齐型空间上的拓扑

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Definition 0.1. A *quasi-metric space* $(\mathcal{X}, d)$ is a non-empty set $\mathcal{X}$ equipped with a *quasi-metric* d, namely, a non-negative function defined on $\mathcal{X} \times \mathcal{X}$, satisfying that, for any $x, y, z \in \mathcal{X}$,

(i) $d(x, y) = 0$ if and only if $x = y$;

(ii) $d(x, y) = d(y, x)$;

(iii) there exists a constant $A_0 \in [1, \infty)$, independent of x, y, and z, such that $d(x, z) \leq A_0[d(x, y) + d(y, z)]$.

The *ball* B on $\mathcal{X}$, centered at $x_0 \in \mathcal{X}$ with radius $r \in (0, \infty)$, is defined by setting

$$B := \{x \in \mathcal{X} : \ d(x, x_0) < r\} =: B(x_0, r).$$

构造齐型空间$\mathcal{X}$ 中的拓扑有两种定义方法, 一种是用球来定义开集, 一种是用收敛性来定义闭集.

Definition 0.2. 称$E \subset \mathcal{X}$ 是开集, 若对$\forall x \in E$, 存在$\delta \in (0, \infty)$ 使得$B(x, \delta) \subset E$. 称$E \subset \mathcal{X}$ 是闭集, 若$\mathcal{X} \setminus E$ 是开集.

Definition 0.3. 称$E \subset \mathcal{X}$ 是闭集, 若对$\forall x \in E$, 存在$\{x_k\}_{k \in \mathbb{N}} \subset E$ 使得$\lim_{k \to \infty} d(x, x_k) \subset E$. 称$E \subset \mathcal{X}$ 是开集, 若$\mathcal{X} \setminus E$ 是闭集.

Remark 0.4. Definition 0.2 和Definition 0.3 是等价的.

等价性的证明来自[1, Theorem 3.1], 但不确定他证明的原创性.

References

[1] G. Rano, and T. Bag. Quasi-metric space and fixed point theorems, International Journal of Mathematics and scientific Computing 3 (2013), 27–31.