1   Fibonacci Series

The nth term a_n of the Fibonacci Series is defined as

$$a_n = a_{n-2} + a_{n-1}. \tag{1}$$

I.e. the nth term is the sum of the 2 previous terms. The first few terms are $1, 1, 2, 3, 5, 8, ...$

Write a python script to generate first 20 terms. Then, find the ratios between each 2 consecutive terms, like $\frac{a_1}{a_2}, \frac{a_2}{a_3}$. You should print out the 20 terms and the 19 ratios. Now look at the 19 ratios, which number do the ratios converge to?

Hint:

- You need to construct lists with python. The initial list of the Fibonacci series is $[1, 1]$. Try to extend this list by the "append" command in python

- You will have to use the last 2 elements of the growing lists. To access the last list element, of the list L for example, use L[-1].

2   Elliptic integral $K(k)$

We looked at the elliptic integral,

$$K(k) = \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1 - k^2 \sin^2(\theta)}}, \tag{2}$$

in the lecture. I also showed you how to numerically obtain a special value $K(\frac{1}{2})$ by using the trapezium method. Obviously, $K(k)$ is well defined for $k \in [0, 1)$ (think why). So, your task is try to find 100 values of $K(k)$ for k ranging from 0 to 0.999, inclusively. Plot your graph and save the 100 values of k in a separate text file.

Hint:

- In calculating $K(\frac{1}{2})$, I used a for loop. But this time, you will need to use the similar for loops 100 times.

- When it comes to so many times, do not worry. You are using python, not just pen and paper!

3   Matrix similarity

Check the following pair of matrices. Are they similar?

$$\begin{pmatrix} 0 & 1 \\ 5 & 3 \end{pmatrix} \text{ VS } \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} \tag{3}$$

$$\begin{pmatrix} 1 & 2 \\ -1 & 4 \end{pmatrix} \text{ VS } \begin{pmatrix} -1 & 6 \\ -2 & 6 \end{pmatrix} \tag{4}$$

Hint:

- Get ready by reading https://numpy.org/doc/stable/reference/generated/numpy.linalg.eig.html

- In numpy, the eigenvalues are collected as an 1d array. If the eigenvalue arrays of 2 matrices contain the same numbers and each number has the same frequency, these 2 matrices are similar.

- Be careful, np.array([1,2,1]) and np.array([2,1,1]) are not the same, but there corresponding matrices are similar. How to resolve this issue to avoid mistakes?