Exercise Guide for *Statistical Inference (2nd Ed.)* by Casella and Berger

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Contents

About

> *"A modern mathematical proof is not very different from a modern machine, or a modern test setup: the simple fundamental principles are hidden and almost invisible under a mass of technical details."*
> - Hermann Weyl

I cannot imagine these notes will be of any use to anyone given that this exists. Still, I wrote them while working through the book, so here they are.

1 Probability Theory

1.1 Exercise 1

(a) Four character strings consisting of H and T (16 total).

(b) $\mathbb{N}$

(c) $\mathbb{R}^+$

(d) $\mathbb{R}^+/0$

(e) $\frac{i}{n}$ for $i = 0, ..., n$.

1.2 Exercise 4

(a) $P(A) + P(B) - P(A \cap B)$

(b) $P(A) + P(B) - 2P(A \cap B)$

(c) Same as (a).

(d) $1 - P(A \cap B)$

1.3 Exercise 6

We have $u + w = 1 \implies u^2 + 2uw + w^2 = 1$. However, it is not possible for $u^2, 2uw, w^2$ to all equal $\frac{1}{3}$; thus, there are no such u, w.

1.4 Exercise 12

(a) Just a case of countable additivity.

(b) Split $\cup_{i=1}^{\infty} A_i$ into $\cup_{i=1}^{n} A_i$ and $\cup_{i=n+1}^{\infty} A_i$. Taking $n \to \infty$ sends the probability of the latter union to 0, leaving the desired result.

1.5 Exercise 13

No, since $P(B^c) = 1 - P(B) \implies P(B) = \frac{3}{4}$ and $\frac{3}{4} + \frac{1}{3} > 1$.

1.6 Exercise 18

This exercise (based on the provided answer) seems to assume that the balls are distinguishable. There are n^n different arrangements of balls in cells (each ball has n options). If exactly one cell must remain empty, then exactly one other cell must have two balls. There are n choices for the empty cell and $n-1$ choices for the double cell. The two balls to go into the double cell can be chosen in $\binom{n}{2}$ ways, and the remaining balls can be ordered in $(n-2)!$ ways (since they are distinguishable), hence giving the provided result.

1.7 Exercise 21

There are $\binom{2n}{2r}$ ways to choose $2r$ shoes from the collection. If we want to ensure that no pairs of shoes are drawn, we should only draw one shoe from each of the n pairs, which we can do in $\binom{n}{2r}$ ways. For each of the $2r$ shoes we can either choose the right or left shoe from the pair (assuming distinguishability).

1.8 Exercise 23

We use Vandermonde's identity with $m = n = r$ to get

$$\sum_{k=0}^{n} \left(\frac{1}{2^n} \binom{n}{k} \right)^2 = \frac{1}{4^n} \binom{2n}{n}$$

1.9 Exercise 25

Depends on how you define the sample space, see Wikipedia for a lengthy discussion.

1.10 Exercise 26

We take the complement of the probability that we roll a 6 in the first 5 rolls: $1 - \sum_{k=0}^{4} \frac{5^k}{6^{k+1}}$.

1.11 Exercise 35

From the definition of conditional probability and the fact that $P(B) > 0$, we have that $P(|B) > 0$ and $P(S|B) = \frac{P(B)}{P(B)} = 1$. Countable additivity follows from $A \cap B$ and $A' \cap B$ being disjoint if A and A' are.

1.12 Exercise 36

To solve the first part, we take the complement of the probability that the target was hit fewer than two times, which is

$$1 - \left(\frac{4}{5}\right)^{10} - 10\left(\frac{4}{5}\right)^{9} \frac{1}{5} \approx 0.624$$

If we let A be the event that the target was hit at least once, and B be the event that the target was hit at least twice, then we see that $A \cap B = B$. The conditional probability is thus

$$\frac{1 - \left(\frac{4}{5}\right)^{10} - 10\left(\frac{4}{5}\right)^{9} \frac{1}{5}}{1 - \left(\frac{4}{5}\right)^{10}} \approx 0.699$$

1.13 Exercise 46

1.14 Exercise 51