Problem Guide for *Fifty Challenging Problems in Probability with Solutions* by Mosteller

Muthu Chidambaram

Last Updated: June 23, 2019

Contents

About

> *"A man must love a thing very much if he not only practices it without any hope of fame or money, but even practices it without any hope of doing it well."* - G.K. Chesterton (Maybe)

Almost certainly my most useless set of notes, as the title of this book says *with Solutions*.

1 The Sock Drawer

Let the number of socks in the drawer be n, and let the number of red socks be k. Then we are given that $\binom{k}{2}/\binom{n}{2}$ is $\frac{1}{2}$. Thus, we can constrain our possibilities as follows

$$\begin{aligned}\binom{k}{2} + \binom{k}{1}\binom{n-k}{1} + \binom{n-k}{2} &= \binom{n}{2} \\ \binom{k}{1}\binom{n-k}{1} + \binom{n-k}{2} &= \frac{1}{2}\binom{n}{2} \\ n(n-1) &= 2k(k-1)\end{aligned}$$

From the last equation, we have that $n = 4, k = 3$ is one solution. Additionally, we can note that $21 * 20 = 3 * 7 * 5 * 2^2 = 2 * 15 * 14$ to get a solution with an even number of black socks. Beyond that, I'll have to get back to you; I haven't started reading number theory books yet.

2 Successive Wins

Let the probability that Elmer beats his dad be p and let the probability that he beats the champion be q. We assume that these two probabilities are independent (which may not be safe, hot streaks are a thing). We are given that $p > q$. The probability that Elmer wins the prize in the dad-champ-dad series is $p^2q + 2pq(1-p) = 2pq - p^2q$ (either Elmer wins all 3, the first 2, or the last 2). Similarly, the probability for the other series is $2pq - pq^2$. Since $p^2q > pq^2$ from our assumption that $p > q$, Elmer should choose the champ-dad-champ series.

3 The Flippant Juror

We see that the probability that the three-man jury succeeds is $p(1-p)+p^2 = p$, since either both non-coin-flipping jurors pick correctly or one of them picks incorrectly but the coin flip is correct. Thus, the one-man and three-man juries both have the same probability of being correct.

4 Trials until First Success

This question is the same as asking for the expected value of a geometric random variable with $p = \frac{1}{6}$, so the answer is 6.

5 Coin in Square

Consider a table consisting of just a single 1-by-1 square. The probability that the coin lands entirely within this square is the same as the probability of picking a point (x, y) with $\frac{3}{8} \leq x, y \leq \frac{5}{8}$ (assuming the position of the center of the coin is uniformly distributed). This is because we can consider the square with opposite corners $(0, 0)$ and $(1, 1)$ and note that the coin is only within this square if its center is at least $\frac{3}{8}$ (its radius) away from each of the boundaries. Since these constraints form a square with sidelength $\frac{1}{4}$, the probability the coin lands in a single 1-by-1 square is $\frac{1}{16}$. Since the squares are all 1-by-1, the total area of the table scales exactly as the probability, so the number of squares does not matter. Thus, the probability is just $\frac{1}{16}$.