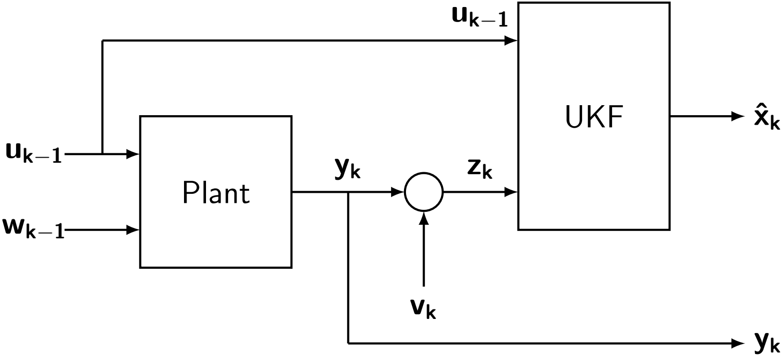
Kalman Filter Flow Diagram



Note: In this scenario, since there is no input u, we assume that there is also no process noise w.

Sigma Point Transformation

- **Step 1: Unscented Transformation (UT)**
  Transform the corrected state and covariance to sigma points:

$$\mathbf{x}_{\mathrm{cor}_{k-1}} \longrightarrow \mathbf{X}_{\mathrm{cor}_{k-1}} (\sigma \text{ points})$$

  We will spread $\mathbf{2N}+\mathbf{1}$ sigma points around the state estimate as follows:

$$\begin{aligned} & = \mathbf{x}_{\mathrm{cor}_{k-1}}, \mathbf{i} = 1 \\ \mathbf{X}_{\mathrm{cor}_{k-1}} & = \mathbf{x}_{\mathrm{cor}_{k-1}} + \sqrt{(\mathbf{N}+\lambda)\mathbf{P}_{\mathbf{cor_{k-1}}}}, \mathbf{i} = \mathbf{i}+1 \\ & = \mathbf{x}_{\mathrm{cor}_{k-1}} - \sqrt{(\mathbf{N}+\lambda)\mathbf{P}_{\mathbf{cor_{k-1}}}}, \mathbf{i} = \mathbf{i}+\mathbf{N}+1 \end{aligned}$$

State Prediction

- **Step 2: Propagation through Process Model**
  Each sigma point is propagated using the nonlinear function:

$$\mathbf{X}_k = \mathbf{f_d}(\mathbf{X}_{\mathrm{cor}_{k-1}})$$

Predicted State Mean

- **Step 3: Predicted State Mean**
  The predicted state is calculated as the weighted sum of the propagated sigma points:

$$\mathbf{x}_{\mathsf{pre}_k} = \sum_{\mathbf{i}=1}^{2\mathbf{n}+1} \mathbf{w_m}^{(\mathbf{i})} \mathbf{X}_k^{(\mathbf{i})}$$

Covariance Prediction

- **Step 4: State Deviation**
  The deviation of each sigma point is computed as:

$$\Delta \mathbf{x}_k = \mathbf{X}_k - \mathbf{x}_{\text{pre}_k}$$

- **Step 5: Predicted Covariance**
  Covariance is calculated as:

$$\mathbf{P}_{\text{pre}_k} = \Delta \mathbf{x}_k \text{diag}(\mathbf{w_c}) \Delta \mathbf{x}_k^T + \mathbf{Q_k}$$

Measurement Update (Part 1)

- **Step 6: UT Transfer for Measurement**
  Transform the predicted state and covariance to sigma points:

  $$\mathbf{x}_{\mathrm{pre}_k} \longrightarrow \mathbf{X}_{\mathrm{pre}_k}(\sigma \text{ points})$$

  We will spread $\mathbf{2N} + \mathbf{1}$ sigma points around the state estimate as follows:

  $$\mathbf{X}_{\mathrm{pre}_k} \begin{aligned} &= \mathbf{x}_{\mathrm{pre}_k}, \mathbf{i} = 1 \\ &= \mathbf{x}_{\mathrm{pre}_k} + \sqrt{(\mathbf{N}+\lambda)\mathbf{P}_{\mathbf{pre_k}}}, \mathbf{i} = \mathbf{i} + 1 \\ &= \mathbf{x}_{\mathrm{pre}_k} - \sqrt{(\mathbf{N}+\lambda)\mathbf{P}_{\mathbf{pre_k}}}, \mathbf{i} = \mathbf{i} + \mathbf{N} + 1 \end{aligned}$$

Measurement Update (Part 2)

- **Step 7: Measurement Prediction**
  Nonlinear measurement model is applied to each sigma point:

$$\mathbf{Z}_k = \mathbf{h}(\mathbf{X}_{\text{pre}_k})$$

- **Step 8: Predicted Measurement**
  The predicted measurement is computed as:

$$\mathbf{z}_{\text{pre}_k} = \sum_{\mathbf{i}=1}^{2\mathbf{n}+1} \mathbf{w_m}^{(\mathbf{i})} \mathbf{Z}_k^{(\mathbf{i})}$$

Measurement Innovation and Covariance Updates I

- **Step 9: Measurement Innovation**
  The measurement innovation (or residual) is calculated as the difference between the actual measurement and the predicted measurement:

$$\Delta \mathbf{z}_k = \mathbf{Z}_k - \mathbf{z}_{\text{pre}_k}$$

- **Step 10: Predicted Measurement Covariance**
  The predicted measurement covariance is calculated using the sigma points:

$$\mathbf{P}_{z,\text{pre}_k} = \Delta \mathbf{z}_k \text{diag}(\mathbf{w_c}) \Delta \mathbf{z}_k^T + \mathbf{R}_k$$

Measurement Innovation and Covariance Updates II

- **Step 11: Cross-Covariance**
  The cross-covariance between the state and the measurement is computed as:

$$\mathbf{P}_{xz,\text{pre}_k} = \Delta\mathbf{x}_k \text{diag}(\mathbf{w_c}) \Delta\mathbf{z}_k^T$$

Kalman Gain and State Correction

- **Step 12: Kalman Gain**
  Kalman gain is calculated as:

$$\mathbf{K}_k = \frac{\mathbf{P}_{xz_{\text{pre}}k}}{\mathbf{P}_{z_{\text{pre}}k}}$$

- **Step 13: State Correction**
  The state is corrected based on the measurement residual:

$$\mathbf{x}_{\text{cor}_k} = \mathbf{x}_{\text{pre}_k} + \mathbf{K}_k(\mathbf{z}_k - \mathbf{z}_{\text{pre}_k})$$

Covariance Correction

- **Step 14: Correct the Covariance**
  The covariance is updated as:

$$\mathbf{P}_{x\mathrm{cor}_k} = \mathbf{P}_{x\mathrm{pre}_k} - \mathbf{K}_k \mathbf{P}_{z_{\mathrm{pre}\,k}} \mathbf{K}_k^T$$