# Polynomial Factorizations with Full Steps (101–120)

## 101) 5x² − 2x + 3

• Check GCF → none.

• Use discriminant to test factorability over integers: Δ = b² − 4ac = (−2)² − 4·5·3 = 4 − 60 = −56 < 0.

• Negative discriminant ⇒ no real linear factors; over integers/rationals this quadratic is prime.

➤ Final factorization: Irreducible over the rationals (prime).

## 102) x³ − 343

• Recognize difference of cubes: x³ − 7³.

• Apply a³ − b³ = (a − b)(a² + ab + b²).

➤ Final factorization: (x − 7)(x² + 7x + 49)

## 103) 40y² + 28y − 48

• GCF 4: 4(10y² + 7y − 12).

• AC method on 10y² + 7y − 12 → AC = 10·(−12) = −120.

• Find integers with product −120 and sum 7 → 15 and −8.

• Split and group: 10y² + 15y − 8y − 12 = 5y(2y+3) − 4(2y+3).

➤ Final factorization: 4(5y − 4)(2y + 3)

## 104) 3ab − 5bc + bd

• Factor out common b.

• Reorder inside if desired.

➤ Final factorization: b(3a + d − 5c)

## 105) 8c⁶ − 125d⁶

• Write as cubes: (2c²)³ − (5d²)³.

• Use a³ − b³ formula.

➤ Final factorization: (2c² − 5d²)(4c⁴ + 10c²d² + 25d⁴)

## 106) 81 − 18z + z²

• Reorder: z² − 18z + 81.

• Perfect square: (z − 9)² since 81=9² and −18=−2·9.

➤ Final factorization: (z − 9)²

## 107) x⁴ + 10x³ + 25x²

• GCF x²: x²(x² + 10x + 25).

• Quadratic is a perfect square: (x + 5)².

➤ Final factorization: x²(x + 5)²

## 108) xz − xw − yz + yw

• Group in pairs: (xz − xw) + (−yz + yw).

• Factor each pair: x(z − w) − y(z − w).

• Common binomial (z − w).

➤ Final factorization: (x − y)(z − w)

## 109) y² + 5y − 36

• AC = 1·(−36) = −36; need sum 5 → 9 and −4.

• Factor.

➤ Final factorization: (y + 9)(y − 4)

## 110) x² − 11x − 42

• AC = 1·(−42) = −42; need sum −11 → −14 and +3.

➤ Final factorization: (x − 14)(x + 3)

## 111) 7a² − 7b²

• GCF 7: 7(a² − b²).

• Difference of squares.

➤ Final factorization: 7(a − b)(a + b)

## 112) 216 − a³

• Recognize difference of cubes: 6³ − a³.

• Use a³ − b³ with a=6, b=a.

➤ Final factorization: (6 − a)(36 + 6a + a²)

## 113) 81 + 18y + y²

• Reorder: y² + 18y + 81.

• Perfect square: (y + 9)².

➤ Final factorization: (y + 9)²

## 114) b² − 5b − 14

• AC = 1·(−14) = −14; need sum −5 → −7 and +2.

➤ Final factorization: (b − 7)(b + 2)

## 115) q⁴ − 10q³ + 21q²

• GCF q²: q²(q² − 10q + 21).

• Factor quadratic: AC = 1·21=21; sum −10 → −3 and −7.

➤ Final factorization: q²(q − 3)(q − 7)

## 116) 9x²y² − 25y⁴

• GCF y²: y²(9x² − 25y²).

• Difference of squares inside: (3x − 5y)(3x + 5y).

➤ Final factorization: y²(3x − 5y)(3x + 5y)

## 117) 105 + 8x − x²

• Reorder: −(x² − 8x − 105).

• Factor x² − 8x − 105: numbers with product −105 and sum −8 → −15 and +7.

• Apply the leading − sign.

➤ Final factorization: −(x − 15)(x + 7) (or (15 − x)(x + 7))

## 118) x² − 3x − 2

• AC = 1·(−2) = −2; integers adding to −3 do not multiply to −2.

• Discriminant Δ = (−3)² − 4·1·(−2) = 9 + 8 = 17 (not a perfect square).

• Therefore irreducible over the integers; over reals it factors with irrational roots.

➤ Final factorization: Prime over ℤ; over ℝ: (x − (3+√17)/2)(x − (3−√17)/2)

## 119) 6y³ + 48

• GCF 6: 6(y³ + 8).

• Sum of cubes: y³ + 2³ = (y + 2)(y² − 2y + 4).

➤ Final factorization: 6(y + 2)(y² − 2y + 4)

## 120) a³ − 14a² + 49a

• GCF a: a(a² − 14a + 49).

• Quadratic is a perfect square: (a − 7)².

➤ Final factorization: a(a − 7)²