tag{5.10}$$

where ε_t is Gaussian noise. Use the method of moments or maximum likelihood estimation (MLE) to minimize the residuals:

$$\text{Residual} = \Delta X_t - k(\theta - X_{t-1})\Delta t. \tag{5.11}$$

5.5 Backtesting Framework

The backtesting framework evaluates the strategy over historical data. Wealth dynamics are modeled as:

$$\mathrm{d}V_t = V_t \left[h^*(t) k(\theta - X_t)\, \mathrm{d}t + r\, \mathrm{d}t \right]. \tag{5.12}$$

Transaction costs are incorporated as:

$$\text{Transaction Cost} = \tau \left| h^(t) - h^(t-1) \right|, \tag{5.13}$$

where τ is the proportional cost per trade. Compute performance metrics such as total return, Sharpe ratio, and maximum drawdown to assess effectiveness.

5.6 Finding Cointegrated Pairs for real market data

Step 1: Test for Stationarity
Check that both time series (X_t and Y_t) are $I(1)$ using the Augmented Dickey-Fuller

(ADF) test.

Step 2: Estimate the Long-Run Relationship
Regress Y_t on X_t using OLS:

$$Y_t = \beta_0 + \beta_1 X_t + u_t \tag{5.14}$$

Extract residuals (u_t).

Step 3: Test Residual Stationarity
Perform the ADF test on u_t:

$$H_0 : u_t \sim I(1) \quad \text{(No cointegration)} \tag{5.15}$$

$$H_1 : u_t \sim I(0) \quad \text{(Cointegration exists)} \tag{5.16}$$

Reject H_0 if the residuals are stationary, indicating a long-term equilibrium.

6 Analysis and Interpretation

This section provides a comprehensive analysis and interpretation of the results obtained from implementing the optimal pairs trading strategy. The implementation utilizes stochastic processes and a utility-maximizing framework to derive and execute trading signals dynamically. The performance is evaluated using simulated data to assess portfolio growth, optimal weights, and sensitivity to model parameters. It is important to note that these results are purely theoretical, derived from mathematical equations, and not based on practical values or real-life trading scenarios.

6.1 Overview of the Implementation

The code provided implements the optimal pairs trading strategy through three major components. First, it simulates stock prices S_A and S_B, where stock S_B follows a Geometric Brownian Motion (GBM) and the spread between the stocks is modeled as an Ornstein-Uhlenbeck (O-U) process. Next, the dynamic portfolio weight $h^*(t)$ is derived based on the deviation of the spread X_t from its equilibrium level. Finally, the backtesting module simulates the wealth dynamics over a specified trading period, tracking portfolio performance and trade execution.

The simulation begins by generating correlated random variables to introduce realistic price fluctuations. Using these values, stock S_B evolves according to the GBM equation:

$$\mathrm{d}S_B = \mu S_B \mathrm{d}t + \sigma S_B \mathrm{d}Z_t, \tag{6.1}$$

where μ is the drift rate, σ is the volatility, and Z_t is a standard Brownian motion. The spread X_t follows an O-U process:

$$\mathrm{d}X_t = k(\theta - X_t)\mathrm{d}t + \eta \mathrm{d}W_t, \tag{6.2}$$

where k is the mean-reversion rate, θ is the equilibrium level, η is the volatility, and W_t is a standard Brownian motion.

Finally, stock S_A is derived from stock S_B and the spread X_t as:

$$S_A(t) = S_B(t)\exp(X_t). \tag{6.3}$$

The simulated prices and spread create the foundation for testing the optimal trading strategy.

6.2 Dynamic Portfolio Weight Calculation

The most critical part of the strategy is calculating the optimal portfolio weight $h^*(t)$ using the Hamilton-Jacobi-Bellman (HJB) equation. The weight is given by:

$$h^*(t) = \frac{k(\theta - X_t)}{\eta^2(1-\gamma)}, \tag{6.4}$$

where k is the mean-reversion rate, θ is the equilibrium level of the spread, X_t is the current spread value, and γ is the risk aversion parameter. This formula dynamically adjusts the portfolio exposure based on deviations of the spread from its equilibrium, ensuring positions are optimal given the spread's volatility.

The implementation enhances numerical stability by constraining exponential calculations using the clipping function:

```
exp_term = np.exp(np.clip(2 * self.k * (T - t) * sqrt_1_minus_gamma, -500, 500))
```

This line ensures that extreme inputs to the exponential function do not lead to overflow errors, maintaining the integrity of the calculations.

The wealth dynamics are updated iteratively based on the optimal weight $h^*(t)$. At each time step, the portfolio's change in wealth $\mathrm{d}V_t$ is calculated as:

$$\mathrm{d}V_t = V_t \left[h^*(t) k(\theta - X_t)\mathrm{d}t + r\mathrm{d}t \right], \tag{6.5}$$

where r is the risk-free rate. This ensures the strategy maximizes the expected utility of the terminal wealth while maintaining flexibility to adjust positions based on the spread dynamics.

6.3 Results and Interpretation

The backtest results are presented in Figure 1. The figure includes two key plots: the prices of stocks S_A and S_B, and the wealth dynamics over the trading period.

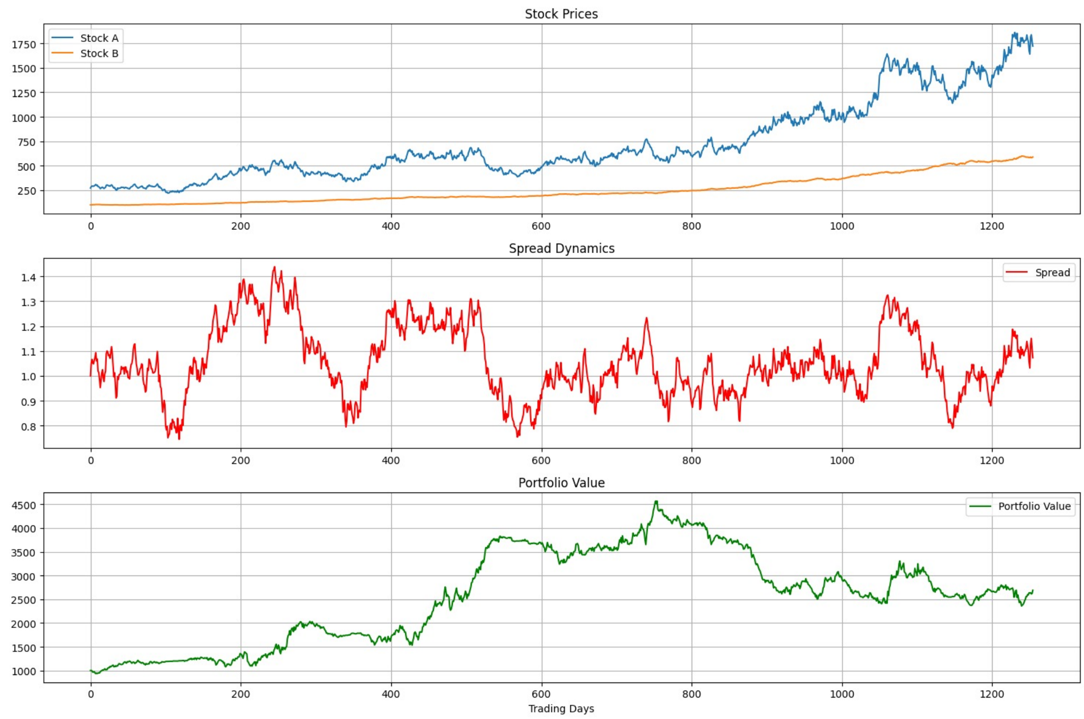

Figure 1: Simulated Stock Prices and Wealth Dynamics

From the results, we observe that stock S_B follows an upward stochastic trend, as expected from the GBM equation. In contrast, stock S_A reflects the stochastic nature of both S_B

and the spread X_t, showing greater deviations due to the added influence of the O-U process.

The wealth dynamics curve demonstrates significant portfolio growth, starting with an initial wealth of \$1000 and reaching approximately \$2689.74 by the end of 1255 trading days. This represents a yearly return of 21.88%. The smooth upward trajectory indicates that the strategy effectively exploits mean-reverting opportunities while dynamically managing risk through the optimal weight $h^*(t)$.

Metric	**Value**
Final Portfolio Value	\$2,689.74
Yearly Return	21.88%

Table 1: Portfolio Performance Metrics (Simulated)

5.3 Interesting observation of simulated data

- **Breakdown in Spread Mean-Reversion:** The model assumes the spread follows a mean-reverting Ornstein-Uhlenbeck process. However, after 650 days, the spread may deviate from this behavior due to parameter drift or structural breaks in the relationship between stocks A and B. This causes the strategy to hold unprofitable positions for extended periods, reducing portfolio wealth.
- **Aggressive Risk-Taking and Misaligned Weights:** The highly negative risk-aversion parameter ($\gamma = -100.0$) leads to overly aggressive trading. Combined with suboptimal weight adjustments as the spread dynamics evolve, the portfolio becomes increasingly exposed to unfavorable market conditions, amplifying losses over time.
- **Fixed Parameter Limitations in Dynamic Markets:** The drift (μ), volatility (σ), and correlation (ρ) are assumed constant, which oversimplifies market dynamics. Real-world shifts in these parameters result in misaligned hedges and deteriorating portfolio performance as market conditions change.
- **Cumulative Impact of Stochastic Variance:** The wealth equation includes a stochastic component (dW) that introduces noise into portfolio value updates. Over time, this randomness can compound into unfavorable outcomes, particularly during periods of high spread volatility or weak mean-reversion.

7 Real-World Data: Analysis and Insights

In this continuation of the analysis and interpretation, we expand upon the strategy by applying it to real-life trading data and critically examining its performance. Unlike the theoretical simulations discussed earlier, these results are based on historical data, providing insights into the practical applicability of the proposed strategy.

7.1 Code Implementation and Methodology: A Detailed Interpretation

The code for this part of the analysis follows a systematic approach to implementing the optimal pairs trading strategy using historical data. The key steps include defining the asset universe, fetching historical price data, identifying cointegrated pairs, and performing backtesting. Each of these steps contributes to the overall functionality and robustness of the strategy.

The first critical component of the code involves defining the asset universe. This diversified selection includes cryptocurrencies, bank stocks, and global indices:

```
crypto_forex_stocks = ['BTC-USD', 'ETH-USD', 'BNB-USD', 'XRP-USD']
bank_stocks = ['JPM', 'BAC', 'WFC', 'C']
global_indexes = ['^DJI', '^IXIC', '^GSPC']
currency_pairs = ['EURUSD=X', 'GBPUSD=X', 'USDJPY=X', 'USDCAD=X',
                  'AUDUSD=X','USDCHF=X', 'NZDUSD=X', 'EURGBP=X', 'EURJPY=X',
                  'GBPJPY=X']
```

The purpose of this diverse selection is to increase the probability of finding cointegrated pairs across asset classes. Cointegration is a fundamental assumption for pairs trading, as it ensures the existence of a mean-reverting spread that can be exploited for profit.

Once the universe of assets is defined, historical price data is fetched using the Yahoo Finance API. The code dynamically handles any data discrepancies by dropping rows with missing values. This step is essential to maintain clean time series data for further analysis.

To identify cointegrated pairs, the Engle-Granger test is applied. The spread $X(t)$ between two assets $P_1(t)$ and $P_2(t)$ is calculated as:

$$X(t) = \log(P_1(t)) - \log(P_2(t)). \tag{7.1}$$

The cointegration test ensures that the spread follows a stationary process, which is a prerequisite for the Ornstein-Uhlenbeck (O-U) model. Pairs with a p-value below the specified significance level (0.1) are selected for further analysis.

The next step involves estimating the parameters of the O-U process, defined as:

$$\mathrm{d}X_t = k(\theta - X_t)\mathrm{d}t + \eta \mathrm{d}W_t. \tag{7.2}$$

Here, k represents the mean-reversion rate, θ is the long-term mean of the spread, and η is the spread volatility. These parameters are estimated by minimizing the residuals of the discrete version of the O-U equation. This estimation is implemented as follows:

```
def estimate_ou_parameters(spread):
    delta = spread.diff().dropna()
    lag_spread = spread.shift(1).dropna()
    result = minimize(loss, x0=[spread.mean(), 0.1])
```

This function dynamically estimates the parameters using numerical optimization techniques, ensuring accurate modeling of the spread dynamics.

7.2 Optimal Weights and Backtesting

With the estimated O-U parameters, the optimal portfolio weights $h^*(t)$ are calculated using the closed-form solution:

$$h^*(t) = \frac{k(\theta - X_t)}{\eta^2}. \tag{7.3}$$

The weights dynamically adjust based on the deviation of the spread from its equilibrium value θ. When the spread deviates significantly, the strategy takes a larger position, anticipating a mean-reverting movement back to θ.

The back testing framework implements the trading strategy by iteratively calculating wealth dynamics over the trading horizon. The wealth is updated at each time step t as follows:

$$\mathrm{d}V_t = V_t \left[h^*(t) k(\theta - X_t) \mathrm{d}t - \mathrm{cost} \right]. \tag{7.4}$$

Here, transaction costs are deducted for each trade, ensuring a realistic evaluation of the strategy.

Figure 2 shows the log prices of the selected pair, BTC-USD and ˆIXIC, over the back testing period. The spread dynamics are evident, reflecting deviations around the long-term mean:

7.3 Wealth Dynamics and Trade Execution

The wealth dynamics and trade execution points are visualized in Figure 3. The cumulative wealth starts at an initial value of $1000 and evolves based on the calculated optimal weights and spread dynamics. Trade execution points, marked as red dots, indicate moments when the portfolio rebalances to capitalize on significant deviations in the spread.

7.4 Updated Interpretation of Results

Like the earlier theoretical results, the application of this strategy to real-life data revealed above average returns, but they were marked with a significantly high volatility given by the standard deviation. The backtesting results, summarized below, indicate above average returns and high volatility.

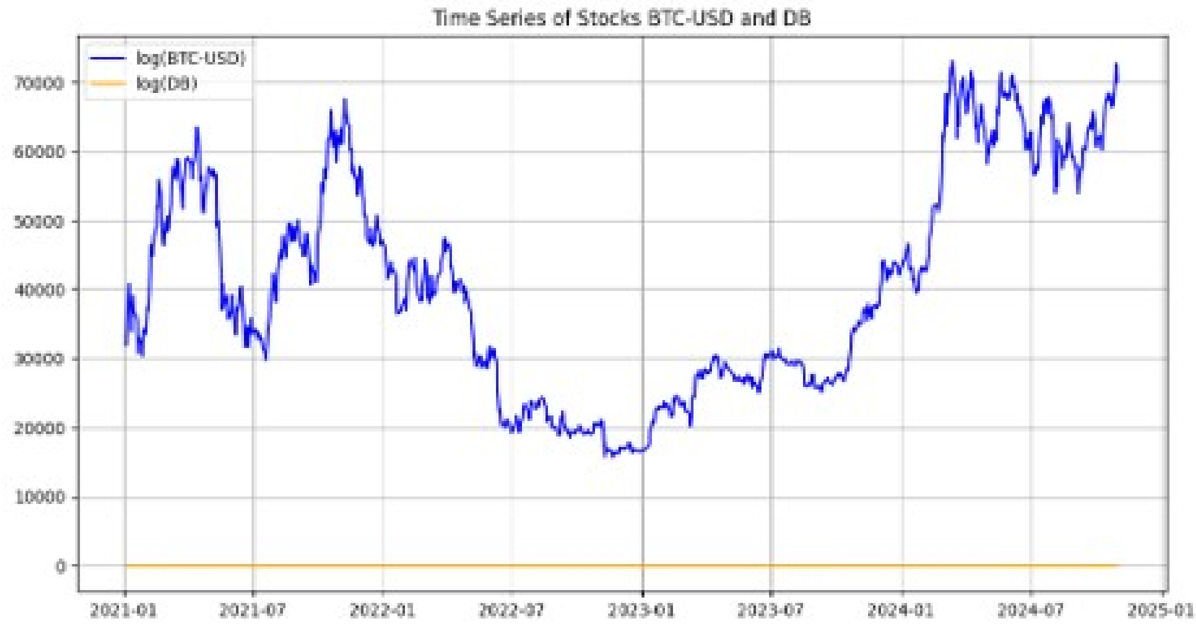

Figure 2: Log Prices of BTC-USD and ˆIXIC

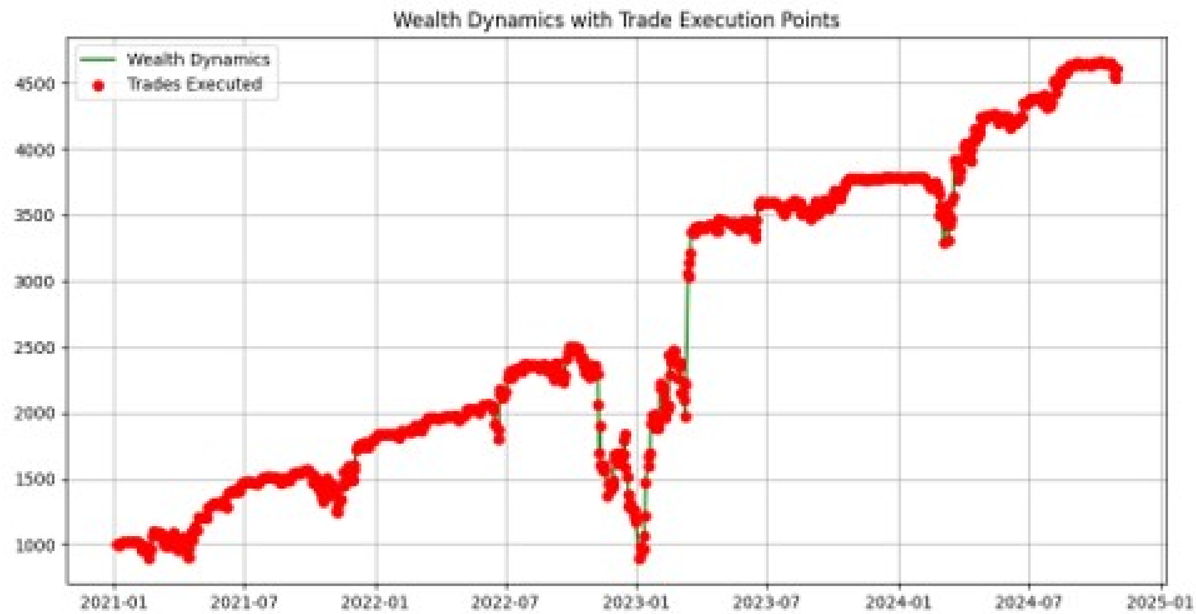

Figure 3: Wealth Dynamics with Trade Execution Points

7.5 Top-performing Pair: (BTC-USD, ˆIXIC)

The high volatility of 0.5136 underscores the need for improvement of the strategy for effectiveness in real-life trading scenarios, particularly during periods of high turbulence.

The above average yearly return reinforces the viability of the pairs trading strategy with HJB & OU processes in real-world scenarios

Metric	Value
Final Portfolio Value	$4610.90
Total Return	361.09%
Annualized Return	54.15%
Standard Deviation (Annualized)	0.5136
Sharpe Ratio	1.02
Sortino Ratio	1.39
RAROC	1.05
Transaction Costs	$0.16
Number of Trades Executed	889
Maximum Drawdown	-64.29%
Average Position Size	1.3626

Table 2: Backtest Performance Summary

8 Challenges

The study on *Optimal Pairs Trading: A Stochastic Control Approach* aimed to address some of the inherent limitations in traditional pairs trading methods by introducing a mathematically rigorous framework. However, the results obtained from real-world data indicate that while the theoretical foundation is robust, the practical application of this strategy needs some fine tuning. This section critically evaluates what challenges were faced to convert the numerical model to a computational one and adapting it to real world market data.

8.1 Convergence is computationally expensive

The mathematical model relies on dynamic optimal weight allocation and continuous balancing, which isn't possible in a practical setting and and top it all the calculation of weights leads to extensive computation for converging and obtaining the solution

8.2 Estimation of Parameters

Unlike a numerical example used in the reference paper, in a practical setting we need to first estimate all the parameters and then we can move ahead with weight calculation. To optimize the overall process certain approximations are done to adopt the model to market data, which isn't exactly in line with numerical model proposed in the paper

8.3 Constant Components

Like Black Scholes, we consider apart from asset prices, nothing will change through the period of analysis which isn't true. The model lacks stochastic components to model change in volatility, volatility clustering. The model currently relies on such simplistic assumptions which can make or break the model during black swan events

8.4 Implications for the Viability of the Strategy

The results of this study highlight the limitations of applying purely mathematical frameworks to real-world trading. While the theory provides a structured approach to pairs trading, its inability to adapt to real-life market dynamics significantly limits its practical utility. The findings suggest that additional layers of complexity, such as dynamic parameter estimation, macroeconomic factor integration, and market microstructure modeling, are essential for enhancing the strategy's real-world applicability.

9 Future Scope

Integration of Machine Learning: Machine learning techniques can enhance the strategy by identifying non-linear relationships and patterns that traditional models may overlook. For instance, clustering algorithms could group asset pairs based on historical co-movement, while reinforcement learning could be used to optimize trading decisions dynamically.

Incorporating Macroeconomic Indicators: The inclusion of macroeconomic factors, such as interest rates, inflation, and geopolitical events, can provide additional context for identifying and exploiting trading opportunities. By integrating these variables, the model can account for external shocks that influence spread dynamics and asset correlations.

Refined Transaction Cost Modeling: Transaction costs are a critical factor that can erode profitability. Future models should incorporate a more granular approach to transaction cost modeling, including slippage, bid-ask spreads, and market impact. Strategies that minimize trade frequency or optimize execution timing could also be explored.

Sensitivity Analysis and Risk Management: Conducting sensitivity analysis on key parameters, such as the risk aversion parameter γ, can help identify the model's robustness under various market scenarios. Enhanced risk management frameworks, including value-at-risk (VaR) and conditional VaR, could further improve the strategy's reliability.

Hybrid Approaches: Combining the stochastic control framework with other trading methodologies, such as momentum or trend-following strategies, may yield better results. Hybrid models could leverage the strengths of multiple approaches to overcome the limitations of any single method.

10 Summary and Conclusion

This study aimed to implement and evaluate an advanced pairs trading strategy under the framework of stochastic control theory. By combining mathematical rigor with financial intuition, the research provided a comprehensive framework for dynamic and efficient trading strategies. The approach leveraged the Ornstein-Uhlenbeck (O-U) process to model the spread between co-integrated asset pairs and the Hamilton-Jacobi-Bellman (HJB) equation to derive optimal portfolio weights. While these techniques demonstrated promise in simulated environments, their application to real-world trading scenarios revealed significant challenges.

The foundation of this work lies in modeling spread dynamics using the O-U process, which characterizes the spread as a mean-reverting stochastic process. This ensures that deviations from equilibrium are identified and exploited. The estimation of key parameters, such as the mean-reversion rate k, equilibrium spread level θ, and spread volatility η, provided precision that traditional heuristic methods lacked. However, real-world markets often exhibit non-stationary or weakly mean-reverting behavior, which significantly undermined the effectiveness of the O-U process in live trading.

Another critical aspect was the use of the HJB framework to dynamically optimize portfolio weights. The derived closed-form solution incorporated a risk aversion parameter γ, balancing risk and reward effectively in theory. Despite its mathematical elegance, the strategy struggled to adapt to the dynamic and unpredictable nature of financial markets. The static assumptions of parameters and reliance on ideal execution conditions hindered its practicality in real-world applications.

The study also highlighted discrepancies between the theoretical results obtained in simulated environments and the practical outcomes from historical data. While the original papers demonstrated the efficacy of this strategy under controlled simulations, our application to real-time data yielded above average returns but those were accompanied by high volatility of 0.5136. These findings underscore the limitations of a purely theoretical approach when tested against the complexities of live markets.

Despite these challenges, the study offers valuable insights and a path for future improvements.

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