Probability Assignment

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***Abstract*—This document contains the solution to Question 8 of Exercise 3 in Chapter 16 of the class 11 NCERT textbook.**

1) Three coins are tossed once. Find the probability of getting
   a) 3 heads
   b) 2 heads
   c) atleast 2 heads
   d) atmost 2 heads
   e) no head
   f) 3 tails
   g) exactly two tails
   h) no tail
   i) atmost two tails

   Solution: Let the random variable X denote one single coin toss, where obtaining a head is considered a success. Then,

$$X \sim \text{Ber}(p) \tag{1}$$

Suppose $X_i, 1 \leq i \leq n$ represent each of the n tosses. Define Y as

$$Y = \sum_{i=1}^{n} X_i \tag{2}$$

Then, since the X_i are iid, the pmf of Y is given by

$$Y \sim \text{Bin}(n, p) \tag{3}$$

The cdf of Y is given by

$$F_Y(k) = \Pr(Y \leq k) \tag{4}$$

$$= \begin{cases} 0 & k < 0 \\ \sum_{i=1}^{k} \binom{n}{i} p^i (1-p)^{n-i} & 1 \leq k \leq n \\ 1 & k \geq n \end{cases} \tag{5}$$

In this case,

$$p = \frac{1}{2}, \ n = 3 \tag{6}$$

a) We require $\Pr(Y = 3)$. Thus, from (3),

$$\Pr(Y = 3) = \binom{n}{3} p^3 (1-p)^{n-3} \tag{7}$$

$$= \frac{1}{8} \tag{8}$$

b) We require $\Pr(Y = 2)$. Thus, from (3),

$$\Pr(Y = 2) = \binom{n}{2} p^2 (1-p)^{n-2} \tag{9}$$

$$= \frac{3}{8} \tag{10}$$

c) We require $\Pr(Y \geq 2)$. Since $n = 3$ in (5),

$$\Pr(Y \geq 2) = 1 - \Pr(Y < 2) \tag{11}$$

$$= F_Y(3) - F_Y(1) \tag{12}$$

$$= \sum_{k=2}^{3} \binom{n}{k} p^k (1-p)^{n-k} \tag{13}$$

$$= \frac{1}{2} \tag{14}$$

d) We require $\Pr(Y \leq 2)$. Thus, from (5),

$$\Pr(Y \leq 2) = \sum_{k=0}^{2} \binom{n}{k} p^k (1-p)^{n-k} \tag{15}$$

$$= \frac{7}{8} \tag{16}$$

e) We require $\Pr(Y = 0)$. Thus, from (3),

$$\Pr(Y = 0) = \binom{n}{0} p^0 (1-p)^n \tag{17}$$

$$= \frac{1}{8} \tag{18}$$

f) Obtaining 3 tails is the same as obtaining no heads. Hence, from (18), we require $\Pr(Y = 0) = \frac{1}{8}$.

g) We require $\Pr(Y = 1)$ (since only one head is obtained). Thus, from (3),

$$\Pr(Y = 1) = \binom{n}{1} p^1 (1-p)^{n-1} \tag{19}$$
$$= \frac{3}{8} \tag{20}$$

h) We require $\Pr(Y = 3) = \frac{1}{8}$ from (8).

i) We require $\Pr(Y \geq 1)$ (since at least one head is obtained). Thus, from (5) and (18),

$$\Pr(Y \geq 1) = 1 - \Pr(Y < 1) \tag{21}$$
$$= 1 - F_Y(0) \tag{22}$$
$$= 1 - \Pr(Y = 0) \tag{23}$$
$$= \frac{7}{8} \tag{24}$$