Probability Assignment

Gautam Singh

***Abstract*—This document contains the solution to Question 18 of Exercise 2 in Chapter 13 of the class 12 NCERT textbook.**

1) Two events A and B will be independent, if
   a) A and B are mutually exclusive
   b) $\Pr(A'B') = (1 - \Pr(A))(1 - \Pr(B))$
   c) $\Pr(A) = \Pr(B)$
   d) $\Pr(A) + \Pr(B) = 1$.

   Solution: Two events A and B are independent if

$$\Pr(AB) = \Pr(A)\Pr(B|A) = \Pr(A)\Pr(B) \quad (1)$$

   using Bayes' Rule.
   We consider the options one by one. Here, let A be the event of rolling a prime number on a fair die and B the event of rolling an odd prime number on a fair die. The joint pmf is shown in Table 1. Notice that A and B are independent,

	A	$\bar{A}$
B	$\frac{1}{3}$	0
$\bar{B}$	$\frac{1}{6}$	$\frac{1}{2}$

TABLE 1: Joint PMF of A and B.

   as

$$\Pr(A) = \frac{1}{2},\ \Pr(B) = \frac{1}{3} \quad (2)$$

$$\Pr(AB) = \frac{1}{6} = \Pr(A)\Pr(B) \quad (3)$$

   thereby satisfying (1)
   a) From (3), $\Pr(AB) > 0$, hence this option is incorrect.
   b) We have,

$$\Pr(A'B') = \Pr((A + B)') \quad (4)$$
$$= 1 - \Pr(A + B) \quad (5)$$
$$= 1 - \Pr(A) - \Pr(B) + \Pr(AB) \quad (6)$$
$$= 1 - \Pr(A) - \Pr(B) + \Pr(A)\Pr(B) \quad (7)$$
$$= (1 - \Pr(A))(1 - \Pr(B)) \quad (8)$$

      where (4) follows from De-Morgan's laws and (7) follows from (1). Thus, this option is correct.
   c) Clearly from the given example, $\Pr(A) \neq \Pr(B)$. Thus, this option is incorrect.
   d) Again, from the given example, $\Pr(A) + \Pr(B) = \frac{5}{6} < 1$. Thus, this option is incorrect.

   Hence, the answer is option **b)**.