Risk-neutral valuation of Swing options in presence of jumps

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Goal of this presentation

- The main goal of this presentation is to show the basic framework for the **risk-neutral pricing of swing options valuation** where the logarithm of the **spot price** is the sum of a deterministic **seasonal trend** and the **Ornstein-Uhlenbeck** process driven by a **Lévy process** (jump-diffusion).

- This presentation has been designed for quants analyst, and **the content of this presentation can hurt your feelings** if you do not used to work with stochastic differential equations, PDE-PIDE and some measure theory.

Questions addressed in this presentation:

- Which is the **stochastic differential equation** that better fits the "most correct" dynamics of the commodities involved in a swing option?
- Can we build a **risk-neutral model** that allow us to model the **forward prices**?
- Could we build an **analytical pricing framework** that allows us to estimate not only the **prices** but also the **Greeks** for the swing option hedging?
- How can we obtain the most **robust parameters** that we have to introduce in the model such that guarantee the **most stable possible pricing and hedging**?

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1. Introduction

Introduction

Definition

A **Swing option** is a financial contract with the following payoff characteristics:

1. **Maturity contract:** runs over $[0, T]$
2. **Strike**: fixed price K Eur/MWh
3. **Swing action times**: finite set of dates $\{T_n\}_{n=1}^{N}$ with $0 \leq T_1 < T_2 < ... < T_N < T$
4. **Swing action**: At each swing action date T_n the holder decides on the **amount of energy** B_n^d MWh to be bought at fixed price K Eur/MWh over each of the D periods $\left(T_n^d, T_n^{d+1}\right]$, $1 \leq d \leq D$.
5. **Total and partial boundaries**: assume that $B_n^d \in \mathcal{O} \subseteq [0, \infty)$ where $\mathcal{O}$ is either a closed interval $\mathcal{O} = [\underline{\mathrm{B}}, \bar{B}]$ or a discrete set. Additionally, the holder must buy at least $\underline{\mathrm{M}}$ MWh, and the most $\bar{\mathrm{M}}$ MWh in total
6. **Settlement**: All swing options are financially settled.

Introduction

The **outline** of this presentation (excluding this introduction) is the following:

2. **Section 1: Deng (2000) spot price model**
   - This section introduce the model developed by **Deng (2000)** where the logarithm of the spot price is the sum of a seasonality term and a Ornstein-Uhlenbeck process driven by a jump diffusion.
3. **Section 2: Model calibration**
   - Section 2 is devoted to the calibration or estimation of the parameters of the model, using a combination between a least-squares method (**Lucia and Schwartz (2002)**) and the Fourier transform based in the maximum likelihood approach by **Singleton (2001)**.
4. **Section 3: Swing option price estimation and numerical algorithm**
   - In the last section, we proceed to develop the swing option pricing algorithm based in the discretization of a PIDE using a finite difference method, similar as **Cont and Tankov (2003)**.

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2. **Section 1:**
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- We consider that power spot prices $S = \{S_t : 0 \leq t \leq T^\star\}$ lives in a **continuous-time trading economy** in $[0, T^\star]$ driven by a compensated **Lévy** process L on a *'sufficiently rich'* stochastic basis $(\Omega, \mathcal{L}, \mathbb{L}, \mathbb{P})$.

Definition

Assume that the **power-spot prices follows the $\mathbb{P}$-dynamics:**

$$\begin{cases} S_t = \exp\left(f\left(t\right) + X_t\right) \\ dX_t = -\alpha X_{t-} dt + dL_t \end{cases} \tag{1}$$

where $\alpha > 0$ is fixed, $f\left(t\right)$ is a **deterministic seasonal trend** and L_t is a **compensated Lévy process** that accept the canonical **Lévy-Ito decomposition** with triplet $(\sigma, v, 0)$ such that

$$\begin{aligned} L(t) &= \int_0^t \sigma W\left(ds\right) + \int_0^t \int_{|x|>\varepsilon} x \cdot \left(J_X - v\right)\left(ds, dx\right) \\ &= \sigma W_t + U_t^x - v\mathbb{E}_P\left[x\right] t \end{aligned} \tag{2}$$

- The solution S_t to the SDE (1) started in t_0 is given by

$$\begin{aligned} S_t &= S_{t_0} e^{-\alpha(t-t_0)} - \frac{\upsilon \mathbb{E}_P[x]}{\alpha}\left(1 - e^{-\alpha(t-t_0)}\right) \\ &\quad + \sigma \int_{t_0}^{t} e^{-\alpha(t-t_0)} dW_s + \sum_{i=N_{t_0}}^{N_t} e^{-\alpha(t-t_i)} X_i \end{aligned} \tag{3}$$

- The density of S_t is generally not known explicitly, but using the FT for any $t \in \mathbb{R}^+$

$$\hat{\mu}_t(z) = e^{(t-t_0)\psi(z)} \text{ with } z \in \mathbb{R}^d \tag{4}$$

where $\psi_j(z)$ is the **Lévy-Khitchine exponent** with the following representation

$$\begin{aligned} \psi(z) &= izxe^{-\alpha(t-t_0)} - \frac{\sigma^2 z^2}{4\alpha}\left(1 - e^{-2\alpha(t-t_0)}\right) \\ &\quad + \int_{\mathbb{R}^d} \left(e^{izx} - 1 - izxe^{-\alpha(t-t_0)}\right) \upsilon(dx) \end{aligned} \tag{5}$$

Fact

Forward prices, under a 'diffusive' risk-neutral measure, has the following expression:

$$\begin{aligned} F(t,T) &= \exp\left(f(T) + (\log S_t - f(t))\, e^{-\alpha(T-t)}\right) \\ &\quad \times \exp\left(\frac{\sigma\lambda}{\alpha}\left(1 - e^{-\alpha(T-t)}\right) + \frac{\sigma^2}{4\alpha}\left(1 - e^{-2\alpha(T-t)}\right)\right) \\ &\quad \times \exp\left(\int_{\mathbb{R}^d}\left(e^x - 1 - xe^{-\alpha(T-t)}\right) \upsilon\,(dx)\right) \qquad (6) \\ &\equiv F_{season} \times F_{diffusion} \times F_{jump} \end{aligned}$$

Proof.

That can be obtained following **Lucia and Schwartz**(2002) □

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Section 2: Model Calibration

1. Let us define a **periodic seasonal trend** $f\left(t|\,\Theta\right)$ with $\Theta = \left(A_0, A_n, B_n\right)_{n=1}^{N}$ such that

$$f\left(t|\,\Theta\right) = A_0 + \sum_{n=1}^{N} A_n \cos\left(2\pi f_n t + B_n\right) \tag{7}$$

where the parameter vector Θ is unknown and is to be estimated from data.

2. Notice that the **stationary covariance function** of the $\ln S_t$ is given by $Cov\left(\ln S_{t+\Delta}, \ln S_t\right) = e^{-\alpha\Delta} Var\left(\ln S_t\right)$ so α is estimated from sample covariances and variances.
3. Next, we subtract the seasonal component $f\left(t|\,\Theta\right)$ from the $\ln S_t$, and we estimate the remaining parameters with the **maximum likelihood method**. Notice that if we unknown the density function we may form the **log likelihood function** using the inverse **Fourier transformation** of the characteristic function, following **Deng**(2000) or **Singleton**(2001).
4. Finally, to estimate the **market price of risk** λ, we minimized the distance between market and model prices, such that

$$\hat{\lambda} = \arg\min \sum_{k=1}^{K} \left|\hat{F}\left(t, T\right) - F\left(t, T\right)\right|^2 \tag{8}$$

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Section 3: Swing option price estimation

- Let us remind that at time T_n, the swing option holder decide to buy B_n^d MWh at K EUR/MWh.
- A **swing action** is described as a D-vector $\left\{B_n^1 ... B_n^D\right\}$, and defined as $\Delta_n = \sum_{d=1}^{D} B_n^d$ such that $\Delta_n \in S = [0, D\bar{B}]$. A swing action means choosing Δ_n without violating the contract constraints, and receiving the amount $g(T_n, s, \Delta_n)$.
- Finally, let us define $Z_t = \sum_{n=1}^{j} \Delta_n$ for $t \in \left(T_j, T_{j+1}\right]$.

Definition

We can define the **value of a swing option** at moment t, when $s = S_t$ and $z = Z_t$, as $V(t, s, z)$ given by

$$V(t,s,z) = \begin{cases} \sup \sum_{n=j}^{N} e^{-r\left(T_n - T_j\right)} \mathbb{E}\left[g\left(T_n, S_{T_n}, \Delta_n\right) \middle| \mathcal{F}_{T_j} \right] & t = T_j \\ e^{-r\left(T_n - T_j\right)} \mathbb{E}\left[V\left(T_j, S_{T_j}, z\right) \middle| \mathcal{F}_{T_j} \right] & \begin{array}{c} T_j < t < T_j,\ j > 1 \\ t < T_1 \end{array} \end{cases} \tag{9}$$

Theorem

There exists at least one optimal swing action strategy $\{\Delta_n^*\}_{n=j}^N$ such that the supremum is attainable.

According with the **Feynman-Kac theorem**, the swing option price $V(t,s,z)$ is the unique solution to the following parabolic partial integro-differential equation (PIDE)

$$\frac{\partial V}{\partial t} + \mathcal{L}_x V - rV = 0 \tag{10}$$

where the operator $\mathcal{L}_x$ ca be splitted into two parts (integral and differential) such that $\mathcal{L}_x = \mathcal{D}_x + \upsilon\mathcal{I}_x$ with

$$\begin{aligned} \mathcal{D}_x V(t,x,z) &= \tfrac{\sigma^2}{2}\tfrac{\partial^2 V}{\partial x^2} - (\sigma\lambda + \alpha x)\tfrac{\partial V}{\partial x} \\ \mathcal{I}_x V(t,x,z) &= \int_{\mathbb{R}} \left[V(t, x_l + y, z) - V(t, x_l, z) - y\tfrac{\partial V}{\partial x} \right] f(y)dy \end{aligned} \tag{11}$$

that can be solved numerically splitting the diffusion or continuous part, from the integral or discontinuous part.

- The **space derivatives** are discretized using **finite differences**:

$$\left(\frac{\partial V}{\partial x}\right)_n \approx \begin{cases} \frac{V_{n+1}-V_n}{\Delta x} & \mathit{if} \quad \sigma\lambda + \alpha x < 0 \\ \frac{V_n - V_{n-1}}{\Delta x} & \mathit{if} \quad \sigma\lambda + \alpha x \geq 0 \end{cases}$$

$$\left(\frac{\partial^2 V}{\partial x^2}\right)_n \approx \frac{V_{n+1} - 2V_n - V_{n-1}}{(\Delta x)^2}$$

- In order to approximate the **integral terms** one can use the **trapezoidal quadrature rule** with the same grid resolution Δx. More specifically, if we define:

$$V_n = \int_{\Delta x\left(n-\frac{1}{2}\right)}^{\Delta x\left(n+\frac{1}{2}\right)} f(y)dy$$

then $\mathcal{I}_x V(t, x, z)$ may approximated as:

$$\mathcal{I}_x V(t_{k+1}, x_l, z) \approx \lambda \sum_{n=-N}^{N} \left[V_{k+1}^{l+n} - V_{k+1}^{l} - \frac{n}{2}\left(V_{k+1}^{l+1} - V_{k+1}^{l-1}\right)\right] \nu_n$$

Answers:

1. We have introduced a **stochastic process** L_t, that has "some good" properties for the commodities price modelling.
2. We have built the **risk-neutral dynamics for forward prices** where we have included **seasonality, mean-reversion, Brownian motion** and **jumps**.
3. We have proposed a **calibration methodology**, that include the **market price of risk** for the diffusion part.
4. We have defined a **pricing framework for swing option** based in a **PIDE**. The solution of the PIDE can be found with a **numerical pricing algorithm** based in **finite differences** (using a combination of **explicit-implicit** schemes).

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... among others.