GRE Quantitative Reasoning

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GRE数学基本假设

1. All numbers used are real numbers. 所有的数都是实数
2. All figures lie on a plane unless otherwise indicated. 除非题目中有专门指出，假设所有图形都在同一平面内
3. All angle measures are positive 所有的测量角度都是正数
4. All line shown as straight are straight. On the computer-based test, lines that appear "jagged" can also be assumed to be straight. 所有显示为直线的线都可以当作直线来处理
5. Figures are intended to provide useful information for answering the questions. However, except where a figure is accomplish by a "Note" stating that the figure is drawn to scale, solve the problem using your knowledge of mathematics, not by visual measurement or estimation. 伴随问题的图形将为解题提供有用的信息。但是，只有在问题中指出相应图形是按比例画出**drawn to scale**）时，才可以用目测或者估计而得到的信息去解题。否则只能运用你的数学知识去回答问题。

Example

n is an integer.

Quantity A	Quantity B
$(-1)^n(-1)^{n+2}$	1

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Practice

Of the following values of n, the value of $(\frac{1}{3})^n$ will be greatest for $n =$

A. -3

B. -2

C. 0

D. 2

E. 3

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Practice

If n is an integer and $5^n > 4,000$, what is the least possible value of n?

A. 3

B. 4

C. 5

D. 6

E. 7

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Practice

$3^x + 3^x + 3^x =$

A. 9^x

B. 3^{x+1}

C. 9^{3x}

D. 3^{2x}

E. 3^{3x}

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Exponents指数式

形如x^a的代数式称之为指数式，其中$x \neq 0$称为base底，a称为exponent指数
当a, b为整数的时候，以下成立。否则请谨慎

运算法则

1. $x^{-a} = \frac{1}{x^a}$
2. $(x^a)(x^b) = x^{a+b}$
3. $\frac{x^a}{x^b} = x^{a-b}$
4. $x^0 = 1$
5. $(xy)^a = (x^a)(y^a)$
6. $(\frac{x}{y})^a = \frac{x^a}{y^a}$
7. $(x^a)^b = x^{ab}$

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Example

What is the sum of the integers between -90 and 95, inclusive?

端点包括

A. 5
B. 185
C. 465
D. 4275
E. 4560

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Example

A certain sequence has 25 terms, all of which are positive. The first term in the sequence is an even integer, and the sum of any two consecutive terms is an odd integer.

Quantity A	Quantity B
number of even integers in the sequence	number of odd integers in the sequence

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Practice

Which of the following could be the sum of three consecutive integers?

A. 29

B. 46

C. 57

D. 92

E. 100

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Practice

If x is an integer and $k = (x-1)(x+2) - (x-1)(x-2)$, then which of the following must be true?

A. k is odd only when x is odd.

B. k is odd only when x is even.

C. k is even only when x is odd.

D. k is even only when x is even.

E. None of the above

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Example

$|2y - 5| < 1$

Quantity A	Quantity B
y	1

选哪一个大，绝对的大，不可以等于

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Example

Which of the following is equivalent to $0 < x < 2$?

A. $x = 1$

B. $|x| < 1$

C. $|x| < 2$

D. $|x - 1| < 1$

E. $|x + 1| > 1$

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Example

$s = |t - 2|$

Quantity A	Quantity B
$s + 2$	t

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Practice

x is some negative number

Quantity A	Quantity B
$\lvert x \rvert$	$\sqrt{x^2}$

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Absolute Value 绝对值

absolute value 绝对值：$|a| = a$ if $a \geq 0$ and $|a| = -a$ if $a < 0$
解带绝对值的不等式：$|ax + b| > (<, \geq, \leq)c \Rightarrow -c < ax + b < c$ 然后解不等式（记得乘除负数不等式要变号）

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Example

n is an odd integer between 2 and 10, and n is not a prime number.

Quantity A	Quantity B
n	9

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Example

If p and n are prime numbers, $p - n = 4$, and $\frac{3}{2} < \frac{p}{n} < 2$, what is the value of p?

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Example

Quantity A	Quantity B
the number of different prime factors of 500	the number of different prime factors of 360

composite合数

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Practice

If p, q are prime numbers, how many divisors does the product p^3q^6 have?

A. 9

B. 12

C. 18

D. 28

E. 36

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Example

If x, y, and z are integers greater than 1 and $xyz = 483$, which of the following could be the value of xy?

A. 15

B. 22

C. 69

D. 91

E. 144

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Example

k is a positive integer and 225 and 216 are both divisors of k . If $k = 2^a 3^b 5^c$, where a , b and c are positive integers, what is the least possible value of $a + b + c$?

A. 4

B. 5

C. 6

D. 7

E. 8

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Practice

If k is an integer, and $\frac{35^2-1}{k}$ is an integer, then k could be each of the following, EXCEPT?

A. 8

B. 9

C. 12

D. 16

E. 17

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Example

x is an integer greater than 3. Quantity A
Quantity B

Quantity A	Quantity B
the number of even factors of $2x$	the number of odd factors of $3x$

factor是因子 = divider，1和本身加上排列组合
不是prime factor

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Example

Quantity A	Quantity B
x^2	$x(x+5)$

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Example

$0 < x < 2$

Quantity A	Quantity B
$2x + 3$	$3x + 2$

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Example

$a > 0$

Quantity A	Quantity B
$(a + a^{-1})^2$	$a^2 + a^{-2}$

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Example

$x > 1$, $y > 1$

Quantity A	Quantity B
x^3y^2	$(xy)^5$

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Example

$(x-y)^3+(x-y)=0$

Quantity A	Quantity B
x	y

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Example

$x^2 + 6x = 7$

Quantity A	Quantity B
$(x+3)^2$	16

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Solving quadratic equations解一元二次方程

形如$ax^2 + bx + c = 0$其中$a \neq 0$的方程我们成为一元二次方程

方程的解我们可以用quadratic formula一元二次方程求根公式求得：$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\Delta = b^2 - 4ac > 0$则方程有俩不同的解

$\Delta = b^2 - 4ac = 0$则方程有一个解

$\Delta = b^2 - 4ac < 0$则方程无解。

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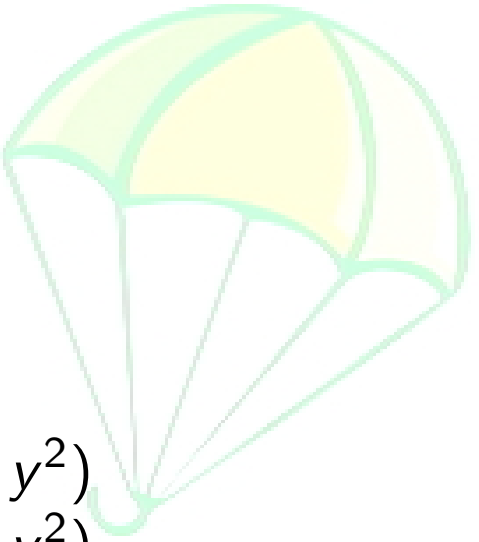
常用公式

$$(x+y)^2 = x^2 + 2xy + y^2$$
$$(x-y)^2 = x^2 - 2xy + y^2$$
$$x^2 - y^2 = (x+y)(x-y)$$
$$x^3 + y^3 = (x+y)(x^2 - xy + y^2)$$
$$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$$



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Example

Sam and Tara together spent a total of $1,400. Tara spent $400 less than twice the amount Sam spent.

Quantity A	Quantity B
The amount that Sam spent	The amount that Tara spent

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Example

If x is 4 more than half of y and if y is 10 more than half of x, what is the value of x?

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Example

A pianist agreed to perform one concert at a fee $12\frac{1}{2}$ percent less than her usual fee and a second concert at a fee 20 percent greater than the first fee. The fee for the second concert was what percent greater than her usual fee?

A. 5%
B. 7.5%
C. 15%
D. 16.25%
E. 32.5%

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Example

140.6 is approximately what percent less than 181.8?

A. 22.7%

B. 24.1%

C. 26.5%

D. 27.2%

E. 29.3%

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Example

For each of the last 5 years, the population of a colony of beetles increased by 8 percent of the preceding year' s population. If P represents the current population of the colony, which of the following best represents the population 5 years ago, in terms of P ?

A. $(5)(1.08P^{-1})$
B. $(1.08)^{-5}P^{-1}$
C. $(1.08P)^{-5}$
D. $(1.08)^{-5}P$
E. $(1.08)^{-5}P^{5}$

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Example

The population of Country X for 1980 was P. The population of Country X increased by 3.8 percent in each of the next two years.

Quantity A	Quantity B
The population of Country X for 1982	$1.076P$

simple单利
compound复利

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Example

If 32 is 40 percent of x, then x is what percent of 320?

A. 10%

B. 25%

C. 40%

D. 65%

E. 80%

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Practice

What percent of 15 is 15 percent of 1?

A. 0.001

B. 0.01

C. 0.1

D. 1

E. 10

前面写了percent

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Example

According to a tax rate formula for a certain year, the amount of tax owed by an individual whose annual income was between \$31,850 and \$77,100 was equal to a base of \$4,386 plus 24 percent of the annual income that exceeded \$31,850. According to this formula, what was the amount of tax owed by an individual whose annual income that year was \$42,000?

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percent 百分数

Example 1

What percent of 150 is 12.9?
Solution: $\frac{12.9}{150} = 0.086 = 8.6\%$

Example 2

Find 30% of 350
Solution: $30\% * 350 = 105$

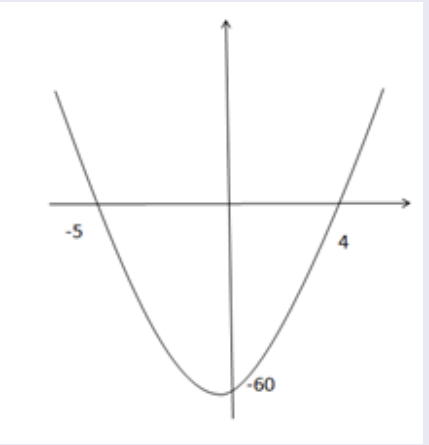
Example 3

15 is 60% of what number?
Solution: $15 \div 60\% = 25$

Amount A increased by p percent is $A(1 + p\%)$
Amount A decreased by p percent is $A(1 - p\%)$
A is what percent larger (smaller) than B:
$|A - B|/B * 100\%$, 两个数的差(大的减小的)除以than后面的数

Example

Which of the following could be the equation of the graph in the xy-plane as shown?



A. $y = x^2 + x - 60$
B. $y = x^2 + x - 20$
C. $y = x^2 + 3x - 60$
D. $y = 3x^2 + x - 60$
E. $y = 3x^2 + 3x - 60$

Example

In the xy-plane, the point $(t, t-1)$ lies on the line with equation $y = -\frac{1}{2}x + \frac{1}{3}$. What is the value of t?
Give your answer as a fraction.

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Example

In the xy-plane, the point (c, c) lies on the graph of the equation $0.3x + 0.3y = 12$

Quantity A	Quantity B
The value of c	20

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Example

In the rectangular coordinate system, (x, y) is a point on a circle that has center $(3, 2)$ and is tangent to x-axis at $(3, 0)$

Quantity A	Quantity B
The least possible value of x	0

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Example

In the rectangular coordinate system, the point $(3, 1)$ is on a circle with center $(0, -3)$. What is the area of the circle?

A. 5π

B. 7π

C. 10π

D. 25π

E. $y\sqrt{7}\pi$

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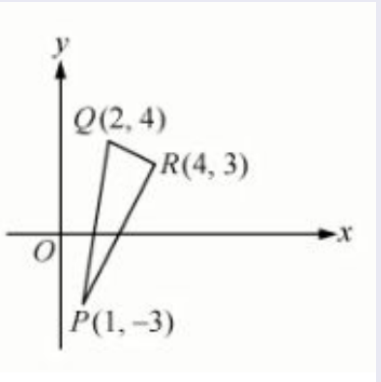
Example

Which of the following statements about triangle PQR shown in the xy-plane are true?



A. PQR is a right triangle.
B. The area of PQR is 15/2.
C. PQR is an isosceles triangle.

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graphs of functions函数图像

$y = ax + b$的图像是一条直线，斜率的a,y截距为b。

$y = ax^2 + bx + c$ 其中$a \neq 0$的图像是一个抛物线parabola
如$y = x^2 - 2x - 3$的顶点vertex是$(1, -4)$,$x = 1$是对称轴line of symmetry，开口向上opens upward

$(x - a)^2 + (y - b)^2 = r^2$的图像是一个圆circle，圆心center为$(a, b)$,radius半径为$r$

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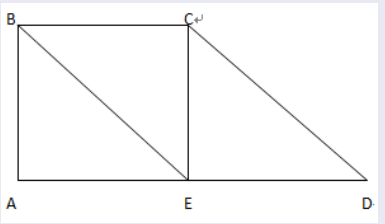
Example

ABCE is a square, and BCDE is a parallelogram.



Quantity A
The area of ABCE

Quantity B
The area of BCDE

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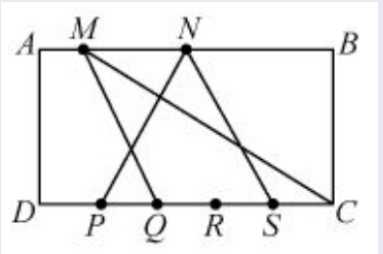
Example

In rectangle ABCD, side DC is divided into five equal segments by points P,Q,R and S.



Quantity A
The area of MCQ

Quantity B
The area of NSP

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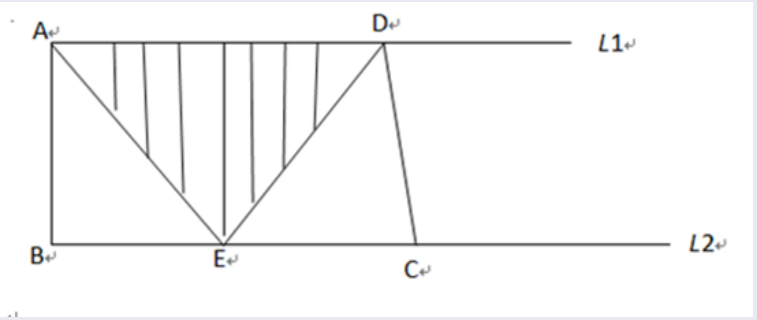
Example



Quantity A
The area of shaded region

Quantity B
The sum of the two unshaded triangular regions

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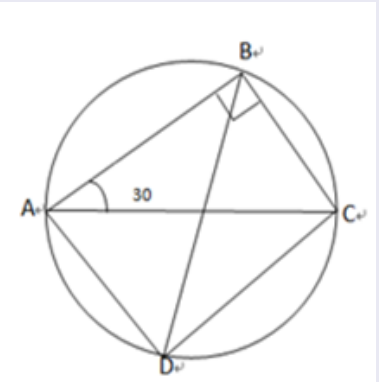
Example

In the figure, quadrilateral ABCD is inscribed in a circle with radius 10. The lengths of which of the following line segments can be determined from the information given? Indicates all such line segments.



A. AB
B. AC
C. AD
D. BC
E. BD
F. CD

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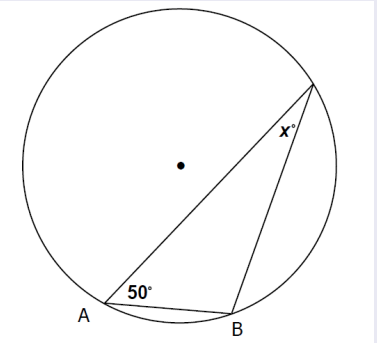
Example

If the radius of the circle above is equal to the chord AB, then what is the value of x ?



A. 25 B. 30
C. 40 D. 45
E. 50

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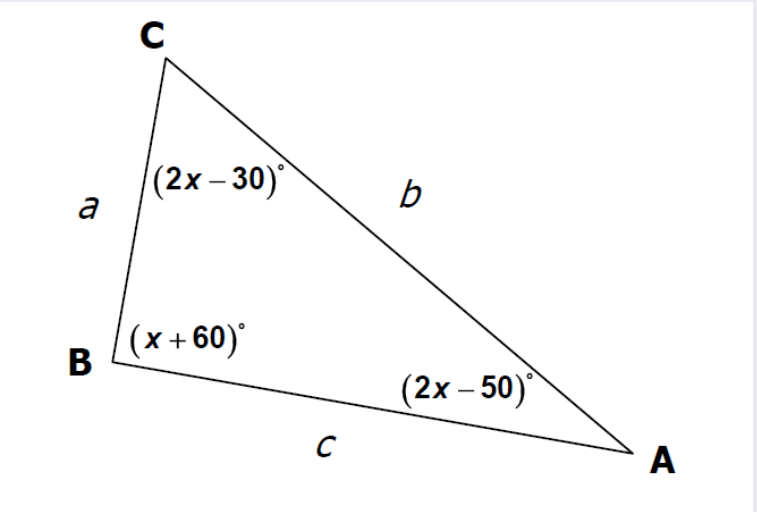
Example

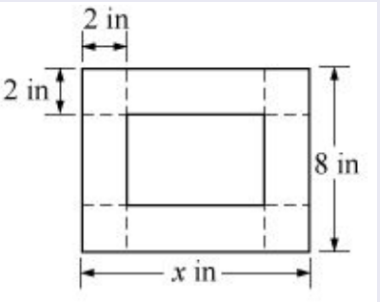


Quantity A
$a^2 + c^2$

Quantity B
b^2

Example

The thin rectangular sheet of metal shown in the figure is 8 inches wide and x inches long. An open box is to be made by cutting a 2-inch square from each corner of the sheet of metal and then folding up the sides. If the volume of the box is to be 48 cubic inches, what is the value of x ?



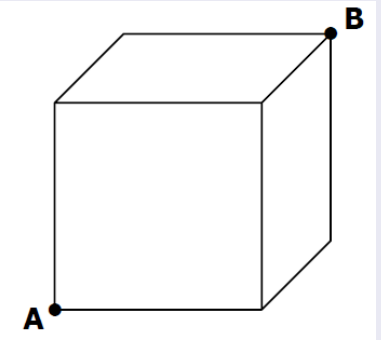
A. 6 B. 8

C. 10 D. 12

E. 14

Example

If the volume of the cube above is 64 cubic centimeters, what is the shortest distance, in centimeters, from point A to point B?



A. $4\sqrt{2}$
B. $4\sqrt{3}$
C. $4\sqrt{6}$
D. $8\sqrt{2}$
E. $8\sqrt{3}$

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Example

The average of three different positive integers is 6.

Quantity A	Quantity B
The product of the three integers	25

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Example

From a set of 100 numbers, half were selected to form group I, and 60 percent of the remaining numbers were selected to form group II. The average (arithmetic mean) of the numbers in group I is 24.4, and the average of the numbers in group II is 31.5. Which of the following is closest to the average of the numbers in groups I and II combined?

A. 27.1
B. 27.6
C. 27.8
D. 28.0
E. 28.3

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Example

List L consists of 7 numbers u, -2u, 3u, -4u, 5u, -6u, and 7u, where $u \neq 0$

Quantity A	Quantity B
The median of L	u

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Example

List L consists of 7 numbers. The range of the numbers in List L is 0.

Quantity A	Quantity B
The mean of L	0

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Example

List K consists of 16 positive numbers. List M is obtained from list K by multiplying each number in list K by -1

Quantity A	Quantity B
The standard deviation of K	The standard deviation of M

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数据集变化，统计量的随之变化

如果A,B两个数据集，其中$B = cA + d$, 对每个A的元素x进行运算$cx + d$就组成了B。如$A = \{1, 2, 3\}$, $B = \{0, -2, -4\}$, 那么$B = 2 - 2A$.
那么$mean(B) = c * mean(A) + d$, $median(B) = c * median(A) + d$,
$mode(B) = c * mode(A) + d$
standard deviation $d(B) = |c| * d(A)$, range $range(B) = |c| * range(A)$

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Example

A jar contains exactly 10 dimes and x quarters and no other coins. If a coin is randomly selected from the jar, the probability that a quarter is selected is 0.6. What is the value of x?

A. 5

B. 6

C. 8

D. 12

E. 15

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Example

There are 30 balls in an urn, each of which is either red or yellow. If one ball is to be selected at random from the urn, the probability that the ball will be red is greater than $\frac{1}{3}$. Which of the following could be the number of yellow balls in the urn? Indicates all such number.

A. 10

B. 14

C. 19

D. 22

E. 26

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Example

In how many ways can Ann, Bob, Chuck, Don and Ed be seated in a row such that Ann and Bob are not seated next to each other?

A. 24

B. 48

C. 56

D. 72

E. 96

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Example

In how many ways can 3 boys and 3 girls be seated in a row of 6 chairs such that the girls are not separated and the boys are not separated?

A. 24

B. 36

C. 72

D. 144

E. 288

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Example

If there are 16 people to choose from, what is the ratio of the number of possible 7-person committees to the number of possible 8-person committees?

A. 7:8

B. 8:7

C. 7:9

D. 8:9

E. 9:8

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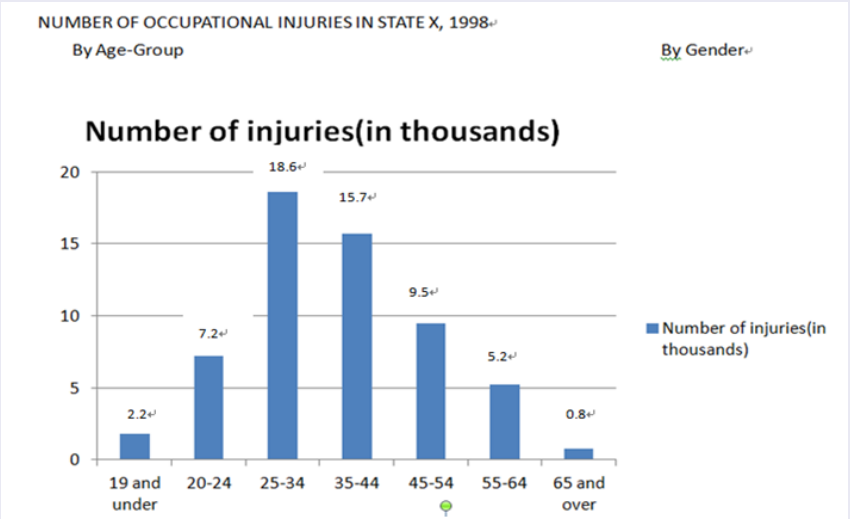
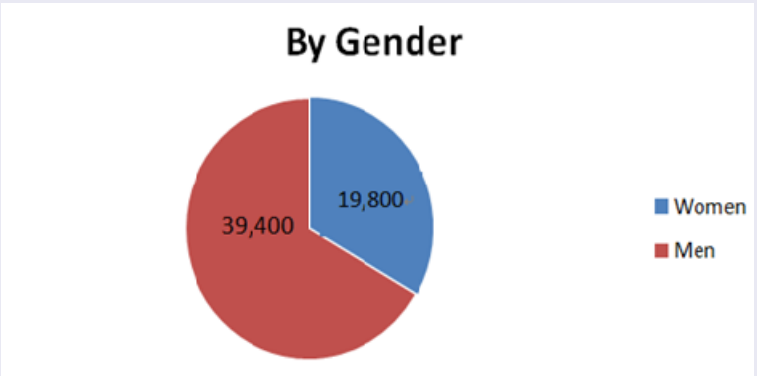
Example

How many of the age-groups each accounted for more than 15 percent of the total number of occupational injuries in State X in 1998?

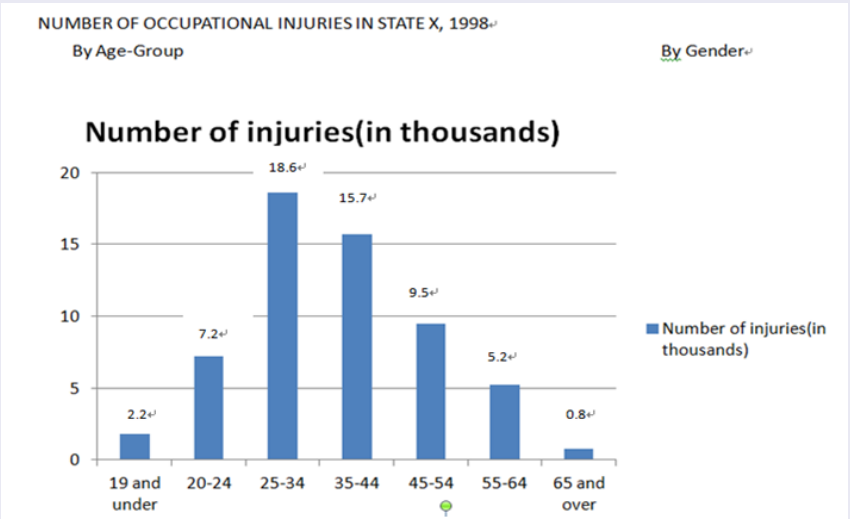


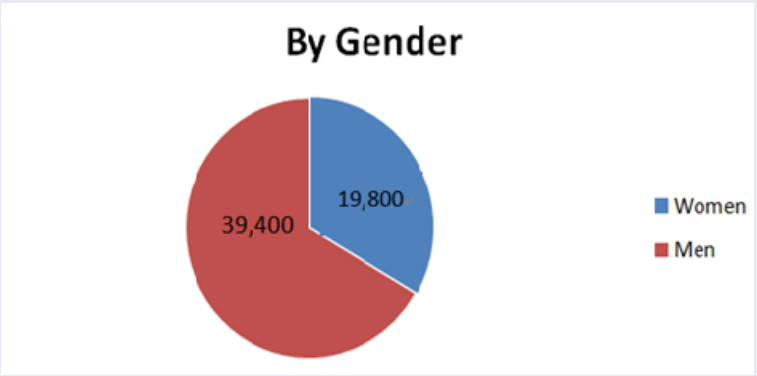


A. One B. Two C. Three D. Four E. Five

Example

In 1998, if one-half of the occupational injuries in the combined 34-and – under age-groups were incurred by men, what was the number of occupational injuries incurred by men in the combined 35-and-over age-groups?

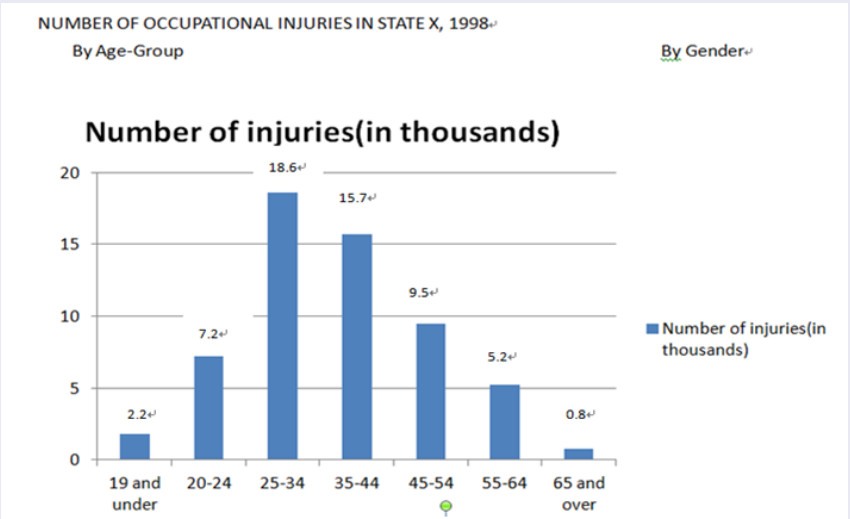


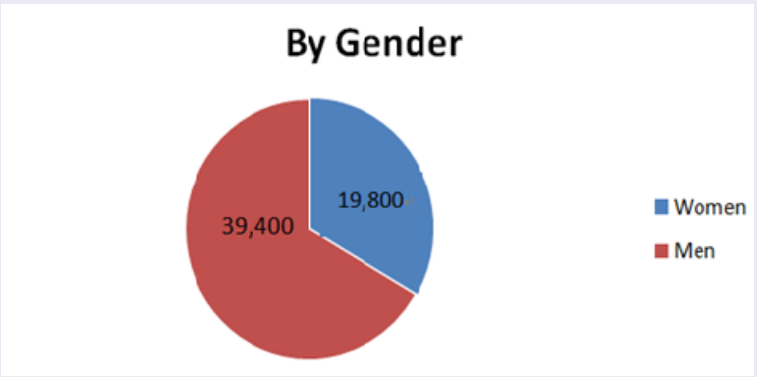


A. 33, 500 B. 31900 C. 30500 D. 25400 E. 21700

Example

For the 55-64 age-group in 1998, the average (arithmetic mean) number of work-hours lost per occupational injury was 48.5. If a workweek is 40 work-hours, which of the following is closest to the total number of workweeks lost due to occupational injuries in the 55-64 age-group in 1998?

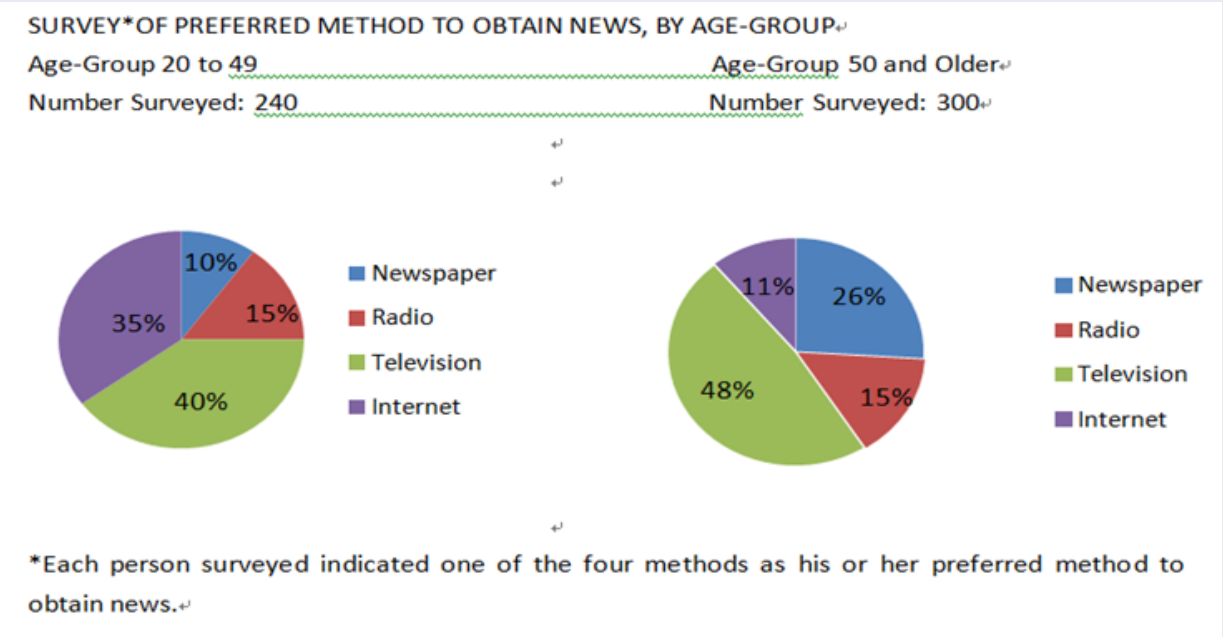




A. 4500 B. 5200 C. 5500 D. 5900 E. 6300

Example

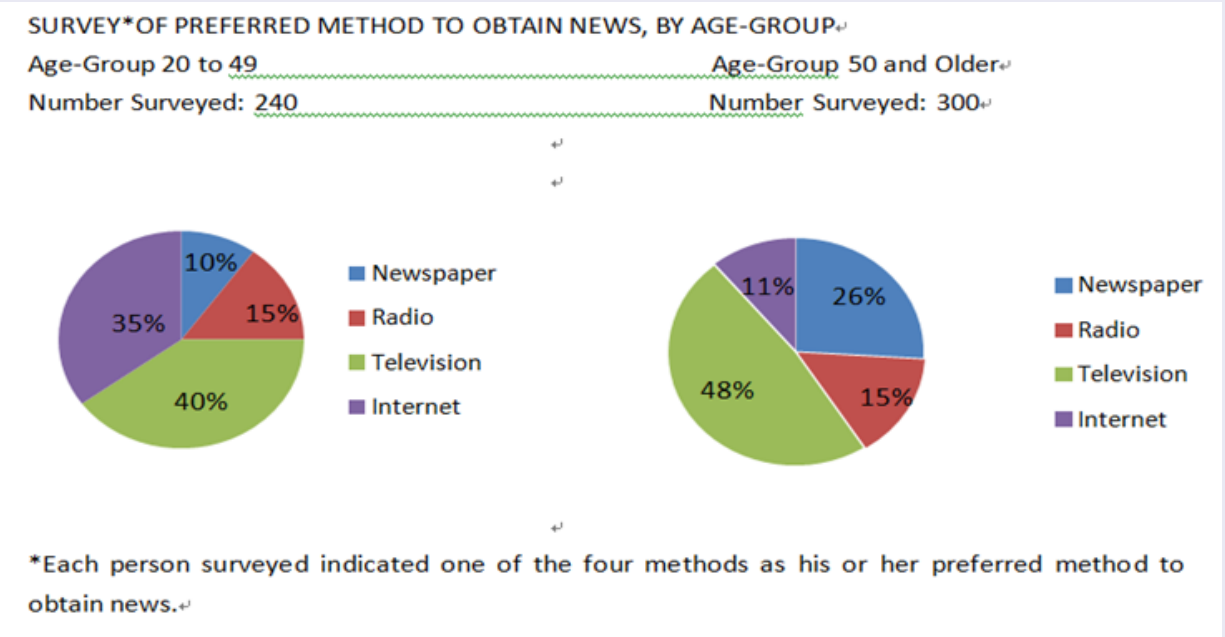
What fraction of the people in the age-group 20 to 49 indicated newspaper or the Internet as their preferred method to obtain news?



Write the answer as a quotient.

Example

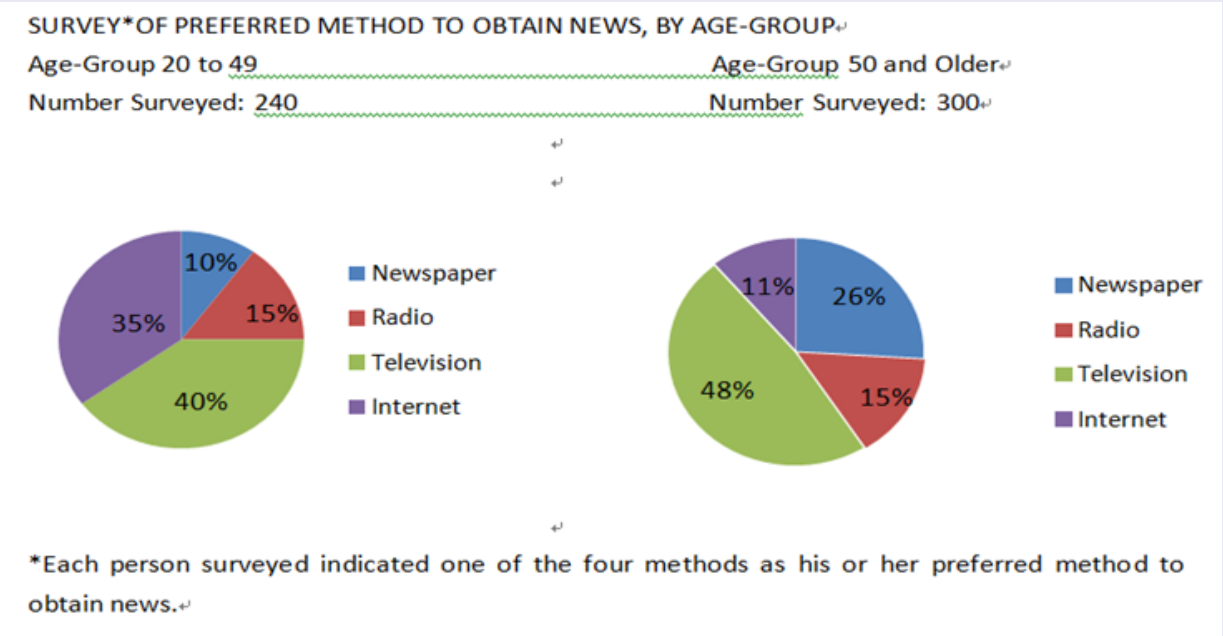
Which of the following is close to the percent of all the people surveyed who indicated the Internet as their preferred method to obtain news?



A. 18.8% B. 21.7% C. 23.0% D. 33.3% E. 46.0%

Example

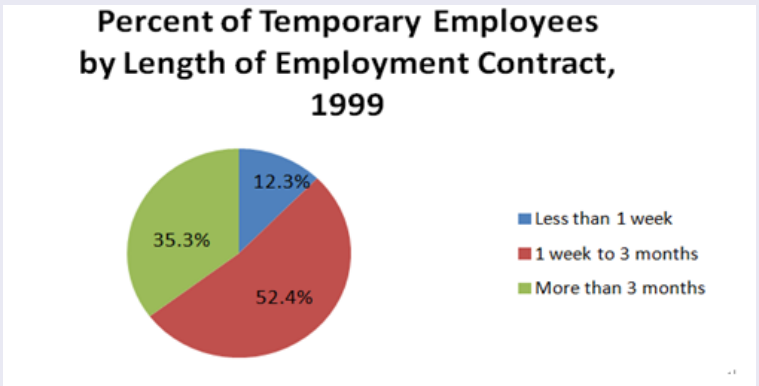
For the age-group 50 and older, the number of people who indicated the Internet as their preferred method to obtain news is approximately what percent less than the number of people who indicated radio?



A. 12% B. 27% C. 36% D. 45% E. 50%

Example

Which of the following is closest to the percent increase in the number of temporary employees from 1993 to 1999?



A. 36% B. 58% C. 136% D. 158% E. 236%

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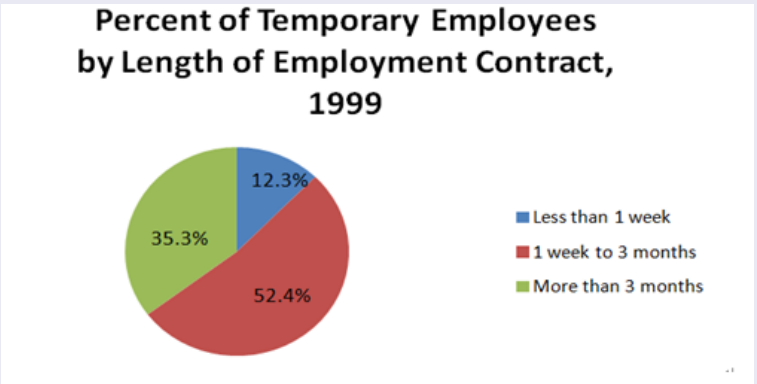
Example

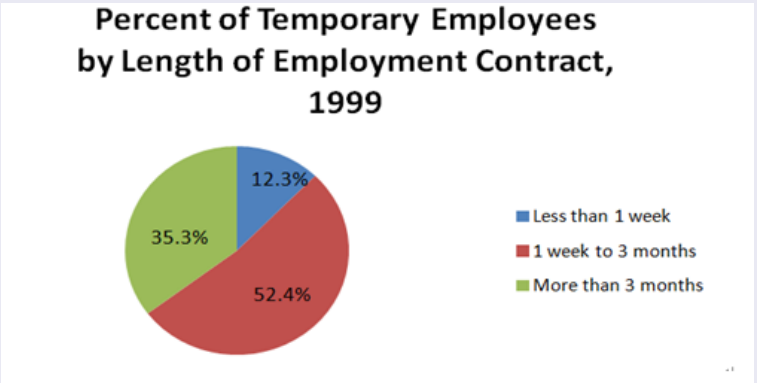
In 1999 approximately how many of the temporary employees had an employment contract with a length of at most 3 months?



A. 185,000 B. 150,000 C. 101,000 D. 35,000 E. 19,000

Example

In 1998 the ratio of the number of female temporary employees to the number of male temporary employees was 1 to x, where x ¿0. In terms of x, what was the number, in thousands, of female temporary employees in 1998?



A. $253(x-1)$ B. $253(x+1)$ C. $253/x$ D. $253/(x-1)$
E. $253/(x+1)$

Example

For how many of the four incentives listed does each of the companies surveyed that offers this incentive also offer a health-insurance benefit?

BENEFITS AND INCENTIVES SURVEY RESULTS FOR 600 COMPANIES

						Incentive	Number of Companies Offering Incentive
	481	327	274	225	198	Bonus Plan	482
	246	103	195	186	112	Profit Sharing	246
	78	60	59	55	40	Stock Ownership	78
	60	41	44	41	24	Stock Options	60
Benefit	Health Insurance	Flex-time	Disability Insurance	Tuition	Tele-commuting		
Number of Companies Offering Benefit	588	426	387	303	207		

Note: Each shaded cell is the intersection of a benefit column and an incentive row and contains the number of companies (out of the 600 surveyed) that offer both that benefit and that incentive. For example, 195 of the companies surveyed offer both a disability-insurance benefit and a profit-sharing incentive.

A. None B. One C. Two D. Three E. Four

Example

A certain benefits and incentives package consists of 2 benefits to be chosen from the benefits offered by more than 1/2 of all the companies surveyed and 1 incentive to be chosen from the incentives offered by more than 1/3 of all the companies surveyed. How many suck packages are possible?

BENEFITS AND INCENTIVES SURVEY RESULTS FOR 600 COMPANIES

Benefit	Health Insurance	Flex-time	Disability Insurance	Tuition	Tele-commuting
Number of Companies Offering Benefit	588	426	387	303	207

					Incentive	Number of Companies Offering Incentive
481	327	274	225	198	Bonus Plan	482
246	103	195	186	112	Profit Sharing	246
78	60	59	55	40	Stock Ownership	78
60	41	44	41	24	Stock Options	60

A. 4 B. 6 C. 8 D. 10 E. 12

Example

The ratio of the number of companies offering both a stock-options incentive and one of the benefits listed to the number of companies offering that benefit is greatest for which of the five benefits?

BENEFITS AND INCENTIVES SURVEY RESULTS FOR 600 COMPANIES

						Incentive	Number of Companies Offering Incentive
	481	327	274	225	198	Bonus Plan	482
	246	103	195	186	112	Profit Sharing	246
	78	60	59	55	40	Stock Ownership	78
	60	41	44	41	24	Stock Options	60
Benefit	Health Insurance	Flex-time	Disability Insurance	Tuition	Tele-commuting		
Number of Companies Offering Benefit	588	426	387	303	207		

A. Health insurance B. Flex-time C. Disability insurance D. Tuition E. Telecommuting

The End

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