Schwartz Space and Tempered Distributions

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在本文中,

$$\mathbb{Z}_+ := \{0, 1, 2, \ldots\}.$$

对$\forall \alpha := (\alpha_1, \ldots, \alpha_n) \in \mathbb{Z}_+^n$,

$$|\alpha| := \sum_{k=1}^{n} |\alpha_k|.$$

对$\forall \alpha := (\alpha_1, \ldots, \alpha_n) \in \mathbb{Z}_+^n$ 和$\forall x := (x_1, \ldots, x_n) \in \mathbb{R}^n$,

$$x^\alpha := \prod_{k=1}^{n} x_k^{\alpha_k}.$$

$\mathcal{S}$ 代表Schwartz 空间.

0.1 Space of Tempered Distributions Modulo Polynomials

本section 的内容来自[1, 1.1.1]

首先介绍多项式空间$\mathcal{P}(\mathbb{R}^n)$.

Definition 0.1.

$$\mathcal{P}(\mathbb{R}^n) := \left\{ \sum_{\alpha \in \mathbb{Z}_+^n, |\alpha| \leq m} c_\alpha x^\alpha : \ m \in \mathbb{Z}_+, \text{ for any } \alpha \in \mathbb{Z}_+^n \text{ and } |\alpha| \leq m, c_\alpha \in \mathbb{C} \right\}.$$

Definition 0.2.

$$\mathcal{S}_0(\mathbb{R}^n) := \left\{ \varphi \in \mathcal{S} : \ \text{ for any } \alpha \in \mathbb{Z}_+^n, \int_{\mathbb{R}^n} x^\alpha \varphi(x) dx = 0 \right\}.$$

References

[1] L. Grafakos, Modern Fourier Analysis, Third edition, Graduate Texts in Mathematics, 249, Springer, New York, 2014.