Hypothesis-Testing Summary 1

1. Comparison of a sample mean with a specific population mean.

Example: H_0: $\mu = 100$

a. Use the z test when σ is known:

$$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

b. Use the t test when σ is unknown:

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}} \qquad \text{with d.f.} = n - 1$$

2. Comparison of a sample variance or standard deviation with a specific population variance or standard deviation.

Example: H_0: $\sigma^2 = 225$

Use the chi-square test:

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} \qquad \text{with d.f.} = n - 1$$

3. Comparison of two sample means.

Example: H_0: $\mu_1 = \mu_2$

a. Use the z test when the population variances are known:

$$z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

b. Use the t test for independent samples when the population variances are unknown and assume the sample variances are unequal:

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

with d.f. = the smaller of $n_1 - 1$ or $n_2 - 1$.

Formula for the t test for comparing two means (independent samples, variances equal):

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$$

with d.f. $= n_1 + n_2 - 2$.

c. Use the t test for means for dependent samples:

Example: H_0: $\mu_D = 0$

$$t = \frac{\bar{D} - \mu_D}{s_D/\sqrt{n}} \qquad \text{with d.f.} = n - 1$$

where n = number of pairs.

4. Comparison of a sample proportion with a specific population proportion.

Example: H_0: $p = 0.32$

Use the z test:

$$z = \frac{X - \mu}{\sigma} \qquad \text{or} \qquad z = \frac{\hat{p} - p}{\sqrt{pq/n}}$$

5. Comparison of two sample proportions.

Example: H_0: $p_1 = p_2$

Use the z test:

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}\,\bar{q}\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$$

where

$$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2} \qquad \hat{p}_1 = \frac{X_1}{n_1}$$

$$\bar{q} = 1 - \bar{p} \qquad \hat{p}_2 = \frac{X_2}{n_2}$$

6. Comparison of two sample variances or standard deviations.

Example: H_0: $\sigma_1^2 = \sigma_2^2$

Use the F test:

$$F = \frac{s_1^2}{s_2^2}$$

where

s_1^2 = larger variance   d.f.N. $= n_1 - 1$

s_2^2 = smaller variance   d.f.D. $= n_2 - 1$

7. Test of the significance of the correlation coefficient.

Example: H_0: $\rho = 0$

Use a t test:

$$t = r\sqrt{\frac{n-2}{1-r^2}} \qquad \text{with d.f.} = n - 2$$

8. Formula for the F test for the multiple correlation coefficient.

Example: H_0: $\rho = 0$

$$F = \frac{R^2/k}{(1 - R^2)/(n - k - 1)}$$

d.f.N. $= n - k$ d.f.D. $= n - k - 1$

9. Comparison of a sample distribution with a specific population.

Example: H_0: There is no difference between the two distributions.

Use the chi-square goodness-of-fit test:

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

d.f. = no. of categories − 1

10. Comparison of the independence of two variables.

Example: H_0: Variable A is independent of variable B.

Use the chi-square independence test:

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

d.f. $= (R - 1)(C - 1)$

11. Test for homogeneity of proportions.

Example: H_0: $p_1 = p_2 = p_3$

Use the chi-square test:

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

d.f. $= (R - 1)(C - 1)$

12. Comparison of three or more sample means.

Example: H_0: $\mu_1 = \mu_2 = \mu_3$

Use the analysis of variance test:

$$F = \frac{s_B^2}{s_W^2}$$

where

$$s_B^2 = \frac{\Sigma n_i(\bar{X}_i - \bar{X}_{GM})^2}{k - 1}$$

$$s_W^2 = \frac{\Sigma(n_i - 1)s_i^2}{\Sigma(n_i - 1)}$$

d.f.N. $= k - 1$ $N = n_1 + n_2 + \cdots + n_k$

d.f.D. $= N - k$ k = number of groups

13. Test when the F value for the ANOVA is significant. Use the Scheffé test to find what pairs of means are significantly different.

$$F_s = \frac{(\bar{X}_i - \bar{X}_j)^2}{s_W^2[(1/n_i) + (1/n_j)]}$$

$$F' = (k - 1)(\text{C.V.})$$

Use the Tukey test to find which pairs of means are significantly different.

$$q = \frac{\bar{X}_i - \bar{X}_j}{\sqrt{s_W^2/n}} \qquad \begin{array}{l}\text{d.f.N.} = k \\ \text{d.f.D.} = \text{degrees of freedom for } s_W^2\end{array}$$

14. Test for the two-way ANOVA.

Example:

H_0: There is no significant difference for the main effects.

H_1: There is no significant difference for the interaction effect.

$$\text{MS}_A = \frac{\text{SS}_A}{a - 1}$$

$$\text{MS}_B = \frac{\text{SS}_B}{b - 1}$$

$$\text{MS}_{A\times B} = \frac{\text{SS}_{A\times B}}{(a - 1)(b - 1)}$$

$$\text{MS}_W = \frac{\text{SS}_W}{ab(n - 1)}$$

$$F_A = \frac{\text{MS}_A}{\text{MS}_W} \qquad \begin{array}{l}\text{d.f.N.} = a - 1 \\ \text{d.f.D.} = ab(n - 1)\end{array}$$

$$F_B = \frac{\text{MS}_B}{\text{MS}_W} \qquad \begin{array}{l}\text{d.f.N.} = (b - 1) \\ \text{d.f.D.} = ab(n - 1)\end{array}$$

$$F_{A\times B} = \frac{\text{MS}_{A\times B}}{\text{MS}_W} \qquad \begin{array}{l}\text{d.f.N.} = (a - 1)(b - 1) \\ \text{d.f.D.} = ab(n - 1)\end{array}$$

15. Test to see whether the median of a sample is a specific value when $n \geq 26$.

Example: H_0: median = 100

Use the sign test:

$$z = \frac{(X + 0.5) - (n/2)}{\sqrt{n}/2}$$

16. Test to see whether two independent samples are obtained from populations that have identical distributions.

Example: H_0: There is no difference in the ages of the subjects.

Use the Wilcoxon rank sum test:

$$z = \frac{R - \mu_R}{\sigma_R}$$

where

$$\mu_R = \frac{n_1(n_1 + n_2 + 1)}{2}$$

$$\sigma_R = \sqrt{\frac{n_1 n_2(n_1 + n_2 + 1)}{12}}$$

*This summary is a continuation of Hypothesis-Tes Chapter 12.

18. Test to see whether three or more samples come from identical populations.

Example: H_0: There is no difference in the weights of the three groups.

Use the Kruskal-Wallis test:

$$H = \frac{12}{N(N+1)}\left(\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \cdots + \frac{R_k^2}{n_k}\right) - 3(N + 1)$$

19. Rank correlation coefficient.

$$r_s = 1 - \frac{6\,\Sigma d^2}{n(n^2 - 1)}$$

20. Test for randomness: Use the runs test.