Probability Assignment

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***Abstract*—This document contains the solution to Question 5 of Exercise 6 in Chapter 13 of the class 12 NCERT textbook.**

1) An urn contains 25 balls of which 10 balls bear a mark 'X' and the remaining 15 bear a mark 'Y'. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that
   a) all will bear 'X' mark.
   b) not more than 2 will bear 'Y' mark.
   c) at least one ball will bear 'Y' mark.
   d) the number of balls with 'X' mark and 'Y' mark will be equal.

   Solution: Let the random variable X denote one single draw with p being the probability that a ball marked 'X' is drawn. Then,

$$X \sim \text{Ber}(p) \tag{1}$$

   Suppose $X_i, 1 \le i \le n$ represent each of the n draws. Define Y as

$$Y = \sum_{i=1}^{n} X_i \tag{2}$$

   Then, since the X_i are iid, the pmf of Y is given by

$$Y \sim \text{Bin}(n, p) \tag{3}$$

   The cdf of Y is given by

$$F_Y(k) = \Pr(Y \le k) \tag{4}$$

$$= \begin{cases} 0 & k < 0 \\ \sum_{i=1}^{k} \binom{n}{i} p^i (1-p)^{n-i} & 1 \le k \le n \\ 1 & k \ge n \end{cases} \tag{5}$$

   In this case,

$$p = \frac{2}{5}, \ n = 6 \tag{6}$$

   a) We require $\Pr(Y = 6)$. Thus, from (3),

$$\Pr(Y = 6) = \binom{n}{6} p^6 (1-p)^0 \tag{7}$$

$$= 0.004096 \tag{8}$$

   b) We require $\Pr(Y \ge 4)$. Thus, from (5),

$$\Pr(Y \ge 4) = 1 - \Pr(Y < 4) \tag{9}$$

$$= F_Y(6) - F_Y(3) \tag{10}$$

$$= \sum_{k=4}^{6} \Pr(Y = k) \tag{11}$$

$$= \sum_{k=4}^{6} \binom{n}{k} p^k (1-p)^{n-k} \tag{12}$$

$$= 0.1792 \tag{13}$$

   c) We require $\Pr(Y \le 5)$. Since $n = 6$ in (5), using (8) gives

$$\Pr(Y \le 5) = F_Y(5) \tag{14}$$

$$= 1 - \Pr(Y = 6) \tag{15}$$

$$= 0.995904 \tag{16}$$

   d) We require $\Pr(Y = 3)$. Thus, from (3),

$$\Pr(Y = 3) = \binom{n}{3} p^3 (1-p)^{n-3} \tag{17}$$

$$= 0.27648 \tag{18}$$