Constraints:

$$X + Y + Z = 1 \tag{1}$$

$$Y = Y_{BBN} + \frac{\Delta Y}{\Delta Z} Z \tag{2}$$

$$\frac{Z}{X} = \left(\frac{Z}{X}\right)_{\odot} 10^{[Fe/H]} \tag{3}$$

Where Y_{BBN}, $\left(\frac{Z}{X}\right)_{\odot}$, $\frac{\Delta Y}{\Delta Z}$, and [Fe/H] are given as input.

Even though we don't know X or Z, we know (Z/X) from (3) and the input [Fe/H]. So we treat (Z/X) as a single variable below.

Begin with (1) and isolate Y.

$$X + Z = 1 - Y \tag{4}$$

Replace Y with (2).

$$X + Z = 1 - (Y_{BBN} + \frac{\Delta Y}{\Delta Z} Z) \tag{5}$$

Replace Z with (Z/X)×X.

$$X(1 + \frac{Z}{X}) = 1 - (Y_{BBN} + \frac{\Delta Y}{\Delta Z} \frac{Z}{X} X) \tag{6}$$

Gather terms with X on the LHS.

$$X(1 + \frac{Z}{X}(1 + \frac{\Delta Y}{\Delta Z})) = 1 - Y_{BBN} \tag{7}$$

Finally make X all alone on the RHS.

$$X = \frac{1 - Y_{BBN}}{1 + \frac{Z}{X}(1 + \frac{\Delta Y}{\Delta Z})} \tag{8}$$

$$Z = \frac{Z}{X} \times X \tag{9}$$

$$Y = 1 - X - Z \tag{10}$$

Done!