Numerical Analysis

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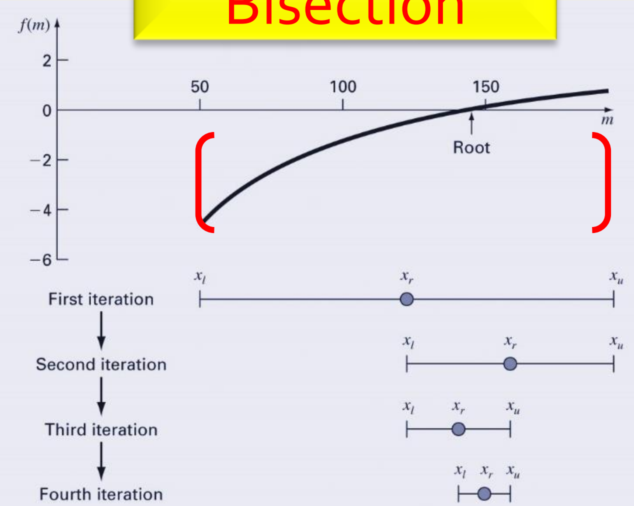
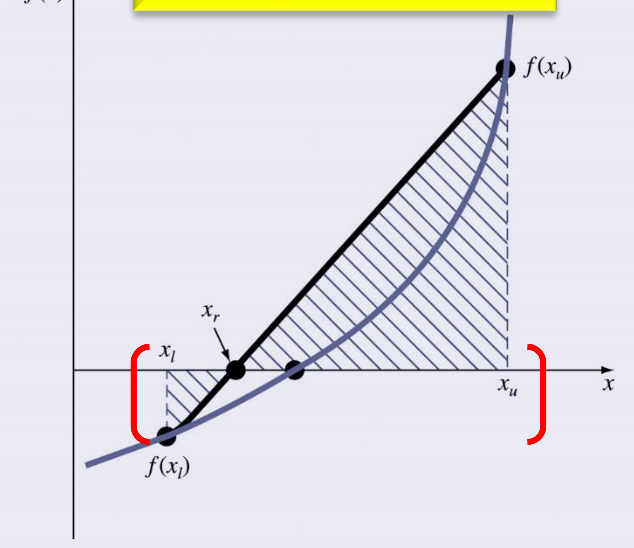
Open Method: Newton-Raphson

구간법(Bracketing)과 개방법(Open)

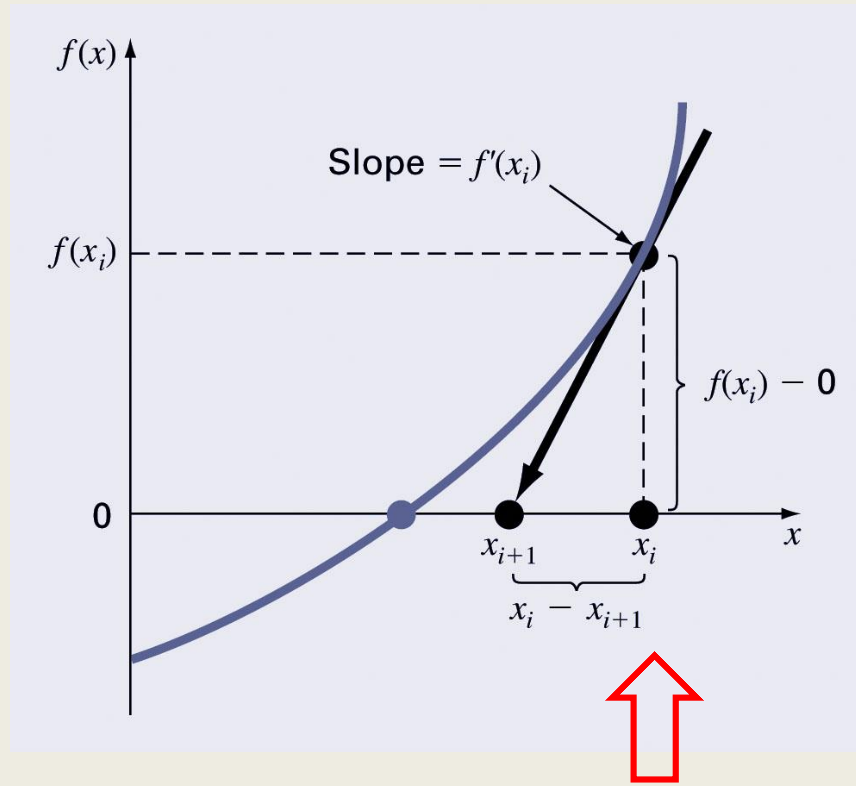
Bisection



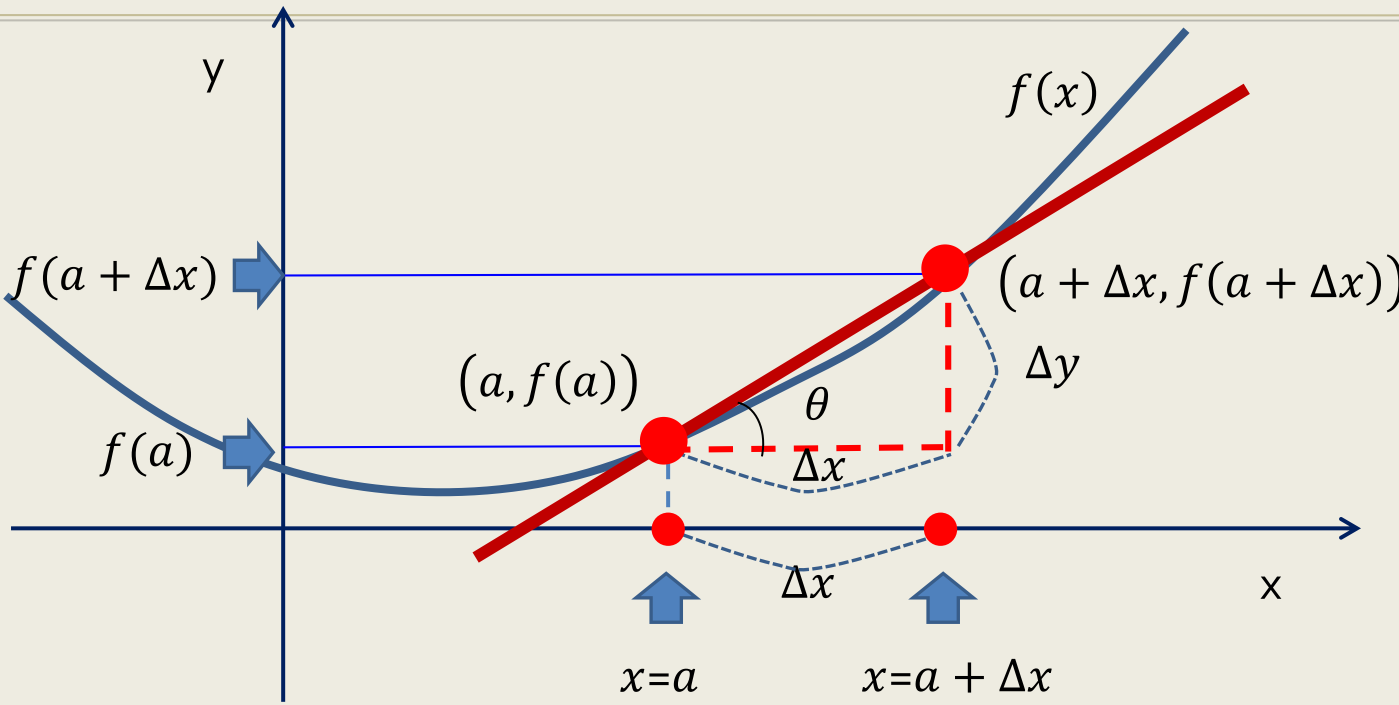
False Position



Newton Raphson



두 점을 지나는 직선의 기울기=평균 변화율



두 점을 지나는 직선 기울기 $= \dfrac{\Delta y}{\Delta x} = \dfrac{f(a+\Delta x)-f(a)}{(a+\Delta x)-a} = \dfrac{f(a+\Delta x)-f(a)}{\Delta x}$

= a 에 대한 평균 변화율

순간 변화율 or 미분계수

- a 에 대한 **평균** 변화율 <u>= **두 점을 지나는 직선 기울기**</u>

$$\frac{\Delta y}{\Delta x} = \frac{f(a+\Delta x)-f(a)}{(a+\Delta x)-a} = \frac{f(a+\Delta x)-f(a)}{\Delta x}$$

- a 에 대한 **순간** 변화율 = 미분계수

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(a+\Delta x)-f(a)}{(a+\Delta x)-a} = \lim_{\Delta x \to 0} \frac{f(a+\Delta x)-f(a)}{\Delta x}$$

$$f'(a) = y'_{x=a} = \left[\frac{dy}{dx}\right]_{x=a}$$

도함수

- a 에 대한 순간 변화율 = 미분계수

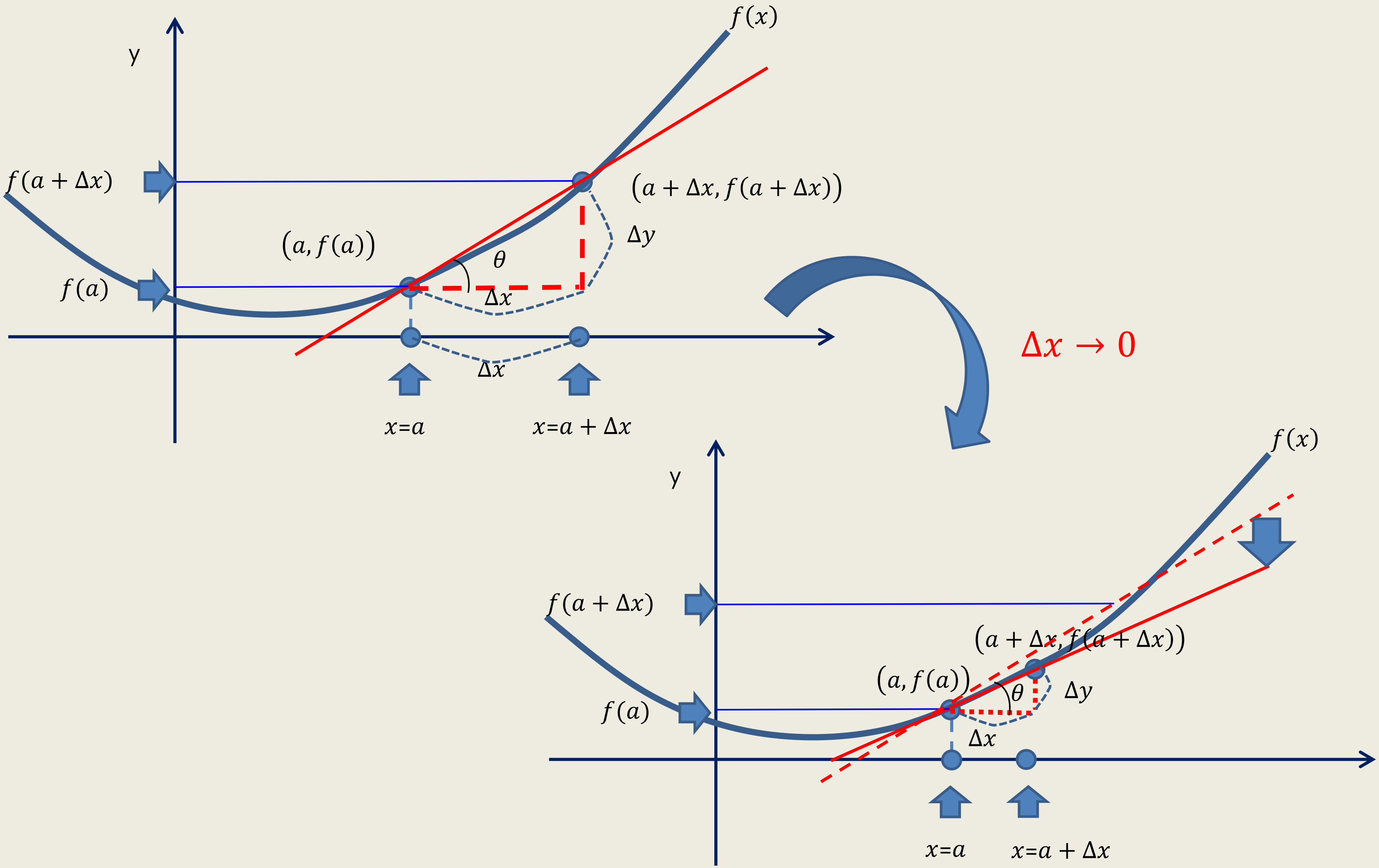
$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(a+\Delta x) - f(a)}{(a+\Delta x) - a} = \lim_{\Delta x \to 0} \frac{f(a+\Delta x) - f(a)}{\Delta x}$$

$$f'(a) = y'_{x=a} = \left[\frac{dy}{dx}\right]_{x=a}$$

- x 에 대한 순간 변화율 = 도함수

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{(x+\Delta x) - x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

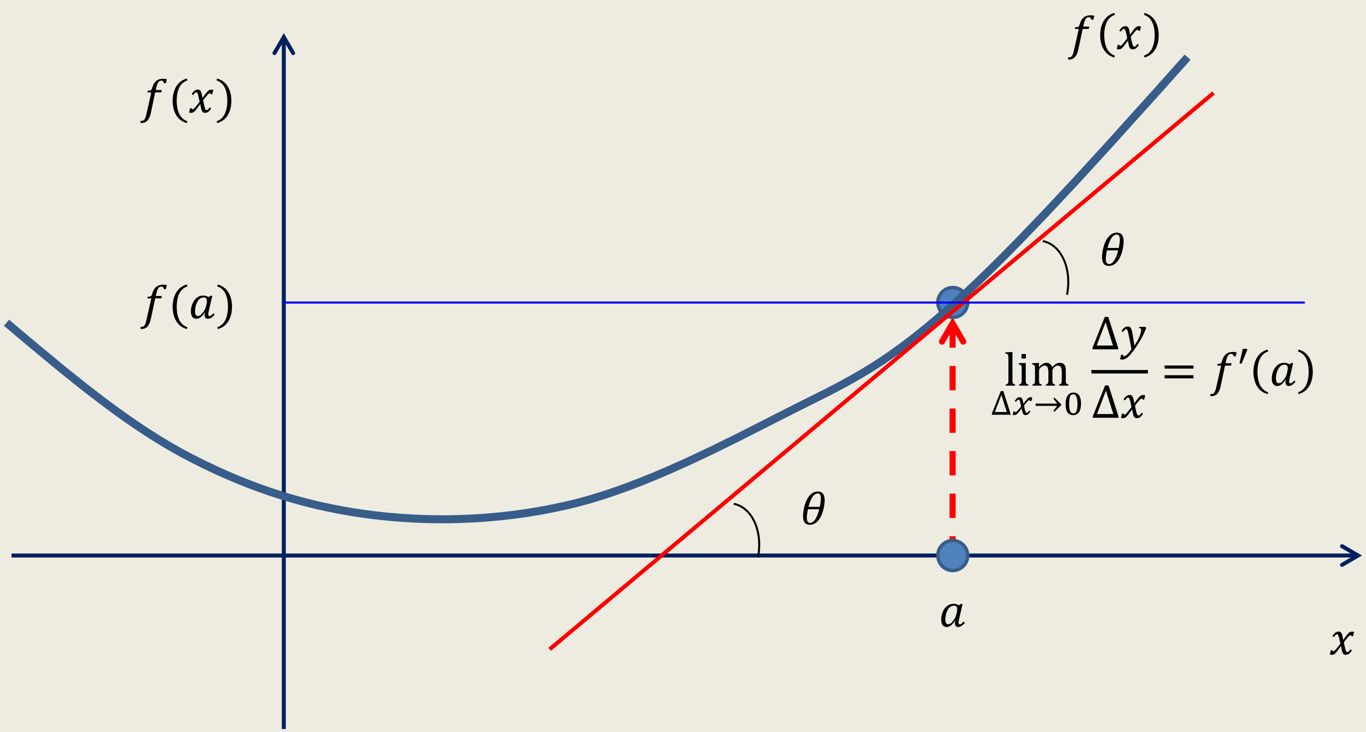
$$f'(x) = y' = \frac{dy}{dx} = \frac{df(x)}{dx} = \frac{d}{dx} f(x)$$



y
$f(x)$
$f(a+\Delta x)$
$(a+\Delta x, f(a+\Delta x))$
Δy
$(a, f(a))$
θ
$f(a)$
Δx
Δx
x=a
x=$a+\Delta x$
$\Delta x \to 0$
y
$f(x)$
$f(a+\Delta x)$
$(a+\Delta x, f(a+\Delta x))$
$(a, f(a))$
θ
Δy
$f(a)$
Δx
x=a
x=$a+\Delta x$

미분계수 $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = f'(a)$

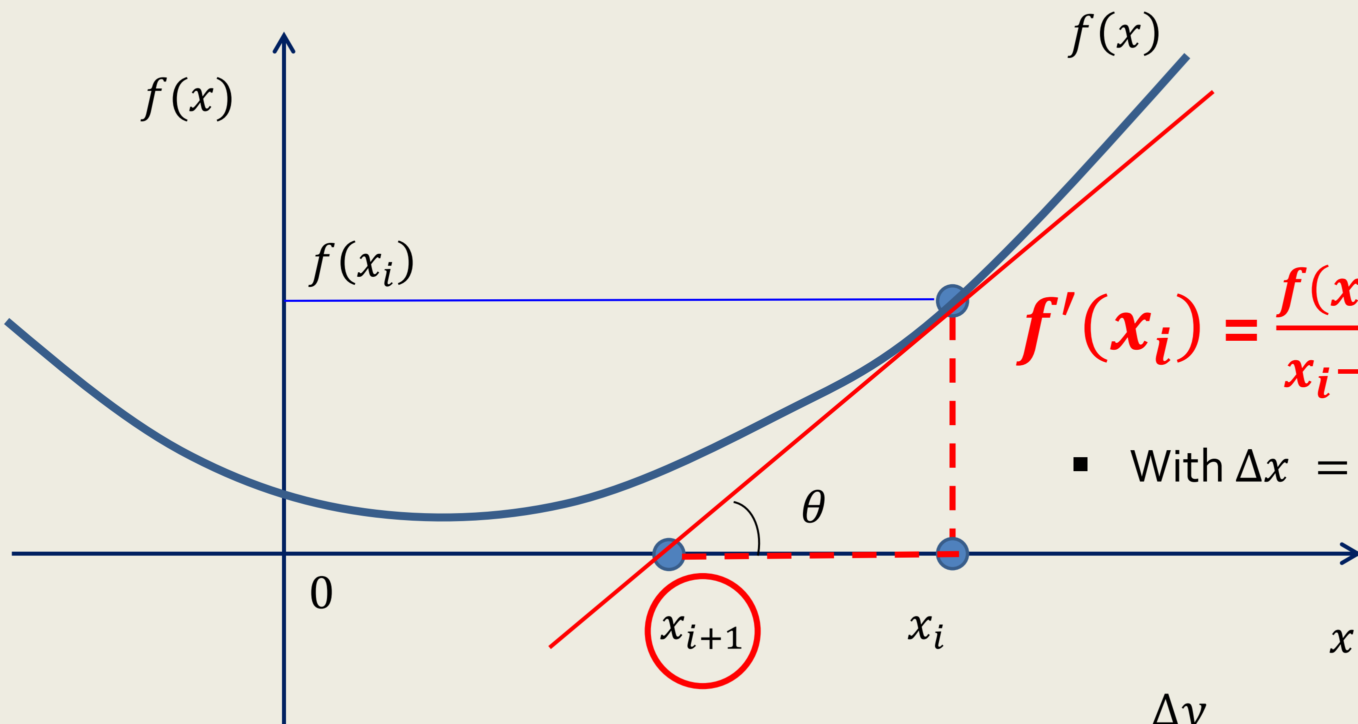
Δx should not be 0 since it is denominator



Newton-Raphson Algorithm

$$f'(x_i) = \frac{f(x_i) - 0}{x_i - x_{i+1}}$$

-. 미분계수 $f'(x_i)$ 는 접점 x_i 에서의 접선의 기울기를 충실히 따르는 알고리즘
-. 구하고자 하는 것은 x_{i+1}



$$f'(x_i) = \frac{f(x_i) - 0}{x_i - x_{i+1}}$$

- With $\Delta x = x_i - x_{i+1} \rightarrow 0$

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = f'(x_i)$$

Calculate $\boldsymbol{x_r}^{\boldsymbol{new}} = \boldsymbol{x_{i+1}}$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$f'(x_i) = \frac{f(x_i) - 0}{x_i - x_{i+1}}$$

$$(x_i - x_{i+1}) \cdot f'(x_i) = f(x_i)$$

$$x_i \cdot f'(x_i) - x_{i+1} \cdot f'(x_i) = f(x_i)$$

$$x_i \cdot f'(x_i) - f(x_i) = x_{i+1} \cdot f'(x_i)$$

$$x_i \cdot f'(x_i) - f(x_i) = x_{i+1} \cdot f'(x_i)$$

$$x_i - \frac{f(x_i)}{f'(x_i)} = x_{i+1}$$

Newton-Raphson Algorithm

$$x_{i+1} = x_i - \frac{f(x_i)}{f\prime(x_i)}$$

$$x_2 = x_1 - \frac{f(x_1)}{f\prime(x_1)}$$, x_1 = 200 (초기값)

$$x_2 = 200 - \frac{f(200)}{f\prime(200)}$$ 계산 필요

$$f(200) = \sqrt{\frac{gm}{c_d}} \cdot tanh\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) - 36$$

$$f(200) = \sqrt{\frac{9.81 \times 200}{0.25}} \cdot tanh\left(\sqrt{\frac{9.81 \times 0.25}{200}} \cdot 4\right) - 36$$

$$f'(200) = ?$$

$$f(m)=\underbrace{\sqrt{\frac{gm}{c_d}}}_{f_1(m)}\cdot\underbrace{tanh\left(\sqrt{\frac{gc_d}{m}}\cdot t\right)}_{f_2(m)}-36$$

$$f'(200)=?$$

$$f(m)=f_1(m)\cdot f_2(m)$$

$$f'(m)=f_1{}'(m)\cdot f_2(m)+f_1(m)\cdot f_2{}'(m)$$

$$f_1(m)=\sqrt{\frac{gm}{c_d}}=\sqrt{\frac{g}{c_d}}\cdot(m)^{\frac{1}{2}}$$

$$f_1{}'(m)=\left(\sqrt{\frac{g}{c_d}}\cdot m^{\frac{1}{2}}\right)'$$

$$f_2(m)=tanh\left(\sqrt{\frac{gc_d}{m}}\cdot t\right)$$

$$f_1{}'(m)=\sqrt{\frac{g}{c_d}}\cdot\left(m^{\frac{1}{2}}\right)'=\sqrt{\frac{g}{c_d}}\cdot\frac{1}{2}\cdot m^{\frac{1}{2}-1}$$

$$f_2{}'(m)\;=tanh'\left(\sqrt{\frac{gc_d}{m}}\cdot t\right)\cdot\left(\sqrt{\frac{gc_d}{m}}\cdot t\right)'$$

$$use\quad y(x)=x^n$$

$$y'(x)=n\cdot x^{n-1}$$

$$use\quad y(x)=f\big(g(x)\big)$$

$$y'(x)=f'\big(g(x)\big)\cdot g'(x)$$

$$f(m) = \sqrt{\frac{gm}{c_d}} \cdot tanh\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) - 36$$

$$f'(m) = f_1{}'(m) \cdot f_2(m) + f_1(m) \cdot f_2{}'(m)$$

$$f_1{}'(m) = \sqrt{\frac{g}{c_d}} \cdot \left(m^{\frac{1}{2}}\right)' = \sqrt{\frac{g}{c_d}} \cdot \frac{1}{2} \cdot m^{-\frac{1}{2}} \qquad f_2{}'(m) = tanh'\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) \cdot \left(\sqrt{\frac{gc_d}{m}} \cdot t\right)'$$

$$f'(m) = \sqrt{\frac{g}{c_d}} \cdot \frac{1}{2} \cdot m^{-\frac{1}{2}} \cdot tanh\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) + \sqrt{\frac{gm}{c_d}} \cdot tanh'\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) \cdot \left(\sqrt{\frac{gc_d}{m}} \cdot t\right)'$$

$$y = tanh(x) \Rightarrow y' = sech^2(x)$$

$$= \frac{1}{2} \cdot \sqrt{\frac{g}{c_d}} \cdot m^{-\frac{1}{2}} \cdot tanh\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) + \sqrt{\frac{gm}{c_d}} \cdot sech^2\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) \cdot \left(\sqrt{gc_d} \cdot t \cdot m^{-\frac{1}{2}}\right)'$$

$$= \frac{1}{2} \cdot \sqrt{\frac{g}{c_d}} \cdot m^{-\frac{1}{2}} \cdot tanh\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) + \sqrt{\frac{gm}{c_d}} \cdot sech^2\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) \cdot \left(-\frac{1}{2} \cdot m^{-\frac{1}{2}-1} \cdot \sqrt{gc_d} \cdot t\right)$$

$$= \frac{1}{2} \cdot \sqrt{\frac{g}{c_d}} \cdot m^{-\frac{1}{2}} \cdot tanh\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) + \sqrt{\frac{gm}{c_d}} \cdot sech^2\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) \cdot \left(-\frac{1}{2} \cdot m^{-\frac{1}{2}-1} \cdot \sqrt{gc_d} \cdot t\right)$$

$$= \frac{1}{2} \cdot \sqrt{\frac{g}{c_d}} \cdot m^{-\frac{1}{2}} \cdot tanh\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) + \sqrt{\frac{gm}{c_d}} \cdot \left(-\frac{1}{2}\right) \cdot \sqrt{gc_d} \cdot m^{-\frac{3}{2}} \cdot t \cdot sech^2\left(\sqrt{\frac{gc_d}{m}} \cdot t\right)$$

$$= \frac{1}{2} \cdot \sqrt{\frac{g}{c_d}} \cdot m^{-\frac{1}{2}} \cdot tanh\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) - \frac{1}{2} \cdot \sqrt{\frac{gm}{c_d}} \cdot \sqrt{gc_d} \cdot \frac{1}{\sqrt{m}} \cdot \frac{1}{m} \cdot t \cdot sech^2\left(\sqrt{\frac{gc_d}{m}} \cdot t\right)$$

$$= \frac{1}{2} \cdot \sqrt{\frac{g}{c_d}} \cdot m^{-\frac{1}{2}} \cdot tanh\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) - \frac{1}{2} \cdot \frac{\sqrt{g}\cancel{\sqrt{m}}}{\cancel{\sqrt{c_d}}} \cdot \sqrt{g} \cdot \cancel{\sqrt{c_d}} \cdot \frac{1}{\cancel{\sqrt{m}}} \cdot \frac{1}{m} \cdot t \cdot sech^2\left(\sqrt{\frac{gc_d}{m}} \cdot t\right)$$

$$= \frac{1}{2} \cdot \sqrt{\frac{g}{c_d}} \cdot m^{-\frac{1}{2}} \cdot tanh\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) - \frac{1}{2} \cdot g \cdot \frac{t}{m} \cdot sech^2\left(\sqrt{\frac{gc_d}{m}} \cdot t\right)$$

$$= \frac{1}{2} \cdot \sqrt{\frac{g}{c_d}} \cdot m^{-\frac{1}{2}} \cdot tanh\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) - \frac{gt}{2m} \cdot sech^2\left(\sqrt{\frac{gc_d}{m}} \cdot t\right)$$

$$= \frac{1}{2} \cdot \sqrt{\frac{g}{mc_d}} \cdot tanh\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) - \frac{gt}{2m} \cdot sech^2\left(\sqrt{\frac{gc_d}{m}} \cdot t\right)$$

$$f'(m) = \frac{1}{2} \cdot \sqrt{\frac{g}{mc_d}} \cdot tanh\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) - \frac{gt}{2m} \cdot sech^2\left(\sqrt{\frac{gc_d}{m}} \cdot t\right)$$

$$f(m) = \sqrt{\frac{gm}{c_d}} \cdot tanh\left(\sqrt{\frac{gc_d}{m}} \cdot t\right) - 36$$