

Computational Thinking

# Discrete Mathematics

Number Theory

## Topic 03 : Collections

Logic

### Lecture 02 : Sequence Collections

Dr Kieran Murphy 

Computing and Mathematics, SETU (Waterford).  
(kieran.murphy@setu.ie)

Graphs and  
Networks

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Collections

#### Outline

- Mathematical concept of a sequence, AP and GP
- Sequence collections
- Lists, tuples, and strings

Enumeration

Relations & Functions

# Outline

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# Sequence

Informally, a **sequence** is just an ordered list of numbers. Since the order is important we can label the values in the list, starting with zero, then one and so on. This gives us the formal definition of a sequence

## Definition 1 (Sequence)

A **sequence** is a function from the set of natural numbers,  $\mathbb{N} = \{0, 1, 2, 3, 4, \dots\}$  to a some set  $A$ . So we have

$$\begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 & & n \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \dots & \downarrow \\ a_0 & a_1 & a_2 & a_3 & a_4 & & a_n \end{array}$$

and

- $a_n$  is the image of  $n$ , and is called the  **$n^{\text{th}}$  term/element** of the sequence.
- To refer to the *entire* sequence at once, we will write  $(a_n)_{n \in \mathbb{N}}$  or  $(a_n)_{n \geq 0}$ , or if we are being sloppy, just  $(a_n)$  (in which case we assume we start the sequence with  $a_0$ ).
- The numbers in the subscripts are called **indices** (the plural of **index**).

## Examples of Sequences

- The sequence  $a_n = n^2$ , where  $n = 1, 2, 3, \dots$  has elements

$1, 4, 9, 16, 25, 36, 49, \dots$

- The sequence  $a_n = (-1)^n$ , where  $n = 0, 1, 2, \dots$  has elements

$1, -1, 1, -1, 1, -1, \dots$

- The sequence  $a_n = 2^n$ , where  $n = 0, 1, 2, \dots$  has elements

$1, 2, 4, 8, 16, 32, 64, 128, \dots$

- The **Fibonacci sequence** has elements

$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, \dots$

## A Quick Look at Fibonacci Sequence

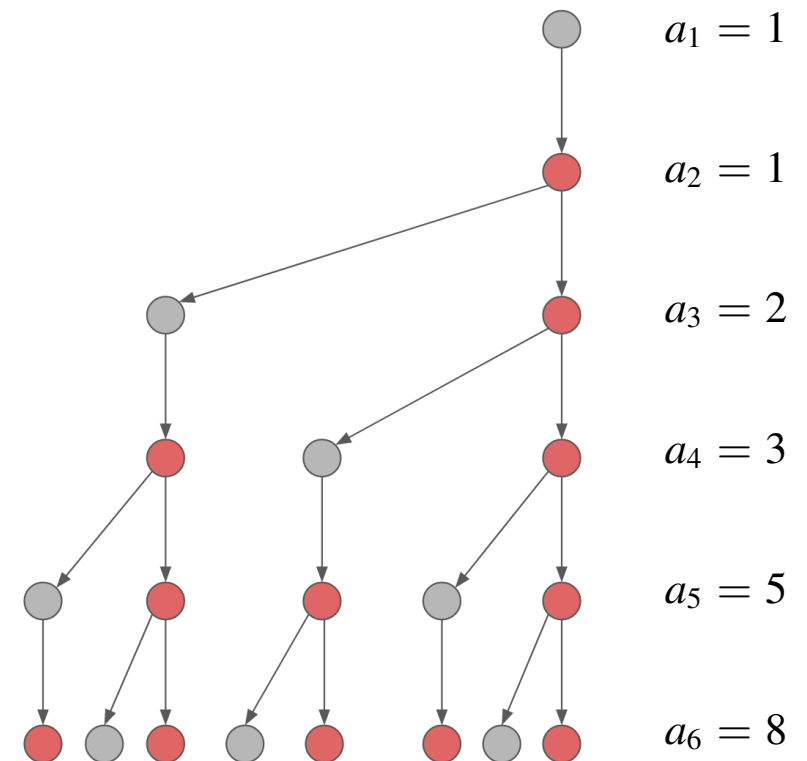
During 13<sup>th</sup> century, in Liber Abaci, Fibonacci\* poses the following question (paraphrasing):

*Suppose we have two newly-born rabbits, one female and one male. Suppose these rabbits produce another pair of female and male rabbits after one month. These newly-born rabbits will, in turn, also mate after one month, producing another pair, and so on. Rabbits never die. How many pairs of rabbits exist after one year?*

The figure to the right illustrates this process.

- Every point denotes one rabbit pair.
- A grey point denotes a newborn pair (and not ready to reproduce).
- A red point denotes a mature, reproducing pair.

*Fibonacci's Rabbits*



\*discovered earlier by Indian scholars (Gopāla, before 1135), studying rhythmic patterns

## Closed vs Recursive Formula for Sequences

We often need to specify a rule for the general term in the sequence — we have two options:

### Definition 2 (Closed Formula and Recursive Definition)

- A **closed formula** for a sequence  $a_n$  is a formula for  $a_n$  using a fixed, finite number of operations on  $n$ .
- A **recursive definition** for a sequence  $(a_n)$  consists of a **recurrence relation**: an equation relating the current term in the sequence,  $(a_n)$ , to earlier terms in the sequence,  $(a_{n-1})$ ,  $(a_{n-2})$ , ... (i.e., terms with smaller index) and **initial/terminal condition(s)**.

### Example

The Fibonacci sequence  $(a_n) = (0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \dots)$  has closed formula

$$a_n = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n}{\sqrt{5}}$$

Hard to obtain, easy to use

and recursive formula

$$\underbrace{a_n = a_{n-1} + a_{n-2}}_{\text{recurrence relation}}$$

and

$$\underbrace{a_0 = 0, \quad a_1 = 1}_{\text{terminal conditions}}$$

Easy to obtain,  
hard to use

# Computing Fibonacci Sequence using Closed Formula

Compute the first 7 terms of the Fibonacci sequence using the closed formula

$$a_n = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n}{\sqrt{5}}$$

```
1 import math
2
3 for n in range(7):
4     tmp_1 = (1 + math.sqrt(5)) / 2
5     tmp_2 = (1 - math.sqrt(5)) / 2
6
7     a_n = (tmp_1**n - tmp_2**n) / math.sqrt(5)
8
9     print(n, round(a_n))
10
```

0	0
1	1
2	1
3	2
4	3
5	5
6	8



# Computing Fibonacci Sequence using Recursive Formula

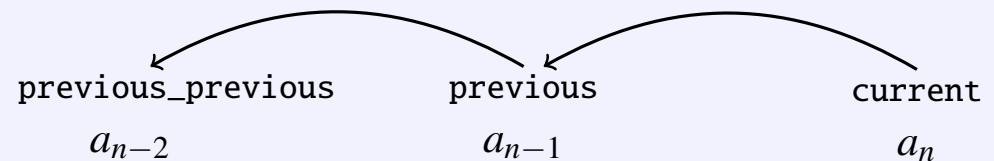
Compute the first 7 terms of the Fibonacci sequence using the recursive formula

```

1 previous_previous = 0
2 previous = 1
3
4 for n in range(7):
5
6     if n == 0:           # terminal condition n=0
7         current = 0
8     elif n == 1:        # terminal condition n=1
9         current = 1
10    else:                # recursive formula n>1
11        current = previous + previous_previous
12
13    # leapfrog values
14    previous_previous = previous
15    previous = current
16
17    print(n, current)

```

0	0
1	1
2	1
3	2
4	3
5	5
6	8





## Example

### Example 3

Find  $a_6$  in the sequence defined by  $a_n = 2a_{n-1} - a_{n-2}$  with  $a_0 = 3$  and  $a_1 = 4$ .

**Solution.** Using  $n = 6$ , we know that  $a_6 = 2a_5 - a_4$ . So to find  $a_6$  we need to find  $a_5$  and  $a_4$ . And we repeat this process down to  $a_0$  and  $a_1$ . We will use the approach when we define functions.

But for now, we will determine  $a_6$  by starting at  $a_0$  and  $a_1$ , and working upwards towards  $a_6$ .

$a_0 = 3$	(given terminal condition)
$a_1 = 4$	(given terminal condition)
$a_2 = 2 \cdot 4 - 3 = 5$	(use $n = 2$ in recursive formula)
$a_3 = 2 \cdot 5 - 4 = 6$	(use $n = 3$ in recursive formula)
$a_4 = 2 \cdot 6 - 5 = 7$	(use $n = 4$ in recursive formula)
$a_5 = 2 \cdot 7 - 6 = 8$	(use $n = 5$ in recursive formula)
$a_6 = 2 \cdot 8 - 7 = 9$	(use $n = 6$ in recursive formula)

Note that in this case a closed formula for  $a_n$  exists. Namely,  $a_n = n + 3$ . A closed formula is easier to use to calculate a general term, but it is often much harder, if not impossible, to derive.

# Computing Sequence using Closed Formula

First 7 terms of the sequence using the closed formula

$$a_n = n + 3$$

```
3 ●  
1 for n in range(7):  
2       
3     a_n = n + 3  
4       
5     print(n, a_n)
```

```
0 3  
1 4  
2 5  
3 6  
4 7  
5 8  
6 9
```



# Computing Sequence using Recursive Formula

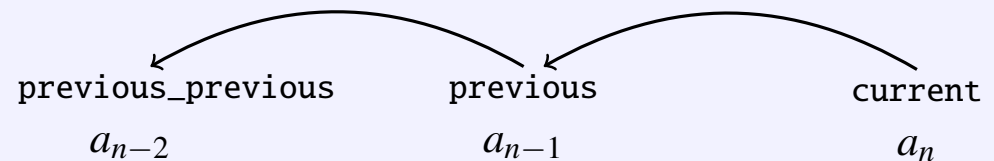
Compute the first 7 terms of the Fibonacci sequence using the recursive formula

```

1 previous_previous = 3
2 previous = 4
3
4 for n in range(7):
5     #
6     if n == 0:           # terminal condition n=0
7         current = 3
8     elif n == 1:        # terminal condition n=1
9         current = 4
10    else:                 # recursive formula n>1
11        current = 2 * previous - previous_previous
12    #
13    # leapfrog values
14    previous_previous = previous
15    previous = current
16
17 print(n, current)

```

0	3
1	4
2	5
3	6
4	7
5	8
6	9



# Summation Notation

- The  $\sum$  operator is used to denote the addition of elements from a sequence/list.
- It can be implemented using a **for** loop in Python/Java/Processing.

## Example 4

$$\underbrace{\sum_{k=1}^{10} \left[ k^2 \right]}_{\text{Determine the value of expression within the brackets as } k = 1, 2, 3, \dots, 10 \text{ and add all the results.}} = \underbrace{1^2}_{k=1} + \underbrace{2^2}_{k=2} + \underbrace{3^2}_{k=3} + \underbrace{4^2}_{k=4} + \dots + \underbrace{10^2}_{k=10}$$

“Determine the value of expression within the brackets as  $k = 1, 2, 3, \dots, 10$  and add all the results.”

$$= 1 + 4 + 9 + 16 + 25 + 36 + \dots + 100 = 385$$

```

5 1 result = 0                # start result with zero - why?
2  for k in range(1,11):
3      term = k*k
4      result += term        # shorthand for result = result + term
5
6  print(result)

```

385



## Product Notation

- The  $\prod$  operator is used to denote the product of elements from a sequence/list.
- It can be implemented using a **for** loop in Python/Java/Processing.

### Example 5

$$\underbrace{\prod_{k=1}^{10} [k^2]} = \underbrace{1^2}_{k=1} \times \underbrace{2^2}_{k=2} \times \underbrace{3^2}_{k=3} \times \underbrace{4^2}_{k=4} \times \cdots \times \underbrace{10^2}_{k=10}$$

“Determine the value of expression within the brackets as  $k = 1, 2, 3, \dots, 10$  and multiply all the results.”

$$= 1 \times 4 \times 9 \times 16 \times 25 \times 36 \times \cdots \times 100 = 13,168,189,440,000$$

```

6 result = 1                # start result with one - why?
1 for k in range(1,11):
2     term = k*k
3     result *= term        # shorthand for result = result * term
4
5
6 print(result)

```

13168189440000



# Review Exercises 1 (Sequences)

## Sequences

### Question 1:

Expand the following sums

(a)  $\sum_{k=4}^7 k$

(b)  $\sum_{k=1}^5 (k^1 - 1)$

(c)  $\sum_{n=1}^4 (10^n)$

(d)  $\sum_{k=1}^5 (k^1 - 1)$

### Question 2:

Write the following expressions using summation notation

(a)  $2 + 4 + 6 + 8 + 10$

(b)  $1 + 4 + 7 + 10$

(c)  $\frac{1}{4} + \frac{1}{2} + 1 + 2 + 4$

### Question 3:

Expand the following sums

(a)  $\prod_{k=-4}^4 k$

(b)  $\prod_{k=1}^4 (k^1 - 1)$

(c)  $\prod_{k \in S} (-1)^k$  where  $S = \{2, 4, 6, 7\}$ .

### Question 4:

For each of the following sequences, determine a recursive definition.

(a)  $2, 4, 6, 10, 16, 26, 42, \dots$

# Outline

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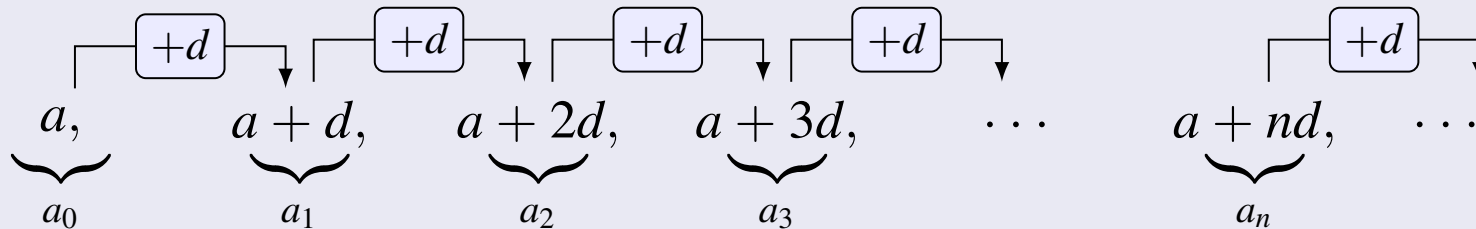
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# Arithmetic Progression/Sequence

## Definition 6 (Arithmetic Progression/Sequence (AP))

A sequence is called **arithmetic** if the terms of the sequence differ by a constant.

Suppose the initial term ( $a_0$ ) of the sequence is  $a$  and the **common difference** is  $d$ , then we have sequence



Recursive definition:  $a_n = a_{n-1} + d$  with  $a_0 = a$ .

Closed formula:  $a_n = a + dn$ .

## Example 7

Find recursive definitions and closed formulas for the sequences below. Assume the first term listed is  $a_0$ .

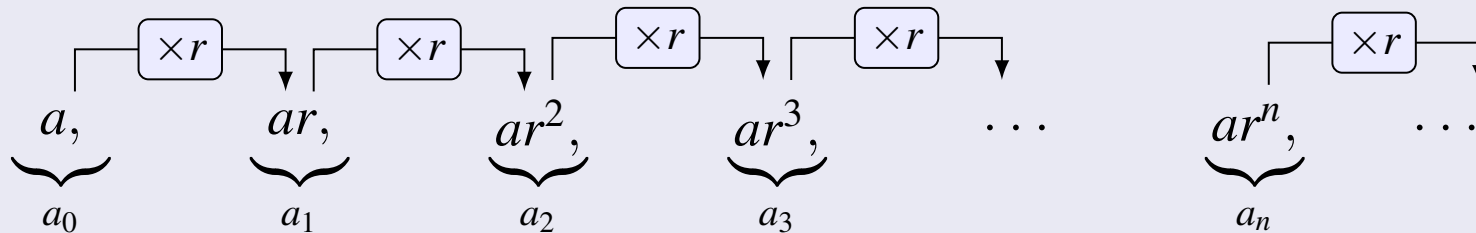
- 2, 5, 8, 11, 14, ...
- 50, 43, 36, 29, ...



# Geometric Progression/Sequence

## Definition 8 (Geometric Progression/Sequence (GP))

A sequence is called **geometric** if the ratio between successive terms is constant. Suppose the initial term  $a_0$  is  $a$  and the **common ratio** is  $r$ . Then we have, sequence



Recursive definition:  $a_n = ra_{n-1}$  with  $a_0 = a$ .

Closed formula:  $a_n = ar^n$ .

## Example 9

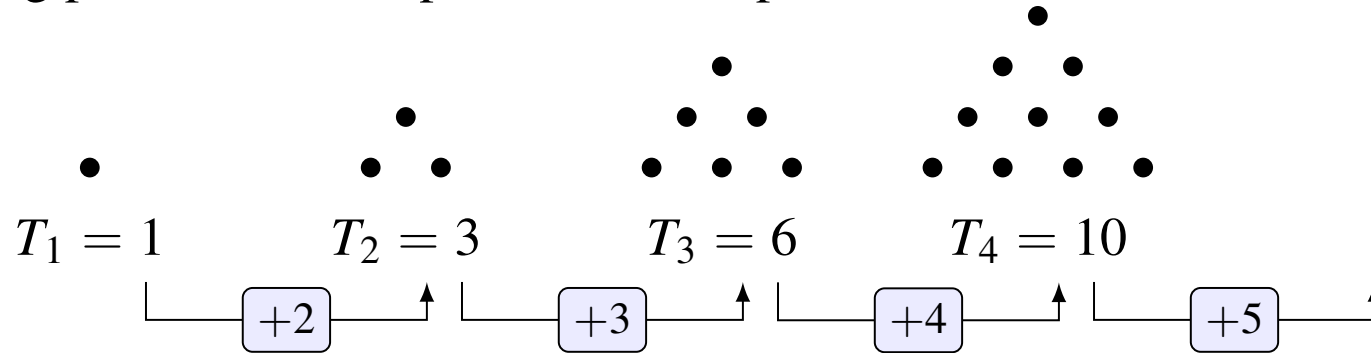
Find the recursive and closed formula for the sequences below. Again, the first term listed is  $a_0$ .

•  $3, 6, 12, 24, 48, \dots$

•  $27, 9, 3, 1, 1/3, \dots$

# Motivation

Look at the sequence  $(T_n)_{n \geq 1}$  which starts 1, 3, 6, 10, 15, ... These are called the **triangular numbers** since they represent the number of dots in an equilateral triangle (think of how you arrange 10 bowling pins: a row of 4 plus a row of 3 plus a row of 2 and a row of 1).



- Is this sequence arithmetic?  
No, since  $3 - 1 = 2$  and  $6 - 3 = 3 \neq 2$ , so there is no common difference.
- Is the sequence geometric?  
No.  $3/1 = 3$  but  $6/3 = 2$ , so there is no common ratio.
- Notice that the differences between terms generate an arithmetic sequence: 2, 3, 4, 5, 6, ...  
This says that the  $n$ th term of the triangular sequence is the sum of the first  $n$  terms in the sequence 1, 2, 3, 4, 5, ..., i.e, the triangular sequence is a **sequence of partial sums**.

# Summing Arithmetic Sequences: Reverse and Add

I

## Example 10

Find the sum:  $2 + 5 + 8 + 11 + 14 + \dots + 470$ .

**Solution.** If we add the first and last terms, we get 472. The second term and second-to-last term also add up to 472. To keep track of everything, we might express this as follows. Call the sum  $S$ . Then,

$$\begin{array}{rcccccccc} S = & 2 & + & 5 & + & 8 & + \dots + & 467 & + & 470 \\ + S = & 470 & + & 467 & + & 464 & + \dots + & 5 & + & 2 \\ \hline 2S = & 472 & + & 472 & + & 472 & + \dots + & 472 & + & 472 \end{array}$$

Hence, to find  $2S$  then we add 472 to itself a number of times. What number?

We need to decide how many terms are in the sum. Since the terms form an arithmetic sequence, the  $n$ th term in the sum (counting 2 as the 0th term) can be expressed as  $2 + 3n$ . If  $2 + 3n = 470$  then  $n = 156$ . So  $n$  ranges from 0 to 156, giving 157 terms in the sum. This is the number of 472's in the sum for  $2S$ . Thus

$$2S = 157 \times 472 = 74104 \quad \implies \quad S = \frac{74104}{2} = 37052$$

## Summing Arithmetic Sequences: Reverse and Add

## II

The process covered in the previous slide will work for any sum of arithmetic sequences.

- STEP 1 Call the sum  $S$ .
- STEP 2 Reverse and add.
- STEP 3 This produces a single number added to itself many times.
- STEP 4 Determine the number of times.
- STEP 5 Multiply. Divide by 2. Done

### Definition 11 (Arithmetic Series)

The sum of the terms of the arithmetic sequence

$$S_n = [a] + [a + d] + [a + 2d] + \cdots + [a + nd]$$

is called an **arithmetic series** and is given by

$$S_n = (n + 1)a + \frac{dn(n + 1)}{2}$$

# Summing Geometric Sequences: Multiply and Subtract

I

To find the sum of a geometric sequence, we cannot just reverse and add. Instead we multiply and subtract:

## Example 12

What is  $3 + 6 + 12 + 24 + \cdots + 12288$ ?

This terms in the sum are from a geometric progression with initial term,  $a_0 = 3$ , and common ratio,  $r = 2$ .

**STEP 1** Call the sum  $S$ .

**STEP 2** Multiply each term by the common ratio,  $r = 2$

**STEP 3** Subtract, and solve for  $S$ .

$$\begin{array}{rcl}
 S & = & 3 + 6 + 12 + 24 + \cdots + 12288 \\
 2S & = & \phantom{3 + } 6 + 12 + 24 + \cdots + 12288 \quad +24576 \\
 \hline
 -S & = & 3 + 0 + 0 + 0 + \cdots + 0 \quad -24576 \\
 -S & = & 3 - 24576 \quad \implies \quad S = 24573
 \end{array}$$

# Summing Geometric Sequences: Multiply and Subtract

## II

## Definition 13 (Geometric Series)

The sum of the terms of the geometric sequence

$$S_n = [a] + [ar] + [ar^2] + \cdots + [ar^n]$$

is called a **geometric series** and is given by

$$S_n = \frac{a(1 - r^{n+1})}{1 - r}$$

- In the special case of  $-1 < r < 1$  the terms in the geometric sequence tends towards zero fast enough that the sum of the series tends to the finite value

$$S_\infty = \lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} \frac{a}{1 - r}$$

since  $r^{n+1} \rightarrow 0$  as  $n \rightarrow \infty$ .

## Review Exercises 2 (Arithmetic and Geometric Progressions)

### Question 1:

Consider the sequence  $5, 9, 13, 17, 21, \dots$  with  $a_1 = 5$

- (a) Give a recursive definition for the sequence.
- (b) Give a closed formula for the  $n$ th term of the sequence.
- (c) Is 2013 a term in the sequence? Explain.
- (d) How many terms does the sequence  $5, 9, 13, 17, 21, \dots, 533$  have?
- (e) Determine the sum:  $5 + 9 + 13 + 17 + 21 + \dots + 533$ . Show your work.
- (f) Use what you found above to find  $b_n$ , the  $n^{\text{th}}$  term of  $1, 6, 15, 28, 45, \dots$ , where  $b_0 = 1$

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# Math vs. Programming (Python/Processing/Java/...)

## Computers are finite

In mathematics we can define a sequence, just like

$$a_n = 2^n, \text{ for } n \geq 0 \qquad 0, 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, \dots$$

and have no concerns that it has infinite length or that the values become arbitrary large. This is not the case when programming — we (or the language designers) need to deal with both of these issues.

Infinite length sequences in Mathematics → (usually) Finite length sequences in Python

## Programmers need standard tasks/operations

- **Indexing** — Each position in a sequence is given a unique position/index, so we can access/change a single element by referring to its index.
- **Slicing** — Given a sequence collection we want to create a copy of part of that sequence.
- **Iterating over** — Looping over all elements (**for** loops and **list comprehensions**).
- **Filtering** — Given a collection construct a new collection containing only elements that satisfy a condition.

# Python Impleme

```
set() []  
{3} [3]  
{3} [3, 3]  
{'Hello', 3} [3, 3, 'Hello']  
{'Hello', 3, 'All'} [3, 3, 'Hello', 'All']
```

```
1 S = set() # cannot  
2 L = [] # here we can use list() or []  
3 print(S, L)  
4  
5 S.add(3) # we ADD to a set  
6 L.append(3) # but we APPEND to END of list  
7 print(S, L)  
8  
9 S.add(3) # elements are distinct  
10 L.append(3)  
11 print(S, L)  
12  
13 S.add("Hello") # can store mixture of data types  
14 L.append("Hello") # can store mixture of data types  
15 print(S, L)  
16  
17 S.add("All") # unordered  
18 L.append("All") # ordered  
19 print(S, L)
```

```
set() []
```

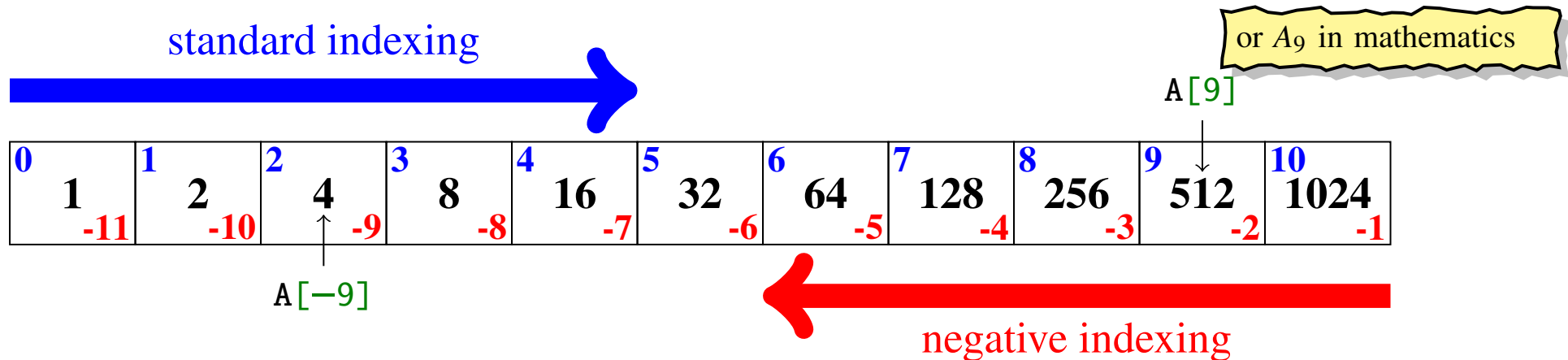


# Indexing

To help illustrate indexing we will define a list containing the powers of 2 up to and including  $2^{10}$ .

```
1 A = [1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024]
```

- The collection (a **list**) is **ordered** — so we can talk about which data value (item/element) comes before/after another data value.
- In addition, each data value has a position, called **index**, which counts from the left of the list. Python is zero-based language so index starts at zero.
- Python, also support **negative indexing** which counts backwards from the end of the list.



# Slicing

## Definition 14 (Slicing)

**Slicing** is a compact syntax to construct a sub-sequence collection from a larger collection. A slice consists of

`[start:end:step]`

where

- `start` — the starting index (inclusive). Defaults to 0 (i.e., start of the collection) if omitted.
- `end` — the ending index (exclusive). Defaults to length of collection if omitted.
- `step` — the amount by which the index increases, defaults to 1. If it's negative, you're slicing over the collection in reverse.

### Some common slices:

Given collection, `A`, then

- `A[:]` creates a copy of the entire collection. (uses default value for start, end, and step)
- `A[::-1]` creates a copy of the entire collection in reverse (step=-1 reverses the collection)

## Slicing

## II

A	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr><tr><td></td><td>1</td><td>2</td><td>4</td><td>8</td><td>16</td><td>32</td><td>64</td><td>128</td><td>256</td><td>512</td><td>1024</td></tr><tr><td></td><td>-11</td><td>-10</td><td>-9</td><td>-8</td><td>-7</td><td>-6</td><td>-5</td><td>-4</td><td>-3</td><td>-2</td><td>-1</td></tr></table>	0	1	2	3	4	5	6	7	8	9	10		1	2	4	8	16	32	64	128	256	512	1024		-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	A[9]
0	1	2	3	4	5	6	7	8	9	10																											
	1	2	4	8	16	32	64	128	256	512	1024																										
	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1																										
A[:5]	<table><tr><td>1</td><td>2</td><td>4</td><td>8</td><td>16</td></tr></table>	1	2	4	8	16	Create a subsequence <i>from start of sequence</i> , from index (inclusive) <b>start</b> (default=0) up to index (but excluding) <b>end=5</b> .																														
1	2	4	8	16																																	
A[5:8]	<table><tr><td>32</td><td>64</td><td>128</td></tr></table>	32	64	128	Create a subsequence <i>from middle of sequence</i> , from index (inclusive) <b>start=5</b> up to index (but excluding) <b>end=8</b> .																																
32	64	128																																			
A[8:]	<table><tr><td>256</td><td>512</td><td>1024</td></tr></table>	256	512	1024	Create a subsequence <i>from end of sequence</i> , from index (inclusive) <b>start=8</b> up to index (but excluding) <b>end</b> (default length of sequence=11).																																
256	512	1024																																			
A[0:9:3]	<table><tr><td>1</td><td>8</td><td>64</td></tr></table>	1	8	64	Create a subsequence from index (inclusive) <b>start=0</b> up to index (but excluding) <b>end=9</b> using a <b>step=3</b> .																																
1	8	64																																			

# Iterating over Collections

- Python's **for** is used to iterate over elements in a collections.
- Function **enumerate** counts the elements during iteration.

```

1 A = [1,2,4,8,16,32,64,128,256,512,1024]
2
3 # loop over all elements
4 for value in A:
5     print(value)
6
7 # count and looping over all elements
8 for pos, value in enumerate(A):
9     print(pos, value)
10
11 # loop over all positions - rarely used in py
12 for pos in range(len(A)):
13     print(pos, A[pos])

```

1  
2  
4  
8  
16  
32  
64  
128  
256  
512  
1024

0 1  
1 2  
2 4  
3 8  
4 16  
5 32  
6 64  
7 128  
8 256  
9 512  
10 1024



# Filtering

## Definition 15 (Filtering)

Build a collection from another by selecting (**filtering**) elements in the collection that satisfy some criteria.

Task — Given a list of powers of 2, select all values that have remainder 4 when divided by 10:

```
10 ●
1 A = [1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024]
```

```
2
3 # old style filtering
```

```
4 B = []
```

```
5 for value in A:
```

```
6     if value % 10 == 4: # remainder is 4
```

```
7         B.append(value)
```

```
8 print(B)
```

```
9
```

```
10 # or using list comprehension
```

```
11 B = [value for value in A if value % 10 == 4]
```

```
12 print(B)
```

Create empty list. Loop over original.  
If element satisfies criteria, then append it to list.

List comprehension

[4, 64, 1024]

[4, 64, 1024]



# List comprehension

## Definition 16 (List comprehension)

**List comprehension** is a compact syntax to construct a new sequence from another collection. It consists of

`[EXPRESSION for value in COLLECTION if CONDITION]`

where

- `EXPRESSION` is any python expression.
  - `COLLECTION` is any python collection (set, list, ...)
  - `CONDITION` — is python expression that results in `True` or `False`
- 
- As a programmer you don't have to use list comprehensions and instead use the longer traditional style, but you will need to be able to read and understand it since it is the default style in modern Python programmers.
  - Replacing `[` and `]` by `{` and `}` will create a `set` instead of a new `list`.



# List Comprehension Example 1

Task — Create list of first 10 square numbers from the set of natural numbers ( $\mathbb{N}$ ).

```
11 •
1  # traditional approach
2  squares = []
3  for k in range(10):
4      squares.append(k**2)
5  print(squares)
6
7  # using list comprehension
8  squares = [k**2 for k in range(10)]
9  print(squares)
```

Create empty list. Loop over original collection, calculate expression ( $k**2$ ) and append result to list.

```
[0, 1, 4, 9, 16, 25, 36, 49, 64, 81]
[0, 1, 4, 9, 16, 25, 36, 49, 64, 81]
```



- The COLLECTION is `range(10)` which generates the list `[0, 1, 2, 3, 4, 5, 6, 7, 8, 9]`.
- The EXPRESSION is `k**2` which generates the required pattern.
- There is no CONDITION so all elements in COLLECTION are used.

## List Comprehension Example 2

Task — Create list of even integers up to but not including 10.

```
12 ●
1  # traditional approach
2  evens = []
3  for k in range(10):
4      if k % 2 == 0:
5          evens.append(k)
6  print(evens)
7
8  # using list comprehension
9  evens = [k for k in range(10) if k % 2 == 0]
10 print(evens)
```

Create empty list. Loop over original collection, if value matches criteria (even), then append value to list.



[0, 2, 4, 6, 8]

[0, 2, 4, 6, 8]

- The COLLECTION is `range(10)` which generates the list `[0, 1, 2, 3, 4, 5, 6, 7, 8, 9]`.
- The EXPRESSION is `k` which generates the required pattern.
- The CONDITION, `k%2==0` selects the even integers only.

## List Comprehension Example 3

Task — Create list of the length of each word in a list of words.

```
13 ●
1  names = ['Alice', 'Bob', 'Charlie']
2
3  # traditional approach
4  lengths = []
5  for name in names:
6      lengths.append(len(name))
7  print(lengths)
8
9  # using list comprehension
10 lengths = [len(name) for name in names]
11 print(lengths)
```

Create empty list. Loop over original collection, calculate length of word, then append result to list.

[5, 3, 7]  
[5, 3, 7]



- The COLLECTION is names, a list of strings.
- The EXPRESSION is `len(name)` which computes the length of the string stored in name.
- There is no CONDITION so all elements in COLLECTION are used.

## Aside - Tuples

### Definition 17 (Tuple)

A **tuple** is ordered, immutable collection.

- A **immutable** collection is unchangeable, meaning that we cannot change, add or remove items after the collection has been created.
- Tuple are denoted by round brackets, ( and ).
- Unfortunately, round brackets are also used in controlling the order of operations in expressions. So a tuple with just one element requires a comma.

```
14 ●  
1 fruits = ("apple", "banana", "cherry")  
2 print(fruits)  
3  
4 fruits = ("apple",)  
5 print(fruits)
```

```
('apple', 'banana', 'cherry')  
( 'apple', )
```