Algebra 2 Honors — Systems of Equations (2 & 3 Variables)

40 Problems with Detailed, Step-by-Step Solutions (No Repeats)

Directions: Solve each system. Use elimination, substitution, or matrices. Each solution shows clean, logically ordered steps.

Part A — Systems in Two Variables (20 problems)

Q1. Solve the system:

$-6x - 2y = -1$

$-5x + 6y = -6$

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Solution Steps:
• Multiply (1) by -5 and (2) by 6 to make x-coefficients opposites.
• New equations: 30x + 10y = 5 and -30x + 36y = -36
• Add the equations: 46y = -31 ⟹ y = -31/46.
• Substitute into (1): -6x + -2(-31/46) = -1 ⟹ x = 9/23.
```

Answer: (x, y) = (9/23, -31/46)

Q2. Solve the system:

$-5x - 6y = 3$

$2y = -4$

```
Solution Steps:
• Multiply (1) by -1 and (2) by -3 to eliminate y.
• New equations: 5x + 6y = -3 and -6y = 12
• Add the equations: 5x = 9 ⟹ x = 9/5.
• Substitute into (1): -5(9/5) + -6y = 3 ⟹ y = -2.
```

Answer: (x, y) = (9/5, -2)

Q3. Solve the system:

$-x - 6y = -4$

$-4x - 4y = 3$

```
Solution Steps:
• Multiply (1) by -4 and (2) by 1 to make x-coefficients opposites.
• New equations: 4x + 24y = 16 and -4x - 4y = 3
• Add the equations: 20y = 19 ⟹ y = 19/20.
• Substitute into (1): -1x + -6(19/20) = -4 ⟹ x = -17/10.
```

Answer: (x, y) = (-17/10, 19/20)

Q4. Solve the system:
 -x + 5y = -14
 -3x - 4y = 14

　　　　Solution Steps:
　　　　• Multiply (1) by -3 and (2) by 1 to make x-coefficients opposites.
　　　　• New equations: 3x - 15y = 42 and -3x - 4y = 14
　　　　• Add the equations: -19y = 56 $\Rightarrow$ y = -56/19.
　　　　• Substitute into (1): -1x + 5(-56/19) = -14 $\Rightarrow$ x = -14/19.

Answer: (x, y) = (-14/19, -56/19)

Q5. Solve the system:
 6y = -2
 -5x + 3y = -12

　　　　Solution Steps:
　　　　• Multiply (1) by 1 and (2) by -2 to eliminate y.
　　　　• New equations: 6y = -2 and 10x - 6y = 24
　　　　• Add the equations: 10x = 22 $\Rightarrow$ x = 11/5.
　　　　• Substitute into (1): 0(11/5) + 6y = -2 $\Rightarrow$ y = -1/3.

Answer: (x, y) = (11/5, -1/3)

Q6. Solve the system:
 -6x - y = -11
 5x + y = 14

　　　　Solution Steps:
　　　　• Multiply (1) by -5 and (2) by -6 to make x-coefficients opposites.
　　　　• New equations: 30x + 5y = 55 and -30x - 6y = -84
　　　　• Add the equations: -1y = -29 $\Rightarrow$ y = 29.
　　　　• Substitute into (1): -6x + -1(29) = -11 $\Rightarrow$ x = -3.

Answer: (x, y) = (-3, 29)

Q7. Solve the system:
 -6x - 5y = -10
 4x - 4y = 11

　　　　Solution Steps:
　　　　• Multiply (1) by -2 and (2) by -3 to make x-coefficients opposites.
　　　　• New equations: 12x + 10y = 20 and -12x + 12y = -33
　　　　• Add the equations: 22y = -13 $\Rightarrow$ y = -13/22.
　　　　• Substitute into (1): -6x + -5(-13/22) = -10 $\Rightarrow$ x = 95/44.

Answer: (x, y) = (95/44, -13/22)

Q8. Solve the system:

-6x - 2y = 1

3y = -6

Solution Steps:
- Multiply (1) by -3 and (2) by -2 to eliminate y.
- New equations: 18x + 6y = -3 and -6y = 12
- Add the equations: 18x = 9 $\Rightarrow$ x = 1/2.
- Substitute into (1): -6(1/2) + -2y = 1 $\Rightarrow$ y = -2.

Answer: (x, y) = (1/2, -2)

Q9. Solve the system:

x - 6y = -4

6x - 2y = 2

Solution Steps:
- Multiply (1) by 6 and (2) by -1 to make x-coefficients opposites.
- New equations: 6x - 36y = -24 and -6x + 2y = -2
- Add the equations: -34y = -26 $\Rightarrow$ y = 13/17.
- Substitute into (1): 1x + -6(13/17) = -4 $\Rightarrow$ x = 10/17.

Answer: (x, y) = (10/17, 13/17)

Q10. Solve the system:

6x + y = 6

-3x + 3y = 2

Solution Steps:
- Multiply (1) by 1 and (2) by 2 to make x-coefficients opposites.
- New equations: 6x + y = 6 and -6x + 6y = 4
- Add the equations: 7y = 10 $\Rightarrow$ y = 10/7.
- Substitute into (1): 6x + 1(10/7) = 6 $\Rightarrow$ x = 16/21.

Answer: (x, y) = (16/21, 10/7)

Q11. Solve the system:

3x + 6y = -2

-x - 6y = 14

Solution Steps:
- Multiply (1) by 1 and (2) by 3 to make x-coefficients opposites.
- New equations: 3x + 6y = -2 and -3x - 18y = 42
- Add the equations: -12y = 40 $\Rightarrow$ y = -10/3.
- Substitute into (1): 3x + 6(-10/3) = -2 $\Rightarrow$ x = 6.

Answer: (x, y) = (6, -10/3)

Q12. Solve the system:
 6x + 4y = 7
 2x = 3

> Solution Steps:
> - Multiply (1) by 1 and (2) by -3 to make x-coefficients opposites.
> - New equations: 6x + 4y = 7 and -6x = -9
> - Add the equations: 4y = -2 ⟹ y = -1/2.
> - Substitute into (1): 6x + 4(-1/2) = 7 ⟹ x = 3/2.

Answer: (x, y) = (3/2, -1/2)

Q13. Solve the system:
 4x + 6y = 14
 4x + 3y = 12

> Solution Steps:
> - Multiply (1) by 1 and (2) by -1 to make x-coefficients opposites.
> - New equations: 4x + 6y = 14 and -4x - 3y = -12
> - Add the equations: 3y = 2 ⟹ y = 2/3.
> - Substitute into (1): 4x + 6(2/3) = 14 ⟹ x = 5/2.

Answer: (x, y) = (5/2, 2/3)

Q14. Solve the system:
 x + 2y = -3
 4x = 2

> Solution Steps:
> - Multiply (1) by 4 and (2) by -1 to make x-coefficients opposites.
> - New equations: 4x + 8y = -12 and -4x = -2
> - Add the equations: 8y = -14 ⟹ y = -7/4.
> - Substitute into (1): 1x + 2(-7/4) = -3 ⟹ x = 1/2.

Answer: (x, y) = (1/2, -7/4)

Q15. Solve the system:
 -5x + y = -9
 4x - 2y = -4

> Solution Steps:
> - Multiply (1) by -4 and (2) by -5 to make x-coefficients opposites.
> - New equations: 20x - 4y = 36 and -20x + 10y = 20
> - Add the equations: 6y = 56 ⟹ y = 28/3.
> - Substitute into (1): -5x + 1(28/3) = -9 ⟹ x = 11/3.

Answer: (x, y) = (11/3, 28/3)

Q16. Solve the system:
-x + 2y = 9
-5y = 0

       ```
Solution Steps:
```
- Multiply (1) by 5 and (2) by 2 to eliminate y.
- New equations: -5x + 10y = 45 and -10y = 0
- Add the equations: -5x = 45 $\Rightarrow$ x = -9.
- Substitute into (1): -1(-9) + 2y = 9 $\Rightarrow$ y = 0.

Answer: (x, y) = (-9, 0)

Q17. Solve the system:
3y = -14
6x - y = -8

       ```
Solution Steps:
```
- Multiply (1) by 1 and (2) by 3 to eliminate y.
- New equations: 3y = -14 and 18x - 3y = -24
- Add the equations: 18x = -38 $\Rightarrow$ x = -19/9.
- Substitute into (1): 0(-19/9) + 3y = -14 $\Rightarrow$ y = -14/3.

Answer: (x, y) = (-19/9, -14/3)

Q18. Solve the system:
3x - 5y = -14
4x - y = 14

       ```
Solution Steps:
```
- Multiply (1) by 4 and (2) by -3 to make x-coefficients opposites.
- New equations: 12x - 20y = -56 and -12x + 3y = -42
- Add the equations: -17y = -98 $\Rightarrow$ y = 98/17.
- Substitute into (1): 3x + -5(98/17) = -14 $\Rightarrow$ x = 84/17.

Answer: (x, y) = (84/17, 98/17)

Q19. Solve the system:
4x - y = -3
-8x + 2y = 6

       ```
Solution Steps:
```
- The second equation is a scalar multiple of the first; elimination yields 0 = 0.

Answer: Infinitely many solutions (coincident lines).

Q20. Solve the system:
x + 2y = 4
3x + 6y = 5

       ```
Solution Steps:
```
- Left sides are proportional but constants differ; elimination yields 0 = nonzero.

Answer: No solution (parallel lines).

Part B — Systems in Three Variables (20 problems)

Q21. Solve the system:

$3y - z = 0$

$4x + 3y - 2z = 12$

$-2x + 4y + 3z = 2$

       Solution Steps:

       • Use Cramer's Rule (or elimination). Compute determinants:

       Δ = -46, Δx = -162, Δy = -32, Δz = -96.

       • Then x = Δx/Δ = 81/23, y = Δy/Δ = 16/23, z = Δz/Δ = 48/23.

Answer: (x, y, z) = (81/23, 16/23, 48/23)

Q22. Solve the system:

$-4x - 4y - 2z = 1$

$3x - y - 4z = 8$

$y = 1$

       Solution Steps:

       • Use Cramer's Rule (or elimination). Compute determinants:

       Δ = -22, Δx = 2, Δy = -22, Δz = 51.

       • Then x = Δx/Δ = -1/11, y = Δy/Δ = 1, z = Δz/Δ = -51/22.

Answer: (x, y, z) = (-1/11, 1, -51/22)

Q23. Solve the system:

$-4x - 3y - 3z = 5$

$4x + y - 3z = 0$

$-3x + 2y - 4z = -10$

       Solution Steps:

       • Use Cramer's Rule (or elimination). Compute determinants:

       Δ = -116, Δx = -110, Δy = 365, Δz = -25.

       • Then x = Δx/Δ = 55/58, y = Δy/Δ = -365/116, z = Δz/Δ = 25/116.

Answer: (x, y, z) = (55/58, -365/116, 25/116)

Q24. Solve the system:

$4x + 2y - 4z = 1$

$x + 3z = -1$

$-4x + 4y + z = -12$

       Solution Steps:

       • Use Cramer's Rule (or elimination). Compute determinants:

       Δ = -90, Δx = -66, Δy = 191, Δz = 52.

       • Then x = Δx/Δ = 11/15, y = Δy/Δ = -191/90, z = Δz/Δ = -26/45.

Answer: (x, y, z) = (11/15, -191/90, -26/45)

Q25. Solve the system:

$-2x + 2y - 3z = 0$

$-2x + y - z = 5$

$-3x - 4y + 4z = -3$

Solution Steps:

- Use Cramer's Rule (or elimination). Compute determinants:

  $\Delta = -11$, $\Delta x = 17$, $\Delta y = -97$, $\Delta z = -76$.

- Then $x = \Delta x / \Delta = -17/11$, $y = \Delta y / \Delta = 97/11$, $z = \Delta z / \Delta = 76/11$.

Answer: $(x, y, z) = (-17/11, 97/11, 76/11)$

Q26. Solve the system:

$-2x + 2z = 8$

$x - 2y + 4z = -7$

$x + 4y - 3z = 6$

Solution Steps:

- Use Cramer's Rule (or elimination). Compute determinants:

  $\Delta = 32$, $\Delta x = -112$, $\Delta y = 88$, $\Delta z = 16$.

- Then $x = \Delta x / \Delta = -7/2$, $y = \Delta y / \Delta = 11/4$, $z = \Delta z / \Delta = 1/2$.

Answer: $(x, y, z) = (-7/2, 11/4, 1/2)$

Q27. Solve the system:

$2x + 2y - 4z = -3$

$-2x + 3y - 4z = 1$

$x + 3y + 3z = 12$

Solution Steps:

- Use Cramer's Rule (or elimination). Compute determinants:

  $\Delta = 82$, $\Delta x = -33$, $\Delta y = 196$, $\Delta z = 143$.

- Then $x = \Delta x / \Delta = -33/82$, $y = \Delta y / \Delta = 98/41$, $z = \Delta z / \Delta = 143/82$.

Answer: $(x, y, z) = (-33/82, 98/41, 143/82)$

Q28. Solve the system:

$-4x - 2y + 4z = 4$

$x + 4y + 4z = -9$

$4x - 2y + z = 5$

Solution Steps:

- Use Cramer's Rule (or elimination). Compute determinants:

  $\Delta = -150$, $\Delta x = -18$, $\Delta y = 340$, $\Delta z = 2$.

- Then $x = \Delta x / \Delta = 3/25$, $y = \Delta y / \Delta = -34/15$, $z = \Delta z / \Delta = -1/75$.

Answer: $(x, y, z) = (3/25, -34/15, -1/75)$

Q29. Solve the system:
x - y + z = -4
-3x + 4z = 0
-x - y + z = -11

```
Solution Steps:
• Use Cramer's Rule (or elimination). Compute determinants:
Δ = 8, Δx = 28, Δy = 81, Δz = 21.
• Then x = Δx/Δ = 7/2, y = Δy/Δ = 81/8, z = Δz/Δ = 21/8.
```

Answer: (x, y, z) = (7/2, 81/8, 21/8)

Q30. Solve the system:
4x - 3y - z = -10
4x - y - 4z = 7
-x - z = -2

```
Solution Steps:
• Use Cramer's Rule (or elimination). Compute determinants:
Δ = -19, Δx = -53, Δy = -139, Δz = 15.
• Then x = Δx/Δ = 53/19, y = Δy/Δ = 139/19, z = Δz/Δ = -15/19.
```

Answer: (x, y, z) = (53/19, 139/19, -15/19)

Q31. Solve the system:
-3x + y + 4z = 11
4x - 2y + 4z = 0
-2x + 3y + 3z = 12

```
Solution Steps:
• Use Cramer's Rule (or elimination). Compute determinants:
Δ = 66, Δx = -54, Δy = 116, Δz = 112.
• Then x = Δx/Δ = -9/11, y = Δy/Δ = 58/33, z = Δz/Δ = 56/33.
```

Answer: (x, y, z) = (-9/11, 58/33, 56/33)

Q32. Solve the system:
-4x + 3z = 7
x + 2y - z = 7
4x - 2y - 4z = 3

```
Solution Steps:
• Use Cramer's Rule (or elimination). Compute determinants:
Δ = 10, Δx = -130, Δy = 25, Δz = -150.
• Then x = Δx/Δ = -13, y = Δy/Δ = 5/2, z = Δz/Δ = -15.
```

Answer: (x, y, z) = (-13, 5/2, -15)

Q33. Solve the system:
 x - 3y - 4z = -6
 x + 3y + z = 12
 2x + y + z = 2

```
Solution Steps:
• Use Cramer's Rule (or elimination). Compute determinants:
Δ = 19, Δx = -6, Δy = 92, Δz = -42.
• Then x = Δx/Δ = -6/19, y = Δy/Δ = 92/19, z = Δz/Δ = -42/19.
```

Answer: (x, y, z) = (-6/19, 92/19, -42/19)

Q34. Solve the system:
 x - y - 2z = -7
 2x - 4y + 2z = 5
 2x + 4y = 0

```
Solution Steps:
• Use Cramer's Rule (or elimination). Compute determinants:
Δ = -44, Δx = 16, Δy = -8, Δz = -142.
• Then x = Δx/Δ = -4/11, y = Δy/Δ = 2/11, z = Δz/Δ = 71/22.
```

Answer: (x, y, z) = (-4/11, 2/11, 71/22)

Q35. Solve the system:
 x - 3y = -11
 -3x + 4z = 6
 x - 4y + 2z = 0

```
Solution Steps:
• Use Cramer's Rule (or elimination). Compute determinants:
Δ = -14, Δx = -140, Δy = -98, Δz = -126.
• Then x = Δx/Δ = 10, y = Δy/Δ = 7, z = Δz/Δ = 9.
```

Answer: (x, y, z) = (10, 7, 9)

Q36. Solve the system:
 -3y + 3z = -5
 x - y + 3z = 10
 3y + z = 2

```
Solution Steps:
• Use Cramer's Rule (or elimination). Compute determinants:
Δ = 12, Δx = 158, Δy = 11, Δz = -9.
• Then x = Δx/Δ = 79/6, y = Δy/Δ = 11/12, z = Δz/Δ = -3/4.
```

Answer: (x, y, z) = (79/6, 11/12, -3/4)

Q37. Solve the system:
 4x + z = 11
 -4x - 2y - 3z = -8
 -4x + 2y + 3z = 3

        Solution Steps:
        • Use Cramer's Rule (or elimination). Compute determinants:
        Δ = -16, Δx = -10, Δy = 160, Δz = -136.
        • Then x = Δx/Δ = 5/8, y = Δy/Δ = -10, z = Δz/Δ = 17/2.

Answer: (x, y, z) = (5/8, -10, 17/2)

Q38. Solve the system:
 4x - 4y + 3z = 2
 4x - 2y + z = 4
 x - y - 4z = 7

        Solution Steps:
        • Use Cramer's Rule (or elimination). Compute determinants:
        Δ = -38, Δx = -44, Δy = 14, Δz = 52.
        • Then x = Δx/Δ = 22/19, y = Δy/Δ = -7/19, z = Δz/Δ = -26/19.

Answer: (x, y, z) = (22/19, -7/19, -26/19)

Q39. Solve the system:
 3x + y + 2z = 2
 2x + y + z = 4
 5x + 2y + 3z = 6

        Solution Steps:
        • One equation is a linear combination of the others with matching constant; elimination yields
        a row 0 = 0.

Answer: Infinitely many solutions (dependent planes).

Q40. Solve the system:
 2x - y + z = 0
 2x + y + z = 1
 4x + 2z = 3

        Solution Steps:
        • One equation is a linear combination of the others but with a different constant; elimination
        yields 0 = nonzero.

Answer: No solution (inconsistent planes).