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Event Rates of EMRIs and IMRIs in Milky Way Galaxy

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Event Rates of EMRIs and IMRIs in Milky Way Galaxy

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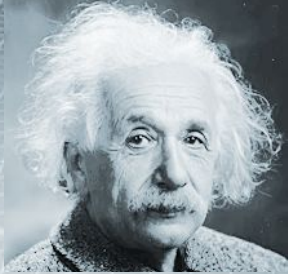
Gravitational Wave - Preliminaries



Wave Generation and Propagation:

- Gravitational waves are the ripples in the fabric of the cosmos
  - Produced by rapidly rotating massive astronomical objects
  - Propagating with speed of light out to infinity from the source
  - The form of periodic perturbations in the space–time

- The effect of gravitational radiation, is measured by strain amplitude h given by $h = \frac{\Delta L}{L}$

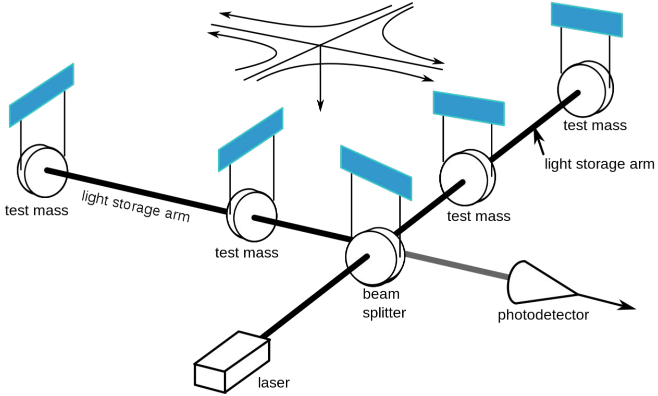
Space-time tells matter how to move; matter tells space-time how to curve .

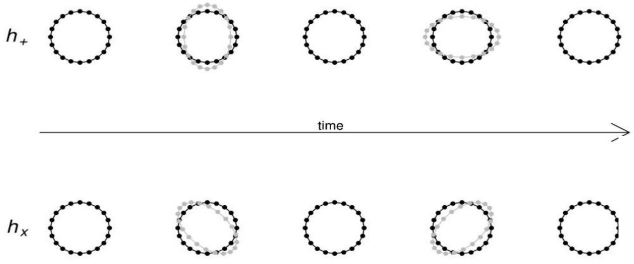
John Archibald Wheeler

Gravitational Wave - Preliminaries

Wave Generation and Propagation:

- 10^{-21} ~ debilitate strength of strain amplitude
- Gravitational waves (GWs) propagates as stretching and squeezing of space–time in two polarizations (plus "**+**" and cross "**×**") propagating in the transverse direction





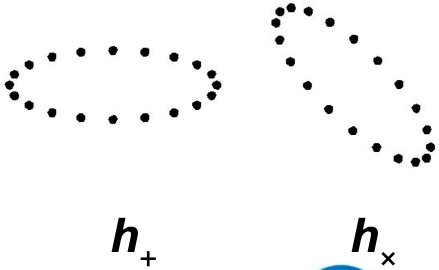
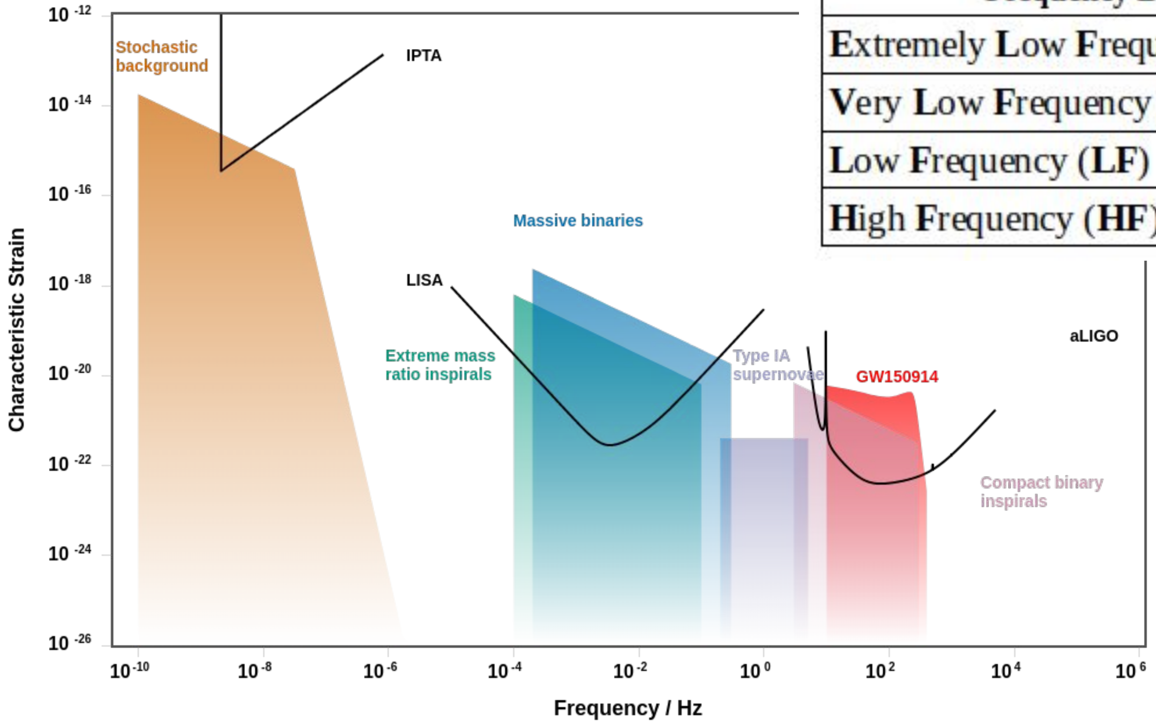


Figure: Ali et al. Bayesian Inference on Gravitational Waves. Pakistan Journal of Statistics and Operation Research, XI(4):645–665, 2015.

Gravitational Wave - Preliminaries

Spectrum of Gravitational waves:

Frequency Band	Frequency (Hz)
Extremely Low Frequency (ELF)	$10^{-18} - 10^{-15}$
Very Low Frequency (VLF)	$10^{-9} - 10^{-6}$
Low Frequency (LF)	$10^{-6} - 1$
High Frequency (HF)	$1 - 10^{4}$



http://gwplotter.com/

Gravitational Wave - Preliminaries

Parameterization of Relativistic binaries:

- Binary systems are parameterized by the field strengths and mass ratio
- Define a dimensionless parameter to parameterize determine gravitational field strength given as $\epsilon = \frac{GM}{rc^2} = \frac{r_g}{r}$

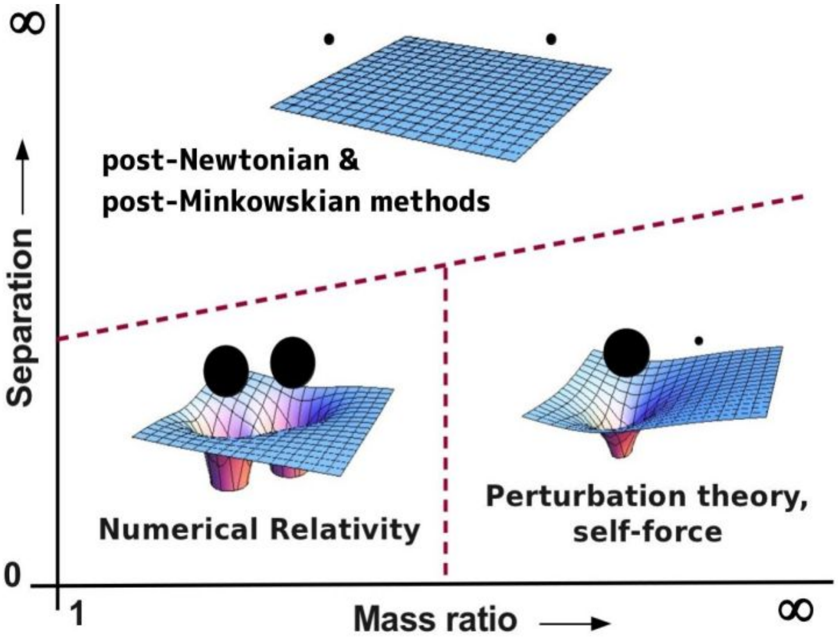
  where gravitational radius r_g is given as $r_g = \frac{GM}{c^2}$
- As potential continually decreases → the field weakens up to the flat space–time metric in the limit
- ϵ << 1 effectively drop to the scale of Newtonian field limit
- Within close range of horizon radii of BHs potential approaches unity

Dimitrios Psaltis. Probes and Tests of Strong-Field Gravity with Observations in the Electromagnetic Spectrum. Living Reviews in Relativity, 11(9), 2008.

Gravitational Wave - Preliminaries

Methods of Analytic Approximation:

- ➢ Analytic approximation methods in GR
  - ○ **Post–Newtonian (PN)**
  - ○ **Numerical Relativity (NR)**
  - ○ **Gravitational Self–Force (GSF)**
- ➢ Based on
  - ○ The mass ratio of the binaries
  - ○ Separation between them



Luc Blanchet. Analytic Approximations in GR and Gravitational Waves. International Journal of Modern Physics D, 28(6):1930011, 2019.
Figure: Deyan P. Mihaylov and Jonathan R. Gair. Transition of EMRIs through resonance: corrections to higher order in the on-resonance flux modification. Journal of Mathematical Physics 58, 112501, 2017.

Inspiralling Sources in the Range of LISA Detector and Detection Methodology

Intermediate Mass Ratio Inspirals (IMRIs):

- Stellar mass BHs ($10M_\odot$ – $100M_\odot$) and MBHs ($> 10^6M_\odot$) in galactic centers.
- **Intermediate Mass Black Holes (IMBH)** of the mass range ~$100M_\odot$ – $10^5M_\odot$.
- A number of formation channels:
  - **Run away collisions**
  - **Primordial population (Pop) III** stars
  - **Repeated mergers of stellar mass BHs** ~ $50M_\odot$
- The binary system formed by the gravitational interaction of IMBHs with other stellar remnants is known as **IMRIs** with mass ratio 10^{-2} – 10^{-4}.
- The event rates of detectable IMRIs is few tens per year in LISA frequency band.

Pau Amaro-Seoane. Detecting Intermediate-Mass Ratio Inspirals From The Ground And Space. Physical Review D, 98(063018), 2018. URL arXiv:1807.03824.

Inspiralling Sources in the Range of LISA Detector and Detection Methodology

Extreme Mass Ratio Inspirals (EMRIs):

- MBH ~ $10^4 - 10^7 M_\odot$
- Exhibiting famously known - Kerr geometry with notable impact of their spins and their masses.
- Inspiralling binaries are formed when a CO is captured by the MBH.
- **EMRIs**: A stellar remnant **compact objects (COs)** can either be
  - **Stellar mass black holes (BHs)**
  - **Neutron stars (NSs)**
  - **White dwarfs (WDs)** with diminishing mass ratio **q << 1** and prolonged cycles ~ $10^4 - 10^5$
- CO sets onto the highly eccentric, relativistic bound orbits shrinking to the **Last Stable Orbit (LSO)**, depends on the initial parameters.
- Due to the dissipation of the orbital energy and angular momentum continuous gravitational waves are emitted in **LISA sensitivity band**.

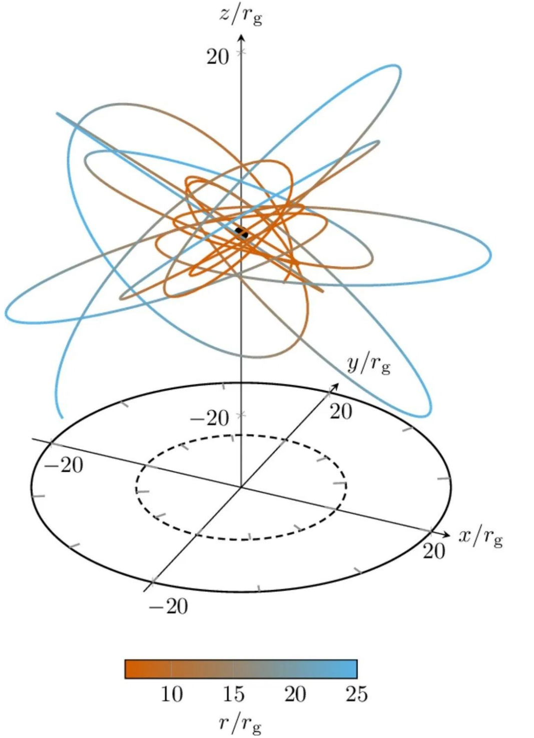
Christopher P. L. Berry et al. The unique potential of extreme mass-ratio inspirals for gravitational-wave astronomy. White paper submitted to Astro2020 (2020 Decadal Survey on Astronomy and Astrophysics).

Inspiralling Sources in the Range of LISA Detector and Detection Methodology

Extreme Mass Ratio Inspirals (EMRIs):

- ➢ Signals from EMRIs depends on of three fundamental frequencies:
  - ○ **f_r** – frequency of radial motion: corresponding to the orbital eccentric motion of CO around the MBH.
  - ○ **f_θ** – frequency of polar motion: arise due to the orbital plane precession emerging due to the spin-orbit coupling.
  - ○ **f_φ** – frequency associated with azimuthal motion that corresponds to the spin axis of MBH.
- ➢ Approximate waveform models of EMRIs
- ➢ Precise measurements of the mass and spin
- ➢ The MBH can be wholly accounted in terms of its mass and spin – **Uniqueness Theorem**.

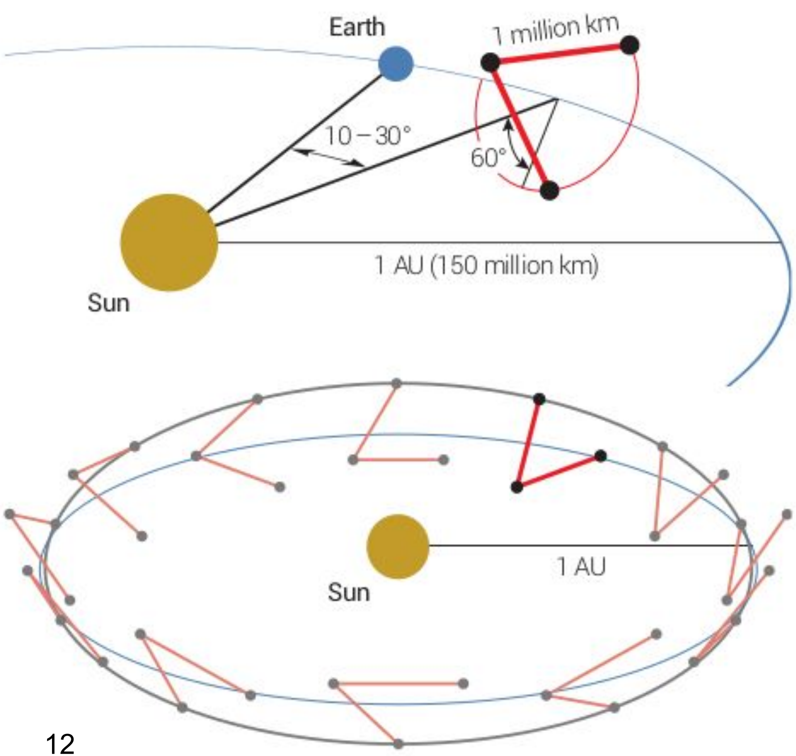
Stanislav Babak et al. Extreme mass ratio inspirals: perspectives for their detection. Fundamental Theories of Physics, 179:783-812, 2015.



Christopher P. L. Berry et al. The unique potential of extreme mass-ratio inspirals for gravitational-wave astronomy. White paper submitted to Astro2020 (2020 Decadal Survey on Astronomy and Astrophysics).

Inspiralling sources in the range of LISA and detection Methodology

LISA — Detector:



THE GRAVITATIONAL UNIVERSE

A science theme addressed by the eLISA mission observing the entire Universe

The last century has seen enormous progress in our understanding of the Universe. We know the life cycles of stars, the structure of galaxies, the remnants of the big bang, and have a general understanding of how the Universe evolved. We have come remarkably far using electromagnetic radiation as our tool for observing the Universe. However, gravity is the engine behind many of the processes in the Universe, and much of its action is dark. Opening a gravitational window on the Universe will let us go further than any alternative. Gravity has its own messenger: Gravitational waves, ripples in the fabric of spacetime. They travel essentially undisturbed and let us peer deep into the formation of the first seed black holes, exploring redshifts as large as z ~ 20, prior to the epoch of cosmic re-ionisation. Exquisite and unprecedented measurements of black hole masses and spins will make it possible to trace the history of black holes across all stages of galaxy evolution, and at the same time constrain any deviation from the Kerr metric of General Relativity. eLISA will be the first ever mission to study the entire Universe with gravitational waves. eLISA is an all-sky monitor and will offer a wide view of a dynamic cosmos using gravitational waves as new and unique messengers to unveil The Gravitational Universe. It provides the closest ever view of the early processes at TeV energies, has guaranteed sources in the form of verification binaries in the Milky Way, and can probe the entire Universe, from its smallest scales around singularities and black holes, all the way to cosmological dimensions.

Astrophysical Dynamics of EMRIs in Galactic Center

Event Rates:

- EMRI events per MBH of GC using LISA sensitivity
- **Astrophysical EMRI Model for Galactic Center:**
  - **MBH population**

    J. R. Gair et al. LISA extreme-mass-ratio inspiral events as probes of the black hole mass function. Physical Review D, 81:104014, 2010.

    - The scaling relation of mass function that is independent of red–shift factor given as
      $$\frac{dn}{d(lnM)} = n_0\left(\frac{M}{3\times10^6 M_\odot}\right)^\beta$$
    - For range MBH falling in LISA sensitivity band n_0 = 0.002 M pc $^{-3}$ and β = 0.3.
  - **Intrinsic Rates of stellar remnants around MBH**
    - Set the range of CO for stellar mass BHs μ ~ [5.5–10]$M_\odot$
    - The intrinsic rates of CO's population in GC is given by power law
      $$\mathcal{R}(M) = \mathcal{R}_0\left(\frac{M}{10^6 M_\odot}\right)^\alpha$$

      Clovis Hopman. Extreme mass ratio inspiral rates: dependence on the massive black hole mass. Classical and Quantum Gravity, 26(9), 2009.
    - The scaling factor α = {−0.15, −0.25, −0.25} with event rates R_0= {400, 7, 20}Gyr $^{-1}$ for BHs, NSs and WDs respectively.

Astrophysical Dynamics of EMRIs in Galactic Center

Event Rates:

- **Event rates** for **detectable EMRIs** using mission life-time of LISA t_{life} ***= 2yrs.***
- The **number density of comoving MBHs *dn/d(lnM)*** and **intrinsic rate of probable EMRIs per MBH** R.
- MBH's **spin** remains **highly uncertain**, hence, the integrating probability of spin distribution ***p(a) da*** is normalized to **1** with uniform range of spins ranging from 0 to 1, considering the prograde spin orbits (aligned).
- Retrograde (anti-aligned) spin ranges between −1 to 0.
- The number of EMRI events falling in LISA frequency band is given by

$$N_{EMRI} = t_{life} \int_{M=M_{min}}^{M_{max}} \mathcal{R} \frac{dn}{d(lnM)} \, d\, lnM$$

J. R. Gair. Probing black holes at low redshift using LISA EMRI observations. Classical and Quantum Gravity, 29:094034, 2009. URL arXiv:0811.0188.

where M_{min} = 10^4 M$_\odot$ and M_{max} = 10^7 M$_\odot$.

- We modify the integration term for MBHs in above equation to Dirac delta function

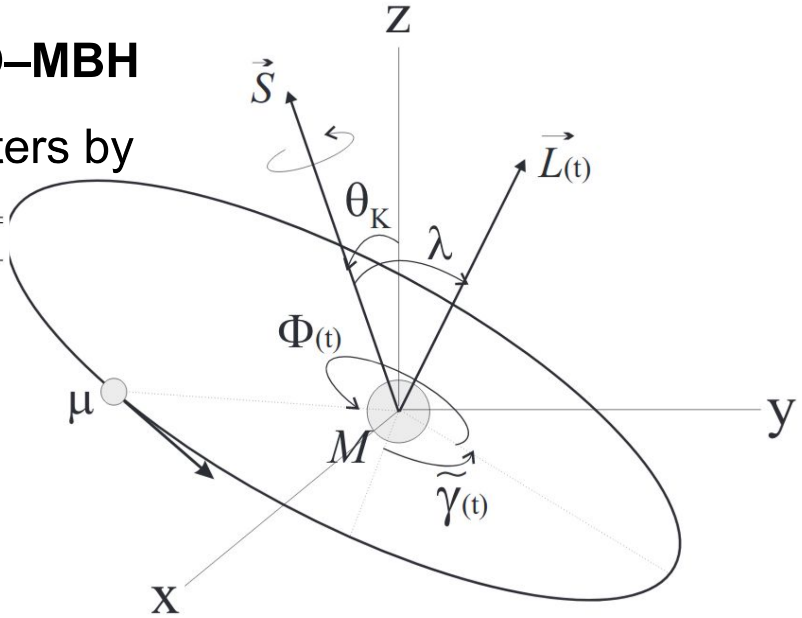
$$N_{EMRI} = t_{life} \int_{M_{min}}^{M_{max}} \mathcal{R} \frac{dn}{d(lnM)} \, d\, lnM \begin{cases} \delta(M - M_\bullet) = 0 & for \quad M \neq M_\bullet \\ \delta(M - M_\bullet) = 1 & for \quad M = M_\bullet \end{cases}$$

EMRI's in Milky Way Galaxy

Waveform Model: Analytical Kludge:

17 physical parameter fully describes the **CO–MBH system** that is further reduced to 14 parameters by

Parameters	*Symbol*	*Unit*
Initial azimuthal orbital frequency	ν_0	Hertz
Mass of CO	μ	$M_\odot$
Mass of MBH	M	$M_\odot$
Spin of MBH	$\tilde{a}$	M^2
Initial eccentricity	e_0	1
Orbital inclination angle	ι	Radian
Ecliptic latitude	θ_s	Radian
Ecliptic longitude	ϕ_s	Radian
Polar spin angle of MBH	θ_k	Radian
Azimuthal spin angle of MBH	ϕ_k	Radian
Distance to the source	D_L	Parsec
Initial direction of pericenter	$\tilde{\gamma}_0$	Radian
Initial azimuthal orbital phase angle	Φ_0	Radian
Initial azimuthal angle of orbital angular momentum	α_0	Radian



Leor Barack and Curt Cutler. LISA Capture Sources: Approximate Waveforms, Signal-to-Noise Ratios, and Parameter Estimation Accuracy. Physical Review D, 69:082005, 2004. URL arXiv:gr-qc/0310125.

EMRI’s in Milky Way Galaxy

Waveform Model: Analytical Kludge

- ➢ EMRI system in which a CO of mass μ is rotating around an SMBH of M ($\mu/M \ll 1$) on initially be on an elliptical orbit.
- ➢ Let n be the number of all possible harmonics associated with the orbital frequency ν of a given EMRI source.
- ➢ For an **n-harmonic** waveform the amplitude coefficients of the two polarizations are defined as:

$$h^{+,\times} = \frac{1}{D_L} \sum_n A_n^{+,\times},$$

$$A_n^{+} = C_a^{+} a_n + C_b^{+} b_n + C_c^{+} c_n,$$

$$A_n^{\times} = C_a^{\times} a_n + C_b^{\times} b_n,$$

$C_{a,b,c}$ be the coefficients that depend on the angular positioning parameters of the EMRI system, such as (ι, γ̃, α, θ k , φ k , θ s , φs)

- ➢ **$A^{+,\times}$ (t)** are the **amplitudes** and coefficients a_n, b_n and c_n by **Peters and Mathews (1963) sum mode approximation**

$$a_n = -n\mathcal{A}[J_{n-2}(ne) - 2eJ_{n-1}(ne) + \frac{2}{n}J_n(ne) + 2eJ_(n+1)(ne) - J_{n+2}(ne)]cos(n\Phi),$$

$$b_n = -n\mathcal{A}(1-e^2)^{1/2}[J_{n-2}(ne) - 2J_n(ne) + J_{n+2}(ne)]sin(n\Phi),$$

$$c_n = 2\mathcal{A}J_n(ne)cos(n\Phi),$$

where, A is the amplitude set to be $A \equiv \tilde{v}^{2/3}\mu/D_L$
J_n be the Bessel function of first kind
Φ(t) represents the azimuthal orbital phase angle
$\nu = (2\pi M)^{-1} (M/a)^{3/2}$ be the orbital frequency for Newtonian CO—MBH system having semi–major axis a and eccentricity e.

Leor Barack and Curt Cutler. LISA Capture Sources: Approximate Waveforms, Signal-to-Noise Ratios, and Parameter Estimation Accuracy. Physical Review D, 69:082005,2004.

EMRI's in Milky Way Galaxy

Waveform Model: Analytical Kludge

➢ The rate of change of orbital parameters Φ(t), ν(t), γ(t), e(t) and α(t) are given by PN formulae

$$\dot{\Phi} = 2\pi\nu,$$

$$\dot{\nu} = \frac{96}{10\pi}\frac{\eta\tilde{\nu}^{11/3}}{M^2(1-e^2)^{9/2}}\left\{\left[1+\frac{73e^2}{24}+\frac{37e^4}{96}\right](1-e^2)+\tilde{\nu}^{2/3}\left[\frac{1273}{336}-\frac{2561e^2}{224}-\frac{3885e^4}{128}-\frac{13147e^6}{5376}\right]-\frac{\tilde{\nu}\tilde{a}cos\iota}{(1-e^2)^{1/2}}\left[\frac{73}{12}+\frac{1211e^2}{24}+\frac{3143e^4}{96}+\frac{65e^6}{64}\right]\right\},$$

$$\dot{\tilde{\gamma}} = \frac{6\pi\nu\tilde{\nu}^{2/3}}{(1-e^2)}\left[1+\frac{1}{4}\tilde{\nu}^{2/3}\frac{(26-15e^2)}{(1-e^2)}\right]-\frac{12\pi\nu\tilde{\nu}\tilde{a}}{(1-e^2)^{3/2}}cos\iota,$$

$$\dot{e} = -\frac{e}{15}\frac{\eta\tilde{\nu}^{8/3}}{M(1-e^2)^{7/2}}\left[(304+121e^2)(1-e^2)(1+12\tilde{\nu}^{2/3})-\frac{1}{56}\tilde{\nu}^{2/3}((8)(16705)+(12)(9082)e^2-25211e^4)\right]+e\frac{\eta\tilde{\nu}^{11/3}}{M(1-e^2)^4}\tilde{a}cos\iota\left[\frac{1364}{5}+\frac{5032e^2}{15}+\frac{263e^4}{10}\right],$$

$$\dot{\alpha} = \frac{4\pi\nu\tilde{\nu}\tilde{a}}{(1-e^2)^{3/2}}.$$

Leor Barack and Curt Cutler. LISA Capture Sources: Approximate Waveforms, Signal-to-Noise Ratios, and Parameter Estimation Accuracy. Physical Review D, 69:082005,2004.

where, we set η = μ/M in

➢ These parameters are set to evolve dynamically forward up until the time of finally CO reaches the event horizon.

EMRI's in Milky Way Galaxy

Waveform Model: Parametric Description

- ➢ The parameter space reduces to 10 parameters by constraining the mass of the MBH, distance to the source and source location and orientation angles
  - MBH's mass $\mathbf{M}_\bullet = 4.28 \pm 0.10|_{\text{stat.}} \pm 0.21|_{\text{sys}} \times 10^6 M_\odot$
  - Astronomical distance $\mathbf{D}_\bullet = 8.32 \pm 0.07|_{\text{stat.}} \pm 0.14|_{\text{sys}}$ kpc
  - Ecliptic latitude θ_s and ecliptic longitude φ_s are set to be, as well calibrated coordinates of Sgr A*
- ➢ We randomly choose parameters related to CO within the range defined by **Mock LISA Data Challenges (MLDC)**, such as mass, eccentricity, orbital frequency and some orientation angles in order to reduce the complexity of waveform
  - The uniform distribution of spin parameter U[0, 1]
  - We took randomized distribution of μ within [5.5, 10.5] $M_\odot$, population of stellar mass BHs
  - Initial eccentricity was chosen in the range [0.15, 0.25] by parameter range
  - Initial orbital frequency at the time instant of CO's capture is taken to be uniform ~ 2 − 5 mHz
  - Orientation vectors of spin are assumed to be isotropic and symmetrically distributed. Three angles corresponding to the initial phase space trajectory can be, naively, taken at the uniform variance between [0, 2π]

Signal Analysis and Event Detection

Signal Analysis:

- Waveform strain amplitude signal *s(t)* is the time dependent linear sum of GW signal symbolized as *h(t)* and noise *n(t)* as a function of time

$$s(t) = h(t) + n(t)$$

where noise *n(t)* is assumed to be uncorrelated Gaussian converging with zero mean, the noise is uncorrelated.

- Strain amplitude *h(t)* can be written as

$$h = hI + hII$$

- This overlapping inner product function, defined as

Leor Barack and Curt Cutler. LISA Capture Sources: Approximate Waveforms, Signal-to-Noise Ratios, and Parameter Estimation Accuracy. Physical Review D, 69:082005,2004.
Lee S. Finn. Detection, measurement, and gravitational radiation. Physical Review D,46:5236-5249, 1992.

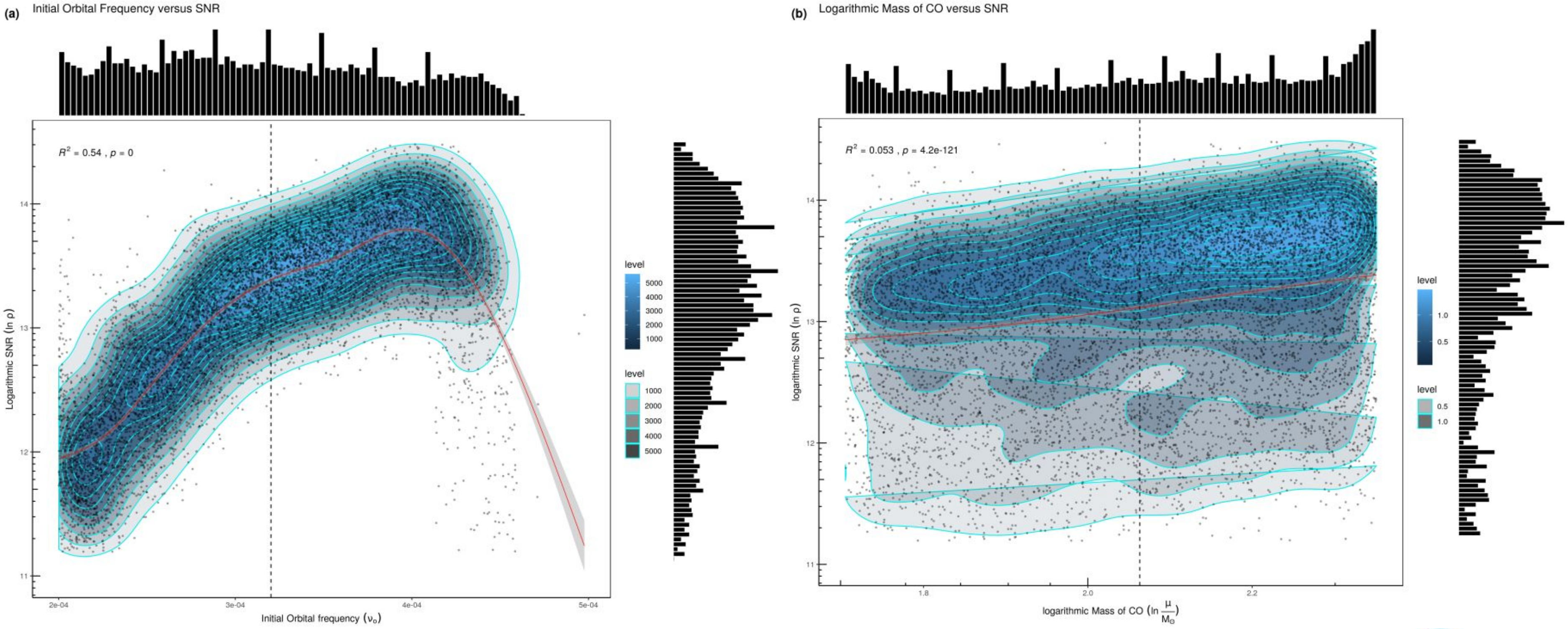
$$\rho^2[h] = (\tilde{h}(f)|\tilde{h}(f)) = 4\Re\left\{ \int_0^\infty \frac{\tilde{h}^*(f)\,\tilde{h}(f)}{S_n(f)}\,df \right\}$$

where $S_n(f)$ is one sided power spectral density, given by sensitivity model of detector's noise.

- For EMRI signals we will follow the theoretical convention to consider the threshold value of SNR ρ = 20.

Results and Applications

Parametric Statistical Inference in the Context of SNRs Estimates:



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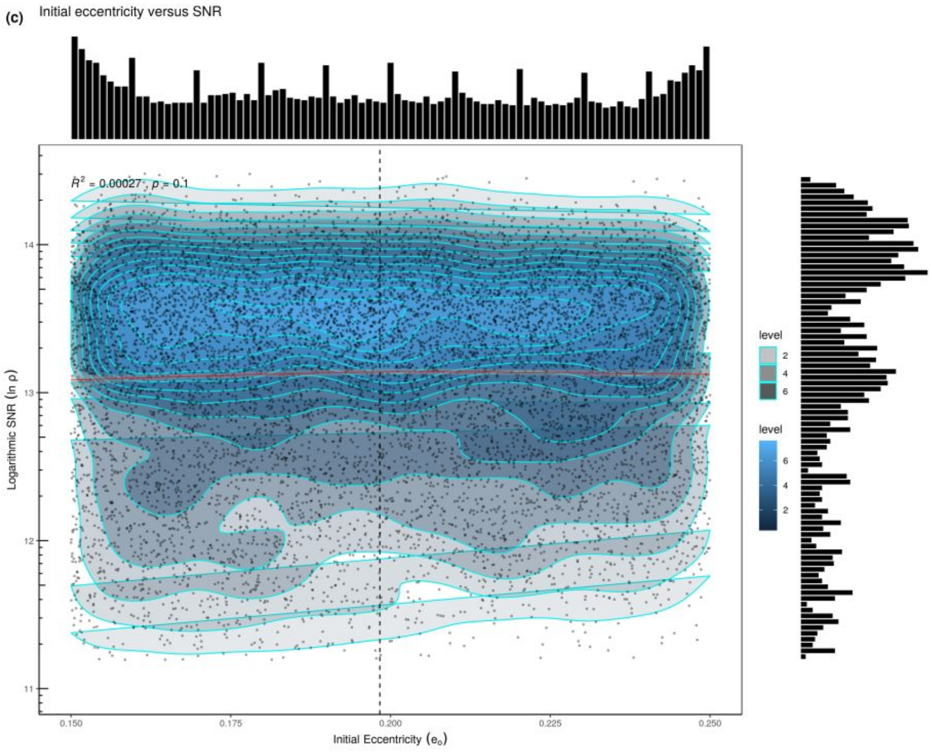

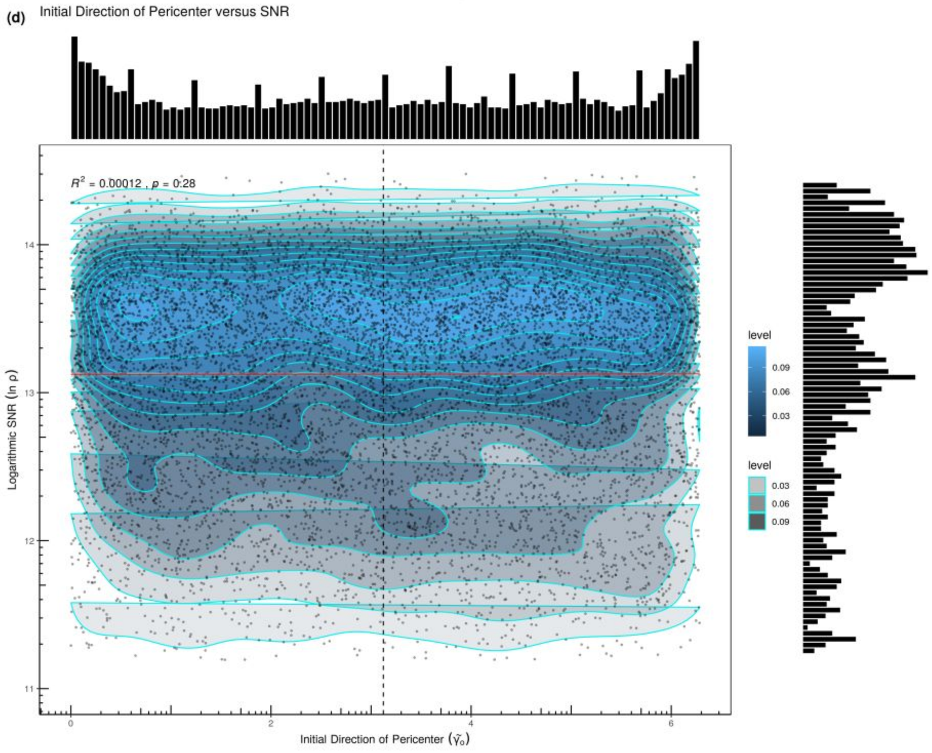

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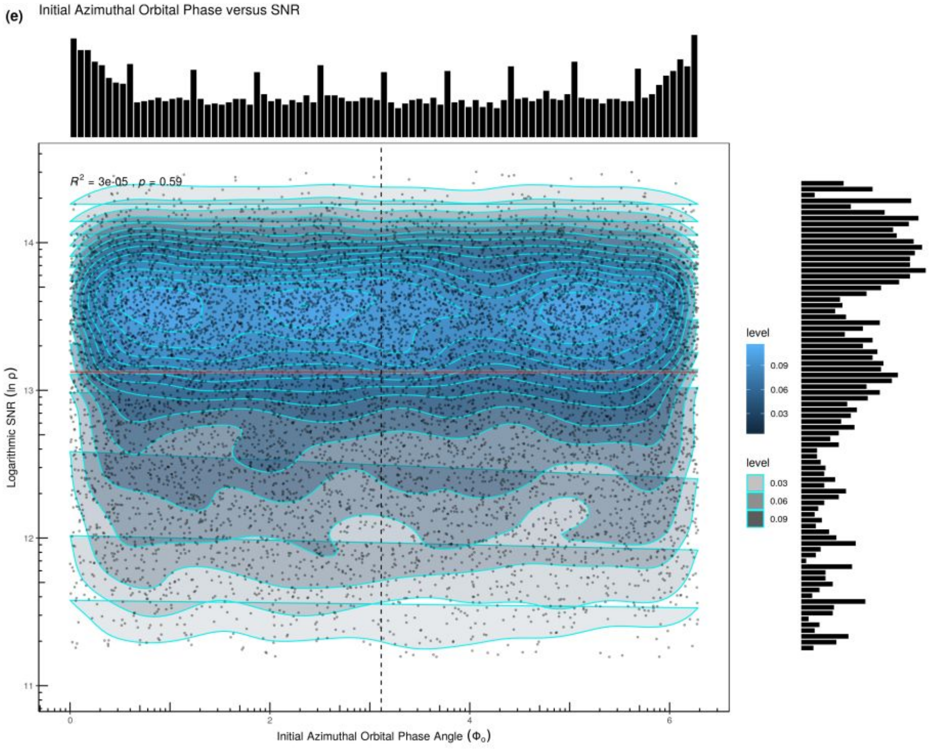

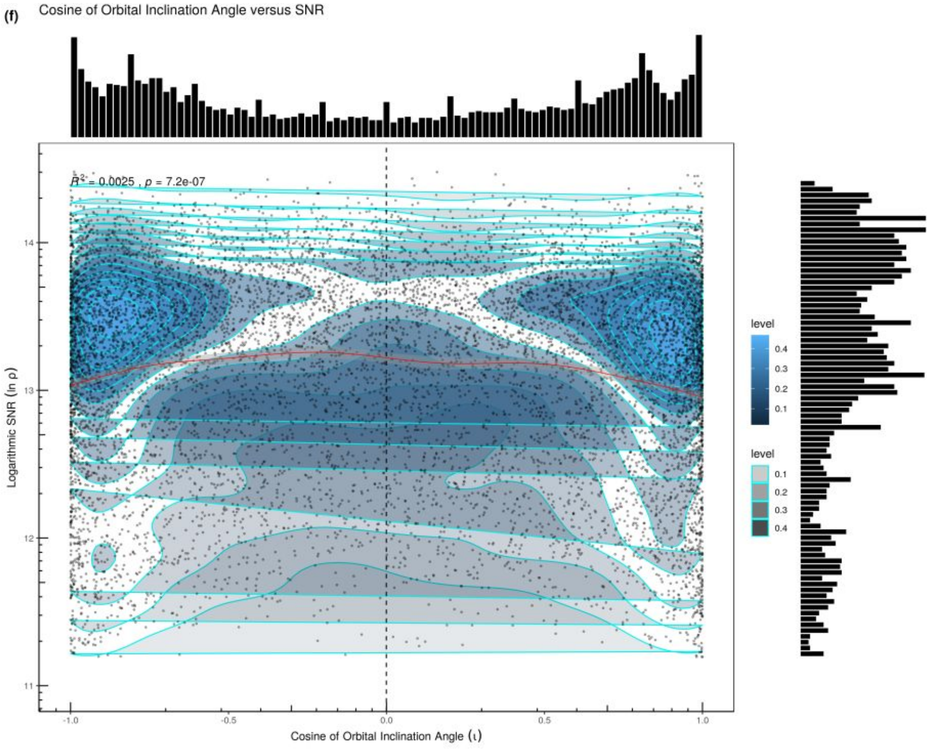

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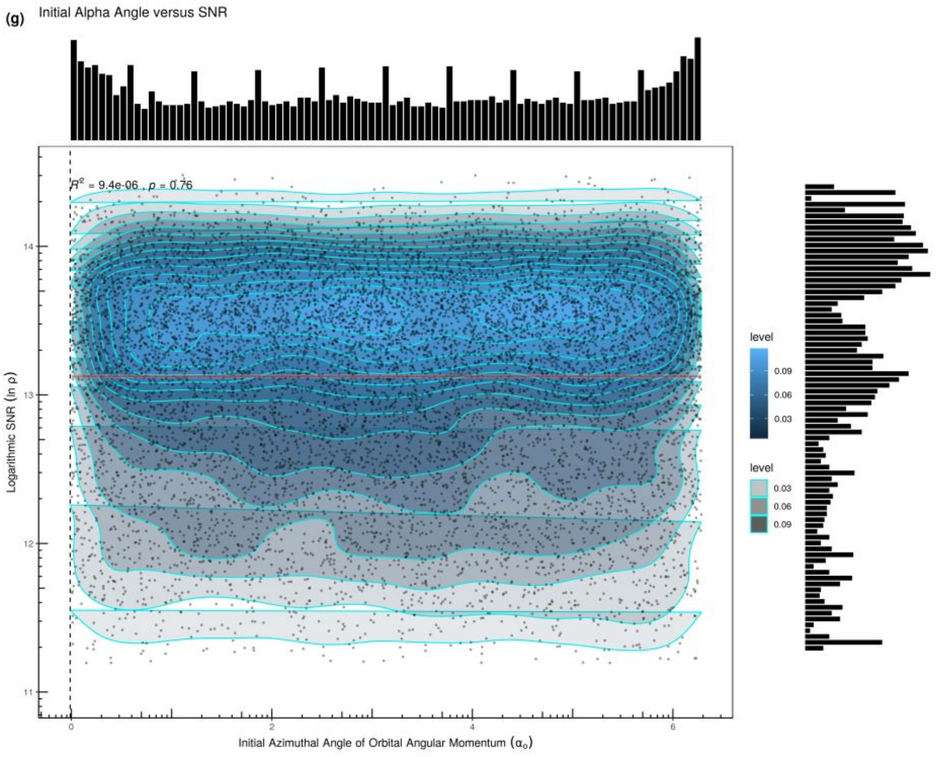

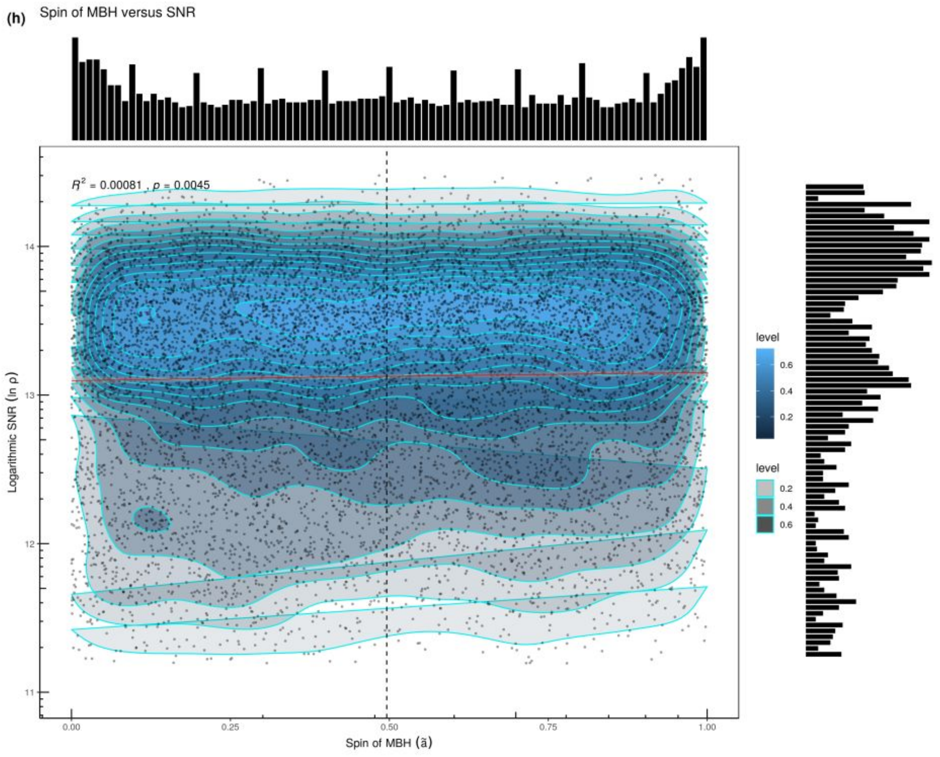

Results and Applications

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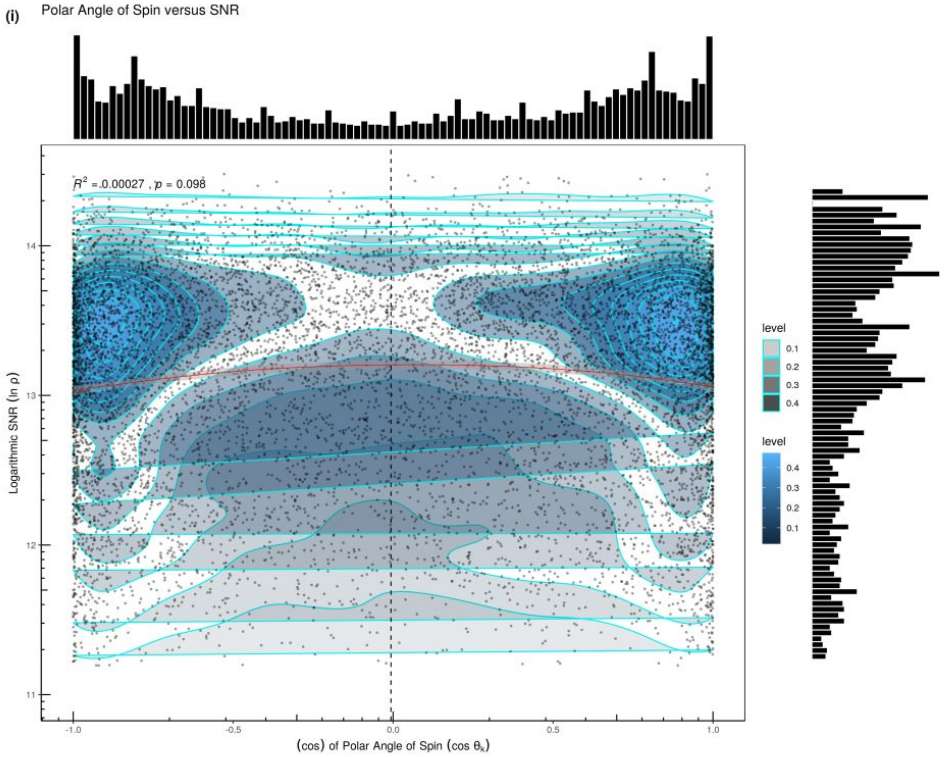
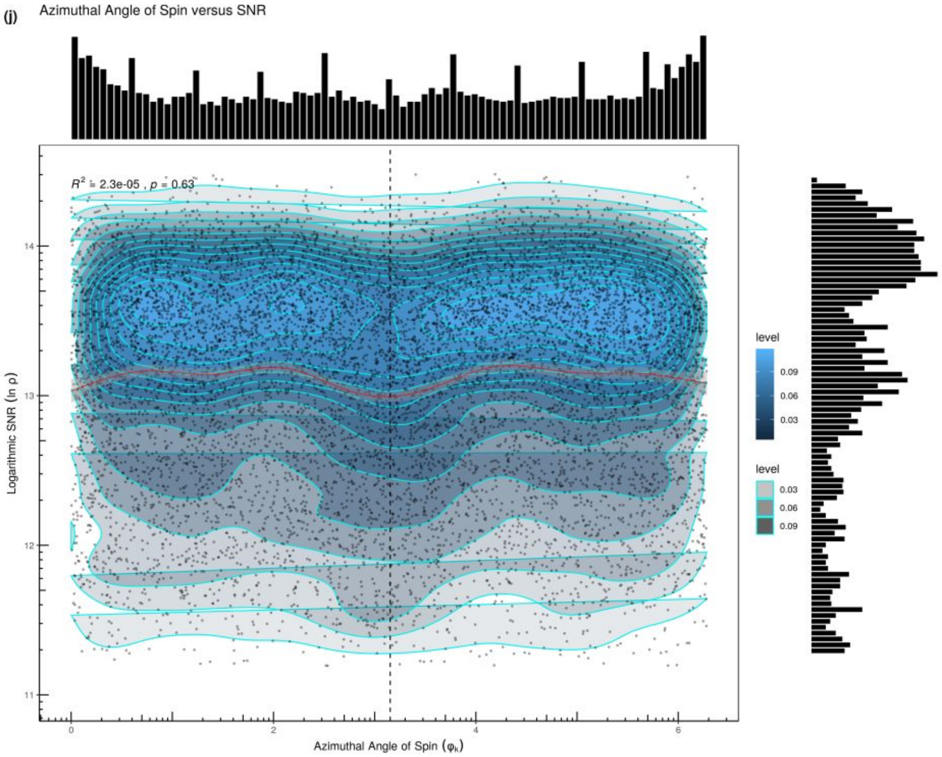




Results and Applications

Parametric Statistical Inference in the Context of SNRs Estimates:

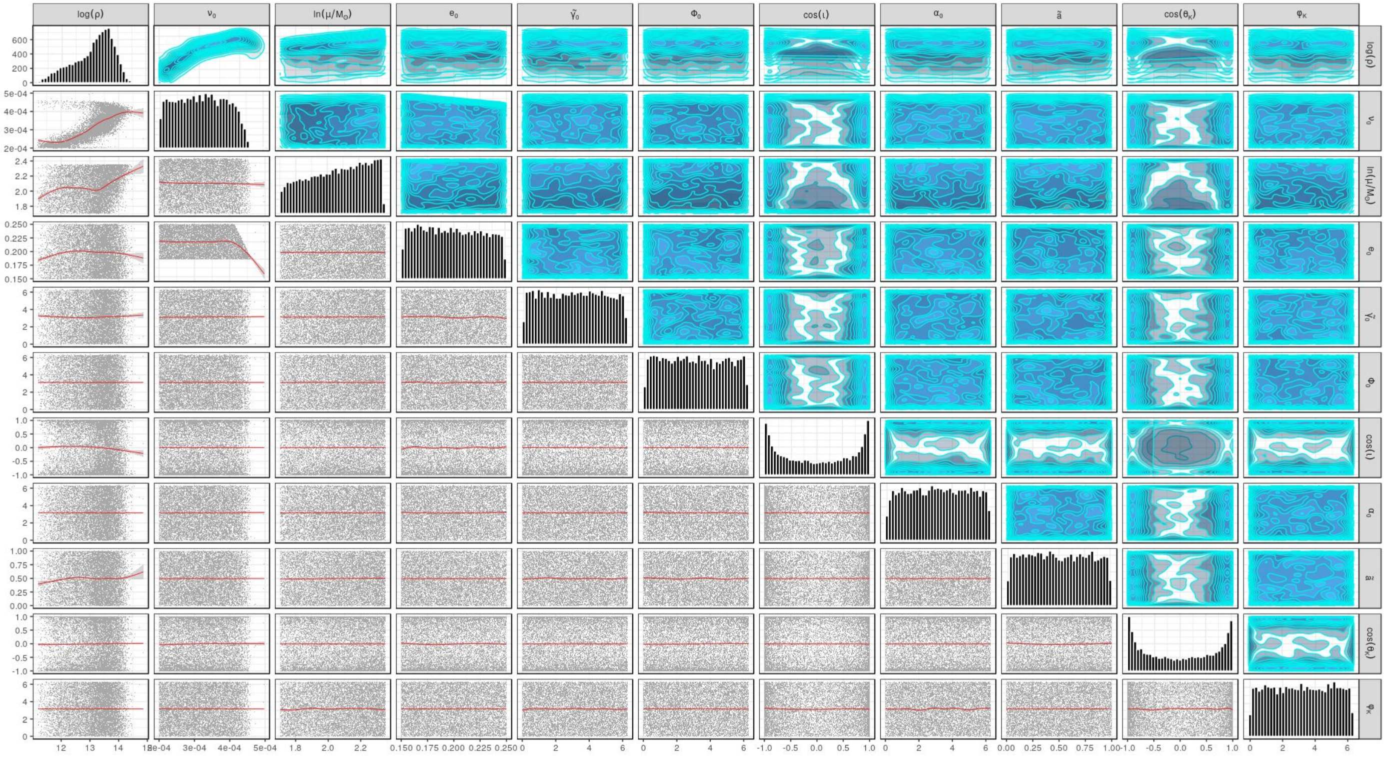
Parameters	*R–squared*	*p–value*
ν_0	0.54	0
$\ln \mu$	0.053	4.2×10^{-121}
e_0	0.00027	0.1
$\tilde{\gamma}_0$	0.00012	0.28
Φ_0	3×10^{-5}	0.59
$\cos \iota$	0.0025	7.2×10^{-7}
α_0	9.4×10^{-6}	0.76
S/M^2	0.0081	0.0045
$\mu_k \equiv \cos \theta_k$	0.00027	0.098
ϕ_k	2.3×10^{-5}	0.63

TABLE 6.1: The SNR correlate strongly with ν_0 and weakly with $\ln \mu$ with statistically significant *p–values* of 0 and 4.2×10^{-121}, respectively. However, SNR is non–correlated to other parameters.

Results and Applications

Parametric Statistical Inference in the Context of SNRs Estimates:

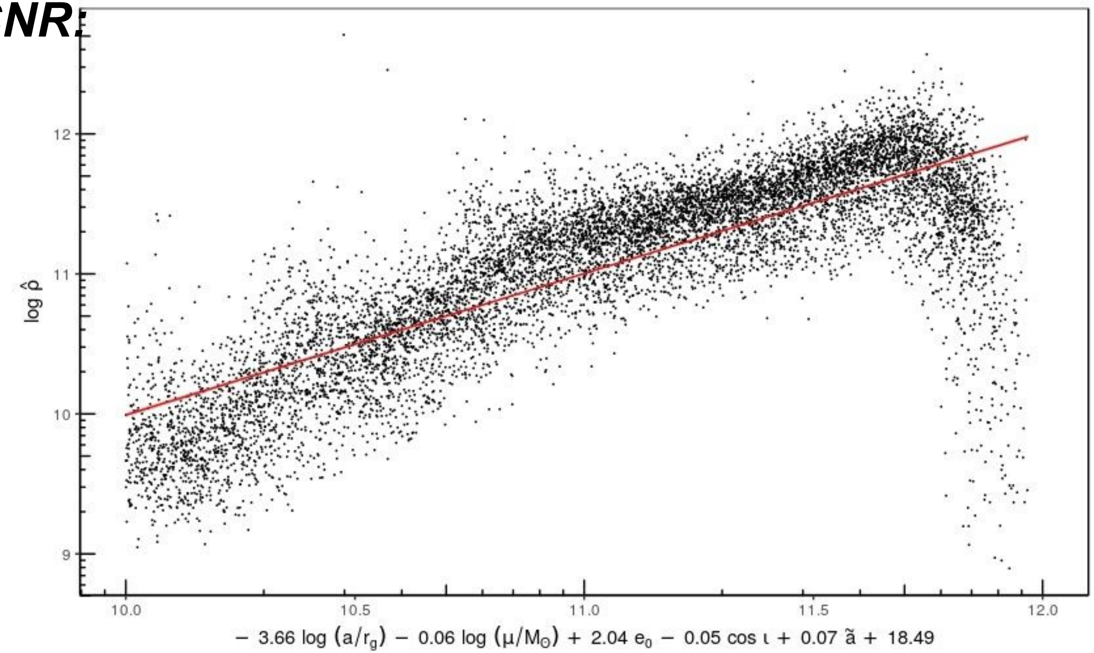
- ➢ Correlogram: Upper panels reads statistical density contour variation.
- ➢ Diagonal represents two dimensional posterior distributions
- ➢ Lower panels illustrates scatter plots and regression lines.



Results and Applications

Back-of-Envelope Estimates of SNR:

- We developed the equation of **back–of–envelope estimates** for EMRIs in GC.
- This power law was fitted making use of **multiple linear regression method** with (forward and backward)-step–wise approach compared to the null model selected with least **variance inflation factor (VIF)** of each parameter indicating the absence of multicollinearity.
- The best fit line was drawn using **generalized linear model (glm–function)** based on gaussian error distribution.
- **R–squared** and **adjusted R–squared** indexes of the regression model are **0.6401** and **0.6399**, respectively with the **p-value** of **2.2×10^{-16}** .
- Individual predictor parameters **semi–major axis a**, **mass of CO μ**, **initial orbital eccentricity e** , **(cosine–) of orbital inclination cos ι** and **spin of MBH ã** contributes to our best fit model.
- It is convenient to work with logarithmic scales of estimated SNR and distance parameter, since the quantities of the interest are positive and definite.
- EMRI signal sensitively varies with each parameter of the model.



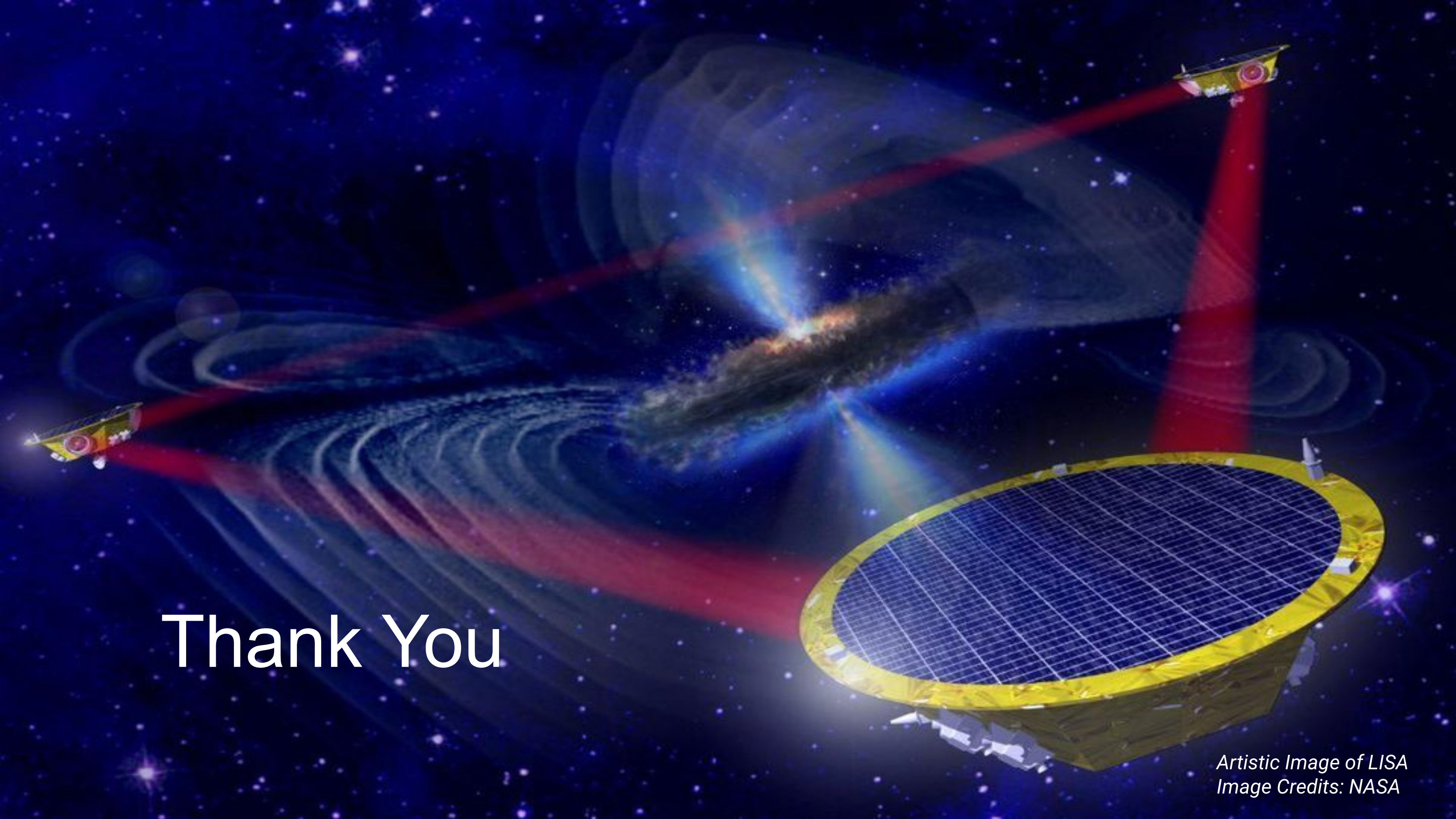
Scatter plot of estimated mass–normalized SNR as a function of orbital parameters i.e. a, μ, e_0, $cos\ \iota$ and $\tilde{a}$. Red line corresponds to the best–fit estimates $\log(\hat{\rho}) = -3.66\log(\frac{a}{r_g}) - 0.06\log(\frac{\mu}{M_\odot}) + 2.04\ e_0 - 0.05\ cos\ \iota + 0.07\ \tilde{a} + 18.49$ of SNR meanwhile the radial parameter sweeps between $6.09r_g \leqslant a \leqslant 11.18r_g$. $\log(\hat{\rho})$ gives good fit for logarithmic $a > 7r_g$ and drops steeply for lower semi–major axis with reduced chi–square value of $\chi^2/\nu = 0.014$.

Results and Applications

Back-of-Envelope Estimates of SNR:

$$\log(\hat{\rho}) = -3.66\log(\frac{a}{r_g}) - 0.06\log(\frac{\mu}{M_{\odot}}) + 2.04\ e_0 - 0.05\ cos\ \iota + 0.07\ \tilde{a} + 18.49$$

- **Semi–major axis** a be the most influential parameter to the estimates of SNR.
  - It anti–correlates with SNR, closer initial orbits will result in louder EMRI event.
- There exists **negative** weak dependency of **CO's mass** upon SNR even though our results are mass–normalized.
  - This correction indicates that **mass–normalized SNR** of an EMRIs still, marginally, varies with CO's mass.
  - There may not necessarily be any reason for this correction here but for sure it is not from the reason of **shorter EMRIs**.
  - It may or may not be the result of mass segregation effect.
- Initial eccentricity increasingly contributes to SNR with relatively stronger **regression coefficient**.
- **Spin** remain highly **uncertain** parameter proportionally relate with positive coefficient but for higher spin distribution term may not be contributing in this fit but may significantly impact SNR upon increasing number of EMRI samples. Likewise, **bimodal distribution** of **inclination angle** confines, typically, at maxima's will remain trivial to the best–fit.



Thank You

Artistic Image of LISA
Image Credits: NASA