Assignment 2

AI1110: Probability and Random Variables
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12.13.1.13 Question: An instructor has a question bank consisting of 300 easy True / False questions, 200 difficult True / False questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question?

Solution: Let X and Y be two random variables in which X describes the difficulty level of question and Y describes the type of question according to the table I :

Variable	Event
$X = 0$	Easy question
$X = 1$	Difficult question
$Y = 0$	True/False question
$Y = 1$	Multiple choice question

TABLE I

From the law of total probability,

$$p_X(0) + p_X(1) = 1 \tag{1}$$
$$p_Y(0) + p_Y(1) = 1 \tag{2}$$

$$p_X(0) = p_{X,Y}(0,0) + p_{X,Y}(0,1) \tag{3}$$
$$p_Y(0) = p_{X,Y}(0,0) + p_{X,Y}(1,0) \tag{4}$$
$$p_X(0) = \frac{300+500}{300+200+500+400} \tag{5}$$
$$= \frac{4}{7} \tag{6}$$
$$p_Y(0) = \frac{300+200}{300+200+500+400} \tag{7}$$
$$= \frac{5}{14} \tag{8}$$

From (1), (2), (6) and (8),

$$p_X(1) = 1 - p_X(0) \tag{9}$$
$$= \frac{3}{7} \tag{10}$$
$$p_Y(1) = 1 - p_Y(0) \tag{11}$$
$$= \frac{9}{14} \tag{12}$$
$$p_{X,Y}(0,0) = \frac{300}{1400} \tag{13}$$
$$= \frac{3}{14} \tag{14}$$
$$p_{X,Y}(1,1) = \frac{400}{1400} \tag{15}$$
$$= \frac{2}{7} \tag{16}$$

From (3), (4), (6), (8) and (14),

$$p_{X,Y}(0,1) = p_X(0) - p_{X,Y}(0,0) \tag{17}$$
$$= \frac{5}{14} \tag{18}$$
$$p_{X,Y}(1,0) = p_Y(0) - p_{X,Y}(0,0) \tag{19}$$
$$= \frac{1}{7} \tag{20}$$

The required probability is $p_{X|Y}(0|1)$

From the definition of conditional probability, We know that,

$$p_{X|Y}(x|y) = \frac{p_{X,Y}(x,y)}{p_Y(y)} \tag{21}$$

From (12) and (18),

$$p_{X|Y}(0|1) = \frac{p_{X,Y}(0,1)}{p_Y(1)} \tag{22}$$
$$= \frac{\frac{5}{14}}{\frac{9}{14}} \tag{23}$$
$$= \frac{5}{9} \tag{24}$$
$$\therefore \quad p_{X|Y}(0|1) = \frac{5}{9} \tag{25}$$