SOCIAL FORCE MODEL HOMEWORK

1. Conceptual Foundation

Q1. Explain the role of each force component in the social force model:

$$\mathbf{F_i = F_i\text{\textasciicircum}desire + \Sigma\ F_{i\,j}\text{\textasciicircum}repulsion + \Sigma\ F_iW\text{\textasciicircum}wall}$$

- What does each term represent?
- Why are these forces vector quantities?

2. Deriving the Desired Force

Q2. The desired force for an agent `i` is defined as:

$$F_i\text{\^{}desire} = m_i (v_i^0 - v_i) / \tau$$

Given:

- mass $m_i = 70$ kg,
- desired speed $v_i^0 = [1.5, 0]$ m/s,
- current velocity $v_i = [0.5, 0]$ m/s,
- relaxation time $\tau = 0.5$ s.

(a) Compute the magnitude and direction of **F_i^desire**.
(b) Interpret what this force physically means in this context.

3. Repulsive Force Between Agents

Q3. The repulsive force between agents i and j is modeled as:

$$\mathbf{F_{ij}\text{\^{}repulsion} = A \cdot exp((r_{ij} - d_{ij}) / B) \cdot n_{ij}}$$

Where:

- $A = 2$ N, $B = 0.5$ m,
- $r_{ij} = 0.6$ m (sum of radii),
- $d_{ij} = 0.4$ m (distance between centers),
- $n_{ij} = (x_i - x_j) / ||x_i - x_j||$ is the unit vector pointing from j to i.
- $x_i = [1, 1]$, $x_j = [0.7, 1]$

(a) Compute the repulsive force vector **F_{ij}^repulsion**.
(b) Explain how this force changes as d_{ij} decreases.

4. Wall Avoidance Force

Q4. Suppose an agent is walking parallel to a wall and the shortest distance from the wall is 0.2 m. The wall exerts a force similar to agent repulsion:

$$\mathbf{F_iW\text{\^{}}wall = A\_w \cdot exp((r_i - d_iW) / B\_w) \cdot n_iW}$$

With parameters:

- $A\_w = 3$ N, $B\_w = 0.3$ m,
- $r_i = 0.3$ m, $d_iW = 0.2$ m,
- $n_iW = [1, 0]$

(a) Compute the wall repulsion force.
(b) Why is it important to include wall forces in crowded navigation?

5. Net Acceleration and Position Update

Q5. An agent has the following total force vector (in Newtons):
F_total = [4, -3] N, and mass `m = 60 kg`. The current velocity is **v = [1.0, 0.5] m/s**, and you use Euler integration with `dt = 0.1 s`.

(a) Compute the acceleration **a = F / m**.
(b) Update the velocity and position after one time step.
Assume the current position is **p = [0, 0]**.