Instructions:

- This assignment is individual task. Please don't copy solutions from any sources.
- Avoid verbosity.
- The assignment can be submitted using MS Word .doc or in latex using the attached text file. The solution for each question should be written in the solution block in space already provided in the text file. **Handwritten/Scanned assignments will not be accepted.**

1. **Partial Derivatives**

(a) Find the derivative of $g(\rho)$ with respect to ρ where $g(\rho)$ is given by,

$$g(\rho) = \frac{1}{2}\rho log\frac{\rho}{\rho+\hat{\rho}} + \frac{1}{2}\hat{\rho} log\frac{\hat{\rho}}{\rho+\hat{\rho}}$$

(You can consider $\hat{}\rho$ as constant)

__Solution :__ $g(\rho) = \frac{1}{2}\rho\log\frac{\rho}{\rho+\hat{\rho}} + \frac{1}{2}\hat{\rho}\log\frac{\hat{\rho}}{\rho+\hat{\rho}}$

$$= \frac{1}{2}\rho\log\frac{\rho}{\rho+\hat{\rho}} + \frac{1}{2}\hat{\rho}\log\hat{\rho} - \frac{1}{2}\hat{\rho}\log(\rho+\hat{\rho})$$

$$\frac{\partial g(\rho)}{\partial\rho} = \frac{\partial}{\partial\rho}\left(\frac{1}{2}\rho\log\frac{\rho}{\rho+\hat{\rho}} + \frac{1}{2}\hat{\rho}\log\hat{\rho} - \frac{1}{2}\hat{\rho}\log(\rho+\hat{\rho})\right)$$

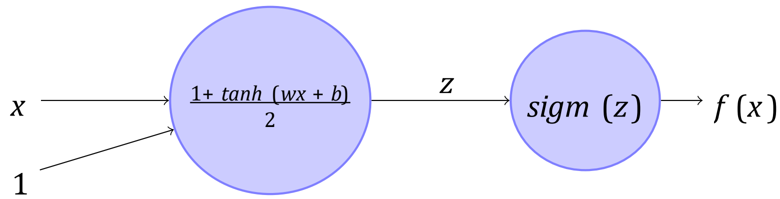
$$= \frac{1}{2}\log\frac{\rho}{\rho+\hat{\rho}} + \frac{1}{2}\rho\left(\frac{\rho+\hat{\rho}}{\rho}\right)\left(\frac{1}{\rho+\hat{\rho}} - \frac{\rho}{(\rho+\hat{\rho})^2}\right) + 0 - \frac{1}{2}\frac{\hat{\rho}}{\rho+\hat{\rho}}$$

$$= \frac{1}{2}\log\frac{\rho}{\rho+\hat{\rho}} + \frac{1}{2}\frac{\hat{\rho}}{\rho+\hat{\rho}} - \frac{1}{2}\frac{\hat{\rho}}{\rho+\hat{\rho}}$$

$$= \frac{1}{2}\log\frac{\rho}{\rho+\hat{\rho}}$$

(b) Consider the following computation ,

by



Where $z = \frac{1+tanh(wx+b)}{2}$ and f(x) = sigm(z)

definition : $sigm(z) = \frac{1}{1+e^{-z}}$ and $tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$

The value *L* is given by,

$$L = -y \log(f(x))$$

Here, *x* and *y* are constants and *w* and *b* are parameters that can be modified. In other words, *L* is a function of *w* and *b*.

Derive the partial derivatives, $\frac{\partial L}{\partial w}$ and $\frac{\partial L}{\partial b}$.

Solution : $\mathrm{L} = -y \log(f(x))$

$$\frac{\partial \mathrm{L}}{\partial \mathrm{w}} = \frac{\partial}{\partial \mathrm{w}}(-y \log(f(x)))$$

$$= -y \frac{\partial}{\partial \mathrm{w}}(\log(f(x)))$$

$$= -y \frac{1}{f(x)} \frac{\partial}{\partial \mathrm{w}}\left(\frac{1}{1+e^{-z}}\right)$$

$$= -y \frac{1}{f(x)} \left(\frac{e^{-z}}{(1+e^{-z})^2}\right) \frac{\partial \mathrm{z}}{\partial \mathrm{w}}$$

$$= -y \frac{1}{f(x)} \left(\frac{e^{-z}}{(1+e^{-z})^2}\right) \frac{\partial}{\partial \mathrm{w}}\left(\frac{1+\tanh(wx+b)}{2}\right)$$

$$= -y \frac{1}{\left(\frac{1}{1+e^{-z}}\right)} \left(\frac{e^{-z}}{(1+e^{-z})^2}\right) \frac{\partial}{\partial \mathrm{w}}\left(\frac{\tanh(wx+b)}{2}\right)$$

$$= \quad -y\left(\frac{e^{-z}}{(1+e^{-z})^1}\right)\frac{\partial}{\partial \mathrm{w}}\left(\frac{\tanh\,(wx+b)}{2}\right)$$

$$= -y \quad \left(\frac{e^{-z}}{1+e^{-z}}\right) \quad \frac{1}{2}(1 - tanh^2(wx+b)) \qquad \frac{\partial}{\partial \mathrm{w}}(wx + b)$$

$$= -\frac{y}{2}\left(\frac{e^{-z}}{1+e^{-z}}\right) \quad (1 - tanh^2(wx+b))x$$

$$= -\frac{yx}{2}\left(\frac{e^{-z}}{1+e^{-z}}\right) \quad (1 - tanh^2(wx+b))$$

Now ,

$$\frac{\partial \mathrm{L}}{\partial b} = \frac{\partial}{\partial b}(-y\log(f(x)))$$

$$= -y \ \frac{\partial}{\partial b}(\log(f(x)))$$

$$= -y\frac{1}{f(x)} \ \frac{\partial}{\partial b}\left(\frac{1}{1+e^{-z}}\right)$$

$$= -y\frac{1}{f(x)} \ \left(\frac{e^{-z}}{(1+e^{-z})^2}\right)\frac{\partial \mathrm{z}}{\partial b}$$

$$= -y\frac{1}{f(x)} \ \left(\frac{e^{-z}}{(1+e^{-z})^2}\right)\frac{\partial}{\partial b}\left(\frac{1+\tanh\,(wx+b)}{2}\right)$$

$$= -y\frac{1}{(\frac{1}{1+e^{-z}})} \ \left(\frac{e^{-z}}{(1+e^{-z})^2}\right)\frac{\partial}{\partial b}\left(\frac{\tanh\,(wx+b)}{2}\right)$$

$$= \quad -y\left(\frac{e^{-z}}{(1+e^{-z})^1}\right)\frac{\partial}{\partial b}\left(\frac{\tanh\,(wx+b)}{2}\right)$$

$$= -y \quad \left(\frac{e^{-z}}{1+e^{-z}}\right) \quad \frac{1}{2}(1 - tanh^2(wx+b)) \qquad \frac{\partial}{\partial b}(wx + b)$$

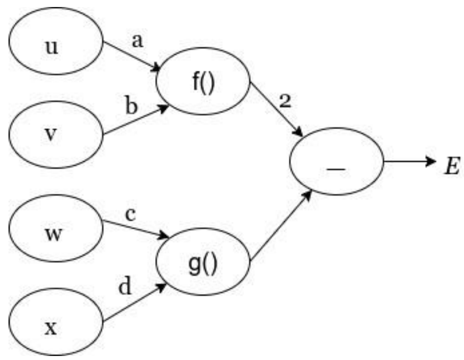
$$= -\frac{y}{2}\left(\frac{e^{-z}}{1+e^{-z}}\right) \quad (1 - tanh^2(wx+b))1$$

$$= -\frac{y}{2}\left(\frac{e^{-z}}{1+e^{-z}}\right) \quad (1 - tanh^2(wx+b))$$

2. **Chain Rule**:

(a) Consider the evaluation of E as given below,

$$E = h(u,v,w,x) = 2 * f(au + bv) - g(cw + dx))$$



Represented as graph:

Here u,v,w,x are inputs (constants) and a,b,c,d are parameters (variables). f and g are the activation functions (with z as input) defined as below:

$f(z) =$ $sigm(z)g(z) = tanh(z)$.

Note that here E is a function of parameters a,b,c,d. Compute the partial derivatives of E with respect to the parameters a, b, c and d *i.e.*, $\frac{\partial E}{\partial a}$, $\frac{\partial E}{\partial b}$, $\frac{\partial E}{\partial c}$ and $\frac{\partial E}{\partial d}$.

__Solution :__ $E= h(u,v,w,x)$

$$=2*f(au+bv) - g(cw+dx)$$

$$\frac{\partial \mathrm{E}}{\partial \mathrm{a}} = 2\frac{\partial f(au+bv)}{\partial \mathrm{a}} - \frac{\partial g(cw+dx)}{\partial \mathrm{a}}$$

$$= 2\,\frac{\partial}{\partial \mathrm{a}}\left(\frac{1}{1+e^{-(\mathrm{au+bv})}}\right) - 0$$

$$= 2\left(\frac{-1}{(1+e^{-(\mathrm{au+bv})})^2}\right).\,e^{-(\mathrm{au+bv})}.(\text{-u})$$

$$=\frac{2u.e^{-(\mathrm{au+bv})}}{(1+e^{-(\mathrm{au+bv})})^2}$$

$$\frac{\partial \mathrm{E}}{\partial \mathrm{b}} = 2\frac{\partial f(au+bv)}{\partial \mathrm{b}} - \frac{\partial g(cw+dx)}{\partial \mathrm{b}}$$

$$=2\,\frac{\partial}{\partial \mathrm{b}}\left(\frac{1}{1+e^{-(\mathrm{au}+\mathrm{bv})}}\right) - 0$$

$$= 2\left(\frac{-1}{(1+e^{-(\mathrm{au}+\mathrm{bv})})^2}\right).\, e^{-(\mathrm{au}+\mathrm{bv})}.(-\mathrm{v})$$

$$= \frac{2ve^{-(\mathrm{au}+\mathrm{bv})}}{(1+e^{-(\mathrm{au}+\mathrm{bv})})^2}$$

$$\frac{\partial \mathrm{E}}{\partial \mathrm{c}} = 2\frac{\partial f(au+bv)}{\partial \mathrm{c}} - \frac{\partial g(cw+dx)}{\partial \mathrm{c}}$$

$$= 0 - \frac{\partial(\tanh(\mathrm{cw}+\mathrm{dx}))}{\partial \mathrm{c}}$$

$$= -(1\text{-}tanh^2(cw+dx))\,\frac{\partial(\mathrm{cw}+\mathrm{dx})}{\partial \mathrm{c}}$$

$$= -w(1\text{-}tanh^2(cw+dx))$$

$$\frac{\partial \mathrm{E}}{\partial \mathrm{d}} = 2\frac{\partial f(au+bv)}{\partial \mathrm{d}} - \frac{\partial g(cw+dx)}{\partial \mathrm{d}}$$

$$= 0 - \frac{\partial(\tanh(\mathrm{cw}+\mathrm{dx}))}{\partial \mathrm{d}}$$

$$= -(1\text{-}tanh^2(cw+dx))\,\frac{\partial(\mathrm{cw}+\mathrm{dx})}{\partial \mathrm{d}}$$

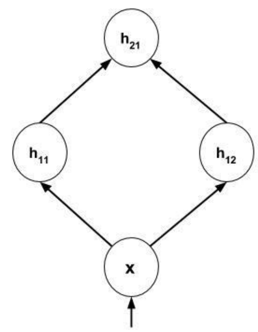
$$= -x(1\text{-}tanh^2(cw+dx))$$

3.Visualize the Plot functions

(a) Consider the variable x and functions $h_{11}(x)$, $h_{12}(x)$ and $h_{21}(x)$ such that

$$h_{11}(x) = \frac{1}{1+e^{-(500x+30)}}$$
$$h_{12}(x) = \frac{1}{1+e^{-(500x-30)}}$$
$$h_{21} = h_{11}(x) - h_{12}(x)$$

The above set of functions are summarized in the graph below.



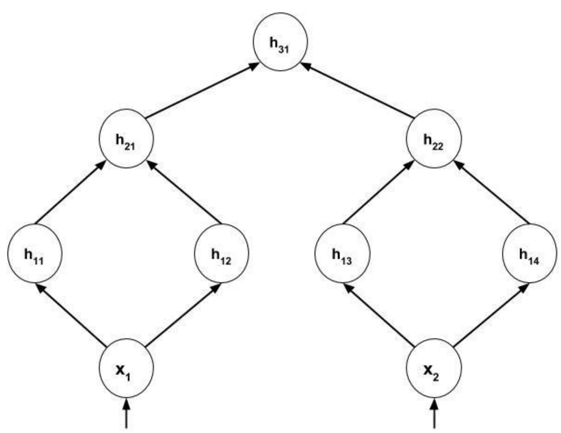
Plot the following functions: $h_{11}(x)$, $h_{12}(x)$ and $h_{21}(x)$ for $x \in (-1,1)$

__Solution :__ https://github.com/Saiananya-25/https-colab.research.google.com-drive-1Nv3WMqoqyds1Le96Eq8V9jqpuBcSrxo4-scrollTo-ztJSPrIX4ryd.git

(b) Now consider the variables x_1, x_2 and the functions $h_{11}(x_1,x_2), h_{12}(x_1,x_2), h_{13}(x_1,x_2), h_{14}(x_1,x_2), h_{21}(x_1,x_2), h_{22}(x_1,x_2), h_{31}(x_1,x_2)$ and $f(x_1,x_2)$ such that

$$
\begin{aligned}
h_{11}(x_1, x_2) &= \frac{1}{1+e^{-(x_1+50x_2+100)}} \\
h_{12}(x_1, x_2) &= \frac{1}{1+e^{-(x_1+50x_2-100)}} \\
h_{13}(x_1, x_2) &= \frac{1}{1+e^{-(50x_1+x_2+100)}} \\
h_{14}(x_1, x_2) &= \frac{1}{1+e^{-(50x_1+x_2-100)}} \\
h_{21}(x_1, x_2) &= h_{11}(x_1, x_2) - h_{12}(x_1, x_2) \\
h_{22}(x_1, x_2) &= h_{13}(x_1, x_2) - h_{14}(x_1, x_2) \\
h_{31}(x_1, x_2) &= h_{21}(x_1, x_2) + h_{22}(x_1, x_2) \\
f(x_1, x_2) &= \frac{1}{1+e^{-(100h_{31}(x)-200)}}
\end{aligned}
$$

The above set of functions are summarized in the graph below.



Plot the following functions: $h_{11}(x_1,x_2), h_{12}(x_1,x_2), h_{13}(x_1,x_2), h_{14}(x_1,x_2), h_{21}(x_1,x_2), h_{22}(x_1,x_2), h_{31}(x_1,x_2)$ and $f(x_1,x_2)$ for $x_1 \in (-5,5)$ and $x_2 \in (-5,5)$

Solution : https://github.com/Saiananya-25/https-colab.research.google.com-drive-1Nv3WMqoqyds1Le96Eq8V9jqpuBcSrxo4-scrollTo-ztJSPrIX4ryd.git