“Sparknotes” for *Principles of Mathematical Analysis* by Walter Rudin

Muthu Chidambaram

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Contents

About

> *"A modern mathematical proof is not very different from a modern machine, or a modern test setup: the simple fundamental principles are hidden and almost invisible under a mass of technical details."*
> \- Hermann Weyl

These notes contain short summaries of (my) proof ideas for exercises and some theorems from the book *Principles of Mathematical Analysis* by Walter Rudin. I have tried to make the summaries as brief as possible, sometimes only one line or one equation. My hope is that the summaries will give enough information to reconstruct a full proof without bogging the reader down with details. In many cases, I am sure that I inadvertently sacrificed clarity in an attempt to obtain brevity, and would greatly appreciate any feedback.

Also, I like when people include (what they presume to be) relevant quotes in their notes, so I have to ask you to forgive my haughtiness in starting these notes with a quote from Hermann Weyl.

1 Numerical Sequences and Series

Definition 3.5

Since $\{p_n\} \to p \implies \forall \epsilon, \exists N | n \geq N \implies |p_n - p| < \epsilon$, we can choose $k | n_k \geq N \implies \{p_{n_k}\} \to p$. The reverse direction can be shown via contradiction of $\{p_n\} \to p$.

Examples 3.18

(a) Density of rationals in reals.

(b) $|s_n| < 1$, take n odd to get -1 and even to get 1.

(c) Every subsequential limit has to converge to s.

Theorem 3.19

For all $\{n_k\}$, we have $\exists K | k \geq K \implies n_k \geq N \implies \lim_{k\to\infty} t_{n_k} - s_{n_k} \geq 0$.

Theorem 3.26

$s_n = 1 + x + ... + x^n \implies xs_n = x + x^2 + ... + x^{n+1} \implies (1-x)s_n = 1 - x^{n+1}$.

Examples 3.40

(a) Root test: $n \to \infty$.

(b) Ratio test: $\frac{1}{n+1} \to 0$.

(c) $1 \to 1$.

(d) Ratio test: $\frac{n}{n+1} \to 1$. $z = 1$ leads to harmonic series.

(e) Ratio test: $\frac{n^2}{(n+1)^2} \to 1$.

Example 3.53

$\sum_{k=1}^{\infty} \frac{1}{4k-3} + \frac{1}{4k-1} - \frac{1}{2k} < \frac{5}{6} + \sum_{k=2}^{\infty} \frac{1}{4k-4} + \frac{1}{4k-4} - \frac{1}{2k}$. The RHS converges since $\frac{1}{4k-4} + \frac{1}{4k-4} - \frac{1}{2k} = \frac{1}{2k^2-2k}$.