NANDHA COLLEGE OF TECHNOLOGY

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DEPARTMENT OF INFORMATION TECHNOLOGY

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ASSIGNMENT / CASE STUDY REPORT – I

TITLE

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MARKS OBTAINED	
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APPROXIMATE INFERENCE IN BN:

INTRODUCTION:

Approximate inference is a technique used in Bayesian networks to calculate the posterior probabilities of variables in the presence of uncertainty or incomplete information. Exact inference is often too computationally expensive, especially for large and complex networks, and approximate inference methods are used instead to provide approximate solutions.

TYPES:

There are several types of approximate inference methods used in Bayesian networks, including:

1. Sampling-based methods
2. Variational methods
3. Approximate message passing

SAMPLING-BASED METHODS:

These methods use Monte Carlo sampling techniques to approximate the posterior probabilities. Examples include likelihood weighting, importance sampling, and Markov Chain Monte Carlo (MCMC) methods such as Gibbs sampling.

VARIATIONAL METHODS:

These methods approximate the posterior probabilities by finding a simpler distribution that is close to the true distribution. Examples include mean-field variational methods and expectation propagation.

APPROXIMATE MESSAGE PASSING:

These methods use iterative algorithms to propagate messages between nodes in the network, approximating the posterior probabilities.

CLARIFICATION:

The choice of method depends on the characteristics of the Bayesian network and the specific inference task at hand. In general, sampling-based methods are more flexible but may require more samples to obtain accurate results, while variational methods are faster but may not be as accurate for complex distributions. Approximate message passing methods can be a good compromise between accuracy and speed, but they may require more tuning and experimentation to get good results.

B) QUESTION :

Consider there are three machines. All of the machines can produce 1000 pins at a time. The rate of producing a faulty pin from Machine 1 be 10%, from Machine 2 be 20% and from Machine 3 be 5%. What is the probability that a produced pin will be faulty and it will be from the first machine?

SOLUTION :

Probability of Faulty Pins

The probability of a produced pin being faulty and coming from the first machine can be calculated using the Bayes' theorem.

Let's define the following events:
A: Pin is produced by the first machine
B: Pin is faulty

We are given that P(B|A) = 0.1 (the probability of a faulty pin being produced by the first machine).

We need to calculate P(A|B) (the probability that the pin was produced by the first machine given that it is faulty).

Using Bayes' theorem, we have:

P(A|B) = P(B|A) * P(A) / P(B)

We need to calculate P(B), the probability that a produced pin is faulty. This can be calculated as the sum of probabilities of producing a faulty pin from each of the three machines, weighted by their production rate:

P(B) = P(B|A) * P(A) + P(B|not A) * P(not A)
= 0.1 * 1/3 + 0.2 * 1/3 + 0.05 * 1/3
= 0.1167

where P(not A) = 2/3 (the probability that the pin was not produced by the first machine).

Now we can substitute the values in the Bayes' theorem formula:

P(A|B) = P(B|A) * P(A) / P(B)
= 0.1 * 1/3 / 0.1167
= 0.2857

Therefore, the probability that a produced pin will be faulty and it will be from the first machine is 0.2857 or approximately 28.57%.

BAYE'S THEOREM:

Bayes' theorem describes the probability of occurrence of an event related to any condition. It is also considered for the case of conditional probability. Bayes theorem is also known as the formula for the probability of "causes". For example: if we have to calculate the probability of taking a blue ball from the second bag out of three different bags of balls, where each bag contains three different colour balls viz. red, blue, black. In this case, the probability of occurrence of an event is calculated depending on other conditions is known as conditional probability. In this article, let us discuss the statement and proof for Bayes theorem, its derivation, formula, and many solved examples.

CONTENTS:

- Statement
- Proof
- Formula
- Derivation
- Examples
- Applications
- Practice problems

BAYE'S THEOREM STATEMENT:

Let $E_1, E_2, \ldots, E_n$ be a set of events associated with a sample space S, where all the events $E_1, E_2, \ldots, E_n$ have nonzero probability of occurrence and they form a partition of S. Let A be any event associated with S, then according to Bayes theorem,

Conditional probability: Bayes' Theorem

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

for any k = 1, 2, 3, …., n

PROOF:

Hypotheses: The events E_1, E_2,… E_n is called the hypotheses

Priori Probability: The probability $P(E_i)$ is considered as the priori probability of hypothesis E_i

Posteriori Probability: The probability $P(E_i|A)$ is considered as the posteriori probability of hypothesis E_i Bayes' theorem is also called the formula for the probability of "causes". Since the E_i's are a partition of the sample space S, one and only one of the events E_i occurs (i.e. one of the events E_i must occur and the only one can occur). Hence, the above formula gives us the probability of a particular E_i (i.e. a "Cause"), given that the event A has occurred.

BAYE'S THEOREM DERIVATION:

Bayes Theorem can be derived for events and random variables separately using the definition of conditional probability and density.

From the definition of conditional probability, Bayes theorem can be derived for events as given below:

$P(A|B) = P(A \cap B)/ P(B)$, where $P(B) \neq 0$

$P(B|A) = P(B \cap A)/ P(A)$, where $P(A) \neq 0$

Here, the joint probability $P(A \cap B)$ of both events A and B being true such that,

$P(B \cap A) = P(A \cap B)$

$P(A \cap B) = P(A \mid B) P(B) = P(B \mid A) P(A)$

$P(A|B) = [P(B|A) P(A)]/ P(B)$, where $P(B) \neq 0$

Similarly,

F(x/y)=y(x)=(f(x,y)(XY))/(f(y)(Y))

EXAMPLES:

Question:

A bag I contains 4 white and 6 black balls while another Bag II contains 4 white and 3 black balls. One ball is drawn at random from one of the bags, and it is found to be black. Find the probability that it was drawn from Bag I.

Solution:

Let E_1 be the event of choosing bag I, E_2 the event of choosing bag II, and A be the event of drawing a black ball.

Then,

$P(E1)=P(E2)=1/2$

Also, $P(A|E_1)$ = P (drawing a black ball from Bag I) = $6/10 = 3/5$

$P(A|E_2)$ = P (drawing a black ball from Bag II) = $3/7$

By using Bayes' theorem, the probability of drawing a black ball from bag I out of two bags,

$P(E1/A)=[P(E1).P(A/E1)] \ / \ [P(E1)/P(A/E1)+P(E2)/P(A/E2)]$

$=7/12$

ANS:7/12

APPLICATIONS :

One of the many applications of Bayes' theorem is Bayesian inference, a particular approach to statistical inference. Bayesian inference has found application in various activities, including medicine, science, philosophy, engineering, sports, law, etc. For example, we can use Bayes' theorem to define the accuracy of medical test results by considering how likely any given person is to have a disease and the test's overall accuracy. Bayes' theorem relies on consolidating prior probability distributions to generate posterior probabilities. In Bayesian statistical inference, prior probability is the probability of an event before new data is collected.