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## Assignment: - 2

# AI1110: Probability and Random Variables Indian Institute of Technology, Hyderabad

### CS22BTECH11001

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**Exemplar 11.16.3.11** The accompanying Venn diagram shows three events, A, B, and C, and also the probabilities of the various intersections (for instance, Pr(AB) = .07). Determine

- (a) Pr(A)
- (b) Pr(BC')
- (c) Pr(A + B)
- (d) Pr(AB')
- (e) Pr(BC)
- (f) Probability of exactly one of the three occurs.

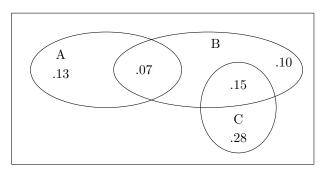


Fig. 0: Question Figure

### Solution.

(a) Clearly,

$$Pr(A) = 0.13 + 0.07 \tag{1}$$

$$= 0.20$$
 (2)

(b) Clearly,

$$Pr(B) = 0.10 + 0.07 + 0.15 \tag{3}$$

$$= 0.32$$
 (4)

Also,

$$A = A(B + B') = AB + AB' \tag{5}$$

$$[:: B + B' = 1] \tag{6}$$

$$Pr(A) = Pr(AB) + Pr(AB')$$

$$[::BB' = 0]$$
(7)

Using (7)

$$Pr(BC') = Pr(B) - Pr(BC)$$
 (8)

$$= 0.32 - 0.15 \tag{9}$$

$$= 0.17$$
 (10)

(c) From Axioms of Probability

$$Pr(A + B) = Pr(A) + Pr(B) - Pr(AB)$$
 (11)

$$= 0.20 + 0.32 - 0.07 \tag{12}$$

$$= 0.45$$
 (13)

(d) Using (7)

$$Pr(AB') = Pr(A) - Pr(AB)$$
 (14)

$$= 0.20 - 0.07 \tag{15}$$

$$= 0.13$$
 (16)

(e) Clearly,

$$Pr(BC) = 0.15$$
 (17)

(f) Let X be the event that exactly one of A, B or C occur.

Let Y be the event that at least one of A, B or C occur.

Using Boolean logic,

$$Y = A + B + C \tag{18}$$

Let Z be the event that at least two of A, B or C occur.

$$Z = AB + BC + CA \tag{19}$$

From (A.2.5)

$$X = AB'C' + A'B'C' + A'B'C$$
 (20)

Now, X has been represented as a union of 3 mutually exclusive events.

As any 2 of them has 0 intersection due of presence of complements.

Therefore, by Axioms of Probability

$$Pr(X) = Pr(AB'C') + Pr(A'BC') + Pr(A'B'C)$$
(21)

Clearly, from the figure

$$Pr(X) = 0.13 + 0.10 + 0.28$$
 (22)

$$= 0.51$$
 (23)