

User's Guide and Developer's Manual Preview August 25, 2017

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# Chapter 1

# Introduction

QMCPACK is an open-source, high-performance electronic structure code that implements numerous Quantum Monte Carlo algorithms. Its main applications are electronic structure calculations of molecular, periodic 2D and periodic 3D solid-state systems. Variational Monte Carlo (VMC), diffusion Monte Carlo (DMC) and a number of other advanced QMC algorithms are implemented. By directly solving the Schrodinger equation, QMC methods offer greater accuracy than methods such as density functional theory, but at a trade-off of much greater computational expense. Distinct from many other correlated many-body methods, QMC methods are readily applicable to both bulk (periodic) and isolated molecular systems.

QMCPACK is written in C++ and designed with the modularity afforded by object-oriented programming. It makes extensive use of template metaprogramming to achieve high computational efficiency. Due to the modular architecture, the addition of new wavefunctions, algorithms, and observables is relatively straightforward. For parallelization QMCPACK utilizes a fully hybrid (OpenMP,CUDA)/MPI approach to optimize memory usage and to take advantage of the growing number of cores per SMP node or graphical processing units (GPUs) and accelerators. High parallel and computational efficiencies are achievable on the largest supercomputers. Finally, QMCPACK utilizes standard file formats for input and output in XML and HDF5 to facilitate data exchange.

This manual currently serves as an introduction to the essential features of QMCPACK and a guide to installing and running it. Over time this manual will be expanded to including a fuller introduction to QMC methods in general and to include more of the specialized features in QMCPACK.

# 1.1 Quickstart and a first QMCPACK calculation

If you are keen to get started this section describes how to quickly build and run QMCPACK on standard UNIX or Linux-like system. The autoconfiguring build system usually works without much fuss on these systems. If C++, MPI, BLAS/LAPACK, FFTW, HDF5, and CMake are already installed, QMCPACK can be built and run within five minutes. For supercomputers, cross-compilation systems, and other computer clusters the build system may require hints on the locations of libraries and which versions to use, typical of any code, see Chapter 2. Section 2.6 includes complete examples for common workstations and supercomputers that you can reuse.

To build QMCPACK:

- 1. Download the latest QMCPACK distribution from http://www.qmcpack.org
- 2. Untar the archive, e.g., tar xvf qmcpack\_v1.3.tar.gz

- 3. Check the instructions in the README
- 4. Run CMake in a suitable build directory to configure QMCPACK for your system: cd qmcpack/build; cmake ...
- 5. If CMake is unable to find all needed libraries, see Chapter 2 for instructions and specific build instructions for common systems.
- 6. Build QMCPACK: make or make -j 16, the latter for a faster parallel build on a system using, e.g., 16 processes.
- 7. The QMCPACK executable is bin/qmcpack

QMCPACK is distributed with examples illustrating different capabilities. Most of the examples are designed to run quickly with modest resources. We'll run a short diffusion Monte Carlo calculation of a water molecule:

- 1. Go to the appropriate example directory: cd ../examples/molecules
- 2. (Optional) Put the QMCPACK binary on your path: export PATH=\$PATH:location-of-qmcpack/build/bin
- 3. Run QMCPACK: ../../build/bin/qmcpack simple-H20.xml or qmcpack simple-H20.xml if you followed the step above.
- 4. The run will output to the screen and generate a number of files:

```
$1s H20*
H20.HF.wfs.xml H20.s001.scalar.dat H20.s002.cont.xml
H20.s002.qmc.xml H20.s002.stat.h5 H20.s001.qmc.xml
H20.s001.stat.h5 H20.s002.dmc.dat H20.s002.scalar.dat
```

5. Partially summarized results are in the standard text files with the suffixes scalar.dat and dmc.dat. They are viewable with any standard editor.

If you have python and matplotlib installed, you can use the qmca analysis utility to produce statistics and plots of the data. See Chapter 11 for information on analysing QMCPACK data.

```
export PATH=$PATH:location-of-qmcpack/nexus/executables
export PYTHONPATH=$PYTHONPATH:location-of-qmcpack/nexus/library
qmca H2O.s002.scalar.dat  # For statistical analysis of the DMC data
qmca -t -q e H2O.s002.scalar.dat # Graphical plot of DMC energy
```

The last command will produce a graph as per Fig. 1.1. This shows the average energy of the DMC walkers at each timestep. In a real simulation we would have to check equilibration, convergence with walker population, timestep etc.

Congratulations, you have completed a DMC calculation with QMCPACK!

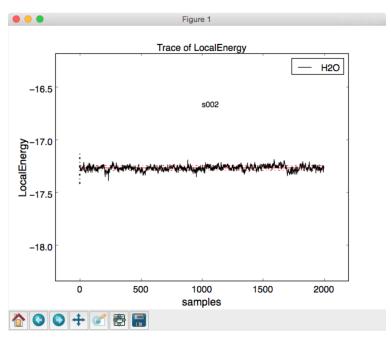


Figure 1.1: Trace of walker energies produced by qmca tool for simple water molecule example.

# 1.2 Authors and History

QMCPACK was initially written by Jeongnim Kim while in the group of Prof. David Ceperley at the University of Illinois at Urbana-Champaign, with later contributations at Oak Ridge National Laboratory. Over the years, many others have contributed, particularly students and researchers in the groups of Prof. David Ceperley and Prof. Richard M. Martin, as well as staff at Lawrence Livermore National Laboratory, Sandia National Laboratories, Argonne National Laboratory, and Oak Ridge National Laboratory.

The primary and original author of the code is Jeongnim Kim. Additional developers, contributors, and advisors include: Anouar Benali, Mark A. Berrill, David M. Ceperley, Simone Chiesa, Raymond C. III Clay, Bryan Clark, Kris T. Delaney, Kenneth P. Esler, Paul R. C. Kent, Jaron T. Krogel, Ying Wai Li, Ye Luo, Jeremy McMinis, Miguel A. Morales, William D. Parker, Nichols A. Romero, Luke Shulenburger, Norman M. Tubman, and Jordan E. Vincent.

If you should be added to this list please let us know.

Development of QMCPACK has been supported financially by several grants, including:

- "Network for ab initio many-body methods: development, education and training" supported through the Predictive Theory and Modeling for Materials and Chemical Science program by the U.S. Department of Energy Office of Science, Basic Energy Sciences.
- "QMC Endstation", supported by Accelerating Delivery of Petascale Computing Environment at the DOE Leadership Computing Facility at ORNL.
- PetaApps, supported by the U. S. National Science Foundation.
- Materials Computational Center, supported by the U.S. National Science Foundation.

# 1.3 Support and Contacting the Developers

Questions about installing, applying or extending QMCPACK can be posted on the QMCPACK Google group https://groups.google.com/forum/#!forum/qmcpack. You may also email any of the developers, but we recommend checking the group first. Particular attention is given to any problem reports.

#### 1.4 Performance

QMCPACK implements modern Monte Carlo algorithms, is highly parallel, and is also written using very efficient code for high per-CPU or on node performance. In particular the code is highly vectorizable, giving high performance on modern CPUs and GPUs. We believe QMCPACK delivers performance either comparable to or better than other QMC codes when similar calculations are run, particularly for the most common QMC methods and for large systems. If you find a calculation where this is not the case, or you simply find performance slower than expected, please post on the Google group or contact one of the developers. These reports are valuable. If your calculation is sufficiently mainstream we will optimize QMCPACK to improve the performance.

# 1.5 Open source license

QMCPACK is distributed under the University of Illinois/NCSA Open Source License.

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Copyright is generally believed to remain with the authors of the individual sections of code. See the various notations in the source code as well as the code history.

# 1.6 Contributing to QMCPACK

QMCPACK is fully open source and we welcome contributions. Please post on the QMCPACK Google group or contact the developers. If you are planning a development, early discussions are encouraged. We will be able to tell you if anyone else if working on a similar feature or if any related work has been done in the past. Credit for your contribution can be obtained, e.g., through citation of a paper, or becoming one of the authors on the next version of the standard QMCPACK reference citation.

A guide to developing for QMCPACK, including instructions on how to work with GitHub and make pull requests (contributions) to the main source are listed on the QMCPACK GitHub wiki https://github.com/QMCPACK/qmcpack/wiki.

Please note the following guidelines for contributions:

- Additions should be fully synchronized with the latest release version and ideally the latest develop branch on github. Merging of code developed on older versions is error prone.
- Code should be cleanly formatted, commented, portable, and accessible to other programmers. i.e. If you need to use any clever tricks, add a comment to note this, why the trick is needed, how it works etc. Although we like high performance, ease of maintenance and accessibility are also considerations.
- Comment your code. You are not only writing it for the compiler for also for other humans! (We know this is a repeat of the previous point, but it is important enough to repeat.)
- Write a brief description of the method, algorithms and inputs and outputs suitable for inclusion in this manual.
- Develop some short tests that exercise the functionality that can be used for validation and for examples. We can help with this and their integration into the test system.

# 1.7 QMCPACK Roadmap

A general outline of the QMCPACK roadmap is given below. Suggestions for improvements are welcome, particularly those that would facilitate new scientific applications. For example, if an interface to a particular quantum chemical or density functional code would help, this would be given strong consideration.

#### 1.7.1 Code

We will to continue improving the accessibility and usability of QMCPACK, by combinations of more convenient input parameters, improved workflow, integration with more quantum chemical and density functional codes, and a wider range of examples.

In terms of methodological development, we expect to significantly increase the range of QMC algorithms in QMCPACK in the near future.

Computationally, we are porting QMCPACK to the next generation of supercomputer systems. The internal changes required to run on these systems efficiently are expected to benefit *all* platforms due to improved vectorization, cache utilization and memory performance.

#### 1.7.2 Documentation

This manual currently describes the core features of QMCPACK that are required for routine research calculations. i.e. the VMC and DMC methods, how to obtain and optimize trial wavefunctions, and simple observables. Over time this manual will be expanded to include a broader introduction to QMC methods and to describe more features of the code.

Due to its history as a research code, QMCPACK contains a variety of additional QMC methods, trial wavefunction forms, potentials (etc.) that, although not critical, may be very useful for specialized calculations or particular material or chemical systems. These "secret features" (every code has these) are not actually secret but simply lack descriptions, example inputs, and tests. You are encouraged to browse and read the source code to find them. New descriptions will be added over time, but can also be prioritized and added on request, e.g. if a specialized Jastrow factor would help or an historical Jastrow form is needed for benchmarking.

# Chapter 2

# Obtaining, installing and validating QMCPACK

This chapter describes how to obtain, build and validate QMCPACK. This process is designed to be as simple as possible and should be no harder than building a modern plane-wave density functional theory code such as Quantum ESPRESSO, QBox, or VASP. Parallel builds enable a complete compilation in under 2 minutes on a fast multicore system. If you are unfamiliar with building codes we suggest working with your system administrator to install QMCPACK.

# 2.1 Installation steps

To install QMCPACK, follow the steps listed below. Full details of each step are given in the referenced sections.

- 1. Download the source code, Sections 2.2 or 2.3.
- 2. Verify that you have the required compilers, libraries and tools installed, Section 2.4.
- 3. Run the cmake configure step and build with make, Section 2.5 and 2.5.1. Some examples for common systems are given in Section 2.6.
- 4. Run the tests to verify QMCPACK, Section 2.7.
- 5. Build the ppconvert utility in QMCPACK, Section 2.9.
- 6. Download and patch Quantum ESPRESSO. This patch adds the pw2qmcpack utility, Section 2.10.

Hints for high performance are in Section 2.11. Troubleshooting suggestions are in Section 2.12. Note that there are two different QMCPACK executables that can be produced: the general one, which is the default, and the "complex" version which support periodic calculations at arbitrary twist angles and k-points. This second version is enabled via a cmake configuration parameter, see Section 2.5.3. The general version only supports wavefunctions that can be made real. If you run a calculation that needs the complex version, QMCPACK will stop and inform you.

# 2.2 Obtaining the latest release version

Major releases of QMCPACK are distributed from http://www.qmcpack.org. These releases undergo the most testing. Unless there are specific reasons we encourage all production calculations to use the latest release versions.

Releases are usually compressed tar files indicating the version number, date, and often the source code revision control number corresponding to the release.

- Download the latest QMCPACK distribution from http://www.qmcpack.org.
- Untar the archive, e.g., tar xvf qmcpack\_v1.3.tar.gz

Releases can also be obtained from the 'master' branch of the QMCPACK git repository, similar to obtaining the development version (Sec. 2.3).

# 2.3 Obtaining the latest development version

The most recent development version of QMCPACK can be obtained anonymously via

git clone https://github.com/QMCPACK/qmcpack.git

Once checked-out, updates can be made via the standard git pull.

The 'develop' branch of the git repository contains the day-to-day development source with the latest updates, bugfixes etc. This may be useful for updates to the build system to support new machines, for support of the latest versions of Quantum ESPRESSO, or for updates to the documentation. Note that the development version may not be fully consistent with the online documentation. We attempt to keep the development version fully working. However, please be sure to run the tests and compare with previous release versions before using for any serious calculations. We try to keep bugs out, but occasionally they crawl in! Reports of any breakages are appreciated.

# 2.4 Prerequisites

The following are required to build QMCPACK. For workstations, these are available via the standard package manager. On shared supercomputers this software is usually installed by default and is often access via a modules environment - check your system documentation.

Use of the latest versions of all compilers and libraries is strongly encouraged, but not absolutely essential. Generally newer versions are faster - see Section 2.11 for performance suggestions.

- C/C++ compilers such as GCC, Intel, IBM XLC. LLVM-based compilers are supported only on a preliminary basis.
- MPI library such at OpenMPI http://open-mpi.org
- BLAS/LAPACK, numerical and linear algebra libraries. Use platform-optimized libraries
  where available, such as Intel MKL. ATLAS or other optimized open-source libraries may
  also be used http://math-atlas.sourceforge.net
- CMake, build utility, http://www.cmake.org

- Libxml2, XML parser, http://xmlsoft.org
- HDF5, portable I/O library, http://www.hdfgroup.org/HDF5/
- BOOST, peer-reviewed portable C++ source libraries, http://www.boost.org
- FFTW, FFT library, http://www.fftw.org/

To build the GPU accelerated version of QMCPACK an installation of NVIDIA CUDA development tools is required. Ensure that this is compatible with the C and C++ compiler versions you plan to use. Supported versions are included in the NVIDIA release notes.

Many of the utilities provided with QMCPACK use python (v2). The numpy and matplotlib libraries are required for full functionality.

Note that the standalone einspline library used by previous versions of QMCPACK is no longer required. A more optimized version is included inside. The standalone version should *not* be on any standard search paths because conflicts between the old and new include files can result.

# 2.5 Building with CMake

The build system for QMCPACK is based on CMake. It will autoconfigure based on the detected compilers and libraries. The most recent version of CMake has the best detection for the greatest variety of systems - at the time of writing this means CMake 3.4.3. The much older CMake 2.8 is known to work, but might not work optimally on your system.

Previously QMCPACK made extensive use of toolchains, but the build system has since been updated to eliminate the use of toolchain files for most cases. The build system is verified to work with GNU, Intel, and IBM XLC compilers. Specific compile options can be specified either through specific environmental or CMake variables. When the libraries are installed in standard locations, e.g., /usr, /usr/local, there is no need to set environmental or cmake variables for the packages.

# 2.5.1 Quick build instructions (try first)

If you are feeling lucky and are on a standard UNIX-like system such as a Linux workstation, the following might quickly give a working QMCPACK:

The safest quick build option is to specify the C and C++ compilers through their MPI wrappers. Here we use Intel MPI and Intel compilers. Move to the build directory, run cmake and make

```
cd build
cmake -DCMAKE_C_COMPILER=mpiicc -DCMAKE_CXX_COMPILER=mpiicpc ..
make -j 8
```

You can increase the "8" to the number of cores on your system for faster builds. Substitute mpicc and mpicxx or other wrapped compiler names to suit your system. e.g. With OpenMPI use

```
cd build
cmake -DCMAKE_C_COMPILER=mpicc -DCMAKE_CXX_COMPILER=mpicxx ..
make -j 8
```

If you are feeling particularly lucky, you can skip the compiler specification:

```
cd build cmake .. make -j 8
```

The complexities of modern computer hardware and software systems are such that you should check that the autoconfiguration system has made good choices and picked optimized libraries and compiler settings before doing significant production. i.e. Check the details below. We give examples for a number of common systems in Section 2.6.

#### 2.5.2 Environment variables

A number of environmental variables affect the build. In particular they can control the default paths for libraries, the default compilers, etc. The list of environmental variables is given below:

CXX C++ compiler
CC C Compiler
MKL\_HOME Path for MKL
LIBXML2\_HOME Path for libxml2
HDF5\_ROOT Path for HDF5
BOOST\_ROOT Path for Boost
FFTW\_HOME Path for FFTW

## 2.5.3 Configuration options

In addition to reading the environmental variables, CMake provides a number of optional variables that can be set to control the build and configure steps. When passed to CMake, these variables will take precedent over the environmental and default variables. To set them add -D FLAG=VALUE to the configure line between the cmake command and the path to the source directory.

#### • Key QMCPACK build options

QMC\_CUDA Enable CUDA and GPU acceleration (1:yes, 0:no)

QMC\_COMPLEX Build the complex (general twist/k-point) version (1:yes, 0:no)

QMC\_MIXED\_PRECISION Build the mixed precision (mixing double/float) version

(1:yes (GPU default), 0:no (CPU default)).

The CPU support is experimental.

Use float and double for base and full precision.

The GPU support is quite mature.

Use always double for host side base and full precision and use float and double for CUDA base and full precision.

#### • General build options

CMAKE\_BUILD\_TYPE A variable which controls the type of build (defaults to Release). Possible values are:

None (Do not set debug/optmize flags, use
CMAKE\_C\_FLAGS or CMAKE\_CXX\_FLAGS)
Debug (create a debug build)
Release (create a release/optimized build)

RelWithDebInfo (create a release/optimized build with debug info)

MinSizeRel (create an executable optimized for size)

CMAKE\_C\_COMPILER Set the C compiler CMAKE\_CXX\_COMPILER Set the C++ compiler

CMAKE\_C\_FLAGS Set the C flags. Note: to prevent default

debug/release flags from being used, set the CMAKE\_BUILD\_TYPE=None

Also supported: CMAKE\_C\_FLAGS\_DEBUG,

CMAKE\_C\_FLAGS\_RELEASE, and CMAKE\_C\_FLAGS\_RELWITHDEBINFO

CMAKE\_CXX\_FLAGS Set the C++ flags. Note: to prevent default

debug/release flags from being used, set the CMAKE\_BUILD\_TYPE=None

Also supported: CMAKE\_CXX\_FLAGS\_DEBUG,

CMAKE\_CXX\_FLAGS\_RELEASE, and CMAKE\_CXX\_FLAGS\_RELWITHDEBINFO

#### • Additional QMCPACK build options

QMC\_INCLUDE Add extra include paths
QMC\_EXTRA\_LIBS Add extra link libraries
QMC\_BUILD\_STATIC Add -static flags to build

QMC\_DATA Specify data directory for QMCPACK (currently

unused, but likely to be used for future performance tests)

#### • libxml related

Libxml2\_INCLUDE\_DIRS Specify include directories for libxml2 Libxml2\_LIBRARY\_DIRS Specify library directories for libxml2

• FFTW related

FFTW\_INCLUDE\_DIRS Specify include directories for FFTW FFTW\_LIBRARY\_DIRS Specify library directories for FFTW

• CTest related

MPIEXEC Specify the mpi wrapper, e.g. srun, aprun, mpirun, etc.
MPIEXEC\_NUMPROC\_FLAG Specify the number of mpi processes flag, e.g. "-n", "-np", etc.

#### 2.5.4 Configure and build using cmake and make

To configure and build QMPACK, move to build directory, run cmake and make

cd build cmake ..

make -j 8

As you will have gathered, cmake encourages "out of source" builds, where all the files for a specific build configuration reside in their own directory separate from the source files. This allows multiple builds to be created from the same source files which is very useful where the filesystem is shared between different systems. You can also build versions with different settings (e.g. QMC\_COMPLEX) and different compiler settings. The build directory does not have to be called build - use something descriptive such as build\_machinename or build\_complex. The ".." in the cmake line refers to the directory containing CMakeLists.txt. Update the ".." for other build directory locations.

# 2.5.5 Example configure and build

• Set the environments (the examples below assume bash, Intel compilers and MKL library)

```
export CXX=icpc
export CC=icc
export MKL_HOME=/usr/local/intel/mkl/10.0.3.020
export LIBXML2_HOME=/usr/local
export HDF5_ROOT=/usr/local
export BOOST_ROOT=/usr/local/boost
export FFTW_HOME=/usr/local/fftw
```

• Move to build directory, run cmake and make

```
cd build
cmake -D CMAKE_BUILD_TYPE=Release ..
make -j 8
```

#### 2.5.6 Build scripts

It is recommended to create a helper script that contains the configure line for CMake. This is particularly useful when avoiding environmental variables, packages are installed in custom locations, or if the configure line is long or complex. In this case it is also recommended to add "rm -rf CMake\*" before the configure line to remove existing CMake configure files to ensure a fresh configure each time that the script is called. Deleting all the files in the build directory is also acceptable. If you do so we recommend to add some sanity checks in case the script is run from the wrong directory, e.g., checking for the existence of some QMCPACK files.

Some build script examples for different systems are given in the config directory. For example, on Cray systems these scripts might load the appropriate modules to set the appropriate programming environment, specific library versions etc.

An example script build.sh is given below. It is much more complex than usually needed for comprehensiveness:

# 2.5.7 Using vendor optimized numerical libraries, e.g. Intel MKL

Although QMC does not make extensive use of linear algebra, use of vendor optimized libraries is strongly recommended for highest performance. BLAS routines are used in the Slater determinant update, the VMC wavefunction optimizer and to apply orbital coefficients in local basis calculations. Vectorized math functions are also beneficial, e.g. for the phase factor computation in solid state calculations. CMake is generally successful in finding these libraries, but specific combinations can require additional hints, as described below:

#### Using Intel MKL with non-Intel compilers

```
To use Intel MKL with, e.g. an MPICH wrapped gcc:
```

```
cmake \
  -DCMAKE_C_COMPILER=mpicc -DCMAKE_CXX_COMPILER=mpicxx \
  -DBLA_VENDOR=Intel10_64lp_seq -DCMAKE_PREFIX_PATH=$MKLROOT/lib \
```

MKLROOT is the directory containing the MKL binary, examples, and lib directories (etc.) and is often /opt/intel/mkl

# 2.6 Installation instructions for common workstations and supercomputers

This section describes how to build QMCPACK on various common systems including multiple Linux distributions, Apple OS X, and various supercomputers. The examples should serve as good starting points for building QMCPACK on similar machines. For example, the software environment on modern Crays is very consistent. Note that updates to operating systems and system software may require small modifications to these recipes. See Section 2.11 for key points to check to obtain highest performance and Section 2.12 for troubleshooting hints.

#### 2.6.1 Installing on Ubuntu Linux or other apt-get based distributions

The following is designed to obtain a working QMCPACK build on e.g. a student laptop, starting from a basic Linux installation with none of the developer tools installed. Fortunately, all the required packages are available in the default repositories making for a quick installation. Note that for convenience we use a generic BLAS. For production a platform optimized BLAS should be used.

```
apt-get cmake g++ openmpi-bin libopenmpi-dev libboost-dev
apt-get libatlas-base-dev liblapack-dev libhdf5-dev libxml2-dev fftw3-dev
export CXX=mpiCC
cd build
cmake ..
make -j 8
ls -l bin/qmcpack
```

For gmca and other tools to function, we install some python libraries:

```
sudo apt-get install python-numpy python-matplotlib
```

# 2.6.2 Installing on CentOS Linux or other yum based distributions

The following is designed to obtain a working QMCPACK build on e.g. a student laptop, starting from a basic Linux installation with none of the developer tools installed. CentOS 7 (Red Hat compatible) is using gcc 4.8.2. The installation is only complicated by the need to install another repository to obtain HDF5 packages which are not available by default. Note that for convenience we use a generic BLAS. For production a platform optimized BLAS should be used.

To setup repoforge as a source for the HDF5 package, go to http://repoforge.org/use. Install the appropriate up to date release package for your OS. By default the CentOS Firefox will offer to run the installer. The CentOS 6.5 settings were still usable for HDF5 on CentOS 7 in 2016, but use CentOS 7 versions when they become available.

```
sudo yum install hdf5 hdf5-devel
    To build QMCPACK

module load mpi/openmpi-x86_64
which mpirun
# Sanity check; should print something like /usr/lib64/openmpi/bin/mpirun
export CXX=mpiCC
cd build
cmake ..
make -j 8
ls -l bin/qmcpack
```

# 2.6.3 Installing on Mac OS X using Macports

These instructions assume a fresh installation of macports and use the gcc 6.1 compiler. Older versions are fine, but it is vital to ensure matching compilers and libraries are used for all packages and to force use of what is installed in /opt/local. Performance should be very reasonable. Note that we utilize the Apple provided Accelerate framework for optimized BLAS.

Follow the Macports install instructions https://www.macports.org/

- Install Xcode and the Xcode Command Line Tools
- Agree to Xcode license in Terminal: sudo xcodebuild -license
- Install MacPorts for your version of OS X

Install the required tools:

```
sudo port install gcc6
sudo port select gcc mp-gcc6
sudo port install openmpi-devel-gcc6
sudo port select --set mpi openmpi-devel-gcc61-fortran
```

Cmake should pickup the versions of HDF5, libxml (etc.) installed in /opt/local by macports. If you have other copies of these libraries installed and wish to force use of a specific version, use the environment variables detailed in Sec. 2.5.2.

This recipe was verified on 1 July 2016 on a Mac running OS X 10.11.5 "El Capitain".

# 2.6.4 Installing on Mac OS X using Homebrew (brew)

Homebrew is a package manager for OS X that provides a convenient route to install all the QMCPACK dependencies. The following recipe will install the latest available versions of each package. This was successfully tested under OS X 10.11 "El Capitain" in February 2016. Note that it is necessary to build the MPI software from source to use the brew-provided gcc instead of Apple CLANG.

1. Install Homebrew from http://brew.sh/

```
/usr/bin/ruby -e "$(curl -fsSL
    https://raw.githubusercontent.com/Homebrew/install/master/install)"
```

2. Install the prerequisites

```
brew install gcc # Builds full gcc 5 from scratch, will take 30 minutes export HOMEBREW_CXX=g++-5 export HOMEBREW_CC=gcc-5 brew install mpich2 --build-from-source # Build from source required to use homebrew compiled compilers as # opposed to Apple CLANG. Check "mpicc -v" indicates Homebrew gcc 5.x.x brew install cmake brew install fftw brew install fftw brew install boost brew install homebrew/science/hdf5 #Note: Libxml2 is not required via brew since OS X already includes it.
```

3. Configure and build QMCPACK

4. Run the short tests. When mpich is used for the first time, OS X will request approval of the network connection.

```
ctest -R short
```

# 2.6.5 Installing on ANL ALCF Mira/Cetus IBM Blue Gene/Q

Mira/Cetus is a Blue Gene/Q supercomputer at Argonne National Laboratory's Argonne Leadership Computing Facility (ANL ALCF). Mira has 49152 compute nodes and each node has a 16-core PowerPC A2 processor with 16 GB DDR3 memory. Due to the fact that the login nodes and the compute nodes have different processors with distinct instruction sets, cross-compiling is required on this platform. See details about using Blue Gene/Q at http://www.alcf.anl.gov/user-guides/compiling-linking. On Mira, compilers are loaded via softenv and users need to add +mpiwrapper-xl and +cmake in \$HOME/.soft. In order to build QMCPACK, a toolchain file is provided for setting up CMake and the cmake command should be executed twice.

```
cd build
cmake -DCMAKE_TOOLCHAIN_FILE=../config/BGQ_XL_ToolChain.cmake ..
cmake -DCMAKE_TOOLCHAIN_FILE=../config/BGQ_XL_ToolChain.cmake ..
make -j 16
ls -l bin/qmcpack
```

In addition, adding a very useful cmake option -DCMAKE\_VERBOSE\_MAKEFILE=TRUE allows printing all the build commands during the make step. Alternatively you can use make VERBOSE=1.

## 2.6.6 Installing on ORNL OLCF Titan Cray XK7 (NVIDIA GPU accelerated)

Titan is a GPU accelerated supercomputer at Oak Ridge National Laboratory's Oak Ridge Leadership Computing Facility (ORNL OLCF). Each compute node has a 16 core AMD 2.2GHz Opteron 6274 (Interlagos) and an NVIDIA Kepler accelerator. The standard Cray software environment is available, with libraries accessed via modules. The only extra settings required to build the GPU version are the cudatoolkit module and specifying -DQMC\_CUDA=1 on the cmake configure line.

Note that on Crays the compiler wrappers "CC" and "cc" are used. The build system checks for these and does not (should not) use the compilers directly.

```
module swap PrgEnv-pgi PrgEnv-gnu # Use gnu compilers
module load cudatoolkit # CUDA for GPU build
module load cray-hdf5
module load cmake
module load fftw
export FFTW_HOME=$FFTW_DIR/..
module load boost
```

```
mkdir build_titan_gpu
cd build_titan_gpu
cmake -DQMC_CUDA=1 ..  # Must enable CUDA capabilities
make -j 8
ls -l bin/qmcpack
```

# 2.6.7 Installing on ORNL OLCF Titan Cray XK7 (CPU version)

As noted in Section 2.6.6 for the GPU, building on Crays requires only loading the appropriate library modules.

```
module swap PrgEnv-pgi PrgEnv-gnu # Use gnu compilers
module unload cudatoolkit # No CUDA for CPU build
module load cray-hdf5
module load cmake
module load fftw
export FFTW_HOME=$FFTW_DIR/..
module load boost
mkdir build_titan_cpu
cd build_titan_cpu
cmake ..
make -j 8
ls -l bin/qmcpack
```

# 2.6.8 Installing on ORNL OLCF Eos Cray XC30

Eos is a Cray XC30 with 16 core Intel Xeon E5-2670 processors connected by the Aries interconnect. The build process is identical to Titan, except that we use the default Intel programming environment. This is usually preferred to GNU.

```
module load cray-hdf5
module load fftw
export FFTW_HOME=$FFTW_DIR/..
module load boost
mkdir build_eos
cd build_eos
cmake ..
make -j 8
ls -l bin/qmcpack
```

## 2.6.9 Installing on ORNL OLCF SummitDev

SummitDev is the development cluster for the next GPU accelerated supercomputer Summit at Oak Ridge National Laboratory's Leadership Computing Facility (ORNL OLCF). It has IBM Power8 CPUs and NVIDIA Pascal GPUs.

#### **Building QMCPACK**

Please note that these build instructions are preliminary as the software environment is subject to change. QMCPACK can be build with the following commands:

```
module load xl
module load essl
module load netlib-lapack
module load hdf5/1.8.18
module load fftw
export FFTW_HOME=$OLCF_FFTW_ROOT
module load python
module load cmake
module load boost
module load cuda
mkdir build_summitdev
cd build_summitdev
cmake -DCMAKE_C_COMPILER="mpix1c" \
      -DCMAKE_CXX_COMPILER="mpix1C" \
      -DBUILD_LMYENGINE_INTERFACE=0 \
      -DQMC_CUDA=1 \
      -DCUDA_ARCH="sm_60" \
make -j 8
ls -l bin/qmcpack
```

## 2.6.10 Installing on NERSC Edison Cray XC30

Edison is a Cray XC30 with dual 12-core Intel "Ivy Bridge" nodes installed at NERSC. The build settings are identical to eos.

```
module load cray-hdf5
module load cmake
module load fftw
export FFTW_HOME=$FFTW_DIR/..
module load boost
mkdir build_edison
cd build_edison
cmake ..
make -j 8
ls -l bin/qmcpack
```

When the above was tested on 1 February 2016, the following module and software versions were present:

4) switch/1.0-1.0502.57058.1.58.ari 19) PrgEnv-intel/5.2.56 5) craype-network-aries 20) craype-ivybridge 6) craype/2.5.0 21) cray-shmem/7.3.0 7) intel/15.0.1.133 22) cray-mpich/7.3.0 23) slurm/edison 8) cray-libsci/13.3.0 9) udreg/2.3.2-1.0502.9889.2.20.ari 24) altd/2.0 10) ugni/6.0-1.0502.10245.9.9.ari 25) darshan/2.3.0 11) pmi/5.0.10-1.0000.11050.0.0.ari 26) subversion/1.7.9 12) dmapp/7.0.1-1.0502.10246.8.47.ari 27) cray-hdf5/1.8.14 13) gni-headers/4.0-1.0502.10317.9.2.ari 28) cmake/2.8.11.2 14) xpmem/0.1-2.0502.57015.1.15.ari 29) fftw/3.3.4.6 15) dvs/2.5\_0.9.0-1.0502.1958.2.55.ari 30) boost/1.54

# 2.6.11 Installing on NERSC Cori, Haswell Partition, Cray XC40

Cori is a Cray XC40 with 16-core Intel "Haswell" nodes installed at NERSC. The build settings are similar to eos at OLCF, but a workround is needed for the newer Cray OS which does not include a static libXml2 library. A NERSC installed version and its dependencies are used:

```
module load cray-hdf5
module load cmake
module load fftw
export FFTW_HOME=$FFTW_DIR/..
module load boost
export LIBXML2_HOME=/usr/common/software/libxml2/2.9.3/hsw
mkdir build_cori
cd build_cori
cd build_cori
cmake -DQMC_EXTRA_LIBS=/usr/common/software/liblzma/20160630/hsw/lib/liblzma.a ..
make -j 8
ls -l bin/qmcpack
```

When the above was tested on 15 July 2016, the following module and software versions were present:

qmcpack@cori05:trunk> module list
Currently Loaded Modulefiles:

```
1) nsg/1.2.0
                                           15) dvs/2.5_0.9.0-1.0502.2188.1.116.ari
                                           16) alps/5.2.4-2.0502.9774.31.11.ari
 2) modules/3.2.10.3
 3) eswrap/1.1.0-1.020200.1231.0
                                           17) rca/1.0.0-2.0502.60530.1.62.ari
4) switch/1.0-1.0502.60522.1.61.ari
                                           18) atp/1.8.3
 5) intel/16.0.0.109
                                           19) PrgEnv-intel/5.2.82
                                           20) craype-haswell
 6) craype-network-aries
7) craype/2.4.2
                                           21) cray-shmem/7.2.5
8) cray-libsci/13.2.0
                                           22) cray-mpich/7.2.5
9) udreg/2.3.2-1.0502.10518.2.17.ari
                                          23) slurm/cori
10) ugni/6.0-1.0502.10863.8.29.ari
                                          24) cray-hdf5/1.8.14
11) pmi/5.0.9-1.0000.10911.0.0.ari
                                          25) gcc/5.1.0
12) dmapp/7.0.1-1.0502.11080.8.76.ari
                                          26) cmake/3.3.2
13) gni-headers/4.0-1.0502.10859.7.8.ari
                                          27) fftw/3.3.4.5
14) xpmem/0.1-2.0502.64982.5.3.ari
                                          28) boost/1.59
```

# 2.6.12 Installing on NERSC Cori, Xeon Phi KNL Knight's Landing partition, Cray XC40

The second phase of NERSC's Cori uses Intel Xeon Phi Knight's Landing (KNL) nodes. Substantive optimizations for QMCPACK on KNL will be forthcoming. The following build recipe ensures that the code generation is appropriate for the KNL nodes:

When the above was tested on 21 December 2016, the following module and software versions were present:

build\_cori\_knl> module list
Currently Loaded Modulefiles:

differitly Loaded Moduleliles:		
1) modules/3.2.6.7	11) dmapp/7.1.0-12.37	21) cray-shmem/7.4
2) nsg/1.2.0	12) gni-headers/5.0.7-3.1	22) cray-mpich/7.4
3) modules/3.2.10.5	13) xpmem/0.1-4.5	23) altd/2.0
4) intel/17.0.1.132	14) job/1.5.5-3.58	24) darshan/3.0.1
<ol><li>5) craype-network-aries</li></ol>	15) dvs/2.7_0.9.0-2.221	25) subversion/1.7
6) craype/2.5.7	16) alps/6.1.3-17.12	26) cray-hdf5/1.8.
7) cray-libsci/16.09.1	17) rca/1.0.0-8.1	27) cmake/3.3.2
8) udreg/2.3.2-4.6	18) atp/2.0.3	28) fftw/3.3.4.10
9) ugni/6.0.12-2.1	19) PrgEnv-intel/6.0.3	29) boost/1.61

.1

#### 2.6.13 Installing on Windows

Install the Windows Subsystem for Linux and Bash on Windows. Open a bash shell and follow the install directions for Ubuntu in Section 2.6.1.

# 2.7 Testing and validation of QMCPACK

We **strongly encourage** running the included tests each time QMCPACK is built. These compare the results from the executable with known-good mean-field, quantum chemical, and other QMC results.

The tests included with QMCPACK currently mainly test the VMC code with single determinant wavefunction and simple spline Jastrow wavefunctions, and for gaussian and periodic spline basis sets. We check that the known mean field results are obtained with no Jastrow. When Jastrow functions are included we test against previous QMC data. The tests are statistical with a generous

3  $\sigma$  tolerance, however the system sizes are small, typically < 10 electrons, so the error bars are typically small. Limited DMC and optimizer tests included and are scheduled for expansion.

The "short" tests only take a few minutes on a 16 core machine. You can run these tests using the command below in the build directory:

## ctest -R short # Run the tests with "short" in their name

The output should be similar to the following:

#### Test project build\_gcc Start 1: short-LiH\_dimer\_ae-vmc\_hf\_noj-16-1 1/44 Test #1: short-LiH\_dimer\_ae-vmc\_hf\_noj-16-1 .......... Passed 11.20 sec Start 2: short-LiH\_dimer\_ae-vmc\_hf\_noj-16-1-kinetic 2/44 Test #2: short-LiH\_dimer\_ae-vmc\_hf\_noj-16-1-kinetic ...... Passed 0.13 sec 42/44 Test #42: short-monoO\_1x1x1\_pp-vmc\_sdj-1-16 ..... Passed 10.02 sec Start 43: short-monoO\_1x1x1\_pp-vmc\_sdj-1-16-totenergy 43/44 Test #43: short-monoO\_1x1x1\_pp-vmc\_sdj-1-16-totenergy ..... 0.08 sec Passed Start 44: short-monoO\_1x1x1\_pp-vmc\_sdj-1-16-samples 44/44 Test #44: short-monoO\_1x1x1\_pp-vmc\_sdj-1-16-samples ...... 0.08 sec

100% tests passed, 0 tests failed out of 44

```
Total Test time (real) = 167.14 sec
```

Note that the number of tests that are run varies between the standard, complex, and GPU compilations.

The full set of tests consist of significantly longer versions of the short tests, as well as tests of the conversion utilities. The runs require several hours each for improved statistics and a much more stringent test of the code. To run all the tests simply run ctest in the build directory:

```
ctest # Run all the tests. This will take several hours.
```

You can also run verbose tests which direct the QMCPACK output to the standard output:

```
ctest -V -R short # Verbose short tests
```

The test system includes specific tests for the complex version of the code.

The data files for the tests are located in the tests directory. The runs occur in build/test-s/system/test\_name. The numerical comparisons and test definitions are in tests/system/CMake-Lists.txt. If all the QMC tests fail it is likely that the appropriate mpiexec (or aprun, srun) is not being called or found. If the QMC runs appear to work but all the other tests fail it is possible that python is not working on your system - we suggest checking some of the test outputs in build/test/system/test\_name.

Note that because most of these tests are very small, consisting of only a few electrons, the performance is not representative of larger calculations. For example, while the calculations might fit in cache, there will be essentially no vectorization due to the small electron counts. **These tests** should therefore not be used for any benchmarking or performance analysis.

Example runs that can be used for testing performance are described in Sec. 2.7.3

#### 2.7.1 Unit tests

QMCPACK has a set of unit tests. All of the unit tests can be run with the following command (in the build directory):

```
ctest -L unit
```

The output should look similar to the following:

```
Test project qmcpack/build
     Start 1: unit_test_numerics
1/11 Test #1: unit_test_numerics ......
                                                  Passed
                                                           0.06 sec
     Start 2: unit_test_utilities
2/11 Test #2: unit_test_utilities ......
                                                  Passed
                                                           0.02 sec
     Start 3: unit_test_einspline
10/11 Test #10: unit_test_hamiltonian ......
                                                           1.88 sec
                                                  Passed
     Start 11: unit_test_drivers
11/11 Test #11: unit_test_drivers .....
                                                 Passed
                                                           0.01 sec
100% tests passed, 0 tests failed out of 11
Label Time Summary:
unit
           2.20 sec
```

2.31 sec

Individual unit test executables can be found in build/tests/bin. The source for the unit tests is located in the tests directory under each directory in src (e.g. src/QMCWavefunctions/tests). See Chapter 19 for more details about unit tests.

#### 2.7.2 Integration tests with Quantum Espresso

As described in Sec. 2.10, it is possible to test entire workflows of trial wavefunction generation, conversion, and eventual QMC calculation. A patched QE must be installed so that the pw2qmcpack converter is available.

By adding -D QE\_BIN=your\_QE\_binary\_path in the cmake command line when building your QMCPACK, tests named with "qe-" prefix will be included in the test set of your build. You can test the whole pw-pw2qmcpack-qmcpack workflow by

```
ctest -R qe
```

This provides a very solid test of the entire QMC toolchain for planewave generated wavefunctions.

#### 2.7.3 Performance tests

Total Test time (real) =

Performance tests representative of real research runs are included in the tests/performance directory. They can be used for benchmarking, comparing machine performance, or assessing optimizations. This is in contrast to the majority of the conventional integration tests where the particle counts are too small to be representative. Care is still needed to remove initiallization, I/O, and compute a representative performance measure.

The ctest integration is sufficient to run the benchmarks and measure relative performance from version to version of QMCPACK and assess proposed code changes. Performance tests are prefixed with "performance". To obtain highest performance on a particular platform, you must run the benchmarks in a standalone manner and tune thread counts, placement, walker count (etc.) This is essential to fairly compare different machines. Check with the developers if you are unsure of what is a fair change.

#### NiO performance tests

Follow the instructions in tests/performance/NiO/README to enable and run the NiO tests.

The NiO tests are for bulk supercells of varying size. The QMC runs consist of short blocks of (i) VMC without drift (ii) VMC with drift term included (iii) DMC with constant population. The tests use spline wavefunctions that must be downloaded as described in the README due to their large size. You will need to set "-DQMC\_DATA=YOUR\_DATA\_FOLDER -DENABLE\_TIMERS=1" when running cmake as described in the README.

Two sets of wavefunction are tested: splined orbitals with a one and two body Jastrow functions, and a more complex form with an additional three body Jastrow function. The Jastrows are the same for each run and are not reoptimized, as might be done for research purposes. Runs in the hundreds of electrons up to low thousands of electrons are representative of research runs performed in 2017. The largest runs target future machines and require very large memory.

Name	Atoms	Electrons	Electrons per spin
S8	32	384	192
S16	64	768	384
S32	128	1536	768
S64	256	3072	1536
S128	512	6144	3072
S256	1024	12288	6144

Table 2.1: System sizes for NiO performance tests

#### 2.7.4 Troubleshooting tests

ctest reports briefly pass or fail of tests in printout and also collects all the standard outputs to help investigating how tests fail. If the ctest execution is completed, look at Testing/Temporary/LastTest.log. If you manually stop the testing (ctrl+c), look at Testing/Temporary/LastTest.log.tmp. You can locate the failing tests by searching for the key word 'Fail'.

# 2.8 Automated testing of QMCPACK

The QMCPACK developers run automatic tests of QMCPACK on several different computer systems, many on a continuous basis. We currently test the following combinations nightly (workstations) and weekly (supercomputers):

- On a Linux Intel Xeon workstation:
  - GCC 4.8.2 with OpenMPI and CUDA 7.0 (GPU build, run on NVIDIA K40s)

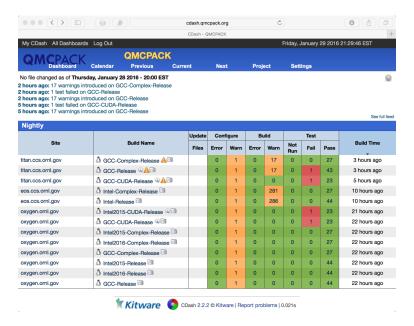


Figure 2.1: Example test results for QMCPACK, showing data for a workstation (Intel, GCC, both CPU and GPU builds) and for two ORNL supercomputers. In this example, 4 errors were found. This dashboard is not yet openly accessible, but the developers hope to make it available at a later date.

- GCC 4.8.2 with OpenMPI with netlib BLAS
- GCC 4.8.2 with OpenMPI with Intel MKL
- Intel 2017 with Intel MPI and MKL
- Intel 2015 with Intel MPI and MKL and CUDA 7.0 (GPU build, run on NVIDIA K40s)
- Intel 2015 with Intel MPI and MKL
- On a Linux Intel Knight's Landing workstation:
  - Intel 2017 with Intel MPI and MKL
  - GCC 4.8.5 with Intel MPI and MKL
- On Eos, a Cray XC30 Intel machine:
  - The default Intel programming environment and compiler with Cray MPI and Intel MKL
- On Titan, a Cray XK7 CPU+GPU machine:
  - The GCC programming environment and compiler with Cray MPI and CUDA
  - The GCC programming environment and compiler with Cray MPI
- On Cetus, an IBM Blue Gene Q:
  - Default programming environment configured using the toolchain config/BGQToolChain.cmake

# 2.9 Building ppconvert, a pseudopotential format converter

QMCPACK includes a utility, ppconvert, to convert between different pseudopotential formats. Examples include effective core potential formats (in gaussians), the UPF format used by Quantum ESPRESSO, and the XML format used by QMCPACK itself. The utility also enables the atomic orbitals to recomputed via a numerical density functional calculation if they need to be reconstructed for use in an electronic structure calculation.

To build ppconvert follow the instructions in src/QMCTools/ppconvert/README. Currently ppconvert is not built automatically although we expect to automate it soon. The makefile must be updated to refer to suitable C++ compiler and link in BLAS. Due to the small size of the calculations, optimal settings are not essential.

# 2.10 Installing and patching Quantum ESPRESSO

For trial wavefunctions obtained in a plane-wave basis we mainly support Quantum ESPRESSO. Note that ABINIT and QBox were supported historically and could be reactivated.

Quantum ESPRESSO currently stores wavefunctions in a non-standard internal "save" format. To convert these to a conventional HDF5 format file we have developed a converter, pw2qmcpack. This is an add on to the Quantum ESPRESSO distribution.

To simplify the process of patching Quantum ESPRESSO we have developed a script that will automatically download and patch the source code. The patches are specific to each version. e.g. To download and patch QE v5.3.0:

```
cd external_codes/quantum_espresso
./download_and_patch_qe5.3.0.sh
```

After running the patch, you must configure Quantum ESPRESSO with the HDF5 capability enabled, i.e.

```
cd espresso-5.3.0
./configure --with-hdf5 HDF5_DIR=/opt/local # Specify HDF5 base directory
```

The complete process is described in external\_codes/quantum\_espresso/README.

The tests involving pw.x and pw2qmcpack.x have been integrated in the test suite of QMC-PACK. By adding -D QE\_BIN=your\_QE\_binary\_path in the cmake command line when building your QMCPACK, tests named with "qe-" prefix will be included in the test set of your build. You can test the whole pw-pw2qmcpack-qmcpack workflow by

```
ctest -R qe
```

See Sec.2.7.2 and the testing section for more details.

# 2.11 How to build the fastest executable version of QMCPACK

To build the fastest version of QMCPACK we recommend the following:

• Use the latest C++ compilers available for your system. Substantial gains have been made optimizing C++ in recent years.

- Use a vendor optimized BLAS library such as Intel MKL and AMD ACML. Although QMC does not make extensive use of linear algebra, it is used in the VMC wavefunction optimizer, to apply the orbital coefficients in local basis calculations, and in the Slater determinant update.
- Use a vector math library such as Intel VML. For periodic calculations, the calculation of the structure factor and Ewald potential benefit from vectorized evaluation of sin and cos. Currently we only autodetect Intel VML, as provided with MKL, but support for MASSV and AMD LibM is included via #defines. See, e.g. src/Numerics/e2iphi.h. For large supercells, this optimization can gain 10% in performance.

Note that greater speedups of QMC calculations can usually be obtained by carefully choosing the required statistics for each investigation. i.e. Do not compute smaller error bars than necessary.

# 2.12 Troubleshooting the installation

Some tips to help troubleshoot installations of QMCPACK:

- First, build QMCPACK on a workstation that you control, or on any system that has a simple and up-to-date set of development tools. You can compare the results of cmake and QMCPACK on this system with any more difficult systems you encounter.
- Use up to date development software, particularly a recent CMake.
- Verify that the compilers and libraries that you expect are being configured. It is common to have multiple versions installed. The configure system will stop at the first version it finds which might not be the most recent. If this occurs, specify the appropriate directories and files directly (Section 2.5.3). e.g. cmake -DCMAKE\_C\_COMPILER=/full/path/to/mpicc -DCMAKE\_CXX\_COMPILER=/full/path/to/mpicx ..
- To monitor the compiler and linker settings, use a verbose build, "make VERBOSE=1". If an individual source file fails to compile you can experiment by hand using the output of the verbose build to reconstruct the full compilation line.

If you still have problems please post to the QMCPACK Google group with full details, or contact a developer.

# Chapter 3

# Running QMCPACK

QMCPACK requires at least one xml input file, and is invoked via: qmcpack [command line options] <XML input file(s)>

# 3.1 Command line options

QMCPACK offers several command line options which affect how calculations are performed. If the flag is absent, then the corresponding option is disabled.

- --async\_swap When enabled, this option will launch a new thread to handle the communication between walkers. If it is not enabled, then the communication happens on the current thread.
- --dryrun Validate the input file without performing the simulation. This is a good way to ensure that QMCPACK will do what you think it will.
- --help Print version information as well as a list of optional command-line arguments.
- --noprint Do not print extra information on Jastrow or pseudopotential. If this flag is not present, QMCPACK will create several .dat files that contain information about pseudopotentials (one file per PP), and jastrow factors (one per jastrow factor). These file may be useful for visual inspection of the jastrow, for example.
- --save\_wfs Write a .h5 file containing the real-space B-spline coefficients of the single particle wave functions. See the manual 7.2.1 for more information.
- --vacuum X For slab or wire boundary conditions (bconds= p p n, or bconds= p n n, respectively), increase the size of the axis with the open boundary condition(s) by a factor X. The default value is 1, *i.e.* no change to the specified axis. For more details on boundary conditions, see ??.
- --version Print version information and optional arguments. Same as --help.

# 3.2 Input files

The input is one or more XML file(s), documented in chapter 5.

# 3.3 Output files

QMCPACK generates multiple files, documented in chapter 10.

# 3.4 Running in parallel

#### 3.4.1 MPI

QMCPACK is fully parallelized with MPI. When performing an ensemble job, all the MPI ranks are first equally divided into groups which perform individual QMC calculations. Within one calculation, all the walkers are fully distributed across all the MPI ranks in the group. Since MPI requires distributed memory, there must be at least one MPI per node. To maximize the efficiency, more facts should be taken into account. When using MPI+threads on compute nodes with more than one NUMA domain (e.g., AMD Interlagos CPU on Titan or a node with multiple CPU sockets), it is recommended to place as many MPI ranks as the number of NUMA domains if the memory is sufficient. On clusters with more than just one GPU per node (NVIDIA Tesla K80), it requires to use the same number of MPI ranks as the number of GPUs per node in order to let each MPI rank take one GPU.

#### 3.4.2 Use of OpenMP threads

Modern processors integrate multiple identical cores even with hardware threads on a single die to increase the total performance and maintain a reasonable power draw. QMCPACK takes advantage of all that compute capability on a processor by using threads via OpenMP programming model as well as threaded linear algebra libraries. By default, QMCPACK is always built with OpenMP enabled. When launching calculations, users should instruct QMCPACK to create the right number of threads per MPI rank by specifying environmental variable OMP\_NUM\_THREADS. Even in the GPU accelerated version, using threads significantly reduces the time spent on the calculations performed by the CPU.

#### Performance consideration

As walkers are the basic units of workload in QMC algorithms, they are loosely coupled and distributed across all the threads. For this reason, the best strategy to run QMCPACK efficiently is to feed enough walkers to the available threads.

In a VMC calculation, the code automatically raises the actual number of walkers per MPI rank to the number of available threads if the user-specified number of walkers is smaller, see "walkers/mpi=XXX" in the VMC output. In a DMC calculation, the target number of walkers should be chosen to be slightly smaller than a multiple of the total number of available threads across all the MPI ranks belongs to this calculation. Since the number of walkers varies from generation to generation, its dynamical value should be slightly smaller or equal to that multiple most of the time.

To achieve better performance, mixed precision version (experimental) has been introduced to the CPU code. The mixed precision CPU code is more aggresive than the GPU version in using single precision (SP) operations. Current implementation utilizes SP on most calculations, except for matrix inversions and reductions where double precision is required to retain high accuracy. All the constant spline data in wavefunction, pseudopotentials and Coulomb potentials are initialized in double precision and later stored in single precision. The mixed precision code is as accurate as the double precision code up to a certain system size. Cross checking and verification of accuracy

are encouraged for systems with more than approximately 1500 electrons. The mixed precision code has been tested on solids with real/complex builds with VMC, VMC using drift and DMC runs with wavefunction including single slater determinant and one- and two-body Jastrow factors. Wavefunction optimization will be fixed in the following updates.

#### Memory consideration

When using threads, some memory objects shared by all the threads. Usually these memory are read-only when the walkers are evolving, for instance the ionic distance table and wavefunction coefficients. If a wavefunction is represented by B-splines, the whole table is shared by all the threads. It usually takes a large chunk of memory when a large primitive cell was used in the simulation. Its actual size is reported as "MEMORY increase XXX MB BsplineSetReader" in the output file. See details about how to reduce it in section 7.2.1.

The other memory objects which are distinct for each walker during random walk need to be associated with individual walkers and can not be shared. This part of memory grows linearly as the number of walkers per MPI rank. Those objects include wavefunction values (Slater determinants) at given electronic configurations and electron related distance tables (electron-electron distance table). Those matrices dominate the  $N^2$  scaling of the memory usage per walker.

# 3.4.3 Running on GPU machines

The GPU version on the NVIDIA CUDA platform is fully incorporated into the main source code. Commonly used functionalities for solid-state and molecular systems using B-spline single-particle orbitals are supported. Use of Gaussian basis sets and three-body Jastrow functions are not yet supported. A detailed description of the GPU implementation can be found in Ref. [?].

The current GPU implementation assumes one MPI process per GPU. To use nodes with multiple GPUs, use multiple MPI processes per node. Vectorization is achieved over walkers, that is, all walkers are propagated in parallel. In each GPU kernel, loops over electrons, atomic cores or orbitals are further vectorized to exploit an additional level of parallelism and to allow coalesced memory access.

#### Supported GPU features

- 1. Quantum Monte Carlo methods:
  - (a) Variational Monte Carlo (VMC).
  - (b) Diffusion Monte Carlo (DMC).
  - (c) Wavefunction optimization.
- 2. Boundary conditions:
  - (a) Periodic and open boundary conditions are fully supported.
  - (b) Twist-averaged boundary conditions and complex wavefunctions are fully supported.
  - (c) Mixed boundary conditions are not yet supported.
- 3. Wavefunctions:

- (a) Single Slater determinants with 3D B-spline orbitals. Only real-valued wavefunctions is supported, but tiling complex orbitals to supercells is supported as long as each k-point is a multiple of half a G-vector of the supercell.
- (b) Mixed basis representation in which orbitals are represented as 1D splines times spherical harmonics in spherical regions (muffin tins) around atoms, and 3D B-splines in the interstitial region.
- (c) One-body and two-body Jastrows represented as 1D B-splines are supported.

#### 4. Interaction types:

- (a) Semilocal (nonlocal and local) pseudopotentials.
- (b) Coulomb interaction (electron-electron, electron-ion).
- (c) Model periodic Coulomb (MPC) interaction.

#### Compiling the GPU code

or

To build the executable qmcpack with GPU support, follow these steps:

- 1. Make sure NVIDIA's CUDA compiler, nvcc, is in the search path. In most cases, CMake should be able to locate the nvcc compiler on the system automatically.
- 2. (a) Run CMake with the argument QMC\_CUDA switched on:

```
cd build cmake -D QMC_CUDA=1 .. make
```

(b) If a CMake toolchain file is used, switch on QMC\_CUDA by including this line in the toolchain file:

```
SET (QMC_CUDA 1)
Then compile the code as before:
cd build
cmake -D CMAKE_TOOLCHAIN_FILE=[toolchain name] ..
make
```

## CMake variables for adjusting CUDA code build features

These values can be changed by passing them as CMake's command line options with the -D flag, or using a toolchain file to overwrite the default values.

```
    QMC_CUDA
        =0 (default): no GPU support, build QMCPACK as a CPU code
        =1 : build QMCPACK with GPU support
    QMC_COMPLEX
        =0 (default): as per the CPU code, build the real version
        =1 : build the complex (general twist/k-point) version
```

3. QMC\_MIXED\_PRECISION (when QMC\_CUDA=1)

```
=1 (default): internally set CUDA_PRECISION=float
=0 : internally set CUDA_PRECISION=double
```

4. CUDA\_PRECISION (deprecated, use QMC\_MIXED\_PRECISION instead)

=float (default): single precision arithmetics and data types will be used for

most GPU kernels. Several precision critical kernels are al-

ways in double precision.

=double : double precision arithmetics and data types will be used for

all the GPU kernels.

#### Performance consideration

The relative speedup of the GPU implementation increases with both the number of electrons and the number of walkers running on a GPU. Typically, 128-256 walkers per GPU utilize sufficient number of threads to operate the GPU efficiently and to hide memory-access latency.

To achieve better performance, current implementation utilizes single precision operations on most GPU calculations, except for matrix inversions and Coulomb interaction where double precision is required to retain high accuracy. The mixed precision GPU code is as accurate as the double precision CPU code up to a certain system size. Cross checking and verification of accuracy are encouraged for systems with more than approximately 1500 electrons.

#### Memory consideration

In the GPU implementation, each walker has an anonymous buffer on the GPU's global memory to store temporary data associated with the wavefunctions. Therefore, the amount of memory available on a GPU limits the number of walkers and eventually the system size that it can process.

If the GPU memory is exhausted, reduce the number of walkers per GPU. Coarsening the grids of the B-splines representation (by decreasing the value of meshfactor in the input file) can also lower the memory usage, at the expense (risk) of obtaining inaccurate results. Proceed with caution if this option has to be considered. It is also possible to distribute the B-spline coefficients table between the host and GPU memory, see option Spline\_Size\_Limit\_MB in Sec. 7.2.1.

# Chapter 4

# Units used in QMCPACK

Internally, QMCPACK uses atomic units throughout. Unless stated, all inputs and outputs are also in atomic units. For convenience the analysis tools offer conversions to eV, Ry, Angstrom, Bohr etc.

# Chapter 5

# Input file overview

This chapter introduces XML as it is used in QMCPACK's input file. The focus is on the XML file format itself and the general structure of the input file rather than an exhaustive discussion of all keywords and structure elements.

QMCPACK uses XML to represent structured data in its input file. Instead of text blocks like

```
begin project
  id = vmc
  series = 0
end project

begin vmc
  move = pbyp
  blocks = 200
  steps = 10
  timestep = 0.4
end vmc
```

QMCPACK input looks like

XML elements start with <element\_name>, end with </element\_name>, and can be nested within each other to denote substructure (the trial wavefunction is composed of a Slater determinant and a Jastrow factor, which are each further composed of ...). id and series are attributes of the cproject/> element. XML attributes are generally used to represent simple values, like names, integers, or real values. Similar functionality is also commonly provided by cparameter/> elements like those shown above.

The overall structure of the input file reflects different aspects of the QMC simulation: the simulation cell, particles, trial wavefunction, Hamiltonian, and QMC run parameters. A condensed version of the actual input file is shown below:

```
<?xml version="1.0"?>
<simulation>
  cproject id="vmc" series="0">
  </project>
  <qmcsystem>
    <simulationcell>
    </simulationcell>
    <particleset name="e">
    </particleset>
    <particleset name="ion0">
    </particleset>
    <wavefunction name="psi0" ... >
      <determinantset>
        <slaterdeterminant>
        </slaterdeterminant>
      </determinantset>
      <jastrow type="One-Body" ... >
      </jastrow>
      <jastrow type="Two-Body" ... >
      </jastrow>
    </wavefunction>
    <hamiltonian name="h0" ... >
      <pairpot type="coulomb" name="ElecElec" ... />
      <pairpot type="coulomb" name="IonIon" ... />
      <pairpot type="pseudo" name="PseudoPot" ... >
      </pairpot>
    </hamiltonian>
```

The omitted portions (...) are more fine-grained inputs such as the axes of the simulation cell, the number of up and down electrons, positions of atomic species, external orbital files, starting Jastrow parameters, and external pseudopotential files.

# Chapter 6

# Specifying the system to be simulated

# 6.1 Specifying the simulation cell

The simulationcell block specifies the geometry of the cell, how the boundary conditions should be handled, and how ewald summation should be broken up.

simulationcell element					
parent elements:	qmcsystem				
child elements:	None				
attribute:					
parameter name	${f datatype}$	values	$\operatorname{default}$	$\operatorname{description}$	
lattice	9 floats	any float	Must be specified	Specification of	
				lattice vectors.	
bconds	string	"p" or "n"	"n n n"	Boundary conditions	
				for each axis.	
$LR\_dim\_cutoff$	float	float	15	Ewald breakup distance.	

An example of a simulationcell block is given below:

```
<simulationcell>
 <parameter name="lattice">
                 3.80000000
                                  0.0000000
                                                   0.0000000
                 0.00000000
                                  3.80000000
                                                   0.0000000
                 0.0000000
                                                   3.80000000
                                  0.0000000
         </parameter>
         <parameter name="bconds">
            ррр
         </parameter>
         <parameter name="LR_dim_cutoff">20</parameter>
</simulationcell>
```

Here, a cubic cell 3.8 bohr on a side will be used. This simulation will use periodic boundary conditions, and the maximum k vector will be  $20/r_{wigner-seitz}$  of the cell.

#### 6.1.1 Lattice

The cell is specified using 3 lattice vectors.

## 6.1.2 Boundary conditions

QMCPACK offers the capability to use a mixture of open and periodic boundary conditions. The bconds parameter expects a single string of three characters separated by spaces, e.g. "p p p" for purely periodic boundary conditions. These characters control the behavior of the x, y, and z, axes, respectively. Examples of valid bconds include:

"p p p" Periodic boundary conditions. Corresponds to 3d crystal.

"n n n" Open boundary conditions. Corresponds to isolated molecule in a vacuum.

" $\mathbf{p}$   $\mathbf{p}$   $\mathbf{n}$ " Slab geometry. Corresponds to 2d crystal.

"p n n" Wire geometry. Corresponds to 1d crystal.

#### 6.1.3 LR\_dim\_cutoff

When using periodic boundary conditions direct calculation of the Coulomb energy is not well behaved. As a result, QMCPACK uses an optimized Ewald summation technique to compute the Coulomb interaction.

In the Ewald summation, the energy is broken into short- and long-ranged terms. The short-ranged term is computed directly in real space, while the long-ranged term is computed in reciprocal space. LR\_dim\_cutoff controls where the short-ranged term ends and the long-ranged term begins. The real-space cutoff, reciprocal-space cutoff, and LR\_dim\_cutoff are related via:

$$LR_dim_cutoff = r_c \times k_c$$

where  $r_c$  is the Wigner-Seitz radius, and  $k_c$  is the length of the maximum k-vector used in the long-ranged term.

# 6.2 Specifying the particle set

The particleset blocks specify the particles in the QMC simulations: their types, attributes (mass, charge, valence), and positions.

#### 6.2.1 Input specification

## 6.2.2 Detailed attribute description

## particleset required attributes

• name/id

Unique name for the particle set. Default is "e" for electrons. "i" or "ion0" is typically used for ions.

particleset element							
parent elements:	simulation	simulation					
child elements:	group, at	trib					
attribute:							
name	${f datatype}$	values	$\mathbf{default}$	$\operatorname{description}$			
$\mathtt{name}/\mathtt{id}$	text	any	e	Name of particle set			
$\mathtt{size}^o$	integer	any	0	Number of particles in set			
${\tt random}^o$	text	yes/no	no	Randomize starting positions			
${ t randomsrc}/$	text	particleset.name	none	Particle set to randomize			
${\tt random\_source}^o$							

group element				
parent elements:	particlese	et		
child elements:	parameter,	attrib		
attribute:				
name	${f datatype}$	values	default	description
name	text	any	e	Name of particle set
$\mathtt{size}^o$	integer	any	0	Number of particles in set
$\mathtt{mass}^o$	real	any	1	Mass of particles in set
$\mathtt{unit}^o$	text	au/amu	au	Units for mass of particles
parameters				
name	${f datatype}$	values	$\mathbf{default}$	description
charge	real	any	0	Charge of particles in set
valence	real	any	0	Valence charge of particles in set
atomicnumber	integer	any	0	Atomic number of particles in set

# particleset optional attributes

#### • size

Number of particles in set

#### random

Randomize starting positions of particles. Each component of each particle's position is randomized independently in the range of the simulation cell in that component's direction.

# • randomsrc/random\_source

Specify source particle set around which to randomize the initial positions of this particle set.

# name required attributes

#### • name/id

Unique name for the particle set group. Typically, element symbols are used for ions and "u" or "d" for spin-up and spin-down electron groups, respectively.

attrib element				
parent elements: attribute:	particles	et,group		
name	datatype	values	$\mathbf{default}$	$\operatorname{description}$
name	string	any	none	Name of attrib
datatype	string	intArray, realArray,	none	Type of data in attrib
		posArray, stringArray		
size <sup>o</sup>	string	any	none	Size of data in attrib

## group optional attributes

- mass
  Mass of particles in set.
- unit Units for mass of particles in set (au[ $m_e = 1$ ] or amu[ $\frac{1}{12}m_{12}C = 1$ ]).

## 6.2.3 Example use cases

Listing 6.1: particleset elements for ions and electrons randomizing electron start positions.

```
<particleset name="i" size="2">
 <group name="Li">
   <parameter name="charge">3.000000</parameter>
   <parameter name="valence">3.000000</parameter>
   <parameter name="atomicnumber">3.000000</parameter>
 <group name="H">
   <parameter name="charge">1.000000</parameter>
   <parameter name="valence">1.000000</parameter>
   <parameter name="atomicnumber">1.000000</parameter>
 <attrib name="position" datatype="posArray" condition="1">
 0.0 0.0 0.0
 0.5 0.5 0.5
 </attrib>
 <attrib name="ionid" datatype="stringArray">
    Li H
 </attrib>
</particleset>
<particleset name="e" random="yes" randomsrc="i">
 <group name="u" size="2">
   <parameter name="charge">-1</parameter>
 <group name="d" size="2">
    <parameter name="charge">-1</parameter>
 </group>
</particleset>
```

Listing 6.2: particleset elements for ions and electrons specifying electron start positions

```
<particleset name="e">
 <group name="u" size="4">
    <parameter name="charge">-1</parameter>
   <attrib name="position" datatype="posArray">
     2.9151687332e-01 -6.5123272502e-01 -1.2188463918e-01
     5.8423636048e-01 4.2730406357e-01 -4.5964306231e-03
     3.5228575807e-01 -3.5027014639e-01 5.2644808295e-01
    -5.1686250912e-01 -1.6648002292e+00 6.5837023441e-01
   </attrib>
 </group>
 <group name="d" size="4">
   <parameter name="charge">-1</parameter>
   <attrib name="position" datatype="posArray">
     3.1443445436e-01 6.5068682609e-01 -4.0983449009e-02
    -3.8686061749e-01 -9.3744432997e-02 -6.0456005388e-01
     2.4978241724e-02 -3.2862514649e-02 -7.2266047173e-01
    -4.0352404772e-01 1.1927734805e+00 5.5610824921e-01
   </attrib>
 </group>
</particleset>
<particleset name="ion0" size="3">
  <group name="0">
   <parameter name="charge">6</parameter>
   <parameter name="valence">4</parameter>
   <parameter name="atomicnumber">8</parameter>
 </group>
 <group name="H">
   <parameter name="charge">1</parameter>
   <parameter name="valence">1</parameter>
   <parameter name="atomicnumber">1</parameter>
 </group>
 <attrib name="position" datatype="posArray">
   0.000000000e+00 0.000000000e+00 0.000000000e+00
   0.000000000e+00 -1.4308249289e+00 1.1078707576e+00
   0.000000000e+00 1.4308249289e+00 1.1078707576e+00
 </attrib>
 <attrib name="ionid" datatype="stringArray">
   ОНН
 </attrib>
</particleset>
```

Listing 6.3: particleset elements for ions specifying positions by ion type

```
<particleset name="ion0">
 <group name="0" size="1">
   <parameter name="charge">6</parameter>
   <parameter name="valence">4</parameter>
   <parameter name="atomicnumber">8</parameter>
   <attrib name="position" datatype="posArray">
     0.000000000e+00 0.000000000e+00 0.000000000e+00
 </group>
 <group name="H" size="2">
   <parameter name="charge">1</parameter>
   <parameter name="valence">1</parameter>
   <parameter name="atomicnumber">1</parameter>
   <attrib name="position" datatype="posArray">
     0.000000000e+00 -1.4308249289e+00 1.1078707576e+00
     0.000000000e+00 1.4308249289e+00 1.1078707576e+00
   </attrib>
 </group>
</particleset>
```

# Chapter 7

# Trial wavefunction specification

## 7.1 Introduction

This section describes the input blocks associated with the specification of the trial wavefunction in a QMCPACK calculation. These sections are contained within the < wavefunction > ... < /wavefunction > xml blocks. Users are expected to rely on converters to generate the input blocks described in this section. The converters and the workflows are designed such that input blocks require minimum modifications from users. Unless the workflow requires modification of wavefunction blocks (e.g. setting the cutoff in a multi determinant calculation), only expert users should directly alter them.

The trial wavefunction in QMCPACK has a general product form:

$$\Psi_T(\vec{r}) = \prod_k \Theta_k(\vec{r}),\tag{7.1}$$

where each  $\Theta_k(\vec{r})$  is a function of the electron coordinates (and possibly ionic coordinates and variational parameters). For problems involving electrons, the overall trial wavefunction must be antisymmetric with respect to electron exchange, so at least one of the functions in the product must be antisymmetric. Notice that, while QMCPACK allows for the construction of arbitrary trial wavefunctions based on the functions implemented in the code (e.g. slater determinants, jastrow functions, etc), the user must make sure that a correct wavefunction is used for the problem at hand. From here on, we assume a standard trial wavefunction for an electronic structure problem,

$$\Psi_T(\vec{r}) = A(\vec{r}) \prod_k J_k(\vec{r}), \tag{7.2}$$

where  $A(\vec{r})$  is one of the antisymmetric functions: 1) slater determinant, 2) multi-slater determinant, or 3) pfaffian, and  $J_k$  is any of the jastrow functions (described in section 7.3). The antisymmetric functions are built from a set of single particle orbitals (sposet). QMCPACK implements 4 different types of sposet, described in the section below. Each sposet is designed for a different type of calculation, so their definition and generation varies accordingly.

# 7.2 Single-particle orbitals

# 7.2.1 Spline basis sets

In this section we describe the use of spline basis sets to expand the sposet. Spline basis sets are designed to work seamless with plane wave DFT code, e.g. Quantum ESPRESSO as a trial wavefunction generator.

In QMC algorithms, all the SPOs  $\{\phi(\vec{r})\}$  need to be updated every time a single electron moves. Evaluating SPOs takes very large portion of computation time. In principle, PW basis set can be used to express SPOs directly in QMC like in DFT. but it introduces an unfavorable scaling due to the fact that the basis set size increases linearly as the system size. For this reason, it is efficient to use a localized basis with compact support and a good transferability from plane wave basis.

In particular, 3D tricubic B-splines provide a basis in which only 64 elements are nonzero at any given point in space [?]. The one-dimensional cubic B-spline is given by,

$$f(x) = \sum_{i'=i-1}^{i+2} b^{i',3}(x) \ p_{i'}, \tag{7.3}$$

where  $b^i(x)$  are the piecewise cubic polynomial basis functions and  $i = \text{floor}(\Delta^{-1}x)$  is the index of the first grid point  $\leq x$ . Constructing a tensor product in each Cartesian direction, we can represent a 3D orbital as

$$\phi_n(x,y,z) = \sum_{i'=i-1}^{i+2} b_x^{i',3}(x) \sum_{j'=j-1}^{j+2} b_y^{j',3}(y) \sum_{k'=k-1}^{k+2} b_z^{k',3}(z) \ p_{i',j',k',n}. \tag{7.4}$$

This allows the rapid evaluation of each orbital in constant time. Furthermore, this basis is systematically improvable with a single spacing parameter, so that accuracy is not compromised compared with plane wave basis.

The use of 3D tricubic B-splines greatly improves the computational efficiency. The gain in computation time from plane wave basis set to an equivalent B-spline basis set becomes increasingly large as the system size grows. On the downside, this computational efficiency comes at the expense of increased memory use, which is easily overcome by the large aggregate memory available per node through OpenMP/MPI hybrid QMC.

The input xml block for the spline SPOs is give in Listing 7.1. A list of options is given in Table 7.1. QMCPACK has a very useful command line option --save\_wfs which allows to dump the real space B-spline coefficient table into a h5 file on the disk. When the orbital transformation from k space to B-spline requires more than available amount of scratch memory on the compute nodes, users can perform this step on fat nodes and transfer back the h5 file for QMC calculations.

Listing 7.1: All electron Hamiltonian XML element.

determinantset element	-			
parent elements:	wavefuncti	ion		
child elements:	slaterdete	erminant		
attribute:				
name	${f datatype}$	values	default	description
type	text	bspline		Type of sposet.
href	text			Path to the h5 file generated by pw2qme
tilematrix	9 integers			Tiling matrix used to expand supercell.
twistnum	integer			Index of the super twist.
twist	3 floats			Super twist.
meshfactor	float	$\leq 1.0$		Grid spacing ratio.
precision	text	single/double		Precision of spline coefficients.
gpu	text	yes/no		GPU switch.
Spline_Size_Limit_MB	integer			Limit the size of B-spline coefficient tab
source	text	any	ion0	Particle set with the position of atom co

Table 7.1: Options for the determinantset xml-block associated with B-spline single particle orbital sets.

#### Additional information:

- precision. Only effective on CPU version without mixed precision, 'single' is always imposed with mixed precision. Using single precision not only saves memory usage but also speeds up the B-spline evaluation. It is recommended to use single precision since we saw little chance of really compromising the accuracy of calculation.
- meshfactor. It is the ratio of actual grid spacing of B-splines used in QMC calculation with respect to the original one calculated from h5. Smaller meshfactor saves memory usage but reduces accuracy. The effects are similar to reducing plane wave cutoff in DFT calculation. Use with caution!
- twistnum. If positive, it is the index. It is recommended not to take this way since the indexing may show some uncertainty. If negative, the super twist is referred by twist.
- Spline\_Size\_Limit\_MB. Allows to distribute the B-spline coefficient table between the host and GPU memory. The compute kernels access host memory via zero-copy. Though the performance penaty introduced by it is significant but allows large calculations to go.

#### 7.2.2 Gaussian basis sets

In this section we describe the use of localized basis sets to expand the **sposet**. The general form of a single particle orbital in this case is given by:

$$\phi_i(\vec{r}) = \sum_k C_{i,k} \, \eta_k(\vec{r}), \tag{7.5}$$

where  $\{\eta_k(\vec{r})\}$  is a set of M atom-centered basis functions and  $C_{i,k}$  is a coefficient matrix. This sposet should be used in calculations of finite systems employing an atom-centered basis set and is typically generated by the convert4qmc converter. (While it is possible to use this sposet on calculations of periodic systems, this feature is not currently implemented in QMCPACK.) Examples include calculations of molecules using gaussian basis sets or slater-type basis functions. Even though this section is called "Gaussian basis set" (by far the most common atom-centered basis set), QMCPACK works with any atom-centered basis set built based on either spherical harmonic angular functions or cartesian angular expansions. The radial functions in the basis set can be expanded in either gaussian functions, slater-type functions or numerical radial functions.

In this section we describe the input sections for the atom-centered basis set and the **sposet** for a single slater determinant trial wavefunction. The input sections for multideterminant trial wavefunctions are described in section 7.4. The basic structure for the input block of a single slater determinant is given in Listing 7.2. A list of options for **determinantset** associated with this **sposet** is given in Table 7.2.

Listing 7.2: Basic input block for a single determinant trial wavefunction using a sposet expanded on an atom-centered basis set.

determinantset element				
parent elements:	wavefunct	ion		
child elements:	basisset,	slaterdeterminan	t,sposet,	multideterminant
attribute:				
name	${f datatype}$	values	$\operatorname{default}$	description
$\mathtt{name}/\mathtt{id}$	text	any	""	Name of determinant set.
type	text	see below	""	Type of sposet.
keyword	text	NMO,GTO,STO	NMO	Type of orbital set generated.
transform	text	yes/no	yes	Transform to numerical radial functions?
source	text	any	ion0	Particle set with the position of atom center
cuspCorrection	text	yes/no	no	Apply cusp correction scheme to sposet?

Table 7.2: Options for the determinantset xml-block associated with atom-centered single particle orbital sets.

The definition of the set of atom-centered basis functions is given by the basisset block, while the sposet is defined within slaterdeterminant. The basisset input block is composed from a collection of atomicBasisSet input blocks, one for each atomic species in the simulation where basis functions are centered. The general structure for basisset and atomicBasisSet are given in Listing 7.3, while the corresponding lists of options are given in Tables 7.3 and 7.4.

Listing 7.3: Basic input block for basisset.

basisset element	t			
parent elements:	determina	ntset		
child elements:	atomicBas	isSet		
attribute:				
name	datatype	values	$\mathbf{default}$	description
$\mathtt{name}/\mathtt{id}$	text	any	" "	Name of atom-centered basis set.

Table 7.3: Options for the basisset xml-block associated with atom-centered single particle orbital sets.

atomicBasisSet element				
parent elements:	basisset			
child elements:	grid,basi	sGroup		
attribute:				
name	${f datatype}$	values	$\mathbf{default}$	description
$\mathtt{name/id}$	text	any	""	Name of atomic basis set.
angular	text	see below	default	Type of angular functions.
expandYlm	text	see below	yes	Expand Ylm shells?
expM	text	see below	yes	Add sign for $(-1)^m$ ?
elementType/species	text	any	e	Atomic species where functions are center
normalized	text	yes/no	yes	Are single particle functions normalized?

Table 7.4: Options for the atomicBasisSet xml-block.

basisGroup elem	basisGroup element						
parent elements:	atomicBas	atomicBasisSet					
child elements:	radfunc						
attribute:							
name	datatype	values	default	description			
$\mathtt{rid}/\mathtt{id}$	$\operatorname{text}$	any	""	Name of the basisGroup.			
type	$\operatorname{text}$	any	""	Type of basisGroup.			
n/1/m/s	integer	anu	0	Quantum numbers of basisGroup.			

Table 7.5: Options for the basisGroup xml-block.

Listing 7.4: Basic input block for slaterdeterminant with an atom-centered sposet.

<pr

element				
parent elements:				
child elements:				
attribute:				
name	datatype	values	$\mathbf{default}$	description
$\mathtt{name}/\mathtt{id}$	text	any	""	Name of determinant set
	text	any	""	

#### Detailed description of attributes:

In the following, we give a more detailed description of all the options presented in the various xml-blocks described in this section. Only non-trivial attributes are described below. Those with simple yes/no options and whose description above is enough to explain the intended behavior are not included.

#### determinantset attributes:

#### • type

Type of sposet. For atom-centered based sposets, use type="MolecularOrbital" or type="MO". Other options describe elsewhere in this manual are "spline", "composite", "pw", "heg", "linearopt", etc.

# • keyword/key

Type of basis set generated, which doesn't necessarily match the type of the basis set on the input block. The three possible options are: NMO (numerical molecular orbitals), GTO (gaussian-type orbitals), STO (slater-type orbitals). The default option is NMO. By default, QMCPACK will generate numerical orbitals from both GTO and STO types and use cubic or quintic spline interpolation to evaluate the radial functions. This is typically more efficient than evaluating the radial functions in the native basis (gaussians or exponents) and allows

for arbitrarily large contractions without any additional cost. To force the use of the native expansion (not recommended), use GTO or STO for each type of input basis set.

#### • transform

Request (or avoid) a transformation of the radial functions to NMO type. The default and recommended behavior is to transform to numerical radial functions. If transform is set to "yes", the option keyword is ignored.

#### • cuspCorrection

Enable (disable) the use of the cusp correction algorithm (CASINO REFERENCE) for a basisset built with GTO functions. The algorithm is implemented as described in (CASINO REFERENCE) and only works with transform="yes" and an input GTO basis set. No further input is needed.

#### atomicBasisSet attributes:

#### • name/id

Name of the basis set. Names should be unique.

#### • angular

Type of angular functions used in the expansion. In general, two angular basis functions are allowed: "spherical" (for spherical Ylm functions) and "cartesian" (for functions of the type  $x^n y^m z^l$ ).

### • expandYlm

Determines whether each basis group is expanded across the corresponding shell of m values (for spherical type) or consistent powers (for cartesian functions). Options:

- "No": Do not expand angular functions across corresponding angular shell.
- "Gaussian": Expand according to Gaussian03 format. This function is only compatible with angular="spherical". For a given input (l,m), the resulting order of the angular functions becomes: (1,-1,0) for l=1 and (0,1,-1,2,-2,...,l,-l) for general l.
- "Natural": Expand angular functions according to (-l,-l+1,...,l-1,l).
- "Gamess": Expand according to Gamess' format for cartesian functions. Notice that this option is only compatible with angular="cartesian". If angular="cartesian" is used, this option is not necessary.

#### • expM

Determines whether the sign of the spherical Ylm function associated with m  $(-1^m)$  is included in the coefficient matrix or not.

#### • elementType/species

Name of the species where basis functions are centered. Only one atomicBasisSet block is allowed per species. Additional blocks are ignored. The corresponding species must exist in the particleset given as the source option to determinantset. Basis functions for all the atoms of the corresponding species are included in the basis set, based on the order of atoms in the particleset.

#### basisGroup attributes:

#### • type

Type of input basis radial function. Notice that this refers to the type of radial function in the input xml-block, which might not match the radial function generated internally and used in the calculation (if transform is set to "yes"). Also notice that different basisGroup blocks within a given atomicBasisSet can have different type.

#### • n/1/m/s

Quantum numbers of the basis function. Notice that if expandYlm is set to "yes" in atomicBasisSet, a full shell of basis functions with the appropriate values of "m" will be defined for the corresponding value of "l". Otherwise a single basis function will be given for the specific combination of "(l,m)".

radfunc attributes for type="Gaussian":

•

slaterdeterminant attributes:

•

# 7.2.3 Plane-wave basis sets

# 7.2.4 Homogeneous electron gas

The interacting Fermi Liquid has its own special determinantset for filling up a Fermi surface. The shell number can be specified seperately for both spin up and spin down. This determines how many electrons to include of each time, only closed shells are currently implemented. The shells are filled according to the rules of a square box, if other lattice vectors are used, the electrons may not fill up a complete shell.

This following example can also be used for Helium simulations too, by specifying the proper pair interaction in the Hamiltonian section.

Listing 7.5: 2D Fermi Liquid example: particle specification

```
<qmcsystem>
<simulationcell name="global">
<parameter name="rs" pol="0" condition="74">6.5</parameter>
<parameter name="bconds">p p p</parameter>
<parameter name="LR_dim_cutoff">15</parameter>
</parameter name="LR_dim_cutoff">15</parameter>
</simulationcell>
<particleset name="e" random="yes">
<parameter name="u" size="37">
<parameter name="usize="37">
<parameter name="mass">1</parameter>
<parameter name="mass">1</parameter>
<parameter name="d" size="37">
<parameter name="charge">-1</parameter>
<parameter name="charge">-1</parameter>
<parameter name="mass">1</parameter>
<parameter name="mass">1</parameter>
<parameter name="mass">1</parameter>
</parameter name="mass">1</parameter>

</parameter name="mass">1</parameter>

<
```

Listing 7.6: 2D Fermi Liquid example (Slater Jastrow wave function)

## 7.3 Jastrow Factors

Jastrow factors are among the simplest and most effective ways of including dynamical correlation in the trial many body wavefunction. The resulting many body wavefunction is expressed as the product of an antisymmetric (in the case of Fermions) or symmetric (for Bosons) part and a correlating jastrow factor like so:

$$\Psi(\vec{R}) = \mathcal{A}(\vec{R}) \exp\left[J(\vec{R})\right] \tag{7.6}$$

In this section we will detail the types and forms of Jastrow factor used in QMCPACK. Note that each type of Jastrow factor needs to be specified using its own individual jastrow XML element. For this reason, we have repeated the specification of the jastrow tag in each section, with specialization for the options available for that given type of jastrow.

# 7.3.1 One-body Jastrow functions

The one-body Jastrow factor is a form that allows for the direct inclusion of correlations between particles that are included in the wavefunction with particles that are not explicitly part of it. The most common example of this are correlations between electrons and ions.

The jastrow function is specified within a wavefunction element and must contain one or more correlation elements specifying additional parameters as well as the actual coefficients. Section 7.3.1 gives examples of the typical nesting of jastrow, correlation, and coefficient elements.

Input	Sp	ecific	ation
TIPGO	$\sim$ $\sim$	CCIIIC	CCLOIL

Jastrow e	lement			
name	datatype	values	defaults	description
name	text		(required)	Unique name for this Jastrow function
type	$\operatorname{text}$	One-body	(required)	Define a one-body function
function	$\operatorname{text}$	Bspline	(required)	BSpline Jastrow
	$\operatorname{text}$	pade2		Pade form
	$\operatorname{text}$			•••
source	$\operatorname{text}$	name	(required)	name of attribute of classical particle set
print	text	yes / no	yes	jastrow factor printed in external file?
elements				
	Correlation			
Contents				
	(None)			

To be more concrete, the one-body jastrow factors used to describe correlations between electrons and ions take the form below

$$J1 = \sum_{I}^{ion0} \sum_{i}^{e} u_{ab}(|r_i - R_I|) \tag{7.7}$$

where I runs over all of the ions in the calculation, i runs over the electrons and  $u_{ab}$  describes the functional form of the correlation between them. Many different forms of  $u_{ab}$  are implemented in QMCPACK. We will detail two of the most common ones below.

#### Spline form

The one-body spline Jastrow function is the most commonly used one-body Jastrow for solids. This form was first described and used in [?]. Here  $u_{ab}$  is an interpolating 1D Bspline (tricuble spline on a linear grid) between zero distance and  $r_{cut}$ . In 3D periodic systems the default cutoff distance is the Wigner Seitz cell radius. For other periodicities including isolated molecules the  $r_{cut}$  must be specified. The cusp can be set.  $r_i$  and  $R_I$  are most commonly the electron and ion positions, but any particlesets that can provide the needed centers can be used.

Correlation element					
name	datatype	values	defaults	description	
elementType	text	name	see below	Classical particle target	
speciesA	$\operatorname{text}$	name	see below	Classical particle target	
speciesB	$\operatorname{text}$	name	see below	Quantum species target	
size	integer	> 0	(required)	number of coefficients	
rcut	real	> 0	see below	distance at which the correlation goes to 0	
$\operatorname{cusp}$	real	$\geq 0$	0	value for use in Kato cusp condition	
spin	$\operatorname{text}$	yes or no	no	spin dependent jastrow factor	
elements					
	Coefficients				
Contents					
	(None)				

# **Input Specification** Additional information:

- elementType, speciesA, speciesB, spin. For a spin independent Jastrow factor (spin = "no") elementType should be the name of the group of ions in the classical particleset to which the quantum particles should be correlated. For a spin dependent Jastrow factor (spin = "yes") set speciesA to the group name in the classical particleset and speciesB to the group name in the quantum particleset.
- rcut. The cutoff distance for the function in atomic units (bohr). For 3D fully periodic systems this parameter is optional and a default of the Wigner Seitz cell radius is used. Otherwise this parameter is required.
- cusp. The one body jastrow factor can be used to make the wavefunction satisfy the electronion cusp condition[?]. In this case, the derivative of the jastrow factor as the electron approaches the nucleus will be given by:

$$\left(\frac{\partial J}{\partial r_{iI}}\right)_{r_{iI}=0} = -Z \tag{7.8}$$

Note that if the antisymmetric part of the wavefunction satisfies the electron-ion cusp condition (for instance by using single particle orbitals that respect the cusp condition) or if a non-divergent pseudopotential is used that the Jastrow should be cuspless at the nucleus and this value should be kept at its default of 0.

Coefficients element						
name	datatype	values	${f defaults}$	description		
id	text		(required)	Unique identifier		
type	$\operatorname{text}$	Array	(required)			
optimize	$\operatorname{text}$	yes or no	yes	if no, values are fixed in optimizations		
elements						
(None)						
Contents						
(no name)	real array		zeros	Jastrow coefficients		

**Example use cases** Specify a spin-independent function with four parameters. Because rcut is not specified, the default cutoff of the Wigner Seitz cell radius is used; this Jastrow must be used with a 3D periodic system such as a bulk solid. The name of the particleset holding the ionic positions is "i".

Specify a spin-dependent function with seven upspin and seven downspin parameters. The cutoff distance is set to 6 atomic units. Note here that the particleset holding the ions is labeled as ion0 rather than "i" in the other example. Also in this case the ion is Lithium with a coulomb potential, so the cusp condition is satisfied by setting cusp="d".

#### Pade form

While the spline Jastrow factor is the most flexible and most commonly used form implemented in QMCPACK, there are times where its flexibility can make it difficult to optimize. As an example, a spline jastrow with a very large cutoff may be difficult to optimize for isolated systems like molecules due to the small number of samples that will be present in the tail of the function. In such cases, a simpler functional form may be advantageous. The second order Pade jastrow factor, given in Eq.7.9 is a good choice in such cases.

$$u_{ab}(r) = \frac{a * r + c * r^2}{1 + b * r} \tag{7.9}$$

Unlike the spline jastrow factor which includes a cutoff, this form has an infinite range and for every particle pair (subject to the minimum image convention) it will be applied. It also is a cuspless jastrow factor, so it should either be used in combination with a single particle basis set that contains the proper cusp or with a smooth pseudopotential.

Correlation element							
name	datatype	values	defaults	description			
elementType	text	name	see below	Classical particle target			
elements							
	Coefficients						
Contents							
	(None)						

parameter element						
name	datatype	values	${f defaults}$	description		
id	string	name	(required)	name for variable		
name	$\operatorname{string}$	A or B or C	(required)	see Eq.7.9		
optimize	$\operatorname{text}$	yes or no	yes	if no, values are fixed in optimizations		
elements						
(None)						
Contents						
(no name)	real	parameter value	(required)	Jastrow coefficients		
	rear	parameter value	(required)	Jastion Coefficients		

#### Input Specification

**Example use case** Specify a spin independent function with independent jastrow factors for two different species (Li and H). The name of the particleset holding the ionic positions is "i".

```
<parameter id="HB" name="B"> 6.88 </parameter>
  <parameter id="HC" name="C"> 0.237 </parameter>
  </correlation>
  </jastrow>
```

# 7.3.2 Two-body Jastrow functions

The two-body Jastrow factor is a form that allows for the explicit inclusion of dynamic correlation between two particles included in the wavefunction. It is almost always given in a spin dependent form so as to satisfy the Kato cusp condition between electrons of different spins[?].

The two body jastrow function is specified within a wavefunction element and must contain one or more correlation elements specifying additional parameters as well as the actual coefficients. Section 7.3.2 gives examples of the typical nesting of jastrow, correlation and coefficient elements.

### **Input Specification**

Jastrow e	lement				
name	datatype	values	defaults	description	
name	text		(required)	Unique name for this Jastrow function	
type	$\operatorname{text}$	Two-body	(required)	Define a one-body function	
function	$\operatorname{text}$	Bspline	(required)	BSpline Jastrow	
print	$\operatorname{text}$	yes / no	yes	jastrow factor printed in external file?	
elements					
	Correlation				
Contents					
	(None)				

The two-body jastrow factors used to describe correlations between electrons take the form

$$J2 = \sum_{i}^{e} \sum_{j>i}^{e} u_{ab}(|r_i - r_j|)$$
(7.10)

The most commonly used form of two body jastrow factor supported by the code is a splined jastrow factor, with many similarities to the one body spline jastrow.

## Spline form

The two-body spline Jastrow function is the most commonly used two-body Jastrow for solids. This form was first described and used in [?]. Here  $u_{ab}$  is an interpolating 1D Bspline (tricuble spline on a linear grid) between zero distance and  $r_{cut}$ . In 3D periodic systems the default cutoff distance is the Wigner Seitz cell radius. For other periodicities including isolated molecules the  $r_{cut}$  must be specified.  $r_i$  and  $r_j$  are typically electron positions. The cusp condition as  $r_i$  approaches  $r_j$  is set by the relative spin of the electrons.

Correlatio	Correlation element					
name	datatype	values	defaults	description		
speciesA	text	u or d	(required)	Quantum species target		
speciesB	$\operatorname{text}$	u or d	(required)	Quantum species target		
size	integer	> 0	(required)	number of coefficients		
rcut	real	> 0	see below	distance at which the correlation goes to 0		
spin	$\operatorname{text}$	yes or no	no	spin dependent jastrow factor		
elements						
	Coefficients					
Contents						
	(None)					

### Input Specification Additional information:

• speciesA, speciesB The scale function u(r) is defined for species pairs uu and ud. There is no need to define ud or dd since uu=dd and ud=du. The cusp condition is computed internally based on the charge of the quantum particles.

Coefficients element						
name	datatype	values	defaults	description		
id	text		(required)	Unique identifier		
type	$\operatorname{text}$	Array	(required)			
optimize	$\operatorname{text}$	yes or no	yes	if no, values are fixed in optimizations		
elements						
(None)						
Contents						
(no name)	real array		zeros	Jastrow coefficients		

**Example use cases** Specify a spin-dependent function with 4 parameters for each channel. In this case, the cusp is set at a radius of 4.0 bohr (rather than to the default of the Wigner Seitz cell radius). Also, in this example, the coefficients are set to not be optimized during an optimization step.

# 7.3.3 Three-body Jastrow functions

Explicit three body correlations can be included in the wavefunction via the three-body jastrow factor.

# 7.4 Multideterminant wavefunctions

# 7.5 Backflow wavefunctions

One can perturb the nodal surface of a single-slater/multi-slater wavefunction through use of a backflow-transformation. Specifically, if we have an antisymmetric function  $D(\mathbf{x}_{0\uparrow}, \dots, \mathbf{x}_{N\uparrow}, \mathbf{x}_{0\downarrow}, \dots, \mathbf{x}_{N\downarrow})$ , and if  $i_{\alpha}$  is the *i*-th particle of species type  $\alpha$ , then the backflow transformation works by making the coordinate transformation  $\mathbf{x}_{i_{\alpha}} \to \mathbf{x}'_{i_{\alpha}}$ , and evaluating D at these new "quasiparticle" coordinates. QMCPACK currently supports quasiparticle transformations given by:

$$\mathbf{x}'_{i_{\alpha}} = \mathbf{x}_{i_{\alpha}} + \sum_{\alpha \le \beta} \sum_{i_{\alpha} \ne j_{\beta}} \eta^{\alpha\beta} (|\mathbf{x}_{i_{\alpha}} - \mathbf{x}_{j_{\beta}}|) (\mathbf{x}_{i_{\alpha}} - \mathbf{x}_{j_{\beta}})$$
(7.11)

Here,  $\eta^{\alpha\beta}(|\mathbf{x}_{i_{\alpha}}-\mathbf{x}_{j_{\beta}}|)$  is a radially symmetric back flow transformation between species  $\alpha$  and  $\beta$ . In QMCPACK, particle  $i_{\alpha}$  is known as the "target" particle and  $j_{\beta}$  is known as the "source". The main types of transformations we'll talk about are so called one-body terms, which are between an electron and an ion  $\eta^{eI}(|\mathbf{x}_{i_{e}}-\mathbf{x}_{j_{I}}|)$ , and two-body terms. Two body terms are distinguished as those between like and opposite spin electrons:  $\eta^{e(\uparrow)e(\uparrow)}(|\mathbf{x}_{i_{e}(\uparrow)}-\mathbf{x}_{j_{e}(\uparrow)}|)$  and  $\eta^{e(\uparrow)e(\downarrow)}(|\mathbf{x}_{i_{e}(\uparrow)}-\mathbf{x}_{j_{e}(\downarrow)}|)$ . Henceforth, we will assume that  $\eta^{e(\uparrow)e(\uparrow)}=\eta^{e(\downarrow)e(\downarrow)}$ .

In the following, I will explain how to describe general terms like Eq. 7.11 in a QMCPACK XML file. For specificity, I will consider a particle set consisting of H and He (in that order). This ordering will be important when we build the XML file, so you can find this out either through your specific declaration of particleset; by looking at the hdf5 file in the case of plane waves, or by looking at the qmcpack output file in the section labelled "Summary of QMC systems".

#### 7.5.1 Input Specifications

All backflow declarations occur within a single <backflow> ... </backflow> block. Backflow transformations occur in <transformation> blocks, and have the following input parameters

Transform	Transformation element				
name	datatype	values	defaults	description	
name	text		(required)	Unique name for this Jastrow function	
$_{\mathrm{type}}$	$\operatorname{text}$	"e-I"	(required)	Define a one-body backflow transformation.	
		"e-e"		Define a two-body backflow transformation.	
function	$\operatorname{text}$	Bspline	(required)	B-spline type transformation. (No other types supported)	
source	text		·	"e" if two-body, ion particle set if one-body.	

Just like one and two-body jastrows, parameterization of the backflow transformations are specified within the <transformation> blocks by <correlation> blocks. Please refer to 7.3.1 for more information.

## 7.5.2 Example Use Case

Having specified the general form, we present a general example of one-body and two-body backflow transformations in a hydrogen-helium mixture. The H and He ions have independent backflow transformations, as do the like and unlike-spin two-body terms. One caveat is in order: ionic backflow transformations must be listed in the order that they appear in the particle set. If in our example, He is listed first, and H is listed second, the following example would be correct. However, switching backflow declaration to H first, then He, will result in an error. Outside of this, declaration of one-body blocks and two-body blocks aren't sensitive to ordering.

```
<backflow>
<!--The One-Body term with independent e-He and e-H terms. IN THAT ORDER -->
<transformation name="eIonB" type="e-I" function="Bspline" source="ion0">
   <correlation cusp="0.0" size="8" type="shortrange" init="no" elementType="He" rcut="3.0">
        <coefficients id="eHeC" type="Array" optimize="yes">
           00000000
        </coefficients>
   </correlation>
   <correlation cusp="0.0" size="8" type="shortrange" init="no" elementType="H" rcut="3.0">
        <coefficients id="eHC" type="Array" optimize="yes">
           0 0 0 0 0 0 0 0
        </coefficients>
   </correlation>
</transformation>
<!--The Two-Body Term with Like and Unlike Spins -->
<transformation name="eeB" type="e-e" function="Bspline" >
   <correlation cusp="0.0" size="7" type="shortrange" init="no" speciesA="u" speciesB="u" rcut="1.2">
       <coefficients id="uuB1" type="Array" optimize="yes">
           0000000
       </coefficients>
   </correlation>
   <correlation cusp="0.0" size="7" type="shortrange" init="no" speciesA="d" speciesB="u" rcut="1.2">
        <coefficients id="udB1" type="Array" optimize="yes">
           0000000
        </coefficients>
   </correlation>
</transformation>
</backflow>
```

Currently, backflow only works with single-slater determinant wavefunctions. When a backflow transformation has been declared, it should be placed within the <determinantset> block, but outside of the <slaterdeterminant> blocks, like so:

#### 7.5.3 Additional Information

• Optimization: Optimizable backflow transformation parameters are notoriously nonlinear, and so optimizing backflow wavefunctions can sometimes be difficult. We direct the reader

to our provided backflow tutorials for more information.

# Chapter 8

# Hamiltonian and Observables

QMCPACK is capable of the simultaneous measurement of the Hamiltonian and many other quantum operators. The Hamiltonian attains a special status among the available operators (also referred to as observables) because it ultimately generates all available information regarding the quantum system. This is evident from an algorithmic standpoint as well since the Hamiltonian (embodied in the the projector) generates the imaginary time dynamics of the walkers in DMC and RMC.

This section covers how the Hamiltonian can be specified, component by component, by the user in the XML format native to QMCPACK . It also covers the input structure of statistical estimators corresponding to quantum observables such as the density, the static structure factor, and forces.

## 8.1 The Hamiltonian

The many-body Hamiltonian in Hartree units is given by

$$\hat{H} = -\sum_{i} \frac{1}{2m_{i}} \nabla_{i}^{2} + \sum_{i} v^{ext}(r_{i}) + \sum_{i < j} v^{qq}(r_{i}, r_{j}) + \sum_{i \ell} v^{qc}(r_{i}, r_{\ell}) + \sum_{\ell < m} v^{cc}(r_{\ell}, r_{m}).$$
(8.1)

Here, the sums indexed by i/j are over quantum particles, while  $\ell/m$  are reserved for classical particles. Often the quantum particles are electrons and the classical particles are ions, though QMCPACK is not limited in this way. The mass of each quantum particle is denoted  $m_i$ ,  $v^{qq}/v^{qc}/v^{cc}$  are pair potentials between quantum-quantum/quantum-classical/classical-classical particles, and  $v^{ext}$  denotes a purely external potential.

QMCPACK is designed modularly so that any potential can be supported with minimal additions to the code base. Potentials currently supported include Coulomb interactions in open and periodic boundary conditions, the modified periodic coulomb (MPC) potential, non-local pseudopotentials, helium pair potentials, and various model potentials such as hard sphere, gaussian, and modified Poschl-Teller.

Reference information and examples for the <a href="hamiltonian">hamiltonian</a> XML element is provided below. Detailed descriptions of the input for individual potentials is given in the sections that follow.

hamiltonian element							
parent elements:	simulation	simulation, qmcsystem					
child elements:	pairpot ex	ctpot estimator cor	$\mathtt{nstant}(\deg$	precated)			
attributes							
name	${f datatype}$	values	$\mathbf{default}$	description			
$\mathtt{name/id}^o$	text	anything	h0	Unique id for this Hamiltonian instance			
$\mathtt{type}^o$	text		generic	No current function			
$\mathtt{role}^o$	text	primary/extra	extra	Designate as primary Hamiltonian or not			
$\mathtt{source}^o$	text	particleset.name	i	Identify classical particleset			
$\mathtt{target}^o$	text	particleset.name	e	Identify quantum particleset			
$\mathtt{default}^o$	boolean	yes/no	yes	Include kinetic energy term implicitly			

#### Additional information:

• target: Must be set to the name of the quantum particeset. The default value is typically sufficient. In normal usage, no other attributes are provided.

Listing 8.1: All electron Hamiltonian XML element.

```
<hamiltonian target="e">
  <pairpot name="ElecElec" type="coulomb" source="e" target="e"/>
  <pairpot name="ElecIon" type="coulomb" source="i" target="e"/>
  <pairpot name="IonIon" type="coulomb" source="i" target="i"/>
  </hamiltonian>
```

Listing 8.2: Pseudopotential Hamiltonian XML element.

```
<hamiltonian target="e">
  <pairpot name="ElecElec" type="coulomb" source="e" target="e"/>
  <pairpot name="PseudoPot" type="pseudo" source="i" wavefunction="psi0" format="xml">
        <pseudo elementType="Li" href="Li.xml"/>
        <pseudo elementType="H" href="H.xml"/>
        </pairpot>
        <pairpot name="IonIon" type="coulomb" source="i" target="i"/>
        </hamiltonian>
```

# 8.2 Pair potentials

Many pair potentials are supported. Though only the most commonly used pair potentials are covered in detail in this section, all currently available potentials are listed briefly below. If a potential you desire is not covered below, or is not present at all, feel free to contact the developers.

pairpot factory el	omont				
paripot factory er	hamiltonia				
type selector:	type attrib				
type options:	coulomb	dic	Coulomb/Ewald potential		
type options.	pseudo		Semilocal pseudopoten		
	mpc			lomb interaction/correction	
	срр		Core polarization pote	•	
	numerical/	*num*	Numerical radial poter		
	skpot	nam	Unknown		
	vhxc		Exchange correlation p	ootential (external)	
	jellium		Atom-centered spheric	` /	
	hardsphere		Hard sphere potential		
	gaussian		Gaussian potential		
	modpostel		Modified Poschl-Teller potential		
	huse		Huse quintic potential		
	modInsKE		Model insulator kinetic energy  Unknown  Helium pair potential  Helium pair potential  Helium pair potential		
	oscillatory				
	LJP_smoot	hed			
	HeSAPT_si	moothed			
	HFDHE2_N	Moroni1995			
	HFDHE2		Helium pair potential		
	eHe		Helium-electron pair potential		
shared attributes:			1 1		
name	datatype	values	default	description	
$type^r$	text	$See\ above$	0	Select pairpot type	
$\mathtt{name}^r$	text	anything	any	Unique name for this pai	
$\mathtt{source}^r$	text	particleset.name	hamiltonian.target	Identify interacting partic	
$\mathtt{target}^r$	text	particleset.name	hamiltonian.target	Identify interacting partic	

#### Additional information:

text

 $\mathtt{units}^o$ 

• **type:** Used to select the desired pair potential. Must be selected from the list of type options above.

hartree

No current function

- name: A unique name used to identify this pair potential. Block averaged output data will appear under this name in scalar.dat and/or stat.h5 files.
- source/target: These specify the particles involved in a pair interaction. If an interaction is between classical (e.g. ions) and quantum (e.g. electrons), source/target should be the name of the classical/quantum particleset.
- Only coulomb, pseudo, mpc are described in detail below. The older or less used types (cpp, numerical, jellium, hardsphere, gaussian, huse, modpostel, oscillatory, skpot, vhxc, modInsKE, LJP\_smoothed, HeSAPT\_smoothed, HFDHE2\_Moroni1995, eHe, HFDHE2) are not covered.
- Available only if QMC\_BUILD\_LEVEL>2 and QMC\_CUDA is not defined: hardsphere, gaussian, huse, modpostel, oscillatory, skpot.

- Available only if OHMMS\_DIM==3: mpc, vhxc, pseudo.
- Available only if OHMMS\_DIM==3 and QMC\_BUILD\_LEVEL>2 and QMC\_CUDA is not defined: cpp,
   LJP\_smoothed, HeSAPT\_smoothed, HFDHE2\_Moroni1995, eHe, jellium, HFDHE2, modInsKE.

## 8.2.1 Coulomb potentials

The bare Coulomb potential is used in open boundary conditions:

$$V_c^{open} = \sum_{i < j} \frac{q_i q_j}{|r_i - r_j|} \tag{8.2}$$

When periodic boundary conditions are selected, Ewald summation is used automatically:

$$V_c^{pbc} = \sum_{i < j} \frac{q_i q_j}{|r_i - r_j|} + \frac{1}{2} \sum_{L \neq 0} \sum_{i,j} \frac{q_i q_j}{|r_i - r_j + L|}$$
(8.3)

The sum indexed by L is over all non-zero simulation cell lattice vectors. In practice, the Ewald sum is broken into short and long ranged parts in a manner optimized for efficiency (see Ref. [?]) for details.

For information on how to set the boundary conditions, consult Sec. 6.1.

pairpot type=co	pairpot type=coulomb element							
parent elements:	hamiltonia	an						
child elements:	None							
attributes								
name	datatype	values	default	$\operatorname{description}$				
$\mathtt{type}^r$	text	coulomb		Must be coulomb				
$\mathtt{name/id}^r$	text	anything	ElecElec	Unique name for interaction				
$\mathtt{source}^r$	text	particleset.name	hamiltonian.target	Identify interacting particles				
$\mathtt{target}^r$	text	particleset.name	hamiltonian.target	Identify interacting particles				
$\mathtt{pbc}^o$	boolean	yes/no	yes	Use Ewald summation				
${\tt physical}^o$	boolean	yes/no	yes	Hamiltonian(yes)/observable(no)				
forces	boolean	yes/no	no	Deprecated				

#### Additional information

- type/source/target See description for the generic pairpot factory element above.
- name: Traditional user-specified names for electron-electron, electron-ion, and ion-ion terms are ElecElec, ElecIon, and IonIon, respectively. While any choice can be used, the data analysis tools expect to find columns in \*.scalar.dat with these names.
- pbc: Ewald summation will not be performed if simulationcell.bconds== n n n, regardless of the value of pbc. Similarly, the pbc attribute can only be used to turn off Ewald summation if simulationcell.bconds!= n n n. The default value is recommended.
- physical: If physical==yes, this pair potential is included in the Hamiltonian and will factor into the LocalEnergy reported by QMCPACK and also in the DMC branching weight. If physical==no, then the pair potential is treated as a passive observable but not as part of the Hamiltonian itself. As such it does not contribute to the outputted LocalEnergy. Regardless of the value of physical output data will appear in scalar.dat in a column headed by name.

Listing 8.3: XML element for Coulomb interaction between electrons.

```
<pairpot name="ElecElec" type="coulomb" source="e" target="e"/>
```

Listing 8.4: XML element for Coulomb interaction between electrons and ions (all-electron only).

```
<pairpot name="ElecIon" type="coulomb" source="i" target="e"/>
```

Listing 8.5: XML element for Coulomb interaction between ions.

```
<pairpot name="IonIon" type="coulomb" source="i" target="i"/>
```

## 8.2.2 Pseudopotentials

QMCPACK supports pseudopotentials in semilocal form, which is local in the radial coordinate and non-local in angular coordinates. When all angular momentum channels above a certain threshold  $(\ell_{max})$  are well approximated by the same potential  $(V_{\bar{\ell}} \equiv V_{loc})$ , the pseudoptential separates into a fully local channel and an angularly-nonlocal component:

$$V^{PP} = \sum_{ij} \left( V_{\bar{\ell}}(|r_i - \tilde{r}_j|) + \sum_{\ell \neq \bar{\ell}}^{\ell_{max}} \sum_{m = -\ell}^{\ell} |Y_{\ell m}\rangle \left[ V_{\ell}(|r_i - \tilde{r}_j|) - V_{\bar{\ell}}(|r_i - \tilde{r}_j|) \right] \langle Y_{\ell m}| \right)$$
(8.4)

Here the electron/ion index is i/j and only one type of ion is shown for simplicity.

Evaluation of the localized pseudopotential energy  $\Psi_T^{-1}V^{PP}\Psi_T$  requires additional angular integrals. These integrals are evaluated on a randomly shifted angular grid. The size of this grid is determined by  $\ell_{max}$ . See Ref. [?] for further detail.

QMCPACK uses the FSAtom pseudopotential file format associated with the "Free Software Project for Atomic-scale Simulations" initiated in 2002 (see http://www.tddft.org/fsatom/manifest.php for general information). The FSAtom format uses XML for structured data. Files in this format do not use a specific identifying file extension; they are simply suffixed with ".xml". The tabular data format of CASINO is also supported.

pairpot type=pseudo element						
parent elements:	hamiltonia	an				
child elements:	pseudo					
attributes						
name	${f datatype}$	values	default	${f description}$		
$\mathtt{type}^r$	text	pseudo		Must be pseudo		
$\mathtt{name/id}^r$	text	anything	PseudoPot	No current function		
$\mathtt{source}^r$	text	particleset.name	i	Ion particleset name		
$\mathtt{target}^r$	text	particleset.name	hamiltonian.target	Electron particleset nam		
$\mathtt{pbc}^o$	boolean	yes/no	$yes^*$	Use Ewald summation		
forces	boolean	yes/no	no	Deprecated		
$\mathtt{wavefunction}^r$	text	wavefunction.name	invalid	Identify wavefunction		
$\mathtt{format}^r$	text	xml/table	table	Select file format		

### Additional information:

• type/source/target See description for the generic pairpot factory element above.

- name: Ignored. Instead default names will be present in \*scalar.dat output files when pseudopotentials are used. The field LocalECP refers to the local part of the pseudopotential. If non-local channels are present, a NonLocalECP field will be added that contains the non-local energy summed over all angular momentum channels.
- pbc: Ewald summation will not be performed if simulationcell.bconds== n n n, regardless of the value of pbc. Similarly, the pbc attribute can only be used to turn off Ewald summation if simulationcell.bconds!= n n n.
- format: If format==table, QMCPACK looks for \*.psf files containing pseudopotential data in a tabular format. The files must be named after the ionic species provided in particleset (e.g. Li.psf and H.psf). If format==xml, additional pseudo child XML elements must be provided (see below). These elements specify individual file names and formats (both the FSAtom XML and CASINO tabular data formats are supported).

Listing 8.6: XML element for pseudopotential electron-ion interaction (psf files).

```
<pairpot name="PseudoPot" type="pseudo" source="i" wavefunction="psi0" format="psf"/>
```

Listing 8.7: XML element for pseudopotential electron-ion interaction (xml files).

```
<pairpot name="PseudoPot" type="pseudo" source="i" wavefunction="psi0" format="xml">
    <pseudo elementType="Li" href="Li.xml"/>
    <pseudo elementType="H" href="H.xml"/>
    </pairpot>
```

Details of <pseudo/> input elements are given below. It is possible to include (or construct) a full pseudopotential directly in the input file without providing an external file via href. The full XML format for pseudopotentials is not yet covered.

pseudo element				
parent elements:	pairpot ty	ype=pseudo		
child elements:	header lo	cal grid		
attributes				
name	${f datatype}$	values	$\mathbf{default}$	${f description}$
$\verb"elementType/symbol"$	text	<pre>group.name</pre>	none	Identify ionic species
$\mathtt{href}^r$	text	file path	none	Pseudopotential file path
$\mathtt{format}^r$	text	xml/casino	xml	Specify file format
$\mathtt{cutoff}^o$	real			Non-local cutoff radius
$\mathtt{lmax}^o$	integer			Largest angular momentum
$\mathtt{nrule}^o$	integer			Integration grid order

Listing 8.8: XML element for pseudopotential of single ionic species.

```
<pseudo elementType="Li" href="Li.xml"/>
```

### 8.2.3 Modified periodic Coulomb interaction/correction

The modified periodic Coulomb (MPC) interaction is an alternative to direct Ewald summation. The MPC corrects the exchange correlation hole to more closely match its thermodynamic limit. Because of this, the MPC exhibits smaller finite size errors than the bare Ewald interaction, though

a few alternative and competitive finite size correction schemes now exist. The MPC is itself often used just as a finite size correction in postprocessing (set physical=false in the input).

pairpot type=m	pc element			
parent elements:	hamiltoni	an		
child elements:	None			
attributes				
name	${f datatype}$	values	default	description
$\mathtt{type}^r$	text	mpc		Must be mpc
$\mathtt{name/id}^r$	text	anything	MPC	Unique name for intera
$\mathtt{source}^r$	text	particleset.name	hamiltonian.target	Identify interacting par
$\mathtt{target}^r$	text	particleset.name	hamiltonian.target	Identify interacting par
${\tt physical}^o$	boolean	yes/no	no	Hamiltonian(yes)/obser
cutoff	real	> 0	30.0	Kinetic energy cutoff

### Remarks

- physical: Typically set to no, meaning the standard Ewald interaction will be used during sampling and MPC will be measured as an observable for finite-size post correction. If physical is yes, the MPC interaction will be used during sampling. In this case an electron-electron Coulomb pairpot element should not be supplied.
- Developer note: Currently the name attribute for the mpc interaction is ignored. The name is always reset to MPC.

Listing 8.9: Modified periodic coulomb for finite size post-correction.

```
<pairpot type="MPC" name="MPC" source="e" target="e" ecut="60.0" physical="no"/>
```

### 8.3 General estimators

A broad range of estimators for physical observables are available in QMCPACK. The sections below contain input details for the total number density (density), number density resolved by particle spin (spindensity), spherically averaged pair correlation function (gofr), static structure factor (sk), energy density (energydensity), one body reduced density matrix (dm1b), S(k) based kinetic energy correction (chiesa), forward walking (ForwardWalking), and force (Force) estimators. Other estimators are not yet covered.

When an  $\langle estimator \rangle$  element appears in  $\langle hamiltonian \rangle$ , it is evaluated for all applicable chained QMC runs (e.g. VMC $\rightarrow$ DMC $\rightarrow$ DMC). Estimators are generally not accumulated during wavefunction optimization sections. If an  $\langle estimator \rangle$  element is instead provided in a particular  $\langle qmc \rangle$  element, that estimator is only evaluated for that specific section (e.g. during VMC only).

estimator factory	element				
parent elements:	hamiltonia	an. dmc		_	
type selector:	type attrib	-			
type options:	density		Density of	on a grid	
	spindensity		Spin density on a grid		
	gofr		-	elation function (quantum species)	
	$\operatorname{sk}$			ructure factor	
	structurefa	ctor	Species re	esolved structure factor	
	momentum	<u>.</u>	Momentu	um distribution	
	energydens	ity	Energy d	ensity on uniform or Voronoi grid	
	dm1b		One body	y density matrix in arbitrary basis	
	chiesa		Chiesa-Ceperley-Martin-Holzmann kinetic energy correcti		
	Force		Family of "force" estimators (see 8.5)		
	ForwardWa	alking	Forward walking values for existing estimators		
	orbitalimag	ges	Create image files for orbitals, then exit		
	flux		Checks sampling of kinetic energy		
	localmomer	nt	Atomic s	pin polarization within cutoff radius	
	numberfluc	tuations	Spatial number fluctuations		
	HFDHE2		Helium pressure		
	NearestNei	$_{ m ghbors}$	Trace nearest neighbor indices		
	Kinetic		No current function		
	Pressure		$No\ curre$	nt function	
	ZeroVarOb	$\mathbf{s}$	No current function		
	DMCCorre	ction	No current function		
shared attributes:					
name	${f datatype}$	values	$\mathbf{default}$	description	
$\mathtt{type}^r$	text	$See\ above$	0	Select estimator type	
$\mathtt{name}^r$	text	anything	any	Unique name for this estimator	

# 8.3.1 Chiesa-Ceperley-Martin-Holzmann kinetic energy correction

This estimator calculates a finite size correction to the kinetic energy following the formalism laid out in Ref. [?]. The total energy can be corrected for finite size effects by using this estimator in conjuction with the MPC correction.

estimator type=	estimator type=chiesa element					
parent elements:	hamiltonia	hamiltonian, qmc				
child elements:	None					
attributes						
name	${f datatype}$	values	$\mathbf{default}$	$\operatorname{description}$		
$\mathtt{type}^r$	text	chiesa		Must be chiesa		
$\mathtt{name}^o$	text	anything	KEcorr	Always reset to KEcorr		
$\mathtt{source}^o$	text	particleset.name	e	Identify quantum particles		
$\mathtt{psi}^o$	text	wavefunction.name	psi0	Identify wavefunction		

Listing 8.10: "Chiesa" kinetic energy finite size post-correction.

<estimator name="KEcorr" type="chiesa" source="e" psi="psi0"/>

## 8.3.2 Density estimator

The particle number density operator is given by

$$\hat{n}_r = \sum_i \delta(r - r_i) \tag{8.5}$$

The density estimator accumulates the number density on a uniform histogram grid over the simulation cell. The value obtained for a grid cell c with volume  $\Omega_c$  is then the average number of particles in that cell:

$$n_c = \int dR |\Psi|^2 \int_{\Omega_c} dr \sum_i \delta(r - r_i)$$
 (8.6)

estimator type=	estimator type=density element					
parent elements:	hamiltonian,	qmc				
child elements:	None					
attributes						
name	${f datatype}$	values	default	description		
$\mathtt{type}^r$	text	density		Must be density		
$\mathtt{name}^r$	text	anything	any	Unique name for estimator		
$\mathtt{delta}^o$	real $array(3)$	$0 \le v_i \le 1$	$0.1\ 0.1\ 0.1$	Grid cell spacing, unit coords		
${\tt x\_min}^o$	real	> 0	0	Grid starting point in x (Bohr)		
${ t x\_{ t max}}^o$	real	> 0	lattice[0]	Grid ending point in x (Bohr)		
${\tt y\_min}^o$	real	> 0	0	Grid starting point in y (Bohr)		
${\tt y\_max}^o$	real	> 0	lattice[1]	Grid ending point in y (Bohr)		
${\tt z\_min}^o$	real	> 0	0	Grid starting point in z (Bohr)		
${\tt z\_max}^o$	real	> 0	lattice[2]	Grid ending point in z (Bohr)		
${\tt potential}^o$	boolean	yes/no	no	Accumulate local potential, Deprecated		
$\mathtt{debug}^o$	boolean	yes/no	no	No current function		

- name: The name provided will be used as a label in the stat.h5 file for the blocked output data. Post-processing tools expect name="Density".
- delta: This sets the histogram grid size used to accumulate the density: delta="0.1 0.1 0.05"→ 10 × 10 × 20 grid, delta="0.01 0.01 0.01"→ 100 × 100 × 100 grid. The density grid is written to a stat.h5 file at the end of each Monte Carlo block. If you request many blocks in a <qmc/> element, or select a large grid, the resulting stat.h5 file may be many GB in size.
- \*\_min/\*\_max: Can be used to select a subset of the simulation cell for the density histogram grid. For example if a (cubic) simulation cell is 20 Bohr on a side, setting \*\_min=5.0 and \*\_max=15.0 will result in a density histogram grid spanning a 10×10×10 Bohr cube about the center of the box. Use of x\_min, x\_max, y\_min, y\_max, z\_min, z\_max is only appropriate for orthorhombic simulation cells with open boundary conditions.
- When open boundary conditions are used, a <simulationcel1/> element must be explicitly provided as the first sub-element of <qmcsystem/> for the density estimator to work. In this case the molecule should be centered around the middle of the simulation cell (L/2) and not the origin (0 since the space within the cell, and hence the density grid, is defined from 0 to L.

Listing 8.11: Density estimator (uniform grid).

<estimator name="Density" type="density" delta="0.05 0.05 0.05"/>

### 8.3.3 Lattice deviation estimator

Record deviation of a group of particles in one particle set (target) from a group of particles in another particle set (source).

estimator type=sk element					
parent elements:	hamiltonia	an, qmc			
child elements:	None				
attributes					
name	${f datatype}$	values	$\mathbf{default}$	description	
$\mathtt{type}^r$	text	latticedeviation		Must be latticedeviation	
$\mathtt{name}^r$	text	anything	any	Unique name for estimator	
$\mathtt{hdf5}^o$	boolean	yes/no	no	Output to stat.h5 (yes)	
$\mathtt{per\_xyz}^o$	boolean	yes/no	no	Directional-resolved (yes)	
$\mathtt{source}^r$	text	e/ion0/	no	source particle set	
$\mathtt{sgroup}^r$	text	$u/d/\dots$	no	source particle group	
$\mathtt{target}^r$	text	e/ion0/	no	target particle set	
$\mathtt{tgroup}^r$	text	$u/d/\dots$	no	target particle group	

- source: The "reference" particle set to measure distances from, actual reference points are determined together with sgroup.
- sgroup: The "reference" particle group to measure distances from.

- source: The "target" particle set to measure distances to.
- sgroup: The "target" particle group to measure distances to. For example, in Listing 8.12, the distance from the up electron ("u") to the origin of the coordinate system is recorded.
- per\_xyz: Record direction-resolved distance. In Listing 8.12, the x,y,z coordinates of the up electron will be recorded separately if per\_xyz=yes.
- hdf5: Record particle-resolved distances in the h5 file if gdf5=yes...

Listing 8.12: Lattice deviation estimator element.

```
<particleset name="e" random="yes">
  <group name="u" size="1" mass="1.0">
     <parameter name="charge"</pre>
                                             >
                                                   -1
                                                                          </parameter>
                                                                           </parameter>
     <parameter name="mass"</pre>
                                                   1.0
  <group name="d" size="1" mass="1.0">
     <parameter name="charge"</pre>
                                                   -1
                                                                          </parameter>
                                                                          </parameter>
     <parameter name="mass"</pre>
                                                   1.0
 </group>
</particleset>
<particleset name="wf_center">
  <group name="origin" size="1">
    <attrib name="position" datatype="posArray" condition="0">
                                0.00000000
             0.00000000
                                                    0.00000000
    </attrib>
  </group>
</particleset>
<estimator type="latticedeviation" name="latdev" hdf5="yes" per_xyz="yes"</pre>
 source="wf_center" sgroup="origin" target="e" tgroup="u"/>
```

### 8.3.4 Spin density estimator

The spin density is similar to the total density described above. In this case, the sum over particles is performed independently for each spin component.

estimator type	spindensity elen	nent		
parent elements:	hamiltonian, q	mc		
child elements:	None			
attributes				
name	datatype	values	default	$\operatorname{description}$
$\mathtt{type}^r$	text	spindensity		Must be spindensity
$\mathtt{name}^r$	text	anything	any	Unique name for estimator
$\mathtt{report}^o$	boolean	yes/no	no	Write setup details to stdout
parameters				
name	datatype	values	default	${f description}$
${\tt grid}^o$	integer $array(3)$	$v_i > 0$		Grid cell count
$\mathtt{dr}^o$	real array(3)	$v_i > 0$		Grid cell spacing (Bohr)
$\mathtt{cell}^o$	real $array(3,3)$	anything		Volume grid exists in
$\mathtt{corner}^o$	real array(3)	anything		Volume corner location
$\mathtt{center}^o$	real array(3)	anything		Volume center/origin location
${\tt voronoi}^o$	text	particleset.name		$Under\ development$
${\tt test\_moves}^o$	integer	>=0	0	Test estimator with random move

### Additional information:

- name: The name provided will be used as a label in the stat.h5 file for the blocked output data. Post-processing tools expect name="SpinDensity".
- grid: Sets the dimension of the histogram grid. Input like cparameter name="grid"> 40
  40 40 requests a 40 × 40 × 40 grid. The shape of individual grid cells is commensurate with the supercell shape.
- dr: Real space dimensions of grid cell edges (Bohr units). Input like <parameter name="dr">
  0.5 0.5 0.5 </parameter> in a supercell with axes of length 10 Bohr each (but of arbitrary shape) will produce a 20 × 20 × 20 grid. The inputted dr values are rounded to produce an integer number of grid cells along each supercell axis. Either grid or dr must be provided, but not both.
- cell: When cell is provided, a user defined grid volume is used instead of the global supercell. This must be provided if open boundary conditions are used. Additionally, if cell is provided, the user must specify where the volume is located in space in addition to its size/shape (cell) using either the corner or center parameters.
- corner:The grid volume is defined as  $corner + \sum_{d=1}^{3} u_d cell_d$  with  $0 < u_d < 1$  ("cell" refers to either the supercell or user provided cell).
- center: The grid volume is defined as  $center + \sum_{d=1}^{3} u_d cell_d$  with  $-1/2 < u_d < 1/2$  ("cell" refers to either the supercell or user provided cell). corner/center can be used to shift the grid even if cell is not specified. Simultaneous use of corner and center will cause QMCPACK to abort.

Listing 8.13: Spin density estimator (uniform grid).

<estimator type="spindensity" name="SpinDensity" report="yes">

```
<parameter name="grid"> 40 40 40 </parameter>
</estimator>
```

Listing 8.14: Spin density estimator (uniform grid centered about origin).

## **8.3.5** Pair correlation function, g(r)

The functional form of the species resolved radial pair correlation function operator is

$$g_{ss'}(r) = \frac{V}{4\pi r^2 N_s N_{s'}} \sum_{i_s=1}^{N_s} \sum_{j_{s'}=1}^{N_{s'}} \delta(r - |r_{i_s} - r_{j_{s'}}|). \tag{8.7}$$

Here  $N_s$  is the number of particles of species s and V is the supercell volume. If s = s', then the sum is restricted so that  $i_s \neq j_s$ .

In QMCPACK, an estimate of  $g_{ss'}(r)$  is obtained as a radial histogram with a set of  $N_b$  uniform bins of width  $\delta r$ . This can be expressed analytically as

$$\tilde{g}_{ss'}(r) = \frac{V}{4\pi r^2 N_s N_{s'}} \sum_{i=1}^{N_s} \sum_{j=1}^{N_{s'}} \frac{1}{\delta r} \int_{r-\delta r/2}^{r+\delta r/2} dr' \delta(r' - |r_{si} - r_{s'j}|), \tag{8.8}$$

where the radial coordinate r is restricted to reside at the bin centers,  $\delta r/2, 3\delta r/2, 5\delta r/2, \dots$ 

estimator type=gof	r element			
parent elements:	hamiltonia	an, qmc		
child elements:	None			
attributes				
name	${f datatype}$	values	default	description
$\mathtt{type}^r$	text	$\operatorname{gofr}$		Must be gofr
$\mathtt{name}^o$	text	anything	any	No current function
$\mathtt{num\_bin}^r$	integer	> 1	20	# of histogram bins
${\tt rmax}^o$	real	> 0	10	Histogram extent (B
$\mathtt{dr}^o$	real	> 0	0.5	No current function
$\mathtt{debug}^o$	boolean	yes/no	no	No current function
$\mathtt{target}^o$	text	particleset.name	hamiltonian.target	Quantum particles
${ t source/sources}^o$	text array	particleset.name	hamiltonian.target	Classical particles

- num\_bin: The number of bins in each species pair radial histogram.
- rmax: Maximum pair distance included in the histogram. The uniform bin width is  $\delta r = \text{rmax/num\_bin}$ . If periodic boundary conditions are used for any dimension of the simulation cell, then the default value of rmax is the simulation cell radius instead of 10 Bohr. For open boundary conditions the volume (V) used is 1.0 Bohr<sup>3</sup>.
- source/sources: If unspecified, only pair correlations between each species of quantum particle will be measured. For each classical particleset specified by source/sources, additional pair correlations between each quantum and classical species will be measured. Typically there is only one classical particleset (e.g. source="ion0"), but there can be several in principle (e.g. sources="ion0 ion1 ion2").
- target: The default value is the preferred usage (i.e. target does not need to be provided).
- Data is outputted to the stat.h5 for each QMC sub-run. Individual histograms are named according to the quantum particleset and index of the pair. For example, if the quantum particleset is named "e" and there are two species (up and down electrons, say), then there will be three sets of histogram data in each stat.h5 file named gofr\_e\_0\_0, gofr\_e\_0\_1, and gofr\_e\_1\_1 for up-up, up-down, and down-down correlations, respectively.

Listing 8.15: Pair correlation function estimator element.

```
<estimator type="gofr" name="gofr" num_bin="200" rmax="3.0" />
```

Listing 8.16: Pair correlation function estimator element with additional electron-ion correlations.

```
<estimator type="gofr" name="gofr" num_bin="200" rmax="3.0" source="ion0" />
```

## 8.3.6 Species kinetic energy

Record species-resolved kinetic energy instead of the total kinetic energy in the Kinetic column of scalar.dat. SpeciesKineticEnergy is arguable the simplest estimator in QMCPACK. The implementation of this estimator is detailed in manual/estimator/estimator\_implementation.pdf.

estimator type=sk element						
parent elements:	hamiltonia	an, qmc				
child elements:	None	None				
attributes						
name	datatype	values	default	description		
$\mathtt{type}^r$	text	specieskinetic		Must be specieskinetic		
$\mathtt{name}^r$	text	anything	any	Unique name for estimator		
${\tt hdf5}^o$	boolean	yes/no	no	Output to stat.h5 (yes)		

Listing 8.17: Species kinetic energy estimator element.

```
<estimator type="specieskinetic" name="skinetic" hdf5="no"/>
```

## 8.3.7 Static structure factor, S(k)

Let  $\rho_{\mathbf{k}}^e = \sum_j e^{i\mathbf{k}\cdot\mathbf{r}_j^e}$  be the Fourier space electron density, with  $\mathbf{r}_j^e$  being the coordinate of the j-th electron.  $\mathbf{k}$  is a wavevector commensurate with the simulation cell. QMCPACK allows the user to accumulate the static electron structure factor  $S(\mathbf{k})$  at all commensurate  $\mathbf{k}$  such that  $|\mathbf{k}| \leq (LR\_DIM\_CUTOFF)r_c$ .  $N^e$  is the number of electrons, LR\_DIM\\_CUTOFF is the optimized breakup parameter, and  $r_c$  is the Wigner-Seitz radius. It is defined as follows:

$$S(\mathbf{k}) = \frac{1}{N^e} \langle \rho_{-\mathbf{k}}^e \rho_{\mathbf{k}}^e \rangle \tag{8.9}$$

estimator type=sk element					
parent elements:	hamiltonia	hamiltonian, qmc			
child elements:	None				
attributes					
name	datatype	values	default	description	
$\mathtt{type}^r$	text	$\operatorname{sk}$		Must be sk	
$\mathtt{name}^r$	text	anything	any	Unique name for estimator	
$\mathtt{hdf5}^o$	boolean	yes/no	no	Output to stat.h5 (yes) or scalar.dat (no	

### Additional information:

- name: Unique name for estimator instance. A data structure of the same name will appear in stat.h5 output files.
- hdf5: If hdf5==yes output data for S(k) is directed to the stat.h5 file (recommended usage). If hdf5==no, the data is instead routed to the scalar.dat file resulting in many columns of data with headings prefixed by name and postfixed by the k-point index (e.g. sk\_0 sk\_1 ...sk\_1037 ...).
- This estimator only works in periodic boundary conditions. Its presence in the input file is ignored otherwise.
- This is not a species resolved structure factor. Additionally, for  $\mathbf{k}$  vectors commensurate with the unit cell,  $S(\mathbf{k})$  will include contributions from the static electronic density, thus meaning it won't accurately measure the electron-electron density response.

Listing 8.18: Static structure factor estimator element.

```
<estimator type="sk" name="sk" hdf5="yes"/>
```

### 8.3.8 Energy density estimator

An energy density operator,  $\hat{\mathcal{E}}_r$ , satisfies

$$\int dr \hat{\mathcal{E}}_r = \hat{H},\tag{8.10}$$

where the integral is over all space and  $\hat{H}$  is the Hamiltonian. In QMCPACK, the energy density is split into kinetic and potential components

$$\hat{\mathcal{E}}_r = \hat{\mathcal{T}}_r + \hat{\mathcal{V}}_r \tag{8.11}$$

with each component given by

$$\hat{\mathcal{T}}_r = \frac{1}{2} \sum_i \delta(r - r_i) \hat{p}_i^2$$

$$\hat{\mathcal{V}}_r = \sum_{i < j} \frac{\delta(r - r_i) + \delta(r - r_j)}{2} \hat{v}^{ee}(r_i, r_j) + \sum_{i\ell} \frac{\delta(r - r_i) + \delta(r - \tilde{r}_\ell)}{2} \hat{v}^{eI}(r_i, \tilde{r}_\ell)$$

$$+ \sum_{\ell < m} \frac{\delta(r - \tilde{r}_\ell) + \delta(r - \tilde{r}_m)}{2} \hat{v}^{II}(\tilde{r}_\ell, \tilde{r}_m).$$
(8.12)

Here  $r_i$  and  $\tilde{r}_\ell$  represent electon and ion positions, respectively,  $\hat{p}_i$  is a single electron momentum operator, and  $\hat{v}^{ee}(r_i, r_j)$ ,  $\hat{v}^{eI}(r_i, \tilde{r}_\ell)$ ,  $\hat{v}^{II}(\tilde{r}_\ell, \tilde{r}_m)$  are the electron-electron, electron-ion, and ionion pair potential operators (including non-local pseudopotentials, if present). This form of the energy density is size consistent, *i.e.* the partially integrated energy density operators of well separated atoms gives the isolated Hamiltonians of the respective atoms. For periodic systems with twist averaged boundary conditions, the energy density is formally correct only for either a set of supercell k-points that correspond to real valued wavefunctions, or a k-point set that has inversion symmetry around a k-point having a real valued wavefunction. For more information about the energy density, see Ref. [?].

In QMCPACK, the energy density can be accumulated on piecewise uniform three dimensional grids in generalized cartesian, cylindrical, or spherical coordinates. The energy density integrated within Voronoi volumes centered on ion positions is also available. The total particle number density is also accumulated on the same grids by the energy density estimator for convenience so that related quantities, such as the regional energy per particle, can be computed easily.

estimator type=EnergyDensity element						
parent elements:	hamiltonia	hamiltonian, qmc				
child elements:	reference	reference_points, spacegrid				
attributes						
name	datatype	values	$\mathbf{default}$	description		
$\mathtt{type}^r$	text	EnergyDensity		Must be EnergyDensity		
$\mathtt{name}^r$	text	anything		Unique name for estimator		
$\mathtt{dynamic}^r$	text	particleset.name		Identify electrons		
$\mathtt{static}^o$	text	particleset.name		Identify ions		

### Additional information:

• name: Must be unique. A dataset with blocked statistical data for the energy density will appear in the stat.h5 files labeled as name.

Listing 8.19: Energy density estimator accumulated on a 20x10x10 grid over the simulation cell.

Listing 8.20: Energy density estimator accumulated within spheres of radius 6.9 Bohr centered on the first and second atoms in the ion0 particleset.

```
<estimator type="EnergyDensity" name="EDatom" dynamic="e" static="ion0">
  <reference_points coord="cartesian">
   r1 1 0 0
   r2 0 1 0
   r3 0 0 1
  </reference_points>
  <spacegrid coord="spherical">
    <origin p1="ion01"/>
    <axis p1="r1" scale="6.9" label="r"</pre>
    <axis p1="r2" scale="6.9" label="phi"</pre>
                                             grid="0 1"/>
    <axis p1="r3" scale="6.9" label="theta" grid="0 1"/>
  </spacegrid>
  <spacegrid coord="spherical">
    <origin p1="ion02"/>
    <axis p1="r1" scale="6.9" label="r"
                                             grid="0 1"/>
    <axis p1="r2" scale="6.9" label="phi"</pre>
                                             grid="0 1"/>
    <axis p1="r3" scale="6.9" label="theta" grid="0 1"/>
  </spacegrid>
</estimator>
```

Listing 8.21: Energy density estimator accumulated within Voronoi polyhedra centered on the ions.

```
<estimator type="EnergyDensity" name="EDvoronoi" dynamic="e" static="ion0">
        <spacegrid coord="voronoi"/>
        </estimator>
```

The <reference\_points/> element provides a set of points for later use in specifying the origin and coordinate axes needed to construct a spatial histogramming grid. Several reference points on the surface of the simulation cell (see Table 8.1) as well as the positions of the ions (see the energydensity.static attribute) are made available by default. The reference points can be used, for example, to construct a cylindrical grid along a bond with the origin on the bond center.

```
reference_points element
parent elements:
                  estimator type=EnergyDensity
child elements:
                  None
attributes
  name
                  datatype values
                                             default
                                                       description
  coord^r
                  text
                              cartesian/cell
                                                       Specify coordinate system
body text
                  The body text is a line formatted list of points with labels
```

- coord: If coord=cartesian, labeled points are in cartesian (x,y,z) format in units of Bohr. If coord=cell, then labeled points are in units of the simulation cell axes.
- body text: The list of points provided in the body text are line formatted, with four entries per line (label coor1 coor2 coor3). A set of points referenced to the simulation cell are available by default (see table 8.1). If energydensity.static is provided, the location of each individual ion is also available (e.g. if energydensity.static=ion0, then the location of the first atom is available with label ion01, the second with ion02, etc.). All points can be used by label when constructing spatial histogramming grids (see the spacegrid element below) used to collect energy densities.

label	point	description
zero	0 0 0	Cell center
a1	$a_1$	Cell axis 1
a2	$a_2$	Cell axis 2
a3	$a_3$	Cell axis 3
f1p	$a_1/2$	Cell face 1+
f1m	$-a_1/2$	Cell face 1-
f2p	$a_2/2$	Cell face 2+
f2m	$-a_2/2$	Cell face 2-
f3p	$a_3/2$	Cell face 3+
f3m	$-a_3/2$	Cell face 3-
cppp	$(a_1 + a_2 + a_3)/2$	Cell corner $+,+,+$
cppm	$(a_1 + a_2 - a_3)/2$	Cell corner $+,+,-$
cpmp	$(a_1 - a_2 + a_3)/2$	Cell corner $+,-,+$
cmpp	$(-a_1 + a_2 + a_3)/2$	Cell corner $-,+,+$
cpmm	$(a_1 - a_2 - a_3)/2$	Cell corner $+,-,-$
cmpm	$(-a_1 + a_2 - a_3)/2$	Cell corner $-,+,-$
cmmp	$(-a_1 - a_2 + a_3)/2$	Cell corner $-,-,+$
cmmm	$(-a_1 - a_2 - a_3)/2$	Cell corner -,-,-

Table 8.1: Reference points available by default. The vectors  $a_1$ ,  $a_2$ , and  $a_3$  refer to the simulation cell axes. The representation of the cell is centered around zero.

The <spacegrid/> element is used to specify a spatial histogramming grid for the energy density. Grids are constructed based on a set of, potentially non-orthogonal, user provided coordinate axes. The axes are based on information available from reference\_points. Voronoi grids are based only on nearest neighbor distances between electrons and ions. Any number of space grids can be provided to a single energy density estimator.

spacegrid elemen	nt			
parent elements: child elements: attributes	estimator origin, a	type=Energ xis	yDensity	
name coord <sup>r</sup>	datatype text	values cartesian cylindrical spherical voronoi	default	description Specify coordinate system

The <origin/> element gives the location of the origin for a non-Voronoi grid. Additional information:

• p1/p2/fraction: The location of the origin is set to p1+fraction\*(p2-p1). If only p1 is provided, the origin is at p1.

origin element				
parent elements:	spacegrid			
child elements:	None			
attributes				
name	${f datatype}$	values	$\mathbf{default}$	${f description}$
$\mathtt{p1}^r$	text	reference_point.label		Select end point
$\mathtt{p2}^o$	text	reference_point.label		Select end point
${\tt fraction}^o$	real		0	Interpolation fraction

The <axis/> element represents a coordinate axis used to construct the, possibly curved, coordinate system for the histogramming grid. Three <axis/> elements must be provided to a non-Voronoi <spacegrid/> element.

axis element				
parent elements:	spacegrid			
child elements:	None			
attributes				
name	${f datatype}$	values	$\mathbf{default}$	${f description}$
$\mathtt{label}^r$	text	$See\ below$		Axis/dimension label
$\mathtt{grid}^r$	text		"0 1"	Grid ranges/intervals
$\mathtt{p1}^r$	text	reference_point.label		Select end point
$\mathtt{p2}^o$	text	reference_point.label		Select end point
$\mathtt{scale}^o$	real			Interpolation fraction

### Additional information:

- label: The allowed set of axis labels depends on the coordinate system (i.e. spacegrid.coord). Labels are x/y/z for coord=cartesian, r/phi/z for coord=cylindrical, r/phi/theta for coord=spherical.
- p1/p2/scale: The axis vector is set to p1+scale\*(p2-p1). If only p1 is provided, the axis vector is p1.
- grid: Specifies the histogram grid along the direction specified by label. The allowed grid points fall in the range [-1,1] for label=x/y/z or [0,1] for r/phi/theta. A grid of 10 evenly spaced points between 0 and 1 can be requested equivalently by grid="0 (0.1) 1" or grid="0 (10) 1". Piecewise uniform grids covering portions of the range are supported, e.g. grid="-0.7 (10) 0.0 (20) 0.5".
- Note that grid specifies the histogram grid along the (curved) coordinate given by label. The axis specified by p1/p2/scale does not correspond one-to-one with label unless label=x/y/z, but the full set of axes provided define the (sheared) space on top of which the curved (e.g. spherical) coordinate system is built.

## 8.3.9 One body density matrix

The N-body density matrix in DMC is  $\hat{\rho}_N = |\Psi_T\rangle\langle\Psi_{FN}|$  (for VMC, substitute  $\Psi_T$  for  $\Psi_{FN}$ ). The one body reduced density matrix (1RDM) is obtained by tracing out all particle coordinates but

one:

$$\hat{n}_1 = \sum_n Tr_{R_n} |\Psi_T\rangle \langle \Psi_{FN}| \tag{8.13}$$

In the formula above, the sum is over all electron indices and  $Tr_{R_n}(*) \equiv \int dR_n \langle R_n | * | R_n \rangle$  with  $R_n = [r_1, ..., r_{n-1}, r_{n+1}, ..., r_N]$ . When the sum is restricted over spin up or down electrons, one obtains a density matrix for each spin species. The 1RDM computed by QMCPACK is partitioned in this way.

In real space, the matrix elements of the 1RDM are

$$n_1(r,r') = \langle r|\hat{n}_1|r'\rangle = \sum_n \int dR_n \Psi_T(r,R_n) \Psi_{FN}^*(r',R_n)$$
 (8.14)

A more efficient and compact representation of the 1RDM is obtained by expanding in the single particle orbitals obtained from a Hartree-Fock or DFT calculation,  $\{\phi_i\}$ :

$$n_{1}(i,j) = \langle \phi_{i} | \hat{n}_{1} | \phi_{j} \rangle$$

$$= \int dR \Psi_{FN}^{*}(R) \Psi_{T}(R) \sum_{n} \int dr'_{n} \frac{\Psi_{T}(r'_{n}, R_{n})}{\Psi_{T}(r_{n}, R_{n})} \phi_{i}(r'_{n})^{*} \phi_{j}(r_{n})$$
(8.15)

The integration over r' in Eq. 8.15 is inefficient when one is also interested in obtaining matrices involving energetic quantities, such as the energy density matrix of Ref. [?] or the related (and more well known) Generalized Fock matrix. For this reason, an approximation is introduced as follows:

$$n_1(i,j) \approx \int dR \Psi_{FN}(R)^* \Psi_T(R) \sum_n \int dr_n' \frac{\Psi_T(r_n', R_n)^*}{\Psi_T(r_n, R_n)^*} \phi_i(r_n)^* \phi_j(r_n')$$
(8.16)

For VMC, FN-DMC, FP-DMC, and RN-DMC the formula above represents an exact sampling of the 1RDM corresponding to  $\hat{\rho}_N^{\dagger}$  (see appendix A of Ref. [?] for more detail).

estimator type=dm1b e	lomont			
parent elements:		am c		
child elements:	hamiltonian	, qmc		
	none			
attributes	1	1	1.6.1	
name	datatype	values	default	description
$\mathtt{type}^r$	text	dm1b		Must be dm1b
$\mathtt{name}^r$	text	anything		Unique name for estimator
parameters				
name	${f datatype}$	values	default	${f description}$
$\mathtt{basis}^r$	text array	sposet.name(s)		Orbital basis
${ t integrator}^o$	text	uniform_grid uniform density	$uniform\_grid$	Integration method
${\tt evaluator}^o$	text	loop/matrix	loop	Evaluation method
$\mathtt{scale}^o$	real	0 < scale < 1	1.0	Scale integration cell
$\mathtt{center}^o$	real array(3)	any point		Center of cell
$\mathtt{points}^o$	integer	> 0	10	Grid points in each dim
${ t samples}^o$	integer	> 0	10	MC samples
$\mathtt{warmup}^o$	integer	> 0	30	MC warmup
$\verb timestep ^o$	real	> 0	0.5	MC time step
${\tt use\_drift}^o$	boolean	yes/no	no	Use drift in VMC
$\verb check_overlap ^o$	boolean	yes/no	no	Print overlap matrix
$\verb check_derivatives ^o$	boolean	yes/no	no	Check density derivatives
$\verb"acceptance_ratio"$	boolean	yes/no	no	Print accept ratio
${ t rstats}^o$	boolean	yes/no	no	Print spatial stats
${ t normalized}^o$	boolean	yes/no	no	basis comes norm'ed
$\verb"energy_matrix"$	boolean	yes/no	no	Energy density matrix

- name: Density matrix results appear in stat.h5 files labeled according to name.
- basis: List of sposet.name's. The total set of orbitals contained in all sposet's comprises the basis (subspace) the one body density matrix is projected onto. This set of orbitals generally includes many virtual orbitals that are not occupied in a single reference Slater determinant.
- integrator: This selects the method used to perform the additional single particle integration. Options are uniform\_grid (uniform grid of points over the cell), uniform (uniform random sampling over the cell), and density (Metropolis sampling of approximate density:  $\sum_{b \in \mathtt{basis}} |\phi_b|^2$ , not well tested, please check results carefully!). Depending on the integrator selected, different subsets of the other input parameters are active.
- evaluator: Select for-loop or matrix multiply implementations. Matrix is preferred for speed. Both implementations should give the same results, but please check as this has not been exhaustively tested.
- scale: Resize the simulation cell by scale for use as an integration volume (active for integrator=uniform/uniform\_grid).

- center: Translate the integration volume to center at this point (active for integrator=uniform/uniform\_g. If center is not provided, the scaled simulation cell is used as is.
- points: The number of grid points in each dimension for integrator=uniform\_grid. For example, points=10 results in a uniform 10x10x10 grid over the cell.
- samples: Sets the number of Monte Carlo samples collected each step (active for integrator=uniform/dense
- warmup: Number of warmup Metropolis steps at the start of the run, prior to data collection (active for integrator=density).
- timestep: Drift-diffusion timestep used in Metropolis sampling (active for integrator=density).
- use\_drift: Enable drift in Metropolis sampling (active for integrator=density).
- check\_overlap: Print the overlap matrix (computed via simple Riemann sums) to the log and then abort. Note that subsequent analysis based on the 1RDM is simplest if the input orbitals are orthogonal.
- check\_derivatives: Print analytic and numerical derivatives of the approximate (sampled) density for several sample points, then abort.
- acceptance\_ratio: Print the acceptance ratio of the density sampling to the log each step.
- rstats: Print statistical information about the spatial motion of the sampled points to the log each step.
- normalized: Declare whether the inputted orbitals are normalized or not. If normalized=no, direct Riemann integration over a 200x200x200 grid will be used to compute the normalizations prior to use.
- energy\_matrix: Also accumulate the one body reduced energy density matrix and write it to stat.h5. This matrix is not covered in any detail here; the interested reader is referred to Ref. [?].

Listing 8.22: One body density matrix with uniform grid integration.

```
<estimator type="dm1b" name="DensityMatrices">
 <parameter name="basis"</pre>
                            > spo_u spo_uv </parameter>
 <parameter name="evaluator"</pre>
                            > matrix
                                            </parameter>
 <parameter name="points"</pre>
                                            </parameter>
                            > 4
                            > 1.0
 <parameter name="scale"</pre>
                                            </parameter>
 <parameter name="center"</pre>
                            > 000
                                            </parameter>
</estimator>
```

Listing 8.23: One body density matrix with uniform sampling.

```
<estimator type="dm1b" name="DensityMatrices">
 <parameter name="basis"</pre>
                                   > spo_u spo_uv
                                                      </parameter>
  <parameter name="evaluator"</pre>
                                   > matrix
                                                      </parameter>
  <parameter name="integrator" > uniform
                                                      </parameter>
 <parameter name="samples"</pre>
                                  > 64
                                                      </parameter>
  <parameter name="scale"</pre>
                                   > 1.0
                                                      </parameter>
  <parameter name="center"</pre>
                                   > 0 0 0
                                                      </parameter>
</estimator>
```

Listing 8.24: One body density matrix with density sampling.

```
<estimator type="dm1b" name="DensityMatrices">
 <parameter name="basis"</pre>
                                   > spo_u spo_uv
                                                       </parameter>
  <parameter name="evaluator"</pre>
                                    > matrix
                                                       </parameter>
 <parameter name="integrator"</pre>
                                                       </parameter>
                                   > density
                                                       </parameter>
 <parameter name="samples"</pre>
                                   > 64
  <parameter name="timestep"</pre>
                                    >
                                      0.5
                                                       </parameter>
  <parameter name="use_drift"</pre>
                                                       </parameter>
                                    > no
</estimator>
```

Listing 8.25: Example sposet initialization for density matrix use. Occupied and virtual orbital sets are created separately, then joined (basis="spo\_u spo\_uv").

```
<sposet_builder type="bspline" href="../dft/pwscf_output/pwscf.pwscf.h5" tilematrix="1 0 0 0 1 0 0 0 1"
    twistnum="0" meshfactor="1.0" gpu="no" precision="single">
    <sposet type="bspline" name="spo_u" group="0" size="4"/>
    <sposet type="bspline" name="spo_d" group="0" size="2"/>
    <sposet type="bspline" name="spo_uv" group="0" index_min="4" index_max="10"/>
    </sposet_builder>
```

Listing 8.26: Example sposet initialization for density matrix use. Density matrix orbital basis created separately (basis="dm\_basis").

```
<sposet_builder type="bspline" href="../dft/pwscf_output/pwscf.pwscf.h5" tilematrix="1 0 0 0 1 0 0 0 1"
    twistnum="0" meshfactor="1.0" gpu="no" precision="single">
    <sposet type="bspline" name="spo_u" group="0" size="4"/>
    <sposet type="bspline" name="spo_d" group="0" size="2"/>
    <sposet type="bspline" name="dm_basis" size="50" spindataset="0"/>
    </sposet_builder>
```

## 8.4 Forward Walking Estimators

Forward walking is a method by which one can sample the pure fixed-node distribution  $\langle \Phi_0 | \Phi_0 \rangle$ . Specifically, one multiplies each walker's DMC mixed estimate for the observable  $\mathcal{O}$ ,  $\frac{\mathcal{O}(\mathbf{R})\Psi_T(\mathbf{R})}{\Psi_T(\mathbf{R})}$ , by the weighting factor  $\frac{\Phi_0(\mathbf{R})}{\Psi_T(\mathbf{R})}$ . As it turns out, this weighting factor for any walker  $\mathbf{R}$  is proportional to the total number of descendants the walker will have after a sufficiently long projection time  $\beta$ .

To forward walk on an observable, one declares a generic forward walking estimator within a <hamiltonian> block, and then specifies the observables to forward walk on and forward walking parameters. Here is a summary.

estimator type=	estimator type=ForwardWalking element						
parent elements:	hamiltonia	hamiltonian, qmc					
child elements:	Observable	Observable					
attributes							
name	datatype	values	$\mathbf{default}$	$\operatorname{description}$			
$\mathtt{type}^r$	text	ForwardWalking		Must be "ForwardWalking"			
$\mathtt{name}^r$	text	anything	any	Unique name for estimator			

### Additional information:

• Cost: Due to having to store histories of observables up to max time-steps, one should multiply the memory cost of storing the non-forward walked observables variables by max. Not an issue

Observable $\operatorname{elem}$	ent				
parent elements:	estimator	, hamilton	ian, qmc		
child elements:	None				ļ
attributes					ļ
name	${f datatype}$	values	$\mathbf{default}$	description	ļ
$\mathtt{name}^r$	text	anything	any	Registered name of existing estimator on which to forw	ard
$\mathtt{max}^r$	integer	> 0		The maximum projection time in steps (max= $\beta/\tau$ ).	
$\mathtt{frequency}^r$	text	$\geq 1$		Dump data only for every frequency-th	
				to scalar.dat file	

for things like the potential energy, but can be prohibitive for observables like density, forces,

• Naming Convention: Forward walked observables are automatically named FWE\_name\_i, where i is the forward walked expectation value at time step i, and name is whatever name appears in the <Observable> block. This is also how it will appear in the scalar.dat file.

In the following example case, QMCPACK forward walks on the potential energy for 300 time steps, and dumps the forward walked value at every time step.

Listing 8.27: Forward walking estimator element.

```
<estimator name="fw" type="ForwardWalking">
  <Observable name="LocalPotential" max="300" frequency="1"/>
  <!--- Additional Observable blocks go here -->
  </estimator>
```

## 8.5 "Force" estimators

QMCPACK supports force estimation by use of the Chiesa-Ceperly-Zhang (CCZ) estimator. Currently, open and periodic boundary conditions are supported, but for all-electron calculations only.

Without loss of generality, the CCZ estimator for the z-component of the force on an ion centered at the origin is given by the following expression:

$$F_z = -Z \sum_{i=1}^{N_e} \frac{z_i}{r_i^3} [\theta(r_i - \mathcal{R}) + \theta(\mathcal{R} - r_i) \sum_{\ell=1}^{M} c_\ell r_i^\ell]$$
 (8.17)

Z is the ionic charge, M is the degree of the smoothing polynomial,  $\mathcal{R}$  is a real-space cutoff of the sphere within which the bare-force estimator is smoothed, and  $c_{\ell}$  are predetermined coefficients. These coefficients are chosen to minimize the weighted mean square error between the bare force estimate and the s-wave filtered estimator. Specifically,

$$\chi^2 = \int_0^{\mathcal{R}} dr \, r^m \left[ f_z(r) - \tilde{f}_z(r) \right]^2 \tag{8.18}$$

Here, m is the weighting exponent,  $f_z(r)$  is the unfiltered radial force density for the z force component, and  $\tilde{f}_z(r)$  smoothed polynomial function for the same force density. The reader is invited to refer to the original paper for a more thorough explanation of the methodology, but with the notation in hand, QMCPACK takes the following parameters.

estimator type=	Force eleme	nt		
parent elements:	hamiltonia	an, qmc		
child elements:	parameter			
attributes				
name	${f datatype}$	values	$\operatorname{default}$	description
$\mathtt{mode}^o$	text	$See\ above$	bare	Select estimator type
$\mathtt{type}^r$	text	Force		Must be "Force"
$\mathtt{name}^o$	text	anything	ForceBase	Unique name for this estimator
$\mathtt{pbc}^o$	boolean	yes/no	yes	Using periodic BC's or not
${\tt addionion}^o$	boolean	yes/no	no	Add the ion-ion force contribution to output force estim
parameters				
name	${f datatype}$	values	$\mathbf{default}$	description
$\mathtt{rcut}^o$	real	> 0	1.0	Real space cutoff $\mathcal{R}$ in bohr.
$\mathtt{nbasis}^o$	integer	> 0	2	Degree of smoothing polynomial $M$
${\tt weightexp}^o$	integer	> 0	2	$\chi^2$ weighting exponent $m$ .

### Additional information:

- Naming Convention: The unique identifier name is appended with name\_X\_Y in the scalar.dat file, where X is the ion ID number, and Y is the component ID (an integer with x=0, y=1, z=2). All force components for all ions are computed and dumped to the scalar.dat file.
- Miscellaneous: Usually, the default choice of weightexp is sufficient. Different combinations of rcut and nbasis should be tested though to minimize variance and bias. There is of course a tradeoff, with larger nbasis and smaller rcut leading to smaller biases and larger variances.

The following is an example use case.

```
<estimator name="myforce" type="Force" mode="cep" addionion="yes">
    <parameter name="rcut">0.1</parameter>
    <parameter name="nbasis">4</parameter>
    <parameter name="weightexp">2</parameter>
</estimator>
```

# Chapter 9

# Quantum Monte Carlo Methods

qmc factory elemen	it							
parent elements:	simulation	simulation, loop						
type selector:	method att	ribute						
type options:	vmc	Variational Monte Carlo						
	linear	Wavefunction	Wavefunction optimization with linear method					
	dmc	Diffusion Monte Carlo						
	rmc	Reptation Monte Carlo						
shared attributes:								
name	datatype	values	$\mathbf{default}$	$\operatorname{description}$				
method	text	listed above	invalid	QMC driver				
move	text	pbyp, alle	pbyp	method used to move electrons				
gpu	text	yes, no	dep.	use the GPU				
trace	text		no	???				
checkpoint	integer	-1, 0, n	-1	checkpoint frequency				
record	integer	n	0	save configuration every n steps				
target	text			???				
completed	text			???				
append	text	yes, no	yes	???				

- move. There are two ways implemented to move electrons. The more used method is the particle-by-particle move. In this method, only one electron is moved for acceptance or rejection. The other method is the all-electron move, namely all the electrons are moved once for testing acceptance or rejection.
- gpu. When the executable is compiled with CUDA, the target computing device can be chosen by this switch. With a regular CPU only compilation, this option is not effective.
- checkpoint. Enable or disable checkpointing and specify the frequency of output. Possible values are:
  - -1 No checkpoint (default setting).
  - **0** Dump after the completion of a qmc section.

**n** Dump after every n blocks. Also dump at the end of the run.

The particle configurations will be written to a .config.h5 file.

Listing 9.1: The following is an example of running a simulation that can be restarted.

The checkpoint flag instructs queepack to output walker configurations. This also works in Variational Monte Carlo. This will output an h5 file with the name "projectid"."runnumber".config.h5. Check that this file exists before attempting a restart.

To continue a run, specify the mcwalkerset element before your VMC/DMC block:

Listing 9.2: Restart (read walkers from previous run)

BH is the project id and s002 is the calculation number to read in the walkers from the previous run.

In the project id section, make sure that the series number is different than any existing one to avoid overwriting it.

## 9.1 Variational Monte Carlo

vmc method				
parameters				
name	datatype	values	$\mathbf{default}$	description
walkers	integer	> 0	dep.	number of walkers per node
blocks	integer	$\geq 0$	1	number of blocks
steps	integer	$\geq 0$	1	number of steps per block
warmupsteps	integer	$\geq 0$	0	number of steps for warming up
substeps	integer	$\geq 0$	1	number of substeps per step
usedrift	text	yes, no	no	use the algorithm with drift
timestep	real	> 0	0.1	time step for each electron mov
samples	integer	$\geq 0$	0	total number of samples
stepsbetweensamples	integer	> 0	1	period of the sample accumulat
samplesperthread	integer	$\geq 0$	0	number of samples per thread
storeconfigs	integer	all values	0	store configurations
blocks_between_recompute	integer	$\geq 0$	dep.	wavefunction recompute frequen

- walkers. The initial default number of walkers is 1 but in the CPU branch this number will be overwritten as the number of OpenMP threads if the user requested number is smaller than the number of threads.
- blocks. This parameter is universal for all the method. At the end of each block, all the statistics accumulated in the block is dumped in to files, e.g. scalar.dat.
- warmupsteps. Warm-up steps are steps used only for equilibration. All the samples generated by warm-up steps are discarded. In practice, there's no need to use many walm-up steps because we can always discard more statistics when we perform the post-process.
- substeps. In a substep, each of the electrons is moved only once by either particle-by-particle or all-electron move. Because the local energy is evaluated not at each substep but at each step, increasing the number of substeps doesn't accumulate more samples. But in order to reduce the correlation between consecutive samples, increasing substeps is a very good option for its cheaper computational cost.
- usedrift. The VMC is implemented in two algorithms with or without drift. In the no-drift algorithm, the move of each electron is proposed with a Gaussian distribution. The standard deviation is chosen as the timestep input. In the drift algorithm, electrons are moved by langevin dynamics.
- timestep. The meaning of timestep depends on whether the drift is used or not. In general, larger timestep reduces the time correlation but might also reduces the accept ratio. Users are required to check the accept ratio of the calculation and make sure it's larger than 0.9 or between 0.2 and 0.8 with or without the drift.
- stepsbetweensamples. Due to the fact that samples generated by consecutive steps might be still correlated. Having stepsbetweensamples larger than 1 reduces that correlation. In practice, using larger substeps is cheaper than using stepsbetweensamples to decorrelate samples.
- samples. This is the total amount of samples generated in the current VMC session. This parameter is not important for VMC only calculation but necessary if optimization or DMC follows.

$$samples = \frac{blocks \cdot steps \cdot walkers}{steps between samples} \cdot number of MPI tasks$$

• samplesperthread. This is an alternative way to set the target amount of samples. More useful in the VMC session preparing the population for the following DMC calculation.

$$sample sperthread = \frac{blocks \cdot steps}{steps between samples}$$

- storeconfigs. If storeconfigs is set to a non-zero value, then electron configurations during the VMC run will be saved to the files.
- blocks\_between\_recompute. For every few blocks, recompute the accuracy critical part of the wavefunction from scratch. =1 by default when using mixed precision. =0 (no recompute) by default when not using mixed precision. Recomputing introduces a performance penalty but it is small using > 1 recompute period.

The following is an example of VMC section.

```
<qmc method="vmc" move="pbyp" gpu="yes">
    <estimator name="LocalEnergy" hdf5="no"/>
    <parameter name="walkers"> 256 </parameter>
    <parameter name="samples"> 2867200 </parameter>
    <parameter name="stepsbetweensamples"> 1 </parameter>
    <parameter name="stepsbetweensamples"> 1 </parameter>
    <parameter name="substeps"> 5 </parameter>
    <parameter name="warmupSteps"> 5 </parameter>
    <parameter name="blocks"> 70 </parameter>
    <parameter name="timestep"> 1.0 </parameter>
    <parameter name="timestep"> 1.0 </parameter>
    <parameter name="usedrift"> no </parameter>
    </qmc>
```

## 9.2 Wavefunction Optimization

Optimizing wavefunction is critical in all kinds of real-space quantum Monte Carlo calculations because it significantly improves both the accuracy and efficiency of computation. However, it is very difficult to directly adopt deterministic minimization approaches due to the stochastic nature of evaluating quantities with Monte Carlo. Thanks to the algorithmic breakthrough during the first decade of this century and the tremendous computer power available, it becomes feasible to optimize tens of thousands of parameters in a wavefunction for a solid or molecule. QMCPACK has multiple optimizers implemented based on the state-of-the-art linear method. We are continually improving our optimizers for the robustness and friendliness and trying to provide a single solution. Due to the large variation of wavefunction types carrying distinct characteristics, using several optimizer may be needed in some cases. It is highly suggested to read the recommendation from the experts maintaining these optimizers.

A typical optimization block looks like the following. It starts with method="linear" and contains three blocks of parameters.

```
<loop max="10">
 <qmc method="linear" move="pbyp" gpu="yes">
   <!-- Specify the VMC options -->
   <parameter name="walkers">
                                 256 </parameter>
   <parameter name="samples">
                                 2867200 </parameter>
   <parameter name="stepsbetweensamples">
                                            1 </parameter>
   <parameter name="substeps"> 5 </parameter>
   <parameter name="warmupSteps"> 5 </parameter>
   <parameter name="blocks"> 70 </parameter>
   <parameter name="timestep"> 1.0 </parameter>
   <parameter name="usedrift"> no </parameter>
   <estimator name="LocalEnergy" hdf5="no"/>
   <!-- Specify the correlated sampling options and define the cost function -->
   <parameter name="usebuffer"> no </parameter>
   <parameter name="nonlocalpp"> yes </parameter>
        <cost name="energy">
                                               0.95 </cost>
        <cost name="unreweightedvariance">
                                               0.00 </cost>
        <cost name="reweightedvariance">
                                               0.05 </cost>
   <!-- Specify the optimizer options -->
   <parameter name="MinMethod">quartic</parameter>
   <parameter name="exp0">-6</parameter>
   <parameter name="alloweddifference"> 1.0e-4 </parameter>
   <parameter name="nstabilizers"> 1 </parameter>
   <parameter name="bigchange">15.0</parameter>
 </amc>
</loop>
```

- loop is helpful to execute identical optimization blocks repeatedly.
- The first part is highly identical to a regular VMC block.
- The second part is to specify the correlated sampling options and define the cost function.
- The last part is used to specify the options of different optimizers. They can be very distinct from one optimizer to another.

## 9.2.1 VMC run for the optimization

The VMC calculation for the wavefunction optimization has a strict requirement that samples or samplesperthread must be specified because of the optimizer needs for the stored samples. The input parameters of this part are identical to the VMC method.

Recommendations:

- Run the inclusive VMC calculation correctly and efficiently, because the this part takes significant amount of time in optimization. For example, make sure the derived steps per block is 1 and use larger substeps to control the correlation between samples.
- A reasonable starting wavefunction is necessary. A lot of optimization fails because of a bad wavefunction starting point. The sign of a bad initial wavefunction includes but not limited to very long equilibration time, low acceptance ratio and huge variance. The first thing to do after a failed optimization is to check the information provided by the VMC calculation via \*.scalar.dat files.

## 9.2.2 Correlated sampling and Cost function

After generating the samples with VMC, the derivatives of the wavefunction with respect to the parameters are computed for proposing a new set of parameters by optimizers. And later, a correlated sampling calculation is performed to quickly evaluate values of the cost function on the old set of parameters and the new set for the further decisions. The input parameters are listed in the following table.

linear method				
parameters				
name	datatype	values	default	description
usebuffer	text	yes, minimum, no	no	buffer info to speed up the correlated sampling
nonlocalpp	text	yes, no	no	include non-local PP energy in the cost function
minwalkers	real	0-1	0.3	lower bound of the effective weight
maxWeight	real	> 1	1e6	Maximum weight allowed in reweighting

- maxWeight. The default should be good.
- usebuffer. It saves a lot of computation especially with the 'quartic' optimizer.
- nonlocalpp. It is recommended to enable it when 3 body Jastrow is on. GPU code has a implementation issue that large amount of memory is consumed with this option.

• minwalkers. A CRITICAL parameter. When the ratio of effective samples to actual number of samples in a reweighting step goes lower than minwalkers, the proposed set of parameters is invalid.

The cost function consists of three components: energy, unreweighted variance and reweighted variance.

### 9.2.3 Optimizers

QMCPACK implements a few optimizers having different preference aiming for different priorities. They can be switched among 'rescale', 'quartic' (default), 'adaptive' and 'OneShiftOnly' by the following line in the optimization block.

```
<parameter name="MinMethod"> THE METHOD YOU LIKE </parameter>
```

### quartic

The quartic optimizer fits a quartic polynomial to 7 values of the cost function obtained using reweighting along chosen direction and determines the optimal move. This optimizer is very robust but a bit conservative to accept new steps especially when large parameters changes are proposed.

${f datatype}$	values	$\mathbf{default}$	$\operatorname{description}$
real	> 0	50.0	Largest parameter change allowed
real	> 0	1e-4	Allowed increased in energy
real	any value	-16.0	Initial value for stabilizer
real	> 0	2.0	Increase in value of exp0 between iterati
integer	> 0	3	Number of stabilizers to try
integer	> 0	1	Number of inner loops with same sample
	real real real integer	$\begin{array}{ll} \text{real} & > 0 \\ \text{real} & > 0 \\ \text{real} & \text{any value} \\ \text{real} & > 0 \\ \text{integer} & > 0 \end{array}$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$

### Additional information:

• exp0. It is the initial value for stabilizer (shift to diagonal of H). The actual value of stabilizer is  $10^{\exp 0}$ .

### Recommendations:

• For hard cases (e.g. simultaneous optimization of long MSD and 3-Body J), set exp0 to 0 and do a single inner iteration (max its=1) per sample of configurations.

```
<!-- Specify the optimizer options -->
<parameter name="MinMethod">quartic</parameter>
<parameter name="exp0">-6</parameter>
<parameter name="alloweddifference"> 1.0e-4 </parameter>
<parameter name="nstabilizers"> 1 </parameter>
<parameter name="bigchange">15.0</parameter></parameter name="bigchange">15.0</parameter></parameter>
```

### adaptive

The default setting of the adaptive optimizer is to construct the linear method Hamiltonian and overlap matrices explicitly and add different shifts to the Hamiltonian matrix as "stabilizers". The generalized eigenvalue problem is solved for each shift to obtain updates to the wave function parameters. Then a correlated sampling is performed for each shift's updated wave function and the initial trial wave function using the middle shift's updated wave function as the guiding function. The cost function for these wave functions is compared, and the update corresponding to the best cost function is selected. In the next iteration, the median magnitude of the stabilizers is set to that that generated the best update in the current iteration, thus adapting the magnitude of the stabilizers automatically.

When the trial wave function contains more than ten thousand parameters, constructing and storing the linear method matrices may become a memory bottleneck. To avoid explicit construction of these matrices, the adaptive optimizer implements the block linear method (BLM) approach. [?] The BLM tries to find an approximate solution  $\vec{c}_{opt}$  to the standard LM generalized eigenvalue problem by dividing the variable space into a number of blocks and making intelligent estimates for which directions within those blocks will be most important for constructing  $\vec{c}_{opt}$ . which is then obtained by solving a smaller, more memory-efficient eigenproblem in the basis of these supposedly important block-wise directions.

linear method				
parameters				
name	${f datatype}$	values	$\operatorname{default}$	description
${\tt max\_relative\_change}$	real	> 0	10.0	Allowed change in cost function
${\tt max\_param\_change}$	real	> 0	0.3	Allowed change in wave function parameter
$\mathtt{shift}_{ extstyle -} \mathtt{i}$	real	> 0	0.01	Initial diagonal stabilizer added to the Hamiltonian
$\mathtt{shift}_{\mathtt{L}}\mathtt{s}$	real	> 0	1.00	Initial overlap-based stabilizer added to the Hamilto
${\tt chase\_lowest}$	text	yes, no	yes	Chase the lowest eigenvector in iterative solver
${\tt chase\_closest}$	text	yes, no	no	Chase the eigenvector closest to initial guess
$block_lm$	text	yes, no	no	Use block linear method
nblocks	integer	> 0		# of blocks in BLM
nolds	integer	> 0		# of old update vectors used in BLM
nkept	integer	> 0		# of eigenvectors to keep per block in BLM

- shift\_i. This is the initial coefficient used to scale the diagonal stabilizer. More stable but slower optimization is expected with a large value. The adaptive method will automatically adjust this value after each linear method iteration.
- shift\_s. This is the initial coefficient used to scale the overlap-based stabilizer. More stable but slower optimization is expected with a large value. The adaptive method will automatically adjust this value after each linear method iteration.
- nblocks. This is the number of blocks used in block LM. The amount of memory required to store LM matrices decreases with increased number of blocks. But the error introduced by BLM would increase with number of blocks.
- nolds. In BLM, the inter-block correlation is accounted for by including a small number of

wave function update vectors outside the block. Larger nolds would include more inter-block correlation and more accurate results, but also higher memory requirements.

• nkept. This is the number of update directions retained from each block in the BLM. If all directions are retained in each block, then the BLM becomes equivalent to the standard LM. Retaining 5 or fewer directions per block is often sufficient.

### Recommendations:

- Default shift\_i, shift\_s should be fine.
- When there are fewer than about 5,000 variables being optimized, the traditional LM is preferred as it has a lower overhead than the BLM when the number of variables is small.
- Initial experience with the BLM suggests that a few hundred blocks and a handful of nolds and nkept often provide a good balance between memory use and accuracy. In general, using fewer blocks should be more accurate but will require more memory.

The adaptive optimizer is also able to optimize individual excited states directly. [?] In this case, it tries to minimize the following function:

$$\Omega[\Psi] = rac{\langle \Psi | \omega - H | \Psi 
angle}{\left\langle \Psi | (\omega - H)^2 | \Psi 
ight
angle}$$

The global minimum of this function corresponds to the state whose energy lies immediately above the shift parameter  $\omega$  in the energy spectrum. For example, if  $\omega$  were placed in between the ground state energy and the first excited state energy and the wave function ansatz was capable of a good description for the first excited state, then the wave function would be optimized for the first excited state. It is important to note that, if the ansatz is not capable of a good description of the excited state in question, the optimization may converge to a different state, as is known to occur in some circumstances for traditional ground state optimizations. Note also that the ground state can be targeted by this method by choosing  $\omega$  to be below the ground state energy, although we should stress that this is not the same thing as a traditional ground state optimization and will in general give a slightly different wave function. Excited state targeting requires two additional parameters, as shown in this table.

Excited state recommendations:

• Due to the finite variance in any approximate wave function, it is recommended to set  $\omega = \omega_0 - \sigma$ , where  $\omega_0$  is placed just below the energy of the targeted state and  $\sigma^2$  is the energy variance.

Excited State Targe	$_{ m eting}$			
parameters				
name	datatype	values	default	description
targetExcited	text	yes, no	no	Whether to use the excited state targeting optimization
omega	real	real numbers	none	Energy shift used to target different excited states

• In order to obtain an unbiased excitation energy, one should optimize the ground state with the excited state variational principle as well by setting omega below the ground state energy. Note that using the ground state variational principle for the ground state and the excited state variational principle for the excited state creates a bias in favor of the ground state.

### **OneShiftOnly**

The OneShiftOnly optimizer targets a fast optimization by moving parameters more aggressively. It works with OpenMP and GPU and can be considered for large systems. This method relies on minwalkers to justify a new set of parameters. If it is valid, the new set is taken no matter if the cost function value decreases or not. If a proposed set is rejected, the standard output prints the measured ratio of effective samples to the total number of samples and adjustment on minwalkers can be made if needed.

linear meth	nod			
parameters				
name	datatype	values	$\mathbf{default}$	description
$\mathtt{shift}_{\mathtt{-}}\mathtt{i}$	real	> 0	0.01	Direct stabilizer added to the Hamiltonian matrix
$\mathtt{shift}_{\mathtt{s}}$	real	> 0	1.00	Initial stabilizer based on the overlap matrix

### Additional information:

- shift\_i. This is the direct term added to the diagonal of the Hamiltonian matrix. More stable but slower optimization with a large value.
- shift\_s. This is the initial value of the stabilizer based on the overlap matrix added to the Hamiltonian matrix. More stable but slower optimization with a large value. The used value is auto-adjusted by the optimizer.

### Recommendations:

- Default shift\_i, shift\_s should be fine.
- For hard cases, increasing shift\_i (factor of 5 or 10) can significantly stabilize the optimization by reducing the pace towards the optimal parameter set.
- If the VMC energy of the last optimization iterations grows significantly, increase minwalkers closer to 1 and make the optimization stable.
- If the first iterations of optimization are rejected on a reasonable initial wavefunction, lower the minwalkers value based on the measured value printed in the standard output to accept the move.

It is recommended to use this optimizer in two sections with a very small minwalkers in the first and a large value in the second like the following. In the very beginning, parameters are far away form optimal values and large changes are proposed by the optimizer. Having a small minwalkers allows accepting these changes much easier. When the energy gradually converges, we can have a large minwalkers to guarantee a stable optimization.

```
<loop max="6">
 <qmc method="linear" move="pbyp" gpu="yes">
   <!-- Specify the VMC options -->
   <!-- Specify the correlated sampling options and define the cost function -->
   <parameter name="minwalkers"> 1e-4 </parameter>
   <!-- Specify the optimizer options -->
   <parameter name="MinMethod">OneShiftOnly</parameter>
 </qmc>
</loop>
<loop max="12">
 <qmc method="linear" move="pbyp" gpu="yes">
   <!-- Specify the VMC options -->
   <!-- Specify the correlated sampling options and define the cost function -->
   <parameter name="minwalkers"> 0.5 </parameter>
   <!-- Specify the optimizer options -->
   <parameter name="MinMethod">OneShiftOnly</parameter>
 </qmc>
</loop>
```

For each optimization step, you will see

```
The new set of parameters is valid. Updating the trial wave function!
```

or

```
The new set of parameters is not valid. Revert to the old set!
```

Occasional rejection is fine. Frequent rejection indicates potential problems and users should inspect the VMC calculation or change optimization strategy. To track the progress of optimization, using command "qmca -q ev \*.scalar.dat" to look at the VMC energy and variance for each optimization step.

### 9.2.4 General recommendations

Here are a few reminders to make wavefunction easier.

- All electron wavefunctions are more difficult to optimize than pseudopotential ones especially when the cusp is not removed.
- Two body Jastrow contributes the largest portion of correlation energy from bare Slater determinants. It's also the easiest part for optimizer and gains most. For this reason, the recommend order optimizing wavefunction components is two-body, one-body, three-body Jastrow factors and MSD coefficients.
- Never use all-zero two-body spline Jastrow and always start from a reasonable one. You might want to bang your head against a brick wall for a bad two-body Jastrow.
- One-body spline Jastrow from old calculations can be good starting point.
- Three-body polynomial Jastrow should start from zero.

# 9.3 Diffusion Monte Carlo

dmc method				
parameters				
name	${f datatype}$	values	default	description
targetwalkers	integer	> 0	dep.	number of walkers per node
blocks	integer	$\geq 0$	1	number of blocks
steps	integer	$\geq 0$	1	number of steps per block
warmupsteps	integer	$\geq 0$	0	number of steps for warming up
timestep	real	> 0	0.1	time step for each electron move
checkproperties	integer	$\geq 0$	100	number of steps between walker upo
maxcpusecs	real	$\geq 0$	3.6e5	maximum allowed walltime in secon
${\tt energyUpdateInterval}$	integer	$\geq 0$	0	trial energy update interval
refEnergy	$\mathrm{AU}$	all values	dep.	reference energy
feedback	double	$\geq 0$	1.0	population feedback on the trial ene
useBareTau	option	yes,no	0	do not use effective time step
${\tt warmupByReconfiguration}$	option	yes,no	0	warm up with a fixed population
sigmaBound	double	$\geq 0$	10	parameter to cutoff large weights
killnode	string	yes/other	no	kill or reject walkers that cross node
reconfiguration	string	yes/pure/other	no	fixed population technique
branchInterval	integer	$\geq 0$	1	branching interval
substeps	integer	$\geq 0$	1	branching interval
nonlocalmoves	string	yes/other	no	run with tmoves
scaleweight	string	yes/other	yes	scale weights (CUDA only)
MaxAge	double	$\geq 0$	10	kill persistent walkers
MaxCopy	double	$\geq 0$	2	limit population growth
fastgrad	$\operatorname{text}$	yes/other	yes	fast gradients
maxDisplSq	real	all values	-1	maximum particle move
storeconfigs	integer	all values	0	store configurations
blocks_between_recompute	integer	$\geq 0$	dep.	wavefunction recompute frequency

- targetwalkers. A DMC run can be considered a restart run or a new run. A restart run is considered to be any method block beyond the first one, such as when a DMC method block that follows a VMC block. Alternatively, if the user reads in configurations from disk it is also considered a restart run. In the case of a restart run, the DMC driver will use the configurations from the previous run, and this variable will not be used. For a new run, if the number of walkers is less than the number of threads, then the number of walkers will be set equal to the number of threads.
- blocks. Number of blocks run during an DMC method block. A block consists of a number of DMC steps (steps), after which all the statistics accumulated in the block are written to disk.
- steps. Number of diffusion Monte Carlo steps in a block.
- warmupsteps. Warm-up steps are steps at the beginning of a DMC run in which the instantaneous average energy is used to update the trial energy. During regular steps,  $E_{ref}$  is used.

- timestep. The timestep determines the accuracy of the imaginary time propagator. Generally, multiple time steps are used to extrapolate to the infinite time step limit. A good range of timesteps in which to perform time step extrapolation will typically have a minimum of 99% acceptance probability for each step.
- checkproperties. When using particle by particle driver, this variable specifies how often to reset all the variables kept in the buffer.
- maxcpusecs. The default is 100 hours. Once the specified time has elapsed, the program will finalize the simulation even if not all blocks are completed.
- energyUpdateInterval. The default is to update the trial energy at every step. Otherwise the trial energy is updated every energyUpdateInterval steps.

$$E_{\text{trial}} = \text{refEnergy} + \text{feedback} \cdot (\text{ln targetWalkers} - \text{ln } N)$$

where N is the current population.

- refEnergy. The default reference energy is taken from the VMC run that precedes the DMC run. This value is updated to the current mean whenever branching happens.
- feedback. Variable used to determine how strong to react to population fluctuations when doing population control. See the equation in energy UpdateInterval for more details.
- useBareTau. The same time step is used whether a move is rejected to not. The default is to use an effective time step when a move is rejected.
- warmupByReconfiguration. Warmup DMC is done with a fixed population
- sigmaBound . Determine the branch cutoff to limit wild weights based on the sigma and sigmaBound
- killnode . When running fixed-node, if a walker attempts to cross a node, the move will normally be rejected. If killnode = "yes", then walkers are destroyed when they cross a node.
- reconfiguration. If reconfiguration is "yes", then run with a fixed walker population using the reconfiguration technique.
- branchInterval. Number of steps between branching. The total number of DMC steps in a block will be BranchInterval\*Steps.
- substeps. Same as BranchInterval.
- nonlocalmoves. DMC driver for running Hamiltonians with non-local moves. An typical usage is to simulate Hamiltonians with non-local psuedopotentials with T-Moves. Setting this equal to false will impose the locality approximation.
- scaleweight. Scaling weight per Umrigar/Nightengale. CUDA only.
- MaxAge. Set the weight of a walker to min(currentweight, 0.5) after a walker has not moved for MaxAge steps. Needed if persistent walkers appear during the course of a run.
- MaxCopy. When determining the number of copies of a walker to branch, set the number of copies equal to min(Multiplicity, MaxCopy).

- fastgrad. Calculates gradients with either the fast version or the full-ratio version.
- maxDisplSq. When running a DMC calculation with particle by particle, this sets the maximum displacement allowed for a single particle move. All distance displacements larger than the max is rejected. If initialized to a negative value, it becomes equal to Lattice(LR/rc).
- sigmaBound . Determine the branch cutoff to limit wild weights based on the sigma and sigmaBound
- storeconfigs. If storeconfigs is set to a non-zero value, then electron configurations during the DMC run will be saved. This option is disabled for the OpenMP version of DMC.
- blocks\_between\_recompute. See details in VMC section 9.1.

Listing 9.3: The following is an example of a very simple DMC section.

```
<qmc method="dmc" move="pbyp" target="e">
  <parameter name="blocks">100</parameter>
  <parameter name="steps">400</parameter>
  <parameter name="timestep">0.010</parameter>
  <parameter name="warmupsteps">100</parameter>
  </qmc>
```

The time step should be adjusted for each problem individually. Please refer to the theory section on diffusion Monte Carlo.

Listing 9.4: The following is an example of running a simulation that can be restarted.

The checkpoint flag instructs queepack to output walker configurations. This also works in Variational Monte Carlo. This will output an h5 file with the name "projectid"."run-number".config.h5. Check that this file exists before attempting a restart. To read in this file for a continuation run, specify the following:

Listing 9.5: Restart (read walkers from previous run)

```
<mcwalkerset fileroot="BH.s002" version="0 6" collected="yes"/>
```

BH is the project id and s002 is the calculation number to read in the walkers from the previous run.

Combining VMC and DMC in a single run (and wave function optimization can be combined in this way too) is the standard way in which QMCPACK is typical run. There is no need to run two separate jobs, as method sections can be stacked, and walkers are transferred between them.

Listing 9.6: Combined VMC and DMC run

```
<qmc method="vmc" move="pbyp" target="e">
    <parameter name="blocks">100</parameter>
    <parameter name="steps">4000</parameter>
    <parameter name="warmupsteps">100</parameter>
    <parameter name="samples">1920</parameter>
    <parameter name="samples">1920</parameter>
    <parameter name="walkers">1</parameter>
    <parameter name="timestep">0.5</parameter>
    </qmc>
    <qmc method="dmc" move="pbyp" target="e"></parameter"></parameter"></parameter"></parameter move="pbyp" target="e"></parameter"></parameter"></parameter move="pbyp" target="e"></parameter"></parameter</p>
```

```
<parameter name="blocks">100</parameter>
  <parameter name="steps">400</parameter>
  <parameter name="timestep">0.010</parameter>
  <parameter name="warmupsteps">100</parameter>
  </qmc>
</qmc>
<qmc method="dmc" move="pbyp" target="e">
  <parameter name="warmupsteps">500</parameter>
  <parameter name="warmupsteps">500</parameter>
  <parameter name="blocks">50</parameter>
  <parameter name="steps">100</parameter>
  <parameter name="steps">100</parameter>
  <parameter name="timestep">0.005</parameter>
  </parameter name="timestep">0.005</parameter>
  </parameter</pre>
```

## 9.4 Reptation Monte Carlo

Like diffusion monte carlo, reptation monte carlo (RMC) is a projector based method, allowing us the ability to sample the fixed-node wavefunction. However, by exploiting the path-integral formulation of Schrödinger's equation, the RMC algorithm can offer some advantages over traditional DMC, such as sampling both the mixed and pure fixed-node distributions in polynomial time, as well as not having population fluctuations and biases. The current implementation does not work with T-moves.

There are two adjustable parameters that affect the quality of the RMC projection: imaginary projection time  $\beta$  of the sampling path (commonly called a "reptile"), and the Trotter time step  $\tau$ .  $\beta$  must be chosen to be large enough such that  $e^{-\beta \hat{H}}|\Psi_T\rangle \approx |\Phi_0\rangle$  for mixed observables, and  $e^{-\frac{\beta}{2}\hat{H}}|\Psi_T\rangle \approx |\Phi_0\rangle$  for pure observables. The reptile is discretized into  $M=\beta/\tau$  beads at the cost of an  $\mathcal{O}(\tau)$  time-step error for observables arising from the Trotter-Suzuki breakup of the short-time propagator.

The following table lists some of the more practical

vmc method				
parameters				
name	${f datatype}$	values	$\operatorname{default}$	description
beta	real	> 0	dep.	reptile projection time $\beta$
timestep	real	> 0	0.1	Trotter time step $\tau$ for each electron move
beads	int	> 0	1	Number of reptile beads $M = \beta/\tau$
blocks	integer	$\geq 0$	1	number of blocks
steps	integer	$\geq 0$	1	number of steps per block
vmcpresteps	integer	$\geq 0$	0	propagates reptile using VMC for given number
warmupsteps	integer	$\geq 0$	0	number of steps for warming up
MaxAge	integer	$\geq 0$	0	force accept for stuck reptile if age exceeds MaxA

### Additional information:

Because of the sampling differences between DMC ensembles of walkers and RMC reptiles, the RMC block should contain the following estimator declaration to ensure correct sampling: <estimator name="RMC" hdf5="no">.

- beta or beads? One can specify one or the other, and from the Trotter time-step, the code will construct an appropriately sized reptile. If both are given, beta overrides beads.
- Mixed vs. Pure observables? For all observables appearing in the scalar.dat file

in either VMC or DMC, RMC appends the suffix \_m or \_p for mixed and pure estimates respectively.

- Sampling. For pure estimators, one should check the traces of both pure and mixed estimates. Ergodicity is a known problem in RMC. Because we use the bounce algorithm, it is possible for the reptile to bounce back and forth without changing the electron coordinates of the central beads. This might not easily show up with mixed estimators, since these are accumulated at constantly regrown ends, but pure estimates are accumulated on these central beads, and so can exhibit strong autocorrelations in pure estimate traces.
- **Propagator**: Our implementation of RMC uses Moroni's DMC link action (symmetrized), with Umrigar's scaled drift near nodes. In this regard, the propagator is identical to the one QMCPACK uses in DMC.
- Sampling: We use Ceperley's bounce algorithm. MaxAge is used in case the reptile gets stuck, at which point the code forces move acceptance, stops accumulating statistics, and requilibrates the reptile. Very rarely will this be required. For move proposals, we use particle-by-particle VMC a total of  $N_e$  times to generate a new all-electron configuration, at which point the action is computed and the move is either accepted or rejected.

# Chapter 10

# Output overview

QMCPACK writes several output files which report information about the simulation (e.g. the physical properties such as the energy), as well as information about the computational aspects of the simulation, checkpoints, and restarts. The types of output files generated depend on the details of a calculation. The list below is not meant to be exhaustive, but rather to highlight some salient features of the more common filetypes. Further detail can be found in the description of the estimator one is interested in computing.

#### 10.1 The .scalar.dat file

The most important output file is the .scalar.dat file. This file contains the output of block averaged properties of the system such as the local energy and other estimators. Each line corresponds to an average over  $N_{walkers} * N_{steps}$  samples. By default, the quantities reported in the .scalar.dat file include:

**LocalEnergy** The local energy.

**LocalEnergy\_sq** The local energy squared.

**LocalPotential** The local potential energy.

**Kinetic** The kinetic energy.

**ElecElec** The electron-electron potential energy.

**IonIon** The ion-ion potential energy.

LocalECP The energy due to the pseudopotential/effective core potential.

NonLocalECP The non-local energy due to the pseudopotential/effective core potential.

**MPC** The modified periodic coulomb potential energy.

**BlockWeight** The number of MC samples in the block.

**BlockCPU** The number of seconds to compute the block.

AcceptRatio The acceptance ratio.

QMCPACK includes a python utility, qmca, which can be used to process these files. Details and examples are given in chapter 11.

### 10.2 The .opt.xml file

This file is generated after a VMC wave function optimization, and contains the part of the input file which lists the optimized optimized jastrow factors. Conveniently, this file is already formatted such it can easily be incorporated into a DMC input file.

### 10.3 The .qmc.xml file

This file contains information about the computational aspects of the simulation, for example, which parts of the code are being executed when. This file is only generated in an ensemble run in which quepack runs multiple input files.

#### 10.4 The .dmc.dat file

This file contains information similar to the .scalar.dat file, but also includes extra information about the details of a DMC calculation. For example, information about the walker population.

Index The block number.

**LocalEnergy** The local energy.

Variance The variance.

Weight The number of samples in the block.

NumOfWalkers The number of walkers times the number of steps.

**AvgSentWalkers** The average number of walkers sent. During a DMC simulation walkers may be created or destroyed. At every step, QMCPACK will do some load balancing to ensure that the walkers are evenly distributed across nodes.

**TrialEnergy** The trial energy. See 9.3 for an explanation of the trial energy.

**DiffEff** The diffusion efficiency.

**LivingFraction** The fraction of the walker population from the previous step that survived to the current step.

#### 10.5 The .bandinfo.dat file

This file contains information from the trial wavefunction about the band structure of the system, including the available k-points. This can be helpful in constructing trial wavefunctions.

# 10.6 Checkpoint and restart files

#### 10.6.1 The .cont.xml file

This file enables continuation of the run. It is mostly a copy of the input XML file with the series number incremented, and the mcwalkerset element added to read the walkers from a config file. The .cont.xml file is always created, but other files it depends on are only present if checkpointing is enabled.

# 10.6.2 The .config.h5 file

This file contains stored walker configurations.

## 10.6.3 The .random.xml file

This file contains the state of the random number generator to allow restarts.

# Chapter 11

# Analysing QMCPACK data

# 11.1 Using the qmca tool to obtain total energies and related quantities

The qmca tool is the primary means of analyzing scalar valued data generated by QMCPACK. Output files that contain scalar valued data are \*.scalar.dat and \*dmc.dat (see chapter 10 for a detailed description of these files). Quantities that are available for analysis in \*scalar.dat files include the local energy and its variance, the kinetic energy, the potential energy (also broken into components), the acceptance ratio, and the average cpu time spent per block, among others. The \*dmc.dat files make available information regarding the DMC walker population in addition to the local energy.

Basic capabilities of qmca include calculating mean values and associated error bars, processing multiple files at once in batched fashion, performing twist averaging, plotting mean values by series, and plotting traces (per block or step) of the underlying data. These capabilities are explained with accompanying examples in the following subsections.

To use qmca, installations of Python and NumPy must be present on the local machine. For plotting, the matplotlib module must also be available.

An overview of all supported input flags to qmca can be obtained by typing "qmca" at the command line with no other inputs (also try "qmca -x" for a short list of examples):

```
-e EQUILIBRATION, --equilibration=EQUILIBRATION
                      Equilibration length in blocks (default=auto).
                      Average over files in each series (default=False).
-a, --average
-w WEIGHTS, --weights=WEIGHTS
                      List of weights for averaging (default=None).
-b, --reblock
                      (pending) Use reblocking to calculate statistics
                      (default=False).
-p, --plot
                      Plot quantities vs. series (default=False).
-t, --trace
                      Plot a trace of quantities (default=False).
-h, --histogram
                      (pending) Plot a histogram of quantities
                      (default=False).
-o, --overlay
                      Overlay plots (default=False).
--legend=LEGEND
                      Placement of legend. None for no legend, outside for
                      outside legend (default=upper right).
--noautocorr
                      Do not calculate autocorrelation. Warning: error bars
                      are no longer valid! (default=False).
                      Alias for -- noautocorr (default=False).
--noac
                      Show autocorrelation of sample data (default=False).
--sac
                      Show variance of sample data (default=False).
--sv
-i, --image
                      (pending) Save image files (default=False).
                      (pending) Write a report (default=False).
-r, --report
-s, --show_options
                      Print user provided options (default=False).
-x, --examples
                      Print examples and exit (default=False).
--help
                      Print help information and exit (default=False).
-d DESIRED_ERROR, --desired_error=DESIRED_ERROR
                      Show number of samples needed for desired error bar
                      (default=none).
-n PARTICLE_NUMBER, --enlarge_system=PARTICLE_NUMBER
                      Show number of samples needed to maintain error bar on
                      larger system: desired particle number first, current
                      particle number second (default=none)
```

#### 11.1.1 Obtaining a statistically correct mean and error bar

A rough guess at the mean and error bar of the local energy can be obtained in the following way with qmca:

```
>qmca -q e qmc.s000.scalar.dat
qmc series 0 LocalEnergy = -45.876150 +/- 0.017688
```

In this case the VMC energy of an 8 atom cell of diamond is estimated to be -45.876(2) Hartrees. This rough guess should not be used for production-level or publication quality estimates.

To obtain production-level results, the underlying data should first inspected visually to ensure that all data included in the averaging can be attributed to a distribution sharing the same mean. The first steps of essentially any Monte Carlo calculation (the "equilibration phase") do not belong to the equilibrium distribution and should be excluded from estimates of the mean and its error bar.

We can plot a data trace ("-t") of the local energy in the following way:

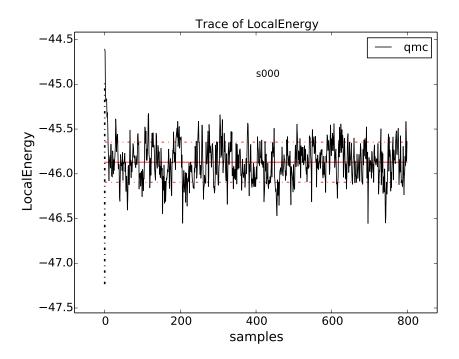


Figure 11.1: Trace of the VMC local energy for an 8 atom cell of diamond generated with qmca. The x-axis ("samples") refers to the VMC block index in this case.

```
>qmca -t -q e -e 0 qmc.s000.scalar.dat
```

The "-e 0" part indicates that we do not want any data to be excluded from the calculation of averages initially. The resulting plot is shown in Fig. 11.1. The unphysical equilibration period is visible on the left side of the plot.

Most of the data fluctuates around a well defined mean (consistent variations around a flat line). This property is important to verify by plotting the trace for each QMC run.

If we exclude none of the equilibration data points, we get an erroneous estimate of -45.870(2) Ha for the local energy:

```
>qmca -q e -e 0 qmc.s000.scalar.dat
qmc series 0 LocalEnergy = -45.870071 +/- 0.018072
```

The equilibration period is typically estimated by eye, though one can check a few conservative values to ensure that the mean remains unaffected. In this dataset, the equilibration appears to have been reached after 100 samples or so. After excluding the first 100 VMC blocks from the analysis we get:

```
>qmca -q e -e 100 qmc.s000.scalar.dat
qmc series 0 LocalEnergy = -45.877363 +/- 0.017432
```

This estimate (-45.877(2) Ha) differs significantly from the -45.870(2) Ha figure obtained from the full set of data, but it agrees with the rough estimate of -45.876(2) Hartrees obtained with

the abbreviated command ("qmca -q e qmc.s000.scalar.dat"). This is because qmca makes a heuristic guess at the equilibration period and got it reasonably correct in this case. There are many cases where the heuristic guess fails and it should not be relied on for quality results.

We have so far obtained a statistically correct mean. To obtain a statistically correct error bar it is best to include  $\sim 100$  or more statistically independent samples. An estimate of the number of independent samples can be obtained by considering the autocorrelation time, which is essentially a measure of the number of samples that must be traversed before an uncorrelated/independent sample is reached. We can get an estimate of the autocorrelation time in the following way:

```
>qmca -q e -e 100 qmc.s000.scalar.dat --sac
qmc series 0 LocalEnergy = -45.877363 +/- 0.017432 4.8
```

The flag "--sac" stands for (s)how (a)uto(c)orrelation. In this case the autocorrelation estimate is  $4.8 \approx 5$  samples. Since the total run contained 800 samples and we have excluded 100 of them, we can estimate the number of independent samples as (800-100)/5=140. In this case, the error bar is expected to be estimated reasonably well.

Please keep in mind that the error bar represents the expected range of the mean with a certainty of only  $\sim 70\%$ . The actual mean value will lie outside the range indicated by the error bar in one out of every three runs and in a set of 20 runs one value can be expected to deviate from its estimate by twice the error bar.

#### 11.1.2 Judging wavefunction optimization

Wavefunction optimization is a highly non-linear and sometimes sensitive process. As such, there is a risk that systematic errors encountered at this stage of the QMC process can be propagated into subsequent (expensive) DMC runs unless they are guarded against with vigilence.

In this section we again consider an 8 atom cell of diamond, but now in the context of Jastrow optimization (one- and two-body terms). In optimization runs it is often preferable to use a large number of warmupsteps ( $\sim 100$ ) so that equilibration bias does not propagate into the optimization process. We can check that the added warmup has had its intended effect by again checking the local energy trace:

```
>qmca -t -q e *scalar*
```

The resulting plot can be found in Fig. 11.2. In this case sufficient warmupsteps were used to exit the equilibration period before samples were collected and we can proceed without using the "-e" option with amca.

After inspecting the trace, we should inspect the text output from qmca, now including the total energy and its variance:

>qmc	a -q ev op	t*scalar.dat		
		${ t LocalEnergy}$	Variance	ratio
opt	series 0	-44.823616 +/- 0.007430	7.054219 +/- 0.041998	0.1574
opt	series 1	-45.877643 +/- 0.003329	1.095362 +/- 0.041154	0.0239
opt	series 2	-45.883191 +/- 0.004149	1.077942 +/- 0.021555	0.0235
opt	series 3	-45.877524 +/- 0.003094	1.074047 +/- 0.010491	0.0234
opt	series 4	-45.886062 +/- 0.003750	1.061707 +/- 0.014459	0.0231
opt	series 5	-45.877668 +/- 0.003475	1.091585 +/- 0.021637	0.0238

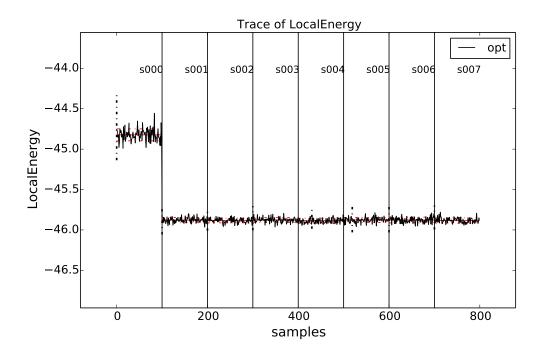


Figure 11.2: Trace of the local energy during one- and two-body Jastrow optimization for an 8 atom cell of diamond generated with qmca. Data for each optimization cycle (QMCPACK series) is separated by a vertical black line.

```
opt series 6 -45.877109 +/- 0.003586 1.069205 +/- 0.009387 0.0233
opt series 7 -45.882563 +/- 0.004324 1.058771 +/- 0.008651 0.0231
```

The flags "-q ev" requested the energy (e) and the variance (v). For this combination of quantities, a third column ("ratio") is printed containing the ratio of the variance and the absolute value of the local energy. The variance/energy ratio is an intensive quantity and is useful to inspect regardless of the system under study. Successful optimization of molecules and solids of any size generally result in comparable values for the variance/energy ratio.

The first line of the output ("series 0") corresponds to the local energy and variance of the system without a Jastrow factor (all Jastrow coefficients were initialized to zero in this case), reflecting the quality of the orbitals alone. For pseudopotential systems, a variance/energy ratio > 0.20 Ha generally indicates there is a problem with the input orbitals that needs to be resolved prior to performing wavefunction optimization.

The subsequent lines correspond to energies and variances of intermediate parameterizations of the trial wavefunction during the optimization process. The output line containing "opt series 1", for example, corresponds to the trial wavefunction parameterized during the "series 0" step (the parameters of this wavefunction would be found in an output file matching \*s000\*opt.xml). The first thing to check about the resulting optimization is again the variance/energy ratio. For pseudopotential systems, a variance/energy ratio < 0.03 Ha is consistent with a trial wavefunction of production quality, and values of 0.01 Ha are rarely obtainable for standard Slater-Jastrow wavefunctions. By this metric, all parameterizations obtained for optimizations performed in series 0-6 are of comparable quality (note that the quality of the wavefunction obtained during optimization

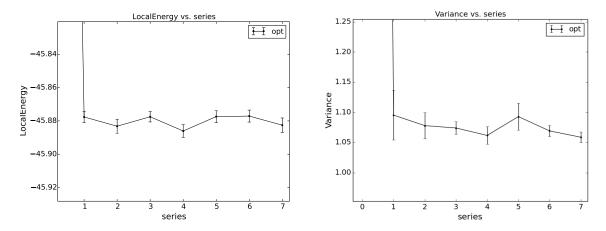


Figure 11.3: Energy and variance vs. optimization series for an 8 atom cell of diamond as plotted by qmca.

series 7 is effectively unknown).

A good way to further discriminate among the parameterizations is to plot the energy and variance as a function of series with qmca:

#### >qmca -p -q ev opt\*scalar.dat

The "-p" option results in plots of means plus error bars vs. series for all requested quantities. The resulting plots for the local energy and variance are shown in Fig. 11.3. In this case the resulting energies and variances are statistically indistinguishable for all optimization cycles.

A good way to choose the optimal wavefunction for use in DMC is to select the one with lowest statistically significant energy within the set of optimized wavefunctions with reasonable variance (e.g. among those with variance/energy ratio < 0.03 Ha). For pseudopotential calculations, minimizing according to the total energy is recommended to reduce locality errors in DMC.

#### 11.1.3 Judging diffusion Monte Carlo runs

Judging the quality of the DMC projection process requires more care than this needed in VMC. In order to reduce bias, a small timestep is required in the approximate projector but this also leads to slow equilibration and long autocorrelation times. Systematic errors in the projection process can also arise from statistical fluctuations due to pseudopotentials or from trial wavefunctions with larger than necessary variance.

To illustrate the problems that can arise with respect to slow equilibration and long autocorrelation times, we consider the 8 atom diamond system with VMC (200 blocks of 160 steps) followed by DMC (400 blocks of 5 steps) with a small timestep (0.002 Ha<sup>-1</sup>). A good first step in assessing the quality of any DMC run is to plot the trace of the local energy:

#### >qmca -t -q e -e 0 \*scalar\*

The resulting trace plot is shown in Fig. 11.4. As always, the DMC local energy decreases exponentially away from the VMC value but in this case it takes a long time to do so. At least half of the DMC run is inefficiently consumed by equilibration. If we are not careful to inspect and remove the transient, the estimated DMC energy will be strongly biased by the transient as shown

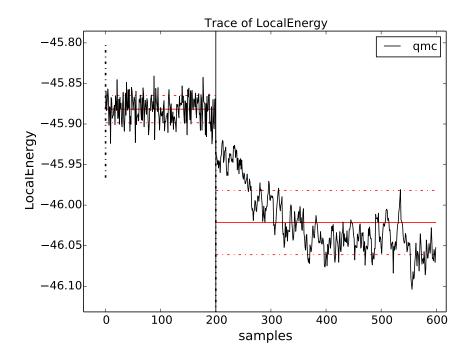


Figure 11.4: Trace of the local energy for VMC followed by DMC with a small timestep  $(0.002 \, \mathrm{Ha^{-1}})$  for an 8 atom cell of diamond generated with qmca.

by the horizontal red line (estimated mean) in the figure. The autocorrelation time is also large ( $\sim 12$  blocks):

```
>qmca -q e -e 200 --sac *s001.scalar*
qmc series 1 LocalEnergy = -46.045720 +/- 0.004813 11.6
```

Of the included 200 blocks, fewer than 20 contribute to the estimated error bar, indicating that we cannot trust the reported error bar. This can also be demonstrated directly from the data. If we halve the number of samples included to 100, we would expect from Gaussian statistics that the error bar would grow by a factor of  $\sqrt{2}$ , but instead we get

```
>qmca -q e -e 300 *s001.scalar*
qmc series 1 LocalEnergy = -46.048537 +/- 0.009280
```

which erroneously shows an estimated increase in the error bar by a factor of about two. Overall this run is simply too short to gain meaningful information.

Consider the case where we are interested in the cohesive energy of diamond and, after having performed a timestep study of the cohesive energy, we have found that the energy difference between bulk diamond and atomic carbon converges to our required accuracy with a larger timestep of 0.01 Ha<sup>-1</sup>. In a production setting, a small cell could be used to determine the appropriate timestep while a larger cell would subsequently be used to obtain a converged cohesive energy, though for purposes of demonstration we still proceed with the 8 atom cell here. The new timestep of 0.01 Ha<sup>-1</sup> will result in a shorter autocorrelation time than the smaller timestep used previously, but we would like to shorten the equilibration time further still. This can be acheived by using a larger

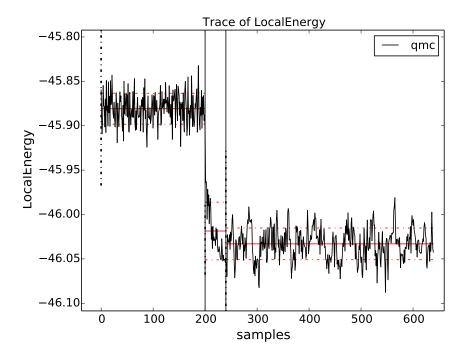


Figure 11.5: Trace of the local energy for VMC followed by a short intermediate DMC with a large timestep  $(0.02~{\rm Ha}^{-1})$  and finally a production DMC run with a timestep of  $0.01~{\rm Ha}^{-1}$ . Calculations were performed in an 8 atom cell of diamond.

timestep (say  $0.02~{\rm Ha^{-1}}$ ) in a short intermediate DMC run used to walk down the transient. The rapidly achieved equilibrium with the  $0.02~{\rm Ha^{-1}}$  timestep projector will be much nearer to the  $0.01~{\rm Ha^{-1}}$  timestep one we seek than the original VMC equilibrium, and so we can expect a shortened secondary equilibration time in the production  $0.01~{\rm Ha^{-1}}$  timestep run. Note that this procedure is fully general, even if one has to deal with an even shorter timestep– $e.g.~0.002~{\rm Ha^{-1}}$ –for a particular problem.

We now rerun the prior example but with an intermediate DMC calculation using 40 blocks of 5 steps with a timestep of  $0.02~{\rm Ha^{-1}}$  followed by a production DMC calculation using 400 blocks of 10 steps with a timestep of  $0.01~{\rm Ha^{-1}}$ . We again plot the local energy trace using qmca

```
>qmca -t -q e -e 0 *scalar*
```

with the result shown in Fig. 11.5. The projection transient has been effectively contained in the short DMC run with a larger timestep. As expected, the production run contains only a short equilibration period. Removing the first 20 blocks as a precaution, we obtain an estimate of the total energy in VMC and DMC:

>qmc	a -q ev -e	20sac qmc.*.scalar.dat				
		LocalEnergy		Variance	ratio	
qmc	series 0	-45.881042 +/- 0.001283	1.0	1.076726 +/- 0.007013	1.0	0.0235
qmc	series 1	-46.040814 +/- 0.005046	3.9	1.011303 +/- 0.016807	1.1	0.0220
qmc	series 2	-46.032960 +/- 0.002077	5.2	1.014940 +/- 0.002547	1.0	0.0220

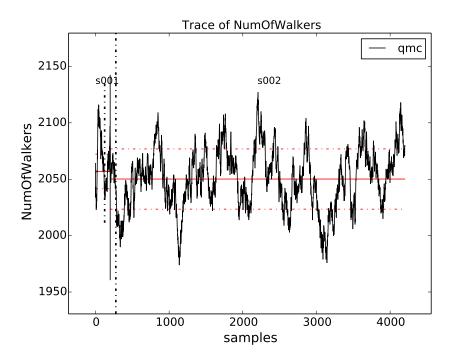


Figure 11.6: Trace of the DMC walker population for an 8 atom cell of diamond obtained with qmca.

Notice that the variance energy ratio in DMC (0.220 Ha) is similar to, but slightly smaller than, what is obtained with VMC (0.235 Ha). If the DMC variance/energy ratio is ever significantly larger than in VMC, this is cause to be concerned about the correctness of the DMC run. Also notice the estimated autocorrelation time ( $\sim 5$  blocks). This leaves us with an estimated  $\sim 76$  independent samples, though we should recall that the autocorrelation time is also a statistical estimate which can be improved with more data. We can gain a better estimate of the autocorrelation time by using the \*.dmc.dat files which contain output data resolved per step rather than per block (there are  $10 \times$  more steps than blocks in this example case):

```
>qmca -q ev -e 200 --sac qmc.s002.dmc.dat
LocalEnergy Variance ratio
qmc series 2 -46.032909 +/- 0.002068 31.2 1.015781 +/- 0.002536 1.4 0.0221
```

This results in an estimated autocorrelation time of  $\sim 31$  steps, or  $\sim 3$  blocks, indicating that we actually have  $\sim 122$  independent samples which should be sufficient to obtain a trustworthy error bar. Our final DMC total energy is estimated to be -46.0329(2) Ha.

Another simulation property that should be explicitly monitored is the behavior of the DMC walker population. Data regarding the walker population is contained in the \*.dmc.dat files. In Fig. 11.6 we show the trace of the DMC walker population for the current run:

Following a DMC run the walker population should be checked for two qualities: 1) that the population is sufficiently large (a number > 2000 is generally sufficient to reduce population control bias) and 2) that the population fluctuates benignly around its intended target value. In this case the target walker count (provided in the input file) was 2048 and we can confirm from the plot that the population is simply fluctating around this value. Also from the text output we have a dynamic population estimate of 2050(5) walkers. Rapid population reductions or increases—population explosions—are indicative of problems with a run. These issues sometimes result from using a considerably poor wavefunction (see comments regarding variance/energy ratio above and in the preceeding subsections). QMCPACK has internal guards in place that prevent the population from exceeding certain maximum and minimum bounds, so in particularly faulty runs one might see the population "stabilize" to a constant value much larger or smaller than the target. In these cases the cause(s) for the divergent population behavior need to be investigated and resolved before proceeding further.

#### 11.1.4 Obtaining other quantities

A number of other scalar valued quantities are available with qmca. To obtain text output for all quantities available, simply exclude the "-q" option used in the prior examples. Below is example output for a DMC calculation of the 8 atom diamond system from the scalar.dat file:

>qmca -e 20 qmc.s002.s	calar.d	lat		
qmc series 2				
LocalEnergy	=	-46.0330 +/-	0.0021	
Variance	=	1.0149 +/-	0.0025	
Kinetic	=	33.851 +/-	0.019	
LocalPotential	=	-79.884 +/-	0.020	
ElecElec	=	-11.4483 +/-	0.0083	
LocalECP	=	-22.615 +/-	0.029	
NonLocalECP	=	5.2815 +/-	0.0079	
IonIon	=	-51.10 +/-	0.00	
LocalEnergy_sq	=	2120.05 +/-	0.19	
BlockWeight	=	20514.27 +/-	48.38	
BlockCPU	=	1.4890 +/-	0.0038	
AcceptRatio	=	0.9963954 +/-	0.0000055	
Efficiency	=	71.88 +/-	0.00	
TotalTime	=	565.80 +/-	0.00	
TotalSamples	=	7795421 +/-	0	

Similarly, for the dmc.dat file we get

>qmca -e 20 qmc.s00	02.dmc.dat			
qmc series 2				
LocalEnergy	=	-46.0329 +/-	0.0020	
Variance	=	1.0162 +/-	0.0025	
TotalSamples	=	8201275 +/-	0	
TrialEnergy	=	-46.0343 +/-	0.0023	
DiffEff	=	0.9939150 +/-	0.0000088	
Weight	=	2050.23 +/-	4.82	

NumOfWalkers	=	2050 +/-	5	
LivingFraction	=	0.996427 +/-	0.000021	
AvgSentWalkers	=	0.2625 +/-	0.0011	

Any subset of desired quantities can be obtained by using the "-q" option with either the full names of the quantities listed above

```
>qmca -q 'LocalEnergy Kinetic LocalPotential' -e 20 qmc.s002.scalar.dat
qmc series 2
LocalEnergy = -46.0330 +/- 0.0021
Kinetic = 33.851 +/- 0.019
LocalPotential = -79.884 +/- 0.020
```

or with their corresponding abbreviations

```
>qmca -q ekp -e 20 qmc.s002.scalar.dat
qmc series 2
LocalEnergy = -46.0330 +/- 0.0021
Kinetic = 33.851 +/- 0.019
LocalPotential = -79.884 +/- 0.020
```

Abbreviations for each quantity can be found by typing qmca at the command line with no other input. A current list is provided below:

```
Abbreviations and full names for quantities:
                  = AcceptRatio
                  = BlockCPU
  bc
                  = BlockWeight
  bw
                  = CorrectedEnergy
  се
                  = DiffEff
  de
                  = LocalEnergy
  е
                  = ElecElec
  ee
                  = Efficiency
  eff
  ii
                  = IonIon
 k
                  = Kinetic
  kc
                  = KEcorr
 1
                  = LocalECP
  le2
                  = LocalEnergy_sq
                  = MPC
  mpc
                  = NonLocalECP
 n
                  = NumOfWalkers
  nw
                  = LocalPotential
 р
                  = AvgSentWalkers
  SW
                  = TrialEnergy
  te
                  = TotalSamples
  ts
                  = TotalTime
  tt
                  = Variance
  V
                  = Weight
```

Please see the output overview for scalar.dat (Sec. 10.1) and dmc.dat (Sec. 10.4) for more information about these quantities. The data analysis aspects for these quantities is essentially the same as for the local energy as covered in the preceding subsections. Quantities that do not belong to an equilibrium distribution (e.g. BlockCPU) are somewhat different, though they still exhibit statistical fluctuations.

#### 11.1.5 Processing multiple files

Batch file processing is a common use case for qmca. If we consider an "equation of state" calculation involving the 8 atom diamond cell we have used so far, we might be interested in the total energy for the various supercell volumes along the trajectory from compression to expansion. After checking the traces ("qmca -t -q e scale\_\*/vmc/\*scalar\*") to settle on a sensible equilibration cutoff as discussed in the preceeding subsections we can obtain the total energies all at once:

```
>qmca -q ev -e 40 scale_*/vmc/*scalar*
                            LocalEnergy
                                                       Variance
                                                                           ratio
scale_0.80/vmc/qmc
                    series 0 -44.670984 +/- 0.006051
                                                       2.542384 +/- 0.019902
                                                                               0.0569
                    series 0 -44.982818 +/- 0.005757
scale_0.82/vmc/qmc
                                                       2.413011 +/- 0.022626
                                                                               0.0536
scale_0.84/vmc/qmc
                                                       2.258577 +/- 0.019322
                    series 0 -45.228257 +/- 0.005374
                                                                               0.0499
scale_0.86/vmc/qmc
                    series 0 -45.415842 +/- 0.005532
                                                       2.204980 +/- 0.052978
                                                                               0.0486
scale_0.88/vmc/qmc
                    series 0 -45.570215 +/- 0.004651
                                                       2.061374 +/- 0.014359
                                                                               0.0452
scale_0.90/vmc/qmc
                    series 0 -45.683684 +/- 0.005009
                                                       1.988539 +/- 0.018267
                                                                               0.0435
scale_0.92/vmc/qmc
                    series 0 -45.751359 +/- 0.004928
                                                       1.913282 +/- 0.013998
                                                                               0.0418
scale_0.94/vmc/qmc
                    series 0 -45.791622 +/- 0.005026
                                                       1.843704 +/- 0.014460
                                                                               0.0403
                    series 0 -45.809256 +/- 0.005053
scale_0.96/vmc/qmc
                                                       1.829103 +/- 0.014536
                                                                               0.0399
scale_0.98/vmc/qmc
                    series 0 -45.806235 +/- 0.004963
                                                       1.775391 +/- 0.015199
                                                                               0.0388
scale_1.00/vmc/qmc
                    series 0 -45.783481 +/- 0.005293
                                                       1.726869 +/- 0.012001
                                                                               0.0377
scale_1.02/vmc/qmc
                    series 0 -45.741655 +/- 0.005627
                                                       1.681776 +/- 0.011496
                                                                               0.0368
scale_1.04/vmc/qmc
                    series 0 -45.685101 +/- 0.005353
                                                       1.682608 +/- 0.015423
                                                                               0.0368
scale_1.06/vmc/qmc
                    series 0 -45.615164 +/- 0.005978
                                                       1.652155 +/- 0.010945
                                                                               0.0362
scale_1.08/vmc/qmc
                    series 0 -45.543037 +/- 0.005191
                                                       1.646375 +/- 0.013446
                                                                               0.0361
scale_1.10/vmc/qmc
                    series 0 -45.450976 +/- 0.004794
                                                       1.707649 +/- 0.048186
                                                                               0.0376
scale_1.12/vmc/qmc
                    series 0 -45.371851 +/- 0.005103
                                                       1.686997 +/- 0.035920
                                                                               0.0372
scale_1.14/vmc/qmc
                    series 0 -45.265490 +/- 0.005311
                                                       1.631614 +/- 0.012381
                                                                               0.0360
scale_1.16/vmc/qmc
                    series 0 -45.161961 +/- 0.004868
                                                       1.656586 +/- 0.014788
                                                                               0.0367
scale_1.18/vmc/qmc
                    series 0 -45.062579 +/- 0.005971
                                                       1.671998 +/- 0.019942
                                                                               0.0371
scale_1.20/vmc/qmc
                    series 0 -44.960477 +/- 0.004888
                                                       1.651864 +/- 0.009756
                                                                               0.0367
```

In this case, we are using a Jastrow factor optimized only at the equilibrium geometry ("scale\_1.00") but with radial cutoffs restricted to the Wigner-Seitz radius of the most compressed supercell ("scale\_0.80") to avoid introducing wavefunction cusps at the cell boundary (QMCPACK would have aborted with a warning in this case, had we tried). It is clear that this restricted Jastrow factor is not an optimal choice as it yields variance/energy ratios between 0.036 and 0.057 Ha. This issue is largely a result of our undersized (8 atom) supercell and larger cells should always be used in real production calculations.

Batch processing is also possible for multiple quantities. If multiple quantities are requested, an additional line is inserted to separate results from different runs:

>qmca -q 'e bc eff'	-e 40 scal	e_*/vmc/*scalar*		
scale_0.80/vmc/qmc	series 0			
LocalEnergy	=	-44.6710 +/-	0.0061	
BlockCPU	=	0.02986 +/-	0.00038	
Efficiency	=	38104.00 +/-	0.00	
scale_0.82/vmc/qmc	series 0			
LocalEnergy	=	-44.9828 +/-	0.0058	
BlockCPU	=	0.02826 +/-	0.00013	
Efficiency	=	44483.91 +/-	0.00	
scale_0.84/vmc/qmc	series 0			
LocalEnergy	=	-45.2283 +/-	0.0054	
BlockCPU	=	0.02747 +/-	0.00030	
Efficiency	=	52525.12 +/-	0.00	
scale_0.86/vmc/qmc	series 0			
LocalEnergy	=	-45.4158 +/-	0.0055	
BlockCPU	=	0.02679 +/-	0.00013	
Efficiency	=	50811.55 +/-	0.00	
scale_0.88/vmc/qmc	series 0			
LocalEnergy	=	-45.5702 +/-	0.0047	
BlockCPU	=	0.02598 +/-	0.00015	
Efficiency	=	74148.79 +/-	0.00	
scale_0.90/vmc/qmc	series 0			
LocalEnergy	=	-45.6837 +/-	0.0050	
BlockCPU	=	0.02527 +/-	0.00011	
Efficiency	=	65714.98 +/-	0.00	

#### 11.1.6 Twist averaging

Twist averaging can be performed straightforwardly for any output quantity listed in Sec. 11.1.4 with qmca. We illustrate these capabilities by repeating the 8 atom diamond DMC runs performed in Sec. 11.1.3 at eight real valued supercell twist angles (a  $2 \times 2 \times 2$  Monkhorst-Pack grid centered at the  $\Gamma$ -point). Data traces for each twist can be overlapped on the same plot:

```
>qmca -to -q e -e '30 20 30' *scalar* --legend outside
```

The "-o" option requests the plots be overlapped; eight separate plots would be generated otherwise. The equilibration input "-e '30 20 30'" cuts out from the analyzed data the first 30 blocks for series 0 (VMC), 20 blocks for series 1 (intermediate DMC), and 30 blocks for series 2 (production DMC). The resulting plot is shown in Fig. 11.7

Twist averaging is performed by providing the "-a" option. If provided on its own, uniform weights are applied to each twist angle. To obtain a trace plot with twist averaging enforced, use

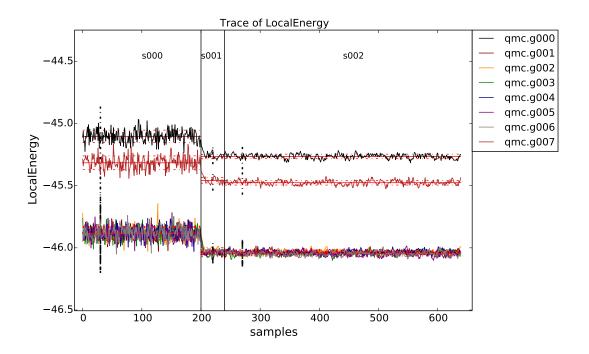


Figure 11.7: Overlapped energy traces from VMC to DMC for an 8 supercell of diamond obtained with qmca. Data for each twist appears in a different color.

a command similar to the following:

```
>qmca -a -t -q e -e '30 20 30' *scalar*
```

The resulting plot is shown in Fig. 11.8. As can be seen from the trace plot, the chosen equilibration lengths are appropriate and we proceed to obtain the twist averaged total energy from the scalar.dat files

```
>qmca -a -q ev -e 30 --sac *s002.scalar*

LocalEnergy Variance ratio

avg series 2 -45.873369 +/- 0.000753 5.3 1.028751 +/- 0.001056 1.3 0.0224
```

and also from the dmc.dat files

yielding a twist averaged total energy of -45.8733(8) Ha.

As can be seen from the Fig. 11.7, some of the twist angles are degenerate. This is seen more clearly in the text output:

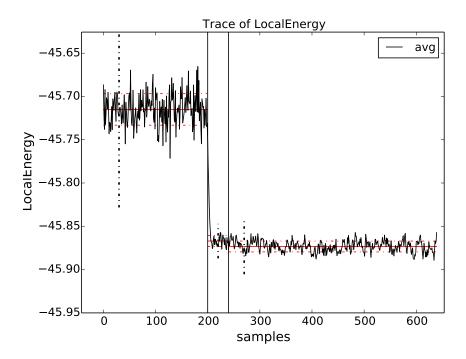


Figure 11.8: Twist averaged energy trace from VMC to DMC for an 8 supercell of diamond obtained with qmca.

>qmca -q	ev -e 30 *	s002.scalar*		
		LocalEnergy	Variance	ratio
qmc.g000	series 2	-45.264510 +/- 0.001942	1.057065 +/- 0.002318	0.0234
qmc.g001	series 2	-46.035511 +/- 0.001806	1.015992 +/- 0.002836	0.0221
qmc.g002	series 2	-46.035410 +/- 0.001538	1.015039 +/- 0.002661	0.0220
qmc.g003	series 2	-46.047285 +/- 0.001898	1.018219 +/- 0.002588	0.0221
qmc.g004	series 2	-46.034225 +/- 0.002539	1.013420 +/- 0.002835	0.0220
qmc.g005	series 2	-46.046731 +/- 0.002963	1.018337 +/- 0.004109	0.0221
qmc.g006	series 2	-46.047133 +/- 0.001958	1.021483 +/- 0.003082	0.0222
qmc.g007	series 2	-45.476146 +/- 0.002065	1.070456 +/- 0.003133	0.0235

The degenerate twists grouped by set are  $\{0\}$ ,  $\{1, 2, 4\}$ ,  $\{3, 5, 6\}$ ,  $\{7\}$ .

Alternatively, the run could have been performed at *only* the four unique (irreducible) twist angles. We will emulate this situation by analyzing data for twists 0, 1, 3, and 7 only. In a production setting with irreducibly weighted twists, run would be performed on these twists alone; we resuse the uniform twist data for illustration purposes only.

We can use qmca to perform twist averaging with different weights applied to each twist

yielding a total energy value of -45.874(1) Ha, in agreement with the uniform weighted twist average performed above.

The decision of whether or not to perform irreducible weighted twist averaging should be made on the basis of efficiency. The relative efficiency of irreducible vs. uniform weighted twist averaging depends on the irreducible weights and the ratio of the lengths of the available sampling and equilibration periods. A formula for the relative efficiency of these two cases is derived and discussed in more detail in Appendix A.

#### 11.1.7 Setting output units

Estimates outputted by qmca are in Hartree units by default. The output units for energetic quantities can be changed by using the "-u" option.

Energy in Hartrees:

```
>qmca -q e -u Ha -e 20 qmc.s002.scalar.dat
qmc series 2 LocalEnergy = -46.032960 +/- 0.002077
```

Energy in electron volts:

```
>qmca -q e -u eV -e 20 qmc.s002.scalar.dat
qmc series 2 LocalEnergy = -1252.620565 +/- 0.056521
```

Energy in Rydbergs:

```
>qmca -q e -u rydberg -e 20 qmc.s002.scalar.dat
qmc series 2 LocalEnergy = -92.065919 +/- 0.004154
```

Energy in kilojoules per mole:

```
>qmca -q e -u kj_mol -e 20 qmc.s002.scalar.dat
qmc series 2 LocalEnergy = -120859.512998 +/- 5.453431
```

#### 11.1.8 Speeding up trace plotting

When working with many files or files with many entries, qmca may take a long time to produce plots. The time delay is actually due to the autocorrelation time estimate used to calculate error bars. The calculation time for the autocorrelation scales as  $\mathcal{O}(M^2)$ , with M being the number of statistical samples. If you are only interested in plotting traces and not in the estimated error bars, the autocorrelation time estimation can be turned off with the "--noac" option:

```
>qmca -t -q e -e 20 --noac qmc.s002.scalar.dat
```

Please note that the resulting error bars printed to the console will be underestimated and are not meaningful. Do *not* use "--noac" in conjunction with the "-p" plotting option as these plots are of no use without meaningful error bars.

#### 11.1.9 Short usage examples

Plotting a trace of the local energy:

```
>qmca -t -q e *scalar*
```

Applying an equilibration cutoff to VMC data (series 0):

```
>qmca -q e -e 30 *s000.scalar*
```

Applying the same equilibration cutoff to VMC and DMC data (series 0, 1, 2):

```
>qmca -q e -e 20 *scalar*
```

Applying different equilibration cutoffs to VMC and DMC data (series 0, 1, 2):

```
>qmca -q e -e '30 20 40' *scalar*
```

Obtaining the energy, variance, and variance/energy ratio for all series:

```
>qmca -q ev -e 30 *scalar*
```

Overlaying plots of mean + error bar for energy and variance for separate two- and three- body Jastrow optimization runs:

```
>qmca -po -q ev ./optJ2/*scalar* ./optJ3/*scalar*
```

Obtaining the acceptance ratio:

```
>qmca -q ar -e 30 *scalar*
```

Obtaining the average DMC walker population:

```
>qmca -q nw -e 400 *s002.dmc.dat
```

Obtaining the Monte Carlo efficiency:

```
>qmca -q eff -e 30 *scalar*
```

Obtaining the total wallclock time per series:

```
>qmca -q tt -e 0 *scalar*
```

Obtaining the average wallclock time spent per block:

```
>qmca -q bc -e 0 *scalar*
```

Obtaining a subset of desired quantities:

Obtaining all available quantities:

```
>qmca -e 30 *scalar*
```

Obtaining the twist averaged total energy with uniform weights:

```
>qmca -a -q e -e 40 *g*s002.scalar.dat
```

Obtaining the twist averaged total energy with specific weights:

```
>qmca -a -w '1 3 3 1' -q e -e 40 *g*s002.scalar.dat
```

Obtaining the local, kinetic, and potential energies in eV:

```
>qmca -q ekp -e 30 -u eV *scalar*
```

#### 11.1.10 Production quality checklist

- 1. Inspect the trace plots ("-t" option) for any oddities in the data. Typical behavior is a short equilibration period followed by benign fluctuations around a clear mean value. There should not be any large spikes in the data. This applies to *all* runs (VMC, optimization, DMC, etc.).
- 2. Remove all equilibration steps ("-e" option) from the data by inspecting the trace plot.
- 3. Check the quality of the orbitals (standalone Jastrow-less VMC or sometimes the first scalar file produced during optimization) by inspecting the variance/energy ratio "qmca -q ev \*scalar\*". For pseudopotential systems without a Jastrow, the variance/energy ratio should not exceed 0.2 Ha, otherwise there is a problem with the orbitals.
- 4. Check the quality of the optimized Jastrow factor by inspecting the variance/energy ratio. For pseudopotential systems with a Jastrow, the variance/energy ratio should not exceed 0.04 Ha for pseudopotential systems. A good Jastrow is indicated by a variance/energy ratio in the range 0.01 0.03 Ha. A value less than 0.01 Ha is difficult to achieve.
- 5. Confirm that the optimization has converged by plotting the energy and variance vs. optimization series ("qmca -p -q ev \*scalar\*"). Do not assume that optimization has converged in only a few cycles. Use at least 10 cycles of with around 100,000 samples unless you already have experience with the system in question.
- 6. Optimize Jastrow factors according to energy minimization to reduce locality errors arising from the use of non-local pseudopotentials in DMC. A good approach is to optimize with a few cycles of variance minimization followed by several cycles of energy minimization.
- 7. Occasionally try optimizing with more samples and/or cycles to see if improved results are obtained.
- 8. If using a B-spline representation of the orbitals, converge the VMC energy and variance with respect to the mesh size (controlled via meshfactor). This is best done in the presence of any Jastrow factor to reduce noise. Consider using the hybrid LMTO representation of the orbitals as this can reduce both the VMC/DMC variance and DMC timestep error in addition to saving memory.

- 9. Check the variance/energy ratio of all production VMC and DMC calculations. In all cases the DMC ratio should be slightly less than the VMC one and both should abide the guidelines above, *i.e.* the ratio should be less than 0.04 Ha for pseudopotential systems. The production ratio should also be consistent with what is observed during wavefunction optimization.
- 10. Be aware of population control bias in DMC. Run with a population of  $\sim 2000$  or greater. Occasionally repeat a run using a larger population to explicitly confirm that population control bias is small.
- 11. Check the stability of the DMC walker population by plotting the trace of the population size ("qmca -t -q nw \*dmc.dat"). Verify that the average walker population is consistent with the requested value provided in the input.
- 12. In DMC, perform a timestep study to either 1) obtain extrapolated results, or 2) obtain a timestep for future production where an energy difference shows convergence (e.g. a band gap or defect formation energy). For pseudopotential systems, converged timesteps for many systems are in the range 0.002 0.01 Ha<sup>-1</sup>, but the actual converged timestep must be explicitly checked.
- 13. In periodic systems, converge the total energy with respect to the size of the twist/k-point grid. Results for smaller systems can easily be transferred to larger ones (e.g. a 2x2x2 twist grid in a 2x2x2 tiled cell is equivalent to a 1x1x1 twist grid in a 4x4x4 tiled cell).
- 14. In periodic systems, perform finite size extrapolation including two body corrections (needed for cohesive energy/phase stability studies) unless it can be shown that finite size effects cancel for the energy difference in question (e.g. some defect formation energies).

# 11.2 Densities and spin-densities

# 11.3 Energy densities

# Chapter 12

# **Examples**

WARNING: THESE EXAMPLES ARE NOT CONVERGED! YOU MUST CONVERGE PARAMETERS (SIMULATION CELL SIZE, JASTROW PARAMETER NUMBER/CUTOFF, TWIST NUMBER, DMC TIME STEP, DFT PLANE WAVE CUTOFF, DFT K-POINT MESH, ETC.) FOR REAL CALCUATIONS!

The following examples should run in serial on a modern workstation in a few hours.

### 12.1 Using QMCPACK directly

In examples/molecules, there are the following examples. Each directory also contains a README file with more details.

Directory Description

H2O molecule from GAMESS orbitalsHe Helium atom with simple wavefunctions

# 12.2 Using Nexus

For more information about Nexus, see the User Guide in nexus/documentation.

For Python to find the Nexus library, the PYTHONPATH environment variable should be set to <QMCPACK source>/nexus/library. For these examples to work properly, the executables for Quantum ESPRESSO and QMCPACK either need to be on the path, or the paths in the script should be adjusted.

These examples can be found under the nexus/examples/qmcpack directory.

Directory Description

diamond Bulk diamond with VMC graphene Graphene sheet with DMC

c20 C20 cage molecule

oxygen\_dimer Binding curve for O2 molecule

 $H_2O$  molecule with Quantum ESPRESSO orbitals LiH crystal with Quantum ESPRESSO orbitals

# Chapter 13

# Lab 1: Monte Carlo Statistical Analysis

### 13.1 Topics covered in this Lab

This lab focuses on the basics of analyzing data from Monte Carlo (MC) calculations. In this lab, participants will use data from VMC calculations of a simple one-electron system with an analytically soluble system (the ground state of the hydrogen atom) to understand how to interpret a MC situation. Most of these analyses will also carry over to diffusion Monte Carlo (DMC) simulations. Topics covered include:

- averaging Monte Carlo variables
- the statistical error bar of mean values
- effects of autocorrelation and variance on the error bar
- the relationship between Monte Carlo timestep and autocorrelation
- the use of blocking to reduce autocorrelation
- ullet the significance of the acceptance ratio
- the significance of the sample size
- how to determine whether a Monte Carlo run was successful
- the relationship between wavefunction quality and variance
- gauging the efficiency of Monte Carlo runs
- the cost of scaling up to larger system sizes

#### 13.2 Lab directories and files

labs/lab1\_qmc\_statistics/
atom - H atom VMC calculation

```
- H atom VMC data
  H.s000.scalar.dat
  H.xml
                                    - H atom VMC input file
autocorrelation
                                  - varying autocorrelation
                                    - data for gnuplot
  H.dat
                                    - gnuplot for time step vs. E_L, tau_c
  H.plt
  H.s000.scalar.dat
                                    - H atom VMC data: time step = 10
  H.s001.scalar.dat
                                    - H atom VMC data: time step = 5
  H.s002.scalar.dat
                                   - H atom VMC data: time step = 2
  H.s003.scalar.dat
                                   - H atom VMC data: time step = 1
  H.s004.scalar.dat
                                   - H atom VMC data: time step = 0.5
  H.s005.scalar.dat
                                   - H atom VMC data: time step = 0.2
  H.s006.scalar.dat
                                   - H atom VMC data: time step = 0.1
  H.s007.scalar.dat
                                   - H atom VMC data: time step = 0.05
  H.s008.scalar.dat
                                   - H atom VMC data: time step = 0.02
  H.s009.scalar.dat
                                   - H atom VMC data: time step = 0.01
  H.s010.scalar.dat
                                   - H atom VMC data: time step = 0.005
  H.s011.scalar.dat
                                   - H atom VMC data: time step = 0.002
  H.s012.scalar.dat
                                   - H atom VMC data: time step = 0.001
  H.s013.scalar.dat
                                   - H atom VMC data: time step = 0.0005
  H.s014.scalar.dat
                                   - H atom VMC data: time step = 0.0002
  H.s015.scalar.dat
                                   - H atom VMC data: time step = 0.0001
  H.xml
                                    - H atom VMC input file
                                  - Python scripts for average/std. dev.
average
                                     - average five E_L from H atom VMC
  average.py
                                      - standard deviation using (E_L)^2
   stddev2.py
   stddev.py
                                      - standard deviation around the mean
basis
                                   - varying basis set for orbitals
                                     - H atom VMC data using STO basis
  H__exact.s000.scalar.dat
  H_STO-2G.s000.scalar.dat
                                     - H atom VMC data using STO-2G basis
                                     - H atom VMC data using STO-3G basis
  H_STO-3G.s000.scalar.dat
   H_STO-6G.s000.scalar.dat
                                     - H atom VMC data using STO-6G basis
                                  - varying block/step ratio
blocking
  H.dat
                                      - data for gnuplot
                                      - gnuplot for N_block vs. E, tau_c
  H.plt
                                     - H atom VMC data 50000:1 blocks:steps
  H.s000.scalar.dat
  H.s001.scalar.dat
                                                        25000:2 blocks:steps
                                                     " 12500:4 blocks:steps
  H.s002.scalar.dat
                                      _ " "
  H.s003.scalar.dat
                                                     " 6250: 8 blocks:steps
                                      _ " "
  H.s004.scalar.dat
                                                     " 3125:16 blocks:steps
                                      _ " "
  H.s005.scalar.dat
                                                11
                                                     " 2500:20 blocks:steps
                                                11
  H.s006.scalar.dat
                                                     " 1250:40 blocks:steps
                                      _ " "
                                                11
  H.s007.scalar.dat
                                                     " 1000:50 blocks:steps
                                                     " 500:100 blocks:steps
  H.s008.scalar.dat
```

```
H.s009.scalar.dat
                                                        250:200 blocks:steps
                                                     " 125:400 blocks:steps
   H.s010.scalar.dat
   H.s011.scalar.dat
                                                     " 100:500 blocks:steps
                                      _ " "
   H.s012.scalar.dat
                                                     " 50:1000 blocks:steps
                                      _ 11 11
                                                     " 40:1250 blocks:steps
   H.s013.scalar.dat
   H.s014.scalar.dat
                                      _ " "
                                                        20:2500 blocks:steps
                                      _ " "
   H.s015.scalar.dat
                                                        10:5000 blocks:steps
   H.xml
                                     - H atom VMC input file
blocks
                                   - varying total number of blocks
                                     - data for gnuplot
  H.dat
   H.plt
                                     - gnuplot for N_block vs. E
                                     - H atom VMC data
   H.s000.scalar.dat
                                                          500 blocks
   H.s001.scalar.dat
                                                         2000 blocks
   H.s002.scalar.dat
                                                         8000 blocks
   H.s003.scalar.dat
                                                        32000 blocks
   H.s004.scalar.dat
                                                    " 128000 blocks
   H.xml
                                     - H atom VMC input file
dimer
                               - comparing no and simple Jastrow factor
   H2_STO___no_jastrow.s000.scalar.dat - H dimer VMC data without Jastrow
   H2_STO_with_jastrow.s000.scalar.dat - H dimer VMC data with Jastrow
docs
                                   - documentation
  Lab_1_MC_Analysis.pdf
                                     - this document
   Lab_1_Slides.pdf
                                     - slides presented in the lab
nodes
                                   - varying number of computing nodes
  H.dat
                                     - data for gnuplot
                                     - gnuplot for N_node vs. E
   H.plt
   H.s000.scalar.dat
                                     - H atom VMC data with 32 nodes
                                     - H atom VMC data with 128 nodes
   H.s001.scalar.dat
   H.s002.scalar.dat
                                     - H atom VMC data with 512 nodes
problematic
                                   - problematic VMC run
   H.s000.scalar.dat
                                     - H atom VMC data with a problem
size
                                    - scaling with number of particles
                                     - H atom VMC data
   01_____H.s000.scalar.dat
   02_____H2.s000.scalar.dat
                                     - H dimer "
                                     - C atom "
   06_____C.s000.scalar.dat
   10_____CH4.s000.scalar.dat
                                     - methane "
   12_____C2.s000.scalar.dat
                                     - C dimer "
   16____C2H4.s000.scalar.dat
                                     - ethene "
                                     - methane dimer VMC data
   18___CH4CH4.s000.scalar.dat
   32_C2H4C2H4.s000.scalar.dat
                                     - ethene dimer
   nelectron_tcpu.dat
                                     - data for gnuplot
```

#### 13.3 Atomic units

QMCPACK operates in Hartree atomic units to reduce the number of factors in the Schrödinger equation. Thus, the unit of length is the bohr (5.291772  $\times 10^{-11}$  m = 0.529177 Å); the unit of energy is the hartree (4.359744  $\times 10^{-18}$  J = 27.211385 eV). The energy of the ground state of the hydrogen atom in these units is -0.5 hartrees.

### 13.4 Reviewing statistics

We will practice taking the average (mean) and standard deviation of some Monte Carlo data by hand to review the basic definitions.

Enter Python's command line by typing **python** [Enter]. You will see a prompt ">>>". The mean of a data set is given by:

$$\overline{x} = \frac{1}{N} \sum_{i=1}^{N} x_i \tag{13.1}$$

To calculate the average of five local energies from a MC calculation of the ground state of an electron in the hydrogen atom, input (truncate at the thousandths place if you cannot copy and paste; script versions are also available in the average directory):

((-0.45298911858) + (-0.45481953564) + (-0.48066105923) + (-0.47316713469) + (-0.462047335)

Then, press [Enter] to get:

```
>>> ((-0.45298911858) + (-0.45481953564) + (-0.48066105923) + (-0.47316713469) + (-0.4620473302))/5.
-0.46473683566800006
```

To understand the significance of the mean, we also need the standard deviation around the mean of the data (also called the error bar), given by:

$$\sigma = \sqrt{\frac{1}{N(N-1)} \sum_{i=1}^{N} (x_i - \overline{x})^2}$$
 (13.2)

To calculate the standard deviation around the mean (-0.464736835668) of these five data points, put in:

```
( (1./(5.*(5.-1.))) * ( (-0.45298911858-(-0.464736835668))**2 + (-0.45481953564-(-0.464736835668))**2 + (-0.48066105923-(-0.464736835668))**2 + (-0.47316713469) + (-0.46204733302-(-0.464736835668))**2 ) ) **0.5 Then, press [Enter] to get:
```

```
>>> ( (1./(5.*(5.-1.))) * ( (-0.45298911858-(-0.464736835668))**2 + (-0.45481953564-(-0.464736835668))**2 + (-0.47316713469-(-0.464736835668))**2 + (-0.46204733302-(-0.464736835668))**2 + (-0.53303187464332066
```

Thus, we might report this data as having a value -0.465 + /-0.005 hartrees. This calculation of the standard deviation assumes that the average for this data is fixed, but we may continually add Monte Carlo samples to the data so it is better to use an estimate of the error bar that does not rely on the overall average. Such an estimate is given by:

$$\tilde{\sigma} = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} \left[ (x^2)_i - (x_i)^2 \right]}$$
(13.3)

To calculate the standard deviation with this formula, input the following, which includes the square of the local energy calculated with each corresponding local energy:

```
( (1./(5.-1.)) * ( (0.60984565298-(-0.45298911858)**2) + (0.61641291630-(-0.45481953564)**2) + (1.35860151160-(-0.48066105923)**2) + (0.78720769003-(-0.47316713469)**2) + (0.56393677687-(-0.46204733302)**2) ) )**0.5
```

```
>>> ((1./(5.-1.))*((0.60984565298-(-0.45298911858)**2)+
(0.61641291630-(-0.45481953564)**2)+(1.35860151160-(-0.48066105923)**2)+
(0.78720769003-(-0.47316713469)**2)+(0.56393677687-(-0.46204733302)**2))
)**0.5
0.84491636672906634
```

This much larger standard deviation, acknowledging that the mean of this small data set is not the average in the limit of infinite sampling more accurately, reports the value of the local energy as -0.5 + /-0.8 hartrees.

Type quit() and press [Enter] to exit the Python command line.

# 13.5 Inspecting Monte Carlo data

and press [Enter] to get:

QMCPACK outputs data from MC calculations into files ending in scalar.dat. Several quantities are calculated and written for each block of Monte Carlo steps in successive columns to the right of the step index.

Change directories to atom, and open the file ending in scalar.dat with a text editor (e.g., vi \*.scalar.dat or emacs \*.scalar.dat. If possible, adjust the terminal so that lines do not wrap. The data will begin as follows (broken into three groups to fit on this page):

```
# index LocalEnergy LocalEnergy_sq LocalPotential ...

0 -4.5298911858e-01 6.0984565298e-01 -1.1708693521e+00
1 -4.5481953564e-01 6.1641291630e-01 -1.1863425644e+00
2 -4.8066105923e-01 1.3586015116e+00 -1.1766446209e+00
```

```
3
    -4.7316713469e-01
                          7.8720769003e-01
                                              -1.1799481122e+00
4
    -4.6204733302e-01
                          5.6393677687e-01
                                              -1.1619244081e+00
5
    -4.4313854290e-01
                          6.0831516179e-01
                                              -1.2064503041e+00
6
    -4.5064926960e-01
                          5.9891422196e-01
                                              -1.1521370176e+00
7
    -4.5687452611e-01
                          5.8139614676e-01
                                              -1.1423627617e+00
8
    -4.5018503739e-01
                          8.4147849706e-01
                                              -1.1842075439e+00
    -4.3862013841e-01
                          5.5477715836e-01
                                              -1.2080979177e+00
```

The first line begins with a #, indicating that this line does not contain MC data but rather the labels of the columns. After a blank line, the remaining lines consist of the MC data. The first column, labeled index, is an integer indicating which block of MC data is on that line. The second column contains the quantity usually of greatest interest from the simulation, the local energy. Since this simulation did not use the exact ground state wave function, it does not produce -0.5 hartrees as the local energy although the value lies within about 10%. The value of the local energy fluctuates from block to block and the closer the trial wave function is to the ground state, the smaller these fluctuations will be. The next column contains an important ingredient in estimating the error in the MC average—the square of the local energy—found by evaluating the square of the Hamiltonian.

```
Kinetic
                    Coulomb
                                         BlockWeight
 7.1788023352e-01
                    -1.1708693521e+00
                                          1.2800000000e+04
 7.3152302871e-01
                    -1.1863425644e+00
                                          1.2800000000e+04
 6.9598356165e-01
                    -1.1766446209e+00
                                          1.2800000000e+04
 7.0678097751e-01
                    -1.1799481122e+00
                                          1.2800000000e+04
 6.9987707508e-01
                    -1.1619244081e+00
                                          1.280000000e+04
                    -1.2064503041e+00
                                          1.2800000000e+04
 7.6331176120e-01
 7.0148774798e-01
                    -1.1521370176e+00
                                          1.2800000000e+04
 6.8548823555e-01
                    -1.1423627617e+00
                                          1.280000000e+04
 7.3402250655e-01
                    -1.1842075439e+00
                                          1.2800000000e+04
 7.6947777925e-01
                    -1.2080979177e+00
                                          1.2800000000e+04
```

The fourth column from the left consists of the values of the local potential energy. In this simulation, it is identical to the Coulomb potential (contained in the sixth column) because the one electron in the simulation has only the potential energy coming from its interaction with the nucleus. In many-electron simulations, the local potential energy contains contributions from the electron-electron Coulomb interactions and the nuclear potential or pseudopotential. The fifth column contains the local kinetic energy value for each MC block, obtained from the Laplacian of the wave function. The sixth column shows the local Coulomb interaction energy. The seventh column displays the weight each line of data has in the average (the weights are identical in this simulation).

```
... BlockCPU AcceptRatio
6.0178991748e-03 9.8515625000e-01
5.8323097461e-03 9.856250000e-01
5.8213412744e-03 9.8531250000e-01
5.8330412549e-03 9.8828125000e-01
5.8108362256e-03 9.8625000000e-01
5.8254170264e-03 9.8625000000e-01
```

```
5.8314813086e-03 9.8679687500e-01

5.8258469971e-03 9.8726562500e-01

5.8158433545e-03 9.8468750000e-01

5.7959401123e-03 9.8539062500e-01
```

The eighth column shows the CPU time (in seconds) to calculate the data in that line. The ninth column from the left contains the acceptance ratio (1 being full acceptance) for Monte Carlo steps in that line's data. Other than the block weight, all quantities vary from line to line.

Exit the text editor ([Esc] :q! [Enter] in vi, [Ctrl]-x [Ctrl]-c in emacs).

## 13.6 Averaging quantities in the MC data

QMCPACK includes the qmca Python tool to average quantities in the scalar.dat file (and also the dmc.dat file of DMC simulations). Without any flags, qmca will output the average of each column with a quantity in the scalar.dat file as follows.

Execute qmca by qmca \*.scalar.dat, which for this data outputs:

H series 0				
LocalEnergy	=	-0.45446 +/-	0.00057	
Variance	=	0.529 +/-	0.018	
Kinetic	=	0.7366 +/-	0.0020	
LocalPotential	=	-1.1910 +/-	0.0016	
Coulomb	=	-1.1910 +/-	0.0016	
LocalEnergy_sq	=	0.736 +/-	0.018	
BlockWeight	=	12800.00000000 +/-	0.00000000	
BlockCPU	=	0.00582002 +/-	0.0000067	
AcceptRatio	=	0.985508 +/-	0.000048	
Efficiency	=	0.00000000 +/-	0.00000000	

After one blank, qmca prints the title of the subsequent data, gleaned from the data file name. In this case, H.s000.scalar.dat became "H series 0". Everything before the first ".s" will be interpreted as the title, and the number between ".s" and the next "." will be interpreted as the series number.

The first column under the title is the name of each quantity qmca averaged. The column to the right of the equal signs contains the average for the quantity of that line, and the column to the right of the plus-slash-minus is the statistical error bar on the quantity. All quantities calculated from MC simulations have and must be reported with a statistical error bar!

Two new quantities not present in the scalar.dat file are computed by qmca from the data–variance and efficiency. We will look at these later in this lab.

To view only one value, **qmca** takes the **-q** (**quantity**) flag. For example, the output of **qmca -q** LocalEnergy \*.scalar.dat in this directory produces a single line of output:

```
H series 0 LocalEnergy = -0.454460 +/- 0.000568
```

Type **qmca** -help to see the list of all quantities and their abbreviations.

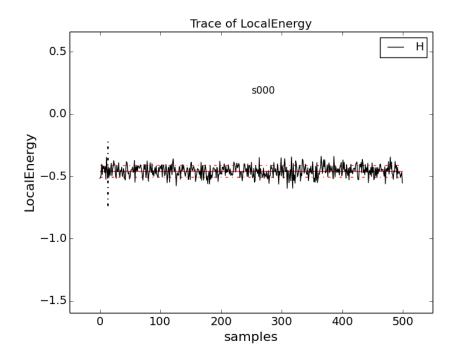
### 13.7 Evaluating MC simulation quality

There are several aspects of a MC simulation to consider in deciding how well it went. Besides the deviation of the average from an expected value (if there is one), the stability of the simulation in its sampling, the autocorrelation between MC steps, the value of the acceptance ratio (accepted steps over total proposed steps), and the variance in the local energy all indicate the quality of a MC simulation. We will look at these one by one.

#### 13.7.1 Tracing MC quantities

Visualizing the evolution of MC quantities over the course of the simulation by a *trace* offers a quick picture of whether the random walk had expected behavior. qmca plots traces with the -t flag.

Type qmca -q e -t H.s000.scalar.dat, which produces a graph of the trace of the local energy:



The solid black line connects the values of the local energy at each MC block (labeled "samples"). The average value is marked with a horizontal, solid red line. One standard deviation above and below the average are marked with horizontal, dashed red lines.

The trace of this run is largely centered around the average with no large-scale oscillations or major shifts, indicating a good quality MC run.

Try tracing the kinetic and potential energies, seeing that their behavior is comparable to the total local energy.

Change to directory problematic and type qmca -q e -t H.s000.scalar.dat to produce this graph:



Here, the local energy samples cluster around the expected -0.5 hartrees for the first 150 samples or so and then begin to oscillate more wildly and increase erratically toward 0, indicating a poor quality MC run.

Again, trace the kinetic and potential energies in this run and see how their behavior compares to the total local energy.

#### 13.7.2 Blocking away autocorrelation

Autocorrelation occurs when a given MC step biases subsequent MC steps, leading to samples that are not statistically independent. We must take this autocorrelation into account in order to obtain accurate statistics. qmca outputs autocorrelation when given the --sac flag.

Change to directory autocorrelation and type qmca -q e --sac H.s000.scalar.dat.

```
H series 0 LocalEnergy = -0.454982 +/- 0.000430 1.0
```

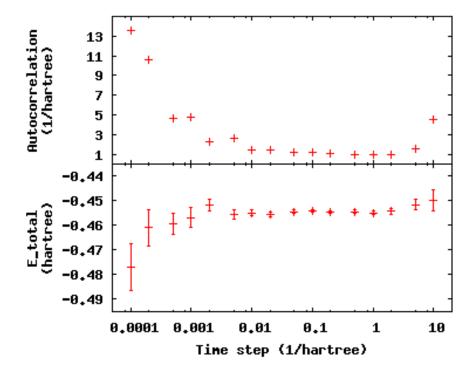
The value after the error bar on the quantity is the autocorrelation (1.0 in this case).

Proposing too small a step in configuration space, the MC *time step*, can lead to autocorrelation since the new samples will be in the neighborhood of previous samples. Type **grep timestep H.xml** to see the varying time step values in this QMCPACK input file (H.xml):

```
<parameter name="timestep">10</parameter>
<parameter name="timestep">5</parameter>
<parameter name="timestep">2</parameter>
<parameter name="timestep">1</parameter>
<parameter name="timestep">0.5</parameter>
<parameter name="timestep">0.2</parameter></parameter name="timestep">0.2</parameter></parameter name="timestep">0.2</parameter></parameter>
```

```
<parameter name="timestep">0.1</parameter>
<parameter name="timestep">0.05</parameter>
<parameter name="timestep">0.02</parameter>
<parameter name="timestep">0.01</parameter>
<parameter name="timestep">0.005</parameter>
<parameter name="timestep">0.002</parameter>
<parameter name="timestep">0.001</parameter>
<parameter name="timestep">0.001</parameter>
<parameter name="timestep">0.0005</parameter>
<parameter name="timestep">0.0005</parameter>
<parameter name="timestep">0.0002</parameter>
<parameter name="timestep">0.0002</parameter>
<parameter name="timestep">0.0001</parameter>
<parameter name="timestep">0.0001</parameter></parameter>
<parameter name="timestep">0.0001</parameter></parameter>
```

Generally, as the time step decreases, the autocorrelation will increase (caveat: very large time steps will also have increasing autocorrelation). To see this, type **qmca -q e --sac \*.scalar.dat** to see the energies and autocorrelation times, then plot with gnuplot by inputting **gnuplot H.plt**:



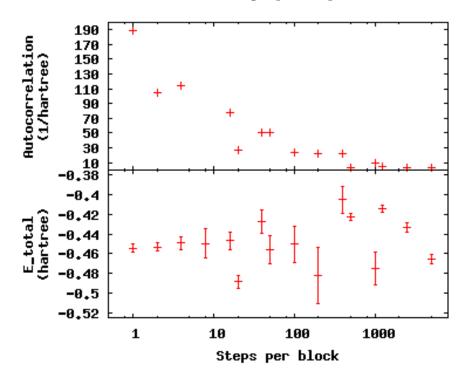
The error bar also increases with the autocorrelation.

Press q [Enter] to quit gnuplot.

To get around the bias of autocorrelation, we group the MC steps into blocks, take the average of the data in the steps of each block, and then finally average the averages in all the blocks. QMCPACK outputs the block averages as each line in the scalar dat file. (For DMC simulations, in addition to the scalar dat, QMCPACK outputs the quantities at each step to the dmc dat file, which permits reblocking the data differently from the specification in the input file.)

Change directories to blocking. Here we look at the time step of the last data set in the autocorrelation directory. Verify this by typing grep timestep H.xml to see that all values are set to 0.001. Now to see how we will vary the blocking, type grep -A1 blocks H.xml. The parameter "steps" indicates the number of steps per block, and the parameter "blocks" gives the

number of blocks. For this comparison, the total number of MC steps (equal to the product of "steps" and "blocks") is fixed at 50000. Now check the effect of blocking on autocorrelation—type **qmca -q e --sac \*scalar.dat** to see the data and **gnuplot H.plt** to visualize the data:



The greatest number of steps per block produces the smallest autocorrelation time. The larger number of blocks over which to average at small step-per-block number masks the corresponding increase in error bar with increasing autocorrelation.

Press q [Enter] to quit gnuplot.

#### 13.7.3 Balancing autocorrelation and acceptance ratio

Adjusting the time step value also affects the ratio of accepted steps to proposed steps. Stepping nearby in configuration space implies that the probability distribution is similar and thus more likely to result in an accepted move. Keeping the acceptance ratio high means the algorithm is efficiently exploring configuration space and not sticking at particular configurations. Return to the autocorrelation directory. Refresh your memory on the time steps in this set of simulations by grep timestep H.xml. Then, type qmca -q ar \*scalar.dat to see the acceptance ratio as it varies with decreasing time step:

```
AcceptRatio = 0.047646 + - 0.000206
Η
   series 0
Η
   series 1
             AcceptRatio = 0.125361 + /- 0.000308
Η
   series 2
             AcceptRatio = 0.328590 + /- 0.000340
             AcceptRatio = 0.535708 + /- 0.000313
   series 3
             AcceptRatio = 0.732537 + - 0.000234
Η
   series 4
Η
             AcceptRatio = 0.903498 + /- 0.000156
   series 5
Η
   series 6
             AcceptRatio = 0.961506 + /- 0.000083
             AcceptRatio = 0.985499 + /- 0.000051
Η
   series 7
```

```
series 8
             AcceptRatio = 0.996251 + /- 0.000025
Η
   series 9
             AcceptRatio = 0.998638 + /- 0.000014
              AcceptRatio = 0.999515 + /- 0.000009
Η
   series 10
Η
   series 11
              AcceptRatio = 0.999884 + /- 0.000004
Η
   series 12
              AcceptRatio = 0.999958 + /- 0.000003
              AcceptRatio = 0.999986 + /- 0.000002
Η
   series 13
   series 14
              AcceptRatio = 0.999995 + /- 0.000001
   series 15
              AcceptRatio = 0.999999 + /- 0.000000
```

By series 8 (time step = 0.02), the acceptance ratio is in excess of 99%.

Considering the increase in autocorrelation and subsequent increase in error bar as time step decreases, it is important to choose a time step that trades off appropriately between acceptance ratio and autocorrelation. In this example, a time step of 0.02 occupies a spot where acceptance ratio is high (99.6%), and autocorrelation is not appreciably larger than the minimum value (1.4 vs. 1.0).

#### 13.7.4 Considering variance

Besides autocorrelation, the dominant contributor to the error bar is the *variance* in the local energy. The variance measures the fluctuations around the average local energy, and, as the fluctuations go to zero, the wave function reaches an exact eigenstate of the Hamiltonian. qmca calculates this from the local energy and local energy squared columns of the scalar.dat.

Type qmca -q v H.s009.scalar.dat to calculate the variance on the run with time step balancing autocorrelation and acceptance ratio:

```
H series 9 Variance = 0.513570 +/- 0.010589
```

Just as the total energy doesn't tell us much by itself, neither does the variance. However, comparing the ratio of the variance to the energy indicates how the magnitude of the fluctuations compares to the energy itself. Type **qmca -q ev H.s009.scalar.dat** to calculate the energy and variance on the run side by side with the ratio:

```
LocalEnergy Variance ratio
H series 0 -0.454460 +/- 0.000568 0.529496 +/- 0.018445 1.1651
```

1.1651 is a very high ratio indicating the square of the fluctuations is on average larger than the value itself. In the next section, we will approach ways to improve the variance that subsequent labs will build upon.

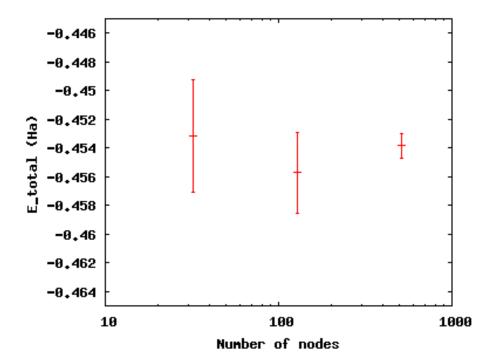
# 13.8 Reducing statistical error bars

#### 13.8.1 Increasing MC sampling

Increasing the number of MC samples in a data set reduces the error bar as the inverse of the square root of the number of samples. There are two ways to increase the number of MC samples in a simulation: running more samples in parallel and increasing the number of blocks (with fixed number of steps per block, this increases the total number of MC steps).

To see the effect of the running more samples in parallel, change to the directory **nodes**. The series here increases the number of nodes by factors of four from 32 to 128 to 512. Type **qmca -q** 

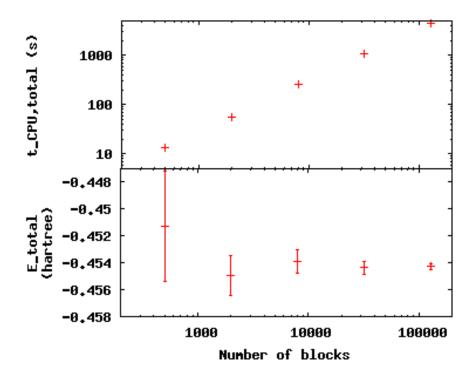
ev \*scalar.dat and note the change in the error bar on the local energy as the number of nodes. Visualize this with gnuplot H.plt:



Increasing the number of blocks, unlike running in parallel, increases the total CPU time of the simulation.

Press q [Enter] to quit gnuplot.

To see the effect of increasing the block number, change to the directory blocks. To see how we will vary the number of blocks, type **grep -A1 blocks H.xml**. The number of steps remains fixed, thus increasing the total number of samples. Visualize the tradeoff by inputting **gnuplot H.plt**:



Press q [Enter] to quit gnuplot.

#### 13.8.2 Improving the basis set

In all of the above examples, we are using the sum of two gaussian functions (STO-2G) to approximate what should be a simple decaying exponential (STO = Slater-type orbital) for the wave function of the ground state of the hydrogen atom. The sum of multiple copies of a function varying each copy's width and amplitude with coefficients is called a *basis set*. As we add gaussians to the basis set, the approximation improves, the variance goes toward zero and the energy goes to -0.5 hartrees. In nearly every other case, the exact function is unknown, and we add basis functions until the total energy does not change within some threshold.

Change to the directory basis and look at the total energy and variance as we change the wave function by typing  $\mathbf{qmca} - \mathbf{q} + \mathbf{ev} + \mathbf{H}_{-}$ :

		LocalEnergy	Variance	ratio
H_STO-2G	series 0	-0.454460 +/- 0.000568	0.529496 +/- 0.018445	1.1651
H_STO-3G	series 0	-0.465386 +/- 0.000502	0.410491 +/- 0.010051	0.8820
H_STO-6G	series 0	-0.471332 +/- 0.000491	0.213919 +/- 0.012954	0.4539
Hexact	series 0	-0.500000 +/- 0.000000	0.000000 +/- 0.000000	-0.0000

qmca also puts out the ratio of the variance to the local energy in a column to the right of the variance error bar. A typical high quality value for this ratio is lower than 0.1 or so—none of these few-gaussian wave functions satisfy that rule of thumb.

Use qmca to plot the trace of the local energy, kinetic energy, and potential energy of H\_exact—the total energy is constantly -0.5 hartree even though the kinetic and potential energies fluctuate from configuration to configuration.

#### 13.8.3 Adding a Jastrow factor

Another route to reducing the variance is the introduction of a Jastrow factor to account for electron-electron correlation (not the statistical autocorrelation of Monte Carlo steps but the physical avoidance that electrons have of one another). To do this, we will switch to the hydrogen dimer with the exact ground state wave function of the atom (STO basis)—this will not be exact for the dimer. The ground state energy of the hydrogen dimer is -1.174 hartrees.

Change directories to dimer and put in qmca -q ev \*scalar.dat to see the result of adding a simple, one-parameter Jastrow to the STO basis for the hydrogen dimer at experimental bond length:

```
LocalEnergy Variance

H2_STO___no_jastrow series 0 -0.876548 +/- 0.005313 0.473526 +/- 0.014910

H2_STO_with_jastrow series 0 -0.912763 +/- 0.004470 0.279651 +/- 0.016405
```

The energy reduces by 0.044 +/- 0.006 hartrees and the variance by 0.19 +/- 0.02. This is still 20% above the ground state energy, and subsequent labs will cover how to improve on this with improved forms of the wave function that capture more of the physics.

#### 13.9 Scaling to larger numbers of electrons

#### 13.9.1 Calculating the efficiency

The inverse of the product of CPU time and the variance measures the *efficiency* of an MC calculation. Use qmca to calculate efficiency by typing  $\mathbf{qmca}$  - $\mathbf{q}$  eff \*scalar.dat to see the efficiency of these two  $\mathbf{H}_2$  calculations:

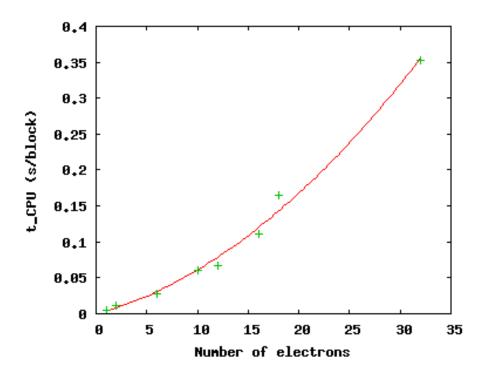
```
H2_STO___no_jastrow series 0 Efficiency = 16698.725453 +/- 0.000000 
H2_STO_with_jastrow series 0 Efficiency = 52912.365609 +/- 0.000000
```

The Jastrow factor increased the efficiency in these calculations by a factor of three, largely through the reduction in variance (check the average block CPU time to verify this claim).

#### 13.9.2 Scaling up

To see how MC scales with increasing particle number, change directories to size. Here are the data from runs of increasing number of electrons for H,  $H_2$ , C,  $CH_4$ ,  $C_2$ ,  $C_2H_4$ ,  $(CH_4)_2$ , and  $(C_2H_4)_2$  using the STO-6G basis set for the orbitals of the Slater determinant. The file names begin with the number of electrons simulated for those data.

Use **qmca** -**q** bc \*scalar.dat to see that the CPU time per block increases with number of electrons in the simulation, then plot the total CPU time of the simulation by **gnuplot Nelectron\_tCPU.plt**:



The green pluses represent the CPU time per block at each electron number. The red line is a quadratic fit to those data. For a fixed basis set size, we expect the time to scale quadratically up to 1000s of electrons, at which point a cubic scaling term may become dominant. Knowing the scaling allows you to roughly project the calculation time for a larger number of electrons.

Press q [Enter] to quit gnuplot.

This isn't the whole story, however. The variance of the energy also increases with a fixed basis set as the number of particles increases at a faster rate than the energy decreases. To see this, type qmca -q ev \*scalar.dat:

		LocalEnergy	Variance	
01H	series 0	-0.471352 +/- 0.000493	0.213020 +/- 0.012950	
02H2	series 0	-0.898875 +/- 0.000998	0.545717 +/- 0.009980	
06C	series 0	-37.608586 +/- 0.020453	184.322000 +/- 45.481193	
10CH4	series 0	-38.821513 +/- 0.022740	169.797871 +/- 24.765674	
12C2	series 0	-72.302390 +/- 0.037691	491.416711 +/- 106.090103	
16C2H4	series 0	-75.488701 +/- 0.042919	404.218115 +/- 60.196642	
18CH4CH4	series 0	-58.459857 +/- 0.039309	498.579645 +/- 92.480126	
32_C2H4C2H4	series 0	-91.567283 +/- 0.048392	632.114026 +/- 69.637760	

The increase in variance is not uniform, but the general trend is upward with a fixed wave function form and basis set. Subsequent labs will address how to improve the wave function in order to keep the variance manageable.

# Chapter 14

# Lab 2: QMC Basics

#### 14.1 Topics covered in this Lab

This lab focuses on the basics of performing quality QMC calculations. As an example participants test an oxygen pseudopotential within DMC by calculating atomic and dimer properties, a common step prior to production runs. Topics covered include:

- converting pseudopotentials into QMCPACK's FSATOM format
- generating orbitals with Quantum ESPRESSO
- converting orbitals into QMCPACK's ESHDF format with pw2qmcpack
- optimizing Jastrow factors with QMCPACK
- removing DMC timestep error via extrapolation
- automating QMC workflows with Nexus
- testing pseudopotentials for accuracy

#### 14.2 Lab outline

- 1. download and conversion of oxygen atom pseudopotential
- 2. DMC timestep study of the neutral oxygen atom
  - (a) DFT orbital generation with Quantum ESPRESSO
  - (b) orbital conversion with pw2qmcpack.x
  - (c) optimization of Jastrow correlation factor with QMCPACK
  - (d) DMC run with multiple timesteps
- 3. DMC timestep study of the first ionization potential of oxygen
  - (a) repetition of a-d above for ionized oxygen atom
- 4. automated DMC calculations of the oxygen dimer binding curve

#### 14.3 Lab directories and files

```
labs/lab2_qmc_basics/
 oxygen_atom
                       - oxygen atom calculations
    0.q0.dft.in
                         - Quantum ESPRESSO input for DFT run
                         - pw2qmcpack.x input for orbital conversion run
    0.q0.p2q.in
    O.qO.opt.in.xml
                         - QMCPACK input for Jastrow optimization run
    0.q0.dmc.in.xml
                         - QMCPACK input file for neutral O DMC
    ip_conv.py
                         - tool to fit oxygen IP vs timestep
    reference
                         - directory w/ completed runs
                       - oxygen dimer calculations
 oxygen_dimer
    dimer_fit.py
                         - tool to fit dimer binding curve
                         - automation script for dimer calculations
    O_dimer.py
    pseudopotentials
                         - directory for pseudopotentials
                         - directory w/ completed runs
    reference
                       - performing calculations for an arbitrary system (yours)
 your_system
     example.py
                          - example nexus file for periodic diamond
     pseudopotentials
                          - directory containing C pseudopotentials
     reference
                          - directory w/ completed runs
```

## 14.4 Obtaining and converting a pseudopotential for oxygen

First enter the oxygen\_atom directory:

```
cd labs/lab2_qmc_basics/oxygen_atom/
```

Throughout the rest of the lab, locations will be specified with respect to labs/lab2\_qmc\_basics (e.g. oxygen\_atom).

We will use a potential from the Burkatzki-Filippi-Dolg pseudopotential database. Although the full database is available in QMCPACK distribution (trunk/pseudopotentials/BFD/), we use a BFD pseudopotential to illustrate the process of converting and testing an external potential for use with QMCPACK. To obtain the pseudopotential, go to http://www.burkatzki.com/pseudos/index.2.html and click on the "Select Pseudopotential" button. Next click on oxygen in the periodic table. Click on the empty circle next to "V5Z" (a large gaussian basis set) and click on "Next". Select the Gamess format and click on "Retrive Potential". Helpful information about the pseudopotential will be displayed. The desired portion is at the bottom (the last 7 lines). Copy this text into the editor of your choice (e.g. emacs or vi) and save it as 0.BFD.gamess (be sure to include a newline at the end of the file). To transform the pseudopotential into the FSATOM XML format used by QMCPACK, use the ppconvert tool:

```
jobrun_vesta ppconvert --gamess_pot 0.BFD.gamess --s_ref "1s(2)2p(4)" \ --p_ref "1s(2)2p(4)" --d_ref "1s(2)2p(4)" --xml 0.BFD.xml
```

Observe the notation used to describe the reference valence configuration for this helium-core PP: 1s(2)2p(4). The ppconvert tool uses the following convention for the valence states: the first s state is labeled 1s(1s, 2s, 3s, ...), the first p state is labeled 2p(2p, 3p, ...), the first p state is labeled p (p, p, p), the first p state is labeled p0 (p0, p0, p0, p0). Copy the resulting xml file into the oxygen\_atom directory.

Note: the command to convert the PP into QM Espresso's UPF format is similar (both formats are required):

```
jobrun_vesta ppconvert --gamess_pot 0.BFD.gamess --s_ref "1s(2)2p(4)" \
--p_ref "1s(2)2p(4)" --d_ref "1s(2)2p(4)" --log_grid --upf 0.BFD.upf
```

For reference, the text of O.BFD.gamess should be:

```
O-QMC GEN 2 1

3

6.00000000 1 9.29793903

55.78763416 3 8.86492204

-38.81978498 2 8.62925665

1

38.41914135 2 8.71924452
```

The full QMCPACK pseudopotential is also included in oxygen\_atom/reference/0.BFD.\*.

# 14.5 DFT with Quantum ESPRESSO to obtain the orbital part of the wavefunction

With the pseudopotential in hand, the next step toward a QMC calculation is to obtain the Fermionic part of the wavefunction, in this case a single Slater determinant constructed from DFT-LDA orbitals for a neutral oxygen atom. If you had trouble with the pseudopotential conversion step, pre-converted pseudopotential files are located in the oxygen\_atom/reference directory.

Quantum ESPRESSO input for the DFT-LDA ground state of the neutral oxygen atom can be found in 0.q0.dft.in and also listing 14.1 below. Setting wf\_collect=.true. instructs Quantum Espresso to write the orbitals to disk at the end of the run. Option wf\_collect=.true. may be a potential problem in large simulations, it is recommended to avoid it and use the converter pw2qmcpack in parallel, see details in Sec. 14.13. Note that the plane-wave energy cutoff has been set to a reasonable value of 300 Ry here (ecutwfc=300). This value depends on the pseudopotentials used, and in general should be selected by running DFT $\rightarrow$ (orbital conversion) $\rightarrow$ VMC with increasing energy cutoffs until the lowest VMC total energy and variance is reached.

Listing 14.1: Quantum ESPRESSO input file for the neutral oxygen atom (0.q0.dft.in)

```
&CONTROL
                      = 'scf'
   calculation
                     = 'from_scratch'
   restart_mode
                     = '0.q0'
   prefix
   outdir
                      = './'
                      = './'
   pseudo_dir
                     = 'low'
   disk io
   wf_collect
                      = .true.
&SYSTEM
                      = 1.0
   celldm(1)
   ibrav
                      = 0
                      = 1
   nat
   ntvp
                      = 1
   nspin
                      = 2
```

```
= 0
  tot_charge
  tot_magnetization = 2
                 = 'lda'
  input_dft
                   = 300
  ecutwfc
  ecutrho
                   = 1200
  nosym
                   = .true.
  occupations
                   = 'smearing'
                   = 'fermi-dirac'
  smearing
  degauss
                   = 0.0001
&ELECTRONS
  diagonalization = 'david'
                   = 'plain'
  mixing_mode
  mixing_beta
                   = 0.7
  conv thr
                   = 1e-08
  electron_maxstep = 1000
ATOMIC_SPECIES
  0 15.999 O.BFD.upf
ATOMIC_POSITIONS alat
        9.44863067
                         9.44863161
                                         9.44863255
K_POINTS automatic
  1 1 1 0 0 0
CELL_PARAMETERS cubic
       18.89726133
                        0.00000000
                                         0.00000000
        0.00000000
                        18.89726133
                                         0.00000000
        0.00000000
                        0.00000000
                                        18.89726133
```

Run Quantum ESPRESSO by typing

```
jobrun_vesta pw.x 0.q0.dft.in
```

The DFT run should take a few minutes to complete. If desired, you can track the progress of the DFT run by typing "tail -f 0.q0.dft.output". Once finished, you should check the LDA total energy in 0.q0.dft.output by typing "grep '! ' 0.q0.dft.output". The result should be close to

```
! total energy = -31.57553905 Ry
```

The orbitals have been written in a format native to Quantum ESPRESSO in the O.qO.save directory. We will convert them into the ESHDF format expected by QMCPACK by using the pw2qmcpack.x tool. The input for pw2qmcpack.x can be found in the file O.qO.p2q.in and also in listing 14.2 below.

Listing 14.2: pw2qmcpack.x input file for orbital conversion (0.q0.p2q.in)

```
&inputpp
  prefix = '0.q0'
  outdir = './'
  write_psir = .false.
/
```

Perform the orbital conversion now by typing the following:

Upon completion of the run, a new file should be present containing the orbitals for QMC-PACK: 0.q0.pwscf.h5. Template XML files for particle (0.q0.ptcl.xml) and wavefunction (0.q0.wfs.xml) inputs to QMCPACK should also be present.

# 14.6 Optimization with QMCPACK to obtain the correlated part of the wavefunction

The wavefunction we have obtained to this point corresponds to a non-interacting Hamiltonian. Once the Coulomb pair potential is switched on between particles, it is known analytically that the exact wavefunction has cusps whenever two particles meet spatially and in general the electrons become correlated. This is represented in the wavefunction by introducing a Jastrow factor containing at least pair correlations

$$\Psi_{Slater-Jastrow} = e^{-J}\Psi_{Slater} \tag{14.1}$$

$$J = \sum_{\sigma\sigma'} \sum_{i < j} u_2^{\sigma\sigma'} (|r_i - r_j|) + \sum_{\sigma} \sum_{iI} u_1^{\sigma I} (|r_i - r_I|)$$

$$\tag{14.2}$$

Here  $\sigma$  is a spin variable while  $r_i$  and  $r_I$  represent electron and ion coordinates, respectively. The introduction of J into the wavefunction is similar to F12 methods in quantum chemistry, though it has been present in essentially all QMC studies since the first applications the method (circa 1965).

How are the functions  $u_2^{\sigma\sigma'}$  and  $u_1^{\sigma}$  obtained? Generally, they are approximated by analytical functions with several unknown parameters that are determined by minimizing the energy or variance directly within VMC. This is effective because the energy and variance reach a global minimum only for the true ground state wavefunction (Energy  $= E \equiv \langle \Psi | \hat{H} | \Psi \rangle$ , Variance  $= V \equiv \langle \Psi | (\hat{H} - E)^2 | \Psi \rangle$ ). For this exercise, we will focus on minimizing the variance.

First, we need to update the template particle and wavefunction information in O.qO.ptcl.xml and O.qO.wfs.xml. We want to simulate the O atom in open boundary conditions (the default is periodic). To do this open O.qO.ptcl.xml with your favorite text editor (e.g. emacs or vi) and replace

with

```
<parameter name="bconds">
    n n n
</parameter>
```

Next we will select Jastrow factors appropriate for an atom. In open boundary conditions, the B-spline Jastrow correlation functions should cut off to zero at some distance away from the atom. Open O.qO.wfs.xml and add the following cutoffs (rcut in Bohr radii) to the correlation factors:

```
...
<correlation speciesA="u" speciesB="u" size="8" rcut="10.0">
...
<correlation speciesA="u" speciesB="d" size="8" rcut="10.0">
```

```
...
<correlation elementType="0" size="8" rcut="5.0">
...
```

These terms correspond to  $u_2^{\uparrow\uparrow}/u_2^{\downarrow\downarrow}$ ,  $u_2^{\uparrow\downarrow}$ , and  $u_1^{\uparrow O}/u_1^{\downarrow O}$ , respectively. In each case, the correlation function  $(u_*)$  is represented by piecewise continuous cubic B-splines. Each correlation function has eight parameters which are just the values of u on a uniformly spaced grid up to rcut. Initially the parameters (coefficients) are set to zero:

```
<correlation speciesA="u" speciesB="u" size="8" rcut="10.0">
    <coefficients id="uu" type="Array">
        0.0 0.0 0.0 0.0 0.0 0.0 0.0
      </coefficients>
      </correlation>
```

Finally, we need to assemble particle, wavefunction, and pseudopotential information into the main QMCPACK input file (0.q0.opt.in.xml) and specify inputs for the Jastrow optimization process. Open 0.q0.opt.in.xml and write in the location of the particle, wavefunction, and pseudopotential files ("<!-- ... -->" are comments):

```
...
<!-- include simulationcell and particle information from pw2qmcpqack -->
<include href="0.q0.ptcl.xml"/>
...
<!-- include wavefunction information from pw2qmcpqack -->
<include href="0.q0.wfs.xml"/>
...
<!-- 0 pseudopotential read from "0.BFD.xml" -->
<pseudo elementType="0" href="0.BFD.xml"/>
...
```

The relevant portion of the input describing the linear optimization process is

```
<qmc method="linear" move="pbyp" checkpoint="-1">
                                       > ECOST
    <cost name="energy"</pre>
                                                     </cost>
    <cost name="unreweightedvariance"> UVCOST
                                                     </cost>
    <cost name="reweightedvariance" > RVCOST
                                                    </cost>
    <parameter name="timestep"</pre>
                                       > TS
                                                     </parameter>
    <parameter name="samples"</pre>
                                       > SAMPLES
                                                    </parameter>
    <parameter name="warmupSteps"</pre>
                                       > 50
                                                     </parameter>
    <parameter name="blocks"</pre>
                                       > 200
                                                     </parameter>
    <parameter name="subSteps"</pre>
                                                     </parameter>
                                       > 1
    <parameter name="nonlocalpp"</pre>
                                       > yes
                                                     </parameter>
    <parameter name="useBuffer"</pre>
                                       > yes
                                                     </parameter>
 </qmc>
</loop>
```

An explanation of each input variable can be found below. The remaining variables control specialized internal details of the linear optimization algorithm. The meaning of these inputs is beyond the scope of this lab and reasonable results are often obtained keeping these values fixed.

**energy** Fraction of trial energy in the cost function.

**unreweightedvariance** Fraction of unreweighted trial variance in the cost function. Neglecting the weights can be more robust.

reweighted variance Fraction of trial variance (including the full weights) in the cost function.

timestep Timestep of the VMC random walk, determines spatial distance moved by each electron during MC steps. Should be chosen such that the acceptance ratio of MC moves is around 50% (30-70% is often acceptable). Reasonable values are often between 0.2 and 0.6 Ha<sup>-1</sup>.

- samples Total number of MC samples collected for optimization, determines statistical error bar of cost function. Often efficient to start with a modest number of samples (50k) and then increase as needed. More samples may be required if the wavefunction contains a large number of variational parameters. MUST be a multiple of the number of threads/cores (use multiples of 512 on Vesta).
- warmupSteps Number of MC steps discarded as a warmup or equilibration period of the random walk. If this is too small, it will bias the optimization procedure.
- **blocks** Number of average energy values written to output files. Should be greater than 200 for meaningful statistical analysis of output data (e.g. via qmca).
- subSteps Number of MC steps in between energy evaluations. Each energy evaluation is expensive so taking a few steps to decorrelate between measurements can be more efficient. Will be less efficient with many substeps.
- nonlocalpp,useBuffer If nonlocalpp="no", then nonlocal part of the pseudopotential is not included when computing the cost function. If useBuffer="yes", then temporary data is stored to speed up nonlocal pseudopotential evaluation at the expense of memory consumption.
- **loop max** Number of times to repeat the optimization. Using the resulting wavefunction from the previous optimization in the next one improves the results. Typical choices range between 8 and 16.

The cost function defines the quantity to be minimized during optimization. The three components of the cost function, energy, unreweighted variance, and reweighted variance should sum to one. Dedicating 100% of the cost function to unreweighted variance is often a good choice. Another common choice is to try 90/10 or 80/20 mixtures of reweighted variance and energy. Using 100% energy minimization is desirable for reducing DMC pseudopotential localization errors, but the optimization process is less stable and should only be attempted after performing several cycles of e.g. variance minimization first (the entire loop section can be duplicated with a different cost function each time).

Replace MAX, EVCOST, UVCOST, RVCOST, TS, and SAMPLES in the loop with appropriate starting values in the O.qO.opt.in.xml input file. Perform the optimization run by typing

```
jobrun_vesta qmcpack 0.q0.opt.in.xml
```

The run should only take a few minutes for reasonable values of loop max and samples.

Log file output will appear in O.qO.opt.output. The beginning of each linear optimization will be marked with text similar to

```
Start QMCFixedSampleLinearOptimize
File Root 0.q0.opt.s011 append = no
```

At the end of each optimization section the change in cost function, new values for the Jastrow parameters, and elapsed wallclock time are reported:

```
OldCost: 7.0598901869e-01 NewCost: 7.0592576381e-01 Delta Cost:-6.3254886314e-05
  <optVariables href="0.q0.opt.s011.opt.xml">
uu_0 6.9392504232e-01 1 1
uu_1 4.9690781460e-01 1 1
uu_2 4.0934542375e-01 1 1
                          0N2
uu_3 3.7875640157e-01 1 1 ON 3
uu_4 3.7308380014e-01 1 1
                          ON 4
uu_5 3.5419786809e-01 1 1
                          ON 5
uu_6 4.3139019377e-01 1 1
                          ON 6
uu_7 1.9344371667e-01 1 1
                          ON 7
ud_0 3.9219009713e-01 1 1
ud_1 1.2352664647e-01 1 1
                          ON 9
ud_2 4.4048945133e-02 1 1 ON 10
ud_3 2.1415676741e-02 1 1 ON 11
ud_4 1.5201803731e-02 1 1
                          ON 12
ud_5 2.3708169445e-02 1 1
                          ON 13
ud_6 3.4279064930e-02 1 1
                          ON 14
ud_7 4.3334583596e-02 1 1
eO_0 -7.8490123937e-01 1 1 ON 16
eO_1 -6.6726618338e-01 1 1 ON 17
eO_2 -4.8753453838e-01 1 1 ON 18
eO_3 -3.0913993774e-01 1 1 ON 19
eO_4 -1.7901872177e-01 1 1 ON 20
eO_5 -8.6199000697e-02 1 1 ON 21
eO_6 -4.0601160841e-02 1 1 ON 22
eO_7 -4.1358075061e-03 1 1 ON 23
  </optVariables>
  QMC Execution time = 2.8218972974e+01 secs
```

The cost function should decrease during each linear optimization (Delta cost < 0). Try "grep OldCost \*opt.output". You should see something like this:

```
OldCost: 1.2655186572e+00 NewCost: 7.2443875597e-01 Delta Cost:-5.4107990118e-01 OldCost: 7.2229830632e-01 NewCost: 6.9833678217e-01 Delta Cost:-2.3961524143e-02 OldCost: 8.0649629434e-01 NewCost: 8.0551871147e-01 Delta Cost:-9.7758287036e-04 OldCost: 6.6821241388e-01 NewCost: 6.6797703487e-01 Delta Cost:-2.3537901148e-04 OldCost: 7.0106275099e-01 NewCost: 7.0078055426e-01 Delta Cost:-2.8219672877e-04 OldCost: 6.9538522411e-01 NewCost: 6.9419186712e-01 Delta Cost:-1.1933569922e-03 OldCost: 6.7709626744e-01 NewCost: 6.7501251165e-01 Delta Cost:-2.0837557922e-03 OldCost: 6.6659923822e-01 NewCost: 6.6651737755e-01 Delta Cost:-8.1860671682e-05 OldCost: 7.7828995609e-01 NewCost: 7.7735482525e-01 Delta Cost:-9.3513083900e-04 OldCost: 7.2717974404e-01 NewCost: 7.2715201115e-01 Delta Cost:-2.7732880747e-05 OldCost: 6.9400639873e-01 NewCost: 6.9257183689e-01 Delta Cost:-1.4345618444e-03 OldCost: 7.0598901869e-01 NewCost: 7.0592576381e-01 Delta Cost:-6.3254886314e-05
```

Blocked averages of energy data, including the kinetic energy and components of the potential

energy, are written to scalar.dat files. The first is named "O.qO.opt.sOOO.scalar.dat", with a series number of zero (sOOO). In the end there will be MAX of them, one for each series.

When the job has finished, use the qmca tool to assess the effectiveness of the optimization process. To look at just the total energy and the variance, type "qmca -q ev 0.q0.opt\*scalar\*". This will print the energy, variance, and the variance/energy ratio in Hartree units:

		LocalEnergy	Variance	ratio
0.q0.opt	series 0	-15.739585 +/- 0.007656	0.887412 +/- 0.010728	0.0564
0.q0.opt	series 1	-15.848347 +/- 0.004089	0.318490 +/- 0.006404	0.0201
0.q0.opt	series 2	-15.867494 +/- 0.004831	0.292309 +/- 0.007786	0.0184
0.q0.opt	series 3	-15.871508 +/- 0.003025	0.275364 +/- 0.006045	0.0173
0.q0.opt	series 4	-15.865512 +/- 0.002997	0.278056 +/- 0.006523	0.0175
0.q0.opt	series 5	-15.864967 +/- 0.002733	0.278065 +/- 0.004413	0.0175
0.q0.opt	series 6	-15.869644 +/- 0.002949	0.273497 +/- 0.006141	0.0172
0.q0.opt	series 7	-15.868397 +/- 0.003838	0.285451 +/- 0.007570	0.0180

Plots of the data can also be obtained with the "-p" option ("qmca -p -q ev 0.q0.opt\*scalar\*"). Identify which optimization series is the "best" according to your cost function. It is likely that multiple series are similar in quality. Note the opt.xml file corresponding to this series. This file contains the final value of the optimized Jastrow parameters to be used in the DMC calculations of the next section of the lab.

#### Questions and Exercises

- 1. What is the acceptance ratio of your optimization runs? (use "qmca -q ar 0.q0.opt\*scalar\*")
  Do you expect the Monte Carlo sampling to be efficient?
- 2. How do you know when the optimization process has converged?
- 3. (optional) Optimization is sometimes sensitive to initial guesses of the parameters. If you have time, try varying the initial parameters, including the cutoff radius (rcut) of the Jastrow factors (remember to change id in the cproject/> element). Do you arrive at a similar set of final Jastrow parameters? What is the lowest variance you are able to achieve?

# 14.7 DMC timestep extrapolation I: neutral O atom

The diffusion Monte Carlo (DMC) algorithm contains two biases in addition to the fixed node and pseudopotential approximations that are important to control: timestep and population control bias. In this section we will focus on estimating and removing timestep bias from DMC calculations. The essential fact to remember is that the bias vanishes as the timestep goes to zero while the needed computer time increases inversely with the timestep.

In the same directory you used to perform wavefunction optimization (oxygen\_atom) you will find a sample DMC input file for the neutral oxygen atom named O.qO.dmc.in.xml. Open this file in a text editor and note the differences from the optimization case. Wavefunction information is no longer included from pw2qmcpack, but instead should come from the optimization run:

```
<!-- OPT_XML is from optimization, e.g. 0.q0.opt.s008.opt.xml --> <include href="OPT_XML"/>
```

Replace "OPT\_XML" with the opt.xml file corresponding to the best Jastrow parameters you found in the last section (this is a file name similar to O.qO.opt.s008.opt.xml).

The QMC calculation section at the bottom is also different. The linear optimization blocks have been replaced with XML describing a VMC run followed by DMC. The input keywords are described below.

timestep Timestep of the VMC/DMC random walk. In VMC choose a timestep corresponding to an acceptance ratio of about 50%. In DMC the acceptance ratio is often above 99%.

warmupSteps Number of MC steps discarded as a warmup or equilibration period of the random walk.

**steps** Number of MC steps per block. Physical quantities, such as the total energy, are averaged over walkers and steps.

blocks Number of blocks. This is also the number of average energy values written to output files. Should be greater than 200 for meaningful statistical analysis of output data (e.g. via qmca). The total number of MC steps each walker takes is blocks×steps.

samples VMC only. This is the number of walkers used in subsequent DMC runs. Each DMC walker is initialized with electron positions sampled from the VMC random walk.

**nonlocalmoves** DMC only. If yes/no, use the locality approximation/T-moves for non-local pseudopotentials. T-moves generally improve the stability of the algorithm and restore the variational principle for small systems (T-moves version 1).

The purpose of the VMC run is to provide initial electron positions for each DMC walker. Setting walkers = 1 in the VMC block ensures there will be only one VMC walker per execution thread. There will be a total of 512 VMC walkers in this case (see 0.q0.dmc.qsub.in). We want the electron positions used to initialize the DMC walkers to be decorrelated from one another. A VMC walker will often decorrelate from its current position after propagating for a few Ha<sup>-1</sup> in imaginary time (in general this is system dependent). This leads to a rough rule of thumb for choosing blocks and steps for the VMC run (VWALKERS = 512 here):

$$VBLOCKS \times VSTEPS \ge \frac{DWALKERS}{VWALKERS} \frac{5 \text{ Ha}^{-1}}{VTIMESTEP}$$
 (14.3)

Fill in the VMC XML block with appropriate values for these parameters. There should be more than one DMC walker per thread and enough walkers in total to avoid population control bias. The general rule of thumb is to have more than  $\sim 2000$  walkers, although the dependence of the total energy on population size should be explicitly checked from time to time.

To study timestep bias, we will perform a sequence of DMC runs over a range of timesteps (0.1 Ha<sup>-1</sup> is too large and timesteps below 0.002 Ha<sup>-1</sup> are probably too small). A common approach is to select a fairly large timestep to begin with and then decrease the timestep by a factor of two in each subsequent DMC run. The total amount of imaginary time the walker population propagates should be the same for each run. A simple way to accomplish this is to choose input parameters in

the following way

```
\begin{aligned} &\texttt{timestep}_n = \texttt{timestep}_{n-1}/2\\ &\texttt{warmupSteps}_n = \texttt{warmupSteps}_{n-1} \times 2\\ &\texttt{blocks}_n = \texttt{blocks}_{n-1}\\ &\texttt{steps}_n = \texttt{steps}_{n-1} \times 2 \end{aligned} \tag{14.4}
```

Each DMC run will require about twice as much computer time as the one preceding it. Note that the number of blocks is kept fixed for uniform statistical analysis. blocks  $\times$  steps  $\times$  timestep  $\sim$  60 Ha<sup>-1</sup> is sufficient for this system.

Choose an initial DMC timestep and create a sequence of N timesteps according to 14.4. Make N copies of the DMC XML block in the input file

```
<qmc method="dmc" move="pbyp">
   <parameter name="warmupSteps"</pre>
                                                    DWARMUP
                                                                       </parameter>
   <parameter name="blocks"</pre>
                                                    DBLOCKS
                                                                       </parameter>
   <parameter name="steps"</pre>
                                               >
                                                    DSTEPS
                                                                       </parameter>
   <parameter name="timestep"</pre>
                                                    DTIMESTEP
                                                                       </parameter>
   <parameter name="nonlocalmoves"</pre>
                                                                       </parameter>
                                                    yes
</qmc>
```

Fill in DWARMUP, DBLOCKS, DSTEPS, and DTIMESTEP for each DMC run according to 14.4. Start the DMC timestep extrapolation run by typing:

```
jobrun_vesta qmcpack 0.q0.dmc.in.xml
```

The run should take only a few minutes to complete.

QMCPACK will create files prefixed with 0.q0.dmc. The log file is 0.q0.dmc.output. As before, block averaged data is written to scalar.dat files. In addition, DMC runs produce dmc.dat files which contain energy data averaged only over the walker population (one line per DMC step). The dmc.dat files also provide a record of the walker population at each step.

Use the PlotTstepConv.pl to obtain a linear fit to the timestep data (type "PlotTstepConv.pl O.qO.dmc.in.xml 40"). You should see a plot similar to fig. 14.1. The tail end of the text output displays the parameters for the linear fit. The "a" parameter is the total energy extrapolated to zero timestep in Hartree units.

#### Questions and Exercises

- 1. What is the  $\tau \to 0$  extrapolated value for the total energy?
- 2. What is the maximum timestep you should use if you want to calculate the total energy to an accuracy of 0.05 eV? For convenience,  $1~\mathrm{Ha} = 27.2113846$  eV.

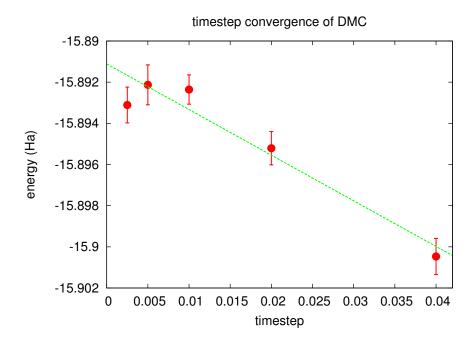


Figure 14.1: Linear fit to DMC timestep data from PlotTstepConv.pl.

- 3. What is the acceptance ratio for this (bias < 0.05 eV) run? Does it follow the rule of thumb for sensible DMC (acceptance ratio > 99%)?
- 4. Check the fluctuations in the walker population (qmca -t -q nw 0.q0.dmc\*dmc.dat --noac).

  Does the population seem to be stable?

# 14.8 DMC timestep extrapolation II: O atom ionization potential

In this section, we will repeat the calculations of the prior two sections (optimization, timestep extrapolation) for the +1 charge state of the oxygen atom. Comparing the resulting 1st ionization potential (IP) with experimental data will complete our first test of the BFD oxygen pseudopotential. In actual practice, higher IP's could also be tested prior to performing production runs.

Obtaining the timestep extrapolated DMC total energy for ionized oxygen should take much less (human) time than for the neutral case. For convenience, the necessary steps are briefly summarized below.

1. Obtain DFT orbitals with Quantum ESPRESSO

- (a) Copy the DFT input (0.q0.dft.in) to 0.q1.dft.in
- (b) Edit O.q1.dft.in to match the +1 charge state of the oxygen atom

prefix = '0.q1'
...
tot\_charge = 1
tot\_magnetization = 3
...

- (c) Perform the DFT run: jobrun\_vesta pw.x O.q1.dft.in
- 2. Convert the orbitals to ESHDF format
  - (a) Copy the pw2qmcpack input (0.q0.p2q.in) to 0.q1.p2q.in
  - (b) Edit O.q1.p2q.in to match the file prefix used in DFT

prefix = '0.q1'

- (c) Perform the orbital conversion run: jobrun\_vesta pw2qmcpack.x O.q1.p2q.in
- 3. Optimize the Jastrow factor with QMCPACK
  - (a) Copy the optimization input (0.q0.opt.in.xml) to 0.q1.opt.in.xml
  - (b) Edit O.q1.opt.in.xml to match the file prefix used in DFT

(c) Edit the particle XML file (O.q1.ptcl.xml) to have open boundary conditions

```
<parameter name="bconds">
    n n n
</parameter>
```

(d) Add cutoffs to the Jastrow factors in the wavefunction XML file (O.q1.wfs.xml)

```
<correlation elementType="0" size="8" rcut="5.0">
...
```

- (e) Perform the Jastrow optimization run: jobrun\_vesta qmcpack 0.q1.opt.in.xml
- (f) Identify the optimal set of parameters with qmca ([your opt.xml]).
- 4. DMC timestep study with QMCPACK
  - (a) Copy the DMC input (0.q0.dmc.in.xml) to 0.q1.dmc.in.xml
  - (b) Edit O.ql.dmc.in.xml to use the DFT prefix and the optimal Jastrow

- (c) Perform the DMC run: jobrun\_vesta qmcpack O.q1.dmc.in.xml
- (d) Obtain the DMC total energy extrapolated to zero timestep with PlotTstepConv.pl.

The process listed above, which excludes additional steps for orbital generation and conversion, can become tedious to perform by hand in production settings where many calculations are often required. For this reason automation tools are introduced for calculations involving the oxygen dimer in section 14.10 of the lab.

#### **Questions and Exercises**

- 1. What is the  $\tau \to 0$  extrapolated DMC value for the 1st ionization potential of oxygen?
- 2. How does the extrapolated value compare to the experimental IP? Go to http://physics.nist.gov/PhysRefData/ASD/ionEnergy.html and enter "O I" in the box labeled "Spectra" and click on the "Retrieve Data" button.
- 3. What can we conclude about the accuracy of the pseudopotential? What factors complicate this assessment?
- 4. Explore the sensitivity of the IP to the choice of timestep. Type "./ip\_conv.py" to view three timestep extrapolation plots: two for the q = 0, 1 total energies and one for the IP. Is the IP more, less, or similarly sensitive to timestep than the total energy?
- 5. What is the maximum timestep you should use if you want to calculate the ionization potential to an accuracy of 0.05 eV? What factor of cpu time is saved by assessing timestep convergence on the IP (a total energy difference) vs. a single total energy?
- 6. Are the acceptance ratio and population fluctuations reasonable for the q=1 calculations?

#### 14.9 DMC workflow automation with Nexus

Production QMC projects are often composed of many similar workflows. The simplest of these is a single DMC calculation involving four different compute jobs:

- 1. Orbital generation via Quantum ESPRESSO or GAMESS.
- 2. Conversion of orbital data via pw2qmcpack.x or convert4qmc.
- 3. Optimization of Jastrow factors via QMCPACK.
- 4. DMC calculation via QMCPACK.

Simulation workflows quickly become more complex with increasing costs in terms of human time for the researcher. Automation tools can decrease both human time and error if used well.

The set of automation tools we will be using is known as Nexus [?], which is distributed with QMCPACK. Nexus is capable of generating input files, submitting and monitoring compute jobs, passing data between simulations (such as relaxed structures, orbital files, optimized Jastrow parameters, etc.), and data analysis. The user interface to Nexus is through a set of functions defined in the Python programming language. User scripts that execute simple workflows resemble input files and do not require programming experience. More complex workflows require only basic programming constructs (e.g. for loops and if statements). Nexus input files/scripts should be easier to navigate than QMCPACK input files and more efficient than submitting all the jobs by hand.

Nexus is driven by simple user-defined scripts that resemble keyword-driven input files. An example Nexus input file that performs a single VMC calculation (with pre-generated orbitals) is shown below. Take a moment to read it over and especially note the comments (prefixed with "#") explaining most of the contents. If the input syntax is unclear you may want to consult portions of appendix 14.12, which gives a condensed summary of Python constructs. An additional example and details about the inner workings of Nexus can be found in the reference publication [?].

```
#! /usr/bin/env python
# import Nexus functions
from nexus import settings,job,get_machine,run_project
from nexus import generate_physical_system
from nexus import generate_qmcpack,vmc
settings(
                                      # Nexus settings
                 = './pseudopotentials', # location of PP files
   pseudo_dir
                                         # root directory for simulations
   runs
                 = '',
   results
                                         # root directory for simulation results
   status_only = 0,
                                         # show simulation status, then exit
   generate_only = 0,
                                         # generate input files, then exit
   sleep
                 = 3.
                                         # seconds between checks on sim. progress
   machine
                 = 'vesta',
                                         # name of local machine
                 = 'QMCPACK-Training'  # charge account for cpu time
   account
vesta = get_machine('vesta')
                                      # allow max of one job at a time (lab only)
vesta.queue_size = 1
qmcjob = job(
                                      # specify job parameters
   nodes = 32,
                                         # use 32 Vesta nodes
   threads = 16.
                                         # 16 OpenMP threads per node (32 MPI tasks)
                                         # wallclock limit of 1 hour
   hours
                                          # use QMCPACK executable
            = '/soft/applications/qmcpack/Binaries/qmcpack
   app
```

```
qmc_calcs = [
                                   # list QMC calculation methods
                                     # VMC
   vmc(
               = 1,
       walkers
                                      #
                                           1 walker
       warmupsteps = 50,
                                           50 MC steps for warmup
               = 200,
       blocks
                                     # 200 blocks
                                          10 steps per block
                  = 10,
                                      #
       steps
       timestep
                  = .4
                                      # 0.4 1/Ha timestep
dimer = generate_physical_system(
                                # make a dimer system
           = 'dimer',
                                    # system type is dimer
   type
             = ('0','0'),
                                      # dimer is two oxygen atoms
   separation = 1.2074,
                                      # separated by 1.2074 Angstrom
   Lbox
           = 15.0,
                                      # simulation box is 15 Angstrom
             = 'A',
   units
                                      # Angstrom is dist. unit
   net_spin = 2,
                                      # nup-ndown is 2
   0
              = 6
                                       # pseudo-oxygen has 6 valence el.
   )
qmc = generate_qmcpack(
                                  # make a qmcpack simulation
                                  # prefix files with 'example'
# run in ./scale_1.0 directory
   identifier = 'example',
               = 'scale_1.0',
   path
               = dimer.
                                     # run the dimer system
   svstem
               = qmcjob,
                                     # set job parameters
   job
   input_type = 'basic',
                                      # basic qmcpack inputs given below
               = ['0.BFD.xml'],
   pseudos
                                      # list of PP's to use
   orbitals_h5 = '02.pwscf.h5',
                                      # file with orbitals from DFT
   bconds = 'nnn',
                                     # open boundary conditions
              = [],
   iastrows
                                      # no jastrow factors
                                      # QMC calculations to perform
   calculations = qmc_calcs
run_project(qmc)
                                    # write input file and submit job
```

## 14.10 Automated binding curve of the oxygen dimer

In this section we will use Nexus to calculate the DMC total energy of the oxygen dimer over a series of bond lengths. The equilibrium bond length and binding energy of the dimer will be determined by performing a polynomial fit to the data (Morse potential fits should be preferred in production tests). Comparing these values with corresponding experimental data provides a second test of the BFD pseudopotential for oxygen.

Enter the oxygen\_dimer directory. Copy your BFD pseudopotential from the atom runs into oxygen\_dimer/pseudopotentials (be sure to move both files: .upf and .xml). Open O\_dimer.py with a text editor. The overall format is similar to the example file shown in the last section. The main difference is that a full workflow of runs (DFT orbital generation, orbital conversion, optimization and DMC) are being performed rather than a single VMC run.

As in the example in the last section, the oxygen dimer is generated with the generate\_physical\_system function:

Similar syntax can be used to generate crystal structures or to specify systems with arbitrary atomic configurations and simulation cells. Notice that a "scale" variable has been introduced to stretch or compress the dimer.

Next, objects representing a Quantum ESPRESSO (PWSCF) run and subsequent orbital conversion step are constructed with respective generate\_\* functions:

```
dft = generate_pwscf(
   identifier = 'dft',
   ...
   input_dft = 'lda',
   ...
  )
sims.append(dft)

# describe orbital conversion run
p2q = generate_pw2qmcpack(
   identifier = 'p2q',
   ...
   dependencies = (dft,'orbitals'),
   )
sims.append(p2q)
```

Note the dependencies keyword. This keyword is used to construct workflows out of otherwise separate runs. In this case, the dependency indicates that the orbital conversion run must wait for the DFT to finish prior to starting.

Objects representing QMCPACK simulations are then constructed with the generate\_qmcpack function:

```
opt = generate_qmcpack(
   identifier = 'opt',
                = [('J1','bspline',8,5.0),
   jastrows
                   ('J2', 'bspline', 8, 10.0)],
   calculations = [
       loop(max=12,
            qmc=linear(
                                    = 0.0.
               energy
               unreweightedvariance = 1.0,
               reweightedvariance = 0.0,
               timestep
                                   = 0.3,
               samples
                                   = 61440.
                                   = 50.
               warmupsteps
               blocks
                                    = 200,
               substeps
                                    = 1.
               nonlocalpp
                                    = True,
                                    = True,
               usebuffer
               walkers
                                   = 1.
               minwalkers
                                   = 0.5,
               maxweight
                                   = 1e9,
               usedrift
                                    = False,
               minmethod
                                   = 'quartic',
                                    = 0.025,
               beta
               exp0
                                   = -16,
               bigchange
                                    = 15.0.
               alloweddifference
                                   = 1e-4,
               stepsize
                                   = 0.2,
                                   = 1.0,
               stabilizerscale
               nstabilizers
                                   = 3,
       ],
   dependencies = (p2q,'orbitals'),
sims.append(opt)
```

```
qmc = generate_qmcpack(
   identifier = 'qmc',
   iastrows
   calculations = [
        vmc(
            walkers
                             1,
            warmupsteps =
                            30.
            blocks
                            20,
            steps
                            10.
            substeps
                            2.
            timestep
            samples
                         = 2048
                          = 100.
            warmupsteps
                           = 400,
            blocks
                           = 32.
            steps
                           = 0.01.
            timestep
            nonlocalmoves = True,
   dependencies = [(p2q, 'orbitals'), (opt, 'jastrow')],
sims.append(qmc)
```

Shared details such as the run directory, job, pseudopotentials, and orbital file have been omitted (...). The "opt" run will optimize a 1-body B-spline Jastrow with 8 knots having a cutoff of 5.0 Bohr and a B-spline Jastrow (for up-up and up-down correlations) with 8 knots and cutoffs of 10.0 Bohr. The Jastrow list for the DMC run is empty and the usage of dependencies above indicates that the DMC run depends on the optimization run for the Jastrow factor. Nexus will submit the "opt" run first and upon completion it will scan the output, select the optimal set of parameters, pass the Jastrow information to the "qmc" run and then submit the DMC job. Independent job workflows are submitted in parallel when permitted (we have explicitly limited this for this lab by setting queue\_size=2 for Vesta). No input files are written or job submissions made until the "run\_project" function is reached:

```
run_project(sims)
```

All of the simulations objects have been collected into a list (sims) for submission.

As written, O\_dimer.py will only perform calculations at the equilibrium separation distance of 1.2074 Angstrom, since the list of scaling factors (representing stretching or compressing the dimer) only contains one value (scales = [1.00]). Modify the file now to perform DMC calculations across a range of separation distances with each DMC run using the Jastrow factor optimized at the equilibrium separation distance. Specifically, you will want to change the list of scaling factors to include both compression (scale<1.0) and stretch (scale>1.0):

```
scales = [1.00,0.90,0.95,1.05,1.10]
```

Note that "1.00" is left in front because we are going to optimize the Jastrow factor first at the equilibrium separation and reuse this Jastrow factor for all other separation distances. This procedure is used because it can reduce variations in localization errors (due to pseudopotentials in DMC) along the binding curve.

Change the "status\_only" parameter in the "settings" function to 1 and type "./O\_dimer.py" at the command line. This will print the status of all simulations:

```
Project starting
  checking for file collisions
  loading cascade images
    cascade 0 checking in
    cascade 10 checking in
    cascade 4 checking in
    cascade 13 checking in
    cascade 7 checking in
  checking cascade dependencies
    all simulation dependencies satisfied
  cascade status
    setup, sent_files, submitted, finished, got_output, analyzed
    000000 dft
                    ./scale_1.0
    000000 p2q
                    ./scale_1.0
    000000 opt
                    ./scale_1.0
                    ./scale_1.0
    000000 qmc
    000000 dft
                   ./scale_0.9
                    ./scale_0.9
    000000 p2q
    000000 qmc
                   ./scale_0.9
                   ./scale_0.95
    000000 dft
    000000 p2q
                    ./scale_0.95
    000000 qmc
                    ./scale_0.95
    000000 dft
                    ./scale_1.05
    000000 p2q
                   ./scale_1.05
    000000 qmc
                    ./scale_1.05
    000000 dft
                    ./scale_1.1
    000000 p2q
                    ./scale_1.1
    000000 qmc
                    ./scale_1.1
    setup, sent_files, submitted, finished, got_output, analyzed
```

In this case, five simulation "cascades" (workflows) have been identified, each one starting and ending with "dft" and "qmc" runs, respectively. The six status flags (setup, sent\_files, submitted, finished, got\_output, analyzed) each show 0, indicating that no work has been done yet.

Now change "status\_only" back to 0, set "generate\_only" to 1, and run O\_dimer.py again. This will perform a dry-run of all simulations. The dry-run should finish in about 20 seconds:

```
Project starting
checking for file collisions
loading cascade images
cascade 0 checking in
cascade 10 checking in
cascade 4 checking in
cascade 4 checking in
cascade 7 checking in
checking cascade dependencies
all simulation dependencies satisfied
```

```
starting runs:
poll 0 memory 91.03 MB
 Entering ./scale_1.0 0
    writing input files 0 dft
 Entering ./scale_1.0 0
    sending required files 0 dft
    submitting job 0 dft
poll 1 memory 91.10 MB
 Entering ./scale_1.0 1
    Would have executed: qsub --mode script --env BG_SHAREDMEMSIZE=32 dft.qsub.in
poll 2 memory 91.10 MB
 Entering ./scale_1.0 0
    copying results 0 dft
  Entering ./scale_1.0 0
    analyzing 0 dft
poll 3 memory 91.10 MB
 Entering ./scale_1.0 1
   writing input files 1 p2q
  Entering ./scale_1.0 1
    sending required files 1 p2q
    submitting job 1 p2q
  Entering ./scale_1.0 2
    Would have executed: qsub --mode script --env BG_SHAREDMEMSIZE=32 p2q.qsub.in
poll 4 memory 91.10 MB
 Entering ./scale_1.0 1
    copying results 1 p2q
 Entering ./scale_1.0 1
    analyzing 1 p2q
poll 5 memory 91.10 MB
 Entering ./scale_1.0 2
    writing input files 2 opt
 Entering ./scale_1.0 2
    sending required files 2 opt
   submitting job 2 opt
  Entering ./scale_1.0 3
    Would have executed: qsub --mode script --env BG_SHAREDMEMSIZE=32 opt.qsub.in
```

```
poll 6 memory 91.16 MB
   Entering ./scale_1.0 2
   copying results 2 opt
   Entering ./scale_1.0 2
   analyzing 2 opt
...

poll 7 memory 93.00 MB
   Entering ./scale_1.0 3
   writing input files 3 qmc
   Entering ./scale_1.0 3
   sending required files 3 qmc
   submitting job 3 qmc
...
   Entering ./scale_1.0 4
   Would have executed: qsub --mode script --env BG_SHAREDMEMSIZE=32 qmc.qsub.in
...

poll 17 memory 93.00 MB
Project finished
```

Nexus polls the simulation status every 3 seconds and sleeps in between. The "scale\_\*" directories should now contain several files:

```
scale_1.0
 dft.in
dft.qsub.in
O.BFD.upf
 O.BFD.xml
 opt.in.xml
opt.qsub.in
p2q.in
p2q.qsub.in
pwscf_output
 qmc.in.xml
 qmc.qsub.in
sim_dft
 analyzer.p
 input.p
 sim.p
 sim_opt
  analyzer.p
 input.p
  sim.p
 sim_p2q
  analyzer.p
  input.p
  sim.p
 sim_qmc
```

```
analyzer.p
input.p
sim.p
```

Take a minute to inspect the generated input (dft.in, p2q.in, opt.in.xml, qmc.in.xml) and submission (dft.qsub.in, p2q.qsub.in, opt.qsub.in, qmc.qsub.in) files. The pseudopotential files (0.BFD.upf and 0.BFD.xml) have been copied into each local directory. Four additional directories have been created: sim\_dft, sim\_p2q, sim\_opt and sim\_qmc. The sim.p files in each directory contain the current status of each simulation. If you run O\_dimer.py again, it should not attempt to rerun any of the simulations:

```
Project starting
    checking for file collisions
loading cascade images
    cascade 0 checking in
    cascade 10 checking in
    cascade 4 checking in
    cascade 13 checking in
    cascade 7 checking in
    checking cascade dependencies
    all simulation dependencies satisfied

starting runs:

poll 0 memory 64.25 MB

Project finished
```

This way one can continue to add to the  $O\_dimer.py$  file (e.g. adding more separation distances) without worrying about duplicate job submissions.

Let's actually submit the jobs in the dimer workflow now. Reset the state of the simulations by removing the sim.p files ("rm ./scale\*/sim\*/sim.p"), set "generate\_only" to 0, and rerun O\_dimer.py. It should take about 20 minutes for all the jobs to complete. You may wish to open another terminal to monitor the progress of the individual jobs while the current terminal runs O\_dimer.py in the foreground. You can begin the first exercise below once the optimization job completes.

#### Questions and Exercises

1. Evaluate the quality of the optimization at scale=1.0 using the qmca tool. Did the optimization succeed? How does the variance compare with the neutral oxygen atom? Is the wavefunction of similar quality to the atomic case?

- 2. Evaluate the traces of the local energy and the DMC walker population for each separation distance with the qmca tool. Are there any anomalies in the runs? Is the acceptance ratio reasonable? Is the wavefunction of similar quality across all separation distances?
- 3. Use the dimer\_fit.py tool located in oxygen\_dimer to fit the oxygen dimer binding curve. To get the binding energy of the dimer, we will need the DMC energy of the atom. Before performing the fit, answer: What DMC timestep should be used for the oxygen atom results? The tool accepts three arguments ("./dimer\_fit.py P N E Eerr"), P is the prefix of the DMC input files (should be "qmc" at this point), N is the order of the fit (use 2 to start), E and Eerr are your DMC total energy and error bar, respectively for the oxygen atom (in eV). A plot of the dimer data will be displayed and text output will show the DMC equilibrium bond length and binding energy as well as experimental values. How accurately does your fit to the DMC data reproduce the experimental values? What factors affect the accuracy of your results?
- 4. Refit your data with a fourth-order polynomial. How do your predictions change with a fourth-order fit? Is a fourth-order fit appropriate for the available data?
- 5. Add new "scale" values to the list in O\_dimer.py that interpolate between the original set (e.g. expand to [1.00,0.90,0.925,0.95,0.975,1.025,1.05,1.075,1.10]). Perform the DMC calculations and redo the fits. How accurately does your fit to the DMC data reproduce the experimental values? Should this pseudopotential be used in production calculations?
- 6. (Optional) Perform optimization runs at the extremal separation distances corresponding to scale=[0.90,1.10]. Are the individually optimized wavefunctions of significantly better quality than the one imported from scale=1.00? Why? What form of Jastrow factor might give an even better improvement?

# 14.11 (Optional) Running your system with QMCPACK

This section covers a fairly simple route to get started on QMC calculations of an arbitrary system of interest using the Nexus workflow management system to setup input files and optionally perform the runs. The example provided in this section uses QM Espresso (PWSCF) to generate the orbitals forming the Slater determinant part of the trial wavefunction. PWSCF is a natural choice for solid state systems and it can be used for surface/slab and molecular systems as well, albeit at the price of describing additional vacuum space with plane waves.

To start out with, you will need pseudopotentials (PP's) for each element in your system in both the UPF (PWSCF) and FSATOM/XML (QMCPACK) formats. A good place to start is the Burkatzki-Filippi-Dolg (BFD) pseudopotential database

(http://www.burkatzki.com/pseudos/index.2.html), which we have already used in our study of the oxygen atom. The database does not contain PP's for the 4th and 5th row transition metals or any of the lanthanides or actinides. If you need a PP that is not in the BFD database, you may need to generate and test one manually (e.g. with OPIUM, http://opium.sourceforge.net/). Otherwise, use ppconvert as outlined in section 14.4 to obtain PP's in the formats used by PWSCF and QMCPACK. Enter the your\_system lab directory and place the converted PP's in your\_system/pseudopotentials.

Before performing production calculations (more than just the initial setup in this section) be sure to converge the plane wave energy cutoff in PWSCF as these PP's can be rather hard, sometimes requiring cutoffs in excess of 300 Ry. Depending on the system under study, the amount

of memory required to represent the orbitals (QMCPACK uses 3D B-splines) becomes prohibitive and one may be forced to search for softer PP's.

Beyond pseudopotentials, all that is required to get started are the atomic positions and the dimensions/shape of the simulation cell. The Nexus file example.py illustrates how to setup PWSCF and QMCPACK input files by providing minimal information regarding the physical system (an 8-atom cubic cell of diamond in the example). Most of the contents should be familiar from your experience with the automated calculations of the oxygen dimer binding curve in section 14.10 (if you've skipped ahead you may want to skim that section for relevant information). The most important change is the expanded description of the physical system:

```
# details of your physical system (diamond conventional cell below)
my_project_name = 'diamond_vmc' # directory to perform runs
             = ['C.BFD.upf']
                                 # pwscf pseudopotentials
my_dft_pps
                                # qmcpack pseudopotentials
my_qmc_pps
               = ['C.BFD.xml']
  generate your system
             : 'A'/'B' for Angstrom/Bohr
    units
               : simulation cell axes in cartesian coordinates (a1,a2,a3)
    elem
               : list of atoms in the system
               corresponding atomic positions in cartesian coordinatesMonkhorst-Pack grid
    pos
    kgrid
   kshift : Monkhorst-Pack shift (between 0 and 0.5)
   net_charge : system charge in units of e
   net_spin : # of up spins - # of down spins
    C = 4
               : (pseudo) carbon has 4 valence electrons
my_system = generate_physical_system(
              = 'A',
   units
   axes
              = [[ 3.57000000e+00, 0.00000000e+00, 0.00000000e+00],
                 [ 0.00000000e+00, 3.57000000e+00, 0.00000000e+00],
                  [ 0.00000000e+00, 0.00000000e+00, 3.57000000e+00]],
              = ['C','C','C','C','C','C','C','C','C'],
   elem
              = [[ 0.0000000e+00, 0.0000000e+00, 0.00000000e+00],
   pos
                  [8.92500000e-01, 8.92500000e-01, 8.92500000e-01],
                  [ 0.00000000e+00, 1.78500000e+00, 1.78500000e+00],
                  [8.92500000e-01, 2.67750000e+00, 2.67750000e+00],
                  [ 1.78500000e+00, 0.00000000e+00, 1.78500000e+00],
                 [ 2.67750000e+00, 8.92500000e-01, 2.67750000e+00],
                  [ 1.78500000e+00, 1.78500000e+00, 0.00000000e+00],
                 [ 2.67750000e+00, 2.67750000e+00, 8.92500000e-01]],
   kgrid
              = (1,1,1),
   kshift
              = (0,0,0),
   net_charge = 0,
   net_spin = 0,
   С
                         # one line like this for each atomic species
   )
my_bconds
               = 'ppp' # ppp/nnn for periodic/open BC's in QMC
                         # if nnn, center atoms about (a1+a2+a3)/2
```

If you have a system you would like to try with QMC, make a copy of example.py and fill in the relevant information about the pseudopotentials, simulation cell axes, and atomic species/positions. Otherwise, you can proceed with example.py as it is.

Set "generate\_only" to 1 and type "./example.py" or similar to generate the input files. All files will be written to "./diamond\_vmc" ("./[my\_project\_name]" if you have changed "my\_project\_name" in the file). The input files for PWSCF, pw2qmcpack, and QMCPACK are scf.in, pw2qmcpack.in, and vmc.in.xml, repectively. Take some time to inspect the generated input files. If you have questions about the file contents, or run into issues with the generation process, feel free to consult with a lab instructor.

If desired, you can submit the runs directly with example.py. To do this, first reset the Nexus simulation record by typing "rm ./diamond\_vmc/sim\*/sim.p" or similar and set "generate\_only"

back to 0. Next rerun example.py (you may want to redirect the text output). Alternatively the runs can be submitted by hand:

```
qsub --mode script --env BG_SHAREDMEMSIZE=32 scf.qsub.in

(wait until JOB DONE appears in scf.output)

qsub --mode script --env BG_SHAREDMEMSIZE=32 p2q.qsub.in
```

Once the conversion process has finished the orbitals should be located in the file diamond\_vmc/pwscf\_output/pwscf.pwscf.h5. Open diamond\_vmc/vmc.in.xml and replace "MISSING.h5" with "./pwscf\_output/pwscf.pwscf.h5". Next submit the VMC run:

```
qsub --mode script --env BG_SHAREDMEMSIZE=32 vmc.qsub.in
```

Note: If your system is large, the above process may not complete within the time frame of this lab. Working with a stripped down (but relevant) example is a good idea for exploratory runs.

Once the runs have finished, you may want to begin exploring Jastrow optimization and DMC for your system. Example calculations are provided at the end of example.py in the commented out text.

### 14.12 Appendix A: Basic Python constructs

Basic Python data types (int, float, str, tuple, list, array, dict, obj) and programming constructs (if statements, for loops, functions w/ keyword arguments) are briefly overviewed below. All examples can be executed interactively in Python. To do this, type "python" at the command line and paste any of the shaded text below at the ">>>" prompt. For more information about effective use of Python, consult the detailed online documentation: https://docs.python.org/2/.

Intrinsic types: int, float, str

```
#this is a comment
i=5
                         # integer
f = 3.6
                         # float
s='quantum/monte/carlo' # string
                         # represents "nothing"
n=None
f+=1.4
                         # add-assign (-,*,/ also): 5.0
2**3
                         # raise to a power: 8
                         # int to string: '5'
str(i)
s+'/simulations'
                         # joining strings: 'quantum/monte/carlo/simulations'
'i={0}'.format(i)
                         # format string: 'i=5'
```

Container types: tuple, list, array, dict, obj

```
from numpy import array # get array from numpy module
from generic import obj # get obj from Nexus' generic module
t=('A',42,56,123.0)
                      # tuple
l=['B',3.14,196]
                    # list
a=array([1,2,3])
                      # array
d={'a':5,'b':6}
                      # dict
o=obj(a=5,b=6)
                       # obj
                       # printing
                       # ('A', 42, 56, 123.0)
print t
                       # ['B', 3.140000000000001, 196]
print 1
                       # [1 2 3]
print a
                       # {'a': 5, 'b': 6}
print d
                                            = 5
print o
                            a
                                            = 6
len(t),len(1),len(a),len(d),len(o) #number of elements: (4, 3, 3, 2, 2)
t[0],1[0],a[0],d['a'],o.a #element access: ('A', 'B', 1, 5, 5)
s = array([0,1,2,3,4]) # slices: works for tuple, list, array
s[:]
                       # array([0, 1, 2, 3, 4])
s[2:]
                       # array([2, 3, 4])
s[:2]
                       # array([0, 1])
s[1:4]
                       # array([1, 2, 3])
                           array([0, 2, 4])
s[0:5:2]
                       # list operations
12 = list(1)
                       # make independent copy
1.append(4)
                       # add new element: ['B', 3.14, 196, 4]
                       # addition: ['B', 3.14, 196, 4, 5, 6, 7]
1+[5,6,7]
3*[0,1]
                           multiplication: [0, 1, 0, 1, 0, 1]
b=array([5,6,7])
                       # array operations
a2 = a.copy()
                           make independent copy
a+b
                       # addition: array([6, 8, 10])
                       # addition: array([ 4, 5, 6])
a+3
                           multiplication: array([ 5, 12, 21])
a*b
                           multiplication: array([3, 6, 9])
3*a
                       # dict/obj operations
d2 = d.copy()
                           make independent copy
```

```
d['c'] = 7  # add/assign element
d.keys()  # get element names: ['a', 'c', 'b']
d.values()  # get element values: [5, 7, 6]

# obj-specific operations
o.c = 7  # add/assign element
o.set(c=7,d=8)  # add/assign multiple elements
```

An important feature of Python to be aware of is that assignment is most often by reference, *i.e.* new values are not always created. This point is illustrated below with an obj instance, but it also holds for list, array, dict, and others.

```
>>> o = obj(a=5,b=6)
>>>
>>> p=o
>>>
>>> p.a=7
>>>
>>> print o
                   = 7
  a
                   = 6
  b
>>> q=o.copy()
>>>
>>> q.a=9
>>>
>>> print o
                   = 7
  a
  b
                   = 6
```

Here p is just another name for o, while q is a fully independent copy of it.

#### Conditional Statements: if/elif/else

```
a = 5
if a is None:
    print 'a is None'
elif a==4:
    print 'a is 4'
elif a<=6 and a>2:
    print 'a is in the range (2,6]'
elif a<-1 or a>26:
    print 'a is not in the range [-1,26]'
elif a!=10:
    print 'a is not 10'
else:
    print 'a is 10'
```

#### #end if

The "#end if" is not part of Python syntax, but you will see text like this throughout Nexus for clear encapsulation.

#### Iteration: for

```
from generic import obj
1 = [1,2,3]
m = [4,5,6]
s = 0
for i in range(len(1)): # loop over list indices
   s += l[i] + m[i]
#end for
                         # s is 21
print s
s = 0
for v in 1:
                        # loop over list elements
   s += v
#end for
print s
                         # s is 6
o = obj(a=5,b=6)
s = 0
for v in o:
                        # loop over obj elements
   s += v
#end for
print s
                         # s is 11
d = \{'a':5,'b':4\}
for n,v in o.iteritems():# loop over name/value pairs in obj
    d[n] += v
#end for
                         # d is {'a': 10, 'b': 10}
print d
```

#### Functions: def, argument syntax

```
f(1,b=2)
                         # prints: 1 2 5
def f(*args,**kwargs):
                         # general function, returns nothing
    print args
                                args: tuple of positional arguments
                         #
                             kwargs: dict of keyword arguments
    print kwargs
#end def f
f('s',(1,2),a=3,b='t')
                         # 2 pos., 2 kw. args, prints:
                            ('s', (1, 2))
                             {'a': 3, 'b': 't'}
1 = [0,1,2]
f(*1,a=6)
                          # pos. args from list, 1 kw. arg, prints:
                              (0, 1, 2)
                             {'a': 6}
o = obj(a=5,b=6)
f(*1,**o)
                         # pos./kw. args from list/obj, prints:
                              (0, 1, 2)
                              {'a': 5, 'b': 6}
f(
                         # indented kw. args, prints
    blocks
             = 200,
                              {'steps': 10, 'blocks': 200, 'timestep': 0.01}
    steps
             = 10,
    timestep = 0.01
    )
o = obj(
                         # obj w/ indented kw. args
    blocks
             = 100,
    steps
             = 5,
    timestep = 0.02
f(**o)
                         # kw. args from obj, prints:
                              ()
                              {'timestep': 0.02, 'blocks': 100, 'steps': 5}
```

# 14.13 Appendix B: pw2qmcpack in parallel

pw2qmcpack is a converter which extracts the orbitals calculated by DFT with Quantum ESPRESSO and packs them into a HDF5 file for QMCPACK. It used to be used in serial without problems in small systems and becomes a severe bottleneck for a large system. Due to the large energy cutoff in the calculation with pw.x required by the pseudopotentials and the increasing sizes of systems we are interested in, one limitation of QE can be easily reached that wf\_collect=.true. results in problems of writing and loading correct plane wave coefficients on disks by pw.x because of the 32bit integer issue. Thus, pw2qmcpack.x fails to convert the orbitals for QMCPACK.

Since Quantum ESPRESSO 5.3.0, the new converter has been fully parallelized to overcome that limitation completely. By removing wf\_collect=.true. (by default wf\_collect=.false.), pw.x doesn't collect the whole wave function into individual files for each k point but write one small file for each processor instead. By just running pw2qmcpack.x in the same parallel setup (MPI tasks and k-pools) as the last scf/nscf calculation with pw.x, the orbitals distributed among processors will first be aggregated by the converter into individual temporal HDF5 files for each k-pool and then merged into the final file. In large calculations, users should benefit from a significant reduction of time in writing the wave function by pw.x thanks to avoiding the wavefunction collection.

pw2qmcpack has been included in the test suite of QMCPACK, see instructions about how to activate the tests in Sec. 2.10. There are tests labeled "no-collect" running the pw.x without setting wf\_collect. The input files are stored at examples/solids/dft-inputs-polarized-no-collect. The scf, nscf, pw2qmcpack runs are performed on 16, 12, 12 MPI tasks with 16, 2, 2 k-pools respectively.

# Chapter 15

# Lab 3: Advanced Molecular Calculations

#### 15.1 Topics covered in this Lab

This lab covers molecular QMC calculations with wavefunctions of increasing sophistication. All of the trial wavefunctions are initially generated with the GAMESS code. Topics covered include:

- Generating single determinant trial wavefunctions with GAMESS (HF and DFT)
- Generating multi-determinant trial wavefunctions with GAMESS (CISD, CASCI, SOCI)
- Optimizing wavefunctions (Jastrow factors and CSF coefficients) with QMC
- DMC timestep and walker population convergence studies
- Systematic progressions of Jastrow factors in VMC
- Systematic convergence of DMC energies with multi-determinant wavefunctions
- Influence of orbitals basis choice on DMC energy

#### 15.2 Lab directories and files

```
labs/lab3_advanced_molecules/exercises
ex1_first-run-hartree-fock
                              - basic work flow from Hatree-Fock to DMC
                            - Hatree-Fock calculation using GAMESS
 gms
    h2o.hf.inp
                             - GAMESS input
    h2o.hf.dat
                             - GAMESS punch file containing orbitals
                             - GAMESS output with orbitals and other info
   h2o.hf.out
                            - Convert GAMESS wavefunction to QMCPACK format
 convert
    h2o.hf.out
                             - GAMESS output
    h2o.ptcl.xml
                             - converted particle positions
    h2o.wfs.xml
                             - converted wave function
                            - VMC optimization
 opt
                             - QMCPACK VMC optimization input
    optm.xml
```

```
dmc_timestep
                            - Check DMC timestep bias
    dmc_ts.xml
                             - QMCPACK DMC input
 dmc_walkers
                            - Check DMC population control bias
                              - QMCPACK DMC input template
     dmc_wk.xml
ex2_slater-jastrow-wf-options - explore jastrow and orbital options
 jastrow
                            - Jastrow options
    12j
                             - no 3-body Jastrow
                             - only 1-body Jastrow
    1j
    2i
                             - only 2-body Jastrow
                            - Orbital options
 orbitals
                              - PBE orbitals
     pbe
                               - DFT calculation using GAMESS
            h2o.pbe.inp
                                  - GAMESS DFT input
     pbe0
                              - PBEO orbitals
                              - BLYP orbitals
    blyp
                               - B3LYP orbitals
     b3lyp
ex3_multi-slater-jastrow
 cisd
                           - CISD wave function
    gms
                            - CISD calculation using GAMESS
       h2o.cisd.inp
                              - GAMESS input
       h2o.cisd.dat
                              - GAMESS punch file containing orbitals
       h2o.cisd.out
                              - GAMESS output with orbitals and other info
    convert
                            - Convert GAMESS wavefunction to QMCPACK format
        h2o.hf.out
                               - GAMESS output
                           - CASCI wave function
 casci
                            - CASCI calculation using GAMESS
    gms
                           - SOCI wave function
 soci
                             - SOCI calculation using GAMESS
     gms
     thres0.01
                             - VMC optimization with few determinants
                             - VMC optimization with more determinants
     thres0.0075
pseudo
H.BFD.gamess
                          - BFD pseudopotential for H in GAMESS format
O.BFD.CCT.gamess
                          - BFD pseudopotential for O in GAMESS format
H.xml
                          - BFD pseudopotential for H in QMCPACK format
 0.xml
                          - BFD pseudopotential for H in QMCPACK format
```

## 15.3 Exercise #1: Basics

The purpose of this exercise is to show how to generate wave-functions for QMCPACK using GAMESS and to optimize the resulting wave-functions using VMC. This will be followed by a study of the time-step and walker population dependence of DMC energies. The exercise will be performed on a water molecule at the equilibrium geometry.

#### 15.3.1 Generation of a Hatree-Fock wave-function with GAMESS

From the top directory, go to "ex1\_first-run-hartree-fock/gms". This directory contains an input file for a HF calculation of a water molecule using BFD ECPs and the corresponding cc-pVTZ basis set. The input file should be named: h2o.hf.inp. Study the input file. If the student wishes, he can refer to section A for a more detailed description of the GAMESS input syntax. There will be a better time to do this soon, so we recommend that the student continues with the exercise at this point. After you are done, execute GAMESS with this input and store the standard output in a file named "h2o.hf.output". Finally, in the "convert" folder, use convert4qmc to generate the QMCPACK particleset and wavefunction files. It is always useful to rename the files generated by convert4qmc to something meaningful, since by default they are called sample.Gaussian-G2.xml and sample.Gaussian-G2.ptcl.xml. In a standard computer (without cross-compilation), these tasks could be accomplished by the following commands.

```
cd ${TRAINING TOP}/ex1_first-run-hartree-fock/gms
jobrun_vesta rungms h2o.hf
cd ../convert
cp ../gms/h2o.hf.output
jobrun_vesta convert4qmc -gamessAscii h2o.hf.output -add3BodyJ
mv sample.Gaussian-G2.xml h2o.wfs.xml
mv sample.Gaussian-G2.ptcl.xml h2o.ptcl.xml
```

The HF energy of the system is -16.9600590022 Ha. To search for the energy in the output file quickly, you can use

```
grep "TOTAL ENERGY =" h2o.hf.output
```

As the job runs on VESTA, it is a good time to review section B, which contains a description on the use of the converter.

## 15.3.2 Optimize the wave-function

When the execution of the previous steps is completed, there should be 2 new files called h2o.wfs.xml and h2o.ptcl.xml. Now we will use VMC to optimize the Jastrow parameters in the wave-function. From the top directory, go to "ex1\_first-run-hartree-fock/opt". Copy the xml files generated in the previous step to the current directory. This directory should already contain a basic QMCPACK input file for an optimization calculation (optm.xml) Open optm.xml with your favorite text editor and modify the name of the files that contain the wavefunction and particleset XML blocks. These files are included with the commands:

```
<include href=ptcl.xml/>
<include href=wfs.xml/>
```

(the particle set must be defined before the wave-function). The name of the particle set and wave-function files should now be h2o.ptcl.xml and h2o.wfs.xml, respectively. Study both files and submit when you are ready. Notice that the location of the ECPs has been set for you, in your own calculations you have to make sure you obtain the ECPs from the appropriate libraries and convert them to QMCPACK format using ppconvert. This is a good time to study section C, which contains a review of the main parameters in the optimization XML block, while this calculation finishes. The previous steps can be accomplished by the following commands:

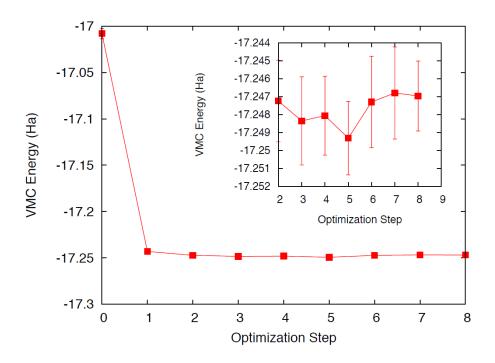


Figure 15.1: VMC energy as a function of optimization step.

```
cd ${TRAINING TOP}/ex1_first-run-hartree-fock/opt
cp ../convert/h2o.wfs.xml ./
cp ../convert/h2o.ptcl.xml ./
# edit optm.xml to include the correct ptcl.xml and wfs.xml
jobrun_vesta qmcpack optm.xml
```

Use the analysis tool qmca to analyze the results of the calculation. Obtain the VMC energy and variance for each step in the optimization and plot it using your favorite program. Remember that qmca has built-in functions to plot the analyzed data.

```
qmca -q e *scalar.dat -p
```

The resulting energy as a function of optimization step should look qualitatively similar to figure 15.1. The energy should decrease quickly as a function of the number of optimization steps. After 6-8 steps, the energy should be converged to ~2-3mHa. To improve convergence, we would need to increase the number of samples used during the optimization. You can check this for yourself on your free time. With optimized wave-functions we are in a position to perform VMC and DMC calculations. The modified wave-function files after each step are written in a file named ID.sNNN.opt.xml, where ID is the identifier of the calculation defined in the input file (this is defined in the project XML block with parameter id) and NNN is a series number which increases with every executable xml block in the input file.

## 15.3.3 Time-step Study

Now we will study the dependence of the DMC energy with time-step. From the top directory, go to ex1\_first-run-hartree-fock/dmc\_timestep. This folder contains a basic xml input file (dmc\_ts.xml)

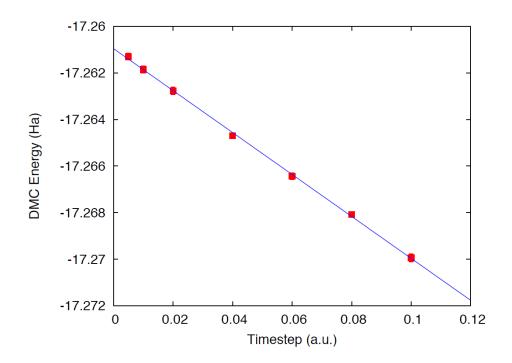


Figure 15.2: DMC energy as a function of timestep.

that performs a short VMC calculation and three DMC calculations with varying time-steps (0.1, 0.05, 0.01). Link the particle set and the last optimization file from the previous folder (the file called jopt-h2o.sNNN.opt.xml with the largest value of NNN). Rename the optimized wave-function to any suitable name if you wish, for example h2o.opt.xml, and change the name of the particle set and wave-function files in the input file. An optimized wave-function can be found in the reference files (same location) in case it is needed.

The main steps needed to perform this exercise are:

```
cd ${TRAINING TOP}/ex1_first-run-hartree-fock/dmc_timestep
cp ../opt/h2o.ptcl.xml ./
cp ../opt/jopt-h2o.s007.opt.xml h2o.opt.wfs.xml
# edit dmc_ts.xml to include the correct ptcl.xml and wfs.xml
jobrun_vesta qmcpack dmc_ts.xml
```

While these runs complete, go to section D and review the basic VMC and DMC input blocks. Notice that in the current DMC blocks, as the time-step is decreased the number of blocks is also increased. Why is this?

When the simulations are finished, use qmca to analyze the output files and to plot the DMC energy as a function of time-step. Results should be qualitatively similar to those presented in figure 15.2, in this case we present more time-steps with well converged results to better illustrate the time-step dependence. In realistic calculations, the time-step must be chosen small enough so that the resulting error is below the desire accuracy. Alternatively, various calculations can be performed and the results extrapolated to the zero time-step limit.

## 15.3.4 Walker Population Study

Now we will study the dependence of the DMC energy with the number of walkers in the simulation. Remember that, in principle, the DMC distribution is reached in the limit of an infinite number of walkers. In practice, the energy and most properties converge to high accuracy with  $\sim 100\text{-}1000$  walkers. The actual number of walkers needed in a calculation will depend on the accuracy of the VMC wave-function and on the complexity and size of the system. Also notice that using too many walkers is not a problem, at worse it will be inefficient since it will cost more computer time than necessary. In fact, this is the strategy used when running QMC calculations on large parallel computers since we can reduce the statistical error bars efficiently by running with large walker populations distributed across all processors.

From the top directory, go to "ex1\_first-run-hartree-fock/dmc\_walkers". Copy the optimized wave-function and particle set files used in the previous calculations to the current folder, these are the ones generated on step 2 of this exercise. An optimized wave-function can be found in the reference files (same location) in case it is needed. The directory contains a sample DMC input file and submission script. Make 3 directories named NWx, with x values 120,240,480 and copy the input file to each one. Go to "NW120", and, in the input file, change the name of the wave-function and particle set files (in this case they will be located one directory above, so use "../dmc\_timestep/h2.opt.xml" for example), change the pseudopotential directory to point to one directory above, change "targetWalkers" to 120, change the number of steps to 100, the time-step to 0.01 and the number of blocks to 400. Notice that "targetWalkers" is one way to set the desired (average) number of walkers in a DMC calculation. One can alternatively set "samples" in the <qmc method=''vmc'' block to carry over de-correlated VMC configurations as DMC walkers. For your own simulations we generally recommend setting ~2\*(#threads) walkers per node (slightly smaller than this value).

The main steps needed to perform this exercise are:

```
cd ${TRAINING TOP}/ex1_first-run-hartree-fock/dmc_walkers
cp ../opt/h2o.ptcl.xml ./
cp ../opt/jopt-h2o.s007.opt.xml h2o.opt.wfs.xml
# edit dmc_wk.xml to include the correct ptcl.xml and wfs.xml and
# use the correct pseudopotential directory
mkdir NW120
cp dmc_wk.xml NW120
# edit dmc_wk.xml to use the desired number of walkers,
# and collect the desired amount of statistics
jobrun_vesta qmcpack dmc_wk.xml
# repeat for NW240, NW480
```

Repeat the same procedure in the other folders by setting (targetWalkers=240, steps=100, timestep=0.01, blocks=200) in NW240 and (targetWalkers=480, steps=100, timestep=0.01, blocks=100) in NW480. When the simulations complete, use qmca to analyze and plot the energy as a function of the number of walkers in the calculation. As always, figure 15.3 shows representative results of the dependence of the energy on the number of walkers for a single water molecule. As shown, less than 240 walkers are needed to obtain an accuracy of 0.1 mHa.

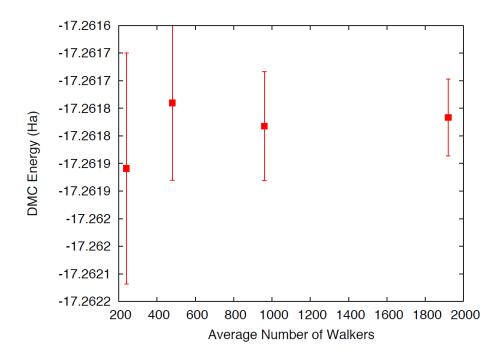


Figure 15.3: DMC energy as a function of the average number of walkers.

# 15.4 Exercise #2 Slater-Jastrow Wave-Function Options

From this point on in the tutorial we assume familiarity with the basic parameters in the optimization, VMC and DMC XML input blocks of QMCPACK. In addition, we assume familiarity with the submission system. As a result, the folder structure will not contain any prepared input or submission files, the student will generate them using input files from exercise 1. In the case of QMCPACK sample files, you will find optm.xml, vmc dmc.xml and submit.csh files. Some of the options in these files can be left unaltered, but many of them will need to be tailored to the particular calculation.

In this exercise we will study the dependence of the DMC energy on the choices made in the wave-function ansatz. In particular, we will study the influence/dependence of the VMC energy with the various terms in the Jastrow. We will also study the influence of the VMC and DMC energies on the single particle orbitals used to form the Slater determinant in single determinant wave-functions. For this we will use wave-functions generated with various exchange-correlation functionals in DFT. Finally, we will optimize a simple multi-determinant wave-function and study the dependence of the energy of the number of configurations used in the expansion. All of these exercises will be performed on the water molecule at equilibrium.

# 15.4.1 Influence of Jastrow on VMC energy with HF wave-function

In this section we will study the dependence of the VMC energy on the various Jastrow terms, e.g. one-body, two-body and three-body. From the top directory, go to "ex2\_slater-jastrow-wf-options/jastrow". We will compare the single determinant VMC energy using a two-body Jastrow term, both one- and two-body terms and finally one-, two- and three-body terms. Since we are interested in the influence of the Jastrow, we will use the HF orbitals calculated in exercise #1.



Figure 15.4: VMC energy as a function of Jastrow type.

Make three folders named 2j,12j,123j. For both 2j and 12j, copy the input file optm.xml from "ex1\_first-run-hartree-fock/opt". This input file performs both wave-function optimization and a VMC calculation. Remember to correct relative paths to the pseudopotential directory. Copy the un-optimized HF wave-function and particle set files from "ex1\_first-run-hartree-fock/convert", if you followed the instructions in exercise #1 these should be named h2o.wfs.xml and h2o.ptcl.xml. Otherwise, you can obtained them from the REFERENCE files. Modify the file h2o.wfs.xml to remove the appropriate jastrow blocks. For example, for a two-body Jastrow (only), you need to eliminate the jastrow blocks named <jastrow name="J1" and <jastrow name="J3". In the case of 12j, remove only <jastrow name="J3". Recommended settings for the optimization run are: nodes=32, threads=16, blocks=250, samples=128000, time-step=0.5, 8 optimization loops, and in the VMC section we recommend walkers=16, blocks=1000, steps=1, substeps=100. Notice that samples should always be set to blocks\*threads per node\*nodes = 32\*16\*250=128000. Repeat the process in both 2j and 12j cases. For the 123j case, the wave-function has already been optimized in the previous exercise. Copy the optimized HF wave-function and the particle set from "ex1\_first-run-hartree-fock/opt". Copy the input file from any of the previous runs and remove the optimization block from the input, just leave the VMC step. In all three cases, modify the submission script and submit the run.

These simulations will take several minutes to complete. This is an excellent opportunity to go to section E and review the wavefunction XML block used by QMCPACK. When the simulation are completed, use qmca to analyze the output files. Using your favorite plotting program (e.g. gnu plot), plot the energy and variance as a function of the Jastrow form. Figure 15.4 shows a typical result for this calculation. As can be seen, the VMC energy and variance depends strongly on the form of the Jastrow. Since the DMC error bar is directly related to the variance of the VMC energy, improving the Jastrow will always lead to a reduction in the DMC effort. In addition, systematic approximations (time-step, number of walkers, etc) are also reduced with improved wave-functions.

#### 15.4.2 Generation of wave-functions from DFT using GAMESS

In this section we will use GAMESS to generate wave-functions for QMCPACK from DFT calculations. From the top folder, go to "ex2\_slater-jastrow-wf-options/orbitals". In order to demonstrate the variation in DMC energies with the choice of DFT orbitals, we will choose the following set of exchange-correlation functionals (PBE, PBE0, BLYP, B3LYP). For each functional, make a directory using your preferred naming convention (e.g. the name of the functional). Go into each folder and copy a GAMESS input file from "ex1\_first-run-hartree-fock/gms" .Rename the file with your preferred naming convention, we suggest using h2o.[dft].inp, where [dft] is the name of the functional used in the calculation. At this point, this input file should be identical to the one used to generate the HF wave-function in exercise #1. In order to perform a DFT calculation we only need to add "DFTTYP" to the \$CONTRL . . . \$END section and set it to the desired functional type, for example "DFTTYP=PBE" for a PBE functional. This variable must be set to (PBE, PBE0, BLYP, B3LYP) to obtain the appropriate functional in GAMESS. For a complete list of implemented functionals, see the GAMESS input manual.

#### 15.4.3 Optimization and DMC calculations with DFT wave-functions

In this section we will optimize the wave-function generated in the previous step and perform DMC calculations. From the top directory, go to ex2\_slater-jastrow-wf-options/orbitals. The steps required to achieve this are identical to those used to optimize the wave-function with HF orbitals. Make individual folders for each calculation and obtain the necessary files to perform optimization, VMC and DMC calculations from "ex1\_first-run-hartree-fock/opt" and "ex1\_first-run-hartree-fock/dmc\_ts", for example. For each functional, make the appropriate modifications to the input files and copy the particle set and wave-function files from the appropriate directory in ex2\_slater-jastrow-wf-options/orbitals/[dft]. We recommend the following settings: nodes=32, threads=16, (in optimization) blocks=250, samples=128000, timestep=0.5, 8 optimization loops, (in VMC) walkers=16, blocks=100, steps=1, substeps=100, (in DMC) blocks 400, targetWalkers=960, timestep=0.01. Submit the runs and analyze the results using qmca.

How do the energies compare against each other? How do they compare against DMC energies with HF orbitals?

# 15.5 Exercise #3: Multi-Determinant Wave-Functions

In this exercise we will study the dependence of the DMC energy on the set of orbitals and the type of configurations included in a multi-determinant wave-function.

#### 15.5.1 Generation of a CISD wave-functions using GAMESS

In this section we will use GAMESS to generate a multi-determinant wave-function with Configuration Interaction with Single and Double excitations (CISD). In CISD, the Schrodinger equation is solved exactly in a basis of determinants including the HF determinant and all its single and double excitations.

Go to "ex3\_multi-slater-jastrow/cisd/gms" and you'll see input and output files named h2o.cisd.inp and h2o.cisd.out. Due to technical problems with GAMESS in the BGQ architecture of VESTA, we are unable to use CISD properly in GAMESS. For this reason, the output of the calculation is already provided in the directory.

There will be time in the next step to study the GAMESS input files and the description in section A. Since the output is already provided, the only thing needed is to use the converter to generate the appropriate QMCPACK files.

```
jobrun_vesta convert4qmc h2o.cisd.out -ci h2o.cisd.out \
-readInitialGuess 57 -threshold 0.0075
```

We used the PRTMO=.T. flag in the GUESS section to include orbitals in the output file. You should read these orbitals from the output (-readInitialGuess 40). The highest occupied orbital in any determinant should be 34, so reading 40 orbitals is a safe choice. In this case, it is important to rename the xml files with meaningful names, for example h2o.cisd.wfs.xml. A threshold of 0.0075 is sufficient for the calculations in the training.

#### 15.5.2 Optimization of Multi-Determinant wave-function

In this section we will optimize the wave-function generated in the previous step. There is no difference in the optimization steps if a single determinant and a multi-determinant wave-function. QMCPACK will recognize the presence of a multi-determinant wavefunction and will automatically optimize the linear coefficients by default. Go to "ex3\_multi-slater-jastrow/cisd" and make a folder called thres0.01. Copy the particle set and wavefunction files created in the previous step to the current directory. With your favorite text editor, open the wave-function file h2o.wfs.xml. Look for the multideterminant XML block and change the "cutoff" parameter in detlist to 0.01. Then follow the same steps used in the subsection "Optimization and DMC calculations with DFT wavefunctions" to optimize the wave-function. Similar to this case, design a QMCPACK input file that performs wave-function optimization followed by VMC and DMC calculations. Submit the calculation.

This is a good time to review the GAMESS input file description in Appendix section A. When the run is completed, go to the previous directory and make a new folder named thres0.0075. Repeat the steps performed above to optimize the wave-function with a cutoff of 0.01, but use a cutoff of 0.0075 this time. This will increase the number of determinants used in the calculation. Notice the "cutoff" parameter in the XML should be less than the "-threshold 0.0075" flag passed to the converted, which is further bounded by the PRTTOL flag in the GAMESS input.

After the wave-function is generated, we are ready to optimize. Instead of starting from an un-optimized wave-function, we can start from optimized wave-function from thres0.01 to speed up convergence. You will need to modify the file and change the cutoff in detlist to 0.0075 with a text editor. Repeat the optimization steps and submit the calculation.

When you are done, use qmca to analyze the results. Compare the energies at these two coefficient cutoffs with the energies obtained with DFT orbitals. Due to the time limitations of this tutorial it is not practical to optimize the wave-functions with a smaller cutoff, since this would require more samples and longer runs due to the larger number of optimizable parameters. Figure 15.5 shows the results of such exercise, the DMC energy as a function of the cutoff in the wave-function. As can be seen, a large improvement in the energy is obtained as the number of configurations is increased.

# 15.5.3 CISD, CASCI and SOCI

Go to ex3\_multi-slater-jastrow and inspect folders for the remaining wave-function types: CASCI and SOCI. Follow steps in the previous exercise and obtain the optimized wave-functions for these

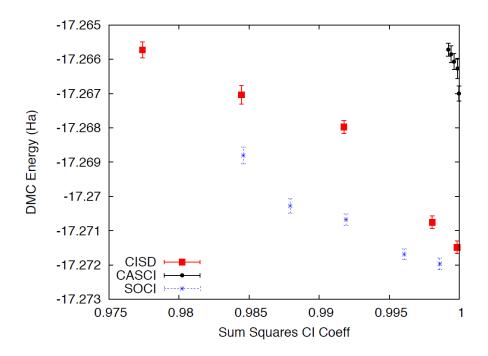


Figure 15.5: DMC energy as a function of the sum of the square of CI coefficients from CISD.

determinant choices. Notice the SOCI GAMESS output is not included because it is large. Already converted XML inputs can be found in "ex3\_multi-slater-jastrow/soci/thres\*".

A CASCI wave-function is produced from a CI calculation that includes all the determinants in a complete active space (CAS) calculation, in this case using the orbitals from a previous CASSCF calculation. In this case we used a CAS(8,8) active space, that includes all determinants generated by distributing 8 electrons in the lowest 8 orbitals. A second-order CI (SOCI) calculation is similar to the CAS-CI calculation, but in addition to the determinants in the CAS it also includes all single and double excitations from all of them, leading to a much larger determinant set. Since we now have considerable experience optimizing wave-functions and calculating DMC energies, we will leave it to the student to complete the remaining tasks on its own. If you need help, refer to previous exercises in the tutorial. Perform optimizations for both wave-functions using cutoffs in the CI expansion of 0.01 an 0.0075. If there is enough time left, try to optimize the wave-functions with a cutoff of 0.005. Analyze the results and plot the energy as a function of cutoff for all three cases, CISD, CAS-CI and SOCI.

Figure 15.5 shows the result of similar calculations using more samples and smaller cutoffs. The results should be similar to those produced in the tutorial. For reference, the exact energy of the water molecule with ECPs is approximately -17.276 Ha. From the results of the tutorial, how does the selection of determinants is related to the expected DMC energy? What about the choice in the set of orbitals?

# 15.6 Appendix A: GAMESS input

In this section we provide a brief description of the GAMESS input needed to produce trial wavefunction for QMC calculations with QMCPACK. We assume basic familiarity with GAMESS input structure, in particular regarding the input of atomic coordinates and the definition of gaussian basis sets. This section will focus on the generation of the output files needed by the converter tool, convert4qmc. For a description of the converter, see B.

Only a subset of the methods available in GAMESS can be used to generate wave-functions for QMCPACK and we restrict our description here to these. For a complete description of all the options and methods available in GAMESS, please refer to the official documentation which could be found in http://www.msg.ameslab.gov/gamess/documentation.html.

Currently, convert4qmc can process output for the following methods in GAMESS (in SCFTYP): RHF, ROHF, and MCSCF. Both HF as well as DFT calculations (any DFT type) could be used in combination with RHF and ROHF calculations. For MCSCF and CI calculations, ALDET, ORMAS and GUGA drivers can be used (see below for details).

#### 15.6.1 HF input

The following input will perform a restricted HF calculation on a closed-shell singlet (multiplicity=1). This will generate RHF orbitals for any molecular system defined in \$DATA ... \$END.

```
$CONTRL SCFTYP=RHF RUNTYP=ENERGY MULT=1
ISPHER=1 EXETYP=RUN COORD=UNIQUE MAXIT=200 $END
$SYSTEM MEMORY=150000000 $END
$GUESS GUESS=HUCKEL $END
$SCF DIRSCF=.TRUE. $END
$DATA
...
Atomic Coordinates and basis set
...
$END
```

#### Main options:

- 1. SCFTYP: Type of SCF method, options: RHF, ROHF, MCSCF, UHF and NONE.
- 2. RUNTYP: Type of run. For QMCPACK wave-function generation this should always be ENERGY.
- 3. MULT: Multiplicity of the molecule.
- 4. ISPHER: Use spherical harmonics (1) or cartesian basis functions (-1).
- 5. COORD: Input structure for the atomic coordinates in \$DATA.

#### 15.6.2 DFT calculations

The main difference between the input for a RHF/ROHF calculation and a DFT calculation is the definition of the DFTTYP parameter. If this is set in the \$CONTROL section, a DFT calculation will be performed with the appropriate functional. Notice that while the default values are usually adequate, DFT calculations have many options involving the integration grids and accuracy settings. Make sure you study the input manual to be aware of these. Refer to the input manual for a list of the implemented exchange-correlation functionals.

#### 15.6.3 Multi-Configuration Self-Consistent Field (MCSCF)

MCSCF calculations are performed by setting SCFTYP=MCSCF in the CONTROL Section. If this option is set, a Messection must be added to the input file with the options for the calculation. An example section for the water molecule used in the tutorial is shown below.

```
$MCSCF CISTEP=GUGA MAXIT=1000 FULLNR=.TRUE. ACURCY=1.0D-5 $END
```

The most important parameter is CISTEP, which defines the CI package used. The only options compatible with QMCPACK are: ALDET, GUGA, and ORMAS. Depending on the package used, additional input sections are needed.

#### 15.6.4 Configuration Interaction (CI)

Configuration interaction (full CI, truncated CI, CAS-CI, etc) calculations are performed by setting SCFTYP=NONE and CITYP=GUGA, ALDET, ORMAS. Each one of this packages requires further input sections, which are typically slightly different to the input sections needed for MCSCF runs.

# 15.6.5 GUGA: Unitary Group CI package

The GUGA package is the only alternative if one wants CSFs with GAMESS. Below we provide a very brief description of input sections needed to perform MCSCF, CASCI, truncated CI and SOCI with this package. For a complete description of these methods and all the options available, please refer to the GAMESS input manual.

#### **GUGA-MCSCF**

The following input section performs a CASCI calculation, with a CAS that includes 8 electrons in 8 orbitals (4 DOC and 4 VAL), e.g. CAS(8,8). NMCC is the number of frozen orbitals (doubly occupied orbitals in all determinants), NDOC is the number of double occupied orbitals in the reference determinant, NVAL is the number of singly occupied orbitals in the reference (for spin polarized cases), and NVAL is the number of orbitals in the active space. Since FORS is set to .TRUE., all configurations in the active space will be included. ISTSYM defines the symmetry of the desired state.

```
$MCSCF CISTEP=GUGA MAXIT=1000 FULLNR=.TRUE. ACURCY=1.0D-5 $END $DRT GROUP=C2v NMCC=0 NDOC=4 NALP=0 NVAL=4 ISTSYM=1 MXNINT= 500000 FORS=.TRUE. $END
```

#### **GUGA-CASCI**

The following input section performs a CASCI calculation, with a CAS that includes 8 electrons in 8 orbitals (4 DOC and 4 VAL), e.g. CAS(8,8). NFZC is the number of frozen orbitals (doubly occupied orbitals in all determinants). All other parameters are identical to those in the MCSCF input section.

```
$CIDRT GROUP=C2v NFZC=0 NDOC=4 NALP=0 NVAL=4 NPRT=2 ISTSYM=1 FORS=.TRUE. MXNINT= 500000 $END $GUGDIA PRTTOL=0.001 CVGTOL=1.0E-5 ITERMX=1000 $END
```

#### **GUGA-Truncated CI**

The following input sections will lead to a truncated CI calculation, in this particular case it will perform a CISD calculation since IEXCIT is set to 2. Other values in IEXCIT will lead to different CI truncations, for example IEXCIT=4 will lead to CISDTQ. Notice that only the lowest 30 orbitals will be included in the generation of the excited determinants in this case. For a full CISD calculation, NVAL should be set to the total number of virtual orbitals.

```
$CIDRT GROUP=C2v NFZC=0 NDOC=4 NALP=0 NVAL=30 NPRT=2 ISTSYM=1 IEXCIT=2 MXNINT= 500000 $END $GUGDIA PRTTOL=0.001 CVGTOL=1.0E-5 ITERMX=1000 $END
```

#### **GUGA-SOCI**

The following input section performs a SOCI calculation, with a CAS that includes 8 electrons in 8 orbitals (4 DOC and 4 VAL), e.g. CAS(8,8). Since SOCI is set to .TRUE., all single and double determinants from all determinants in the CAS(8,8) will be included.

```
$CIDRT GROUP=C2v NFZC=0 NDOC=4 NALP=0 NVAL=4 NPRT=2 ISTSYM=1 SOCI=.TRUE. NEXT=30 MXNINT= 500000 $END $GUGDIA PRTTOL=0.001 CVGTOL=1.0E-5 ITERMX=1000 $END
```

#### 15.6.6 ECP

To use Effective Core Potentials (ECP) in GAMESS, you must define a {\$ECP ... \$END} block. There must be a definition of a potential for every atom in the system, including symmetry equivalent ones. In addition, they must appear in the particular order expected by GAMESS. Below is an example of an ECP input block for a single water molecule using BFD ECPs. To turn on the use of ECPs, the option ECP=READ must be added to the CONTROL input block.

```
$ECP

0-QMC GEN 2 1

3

6.00000000 1 9.29793903

55.78763416 3 8.86492204

-38.81978498 2 8.62925665

1

38.41914135 2 8.71924452

H-QMC GEN 0 0

3

1.000000000000 1 25.00000000000

25.00000000000 3 10.821821902641

-8.228005709676 2 9.368618758833

H-QMC

$END
```

# 15.7 Appendix B: convert4qmc

To generate the particleset and wavefunction XML blocks required by QMCPACK in calculations with molecular systems, the converter convert4qmc must be used. The converter will read the standard output from the appropriate Quantum Chemistry calculation and will generate all the necessary input for QMCPACK. Below we describe the main options of the converter for GAMESS output. In general, there are 3 ways to use the converter depending on the type of calculation performed. The minimum syntax for each option is found below. For a description of the xml files produced by the converter, see section E.

1. For all single determinant calculations (HF and DFT with any DFTTYP):

```
convert4qmc -gamessAscii single det.out
```

- single det.out is the standard output generated by GAMESS.
- 2. (This option is not recommended. Use option below to avoid mistakes.) For multi-determinant calculations where the orbitals and configurations are read from different files (for example when using orbitals from a MCSCF run and configurations from a subsequent CI run):

```
convert4qmc -gamessAscii orbitals multidet.out -ci cicoeff multidet.out
```

- orbitals\_multidet.out is the standard output from the calculation that generates the orbitals. cicoeff multidet.out is the standard output from the calculation that calculates the CI expansion.
- 3. For multi-determinant calculations where the orbitals and configurations are read from the same file, using PRTMO=.T. in the GUESS input block:

```
convert4qmc -gamessAscii multi det.out -ci multi det.out -readInitialGuess Norb
```

• multi\_det.out is the standard output from the calculation that calculates the CI expansion.

#### Options:

- -gamessAscii file.out: Standard output of GAMESS calculation. With the exception of determinant configurations and coefficients in multi-determinant calculations, everything else is read from this file including: atom coordinates, basis sets, single particle orbitals, ECPs, number of electrons, multiplicity, etc.
- -ci file.out: In multi-determinant calculations, determinant configurations and coefficients are read from this file. Notice that single particle orbitals are NOT read from this file. Recognized CI packages are: ALDET, GUGA and ORMAS. Output produced with the GUGA package MUST have the option NPRT=2 in the CIDRT or DRT input blocks.

- **-threshold cutoff**: Cutoff in multi-determinant expansion. Only configurations with coefficients above this value are printed.
- -zeroCI: Sets to zero the CI coefficients of all determinants, with the exception of the first one.
- -readInitialGuess Norb: Reads Norb initial orbitals (INITIAL GUESS ORBITALS) from GAMESS output. These are orbitals generated by the GUESS input block and printed with the option PRTMO=.T.. Notice that this is useful only in combination with the option GUESS=MOREAD and in cases where the orbitals are not modified in the GAMESS calculation, e.g. CI runs. This is the recommended option in all CI calculations.
- -NaturalOrbitals Norb: Read Norb NATURAL ORBITALS from GAMESS output. The natural orbitals must exists in the output, otherwise the code aborts.
- -add3BodyJ: Adds three-body Jastrow terms (e-e-I) between electron pairs (both same spin and opposite spin terms) and all ion species in the system. The radial function is initialized to zero and the default cutoff is 10.0 bohr. The converter will add a one- and two-body Jastrow to the wavefunction block by default.

#### Useful notes:

- The type of single particle orbitals read by the converter depends on the type of calculation and on the options used. By default, when neither -readInitialGuess or -NaturalOrbitals are used, the following orbitals are read in each case (notice that -readInitialGuess or -NaturalOrbitals are mutually exclusive):
  - RHF and ROHF: EIGENVECTORS
  - MCSCF: MCSCF OPTIMIZED ORBITALS
  - GUGA, ALDET, ORMAS: Cannot read orbitals without -readInitialGuess or -NaturalOrbitals options.
- The single particle orbitals and printed CI coefficients in MCSCF calculations are not consistent in GAMESS. The printed CI coefficients correspond to the next-to-last iteration, they are not recalculated with the final orbitals. So in order to get appropriate CI coefficients from MCSCF calculations, a subsequent CI (no SCF) calculation is needed to produce consistent orbitals. In principle, it is possible to read the orbitals from the MCSCF output and the CI coefficients and configurations from the output of the following CI calculations. This could lead to problems in principle, since GAMESS will rotate initial orbitals by default in order to obtain an initial guess consistent with the symmetry of the molecule. This last step is done by default and can change the orbitals reported in the MCSCF calculation before the CI is performed. In order to avoid this problem, it is highly recommended to use option #3 above to read all the information from the output of the CI calculation, this requires the use of PRTMO=.T. in the GUESS input block. Since the orbitals are printed after any symmetry rotation, the resulting output will always be consistent.

# 15.8 Appendix C: Wave-function Optimization XML block

```
<loop max="10">
 <qmc method="linear" move="pbyp" checkpoint="-1" gpu="no">
  <parameter name="blocks"> 10 </parameter>
  <parameter name="warmupsteps"> 25 </parameter>
  <parameter name="steps"> 1 </parameter>
  <parameter name="substeps"> 20 </parameter>
  <parameter name="timestep"> 0.5 </parameter>
  <parameter name="samples"> 10240 </parameter>
  <cost name="energy">
                                         0.95 </cost>
  <cost name="unreweightedvariance">
                                         0.0 </cost>
  <cost name="reweightedvariance">
                                         0.05 </cost>
  <parameter name="useDrift"> yes </parameter>
  <parameter name="bigchange">10.0</parameter>
  <estimator name="LocalEnergy" hdf5="no"/>
  <parameter name="usebuffer"> yes </parameter>
  <parameter name="nonlocalpp"> yes </parameter>
  <parameter name="MinMethod">quartic</parameter>
  <parameter name="exp0">-6</parameter>
  <parameter name="alloweddifference"> 1.0e-5 </parameter>
  <parameter name="stepsize"> 0.15 </parameter>
  <parameter name="nstabilizers"> 1 </parameter>
 </gmc>
</loop>
```

Figure 15.6: Sample XML optimization block.

#### Options:

- bigchange: (default 50.0) largest parameter change allowed
- usebuffer: (default no) Save useful information during VMC
- nonlocalpp: (default no) Include non-local energy on 1-D min
- MinMethod: (default quartic) Method to calculate magnitude of parameter change quartic: fit quartic polynomial to 4 values of the cost function obtained using reweighting along chosen direction linemin: direct line minimization using reweighting rescale: no 1-D minimization. Uses Umrigars suggestions.
- stepsize: (default 0.25) step size in either quartic or linemin methods.
- alloweddifference: (default 1e-4) Allowed increased in energy
- exp0: (default -16.0) Initial value for stabilizer (shift to diagonal of H) Actual value of stabilizer is 10 exp0
- nstabilizers: (default 3) Number of stabilizers to try
- stabilizaterScale: (default 2.0) Increase in value of exp0 between iterations.
- max its: (default 1) number of inner loops with same sample

- minwalkers: (default 0.3) minimum value allowed for the ratio of effective samples to actual number of walkers in a reweighting step. The optimization will stop if the effective number of walkers in any reweighting calculation drops below this value. Last set of acceptable parameters are kept.
- maxWeight: (defaul 1e6) Maximum weight allowed in reweighting. Any weight above this value will be reset to this value.

#### Recommendations:

- Set samples to equal to (#threads)\*blocks.
- Set steps to 1. Use substeps to control correlation between samples.
- For cases where equilibration is slow, increase both substeps and warmupsteps.
- For hard cases (e.g. simultaneous optimization of long MSD and 3-Body J), set exp0 to 0 and do a single inner iteration (max its=1) per sample of configurations.

# 15.9 Appendix D: VMC and DMC XML block

```
<qmc method="vmc" move="pbyp" checkpoint="-1">
  <parameter name="useDrift">yes</parameter>
  <parameter name="warmupSteps">100</parameter>
  <parameter name="blocks">100</parameter>
  <parameter name="steps">1</parameter>
  <parameter name="subSteps"> 20 </parameter>
  <parameter name="walkers">30</parameter>
  <parameter name="timestep">0.3</parameter>
  <estimator name="LocalEnergy" hdf5="no"/>
</gmc>
<qmc method="dmc" move="pbyp" checkpoint="-1">
  <parameter name="nonlocalmoves">yes</parameter>
  <parameter name="targetWalkers">1920</parameter>
  <parameter name="blocks">100</parameter>
  <parameter name="steps">100</parameter>
  <parameter name="timestep">0.1</parameter>
  <estimator name="LocalEnergy" hdf5="no"/>
</gmc>
```

Figure 15.7: Sample XML blocks for VMC and DMC calculations.

#### General Options:

- move: (default walker) Type of electron move. Options: pbyp and walker.
- **checkpoint**: (default -1) (If 0) Generate checkpoint files with given frequency. The calculations can be restarted/continued with the produced checkpoint files.
- useDrift: (default yes) Defines the sampling mode. useDrift = yes will use Langevin acceleration to sample the VMC and DMC distributions, while useDrift=no will use random displacements in a box.
- warmupSteps: (default 0) Number of steps warmup steps at the beginning of the calculation. No output is produced for these steps.
- blocks: (default 1) Number of blocks (outer loop).
- steps: (default 1) Number of steps per blocks (middle loop).
- sub steps: (default 1) Number of substeps per step (inner loop). During sub steps, the local energy is not evaluated in VMC calculations, which leads to faster execution. In VMC calculations, set sub steps to the average autocorrelation time of the desired quantity.
- time step: (default 0.1) Electronic time step in bohr.
- samples: (default 0) Number of walker configurations saved during the current calculation.

• walkers: (default #threads) In VMC, sets the number of walkers per node. The total number of walkers in the calculation will be equal to walkers\*(# nodes).

## Options unique to DMC:

- target Walkers: (default #walkers from previous calculation, e.g. VMC.) Sets the target number of walkers. The actual population of walkers will fluctuate around this value. The walkers will be distributed across all the nodes in the calculation. On a given node, the walkers are split across all the threads in the system.
- nonlocalmoves: (default no) Set to yes to turns on the use of Casulas T-moves.

# 15.10 Appendix E: Wave-function XML block

```
<wavefunction id="psi0" target="e">
  <determinantset name="LCAOBSet" type="MolecularOrbital" transform="yes" source="ion0">
    <basisset name="LCAOBSet">
      <atomicBasisSet name="Gaussian-G2" angular="cartesian" type="Gaussian"</pre>
           elementType="0" normalized="no">
      </atomicBasisSet>
      . . .
    </basisset>
    <slaterdeterminant>
      <determinant id="updet" size="4">
        <occupation mode="ground"/>
        <coefficient size="57" id="updetC">
        </coefficient>
      determinant>
      <determinant id="downdet" size="4">
        <occupation mode="ground"/>
        <coefficient size="57" id="downdetC">
        </coefficient>
      </determinant>
    </slaterdeterminant>
  </determinantset>
  <jastrow name="J2" type="Two-Body" function="Bspline" print="yes">
  </jastrow>
</wavefunction>
```

Figure 15.8: Basic framework for a single determinant determinantset XML block.

In this section we describe the basic format of a QMCPACK wavefunction XML block. Everything listed in this section is generated by the appropriate converter tools. Little to no modification is needed when performing standard QMC calculations. As a result, this section is meant mainly for illustration purposes. Only experts should attempt to modify these files (with very few exceptions like the cutoff of CI coefficients and the cutoff in Jastrow functions) since changes can lead to unexpected results.

A QMCPACK wavefunction XML block is a combination of a determinantset, which contains the anti-symmetric part of the wave-function, and one or more jastrow blocks. The syntax of the anti-symmetric block depends on whether the wave-function is a single determinant or a multi-determinant expansion. Figure 15.8 shows the general structure of the single determinant case. The determinantset block is composed of a basisset block, which defines the atomic orbital basis set, and a slater-determinant block, which defines the single particle orbitals and occupation numbers of the Slater determinant. Figure 15.9 shows a section of a basisset block for an oxygen atom. The

```
<basisset name="LCAOBSet">
 <atomicBasisSet name="Gaussian-G2" angular="cartesian" type="Gaussian"</pre>
      elementType="0" normalized="no">
   <grid type="log" ri="1.e-6" rf="1.e2" npts="1001"/>
   <basisGroup rid="000" n="0" l="0" type="Gaussian">
      <radfunc exponent="1.253460000000e-01" contraction="5.574095889400e-02"/>
      <radfunc exponent="2.680220000000e-01" contraction="3.048477751890e-01"/>
      <radfunc exponent="5.730980000000e-01" contraction="4.537516653790e-01"/>
      <radfunc exponent="1.225429000000e+00" contraction="2.959257817680e-01"/>
      <radfunc exponent="2.620277000000e+00" contraction="1.956698557000e-02"/>
      <radfunc exponent="5.602818000000e+00" contraction="-1.286269051440e-01"/>
      <radfunc exponent="1.198024500000e+01" contraction="1.202399113300e-02"/>
      <radfunc exponent="2.561680100000e+01" contraction="4.069997000000e-04"/>
      <radfunc exponent="5.477521600000e+01" contraction="-7.599994400000e-05"/>
   </basisGroup>
    <basisGroup rid="010" n="1" l="0" type="Gaussian">
      <radfunc exponent="1.686633000000e+00" contraction="1.000000000000e+00"/>
    <basisGroup rid="020" n="2" l="0" type="Gaussian">
      <radfunc exponent="2.379970000000e-01" contraction="1.000000000000e+00"/>
    </basisGroup>
```

Figure 15.9: Sample XML block for an atomic orbital basis set.

structure of this block is rigid and should not be modified. Figure 15.10 shows a (piece of a) sample of a slaterdeterminant block. The slaterdeterminant block consists of 2 determinant blocks, one for each electron spin. The parameter size in the determinant block refers to the number of single particle orbitals present while the size parameter in the coefficient block refers to the number of atomic basis functions per single particle orbital.

```
<slaterdeterminant>
      <determinant id="updet" size="4">
        <occupation mode="ground"/>
        <coefficient size="57" id="updetC">
 8.5631900000000e-01 -1.0675000000000e-02 -9.2455000000000e-02 0.0000000000000e+00
0.000000000000e+00 1.2639300000000e-01 0.000000000000e+00 0.000000000000e+00
-3.6884000000000e-02 0.0000000000000e+00 0.00000000000e+00 -2.0773000000000e-02
-1.120000000000000e-04
                     9.06000000000000e-04 -7.940000000000e-04 0.000000000000e+00
 0.000000000000000e+00
                     0.00000000000000e+00 -3.5130000000000e-03
                                                               2.29100000000000e-03
 1.22100000000000e-03
                     0.00000000000000e+00 0.000000000000e+00
                                                               0.00000000000000e+00
0.000000000000000e+00
                     0.000000000000000e+00 -1.6340000000000e-03
                                                               0.00000000000000000+00
-1.8190000000000e-03 0.000000000000e+00 4.011000000000e-03
                                                               0.0000000000000000e+00
0.0000000000000e+00 0.000000000000e+00 1.5277800000000e-01
                                                               3.87120000000000e-02
-8.8400000000000e-04 0.0000000000000e+00 2.80600000000e-02 -1.6863000000000e-02
0.000000000000e+00 1.0741000000000e-02 -6.633000000000e-03 -3.923000000000e-03
4.7270000000000e-03 -8.040000000000e-04 0.000000000000e+00
                                                              0.0000000000000000e+00
-5.740000000000e-03 1.5277800000000e-01 3.871200000000e-02 -8.840000000000e-04
0.00000000000000e+00 -2.8060000000000e-02 -1.6863000000000e-02
                                                                0.000000000000000e+00
-1.07410000000000e-02 -6.6330000000000e-03 -3.9230000000000e-03
                                                                4.727000000000000e-03
-8.0400000000000e-04 0.000000000000e+00 0.000000000000e+00
                                                               5.74000000000000e-03
```

Figure 15.10: Sample XML block for the single slater determinant case.

Figure 15.11 shows the general structure of the multi-determinant case. Similar to the single determinant case, the determinantset must contain a basisset block. This definition is identical to

the one described above. In this case, the definition of the single particle orbitals must be done independently from the definition of the determinant configurations, the latter is done in the sposet block while the former is done on the multideterminant block. Notice that 2 sposet sets must be defined, one for each electron spin. The name of reach sposet set is required in the definition of the multideterminant block. The determinants are defined in terms of occupation numbers based on these orbitals.

```
<wavefunction id="psi0" target="e">
 <determinantset name="LCAOBSet" type="MolecularOrbital" transform="yes" source="ion0">
    <basisset name="LCAOBSet">
      <atomicBasisSet name="Gaussian-G2" angular="cartesian" type="Gaussian"</pre>
        elementType="0" normalized="no">
      </atomicBasisSet>
    </basisset>
    <sposet basisset="LCAOBSet" name="spo-up" size="8">
      <occupation mode="ground"/>
      <coefficient size="40" id="updetC">
      </coefficient>
    </sposet>
    <sposet basisset="LCAOBSet" name="spo-dn" size="8">
      <occupation mode="ground"/>
      <coefficient size="40" id="downdetC">
      </coefficient>
    </sposet>
    <multideterminant optimize="yes" spo_up="spo-up" spo_dn="spo-dn">
      <detlist size="97" type="CSF" nca="0" ncb="0" nea="4" neb="4" nstates="8" cutoff="0.01">
        <csf id="CSFcoeff_0" exctLvl="0" coeff="0.984378" qchem_coeff="0.984378" occ="22220000">
          <det id="csf_0-0" coeff="1" alpha="11110000" beta="11110000"/>
        </csf>
      </detlist>
    </multideterminant>
 </determinantset>
 <jastrow name="J2" type="Two-Body" function="Bspline" print="yes">
  </jastrow>
</wavefunction>
```

Figure 15.11: Basic framework for a multi-determinant determinantset XML block.

There are various options in the multideterminant block that users should be aware of.

- cutoff: (IMPORTANT!) Only configurations with (absolute value) qchem coeff larger than this value will be read by QMCPACK.
- optimize: Turn on/off the optimization of linear CI coefficients.
- coeff: (in csf ) Current coefficient of given configuration. Gets updated during wavefunction optimization.

- qchem coeff: (in csf ) Original coefficient of given configuration from GAMESS calculation. This is used when applying a cutoff to the configurations read from the file. The cutoff is applied on this parameter and not on the optimized coefficient.
- nca and nab: number of core orbitals for up/down electrons. A core orbital is an orbital that is doubly occupied in all determinant configurations, not to be confused with core electrons. These are not explicitly listed on the definition of configurations.
- nea and neb: number of up/down active electrons (those being explicitly correlated).
- nstates: number of correlated orbitals
- size (in detlist): contains the number of configurations in the list.

The remaining part of the determinantset block is the definition of jastrow factor. Any number of these can be defined. Figure 15.12 shows a sample jastrow block including one-, two- and three-body terms. This is the standard block produced by convert4qmc with the option -add3BodyJ (this particular example is for a water molecule). Optimization of individual radial functions can be turned on/off using the optimize parameter. It can be added to any coefficients block, even though it is currently not present in the J1 and J2 blocks.

```
<jastrow name="J2" type="Two-Body" function="Bspline" print="yes">
 <correlation rcut="10" size="10" speciesA="u" speciesB="u">
   </correlation>
 <correlation rcut="10" size="10" speciesA="u" speciesB="d">
   </jastrow>
<jastrow name="J1" type="One-Body" function="Bspline" source="ion0" print="yes">
 <correlation rcut="10" size="10" cusp="0" elementType="0">
   </correlation>
 <correlation rcut="10" size="10" cusp="0" elementType="H">
   </correlation>
</iastrow>
<jastrow name="J3" type="eeI" function="polynomial" source="ion0" print="yes">
 <correlation ispecies="0" especies="u" isize="3" esize="3" rcut="10">
   <coefficients id="uu0" type="Array" optimize="yes">
   </coefficients>
 </correlation>
 <correlation ispecies="0" especies1="u" especies2="d" isize="3" esize="3" rcut="10">
   <coefficients id="ud0" type="Array" optimize="yes">
   </coefficients>
 </correlation>
 <correlation ispecies="H" especies="u" isize="3" esize="3" rcut="10">
   <coefficients id="uuH" type="Array" optimize="yes">
   </coefficients>
 </correlation>
 <correlation ispecies="H" especies1="u" especies2="d" isize="3" esize="3" rcut="10">
   <coefficients id="udH" type="Array" optimize="yes">
   </coefficients>
 </correlation>
</jastrow>
```

Figure 15.12: Sample Jastrow XML block.

This training assumes basic familiarity with the UNIX operating system. In particular, we use

simple scripts written in csh. In addition, we assume that the student has obtained all the necessary files and executables, and that the location of the training files are located at \${TRAINING TOP}.

The goal of the training not only to familiarize the student with the execution and options in QMCPACK, but also to introduce him/her to important concepts in quantum Monte Carlo calculations and many-body electronic structure calculations.

# Chapter 16

# Lab 4: Condensed Matter Calculations

# 16.1 Topics covered in this Lab

- tiling DFT primitive cells into QMC supercells
- reducing finite size errors via extrapolation
- reducing finite size erors via averaging over twisted boundary conditions
- using the B-spline mesh factor to reduce memory requirements
- using a coarsely resolved vacuum buffer region to reduce memory requirements
- calculating the DMC total energies of representative 2D and 3D extended systems

#### 16.2 Lab directories and files

```
labs/lab4_condensed_matter/
Be-2at-setup.py
                          - DFT only for prim to conv cell
                          - QMC only for prim to conv cell
Be-2at-qmc.py
                          - DFT and QMC for prim to 16 atom cell
Be-16at-qmc.py
 graphene-setup.py
                          - DFT and OPT for graphene
graphene-loop-mesh.py
                          - VMC scan over orbital bspline mesh factors
 graphene-loop-buffer.py - VMC scan over orbital bspline buffer region size
 graphene-final.py
                           - DMC for final meshfactor and buffer region
 pseudopotentials
                          - pseudopotential directory
    Be.ncpp
                            - Be PP for Quantum ESPRESSO
                             - Be PP for QMCPACK
    Be.xml
                            - C PP for Quantum ESPRESSO
    C.BFD.upf
    C.BFD.xml
                             - C PP for QMCPACK
```

The goal of this lab will be to introduce you to the somewhat specialized problems involved in performing diffusion Monte Carlo calculations on condensed matter as opposed to the atoms and molecules that were the focus of earlier labs. Calculations will be performed on two different systems. Firstly, we will perform a series of calculations on BCC beryllium focusing on the necessary methodology to limit finite size effects. Secondly, we will perform calculations on graphene as an example of a system where qmcpacks ability to handle cases with mixed periodic and open boundary conditions is useful. This example will also focus on strategies to limit memory usage for such systems. All of the calculations performed in this lab will utilize the Nexus workflow management system that vastly simplifies the process by automating the steps of generating trial wavefunctions and performing DMC calculations.

#### 16.3 Preliminaries

For any DMC calculation, we must start with a trial wavefunction. As is typical for our calculations of condensed matter, we will produce this wavefunction using density functional theory. Specifically, we will use quantum espresso to generate a slater determinant of single particle orbitals. This is done as a three step process. First, we calculate the converged charge density by performing a DFT calculation with a fine grid of k-points to fully sample the Brilloiun zone. Next, a non-self consistent calculation is performed at the specific k-points needed for the supercell and twists needed in the DMC calculation (more on this later). Finally, a wavefunction is converted from the binary representation used by quantum espresso to the portable hdf5 representation used by quency.

The choice of k-points necessary to generate the wavefunctions dependes on both the supercell chosen for the DMC calculation and by the supercell twist vectors needed. Recall that the wavefunction in a plane wave DFT calculation is written using Bloch's theorem as:

$$\Psi(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}u(\vec{r}) \tag{16.1}$$

Where  $\vec{k}$  is confined of the first Brillouin zone of the cell chosen and  $u(\vec{r})$  is periodic in this simulation cell. A plane wave DFT calculation stores the periodic part of the wavefunction as a linear combination of plane waves for each single particle orbital at all k-points selected. The symmetry of the system allows us to generate an arbitrary supercell of the primitive cell as follows: Consider the set of primitive lattice vectors,  $\{\mathbf{a}_1^p, \mathbf{a}_2^p, \mathbf{a}_3^p\}$ . We may write these vectors in a matrix,  $\mathbf{L}_p$ , whose rows are the primitive lattice vectors. Consider a non-singular matrix of integers,  $\mathbf{S}$ . A corresponding set of supercell lattice vectors,  $\{\mathbf{a}_1^s, \mathbf{a}_2^s, \mathbf{a}_3^s\}$ , can be constructed by the matrix product

$$\mathbf{a}_i^s = S_{ij}\mathbf{a}_j^p \tag{16.2}$$

If the primitive cell contains  $N_p$  atoms, the supercell will then contain  $N_s = |\det(\mathbf{S})| N_p$  atoms.

Now, the wavefunction at any point in this new supercell can be related to the wavefunction in the primitive cell by finding the linear combination of primitive lattice vectors that maps this point back to the primitive cell:

$$\vec{r}' = \vec{r} + x\mathbf{a}_1^p + y\mathbf{a}_2^p + z\mathbf{a}_3^p = \vec{r} + \vec{T}$$
(16.3)

where x, y, z are integers. Now the wavefunction in the supercell at point  $\vec{r}'$  can be written in terms of the wavefunction in the primitive cell at  $\vec{r}'$  as:

$$\Psi(\vec{r}) = \Psi(\vec{r}')e^{i\vec{T}\cdot\vec{k}} \tag{16.4}$$

where  $\vec{k}$  is confined to the first Brillouin zone of the primitive cell. We have also chosen the supercell twist vector which places a constraint on the form of the wavefunction in the supercell.

The combination of these two constraints allows us to identify family of N k-points in the primitive cell that satisfy the constraints. Thus for a given supercell tiling matrix and twist angle, we can write the wavefunction everywhere in the supercell by knowing the wavefunction a N k-points in the primitive cell. This means that the memory necessary to store the wavefunction in a supercell is only linear in the size of the supercell rather than the quadratic cost if symmetry were neglected.

# 16.4 Total energy of BCC beryllium

As was discussed in this mornings lectures when performing calculations of periodic solids with QMC, it is essential to work with a reasonable size supercell rather than the primitive cells that are common in mean field calculations. Specifically, all of the finite size correction schemes discussed in the morning require that the exchange-correlation hole be considerably smaller than the periodic simulation cell. Additionally, finite size effects are lessened as the distance between the electrons in the cell and their periodic images increases, so it is advantageous to generate supercells that are as spherical as possible so as to maximize this distance. However, there is a competing consideration in that for calculating total energies we often want to be able to extrapolate the energy per particle to the thermodynamic limit by means of the following formula in 3 dimensions:

$$E_{\rm inf} = C + E_N/N \tag{16.5}$$

This formula derived assuming the shape of the supercells is consistent (more specifically that the periodic distances scale uniformly with system size), meaning we will need to do a uniform tiling, ie, 2x2x2, 3x3x3 etc. As a 3x3x3 tiling is 27 times larger than the supercell and the practical limit of DMC is on the order of 200 atoms (depending on Z), sometimes it is advantagous to choose a less spherical supercell with fewer atoms rather than a more spherical one that is too expensive to tile.

In the case of a BCC crystal, it is possible to tile the one atom primitive cell to a cubic supercell by only doubling the number of electrons. This is the best possible combination of a small number of atoms that can be tiled and a regular box that maximizes the distance between periodic images. We will need to determine the tiling matrix S that generates this cubic supercell by solving the following equation for the coefficients of the S matrix:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} \cdot \begin{bmatrix} 0.5 & 0.5 & -0.5 \\ -0.5 & 0.5 & 0.5 \\ 0.5 & -0.5 & 0.5 \end{bmatrix}$$
(16.6)

We will now use Nexus to generate the trial wavefunction for this BCC beryllium.

Fortunately, the Nexus will handle determination of the proper k-vectors given the tiling matrix. All that is needed is to place the tiling matrix in the Be-2at-setup.py file. Now the definition of the physical system is:

```
bcc_Be = generate_physical_system(
    lattice = 'cubic',
    cell = 'primitive',
    centering = 'I',
    atoms = 'Be',
    constants = 3.490,
    units = 'A',
    net_charge = 0,
    net_spin = 0,
    Be = 2,
    tiling = [[a,b,c],[d,e,f],[g,h,i]],
    kgrid = kgrid,
```

```
kshift = (.5,.5,.5)
)
```

Where the tiling line should be replaced with the row major tiling matrix from above. This script file will now perform a converged DFT calculation to generate the charge density in a directory called bcc-beryllium/scf and perform a non self consistend DFT calculation to generate single particle orbitals in the directry bcc-beryllium/nscf. Fortunately, Nexus will calculate the required k-points needed to tile the wavefunction to the supercell, so all that is necessary is the granularity of the supercell twists and whether this grid is shifted from the origin. Once this is finished, it performs the conversion from pwscf's binary format to the hdf5 format used by qmcpack. Finally, it will optimize the coefficients of one-body and two-body jastrow factors in the supercell defined by the tiling matrix.

Run these calculations by executing the script Be-2at-setup.py. You will notice that such small calculations as are required to generate the wavefunction of Be in a one atom cell are rather inefficent to run on a high performance computer such as vesta in terms of the time spent doing calculations versus time waiting on the scheduler and booting compute nodes. One of the benefits of the portable hdf format that is used by qmcpack is that you can generate data like wavefunctions on a local workstation or other convenient resource and only use high performance clusters for the more expensive QMC calculations.

In this case, the wavefunction is generated in the directory bcc-beryllium/nscf-2at\_222/pwscf\_output in a file called pwscf.pwscf.h5. It can be useful for debugging purposes to be able to verify the contents of this file are what you expect. For instance, you can use the tool h5ls to check the geometry of the cell where the dft calculations were performed, or number of k-points or electrons in the calculation. This is done with the command: h5ls -d pwscf.pwscf.h5/supercell or h5ls -d pwscf.pwscf.h5/electrons.

In the course of running Be-2at-setup.py, you will get an error when attempting to perform the vmc and wavefunction optimization calculations. This is due to the fact that the wavefunction has been generated supercell twists of the form (+/-1/4, +/-1/4, +/-1/4). In the case that the supercell twist contains only 0 or 1/2, it is possible to operate entirely with real arithmetic. The executabe that has been indicated in Be-2at-setup.py has been compiled for this case. Note that where this is possible, the memory usage is a factor of two less than the general case and the calculations are somewhat faster. However, it is often necessary to perform calculations away from these special twist angles in order to reduce finite size effects. To fix this, delete the directory bcc-beryllium/opt-2at, change the line in near the top of Be-2at-setup.py from

```
qmcpack = '/soft/applications/qmcpack/Binaries/qmcpack'
```

to

```
qmcpack = '/soft/applications/qmcpack/Binaries/qmcpack_comp'
```

and rerun the script.

When the optimization calculation has finished, check that everything as proceeded correctly by looking at the output in the opt-2at directory. Firstly, you can grep the output file for Delta to see if the cost function has indeed been decreasing during the optimization. You should find something like:

```
OldCost: 4.8789147e-02 NewCost: 4.0695360e-02 Delta Cost:-8.0937871e-03
OldCost: 3.8507795e-02 NewCost: 3.8338486e-02 Delta Cost:-1.6930674e-04
OldCost: 4.1079105e-02 NewCost: 4.0898345e-02 Delta Cost:-1.8076319e-04
OldCost: 4.2681333e-02 NewCost: 4.2356598e-02 Delta Cost:-3.2473514e-04
```

```
OldCost: 3.9168577e-02 NewCost: 3.8552883e-02 Delta Cost:-6.1569350e-04
OldCost: 4.2176276e-02 NewCost: 4.2083371e-02 Delta Cost:-9.2903058e-05
OldCost: 4.3977361e-02 NewCost: 4.2865751e-02 Delta Cost:-1.11161830-03
OldCost: 4.1420944e-02 NewCost: 4.0779569e-02 Delta Cost:-6.4137501e-04
```

Which shows that the starting wavefunction was fairly good and that most of the optimizaiton occurred in the first step. Confirm this by using qmca to look at how the energy and variance changed over the course of the calculation with teh comand: qmca -q ev -e 10 \*.scalar.dat executed in the opt-2at directory. You should get output like the following:

		LocalEnergy	Variance	ratio
opt	series 0	-2.159139 +/- 0.001897	0.047343 +/- 0.000758	0.0219
opt	series 1	-2.163752 +/- 0.001305	0.039389 +/- 0.000666	0.0182
opt	series 2	-2.160913 +/- 0.001347	0.040879 +/- 0.000682	0.0189
opt	series 3	-2.162043 +/- 0.001223	0.041183 +/- 0.001250	0.0190
opt	series 4	-2.162441 +/- 0.000865	0.039597 +/- 0.000342	0.0183
opt	series 5	-2.161287 +/- 0.000732	0.039954 +/- 0.000498	0.0185
opt	series 6	-2.163458 +/- 0.000973	0.044431 +/- 0.003583	0.0205
opt	series 7	-2.163495 +/- 0.001027	0.040783 +/- 0.000413	0.0189

Now that the optimization has completed successfully, we can perform dmc calculations. The first goal of the calculations will be to try to eliminate the one body finite size effects by twist averaging. The script Be-2at-qmc.py has the necessary input. Note on line 42 two twist grids are specified, (2,2,2) and (3,3,3). Change the tiling matrix in this input file as in Be-2at-qmc.py and start the calculations. Note that this workflow takes advantage of qmcpack's ability to group jobs. If you look in the directory dmc-2at-222 at the job submission script, (dmc.qsub.in) you will note that rather than operating on an xml input file, qmcapp is targeting a text file called dmc.in. This file is a simple text file that contains the names of the 8 xml input files needed for this job, one for each twist. When operated in this mode, qmcpack will use mpi groups to run multiple copies of itself within the same mpi context. This is often useful both in terms of organizing calculations and also for taking advantage of the large job sizes that computer centers often encourage.

The dmc calculations in this case are designed to complete in a few minutes. When they have finished running, first look at the scalar dat files corresponding to the dmc calculations at the various twists in dmc-2at\_222. Using a command like 'qmca -q ev -e 32 \*.s001.scalar.dat' (with a suitably chosen number of blocks for the equilibration), you will see that the dmc energy in each calcuation is nearly identical within the statistical uncertainty of the calculations. In the case of a large supercell, this is often indicative of a situation where the Brilloiun zone is so small that the one body finite size effects are nearly converged without any twist averaging. In this case, however, this is because of the symmetry of the system. For this cubic supercell, all of the twist angles chosen in this shifted 2x2x2 grid are equivalent by symmetry. In the case where substantial resources are required to equilibrate the dmc calculations, it can be beneficial to avoid repeating such twists and instead simply weight them properly. In this case however where the equilibration is inexpensive, there is no benefit to adding such complexity as the calculations can simply be averaged together and the result is equivalent to performing a single longer calcuation.

Using the command qmc -a -q ev -e 16 \*.s001.scalar.dat, average the dmc energies in dmc- $2at_222$  and dmc- $2at_333$  to see whether the one body finite size effects are converged with a 3x3x3 grid of twists. As beryllium as a metal, the convergence is quite poor (0.025 Ha / Be or 0.7 eV

/ Be). If this were a production calculation it would be necessary to perform calculations on much larger grids of supercell twists to eliminate the one body finite size effects.

In this case there are several other calculations that would warrent a high priority. A script Be-16at-qmc.py has been provided where you can imput the appropriate tiling matrix for a 16 atom cell and perform calculations to estimate the two body finite size effects which will also be quite large in the 2 atom calculations. This script will take approximately 30 minutes to run to completion, so depending on interest, you can either run it, or also work to modify the scripts to address the other technical issues that would be necessary for a production calculation such as calculating the population bias or the timestep error in the dmc calculations.

Another useful exercise would be to attempt to validate this pseudopotential by calculating the ionization potential and electron affinity of the isolated atom and comparing to the experimental values:  $IP = 9.3227 \; eV$ ,  $EA = 2.4 \; eV$ .

# 16.5 Handling a 2D system: graphene

In this section we will examine a calculation of an isolated sheet of graphene. As graphene is a two dimensional system, we will take advantage of qmcpack's ability to mix periodic and open boundary conditions to eliminate and spurious interaction of the sheet with its images in the z direction. Run the script graphene-setup.py which will generate the wavefunction and optimize one and two body jastrow factors. In the script, notice line 160: bconds = 'ppn' in the generate\_qmcpack function which specifies this mix of open and periodic boundary conditions. As a consequence of this, the atoms will need to be kept away from this open boundary in the z direction as the electronic wavefunction will not be defined outside of the simulation box in this direction. For this reason, all of the atom positions in at the beginning of the file have z coordinates 7.5. At this point, run the script graphene-setup.py.

Aside from the change in boundary conditions, the main thing that distinguished this kind of calculation from the beryllium example above is the large amount of vacuum in the cell. While this is a very small calculation designed to run quickly in the tutorial, in general a more converged calculation would quickly become memory limited on an architecture like BG/Q. When the initial wavefunction optimization has completed to your satisfaction, run the scripts grapheneloop-buffer.py and graphene-loop-mesh.py. These examine within variational Monte Carlo two approaches to reducing the memory required to store the wavefunction. In graphene-loop-mesh.py, the spacing between the b-spline points is varied uniformly. The mesh spacing is a prefactor to the linear spacing between the spline points, so the memory usage goes as the cube of the meshfactor. When you run the calculations, examine the .s000.scalar.dat files with qmca to determine the lowest possible mesh spacing that preserves both the vmc energy and the variance. Similarly, the script graphene-loop-buffer.py uses a feature which generates two spline tables for the wavefunction. One will have half of the mesh spacing requested in the input file and will be valid everywhere. The second one will only be defined in the smallest parallelpiped that contains all of the atoms in the simulation cell with minimum distance given by the buffer size. Again, see what the smallest possible buffer size is that preserves the vmc energy and variance.

Finally, edit the file graphene-final.py which will perform two DMC calculations. In the first, (qmc1) replace the following lines:

```
meshfactor = xxx,
precision = '---',
truncate = False,
buffer = 0.0,
```

using the values you have determined to perform the calculation with as small as possible of wavefunction. Note that we can also use single precision arithmetic to store the wavefunction by specifying precision='single'. When you run the script, compare the output of the two DMC calculations in terms of energy and variance. Also see if you can calculate the fraction of memory that you were able to save by using a meshfactor other than 1, a buffer table and single precision arithmetic.

## 16.6 Conclusion

Upon completion of this lab, you should be able to use Nexus to perform DMC calculations on periodic solids when provided with a pseudopotential. You should also be able to reduce the size of the wavefunction in a solid state calculation in cases where memory is a limiting factor.

# Chapter 17

# **Additional Tools**

QMCPACK provides a set of lightweight executables that address certain common problems in QMC workflow and analysis. These range from conversion utilities between different file formats and QMCPACK (e.g. ppconvert and convert4qmc), (extract-eshdf-kvectors), to post-processing utilities (trace-density and qmcfinitesize), and to many others. In this chapter, we cover the use cases, syntax, and features of all additional tools provided with QMCPACK.

#### 17.1 Initialization Tools

## 17.1.1 getSupercell

# 17.2 Post-Processing

#### 17.2.1 qmca

qmca is a versatile tool to analyze and plot the raw data from QMCPACK \*.scalar.dat files. It is a python executable and part of the Nexus suite of tools. It can be found in trunk/nexus/executables.

#### 17.2.2 qmcfit

#### 17.2.3 qmcfinitesize

qmcfinitesize is a utility to compute many-body finite-size corrections to the energy. It is a C++ executable that is built alongside the qmcpack executable. It can be found in trunk/build/bin

#### 17.2.4 trace-density

#### 17.3 Converters

#### 17.3.1 convert4qmc

convert4qmc allows conversion of orbitals and wavefunctions from quantum chemistry output files to QMCPACK xml input files. It is a small C++ executable that is built alongside the qmcpack executable, and can be found in trunk/build/bin.

#### 17.3.2 wfconvert

wfconvert allow conversion between

#### 17.3.3 pw2qmcpack.x

pw2qmcpack.x is an executable that converts PWSCF wavefunctions to QMCPACK readable HDF5 format. This utility is built alongside the Quantum Espresso post-processing utilities. This utility is written in Fortran90, and is distributed as a patch of the Quantum Espresso source code. The patch, as well as automated QE download and patch scripts, can be found in trunk/external\_codes/quantum\_espresso.

### 17.3.4 ppconvert

ppconvert is a utility to convert pseudopotentials between different commonly used formats. It is a stand alone C++ executable that is not automatically built, but the source is included in trunk/src/QMCTools/ppconvert. BLAS/LAPACK libraries are required.

## 17.4 Miscellaneous

#### 17.4.1 extract-eshdf-kvectors

# Chapter 18

# Contributing to the Manual

This section briefly describes how to contribute to the manual. It is primarily "by developers, for developers". This section should iterate until a consistent view on style/contents is reached.

#### Desirable:

- Use the table templates below when describing XML input.
- Place unformatted text targeted at developers in comments. Include generously.
- Encapsulate formatted text aimed at developers (like this entire chapter), in \dev{}. Text encapsulated in this way will be removed from the user version of the manual by editing the definition of \dev in qmcpack\_manual.tex. Existing but deprecated or partially functioning features fall in this category.

#### Missing sections (these are opinions, not decided priorities):

- Overview of the input file in general, broad structure, and at least one full example that works in isolation.

#### Information currently missing for a complete reference specification:

• Noting how many instances of each child element are allowed. Examples: simulation-1 only, method-1 or more, jastrow-0 or more.

Below are template tables for describing XML elements in reference fashion. A number of examples can be found in e.g. Chapter 8. Preliminary style is (please weigh in with opinions): typewriter text (\texttt{}) for XML element, attribute, and parameter names, normal text for literal information in datatype/values/default columns, bold (\textbf{}) text if an attribute/parameter must take on a particular value (values column), italics (\textit{}) for descriptive (non-literal) information in the values column (e.g. anything, non-zero, etc.), required/optional attributes/parameters noted by some\_attr<sup>r</sup>/some\_attr<sup>o</sup> superscripts. Valid datatypes are text, integer, real, boolean, and arrays of each. Fixed lengh arrays can be noted, e.g. by "real array(3)".

Template for a generic XML element:

generic element						
parent elements:	parent1 pare	nt2				
child elements:	child1 child2	2 child3				
attributes						
name	datatype	values	$\mathbf{default}$	${f description}$		
$\mathtt{attr1}^r$	text					
$\mathtt{attr2}^r$	integer					
$\mathtt{attr3}^o$	real					
$\mathtt{attr4}^o$	boolean					
$\mathtt{attr5}^o$	text array					
$\mathtt{attr6}^o$	integer array					
$\mathtt{attr7}^o$	real array					
$\mathtt{attr8}^o$	boolean array					
parameters						
name	datatype	values	$\mathbf{default}$	${f description}$		
$\mathtt{param1}^r$	text					
$\mathtt{param2}^r$	integer					
$\mathtt{param3}^o$	real					
$\mathtt{param4}^o$	boolean					
$\mathtt{param5}^o$	text array					
$\mathtt{param6}^o$	integer array					
$\mathtt{param7}^o$	real array					
$\mathtt{param8}^o$	boolean array					
body text						
	Long form description of body text format					

"Factory" elements are XML elements that share a tag, but whose contents change based on the value an attribute (or sometimes multiple attributes take). The attribute(s) that determine the allowed contents is referred to below as the "type selector" (e.g. for <estimator/> elements, the type selector is usually the type attribute). These types of elements are frequently encountered as they correspond (sometimes loosely, sometimes literally) to polymorphic classes in QMCPACK that are built in "factories". This name is true to the underlying code, but may be obscure to the general user (is there a better name to retain the general meaning?).

The template below should be provided each time a new "factory" type is encountered (like <estimator/>). The table lists all types of possible elements (see "type options" below) and any attributes that are common to all possible related elements. Specific "derived" elements are then described one at a time with the template above, noting the type selector in addition to the XML tag (e.q. "estimator type=density element").

Template for shared information about "factory" elements.

# generic factory element

parent elements: parent1 parent2

child elements: child1 child2 child3 ...

type selector: some attribute

 $\begin{array}{c} \text{type options:} & \text{Selection1} \\ & \text{Selection2} \\ & \text{Selection3} \end{array}$ 

. . .

shared attributes:

name datatype values default description

attr1 text attr2 integer

• • •

# Chapter 19

# Unit Testing

Unit testing is a standard software engineering practice to aid in ensuring a quality product. A good suite of unit tests provides confidence in refactoring and changing code, provides some documentation on how classes and functions are used, and can drive a more decoupled design.

If unit tests do not already exist for a section of code, you are encouraged to add them when modifying that section of code. New code additions should also include unit tests. When possible, fixes for specific bugs should also include a unit test that would have caught the bug.

# 19.1 Unit testing framework

The Catch framework is used for unit testing. See the project site for a tutorial and documentation: https://github.com/philsquared/Catch

Catch consists solely of header files. It is distributed as a single include file about 400KB in size. In QMCPACK, it is stored in external\_codes/catch.

# 19.2 Unit test organization

The source for the unit tests is located in the tests directory under each directory in src (e.g. src/QMCWavefunctions/tests). All of the tests in each tests directory get compiled into an executable. After building the project, the individual unit test executables can be found in build/tests/bin. For example, the tests in src/QMCWavefunctions/tests are compiled into build/tests/bin/test\_wavefunction.

All the unit test executables are collected under ctest with the unit label. When checking the whole code, it's useful to run through cmake (cmake -L unit). When working on an individual directory, it's useful to run the individual executable.

Some of the tests reference input files. The unit test CMake setup places those input files in particular locations under the tests directory (e.g. tests/xml\_test). The individual test needs to be run from that directory to find the expected input files.

Command line options are available on the unit test executables. Some of the more useful ones are

- -h List command line options.
- --list-tests List all the tests in the executable.

A test name can be given on the command line to execute just that test. This is useful when iterating on a particular test, or when running in the debugger. Test names often contain spaces, so most command line environments require enclosing the test name in single or double quotes.

## 19.3 Example

The first example is one test from src/Numerics/tests/test\_grid\_functor.cpp

Listing 19.1: Unit test example using Catch

```
TEST_CASE("double_1d_grid_functor", "[numerics]")
{
    LinearGrid<double> grid;
    OneDimGridFunctor<double> f(&grid);

grid.set(0.0, 1.0, 3);

REQUIRE(grid.size() == 3);
    REQUIRE(grid.rmin() == 0.0);
    REQUIRE(grid.rmax() == 1.0);
    REQUIRE(grid.dh() == Approx(0.5));
    REQUIRE(grid.dr(1) == Approx(0.5));
}
```

The test function declaration is TEST\_CASE("double\_1d\_grid\_functor","[numerics]"). The first argument is the test name, and it must be unique in the test suite. The second argument is an optional list of tags. Each tag is a name surrounded by brackets ("[tag1][tag2]"). It can also be the empty string.

The REQUIRE macro accepts expressions with C++ comparison operators and records an error if the value of the expression is false.

Floating point numbers may have small differences due to roundoff, etc. The Approx class adds some tolerance to the comparison. Place it on either side of the comparison (e.g. Approx(a) == 0.3 or a = Approx(0.3)). To adjust the tolerance, use the epsilon and scale methods to Approx (REQUIRE(Approx(a).epsilon(0.001) = 0.3);.

#### 19.3.1 Expected output

When running the test executables individually, the output of a run with no failures should look like

```
All tests passed (26 assertions in 4 test cases)
```

A test with failures will look like

```
test_numerics is a Catch v1.4.0 host application.

Run with -? for options

double_1d_grid_functor

/home/user/qmcpack/src/Numerics/tests/test_grid_functor.cpp:29

/home/user/qmcpack/src/Numerics/tests/test_grid_functor.cpp:39: FAILED:

REQUIRE( grid.dh() == Approx(0.6) )

with expansion:
    0.5 == Approx( 0.6 )

test cases: 4 | 3 passed | 1 failed

assertions: 25 | 24 passed | 1 failed
```

## 19.4 Adding tests

There are three scenarios covered here: adding a new test in an existing file, adding a new test file, or adding a new test directory.

### 19.4.1 Adding a test to existing file

Copy an existing test, or from the example shown here. Be sure to change the test name.

#### 19.4.2 Adding a test file

When adding a new test file, create a file in the test directory, or copy from an existing file. Add the file name to the ADD\_EXECUTABLE in the CMakeLists.txt file in that directory.

One (and only one) file must define the main function for the test executable by defining CATCH\_CONFIG\_MAIN before including the Catch header. If more than one file defines this value, there will be linking errors about multiply defined values.

Some of the tests need to shut down MPI properly to avoid extraneous error messages. Those tests include Message/catch\_mpi\_main.hpp instead of defining CATCH\_CONFIG\_MAIN.

#### 19.4.3 Adding a test directory

Copy the CMakeLists.txt file from an existing tests directory. Change the SRC\_DIR name and the files in the ADD\_EXECUTABLES line. The libraries to link in TARGET\_LINK\_LIBRARIES may need to be updated.

Add the new test directory to src/CMakeLists.txt in the BUILD\_UNIT\_TESTS section near the end.

## 19.5 Testing with random numbers

Many algorithms and parts of the code depend on random numbers, which makes validating the results difficult. One solution is to verify that certain properties hold for any random number. This approach is valuable at some levels of testing, but is unsatisfying at the unit test level.

The Utilities directory contains a 'fake' random number generator that can be used for deterministic tests of these parts of the code. Currently it outputs a single, fixed value every time it is called, but it could be expanded to produce more varied, but still deterministic, sequences. See src/QMCDrivers/test\_vmc.cpp for an example of using the fake random number generator.

# Chapter 20

# Development Guide

The section gives guidance on how to extend the functionality of QMCPACK. Future examples will likely include topics such as the addition of a jastrow function or add a new QMC method.

## 20.1 Scalar Estimator Implementation

#### 20.1.1 Introduction: Life of a Specialized QMCHamiltonianBase

Almost all observables in QMCPACK are implemented as specialized derived classes of the QM-CHamiltonianBase base class. Each observable is instantiated in HamiltonianFactory and added to QMCHamiltonian for tracking. QMCHamiltonian tracks two types of observables: main and auxiliary. Main observables contribute to the local energy. These observables are elements of the simulated Hamiltonian such as kinetic or potential energy. auxiliary observables are expectation values of matrix elements that do not contribute to the local energy. These Hamiltonians do not affect the dynamics of the simulation. In the code, the main observables are labeled by "physical" flag, the auxiliary observables have "physical" set to false.

#### Initialization

When an <estimator type="est\_type" name="est\_name" other\_stuff="value"/> tag is present in the <hamiltonian/> section, it is first read by HamiltonianFactory. In general, the type of the estimator will determine which specialization of QMCHamiltonianBase to be instantiated, and a derived class with myName="est\_name" will be constructed. Then, the put() method of this specific class will be called to read any other parameters in the <estimator/> xml node. Sometimes these parameters will be read by HamiltonianFactory instead, because it can access more objects than QMCHamiltonianBase.

#### Cloning

When OpenMP threads are spawned, the estimator will be cloned by the CloneManager, which is a parent class of many QMC drivers.

```
// In CloneManager.cpp
#pragma omp parallel for shared(w,psi,ham)
for(int ip=1; ip<NumThreads; ++ip)
{
   wClones[ip]=new MCWalkerConfiguration(w);
   psiClones[ip]=psi.makeClone(*wClones[ip]);
   hClones[ip]=ham.makeClone(*wClones[ip],*psiClones[ip]);</pre>
```

}

In the above snippet, ham is the reference to the estimator on the master thread. If the implemented estimator does not allocate memory for any array, then the default constructor should suffice for the makeClone method.

```
// In SpeciesKineticEnergy.cpp
QMCHamiltonianBase* SpeciesKineticEnergy::makeClone(ParticleSet& qp, TrialWaveFunction& psi)
{
   return new SpeciesKineticEnergy(*this);
}
```

If memory is allocated during estimator construction (usually when parsing the xml node in the put method), then the makeClone method should perform the same initialization on each thread.

```
QMCHamiltonianBase* LatticeDeviationEstimator::makeClone(ParticleSet& qp, TrialWaveFunction& psi)
{
   LatticeDeviationEstimator* myclone = new LatticeDeviationEstimator(qp,spset,tgroup,sgroup);
   myclone->put(input_xml);
   return myclone;
}
```

#### **Evaluate**

After the observable class (derived class of QMCHamiltonianBase) is constructed and prepared (by the put() method), it is ready to be used in a QMCDriver. A QMCDriver will call H.auxHevaluate(W,thisWalker) after every accepted move, where H is the QMCHamiltonian that holds all main and auxiliary Hamiltonian elements, W is a MCWalkerConfiguration and thisWalker is a pointer to the current walker being worked on. As shown below, this function goes through each auxiliary Hamiltonian element and evaluate it using the current walker configuration. Under the hood, observables are calculated and dumped to the main particle set's property list for later collection.

```
// In QMCHamiltonian.cpp
// This is more efficient
// Only calculate auxH elements if moves are accepted.
void QMCHamiltonian::auxHevaluate(ParticleSet& P, Walker_t& ThisWalker)
#if !defined(REMOVE_TRACEMANAGER)
  collect_walker_traces(ThisWalker,P.current_step);
#endif
 for(int i=0; i<auxH.size(); ++i)</pre>
    auxH[i]->setHistories(ThisWalker);
    RealType sink = auxH[i]->evaluate(P);
    auxH[i]->setObservables(Observables);
#if !defined(REMOVE_TRACEMANAGER)
    auxH[i]->collect_scalar_traces();
    auxH[i]->setParticlePropertyList(P.PropertyList,myIndex);
 }
}
```

For estimators that contribute to the local energy (main observables), the return value of evaluate() is used in accumulating the local energy. For auxiliary estimators, the return value is not used (sink local variable above), only the value of Value is recorded property lists by the setObservables() method as shown in the above code snippet. By default, the setObservables() method will transfer auxH[i]->Value to P.PropertyList[auxH[i]->myIndex]. The same property list is also kept by the particle set being moved by QMCDriver. This list is updated by auxH[i]->setParticlePropertyList(P.PropertyList,myIndex), where myIndex is the starting

index of space allocated to this specific auxiliary Hamiltonian in the property list kept by the target particle set P.

#### Collection

The actual statistics are collected within the QMCDriver, which owns an Estimator Manager object. This object (or a clone in the case of multithreading) will be registered with each mover it owns. For each mover (such as VMCUpdatePbyP derived from QMCUpdateBase), an accumulate() call is made, which by default, makes an accumulate(walkerset) call to the Estimator Manager it owns. Since each walker has a property set, Estimator Manager uses that local copy to calculate statistics. The Estimator Manager performs block averaging and file I/O.

## 20.1.2 Single Scalar Estimator Implementation Guide

Almost all of the defaults can be utilized for a single scalar observable. With any luck, only the put() and evaluate() methods need to be implemented. As an example, this section will present a step-by-step guide to implement a SpeciesKineticEnergy estimator that calculates the kinetic energy of a specific species instead of the entire particle set. For example, a possible input to this estimator can be:

```
<estimator type="specieskinetic" name="ukinetic" group="u"/>
<estimator type="specieskinetic" name="dkinetic" group="d"/>.
```

This should create two extra columns in the scalar.dat file that contains the kinetic energy of the up and down electrons in two separate columns. If the estimator is properly implemented, then the sum of these two columns should be equal to the default Kinetic column.

#### Barebone

The first step is to create a barebone class structure for this simple scalar estimator. The goal is to be able to instantiate this scalar estimator with an xml node and have it print out a column of zeros in the scalar dat file.

To achieve this, first create a header file "SpeciesKineticEnergy.h" in QMCHamiltonians folder, with only the required functions declared as follows:

```
// In SpeciesKineticEnergy.h
#ifndef QMCPLUSPLUS_SPECIESKINETICENERGY_H
#define QMCPLUSPLUS_SPECIESKINETICENERGY_H
#include <Particle/WalkerSetRef.h>
#include <QMCHamiltonians/QMCHamiltonianBase.h>
namespace qmcplusplus
class SpeciesKineticEnergy: public QMCHamiltonianBase
public:
 SpeciesKineticEnergy(ParticleSet& P):tpset(P){ };
 bool put(xmlNodePtr cur);
                                   // read input xml node, required
 bool get(std::ostream& os) const; // class description, required
 Return_t evaluate(ParticleSet& P);
 inline Return_t evaluate(ParticleSet& P, std::vector<NonLocalData>& Txy)
  { // delegate responsity inline for speed
   return evaluate(P);
```

Notice a local reference tpset to the target particle set P is saved in the constructor. The target particle set carries much information useful for calculating observables. Next, make "SpeciesKineticEnergy.cpp" and make vacuous definitions

```
// In SpeciesKineticEnergy.cpp
#include <QMCHamiltonians/SpeciesKineticEnergy.h>
namespace qmcplusplus {
   bool SpeciesKineticEnergy::put(xmlNodePtr cur) {
      return true;
   }
   bool SpeciesKineticEnergy::get(std::ostream& os) const
   {
      return true;
   }
   SpeciesKineticEnergy::Return_t SpeciesKineticEnergy::evaluate(ParticleSet& P) {
      Value = 0.0;
      return Value;
   }
   QMCHamiltonianBase* SpeciesKineticEnergy::makeClone(ParticleSet& qp, TrialWaveFunction& psi) {
      // no local array allocated, default constructor should be enough
      return new SpeciesKineticEnergy(*this);
   }
} // namespace qmcplusplus
```

Now, head over to Hamiltonian Factory and instantiate this observable if an xml node is found requesting it. Look for "gofr" in HamiltonianFactory.cpp, for example, and follow the if block

```
// In HamiltonianFactory.cpp
#include <QMCHamiltonians/SpeciesKineticEnergy.h>
else if(potType =="specieskinetic")
{
    SpeciesKineticEnergy* apot = new SpeciesKineticEnergy(*target_particle_set);
    apot->put(cur);
    targetH->addOperator(apot,potName,false);
}
```

The last argument of addOperator(), i.e. the false flag, is crucial. This tells QMCPACK that the observable we implemented is not a physical Hamiltonian, thus it will not contribute to the local energy. Changes to the local energy will alter the dynamics of the simulation. Finally, add "SpeciesKineticEnergy.cpp" to HAMSRCS in "CMakeLists.txt" located in the QMCHamiltonians folder. Now, recompile QMCPACK and run it on an input that requests <estimator type="specieskinetic" name="ukineti"

in the hamiltonian block. A columns of zeros should appear in the scalar dat file under the name "ukinetic".

#### **Evaluate**

The evaluate() method is where we perform the calculation of the desired observable. In a first iteration, we will simply hard-code the name and mass of the particles

```
// In SpeciesKineticEnergy.cpp
#include <QMCHamiltonians/BareKineticEnergy.h> // laplaician() defined here
SpeciesKineticEnergy::Return_t SpeciesKineticEnergy::evaluate(ParticleSet& P)
  std::string group="u";
  RealType minus_over_2m = -0.5;
  SpeciesSet& tspecies(P.getSpeciesSet());
  Value = 0.0;
  for (int iat=0; iat<P.getTotalNum(); iat++)</pre>
    if (tspecies.speciesName[ P.GroupID(iat) ] == group)
    {
      Value += minus_over_2m*laplacian(P.G[iat],P.L[iat]);
   }
 }
  return Value;
  // Kinetic column has:
  // Value = -0.5*(Dot(P.G,P.G) + Sum(P.L));
```

Voila, you should now be able to compile QMCPACK, rerun, and see that the values in the "ukinetic" column are no longer zero. Now, the only task left to make this basic observable complete is to read in the extra parameters instead of hard-coding them.

#### Parse Extra Input

The preferred method to parse extra input parameters in the xml node is to implement the put() function of our specific observable. Suppose we wish to read in a single string that tells us whether to record the kinetic energy of the up electron (group="u") or the down electron (group="d"). This is easily achievable using the OhmmsAttributeSet class

```
// In SpeciesKineticEnergy.cpp
#include <OhmmsData/AttributeSet.h>
bool SpeciesKineticEnergy::put(xmlNodePtr cur)
{
    // read in extra parameter "group"
    OhmmsAttributeSet attrib;
    attrib.add(group, "group");
    attrib.put(cur);

    // save mass of specified group of particles
    SpeciesSet& tspecies(tpset.getSpeciesSet());
    int group_id = tspecies.findSpecies(group);
    int massind = tspecies.getAttribute("mass");
    minus_over_2m = -1./(2.*tspecies(massind,group_id));
    return true;
}
```

where we may keep "group" and "minus\_over\_2m" as local variables to our specific class.

```
// In SpeciesKineticEnergy.h
private:
  ParticleSet& tpset;
  std::string group;
  RealType minus_over_2m;
```

Notice, the above operations are made possible by the saved reference to the target particle set. Last but not least, compile and run a full example (i.e. a short DMC calculation) with the following xml nodes in your input

```
<estimator type="specieskinetic" name="ukinetic" group="u"/>
<estimator type="specieskinetic" name="dkinetic" group="d"/>.
```

Make sure the sum of the "ukinetic" and "dkinetic" columns is **exactly** the same as the Kinetic columns at **every block**.

For easy reference, the complete list of changes is summarized bellow

```
// In SpeciesKineticEnergy.h
#include <Particle/WalkerSetRef.h>
#include <QMCHamiltonians/QMCHamiltonianBase.h>
namespace qmcplusplus
class SpeciesKineticEnergy: public QMCHamiltonianBase
public:
 SpeciesKineticEnergy(ParticleSet& P):tpset(P){ };
  // xml node is read by HamiltonianFactory, eg. the sum of following should be equivalent to Kinetic
 // <estimator name="ukinetic" type="specieskinetic" target="e" group="u"/>
 // <estimator name="dkinetic" type="specieskinetic" target="e" group="d"/>
 bool put(xmlNodePtr cur);
                                  // read input xml node, required
 bool get(std::ostream& os) const; // class description, required
 Return_t evaluate(ParticleSet& P);
 inline Return_t evaluate(ParticleSet& P, std::vector<NonLocalData>& Txy)
 { // delegate responsity inline for speed
   return evaluate(P);
 // pure virtual functions require overrider
 void resetTargetParticleSet(ParticleSet& P) { }
                                                                          // required
 QMCHamiltonianBase* makeClone(ParticleSet& qp, TrialWaveFunction& psi); // required
private:
                    tpset; // reference to target particle set
 ParticleSet&
 std::string
                    group; // name of species to track
 RealType minus_over_2m; // mass of the species !! assume same mass
 // for multiple species, simply initialize multiple estimators
}; // SpeciesKineticEnergy
} // namespace qmcplusplus
#endif
```

```
// In SpeciesKineticEnergy.cpp
#include <QMCHamiltonians/SpeciesKineticEnergy.h>
#include <QMCHamiltonians/BareKineticEnergy.h> // laplaician() defined here
#include <OhmmsData/AttributeSet.h>
namespace qmcplusplus
bool SpeciesKineticEnergy::put(xmlNodePtr cur)
  // read in extra parameter "group"
 OhmmsAttributeSet attrib;
 attrib.add(group, "group");
 attrib.put(cur);
 // save mass of specified group of particles
 int group_id = tspecies.findSpecies(group);
 int massind = tspecies.getAttribute("mass");
 minus_over_2m = -1./(2.*tspecies(massind,group_id));
 return true;
}
bool SpeciesKineticEnergy::get(std::ostream& os) const
{ // class description
 os << "SpeciesKineticEnergy: " << myName << " for species " << group;
 return true;
SpeciesKineticEnergy::Return_t SpeciesKineticEnergy::evaluate(ParticleSet& P)
 Value = 0.0;
 for (int iat=0; iat<P.getTotalNum(); iat++)</pre>
   if (tspecies.speciesName[ P.GroupID(iat) ] == group)
     Value += minus_over_2m*laplacian(P.G[iat],P.L[iat]);
   }
 }
 return Value;
QMCHamiltonianBase* SpeciesKineticEnergy::makeClone(ParticleSet& qp, TrialWaveFunction& psi)
{ //default constructor
 return new SpeciesKineticEnergy(*this);
} // namespace qmcplusplus
```

#### 20.1.3 Multiple Scalars

It is fairly straight-forward to created more than one column in the scalar.dat file with a single observable class. For example, if we want a single SpeciesKineticEnergy estimator to record the kinetic energies of all species in the target particle set simultaneously, we only have to write two new methods: addObservables() and setObservables(), then tweak the behavior of evaluate(). Firstly, we will have to override the default behavior of addObservables() to make room for more than one column in the scalar.dat file as follows

```
// In SpeciesKineticEnergy.cpp
void SpeciesKineticEnergy::addObservables(PropertySetType& plist, BufferType& collectables)
{
   myIndex = plist.size();
   for (int ispec=0; ispec<num_species; ispec++)</pre>
```

```
{ // make columns named "$myName_u", "$myName_d" etc.
   plist.add(myName + "_" + species_names[ispec]);
}
}
```

where "num\_species" and "species\_name" can be local variables initialized in the constructor. We should also initialize some local arrays to hold temporary data

```
// In SpeciesKineticEnergy.h
private:
 int num_species;
  std::vector<std::string> species_names;
  std::vector<RealType> species_kinetic,vec_minus_over_2m;
// In SpeciesKineticEnergy.cpp
SpeciesKineticEnergy::SpeciesKineticEnergy(ParticleSet& P):tpset(P)
  SpeciesSet& tspecies(P.getSpeciesSet());
  int massind = tspecies.getAttribute("mass");
  num_species = tspecies.size();
  species_kinetic.resize(num_species);
  vec_minus_over_2m.resize(num_species);
  species_names.resize(num_species);
  for (int ispec=0; ispec<num_species; ispec++)</pre>
    species_names[ispec] = tspecies.speciesName[ispec];
    vec_minus_over_2m[ispec] = -1./(2.*tspecies(massind,ispec));
}
```

Next, we need to override the default behavior of setObservables() to transfer multiple values to the property list kept by the main particle set, which eventually go into the scalar.dat file

```
// In SpeciesKineticEnergy.cpp
void SpeciesKineticEnergy::setObservables(PropertySetType& plist)
{ // slots in plist must be allocated by addObservables() first
   copy(species_kinetic.begin(),species_kinetic.end(),plist.begin()+myIndex);
}
```

Finally, we need to change the behavior of evaluate() to fill the local vector "species\_kinetic" with appropriate observable values

```
SpeciesKineticEnergy::Return_t SpeciesKineticEnergy::evaluate(ParticleSet& P)
{
   std::fill(species_kinetic.begin(),species_kinetic.end(),0.0);

   for (int iat=0; iat<P.getTotalNum(); iat++)
   {
      int ispec = P.GroupID(iat);
      species_kinetic[ispec] += vec_minus_over_2m[ispec]*laplacian(P.G[iat],P.L[iat]);
   }

   Value = 0.0; // Value is no longer used
   return Value;
}</pre>
```

That's it! SpeciesKineticEnergy estimator no longer needs the "group" input and will automatically output the kinetic energy of every species in the target particle set in multiple columns. You should now be able to run with <estimator type="specieskinetic" name="skinetic"/> and check that the sum of all columns that starts with "skinetic" is equal to the default "Kinetic" column.

#### 20.1.4 HDF5 Output

If we desire an observable that will output hundreds of scalars per simulation step (eg. SkEstimator), then it is preferred to output to the stat.h5 file instead of the scalar.dat file for better organization. A large chunk of data to be registered in the stat.h5 file is called a "Collectable" in QMCPACK. In particular, if a QMCHamiltonianBase object is initialized with UpdateMode.set(COLLECTABLE,1) then the "Collectables" object carried by the main particle set will be processed and written to the stat.h5 file, where "UpdateMode" is a bit set (i.e. a collection of flags) with the following enumeration

```
// In QMCHamiltonianBase.h
///enum for UpdateMode
enum {PRIMARY=0,
    OPTIMIZABLE=1,
    RATIOUPDATE=2,
    PHYSICAL=3,
    COLLECTABLE=4,
    NONLOCAL=5,
    VIRTUALMOVES=6
};
```

As a simple example, to put the two columns we produced in the previous section into the stat.h5 file, we will first need to declare that our observable uses "Collectables"

```
// In constructor add:
hdf5_out = true;
UpdateMode.set(COLLECTABLE,1);
```

Then make some room in the "Collectables" object carried by the target particle set.

```
// In addObservables(PropertySetType& plist, BufferType& collectables) add:
if (hdf5_out)
{
   h5_index = collectables.size();
   std::vector<RealType> tmp(num_species);
   collectables.add(tmp.begin(),tmp.end());
}
```

Next make some room in the stat.h5 file by overriding the registerCollectables() method

```
// In SpeciesKineticEnergy.cpp
void SpeciesKineticEnergy::registerCollectables(std::vector<observable_helper*>& h5desc, hid_t gid) const
{
   if (hdf5_out)
   {
     std::vector<int> ndim(1,num_species);
     observable_helper* h5o=new observable_helper(myName);
     h5o->set_dimensions(ndim,h5_index);
     h5o->open(gid);
     h5desc.push_back(h5o);
   }
}
```

Finally, edit evaluate() to use the space in the "Collectables" object.

```
// In SpeciesKineticEnergy.cpp
SpeciesKineticEnergy::Return_t SpeciesKineticEnergy::evaluate(ParticleSet& P)
{
   RealType wgt = tWalker->Weight; // MUST explicitly use DMC weights in Collectables!
   std::fill(species_kinetic.begin(),species_kinetic.end(),0.0);

for (int iat=0; iat<P.getTotalNum(); iat++)
   {
    int ispec = P.GroupID(iat);
      species_kinetic[ispec] += vec_minus_over_2m[ispec]*laplacian(P.G[iat],P.L[iat]);
}</pre>
```

```
P.Collectables[h5_index + ispec] += vec_minus_over_2m[ispec]*laplacian(P.G[iat],P.L[iat])*wgt;
}
Value = 0.0; // Value is no longer used
  return Value;
}
```

That should be it. There should now be a new entry in the stat.h5 file containing the same columns of data as the scalar.dat file. After this check, we should clean up the code by

- 1. make "hdf5\_out" and input flag by editing the put() method
- 2. disable output to scalar.dat when "hdf5\_out" flag is on

# Appendix A

# Derivation of twist averaging efficiency

In this appendix we derive the relative statistical efficiency of twist averaging with an irreducible (weighted) set of k-points versus using uniform weights over an unreduced set of k-points (e.g. a full Monkhorst-Pack mesh).

Consider the weighted average of a set of statistical variables  $\{x_m\}$  with weights  $\{w_m\}$ :

$$x_{TA} = \frac{\sum_{m} w_m x_m}{\sum_{m} w_m} \tag{A.1}$$

If produced by a finite quantum Monte Carlo run at a set of twist angles/k-points  $\{k_m\}$ , each variable mean  $\langle x_m \rangle$  has a statistical error bar  $\sigma_m$  and we can also obtain the statistical error bar of the mean of the twist averaged quantity  $\langle x_{TA} \rangle$ :

$$\sigma_{TA} = \frac{\left(\sum_{m} w_m^2 \sigma_m^2\right)^{1/2}}{\sum_{m} w_m} \tag{A.2}$$

The error bar of each individual twist  $\sigma_m$ , is related to the autocorrelation time  $\kappa_m$ , intrinsic variance  $v_m$ , and number of post-equilibration Monte Carlo steps  $N_{step}$  in the following way:

$$\sigma_m^2 = \frac{\kappa_m v_m}{N_{step}} \tag{A.3}$$

In the setting of twist averaging, the autocorrelation time and variance for different twist angles are often very similar across twists and we have

$$\sigma_m^2 = \sigma^2 = \frac{\kappa v}{N_{step}} \tag{A.4}$$

If we define the total weight as W, i.e.  $W \equiv \sum_{m=1}^{M} w_m$ , for the weighted case with M irreducible twists the error bar is

$$\sigma_{TA}^{weighted} = \frac{\left(\sum_{m=1}^{M} w_m^2\right)^{1/2}}{W} \sigma \tag{A.5}$$

For uniform weighting with  $w_m = 1$  the number of twists is W and we have

$$\sigma_{TA}^{uniform} = \frac{1}{\sqrt{W}}\sigma\tag{A.6}$$

We are interested in comparing the efficiency of choosing weights uniformly or based on the irreducible multiplicity of each twist angle for a given target error bar  $\sigma_{target}$ . The number of Monte Carlo steps required to reach this target for uniform weighting is

$$N_{step}^{uniform} = \frac{1}{W} \frac{\kappa v}{\sigma_{target}^2} \tag{A.7}$$

while for non-uniform weighting we have

$$N_{step}^{weighted} = \frac{\sum_{m=1}^{M} w_m^2}{W^2} \frac{\kappa v}{\sigma_{target}^2}$$

$$= \frac{\sum_{m=1}^{M} w_m^2}{W} N_{step}^{uniform}$$
(A.8)

The Monte Carlo efficiency is defined as

$$\xi = \frac{1}{\sigma^2 t} \tag{A.9}$$

where  $\sigma$  is the error bar and t is the total cpu time required for the Monte Carlo run.

The main advantage made possible by irreducible twist weighting is to reduce the equilibration time overhead by having fewer twists, and hence fewer Monte Carlo runs to equilibrate. In the context of twist averaging, the total cpu time for a run can be considered to be

$$t = N_{twist}(N_{eq} + N_{step})t_{step} (A.10)$$

where  $N_{twist}$  is the number of twists,  $N_{eq}$  is the number of Monte Carlo steps required to reach equilibrium,  $N_{step}$  is the number of Monte Carlo steps included in the statistical averaging as before, and  $t_{step}$  is the wall clock time required to complete a single Monte Carlo step. For uniform weighting  $N_{twist} = W$  while for irreducible weighting  $N_{twist} = M$ .

We can now calculate the relative efficiency  $(\eta)$  of irreducible vs. uniform twist weighting with the aim of obtaining a target error bar  $\sigma_{target}$ :

$$\eta = \frac{\xi_{TA}^{weighted}}{\xi_{TA}^{uniform}} 
= \frac{\sigma_{target}^{2} t_{TA}^{uniform}}{\sigma_{target}^{2} t_{TA}^{weighted}} 
= \frac{W(N_{eq} + N_{step}^{uniform})}{M(N_{eq} + N_{step}^{weighted})} 
= \frac{W(N_{eq} + N_{step}^{weighted})}{M(N_{eq} + \frac{\sum_{m=1}^{M} w_m^2}{W} N_{step}^{uniform})} 
= \frac{W}{M} \frac{1 + f}{1 + \frac{\sum_{m=1}^{M} w_m^2}{W} f}$$
(A.11)

In this last expression, f is the ratio of the number of usable Monte Carlo steps to the number that must be discarded during equilibration  $(f = N_{step}^{uniform}/N_{eq})$  and as before  $W = \sum_{m} w_m$ , which is the number of twist angles in the uniform weighting case. It is important to recall that  $N_{step}^{uniform}$  in

f is defined relative to uniform weighting; it is the number of Monte Carlo steps required to reach a target accuracy in the case of uniform twist weights.

The formula for  $\eta$  above can be easily changed with the help of Eq. A.8 to reflect the number of Monte Carlo steps obtained in an irreducibly weighted run instead. A good exercise is to consider runs one has already completed with either uniform or irreducible weighting and calculate the expected efficiency change had the opposite type of weighting been used.

The break even point  $(\eta = 1)$  can be found at a usable step fraction of

$$f = \frac{W - M}{M^{\frac{\sum_{m=1}^{M} w_m^2}{W}} - W}$$
 (A.12)

The relative efficiency  $(\eta)$  above is useful to consider in view of certain scenarios. An important case is where the number of required sampling steps is no larger than the number of equilibration steps, i.e.  $f \approx 1$ . For a very simple case with 8 uniform twists with irreducible multiplicities of  $w_m \in \{1, 3, 3, 1\}$  (W = 8, M = 4), the relative efficiency of irreducible vs. uniform weighting is  $\eta = \frac{8}{4} \frac{2}{1+20/8} \approx 1.14$ . In this case, irreducible weighting is about 14% more efficient than uniform weighting.

Another interesting case is where the number of sampling steps you can reach with uniform twists before wall clock time runs out is small relative to the number of equilibration steps  $(f \to 0)$ . In this limit  $\eta \approx W/M$ . For our 8 uniform twist example, this would result in a relative efficiency of  $\eta = 8/4 = 2$  making irreducible weighting twice as efficient.

A final case of interest is where the equilibration time is short relative to the available sampling time  $(f \to \infty)$ , giving  $\eta \approx W^2/(M\sum_{m=1}^M w_m^2)$ . Again for our simple example we find  $\eta = 8^2/(4 \times 20) \approx 0.8$ , with uniform weighting being 25% more efficient than irreducible.

For this simple example, the crossover point for irreducible weighting being more efficient than uniform weighting is f < 2, *i.e.* when the available sampling period is less than twice the length of the equilibration period. The expected efficiency ratio and crossover point should be checked for the particular case under consideration to inform the choice between twist averaging methods.

# Appendix B

# References