Algebra 2 Honors — Absolute Value Equations & Inequalities (Challenge Set)

25 Hard/Tricky Problems with Full Solution Steps (No Repeats)

Each problem is designed to be non-routine: multiple absolute values, piecewise casework, sign charts, bands, and near-edge cases. Solutions are detailed and justified.

Q1. Solve: $5|-7x - | = 8$

Solution Steps:

Divide both sides by 5 (>0): $|-7x - 1| = 8/5$.
For $|u| = c$ with $c \geq 0$, set $u = c$ or $u = -c$.
$-7x - 1 = 8/5 \Rightarrow x = -13/35$.
$-7x - 1 = -8/5 \Rightarrow x = 3/35$.

Answer: $x = -13/35$ or $x = 3/35$

Q2. Solve: $3|4x - 7| = 5$

Solution Steps:

Divide both sides by 3 (>0): $|4x - 7| = 5/3$.
For $|u| = c$ with $c \geq 0$, set $u = c$ or $u = -c$.
$4x - 7 = 5/3 \Rightarrow x = 13/6$.
$4x - 7 = -5/3 \Rightarrow x = 4/3$.

Answer: $x = 13/6$ or $x = 4/3$

Q3. Solve: $5|4x + 2| = 4$

Solution Steps:

Divide both sides by 5 (>0): $|4x + 2| = 4/5$.
For $|u| = c$ with $c \geq 0$, set $u = c$ or $u = -c$.
$4x + 2 = 4/5 \Rightarrow x = -3/10$.
$4x + 2 = -4/5 \Rightarrow x = -7/10$.

Answer: $x = -3/10$ or $x = -7/10$

Q4. Solve: $3|-7x + 10| = 4$

Solution Steps:

Divide both sides by 3 (>0): $|-7x + 10| = 4/3$.
For $|u| = c$ with $c \geq 0$, set $u = c$ or $u = -c$.
$-7x + 10 = 4/3 \Rightarrow x = 26/21$.
$-7x + 10 = -4/3 \Rightarrow x = 34/21$.

Answer: $x = 26/21$ or $x = 34/21$

Q5. Solve: $2|-3x - 5| = -3$

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Solution Steps:

Divide both sides by 2 (>0): |-3x - 5| = -3/2.
Absolute value cannot be negative ⇒ no solution.
```

Answer: $\varnothing$

Q6. Solve: $|7x - 5| = 11$

```
Solution Steps:

For |u| = c with c ≥ 0, set u = c or u = −c.
7x - 5 = 11 ⇒ x = 16/7.
7x - 5 = -11 ⇒ x = -6/7.
```

Answer: $x = 16/7$ or $x = -6/7$

Q7. Solve: $|-5x - 8| = 9$

```
Solution Steps:

For |u| = c with c ≥ 0, set u = c or u = −c.
-5x - 8 = 9 ⇒ x = -17/5.
-5x - 8 = -9 ⇒ x = 1/5.
```

Answer: $x = -17/5$ or $x = 1/5$

Q8. Solve: $|-6x + 2| = |-6x + 11|$

```
Solution Steps:

For |U| = |V|, solve U = V or U = −V.
Case 1 yields no finite solution (parallel lines).
Case 2: -6x + 2 = -(-6x + 11) ⇒ x = 13/12.
```

Answer: $x = 13/12$

Q9. Solve: $|6x - 5| = |6x - 11|$

```
Solution Steps:

For |U| = |V|, solve U = V or U = −V.
Case 1 yields no finite solution (parallel lines).
Case 2: 6x - 5 = -(6x - 11) ⇒ x = 4/3.
```

Answer: $x = 4/3$

Q10. Solve: |-4x - 3| = |3x + 12|

```
Solution Steps:

For |U| = |V|, solve U = V or U = –V.
Case 1: -4x - 3 = 3x + 12 ⇒ x = -15/7.
Case 2: -4x - 3 = -(3x + 12) ⇒ x = 9.
```

Answer: x = -15/7 or x = 9

Q11. Solve: |5x - 5| = |3x + 5|

```
Solution Steps:

For |U| = |V|, solve U = V or U = –V.
Case 1: 5x - 5 = 3x + 5 ⇒ x = 5.
Case 2: 5x - 5 = -(3x + 5) ⇒ x = 0.
```

Answer: x = 0 or x = 5

Q12. Solve: |3x + 7| = |5x|

```
Solution Steps:

For |U| = |V|, solve U = V or U = –V.
Case 1: 3x + 7 = 5x ⇒ x = 7/2.
Case 2: 3x + 7 = -(5x) ⇒ x = -7/8.
```

Answer: x = -7/8 or x = 7/2

Q13. Solve: |x - 6| + |-4x - 8| = 9

```
Solution Steps:

Breakpoints where expressions change sign: x = -2, 6.
Solve piecewise on each interval using fixed signs of the expressions.
Candidate solutions (validated): -11/5, -5/3.
```

Answer: x = -11/5 or x = -5/3

Q14. Solve: |6x - 3| + |x + 9| = 6

```
Solution Steps:

Breakpoints where expressions change sign: x = -9, 1/2.
No interval produces a valid solution after checking.
```

Answer: ∅

Q15. Solve: |-2x - 8| + |6x + 9| = 15

```
Solution Steps:

Breakpoints where expressions change sign: x = -4, -3/2.
Solve piecewise on each interval using fixed signs of the expressions.
Candidate solutions (validated): -4, -1/4.
```

Answer: x = -4 or x = -1/4

Q16. Solve: |-x - 3| + |-x + 6| = 8

```
Solution Steps:

Breakpoints where expressions change sign: x = -3, 6.
No interval produces a valid solution after checking.
```

Answer: ∅

Q17. Solve: |3x + 7| < -4

```
Solution Steps:

Absolute value ≥ 0 cannot be less than a negative ⇒ ∅.
```

Answer: ∅

Q18. Solve: |5x - 3| <= 7

```
Solution Steps:

−7 ≤ 5x - 3 ≤ 7.
Intersect the two linear inequalities ⇒ [-4/5, 2].
```

Answer: [-4/5, 2]

Q19. Solve: |3x - 4| >= 7

```
Solution Steps:

3x - 4 >= 7 or 3x - 4 <= -7.
Take the union of the two solution sets ⇒ (-∞, -1] ∪ [11/3, ∞).
```

Answer: (-∞, -1] ∪ [11/3, ∞)

Q20. Solve: $|-4x + 6| > -1$

```
Solution Steps:

Absolute value is always ≥ 0 and any ≥ negative is true ⇒ all reals.
```

Answer: $(-\infty, \infty)$

Q21. Solve: $|6x - 8| < -3$

```
Solution Steps:

Absolute value ≥ 0 cannot be less than a negative ⇒ ∅.
```

Answer: $\varnothing$

Q22. Solve: $|2x + 4| < |-4x - 6|$

```
Solution Steps:

Square both sides (nonnegative) to compare: (ax+b)^2 < (cx+d)^2.
Compute difference of squares to reduce to sign of the product of two linear factors.
Critical points: x = -5/3, -1.
Use a sign chart of the factors to determine where the product satisfies '< 0'.
Solution set: (-∞, -5/3) ∪ (-1, ∞).
```

Answer: $(-\infty, -5/3) \cup (-1, \infty)$

Q23. Solve: $|4x + 1| < |-3x + 4|$

```
Solution Steps:

Square both sides (nonnegative) to compare: (ax+b)^2 < (cx+d)^2.
Compute difference of squares to reduce to sign of the product of two linear factors.
Critical points: x = -5, 3/7.
Use a sign chart of the factors to determine where the product satisfies '< 0'.
Solution set: (-5, 3/7).
```

Answer: $(-5, 3/7)$

Q24. Solve: $0 < |-7x - 9| < 6$

```
Solution Steps:

Intersect the sets |...| < 6 and |...| > 0.
Solution set: [-15/7, -9/7) ∪ (-9/7, -3/7].
```

Answer: $[-15/7, -9/7) \cup (-9/7, -3/7]$

Q25. Solve: $5 \le |-5x + 6| < 12$

```
Solution Steps:

Intersect the sets |...| < 12 and |...| >= 5.
Solution set: [-6/5, 1/5] ∪ [11/5, 18/5].
```

Answer: $[-6/5, 1/5] \cup [11/5, 18/5]$