

BEE ଟଙ୍କା

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Basic Electrical Engineering - BEE - EE101 - 3 credits

Lecture Plan:

- 1) D.C. circuits
 - 2) A.C. circuits
 - 3) Magnetic circuits
 - 4) Single Phase Transformer
 - 5) D.C. Machines.
 - 6) 3-Phase induction motor.
 - 7) Measuring Instruments
 - 8) Illumination.
- 1) st minor Exam → 10m, 1 hr
- Mid Exam → 30m, 2 hrs
- 2) nd minor Exam, 1 hr

Total Syllabus for → End Exam → 50m, 3 hrs

Reference Books

1) Edward Hughes

"ELECTRICAL TECHNOLOGY
ADVANCED VERSION"

2) V.N. Mittel.

"BASIC ELECTRICAL
ENGINEERING
ADVANCED VERSION"

3) VINCENT DEL TORO,

"ELECTRICAL ENGINEERING
FUNDAMENTALS"

Chapter-I: (D.C. Circuits)

Electrical network :- It's an interconnection of two or more elements (electrical).

Basic Elements in Electrical :-

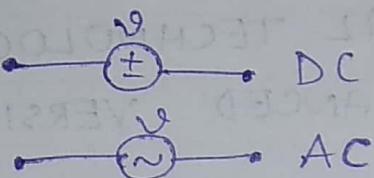
- 1) voltage source
- 2) current source } active elements
- 3) resistor
- 4) Inductor and } passive elements
- 5) capacitor.

Active elements :- They delivers or absorb the energy.

Passive elements :- They only absorb the energy.

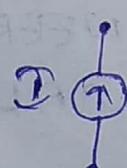
- a) Voltage source:- The source for voltage is electrical generating station \rightarrow power houses

Representation:



- b) Current source:- The current sources are made available. The capacity of current source is small compared to voltage sources. These are used in control circuits, electronic circuits, etc.

Representation:



- c) Resistance:- It opposes the flow of current through a substance. The representation

of resistance is given by $\frac{V}{I}$ or R .

* Denoted by 'R'. units are Ω , ohms.

d) Inductor :- It opposes the sudden change in current. It stores energy in electromagnetic form. Energy stored is $\frac{1}{2} Li^2 = E$

Representation:

(L)

Denoted by :- "L"

units:- ~~ohm~~, Henry (H)

e) Capacitor: It doesn't allow sudden change in voltage. It stores energy in electrostatic form.

Energy stored is $\frac{1}{2} CV^2 = E$

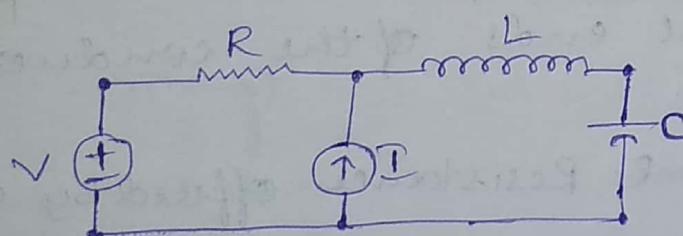
Representation:

C

Denoted by: C

units: Farad

Example for electrical network:-



ELECTRICAL CIRCUIT :- It is an interconnection of two or more elements but provided with atleast one closed path.

ELECTRIC CURRENT:- Flow of electric charges in time called electric current

$$I = \frac{Q}{t}$$

$$I = \frac{dQ}{dt}$$

Units: Ampere

through a cross section of conductor per unit time

(here electrons)

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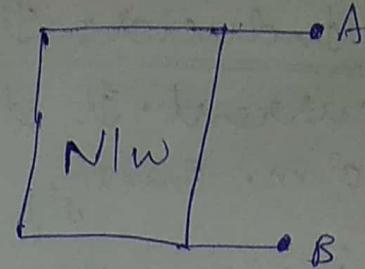
Properties of Open circuit and short circuit:-

a) OPEN CIRCUIT :-

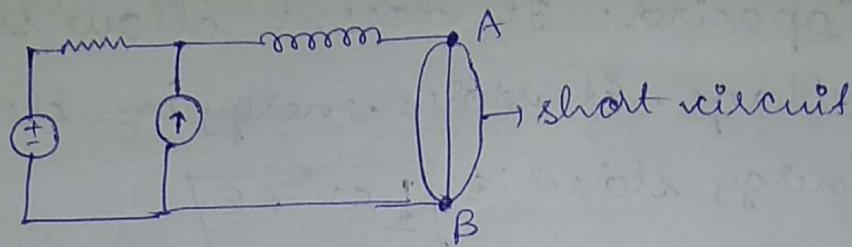
1) current flow through open circuit = 0

2) Resistance offered by open circuit = ∞

3) voltage across open two terminals may be any value.



b) SHORT CIRCUIT :-



1) voltage difference $V=0$

2) Resistance = 0

3) current may be any value.

OHMS LAW :- The current flowing through the conductor is directly proportional to the potential difference across the ends of the conductor

$$\therefore I \propto V$$

$$V = IR$$

R = Resistance offered by conductor

Here $V \rightarrow$ volts

$I \rightarrow$ amp

$R \rightarrow$ ohms

LAW OF RESISTANCE :- The resistance of the wire depends on its length, area of cross section and type of material used.

\therefore Resistance of wire is \propto length of wire
 $\propto \frac{1}{\text{cross section area}}$
 \propto material of conductor

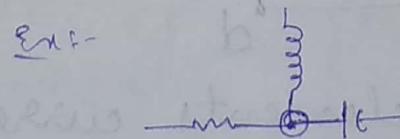
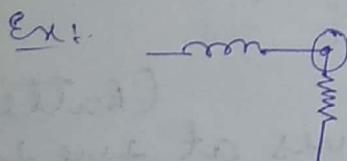
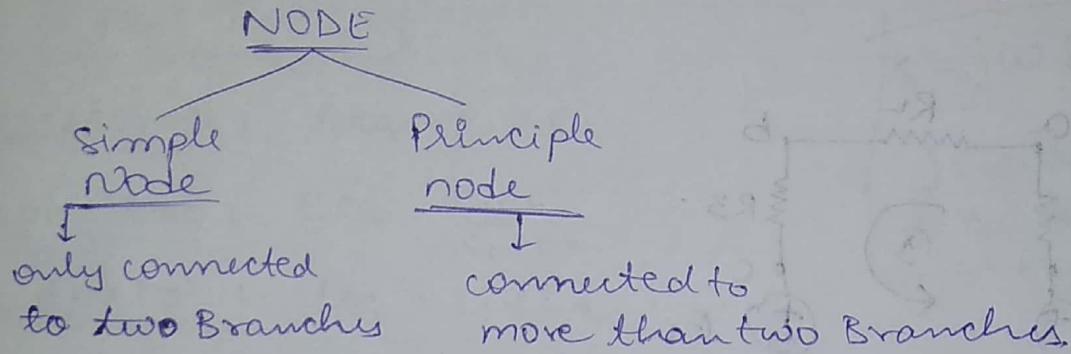
$$\therefore R \propto l, R \propto \frac{l}{A}, R =$$

$$\therefore R = \frac{\rho l}{A} \quad \text{here } \rho = \text{Resistivity of material.}$$

KIRCHHOFF'S LAWS \therefore It is of two types

- i) KIRCHHOFF'S CURRENT LAW (KCL)
- ii) KIRCHHOFF'S VOLTAGE LAW (KVL)

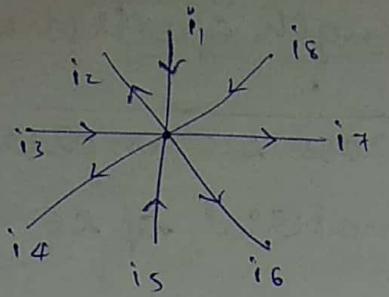
a) K.C.L:- It is only defined at node / junction.
connection of two or more branches.



Defn:- In any electrical network, the algebraic sum of branch currents leaving node is zero.

(*)

In any electrical network (linear, bilateral, Temp is const), the currents outgoing & incoming a node is always equal.



Apply K.C.L at node

$$i_1 + i_3 + i_5 + i_8 = i_2 + i_4 + i_6 + i_7$$

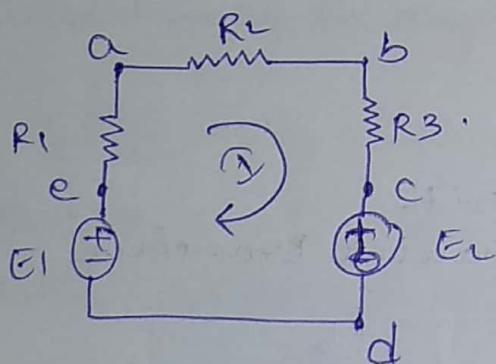
* If current enters the node → -ve

If current leaves the node → +ve

$$-i_1 + i_2 - i_3 + i_4 - i_5 + i_6 + i_7 - i_8 = 0$$

∴ Current leaving the node is taken as +ve,
and current entering the node is taken as -ve

K.V.L: In any electrical network, the algebraic sum of branch voltages around the loop is equal to zero



In active elements current leaves at +ve terminal (Battery)

In passive elements current leaves at -ve terminal

(Resistor)

Apply the K.V.L in the loop ABCDEA :-

$$+E_1 - IR_1 - IR_2 - IR_3 + E_2 = 0$$

$$V_1 = IR_1$$

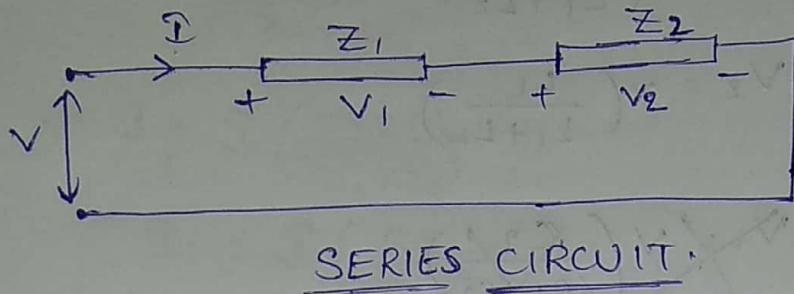
$$V_2 = IR_2$$

$$V_3 = IR_3$$

$$\therefore E_1 + E_2 = V_1 + V_2 + V_3$$

Series and parallel circuit :-

a) Series circuit :- If the current flowing through the elements are same, then the circuit is said to be in series connection.



Z = impedance

$$Z = R + j\omega L$$

$[2\pi f]$ \therefore in D.C, $\omega = 0$

\therefore in D.C, $Z = R$

$$Z_{eq} = Z_1 + Z_2$$

$$X_L = j\omega L = \frac{1}{j\omega C} = X_C$$

j = current leading or current lagging

Resistance : $R_{eq} = R_1 + R_2$

Capacitor : $C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$

Inductor : $L_{eq} = L_1 + L_2$

$$\begin{aligned} \because Z &= Z_1 + Z_2 \\ &= j\omega L_1 + j\omega L_2 \\ &= j\omega (L_1 + L_2) \end{aligned}$$

$$\begin{aligned} \because Z &= Z_1 + Z_2 \\ &= \frac{j\omega}{j\omega C_1} + \frac{j\omega}{j\omega C_2} \\ \frac{1}{j\omega C_{eq}} &= \frac{j\omega}{j\omega} \left(\frac{C_1 C_2}{C_1 + C_2} \right) \\ \therefore C_{eq} &= \frac{C_1 C_2}{C_1 + C_2} \end{aligned}$$

Voltage Division Principle :-

$$V_1 = I Z_1$$

$$I = \frac{V}{Z_1 + Z_2}$$

$$\begin{aligned} V_1 &= \frac{Z_1 V}{Z_1 + Z_2} \\ V_2 &= \frac{Z_2 V}{Z_1 + Z_2} \end{aligned}$$

Resistor :-

$$V_1 = V \left(\frac{R_1}{R_1 + R_2} \right)$$

$$V_2 = V \left(\frac{R_2}{R_1 + R_2} \right)$$

Inductor :-

$$V_1 = V \left(\frac{L_1}{L_1 + L_2} \right)$$

$$V_2 = V \left(\frac{L_2}{L_1 + L_2} \right)$$

Capacitor :-

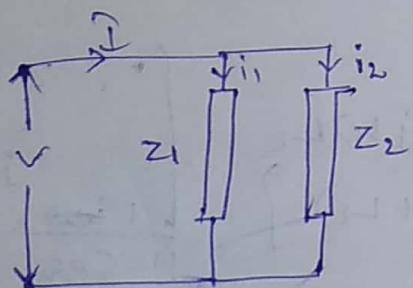
~~$$V_1 = V \left(\frac{C_2}{C_1 + C_2} \right)$$

$$V_2 = V \left(\frac{C_1}{C_1 + C_2} \right)$$~~

$$\boxed{V_1 = \frac{V \cdot C_2}{\frac{C_1 C_2}{C_1 + C_2}} = \frac{V(C_1 + C_2)}{C_2}}$$

$$\boxed{V_2 = \frac{V \cdot C_1}{C_1 C_2} \left(\frac{1}{C_1} + \frac{1}{C_2} \right) = \frac{V(C_1 + C_2)}{C_1}}$$

Parallel circuit :- If the voltage across the elements are same, then circuit is connected in parallel fashion.



$$\frac{1}{Z_{eq}} = \frac{1}{z_1} + \frac{1}{z_2}$$

$$\boxed{Z_{eq} = \frac{z_1 z_2}{z_1 + z_2}}$$

fig :- Parallel circuit

Resistor :- $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

capacitor :- $C_{eq} = C_1 + C_2$

Inducto:- $L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$

Current Division Rule:-

$$I = \frac{V}{Z_1}$$

$$V = I Z_{eq}$$

$$V = \frac{I Z_1 Z_2}{Z_1 + Z_2}$$

$$\therefore I_1 = I \left(\frac{Z_1 Z_2}{Z_1 + Z_2} \right) \times \frac{1}{Z_2}$$

$$I_1 = I \left(\frac{Z_2}{Z_1 + Z_2} \right)$$

$$\text{Hence } I_2 = I \left(\frac{Z_1}{Z_1 + Z_2} \right)$$

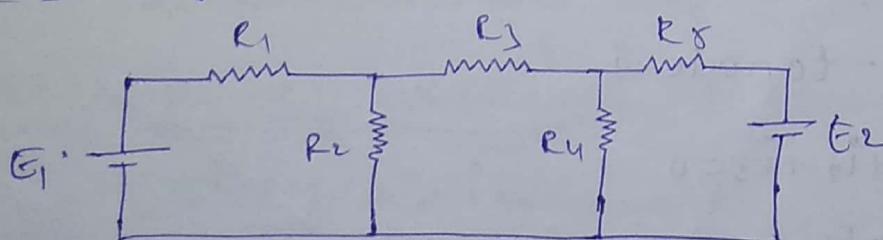
Resistor: $I_1 = I \left(\frac{R_2}{R_1 + R_2} \right), I_2 = I \left(\frac{R_1}{R_1 + R_2} \right)$

Inductor: $I_1 = I \left(\frac{L_2}{L_1 + L_2} \right), I_2 = I \left(\frac{L_1}{L_1 + L_2} \right)$

Capacitor:

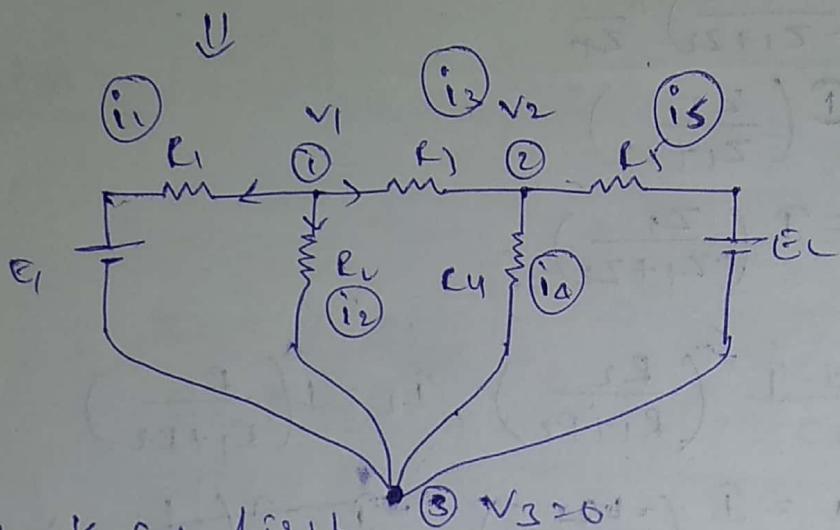
$$\begin{aligned} I_1 &= I \left(\frac{C_2 (C_1 + C_2)}{C_1 C_2} \right) \\ I_1 &= I \left(\frac{C_1 + C_2}{C_1} \right), I_2 = I \left(\frac{C_1 + C_2}{C_2} \right) \\ I_1 &= \frac{I (C_1)}{C_1 + C_2}, I_2 = \frac{I (C_2)}{C_1 + C_2} \end{aligned}$$

Nodal Analysis:-



Step - I: Identify no. of nodes where we don't know voltages.

Step - II: Assign the node voltages with respect to ground node whose voltage is always equal to zero.



Apply K.C.L first

followed by ohms law, write the node equations

At Node - 2: K.C.L.

∴ Assume that when we are solving a node, we assume that Node voltage is greater than any other

$$\begin{pmatrix} V_1 > E_1 \\ V_2 > V_1 \\ V_2 > V_3 \end{pmatrix}$$

Apply K.C.L:- to node - 2.

$$i_1 + i_2 + i_3 = 0$$

$$\frac{V_1 - E_1}{R_1} + \frac{V_1 - 0}{R_2} + \frac{V_1 - V_2}{R_3} = 0,$$

Apply KVL at node-2:

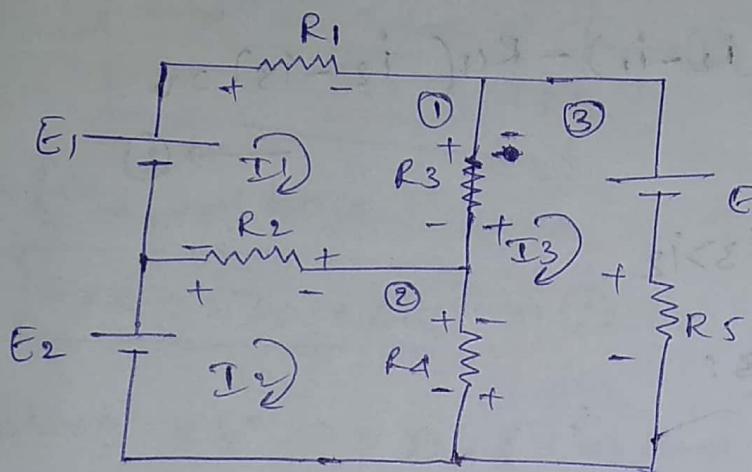
$$\begin{cases} V_2 > E_2 \\ V_2 > V_3 \\ V_2 > V_1 \end{cases}$$

$$i_1 + i_2 + i_3 = 0$$

$$\frac{V_2 - V_1}{R_3} + \frac{V_2 - 0}{R_4} + \frac{V_2 - E_2}{R_5} = 0$$

After simplifying eqns we get V_1, V_2 values
then we can conclude direction of current.

Loop Analysis:-



Step-I: Identify min no. of loops (meshes)

Step-II: Assign loop currents in clockwise direction.

Step-III: Apply KVL in loop followed by Ohm's law and write the loop equations.

loop-I: Assume: whenever we solve a loop a
they $i_1 > i_2, i_1 > i_3$

\therefore here $i_1 > i_2, i_1 > i_3$.

Apply KVL in Loop - I

$$E_1 - V_1 - V_3 - V_2 = 0$$

② Apply Ohms Law:

$$E_1 - i_1 R_1 - R_3(i_1 - i_3) - R_2(i_1 - i_2) = 0 \quad \rightarrow ①$$

Loop - II:

$$i_2 > i_1, i_2 > i_3$$

Apply KVL in loop - II

$$E_2 - V_2 - V_4 = 0$$

$$E_2 - R_2(i_2 - i_1) - R_4(i_2 - i_3) = 0$$

$\rightarrow ②$

Loop - III:

$$i_3 > i_1, i_3 > i_2$$

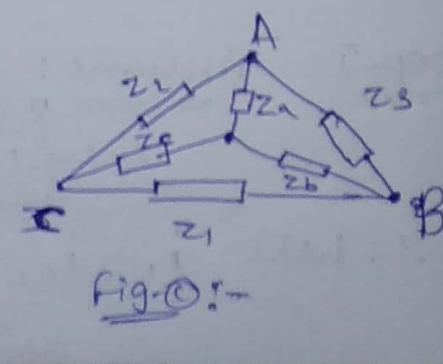
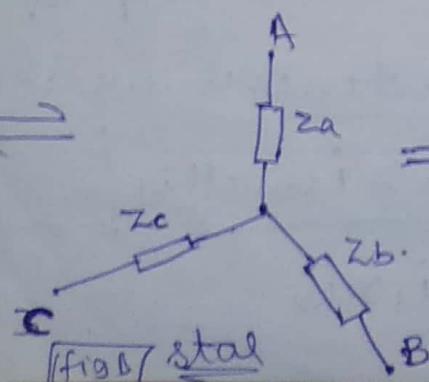
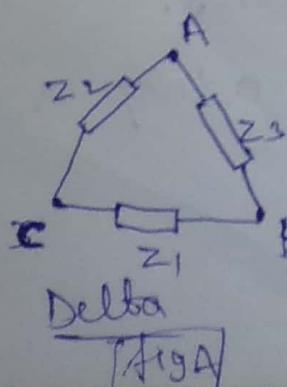
Apply KVL in loop - III

$$-E_3 - V_5 - V_4 - V_3 = 0$$

$$-E_3 - R_5(i_3) - R_4(i_3 - i_2) - R_3(i_3 - i_1) = 0$$

$\rightarrow ③$

Delta - Star Transformation



case-I! Convert Delta to star

figure ① shows impedances z_1, z_2, z_3 are connected in a closed loop (d) mesh (c) delta to three terminals ①, ② ③. Let us consider terminal ④ and ⑤, in fig A, we have a circuit having a impedance z_3 in parallel with a circuit having impedance $z_1 + z_2$ in series. Hence the Equivalent impedance b/w AB is

$$Z_{AB} = (z_1 + z_2) // z_3$$

$$\boxed{Z_{AB} = \frac{(z_1 + z_2) z_3}{z_1 + z_2 + z_3}} \rightarrow ①$$

From fig B, we have

$$\boxed{Z_{AB} = z_a + z_b} \rightarrow ②$$

Equating eqn ① & ②, (star and Delta Both are equal to each other)

$$\boxed{z_a + z_b = \frac{z_1 z_3 + z_2 z_3}{z_1 + z_2 + z_3}} \rightarrow ③$$

Now

$$\boxed{z_b + z_c = \frac{z_2 z_1 + z_3 z_1}{z_1 + z_2 + z_3}} \rightarrow ④$$

Now

$$\boxed{z_c + z_a = \frac{z_1 z_2 + z_1 z_3}{z_1 + z_2 + z_3}} \rightarrow ⑤$$

Subtracting ④ from ③

$$\text{③} - \text{④} \Rightarrow \boxed{z_a - z_c = \frac{z_2 z_3 - z_1 z_2}{z_1 + z_2 + z_3}} \rightarrow \text{⑥}$$

Adding eqⁿ ⑤ and ⑥ and diving by $\frac{z_a - z_c}{2}$ we have

$$\begin{aligned} z_a &= \frac{z_2 z_3}{z_1 + z_2 + z_3} & \rightarrow \text{⑦} \\ \text{Hence } z_b &= \frac{z_1 z_3}{z_1 + z_2 + z_3} & \rightarrow \text{⑧} \\ \text{Hence } z_c &= \frac{z_1 z_2}{z_1 + z_2 + z_3} & \rightarrow \text{⑨} \end{aligned}$$

star-Delta:- Let us consider how to Replace star connected network of fig-B by Equivalent delta connected network of fig-A.

Dividing eqⁿ ⑦ by eqⁿ ⑧

$$\frac{z_a}{z_b} = \frac{z_2}{z_1} \Rightarrow \boxed{z_2 = \frac{z_1 z_a}{z_b}}$$

Hence Dividing ⑨ by ⑦ ,

$$\frac{z_a}{z_c} = \frac{z_3}{z_1} \Rightarrow \boxed{z_3 = \frac{z_a z_1}{z_c}}$$

substituting z_2, z_3 value in eqⁿ ⑦ , we have

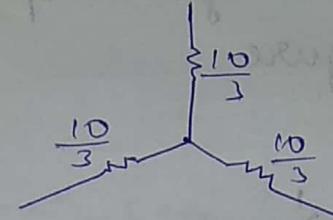
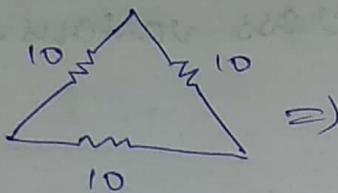
$$Z_1 = Z_a Z_b Z_c + Z_b Z_c$$

$$Z_1 = Z_b + Z_c + \frac{Z_b Z_c}{Z_a}$$

$$Z_2 = Z_a + Z_c + \frac{Z_a Z_c}{Z_b}$$

$$Z_3 = Z_a + Z_b + \frac{Z_a Z_b}{Z_c}$$

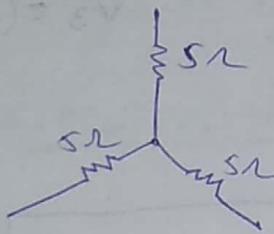
Q.M
sol:



$$Z_a = \frac{(10)(10)}{10 + 10 + 10}$$

$$\Rightarrow \frac{100}{30} = \frac{10}{3}$$

Q.M



$$Z_1 = 5 + 5 + \frac{(5)(5)}{5}$$

$$\underline{Z_1 = 15}$$

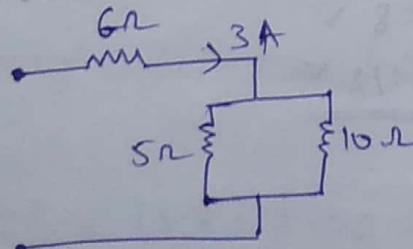
* If all impedances are equal

star \rightarrow Delta ($\times 3$)

Delta \rightarrow star ($\div 3$)

Problems: Find the current through 5Ω and 10Ω

resistor shown in circuit diagram



$$i_1 + i_2 = 3A$$

$$R_1 = 5\Omega \quad R_2 = 10\Omega$$

$$\therefore i_1 R_1 = i_2 R_2$$

$$i_1(5) = (3 - i_1)(10)$$

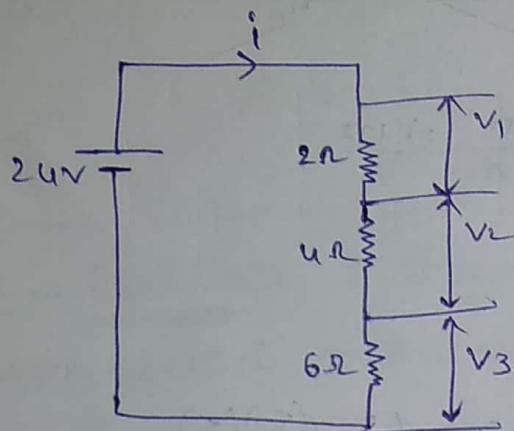
$$i_1 = 6 - 2i_1$$

$$3i_1 = 6$$

$$\underline{i_1 = 2A} \Rightarrow \underline{i_2 = 1A}$$

$$(OR) \quad I_{SR} = \frac{(3)(10)}{15} = 2A$$

Q) Find the voltage drop across various resistors shown in figure.



$$V_1 = \frac{(24)(2)}{12} = 4V$$

$$(OR) \quad V_2 = \frac{(24)(4)}{12} = 8V$$

$$V_3 = \frac{(24)6}{12} = 12V$$

$$i = \text{const}$$

$$\frac{V_1}{2} = \frac{V_2}{4} = \frac{V_3}{6} = k$$

$$V_1 = 2k$$

$$V_2 = 4k$$

$$V_3 = 6k$$

$$\underline{V_1 + V_2 + V_3 = 24}$$

$$12k = 24$$

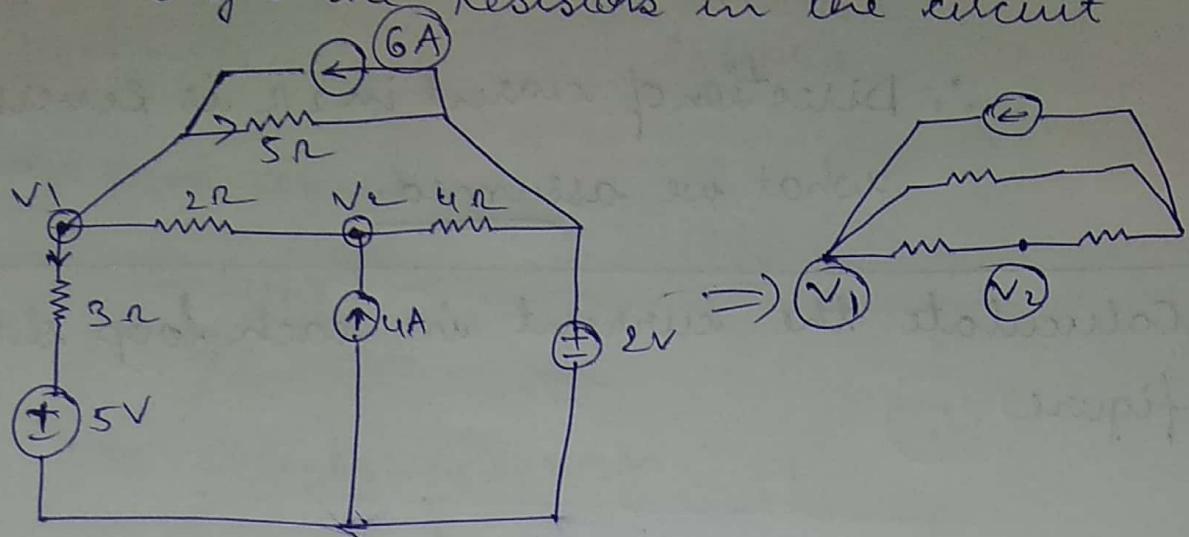
$$\underline{k = 2}$$

$$\Rightarrow V_1 = 4V$$

$$V_2 = 8V$$

$$\underline{V_3 = 12V}$$

* Using Nodal Analysis method, find the current through the resistors in the circuit shown.



At Node -I :-

$$\frac{V_1 - 5}{3} + \frac{V_1 - V_2}{2} + \frac{V_1 - 2}{5} - 6 = 0 \quad \rightarrow ①$$

At node -II:-

$$\frac{V_2 - V_1}{2} + \frac{V_2 - 2}{4} - 4 = 0 \quad \rightarrow ②$$

$$\frac{10(V_1 - 5) + 15(V_1 - V_2) + 6(V_1 - 2) - 180}{30} = 0$$

$$10V_1 - 50 + 15V_1 - 15V_2 + 6V_1 - 18 - 180 = 0$$

~~$$31V_1 - 15V_2 - 50 - 18 - 180 = 0$$~~

$V_1 = 15.8 \text{ V}$
$V_2 = 16.53 \text{ V}$

By solving, we get

$$V_2 > V_1$$

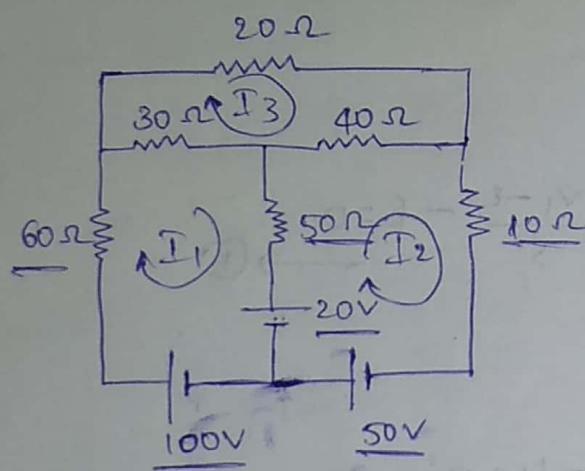
∴ Current through 3Ω Resistor = $\frac{15.8 - 5}{3} = 3.6 \text{ A}$

$$I_{5\Omega} = \frac{15.8 - 2}{5} = \frac{13.8}{5} = 2.76 \text{ A}$$

$$I_{22} = \frac{15.8 - 16.5}{2} = -0.35 \text{ A}$$

\therefore Direction of current in I_{22} is reverse of what we assumed.

Calculate the current in each loop shown in figure.



Loop - 1 :-

$$100 - 60i_1 - 30(i_1 - i_3) - 50(i_1 - i_2) - 20 = 0$$

Loop - 2 :-

$$50 + 20 - 50(i_2 - i_1) - 40(i_2 - i_3) - 10i_2 = 0$$

Loop - 3 :-

$$-40(i_3 - i_2) - 30(i_3 - i_1) - 20i_3 = 0$$

Simplify :-

$$140i_1 - 50i_2 - 30i_3 = 80$$

$$-50i_1 + 100i_2 - 40i_3 = 70$$

$$-30i_1 - 40i_2 + 90i_3 = 0$$

$$\begin{aligned}
 3i_1 + 4i_2 - 9i_3 &= 0 \\
 -5i_1 + 10i_2 - 4i_3 &= 7 \\
 14i_1 - 5i_2 - 3i_3 &= 8
 \end{aligned}$$

$$\begin{aligned}
 i_1 &= 1.65 \text{ A} \\
 i_2 &= 2.16 \text{ A} \\
 i_3 &= 1.5 \text{ A}
 \end{aligned}$$

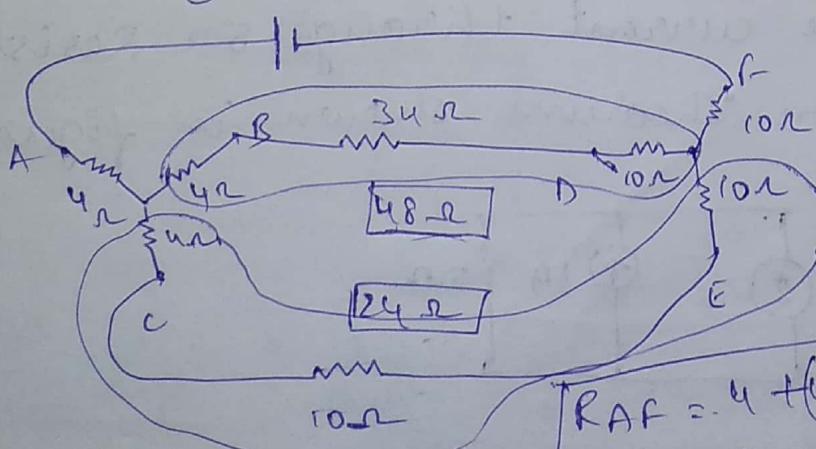
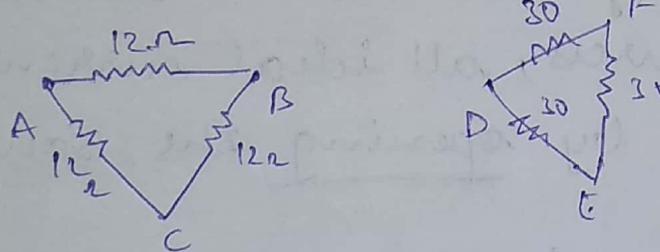
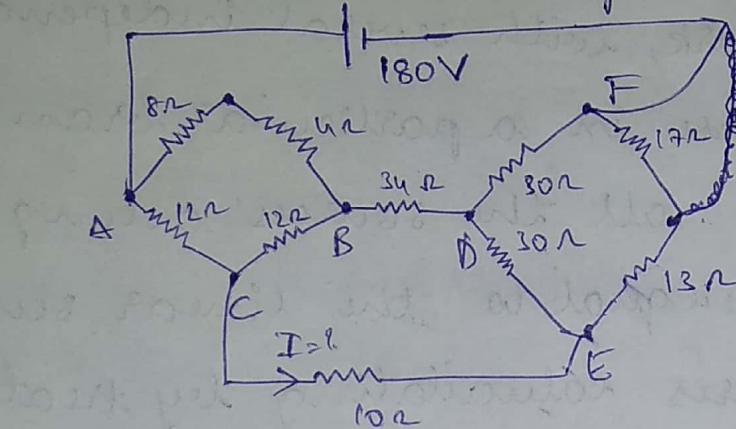
Drop across 50Ω Resistor

$$\begin{aligned}
 V_{50\Omega} &= (60)(1.65) \\
 &= 99 \text{ V}
 \end{aligned}$$

$$V_{30} = 30(1.65 - 1.5) = 4.5 \text{ V}$$

$$V_{50\Omega} = 50(2.16) = 25.5 \text{ V}$$

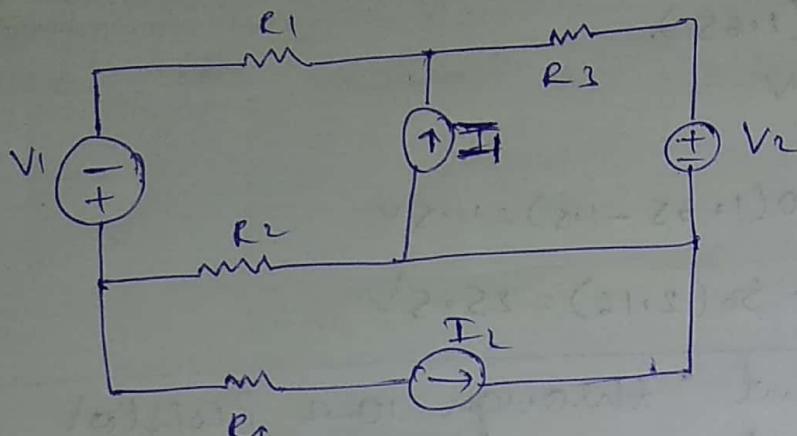
Calculate current through 10Ω Resistor



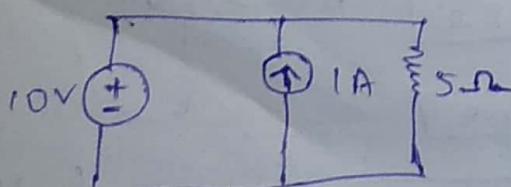
$$\begin{aligned}
 R_{AF} &= 4 + (48 \parallel 24) + 10 \\
 &= 30 \Omega \Rightarrow I = \frac{180}{30} = 6 \text{ A}
 \end{aligned}$$

$$I_{10\Omega} = \frac{(6)(48)}{24+48} = 4A$$

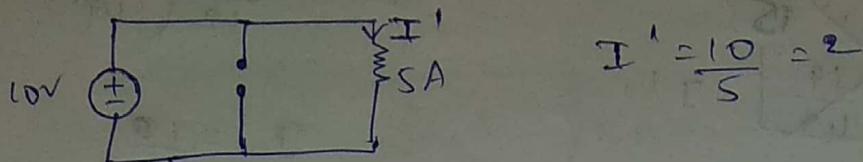
Super Position Theorem :-



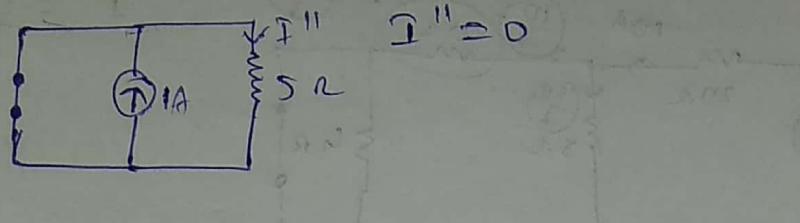
- * In a linear network, with several independent sources, the response in a particular branch (of any branch), then all the sources acting simultaneously is equal to the linear sum of individual responses calculating by treating one independent source at a time.
- * All Ideal voltage sources are eliminated by shorting the source, all ideal current sources are eliminated by opening the source.
- Q) Find the current through 5Ω Resistor using superposition Theorem shown in figure



Due to 10V source



Due to 1A source



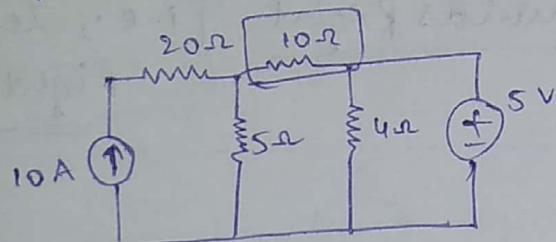
$$\therefore I = I' + I''$$

$$= 2 + 0$$

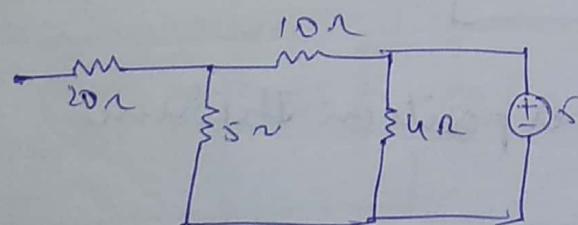
$$= 2 \text{ Amp}$$

∴ According to superposition Theorem, net current flowing through 5Ω Resistor is 2A.

Q) Find the current through 10Ω resistor using superposition Theorem.



Due to 5V source



$$I_{\text{eff}} = (10 + 5) / 4 = 15 / 4 = 3.75 \text{ A}$$

$$I_{\text{net}} = \frac{5}{R_{\text{eff}}} = \frac{5}{3.75} = 1.33 \text{ A}$$

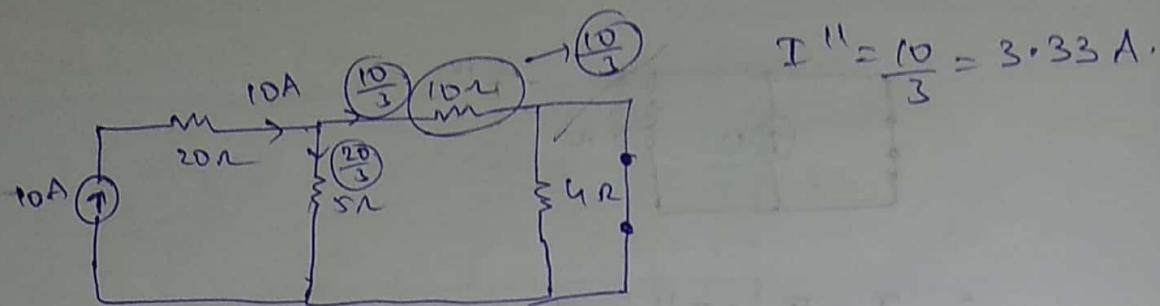
$$= 1.33 \text{ A}$$

$$i_{10\Omega} = \frac{10}{12} \left(\frac{10}{15} \right) = \frac{10}{18}$$

$$I_{10\Omega} = \frac{10}{12} \left(\frac{4}{10} \right) = \frac{1}{3}$$

$$I' = \frac{10}{3}$$

Due to 10 A



$$I'' = \frac{10}{3} = 3.33 \text{ A.}$$

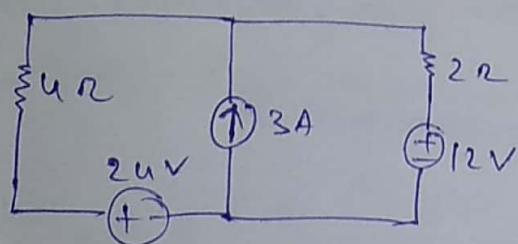
$$i_{\text{net}} = \frac{10}{3} - \frac{10}{45}$$

∴ Net current passing from 10Ω Resistor i.e.,

$$\begin{aligned} I &= I' + I'' \\ &= 3.33 - 0.33 \end{aligned}$$

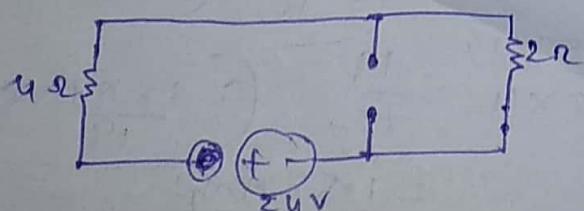
$= 3 \text{ A.}$ towards Right [i.e., left to right].

Q)



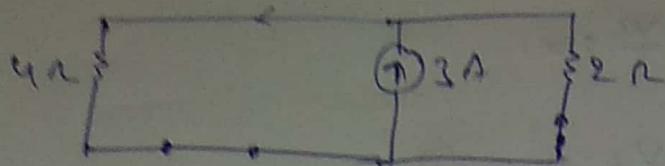
find $I_{4\Omega}$ using superposition Theorem

80) $I' = ?$ through $24V$



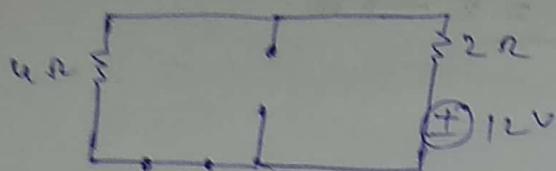
$$I' = \frac{24}{6} = 4 \text{ A}$$

$I'' \rightarrow$ through S.A.



$$\cancel{I''} \text{ through S.A.} \quad I''_{4\Omega} = \frac{3 \times 2}{6} = 1 \text{ A } \downarrow$$

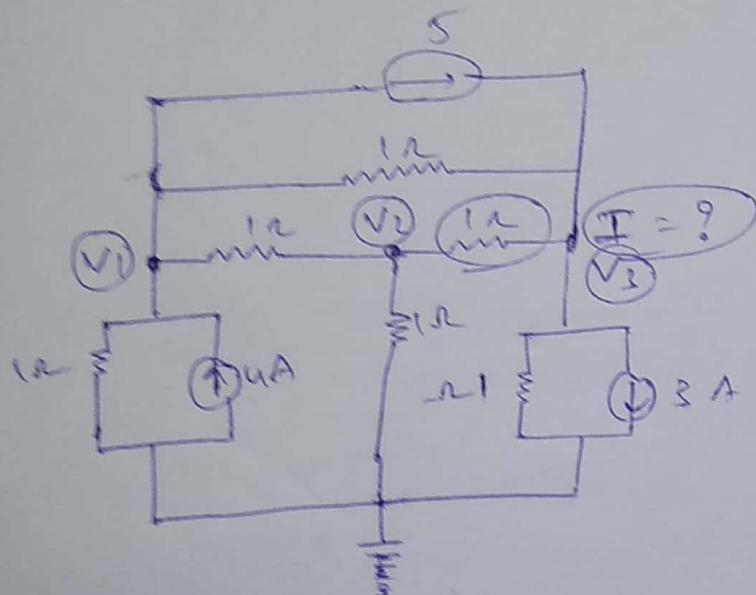
$I''' \Rightarrow$ through 12V



$$I''' = \frac{12}{6} = 2 \text{ A } \downarrow$$

$$I_{\text{net}} = \underline{\underline{4 - 2}} = 1 \text{ A } \uparrow \text{ upward}$$

Q) Find I shown in figure using nodal analysis method or Technique



at Node 1:

$$\frac{V_1 - V_2}{1} + \frac{V_1 - V_3}{1} + 5 - 4 = 0 \rightarrow ①$$

$$3V_1 - V_2 - V_3 = -1$$

Node-III

$$\frac{V_2 - V_1}{1} + \frac{V_2 - V_3}{1} + \frac{V_2}{1} = 0,$$

$$3V_2 - V_1 - V_3 = 0 \rightarrow \textcircled{2}$$

At Node - II

$$\frac{V_3 - V_2}{1} + \frac{V_3 - V_1}{1} + \frac{V_3}{1} + 3 - 5 = 0$$

$$3V_3 - V_1 - V_2 = 2 \rightarrow \textcircled{1}$$

$$3V_1 - V_2 - V_3 = -1$$

$$-V_1 + 3V_2 - V_3 = 0$$

$$-V_1 - V_2 + 3V_3 = 2$$

$$V_1 = 0$$

$$V_2 = 0.25$$

$$V_3 = 0.75$$

$$\therefore I_{12} \text{ resistor is } \Delta - \frac{(V_2 - V_3)}{1} = \underline{\underline{0.5}}$$

$0.75 - 0.25 \uparrow$

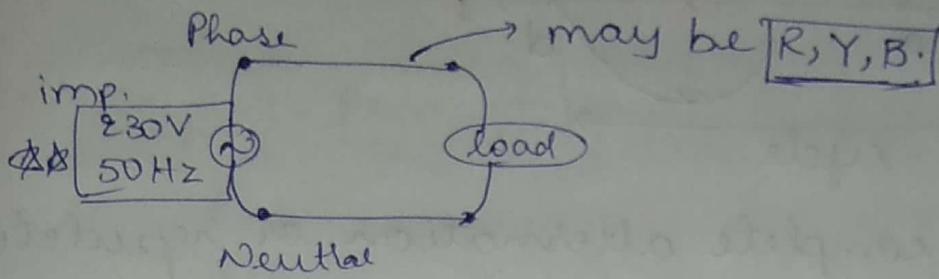
Right to left



Chapter - 2

A.C. Circuits

* 1-Φ (Phase) A.C. Circuit :- Generally for domestic purposes. Only two conductors are required.



In single phase, voltage is 230V & frequency is 50Hz.

* Any quantity which is varying w.r.t time is called an alternating quantity.

Alternating Voltage :- Alternating voltage is any voltage that varies both in magnitude and polarity.

Alternating current :- Alternating current is any current that varies both in magnitude and direction w.r.t time is called alternating current.

→ General exp for sinusoidally varying voltage is given by

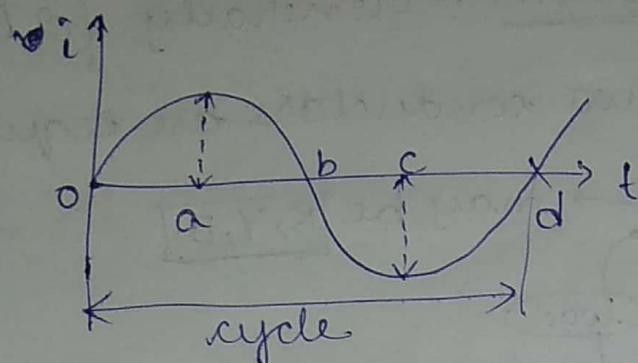
$$V_t = V_m \sin(\omega t + \phi)$$

here V_m = max voltage,

ω = angular frequency
(rad/sec)

ϕ = phase diff b/w voltage & current

* The graphical representation of alternating quantities as shown in figure.



Cycle:- One complete alternation or repetition of current is called cycle. In above figure from point O to D(d) is one complete cycle. Each cycle is having one positive half cycle above the horizontal axis and one negative half cycle below the horizontal axis.

Frequency:- The no. of cycles completed in one second is called frequency. It is represented or denoted by "f". Its unit is Hertz (Hz).

④ In India, the power system frequency is (50 Hz).

Period or Time Period:- The time required to complete one cycle is called time period (or) Periodic Time (or) Period. Represented by 'T'.

Units = sec.

→ Time is inversely proportional to frequency

$$T = \frac{1}{f}$$

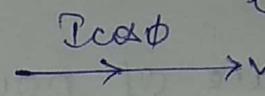
To complete one cycle $\frac{1}{50} = 0.02$ sec are required.

Power :-

- 1) Real power (P)
- 2) Reactive power (Q)
- 3) Apparent power (S)
a) Total Power (S).

a) Real Power :- Product of RMS voltage & inphase current

$$P = V(I \cos \phi)$$



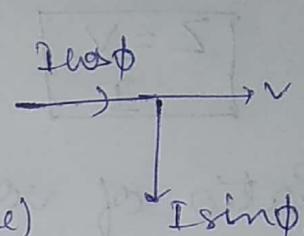
$$P = VI \cos \phi \text{ watts}$$

b) Reactive power :- Product of RMS voltage & quartered the component of current.

$$Q = (V)(I \sin \phi)$$

units: VAR

(volt ampere Reactive)



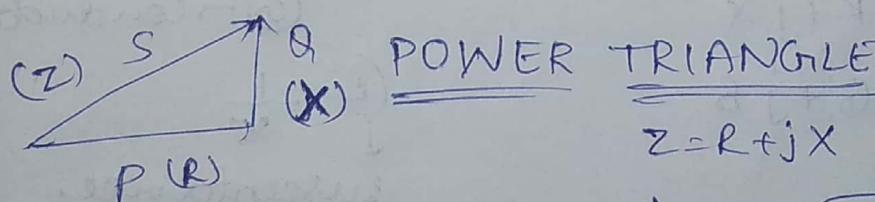
c) Total power! vector sum of above two.

$$S = VI$$

units: VA

(volt Ampere)

Relationship b/w Real, Reactive & Apparent power:-



POWER TRIANGLE

$$Z = R + jX$$

$$S = \sqrt{P^2 + Q^2}$$

$$= \sqrt{(V I \cos \phi)^2 + (V I \sin \phi)^2}$$

$$= VI \text{ VA}$$

$$Z = \sqrt{R^2 + X^2}$$

Power factor :- $(\cos \phi)$

$$P = VI \cos \phi$$

$$\boxed{\cos \phi = \frac{P}{Z}}$$

Defⁿ:

- ① The cosine angle between voltage and current.
- ② It is the ratio of Real power to Apparent power.
- ③ It is the ratio of Resistance to Impedance.
- ④ The value of power factor varies in between 0 to 1.

Impedance :- Total resistance offered by the conductor.

$$V = IZ$$

$$\boxed{Z = \frac{V}{I}}$$

* The Ratio of voltage phasor to current phasor
units: Ω

$$Z = R + jX$$

$$Z = \sqrt{R^2 + X^2}$$

Admittance :- The reciprocal of Impedance is admittance.

$$Z = R + jX$$

$$Y = G + jB$$

$$R = \frac{1}{G} \rightarrow \text{conductance}$$

$$B = \frac{1}{X}$$

susceptance

Values of alternating Quantities :-

- ① Instantaneous values
- ② Maximum value / Peak value

③ Average value / mean value

④ R.M.S. value.

① Form factor:- The form factor of a particular wave is defined as the Ratio of RMS value to average value. It is denoted by k_f . Therefore form factor is equal to RMS value divided by Avg value

$$FF(k_f) = \frac{\text{R.M.S value}}{\text{Avg value}} = 1.11$$

② Peak factor:- The Peak factor of a given wave is defined as the Ratio of maximum value to R.M.S. value. It is denoted by k_p .

$$\therefore PF(k_p) = \frac{\text{Max value}}{\text{R.M.S. value}}$$

A.C. through a Resistor:-

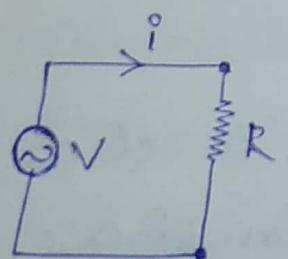


Fig: A.C.T.R.

Consider the circuit shown in figure. It consists of a pure ohming Resistance of R_2 connected across an A.C. supply of V .volts.

The supply voltage is given by

$$V = V_m \sin(\omega t) \rightarrow ①$$

As a result of this voltage, an alternating current i flows through the circuit, causing ohmic drop across the resistor. Hence, the entire supply voltage appears as ohmic drop across the Resistor.

$$V = iR \rightarrow ②$$

$$i = \frac{V}{R} = \frac{V_m \sin \omega t}{R} \rightarrow ③$$

* The current i goes maximum when $\sin \omega t = 1$

i.e., $i_{\max} = \frac{V_{\max}}{R} \rightarrow ④$

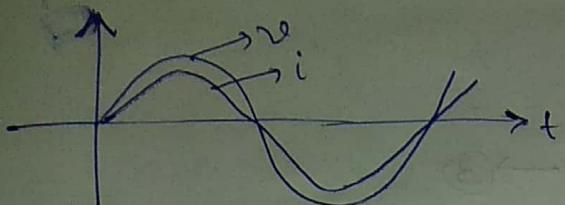
① \rightarrow ③ By substituting eqⁿ ④ in ③

$$i = i_{\max} \sin \omega t \rightarrow ⑤$$

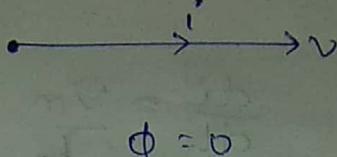
Compare eqⁿ ① & ⑤

Observation: From eqⁿ ① & ⑤ it is clear that the current and voltage are in phase with each other (i.e.) Phase angle b/w these two is 0.

⊕ Fig b and c shows, wave form and phasor Representation



Fig(b) wave form



Fig(c) Phasor Diagram.

* With reference to one vector, if we draw another vector, it is called phasor diagram.

A.C. Through inductor :-

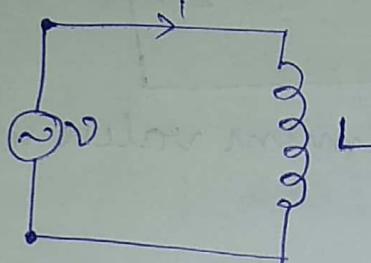


Fig @: A.C.T.I

* Consider a coil of pure inductance (L Henry) connected across an alternating voltage source, as shown in figure. The supply voltage is given by $V = V_m \sin \omega t \rightarrow ①$

As a result, an alternating current i flows through the circuit. This induces an EMF in the coil which is given by Eqⁿ:

$$e_L = L \frac{di}{dt} \rightarrow ②$$

The E.M.F induced in the coil tries to oppose the change in current through it. Hence the entire supply voltage is utilized to overcome this opposition i.e., $V = e_L$

$$V_m \sin \omega t = L \cdot \frac{di}{dt} -$$

$$\frac{di}{dt} = \frac{V_m}{L} \sin \omega t \rightarrow ③$$

Integrating both the sides.

$$\int di = \frac{V_m}{L} \int \sin \omega t dt$$

$$i = \frac{V_m}{L} (\cos \omega t) \left(+ \frac{1}{\omega} \right)$$

$$i = \frac{V_m}{L \omega} (\sin(\omega t - \frac{\pi}{2})) \rightarrow ④$$

Equation ④ attains maximum value when
 $\sin(\omega t - \pi/2) = 1$

$$I_m = \frac{V_m}{L \omega} \rightarrow ⑤$$

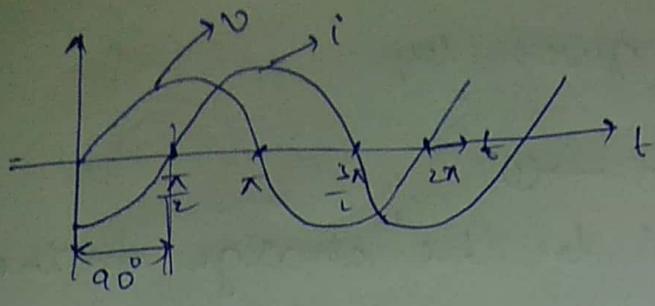
substituting eq^n ⑤ in ④ we have

$$I = I_m \sin(\omega t - \pi/2) \rightarrow ⑥$$

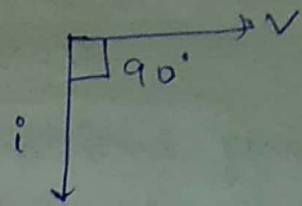
Compare eq^n ① & ⑥

Observation: From eq^n ① & ⑥ it is clear that the current in an inductive circuit lags the applied voltage by $\pi/2$ radians (or) 90° .

Figure b and c shows wave form and Phasor representation.



fig(b) :- waveform



fig(c) :- P.D.

Inductive Reactance (X_L) :- We have an expression for maximum current $I_m = \frac{V_m}{\omega L}$. The denominator ωL plays the same role as that of Resistance in D.C circuits and it is called Inductive Reactance

$$X_L = \omega L$$

$$= 2\pi f L \quad \underline{\Omega}$$

A.C through Capacitor :-

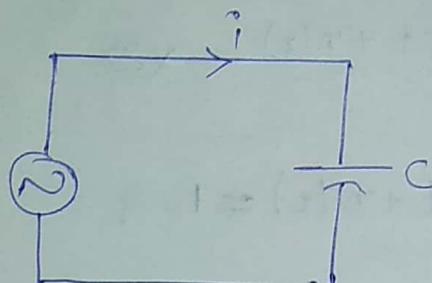


fig @ A.C. through capacitor.

When the alternating voltage (V) is applied across a pure capacitor of "C" farads (shown in figure), It charges alternatively both the directions. As a result, there will be a continuous charge transfer from one plate to another plate and hence a current i through the circuit.

∴ voltage applied is given by

$$V = V_m \sin \omega t \rightarrow ①$$

At any instant let 'q' be the charge on the plate in coulombs.

∴ Charge on capacitor $q = CV$

$$\text{But } i = \frac{dq}{dt}$$

$$\text{substitute } q = CV \text{ in } i = \frac{dq}{dt}$$

$$i = \frac{d(CV)}{dt}$$

substitute value of V

$$i = \frac{d(C \cdot V_m \sin \omega t)}{dt}$$

$$i = \omega C V_m \cos \omega t$$

$$i = \frac{V_m}{(1/\omega C)} \sin(\omega t + \pi/2) \rightarrow ②$$

i attains max if $\sin(\omega t + \pi/2) = 1$

$$i_{\max} = \frac{V_m}{1/\omega C} \rightarrow ③$$

③ in ② \Rightarrow

$$\underline{i = i_{\max} \sin(\omega t + \phi)} \rightarrow ④$$

Comparing eqn ① & ④ \Rightarrow It is clear that current in a capacitive circuit leads the voltage by $\pi/2^c$ (or) 90°

Figure b and c shows wave form and phase representation.

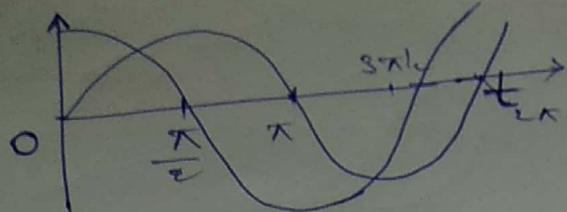
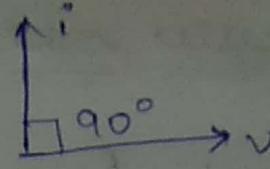


fig (b) :- waveform.



fig(c) :- Phasor diagram.

Capacitive Reactance:- We have an expⁿ for i_{max}

i.e., $I_m = \frac{V_m}{(1/w_c)}$. Here in denominator, $\frac{1}{w_c}$ is

called capacitive Reactance i.e.,

$$X_C = \frac{1}{w_c}$$

$$X_C = \frac{1}{w_c} = \frac{1}{2\pi f C}$$

Series circuit:-

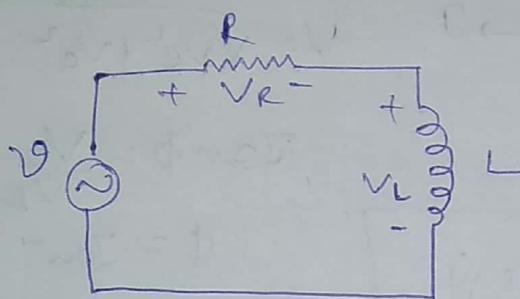


fig R-L (series ckt)

$$\underline{V_R} = I R \angle 0^\circ$$

$$\underline{V_L} = I Z$$

$$Z = R + j X_L$$

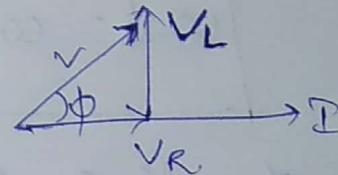
$$Z = j X_L$$

$$\underline{V_L} = I j X_L$$

$$\underline{V_L} = I X_L \angle 90^\circ$$

\because current is const

↓
it is Reference vector.



$$V = \sqrt{V_R^2 + V_L^2}$$

$$\cos \phi = \frac{V_R}{V}$$

lagging power factor

R-C series ckt:

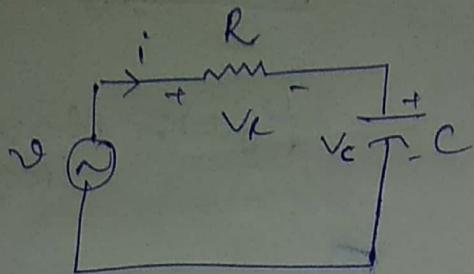


fig :- R-C-ckt

$$V_R = IR \angle 0^\circ$$

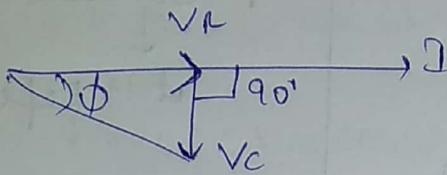
$$V_C = IX_C$$

$$V_C = IX_C \angle -90^\circ$$

$$Z = R + jX_C$$

$$= -jX_C$$

phasor diagram :-



$$V = \sqrt{V_R^2 + V_C^2}$$

$$\tan \phi = \frac{V_C}{V_R}$$

$$\phi = \tan^{-1} \left(\frac{V_C}{V_R} \right)$$

Leading power factor.

RLC series ckt:

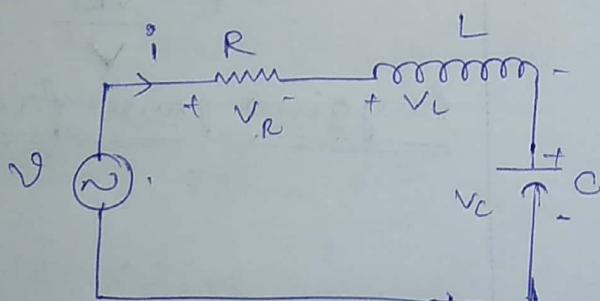


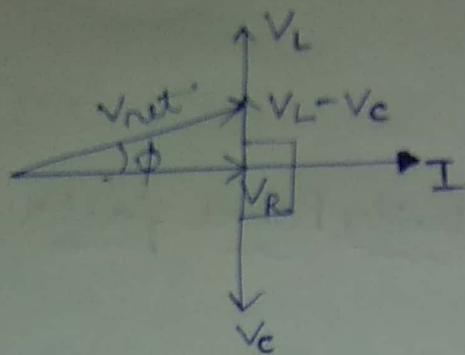
fig: RLC series ckt

$$V_R = IR \angle 0^\circ$$

$$V_L = IX_L \angle 90^\circ$$

$$V_C = IX_C \angle -90^\circ$$

If $V_L > V_C$:-



$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

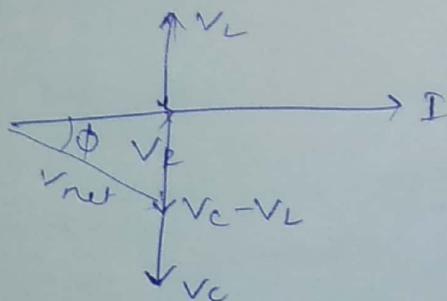
$$\phi = \tan^{-1} \left(\frac{V_L - V_C}{V_R} \right)$$

Power factor = $\cos\phi$

$$= \cos(\tan^{-1} \left(\frac{V_L - V_C}{V_R} \right))$$

=) Lagging power factor.

If $V_C > V_L$:-



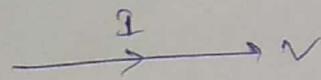
$$V_{net} = \sqrt{V_R^2 + (V_C - V_L)^2}$$

$$\phi = \tan^{-1} \left(\frac{V_C - V_L}{V_R} \right)$$

$$\cos\phi = \cos \left(\tan^{-1} \left(\frac{V_C - V_L}{V_R} \right) \right)$$

Leading power factor

$$V_C = V_L$$



$$\phi = 0$$

$$\underline{\cos \phi = 1}$$

Unity Power factor.

R-L Parallel circuit:-

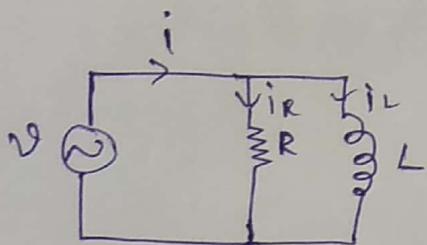
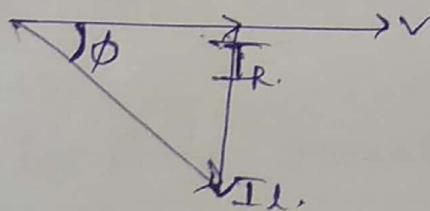


fig: RL P.C.

$$I_R = \frac{V}{Z} = \frac{V}{R} \angle 0^\circ$$

$$I_L = \frac{V}{Z} = \frac{V}{jX_L} = \frac{V}{X_L} \angle -90^\circ$$

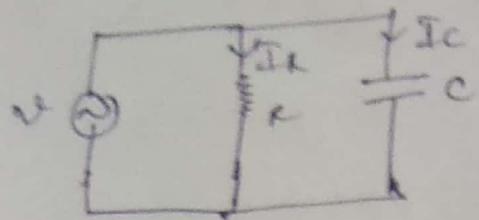


$$\text{Inet} = \sqrt{I_R^2 + I_L^2}$$

$$\phi = \tan^{-1} \left(\frac{I_L}{I_R} \right)$$

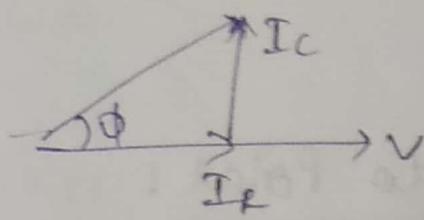
$$\cos \phi = \cos \left(\tan^{-1} \left(\frac{I_L}{I_R} \right) \right)$$

R-C Parallel ckt



$$I_R = \frac{V}{R}$$

$$I_C = \frac{V}{jX_C} = \frac{V}{j\omega C} = \frac{V}{\omega C}$$



$$I_{\text{net}} = \sqrt{I_R^2 + I_C^2}$$

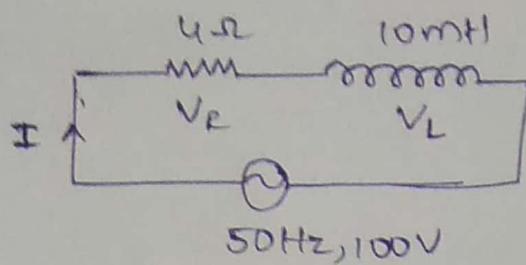
$$\phi = \tan^{-1}\left(\frac{I_C}{I_R}\right)$$

$$\cos\phi = \cos\left(\tan^{-1}\left(\frac{I_C}{I_R}\right)\right)$$

Solving Problems:

Q) 4 ohm Resistor connected to a 10mH inductor across 100V, 50Hz supply. Find

- ① impedance of the ckt
- ② input current
- ③ drop across the Resistor and inductor
- ④ Power factor of the ckt
- ⑤ Real power consumed in the ckt
- ⑥ Total power supplied



Sol) $Z = R + jX_L \Rightarrow Z = 4 + j(3 \cdot 14) \rightarrow \text{Rectangular form}$

$$X_L = 2\pi f(L)$$

$$= 2\pi \times 50 \times 10^{-3} = 5.085 \angle 38.13^\circ \Omega$$

$$= 3.14 \Omega$$

Converting Rectangular to Polar:

Mode \rightarrow Shift \rightarrow Pol($4, 3.14$)

$$Z = 5.085 \angle 38.13^\circ \Omega$$

b) $I = \frac{V}{Z} = \frac{100}{5.085} = 19.66 \angle -38.13^\circ$

c) $V_R = I_{\text{net}} R$

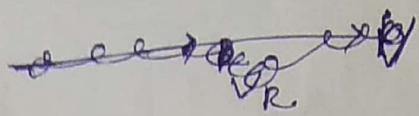
$$= 19.66 (4)$$

$$= 78.64 \angle -38.13^\circ \text{ V}$$

d) $V_L = I_{\text{net}} j X_L$

$$= 19.66 \angle -38.13^\circ \times 19.6 \angle 3.14^\circ$$

$$= 61.73 \angle 51.87^\circ$$



$$\phi = -38.13^\circ$$

$P_f = 0.787$ (Lagging power factor)

i) $P = VI \cos \phi$

$$= 100 \times 19.66 \times 0.787$$

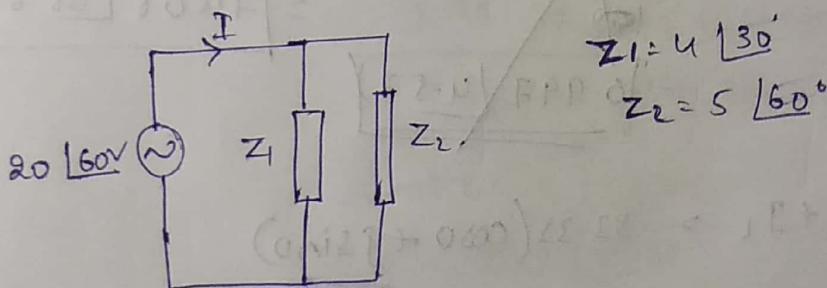
$$= 1547 \text{ W}$$

g) $S = VI$

$$= 100(19.66)$$

$$\boxed{1966 \text{ VA}}$$

Q) Find I shown in figure



$$I_1 = \frac{20 \angle 60^\circ}{4 \angle 130^\circ} = 5 \angle 30^\circ$$

$$I_2 = \frac{20 \angle 60^\circ}{5 \angle 60^\circ} = 4 \angle 0^\circ \text{ A.}$$

$$I = I_1 + I_2$$

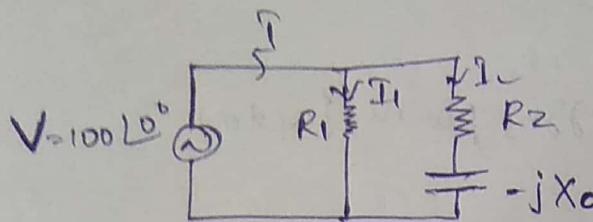
$$= 5 \angle 30^\circ + 4 \angle 0^\circ$$

$$= 5(\cos 30^\circ + j \sin 30^\circ) + 4(\cos 0^\circ + j \sin 0^\circ)$$

$$= 8.33 + j(2.5)$$

$$= 8.7 \angle 16.7^\circ \text{ A}$$

Q) In figure $R_1 = 3 \Omega$, $R_2 = 10 \Omega$, $-jX_C = -j8 \Omega$. Find I_1 , I_2 and I .



$$Z_C = R - jX_C$$

$$I_1 = \frac{V}{R_1} = \frac{100 \angle 0^\circ}{3 \Omega} = 33.33 \angle 0^\circ = 10 - 8j$$

$$I_2 = \frac{V}{R_2} = \frac{100 \angle 0^\circ}{10 - 8j}$$

$$\begin{aligned} &= \frac{100 \angle 0^\circ}{100 \cdot 3 \angle -45^\circ} \\ &= \frac{100 \angle 0^\circ}{12.806 \angle -38.65^\circ} \\ &= 7.808 \angle 38.65^\circ \end{aligned}$$

$$I = I_1 + I_2 = 33.33(\cos 0 + j \sin 0)$$

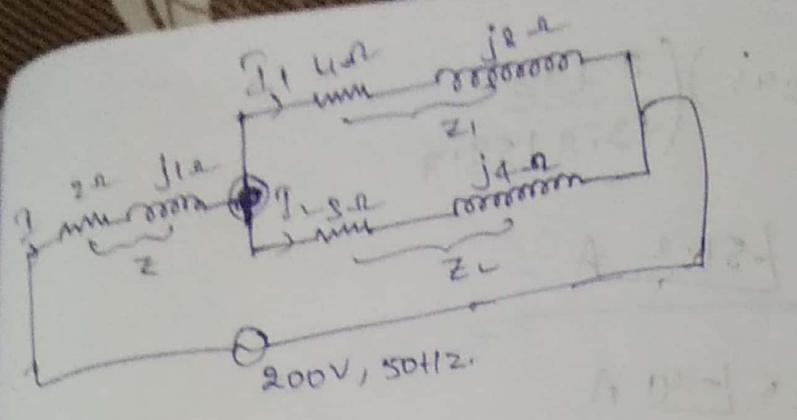
$$+ 7.808 (\cos 38.65 + j \sin 38.65)$$

$$= 33.3 + 6.697 + j(7.808 \sin 38.65)$$

$$= 39.723 + j(4.88)$$

$$= 39.723 \angle 7.055^\circ$$

Q) In the network shown in the figure, find I_1 , I_2 and I when it is connected to 200V, 50Hz supply.



$$Z_1 = 4+8j \Rightarrow 8.95 \angle 63.45^\circ$$

$$Z_2 = 3+4j \Rightarrow 5 \angle 53^\circ$$

$$Z_{BL} = \frac{Z_1 Z_L}{Z_1 + Z_L} \Rightarrow \frac{(8.95 \angle 63.45^\circ)(5 \angle 53^\circ)}{8.95 \angle 63.45^\circ + 5 \angle 53^\circ}$$

$$\Rightarrow \frac{44.75 \angle 116.45^\circ}{8.95(\cos 63.45^\circ + j \sin 63.45^\circ) + 5(\cos 53^\circ + j \sin 53^\circ)}$$

$$\Rightarrow \frac{44.7 \angle 116.45^\circ}{13.9 \angle 59.75^\circ}$$

$$= 3.219 \angle 56.7^\circ$$

$$= [1.76 + j 2.69]$$

$$Z_{eq} = Z_{tot} = Z_{AB} + Z_{BL}$$

$$= (2+j) + (1.76 + j 2.69)$$

$$\Rightarrow 3.76 + j (3.69)$$

$$\Rightarrow 5.268 \angle 44.45^\circ$$

$$I = \frac{V}{Z} = \frac{200 \angle 0^\circ}{5.268 \angle 44.45^\circ} \Rightarrow 37.965 \angle -44.45^\circ$$

$$I_1 = I \frac{(Z_L)}{Z_1 + Z_L} \Rightarrow (37.965) \angle -44.45^\circ \left(\frac{3+4j}{7+12j} \right)$$

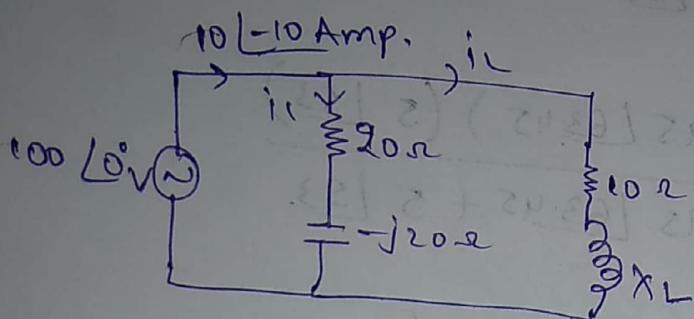
$$\Rightarrow (37.955 \angle -44.45^\circ) \left(\frac{5 \angle 53^\circ}{13.9 \angle 59.7^\circ} \right)$$

$$\Rightarrow 13.654 \angle -51.2^\circ A$$

IIIrd

$$I_2 = 24.45 \angle -41^\circ A$$

Q1 In the network shown in figure, find $X_{L\text{val}}$



$$100 \angle 0^\circ = i_1 (20 - 20j)$$

$$i_1 = \frac{100 \angle 0^\circ}{20 - 20j} = \frac{100 \angle 0^\circ}{28.2 \angle -45^\circ}$$

$$= 3.546 \angle 45^\circ$$

$$I = I_1 + I_L$$

$$10 \angle 0^\circ = 3.546 \angle 45^\circ + I_L$$

$$I_L = 10 (\cos(-10) + j \sin(-10)) - 3.54 (\cos 45^\circ - j \sin 45^\circ)$$

$$= 10 \cos -10 - 3.54 \cos 45^\circ$$

$$+ j (10 \sin(-10) + 3.54 \sin 45^\circ)$$

$$= 4.841 + j(0.7666) = 4.9 \angle 8.99^\circ$$

$$T_1 = 4.84 + j(0.766) \quad T_2 = 8.476 \angle -30^\circ$$

$$V = I_2 Z_L$$

$$\frac{100 \angle 0^\circ}{4.84 + j(0.766)} = 10 + j(?)$$

$$= 10 + j(8.99) = 10 + j8.99$$

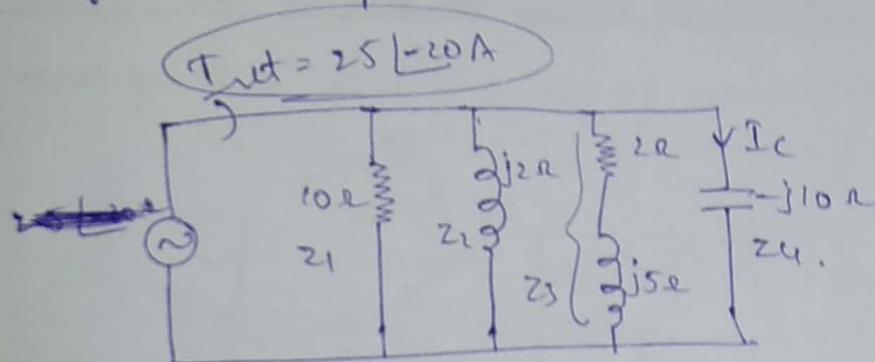
$$= 10 + j8.99 \angle -30^\circ = 10 + j8.99$$

$$\Rightarrow 11.79 \angle 30^\circ = 10 + j(?)$$

$$\Rightarrow 10.1 + j(5.8) = 10 + j(?)$$

$$\boxed{j\Delta = 5.8}$$

Q) Find the Equivalent impedance of given network shown in figure. Calculate current through the capacitor



$$\begin{aligned} Z_1 &= 10 \Omega \\ Z_2 &= j2 \Omega \\ Z_3 &= \frac{1}{j2}, \frac{1}{j5}, \frac{1}{j10} \Omega \\ Z_4 &= -j10 \Omega \end{aligned}$$

$$Y_1 = \frac{1}{Z_1} = 0.1 \text{ mho}$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{j2} = -j0.5 \text{ mho}$$

$$Y_3 = \frac{1}{Z_3} = \frac{1}{\frac{1}{j2} + \frac{1}{j5} + \frac{1}{j10}} = \frac{1}{\frac{1}{\sqrt{21+25}} + \frac{5}{\sqrt{21+25}}} = 0.062 - j(0.171) \text{ mho}$$

$$Y_4 = \frac{1}{Z_4} = 0.1 \text{ mho}$$

$$Y_{\text{net}} = Y_1 + Y_2 + Y_3 + Y_4 \quad (\because \text{They're in parallel})$$

$$Y_{\text{net}} = 0.169 - j 0.572$$

$$Z_{\text{net}} = \frac{1}{Y} = 0.474 + j (1.61)$$

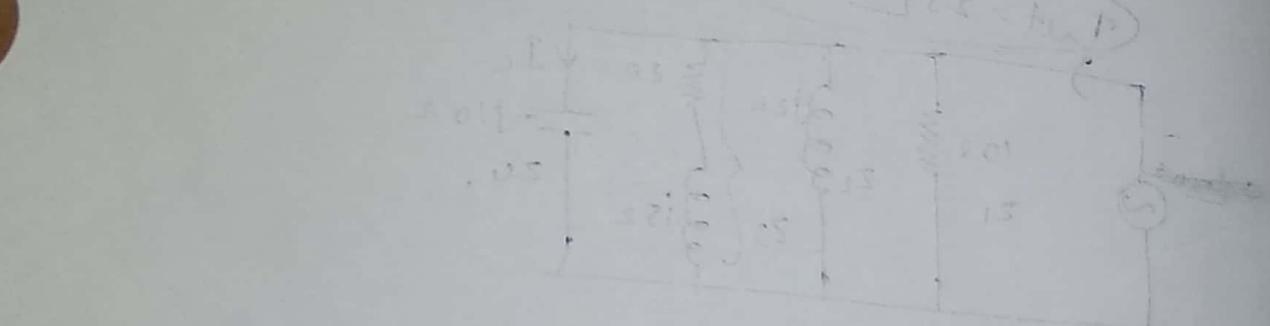
$$\text{Equivalent impedance} = 0.474 + j 1.61 = 1.675 \angle 72.56^\circ$$

$$V_{\text{net}} = (25 \angle 0^\circ) (1.675 \angle 72.56^\circ)$$

$$V_{\text{net}} \Rightarrow 41.875 \angle 52.56^\circ$$

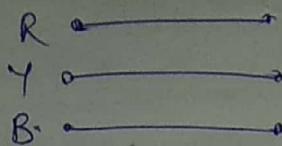
$$I_C = \frac{V_{\text{net}}}{-10j} = 41.875 \angle 52.56^\circ \xrightarrow{-10 \angle -90}$$

$$\Rightarrow 4.1875 \angle 142.56^\circ \text{ Amperes}$$

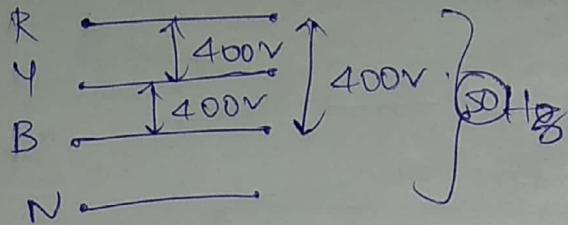


3-Φ - A.C. crcts

* Delta connection we have 3 connections

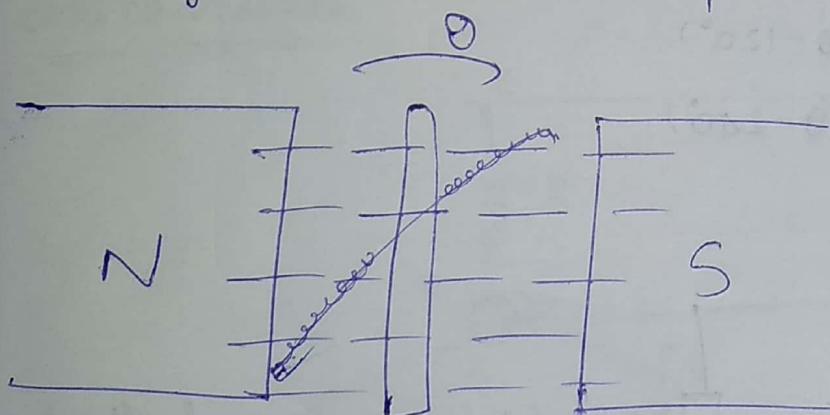


* For star, we have one (neutral) entia.



How to The minimum Requirement of generation of EMF:-

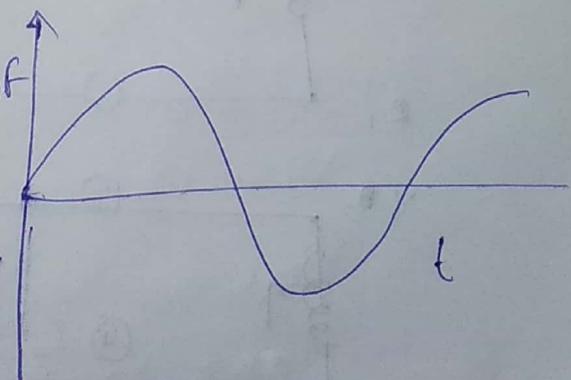
- ① Magnetic field
- ② conductors or set of conductors.
- ③ Time variation or space variation between magnetic field & set of conductors.



$$\textcircled{e} = E_m \sin \theta$$

instantaneous
EMF generated

fig:- gen of 1Φ EMF



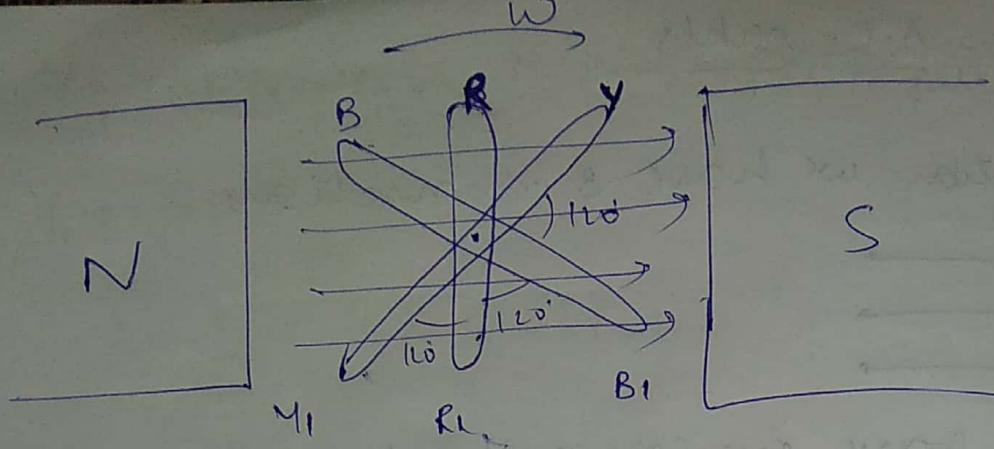
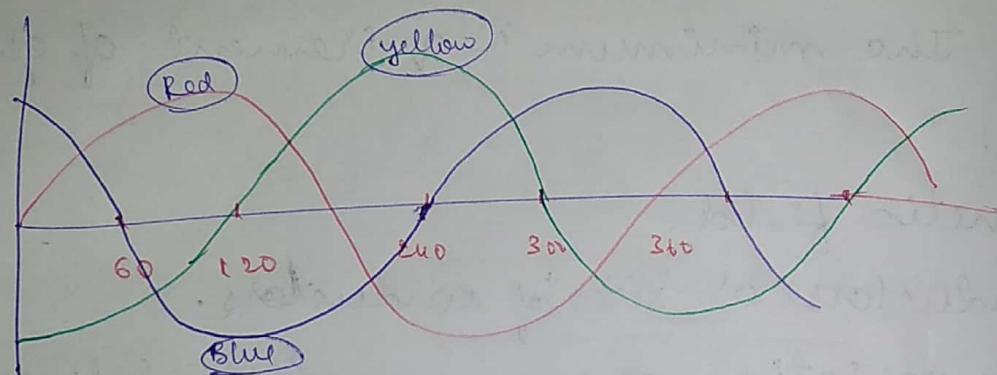


Fig: gen of 3-Φ EMF.

Y lags Red by 120°

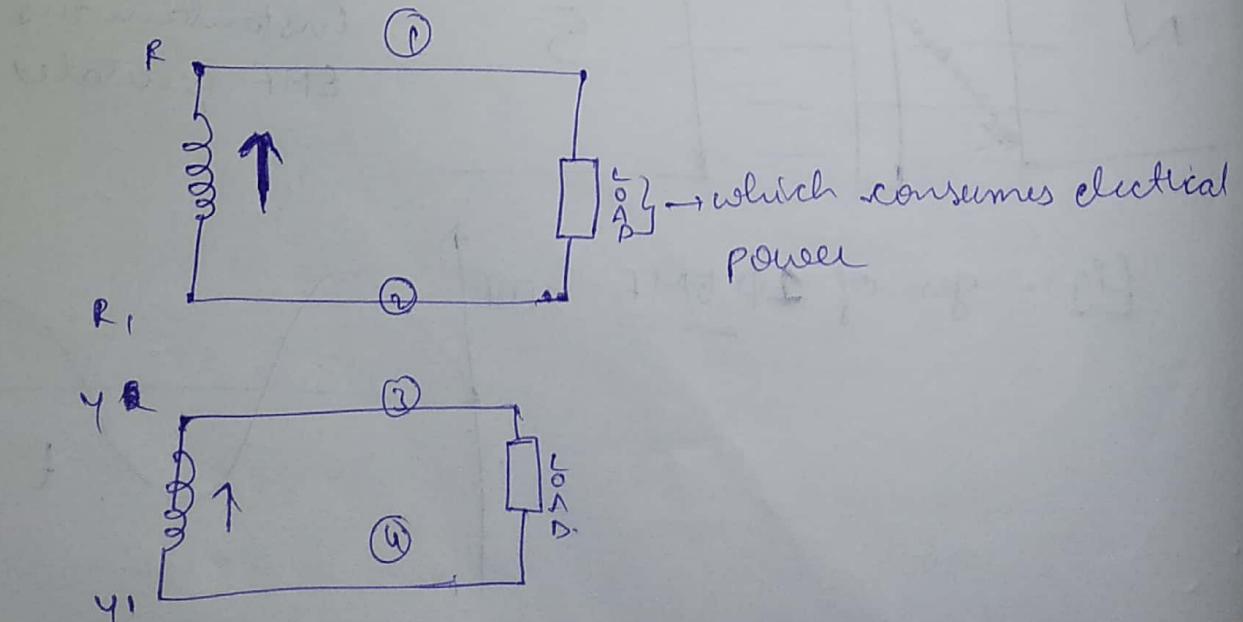
B lags Red by 240°

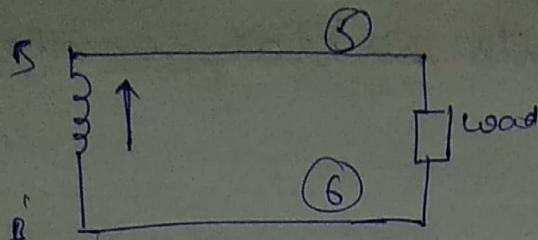


$$e_R = E_R \sin \theta$$

$$e_Y = E_Y \sin(\theta - 120^\circ)$$

$$e_B = E_B \sin(\theta - 240^\circ)$$





for $3\phi \rightarrow$ 6 conductors are required \Rightarrow more cost

To reduce no of conductors we use

star connection

L

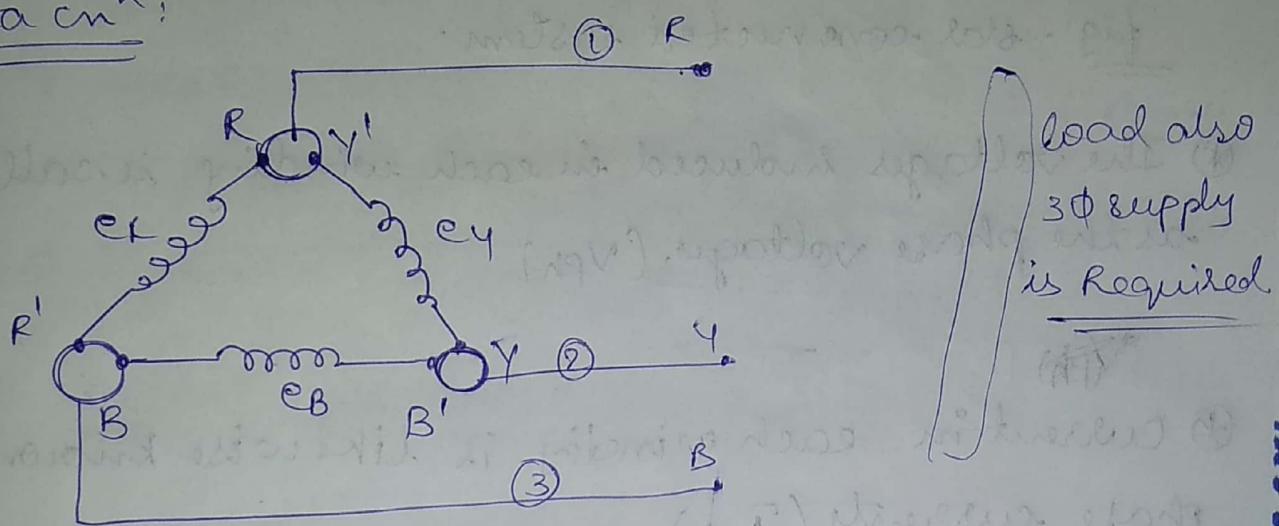
4 Reg

Delta connection

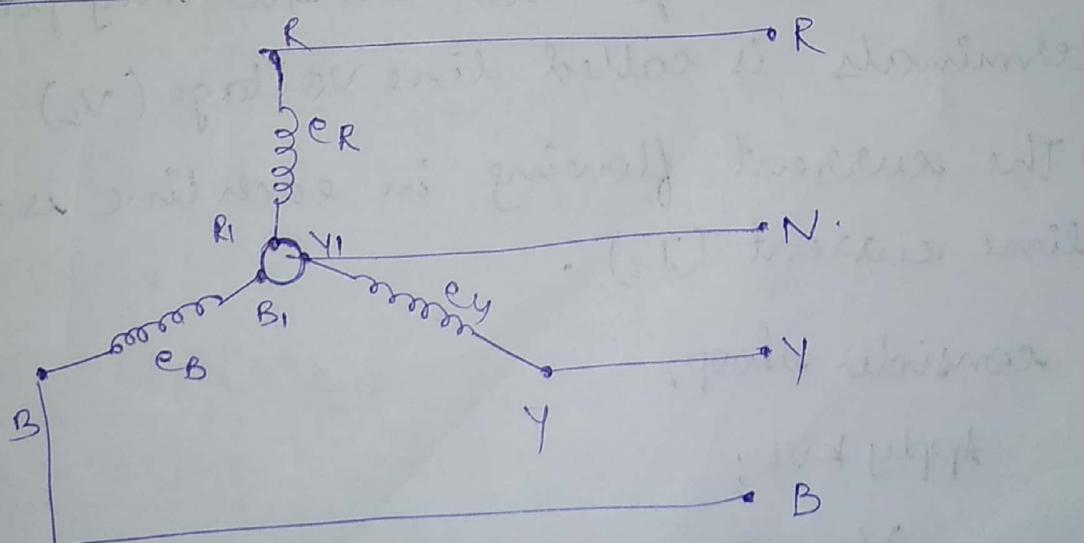
L

only 3 Reg

Delta cnⁿ:



star connection:



line to line $\rightarrow 3\phi \rightarrow [400V]$

line to neutral $\rightarrow 1\phi \rightarrow [230V]$

Voltages and currents in star connected systems :

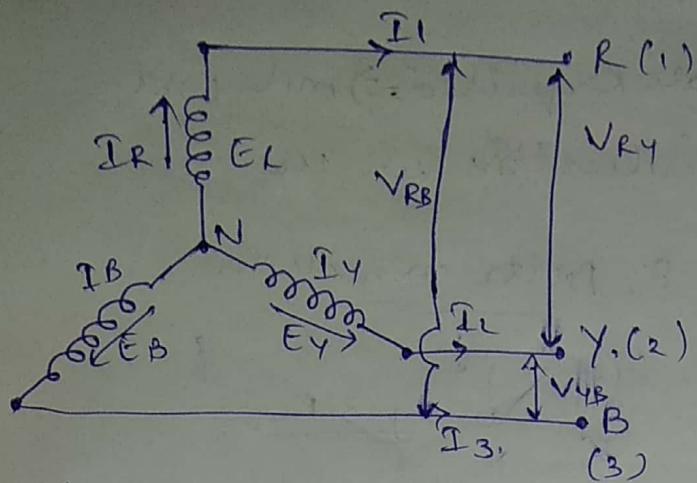


fig:- star connected system.

- ✳ The voltages induced in each winding is called as the phase voltages, (V_{ph})
- ✳ Current in each winding is likewise known as phase currents (I_{ph}).
- ✳ However voltage available b/w any pairs of terminals is called line voltage (V_L)
- ✳ The current flowing in each line is called line current (I_L).

consider a loop;

apply KVL;

$$V_{RY} = E_R - E_Y \quad ||| \quad V_Y$$

$$V_{BR} = V_B - V_R$$

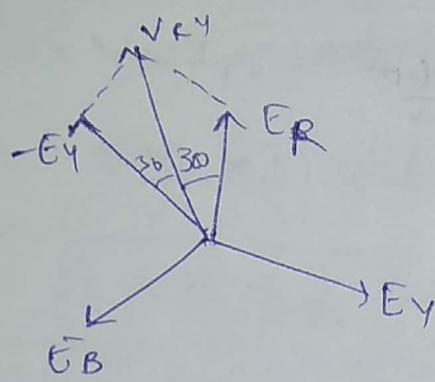
$$V_{BY} = V_Y - V_B$$

The vector diagram for phase voltage & current in star diagram is shown in fig where a Balanced system has been assumed con, considered.

$$\begin{cases} E_R = E_B = E_Y = E_{ph} \\ I_R = I_B = I_Y \\ I_1 = I_2 = I_3 \\ V_{RY} = V_{RB} = V_{YB} \end{cases}$$

* The line voltage V_{RY} b/w line ① & line ② is the vector difference of E_R & E_Y , similarly, the line voltage V_{YB} b/w line ② & line ③ is vector difference of E_Y & E_B vector. and line voltage E_{BR} b/w line ③ & line ① is vector difference of E_B & E_R vector.

a) LINE VOLTAGE AND PHASE VOLTAGE



$$V_{RY} = E_R - E_Y$$

$$V_{RY} = 2 \times E_{ph} (\cos 60^\circ)$$

$$V_L = \sqrt{3} E_{ph}$$

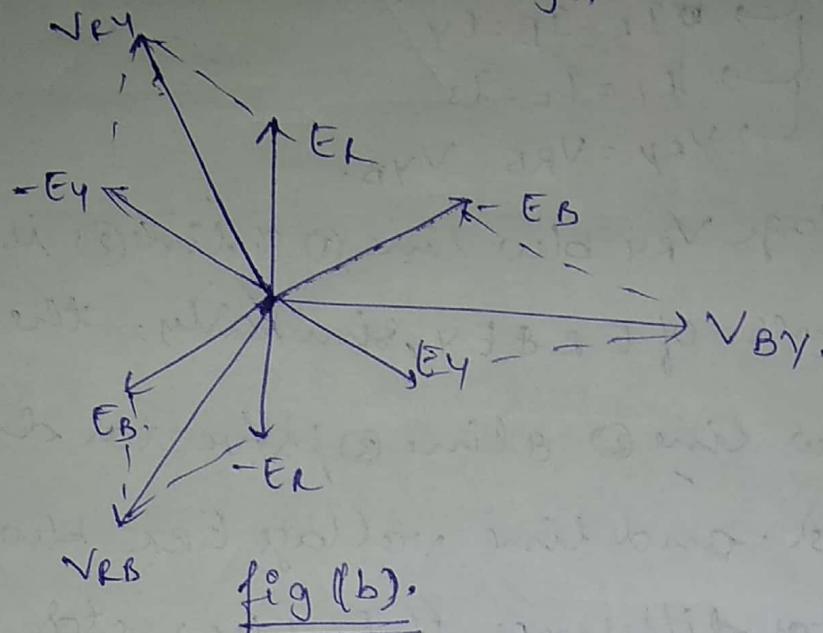
* Potential diff b/w line ① & ② is V_{RY} this is equal to $E_R - E_Y$.

Hence V_{RY} is found by compounding E_R and E_Y Reversed

and its value is given by diagonal of the llgs shown in figure.

* Obviously, the angle between E_R and E_Y (or V_B) is 60° .

Hence if $E_R = E_Y = E_B = E_{ph}$ (say),



from phasor diagram

$$V_{R4} = E_R - E_Y$$

$$V_{R4} = 2 E_{ph} \cos \frac{60}{2}$$

$$= \sqrt{3} E_{ph}$$

$$\therefore \boxed{V_L = \sqrt{3} E_{ph}}$$

$$\text{Hence } V_{YB} = \sqrt{3} E_{ph}$$

$$V_{BR} = \sqrt{3} E_{ph},$$

\therefore In the star connected system, the relationship between line voltage and phase voltage i.e., line voltage is equal to $\sqrt{3}$ times of phase voltage.

$$\therefore \boxed{V_L = \sqrt{3} E_{ph}} \quad \underline{\text{in star.}}$$

b) LINE CURRENT AND PHASE CURRENT

The current in line 1 = I_R .

The current in line 2 = I_Y .

The current in line 3 = I_B .

→ Hence if $I_R = I_B = I_Y = I_{ph}$. (say), then

$$I_L = I_{ph}$$

Therefore in starconnected system, the relationship b/w line current & phase current is both are equal

POWER:

i) The total active power in the crkt is the sum of three phase power. Hence total active power

$$P = 3 \times \text{Phase power}$$

$$\Rightarrow P = 3 \times V_{ph} I_{ph} \cos \phi$$

$$\Rightarrow P = 3 \times \frac{V_L \times I_L \cos \phi}{\sqrt{3}}$$

$$P = \sqrt{3} V_L I_L \cos \phi \quad [\text{watts}]$$

$$\text{Now } Q = \sqrt{3} V_L I_L \sin \phi \quad [\text{VAR}]$$

Reactive power ↑

$$S = \sqrt{3} V_L I_L \text{ VA} \quad [\text{apparent power}]$$

Voltages and currents in Δ connected system.

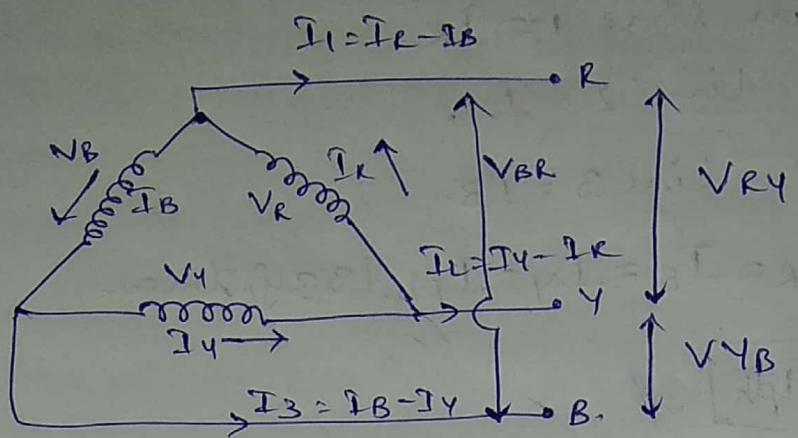
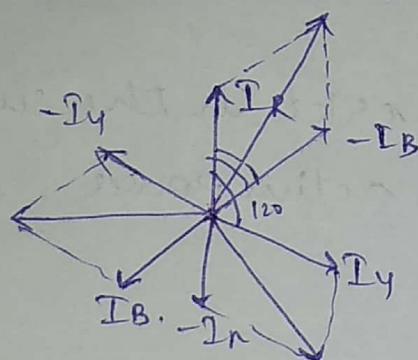


fig Δ -connected system

line currents and phase currents :-



$$I_1 = I_R - I_B$$

$$I_1 = 2 I_{ph} \cos \frac{60}{2}$$

$$I_1 = 2 I_{ph} \frac{\sqrt{3}}{2}$$

$$\boxed{I_1 = \sqrt{3} I_{ph}}$$

from fig ①, current in each line is the vector difference of two phase currents flowing through that line.

For Ex:- current in line ① is

$$I_1 = I_R - I_B$$

$$I_1 = 2 I_{ph} \cos(60)$$

$$I_1 = 2 I_{ph} \frac{\sqrt{3}}{2}$$

$$\boxed{I_1 = \sqrt{3} I_{ph}}$$

current in line ② is

$$\boxed{I_2 = I_B - I_Y}$$

current in line ③ is

$$\boxed{I_3 = I_Y - I_R}$$

current in line ① is found by compounding
I_R and I_B reversed and its value is given by
diagonal of 11 m in fig ②. The angle b/w I_R &
I_B rev is 60°. If I_R = I_y = I_B = I_{ph}. (say).

From phasor diagram, current in line ① is
equal to I₁ = I_R - I_B

$$I_1 = \sqrt{2} I_{ph} \cos(60^\circ)$$

$$\boxed{I_1 = \sqrt{3} I_{ph}}$$

$$\text{Hence } \boxed{I_2 = \sqrt{3} I_{ph}} \text{ and } \boxed{I_3 = \sqrt{3} I_{ph}}$$

Therefore in delta connected system, the
relationship between line current and phase current
is line current is equal to $\sqrt{3}$ times of phase current

$$\boxed{I_L = \sqrt{3} I_{ph}}$$

(b) Line voltage and phase voltage :-

Therefore, in Δ connected systems, the relationship
between line voltage & phase voltage is

$$\boxed{\text{line voltage} = \text{phase voltage}}$$

Power:-

i) Active power :- Hence the total active power
is equal to $3 \times$ phase power.

$$P = 3 \times \text{ph power}$$

$$\Rightarrow P = 3 \times V_{ph} I_{ph} \cos\phi$$

$$P = 3 \times V_L \cdot \frac{I_L}{\sqrt{3}} \cos\phi \Rightarrow \boxed{P = \sqrt{3} V_L I_L \cos\phi}$$

III^{by} Reactive power.

$$Q = \sqrt{3} V_L I_L \sin\phi \quad \text{VAR.}$$

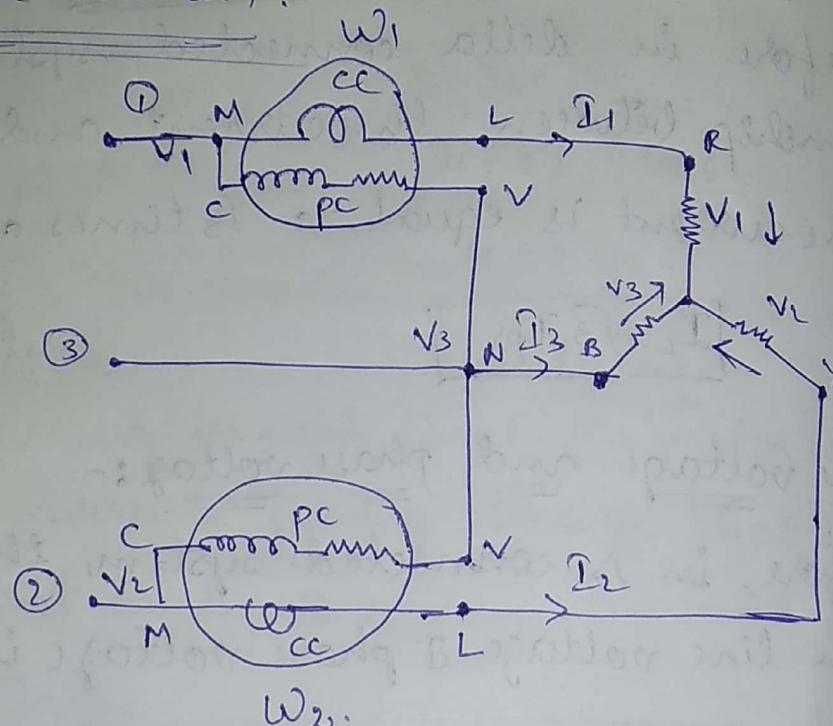
Apparent power:

$$S = \sqrt{3} V_L I_L \quad \text{VA.}$$

MEASUREMENT OF POWER IN 3-Φ

We use 2 wattmeters to measure power in 3Φ.

(2-Watt Meter method):



Supply
M → main

L → load

V → voltage

C → common

The instantaneous reading of W_1 wattmeter

$$P_1 = VI$$

$$= V_{13} I_1$$

$$= (V_1 - V_3) I_1 \rightarrow ①$$

The instantaneous reading of W_2 wattmeter is

$$\begin{aligned} P_2 &= VI \\ &= V_{23} I_2 \\ &= \underline{(V_2 - V_3) I_2} \rightarrow ② \end{aligned}$$

* The sum of the instantaneous powers ^{powers} ~~currents~~ in wattmeter reading is

$$P = P_1 + P_2$$

$$= (V_1 - V_3) I_1 + (V_2 - V_3) I_2$$

$$P = V_1 I_1 + V_2 I_2 - V_3 (I_1 + I_2)$$

Apply KCL at node

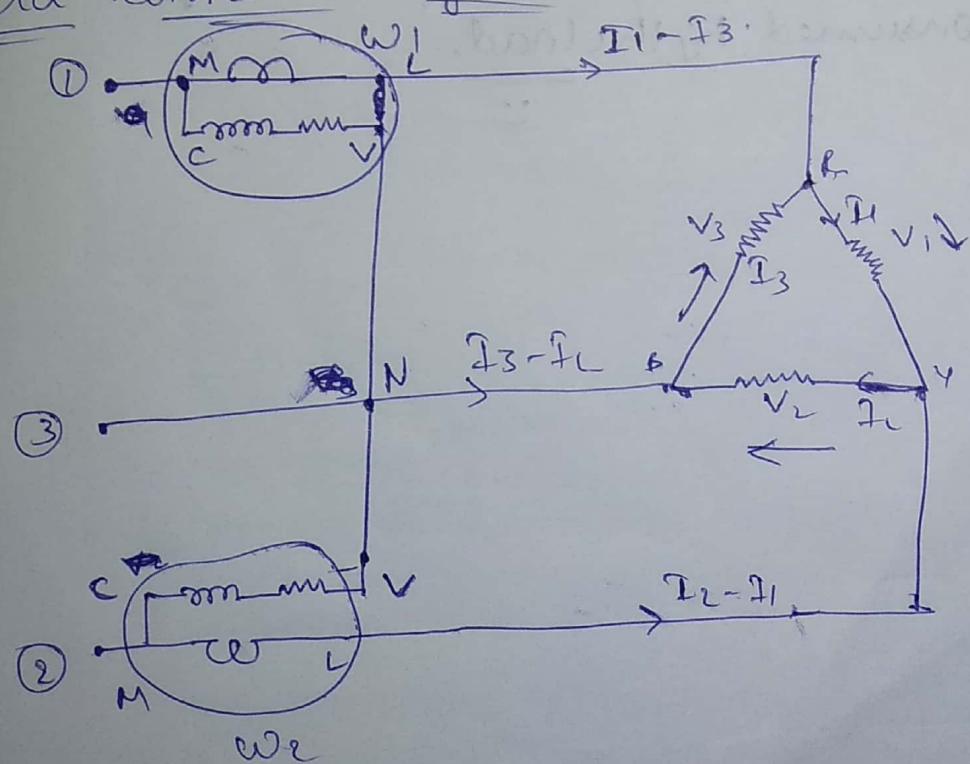
$$I_1 + I_2 + I_3 = 0$$

$$\boxed{I_1 + I_2 = -I_3}$$

$$\boxed{P = V_1 I_1 + V_2 I_2 + V_3 I_3}$$

* Therefore, the sum of the 2w meter reading is equal to the total power consumed by the load

Beta connected system :-



The instantaneous values of w_1 wattmeter

$$P_1 = VI$$

$$= V_3 (I_1 - I_3) \rightarrow ①$$

The instantaneous values of w_2 wattmeter

$$P_2 = V I$$

$$= (I_2 - I_1)V_2 \rightarrow ②$$

sum of wattmeter readings = $P_1 + P_2$

$$P_{\text{alt}} = V_2 I_2 + V_3 I_3 - I_1 (V_2 + V_3)$$

Apply KVL in Loop

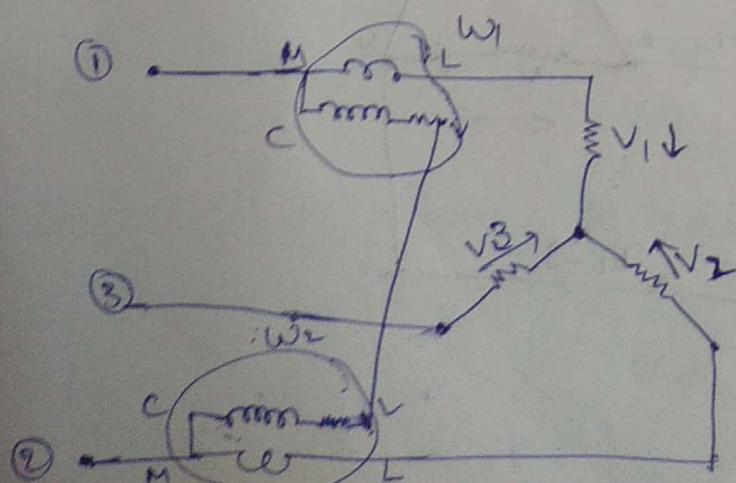
$$V_1 + V_2 + V_3 = 0$$

$$\underline{V_2 + V_3 = -V_1}$$

$$\therefore P_{\text{alt}} = V_1 I_1 + V_2 I_2 + V_3 I_3$$

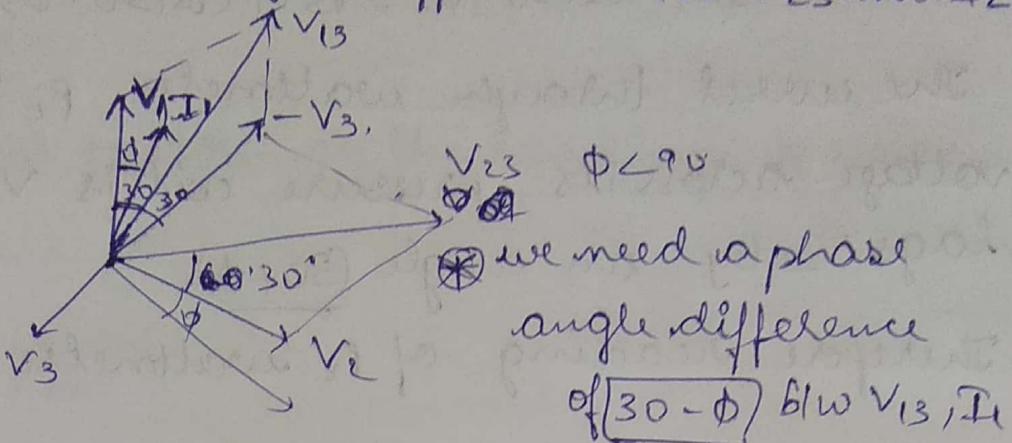
Therefore the sum of 2W meter reading is equal to power consumed by the load.

Measurement of Power factor (a) Exp'n for power factor
[STAR CONNECTED system].



$$P_f = \cos \phi$$

$P = V_{13} I_1$ phase angle difference b/w V_{13} and I_1 ,
 $P = V_{23} I_2$ phase angle difference b/w V_{23} and I_2 .



we also need a phase angle diff of $V_{23} \text{ & } I_2$ is $30 + \phi$

* Figure shows the phasor diagram for a balanced \star connected load. Let V_1, V_2, V_3 be the RMS values of phase voltages and I_1, I_2, I_3 be RMS values of phase current.

The load is balanced;

Therefore phase voltages $V_1 = V_2 = V_3 = \text{V (say)}$

& phase currents $I_1 = I_2 = I_3 = I \text{ (say)}$.

& Line voltage $V_{12} = V_{23} = V_{31} = \sqrt{3} V$

& Line currents $I_L = I_1 = I_2 = I_3 = I$

* Power factor $\cos \phi$.

The phase currents lags the corresponding phase voltages by an angle ϕ .

The current through the wattmeter P_1 is I_1 & the voltage across pressure coil is V_{13} . I_1 leads V_{13} by an

angle $30 - \phi$.

Therefore Reading of P_1 wattmeter is $V_{13} I_1 \cos(30 - \phi)$

$$\therefore P_1 = V_{13} I_1 \cos(30 - \phi) = \sqrt{3} V_1 \cos(30 - \phi) \rightarrow ①$$

The current through wattmeter P_2 is I_2 & voltage across its pressure coil is V_{23} . Here I_2 lags V_{23} by an angle $(30 + \phi)$.

Therefore reading of P_2 wattmeter is

$$\therefore P_2 = V_{23} I_2 \cos(30 + \phi) = \sqrt{3} V_1 \cos(30 + \phi) \rightarrow ②$$

The sum of Readings of 2-wattmeters is $P_1 + P_2$

$$P_1 + P_2 = \sqrt{3} V_1 (\cos(30 - \phi) + \cos(30 + \phi))$$

$$\boxed{P_1 + P_2 = 3V_1 \cos \phi,} \rightarrow ③$$

The difference of Readings of two wattmeters

$$\text{i.e., } P_1 - P_2 = \sqrt{3} V_1 [\cos(30 - \phi) - \cos(30 + \phi)]$$

$$\boxed{P_1 - P_2 = \sqrt{3} V_1 \sin \phi} \rightarrow ④$$

Dividing eqⁿ ④ / ③ we get

$$\frac{P_1 - P_2}{P_1 + P_2} = \frac{\tan \phi}{\sqrt{3}}$$

$$\boxed{\tan \phi = \sqrt{3} \left(\frac{P_1 - P_2}{P_1 + P_2} \right)}$$

$$\boxed{\phi = \tan^{-1} \left(\sqrt{3} \left(\frac{P_1 - P_2}{P_1 + P_2} \right) \right)}$$

$$\text{Power factor} = \cos \left(\tan^{-1} \left(\frac{\sqrt{3}(P_1 - P_2)}{P_1 + P_2} \right) \right)$$

Here P_1 & P_2 are readings of wattmeter 1 & wattmeter 2.

Effect of power factor on the Readings of wattmeter:

1) Unity power factor:

$$\cos\phi = 1 \Rightarrow P_1 = \sqrt{3}VI \cos(30-0)$$

$$\phi = 0 \Rightarrow P_2 = \sqrt{3}VI \cos(30+0)$$

$$\therefore P_1 = P_2 = \frac{3VI}{2}$$

$$P_1 + P_2 = \sqrt{3}VI$$

Thus, at unity power factor, the Readings of the two wattmeters are equal, each wattmeter Reads half of total power

2) case-II :- $\cos\phi = 0.5$

$$\boxed{\phi = 60^\circ}$$

$$\Rightarrow P_1 = \sqrt{3}VI \cos(30-60) = \frac{3VI}{2}$$

$$P_2 = \sqrt{3}VI \cos(30+60) = 0$$

$$\boxed{P_1 + P_2 = \frac{3VI}{2}}$$

Thus, when power factor is 0.5, one of the wattmeter Reads 0 and other Reads total power

3) Case-III :- $\cos\phi = 0 \Rightarrow \phi = 90^\circ$

$$P_1 = \sqrt{3}VI \cos(30+90) =$$

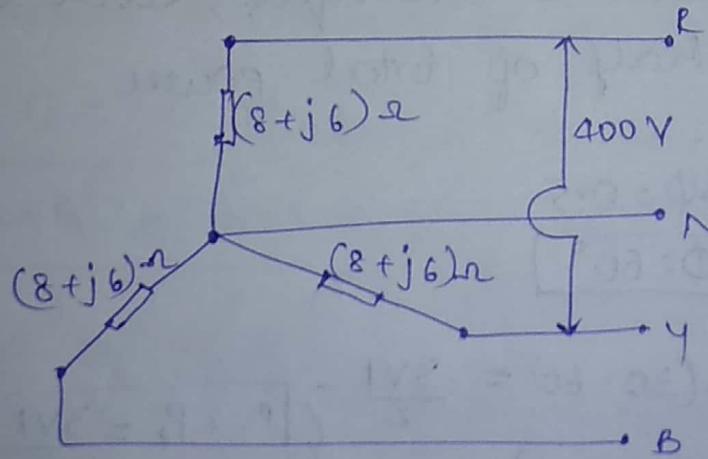
$$P_2 = \sqrt{3}VI \cos(30+90) =$$

$$\begin{cases} \frac{\sqrt{3}VI}{2} \\ -\frac{\sqrt{3}VI}{2} \end{cases}$$

$$\begin{cases} P_1 + P_2 = 0 \\ (P_1 = -P_2) \end{cases}$$

Therefore with zero power factor the readings of the two wattmeters are equal but opposite sign. It should be noted that when the power factor is below 0.5, one of the wattmeter will give -ve indication. Under this conditions, in order to read the wattmeter, we must either reverse the current coil or pressure coil connection. The wattmeter will then give a +ve reading. But this must be taken as -ve for calculating total power.

Prob: A balanced \star connected load of $(8+j6)\Omega$ per phase is connected to a balanced 3ϕ -400V supply. Find line current, power factor, power and total Volt Amperes.



$$\cos \phi = \frac{R}{Z}$$

$$Z = \sqrt{8^2 + 6^2} = 10\Omega$$

$$\text{Power} = \sqrt{3}V_L I_L \cos \phi$$

$$\text{line current} = \frac{\text{line voltage}}{\text{line impedance}}$$

$$V_L = \sqrt{3} V_{ph}$$

$$V_{ph} = \frac{400}{\sqrt{3}} = 231V$$

1) $\therefore I_L = \frac{231}{10} = 23.1$

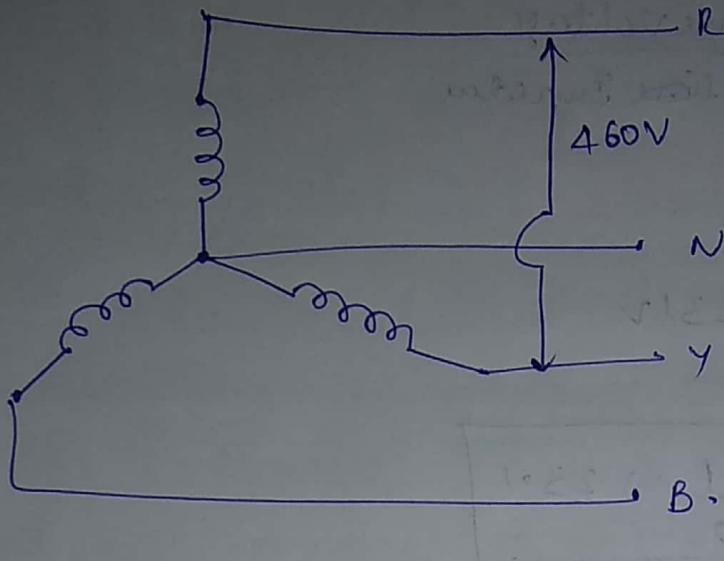
2) Power factor $\cos \phi = \frac{P}{S} = \frac{8}{10} = 0.8$ $\cos \phi = \frac{8}{10} = 0.8$ (log)
 $\phi = \cos^{-1}\left(\frac{3}{5}\right) = 53^\circ$

always
if capacity
is not
mentioned

3) Power $= \sqrt{3} V_L I_L \cos \phi$
 $= \sqrt{3} \times 400 \times 23.1 \times 0.8$
 $> 12.8 \text{ kW}$

4) $S = \sqrt{3} V_L I_L$
 $= \sqrt{3} \times 400 \times 23.1$
 $= 16 \text{ kVA}$

Prob: 3 equal star connected inductors takes 8kWatts at powerfactor 0.8 when connected across a 400V, 3φ supply. Find circuit constants of the load per phase



given
 $\cos\phi = 0.8$
 $P = 8 \text{ kW}$

Sol) given $V_L = 460V$

$$V_{Ph} = \frac{460}{\sqrt{3}} = 265.58 \text{ V}$$

$$\text{Power} = \sqrt{3} V_L I_L \cos\phi$$

$$8 \text{ k} = (\sqrt{3})(460)(I_L)(0.8)$$

$$I_L = 12.55 \text{ Amp} = I_{Ph}$$

$$Z_{Ph} = \frac{V_{Ph}}{I_{Ph}} = \frac{265}{12.5} = 21.16 \Omega$$

$$\cos\phi = \frac{R_{Ph}}{Z_{Ph}}$$

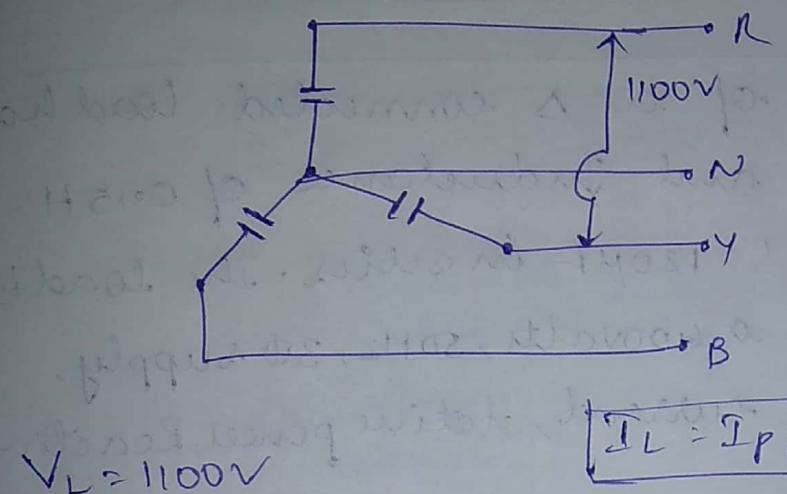
$$0.8 = \frac{R_{Ph}}{21.16} \Rightarrow R_{Ph} = 16.9$$

$$Z_{Ph} = \sqrt{R_{Ph}^2 + X_{Ph}^2}$$

$$X_{Ph} = \sqrt{Z_{Ph}^2 - R_{Ph}^2} = 12.733$$

$$X_{Ph} = 12.733$$

Prob: A balanced 3 ϕ star connected load of 120 kWatts takes a leading current of 85 Amperes when connected across a 3 ϕ 1100 V, 50 Hz supply. obtain the values of the Resistance, Impedance and capacitance of the load per phase and also calculate power factor of the load



$$P = \sqrt{3} V_L I_L \cos \phi$$

$$120 \times 10^3 \text{ W} = \sqrt{3} \times 1100 \text{ V} \times 85 \text{ A} \cos \phi$$

$$\cos \phi = \frac{1200}{85 \sqrt{3} \times 11} = \frac{8.150}{11} = 0.741$$

$$\therefore \text{Power factor} = \frac{8.150}{11} = 0.741$$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{1100}{\sqrt{3} \times 85} = 7.471 \Omega$$

$$\cos \phi = \frac{R_{ph}}{Z_{ph}} \Rightarrow R_{ph} = (0.741) (7.471) = 5.536 \Omega$$

$$Z_{ph}^2 = R_{ph}^2 + X_{ph}^2 \Rightarrow X_{ph} = \sqrt{(7.471)^2 - (5.536)^2}$$

$$X_{ph} = 4.917 + 5.02 \text{ approx}$$

$$X_C = \frac{1}{\omega C}$$

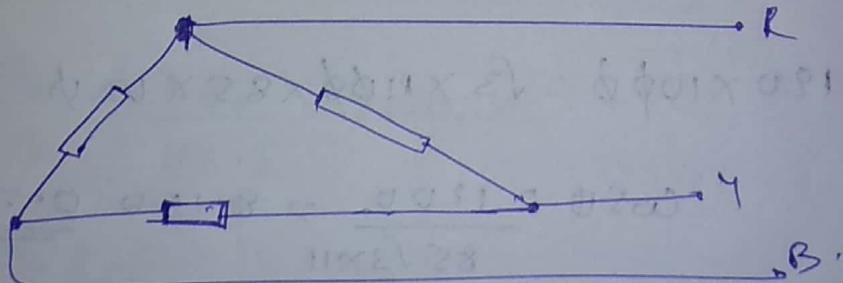
$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 5.02}$$

$$= \frac{1}{100\pi(5.02)}$$

$$\Rightarrow \boxed{634.3 \mu F}$$

Prob: Each phase of a Δ connected load has a Resistance of 25Ω and Inductance of $0.15H$ and capacitance of $C = 120\mu F$ in series. The load is connected across a 400volts, 50Hz, 3ϕ supply. Determine the line current, active power, Reactive volt Amperes.

(sol)



$$X_C = \frac{1}{\omega C} = \frac{10^6}{2\pi \times 50 \times 120} = \frac{10^6}{100\pi \times 120} = \frac{10^3}{12\pi} = 26.53$$

$$X_L = \omega L = 2\pi \times 50 \times 0.15 = 15\pi = 47.1$$

here $X_L > X_C \Rightarrow$ Lagging power factor.

$$Z = R + j X_L - j X_C$$

$$= 25 + j(47.1 - 26.53) = \boxed{25 + j(20.57)}$$

here $V_{ph} = V_L = 400 \text{ V}$

$$Z_{ph} = \sqrt{(25)^2 + (20\sqrt{3})^2}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}}$$

$$Z_{ph} = 32.37 \Omega$$

$$I_{ph} = \frac{400}{32.37} = 12.357$$

$$\text{we know } I_L = \sqrt{3} I_{ph}$$

$$= (12.35)(\sqrt{3})$$

$$I_L = 21.424$$

$$\cos\phi = \frac{1}{2} = \frac{25}{32.37} = 0.7723$$

$$\phi = 39.84^\circ$$

$$\text{Power} = \sqrt{3} V_L I_L \cos\phi$$

$$\text{Power} = \sqrt{3} \times 400 \times 21.42 \times 0.7723 \approx 11426.62 \text{ watts}$$

$$S = \sqrt{3} V_L I_L$$

$$= 14.8 \text{ kVA}$$

Prob:- The power flowing in a 3φ were balanced system measuring by 2wattmeter method. The reading of wattmeter A is 750 watts and of the wattmeter B is 1500 watts.

Find

- Total power
- Power factor

$$\text{Sol) we know } \frac{\tan \phi}{\sqrt{3}} = \frac{P_1 - P_L}{P_1 + P_L}$$

$$\begin{aligned} \tan \phi &= \frac{\sqrt{3}(-7500 - (-1500))}{-750 + 1500} \\ &= \frac{\sqrt{3}(2250)}{-750} \end{aligned}$$

$$\text{Total power} = P_1 + P_L \Rightarrow 7500 - 1500 = 6000 \text{ watts}$$

$$\frac{\tan \phi}{\sqrt{3}} \Rightarrow \frac{P_1 - P_L}{P_1 + P_L} \Rightarrow \frac{9000}{6666\phi} = 3/2$$

$$\tan \phi = \frac{3\sqrt{3}}{2}$$

$$\phi = \tan^{-1}\left(\frac{3\sqrt{3}}{2}\right) = \boxed{2.598} = 68.9^\circ$$

$$\cos \phi = 0.359$$

[HW] A 400V, 3φ voltage is applied to a balanced 3φ Δ connected load of $[15 + j(20)] \Omega$. Find

- ① The phasor currents in each line
- ② What is the power consumed per phase
- ③ what is the phasor sum of 3 line currents

$$2P_h = 25$$

$$I_{ph} = 16A$$

$$2L = 27.7A$$

$$\text{Power per phase} = 2840W$$

Answers

Magnetic Circuits

The route / path followed by magnetic flux is called magnetic circuit.

Defⁿ: It may be defined as the route / path which is followed by the magnetic flux.

(OR)

The closed path followed by the magnetic flux is called magnetic circuit.

* The concept of magnetic circuit is useful in the design, analysis and applications of electro-magnetic devices like rotating machines and transformer.

Magnet:- A substance which when suspended freely points in the direction of North and South is called magnet. They're classified into 2 categories

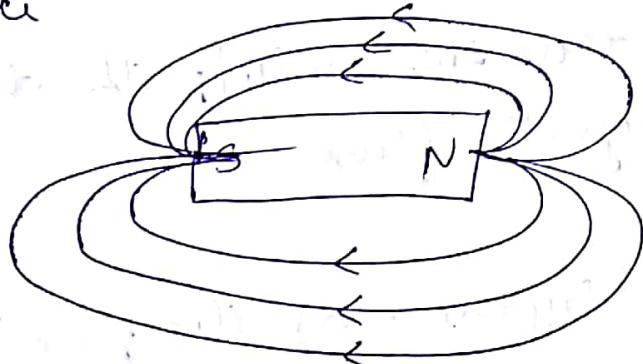
① Permanent magnet

② Temporary / Electromagnet

Permanent magnet:- It is made of steel, cobalt, Thungsten steel

Temporary magnet:- Take a silicon steel or soft iron wound with a coil. It acts as a magnet as long as the current flowing through the coil.

Magnetic lines of force :- The imaginary lines travels from north pole to south pole outside the magnet are called magnetic lines of force.



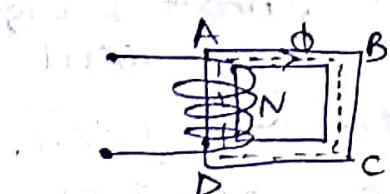
Magnetic field :- The space around magnetic lines of force acts is called magnetic field.

Magnetic flux :- The lines of force in magnetic field is called magnetic flux. It is denoted by Φ . Units \rightarrow [webers].

Magnetic flux density :- The flux passing through unit crosssection area is called magnetic flux density. It is denoted by B .

$$B = \frac{\Phi}{A} \rightarrow \text{[webes/m}^2\text{]}$$

Magnetomotive force :- (MMF)



Consider a coil of N turns wound on a iron core as shown in figure. When current i is passed through the coil, the magnetic flux Φ is setup

in the core. The magnetic flux follows the closed path ABCDA and hence ABCDA is magnetic circuit.

i) The amount of flux setup in the core depends on current i and number of turns "N". The product of N and i is called magnetomotive force (MMF)

Then $\boxed{MMF = Ni}$ units: Ampere turn.
(AT)

ii) Reluctance: The opposition that magnetic crkt offers to the magnetic flux is called reluctance denoted with "S":

→ It depends upon length of magnetic crkt (In this case length = ABCDA), area of cross section of the circuit and the nature of material that makes up the magnetic circuit

$$\boxed{S = \frac{l}{\mu_0 \mu_R}}$$

$$\boxed{S = \frac{mmf}{\Phi} = AT/wb}$$

l = length of mag crkt

a = cross section area

μ_R = permeability of core material

$\mu_0 = 4\pi \times 10^{-7}$ Henry/m²

∴ The magnetic flux

$$\boxed{\Phi = \frac{MMF}{\text{Reluctance}}}$$

$$\boxed{\Phi = \frac{A \cdot T}{\frac{l}{\mu_0 \mu_R}}}$$

Permeance: It is the Reciprocal of Reluctance

∴ i.e., $\boxed{\text{Permeance} = \frac{1}{\text{Reluctance}}}$

Magnetic field strength (H)

It is the force exerted by a north pole of 1 Wb placed at that point. It is denoted by H and its unit is Newton / Weber.

→ If Φ is flux in webers and F is force in Newton

then
$$H = \frac{F}{\Phi} \text{ Newton/weber.}$$

* H is also given by
$$H = \frac{NI}{L} \text{ AT/mt}$$

Permeability (μ) :- The absolute permeability of medium is given by ratio of flux density to magnetic field strength.

$$\mu = \frac{B}{H}$$

The Relative permeability

$$\mu_r = \frac{B}{B_0}$$

$$\mu = \mu_0 \mu_r$$

Similarities between electrical and magnetic circuit :-

Electric	Magnetic
① current	flux.
② EMF	MMF
③ resistance $\frac{l}{A}$	Reluctance $\frac{l}{\mu_0 \mu_r A}$

Electrical

- 1) I, amps
- 2) EMF, volts
- 3) Resistance, Ω
- 4) $i = \frac{V}{R}$
- 5) $R = \frac{\rho l}{A}$
- 6) electric field Intensity
 $E = \frac{V}{d}$
- 7) current density (J) $\rightarrow \text{Am/m}^2$
- 8) permittivity
 $E = \epsilon_0 \epsilon_r$

Magnetic

- Φ , wb; MMF, NI, (AT).
 S , AT/wb.
 $\Phi = \frac{NI}{S}$

8) $S = \frac{l}{\mu_0 \mu_r}$

Magnetic field density

$B = \frac{\Phi}{l}$

Magnetic flux density

$B = \Phi / A \rightarrow \text{wb/m}^2$

permeability

$\mu = \mu_0 \mu_r$

Differences between electrical and magnetic circuit

Electrical

- 1) electric current flows to the coil
- 2) It wants cont supply of energy.
- 3) $R = \text{cost}$ (practically).

Magnetic

- Magnetic flux doesn't flow. It links only with coil.
- Energy Required only to create magnetic flux. Not for maintaining it.
- "S" varies with flux density

Composite magnetic ckt.

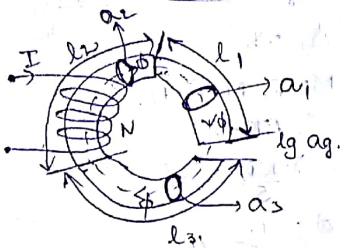


fig:- Composite magnetic ckt.

* A series magnetic ckt that has parts of different directions and materials is called as composite magnetic ckt as shown in fig. Each part will have its own reluctance. Total Reluctance is equal to sum of Reluctances of individual parts

$$\therefore \text{Total Reluctance, } = \frac{l_1}{\alpha_1 \mu_0 \mu_r} + \frac{l_2}{\alpha_2 \mu_0 \mu_r} + \frac{l_3}{\alpha_3 \mu_0 \mu_r} + \frac{l_{lg}}{\alpha_0 \mu_0} \quad (\text{T.R.})$$

Main :-

(*) The total MMF is equal to $= [\text{magnetic flux}] (\text{Reluctance})$

$$= \phi [\text{T.R.}]$$

$$\begin{aligned} &= \phi \frac{l_1}{\alpha_1 \mu_0 \mu_r} + \phi \frac{l_2}{\alpha_2 \mu_0 \mu_r} + \phi \frac{l_3}{\alpha_3 \mu_0 \mu_r} + \phi \frac{l_{lg}}{\alpha_0 \mu_0} \\ &= \frac{B_1 l_1}{\mu_0 \mu_r} + \frac{B_2 l_2}{\mu_0 \mu_r} + \frac{B_3 l_3}{\mu_0 \mu_r} + \frac{B_{lg}}{\mu_0} \end{aligned}$$

$$\mu_0 \mu_r \mu_i$$

$$B/\mu_0 H$$

$$\text{Total MMF} = H_1 l_1 + H_2 l_2 + H_3 l_3 + H_{lg} l_{lg}$$

Parallel magnetic ckt.

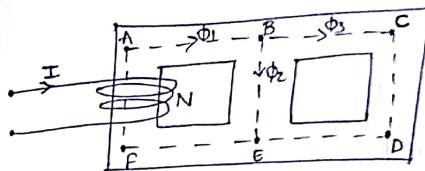


fig:- parallel magnetic circuit.

A magnetic ckt which has more than one path for magnetic flux is called a parallel magnetic ckt.

* The concept of II magnetic ckt is shown in figure. Here a coil of n turns wound on limb AF carry a current of I ampere. The magnetic flux ϕ_1 setup by the coil dividing at point B into 2 paths namely

- ① Magnetic flux ϕ_2 passes along the path BE
- ② magnetic flux ϕ_3 flows along path BCDE

From the figure $\boxed{\phi_1 = \phi_2 + \phi_3}$.

* Therefore the magnetic path BE and BCDE are in parallel and form a parallel magnetic ckt. The ampere turn (MMF) required for this II magnetic ckt is equal to ampere turn required for any one of the path.

Let S_1 = Reluctance of the path EFAB.

S_2 = Reluctance of the path BE

S_3 = Reluctance of path BCE

Therefore the total MMF Required is equal to
MMF for path EFAB + MMF for path BE (or) path BCE

$$NI = S_1 \Phi_1 + S_2 \Phi_2$$

(i)

$$S_1 \Phi_1 + S_3 \Phi_3$$

finally

$$NI = H_1 l_1 + H_2 l_2$$

(ii)

$$H_1 l_1 + H_2 l_3$$

Q) A magnetic flux density of 1.2 Wb/meter^2 is required in the two millimeter air gap of an electromagnet having an iron path of 1 meter long. Calculate the MMF Required assuming a relative permeability of Iron as 1500. Neglect leakage.

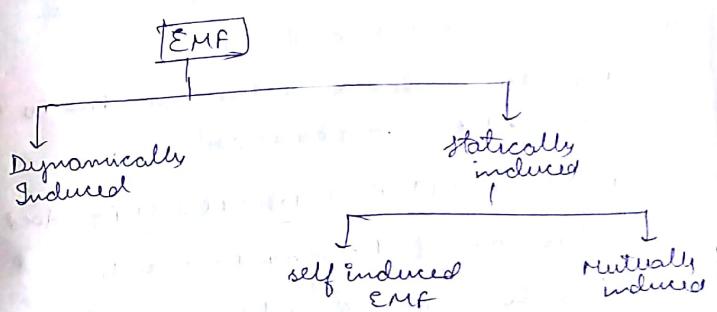
Sol) $\text{MMF} = \cancel{NI} = H_1 l_1 + H_2 l_2$

$$= \frac{B_1 l_1}{\mu_0 \mu_r} + \frac{(B_2 l_2)}{\mu_0 \mu_r}$$
$$= \frac{(1.2)(0.002)}{4\pi \times 10^{-7} \times 1500} + \frac{1.2 \times 1}{4\pi \times 10^{-7} \times 1} = 1910 + 63 \Rightarrow 254 \text{ A}$$

Electromagnetic Induction:-

Basic Requirement of Induced EMF:

- 1) Magnetic field
- 2) conductors / set of conductors
- 3) Time variation / space variation between above two.



Method-I: The dynamically / motionally induced EMF

→ The magnetic field is steady and set of conductors are being moved inside the magnetic field then EMF induced is dynamically induced EMF.

→ This concept is used in D.C. machines. (M&A)

Method-II: D.C. machine (M&A) statically induced EMF.

→ The magnetic field is time varying and the set of conductors are stationary. Then statically

EMF induces in coil

- This concept is used in Transformer.
- The statically induced EMF can be further subdivided into

i) Self Induced EMF :-

→ The EMF induced in the coil due to change of its own flux linkages.

ii) Mutually Induced EMF:

→ The EMF induced in the coil due to the change in flux in neighbouring coil.

Magnitude of statically induced EMF:- According to Faraday's law of Electromagnetic induction

a) Whenever conductor cuts the magnetic flux or flux linked with conductor changes, an EMF is induced in the conductor.

The expⁿ for induced EMF is given by

$$e = N \frac{d\phi}{dt} \text{ Volts}$$

Here e = Induced EMF

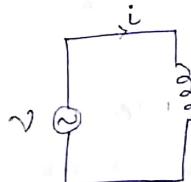
N = No. of turns of coil

$\frac{d\phi}{dt}$ = Rate of change of flux.

Lenz's law: The direction of induced EMF is given by Lenz's law. According to this law, the induced EMF will be acting in such a way as to oppose the cause of production of it.

$$\therefore e = -N \frac{d\phi}{dt} \text{ Volts}$$

Self Inductance :- (L)



④ The property of coil that opposes any change in the current flowing through it is called its self inductance (L). Inductance

Expⁿ for self inductance:- The self inductance (L) of the coil or circuit can be determined by one of the following methods.

Method(1): If the magnitude of self induced EMF (e) and a rate of change of current $\frac{di}{dt}$ are known, then L can be determined by following relationship,

$$e = L \frac{di}{dt}$$

$$L = e / \left(\frac{di}{dt} \right) \text{ Henry.}$$

Method - II: If the flux linkages of the coil and the current are known, the inductance can be determined.

$$e = \frac{d(Li)}{dt} \rightarrow \textcircled{1}$$

$$e = N \frac{d\phi}{dt} \rightarrow \textcircled{2}$$

Equating equation $\textcircled{1}$ and $\textcircled{2}$, we can write

$$\frac{d(Li)}{dt} = \frac{d(N\phi)}{dt}$$

$$Li = N\phi \quad \text{VVV IMP.}$$

$$L = \frac{N\phi}{i} \quad \begin{matrix} \text{\$} \\ \text{flux linkage} \end{matrix}$$

current flowing

$$L = N \left(\frac{\phi}{i} \right) \quad \text{(Henry)}$$

Mutual Inductance (M) :-

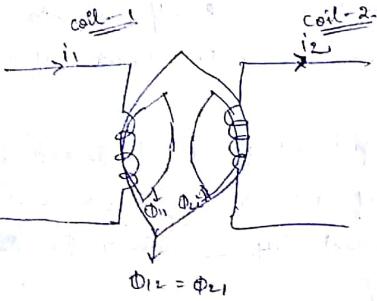


fig : flux linkages

Definition of Mutual Inductance :- Due to the change of current in one coil, the current changes in another coil. This property is called mutual inductance.

$$\text{Total flux of coil } \textcircled{1} \Rightarrow \Phi_1 = \Phi_{11} + \Phi_{12}$$

$$\text{Total flux of coil } \textcircled{2} \Rightarrow \Phi_2 = \Phi_{21} + \Phi_{22}$$

In fig, coils carry current i_1 and i_2 (both AC). Each coil will have linkage flux (self flux). ($\Phi_{11}, \Phi_{22} \rightarrow$ for coil 1, 2)

As well as mutual flux (Φ_{12}, Φ_{21}).

\downarrow
flux of coil $\textcircled{1}$ linked with coil $\textcircled{2}$,
flux of coil $\textcircled{2}$ linked with coil $\textcircled{1}$.

*The induced voltage (EMF) is given by

$$V_{12} = N_2 \frac{d(\Phi_{12})}{dt} \rightarrow \textcircled{1} \quad [\because \text{Due to mutual flux effect}]$$

Since Φ_{12} is related to the current of coil $\textcircled{1}$ & induced voltage is proportional to Rate of change of i_1 then

$$V_{12} = M \frac{di_1}{dt} \rightarrow \textcircled{2} \quad \begin{matrix} \text{proportionally const} \\ \text{mutual inductance} \end{matrix}$$

Equating eqn $\textcircled{1}$ & $\textcircled{2}$

$$M \left(\frac{di_1}{dt} \right) = N_2 \left(\frac{d(\Phi_{12})}{dt} \right)$$

$$M = N_2 \frac{d(\Phi_{12})}{di_1} \quad \textcircled{3}$$

③ ⇒ Exp' for mutual inductance

$$M_{12} = N_2 \frac{d(\Phi_{12})}{di_1}$$

$$M_{21} = N_1 \frac{d(\Phi_{21})}{di_2}$$

Coupling coefficient: The coefficient of coupling is the Ratio of mutual flux to total flux

(OR)

A number that expresses the degree of electrical coupling existing b/w two coils/circuits.

$$k_{12} = \frac{\Phi_{12}}{\Phi_1} \text{ and } k_{21} = \frac{\Phi_{21}}{\Phi_2}$$

→ As the coupling is bilateral, then $k_{12} = k_{21} = k$

$$\Phi_{12} = k\Phi_1$$

$$\Phi_{21} = k\Phi_2$$

Mutual inductance $M = M_{12} M_{21}$

$$M^2 = N_2 \frac{d(\Phi_{12})}{d(i_1)} \cdot N_1 \frac{d(\Phi_{21})}{d(i_2)}$$

$$M^2 = N_1 N_2 \frac{d(k\Phi_1)}{d(i_1)} \cdot \frac{d(k\Phi_2)}{d(i_2)}$$

$$M^2 = k^2 [N_1 \frac{d\Phi_1}{d_i}] \times [N_2 \frac{d\Phi_2}{d_i}]$$

$$M^2 = k^2 \cdot L_1 L_2$$

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

④ K lies b/w 0 & 1

$$0 \leq k \leq 1$$

∴ For $k=0$, $M=0 \Rightarrow$ isolated system

$k=1$, $M=1$, $L_1=L_2=1 \Rightarrow$ tightly coupled system (or)

strongly coupled system.

Types of Coupling:

1. Electrical coupling (E.C)
2. Magnetic coupling (M.C)

a) Electric coupling :- Physical connection is Existing (or) possible.

- i) Series → Aiding
→ opposing
- ii) Parallel → Aiding
→ opposing

b) Magnetic coupling :- Physical connection is not possible.

- M.C → Magnetic aiding
→ Magnetic opposing

Case-I:

Series connection of coupled coils (Aiding)

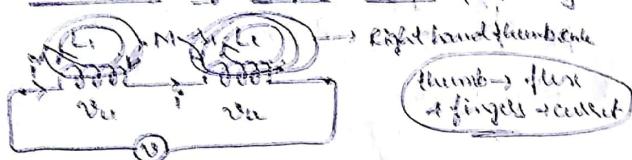


fig:- Mutually coupled coils in series (here the flux of both coils mutually assist each other)

Let two coils of self inductances L_1 and L_2 are connected in series such that voltage in coil \oplus is v_{L_1} and that in coil \ominus is v_{L_2} while current i flowing in them. The mutual induction b/w coil is M_{12} .

$$v_{L_1} = L_1 \frac{di}{dt} + M_{12} \frac{di}{dt}$$

$$= (L_1 + M_{12}) \frac{di}{dt} \rightarrow \textcircled{1}$$

voltage induced in coil \ominus

$$v_{L_2} = L_2 \frac{di}{dt} + M_{21} \frac{di}{dt}$$

$$= (L_2 + M_{21}) \frac{di}{dt} \rightarrow \textcircled{2}$$

$$v_{\text{net}} = v_{L_1} + v_{L_2}$$

$$= (L_1 + L_2 + M_{12} + M_{21}) \frac{di}{dt}$$

if $M_{12} = M_{21} = M$ (bilateral) then

$$v_{\text{net}} = (L_1 + L_2 + 2M) \frac{di}{dt} \rightarrow \textcircled{3}$$

$$\therefore L_{\text{eq}} = L_1 + L_2 + 2M$$

Leg changes from series, parallel, aiding & opposing

Case-II:

series connection of coupled coils (Opposing):

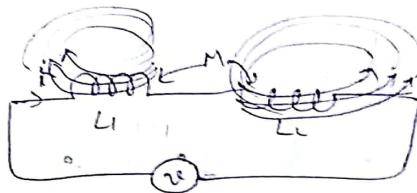


fig:- Mutually coupled coils in series (here fluxes of both coils mutually oppose each other)

Let two coils of self inductances L_1 and L_2 are connected in series such that voltage in coil \oplus is v_{L_1} and that in coil \ominus is v_{L_2} while current i flowing in them. The mutual induction b/w coil is M

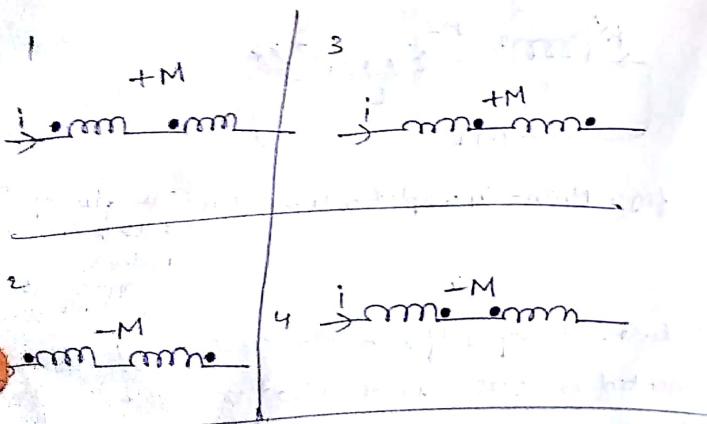
$$v_{L_1} = L_1 \frac{di}{dt} - M_{12} \frac{di}{dt} \rightarrow \textcircled{1}$$

$$V_{L2} = L_2 \frac{di}{dt} - M \frac{di}{dt} \quad \text{--- (2)}$$

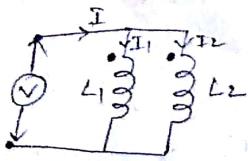
$$V_{\text{net}} = V_U + V_{L2} = (L_1 + L_2 - \epsilon M) \frac{di}{dt} \quad (\because M_{12} = M_{21} = M)$$

\therefore Left for series opposing is $L_{\text{eff}} = L_1 + L_2 - \epsilon M$

Finding using Dot connection:



Mutually coupled coils in parallel (tidying):



* Let L_1, L_2 are the inductances of coil 1 and coil 2. M is the mutual inductance between the coils. Apply KCL at node

$$I = I_1 + I_2$$

$$\frac{di}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt} \quad \text{--- (1)}$$

\therefore Voltage across the parallel branches is given by

$$V = L_1 \frac{dI_1}{dt} + M \frac{dI_2}{dt} \quad \text{--- (2)}$$

$$V = L_2 \frac{dI_2}{dt} + M \frac{dI_1}{dt} \quad \text{--- (3)}$$

Equating eqⁿ (2) & (3) we get

$$L_1 \frac{dI_1}{dt} + M \frac{dI_2}{dt} = L_2 \frac{dI_2}{dt} + M \frac{dI_1}{dt}$$

$$L_1 \frac{dI_1}{dt} - M \frac{dI_2}{dt} = L_2 \frac{dI_2}{dt} - M \frac{dI_1}{dt}$$

$$(L_1 - M) \frac{dI_1}{dt} = (L_2 - M) \frac{dI_2}{dt}$$

$$\frac{dI_1}{dt} = \frac{(L_2 - M)}{(L_1 - M)} \frac{dI_2}{dt} \quad \text{--- (4)}$$

Substitute equation (4) in eqⁿ (2) we have

$$\frac{dI_1}{dt} = \frac{(L_2 - M)}{(L_1 - M)} \frac{dI_2}{dt} + \frac{dI_2}{dt}$$

$$\frac{di}{dt} = \left(\frac{L_2 - M + L_1 - M}{L_1 - M} \right) \frac{dI_2}{dt} \rightarrow (4A)$$

If L_{eq} is the equivalent inductance of parallel circuit shown in figure. Then

$$V = L_{eq} \frac{di}{dt} \rightarrow ④$$

Substituting eqn ④ in equation ③

$$L_{eq} \frac{di}{dt} = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$\frac{di}{dt} = \frac{1}{L_{eq}} \left[L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \right] \rightarrow ⑤$$

Subtract eqn ④ from eqn ⑤

$$\frac{di}{dt} = \frac{1}{L_{eq}} \left[\frac{(L_2 - M)}{L_1 + M} \frac{di_2}{dt} + M \left(\frac{di_2}{dt} \right) \right]$$

$$\frac{di}{dt} = \frac{1}{L_{eq}} \left[\frac{L_2 - M}{L_1 + M} + M \right] \frac{di_2}{dt} \rightarrow ⑥$$

Equating equation ④ and ⑥, we get

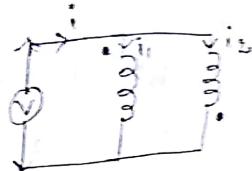
$$\left[\frac{L_2 - M}{L_1 + M} + 1 \right] \frac{di}{dt} = \frac{1}{L_{eq}} \left[\frac{L_2 - M}{L_1 + M} + M \right] \frac{di_2}{dt}$$

Rearranging and simplifying about eqn,

$$\star \boxed{L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}} \star$$

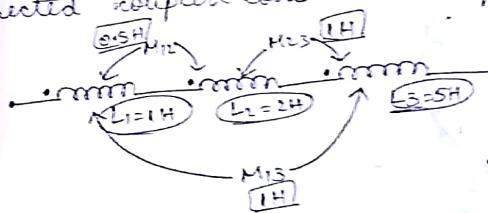
Homework:-

Mutually coupled coils in parallel [Opposite]:



$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

Q) Find the total inductance of 3 series connected coupled coils shown in figure.



Ans) For coil-I:

$$L_1 + M_{12} + M_{13} = 1 + 0.5 + 1 = 2.5 \text{ H}$$

For coil-II:

$$L_2 + M_{21} + M_{23} = 2 + 0.5 + 1 = 3.5 \text{ H}$$

For coil-III:

$$L_3 + M_{31} + M_{32} = 5 + 1 + 1 = 7 \text{ H}$$

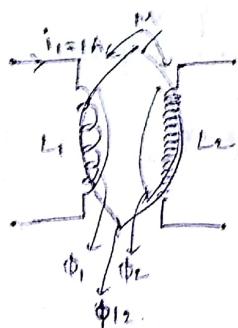
$$L_{\text{Total}} = 2.5 + 3.5 + 7 \Rightarrow 13 \text{ H}$$

Coil 1 + 2 + 3

Q) The following data refers to two coupled coils ① and ② shown in figure. Here $\Phi_{12} = 0.3 \text{ mWb}$,

$N_1 = 100 \text{ turns}$, $N_2 = 500 \text{ turns}$, $I_1 = 1 \text{ A}$.

Find coefficient of coupling (k), self Inductance of coil 1 (L_1), self inductance of coil 2 (L_2) and Mutual Inductance b/w coils (M).



$$\text{Q) Total flux of coil 1 } \Phi_1 = \Phi_{11} + \Phi_{12} \quad | \\ \text{Leakage Flux} \\ = 0.5 + 0.3$$

$$| \quad \Phi_1 = 0.8 \text{ milli wb}$$

$$L_1 = \frac{N_1 \Phi_1}{I_1} = \frac{100 \times 0.8 \times 10^{-3}}{1} = 0.08 \text{ H}$$

$$\text{coeff of coupling } k = \frac{\Phi_{12}}{\Phi_1} = \frac{0.3}{0.8} = 0.375$$

$$M = \frac{N_2 \Phi_{12}}{I_1} = \frac{500 \times 0.3 \times 10^{-3}}{1}$$

$$= 0.15 \text{ H}$$

$$\textcircled{2} \quad k = \frac{M}{\sqrt{L_1 L_2}}$$

$$0.375 = \frac{0.15}{\sqrt{0.08} (L_2)}$$

$$\sqrt{0.08} L_2 = \frac{0.15}{0.375} = 0.4$$

$$0.08 L_2 = 0.16$$

$$| \quad L_2 = 2 \text{ H}$$

Q) Befdl (EMI) or A is given.

~~Q) H & B are given. $H = \frac{B}{\mu_0} = \frac{1.2}{4 \pi \times 10^{-7}} = 9.55 \times 10^3 \text{ A/m}$~~

Q) An iron ring of cross sectional area 6 cm^2 is wound with a wire of 100 turns and has a saw cut of 2 mm . Calculate the magnetizing current required to produce a magnetic flux of 0.1 milli wb if mean length of magnetic path is 30cm and Relative permeability of iron is 470 .

Answer
$H_i L_i = 84.83$
$H_g L_g = 265.8$

$$\text{MMF} = NI$$

CHAPTER - 4

TRANSFORMERS

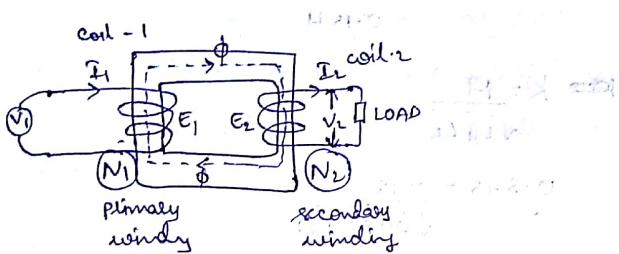


Fig:- Transformer.

What is Transformer:- It is a static device by means of which electric power in one winding transfers into electric power in another ckt without changing the frequency.

Operating principle:- In fig coil 1 and coil 2 are wound on magnetic ckt. One coil is connected to a source of alternating voltage; an alternating flux sets up in the laminated core, most of which linked with other coil in which produces mutually induced EMF [according to farady law of electromagnetic induction] ($e = M \frac{di}{dt}$). If the 2nd coil ckt is closed, a current flows in it so the electrical energy is transferred from primary winding to secondary winding.

The first coil, in which electrical energy is fed from the AC supply mains is called primary winding and the other one (load) draws energy. This load is called secondary winding.

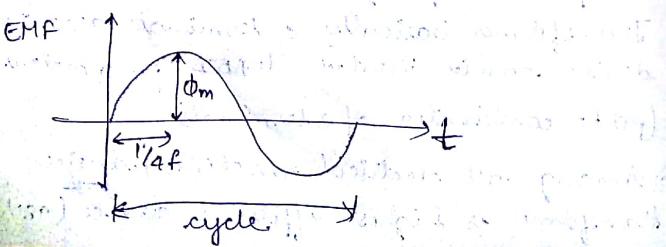
Some Important points about Transformer-

- ① Transformer is static device
- ② It is a constant frequency device
- ③ It is a constant power Device
- ④ It is electrically isolated [separable] and magnetically coupled device.
- ⑤ Transformer is a singly excited device i.e., one source is enough to energise both the windings [coils].
- ⑥ Transformer is an electromagnetic energy conversion device.
- ⑦ The overall transformer is not a energy conversion device. [i.e., input & output is electric power].
- ⑧ It is phase shifting device
- ⑨ Transformer basically 4 terminals and 2 port device can be related to ABCD parameters ($pdlt$ = combination of 2 terminals)
- ⑩ Among all electrical machines / devices Transformer is highest efficient device (98% above)

- (11) Transformer is lowest (~~Regulation~~) device.
 Regulation device.
- (12) It works based on the principle of Faraday's law of electromagnetic induction.
- (13) Primary function of transformer is it converts the one level of voltage to another level of voltage.
 i.e., if low voltage \rightarrow High voltage [STEP UP TRANSFORMER]
 if high voltage \rightarrow Low voltage [STEP DOWN].

- (*) From fig;
- V_1 = primary voltage or supply voltage
 I_1 = primary current
 N_1 = primary no. of turns
 E_1 = EMF induced in primary winding,
 similarly
 $V_2, I_2, N_2, E_2 \rightarrow$ all refer to secondary winding.

EMF equation of transformer:-



Let N_1 = no. of turns in 1^o winding

N_2 = no. of turns in 2^o winding

Φ_m = max flux in transformer core.

$$\Phi_m = B_m A$$

- (*) From fig flux increases from its 0 value to its max value in one quarter of the cycle
 \rightarrow Therefore average rate of change of flux = $\frac{\Phi_m}{1/4f}$

$$\Rightarrow 4f\Phi_m \text{ Wb/sec}$$

* The rate of change of flux per turn is
 = induced EMF in volts

$$\therefore \text{Avg EMF/turm} = 4f\Phi_m \text{ Volts.}$$

- (*) If flux Φ varies instantaneously then R.M.S value of induced EMF is obtained by multiplying the average value with form factor

$$F.F = \frac{\text{R.M.S. value}}{\text{avg value}} = 1.11$$

$$\therefore \text{R.M.S. value of EMF/turm} = 1.11 \times 4f\Phi_m \text{ Wb/sec.}$$

$$\therefore \text{R.M.S. value of EMF/turm} = 4.44 f\Phi_m \text{ Volts.}$$

Now, the R.M.F value of induced EMF in whole primary winding = R.M.F value of induced EMF per turn \times (No. of primary turns) i.e.,

$$* 4.44 f\Phi_m N_1 \quad * \quad V_1 = 4.44 f\Phi_m N_1$$

Similarly, EMF induced in 2nd winding is $E_2 = 4.44 f \Phi m N_2$

$$E_2 = 4.44 f \Phi m N_2 \quad \text{--- (2)}$$

From equation (1) and (2) we have

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = 4.44 f \Phi m.$$

It means that EMF per turn is same in both primary and secondary winding

VOLTAGE TRANSFORMATION RATIO (K) :-

from (1) & (2)

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K. \quad \text{where } K \text{ is voltage transformation ratio}$$

(i) if $N_2 > N_1$, then $K > 1$, then transformer is said to be STEP UP TRANSFORMER.

(ii) if $N_2 < N_1$, then $K < 1$, the transformer is said to be STEP DOWN TRANSFORMER.

* In case of ideal transformer.

$$\text{input power (VA)} = \text{o/p power (VA)}$$

Total power units

$$V_1 I_1 = V_2 I_2$$

$$\frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{I_1}{I_2} = K$$

Observation: current is inversely proportional to voltage transformation Ratio.

finally

voltage transformation ratio K is equal to :-

$$K = \frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

Ideal Transformer :-

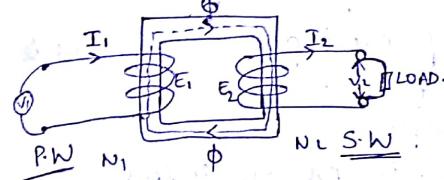


fig : (a) Ideal transformer

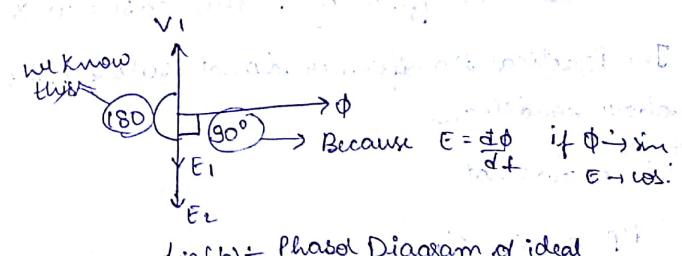


fig (b) + Phasor Diagram of ideal transformer

Properties of Ideal Transformer: It is an imaginary transformer having the foll prop

- ① The Resistance of 1^o & 2^o winding is negligible.
- ② There is no leakage flux and no leakage inductance.
- ③ The core has infinite permeability so that the MMF required to establish the flux in the core is negligible.
- ④ The entire flux is confined to core.
- ⑤ The coefficient of coupling of transformer is $(k=1)$ (\because mutual flux = 1) Total flux.
- ⑥ There are no losses, i.e., no ohmic loss (copper loss $\propto I^2 R$ loss), no hysteresis loss, eddy current loss. \therefore Efficiency in this case = 100%.
- \therefore The Practical transformer doesn't satisfy the above conditions.

* We know that

$$\text{if power} = 0 \text{ p. power}$$

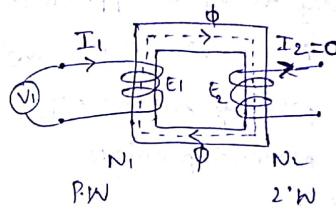
$$V_1 I_1 = V_2 I_2 \quad \text{and} \quad \frac{V_1 I_1}{1000} = \frac{V_2 I_2}{1000}$$

$$(VA)_1 = (VA)_2$$

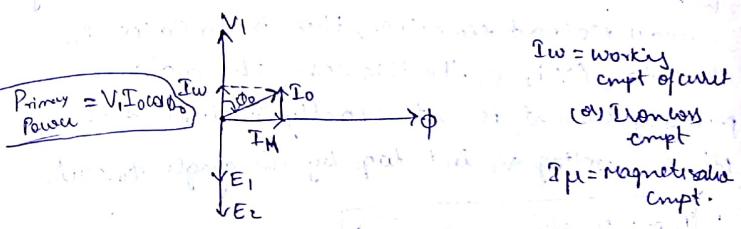
$$(KVA)_1 = (KVA)_2$$

thus in an ideal transformer input KVA and output KVA are identical.

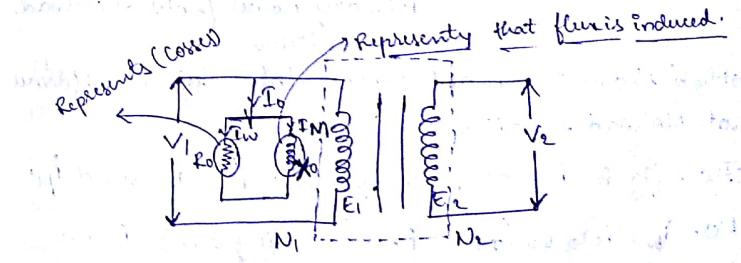
Transformer at No load:



fig(a): T/F at Noload



fig(b):- Phase Diagram of Transformer at No load.



fig(c): Equivalent circuit of Transformer at No load

When the secondary winding of T.F is open circuited, i.e., No load is connected to it, the T.F is said to

be in no load condition. In this case secondary current = 0. When alternating voltage is applied to primary, a small current i_0 flows in the primary. Here i_0 is called no load primary current. The primary current i_0 at no load provides

- Iron loss \rightarrow Hysteresis loss
- Eddy current loss

(2) A very small amount of current in the primary under no load condition, there is no current in secondary ($i_2 = 0$). In this case, the no load primary current is not exactly 90° behind the primary voltage V_1 , but lags by an angle $\phi_0 < 90^\circ$.

$$\star \text{Input power} @ \text{No load} = V_1 I_0 \cos \phi_0$$

Primary power factor at no load condition

\Rightarrow Fig(b) shows the phasor representation of transformer at no load condition.

* From fig b, i_0 has 2 components i.e., $i_{0\alpha}$ and $i_{0\beta}$

Here $i_W = i_{0\alpha} + i_{0\beta}$ (from phasor diagram) is in phase with V_1 is known as active / working iron

loss component. It mainly supplies the iron loss + small Q of primary current.

The secondary component $i_\mu = i_0 \sin \phi_0$ is lagging the applied voltage by 90° and is known as magnetic component of current. It maintains the altered flux in the core. The equivalent circuit is shown in figure "C".

Transformer with load :-

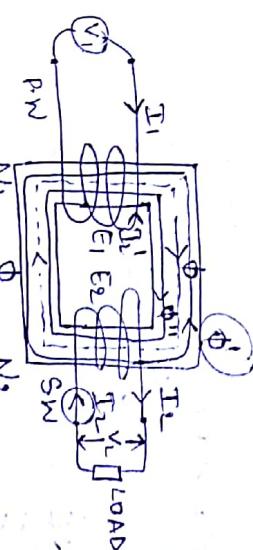


fig: Transformer on-load.

Fig shows that 2nd winding is connected to load.

Before secondary is loaded, the original flux was ϕ . When it is loaded, the secondary current will flow &

it will be setting its own flux ϕ' in the direction opposite to ϕ . Hence total flux will be $\phi - \phi' > 0$

$V_1 > E_1$ and primary will draw more current to compensate ϕ' . Let this current be I_2' . It will be possible iff $\frac{I_2'}{N_1} = \frac{I_2}{N_2} \Rightarrow I_2' = \frac{N_2}{N_1} I_2 \Rightarrow I_2' = k I_2$

Practical transformer:

- 1) It having some 1° & 2° winding resistances.
- 2) Finite permeability.
- 3) It consisting iron and core losses.
- 4) Flux is not only confined to transformer core.
- 5) Coefficient of coupling of transformer ($K \neq 1$)
- 6) Due to Above conditions, the efficiency of the transformer not equal to 100%.

a) Transformer with winding Resistances:

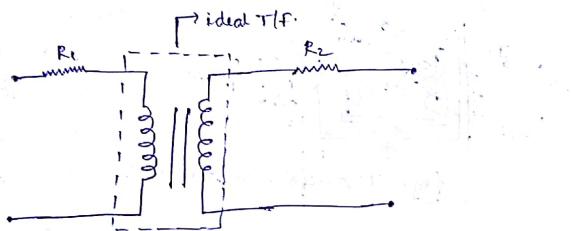


fig: T/F with wdg Resistances.

Any practical or non ideal transformer posses or offers resistance in primary & secondary winding. Due to the presence of Resistance, the practical transformer is equivalent to an Ideal T/F in which the Resistances are connected in series with each windings as shown in figure. R_1 and R_2 in figure shows (d) indicates the resistance of 1° & 2° windings.

of actual transformer. (a) practical T/F

b) Transformer with leakage Reactants:-

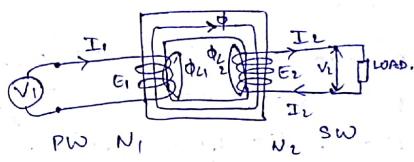


fig a: T/F with leakage flux.

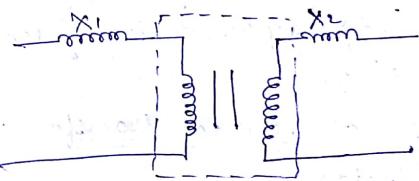


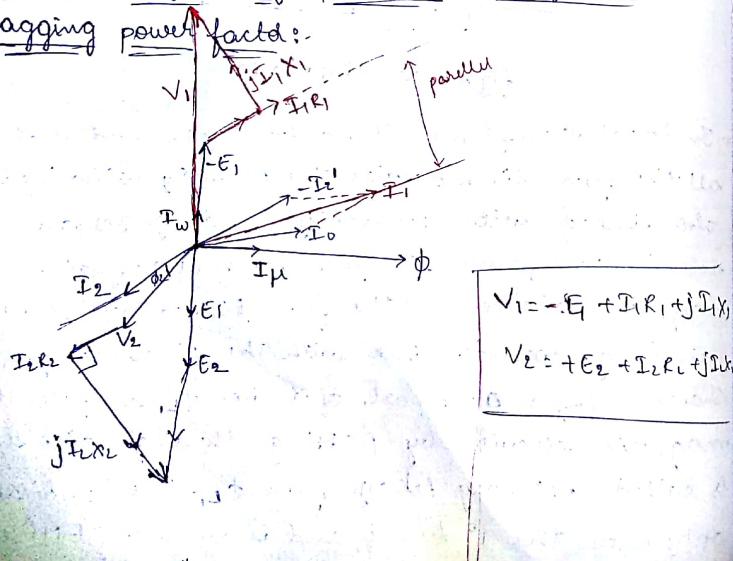
fig b: winding Reactance

* In ideal transformer, it has been assumed that all the flux linked with the primary winding is also linked with secondary winding. For an actual transformer the flux does not remain in the magnetic core i.e., all the flux linked with the 1° winding does not link with the 2° winding. As shown in figure a. Apart of it, i.e., Φ_L , completes its magnetic circuit by passing through air. This is called primary leakage flux Φ_L . The Φ_L links only with the 1° turns and induces an EMF of e_L .

in the primary winding. Similarly 2° ampere turns sets up leakage flux ϕ_L , which is linked with the 2° turns and induces an EMF e_{L2} .

* The leakage flux in each winding produces a self induced EMF in that each winding. It can be also said that a transformer with magnetic leakage is equivalent to an ideal transformer only the inductive coils appear in both primary and 2° windings. In fig (b) the parameters x_1 and x_2 represents leakage reactance of 1° and 2° windings.

Phasor diagram of practical transformer at lagging power factor :-



Referred Values :-

In order to simplify the calculations, it is theoretically possible to transfer voltage, current and impedance of one winding to other and combine them into single values for each quantity. Thus we have to work in one winding only which is more convenient.

a) Transformer with winding Resistance :-

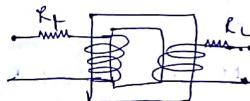
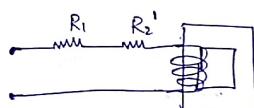


fig:- TIF with winding Resistance.

Secondary referred to primary



Let the TIF the resistance of the secondary winding R_2 to 1° side. Suppose that R_2' is a resistance of 2° winding referred (or) reflected to the primary winding. This Reflected Resistance R_2' should produce the same effect in primary as R_2 produced in secondary.

\therefore Power consumed by R_2' when carrying the 1° current

is equal to power consumed by R_2 due to the secondary current.

SEC TO PRI: i.e. we want to find equivalent circuit before Transf.

'Cu' loss \downarrow \Rightarrow Cu loss \downarrow after Transf

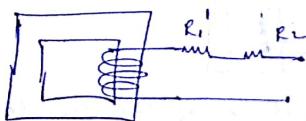
$$I_2^2 R_2 = I_1^2 R_1'$$

$$R_2' = \left(\frac{I_1}{I_2}\right)^2 R_2$$

$$\boxed{R_2' = \frac{R_2}{K_L}}$$

$$\begin{aligned} R_{\text{eff}} &= R_1 + R_2 \\ \text{cov} &= R_1 + R_2' \\ &= R_1 + \frac{R_2}{K_L} \end{aligned}$$

PRI TO SEC:



Cu loss before Transf = Cu loss after Transf

$$I_1^2 R_1 = I_2^2 R_1'$$

$$R_1' = \left(\frac{I_1}{I_2}\right)^2 R_1$$

$$\boxed{R_1' = K_L R_1}$$

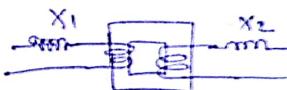
The total resistance referred to 2° side (R_{02})

$$R_{02} = R_1' + R_2$$

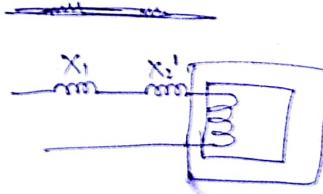
$$\boxed{R_{02} = R^2 R_1 + R_2}$$

* The total Cu loss referred to 2° side is equal to $\boxed{(I_2^2 R_{02})}$

b) Transform with winding reactance:



Secondary Referred optimally:



Reactive power:

$$\begin{aligned} Q &= V_1 I_1 \sin \phi \\ &= I_1^2 X_1 \sin \phi \\ &= I_2^2 \left(\frac{X_2}{K_L}\right) \sin \phi \\ &= I_2^2 X_2 \end{aligned}$$

SEC TO PRI:

'Cu' loss before = Cu loss after

$$I_2^2 X_2 = I_1^2 X_2'$$

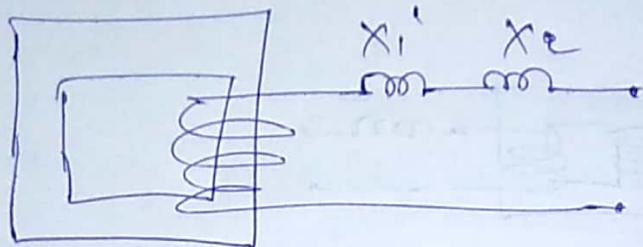
$$\boxed{X_2' = \frac{X_2}{K_L}}$$

$$X_{01} = X_1 + X_2$$

$$= X_1 + \frac{X_2}{k^2}$$

Total cu loss in 1° winding = $I^2 X_{01}$

PRI to SEC:



Cu loss before = Cu loss after

$$I_1^2 X_1 = I_2^2 X_1'$$

$$X_1' = \left(\frac{I_1}{I_2}\right)^2 X_1$$

$$\boxed{X_1' = k^2 X_1}$$

Total $X_{02} = X_2 + X_1'$

$$= \boxed{X_2 + k^2 X_1}$$

Total cu loss Ref to 2° side = $\underline{I_2^2 X_{02}}$