Probability Assignment

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***Abstract*—This document contains the solution to Question 17 of Exercise 1 in Chapter 13 of the class 12 NCERT textbook.**

1) If A and B are events such that

$$\Pr(A|B) = \Pr(B|A) \tag{1}$$

then

a) $A \subset B$ but $A \neq B$
b) $A = B$
c) $A \cap B = \phi$
d) $\Pr(A) = \Pr(B)$

Solution: Using Bayes' Rule,

$$\Pr(A,B) = \Pr(A)\Pr(B|A) \tag{2}$$
$$= \Pr(B)\Pr(A|B) \tag{3}$$

Using (1) in (2) and (3),

$$\Pr(A) = \Pr(B) \tag{4}$$

We consider the options one by one.

a) If $A \subset B$ and $A \neq B$, then we can write $B = A + C$, where $AC = 0$ and $C \neq 0$. Thus,

$$\Pr(B) = \Pr(A + C) \tag{5}$$
$$= \Pr(A) + \Pr(C) - \Pr(AC) \tag{6}$$
$$= \Pr(A) + \Pr(C) > \Pr(A) \tag{7}$$

However, (7) contradicts (4).

b) We give a counterexample to show this is wrong. Consider A as the event that an even number shows on rolling a fair die and B as the event that a prime number shows on rolling a fair die. The joint pmf is shown in Table 1. Clearly,

$$\Pr(A|B) = \frac{\Pr(AB)}{\Pr(B)} = \frac{\frac{1}{6}}{\frac{1}{6}+\frac{1}{3}} = \frac{1}{2} \tag{8}$$

$$\Pr(B|A) = \frac{\Pr(AB)}{\Pr(A)} = \frac{\frac{1}{6}}{\frac{1}{6}+\frac{1}{3}} = \frac{1}{2} \tag{9}$$

c) The same example as before provides the required counterexample, as $\Pr(AB) = \frac{1}{6}$.

d) This is the correct answer, as discussed above.

	A	$\bar{A}$
B	$\frac{1}{6}$	$\frac{1}{3}$
$\bar{B}$	$\frac{1}{3}$	$\frac{1}{6}$

TABLE 1: Joint pmf for events A and B.