Algebra 2 Honors — Absolute Value Equations & Inequalities

45 Problems with Detailed, Step-by-Step Solutions (No Repeats)

Directions: Solve each problem. Show all work. Solutions include clear reasoning for equations and inequalities (compound, union/intersection, and sign charts).

Q1. Solve: 4|3x - 10| = 0

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Divide both sides by 4 (k>0 preserves equality): |3x - 10| = 0.
For |u| = c with c ≥ 0, solve u = c or u = −c.
3x - 10 = 0 ⇒ x = 10/3.
3x - 10 = -0 ⇒ x = 10/3.
```

Answer: x = 10/3 or x = 10/3

Q2. Solve: 3|3x - 12| = 7

```
Divide both sides by 3 (k>0 preserves equality): |3x - 12| = 7/3.
For |u| = c with c ≥ 0, solve u = c or u = −c.
3x - 12 = 7/3 ⇒ x = 43/9.
3x - 12 = -7/3 ⇒ x = 29/9.
```

Answer: x = 43/9 or x = 29/9

Q3. Solve: |4x - 5| = 7

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For |u| = c with c ≥ 0, solve u = c or u = −c.
4x - 5 = 7 ⇒ x = 3.
4x - 5 = -7 ⇒ x = -1/2.
```

Answer: x = 3 or x = -1/2

Q4. Solve: |1x - 9| = -2

```
Absolute value is never negative ⇒ no solution.
```

Answer: ∅

Q5. Solve: |-3x + 0| = 8

```
For |u| = c with c ≥ 0, solve u = c or u = −c.
-3x + 0 = 8 ⇒ x = -8/3.
-3x + 0 = -8 ⇒ x = 8/3.
```

Answer: x = -8/3 or x = 8/3

Q6. Solve: |-6x + 7| = 12

For |u| = c with c ≥ 0, solve u = c or u = −c.
-6x + 7 = 12 ⇒ x = -5/6.
-6x + 7 = -12 ⇒ x = 19/6.

Answer: x = -5/6 or x = 19/6

Q7. Solve: 2|3x - 10| = 1

Divide both sides by 2 (k>0 preserves equality): |3x - 10| = 1/2.
For |u| = c with c ≥ 0, solve u = c or u = −c.
3x - 10 = 1/2 ⇒ x = 7/2.
3x - 10 = -1/2 ⇒ x = 19/6.

Answer: x = 7/2 or x = 19/6

Q8. Solve: 2|-6x + 10| = 15

Divide both sides by 2 (k>0 preserves equality): |-6x + 10| = 15/2.
For |u| = c with c ≥ 0, solve u = c or u = −c.
-6x + 10 = 15/2 ⇒ x = 5/12.
-6x + 10 = -15/2 ⇒ x = 35/12.

Answer: x = 5/12 or x = 35/12

Q9. Solve: 3|1x + 2| = 11

Divide both sides by 3 (k>0 preserves equality): |1x + 2| = 11/3.
For |u| = c with c ≥ 0, solve u = c or u = −c.
1x + 2 = 11/3 ⇒ x = 5/3.
1x + 2 = -11/3 ⇒ x = -17/3.

Answer: x = 5/3 or x = -17/3

Q10. Solve: |-3x - 6| = 2

For |u| = c with c ≥ 0, solve u = c or u = −c.
-3x - 6 = 2 ⇒ x = -8/3.
-3x - 6 = -2 ⇒ x = -4/3.

Answer: x = -8/3 or x = -4/3

Q11. Solve: |4x - 1| = -1

Absolute value is never negative ⇒ no solution.

Answer: ∅

Q12. Solve: |3x - | = 13

For |u| = c with c ≥ 0, solve u = c or u = −c.
3x - 1 = 13 ⇒ x = 14/3.
3x - 1 = -13 ⇒ x = -4.

Answer: x = 14/3 or x = -4

Q13. Solve: 3|2x + 12| = 9

Divide both sides by 3 (k>0 preserves equality): |2x + 12| = 3.
For |u| = c with c ≥ 0, solve u = c or u = −c.
2x + 12 = 3 ⇒ x = -9/2.
2x + 12 = -3 ⇒ x = -15/2.

Answer: x = -9/2 or x = -15/2

Q14. Solve: 3|4x - 7| = 11

Divide both sides by 3 (k>0 preserves equality): |4x - 7| = 11/3.
For |u| = c with c ≥ 0, solve u = c or u = −c.
4x - 7 = 11/3 ⇒ x = 8/3.
4x - 7 = -11/3 ⇒ x = 5/6.

Answer: x = 8/3 or x = 5/6

Q15. Solve: 4|2x + 5| = 7

Divide both sides by 4 (k>0 preserves equality): |2x + 5| = 7/4.
For |u| = c with c ≥ 0, solve u = c or u = −c.
2x + 5 = 7/4 ⇒ x = -13/8.
2x + 5 = -7/4 ⇒ x = -27/8.

Answer: x = -13/8 or x = -27/8

Q16. Solve: 4|6x - 10| = 15

Divide both sides by 4 (k>0 preserves equality): |6x - 10| = 15/4.
For |u| = c with c ≥ 0, solve u = c or u = −c.
6x - 10 = 15/4 ⇒ x = 55/24.
6x - 10 = -15/4 ⇒ x = 25/24.

Answer: x = 55/24 or x = 25/24

Q17. Solve: 3|4x - 12| = 14

Divide both sides by 3 (k>0 preserves equality): |4x - 12| = 14/3.
For |u| = c with c ≥ 0, solve u = c or u = −c.
4x - 12 = 14/3 ⇒ x = 25/6.
4x - 12 = -14/3 ⇒ x = 11/6.

Answer: x = 25/6 or x = 11/6

Q18. Solve: 2|-3x - | = 12

```
Divide both sides by 2 (k>0 preserves equality): |-3x - 1| = 6.
For |u| = c with c ≥ 0, solve u = c or u = −c.
-3x - 1 = 6 ⇒ x = -7/3.
-3x - 1 = -6 ⇒ x = 5/3.
```

Answer: x = -7/3 or x = 5/3

Q19. Solve: |4x + 9| = |-1x - 5|

```
For |U| = |V|, solve U = V or U = −V.
Case 1: 4x + 9 = -1x - 5 ⇒ x = -14/5.
Case 2: 4x + 9 = -(-1x - 5) ⇒ x = -4/3.
```

Answer: x = -14/5 or x = -4/3

Q20. Solve: |-1x + 0| = |-1x + 1|

```
For |U| = |V|, solve U = V or U = −V.
Case 1: coefficients yield no finite solution.
Case 2: -1x + 0 = -(-1x + 1) ⇒ x = 1/2.
```

Answer: x = 1/2

Q21. Solve: |-4x + 7| = |-3x - 2|

```
For |U| = |V|, solve U = V or U = −V.
Case 1: -4x + 7 = -3x - 2 ⇒ x = 9.
Case 2: -4x + 7 = -(-3x - 2) ⇒ x = 5/7.
```

Answer: x = 5/7 or x = 9

Q22. Solve: |4x - 2| = |5x + 10|

```
For |U| = |V|, solve U = V or U = −V.
Case 1: 4x - 2 = 5x + 10 ⇒ x = -12.
Case 2: 4x - 2 = -(5x + 10) ⇒ x = -8/9.
```

Answer: x = -12 or x = -8/9

Q23. Solve: |-3x - 10| = |1x + 9|

```
For |U| = |V|, solve U = V or U = −V.
Case 1: -3x - 10 = 1x + 9 ⇒ x = -19/4.
Case 2: -3x - 10 = -(1x + 9) ⇒ x = -1/2.
```

Answer: x = -19/4 or x = -1/2

Q24. Solve: |4x + 8| = |4x + 7|

For |U| = |V|, solve U = V or U = −V.
Case 1: coefficients yield no finite solution.
Case 2: 4x + 8 = -(4x + 7) ⇒ x = -15/8.

Answer: x = -15/8

Q25. Solve: |5x - 6| = |4x + 2|

For |U| = |V|, solve U = V or U = −V.
Case 1: 5x - 6 = 4x + 2 ⇒ x = 8.
Case 2: 5x - 6 = -(4x + 2) ⇒ x = 4/9.

Answer: x = 4/9 or x = 8

Q26. Solve: |-1x + 6| > 2

For |u| > c with c ≥ 0 ⇒ u > 2 or u < -2.
Solve each inequality and take the union: (-∞, 4) ∪ (8, ∞).

Answer: (-∞, 4) ∪ (8, ∞)

Q27. Solve: |4x - 10| >= 11

For |u| >= c with c ≥ 0 ⇒ u >= 11 or u <= -11.
Solve each inequality and take the union: (-∞, -1/4] ∪ [21/4, ∞).

Answer: (-∞, -1/4] ∪ [21/4, ∞)

Q28. Solve: |5x - 9| <= 1

For |u| <= c with c ≥ 0 ⇒ −1 ≤ u ≤ 1.
Solve the two inequalities and intersect them: [8/5, 2].

Answer: [8/5, 2]

Q29. Solve: |-3x - 4| >= -1

Absolute value is always ≥ 0 and any 0 ≥ negative is true ⇒ all real numbers.

Answer: (-∞, ∞)

Q30. Solve: |4x - 6| < 3

For |u| < c with c ≥ 0 ⇒ −3 < u < 3.
Solve the two inequalities and intersect them: (3/4, 9/4).

Answer: (3/4, 9/4)

Q31. Solve: |-5x + 8| <= 8

```
For |u| <= c with c ≥ 0 ⇒ −8 ≤ u ≤ 8.
Solve the two inequalities and intersect them: [0, 16/5].
```

Answer: [0, 16/5]

Q32. Solve: |-4x - 5| > 2

```
For |u| > c with c ≥ 0 ⇒ u > 2 or u < -2.
Solve each inequality and take the union: (-∞, -7/4) ∪ (-3/4, ∞).
```

Answer: (-∞, -7/4) ∪ (-3/4, ∞)

Q33. Solve: |-2x - 6| > 5

```
For |u| > c with c ≥ 0 ⇒ u > 5 or u < -5.
Solve each inequality and take the union: (-∞, -11/2) ∪ (-1/2, ∞).
```

Answer: (-∞, -11/2) ∪ (-1/2, ∞)

Q34. Solve: |-4x + 2| >= 2

```
For |u| >= c with c ≥ 0 ⇒ u >= 2 or u <= -2.
Solve each inequality and take the union: (-∞, 0] ∪ [1, ∞).
```

Answer: (-∞, 0] ∪ [1, ∞)

Q35. Solve: |-4x - 8| <= 11

```
For |u| <= c with c ≥ 0 ⇒ −11 ≤ u ≤ 11.
Solve the two inequalities and intersect them: [-19/4, 3/4].
```

Answer: [-19/4, 3/4]

Q36. Solve: |-3x + 3| <= 5

```
For |u| <= c with c ≥ 0 ⇒ −5 ≤ u ≤ 5.
Solve the two inequalities and intersect them: [-2/3, 8/3].
```

Answer: [-2/3, 8/3]

Q37. Solve: |-6x + 1| > 11

```
For |u| > c with c ≥ 0 ⇒ u > 11 or u < -11.
Solve each inequality and take the union: (-∞, -5/3) ∪ (2, ∞).
```

Answer: (-∞, -5/3) ∪ (2, ∞)

Q38. Solve: |-1x + 6| > 10

```
For |u| > c with c ≥ 0 ⇒ u > 10 or u < -10.
Solve each inequality and take the union: (-∞, -4) ∪ (16, ∞).
```

Answer: (-∞, -4) ∪ (16, ∞)

Q39. Solve: |-2x - 8| > -1

Absolute value is always ≥ 0 and any 0 ≥ negative is true ⇒ all real numbers.

Answer: (-∞, ∞)

Q40. Solve: |6x + 6| >= 9

For |u| >= c with c ≥ 0 ⇒ u >= 9 or u <= -9.
Solve each inequality and take the union: (-∞, -5/2] ∪ [1/2, ∞).

Answer: (-∞, -5/2] ∪ [1/2, ∞)

Q41. Solve: |5x + 5| < |1x + 2|

Both sides are nonnegative. For absolute values, |U| ◊ |V| ⇔ U² ◊ V² (same ◊).
Compute (ax+b)² − (cx+d)² = (4x + 3)·(6x + 7).
Critical points (where product changes sign): x = -7/6, -3/4.
Use a sign chart on the two linear factors to find where the product satisfies '< 0'.
Solution set: (-7/6, -3/4).

Answer: (-7/6, -3/4)

Q42. Solve: |-2x - 4| > |-1x - 5|

Both sides are nonnegative. For absolute values, |U| ◊ |V| ⇔ U² ◊ V² (same ◊).
Compute (ax+b)² − (cx+d)² = (-1x + 1)·(-3x - 9).
Critical points (where product changes sign): x = -3, 1.
Use a sign chart on the two linear factors to find where the product satisfies '> 0'.
Solution set: (-∞, -3) ∪ (1, ∞).

Answer: (-∞, -3) ∪ (1, ∞)

Q43. Solve: |-2x - 4| < |3x - 3|

Both sides are nonnegative. For absolute values, |U| ◊ |V| ⇔ U² ◊ V² (same ◊).
Compute (ax+b)² − (cx+d)² = (-5x - 1)·(1x - 7).
Critical points (where product changes sign): x = -1/5, 7.
Use a sign chart on the two linear factors to find where the product satisfies '< 0'.
Solution set: (-∞, -1/5) ∪ (7, ∞).

Answer: (-∞, -1/5) ∪ (7, ∞)

Q44. Solve: |-4x - 3| < |-1x + 3|

Both sides are nonnegative. For absolute values, |U| ◊ |V| ⇔ U² ◊ V² (same ◊).
Compute (ax+b)² − (cx+d)² = (-3x - 6)·(-5x + 0).
Critical points (where product changes sign): x = -2, 0.
Use a sign chart on the two linear factors to find where the product satisfies '< 0'.
Solution set: (-2, 0).

Answer: (-2, 0)

Q45. Solve: |-1x + 6| < |-5x + 3|

Both sides are nonnegative. For absolute values, |U| ◊ |V| ⇔ U² ◊ V² (same ◊).
Compute (ax+b)² − (cx+d)² = (4x + 3)·(-6x + 9).
Critical points (where product changes sign): x = -3/4, 3/2.
Use a sign chart on the two linear factors to find where the product satisfies '< 0'.
Solution set: (-∞, -3/4) ∪ (3/2, ∞).

Answer: (-∞, -3/4) ∪ (3/2, ∞)