

“Sparknotes” for *Linear Algebra* by Peter Lax

Muthu Chidambaram

Last Updated: May 23, 2019

Contents

1	Fundamentals	3
1.1	Exercise 1	3
1.2	Exercise 2	3
1.3	Exercise 3	3
1.4	Exercise 4	3
1.5	Exercise 5	3
1.6	Exercise 6	3
1.7	Exercise 7	3
1.8	Exercise 8	3
1.9	Exercise 9	3
1.10	Exercise 10	3
1.11	Exercise 11	3
1.12	Exercise 12	4
1.13	Exercise 13	4
1.14	Exercise 14	4
1.15	Exercise 15	4
1.16	Exercise 16	4
1.17	Exercise 17	4

Preface

“A modern mathematical proof is not very different from a modern machine, or a modern test setup: the simple fundamental principles are hidden and almost invisible under a mass of technical details.” - Hermann Weyl

These notes contain short summaries of (my) proof ideas for exercises and some theorems from the book *Linear Algebra* by Peter Lax. I have tried to make the summaries as brief as possible, sometimes only one line or one equation. My hope is that the summaries will give enough information to reconstruct a full proof without bogging the reader down with details. In many cases, I am sure that I inadvertently sacrificed clarity in an attempt to obtain brevity, and would greatly appreciate any feedback.

Also, I like when people include (what they presume to be) relevant quotes from mathematicians of past generations in their notes, so I have to ask you to forgive my haughtiness in starting these notes with a quote from Hermann Weyl.

1 Fundamentals

1.1 Exercise 1

$$x + z = x = x + z' \implies z = z'.$$

1.2 Exercise 2

$$0x + x = (0 + 1)x = x.$$

1.3 Exercise 3

Coefficients can be represented as row vectors.

1.4 Exercise 4

Function can be represented as row vector by letting $a_i = f(s_i)$ for each $s_i \in S$.

1.5 Exercise 5

Follows from exercises 3 and 4.

1.6 Exercise 6

$$y_1 + z_1 + y_2 + z_2 = (y_1 + y_2) + (z_1 + z_2) \text{ and } k(y_1 + z_1) = ky_1 + kz_1.$$

1.7 Exercise 7

$$a \in Y \cap Z \implies ka \in Y, ka \in Z \implies ka \in Y \cap Z.$$

1.8 Exercise 8

$$k0 = 0, 0 + 0 = 0.$$

1.9 Exercise 9

If S contains x_i then it must contain kx_i .

1.10 Exercise 10

If $x_i = 0$, k_i can be anything.

1.11 Exercise 11

$$x = \sum_{i=1}^m \sum_{j=1}^{\dim Y_i} y_j^{(i)}.$$

1.12 Exercise 12

Complete basis for W to U and V . Use W basis vectors and additional U and V basis vectors to get $\dim X = \dim U - \dim W + \dim V - \dim W + \dim W$.

1.13 Exercise 13

Send i^{th} basis vector to e_i , where e_i is vector of all zeroes except a one in the i^{th} place. Can permute mapping to get different isomorphisms.

1.14 Exercise 14

$$x_1 - x_2 + x_2 - x_3 = x_1 - x_3.$$

1.15 Exercise 15

$$x' = x + z_x, y' = y + z_y \implies x' + y' = x + y + (z_x + z_y).$$

1.16 Exercise 16

$$x \in X_1 \oplus X_2 \implies x = (x_1, x_2) = (x_1, 0) + (0, x_2).$$

1.17 Exercise 17

Construct a basis for X from Y : $y_1, \dots, y_j, x_{j+1}, \dots, x_n$.
Then $X/Y = \text{span}\{x_{j+1}, \dots, x_n\}$.