Probability Assignment

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Abstract—This document contains the solution to Question 6 of Exercise 3 in Chapter 13 of the class 12 NCERT textbook.

1) There are three coins. One is a two headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the two headed coin?

   Solution: Define the random variable X as in Table 1. Clearly, the pmf of X is

$X = 1$	Two-headed coin is selected.
$X = 2$	75% biased coin is selected.
$X = 3$	Fair coin is selected.

TABLE 1: Definition of X.

$$\Pr(X = k) = \begin{cases} \frac{1}{3} & 1 \le k \le 3 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Let the random variables Y_1, Y_2 and Y_3 (one for each coin) be defined as

$$Y_1 \sim \text{Ber}(1) \tag{2}$$

$$Y_2 \sim \text{Ber}\left(\frac{3}{4}\right) \tag{3}$$

$$Y_3 \sim \text{Ber}\left(\frac{1}{2}\right) \tag{4}$$

Define Y as

$$Y \triangleq \sum_{i=1}^{3} \mathbf{1}_i(X) \, Y_i \tag{5}$$

where $\mathbf{1}$ denotes the indicator random variable, defined as

$$\mathbf{1}_i(X) = \begin{cases} 1 & \text{if } X = i \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

We are required to find $\Pr(X = 1|Y = 1)$. However, from Bayes' Rule,

$$\Pr(X = 1, Y = 1) = \Pr(X = 1)\Pr(Y = 1|X = 1) \tag{7}$$

$$= \Pr(Y = 1)\Pr(X = 1|Y = 1) \tag{8}$$

Note from (5) that

$$X = 1 \implies Y = Y_1 \tag{9}$$

and also,

$$\Pr(Y = 1) = \sum_{i=1}^{3} \Pr(X = i)\Pr(Y_i = 1) \tag{10}$$

$$= \frac{1}{3}\left(1 + \frac{3}{4} + \frac{1}{2}\right) = \frac{3}{4} \tag{11}$$

Thus, from (7), (8) and (11), we see that

$$\Pr(X = 1|Y = 1) = \frac{\Pr(X = 1)\Pr(Y_1 = 1)}{\Pr(Y = 1)} \tag{12}$$

$$= \frac{\frac{1}{3}}{\frac{3}{4}} = \frac{4}{9} \tag{13}$$