

Assignment 1

Gunnam Sri Satya Koushik
CS22BTECH11026

12.13.1.12 Question : Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that

- 1) The youngest is a girl
- 2) At least one is a girl

Answer : (i) $\frac{1}{2}$ (ii) $\frac{1}{3}$

Solution : Lets us random variables X_i where $i \in \{1, 2\}$ as follows

Variable	Description	Probability
$X_i = 1$	ith born child is a boy	$\Pr(X_i = 1) = 0.50$
$X_i = 0$	ith born child is a girl	$\Pr(X_i = 0) = 0.50$

TABLE I

RANDOM VARIABLE DEFINITIONS.

- 1) The required probability is the conditional probability that both the children are girls given that the youngest is a girl i.e,

$$\Pr((X_1 + X_2)' | X_2') = \frac{\Pr((X_1' X_2') X_2')}{\Pr(X_2')} \quad (1)$$

$$= \frac{\Pr(X_1') \Pr(X_2')}{\Pr(X_2')} \quad (2)$$

$$= \Pr(X_1') \quad (3)$$

$$= \frac{1}{2} \quad (4)$$

that at least one a girl i.e,

$$\Pr((X_1 + X_2)' | (X_1 X_2)') = \frac{\Pr((X_1' X_2')(X_1' + X_2'))}{1 - \Pr(X_1 X_2)} \quad (5)$$

$$= \frac{\Pr(X_1' X_2')}{1 - \Pr(X_1 X_2)} \quad (6)$$

$$= \frac{\Pr(X_1') \Pr(X_2')}{1 - \Pr(X_1) \Pr(X_2)} \quad (7)$$

$$= \frac{\frac{1}{2} \times \frac{1}{2}}{1 - (\frac{1}{2} \times \frac{1}{2})} = \frac{1}{3} \quad (8)$$

- 2) The required probability is the conditional probability that both the children are girls given