

GRE Quantitative Reasoning (Step 2)

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Overview

1 Arithmetic

2 Algebra

3 Geometry

4 Data Analysis

5 Problem-solving Techniques



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课程简介

课程内容

全面的数学核心词汇串讲
新GRE核心考点归纳
各种题型做题技巧的总结
大量习题

适用对象

完成了GRE数学强化一阶段的学生
对于GRE数学获得满分有强烈需求的学生
题目会做却经常做错的学生

与一阶段的区别

更注重核心考点和做题技巧的总结
例题和练习题难度较大(等同于甚至略高于GRE数学的最高难度)

上课要求



- ① 准备笔记本一个（记录讲课内容的重点）
- ② 草稿纸若干（用于习题练习）
- ③ 最简单功能的计算器（windows自带计算器或者手机里的计算器）

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GRE数学复习流程

- ① 全面复习数学知识点
- ② 熟悉数学词汇和英语中的表达习惯
- ③ 熟悉做题技巧
- ④ 总结归纳易错点
- ⑤ 大量练习巩固



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我们的目标是

数学满分170!



GRE数学简介

2011年ETS对GRE进行全面改革，GRE难度大大增加

中国大陆有一半考生在新GRE数学部分没有上162.对！就是162！

改革后的GRE数学部分，增加了四分位数，百分位数，箱线图，标准正态分布等高难度内容，在概率统计方面难度大增

ETS出版的OG中，有一半的篇幅是在介绍数学知识点

太多信心满满的考生裸考GRE数学，结果...

题型介绍

新GRE数学考试部分，会遇到4种题型，分别是：

- ① 数量比较题quantitative comparison questions
- ② 单项选择题Multiple-choice questions-select one answer choice
- ③ 不定项选择题Multiple-choice questions-select one or more answer choices
- ④ 数字填空题numeric entry questions

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数量比较题

Questions of this type ask you to compare two questions—Quantity A and Quantity B, and then determine which of the following statements describes the comparison.

- A.Quantity A is greater
- B.Quantity B is greater
- C.The two quantities are equal
- D.The relationship cannot be determined from the information given

如果出现A大于或等于B的情况，应该选什么？

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Example

$$x - 3 = 1$$

Quantity A

x

Quantity B

3

Solution: $x = 4$ 所以quantity A大, 选A

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单项选择题

Example

$x = 2^3$, then what is the value of x ?

- A.2
- B.3
- C.5
- D.8
- E.9

Solution: $x = 8$, 选D

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不定项选择题

Example

$x^2 - 1 = 0$, then what is the value of x ? Indicate all possible values.

- A.1
- B.2
- C.0
- D.-1
- E.-2

Solution: $x = \pm 1$, 选AD

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数字填空题



Example

If $x^2 - 1 = 0$ and x is a negative number, what is the value of x ?

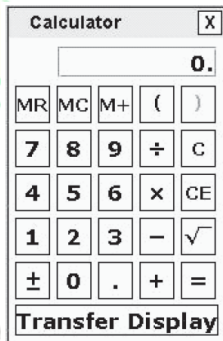
Solution: $x = -1$, 给的空格里填-1

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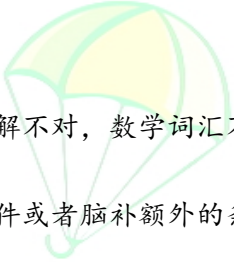
计算器

计算器的使用

- ① 大多数计算不需要计算器
- ② 简单计算尽量不要用计算器，浪费时间
- ③ 有时候可以巧妙的回避计算
- ④ 如有必要用计算器，请注意四舍五入
- ⑤ 尽量在关键步骤时使用计算器



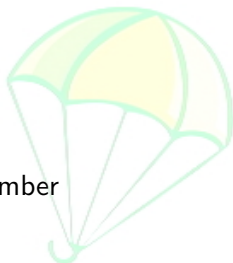
常见错误

- 
- ① 题目看不懂（应用题理解不对，数学词汇不认识）
 - ② 考点遗忘
 - ③ 审题有问题（没看见条件或者脑补额外的条件）
 - ④ 手抖选错了

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Arithmetic

- 1 Integers
- 2 Factors and multiples
- 3 quotients and remainders
- 4 odd and even number
- 5 prime and composite number
- 6 fractions
- 7 exponents and roots
- 8 decimals
- 9 real number
- 10 ratio
- 11 percent



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Integers 整数

Integers 整数, Positive Integers 正整数, Negative Integers 负整数

核心考点: 正+正=正, 正*正=正, 负+负=负, 负*负=正

Example 1

a, b, c are all integers, suppose $a^3 b^4 c^5$ is negative then which of the following CANNOT be true?

- A $a + b > 0$
- B $b + c > 0$
- C $a + c > 0$
- D $ac > 0$
- E $abc > 0$

Factors and Multiples 因子和倍数

factors (divisors) 因子, multiple 倍数, least common multiple 最小公倍数, greatest common divisor 最大公约数

核心考点：因数分解，最大公约数，最小公倍数

Example 2

Let S be the set of all positive integers n such that n^2 is a multiple of both 24 and 108. Which of the following integers are divisor of every integer n in S ? Indicate all such integers

- A 12
- B 24
- C 36
- D 72

Quotients and Remainders 整数带余除法

a divided by b is q remainder $r \Leftrightarrow a = bq + r$ where $(0 \leq r < b)$ a 除以 b 等于 q 余 r
quotient 商; remainder 余数

核心考点：求余数

Example 3

An amazing fact is that the number 2^{29} has 9 digits, and all of them are different. Find the remainder of 2^{29} divided by 9. And find the missing digit.

odd and even number 奇偶数

odd number 奇数, even number 偶数

核心考点：如果一堆整数之积是奇数，那么所有都是奇数
如果一堆整数之积是偶数，那么至少有一个偶数

Example 4

a, b, c are all integers, suppose $a^3 b^4 c^5$ is an odd number then which of the following is true? Indicate all such statements.

- A $a + b$ is odd
- B $b + c$ is odd
- C $a + c$ is even
- D ac is odd
- E abc is even

prime and composite number 质数与合数

prime number 质数, composite 合数, prime factorization 质因数分解

核心考点：质数之积为合数，质数与奇偶性的关系

Example 5

a, b, c are different prime numbers and $a < b < c$ then which of the following CANNOT be a prime number? Indicates all such statements.

A $a + b$

B bc

C $bc + 1$

D $a - 5$

E $b + 3$

fractions 分数

fraction 分数, rational numbers 有理数, irrational numbers 无理数, 分子 numerator; 分母 denominator, common denominator 通分, reciprocal 倒数

核心考点: 分数的化简与通分

Example 6

If $\frac{N}{2}$ and $\frac{M}{5}$ are both integers, which of the following expression must also be an integer?

- A $\frac{N+M}{7}$
- B $\frac{NM}{7}$
- C $\frac{\frac{N}{2} + \frac{M}{5}}{2}$
- D $\frac{N+M}{10}$
- E $\frac{NM}{10}$

exponent and roots 指数和根

底base, 指数exponent, square root 平方根

核心考点：指数的化简

Example 7

Quantity A
 9^{-18}

Quantity B
 $(\frac{1}{27})^{12}$

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decimals 十进制数

个位数, 十位数, 百位数, 千位数, ... 十分位数, 百分位数, 千分位数, ... ones/units, tens, hundreds, thousands, ... tenths, hundredths, thousandths

核心考点: 分数小数的互换

Example 8

Which of the following fractions is equal to the repeating decimal $0.36363636 \dots$

- A $\frac{1}{275}$
- B $\frac{2}{55}$
- C $\frac{4}{99}$
- D $\frac{4}{11}$
- E $\frac{4}{9}$

分数与小数互换

分数化成小数请用**计算器**

有限小数化成分数（上下同时乘以1000...使得上下变成整数）

无限循环小数（只讨论0.···的情况）：循环节有几位就在分母写几个9，有几位没加入循环就写几个0，再用第二个循环节以前的小数部分组成的数与小数部分中不循环部分组成的数的差做分子，如：

$$0.4\bar{3} = \frac{43-4}{90} = \frac{39}{90} = \frac{13}{30}$$

$$0.14\bar{5} = \frac{145-14}{900} = \frac{131}{900}$$

$$0.54\bar{9} = \frac{549-5}{990} = \frac{544}{990} = \frac{272}{495}$$

无限不循环小数没办法化为分数

real number 实数

real numbers 实数,实数轴real line, interval 区间, absolute value 绝对值

核心考点：绝对值，特殊运算

Example 9

$\lfloor x \rfloor$ represents the greatest integer less than or equal to x and $\lceil x \rceil$ represents the least integer greater than or equal to x .

Quantity A

$$\frac{\lfloor x \rfloor + \lceil x \rceil}{2}$$

Quantity B

$$x$$

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ratio 比例, percent 百分数

ratio of s to t : $\frac{s}{t}$, $s : t$

核心考点：百分比增长/减少

Example 10

In 1988 Mr Smith's annual income was greater than Mrs Smith's annual income. In 1989 Mr Smith's annual income decreased by p percent, whereas Mrs Smith annual income increased by p percent. ($p > 0$)

Quantity A

Mr and Mrs Smith's
combined annual
income in 1988

Quantity B

Mr and Mrs Smith's
combined annual
income in 1989

Exercise1

The "reflection" of a positive integer is obtained by reversing its digits. For example, 321 is the reflection of 123. The difference between a five-digit integer and its reflection must be divisible by which of the following?

- A 2
- B 4
- C 5
- D 6
- E 9

Exercise2

Suppose $a^2 + b^2 = c^2$, and a, b, c are all integers. Which of the following CANNOT be the value of $a + b + c$?

- A 2
- B 1
- C -2
- D 4
- E 6

Exercise3

When a positive integer n is divided by 3, the remainder is 2 and when n is divided by 5, the remainder is 1. What is the least possible value of n ? (n 可能的值里面最小的, 不是最不可能的 n 的值)

Exercise4

At Megalomania Industries, factory workers were paid \$20 per hour in 1990 and \$10 per hour in 2000. The CEO of Megalomania Industries was paid \$5 million per year in 1990 and \$50 million per year in 2000. The percent increased in the pay of Megalomania's CEO from 1990 to 2000 was what percent greater than the percent decreased in the hourly pay of Megalomania's factory workers over the same period?

- A 850%
- B 900%
- C 950%
- D 1700%
- E 1900%

Exercise5

The jewels in a certain tiara consist of diamonds, rubies, and emeralds. If the ratio of diamonds to rubies is $\frac{5}{6}$ and the ratio of rubies to emeralds is $\frac{8}{3}$, what is the least number of jewels that could be in the tiara?

- A 16
- B 22
- C 40
- D 53
- E 67

WEICHEN EDUCATION

- 1 algebraic expression
- 2 exponents
- 3 linear equations
- 4 quadratic equations
- 5 linear inequalities
- 6 functions
- 7 interests
- 8 coordinate geometry
- 9 straight lines
- 10 graph of functions



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algebraic expression 代数式

algebraic expression 代数式, terms (代数) 项, like terms or similar terms (同类项), constant term 常数项, coefficient 系数, equation 等式

核心考点：代数式化简

Example 11

Quantity A

$$y^2 - 2y + 3x$$

Quantity B

$$-\frac{3}{2} + 3x$$

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Exponents指数式

形如 x^a 的代数式称之为指数式，其中 x 称为base底， a 称为exponent指数

laws运算法则

- ① $x^{-a} = \frac{1}{x^a}$
- ② $(x^a)(x^b) = x^{a+b}$
- ③ $\frac{x^a}{x^b} = x^{a-b}$
- ④ $x^0 = 1$
- ⑤ $(xy)^a = (x^a)(y^a)$
- ⑥ $(\frac{x}{y})^a = \frac{x^a}{y^a}$
- ⑦ $(x^a)^b = x^{ab}$

常见的错误运算

- ① $x^a y^b \neq (xy)^{a+b}$
- ② $(x^a)^b \neq x^a x^b$
- ③ $(x+y)^a \neq x^a + y^a$
- ④ $(-x)^2 \neq -x^2$
- ⑤ $\sqrt{x^2 + y^2} \neq x + y$
- ⑥ $\frac{a}{x+y} \neq \frac{a}{x} + \frac{a}{y}$

核心考点：指数式化简



Example 12

Quantity A

$$\frac{4^{x+1} - 2^{2x}}{3}$$

Quantity B

$$4^x$$

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核心考点：解方程（二元一次方程）

Example 13

If $x + 2y = 2$ and $(x + y)^2 = 9$, which of the following is a possible value of y ?

A -8

B -5

C $-\frac{3}{2}$

D $\frac{3}{2}$

E 5

Solving quadratic equations 解一元二次方程

形如 $ax^2 + bx + c = 0$ 其中 $a \neq 0$ 的方程我们成为一元二次方程

方程的解我们可以用quadratic formula一元二次方程求根公式求得： $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Delta = b^2 - 4ac > 0$ 则方程有俩不同的解

$\Delta = b^2 - 4ac = 0$ 则方程有一个解

$\Delta = b^2 - 4ac < 0$ 则方程无解。

核心考点：解二次方程

Example 14

$$\frac{x^2 - 3x + 2}{x^4 + 3x^2 + 4x - 3} = 0$$

Quantity A

x

Quantity B

2

Solving linear inequality 解不等式

核心考点：解不等式

Example 15

How many integers are there in the set $-4 < x^2 + 5x < 14$

- A 0
- B 1
- C 2
- D 3
- E 4

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functions函数

$y = f(x)$ 称为一个函数

domain 定义域为函数有定义的所有 x 值

range 值域为函数所有可能的取值

如 $y = f(x) = x^2 - 1$, 那么定义域就是所有实数, 值域为 $y \geq -1$

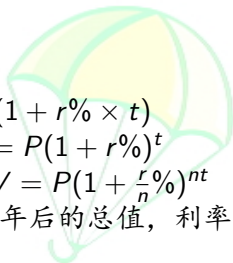
核心考点: 抽象函数的性质

Example 16

$f(2x) = 2f(x) + 1$ for all real number x . And $f(0.5) = 1$ then what is the value of $f(4)$?

- A 1
- B 5
- C 10
- D 15
- E 19

interest利息



simple interest单利: $V = P(1 + r\% \times t)$

compound interest复利: $V = P(1 + r\%)^t$

compound n times per year $V = P(1 + \frac{r}{n}\%)^{nt}$

其中P为principle本金, V为t年后的总值, 利率为r%每年

核心考点:

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rectangular coordinate system, xy-coordinate system or xy-plane都是坐标平面的意思

x轴x-axis; y轴y-axis; 原点origin; 第1234象限quadrants I,II,III,IV
P和P'两点关于x轴(y轴, 原点)对称: P' is reflection of P about x-axis(y-axis,origin), or P' and P are symmetric about the x-axis(y-axis,origin)

距离公式 (x_1, y_1) 和 (x_2, y_2) 的距离为 $\sqrt{(x_2 - x_1)^2 + (y_1 - y_2)^2}$

intercept截距: $y = x^2 - 4$,那么该函数图像与x轴交于 $(-2, 0)$ 和 $(2, 0)$,与y轴交于 $(0, -4)$,那么我们称x截距为2和-2, y截距为-4

straight lines 直线

一条直线可以由slope斜率和一个点 (x_0, y_0) 确定，也可以由两个点 $(x_1, y_1), (x_2, y_2)$ 确定

给定两个点 $(x_1, y_1), (x_2, y_2)$ ，那么通过这两点的直线的斜率为 $m = \frac{y_1 - y_2}{x_1 - x_2}$ ，如果斜率为正，那么直线是斜向上的，否则直线为斜向下的

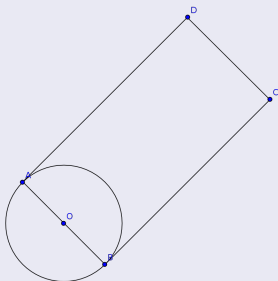
两条直线平行parallel当且仅当斜率相同；两条直线垂直perpendicular当且仅当斜率之积为-1. 核心考点：平行和垂直

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Example 17

suppose $ABCD$ is a rectangle, O is the center of the circle. If the coordinates of O are $(1, 2)$, coordinates of A are $(0, 3)$ and coordinates of D are $(4, 7)$ then what are the coordinates of C ?

- A $(5, 6)$
- B $(6, 5)$
- C $(6, 4)$
- D $(4, 6)$
- E $(5, 4)$



graphs of functions 函数图像

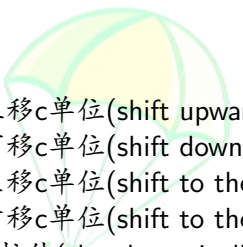
$y = ax + b$ 的图像是一条直线，斜率的 a , y 截距为 b 。

$y = ax^2 + bx + c$ 其中 $a \neq 0$ 的图像是一个抛物线 parabola
如 $y = x^2 - 2x - 3$ 的顶点 vertex 是 $(1, -4)$, $x = 1$ 是对称轴 line of symmetry, 开口向上 opens upward

$(x - a)^2 + (y - b)^2 = r^2$ 的图像是一个圆 circle, 圆心 center 为 (a, b) , radius 半径为 r

$y = f(x)$ 和 $y = g(x)$ 的图像的交点就是 $f(x) = g(x)$ 的解

graphs of functions 函数图像



$f(x) + c$ 就是把 $f(x)$ 的图像上移 c 单位(shift upward by c units)
 $f(x) - c$ 就是把 $f(x)$ 的图像下移 c 单位(shift downward by c units)
 $f(x + c)$ 就是把 $f(x)$ 的图像左移 c 单位(shift to the left by c units)
 $f(x - c)$ 就是把 $f(x)$ 的图像右移 c 单位(shift to the right by c units)
 $cf(x)$ 就是把 $f(x)$ 的图像纵向拉伸(sketch vertically) c 倍($c > 1$)
 $cf(x)$ 就是把 $f(x)$ 的图像纵向压缩(shrink vertically) c 倍($0 < c < 1$)

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核心考点：函数图像和代数方程的关系

Example 18

Which of the following is in the region bounded by $y = x^2$ and $x + 6$?

Indicate all such points

A (0, 4)

B (-1, 0)

C (1, 3)

D (4, 6)

E (5, 4)

Exercise6

$m > n$ are positive integers

Column A

$$m! - n!$$

Column B

$$(m - n)!$$

Exercise7

$$n = \frac{k + \frac{r}{s}}{\frac{t}{v}}$$

In the equation above, k, r, s, t, v represent positive numbers. Multiplying which one of these numbers by 2 will reduce the value of n to $\frac{1}{2}$ of its present value?

A k

B r

C s

D t

E v

Exercise8

$$3 < x < 5$$

Column A

$$\frac{3+x}{3}$$

Column B

$$\frac{5+x}{x}$$

Exercise9

The cost of sending a package special delivery is x cents per ounce up to 10 ounces and y cents for each ounce in excess of 10. Which of the following represents the total cost, in cents, of sending special delivery a package weighing w ounces, if $w > 10$

A $x + (w - 10)y + 10$

B $x + (w - 10) + y$

C $10x + 10(w - y)$

D $10x + y(w - 10)$

E $10x + wy$

Exercise10

T is no more than 20 percent greater than the integer x , and T is no smaller than a number 20 percent less than the integer x . If $T = 60$, which of the following indicates the smallest possible interval for x ?

A $40 \leq x \leq 80$

B $48 \leq x \leq 75$

C $48 \leq x \leq 72$

D $50 \leq x \leq 78$

E $50 \leq x \leq 75$

Exercise11

$$0 < p < 1$$

Column A

The greatest value of $p(1 - p)$

Column B

$$\frac{1}{2}$$

Exercise12

In the xy -coordinate system, if line k intersects the y -axis at $(0, 6)$ and has a slope of -3 , then k intersects the x -axis at

A $(-2, 0)$

B $(-\frac{1}{2}, 0)$

C $(\frac{1}{2}, 0)$

D $(2, 0)$

E $(3, 0)$

WEICHEN EDUCATION

- ① Line and Angles
- ② Polygons
- ③ Triangles
- ④ Quadrilaterals
- ⑤ Circles
- ⑥ Three-dimensional Figures



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line and angles 直线和夹角

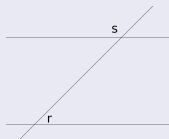
point点, line直线, line segment线段, 线段的两个端点endpoints. 线段的长度length. 相同长度的线段我们成为congruent line segments. 中点midpoint为C。

角angles, vertex P, 对顶角opposite angles, 锐角acute angle, 钝角obtuse angle

垂直perpendicular, 直角right angle, 平行parallel 核心考点: 根据关系求角度

Example 19

Suppose $\frac{s}{r+s} = \frac{3}{5}$ then $r = \text{----}$



polygons 多边形

多边形就是有多个边的图形。

边sides, 顶点vertices

triangle三角形, quadrilateral四边形, pentagon五边形, hexagon六边形, octagon八边形

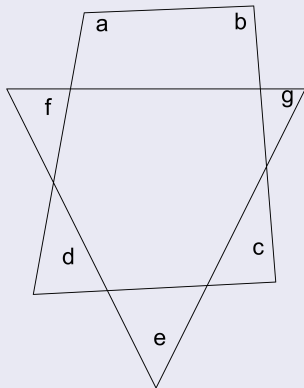
n 条边的多边形内角和为 $180(n-2)^\circ$

正多边形regular polygon, perimeter周长, area面积核心考点：内角和

Example 20

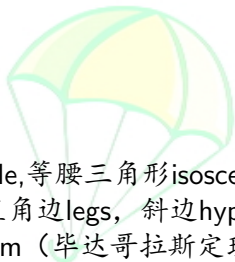
What is the maximum possible number of interior angles that are right angles of a decagon (10-side polygon)?

Example 21



What is $a + b + c + d + e + f + g$?

triangles 三角形



三角形内角和为 180°

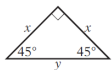
等边三角形equilateral triangle, 等腰三角形isosceles triangle

直角三角形right triangle, 直角边legs, 斜边hypotenuse

勾股定理Pythagorean theorem (毕达哥拉斯定理)

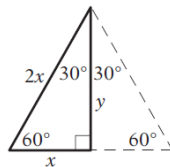
WEICHEN EDUCATION

两个特殊的三角形



$$\begin{aligned}y^2 &= x^2 + x^2 \\y^2 &= 2x^2 \\y &= \sqrt{2}x\end{aligned}$$

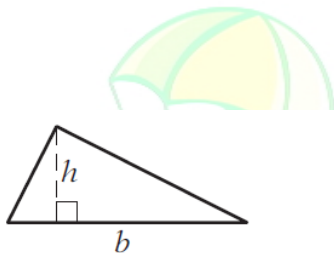
三边比例为 $1 : 1 : \sqrt{2}$



三边比例为 $1 : \sqrt{3} : 2$

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三角形的面积



$$A = \frac{bh}{2}$$

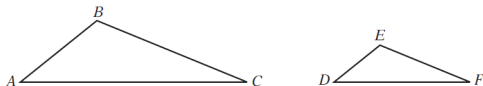
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全等三角形和相似三角形

全等三角形就是形状大小完全一样的三角形

相似三角形就是形状一样但是大小不一定一样的三角形

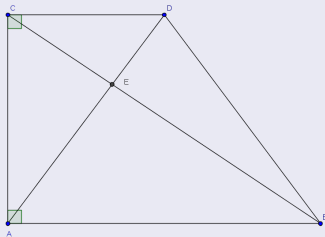
如ABC和DEF就是相似三角形，他们的边长成比例 $\frac{AB}{CD} = \frac{BC}{EF} = \frac{AC}{DF}$



核心考点：以上所有

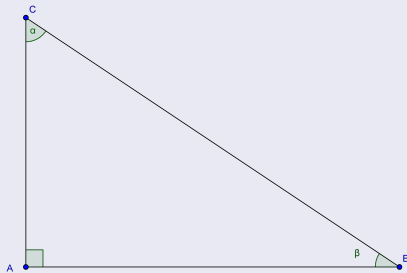
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Example 22



In the figure above, what is the area of triangle BDE and ABE given that the area of triangle ACE is 4, the area of triangle CDE is 3?

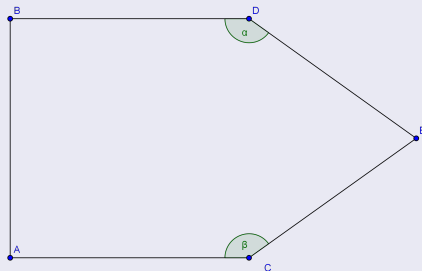
Example 23



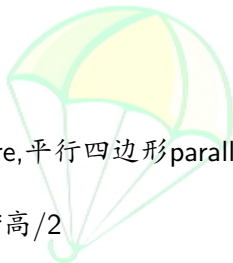
suppose $2\alpha + \beta > 100$ then which of the following could be the value of β ? Indicates all possible values

- A) 60 B) 89 C) 80 D) 45 E) 50 F) 51

Example 24



What is the perimeter of $ABCDE$ if both α and β are 150° , A, B are right angles and $AC=AB=BD=1$?

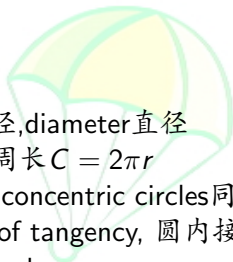


长方形rectangle, 正方形square, 平行四边形parallelogram, 梯形trapezoid

平行四边形面积: 底*高

梯形面积: $(\text{上底} + \text{下底}) * \text{高} / 2$

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circle 圆, center 圆心, radius 半径, diameter 直径

chord 弦, arc 弧, circumference 周长 $C = 2\pi r$

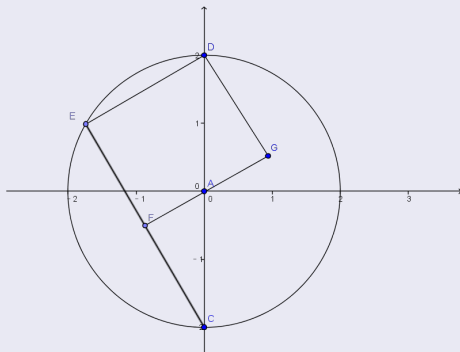
area 面积 $A = \pi r^2$, sector 扇形, concentric circles 同心圆

切线 tangent line, 切点 point of tangency, 圆内接多边形 inscribed polygon,

圆外接多边形 circumscribed polygon

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Example 25

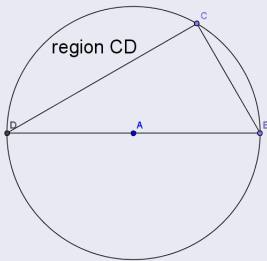


suppose $A(o)$ is the origin and the radius of the circle is 2. suppose $CE=3$, $DEFG$ is a rectangle.

Quantity A
area of $DEFG$

Quantity B
7

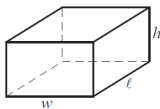
Example 26



Quantity A
area of ABC

Quantity B
area of region CD

three-dimensional figures 立体图形



rectangular solid 长方体
cube 立方体, faces 面
edges 边, vertex 顶点
volume 体积 $V = lwh$
surface area 表面积
积 $A = 2(lw + lh + wh)$



circular cylinder 圆柱, axis 轴(PQ)
volume 体积 $V = \pi r^2 h$
surface area 表面积 $A = 2\pi r^2 + 2\pi rh$

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核心考点：表面积或体积

Example 27

An ice cube is 1cm long, if we slice it into two pieces.

Quantity A

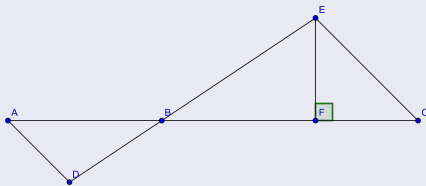
Resulting surface area of all the slices

Quantity B

8 cm^2

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Exercise13

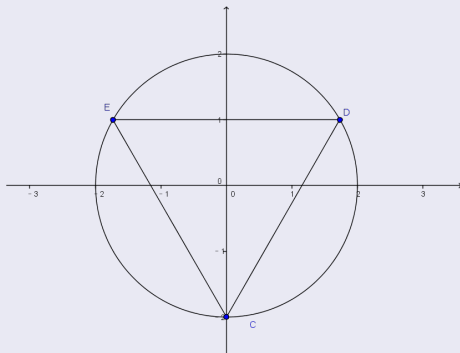


$AC=40$, $BC=25$, $EF=10$, and AD is parallel to EC .

Quantity A
area of ABD

Quantity B
90

Exercise14



triangle CDE is equilateral.

Column A

area of CDE

Column B

total area of the other parts of the circle

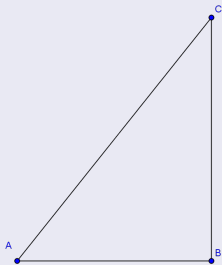
Exercise15



In the figure above $\frac{AB}{AC} = \frac{AC}{BC}$
What is $\frac{AB}{BC}$?

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Exercise16



$AB=3$, $AC=6$

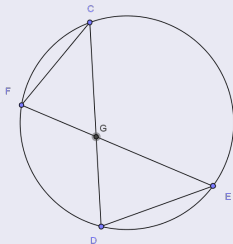
Column A

area of ABC

Column B


8

Exercise17



In the above circle $CG = 10$, $FG = 8$, $CF = 9$ and $DG = 12$. What is the length of DE ?

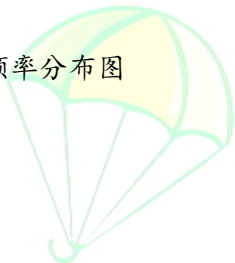
- A) 10 B) 11.5 C) 12 D) 13.5 E) 15

- 
- ① Graphical Methods for Describing Data
 - ② Numerical Methods for Describing Data
 - ③ Counting Methods
 - ④ Probability
 - ⑤ Distributions of Data, Random Variables, and Probability Distributions
 - ⑥ Data Interpretation

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Graphical Methods for Describing Data 用图形描述数据

- ① Frequency Distribution 频率分布图
- ② Bar Graphs 条形图
- ③ Circle Graphs 饼状图
- ④ Histograms 直方图
- ⑤ Scatterplots 散点图
- ⑥ Time Plots 时间点图



区分题目给的图表是哪一种毫无意义，主要要看清楚横纵坐标或者图形代表的意义

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measure of central tendency 数据的中心

arithmetic mean(average) 算术平均值

如给定五个数：6,4,7,10,7和4，算术平均值为 $\frac{6+4+7+10+7+4}{6} = \frac{38}{6}$

median 中位数

将数据从小到大排列，如总共有奇数个数据，中位数就是中间的那个数，如有偶数个数据，中位数就是中间两个数据的平均

如给定：2,4,4,5,7,7,7,7,7,7,8,8,9,9,9,9 共16个数据，那么中位数为7

mode 众数

出现次数最多的数，如2,4,4,5,7,7,7,7,7,7,8,8,9,9,9,9中7就是众数。一个数据集中可以有多个众数

measure of position 数据的位置

数据集中最有用的几个位置就是：最小值，第一个4分位数，第二个4分位数，第三个4分位数，最大值

其中第一个4分位数 Q_1 代表大约有 $\frac{1}{4}$ 的数据比这个数小，第二个4分位数 Q_2 代表大约有 $\frac{2}{4}$ 的数据比这个数小，第三个4分位数 Q_3 代表大约有 $\frac{3}{4}$ 的数据比这个数小。

一般就是把数据从小到大排列，大体上排在 $\frac{1}{4}$ 位置上的数字就是 Q_1 ，以此类推。

另外还有百分位数 P_1, P_2, \dots, P_{99} ，分别表示有大约1%, 2%, \dots , 99% 的数据比这个数小。也就是将数据从小到大排列，大体上排在1% 位置上的数字就是 P_1 ，以此类推。

注意 median, Q_2 和 P_{50} 表示的意思大致相同

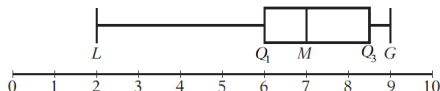
measure of dispersion 数据的分散度

range 极差 (数据中最大值减去最小值)

outliers 离群值 (就是距离大部分数据都很远的值)

interquartile range 四分位差 ($Q_3 - Q_1$)

箱线图 boxplots (box-and-whisker plots)



如图所示L表示最小值，G表示最大值， Q_1 为第一四分位数，M为中位数， Q_3 为第三四分位数，G为最大值

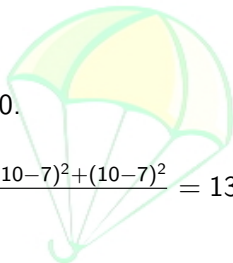
standard deviation 标准差

如给定五个数据: 0, 7, 8, 10, 10.

平均值为 $\frac{0+7+8+10+10}{5} = 7$

方差为 $\frac{(0-7)^2 + (7-7)^2 + (8-7)^2 + (10-7)^2 + (10-7)^2}{5} = 13.6$

标准差为 $\sqrt{13.6} \approx 3.7$

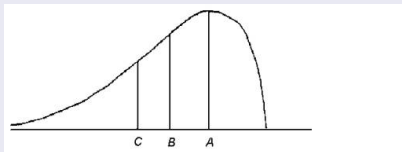


一般说来, 绝大多数的数据会落在平均数的左右3个标准差内

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核心考点: mean, median, mode

Example 28



Find which of the ABC is the mean, median and mode.

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核心考点: statistics



Example 29

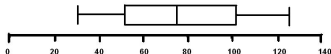
If the average GRE score of Bo, Uncle Qi, Jialing, Glee, Jimmy is 330. Jialing has a score of 333, Glee is 3 points higher than Jimmy. Bo is 10 points lower than Uncle Qi. Then what is the mean of Bo and Jimmy?

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核心考点: mean, median

Example 30

Annual snow depth at Mathsville Ski Resort



- Suppose here is the chart describing the annual snow depth of Mathsville ski resort in 40 years.
1. What is the size of the IQR (interquartile range)?
 2. How many of years are there that has a snow depth higher than 100?
 3. Suppose in 2014 the snow depth is 90; this is the 60th percentile value on this chart. How many years has a snow depth higher than 50 and lower than 90?

核心考点: Percentiles and Quartiles

Example 31

suppose all the 10 teachers of Weichen took the GRE test and Bo got a very good score of 335. Unluckily Uncle Qi (still one of the 10 teachers) had an even better score of 339. And everybody else is lower than Bo. Suppose now Weichen hires 10 more teachers who have GRE scores more than 335, than what is the percentile of Bo now?

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核心考点: Percentiles and Quartiles

Example 32

Bo took a test that is graded on a scale from 0 to 100. Quantity A

Quantity B

Bo's score

the percentile of Bo's score

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set集合是一堆东西(无重复)的总称, 集合里面的每个东西称为element元素。有一些集合是有限的finite, 也有一些无限的infinite。

空集合 \emptyset 称为empty set, 非空的集合nonempty set。子集合为集合的一部分, 如 $\{2, 8\}$ 为 $\{0, 2, 4, 6, 8\}$ 的子集。

list列表基本和集合意义一样, 不过列表可以由重复。

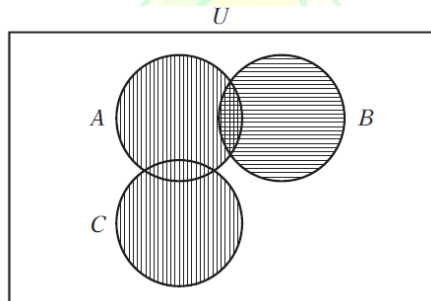
如 $\{1, 1, 1, 1, 1, 2\}$ 不是set, 是list

两个集合的intersection交为两个集合共有的元素形成的子集, 两个集合的union并为至少属于某一个集合的元素的集合。如 $\{0, 1\}$ 和 $\{1, 2\}$ 的交为 $\{1\}$, 并为 $\{0, 1, 2\}$

若两个集合的交为空我们称两个集合disjoint(mutually exclusive)不相交

Venn diagram 维恩图

我们可以用维恩图来表示集合之间的关系



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对于一般的集合S和T, $|S \cup T| = |S| + |T| - |S \cap T|$

Theorem

Theorem (加法原理)

做一件事情，完成它有 N 类方式，第一类方式有 M_1 种方法，第二类方式有 M_2 种方法，……，第 N 类方式有 M_N 种方法，那么完成这件事情共有 $M_1 + \cdots + M_N$ 种方法。

Theorem (乘法原理)

做一件事，完成它需要分成 N 个步骤，做第一步有 M_1 种不同的方法，做第二步有 M_2 种不同的方法，……，做第 N 步有 M_N 种不同的方法。那么完成这件事共有 $M_1 * \cdots * M_N$ 种不同的方法。

从n个不同元素中，任取m个元素按照一定的顺序排成一列，叫做从n个不同元素中取出m个元素的一个排列；所有可能的排列的个数称为排列数 $\frac{n!}{(n-m)!}$

从n个不同元素中，任取m个元素并成一组，叫做从n个不同元素中取出m个元素的一个组合；所有可能的组合的个数称为组合数 $\frac{n!}{(n-m)!m!}$ (n choose m)

其中 $m!$ 称为m的阶乘factorial, $m! = m(m-1)(m-2) \cdots (2)(1)$

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$$P(E) = \frac{\text{thenumberofoutcomesintheeventE}}{\text{thenumberofpossibleoutcomes}}$$

如给定一个普通骰子，那么掷出偶数的概率为 $\frac{3}{6} = 0.5$ 因为总共有6种可能，而其中只有3种为偶数。

概率有一下的性质

- ① 如果E一定发生，那么 $P(E) = 1$
- ② 如果E一定不发生，那么 $P(E) = 0$
- ③ 如果E有可能发生但不是一定发生，那么 $0 < P(E) < 1$
- ④ E不发生的概率为 $1 - P(E)$
- ⑤ $P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F)$
- ⑥ $P(E \text{ or } F) = P(E) + P(F)$ if E and F are mutually exclusive
- ⑦ $P(E \text{ and } F) = P(E)P(F)$ if E and F are independent

核心考点：求概率

Example 33

If a fair coin is tossed 5 times, what is the probability of getting 3 heads in a row?

- A) $1/16$
- B) $3/16$
- C) $1/4$
- D) $3/8$
- E) $1/2$

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核心考点：求概率

Example 34

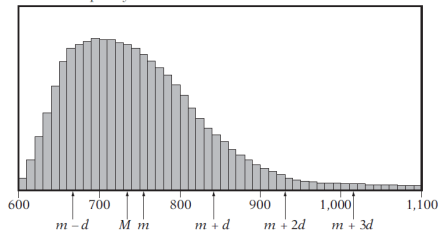
Bo has 10 pets: 4 dogs, 3 cats, 2 alpacas and 1 bunny. If he wants to arrange them in a row and make sure the pets are always grouped according to species, how many ways can he arrange the pets?

- A) 24
- B) 24^2
- C) 288
- D) $(24)(288)$
- E) 144^2

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Distributions of Data

Relative Frequency Distribution for Lifetimes of 800 Electric Devices



其中 M 是中位数， m 是平均值， d 是标准差
我们称之为distribution curve, density curve和frequency curve

Random Variables 随机变量

简单地说，随机变量 X 就是每次取值有可能都不一样的变量，如掷骰子出现的点数，抛硬币正面或者反面。

虽然每次出现的值可能不一样，但是总体上来说是从一定的分布的，如抛硬币只有可能是正面或者反面，而且两种可能性差不多。

Example

Let X represent the number of children in a randomly chosen family among the 25 families. What is the probability that $X = 3$? That $X > 3$? That X is less than the mean of the distribution?

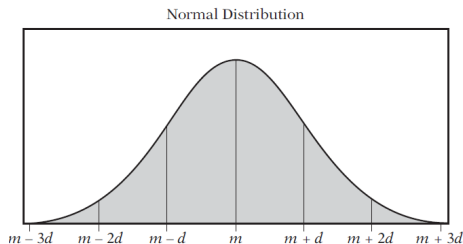
Frequency Distribution	
Number of Children	Frequency
0	3
1	5
2	7
3	6
4	3
5	1
Total	25

$$P(X = 3) = \frac{6}{25}, P(X > 3) = P(X = 4) + P(X = 5) = \frac{4}{25}$$

$$\text{mean} = \frac{0 \cdot 3 + 1 \cdot 5 + 2 \cdot 7 + 3 \cdot 6 + 4 \cdot 3 + 5 \cdot 1}{25} = 2.16,$$

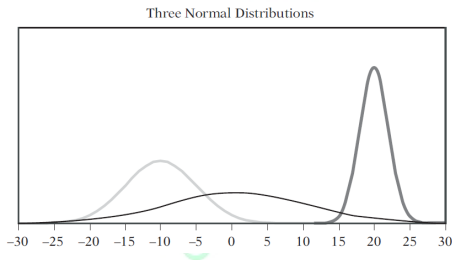
$$P(X < \text{mean}) = P(0) + P(1) + P(2) = \frac{15}{25} = 0.6$$

The Normal Distribution 正态分布



- The mean, median, and mode are equal.
- The data are grouped symmetrically about the mean.
- About two-thirds of the data are within 1 standard deviation of the mean.
- About 95% of the data are within 2 standard deviations of the mean.
- About 99.5% of the data are within 3 standard deviations of the mean.

The Normal Distribution 正态分布



一个正态分布有两个参数： m （平均数）和 d （标准差）

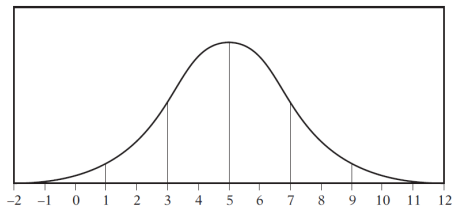
m 决定了分布的中心， d 决定了分布的高矮胖瘦（ d 越大越矮胖，反之越高瘦）

其中有一个特殊的正态分布我们称为standard normal distribution标准正态分布（ $m=0$ ， $d=1$ ）

The Normal Distribution 正态分布

Example

If W is a random variable that is normally distributed with a mean of 5 and a standard deviation 2. What is $P(W > 5)$ Approximately what is $P(3 < W < 7)$? Which of the four numbers 0.5, 0.1, 0.05 or 0.01 is the best estimate of $P(W < -1)$?



$$P(W > 5) = \frac{1}{2}, P(3 < W < 7) \approx \frac{2}{3}, P(W < -1) \approx \frac{1}{2} \cdot 5\% \approx 2.5\%$$

核心考点：正态分布的性质

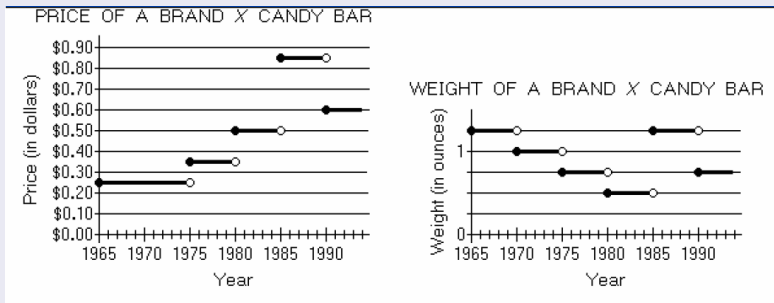
Example 35

Weichen has conducted a survey to determine how Chinese students behave on GRE Math test. The data indicate that the score is approximately normally distributed with a mean of 158 and a standard deviation of 8. Suppose the data size is 1000 then according to the survey, approximately how many of students have a score more than 166?

- A) 160
- B) 280
- C) 500
- D) 680
- E) 840

Exercise 18

These questions refer to the following graphs, which are drawn to scale. An open dot means the point is not included.



Exercise 18

- 1) From 1982 to 1987 the price of a Brand X candy bar increased by approximately what percent?
(A) 40% (B) 50% (C) 70% (D) 150% (E) 250%
- 2) How many times between 1971 and 1991 did the price of a Brand X candy increase while the weight decrease?
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5
- 3) In which year is the unit price cheapest?
(A) 1968 (B) 1973 (C) 1978 (D) 1983 (E) 1988 (F) 1993

Exercise19

Suppose we are trying to select 7 members of the 330 club. We have 14 candidates and half of them are girls (of course the other half are boys.). If the club must contain at least four girls and one boy, then how many ways can we select?

- (A) 441 (B) 1225 (C) 1666 (D) 1715 (E) 1820

Exercise20

Suppose now we have the 330 club members already. We want to build an even smaller group. A small group results as long as the we combine at least two members. If the number of possible groups is less than 500, what is the largest number of members in the club?

- (A) 8 (B) 9 (C) 10 (D) 11 (E) 12

Exercise21

suppose the set A is $\{a, b, b, a, a, b, 3a + b, a - b\}$. If the median of the set is 10 and $b \geq a > 0$, what is the range of the set?

Exercise22

A large distribution of score is normally distributed

Quantity A

score that's one standard
deviation above the mean

Quantity B

score that has
the 80th percentile

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Exercise23

There are n sweets in a bag. 6 of the sweets are orange. The rest of the sweets are yellow. Hannah takes at random a sweet from the bag and eats it. Hannah then takes at random another sweet from the bag and eats it. Suppose the probability that Hannah eats two orange sweets is $\frac{1}{3}$ then what is n ?

WEICHEN EDUCATION

Problem-solving Techniques

- ① 活用计算器
- ② 数值比较
- ③ 代入法



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活用计算器

Example

Column A

$$\sqrt{(79)(80)(81)(82)}$$

Column B

6400

Example

Column A

$$\frac{1}{1-0.03}$$

Column B

1.03

Example

Column A

$$100210 \times 90021$$

Column B

$$100021 \times 90210$$

数值比较

- 算出A,B的值然后直接进行比较
- 求 $A - B$, 如果 > 0 那么A大, 如果 < 0 那么B大, 如果 $= 0$ 那么一样大, 其他情况都选D不能确定
- 如果AB都是正数的时候, 求 $\frac{A}{B}$, 如果 > 1 那么A大, 如果 < 1 那么B大, 如果 $= 1$ 那么一样大, 其他情况都选D不能确定

Example

Column A

$$\frac{1}{1-0.03}$$

Column B

1.03

代入法

帶入不同的值尝试(如有变量 x, y), 选择题就把选项帶入尝试

Example

Column A

$$(x - 2)^2$$

Column B

$$x^2 - 4$$

Example

$$0 > x > y > z > -1$$

Column A

$$\frac{x}{y}$$

Column B

$$\frac{y}{z}$$

Answers

Examples: 1 D 2 AC 3 C 4 CD 5 BCE 6 E 7 C 8 D 9 D 10 A
11 A 12 C 13 E 14 D 15 E 16 D 17 B 18 AC
19 72 20 3 21 540 22 4 $16/3$ 23 ADEF 24 5 25 B 26 D 27 D
28 mode A median B mean C 29 326
30 50 10 14 31 45 32 D 33 C 34 D 35 A

Exercise: 1 E 2 B 3 11 4 D 5 D 6 D
7 D 8 D 9 D 10 E 11 B 12 D
13 B 14 B 15 $\frac{2}{3-\sqrt{5}}$ 16 D 17 D
18 C B A 19 D 20 A 21 40 22 A 23 10



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