학습 내용(1주 / 2):
Part One Modeling, Computers, and Error Analysis, 교안: 가속도

Basic Concept
of Numerical Analysis

수치 해석 기본 개념

2017.03.07
김상철

수치해석은 머신러닝, 알고리즘, 네트워크 분야와 관련

■ 프로그램일정

3월 9일(목)

시 간	내 용	발표자
09:30~10:00	등 록	
10:00~12:00	**딥러닝 기초** [요약] 본 발표에서는 최근의 딥러닝에 대한 이론적 기초를 전반적으로 다루고자 한다. 머신러닝과 딥러닝의 역사 및 기본적인 원리, 그리고 다양한 딥러닝 모델 등을 소개한다. 특별히, 현재 가장 널리 쓰이는 모델 중 하나인 컨벌루션신경회로망(CNN: convolutional neural network)의 구조와 학습에 관한 기초적 이론 및 각종 응용에 대해 살펴보고자 한다.	이종석 교수/연세대
12:00~13:30	점 심	
13:30~15:30	**머신 러닝을 위한 기초 수학** [요약] 본 강좌에서는 기계학습을 위한 기초 수학을 강의한다. 딥러닝을 비롯한 기계학습 알고리즘들은 고차원의 데이터와 고차원 공간 상에서의 최적화 문제를 다루므로 고차원 벡터와 행렬을 이용한 수식 표현 방법을 익혀야 하고 수치해석에 있어서 오류를 피하는 구현 방법을 숙지해야 한다. 이 강좌는 기계학습 알고리즘을 구현하기 위한 벡터, 행렬 연산의 특성과 최적화 문제를 만들고 푸는 데 있어서 고려해야 할 기본적인 지식들을 다룬다.	노영균 교수/서울대
16:00~18:00	**딥러닝 실전 – 오픈소스 툴 중심으로** [요약] 본 강의에서는 오픈소스 딥러닝 라이브러리들을 비교하여 소개한다. 딥러닝 라이브러리들은 각기 다른 기관을 중심으로 다른 언어로 개발되었으나 딥러닝 모델 구현에 핵심적인 추상화된GPU 컴퓨팅 API와 자동 미분(Auto-Diff) 기능을 공통적으로 가지고 있다. 강의에서는 이러한 기능들의 의미를 알아보고, 최근 널리 사용되고 있는 TensorFlow 라이브러리를 이용하여 상세히 설명한다. 또한 이를 이용하여 딥러닝 모델을 실제로 구현하고 테스트하는 방법에 대해서 알아본다.	김지섭 박사과정/서울대

가속도 (Acceleration)

- 가속도
  - 물체의 속도가 변화하는 비율이다.
  - The rate at which the velocity of an object changes
- 물체가 운동하는 경우
  - (in the case of object movement)
  - 정해진 시간 동안에 얼마나 속도가 변했나를 나타내는 값이다.
  - How much speed has been changed for the specified amount of time
- 가속도가 10m/s/s라는 것은?
  - 1초 동안에 10 m/s 씩 속도가 증가한다는 것이다.
  - During 1sec, the speed increase by 10 m/s

가속도 (Acceleration)

- 자동차 급 발진할 때
- <u>Sudden start </u>when driving a car
  - 속도의 변화가 **<u>있는</u>** 구간 (**Velocity Change)**
  - Increase in velocity
    - 느린 속도→ 빠른 속도
    - Slow Velocity → Rapid Velocity
  - **<u>가속도</u>** 구간 (**<u>acceleration</u>**)
- 운전자는 뒤로 쏠리는 **<u>힘</u>** 받음
  - Driver receives the **<u>force</u>** leaning **<u>back</u>**
- **Force (F) ∝ Acceleration (a) = Velocity Change**
- **F=ma (Newton's Second Law of Motion)**

감속도 (Deceleration)

- 자동차 급 정거할 때
- Sudden braking when driving a car
  - 속도의 변화가 **있는** 구간 (**Velocity Change)**
  - Decrease in velocity
    - 빠른 속도 → 느린 속도
    - Rapid Velocity → Slow Velocity
  - **감속도** 구간 (**Deceleration**)
- 운전자는 앞으로 쏠리는 힘 받음
  - Driver receives the **force** leaning **forward**
- **Force (F) ∝ Deceleration (-a) = Velocity Change**
- **F=m(-a) (Newton's Second Law of Motion)**

등속도 (Constant Velocity)

- 자동차가 순항할 때
- When car is cruising, there is no velocity change
  - 속도의 변화가 **<u>없는</u>** 구간 (**No Velocity Change)**
    - rapid velocity (200km/h) → rapid velocity (200km/h)
    - slow velocity (50km/h) → slow velocity (50km/h)
  - **<u>No acceleration, No deceleration</u>**
- 뒤로 또는 앞으로 쏠리는 **<u>힘 발생 없음</u>**
  - Driver receives **no force** leaning **back or no force** leaning **forward.**
  - 속도 변화가 없음 → 가속도, 감속도 구간이 아님
  - **Force (F) ∝ Acceleration (a) = Deceleration (-a)** = zero **= No Velocity Change**
  - $a = \frac{dv}{dt} = 0$ **F=ma =0 (Newton's Second Law of Motion)**

Interstellar, Gravity Movie

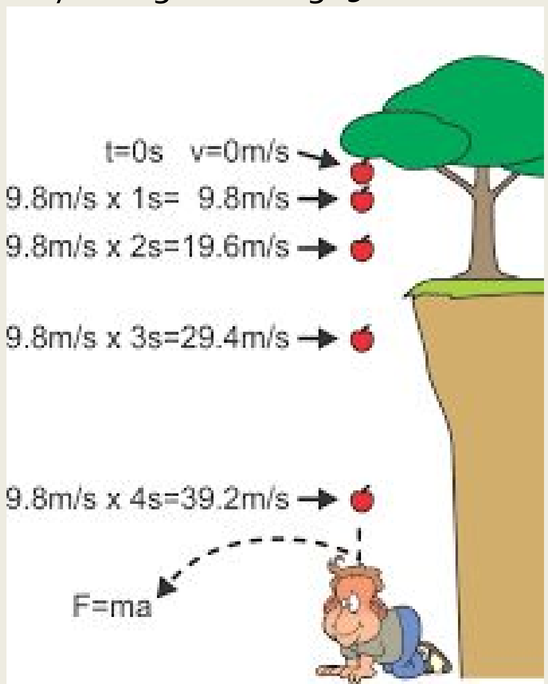
http://www.youtube.com/watch?v=OiTiKOy59o4

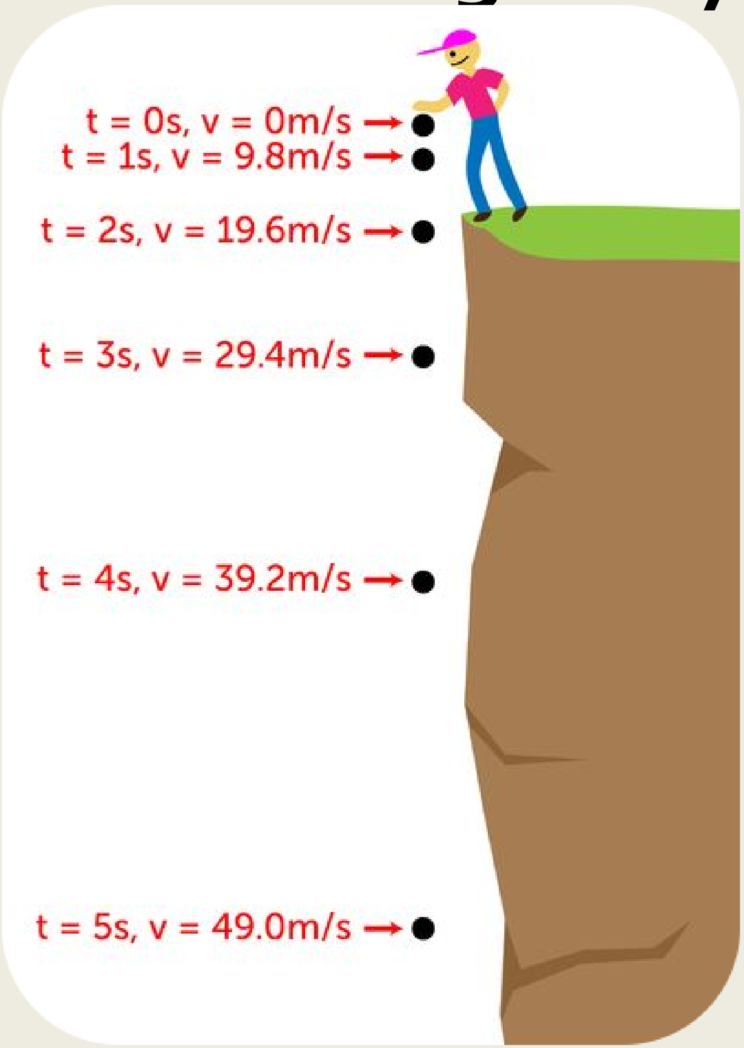
http://www.imdb.com/video/imdb/vi4172195865/
http://www.imdb.com/video/imdb/vi2340006169/

가속도의 예

중력 가속도: Acceleration due to gravity

- 중력 가속도
  - 지구의 중력에 의해 운동하는 물체가 지니는 가속도이다
  - When the object is moving down toward Earth, **Earth's gravity affects (changes, draws)** to the velocity of the moving object
  - With velocity change rate of g=9.8m/s/s
- F=mg





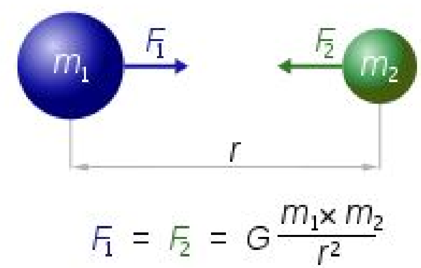
Newton's law of universal gravitation

- 뉴턴의 만유인력의 법칙
  - Newton's law of universal gravitation states that a particle attracts every other particle in the universe using a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

Modern form [edit]

In modern language, the law states the following:

Every point mass attracts every single other point mass by a force pointing along the line intersecting both points. The force is proportional to the product of the two masses and inversely proportional to the square of the distance between them:[2]



$$F = G\frac{m_1 m_2}{r^2}$$

where:

- F is the force between the masses;
- G is the gravitational constant ($6.674 \times 10^{-11}\ \mathrm{N \cdot (m/kg)^2}$);
- m_1 is the first mass;
- m_2 is the second mass;
- r is the distance between the centers of the masses.

중력 가속도: Acceleration due to gravity

Acceleration due to gravity (g)

Acceleration due to gravity is defined as the uniform acceleration produced in a freely falling body due to the gravitational force of the earth.

Acceleration due to gravity (g) = 9.8 m/s² = 980 cm/s².

Calculation of acceleration due to gravity (g)

Suppose a body of mass 'm' is placed on the earth of mass 'M' and radius 'R'.

According to Newton's universal law of gravitation,

Force exerted by the earth on the body is given by

$$F = G\frac{M \times m}{R^2}$$

This force exerted by the earth produces an acceleration on the body.

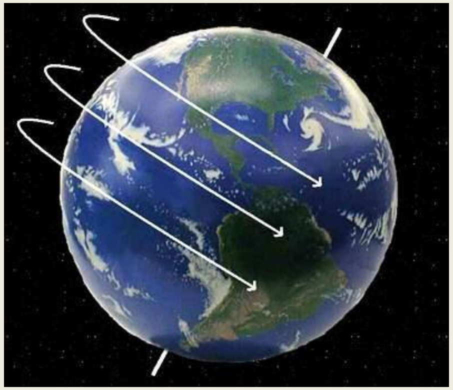
Therefore, $F = mg$ (g - acceleration due to gravity)

From the two equations, we have

$$mg = G\frac{M \times m}{R^2} \quad \text{or} \quad \boxed{g = \frac{G M}{R^2}}$$

F=ma

- The mass of the earth (m)
  - 약 59조 8천억톤
  - 59,800,000,000,000 t
  - Around 60,000 billion (10^9), 60 trillion (10^{12}) ton
- The earth's rotation velocity
  - 초속 30 km
  - 30km/sec
  - Constant Velocity
  - Person on the Earth receives no force leaning back or no force leaning forward.
  - F=ma=0



Earth's axial tilt is about 23.4°.

F=ma

- 지구상에 살고 있는 우리는 지구의 자전 속도를 전혀 느끼지 못하고 있다.
  - 그 이유는 지구의 자전은 가속도 또는 감속도가 없는 등속운동($a = 0, \frac{dv}{dt} = 0$)이기 때문이다.
  - m이 59조 8천억톤이나 되더라도 F는 zero가 된다.
  - 우리가 지구의 자전을 느낄 수 없는 이유가 증명된 것이다.

Newton's Second Law of Motion, F=ma

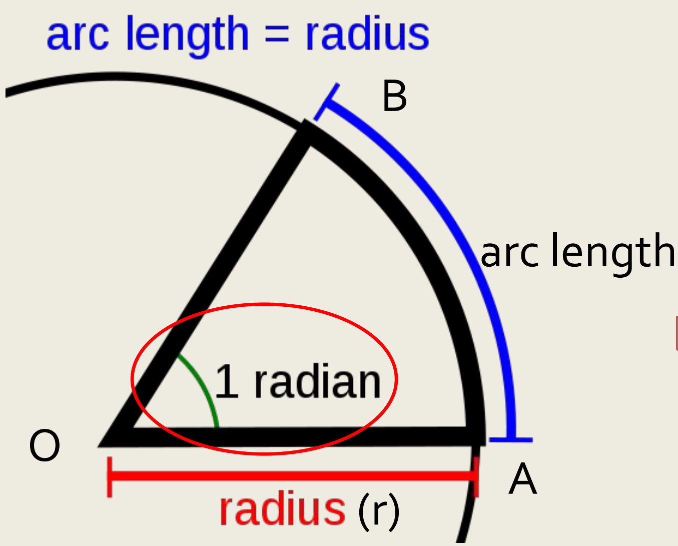
- F=ma
  - m: Mass (질량)
  - a : Acceleration (가속도)
  - 지구의 자전은 등속운동으로, 가속도가 없기 때문에 $\frac{dv}{dt}=0$, 즉 $a=0$ 이다.
  - 따라서 질량(m)이 59조 8천억톤이지만 질량이 m인 지구 위에 있는 우리가 자전에 의해 받는 힘 (F)은 zero이다.

학습내용 (2주 / 1)

Part One Modeling, Computers, and Error Analysis, 교안: 라디안, 삼각 함수

Radian

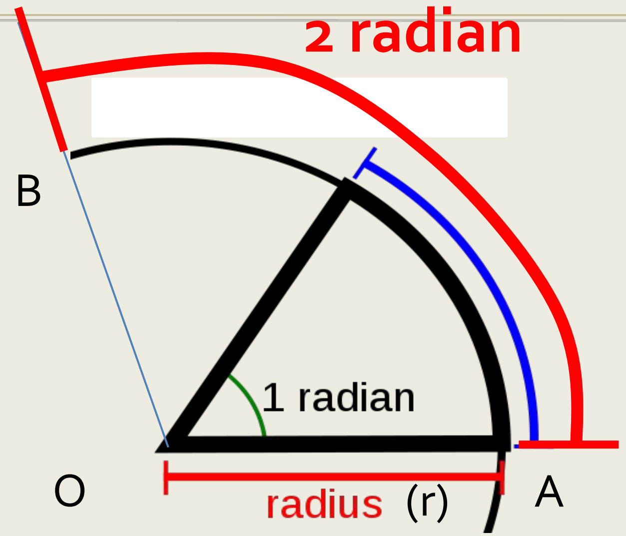
http://en.wikipedia.org/wiki/Radian

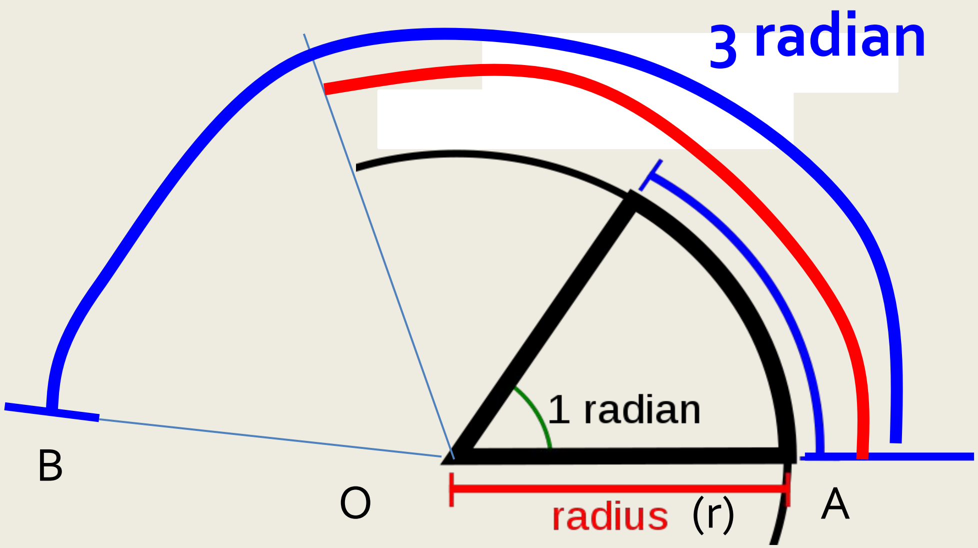


- 1 radian = 57.29 degree
  - $1 \times \frac{180^\circ}{\pi} = 57.29^\circ$

- **Radian**
  - Ratio between the length of an arc (AB) and its radius (r).
  - Radian is the standard unit of angular measure
- 1 radian = 57 degree = angle AOB
  - 반지름의 길이 (radius)와 호 AB의 길이가 같을 때, 각도 AOB의 크기를 1호도 (radian)이라 한다.
  - When the length of the radius (r) and the length of the arc AB is same, The size of the angle AOB is called as 1 radian.

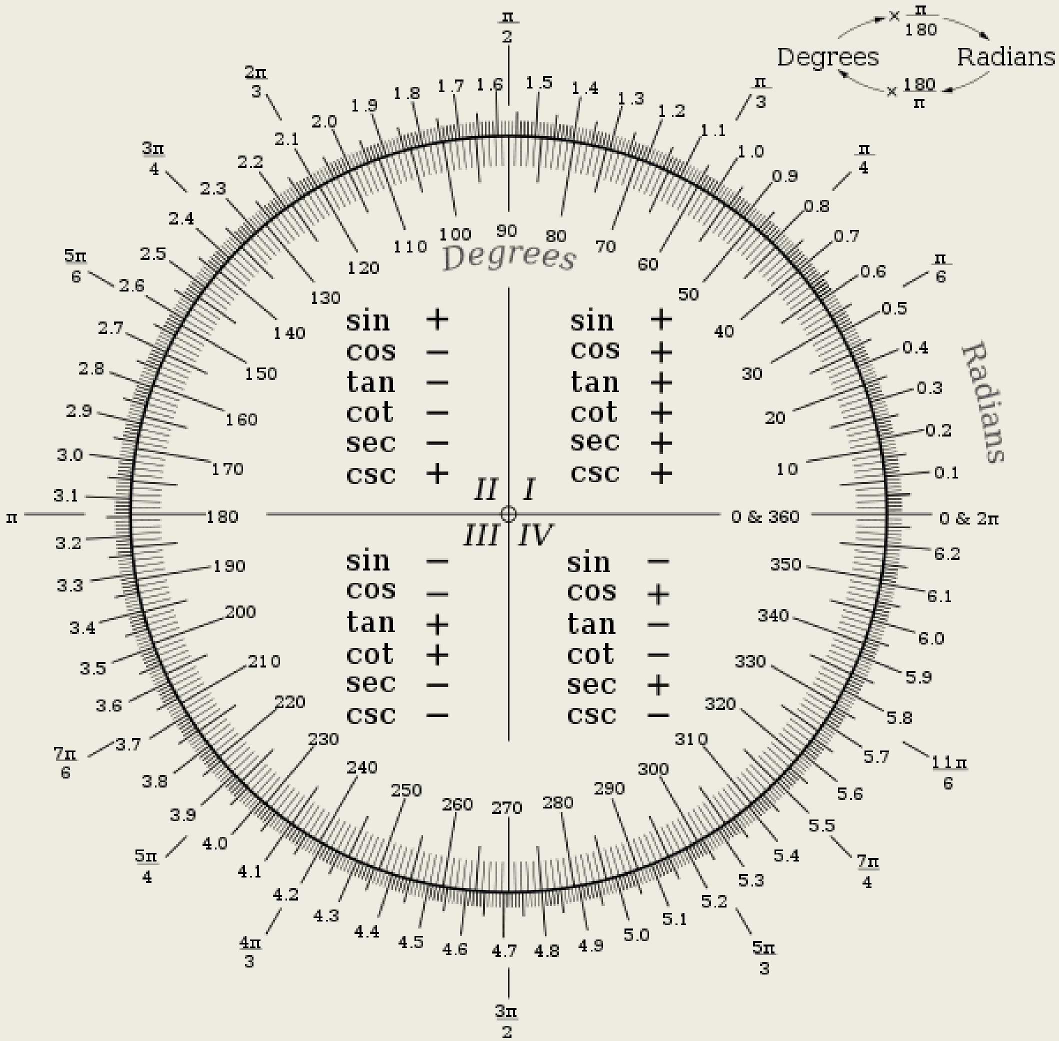
2 Radian, $\pi(3.14..)$ Radian





- 1 radian = 57.29 degree
  - $1 \times \frac{180^\circ}{\pi} = 57.29^\circ$
- 2 radian = 114.59 degree = angle AOB
  - $2 \times \frac{180^\circ}{\pi} = 114.59^\circ$
- 3 radian = 171.89 degree
  - $3 \times \frac{180^\circ}{\pi} = 171.89^\circ$
  - which is not 180°
- 3.14159 radian = 180 degree = π radian

Degree and Radian

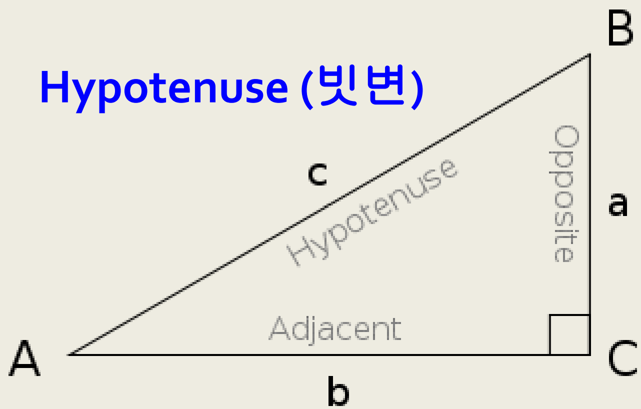


Trigonometry (삼각법)

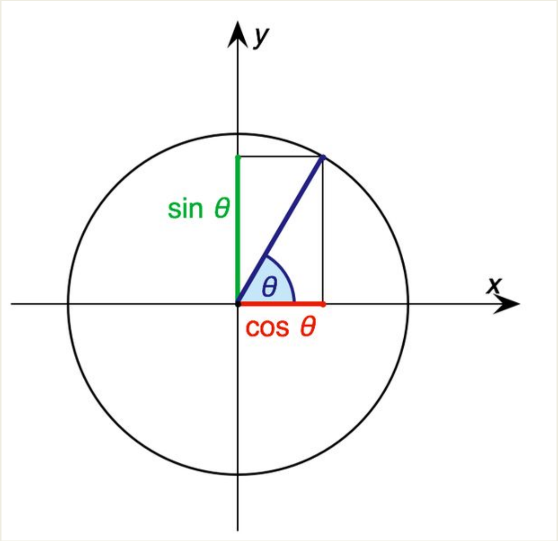
- A branch of mathematics that studies triangles and the relationships between their sides and the angles between these sides

Hypotenuse (빗변)

Height (높이)



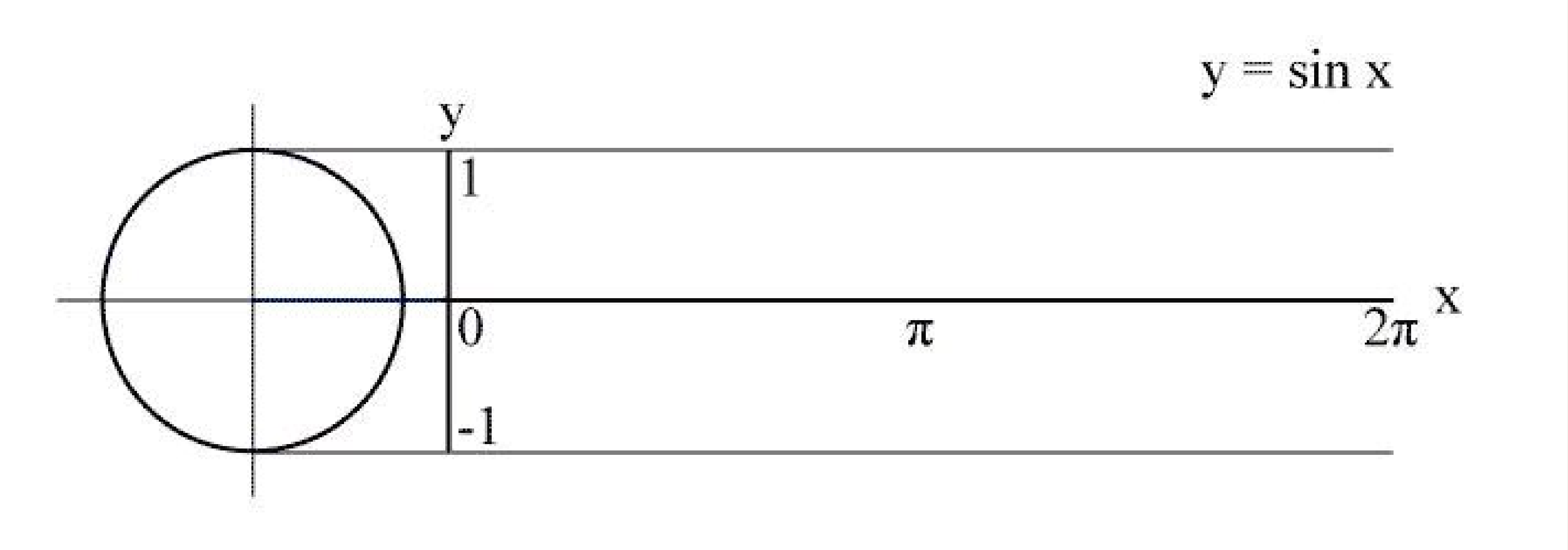
Base line (밑변)



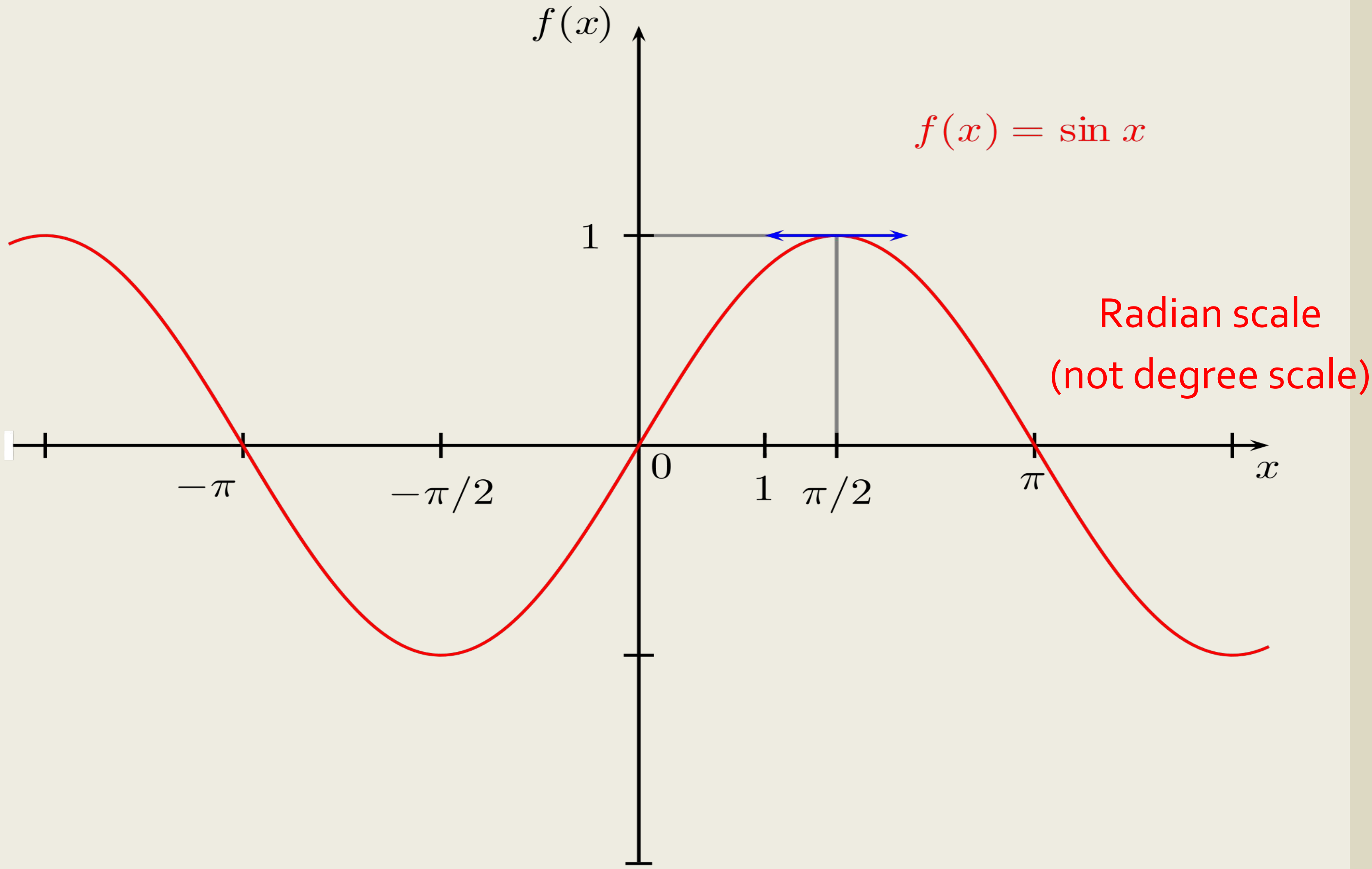
$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}\,.$$

$$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b} = \frac{\sin A}{\cos A}\,.$$

Trigonometric Function (삼각 함수)

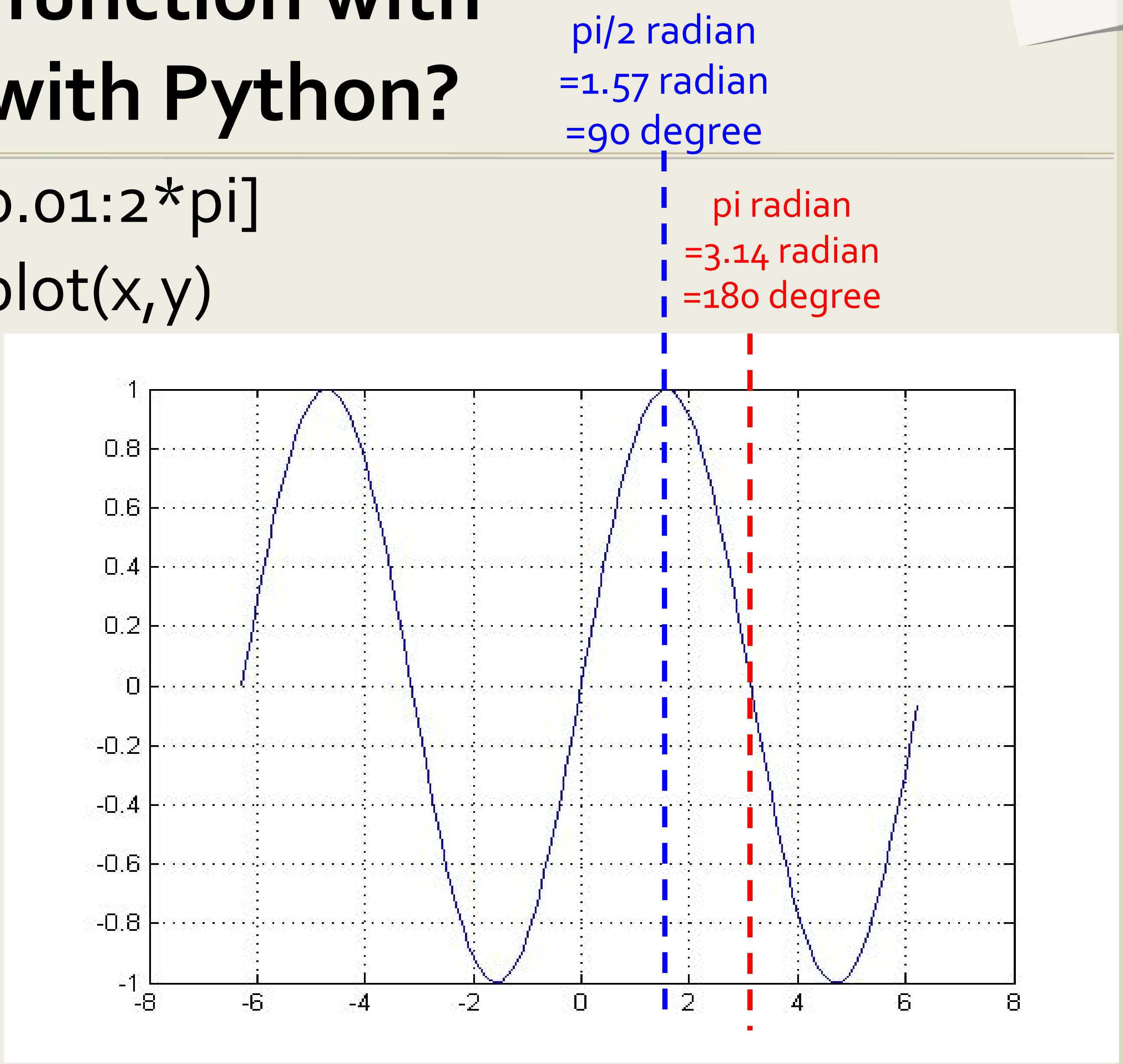


Sine function



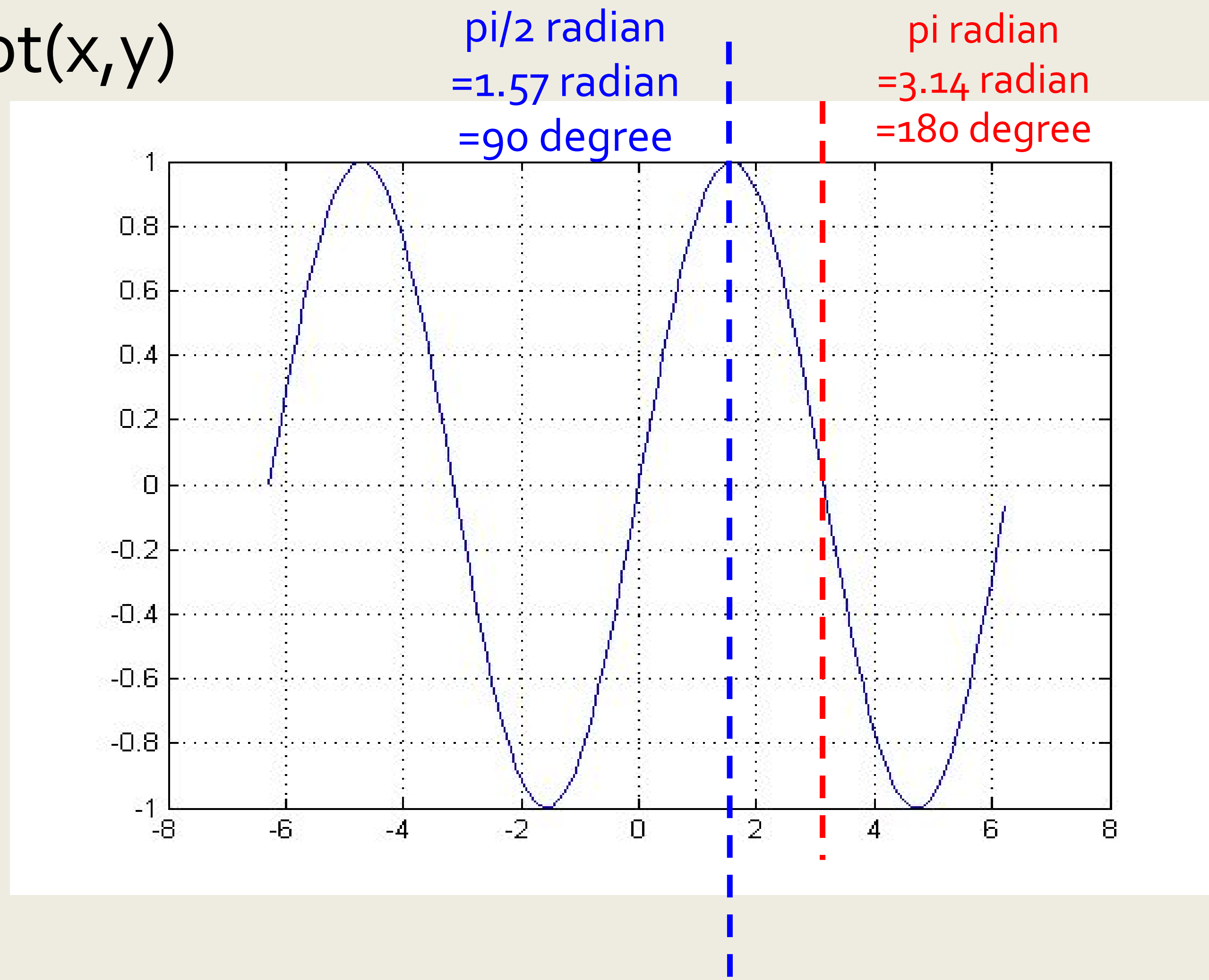
Draw Sin function with Matlab, with Python?

- x=[-2*pi: 0.01:2*pi]
- y=sin(x) , plot(x,y)



Draw Sin function with Matlab, How about Python?

- x=[-2*pi: 0.01:2*pi]
- y=sin(x) , plot(x,y)



학습내용(2주 / 2)
Part One Modeling, Computers, and Error Analysis, 교안: Python 설치

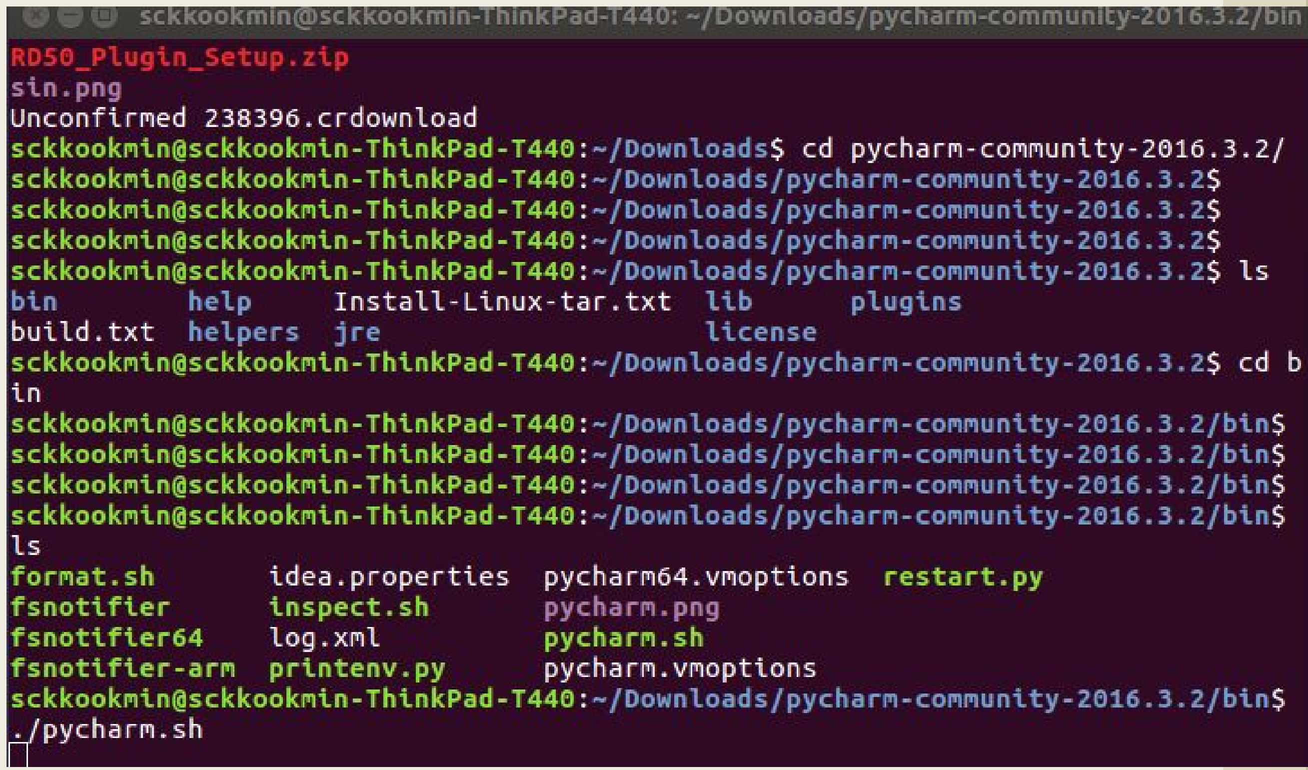
Prof. Sang-Chul Kim

2017.03.07

Drawing using Python

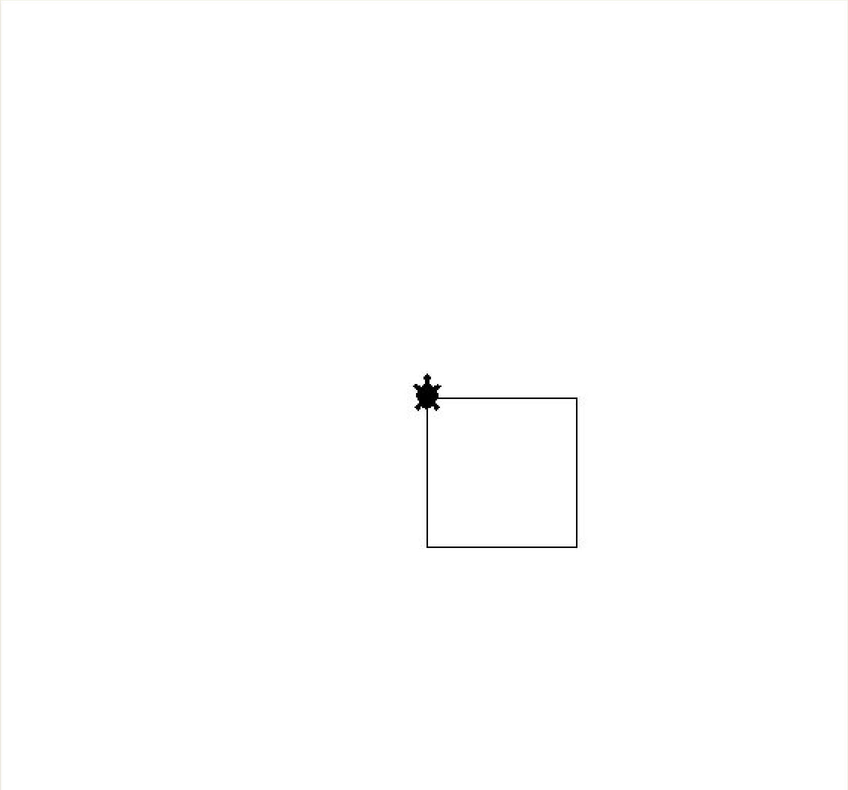
Pycharm in Ubuntu

- cd pycharm-community-2016.3.2/
- cd bin
- ./pycharm.sh



Turtle Python

- import turtle
- t=turtle.Pen()
- t.shape("turtle")
- t.forward(100)
- t.right(90)
- t.forward(100)
- t.right(90)
- t.forward(100)
- t.right(90)
- t.forward(100)

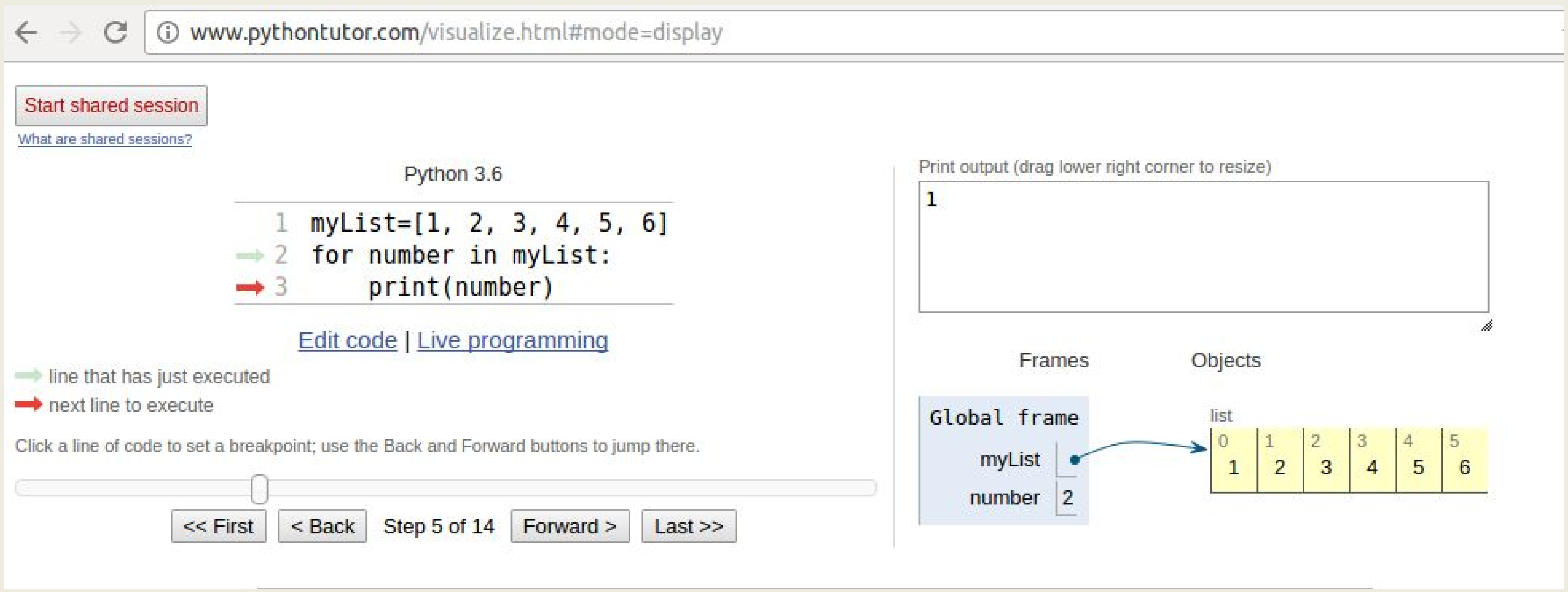


tKinter Python

- from tkinter import *
- window=Tk()
- label=Label(window, text="Hello WOrld")
- label.pack()
- window.mainloop()

Debugger Python

- python tutor
- www.pythontutor.com



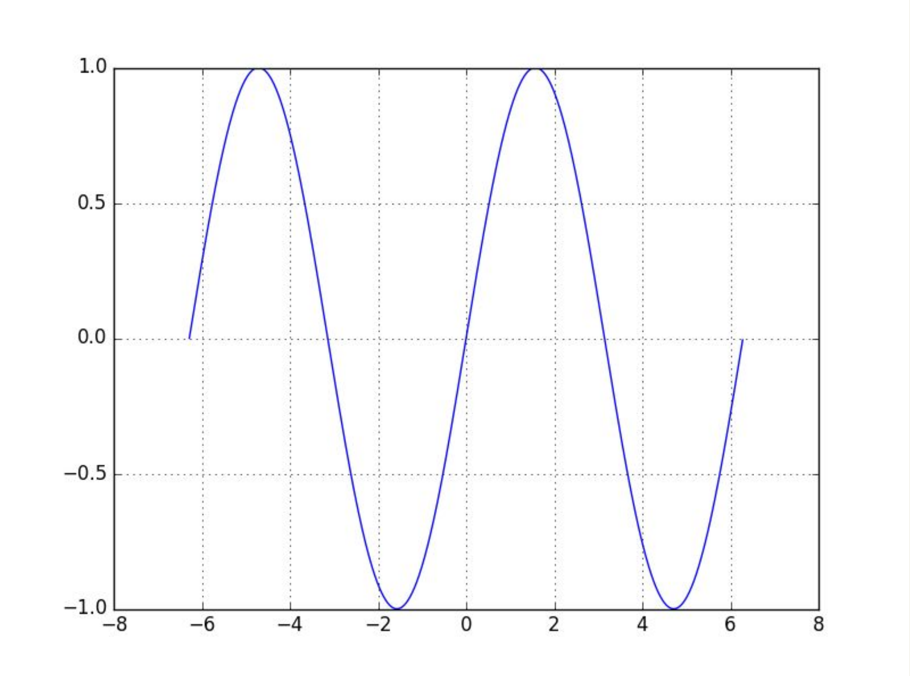
Prof. Sang-Chul Kim

2017.03.07

Plotting Sine wave using Python

Plotting Sine wave using Python

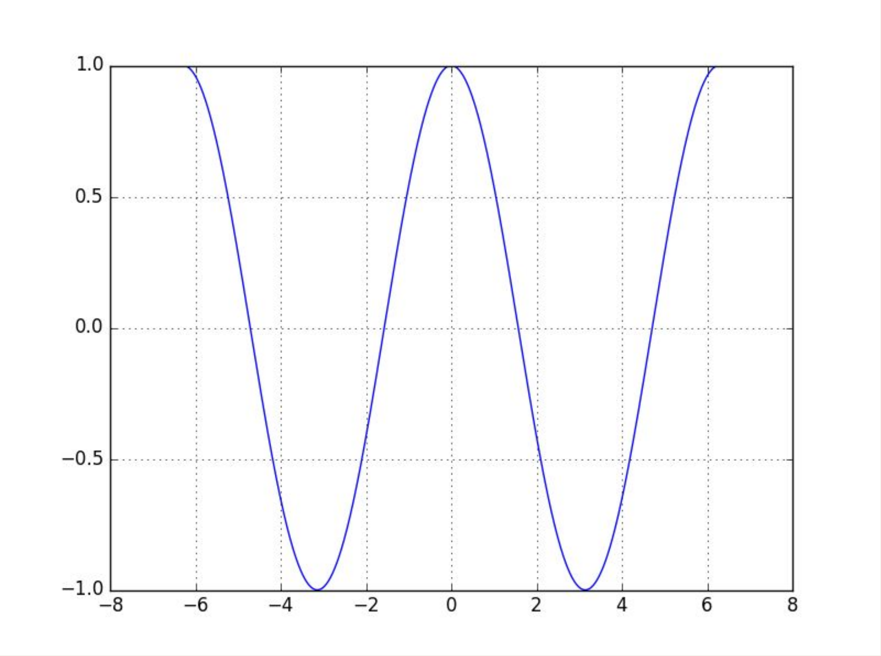
- import numpy as np
- import matplotlib.pyplot as plt
- plt.figure(1)
- t=np.arange(-2*np.pi, 2*np.pi, 0.01)
- y=np.sin(t)
- plt.plot(t,y)
- plt.grid(True)
- plt.show()



Plotting Cosine wave using Python

- import numpy as np
- import matplotlib.pyplot as plt

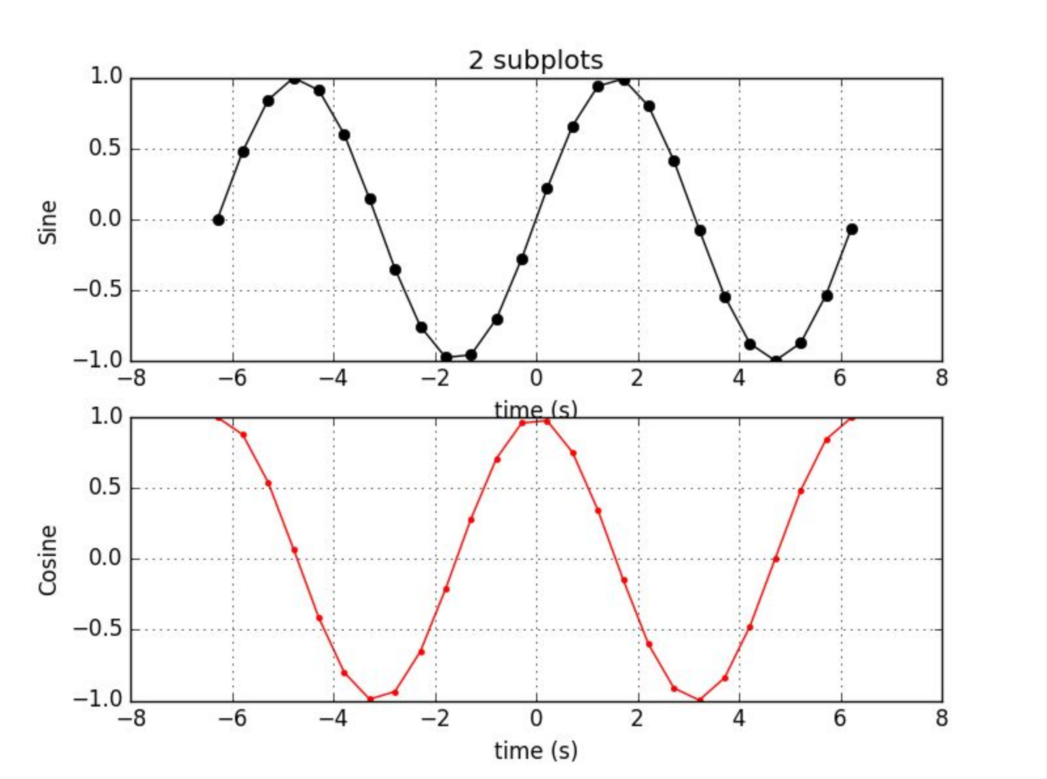
- plt.figure(2)
- t=np.arange(-2*np.pi, 2*np.pi, 0.01)
- y1=np.cos(t)
- plt.plot(t,y1)
- plt.grid(True)
- plt.show()



Two Sub-Plot for Sin and Cos

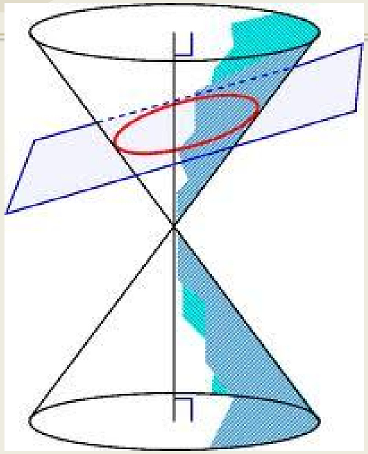
- import numpy as np
- import matplotlib.pyplot as plt
- t=np.arange(-2*np.pi, 2*np.pi, 0.5)
- y1=np.sin(t)
- y2=np.cos(t)
- plt.subplot(2, 1, 1)
- plt.plot(t, y1, 'ko-')
- plt.title('2 subplots')
- plt.xlabel('time (s)')
- plt.ylabel('Sine')
- plt.grid(True)

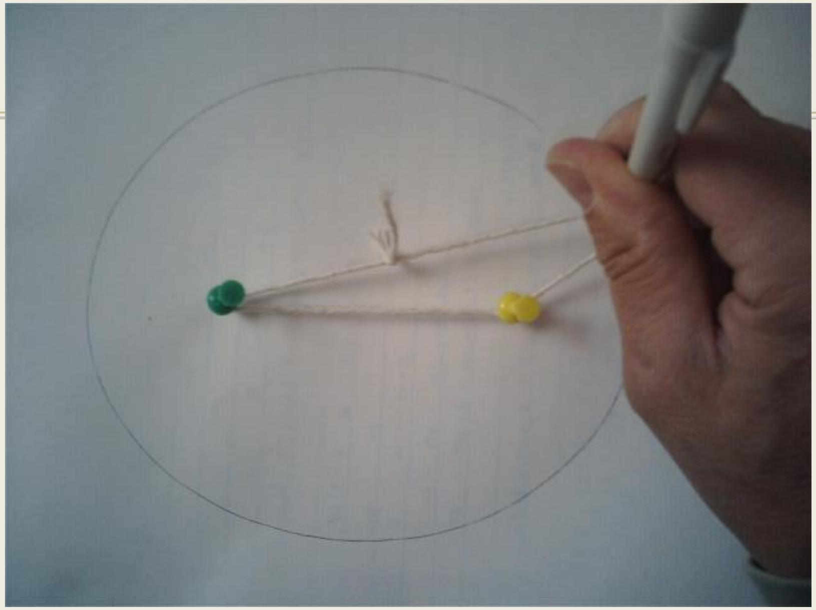
- plt.subplot(2, 1, 2)
- plt.plot(t, y2, 'r.-')
- plt.xlabel('time (s)')
- plt.ylabel('Cosine')
- plt.grid(True)
- plt.show()



학습내용(3주 / 1)
Part One Modeling, Computers, and Error Analysis,
교안: 타원, 쌍곡선, 쌍곡선 함수

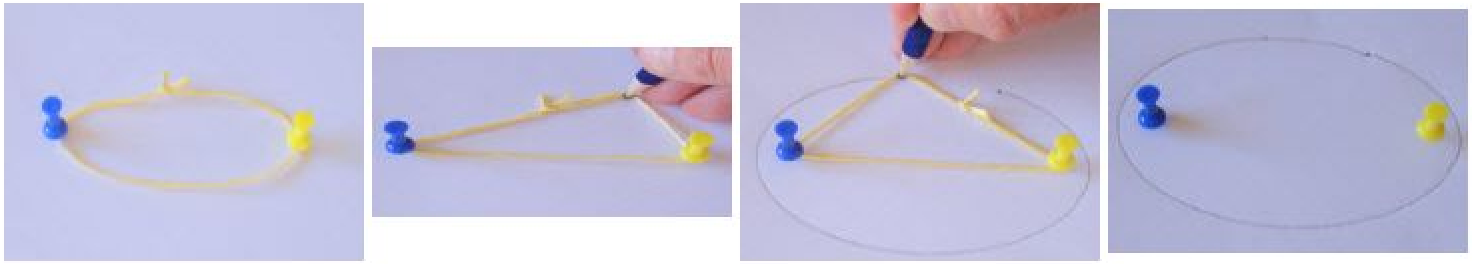
타원 (Ellipse)



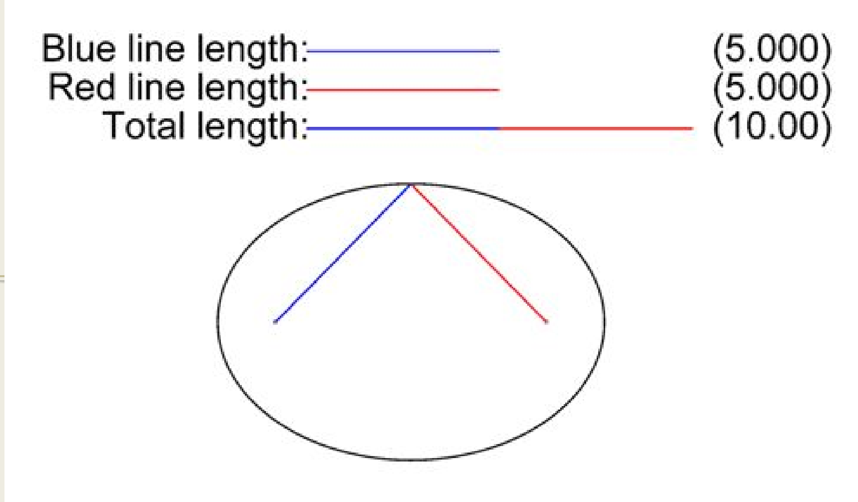


You Can Draw It Yourself

Put two pins in a board, put a loop of string around them, and insert a pencil into the loop. Keep the string stretched so it forms a triangle, and draw a curve ... you will draw an ellipse.



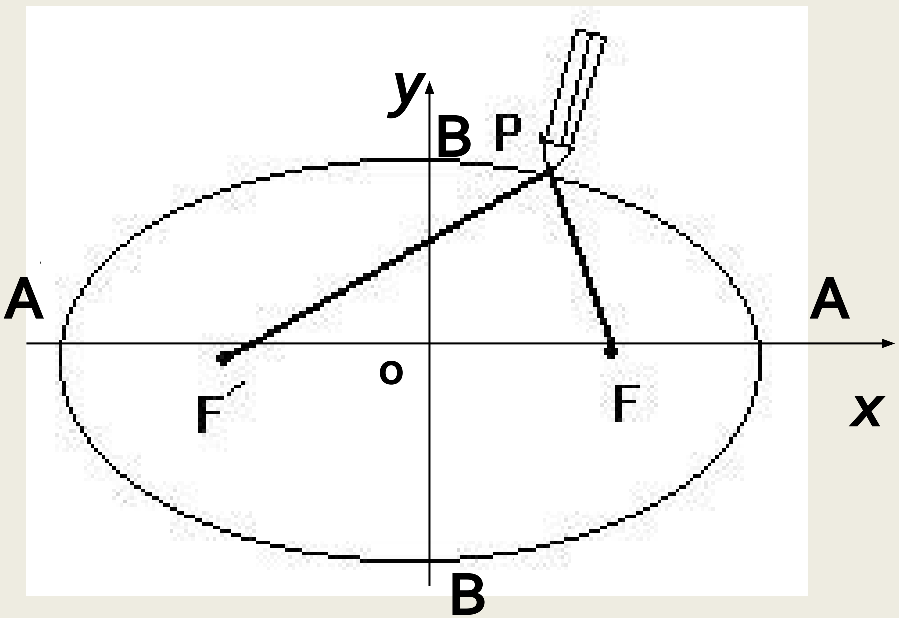
It works because the string naturally forces the **same distance** from **pin-to-pencil-to-other-pin**.



타원 (Ellipse)

A set of points providing constant length **sum** (major axis length) from two vertices F, F ' above the plane

평면 위에서 두 정점 F, F'으로 부터의 거리의 합(장축의 길이)이 일정한 점들의 집합.



F, F' : 타원의 **초점 (focus)**

네 점 A, A', B, B' : 타원의 **꼭지점 (vertex)**

$\overline{AA'}$: 장축 (major axis)

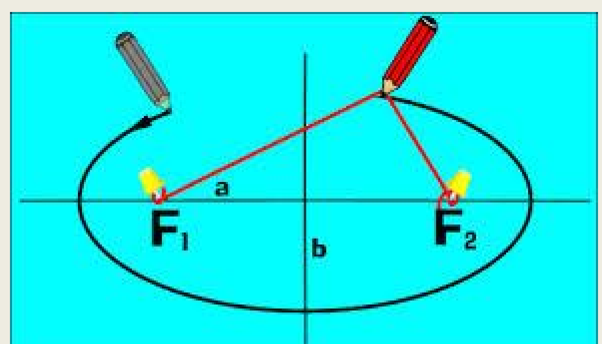
$\overline{BB'}$: 단축 (minor axis)

O : 타원의 중심 (center)

$\overline{PF} + \overline{PF'} =$ (□□□ □□)

$=$ constantlengthsum $= \overline{AA'}$

$=$ majoraxislength



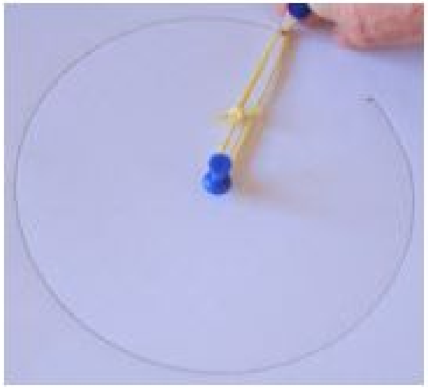
Circle

A circle is a simple closed shape in Euclidean geometry. It is the set of all points in a plane that are at a given distance from a given point, the centre; equivalently it is the curve traced out by a point that moves so that its distance from a given point is constant. The distance between any of the points and the centre is called the radius.

A Circle is an Ellipse

In fact a Circle **is** an Ellipse, where both foci are at the same point (the center).

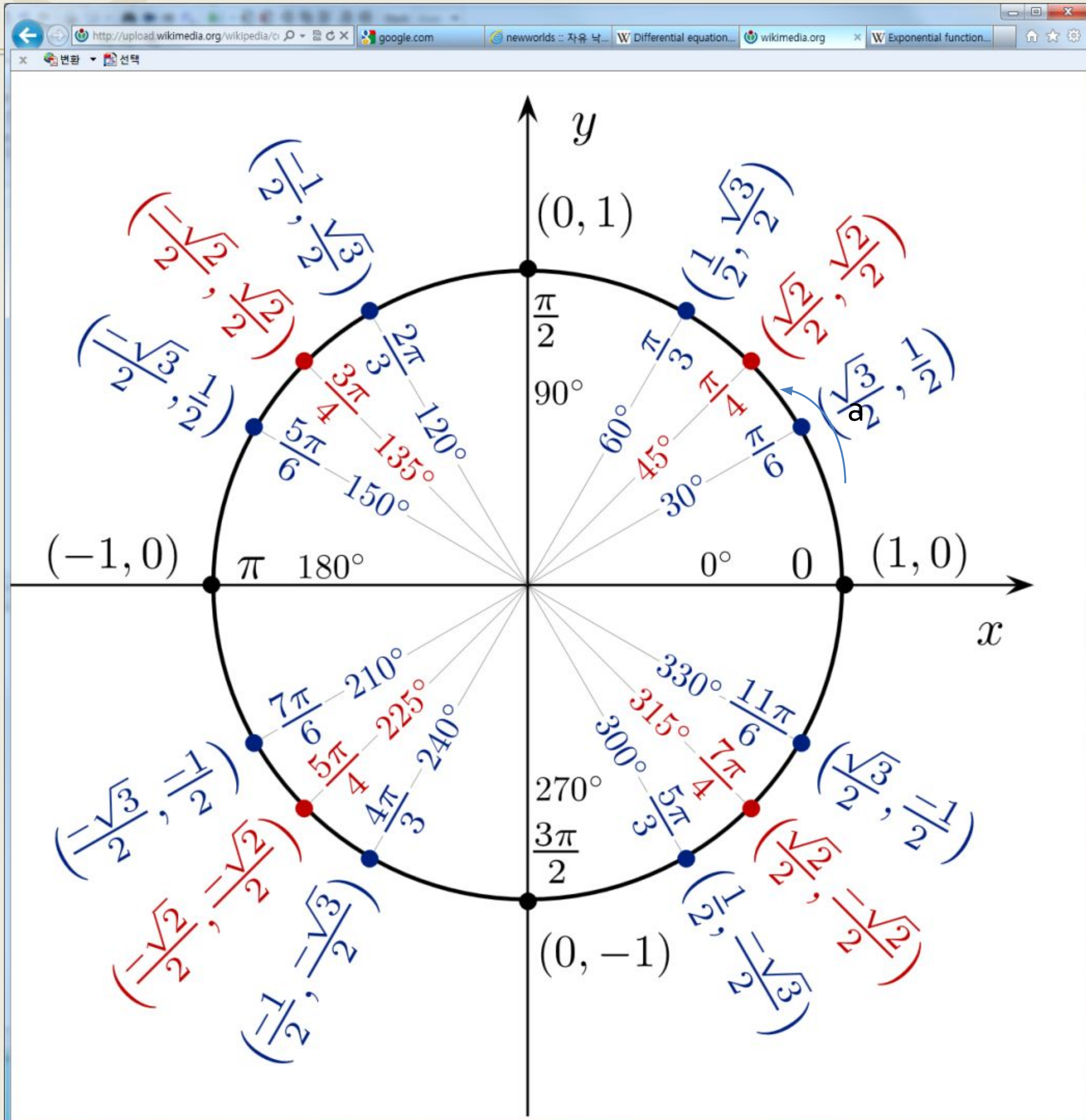
In other words, a circle is a "special case" of an ellipse. Ellipses Rule!



Definition

An ellipse is the **set of all points** on a plane whose distance from two fixed points F and G add up to a constant.

Unit Circle



Trigonometric
function
: sin, cos, tan

$(cosa, sina)$

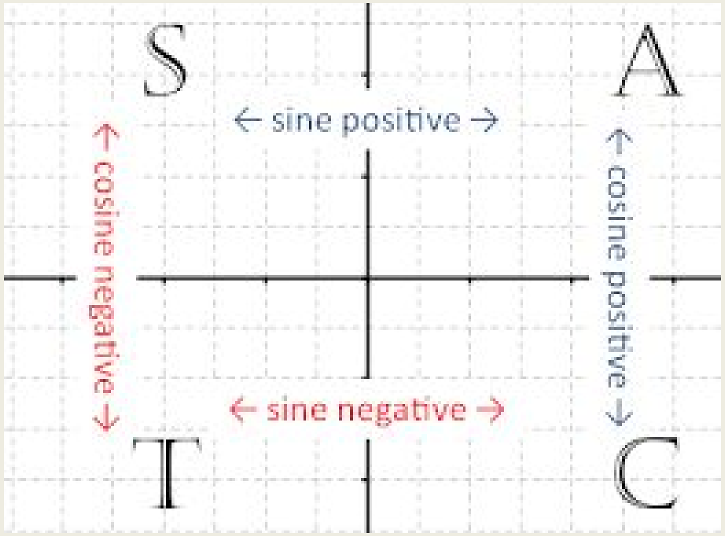
$x^2+y^2=1$

$cosa^2+sina^2=1$

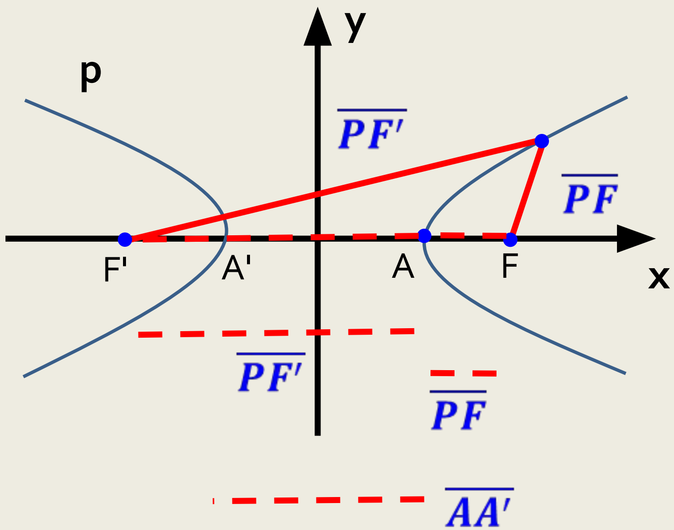
원 위의 점 x좌표, y좌표의 값을 제곱해서 플러스한 값이 1이 되는 원의 x좌표를 cos으로 표시하고, 원의 y좌표를 sin으로 표시한다.

Unit Circle

$\theta°$	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined



쌍곡선 (Hyperbolic)



$$\overline{PF} - \overline{PF'} = (\square\square\square \quad \square\square \quad)$$
$$= \text{constant length sum} = \overline{AA'}$$
$$= \text{principal axis length}$$

평면 위에서 두 정점 F, F' 으로 부터의 거리의 차(주축의 길이)가 일정한 점들의 집합.

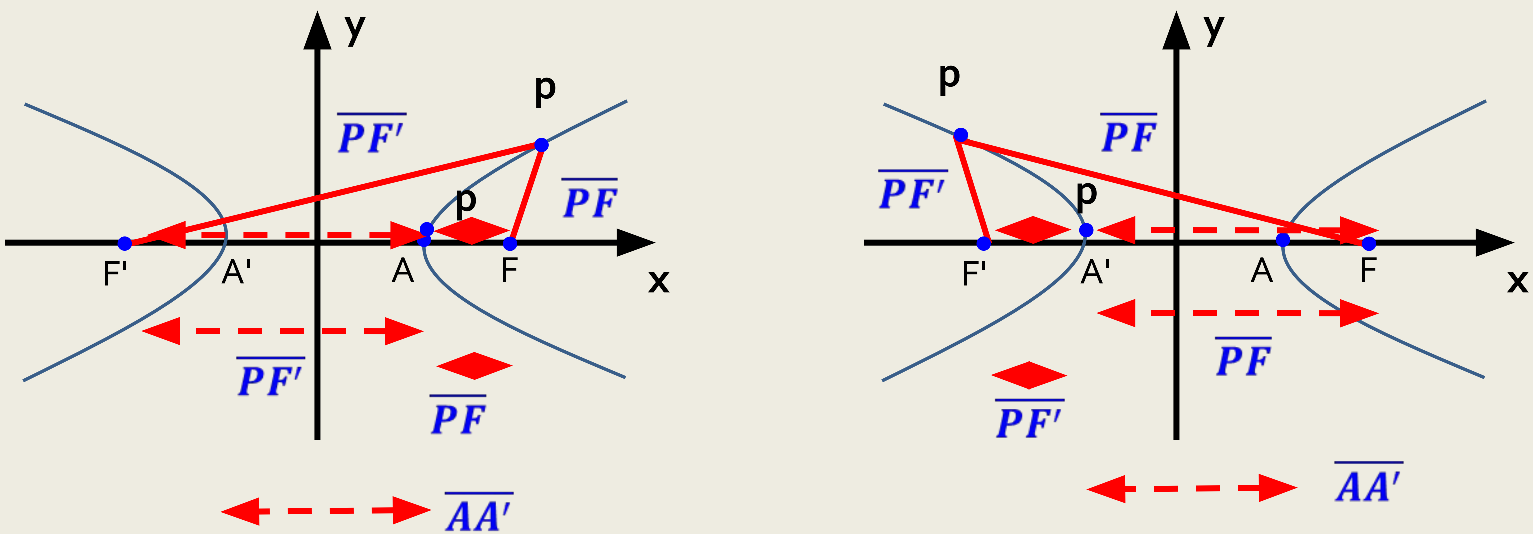
A set of points providing constant length **difference** (principal axis length) from two vertices F, F ' above the plane

F, F' : 초점(focus)

A, A' : 꼭지점 (vertex)

$\overline{AA'}$: 주축 (principal axis)

쌍곡선 (Hyperbolic)

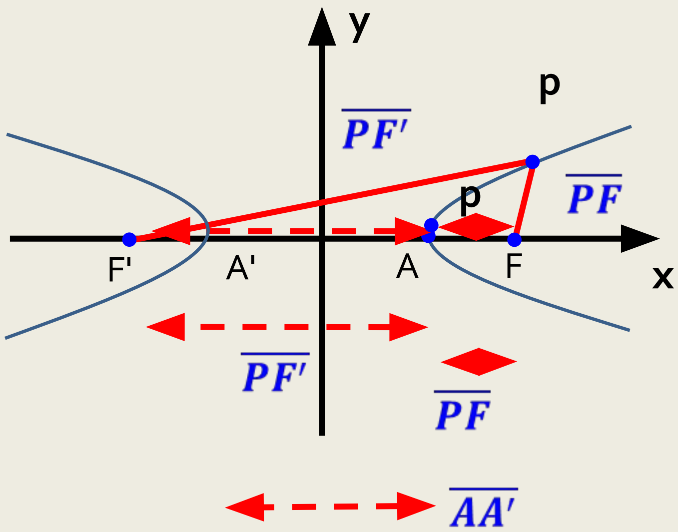


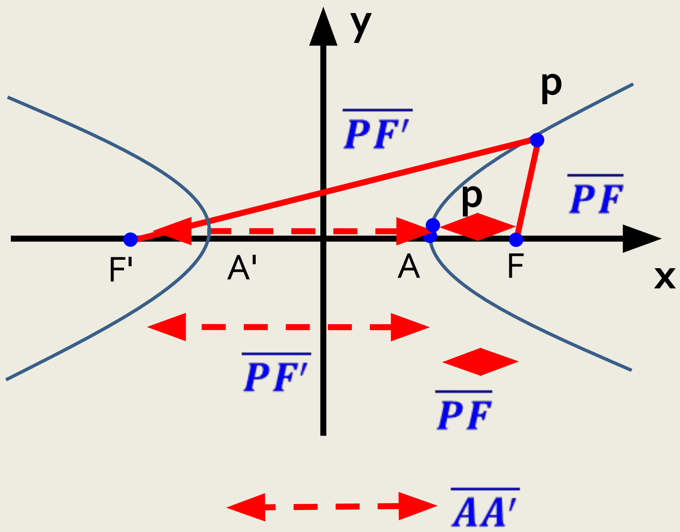
$\overline{PF} - \overline{PF'}$ = (□□□ □□)

$=$ constant length sum $= \overline{AA'}$

$=$ principal axis length

쌍곡선 (Hyperbolic)

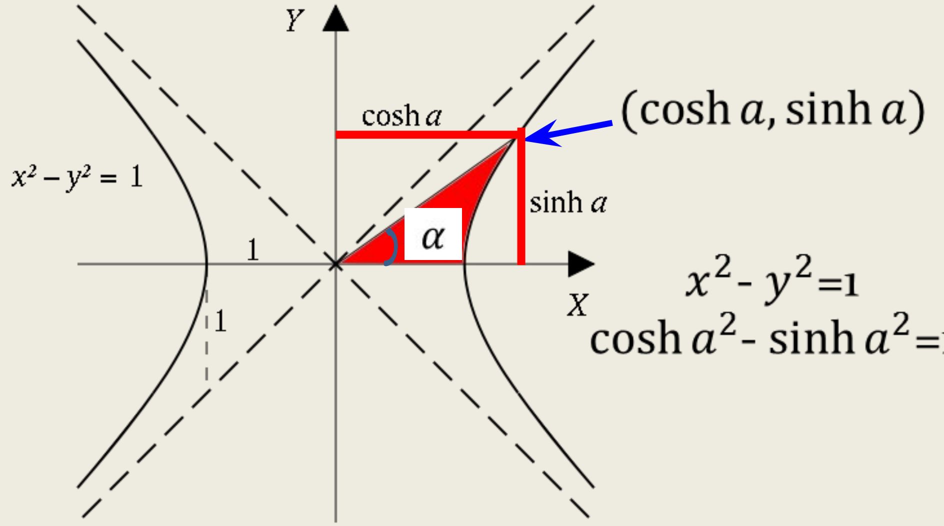




쌍곡선 함수 (Hyperbolic function)

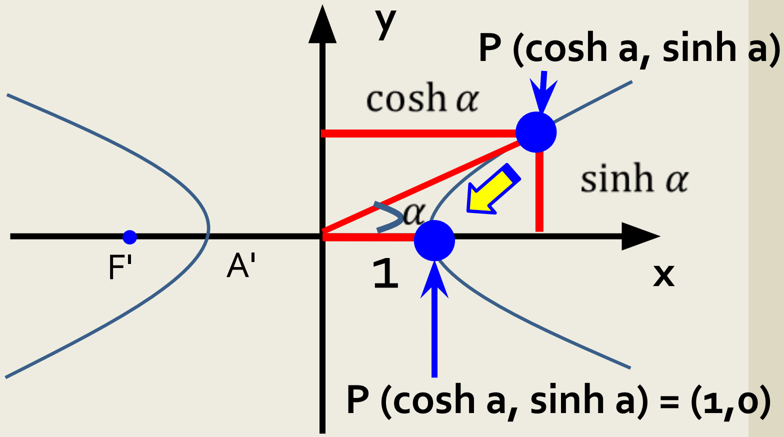
쌍곡선 위의 점 x좌표, y좌표의 값을 제곱해서 마이너스 한 값이 1이 되는 쌍곡선의 x좌표를 cosh으로 표시하고, 쌍곡선의 y좌표를 sinh으로 표시한다.

Hyperbolic: The difference of squared vales of x and y coordinate gives one
A set of x coordinate point of hyperbolic is called as **cosine hyperbolic (cosh)**.
A set of y coordinate point of hyperbolic is called as **sine hyperbolic (sinh)**.



$x^2 - y^2 = 1$
$\cosh a^2 - \sinh a^2 = 1$

Asymptote



$\cosh\alpha = \cosh 0 = 1$

$\sinh\alpha = \sinh 0 = 0$

쌍곡선 함수

종류 [편집]

삼각함수(*원함수*)의 사인, 코사인, 탄젠트 등에서 유추되어 각각에 대응되는 다음과 같은 함수가 있다.

- 쌍곡사인(hyperbolic sine)

$$\sinh x = \frac{e^x - e^{-x}}{2} = -i \sin ix$$

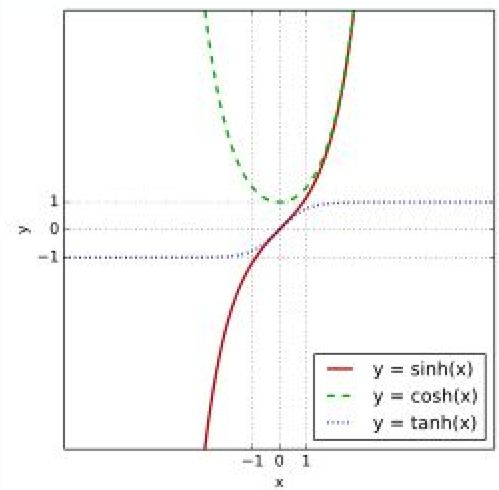
- 쌍곡코사인(hyperbolic cosine)

$$\cosh x = \frac{e^x + e^{-x}}{2} = \cos ix$$

- 쌍곡탄젠트(hyperbolic tangent)

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$= \frac{\frac{e^x - e^{-x}}{2}}{\frac{e^x + e^{-x}}{2}} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1} = -i \tan ix$$



sinh, cosh, tanh

$$\sinh x = \frac{e^x - e^{-x}}{2} = \frac{e^{2x} - 1}{2e^x}$$

$$\int \frac{du}{a^2 - u^2} = a^{-1} \tanh^{-1}\left(\frac{u}{a}\right) + C; u^2 < a^2$$

$$\cosh x = \frac{e^x + e^{-x}}{2} = \frac{e^{2x} + 1}{2e^x}$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1}$$

쌍곡선 함수

삼각함수와의 관계 [편집]

2차원 평면상에서 매개변수 t를 사용한 자취 $(\cos t, \sin t)$가 단위원 $x^2 + y^2 = 1$을 그리는 것처럼, $(\cosh t, \sinh t)$은 쌍곡선 $x^2 - y^2 = 1$ 을 그린다. 이는 다음과 같은 간단한 관계를 통해 쉽게 알 수 있다.

$$\cosh^2 t - \sinh^2 t = 1$$

그러나 쌍곡선함수는 삼각함수와 달리 주기함수가 아니라는 차이가 있다.

$\cosh x$는 짝함수 즉 y축에 대해 대칭이며, $\cosh 0 = 1$ 이다.

$\sinh y$는 홀함수 즉 원점에 대해 대칭이며, $\sinh 0 = 0$ 이다.

쌍곡선함수는 삼각함수 공식과 매우 유사한 항등식을 만족한다. 실제로 **오스본 법칙**에 따라 어떤 삼각함수 항등식이라도 쌍곡선 항등식으로 변환될 수 있다. 예를 들어 삼각함수의 덧셈정리와 반각공식은 다음과 같은 쌍곡선함수의 덧셈 정리와 반각 공식으로 바뀐다.

- 덧셈 정리

$$\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y$$
$$\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$$
$$\tanh(x+y) = \frac{\tanh x + \tanh y}{1 + \tanh x \tanh y}$$

- 반각 공식

$$\cosh^2 \frac{x}{2} = \frac{\cosh x + 1}{2}$$
$$\sinh^2 \frac{x}{2} = \frac{\cosh x - 1}{2}$$

sinh, cosh 특성

- Matlab

(cosh(1.2*pi))^2-(sinh(1.2*pi))^2=1

(cosh(2*pi))^2-(sinh(2*pi))^2=1

- Python

```
>>> (np.cosh(1.2*np.pi))**2-(np.sinh(1.2*np.pi))**2
0.99999999999994316

>>> (np.cosh(2*np.pi))**2-(np.sinh(2*np.pi))**2
1.0000000000145519
```

$$\sinh x = \frac{e^x - e^{-x}}{2} = \frac{e^{2x} - 1}{2e^x}$$

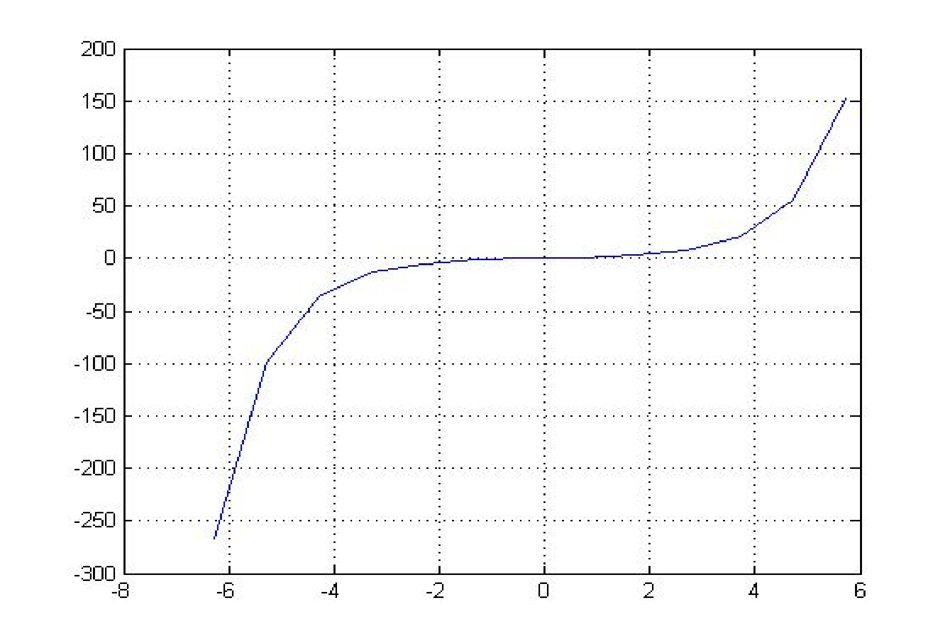
$$\int \frac{du}{a^2 - u^2} = a^{-1} \tanh^{-1}\left(\frac{u}{a}\right) + C; u^2 < a^2$$

$$\cosh x = \frac{e^x + e^{-x}}{2} = \frac{e^{2x} + 1}{2e^x}$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1}$$

sinh(x) function

- x=[-2*pi: 2*pi] (or x=[-2*pi: 0.01: 2*pi] for more detail)
  - [-6.2832 -5.2832 -4.2832 -3.2832 -2.2832 -1.2832 -0.2832 0.7168 1.7168 2.7168 3.7168 4.7168 5.7168]
- y1=sinh(x), plot(x, y1)
  - [-267.7449 -98.4956 -36.2286 -13.3115 -4.8530 -1.6655 -0.2870 0.7798 2.6936 7.5330 20.5544 55.9013 151.9660]



지수 (Exponential)

- $e^0 = 1$
- $e^1 = 2.71828^{1}$
- $e^2 = 2.71828^{2} = 7.3891...$
- $e^3 = 2.71828^{3} = 20.0855...$
- $e^4 = 2.71828^{4} = 54.5982...$

- Matlab

- exp(1)
- exp(1)^2
- exp(1)^3

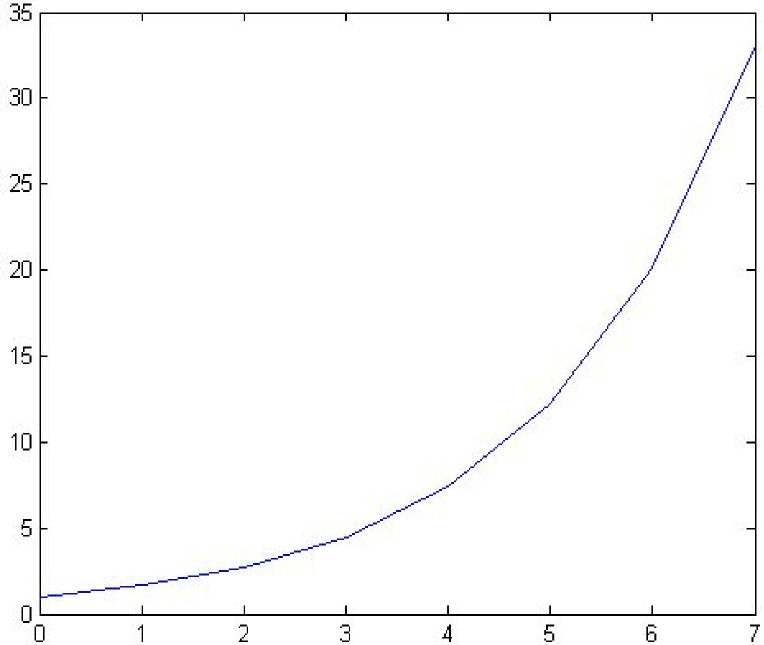
- Python

```
>>> np.exp(0)
1.0
>>> np.exp(1)
 2.718281828459045
>>>  np.exp(2)
 7.38905609893065
```

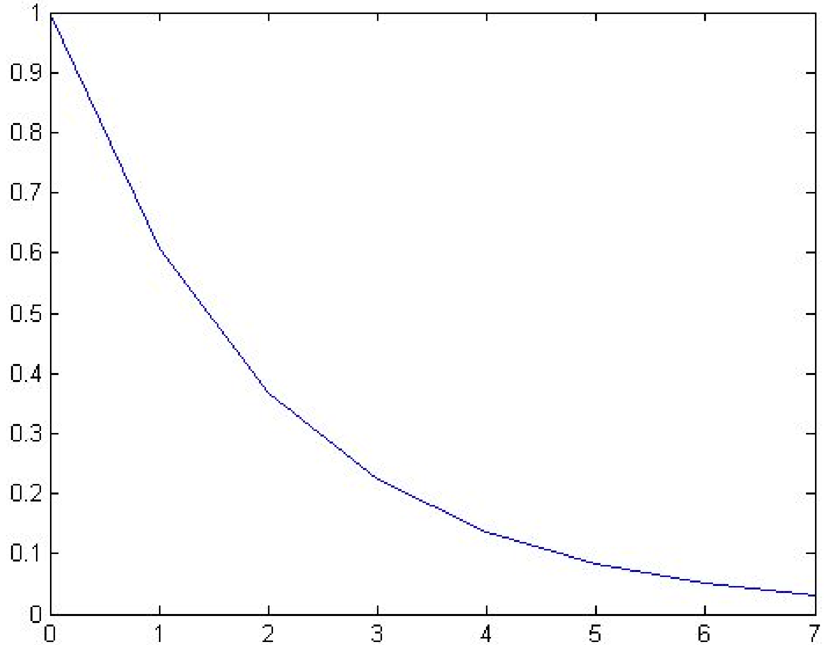
지수 함수 (Exponential function)

- $P=e^{0.5t}$
- $P=e^{-0.5t}$

P



P



t

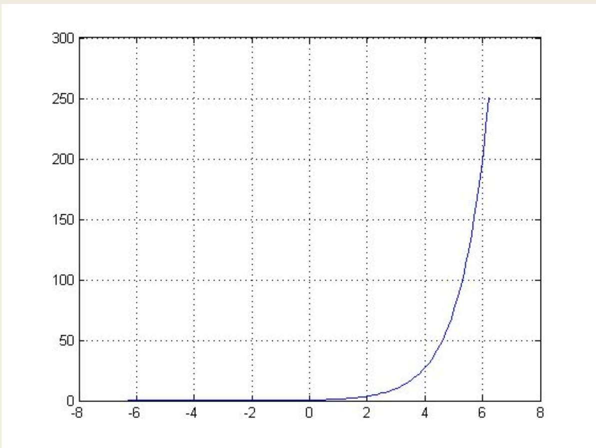
t

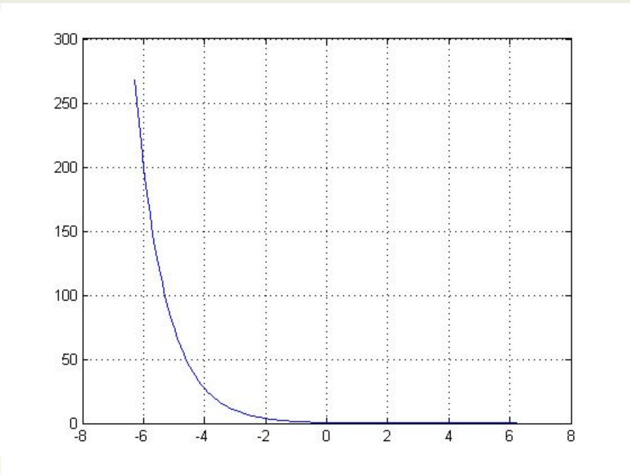
t=[0:7] p=exp(0.5*t) plot(t,p)

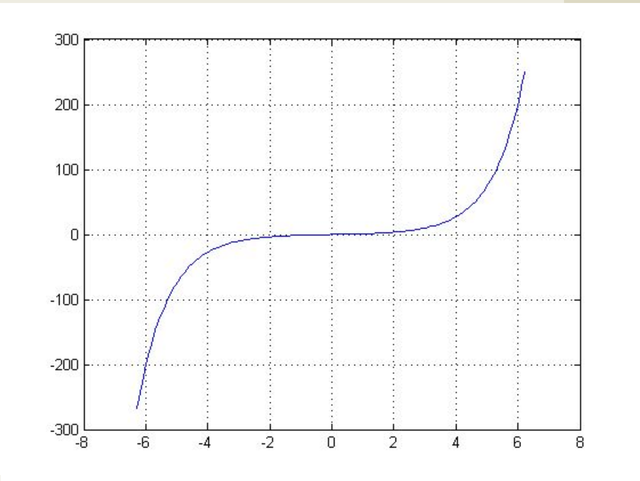
t=[0:7] p=exp(-0.5*t) plot(t,p)

쌍곡선 함수와 지수 함수의 관계

$$\sinh x = \frac{e^{x} - e^{-x}}{2}$$







$\cosh \alpha$

$\cosh \alpha = \cosh 0 = 1$

α

x=[-8: 8]

y1=exp(x)/2

y2=exp(-x)/2

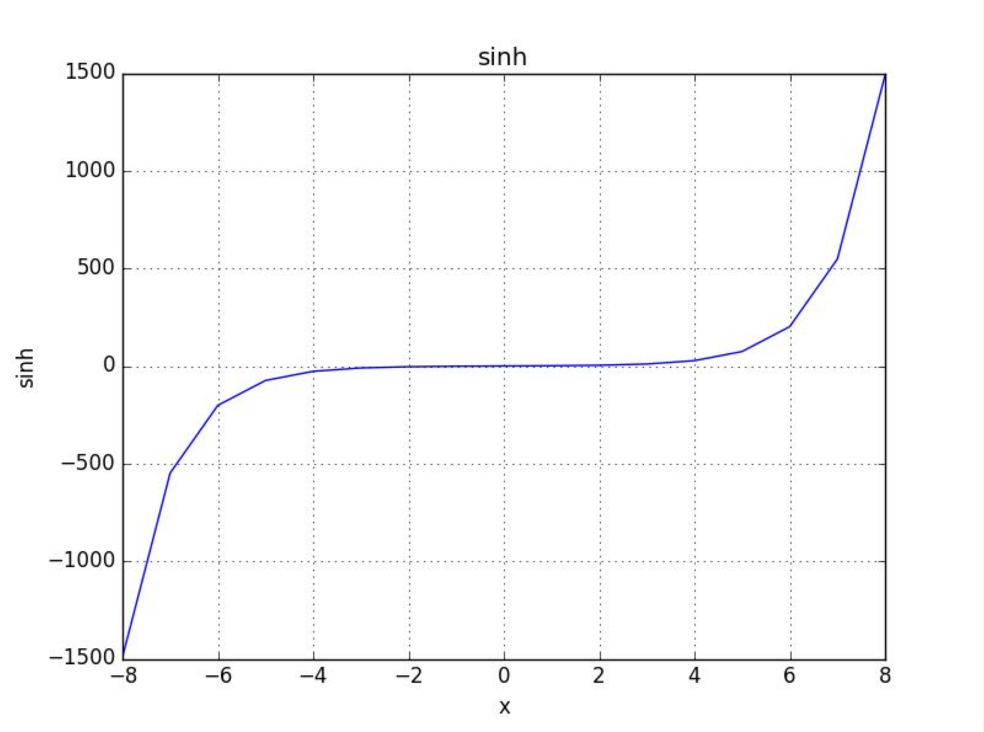
y3=y1-y2

plot(x,y3)

Plot Sinh

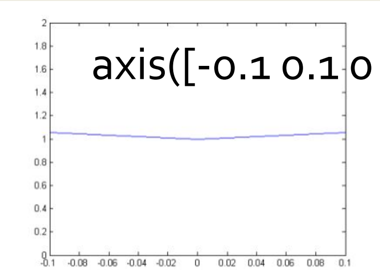
- import numpy as np
- import matplotlib.pyplot as plt
- x=np.arange(-8,8+1,1)
- y1=np.exp(x)/2
- y2=np.exp(-x)/2
- y3=y1-y2

- plt.figure(1)
- plt.plot(x, y3)
- plt.xlabel('x')
- plt.ylabel('sinh')
- plt.title('sinh')
- plt.grid(True)
- plt.show()

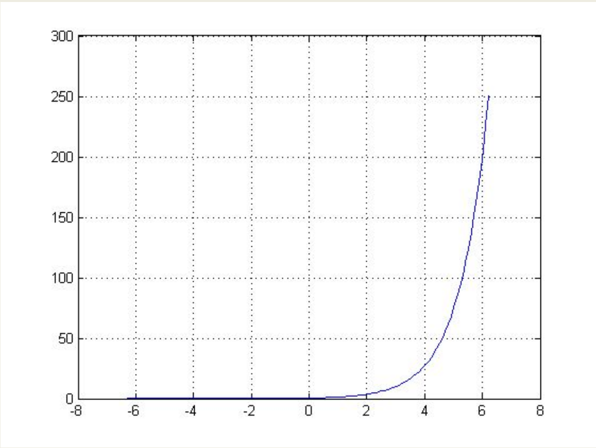


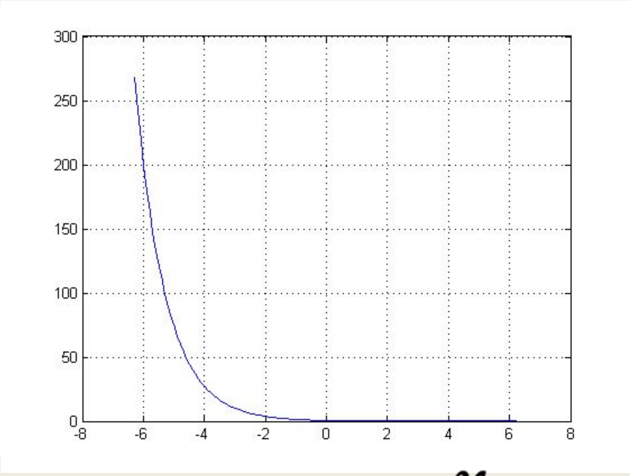
쌍곡선 함수와 지수 함수의 관계

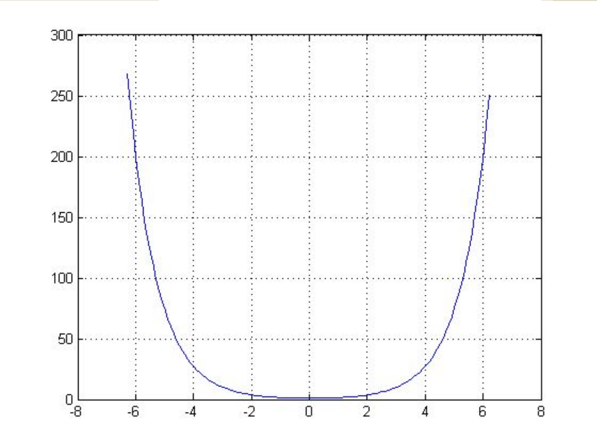
$$\cosh x = \frac{e^x + e^{-x}}{2}$$



axis([-0.1 0.1 0 2])







$$\frac{e^x}{2}$$

$$\frac{e^{-x}}{2}$$

$$\frac{e^x + e}{2}$$

x=[-8: 8]

y1=exp(x)/2

y2=exp(-x)/2

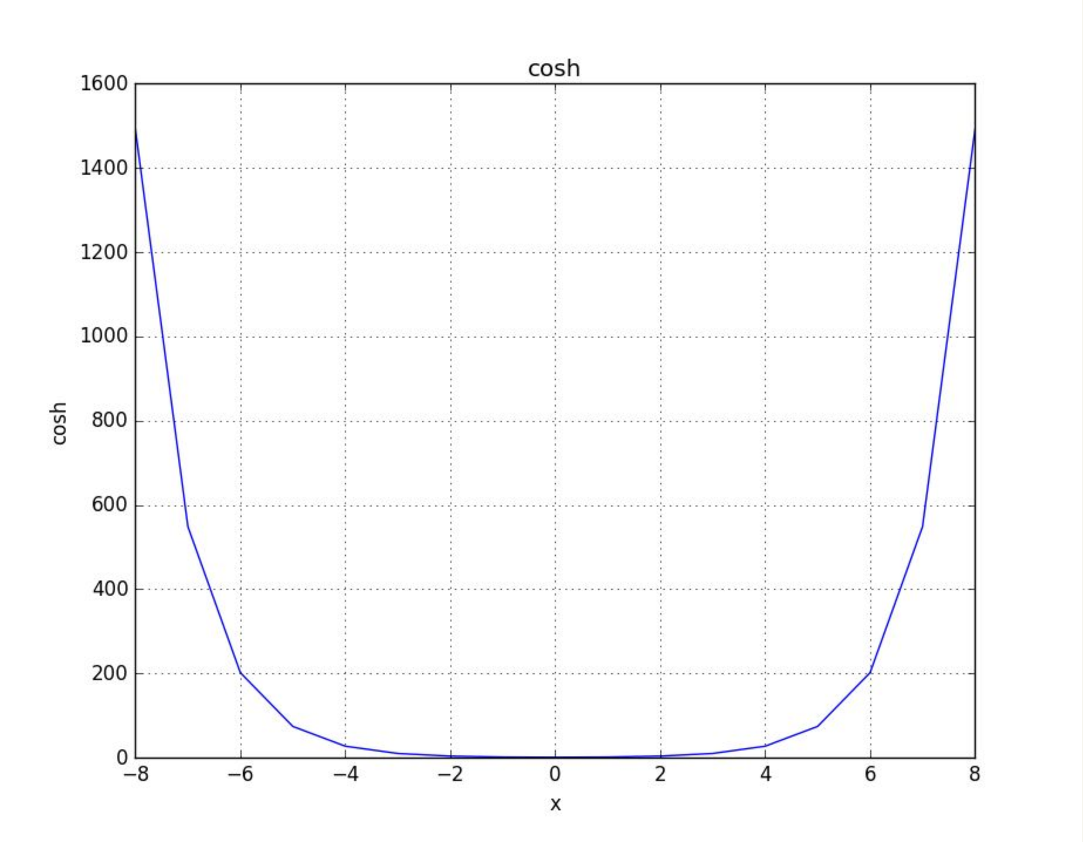
y4=y1+y2

plot(x,y4)

Plot cosh()

- import numpy as np
- import matplotlib.pyplot as plt
- x=np.arange(-8,8+1,1)
- y1=np.exp(x)/2
- y2=np.exp(-x)/2
- y4=y1+y2
- plt.figure(1)
- plt.plot(x, y4)
- plt.xlabel('x')
- plt.ylabel('cosh')
- plt.title('cosh')
- plt.grid(True)

- plt.show()

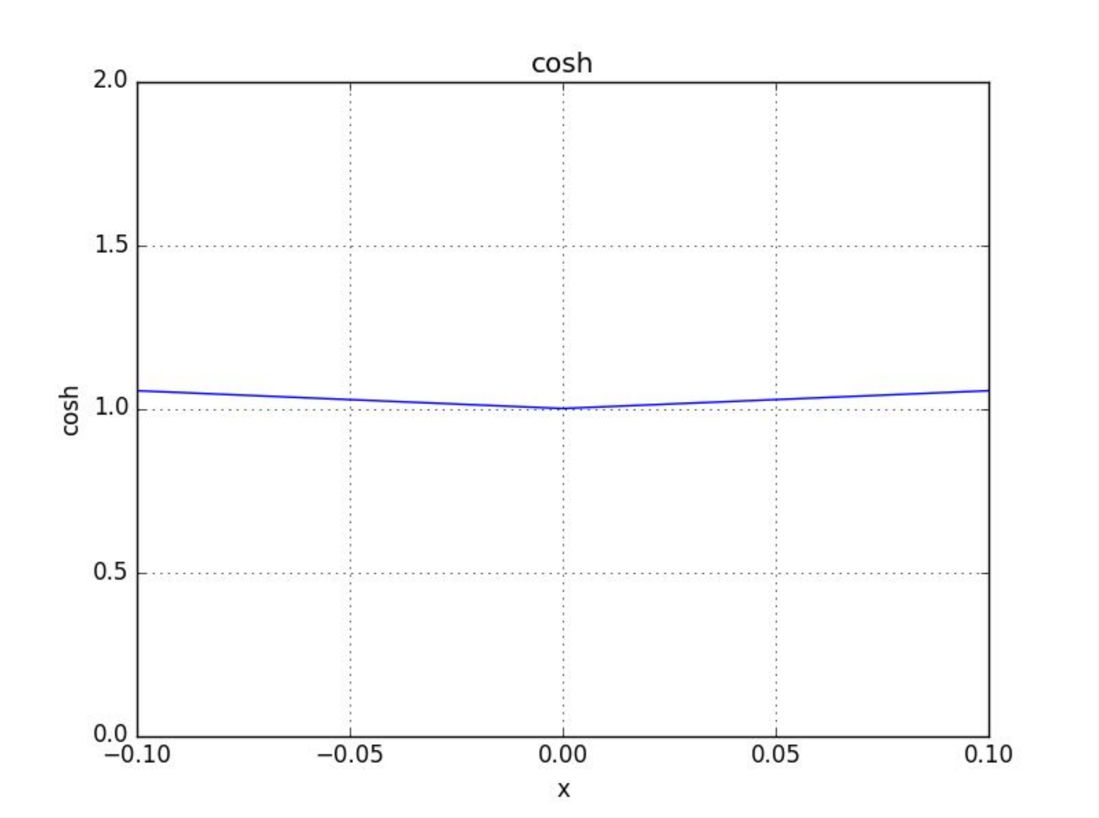


xlim & ylim

- import numpy as np
- import matplotlib.pyplot as plt
- x=np.arange(-8,8+1,1)
- y1=np.exp(x)/2
- y2=np.exp(-x)/2
- y4=y1+y2
- plt.figure(1)
- plt.plot(x, y4)
- plt.xlabel('x')
- plt.ylabel('cosh')
- plt.title('cosh')
- plt.grid(True)

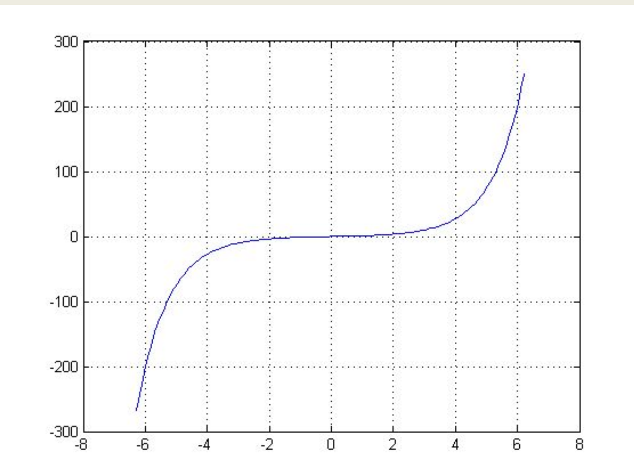
- plt.xlim(-0.1, 0.1)
- plt.ylim(0, 2)

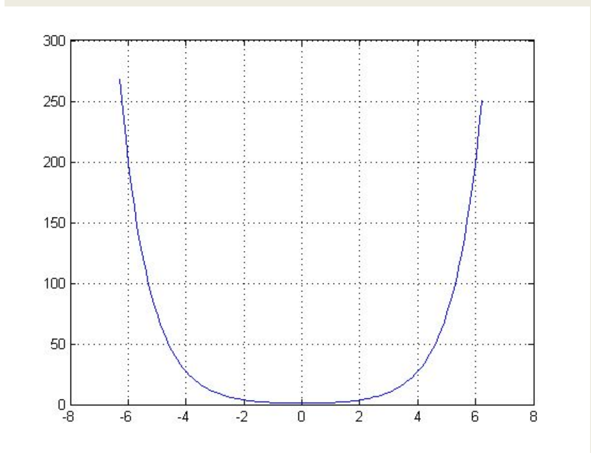
- plt.show()

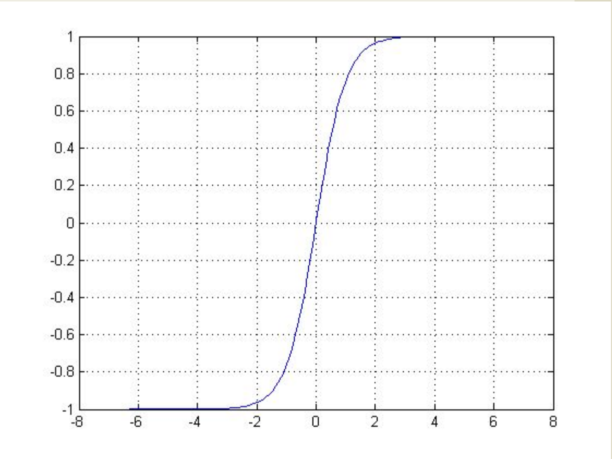


쌍곡선 함수와 지수 함수의 관계

$$\tanh x = \frac{sinhx}{coshx} = \frac{\frac{e^x - e^{-x}}{2}}{\frac{e^x + e^{-x}}{2}} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$







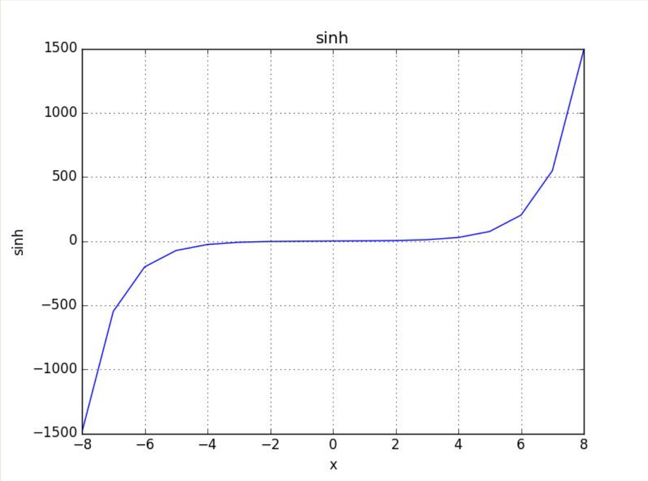
$$\frac{e^x - e}{2}$$

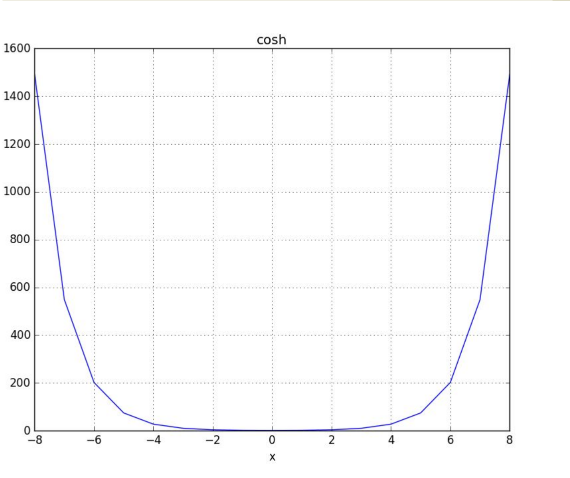
$$\frac{e^x + e}{2}$$

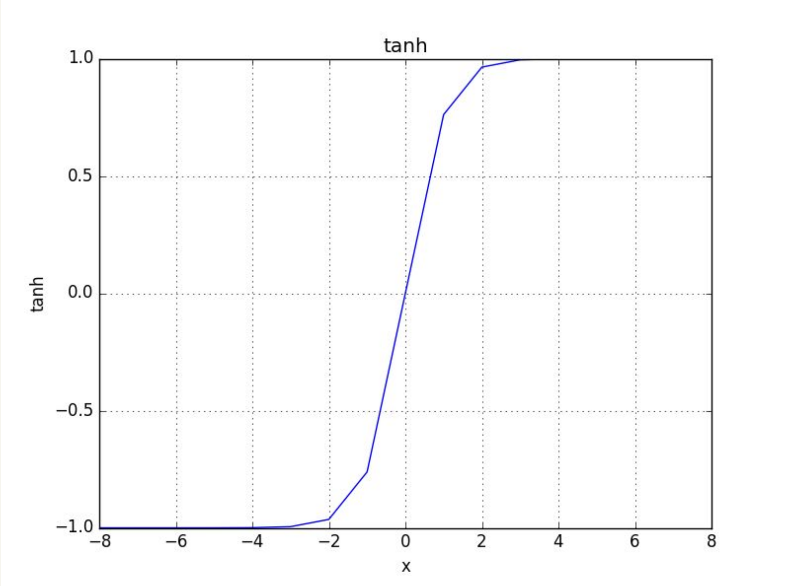
$$\frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Draw tanh

- import numpy as np
- import matplotlib.pyplot as plt
- x=np.arange(-8,8+1,1)
- y1=np.exp(x)/2
- y2=np.exp(-x)/2
- y3=y1-y2
- y4=y1+y2
- y5=y3/y4
- plt.figure(1)
- plt.plot(x, y5)
- plt.xlabel('x')
- plt.ylabel('tanh')
- plt.title('tanh')
- plt.grid(True)
- plt.show()







3주 / 2
Part One Modeling, Computers, and Error Analysis,
교재 Ch.1: 번지 점프 시의 체감 속도 수식 유도
(Mathematical Deviation)

Falling Velocity of Bungee Jumper (Chapter 1)

- where Force(F) is composed of downward force due to gravity and upward force due to air resistance

$$F = F_d + F_u = mg - c_d v^2$$

$$F_d = +mg \quad (downward \ \ force)$$

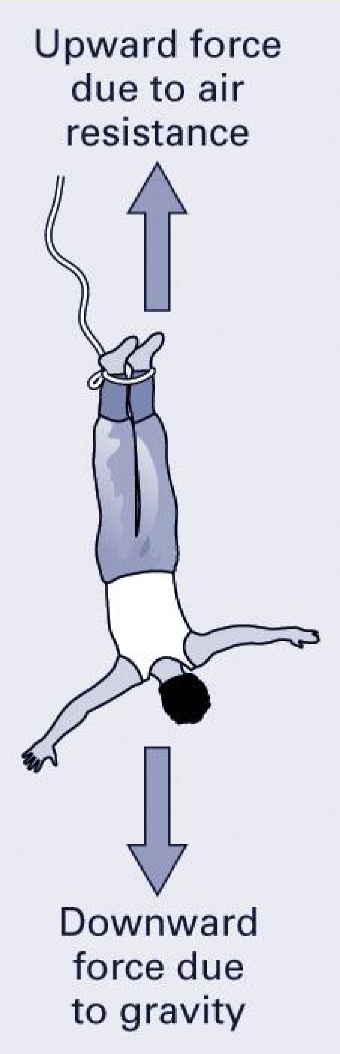
$$F_u = -c_d v^2 \ (upward \ \ force)$$

We can get

$$mg - c_d v^2 = m \cdot \frac{dv}{dt}$$

since velocity has the derivative based on time, $\frac{dv}{dt}$ is called as a $differential\ equation$

$$g - \frac{c_d}{m} v^2 = \frac{dv}{dt}$$



$$\frac{mg}{c_d} - v^2 = \frac{m}{c_d} \cdot \frac{dv}{dt}$$

put $\frac{mg}{c_d} = k^2$ $\quad$ $therefore,\ k = \sqrt{\frac{mg}{c_d}}$

$$k^2 - v^2 = \frac{m}{c_d} \cdot \frac{dv}{dt}$$

When we take the inverse on both sides, we get $\frac{1}{k^2 - v^2} = \frac{1}{\frac{m}{c_d} \cdot \frac{dv}{dt}}$

https://en.wikipedia.org/wiki/Hyperbolic_function

$$\frac{c_d}{m} \cdot \frac{dt}{dv} = \frac{1}{k^2 - v^2}$$

$$\int \frac{du}{a^2 - u^2} = a^{-1} \tanh^{-1}\left(\frac{u}{a}\right) + C; u^2 < a^2$$

Take integration on both sides, we get $\int \frac{c_d}{m} \cdot dt = \int \frac{1}{k^2 - v^2} \cdot dv$

Use $\int \frac{1}{k^2 - v^2} \cdot dx = \frac{1}{k} tanh^{-1} \frac{x}{v} + c$ to the right side of equation,

which comes $\int \frac{1}{a^2 - u^2} \cdot du = \frac{1}{a} tanh^{-1} \frac{u}{a} + c$ from the property of

tan hyperbolic function

we get $\frac{c_d}{m} \cdot t + c_1 = \frac{1}{k} tanh^{-1} \frac{v}{k} + c_2$

$$\frac{c_d}{m} \cdot t = \frac{1}{k} tanh^{-1} \frac{v}{k} + c_2 - c_1$$

Since the velocity equals zero when the time is zero, we can say that

$t = 0 , v = 0$

$$0 = 0 + c_2 - c_1$$

$$c_2 = c_1$$

Since c_1 and c_2 are same, the c_1 and c_2 can be cancelled out.

$$\therefore \frac{c_d}{m} \cdot t + c_1 = \frac{1}{k} tanh^{-1} \frac{v}{k} + c_2$$

$$\therefore \frac{c_d}{m} \cdot t = \frac{1}{k} tanh^{-1} \frac{v}{k}$$

When k is moved to left side, we can get

$$\frac{kc_d}{m} \cdot t = tanh^{-1} \frac{v}{k}$$

tanh inverse function can be moved to the left side and the inverse is no more existed in the right side.

$$tanh\left(\frac{kc_d}{m} \cdot t\right) = \frac{v}{k}$$

$$v = k \cdot tanh\left(\frac{kc_d}{m} \cdot t\right)$$

$v = k \cdot tanh\left(\frac{kc_d}{m} \cdot t\right),$ since $k = \sqrt{\frac{mg}{c_d}}$

We can get

$$v = \sqrt{\frac{mg}{c_d}}\ tanh\left(\sqrt{\frac{mg}{c_d}} \cdot \frac{c_d}{m} \cdot t\right)$$

$$= \sqrt{\frac{mg}{c_d}}\ tanh\left(\frac{\sqrt{g}}{\sqrt{c_d}}\ \frac{\sqrt{c_d}}{\sqrt{m}} \cdot t\right)$$

$$= \sqrt{\frac{mg}{c_d}}\ tanh\left(\sqrt{\frac{gc_d}{m}} \cdot t\right)$$

Therefore, we get the falling velocity v(t), where v is dependent on time t.

$$v(t) = \sqrt{\frac{mg}{c_d}}\ \tanh\left(\sqrt{\frac{gc_d}{m}} \cdot t\right)$$

Euler's method

$$g - \frac{c_d}{m}v^2 = \frac{dv}{dt} \cong \frac{\Delta v}{\Delta t} = \frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i}$$

$$\frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i} = g - \frac{c_d}{m}v(t_i)^2$$

$$v(t_{i+1}) = v(t_i) + \left[g - \frac{c_d}{m}v(t_i)^2\right](t_{i+1} - t_i)$$

Euler's method

```
g=9.8
cd=0.25
m=68

% euler's method
v0=0
v1=(1-0)*(g-cd/m*v0^2)+v0
v2=(2-1)*(g-cd/m*v1^2)+v1
v3=(3-2)*(g-cd/m*v2^2)+v2
v4=(4-3)*(g-cd/m*v3^2)+v3

vel_eulor=[v0 v1 v2 v3 v4]
```

Original method

```
% original method
vel=[];
for(t=0:4)
   v=sqrt(g*m/cd)*tanh(sqrt(g*cd/m)*t)
   vel=[vel v]
end

time=0:4

plot(time,  vel, '-ro', time, vel_eulor, '-b>')
grid on

legend('original', 'eulor')
```