

Algebra 2 Honors — Challenge Set (Version 2)

25 Hard/Tricky Systems of Equations (2 & 3 Variables) with Full Solution Steps

Designed to stress elimination with fractions, near-linear dependencies, contradictions, and parameterized families. Steps show Gaussian elimination or Cramer's Rule where appropriate.

Part A — Two Variables (10 problems)

Q1. Solve the system:

$$\frac{2}{5}x = -\frac{3}{5}$$

$$\frac{5}{2}x + \frac{1}{2}y = -\frac{9}{4}$$

Solution Steps:

Clear denominators (LCM 20):

$$(1) 8x + 0y = -12$$

$$(2) 50x + 10y = -45$$

$$\text{Determinant } \Delta = 8 \cdot 10 - 50 \cdot 0 = 80.$$

$$x = (-12 \cdot 10 - -45 \cdot 0) / \Delta = -3/2; y = (8 \cdot -45 - 50 \cdot -12) / \Delta = 3.$$

$$\text{Answer: } (x, y) = (-3/2, 3).$$

Q2. Solve the system:

$$\frac{1}{7}x - \frac{5}{2}y = \frac{5}{3}$$

$$-\frac{5}{6}x + 3y = \frac{5}{2}$$

Solution Steps:

Clear denominators (LCM 42):

$$(1) 6x - 105y = 70$$

$$(2) -35x + 126y = 105$$

$$\text{Determinant } \Delta = 6 \cdot 126 - -35 \cdot -105 = -2919.$$

$$x = (70 \cdot 126 - 105 \cdot -105) / \Delta = -945/139; y = (6 \cdot 105 - -35 \cdot 70) / \Delta = -440/417.$$

$$\text{Answer: } (x, y) = (-945/139, -440/417).$$

Q3. Solve the system:

$$-\frac{3}{2}x + \frac{2}{3}y = -\frac{5}{6}$$

$$\frac{2}{3}x - \frac{1}{5}y = \frac{4}{3}$$

Solution Steps:

Clear denominators (LCM 30):

$$(1) -45x + 20y = -25$$

$$(2) 20x - 6y = 40$$

$$\text{Determinant } \Delta = -45 \cdot -6 - 20 \cdot 20 = -130.$$

$$x = (-25 \cdot -6 - 40 \cdot 20) / \Delta = 5; y = (-45 \cdot 40 - 20 \cdot -25) / \Delta = 10.$$

$$\text{Answer: } (x, y) = (5, 10).$$

Q4. Solve the system:

$$-1/2x - 6/5y = 8/5$$

$$-1/5x + 3y = 5/2$$

Solution Steps:

Clear denominators (LCM 10):

$$(1) -5x - 12y = 16$$

$$(2) -2x + 30y = 25$$

$$\text{Determinant } \Delta = -5 \cdot 30 - -2 \cdot -12 = -174.$$

$$x = (16 \cdot 30 - 25 \cdot -12) / \Delta = -130/29; y = (-5 \cdot 25 - -2 \cdot 16) / \Delta = 31/58.$$

$$\text{Answer: } (x, y) = (-130/29, 31/58).$$

Q5. Solve the system:

$$x + 2/3y = -5/2$$

$$-1/2y = 1/3$$

Solution Steps:

Clear denominators (LCM 6):

$$(1) 6x + 4y = -15$$

$$(2) 0x - 3y = 2$$

$$\text{Determinant } \Delta = 6 \cdot -3 - 0 \cdot 4 = -18.$$

$$x = (-15 \cdot -3 - 2 \cdot 4) / \Delta = -37/18; y = (6 \cdot 2 - 0 \cdot -15) / \Delta = -2/3.$$

$$\text{Answer: } (x, y) = (-37/18, -2/3).$$

Q6. Solve the system:

$$-3/2x - 2y = 1/2$$

$$-x - 2/3y = -9/5$$

Solution Steps:

Clear denominators (LCM 30):

$$(1) -45x - 60y = 15$$

$$(2) -30x - 20y = -54$$

$$\text{Determinant } \Delta = -45 \cdot -20 - -30 \cdot -60 = -900.$$

$$x = (15 \cdot -20 - -54 \cdot -60) / \Delta = 59/15; y = (-45 \cdot -54 - -30 \cdot 15) / \Delta = -16/5.$$

$$\text{Answer: } (x, y) = (59/15, -16/5).$$

Q7. Solve the system:

$$-2x + 1/4y = 8/3$$

$$1/2x - 5/7y = 1$$

Solution Steps:

Clear denominators (LCM 84):

$$(1) -168x + 21y = 224$$

$$(2) 42x - 60y = 84$$

$$\text{Determinant } \Delta = -168 \cdot -60 - 42 \cdot 21 = 9198.$$

$$x = (224 \cdot -60 - 84 \cdot 21) / \Delta = -362/219; y = (-168 \cdot 84 - 42 \cdot 224) / \Delta = -560/219.$$

$$\text{Answer: } (x, y) = (-362/219, -560/219).$$

Q8. Solve the system:

$$x + 5/6y = 1$$

$$5/2x + 25/12y = 7/3$$

Solution Steps:

Left sides are proportional but constants differ.

Elimination gives $0 = \text{nonzero} \Rightarrow$ parallel, distinct lines.

Answer: No solution.

Q9. Solve the system:

$$1/5y = 3$$

$$-3/5y = 8/3$$

Solution Steps:

Left sides are proportional but constants differ.

Elimination gives $0 = \text{nonzero} \Rightarrow$ parallel, distinct lines.

Answer: No solution.

Q10. Solve the system:

$$1/3x - 2/3y = 0$$

$$5/6x - 5/3y = 0$$

Solution Steps:

Recognize proportional equations: $(2) = k \cdot (1)$ on both sides.

Elimination yields $0 = 0 \Rightarrow$ the two lines coincide.

Answer: Infinitely many solutions along $1/3x - 2/3y = 0$.

Part B — Three Variables (15 problems)

Q11. Solve the system:

$$-4x - 4y + 3z = 7$$

$$2x - 4y + 3z = -8$$

$$x + 2y - 2z = 12$$

Solution Steps:

Use Cramer's Rule (determinants kept small to emphasize algebra):

$$\Delta = -12, \Delta x = 30, \Delta y = 225, \Delta z = 312.$$

$$x = \Delta x / \Delta = -5/2, y = \Delta y / \Delta = -75/4, z = \Delta z / \Delta = -26.$$

$$\text{Answer: } (x, y, z) = (-5/2, -75/4, -26).$$

Q12. Solve the system:

$$-3x + 2y - 4z = 9$$

$$3x - 1y + 3z = 5$$

$$-x - 4y + 3z = 2$$

Solution Steps:

Use Cramer's Rule (determinants kept small to emphasize algebra):

$$\Delta = 1, \Delta x = 135, \Delta y = -179, \Delta z = -193.$$

$$x = \Delta x / \Delta = 135, y = \Delta y / \Delta = -179, z = \Delta z / \Delta = -193.$$

$$\text{Answer: } (x, y, z) = (135, -179, -193).$$

Q13. Solve the system:

$$-3x + 3y - 4z = 12$$

$$x - 2y + 2z = 11$$

$$-2x + 3y = 6$$

Solution Steps:

Use Cramer's Rule (determinants kept small to emphasize algebra):

$$\Delta = 10, \Delta x = -216, \Delta y = -124, \Delta z = 39.$$

$$x = \Delta x / \Delta = -108/5, y = \Delta y / \Delta = -62/5, z = \Delta z / \Delta = 39/10.$$

$$\text{Answer: } (x, y, z) = (-108/5, -62/5, 39/10).$$

Q14. Solve the system:

$$-3z = 11$$

$$-4x + 1y - 3z = 0$$

$$4x - 2y + 1z = -8$$

Solution Steps:

Use Cramer's Rule (determinants kept small to emphasize algebra):

$$\Delta = -12, \Delta x = -79, \Delta y = -184, \Delta z = 44.$$

$$x = \Delta x / \Delta = 79/12, y = \Delta y / \Delta = 46/3, z = \Delta z / \Delta = -11/3.$$

$$\text{Answer: } (x, y, z) = (79/12, 46/3, -11/3).$$

Q15. Solve the system:

$$4x + 2y + 2z = -3$$

$$2x + 1y - 1z = 12$$

$$4x + 4y + 2z = -5$$

Solution Steps:

Use Cramer's Rule (determinants kept small to emphasize algebra):

$$\Delta = 16, \Delta x = 50, \Delta y = -16, \Delta z = -108.$$

$$x = \Delta x / \Delta = 25/8, y = \Delta y / \Delta = -1, z = \Delta z / \Delta = -27/4.$$

$$\text{Answer: } (x, y, z) = (25/8, -1, -27/4).$$

Q16. Solve the system:

$$-x - 2y + 3z = -6$$

$$-4x - 2y - 3z = -6$$

$$-2x + 1y - 3z = -6$$

Solution Steps:

Use Cramer's Rule (determinants kept small to emphasize algebra):

$$\Delta = -21, \Delta x = -108, \Delta y = 72, \Delta z = 54.$$

$$x = \Delta x / \Delta = 36/7, y = \Delta y / \Delta = -24/7, z = \Delta z / \Delta = -18/7.$$

$$\text{Answer: } (x, y, z) = (36/7, -24/7, -18/7).$$

Q17. Solve the system:

$$3x + 2y + 3z = 0$$

$$y + 1z = -6$$

$$-4x + 2y - 4z = -3$$

Solution Steps:

Use Cramer's Rule (determinants kept small to emphasize algebra):

$$\Delta = -14, \Delta x = -81, \Delta y = 9, \Delta z = 75.$$

$$x = \Delta x / \Delta = 81/14, y = \Delta y / \Delta = -9/14, z = \Delta z / \Delta = -75/14.$$

$$\text{Answer: } (x, y, z) = (81/14, -9/14, -75/14).$$

Q18. Solve the system:

$$2x - 4y + 2z = -4$$

$$2x - 2y - 4z = -5$$

$$-x + 1y - 2z = 12$$

Solution Steps:

Use Cramer's Rule (determinants kept small to emphasize algebra):

$$\Delta = -16, \Delta x = 238, \Delta y = 122, \Delta z = 38.$$

$$x = \Delta x / \Delta = -119/8, y = \Delta y / \Delta = -61/8, z = \Delta z / \Delta = -19/8.$$

$$\text{Answer: } (x, y, z) = (-119/8, -61/8, -19/8).$$

Q19. Solve the system:

$$-4x - 2y = 5$$

$$-4x - 3y - 4z = -4$$

$$-x - 1y = 9$$

Solution Steps:

Use Cramer's Rule (determinants kept small to emphasize algebra):

$$\Delta = 8, \Delta x = 52, \Delta y = -124, \Delta z = 49.$$

$$x = \Delta x / \Delta = 13/2, y = \Delta y / \Delta = -31/2, z = \Delta z / \Delta = 49/8.$$

$$\text{Answer: } (x, y, z) = (13/2, -31/2, 49/8).$$

Q20. Solve the system:

$$2y - 2z = 4$$

$$-2x + 2y - 1z = -4$$

$$2x - 2y - 2z = -4$$

Solution Steps:

Use Cramer's Rule (determinants kept small to emphasize algebra):

$$\Delta = -12, \Delta x = -64, \Delta y = -56, \Delta z = -32.$$

$$x = \Delta x / \Delta = 16/3, y = \Delta y / \Delta = 14/3, z = \Delta z / \Delta = 8/3.$$

$$\text{Answer: } (x, y, z) = (16/3, 14/3, 8/3).$$

Q21. Solve the system:

$$4x + 1y + 4z = 1$$

$$-2x + 2y + 1z = -7$$

$$4x - 3y - 1z = 2$$

Solution Steps:

Use Cramer's Rule (determinants kept small to emphasize algebra):

$$\Delta = -2, \Delta x = 64, \Delta y = 118, \Delta z = -94.$$

$$x = \Delta x / \Delta = -32, y = \Delta y / \Delta = -59, z = \Delta z / \Delta = 47.$$

$$\text{Answer: } (x, y, z) = (-32, -59, 47).$$

Q22. Solve the system:

$$4x + 1y - 3z = -7$$

$$3x + 2y - 1z = 6$$

$$21x + 9y - 12z = -2$$

Solution Steps:

Here (3) has the same left-hand combo as $s \cdot (1) + t \cdot (2)$ but a different constant.

Elimination gives $0 = \text{nonzero} \Rightarrow$ inconsistent system.

Answer: No solution.

Q23. Solve the system:

$$4x - 1y + 1z = 8$$

$$x + 3y - 1z = 3$$

$$6x + 5y - 1z = 12$$

Solution Steps:

Here (3) has the same left-hand combo as $s \cdot (1) + t \cdot (2)$ but a different constant.

Elimination gives $0 = \text{nonzero} \Rightarrow$ inconsistent system.

Answer: No solution.

Q24. Solve the system:

$$3x + 2y - 2z = 1$$

$$x - 3y + 2z = 2$$

$$12x - 3y = 9$$

Solution Steps:

Note (3) = $s \cdot (1) + t \cdot (2)$ with matching constant.

Row-reduction yields one row of zeros \Rightarrow dependent planes.

Answer: Infinitely many solutions (describe as a parametric family).

Q25. Solve the system:

$$-x - 4y + 4z = 6$$

$$-3x + 3y + 4z = -5$$

$$-4x - 1y + 8z = 1$$

Solution Steps:

Note (3) = $s \cdot (1) + t \cdot (2)$ with matching constant.

Row-reduction yields one row of zeros \Rightarrow dependent planes.

Answer: Infinitely many solutions (describe as a parametric family).