

# Exercise Guide for *Statistical Inference (2nd Ed.)* by Casella and Berger

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## About

*“If you torture the data enough, nature will always confess.”* -  
Ronald Coase

There are actual solutions available here. What follows are notes I took as I worked through some of the exercises in the book on my own.

# 1 Probability Theory

## 1.1 Exercise 1

- (a) Four character strings consisting of H and T (16 total).
- (b)  $\mathbb{N}$
- (c)  $\mathbb{R}^+$
- (d)  $\mathbb{R}^+/0$
- (e)  $\frac{i}{n}$  for  $i = 0, \dots, n$ .

## 1.2 Exercise 4

- (a)  $P(A) + P(B) - P(A \cap B)$
- (b)  $P(A) + P(B) - 2P(A \cap B)$
- (c) Same as (a).
- (d)  $1 - P(A \cap B)$

## 1.3 Exercise 6

We have  $u + w = 1 \implies u^2 + 2uw + w^2 = 1$ . However, it is not possible for  $u^2, 2uw, w^2$  to all equal  $\frac{1}{3}$ ; thus, there are no such  $u, w$ .

## 1.4 Exercise 12

- (a) Just a case of countable additivity.
- (b) Split  $\cup_{i=1}^{\infty} A_i$  into  $\cup_{i=1}^n A_i$  and  $\cup_{i=n+1}^{\infty} A_i$ . Taking  $n \rightarrow \infty$  sends the probability of the latter union to 0, leaving the desired result.

## 1.5 Exercise 13

No, since  $P(B^c) = 1 - P(B) \implies P(B) = \frac{3}{4}$  and  $\frac{3}{4} + \frac{1}{3} > 1$ .

## 1.6 Exercise 18

This exercise (based on the provided answer) seems to assume that the balls are distinguishable. There are  $n^n$  different arrangements of balls in cells (each ball has  $n$  options). If exactly one cell must remain empty, then exactly one other cell must have two balls. There are  $n$  choices for the empty cell and  $n-1$  choices for the double cell. The two balls to go into the double cell can be chosen in  $\binom{n}{2}$  ways, and the remaining balls can be ordered in  $(n-2)!$  ways (since they are distinguishable), hence giving the provided result.

### 1.7 Exercise 21

There are  $\binom{2n}{2r}$  ways to choose  $2r$  shoes from the collection. If we want to ensure that no pairs of shoes are drawn, we should only draw one shoe from each of the  $n$  pairs, which we can do in  $\binom{n}{2r}$  ways. For each of the  $2r$  shoes we can either choose the right or left shoe from the pair (assuming distinguishability).

### 1.8 Exercise 23

We use Vandermonde's identity with  $m = n = r$  to get

$$\sum_{k=0}^n \left( \frac{1}{2^n} \binom{n}{k} \right)^2 = \frac{1}{4^n} \binom{2n}{n}$$

### 1.9 Exercise 25

Depends on how you define the sample space, see Wikipedia for a lengthy discussion.

### 1.10 Exercise 26

We take the complement of the probability that we roll a 6 in the first 5 rolls:  $1 - \sum_{k=0}^4 \frac{5^k}{6^{k+1}}$ .

### 1.11 Exercise 35

From the definition of conditional probability and the fact that  $P(B) > 0$ , we have that  $P(\cdot|B) > 0$  and  $P(S|B) = \frac{P(S \cap B)}{P(B)} = 1$ . Countable additivity follows from  $A \cap B$  and  $A' \cap B$  being disjoint if  $A$  and  $A'$  are.

### 1.12 Exercise 36

To solve the first part, we take the complement of the probability that the target was hit fewer than two times, which is

$$1 - \left(\frac{4}{5}\right)^{10} - 10 \left(\frac{4}{5}\right)^9 \frac{1}{5} \approx 0.624$$

If we let  $A$  be the event that the target was hit at least once, and  $B$  be the event that the target was hit at least twice, then we see that  $A \cap B = B$ . The conditional probability is thus

$$\frac{1 - \left(\frac{4}{5}\right)^{10} - 10 \left(\frac{4}{5}\right)^9 \frac{1}{5}}{1 - \left(\frac{4}{5}\right)^{10}} \approx 0.699$$

## 2 Transformations and Expectations

### 2.1 Exercise 1

(a)  $F_Y(y) = P(X^3 \leq y) = F_X(y^{\frac{1}{3}})$ . Differentiating, we get that  $f_Y(y) = 14y(1 - y^{\frac{1}{3}})$  which integrates to 1 over  $(0, 1)$ .

(b)  $f_Y(y) = \frac{d}{dy}F_X(\frac{y-3}{4}) = \frac{7}{4} \exp\left(\frac{-7(y-3)}{4}\right)$ , which again integrates to 1 over  $(3, \infty)$ . The sample space of  $Y$  consists of  $(3, \infty)$  because  $4X+3$  is monotonically increasing and  $4(0) + 3 = 3$ .

(c) We need only consider positive square roots, since  $x \in (0, 1)$ . Thus,  $f_Y(y) = \frac{1}{2\sqrt{y}} 30y(1 - \sqrt{y})^2$ , which integrates to 1 over  $(0, 1)$ .

### 2.2 Exercise 2

(a)  $f_Y(y) = \frac{1}{2\sqrt{y}} f_X(\sqrt{y}) = \frac{1}{2\sqrt{y}}$

(b) Plug  $x = e^{-y}$  into  $f_X(x)$  and multiply by  $e^{-y}$  (the negative of the derivative, since  $-\log X$  is decreasing).

(c) Plug in  $x = \log y$  and multiply by  $\frac{1}{y}$ .

### 2.3 Exercise 3

From the definition of  $Y$ , we can see that the sample space is  $\mathcal{Y} = (\frac{n}{n+1})_{\mathbb{N}}$ . Solving  $y = \frac{x}{x+1}$  for  $y$ , we find that  $f_Y(y) = f_X(\frac{y}{1-y})$ .

### 2.4 Exercise 4

(a) Integrating from  $-\infty$  to 0 gives  $\frac{1}{2}$  and likewise for 0 to  $\infty$ , so  $f$  is a pdf.

(b) If  $t \leq 0$ , then we have that  $P(X < t) = \frac{1}{2}e^{\lambda t}$ . Otherwise,

$$\begin{aligned} P(X < t) &= \int_{-\infty}^t f(x) = \int_{-\infty}^0 f(x) + \int_0^t f(x) \\ &= \frac{1}{2} + \frac{1}{2} - \frac{1}{2}e^{-\lambda t} \\ &= 1 - \frac{1}{2}e^{-\lambda t} \end{aligned}$$

(c) If  $t \leq 0$ ,  $P(|X| < t)$  is clearly 0. Otherwise,

$$\begin{aligned} P(|X| < t) &= P(-t < X < t) = P(X < t) - P(X < -t) \\ &= 1 - e^{-\lambda t} \end{aligned}$$

from the CDFs computed in part (b).