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연습문제 5-2

```
function [root,fx,Ea,n] = bisectnew(func,xl,xu,Ead,varargi

if func(xl,varargin{:})*func(xu,varargin{:})>0
sign change
disp('no bracket')
return
end
if nargin<4|isempty(Ead),Ead=0.000001;end
xr = xl;
n = round(log2((xu - xl)/Ead) + 0.5);
for i = 1:n
xrold = xr;
xr = (xl + xu)/2;
if xr ~= 0, ea = abs((xr - xrold)/xr) * 100; end
Ea = abs(xr - xrold);
test = func(xl,varargin{:})*func(xr,varargin{:});
if test < 0
xu = xr;
elseif test > 0
xl = xr;
else
Ea = 0;
end
end
root = xr; fx = func(xr,varargin{:});
```

```
>> fcd=@(cd,m,t,v) sqrt(9.81*m/cd)*tanh(sqrt(9.81*cd/m)*t)-v;
>> [root,fx,Ea,n] =bisectnew(fcd,0.1,0.2,0.0001,80,4,36)
root =    1.401367187500000e-01
fx =   -4.063639939673180e-04
Ea =    9.765625000002220e-05
n =  10
```

연습문제 5-3

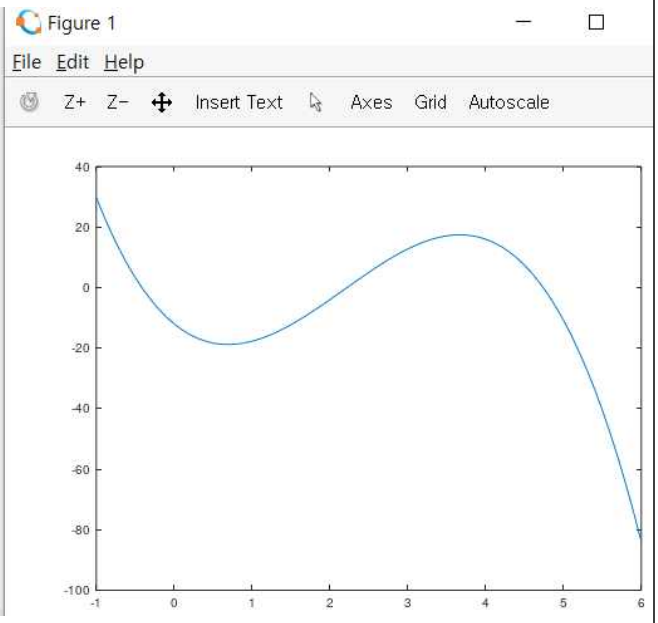
```
function [root,fx,ea,iter]=falsepos(func,xl,xu,es,maxit,varargin)

if nargin<3,error('at least 3 input arguments required'),end
test = func(xl,varargin{:})*func(xu,varargin{:});
if test>0,error('no sign change'),end
if nargin<4|es<=0, es=0.0001;end
if nargin<5|maxit<=0, maxit=50;end
iter = 0; xr = xl;
while (1)
xrold = xr;
fl=func(xl,varargin{:});
fu=func(xu,varargin{:});
xr = xu - fu*(xl - xu)/(fl - fu);
iter = iter + 1;
if xr ~= 0,ea = abs((xr - xrold)/xr) * 100;end
test = fl*func(xr,varargin{:});
if test < 0
xu = xr;
elseif test > 0
xl = xr;
else
ea = 0;
end
if ea <= es | iter >= maxit,break,end
end
root = xr; fx = func(xr,varargin{:});
```

```
>> fcd=@(cd) sqrt(9.81*80/cd)*tanh(sqrt(9.81*cd/80)*4)-36;
>> [root,fx,ea,iter]=falsepos(fcd,0.1,0.2,2)
warning: Matlab-style short-circuit operation performed for op
warning: called from
    falsepos at line 7 column 1
warning: Matlab-style short-circuit operation performed for op
warning: called from
    falsepos at line 24 column 1
root =    1.401650374128185e-01
fx =   -9.964474382755384e-04
ea =  1.172845361902879
iter =  2
```

연습문제 5-5

(A)

```
>> x=[-1:0.1:6];
>> f=-12-21*x+18*x.^2-2.75*x.^3;
>> plot(x,f)
>>
```



(B)

연습문제 5-5

(b) 이분법을 사용

① $x_r = \frac{-1+0}{2} = -0.5$

$f(-1)f(-0.5) = 29.75(3.34375) = 99.47656$

연습문제 5-6

① $x_r = \frac{0.5+1}{2} = 0.75$

$f(0.5)f(0.75) = 0.229426(0.119388) = 0.027333$

② $\frac{0.75+1}{2} = 0.875 = x_r$

$\varepsilon_a = \left|\frac{0.875-0.75}{0.875}\right| \cdot 100\% = 14.29\%$

$f(0.75)f(0.875) = 0.119139(0.0019185) = 0.0002286$

⋮

⑤ 번째 $|\varepsilon_a| = 1.75\%$ (④번째 $|\varepsilon_a| = 3.45\%$)

연습문제 6-1

연습문제 6.1

$x_{i+1} = \sin(\sqrt{x_i})$, $x_0 = 0.5$

① $x_1 = \sin(\sqrt{0.5}) = 0.649637$

$|\varepsilon_a| = \left|\frac{0.649637 - 0.5}{0.649637}\right| \times 100\% = 23\%$

② $x_2 = \sin(\sqrt{0.649637}) = 0.721524$

$|\varepsilon_a| = \left|\frac{0.721524 - 0.649637}{0.721524}\right| \times 100\% = 9.96\%$

... 9회반복하면 근은 0.768606, 근사오차는 0.0097%

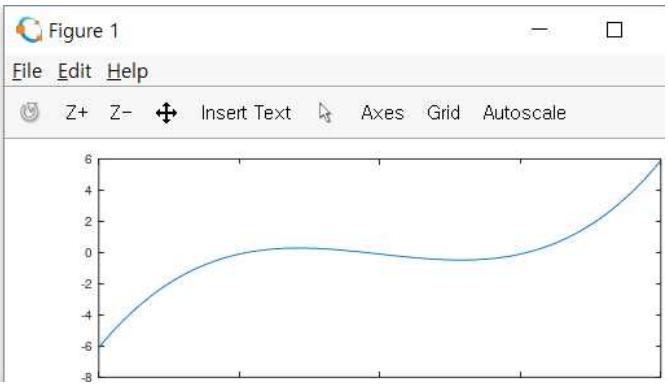
$\frac{E_{i+1}}{E_i} = g'(\xi) = \frac{1}{2\sqrt{\xi}}\cos(\sqrt{\xi})$

ξ에 근을 대입 → 0.365 → 마지막 값과 DH옥근사

연습문제 6-3

(A)

```
>> x = linspace(0,4);
>> y = x.^3-6*x.^2+11*x-6.1;
>> plot(x,y)
>>
```



(B)

예습문제 6-3

(b) 뉴턴 랩슨의 공식에 의해서

$x_{i+1} = x_i = \dfrac{x_i^3 - 6x_i^2 + 11x_i - 6.1}{3x_i^2 - 12x_i + 11}$ 초기추정치는 사용하여

① $x_1 = 3.5 - \dfrac{(3.5)^3 - 6(3.5)^2 + 11(3.5) - 6.1}{3(3.5)^2 - 12(3.5) + 11} = 3.191304$

$|\varepsilon_a| = \left| \dfrac{3.191304 - 3.5}{3.191304} \right| \times 100\% = 9.673\%$

② $x_2 = 3.191304 - \dfrac{(3.191304)^3 \cdots}{\cdots} = 3.068699$

$|\varepsilon_a| = \left| \dfrac{3.068699 - 3.191304}{3.068699} \right| \times 100\% = 3.995\%$

③ $x_3 = 3.068699 = \cdots = 3.047317$

$|\varepsilon_a| = \left| \dfrac{3.047317 - 3.068699}{3.047317} \right| \times 100\% = 0.702\%$

(c)

① $x_1 = 3.5 - \dfrac{1.795(2.5 - 3.5)}{-0.475 - 1.795} = 2.711111$

$|\varepsilon_a| = \left| \dfrac{2.711111 - 3.5}{2.711111} \right| \times 100\% = 29.098\%$

② $x_2 = 2.711111 - \dfrac{-0.45152(3.5 - 2.711111)}{1.795 - (-0.45152)} = 2.871091$

$|\varepsilon_a| = \left| \dfrac{2.871091 - 2.711111}{2.871091} \right| \times 100\% = 5.572\%$

③ $x_3 = 2.871091 - \dfrac{-0.31011(2.711111 - 2.871091)}{-0.45152 - (-0.31011)} = 3.221923$

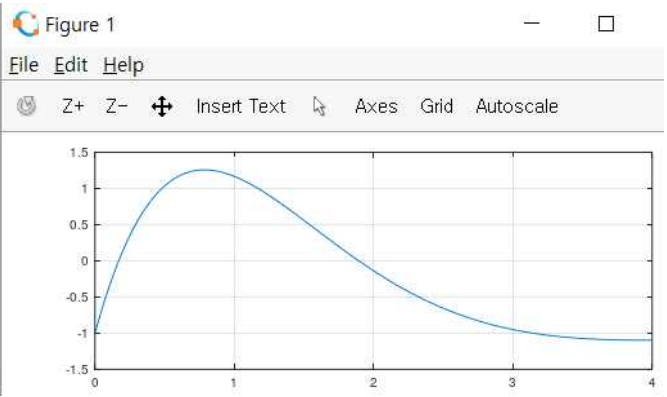
$|\varepsilon_a| = \left| \dfrac{3.221923 - 2.871091}{3.221923} \right| \times 100\% = 10.889\%$

(e) = 3.0467 1.8990 (, 0544

연습문제 6-4

(A)

```
>>
>>
>> x = linspace(0,4);
>> y = 7*sin(x).*exp(-x)-1;
>> plot(x,y)
>>
>>
>>
>> grid
>>
>>
>>
>>
```



(B,C,D)

연습문제 6.4

(b) Gauss-Seidel

$$x_{i+1} = x_i - \frac{7\sin(x_i)e^{-x_i} - 1}{7e^{-x_i}(\cos(x_i) - \sin(x_i))}$$

① $x_1 = 0.3 - \dfrac{7\sin(0.3)e^{-0.3} - 1}{7e^{-0.3}(\cos(0.3) - \sin(0.3))} = 0.144376$

$|\varepsilon_a| = \left|\dfrac{0.144376 - 0.3}{0.144376}\right| \times 100\% = 107.8\%$

② $x_2 = 0.169409$

$|\varepsilon_a| = \left|\dfrac{0.169409 - 0.144376}{0.169409}\right| \times 100\% = 14.776\%$

③ $x_3 = 0.170179$

$|\varepsilon_a| = \left|\dfrac{0.170179 - 0.169409}{0.170179}\right| \times 100\% = 0.453\%$

(c)

① $x_1 = 0.4 - \dfrac{0.827244(0.5 - 0.4)}{1.03550 - 0.827244} = 0.002782$

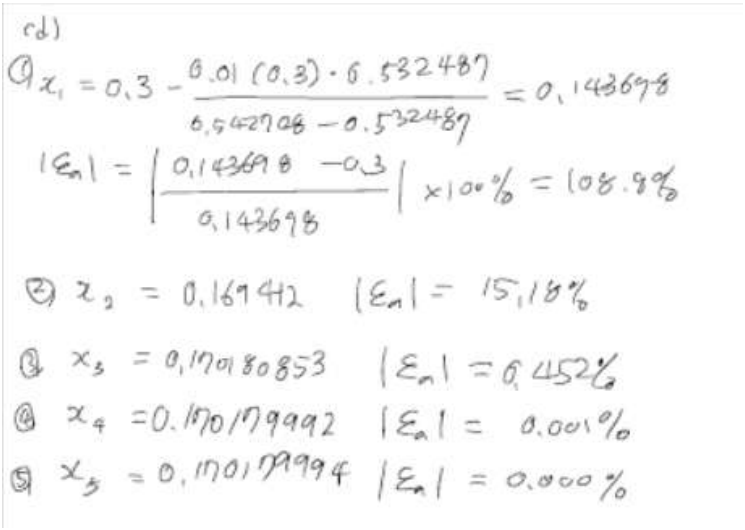
$|\varepsilon_a| = \left|\dfrac{0.002782 - 0.4}{0.002782}\right| \times 100\% = 14.278\%$

② $x_2 = 0.218237$

$|\varepsilon_a| = \left|\dfrac{0.21837 - 0.002782}{0.218237}\right| \times 100\% = 98.725\%$

③ $x_3 = 0.178989$

$|\varepsilon_a| = \left|\dfrac{0.178989 - 0.218237}{0.178989}\right| \times 100\% = 21.927\%$



연습문제 6-6

```
function root = secant(func,xrold,xr,es,maxit)

if nargin<5, maxit=50; end
if nargin<4, es=0.001; end
iter = 0;
while (1)
xrn = xr - func(xr)*(xrold - xr)/(func(xrold) - func(xr));
iter = iter + 1;
if xrn ~= 0, ea = abs((xrn - xr)/xrn) * 100; end
if ea <= es | iter >= maxit, break, end
xrold = xr;
xr = xrn;
end
root = xrn;
```

```
>> format long
>> f=@(x) x^3-6*x^2+11*x-6.1;
>> secant(f,2.5,3.5)
warning: Matlab-style short-cir
warning: called from
    secant at line 10 column 1
ans =  3.046680527126297
```

예제 6-1

예제 6.1

함수를 $x_{i+1} = e^{-x_i}$ 로 나타낼 수 있다.

초기 가정값으로 $x_0 = 0$을 사용하여

$i = 1$, $x_1 = 1$, $|\varepsilon_t|_1 \% = 100$, $|\varepsilon_t|_1 \% = 76.3$

$|\varepsilon_t|_i / |\varepsilon_t|_{i-1} = 0.763$

⋮

$i = 10$ 일때 $|\varepsilon_t|_i / |\varepsilon_t|_{i-1}$ 이 0.566으로

0.567143290에 가까운 추정값을 얻는다.

예제 6-2

예제 6.2

1차 함수는 $f(x) = e^{-x} - x$ 이고 1차 함수와 처음의 함수를

대입하면

$x_{i+1} = x_i - \dfrac{e^{-x_i} - x_i}{-e^{-x_i} - 1}$ 로 반복계산하면

빠르게 참값에 수렴한다. $|\varepsilon_t|_i \%$ $(i = 4)$

$= < 10^{-8}$.

예제 6-3

예제 6.3

Newton-Raphson 방법 $x_{i+1} = x_i - \frac{x_i^{10} - 1}{10x_i^9}$

i	x_i	$\lvert\epsilon_a\rvert$ %
0	0.5	
1	51.65	99.032
2	46.485	11.111
⋮	⋮	⋮
42	1	0.002

예제 6-4

```
>> y = @(m) sqrt(9.81*m/0.25)*tanh(sqrt(9.81 * 0.25/m)*4)-36;
>> dy = @(m) 1/2*sqrt(9.81/(m*0.25))*tanh((9.81*0.25/m)^(1/2)*4
)^2;
>> newtraph(y,dy,140,0.00001)
parse error near line 11 of file C:\Users\micke\newtraph.m

  syntax error

>>>   if ea <= es | iter > = maxit, break, end
                           ^

>> newtraph(y,dy,140,0.00001)
warning: Matlab-style short-circuit operation performed for ope
warning: called from
    newtraph at line 4 column 1
warning: Matlab-style short-circuit operation performed for ope
warning: called from
    newtraph at line 11 column 3
ans =  142.74
```

예제 6-5

① $x_0 = 50$ $\quad f(x_0) = -4.579387078$

$x_0 + \delta x_0 = 50.00005 \quad f(x_0 + \delta x_0) = -4.579387118$

$$x_1 = 50 - \frac{10^{-6}(50)(-4.579387078)}{-4.579387118 - (-4.579387078)}$$

$= 88.39931$ ($|\varepsilon_t| = 38.1\%$, $|\varepsilon_a| = 43.4\%$)

반복하면 $i = 6$, $x_i = 142.7376$, $|\varepsilon_a|_r\% = 3.4 \times 10^{-12}$

$|\varepsilon_a|_r\% = 4.1 \times 10^{-6}$

예제 6-7

```
>> options = optimset('display', 'iter');
>> [x,fx] = fzero(@(x) x^10 -1,0.5,options)
x =  1.0000
fx =    8.8818e-15
>> options = optimset('tolx', 1e-3);
>> [x,fx] = fzero(@(x) x^10 -1,0.5,options)
x =  1.0013
fx =  0.012634
```

예제 7-2

예제 7-2

황금분할 내적성을 구한다

$d = 0.61803\,(4-0) = 2.4721$

$x_1 = 0 + 2.4721 = 2.4721$

$x_2 = 4 - 2.4721 = 1.5279$

$f(x_2) = \frac{1.5279^2}{10} - 2\sin(1.5279) = -1.7641$

$f(x_1) = \frac{2.4721^2}{10} - 2\sin(2.4721) = -0.6300$

$f(x_2) < f(x_1)$ 이므로 극점 최소값은 $x = 1.5279$에 위치,

x_2에서의 함수값은 $f(0.9443) = -1.5310$ 이다.

최소값은 $f(1.5279) = -1.7641$ 이다.

반복시 최소값은 $x = 1.4427$ 이며 함수값은 -1.7655 다.

따라서 $x = 1.4296$에서의 함수값은 -1.7757로 수렴

예제 7-3

예제 7-3

$x_1 = 0 \quad f(x_1) = 0$

$x_2 = 1 \quad f(x_2) = -1.5829$

$x_3 = 4 \quad f(x_3) = 3.1136$

$$x_4 = 1 - \frac{1}{2}\cdot\frac{(1-0)^2[-1.5829 - 3.1136] - (1-4)^2[-1.5829 - 0]}{(1-0)[\quad\quad] + (1-4)[\quad\quad]}$$

$= 1.5055$

$f(1.5055) = -1.0671$

황금분할탐색법을 이용하면 $x_4 = 1.4903$

$\therefore f(1.4903) = -1.7714$

결과는 $x = 1.4296$ 에서의 함수값은 -1.7757로 수렴

예제 7-4

```
>> xlabel('x_1'); ylabel ('x_2'); zlabel('f(x_1,x_2)');
>> title('(b) Mesh plot');
>> x=linspace(-2,0,40); y=linspace(0,3,40);
>> [X,Y] = meshgrid(x,y);
>> Z=2+X-Y+2*X.^2+2*X.*Y+Y.^2;
>> subplot(1,2,1);
>> cs = contour(X,Y,Z);clabel(cs);
>> xlabel('x_1'); ylabel('x_2');
>> title('(A) Contour plot'); grid;
>> subplot(1,2,2);
>> cs = surfc(X,Y,Z);
>> zmin= floor(min(Z));
>> zmax= ceil(max(Z));
>> xlabel('x_1'); ylabel ('x_2'); zlabel('f(x_1,x_2)');
>> title('(b) Mesh plot');
>>
>>
>>
>>
>>
>>
>>
```