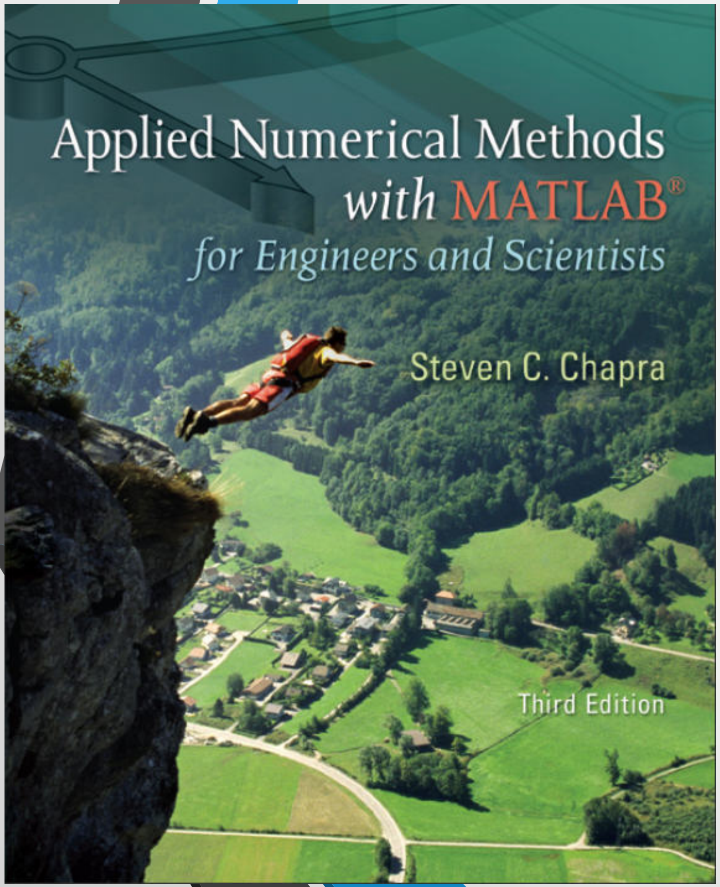
[5주차] 2강: Part Two Roots and Optimization, 교재 Ch.5: Roots (근: 구간법), 이분법 (Bisection) 및 가위치법(False Position) 오차 개념 소개 및 오프 소스 프로그래밍을 통한 알고리즘 학습

학습목표:
-이분법, 가위치법의 오차에 대해 학습한다

수치해석, 김상철 교수



Part 2
Chapter 5

Roots: Bracketing Methods

Error Estimates

Prof. Sang-Chul Kim

Get the real root in Python

```
import numpy as np

from scipy.optimize import fsolve

fm=lambda m:
np.sqrt(9.81*m/0.25)*np.tanh(np.sqrt(9.81*
0.25/m)*4)-36
m=fsolve(fm, 1)


print("Real Root= ", m)
```

Real Root= [142.73763311]

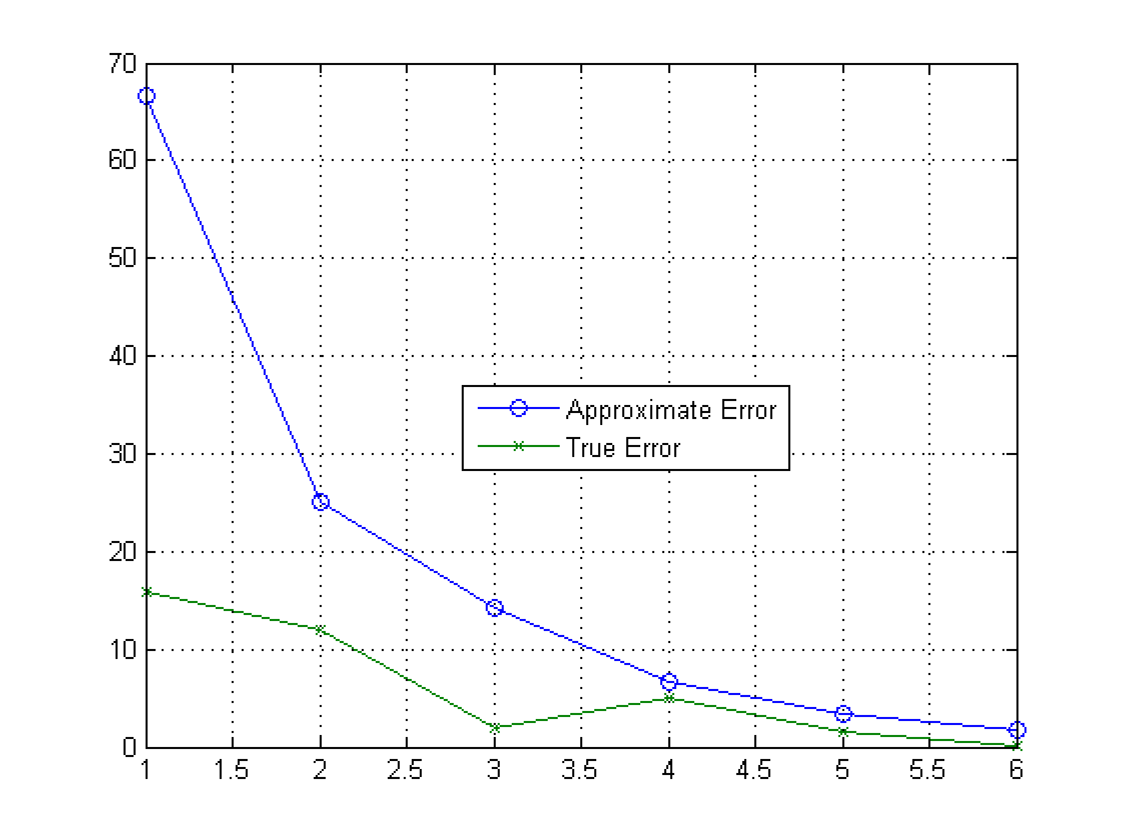
Error in bisection

ea=abs((xr-xrold)/xr)*100;   et=abs((xr-xt)/xt)*100;

xrold	xr	Iter	ea	et	test	xl	xu	ea <= es	iter > maxit
40	120	1	**66.7**	**15.9**	3.25	120	200	0	0
120	160	2	**25**	**12**	-0.1729	120	160	0	0
160	140	3	**14.3**	**1.9**	0.0309	140	160	0	0
140	150	4	**6.7**	**5.1**	-0.0081	140	150	0	0
150	145	5	**3.45**	**1.6**	-0.0026	140	145	0	0
145	142.5	6	**1.75**	**0.016**	2.7742e-004	142.5	145	0	0

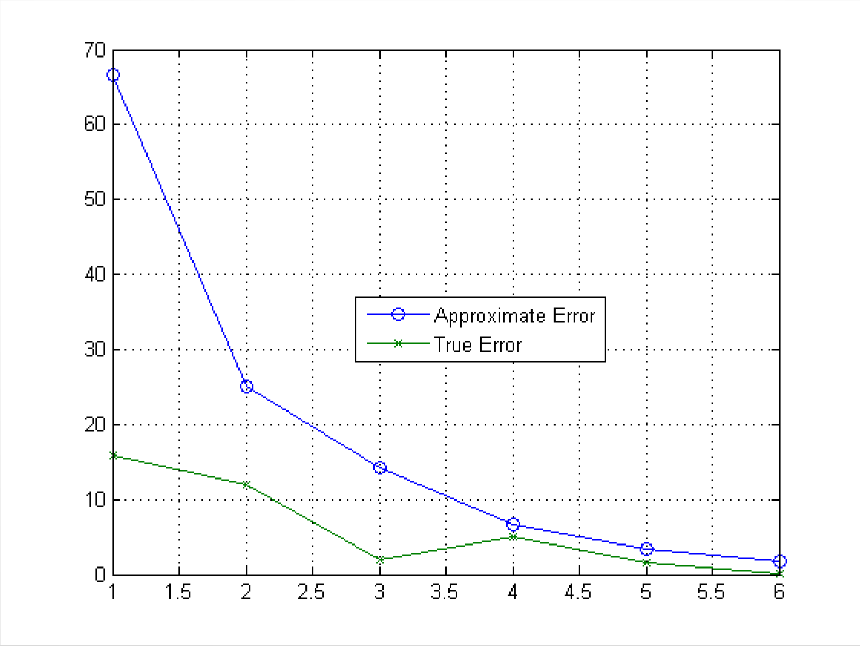
$$|\epsilon_a| = \left| \frac{x_r{}^{new} - x_r{}^{old}}{x_r{}^{new}} \right| \times 100\ (\%) \qquad |\epsilon_t| = \left| \frac{x_t - x_r}{x_t} \right| \times 100\ (\%)$$

```
>> ea=[66.7 25 14.3 6.7 3.45 1.75]
>> et=[15.9 12 1.9 5.1 1.6 0.016]
>> x=[1 2 3 4 5 6]
>> plot(x, ea, 'o-', x, et, 'x-')
>> grid
>> legend('Approximate Error', 'True Error')
```



Downward trend

- Although the approximate error does not provide an exact estimate of the true error, this figure suggests that <u>approximate error captures the general downward trend of true error</u>



Error in Bisection

Dr. Sang-Chul Kim

Error in Bisection

$$e_a = abs\left(\frac{x_r - x_{r\_old}}{x_r}\right) \times 100$$

Stop Error es= 0.0001%

Iter	1. xrold	2. xr	3. Error ea	4. Test sign	5. xl	6. xu	7. ea <= es	8.iter >= maxit
1	40	120.0	66.6%	+	120	200	0	0
2	120.0	160.0	25.0%	-	120.0	160.0	0	0
3	160	140	14.3%	+	140	160	0	0
4	140	150	6.7%	-	140	150	0	0
5	150	145	3.4%	-	140	145	0	0
6	145	142.5	1.75%	+	142.5	145	0	0
7	142.5	143.75	0.89%	-	142.5	143.75	0	0
8	143.75	143.125	0.44%	-	142.5	143.125	0	0
9	143.125	142.8125	0.22%	-	142.5	142.8125	0	0
10	142.8125	142.6563	0.11%	+	142.5	142.6563	0	0

Error in Bisection

$$e_a = abs\left(\frac{x_r - x_{r\_old}}{x_r}\right) \times 100$$

Stop Error es= 0.0001%

Iter	1. xrold	2. xr	3. Error ea	4. Test sign	5. xl	6. xu	7. ea <= es	8.iter >= maxit
1	40	120.0	66.6%	+	120	200	0	0
2	120.0	160.0	25.0%	-	120.0	160.0	0	0
3	160	140	14.3%	+	140	160	0	0
4	140	150	6.7%	-	140	150	0	0
5	150	145	3.4%	-	140	145	0	0
6	145	142.5	1.75%	+	142.5	145	0	0
7	142.5	143.75	0.89%	-	142.5	143.75	0	0
8	143.75	143.125	0.44%	-	142.5	143.125	0	0
9	143.125	142.8125	0.22%	-	142.5	142.8125	0	0
10	142.8125	142.6563	0.11%	+	142.5	142.6563	0	0

Error in False Position

Dr. Sang-Chul Kim

Error in False position

$$e_a = abs\left(\frac{x_r - x_{r\_old}}{x_r}\right) \times 100$$

Stop Error es= 0.0001%

Iter	1. xrold	2. xr	3. Error ea	4. Test sign	5. xl	6. xu	7. ea <= es	8.iter >= maxit
1	40.0	179.8977	77.76%	-	40.0	200.0	0	0
2	179.89	166.85	7.81%	-	40.0	166.85	0	0
3	166.85	158.38	5.34%	-	40.0	158.38	0	0
4	158.38	152.89	3.59%	-	40.0	152.89	0	0
5	152.89	149.32	2.38%	-	40.0	149.32	0	0
6	149.32	147.01	1.57%	-	40.0	147.01	0	0
7	147.01	145.51	1.03%	-	40.0	145.51	0	0
8	145.51	144.53	0.67%	-	40.0	144.53	0	0
9	144.53	143.90	0.43%	-	40.0	143.90	0	0
10	143.90	143.49	0.28%	-	40.0	143.49	0	0