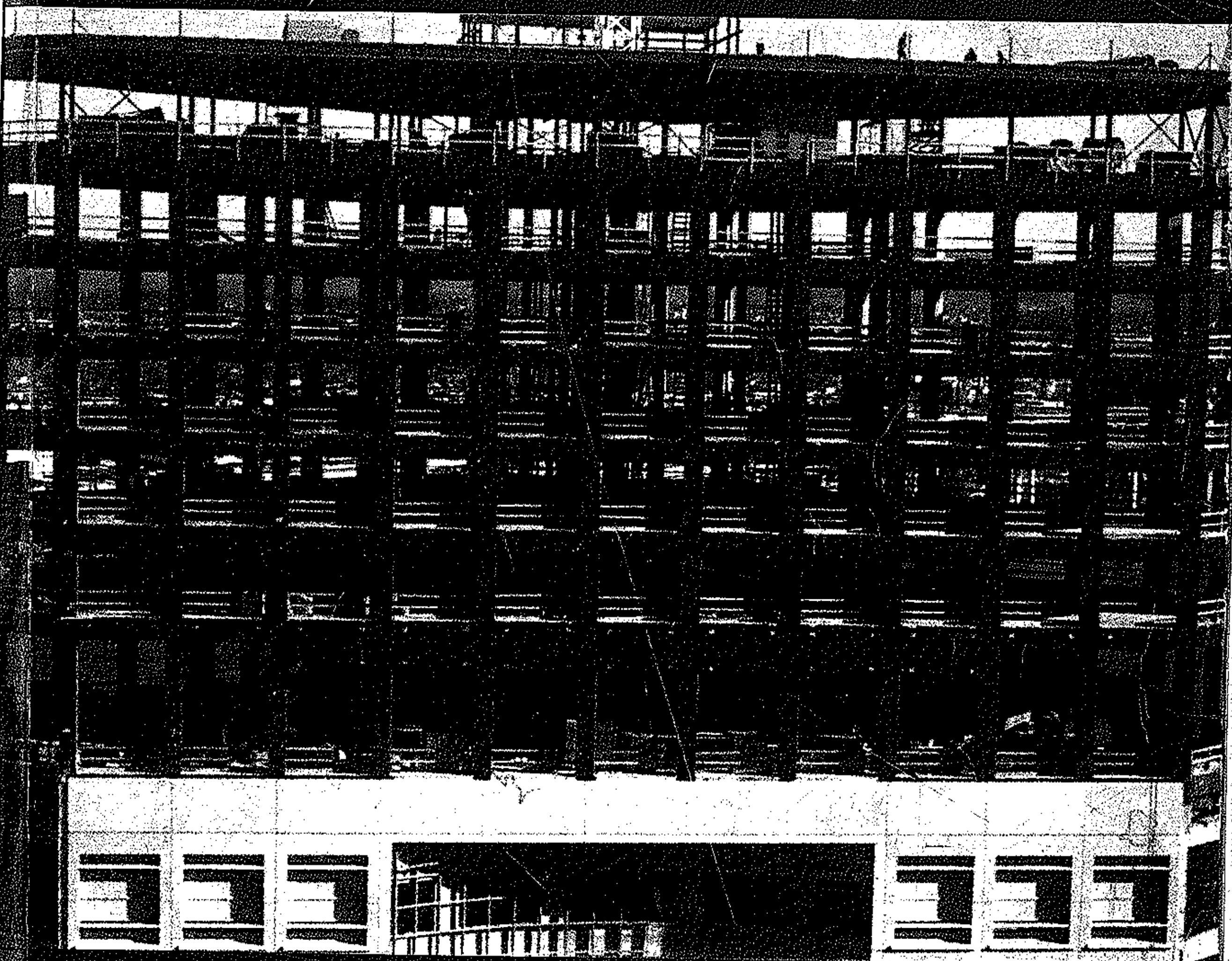
Eastern Economy Edition

Limit State Design of Reinforced Concrete



P.C. Varghese

PHI

श्री गणेशाय नमः

Limit State Design of Reinforced Concrete

P.C. VARGHESE
Honorary Professor, Anna University, Madras
Formerly
Professor and Head, Dept. of Civil Engineering, Indian Institute of Technology Madras
and
UNESCO Chief Technical Adviser, University of Moratuwa, Sri Lanka

Prentice-Hall of India Private Limited
New Delhi-110 001
1999

Contents

List of

List of Illustrations Showing Detailing of Reinforcements

List of Tables

Foreword

Modern reinforced concrete structures however complex they may appear to a novice, are usually designed as an assembly of structural elements such as beams, columns, walls, slabs, and footings. Each of these may be subjected to various combinations of forces with the material itself undergoing effects of creep, shrinkage, temperature variations as well as environmental influences that affect the durability of the structure.

Design of a reinforced concrete structure is carried in many stages, for instance, the empirical apportionment of economical sizes to the various elements, the detailed calculation of the strength and stability of the structure as a whole, and each of the elements under the various forces it is subjected to, the estimation of the economical amount of reinforcements to be provided for safety, as also the detailing of the steel in various parts for integrated action. In addition, serviceability aspects (e.g. deflection and cracking) and durability aspects (e.g. corrosion and deterioration of concrete) should also be given due consideration in the design.

Starting from a purely empirical approach adopted at the turn of this century, reinforced concrete construction has undergone a phase of apparently rigorous elastic theory. Since then we have realised that the semi-empirical approach as advocated by Limit State Design is the best method for design of concrete structures. Thus, after the CEB-FIP recommendations on Limit State Design were published in 1970, Limit State Design approach has been adopted internationally, in the USA by ACI-318-71, in the UK by CP 110-1972, in Australia by AS 1480-1974, and in India by IS-456-78. It should, however, be noted that even though the various aspects of R.C. design are controlled by these codes and regulations, the structural engineer must exercise caution and use his judgement in addition to calculations in the interpretation of the various provisions of the code to obtain an efficient and economical structure. Associated design charts and tables offer great help to shorten the lengthy calculations required. Reference to more than one code brings in a deeper insight into the current state of knowledge on the subject. Besides, detailing of reinforcement, which is an art, has to be carried out according to the recommendations given in approved manuals.

It is evident from the foregoing discussion that to write a book incorporating all the above components of an efficient reinforced concrete design is not an easy task. Yet, Prof. P.C. Varghese has been able to bring out a text book by combining admirably all these elements. I consider this book to be one of the most comprehensive and yet simple text books that has been published so far in India on the subject.

Professor Varghese had himself gained knowledge of Reinforced Concrete Design from his teachers at Harvard University, and Imperial College, London. His subsequent teaching and research career at the Indian Institute of Technology (IIT) Kharagpur and IIT Madras lasting over two decades during the time R.C. design was being revolutionised has reinforced his knowledge on the subject. As UNESCO Technical Adviser at the University of Moratuwa, Sri Lanka, he had the opportunity to introduce Limit State Design based on the British code. No wonder then, he has been able to integrate the best Indian, British and American practices in this text. In addition to explaining the theoretical aspects of the design calculations, he has worked out adequate number of examples to bring out the salient features of R.C. design. It will be advisable if educational institutions inculcate in the students the habit of working out design problems in the professional format presented in the book from the beginning. The review questions given at the end of each chapter will ensure, if answered completely by the students, a thorough comprehension of the subject.

This book should prove to be an ideal text book for the students, as well as an able companion for teachers and those interested in updating their knowledge and expertise on the subject.

It is an honour for anyone to write the Foreword for such a commendable text book, and this is particularly so to me who has been an ardent follower of Prof. P.C. Varghese for the past many years in his different activities—teaching, research and consultancy.

P. Purushothaman
Fomerly Professor of Structural Engineering
and Dean, P.G. Studies
College of Engineering, Guindy
Anna University, Madras

Prefac

From my l
that teachi
Method is
average st
After th
Concrete S
Design fo
was the fi
(1969) by
It deals wi
The firs
1953. It w
Reinforce
the presen
of CEB-FI
as "Code
with desig
sections w
Method h
Limit Stat
As man
engineers
mandatory
should be
Ultimate I
states such
Limit Stat
This bo
Indian Sta
and make
has to be

1. *IS 4*
2. *SP*
3. *SP*

All studen
etc. have
for design
should co

Preface

From my long years of experience in teaching engineering students and practising engineers, I feel that teaching of Reinforced Concrete Design first by Working Stress Method and then by Limit State Method is a sheer waste of time and effort. Such teaching also creates confusion in the minds of the average students. A design-oriented subject should be taught as it is professionally practised.

After the publication of the International Recommendations for the Design and Construction of Concrete Structures by CEB-FIP in 1970, the whole world has accepted the principles of Limit State Design for design of concrete structures in their various national codes. The United Kingdom (UK) was the first country to completely switch over to the new design practice by replacing CP 114 (1969) by CP 110 (1972), which was again revised as the present code of practice, BS 8110 (1985). It deals with both reinforced concrete and prestressed concrete structures.

The first Indian code, the Code of Practice for Plain and Reinforced Concrete, was published in 1953. It was revised first in 1957 and then in 1964 under the title "Code of Practice for Plain and Reinforced Concrete". In 1978 India also accepted the recommendations of CEB-FIP and published the present code IS 456 (1978). As both the British and Indian Codes follow the recommendations of CEB-FIP, many of the ideas of the two codes are similar in nature. IS 456 (1978) retains its title as "Code of Practice for Plain and Reinforced Concrete", and a separate code IS 1343 (1980) deals with design of Prestressed Concrete. IS 456 (1978) is divided into six sections, with the first five sections written along the Limit State Design principles, and the last section on the Working Stress Method has been retained as an alternative method of design so that a gradual changeover to the Limit State Method can take place in the profession.

As many years have already elapsed since the publication of IS 456 (1978), most of the practising engineers in India have already adopted the new method of design and it has obviously become mandatory for the educational institutions also to switch over to teaching the Limit State Design. It should be pointed out that (as explained in detail in the book), Limit State Design is not simply Ultimate Load Design, which is only one of the limit states to be considered. Many additional limit states such as deflection, cracking and durability have to be accounted for in the total design by the Limit State Method.

This book is not written to replace the code and the other valuable publications of the Bureau of Indian Standards. It is meant only to explain the provisions of these publications from fundamentals and make the publication more familiar to the students. Hence to get the maximum benefit, this book has to be used along with the following publications of the Bureau of Indian Standards:

1. *IS 456 (1978) Code of Practice for Plain and Reinforced Concrete.*
2. *SP 16 (1980) Design Aids to IS 456 (1978).*
3. *SP 34 (1987) Handbook on Concrete Reinforcement and Detailing.*

All students should buy copies of these very useful documents. Only sample charts, tables, figures, etc. have been reproduced here with permission from BIS to illustrate the use of these publications for design. Many definitions, list of symbols etc. have been purposely omitted in this text. Students should consult the BIS publications in this regard. In general, easily understandable internationally accepted symbols are used throughout the book. The readers are advised to refer to SP-24 —Explanatory Handbook on IS 456 for a better understanding of the various provisions of the code.

As this text is the outcome of the lectures I delivered for several years for the first compulsory course on Reinforced Concrete Design, it does not deal with all the provisions of IS 456 (1978), but with only those topics that all civil engineers should know. A second course covering advanced topics such as deflection, crack width, flat slabs, deep beams, ribbed floors, beam column connections etc. are offered as an elective to selected undergraduate students or as a basic course to all postgraduate students in Structural Engineering. It is hoped that these will be published as a separate volume at a later date.

The text has been class tested and was well received by many batches of my students. I fervently hope that it will benefit a large number of students and professionals who are interested in the subject.

This year, the College of Engineering, Guindy, Anna University—an institution which has been in the vanguard of education in engineering and technology—is celebrating its two hundredth year of existence. As an Honorary Professor of the University, I have great pleasure in presenting this book during the bicentennial celebration of this great institution.

P.C. Varghese

Ackno

I wish to ack
and publicati
I studied t
Jr. at Harvar
of them I an
It was dur
at IIT Madr
Reinforced C
Moratuwa, S
Code on Lim
I owe a debt
over several
I am indel
for his invita
Prof. M. Ana
with the Uni
help by readi
Suresh Math
while they w
Acknowle
to reproduce
IS 456 (1978
of these ver
designers in
Finally, I
Publishers, P

Acknowledgements

I wish to acknowledge the help received from various individuals and institutions during the preparation and publication stages of the manuscript.

I studied the fundamentals of modern Reinforced Concrete Design first under Prof. Dean Peabody, Jr. at Harvard University and then under Prof. A.L.L. Baker at Imperial College, London. To both of them I am indebted for creating in me an interest in the subject.

It was during my teaching career at the Indian Institute of Technology (IIT) Kharagpur and then at IIT Madras, lasting over twenty years, that I took up postgraduate teaching and research in Reinforced Concrete. While I was working as UNESCO Technical Adviser at the University of Moratuwa, Sri Lanka, I got the opportunity to teach Reinforced Concrete Design based on the British Code on Limit State Design for nearly ten years. To these institutions and the students I taught there, I owe a debt of gratitude for the help I received to evolve this textbook from the lectures delivered over several years.

I am indebted to Prof. V.C. Kulandaiswamy, former Vice-Chancellor, Anna University, Madras for his invitation to work with the University as Honorary Professor after my retirement and to Prof. M. Anandakrishnan, the present Vice-Chancellor for his encouragement to continue my association with the University. Also, Professor P. Purushothaman of Anna University has rendered valuable help by reading as well as correcting the manuscript and using it in his classes at the University. Suresh Mathen and M.A. Abraham have contributed by checking the examples given in the text while they were working with me as engineer-trainees.

Acknowledgement is also due to the Bureau of Indian Standards for liberally granting permission to reproduce in this book typical tables, charts, figures and other materials from their publications, IS 456 (1978), SP 16, SP 24, and SP 34. It is hoped that explanations and illustrations of the use of these very useful publications in this book will lead to their wider use by the students and designers in India.

Finally, I wish to put on record my appreciation for the excellent cooperation received from the Publishers, Prentice-Hall of India, New Delhi, both during editorial and production stages.

P.C. Varghese

Introduction

Before the last two decades reinforced concrete designers were concerned more with the safety against failure of their structures than with durability under service conditions. Theoretical calculations for design of reinforced concrete members were based on classical elastic theory, fictitious modulus of elasticity of concrete, and permissible working stresses. A start on the recent development leading to limit state design, otherwise called strength and performance criterion, can be said to have been made from the date of creation of the European Committee for Concrete (Comite European du Beton) called CEB, in 1953. The initiative for this came from the reinforced concrete contractors of France. The Committee has its headquarters at Luxembourg. Its objectives are the coordination and synthesis of research on safety, durability and design calculation procedures, for practical application to construction. Their first recommendations for reinforced concrete design were published in 1964.

Later, under the leadership of Yves Guyon (well known for his expertise on prestressed concrete), the CEB established technical collaboration with the International Federation for prestressing (Federation International de la Preconstrainte), called FIP. Recommendations for international adoption for design and construction of concrete structures were published by them in June 1970 and the "CEB-FIP Model Code for Concrete Structures" was proposed in 1977. These efforts formed the solid bases for the creation of an "International Code of Practice". Through these publications a unified code for design of both reinforced and prestressed concrete structures was developed.

According to the above model code, structural analyses, for determination of bending moments, shears etc. are to be carried by elastic analysis, but the final design of the concrete structures is to be done by the principles of limit state theory.

The model code was to be a model from which each country was to write its national code, based on its stage of development but agreeing on important points, like method of design for bending, shear, torsion etc., to the model code. The basis had to be scientifically rigorous, but compromises could be made because of inadequacy of data on the subject for any region.

The British were the first to bring out a code based on limit state approach as recommended by the CEB-FIP in 1970. This code was published as Unified Code for structural concrete (CP 110: 1972). Other countries in Europe and the United States adopted similar codes, and today most countries follow codes based on the principles of Limit State Design.

India followed suit during the revision of IS code 456 in 1978, and the provisions of the limit state design (as regards concrete strength, durability and detailing) were incorporated in the revised code IS 456 (1978) in Sections 1–4. However, for design calculations to assess the strength of an R.C. member, the choice of either limit state method or working stress method has been left to the designer (Sections 5 and 6) with the hope that with time, the working stress method will be completely replaced by the limit state method. Many of the Provisions of IS code are very similar to the BS approach.

A uniform approach to design, with reference to the various criteria, is the dream of all reinforced concrete designers with an international outlook, but it is bound to take many more years to come into effect. In the USA the code used for general design of reinforced concrete structures is the "Building Code Requirement for Reinforced Concrete" ACI 318 (1983). The general principles of limit state design are named as *strength and serviceability method* in the above code. It is also interesting to note that among the European Common Market countries there is a move to unify the codes of the various member countries.

As research in various aspects of concrete design is still being carried out in many countries and these countries are anxious that the results of these latest research are reflected in their national codes, it will take a long time for all the codes in the world to be the same. It is therefore advisable that a student be aware of at least the general provisions of the codes of other countries too. It is for this purpose that at many places in this book, IS, BS and ACI provisions are briefly discussed and compared.

As has happened in other scientific fields, new ways of thinking replace the old ways. In scientific circles this is generally referred to as a *paradigm shift*. Limit state design should therefore be looked upon as a "paradigm", a better way of explaining certain aspects of reality and a new way of thinking about old problems. Thus, it should be learned and taught with its own philosophy, and not as an extension of the old elastic theory. This book is therefore exclusively devoted to the study of 'Limit State Philosophy' and is written with the hope that it will give the reader insight into the philosophy of Limit State Method for design of concrete structures.

1

Methods of Design of Concrete Structures

1.1 INTRODUCTION

Reinforced concrete members are allowed to be designed according to existing codes of practice by one of the following two methods (IS 456: clauses 18.2 and 18.3):

1. The method of theoretical calculations using accepted procedures of calculations.
2. The method of experimental investigations.

The theoretical methods are employed for design of the commonly used structures. These methods consist of numerical calculations based on the procedures prescribed in codes of practices prevailing in the country. Such procedures are based on one of the following methods of design:

1. The modular ratio or the working stress method, also known as the elastic method.
2. The load factor method.
3. The limit state method.

The experimental methods are used only for unusual structures and are to be carried out in a properly equipped laboratory by (a) tests on scaled models according to model analysis procedures, and (b) tests on prototype of the structure. This book deals only with the methods based on theoretical calculations, and hence reference should be made to other published literature for methods of design by experimental investigations. As already mentioned, experimental methods arise only when one has to deal with unusual structures about which sufficient data on theoretical methods of calculations is not available.

The theoretical methods themselves are the result of extensive laboratory tests and field investigations. Safe and universally accepted methods of calculations based on strength of materials and applied mechanics have been derived from these laboratory investigations and are codified into the national codes.

The code of practice to be used in India at present is the one published by the Bureau of Indian Standards IS 456-1978. All reinforced concrete structures built in India are required to follow the provisions of these codes. IS 456 is very similar to the British Codes CP 110 (1973) and its revised version BS 8110 (1985). The American practice follows the ACI Code 318 (1983), the German practice, DIN 1045, and the Australian practice, AS 1480.

The Indian Code at present allows the use of both the working stress and the limit state methods of design. However, as more and more countries are adopting only the limit state method of design, we can expect that India will also, in the near future, discard the modular ratio method and follow the limit state method for design of reinforced concrete structures.

This chapter deals briefly with the various theoretical methods of design mentioned above.

1.2 MODULAR RATIO OR WORKING STRESS METHOD (WSM)

This method of design was evolved around 1900 and was the first theoretical method accepted by national codes of practice for design of reinforced concrete sections. It assumes that both steel and concrete act together and are perfectly elastic at all stages so that the modular ratio (ratio between moduli of elasticity of steel and concrete) can be used to determine the stresses in steel and concrete. This method adopts permissible stresses which are obtained by applying specific factors of safety on material strength for design. It uses a factor of safety of about 3 with respect to cube strength for concrete and a factor of safety of about 1.8 (with respect to yield strength) for steel.

Even though structures designed by this method have been performing their functions satisfactorily for many years, it has three major defects. *First*, since the method deals only with the elastic behaviour of the member, it neither shows its real strength nor gives the true factor of safety of the structure against failure. *Second*, modular ratio design results in larger percentages of compression steels than is the case while using limit state design, thus leading to uneconomic sections while dealing with compression members or when compression steel is used in bending members. *Third*, the modular ratio itself is an imaginary quantity. Because of creep and nonlinear stress-strain relationship, concrete does not have a definite modulus of elasticity as in steel.

In the modular ratio method of design, the design moments and shears in the structure are calculated by elastic analysis with the characteristic loads (service loads) applied to the structure; the stresses in concrete and steel in the sections are calculated on the basis of elastic behaviour of the composite section. An imaginary modular ratio which may be either a constant in value for all strengths of concrete or one which varies with the strength of concrete is used for calculation of the probable stresses in concrete and steel.

CP 114, the code used in U.K. till 1973, recommended the use of a constant modular ratio of 15, independent of the strength of concrete and steel. Other codes like IS 456 recommend a modulus of elasticity of concrete which varies with the strength of concrete.

It should, however, be noted that modular ratio method with due allowance for change of the value of modulus of concrete to allow for creep, shrinkage etc. is the only method available when one has to investigate the R.C. section for service stresses and for the serviceability states of deflection and cracking. Hence a knowledge of working stress method is essential for the concrete designer and forms part of limit state design for a serviceability condition. This method is explained in detail in Appendix A at the end of the text.

1.3 LOAD FACTOR METHOD (LFM)

A major defect of the modular ratio method of design is that it does not give a true factor of safety against failure. To overcome this, the ultimate load method of design was introduced in R.C. design. This method, later modified as the Load Factor Method (LFM), was introduced in U.S.A. in 1956, in U.K. in 1957, and later on in India. In this method, the strength of the R.C. section at working load is estimated from the ultimate strength of the section. The concept of load factor, which is defined as the ratio of the ultimate load the section can carry to the working load it has to carry, was also introduced in U.K. Usually, R.C. structures are designed for suitable separate load factors for dead loads and for live loads with additional safety factors for strength of concrete.

After the introduction of the load factor method, in order to make the calculations comparable with the modular ratio method, some codes like the British and the Indian codes adopted the Modified Load Factor Method. This method used the *ultimate load principles* for design, but retained the *allowable service stresses* concept in the calculations. Thus CP 114 (the code used earlier in

U.K.) used a load factor (ratio of ultimate load to working load) of 2 with additional safety factor applied to material strength, to arrive at the permissible service stresses. As the variation of strength of concrete is much more than that in steel, an additional factor of safety of 1.5 (i.e. 3/2) for designed mixes and 1.67 (i.e. 5/3) for nominal mixes were used when calculating the permissible concrete stresses. This additional factor of safety for concrete also ensured that failure always took place due to tension failure of steel, and not by sudden compression failure of concrete. It should be noted that historically the load factor method was the first method which did not use the imaginary modular ratio for design of reinforced concrete members. As this method has since been superseded by the limit state method in codes of practice, today it is not necessary for the student to make a separate study of the load factor method of design in great detail.

1.4 LIMIT STATE METHOD (LSM)

Even though the load factor method based on ultimate load theory at first tended to discredit the traditional elastic approach to design, the engineering profession did not take to such design very readily. Also, steadily increasing knowledge brought the merits of both elastic and ultimate theories into perspective. It has been shown that whereas ultimate theory gives a good idea of the strength aspect, the serviceability limit states are better shown by the elastic theory only.

Since a rational approach to design of reinforced concrete did not mean simply adopting the existing elastic and ultimate load theories, new concepts with a semi-probabilistic approach to design were found necessary. The proposed new method had to provide a framework which would allow designs to be economical and safe. This new philosophy of design was called the *Limit State Method* (LSM) of design. It has been already adopted by many of the leading countries of the world in their codes as the only acceptable method of design of reinforced concrete structures.

1.5 LIMIT STATE METHOD IN NATIONAL CODES

Designs based on limit state principles are nowadays internationally accepted for routine design of reinforced concrete structures. In the U.K., BS 8110 (1985), which is the revised version of CP 110 (1973), follows limit state methods. The Indian Code IS 456 (1978) has adopted limit state method along with working stress method for design of R.C. members.

Provisions in both the Indian and the British Codes for limit design are very similar, and many of the coefficients and tables recommended for design have the same value. Both of them were evolved from the "Recommendations for an International Code of Practice for Reinforced Concrete" published by CEB (the European Committee for Concrete) in 1963, generally known as the *Blue Book*, and the complementary report "International recommendation for the Design of Concrete Structures" published in 1970 by the CEB along with FIP [The International Federation for Prestressing], commonly known as the *Red Book*. These were revised in 1978 by CEB-FIP as the "Model Code for Concrete Structure" as a model for the national codes to follow. BS and IS codes have taken many of their provisions from these publications.

In the U.S.A., the Code of Practice published by the American Concrete Institute ACI 318-83, called Building Code Requirements for Reinforced Concrete, is currently used for design. The philosophy of design used in this code is sometimes referred to as *strength and serviceability design*, and has the same basic philosophy as the BS and IS codes.

1.6 DESIGN BY MODEL AND LOAD TESTS

As pointed out earlier, designs can also be made on the basis of results of load tests on models or prototypes. In that case, instead of theoretical structural analysis of complicated structural combinations, tests are conducted on models made of materials like perspex or microconcrete. Thus:

1. these tests can be used to give a very good physical idea of the action of these structures; or

2. the results of observations of deflections and strains interpreted by principles of model analyses can be directly used for design; or

3. the results of the experimental model tests can be used to determine the boundary conditions and form the basis for complex computer analysis of the whole structure.

The structural adequacy of reinforced concrete members which are factory made or precast in large quantities can also be tested for performance by means of laboratory tests on prototypes. These tests give not only the strength but also the deflection and cracking performance of the structure under any given loading. Many factory made products like prestressed concrete sleepers have been developed by prototype testing.

In those cases where the design and construction are to be finally passed on the basis of experimental (load) tests on prototypes, they should satisfy the necessary requirements of deflection and cracking, depending on conditions under which the product is likely to be used. Thus, prestressed concrete sleepers which will be subjected to a large number of repetitive loading during their life should be tested under millions of cyclic loads in addition to static tests.

IS 456: clause 18.3 gives the following recommendations for designs based on experimental basis:

1. The structure should satisfy the specified requirements for deflection and cracking when subjected to a load of 1.33 times the "factored design load for serviceability conditions" for 24 hours. In addition, there should be 75 per cent recovery of deflection after 24 hours of loading.

2. The structure should have sufficient strength to sustain 1.33 times "the factored load for collapse" for 24 hours.

These tests on prototypes are different from model tests described earlier. Both these tests should be conducted by competent persons with reliable equipments. Testing of structures for acceptance should be carried out according to IS 456: clause 16.

1.7 PUBLICATIONS BY BUREAU OF INDIAN STANDARDS

The Indian Standard Code of Practice IS 456 (1978) published by the Bureau of Indian Standards (formerly known as the Indian Standards Institution) is the main text to be followed by designers in India for limit state method of design of R.C. members. The Bureau has also brought out the following publications to supplement the code and make its use easy and popular:

1. Design Aids for Reinforced Concrete to IS 456 (1978): SP 16 (1980)—Special publication No. 16.

2. Explanatory Handbook on Indian Standard Code of Practice for Plain and Reinforced Concrete IS 456 (1978): SP 24 (1984).

3. Handbook on Concrete Reinforcement and Detailing as SP 34 (1987).

By making use of these special publications, one will be able to design R.C. structures with great speed and accuracy.

REVIEW QUESTIONS

1.1 Enumerate the different methods of design of reinforced concrete members which are accepted in practice.

1.2 Name the codes of practice used for design of concrete structures for general building purposes in (a) India, (b) U.K., (c) U.S.A., and (d) Germany.

1.3 Give a short description of the following methods of design of reinforced concrete structures:

(a) Working stress method
(b) Ultimate strength method
(c) Load factor method
(d) Limit state method
(e) Strength and serviceability method.

State the differences between the load factor method and the limit state method.

1.4 What is meant by modular ratio? Why is it considered to be an unreliable quantity? What is the difference in the value assumed for this quantity between IS and BS?

1.5 Explain the terms *model* and *prototype* of a structure.

1.6 When will one use model studies for the design of a structure as different from theoretical calculations? How can model analysis be used for design of concrete structures?

1.7 Explain the use of prototype testing in structural design. Give examples as to where you will recommend them.

1.8 Give the IS specifications for load testing of prototypes for design based on experiments, stating the conditions to be satisfied. Can field tests on a completed bridge be considered as prototype testing? What are the loadings to be used for these acceptance tests?

1.9 What organisations are referred to as CEB and FIP and in what way is IS 456 (1978) related to their publications?

1.10 Is the limit state method in any way a better method of design of concrete structures than the working stress design? Give reasons for your answer.

1.11 Name the special publications by the Bureau of Indian Standards to supplement IS 456 (1978).

2

Partial Safety Factors in Limit State Design

2.1 INTRODUCTION

A structure is said to have reached its limit state, when the structure as a whole or in part becomes unfit for use, for one reason or another, during its expected life. The limit state of a structure is the condition of its being not fit for use, and limit state design is a philosophy of design where one designs a structure so that it will not reach any of the specified limit states during the expected life of the structure.

Many types of limit states or failure conditions can be specified. The two major limit states which are usually considered are the following:

1. The ultimate strength limit state, or the limit state of collapse, which deals with the strength and stability of the structure under the maximum overload it is expected to carry. This implies that no part or whole of the structure should fall apart under any combination of expected overload.

2. The serviceability limit state which deals with conditions such as deflection, cracking of the structure under service loads, durability (under a given environment in which the structure has been placed), overall stability (i.e. resistance to collapse of the structure due to an accident such as a gas explosion), excessive vibration, fire resistance, fatigue, etc.

2.2 PRINCIPLES OF LIMIT STATE DESIGN

Limit state design should ensure that the structure will be safe as regards the various limit state conditions, in its expected period of existence. Hence the limit state method of design is also known in American terminology as *strength and serviceability* method of design.

The two major limit state conditions to be satisfied namely, the ultimate limit state and the serviceability limit state, are again classified into the following major limit states which are given in the various clauses in IS 456 (1978).

Limit States

(1) Limit State of Collapse or Ultimate Limit State
- (i) Flexure (≠ 37)
- (ii) Compression (≠ 38)
- (iii) Shear (≠ 39)
- (iv) Torsion (≠ 40)
- (v) Tension

(2) Serviceability Limit State
- (i) Durability (≠ 7)
- (ii) Deflection (≠ 41)
- (iii) Cracking (≠ 42)
- (iv) Overall stability

[≠ refer to clause in IS 456 (1978)]

The usual practice of design of concrete structure by limit state principles consists in taking up each of the above conditions and providing for them separately so that the structure is safe under all the limit states of strength and stability.

2.3 PROCEDURE FOR DESIGN FOR LIMIT STATES

The design should provide for all the above limit state conditions; each of these conditions is carried out as described now.

1. Ultimate strength condition

The ultimate strength of the structure or member should allow an overload. For this purpose, the structure should be designed by the accepted ultimate load theory to carry the specified overload. This may be in-flexure, compression, shear, torsion or tension.

2. Durability condition

The structure should be fit for its environment. The cover for the steel as well as the cement content and water-cement ratio of the concrete that is provided in the structure should satisfy the given environmental conditions mentioned in Chapter 3.

3. Deflection condition

The deflection of the structure under service load condition should be within allowable limits. This can be done by two methods:

(i) *Empirical method.* Since the most important empirical factor that controls deflection is span/depth ratio, deflection can be controlled by limiting the span-depth ratios as specified by the codes.

(ii) *Theoretical method.* Deflection can also be calculated by theoretical methods and controlled by suitable dimensioning of the structure.

4. Cracking condition

The structure should not develop cracks of more than the allowable widths under service load condition. This can be taken care of by employing two methods:

(i) *Empirical method.* By strictly following the empirical bar detailing rules as specified in the codes.

(ii) *Theoretical method.* The probable crack width is checked by theoretical calculations.

5. Lateral stability against accidental horizontal loads (overall stability)

This condition is met by observing the empirical rules given in codes for designing and detailing the vertical, horizontal, peripheral and internal ties in the structure.

2.4 CHARACTERISTIC LOAD AND CHARACTERISTIC STRENGTHS

Structures have to carry dead and live loads. The maximum working load that the structure has to withstand and for which it is to be designed is called the *characteristic load.* Thus there are characteristic dead loads and characteristic live loads.

The strengths that one can safely assume for the materials (steel and concrete) are called their *characteristic strengths.*

For the sake of simplicity, it may be assumed that the variation of these loads and strengths follows normal distribution law so that the laws of statistics can be applied to them (see Fig. 2.1).

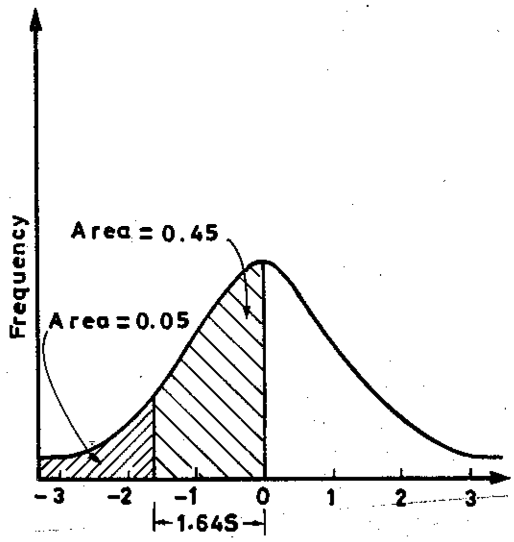


Fig. 2.1 Areas under the normal probability curve.

As the design load should be more than the average load obtained from statistics (Fig. 2.2), we have

Characteristic design load = [Mean load] + K [Standard deviation for load]

As the design strength should be lower than the mean strength,

Characteristic strength = [Mean strength] – K [Standard deviation for strength]

The value of the constant K is taken by common consent as that corresponding to 5 per cent chance so that K will be equal to 1.64 as shown in Fig. 2.1. (This is taken as 1.65 in Indian Standards.)

Even though the design load has to be calculated statistically as indicated above, research for determining the actual loading on structures has not yet yielded adequate data to enable one to calculate theoretical values of variations for arriving at the actual loading on a structure. Loads that have been successfully used so far in the elastic design procedures are at present accepted as the characteristic loads. The specified values to be used are laid down in IS 875.

As stated earlier, the strengths that one can safely assume for steel and concrete are called their characteristic strengths. Sufficient experimental data is already available about characteristic strengths of steel and concrete. These strengths are calculated from the theory of statistics and are related to the standard deviation of the results of strength tests on the constituent materials.

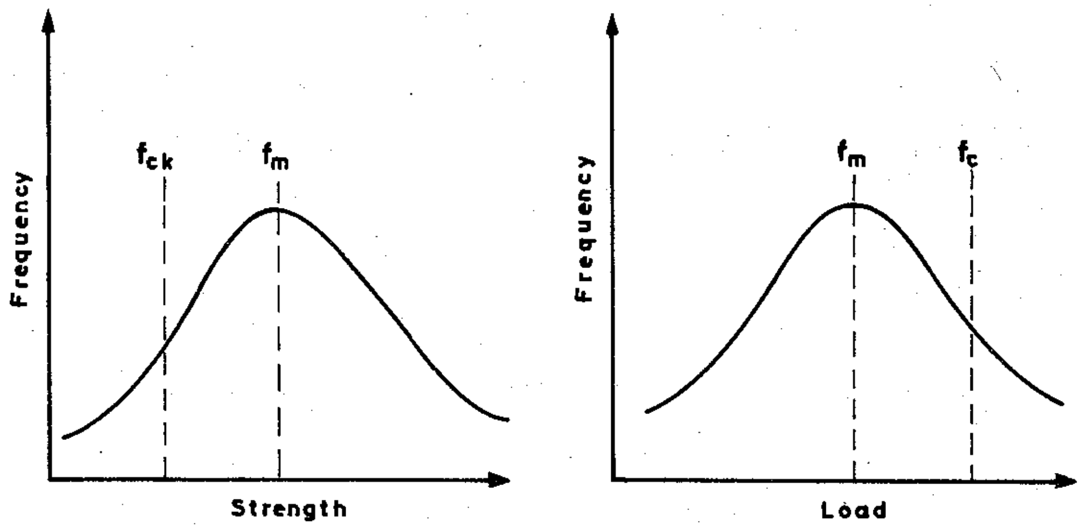


Fig. 2.2 Characteristic strengths and characteristic loads.

2.5 PARTIAL SAFETY FACTORS FOR LOADS AND MATERIAL STRENGTHS

Having obtained the characteristic loads and characteristic strengths, the design loads and design strengths are obtained by the concept of partial safety factors. Partial safety factors are applied both to loads on the structure and to strength of materials. These factors are now explained.

2.5.1 PARTIAL SAFETY FACTOR FOR LOAD γ_f

The load to be used for ultimate strengths design is also termed as *factored load.* In IS Code the symbol DL is used for dead load, LL for live load, WL for wind loads and EL for earthquake loads (Table 12 of IS 456 and Table 2.1 of the text). It may be noted that the use of partial safety factor for load simply means that for calculation of the ultimate load for design, the characteristic load has to be multiplied by a partial safety factor denoted by the symbol γ_f. This may be regarded as the overload factor for which the structure has to be designed. Thus the load obtained by multiplying the characteristic load by the partial safety factor is called the *factored load*, and is given by

$$\text{Factored load} = (\text{Characteristic load}) \times (\text{Partial safety factor for load } \gamma_f)$$

Structures will have to be designed for this factored load.

It is extremely important to remember that in limit state design, the design load is different from that used in elastic design. It is the factored loads, and not the characteristic loads, which are used for the calculation of reactions, bending moments, and shear forces. The partial safety factors to be used for calculating the factored loads as specified in IS 456 for various types of loads are given in Table 2.1.

It may be noted that by adopting a partial safety factor of 1.5, both for dead and live loads, the value of the moment, shear force etc. to be used in limit state design by IS Code is 1.5 times the moments, shear etc. that would have been used for elastic (working stress) design.

Theoretically, the partial safety factors should be different for the two types of loads. The British Code BS 8110 uses a factor of 1.4 for DL and 1.6 for LL for strength considerations. It is

TABLE 2.1 FACTORED LOADS FOR LIMIT STATE DESIGN
(Partial safety factors for loads)
(IS 456: Table 12)

Load combination	Ultimate limit state	Serviceability limit state
1. Dead and imposed	1.5 DL + 1.5 LL	DL + LL
2. Dead and wind		
Case (i): Dead load contributes to stability	0.9 DL + 1.5 WL	DL + WL
Case (ii): Dead load assists overturning	1.5 DL + 1.5 WL	DL + WL
3. Dead, imposed and wind	(1.2 DL + 1.2 LL + 1.2 WL)	(1.0 DL + 0.8 LL + 0.8 WL)

Note: While considering earthquake effects, substitute EL for WL.

only for convenience of using the same structural analysis for both elastic design and limit state design that IS recommends the same partial safety factor for dead and live loads. Thus in IS 456 the factored load, shear, moment etc. in limit state design will be 1.5 times the value used for elastic design.

2.5.2 PARTIAL SAFETY FACTORS FOR MATERIAL STRENGTHS γ_m

The grade strength of concrete is the characteristic strength of concrete, and the guaranteed yield strength of steel is the characteristic strength of steel. Calculation to arrive at the characteristic material strength of materials by using statistical theory takes into account only the variation of strength between the test specimens. It should be clearly noted that the above procedure does not allow for the possible variation between the strength of the test specimen and the material in the structure which, as will be seen in Section 2.6, is taken separately by a factor 0.67. The concept of partial safety factor for material strength due to variations in strength between samples is given by the relation

$$\text{Design strength} = \frac{\text{Characteristic strength}}{\text{Partial safety factor for strength, } \gamma_m}$$

This simply means that the strength to be used for design should be the reduced value of the characteristic strength by the factor denoted by the partial safety factor for the material. The recommended values for these partial safety factors are given in Table 2.2.

TABLE 2.2 PARTIAL SAFETY FACTORS FOR STRENGTH, γ_m
(IS 456: clause 35.4.2)

Material	Ultimate limit state	Serviceability deflection	Limit state cracking
Concrete	1.50	1.00	1.30
Steel	1.15	1.00	1.00

It should be clearly understood that the partial safety factors used in limit state design are different from the factors of safety used in elastic design. The values recommended for factors of safety in elastic design as mentioned in Chapter 1 are usually 3 over the cube strength of concrete for bending compression and 1.8 over the yield strength for steel stresses. Thus in designing by working stress method, one works at stress levels well below the failure strength of concrete and steel.

It should also be remembered that the tables and formulae derived for limit state design and those used in the Design Aids SP16 are derived with values of γ_m already incorporated in them. Hence, unlike the partial safety factor for load, these partial safety factors for material strengths need not be considered in routine design when using these formulae, charts and tables.

2.6 STRESS-STRAIN CHARACTERISTICS OF CONCRETE

The mechanical properties of concrete, such as its stress-strain curve, depend on a number of factors like rate of loading (creep), type of aggregate, strength of concrete, age of concrete, curing conditions, etc. Figure 2.3a shows the typical stress-strain curves for concrete tested under standard conditions. It can be seen that the failure strain is rather high and it is of the order of 0.4 per cent when tested under constant strain rate.

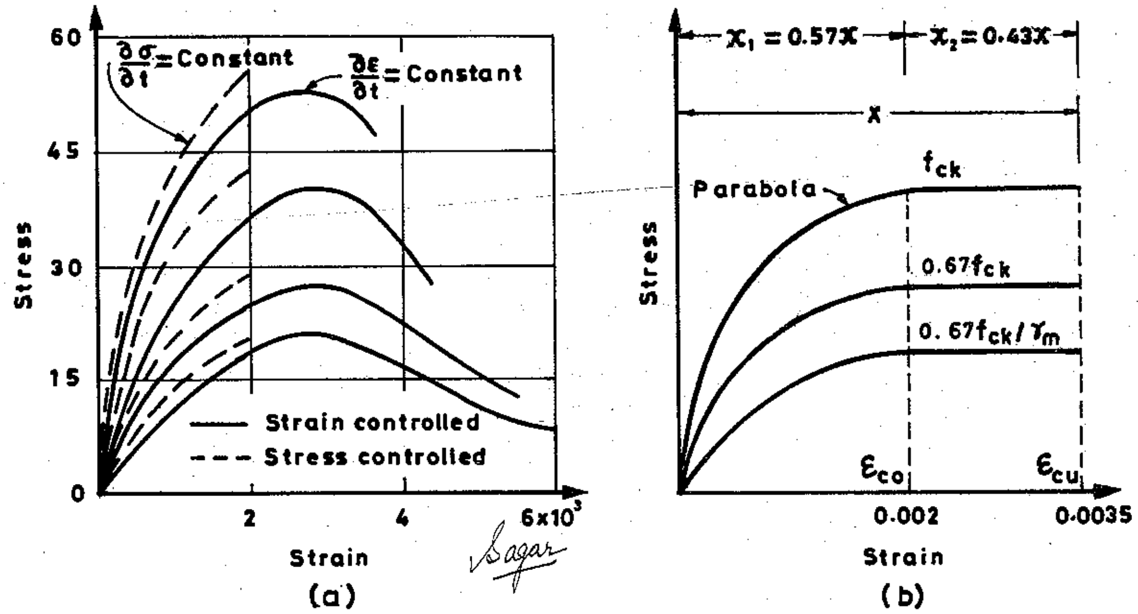


Fig. 2.3 Design stress-strain curves for concrete in compression: (a) Laboratory test curves, and (b) Idealised curves.

However, to derive an analytical expression for the stress-strain curve, it is necessary to idealise the curve. By common consent, a rectangular parabolic curve (Fig. 2.3b) has been accepted as the stress-strain curve for concrete with the ultimate strain at failure as 0.0035. Codes differ with respect to the strain ε_{co} at which the strength becomes constant. In IS it is taken as a constant value of 0.002, and in BS, as a function of the strength of concrete and equal to $2.4 \times 10^{-4}\sqrt{f_{ck}/\gamma_m}$. Thus, the IS curve simplifies the distance at which the parabola ends and the rectangle begins. Its value can be deduced as follows: If x is taken as the depth of the neutral axis corresponding to the strain 0.0035, the distance from the origin for the 0.002 strain is given by

$$x_1 = \frac{0.002}{0.0035} \text{ (distance of neutral axis)}$$

$$= 0.57x$$

Thus the parabola extends to a distance $(0.57x)$ and the rectangle for a distance $0.43x$, as shown in Fig. 2.3.

The short term, static modulus for concrete, E_c is assumed by IS code clause 5.2.3.1 as

$$E_c = 5700\sqrt{f_{ck}} \ (\text{N/mm}^2)$$

In most calculations this value has to be modified for creep and other long term effects.

In order to distinguish between the concrete as tested in a cube and the concrete that exists in the structure (size effect), it is assumed that the concrete in the structure develops a strength of only 0.67 times the strength of the cube. Hence the theoretical stress-strain curve of the concrete in the design of structures is correspondingly reduced by the factor 0.67, as indicated in Fig. 2.3.

In addition to the above and as explained earlier, a partial safety factor of 1.5 is applied on the concrete in the structure so that the design stress-strain curve for concrete in a structure will be as shown in Fig. 2.3 (IS 456; clause 37, Fig. 20).

2.7 STRESS-STRAIN CHARACTERISTICS OF STEEL

The stress-strain curve for steel according to IS 456: clause 37.1 is assumed to depend on the type of steel.

Mild steel bar (f_y = 250) is assumed to have a stress-strain curve as shown in Fig. 2.4a and cold worked deformed bar (Fe 415) a stress-strain curve as shown in Fig. 2.4b (Fig. 22 of IS 456).

The stress-strain curves for steel, both in tension and compression in the structure, are assumed to be the same as obtained in the tension test. As the yield strength of IS grade steel has a minimum guaranteed yield strength, the partial safety factor to be used for steel strength need not be as large as that for concrete. The partial safety factor recommended for steel is 1.15, and this is to be applied to the stress-strain curve as shown in Fig. 2.4 (IS 456, Fig. 22). It should be noted that for cold worked deformed bars the factor 1.15 is applied to points on the stress-strain curve from $0.8f_y$ to f_y only. The value of E_s is assumed as 200 kN/mm^2 for all types of steels.

In the revised BS 8110 (1985), the stress-strain curves for all steels used in reinforced concrete are simplified and assumed to be bilinear as in the case of mild steel bars. The stress-strain curves for compression and tension are also assumed to be the same. The curve used in BS 8110 is shown in Fig. 2.5.

2.8 SUMMARY OF DESIGN BY LIMIT STATE METHOD

The procedure to be followed in design by limit state method consists in examining the safety of the structure for at least all the important limit state conditions explained in this chapter (strength, durability, deflection cracking and overall stability).

Concepts of characteristic strengths and characteristic loads are used for design for strength. Separate partial safety factors for strength and loads are also introduced. The design strengths of steel and concrete are taken as the characteristic strength divided by their respective partial

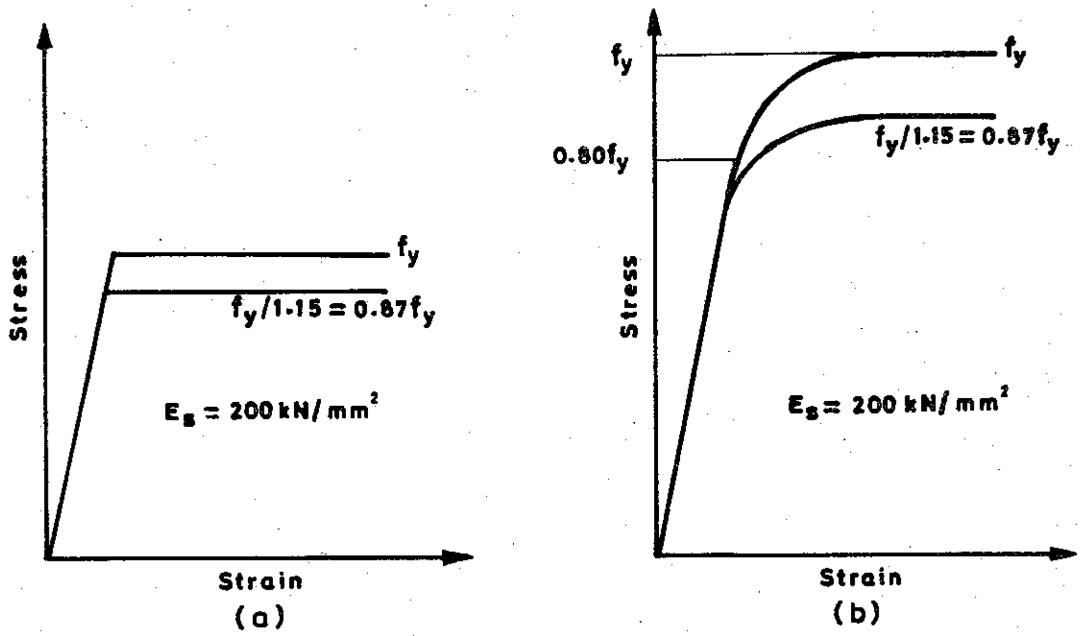


Fig. 2.4 Stress-strain curves for steel reinforcements: (a) Mild steel and (b) Cold worked bars.

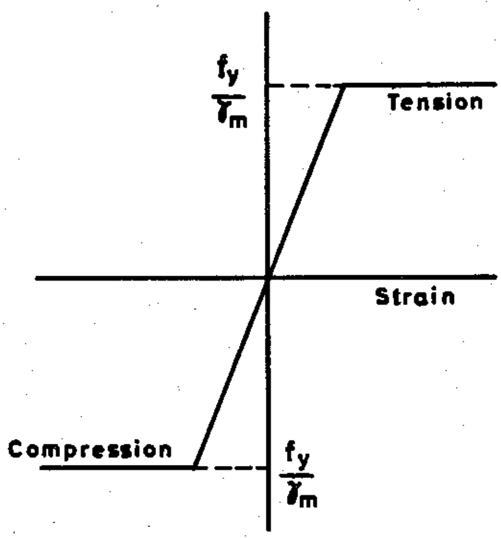


Fig. 2.5 Stress-strain curve for steel (BS 8110).

safety factors for strengths. Similarly, the factored load to be resisted by the structure is taken as the product of the characteristic load and the partial safety factor for loads. The stress-strain curve for concrete and steel are assumed to be of fixed shape, for convenience in mathematical computation.

EXAMPLE 2.1 **(Calculation of factored or design loads)**

A one-way slab for a public building (loading class 300) is 200 mm in overall thickness. It is simply supported on a span of 4 m. Determine the factored moment and factored shear for strength design and the loads for checking serviceability.

Ref.	Step	Calculations	Output
	1.	*Characteristic loads*	
		Assume unit weight of R.C.C. = 25 kN/m^3	
		DL = $0.20 \times 1 \times 25 = 5.0$ kN/m^2	
IS 875		For Class 300, LL = 300 kg/m^2 = 3 kN/m^2	
	2.	*Factored load, moment and shear*	
IS 456 Table 12		Factored load (w) = 1.5 (DL + LL)	
		$w = 1.5\,(5 + 3) = 12$ kN/m^2	$w = 12$ kN/m^2
		Factored (design) moment M_u	
		$M_u = \dfrac{wL^2}{8} = \dfrac{12 \times 4 \times 4}{8} = 24$ kNm	$M_u = 24$ kNm
		Design shear	
		$V_u = \dfrac{wL}{2} = \dfrac{12 \times 4}{2} = 24$ kN	$V_u = 24$ kN
	3.	*Load for serviceability conditions (w_s)*	
		$w_s = 1.0$ (DL + LL)	
		$= 1.0\,(5 + 3) = 8$ kN/m^2	$w_s = 8$ kN/m^2

EXAMPLE 2.2 **(Calculation of factored loads)**

A column 4 m high is fixed at the base and the top end is free. It is subjected to the following loads:

Total DL = 40 kN

Total imposed (gravity) load = 100 kN

Wind load = 4 kN per metre height

Determine the factored loads (a) for strength, and (b) serviceability limit states.

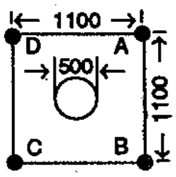
Ref.	Step	Calculations	Output
	1.	*Total wind load*	(Loads in kN)
		WL = 4 × height = 4 × 4 = 16 kN	WL = 16
		This acts at 2 m from the bottom.	

EXAMPLE 2.2 (cont.)

Ref.	Step	Calculations	Output
	2.	*Loading for ultimate strength design*	
IS 456 Table 12		(i) Dead and live = 1.5 (DL + LL) = 1.5 (40 + 100)	
		= 210 kN (Vertical)	$P = 210$
		(ii) Dead + wind (Dead load assists overturning)	
		Vertical = 0.9 DL = 0.9 × 40 = 36 kN	$P = 36$
		Horizontal = 1.5 WL = 1.5 × 16 = 24 kN	$H = 24$
		(iii) Dead + imposed + wind	
		Vertical = 1.2 (DL + LL) = 1.2 (40 + 100)	
		= 168 kN	$P = 168$
		Horizontal = 1.2 × 16 = 19.2 kN	$H = 19.2$
	3.	*Loading for serviceability design*	
		(i) Dead + live = 1.0 (DL + LL) = 140 kN	$P = 140$
		(ii) Dead + wind = 1.0 (DL + WL)	
		Vertical load = 1.0 DL = 40 kN	$P = 40$
		Horizontal load = 1.0 WL = 16 kN	$H = 16$
		(iii) Dead + live + wind	
		Vertical = 1.0 DL + 0.8 LL	
		= 1.0 × 40 + 0.8 × 100 = 120 kN	$P = 120$
		Horizontal = 0.8 WL = 0.8 × 16 = 12.8 kN	$H = 12.8$

EXAMPLE 2.3 **(Calculation of design loads)**

An R.C. column of 500 mm dia has to carry a direct load of 900 kN and a moment of 100 kNm about the *YY*-axis due to characteristic dead and live loads. A seismic moment of 500 kNm is estimated to be felt on the column in any direction. Calculate the design loads for the pile and pile caps for a layout of the piles, as shown in Fig. E.2.3.

Ref.	Step	Calculations	Output
	1.	*Gravity load on each pile* (VL)	
		$= \frac{900}{4} = 225$ kN	
	2.	*Load due to moment with gravity loads* (ML)	**Fig. E. 2.3**
		$= \frac{100}{1.1} = 90.9$ kN	

EXAMPLE 2.3 (cont.)

Ref.	Step	Calculations	Output
		(Say, upwards on D and C and downwards on A and B)	
		Load on each pile = 45.45 kN	
	3.	*Load due to earthquake moment* (EL)	
		$= \frac{500}{1.1} = 454.5$ kN	
		(upwards or downwards)	
		Load on each pile = 227.25 kN	(Loads in kN)
	4.	*Maximum load on the pile due to* VL + ML	
IS 456 Cl. 35.4		= 1.5 (225 + 45.45) = 405.7 kN (max)	$P = 405.7$
	5.	*Maximum load on one pile due to* VL + ML + EL	
		= 1.2 (225 + 45.45 + 227.25) = 597.2 kN	$P = +597.2$
	6.	*Maximum load (Check for uplift)*	
		= 1.2 (VL + ML – EL)	
IS 456 Table 12		= 1.2 (225 – 45.45 – 227.25) = – 57.2 kN	$P = -57.2$
		Piles should be designed for a minimum capacity of 597.2 kN in bearing and 57.2 kN in uplift.	

REVIEW QUESTIONS

2.1 Explain the term 'limit state design'.

2.2 Enumerate the five limit states commonly used in limit state design and state briefly how they are provided for in design.

2.3 Explain how IS limit state design is very similar to the strength and serviceability design of the American Code.

2.4 What is meant by characteristic strength of a material as used in IS 456 (1978)?

2.5 What is meant by normal distribution in statistics and what is the relationship between mean value and characteristic value in such a distribution assuming 5 per cent confidence limit?

2.6 Define the term 'partial safety factors' as used in limit state design. Identify the various factors and state the values recommended in IS 456.

2.7 Explain the terms 'factored load' and 'characteristic loads'. Why does IS 456 specify the same partial safety factor for dead and live loads? Is it technically correct? What values are recommended in BS code?

2.8 Distinguish between the terms 'factor of safety' and 'partial safety factor' for material strength. What are the usual factors of safety used in elastic design of R.C. members?

2.9 How are the following factors incorporated in the design formulae for limit design:

(a) partial safety factor for load,
(b) partial safety factor for material strength,
(c) difference between cube strength and strength of concrete in structure.

2.10 Draw a curve as done in a laboratory and an assumed design stress-strain curve for concrete and Fe 415 steel according to IS 456.

3

Limit State of Durability of Reinforced Concrete to Environment

3.1 INTRODUCTION

Codes based on Limit State have made checking the structure for durability as an integral part of the design requirements. Durability is as important as strength in the long term performance of a concrete structure. A structure which is very strong when it is constructed, but cannot withstand for a long time the effects of the environment in which it has to exist, is not really useful.

When checking for durability, the durability of steel and concrete should be considered separately. Lack of appreciation of the requirements for durability considerably reduces the life expectancy of structures. Often we come across buildings and bridges which have to be repaired soon after they are constructed. This is specially true of structures built near the sea coasts and those located in industrial areas with high rates of pollution.

Experience and experimentation have confirmed that durability of reinforced concrete is ensured by taking the following precautions:

1. Providing proper amount of cover to the reinforcement bars (see Fig. 3.1).

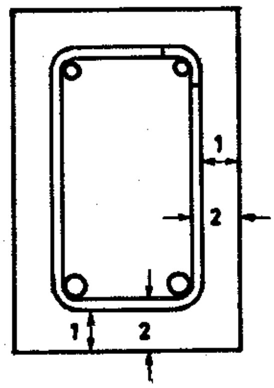


Fig. 3.1 Covers to be satisfied. (1, clear cover to stirrups; 2, clear cover to main steel.)

2. Specifying the minimum cement content and the maximum water-cement ratio that can be used for the given conditions of exposure, to control permeability of concrete.

3. Using proper type of cement and water-cement ratio to withstand special conditions like sulphate action.

4. Using suitable fine and coarse aggregates which will not produce any alkali aggregate reaction.

5. Using the ingredients of concrete (water and aggregates) which do not contain chlorides and sulphates beyond the permissible limits.

6. Compacting the concrete by suitable means, during its placement, so as to form a dense mass without voids.

7. Using proper curing methods after placing of the concrete.

The main factors that affect durability of R.C. structures are discussed in this chapter.

3.2 CORROSION OF STEEL

When the steel in reinforced concrete corrodes, the iron in steel is turned into rust (iron oxide) which occupies up to two and one and a half times more volume than its original volume as steel. This increase in volume causes cracking and breaking up of the concrete cover. The area of steel available to take up forces is also reduced. The rate of corrosion is further accelerated by moisture that gets into the above cracks. Unless proper care is taken in new structures to prevent corrosion and remedial measures are quickly taken in structures that show any distress like cracking, the life expectancy of the concrete structure will be considerably reduced. In addition, there seems to be a tendency for the steel produced by cold working to corrode faster than ordinary mild steel produced by hot rolling. With the wide use of cold worked bars in most of the works done in India, it is all the more necessary that there should be a proper awareness of the urgency to consider and provide for durability during the design stage itself of reinforced concrete structures.

Corrosion of steel is a complicated electrochemical process. Differences in electrochemical potential on the surface of the steel form anodic and cathodic areas on the steel surface. Metal oxidises at the anodes and this leads to corrosion. There are many types of corrosion that can take place in reinforced concrete. Of these, the most important types are the following:

1. Atmospheric corrosion that occurs in normal atmospheric condition.

2. Chloride corrosion that occurs in the presence of chlorides.

Even though other types of actions like stress corrosion and hydrogen embrittlement are important in special situations, they do not occur in ordinary condition in R.C. structures and are not dealt with here. They are more important in prestressed concrete members.

The mechanism of atmospheric corrosion is generally explained as follows: Steel embedded in fresh portland cement concrete is initially in a high alkaline environment produced by the hydration of cement. The pH value of the above matrix is of the order of 12 to 13, and in this range of pH, the steel is in a passive state. However, in normal atmospheric conditions, reduction of the pH value takes place either by the leaching out of the alkali or by the action of atmospheric carbon dioxide with the alkali of the cement paste. This process is called *carbonation*. This can be easily demonstrated by the action of an alcoholic solution of phenolphthalene on a broken concrete surface where the carbonated portion turns pink. When the pH value becomes less than about 9 to a depth equal to the concrete cover, the passivity of steel is destroyed and corrosion starts. The rate of penetration of carbonation, to a certain extent, depends on the permeability of concrete which in turn depends on the cement content and the water-cement ratio. Hence in modern codes like BS 8110, the amount of cover prescribed is not only a function of the environment but also the strength of concrete. In IS 456, however, nominal cover still remains independent of the strength of concrete.

Chloride corrosion takes place in the presence of chlorides, and chlorides present in the constituents of the water or aggregates used for the mix or as chlorides to which the concrete is exposed (as in structures exposed to sea water spray). Corrosion, in this case, takes place irrespective of the pH value. Even with high pH value, the presence of chlorides destroys the passivity of steel. Care should therefore be taken to see that the total chlorides of the constituents of the materials, out of which the concrete is made, are low. Also, in those cases where the finished concrete is exposed to sea water, the depth of cover and *its* permeability are suitably adjusted to control the rate of chloride penetration to the reinforcement. Thus the factors to be controlled in chloride penetration are the amount of chlorides in the constituents of concrete, the permeability of concrete, and the amount of cover to steel provided. The usual limits specified for total chlorides allowed in concrete are given in IS 456 Appendix A, clause A2.

Even though some authorities claim that corrosion can be controlled by addition of corrosion inhibitors into the concrete mix, or by dipping and coating the steel reinforcement by suitable resins or bituminous compounds, the conventional methods of controlling both types of corrosion (oxide and chloride corrosions) are by selecting suitable values of the following:

1. Cover to reinforcement (IS 456: clause 25.4)
2. Cement content (IS 456, Table 19)
3. Water content or water-cement ratio of concrete
4. Proper compaction and curing.

3.2.1 ENVIRONMENT AND CHOICE OF TYPE OF STRUCTURE

Yet another major decision that should be made when considering corrosion is to choose the type of structure to suit the environment. In reinforced concrete structures the mean crack width can be reduced to zero by full prestressing or to 0.1 to 0.2 mm by partial or limited prestressing. The specification for good performance of concrete structures in adverse environment is that the crack width should not exceed 0.1 mm. Thus by choosing a suitable method of construction to limit the crack width, it can be made to withstand the given environment.

The foregoing principles form the basis of (a) choosing fully prestressed concrete structures for marine conditions as in harbour works, warehouses in ports, etc. (b) using low strength steel (like Fe 250) with reduced working stress to limit crack width in reinforced structures like water tanks.

3.3 DETERIORATION OF CONCRETE

Plain concrete without any steel in it can deteriorate under the action of chemicals, e.g. sulphates which may be present in industrial atmosphere and in the soils (ground water) to which the concrete is exposed. Also, structures which have to carry chemically active substances like sewage in sewers or sewage purification plants have to be made to withstand the effect of sulphates.

The ground water sulphates may be that of calcium, potassium, sodium or magnesium. They react with the calcium hydroxide and the hydrated calcium aluminate of the cement to produce products like gypsum and calcium sulphoaluminates respectively, which are also greater in volume than their constituents. This volume increase breaks up the concrete. Under these conditions, ordinary portland cement with low percentage of C_3A (tri-calcium aluminates) and special cement called *super sulphated cement* are found to give good resistance against sulphate action. Sulphates of magnesium present in soils are more aggressive than those of potassium and sodium. As calcium sulphates are less soluble in water than the others, their reactions tend to be slower. Hence higher values than those given in the tables can be tolerated, with calcium sulphates. It is said that the susceptibility of concrete to sulphate attack should be judged not from total sulphates present as SO_4, but from the soluble sulphates present as SO_3. This consideration may result in the classification of a site to a lower class than that resulting from considering total sulphates as the index. The method of determination of sulphates is given in Section 3.12.

Whenever concrete as plain or reinforced is laid in soils or exposed to ground water containing sulphates, precaution should be taken to ensure the use of

1. proper type of cement (low C_3A content or sulphate resistant cement);
2. the minimum cement content specified;
3. the proper water-cement ratio specified;
4. using protective coating to concrete buried in the soil.

The concentration of sulphates is expressed as SO_3 in the soil or in the ground water. The normal values which are found in practice are given in IS 456, Table 20 (Appendix A) and in BS 8110, Table 6.1. Both the tables give more or less the same values for use. These values are shown in Table 3.1. The corresponding table in ACI is discussed in Section 3.2 and is given as Table 3.1a here.

TABLE 3.1 TREATMENT OF CONCRETE EXPOSED TO SULPHATES
(Ref. IS 456, Table 20 and BS 8110, Table 6.1)

Class	Sulphate concentration SO_3		Details of mix		
	in soil total SO_3 as per cent	in ground water as mg/L	type of cement	cement content (kg/m^3) (minimum)	water-cement ratio (maximum)
1	< 0.2	300	O.P.C.	280	0.55
2	0.2–0.5	300–1200	O.P.C.	330	0.50
3	0.5–1.0	1200–2500	O.P.C.* 20% (min) p.f.a.**	380	0.45
			S.R.P.C.† S.S.C.††	330	0.50
4	1.0–2.0	2500–5000	S.R.P.C. S.S.C.	370	0.45
5	Over 2	Over 5000	S.R.P.C. S.S.C. with protective coating	370	0.45

*O.P.C. Ordinary portland cement
**p.f.a. Pulverised fuel ash
†S.R.P.C Sulphate resistant portland cement
††S.S.C. Super sulphated cement

TABLE 3.1(a) CLASSIFICATION OF EXPOSURE CONDITIONS
(ACI Table 4.5.3)

Type of exposure	Soluble SO_4 in soil per cent by weight	SO_4 in water (ppm)
Negligible	Up to 0.1	Up to 150
Moderate	0.1–0.2	150–1500
Severe	0.2–2.0	1500–10,000
Very severe	Over 2.0	Over 10,000

It should be noted that prestressed concrete may not tolerate environments of higher sulphate content than reinforced concrete. The sulphate tolerances allowed by Indian Standards for prestressed concrete work as given in IS 1343, Table 10 are the same as those given in IS 456, Table 20.

When concrete is likely to come into contact with sewage or gases evolved from sewage, special care should be taken in the selection of the type of cement and cover to reinforcements, to withstand the effects of such harmful environments.

3.4 PRESCRIBED COVER TO REINFORCEMENTS

Nominal cover is defined as the distance measured from the concrete surface (without taking into account the plaster or other decorative finishes) to the nearest surface of the reinforcing bar (see Fig. 3.2).

The cover requirement specified in the code applies to all reinforcement including links. Nominal cover should

1. prevent corrosion of steel by carbonation and penetration of moisture and air from the surface; and

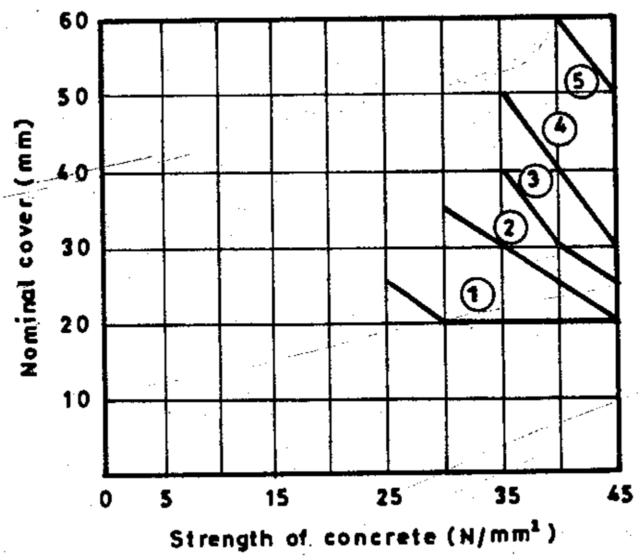


Fig. 3.2 Recommended cover (BS 8110). (1–5: Exposure conditions. 1, mild; 2, moderate; 3, severe; 4, very severe; 5, extreme.)

2. give the reinforcing bar the necessary embedment to be stressed without slipping. (This depends on the diameter of the bar stressed as shown in Table 3.2).

TABLE 3.2 NOMINAL COVER TO REINFORCEMENT FOR MILD CONDITIONS OF EXPOSURE
(IS 456: clause 25.4)

Cover for	Minimum cover against	
	corrosion	slippage
column bars $\phi < 12$ mm	25 mm	ϕ*
column bars $\phi > 12$ mm	40 mm	ϕ
main bars in beams	25 mm	ϕ
other bars like stirrups	15 mm	ϕ
main bars in slabs	15 mm	ϕ
foundation members	40 mm	ϕ
ends of main bars	25 mm (or cover)	2ϕ

*ϕ = diameter of bar

For its first function, namely protection against corrosion, the cover to be provided depends on the environment to which it is exposed. The usual environments met with are classified in codes as shown in Table 3.3.

TABLE 3.3 CLASSIFICATION OF EXPOSURE CONDITIONS ACCORDING TO IS 456
(Ref. IS 456, Table 19)

Type of exposure	Environment description	Allowable maximum crack width
Mild	Protected concrete surface	0.3 mm
Moderate	Sheltered from rain or permanently under water or in contact with non-aggressive soils	0.25 mm
Severe	Alternate wetting and drying or exposed to sea water	0.20 mm
Very severe	Exposed to see water spray or corrosive fumes	0.10 mm
Extreme	Exposed to abrasive action like sea water carrying solids	nil

It should be noted that the actual cover in a construction will be the nominal cover minus the tolerance allowed in construction. A tolerance of 5 mm is usually allowed for cover in the field, so that actual cover may be up to 5 mm less than nominal cover. Hence one should be very careful to specify liberal cover requirement in the design stage itself of reinforced concrete. The cover specified in modern codes like BS 8110 depends both on exposure condition and the strength of concrete as shown in Table 3.4. However, the Indian Standared still continues the old practice of specifying cover, irrespective of the strength of concrete, based on field experience and applies to the usual mixes used in the field. This nominal cover (Table 3.2) specified is for mild conditions (exposure condition 1 of Table 3.3).

TABLE 3.4 NOMINAL COVER FOR DURABILITY
(BS 8110)

Exposure condition (Type of exposure)	Concrete grade (size of aggregate: 20 mm)				
	25	30	35	40	45
Mild	25	20	20+	20+	20+
Moderate	—	35	30	25	20
Severe	—	—	40	30	25
Very severe	—	—	50	40	30
Extreme	—	—	—	60	50

Notes: (i) Minimum grade of concrete for R.C. work is grade 25 (According to BS 8110).
(ii) Grades shown in Table include a relaxation of 5 N/mm^2.
(iii) The cover specified is the nominal cover to reinforcement, including links.
(iv) Cover may be reduced to 15 mm in places marked by (+) when the aggregate size does not exceed 15 mm.

3.4.1 ADDITIONAL COVER FOR EXPOSURE OTHER THAN MILD

It is to be clearly understood that the above table is only for mild exposure conditions. The Indian Code specifies *additional cover*, which is left to the discretion of the engineer for conditions of exposure other than mild (IS 456: clause 25.4.2). This is given in Table 3.5, for concrete whose strength is less than M25. For concretes of strengths M25 and above, the additional cover can be reduced by one-half of that specified in Table 3.5. Thus the cover to be provided according to IS 456 for special conditions of exposure for concrete strength less than M25 is given by the relation

Design cover = (cover for mild condition in Table 3.4)
+ (increased cover for special condition given in Table 3.5)

TABLE 3.5 INCREASED COVER FOR SPECIAL CONDITIONS FOR CONCRETE BELOW M25
(IS 456: clause 25.4)

Condition	Additional cover (mm)
Members totally immersed in sea water	40
Members periodically immersed in sea water or subjected to sea spray	50
Members exposed to harmful chemicals and in contact with earth faces contaminated with cbemicals	15 to 50 (average value = 35)

Notes: 1. For concrete of grade M25 and above, the additional cover specified may be reduced to half.
2. In all cases the cover should not exceed 75 mm.

3.5 CONTROL OF PERMEABILITY OF CONCRETE

It has already been pointed out that one of the desirable properties of concrete is its low permeability which inhibits corrosion of steel and assists in the durability of concrete. The main factors that affect the permeability of concrete are the cement content and water-cement ratio. It is preferable to specify both, a minimum cement content and a maximum water-cement ratio in order to control permeability and thus ensure durability. These recommendations as given in IS 456 (Table 19), Appendix A are summarised in Table 3.6 and compared with the BS values.

TABLE 3.6 MINIMUM CEMENT CONTENT AND WATER-CEMENT RATIO FOR DURABILITY
(Ref. IS 456, Table 19)

Exposure condition	Minimum cement content		Maximum water-cement ratio	
	IS	BS	IS	BS
	(kg/m^3)	(kg/m^3)		
Mild	250	275	0.65	0.65
Moderate	290	300	0.55	0.60
Severe	360	385	0.45	0.55

Note: The minimum cement content is based on 20 mm aggregate. For 40 mm aggregate it should be reduced by 10 per cent and for 12.5 mm aggregate it should be increased by about 10 per cent.

Even though it was thought for a long time that addition of pozzolanic cements (cement blended with fly ash etc.) decreased permeability of concrete, and this would lead to increased resistance to corrosion, it has been brought to light by recent research that these cements have lower initial gain in strength and greater sensitivity to poor curing. Hence even though under ideal conditions of curing, that exist in a laboratory, pozzuolanic cement seems to give encouraging results, under conditions of poor curing it produces an inferior product compared to ordinary portland cement. In addition, the pH value of the pozzuolanic concrete is much lower than that of ordinary concrete due to the absence of free lime in the concrete mix, and this does not favour resistance of reinforcement to corrosion.

It should, however, be noted that the maximum cement content that can be used in concrete of any grade is also specified in all codes as about 400 to 500 kg/cubic metre. This is meant to reduce the risk of cracking that can occur in this section due to drying shrinkage and internal thermal stresses in thicker sections. BS 5337 for liquid retaining structures specifies that for reinforced concrete the maximum cement should be only 400 kg/cubic metre. IS 1343 (for prestressed concrete work): clause 8.1.1 recommends that the cement content should not exceed 530 kg/m^3.

3.6 COMPACTION DURING PLACEMENT OF CONCRETE

Concrete mix should be so designed that it compacts well without segregation and bleeding. As reinforcements are usually situated at the outer face of the structure, it is the compaction of this periphery that is most important as regards durability. Segregation and bleeding affects these parts most and should be always avoided.

3.7 CURING METHODS

Curing of concrete helps proper hydration of cement in the freshly placed concrete. The quality of the skin of the concrete depends, to a great extent, on proper curing of the concrete surface. Durability of concrete is considerably affected by the quality of this skin. The compressive strength of concrete is also a function of its curing. Curing should start as soon as initial setting of concrete takes place and it should be continued uninterrupted to the end of the specified period. Any long delay in commencement of curing or stoppage of curing before the specified period will adversely affect durability. Intermittently cured concrete is not as durable as continuously cured concrete.

The curing methods used in practice can be classified into two main groups: (a) the water adding methods, and (b) the water retaining methods like concrete coatings which seal off the water in the concrete. The former produce better concrete than the latter.

The curing methods commonly used in India are:

1. Maintaining form work in place (water retaining methods) so that the mixed water in the concrete does not escape.

2. Continuous or frequent direct application of water to concrete, totally avoiding alternate wetting and drying (water adding method).

3. Keeping the concrete surface covered with damp absorbent material so that the concrete is always kept moist and protected from sun (water adding method).

Curing methods can be described as follows:

(i) *Good*—when the work is protected from sun and the relative humidity is kept greater than 80%.

(ii) *Poor*—when the work is not protected from the sun and the relative humidity is kept less than 50%.

(iii) *Average.*

The stripping time of form work according to IS 456 is specified in clause 10.3 of the code.

3.8 QUALITY OF AGGREGATES

As already mentioned, care should be taken to see that the coarse and fine aggregates are of proper quality and free from harmful substances like chlorides and sulphates. The maximum amounts of these substances allowed to be in the concrete at the time of placing are specified in IS 456, Appendix A: clause A2. Accordingly, the total amount of chlorides (as Cl) should not be more than 0.15 per cent by mass of cement and the amount of sulphates (as SO_3) should not be more than 4 per cent by mass of cement. Even though for high strength concrete crushed rock coarse aggregates are necessary, ordinary grade concrete can be made with gravel or crushed rock as coarse aggregates.

3.9 CONCRETE IN FOUNDATIONS

Sulphates and chlorides are main cause of deterioration of concrete below ground. Sulphates occur in soils and in peats. The requirements for concrete exposed to sulphate attack have been specified in Section 3.3.

Sea water contains sulphates as much as 250 parts in 100,000. This quantity is considered to be unsafe (IS 456, Table 20) under normal conditions. However, as sea water also contains chlorides which inhibit the action of sulphates, one does not usually find sulphate deterioration with sea water. This is especially true if the concrete is dense and is of good quality. The general precautions to be taken for concreting work in sea water are given in IS 456: clause 13.3.

Acidic ground conditions with low pH value (which can be as low as 3.5) also cause trouble in foundation. In these situations, supersulphated cement can give greater durability than ordinary portland cement.

3.10 CHECKING FOR LIMIT STATE OF DURABILITY

There have been a number of recent failures of concrete structures due to corrosion of steel and deterioration of concrete. It should be noted that present day construction tends to use less and less cement on works (as modern cements can produce high strength with low cement contents). In India, most of the steel used in construction consists of cold worked bars, and hence greater attention should be given to durability requirements. It is therefore necessary to include check for limit state of durability at the design stage itself. The procedure consists of examining the environmental conditions and prescribing the necessary cover, type of cement, minimum cement content, maximum water-cement ratio to be used etc. These should be specified in the design sheets and on the construction drawings.

3.11 COATED REINFORCEMENTS TO PREVENT CORROSION [IS 9077 (1979)]

The Central Electro Chemical Research Institute (CECRI) at Karaikudi, Tamil Nadu, has developed a method for coating individual bars with a phosphate jelly to inhibit corrosion. The process consists of treating the bar at the site with a derusting solution, washing it with an alkaline cleaning powder and then coating it successively with phosphate jelly, inhibitor solution, cement slurry, sealing solution, and again cement slurry. The process is quite tedious and time consuming since each bar has to be treated as described above. These bars are to be used soon after coating. This specification is nowadays prescribed for many major works along the sea coasts in India. The materials for the above process have been patented by CECRI and are available in the market from specified manufacturers. In other countries, reinforcement rods coated with epoxy resins are sometimes used. However, as these coatings tend to be destroyed at places during transport, factory treatment of the reinforcement may not always produce the desired effect, unless great care is taken in handling these bars. They are also quite expensive.

3.12 DETERMINATION OF SOLUBLE SULPHATES

The standard test for estimating total sulphates in soil is to extract it with hot dilute hydrochloric acid; for the determination of water soluble sulphate, one has to obtain the extract by adding water equal to the weight of the soil sample. These procedures will distinguish between the highly soluble sodium and magnesium sulphates from calcium sulphates (gypsum) which have low solubility. As already stated, only the water soluble sulphates of soil have to be considered while estimating aggressiveness of soil or the ground water.

The total sulphates of the extract from soils are estimated in two steps: first N/4 solution of $BaCl_2$ is added to precipitate the sulphates and it is separately estimated; the sulphites are then estimated by titration with acidic K_2CrO_4. The sum of the two quantities gives the total sulphates.

It should be noted that IS 456 and BS 8110 express the severity of the exposure condition in terms of SO_3 only and ACI 318 (83) expresses it in terms of SO_4 (Table 3.1a). Sulphites are more injurious than sulphates.

EXAMPLE 3.1 (Durability requirements)

A concrete beam 300 × 510 mm has to be reinforced with 25 mm diameter. Fe 415 steel for longitudinal steel and 12 mm dia Fe 250 stirrups. If the structure is to be constantly under fresh water, determine the following: (a) cover to be provided, (b) effective depth, (c) minimum cement content, and (d) maximum water-cement ratio to be adopted.

Ref.	Step	Calculations	Output
IS 456 Table 19	1.	*Exposure condition* Continuous under water: Moderate	
	2.	*Calculation of cover*	
IS 456 Clause 25.4		(a) Normal cover for mild conditions Main steel = 25 mm Shear steel = 15 mm For provision of clear cover of 15 mm for steel, cover to main steel = 15 + 12 = 27 mm > 25 mm	
		(b) Increased cover for moderate condition cover to stirrups = normal cover + 25 mm = 15 + 25 = 40 mm	Cover = 40 mm
		cover to main steel = 40 + 12 = 52 mm This cover is satisfactory.	
	3.	*Effective depth* $d = 510 - \left(52 + \frac{25}{2}\right) = 445.5 \text{ mm}$	$d = 445.5$ mm
	4.	***Minimum cement content***	
IS 456 Table 19		Minimum cement content for moderate condition of exposure: 290 kg/m^3 Maximum water-cement ratio not more than 0.55	CC = 290 kg/m^3 Water-cement ratio = 0.55

EXAMPLE 3.2 **(Durability requirements)**

Determine the cement content and type of cement to be used if the structure in Example 3.1 is exposed to sulphate action with total SO_3 content of 0.4%.

Ref.	Step	Calculations	Output
	1.	*Class with respect to SO_3 content*	
IS 456 Table 20		For SO_3 content = 0.4	Class 2 of Table 20
	2.	*For ordinary portland cement (O.P.C.)* Min. cement content = 330 kg/m^3 Max. water-cement ratio = 0.50	CC = 330 kg/m^3 water-cement ratio = 0.50
	3.	*Use of sulphate resisting cement or O.P.C. with controlled C_3A*	
IS 456 Table 20 (Note 6)		Min. cement content = 310 kg/m^3 Max. water-cement ratio = 0.50	CC = 310 kg/m^3 water-cement ratio = 0.50

REVIEW QUESTIONS

3.1 What is meant by limit state of durability? Name the factors that affect durability of reinforced concrete structures.

3.2 Define the term 'clear cover' for steel in R.C. construction. Can cover be made up by plastering after the removal of the shuttering of the R.C. members?

3.3 Explain the terms 'mild', 'moderate', 'severe' and 'very severe' as applied to exposure condition.

3.4 Write a brief note on corrosion of steel in R.C. members and enumerate the precautions to be taken for reducing the chances of corrosion of steel.

3.5 What is meant by sulphate attack on concrete? Which of the sulphate salts are most aggressive? How does one provide for conditions where the subsoil water has a high sulphate content?

3.6 State the differences between the specifications for cover between Indian and British codes. Make critical comments on these differences.

3.7 Distinguish the terms 'nominal cover' (for normal conditions) and 'additional cover' (under special conditions) as given in the IS Code.

3.8 Give reasons why the minimum and maximum cement contents are specified in IS Code. What are the values specified?

3.9 On what factors do permeability of concrete depend on, and why is it necessary to reduce the permeability of concrete in R.C. construction?

3.10 Discuss the importance of curing the concrete on durability characteristics of concrete. What precautions in curing are necessary when using pozzuolanic cements for R.C. construction?

4

Theory of Singly Reinforced Members in Bending (Limit State of Collapse–Flexure)

4.1 INTRODUCTION

Beams and slabs carry loads by bending action. In the limit state method, these members are first designed for strength and durability, and their performance is then checked with regard to other limit states of serviceability, e.g. deflection and cracking. This chapter deals with the formulae for computation of the limit state for strength (collapse) of singly reinforced concrete members in bending. Many of these formulae are given in IS 456 Appendix E. Chapter 5 deals with the practical application of the theory and the formulae derived in this chapter.

Even though it may not be necessary for a designer to know the derivation of all the formulae, it is advisable for the student who is studying the subject for the first time to become familiar with these derivations as it gives a better insight into the design process. For practical designs, however, one may make use of the tables and charts in the "Design Aids to IS 456 (1978)" published as SP16 by the Bureau of Indian Standards or other publications.

4.2 ULTIMATE STRENGTH OF R.C. BEAMS (LIMIT STATE OF COLLAPSE BY FLEXURE)

The following assumptions are made for calculating the ultimate moment of resistance or the strength at limit state of flexural collapse of reinforced concrete beams (IS 456: clause 37.1):

1. Plane sections remain plane in bending up to the point of failure (i.e.) strains are proportional to distance from the neutral axis), see Fig. 4.1.

2. Ultimate limit state of bending failure is deemed to have been reached when the strain in concrete at the extreme bending compression fibre ε_{cu} reaches 0.0035. (See Chapter 14 for general expression for the strain at failure of concrete.)

3. The stress distribution across the compression face will correspond to the stress-strain diagram for concrete in compression. Any suitable shape like parabolic, rectangular or any combinations of shapes that give results which are in substantial agreement with tests may be assumed for this compression block. For design purpose, the maximum compressive strength in the structure is assumed as 0.67 times the characteristic laboratory cube strength (i.e. $2/3f_{ck}$), see Fig. 2.3. With an additional partial safety factor of γ_m = 1.5 applied to concrete strength (see Fig. 2.3), the values of the maximum concrete stress in a beam will be $0.446f_{ck}$, which can be taken as equal to $0.45f_{ck}$ for all practical purposes. In Fig. 2.3, it should be noted that γ_m = 1.5 is applied over the whole stress-strain curve to obtain the design stress-strain curve for concrete.

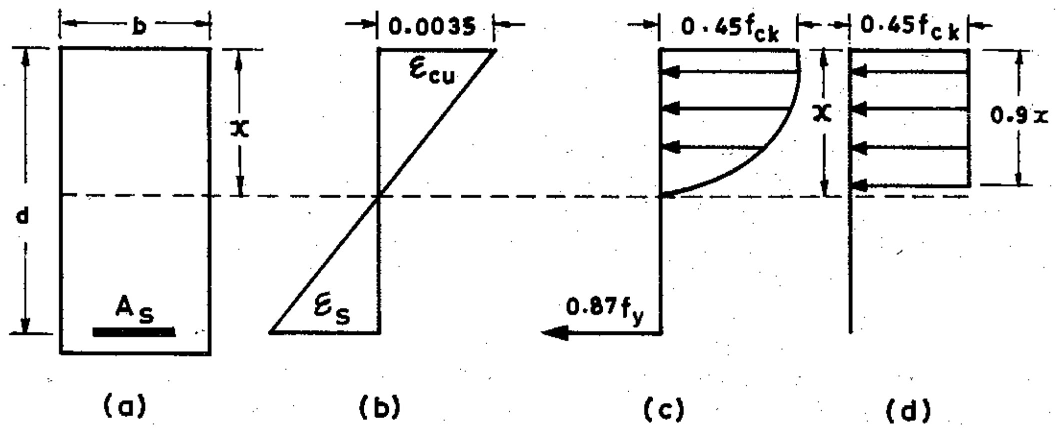


Fig. 4.1 **Strain diagram and stress blocks: (a) Section, (b) Strain diagram (plane sections remain plane), (c) Stress block with partial safety factors, and (d) Simple rectangular stress block (BS).**

4. The tensile strength of concrete is neglected as the section is assumed to be cracked up to the neutral axis.

5. The stress in steel will correspond to the corresponding strain in the steel ε_s, and can be read off from the stress-strain diagram of the steel. For design purposes, a partial safety factor of 1.15 is used for strength of steel so that the maximum stress in steel is limited to $f_y/1.15 = 0.87f_y$. It should be noted that the design stress-strain curve for cold worked steel is obtained by applying partial safety factor $\gamma_m = 1.15$ over the region beyond $0.8f_y$ of the actual stress-strain curve for steel (see Fig. 2.4).

6. In order to avoid sudden and brittle compression failure in singly reinforced beams, the limiting value of the depth of compression block is to be obtained according to IS 456 by assuming the strain of tension steel at failure (ε_{su}) to be not be less than the following:

$$\varepsilon_{su} = \frac{f_y}{1.15E_s} + 0.002 = \frac{0.87f_y}{E_s} + 0.002$$

where

ε_{su} = strain in steel at ultimate failure

f_y = characteristic strength of steel

E_s = modulus of elasticity of steel

4.3 BALANCED, UNDERREINFORCED AND OVERREINFORCED SECTIONS

As given in assumption 2 in Section 4.2, reinforced concrete sections in bending are assumed to fail when the compression strain in concrete reaches the failure strain in bending compression equal to 0.0035. Sections, in which the tension steel also reaches yield strain simultaneously as the concrete reaches the failure strain in bending, are called *balanced sections*. Sections, in which tension steel reaches yield strain at loads lower than the load at which concrete reaches failure strain, are called *underreinforced sections.* It should be remembered that yielding of steel does not mean ultimate failure of the beam. When steel yields, there will be excessive deflection and consequent cracking but complete rupture of steel takes place at a much higher strain, of the order of 0.20 to 0.25 (i.e. 20 to 25 per cent elongation based on the original length) compared to the actual steel yield strain of 0.0038. The latter is only of the same order as failure strain of concrete. The ultimate failure of underreinforced beams in all practical cases is therefore finally due to the concrete reaching the ultimate failure strain of 0.0035.

It is preferable that a beam be designed as an *underreinforced beam,* where 'failure' will take place after yielding of steel, with enough warning signals like excessive cracking and deflection taking place before ultimate failure.

R.C. sections, in which the failure strain in concrete is reached earlier than the yield strain of steel is reached, are called *overreinforced sections.* Such beams, if loaded to full capacity, will again fail by compression failure of concrete but without warning. Such designs are not recommended in practice.

4.4 EQUIVALENT COMPRESSION BLOCK IN CONCRETE

If an idealised stress-strain curve of concrete is used as in the third assumption in Section 4.2, the magnitude of total compression which is given by the area of the stress block in the beam will be as shown in Fig. 4.2 and can be expressed as

$$C = k_1 f_{ck}(x)b$$

where x = depth of the neutral axis.

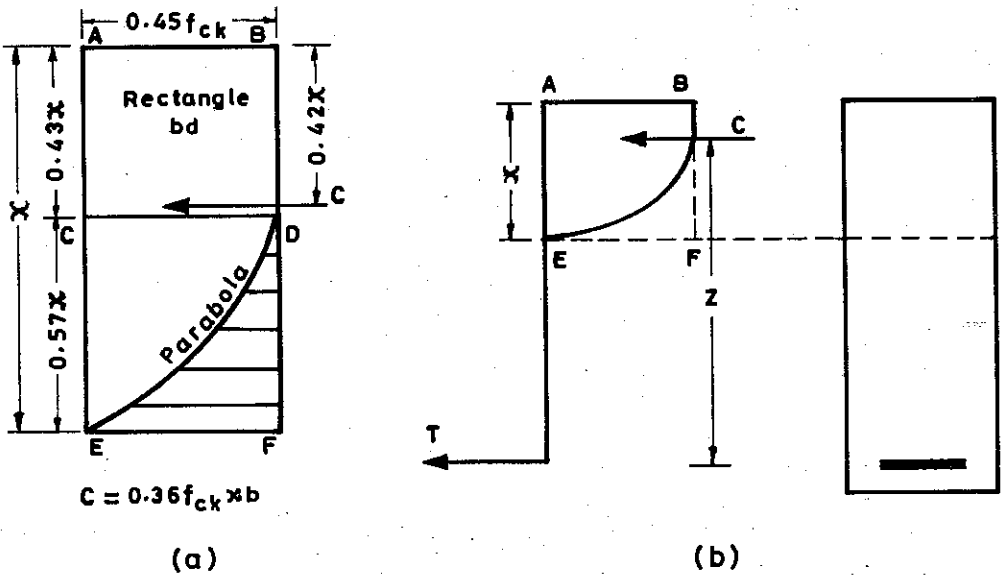


Fig. 4.2 Calculation of resisting moment of a section: (a) Properties of I.S. Stress block in compression and (b) Forces acting on a section subjected to bending.

The point of application of this resultant compression or the depth of centre of compression can be taken as $k_2(x)$ from the extreme compression fibre. The constants k_1, k_2 are called the constants for the stress block. It was pointed out in Chapter 2 that the strain ε_{co} (Fig. 2.3b) at which concrete becomes plastic is in reality a function of the characteristic strength of concrete—and it is so assumed in some of the codes like the British Code. However, much simplicity without loss of accuracy can be achieved by assuming this strain to be a constant value irrespective of strength of concrete.

IS 456 assumes this strain as a constant value equal to 0.002, so that the values of k_1 and k_2 will become independent of the strength of the concrete and are constants for all strengths of concrete.

4.5 DETERMINATION OF CONSTANTS k_1 AND k_2 FOR COMPRESSION STRESS BLOCK

Codes allow the compression block of any reasonable shape. The two most commonly used blocks are the parabolic rectangular block used in IS 456, and a simplified rectangular stress block used in BS 8110 (see Fig. 4.1). The values of k_1 and k_2 for the parabolic rectangular block assumed in IS 456 can be derived as given in the following subsections.

4.5.1 VALUE OF k_1

The areas of the stress block can be calculated by assuming that the parabola extends to x_1 (where the strain is 0.002) from the neutral axis and the rest of the stress block to the top fibre is a rectangle (see Fig. 4.2).

It was shown in Section 2.6 and Fig 2.3 that the value of $x_1 = 0.57x$.

This geometry of the stress-strain curve is important and useful in solving problems from basic considerations. Let the total compression be equal to C. From Fig. 4.2,

$$\begin{aligned} C &= \text{area of (rectangle} - \text{outer parabola)} \\ &= 0.45f_{ck}(x)b - 0.45f_{ck}\,\frac{0.57}{3}(x)b \\ &= 0.45f_{ck}(x)b - 0.086f_{ck}(x)b \\ &= 0.364f_{ck}(x)b \end{aligned}$$

Therefore,

$$k_1 = 0.364 \tag{4.1}$$

Thus, k_1 is a constant and its value may be assumed as 0.36 for all design calculations (IS 456: clause 37.1). Thus we get

$$C = 0.36f_{ck}(x)b \tag{4.1a}$$

It is also sometimes expressed as a function of the area bd as

$$C = 0.36f_{ck}\,\frac{x}{d}(bd) \tag{4.1b}$$

4.5.2 VALUE OF k_2

Let $k_2(x)$ be the distance of the centre of compression in the concrete from the top compression fibre. Taking x_1 as the distance to which the parabola extends, which is equal to $0.57x$, and equating the moment of the total forces about the neutral axis to the moment of the separate forces, we obtain

$$[0.36 f_{ck}(x)b]\,[x - k_2(x)] = 0.45 f_{ck}(x)b\,\frac{x}{2} - 0.45\,f_{ck}\,\frac{(x_1)^2\,b}{12}$$

where $x_1 = 0.57x$, the point where $\varepsilon_{co} = 0.002$. Simplifying the equation, we get

$$(1 - k_2) = 0.584$$

Hence,

$$k_2 = 0.416 = 0.42 \text{ (approx.)} \tag{4.2}$$

Thus, according to IS 456, k_2 will also be a constant of value 0.42, irrespective of the strength of concrete. The position of the centre of compression is thus taken as $0.42x$ from the compression edge (see IS 456, notes to clause 37.1c).

4.6 DEPTH OF NEUTRAL AXIS OF A GIVEN BEAM

4.6.1 GENERAL EXPRESSION FOR DEPTH OF NEUTRAL AXIS x

Beams are assumed to fail when the concrete reaches failure compression strain. But in all cases of design, the steel need not have reached its yield point at the same time, unless it is so designed. If the section is designed as a balanced or underreinforced one, the steel also reaches yield as concrete fails. But in overreinforced beams, the steel stress at failure will be below its yield strength. As equilibrium of forces in bending requires that at all times tension be equal to compression, we have

$$\text{Total tension } T = f_{st}A_{st}$$

$$\text{Total compression } C = 0.36 f_{ck}\,b(x)$$

where f_{st} = actual tension in steel corresponding to the strain in steel. Equating the two expressions, we obtain

$$0.36 f_{ck}b(x) = f_{st}A_{st}$$

$$x = \frac{f_{st}\,A_{st}}{0.36 f_{ck}\,b} \tag{4.3}$$

For underreinforced beams, steel first reaches yield stress of $0.87f_y$. Substituting its value and dividing both sides by the effective depth d (IS 456 Appendix E), we get

$$\frac{x_u}{d} = \frac{0.87 f_y (A_{st})}{0.36 f_{ck}(bd)} \tag{4.3a}$$

where x_u is the depth of neutral axis at ultimate failure of the beam.

4.6.2 LIMITING VALUES OF x/d IN IS 456

The sixth assumption in IS code states that in order to avoid brittle failure, the limiting value of

x/d is to be determined from the condition that the steel strain ε_{su} at failure should not be less than the following:

$$\varepsilon_{su} = \frac{f_y}{1.15E_s} + 0.002 = \frac{0.87f_y}{E_s} + 0.002$$

Assuming $E_s = 2.0 \times 10^5$ N/mm^2, the yield strain for design purposes for different steels works out to the values given in Table 4.1. From proportionality of strains, we have

$$\frac{x_u}{d} = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{su}} = \frac{0.0035}{0.0035 + \varepsilon_{su}} \tag{4.4}$$

Substituting the various values of ε_{su} for different grades of steel, the maximum limiting values of x_u/d for various grades of steel are obtained as in Table 4.1.

TABLE 4.1 LIMITING VALUES OF x/d

Type of steel	f_y	Yield strain (ε_{su})	x_u/d
Mild steel	250	0.0031	0.53
High yield strength	415	0.0038	0.48
High yield strength	500	0.0042	0.46

Note: A mean value of $x_u/d = 0.5$ recommended in BS code and rectangular block may be used for purposes of approximate calculations.

When, as a result of redistribution the moment at a section is reduced, the neutral axis depth x at the section, where the moment has been reduced, should not be greater than the following expression given in IS 456: clause 36.1.1:

$$x = (0.6 - \beta_{\text{red}})d$$

where

$$\beta = \frac{\Delta M}{M} = \frac{\text{reduction of moment}}{\text{original value of moment}}$$

$$d = \text{effective depth}$$

4.6.3 MAXIMUM VALUE OF C WITH GIVEN GRADE OF CONCRETE

The maximum or limiting value of compression, C_L, can be obtained by substituting the limiting values of x_u/d into equation (4.1b):

$$C_L = k_1\left(\frac{x_u}{d}\right)f_{ck}bd$$

$$= 0.364\left(\frac{x_u}{d}\right)f_{ck}bd$$

Putting $F = 0.364\ (x_u/d)$ and taking x_u/d as constant already derived for a given grade of steel, we get

$$C_L = Ff_{ck}bd \tag{4.5}$$

Substituting the limiting value of x_u/d from Table 4.1 for F, the values for the various constants obtained are shown in Table 4.2.

TABLE 4.2 VALUES OF CONSTANTS FOR MAXIMUM COMPRESSION BLOCK

Steel	x_u/d	k_1	k_2	F
Fe 250	0.53	0.364	0.42	0.192
Fe 415	0.48	0.364	0.42	0.175
Fe 500	0.46	0.364	0.42	0.167

Note: $C_L = Ff_{ck}bd$ acting at $0.42\,x_u$ from compression face.

Thus, for example, from Table 4.2 the maximum value of compression C_L with Fe 415 steel is given by

$$C_L = 0.175 f_{ck} bd$$

acting at $(0.42x_u)$ from the compression face. The value of $x_u/d = 0.48$.

4.7 IMPORTANCE OF LIMITING x/d RATIOS

Even though one may assume a parabolic rectangular stress-strain curve for concrete, in reality concrete has a drooping stress-strain curve (Fig. 2.3) so that compression failure in singly reinforced beam will take place with a sudden release of energy. Hence if failure of concrete without yielding of steel is allowed to take place, the collapse occurs without warning. This should be always avoided. On the other hand, steel undergoes large deformation and increases in strength (strain hardening) before it fails. Failure of beams by general yielding of steel takes place gradually and with ample warning before total failure. Consequently, good design practice of R.C. structures should guarantee ultimate failure by first yielding of steel in tension followed by compression failure of concrete.

As already explained, this can be ensured if the value of x/d for singly reinforced concrete beam is not allowed to exceed the limiting values of x_u/d specified in Table 4.1. However, when steel is provided in the compression zone also, the brittleness in the compression zone at failure can be minimised, so that the rule that x/d should not exceed 0.5 (approx.) may be relaxed in such cases.

4.8 CALCULATION OF M_u BY STRAIN COMPATIBILITY METHOD

The most basic method of determining the value of M_u, the ultimate moment capacity, is by using the following assumptions:

1. At failure there should be strain compatibility, with the failure strain of concrete being equal to 0.0035.

2. The value of compression in concrete and tension or compression in steel can be derived from their respective stress-strain curves.

3. For pure bending, total tension in tension steel should be equal to total compression in the compression zone.

4. M_u (Moment of internal couple) = Tension or Compression × Lever arm (LA).

From these assumptions, certain formulae can be derived for analysis and design of singly reinforced concrete beams and slabs as explained now.

4.8.1 EXPRESSION OF RESISTANCE MOMENT FOR A BALANCED SECTION IN TERMS OF f_y AND p (REFER IS 456, APPENDIX E)

The following formulae given in IS 456, Appendix E have already been derived. Thus Eq. (4.3a) gives

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

Dividing both sides by d, we get

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} bd}$$

The ultimate moment of resistance M_u in terms of steel and concrete strength can be found by taking moments of the tension forces about the centre of compression. Thus,

$$M_u = (0.87 f_y A_{st}) \times (\text{Lever arm})$$

$LA = (d - 0.416 x_u)$ as given in Fig. 4.2. Taking the value of x_u from Eq. (4.3a), we obtain

$$LA = d - \left(\frac{0.416(0.87 f_y A_{st})}{0.36 f_{ck} b}\right)$$

$$= \left(d - \frac{1.005 f_y A_{st}}{f_{ck} b}\right)$$

$$M_u = 0.87 f_y A_{st}\left(d - \frac{1.005 f_y A_{st}}{f_{ck} b}\right) \tag{4.6}$$

which can be written for all practical purposes as given in IS 456, Appendix E1 as follows:

$$M_u = 0.87 f_y A_{st} d\left(1 - \frac{f_y A_{st}}{f_{ck} bd}\right) \tag{4.6a}$$

Expressing the area of steel as a percentage of the effective area, we get

$$p = \left(\frac{A_{st}}{bd}\right) \times 100$$

where p is the percentage of steel.

Substituting for A_{st}/bd from the above expression for p into Eq. (4.6), we get

$$M_u = 0.87 f_y \frac{p}{100}\left(1 - \frac{1.005 f_y}{f_{ck}} \frac{p}{100}\right) bd^2 \tag{4.7}$$

Dividing both sides by bd^2, we obtain

$$\frac{M_u}{bd^2} = 0.87f_y \frac{p}{100}\left(1 - \frac{1.005f_y}{f_{ck}} \frac{p}{100}\right)$$

which, for all practical purposes, can be taken as

$$\frac{M_u}{bd^2} = 0.87f_y \frac{p}{100}\left(1 - \frac{f_y}{f_{ck}} \frac{p}{100}\right) \tag{4.7a}$$

It is important to note that the value of p obtained from this quadratic in p is not directly proportional to $M_u/(bd^2)$. Tables showing the value of p for $M_u/(bd^2)$ can be made for speedy determination of p for a given value of M_u. This is presented in IS publication SP 16 as Tables 1 to 4. (This is discussed in more detail in Section 4.13.4). It should be clearly noted that in Eq. (4.7a) p is the percentage of tension steel in the beam, assuming that the tensile stress in steel has reached the yield stress at failure of the beam.

4.8.2 EXPRESSION FOR RESISTANCE MOMENT IN TERMS OF CONCRETE STRENGTH f_{ck}

The ultimate moment of resistance in terms of concrete strength can be derived by taking the moment of compression force about the tension force in steel. Taking x as the depth of neutral axis and taking the lever arm depth from Fig. 4.2, we obtain

$$M_u = 0.36f_{ck}(x)\,b(d - 0.42x)$$

$$= 0.36f_{ck}(x)bd\left(1 - 0.42\,\frac{x}{d}\right)$$

$$= 0.36f_{ck}\,\frac{x}{d}\left(1 - 0.42\,\frac{x}{d}\right)bd^2$$

$$\frac{M_u}{bd^2} = 0.36f_{ck}\,\frac{x}{d}\left(1 - 0.42\,\frac{x}{d}\right) \tag{4.8}$$

This is the formula given in clause E.1.1, see Appendix E of IS 456.

For obtaining the maximum value of M_u, the limiting values of x_u/d as obtained in Table 4.1 is substituted in Eq. (4.8). Thus we get

$$\frac{M_u}{f_{ck}} = (\text{constant})\,bd^2$$

$$M_u = Kf_{ck}bd^2 \tag{4.9}$$

where K is the design constant for ultimate design of the section. From the above procedure, Table 4.3 is obtained.

With the limiting values of x/d from Table 4.1, we get the values for K as given in Table 4.3. It is worth remembering that the expression for M_u with Fe 415 grade steel (which is commonly used in India as reinforcement) is given by the formula

$$M_u = 0.138f_{ck}bd^2 = 0.14f_{ck}bd^2 \text{ (approx.)}$$

TABLE 4.3 TABLE OF VALUES OF RESISTANCE MOMENT FOR LIMITING VALUES OF x/d
(Refer Table C of SP 16)

Steel	x_u/d	Expression for M_u	Value of K
Fe 250	0.53	$M_u = 0.149 f_{ck} bd^{2*}$	0.15
Fe 415	0.48	$M_u = 0.138 f_{ck} bd^{2*}$	0.14
Fe 500	0.46	$M_u = 0.133 f_{ck} bd^{2*}$	0.13

*The general expression is $M_u = Kbd^2$

This is the maximum allowable resisting moment of a given concrete beam section in terms of the strength of the concrete. Even if the beam is reinforced with more steel than that required for a balanced section, there will be only marginal increase in the strength of the beam. Such a beam will be an overreinforced beam, which is not usually accepted in practice.

4.9 MINIMUM DEPTH FOR GIVEN M_u

From Eq. (4.9) the minimum depth of concrete of a singly reinforced beam to carry a given moment can be derived as

$$M_u = Kf_{ck}bd^2$$

Therefore,

$$d = \sqrt{\frac{M_u}{Kf_{ck}b}} \tag{4.10}$$

For Fe 250, the value of $K = 0.15$ is

$$d = \sqrt{\frac{M_u}{0.15 f_{ck} b}} = \sqrt{\frac{6.66 M_u}{f_{ck} b}} \tag{4.10a}$$

For Fe 415, the value of $K = 0.14$ is

$$d = \sqrt{\frac{7.1 M_u}{f_{ck} b}} \tag{4.10b}$$

4.10 EXPRESSION FOR STEEL AREA FOR BALANCED SINGLY REINFORCED SECTION

Equating tension in steel to compression in concrete at failure stage (x_u is the depth of neutral axis at ultimate stage), we get

$$0.87 f_y A_{st} = 0.36 f_{ck} b x_u$$

This can be rewritten as

$$\frac{A_{st}}{bd} = \frac{0.36 f_{ck}}{0.87 f_y} \frac{x_u}{d} = (\text{constant}) \frac{f_{ck}}{f_y}$$

as x_u/d is also a constant for given values of f_y. In this expression, A_{st} gives the limiting area of steel for balanced failure for the section. Denoting this limiting percentage of tension steel as p_t, we have

$$p_t = \left(\frac{A_{st}}{bd}\right)100$$

Rewriting this equation for A_{st}/bd in terms of p_t, we get

$$p_t\,(\text{lim}) = \text{constant}\left(\frac{f_{ck}}{f_y}\right)$$

$$\frac{p_t f_y}{f_{ck}} = \frac{0.36(100)}{0.87}\frac{x_u}{d} = 41.3\frac{x_u}{d} = \text{constant}$$

so that the value of p_t can be put in the form

$$\frac{p_t f_y}{f_{ck}} = \text{constant} = 41.3\frac{x_u}{d} \quad \text{for a given grade of steel} \qquad (4.11)$$

Substituting the limiting values of x_u/d already obtained in Table 4.1 into Eq. (4.11), one gets the constants for the value of the limiting percentage of steel for different grades of steel. This is shown in Table 4.4. The values of p_t and A_{st} required may be then calculated from these

TABLE 4.4 PERCENTAGE OF LIMITING STEEL AREAS (p_t) FOR BALANCED DESIGN
(Refer Table C of SP 16)

Steel	x/d	$p_t\,(f_y/f_{ck})$ = constant
Fe 250	0.53	21.97
Fe 415	0.48	19.82
Fe 500	0.46	18.87

Note: According to ACI and the Australian Code AS 1480, normal structures should have $p_{\max} \not> 0.75p_t$.

values. For example, with Fe 415 steel and Grade 25 concrete, the value of p_t as obtained from Table 4.4 is

$$p_t\left(\frac{f_y}{f_{ck}}\right) = 19.82$$

Hence the limiting steel percentage p_t is given by

$$p_t\,(\text{lim}) = \frac{19.82}{f_y}f_{ck} = \frac{19.82(25)}{415} = 1.19\%$$

4.11 EXPRESSION FOR x/d FOR GIVEN b, d AND M_u

We can obtain the value of x/d for given values of b, d and M_u as follows: From Eq. (4.8),

$$M_u = 0.36 f_{ck}\frac{x}{d}\left(1 - 0.416\frac{x}{d}\right)bd^2$$

Dividing both sides of this equation by (0.36) (0.416)$f_{ck}bd^2$, the above expression reduces to

$$\frac{6.68M_u}{f_{ck}bd^2} = 2.40\left(\frac{x}{d}\right) - \left(\frac{x}{d}\right)^2$$

Rearranging the terms, we get

$$\left(\frac{x}{d}\right)^2 - 2.40\left(\frac{x}{d}\right) + \frac{6.68M_u}{f_{ck}bd^2} = 0$$

Solving for x/d and taking the possible value, we obtain

$$\frac{x}{d} = 1.2 - \left[(1.2)^2 - \frac{6.68M_u}{f_{ck}bd^2}\right]^{1/2} \tag{4.12}$$

This expression is very useful for determining the value of x/d for a given value of M_u and specified b, d and f_{ck}.

4.12 EXPRESSIONS FOR LEVER ARM DEPTH (z)

From Fig. 4.2, the lever arm depth, which is represented by symbol z, is given by

$$z = d\left(1 - 0.416\,\frac{x}{d}\right)$$

If the stress in steel is f_{st}, a value less than the yield stress, the expression for (x/d) will be given by

$$\frac{x}{d} = \frac{f_{st}A_{st}}{0.36f_{ck}bd}$$

Substituting the above values in the expression

$$z = d\left(1 - 0.416\,\frac{x}{d}\right)$$

we obtain

$$z = d\left(1 - \frac{0.416f_{st}A_{st}}{0.36f_{ck}bd}\right)$$

Simplifying this equation, we get

$$z = d\left(1 - \frac{1.155f_{st}A_{st}}{f_{ck}bd}\right) \tag{4.13}$$

When $f_{st} = 0.87f_y$, this equation reduces to

$$z = d\left(1 - \frac{1.005f_y A_{st}}{f_{ck}bd}\right) \tag{4.13a}$$

For all practical purposes the lever arm depth is given by

$$z = d\left(1 - \frac{f_y A_{st}}{f_{ck}bd}\right) \text{ (approx.)} \tag{4.13b}$$

4.13 CALCULATION OF STEEL AREA FOR GIVEN b, d AND M_u FOR DEPTHS LARGER THAN THE MINIMUM REQUIRED

Most of the actual beams and slabs met with in practice have depths larger than those required for the balanced section. This increased depth is obligatory for satisfying deflection and other requirements. Hence these sections are to be designed as underreinforced sections. The steel necessary for the slab or beam with larger than the minimum required depth can be less than that required for balanced failure. If this fact is not incorporated in designs, it will result in provision of excessive steel.

Calculations for the required steel in these cases can be made by any of the four procedures described.

4.13.1 PROCEDURE 1

From Eq. (4.11) one may calculate z from the relation

$$z = d\left(1 - 0.416\frac{x}{d}\right)$$

where the value of x/d is obtained from Eq. (4.12) as

$$\frac{x}{d} = (1.2) - \left[(1.2)^2 - \frac{6.68M_u}{f_{ck}bd^2}\right]^{1/2}$$

The value of x/d should also satisfy the prescribed limiting values. The value of the area of steel is then calculated from the equation

$$A_s = \frac{M_u}{0.87f_y z}$$

4.13.2 PROCEDURE 2 (FROM LEVER ARM FACTOR)

Alternatively, a direct expression may be derived for the lever arm depth z as follows:

$$\frac{z}{d} = \left(1 - 0.416\frac{x}{d}\right)$$

Rearranging the terms, we get

$$\frac{x}{d} = \left(1 - \frac{z}{d}\right)\left(\frac{1}{0.416}\right)$$

Putting $z/d = L_a$ and designating it as *lever arm factor*, we obtain

$$\frac{x}{d} = (1 - L_a)/0.416$$

The expression for moment of resistance in terms of f_{ck} is given by Eq. (4.8) as

$$M_u = 0.36f_{ck}\,\frac{x}{d}\left(\frac{z}{d}\right)(bd^2)$$

Using expressions for x/d above, we get

$$\frac{M_u}{f_{ck}\,bd^2} = 0.36\left(\frac{1 - L_a}{0.416}\right)L_a$$

$$\frac{1.16M_u}{f_{ck}\,bd^2} = (1 - L_a)L_a$$

Putting $\dfrac{1.16M_u}{f_{ck}\,bd^2} = f_1$, we obtain

$$L_a^2 - L_a + f_1 = 0$$

Solving this equation and taking the larger of the two values, we get

$$L_a = 0.5 + (0.25 - f_1)^{1/2} \tag{4.14}$$

where

$$f_1 = \frac{1.16M_u}{f_{ck}\,bd^2}$$

The value of the lever arm factor L_a can be calculated from Eq. (4.14) or read off from a graph constructed for $M_u/(f_{ck}bd^2)$ vs. L_a as shown in Fig. 4.3. This graph has to be plotted for a range of values starting from a minimum value of $M_u/(f_{ck}bd^2)$ (corresponding to a maximum value of $L_a = 0.95$ and a minimum percentage of tension steel) to a maximum value of $M_u/(f_{ck}bd^2) = 0.15$

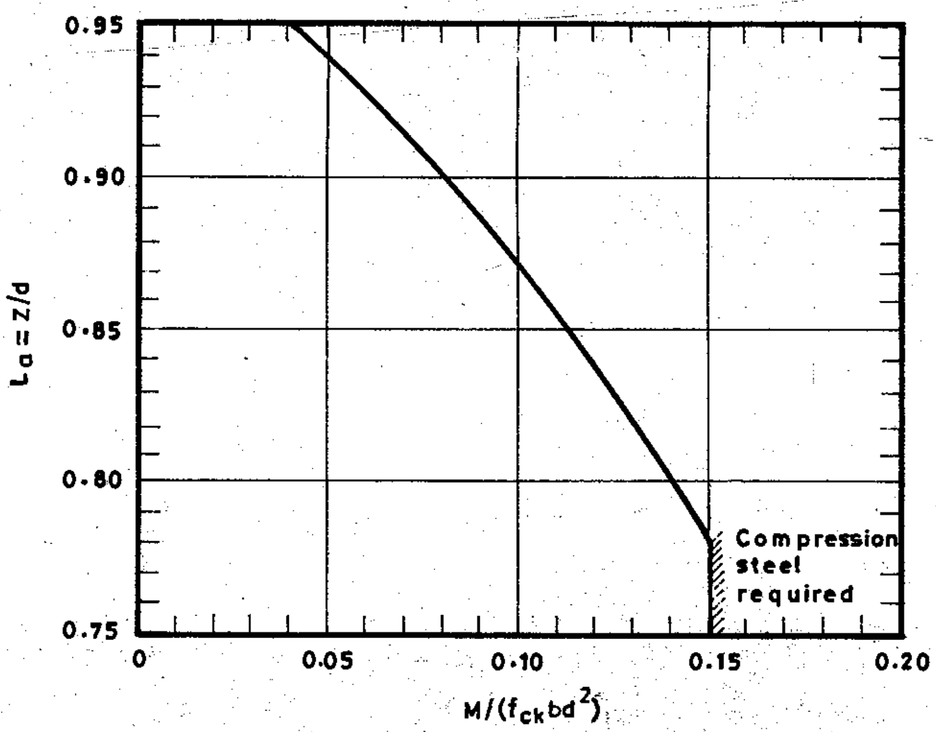


Fig. 4.3 Lever arm depth values (see Table 4.5).

corresponding to the limiting value of M_u for Fe 250 steel. The values are given in Table 4.5. These can be plotted as shown in Fig. 4.3. The area of steel is determined by calculating L_a or reading it off from the figure.

TABLE 4.5 VALUES FOR LEVER ARM DEPTH FACTOR (see Fig. 4.3)

$M_u/f_{ck}bd^2$	0.04	0.05	0.08	0.10	0.11	0.14	0.15
$L_a = z/d$	0.95	0.94	0.90	0.87	0.85	0.80	0.78
x/d	0.12	0.15	0.25	0.32	0.36	0.48	0.53

$$z = dL_a$$

$$A_s = \frac{M_u}{0.87 f_y z}$$

4.13.3 PROCEDURE 3 (DIRECT CALCULATION OF PER CENT STEEL)

Direct calculation of A_s can be done as

$$M_u = 0.87 f_y A_s z$$

However,

$$z = d\left(1 - 0.416\,\frac{x}{d}\right)$$

As the steel reaches yield state, substituting for x/d from Eq. (4.3a), we get

$$z = d\left(1 - \frac{f_y A_{st}}{f_{ck} bd}\right)$$

Hence,

$$M_u = 0.87 f_y A_s d\left(1 - \frac{f_y A_s}{f_{ck} bd}\right)$$

Putting

$$\frac{A_s f_y}{bdf_{ck}} = q$$

we get

$$M_u = 0.87 q f_{ck}(1 - q) bd^2$$

Rearranging the terms, we obtain

$$q(1-q) = \frac{M_u}{0.87 f_{ck} bd^2} = \frac{1.15 M_u}{f_{ck} bd^2} = f_2$$

It may be noted that f_2 has the same magnitude as f_1 in Procedure 2. Simplifying, we have

$$q^2 - q + f_2 = 0$$

Solving q and taking the lesser of the two values (as different from Procedure 1), we get

$$q = 0.50 - (0.25 - f_2)^{1/2} \tag{4.15}$$

The area of steel is obtained by substituting back for q in the expression

$$A_s = \frac{qbdf_{ck}}{f_y}$$

4.13.4 PROCEDURE 4 (USE OF DESIGN AID SP 16 WITHOUT CALCULATIONS)

The special publication No. 16 by Indian Standards Institution gives charts and tables for quick design of R.C. sections. Charts 1 to 18 and Tables 1 to 4 are for singly reinforced beams. These charts and tables are derived from Eq. (4.7) which is

$$M_u = 0.87 f_y \left(\frac{p}{100}\right)\left(1 - \frac{1.005 f_y}{f_{ck}}\frac{p}{100}\right) bd^2$$

Charts 1 to 18 have been prepared by assigning different values of M_u per unit width and plotting d vs. p_t. Tables 1–4 which cover a wider range give the value of p_t for various values of M/bd^2.

To determine the percentage of steel required for a given value of M, b, d, f_{ck}, f_y from Tables 1 to 4 of SP 16, proceed as follows:

1. Calculate M/bd^2.
2. Enter the table corresponding to the given value of f_{ck} and f_y in SP 16.
3. Read off the percentage of steel required.

4.14 GUIDELINES FOR CHOOSING WIDTH, DEPTH AND REINFORCEMENT OF BEAMS

The following guidelines may be used to arrive at the dimensions of R.C. beams:

1. The minimum percentage of tension steel used in beams should be around 0.3 per cent (as we shall see in Section 9.10). Usually, the depth of singly reinforced beams is so arranged that the percentage of steel required is only around 75 per cent of the balanced steel.

2. At least two bars should be used as tension steel, and not more than six bars should be used in one layer in a beam.

3. The diameter of hanger bars should not be less than 10 mm and that of main tension bars 12 mm. The usual diameters of bars chosen for beams are 10, 12, 16, 20, 22, 25 and 32 mm. When using different sized bars in one layer, place the largest diameter bars near the beam faces. The areas of steel should be symmetrical about the centre line of the beam.

4. The width of the beam necessary for accommodating the required number of rods will depend on the specification for cover and minimum spacing. IS 456: clause 25.4 gives the required cover to main steel for beams as 25 mm or diameter of bar, whichever is larger and 15 mm to stirrups. These mean cover values can be adopted for most parts of India, but these should be increased by at least 5 to 10 mm for coastal areas of India, as explained in Section 3.4. IS 456: clause 25.3.1 gives the minimum distance between bars as the diameter of bar or maximum size

of aggregate plus 5 mm. The maximum size of aggregates normally used in India is 20 mm so that clear maximum distance between bars should be 25 mm. Accordingly, the minimum width for placement of steel can be easily worked out. Assuming use of 8 mm steel as stirrups, the minimum widths required for different steels are given in Table 4.6.

TABLE 4.6 MINIMUM BEAM WIDTHS (mm) FOR REINFORCEMENTS
(According to IS 456)

Diameter of bars	No. of bars of given diameter				
1	2	3	4	5	6
10	95	130	165	200	235
12	99	136	173	210	247
16	107	148	189	230	271
20	115	160	205	250	295
22	119	166	213	260	307
25	125	175	225	275	325

Note: Add 10 to 20 mm for coastal regions.

5. The depth of the beam should satisfy the deflection requirements with respect to *L/d* ratios. In addition, for economy, the ratio of overall depth to width should be between 1.5 and 2.0.

6. In *T* beams the depth of the slab is usually taken as about 20 per cent of the overall depth of the beam.

7. For main steel bars, choose one size if possible. In any case, limit the main bars to two sizes and that too without much variation in diameter between the two.

8. The usual widths of beams adopted in mm are 150, 200, 230, 250 and 300. These widths should be equal to or less than the dimension of the columns into which they frame. For example, 300 mm wide beams can frame into 300 mm or 400 mm dimensions of columns.

REVIEW QUESTIONS

4.1 Draw the shape of the actual laboratory stress-strain curve for concrete as obtained from compression tests on cylinder specimen.

4.2 Alongside the curve mentioned in Question 4.1, draw the stress-strain curve assumed in bending compression in beams (bending members). Draw also the curve used for limit state design (ultimate failure) of reinforced concrete section in bending, using partial safety factor of concrete equal to 1.5.

4.3 Draw the laboratory stress-strain for (a) mild steel (grade 250), (b) high yield steel (Grade 415), and alongside these curves draw the design curves using a partial safety factor of 1.15 for steel.

4.4 An examination of the design stress-strain curves of concrete and steel (Figs. 20 and 22 of IS 456) shows that whereas in concrete a reduction by γ_m is applied all along the curve, in steel the reduction is applied only from the yield point ($0.87f_y$). Give reasons for this difference.

4.5 IS 456 clause 37.1(c) states that the stress-strain curve for concrete can be of any shape that is in agreement with test results. Some codes allow a rectangular stress distribution with f_{ck} acting over the entire depth of neutral axis obtained by strain compatibility. Determine the value of k that should be used to get the same moment of resistance as in the parabolic distribution of IS 456.

4.6 Why is it necessary to put a limit on the x/d allowed in singly reinforced beams as stipulated in IS 456 clause 37.1? Can this condition be relaxed for beams with compression steel also? Give reasons for your answer.

4.7 Explain the terms 'balanced', 'overreinforced' and 'underreinforced' sections in bending. Explain which of these terms should be recommended in design. How is this ensured in design of beams according to IS 456?

4.8 Explain how in most designs the depths of slabs or beams are taken larger than those obtained from bending moment considerations only. Does this produce an underreinforced, overreinforced or balanced section?

4.9 Indicate how to determine the area of steel required to carry a given bending moment by a beam where depth is mucb larger than that required from strength consideration only (balanced section).

4.10 Sketch the curve showing the relationship between M/bd^2 and percentage of steel required and explain the salient points on the curve.

5

Examples in Design and Analysis of Singly Reinforced Beams

5.1 INTRODUCTION

The economic execution of a concrete structure depends more on the overall layout of the structure with respect to construction feasibility and cost, (called constructability) than on its theoretical analysis. This knowledge regarding economy is generally acquired only through experience and study of projects already carried out. On the other hand, the structural safety of the individual members depends primarily on theoretical analysis and design. The best way to ensure this safety is to design the structure according to the relevant codes of practice and construct the structure according to accepted practice. As civil engineers will be called upon to carry out design of structures to be constructed as well as analyse (or review) structures already constructed, they should be familiar with the current codes and methods of design and analysis.

In Chapter 4 the theoretical derivations of the design equations for ultimate load were developed. In this chapter, these are reviewed from a designer's viewpoint and their uses for analysis and design are illustrated with the help of examples.

5.2 DESIGN AND ANALYSIS

Design problems in Reinforced Concrete are those in which an engineer is called upon to plan and determine the dimensions and areas of steel required in a member to carry a given load. Analysis problems are those in which concrete dimensions and areas of steel are given and one is required to determine either the maximum stresses in steel and concrete, or the ultimate load carrying capacity of the given beam. It should be borne in mind that the stresses in concrete and steel depend on whether the loading is within the elastic or post-elastic stage, which has to be determined by trial and error. Hence, analysis of a section by limit state theory usually means the determination of the ultimate carrying capacity of the section, and analysis by working stress theory implies estimating for stresses at the service loads by elastic theory.

5.3 METHODS OF DESIGN AND ANALYSIS

Three procedures or methods can be used for analysis and design of reinforced concrete sections for strength by limit state method. These are:

1. The strain compatibility method (i.e. working from fundamentals using the basic assumptions of ultimate limit state design).

2. The formulae method derived from the basic assumptions made in Chapter 4.

3. The design charts and tables method published by the ISI in its publication SP 16—Design-Aids for Reinforced Concrete to IS 456 (1978).

Of these, methods 2 and 3 can be used for rectangular sections only and hence are limited to these profiles. However, the strain compatibility method uses fundamental concepts and is useful for analysis and review of not only rectangular sections, but also for non-rectangular sections, for which the other two methods are not applicable. The basic theory of these three procedures has been dealt with in Chapter 4. These procedures are briefly reviewed in this chapter and examples are worked out to illustrate the procedures.

5.4 PROCEDURE FOR ANALYSIS OF SECTION BY STRAIN COMPATIBILITY (TRIAL AND ERROR METHOD)—METHOD 1

The basic principle underlying the strain compatibility method of analysis of a section is that one can find by trial and error the depth of neutral axis, which will be compatible with the equilibrium of forces, i.e. the total tension in steel is equal to the total compression in concrete plus the compression in steel, if compression reinforcement is also provided. The concrete is assumed to reach failure with a compression strain of 0.0035 and the stress in the steel will depend on the strain level reached in steel corresponding to the maximum concrete strain of 0.0035. The forces in the steel are determined from strain obtained by using the assumption that plane section remains plane till failure so that the strain profile is linear. The moment of resistance of the section in bending is then the moment of the couple thus formed. Example 5.3 illustrates the strain compatibility method. Calculations with parabolic stress block are cumbersome and in order to make the calculations from fundamentals simpler when dealing with non-rectangular sections, it is customary to use a rectangular compression stress block instead of the theoretical parabolic rectangular block. The commonly used blocks are shown in Fig. 5.1. The depth of neutral axis should also be restricted to the limiting x_u/d values given in Table 4.1 or $0.5d$ (approx.) to avoid brittle failures of concrete.

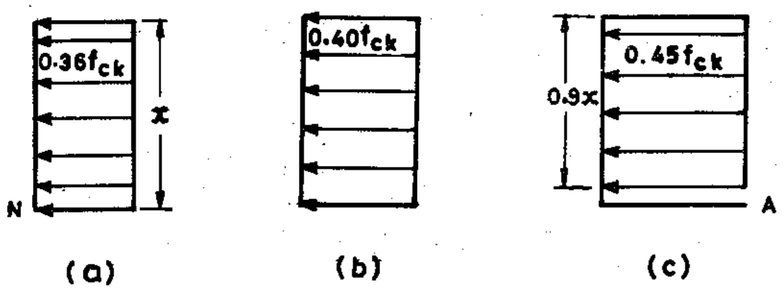


Fig. 5.1 Rectangular stress blocks.

For comparison, the ultimate moments of resistance obtained by the various stress blocks are as follows:

$$M_u \text{ due to stress block in Fig. 5.1(a)} = 0.36 f_{ck} b (0.5d)(0.75d)$$
$$= 0.135 f_{ck} b d^2$$

$$M_u \text{ due to stress block in Fig. 5.1(b)} = 0.40 f_{ck} b(0.5d)(0.75d)$$

$$= 0.150 f_{ck}\, bd^2$$

$$M_u \text{ due to stress block in Fig. 5.1(c)} = 0.45 f_{ck} b(0.45d)(0.77d)$$

$$= 0.156 f_{ck}\, bd^2$$

$$M_u \text{ from IS stress block} = 0.149 f_{ck}\, bd^2$$

These values compare well with one another so that for calculations from fundamentals any of them may be conveniently used by the designer.

5.5 ANALYSIS AND DESIGN BY FORMULAE (METHOD 2)

The formulae derived in Chapter 4 for design and analysis of the ultimate strength of singly reinforced sections in bending are based on rectangle-parabolic stress block. The more important formulae for a singly reinforced rectangular section are the following:

1. From consideration of failure of concrete, we have

$$M_u = K f_{ck} bd^2$$

where from Table 4.3,

$$K = 0.149 \quad \text{for Fe 250}$$

$$= 0.138 \quad \text{for Fe 415}$$

$$= 0.133 \quad \text{for Fe 500}$$

2. From the assumption that steel reaches yield at failure, the ultimate moment of resistance for a given percentage of tension steel p is given by [(IS 456, Appendix E), Eq. (4.6a)]

$$M_u = 0.87 f_y A_{st} d\left(1 - \frac{A_{st} f_y}{bd f_{ck}}\right)$$

or, from Eq. (4.7),

$$M_u = 0.87 f_y \frac{p}{100}\left(1 - \frac{f_y}{f_{ck}} \frac{p}{100}\right) bd^2$$

3. For a given M_u, f_{ck}, b and d from Eq. (4.12), we get

$$\left(\frac{x}{d}\right) = 1.20 - \left[(1.20)^2 - \frac{6.68 M_u}{f_{ck} bd^2}\right]^{1/2}$$

and x/d should not exceed the limiting value given in Table 4.1.

4. The balanced steel percentage (denoting the corresponding steel at p_t limit) as given by Eq. (4.11) is

$$p_t \text{ (lim)} = 41.3 \frac{f_{ck}}{f_y} \frac{x_u}{d}$$

5. The lever arm depth as given by Eq. (4.13) is

$$z = d\left(1 - 0.416\frac{x}{d}\right)$$

$$= d\left(1 - \frac{1.155 f_{st} A_{st}}{f_{ck} bd}\right)$$

With yield of steel being $f_{st} = 0.87 f_y$, from Eq. (4.13),

$$z = d\left(1 - \frac{1.005 f_y A_{st}}{f_{ck} bd}\right)$$

$$= d\left(1 - \frac{f_y A_{st}}{f_{ck} bd}\right)$$

The meanings of the terms *overreinforced, underreinforced* and *balanced section* have already been explained in Chapter 4. In R.C.C. beam design problems, one is given the value of M_u and asked to determine the dimension of the concrete section and the area of the steel required. As already explained, when one designs an R.C.C. section, one can adopt a balanced section, but more often one chooses an underreinforced section and never an overreinforced section.

Thus, in all cases of design of a section for a given moment by limit state, the stress in steel is assumed to reach yield limit before concrete failure. In practice, the depth of the concrete section selected will be from various considerations other than strength. It is usually more than the minimum required from strength considerations only in concrete failure. For example, to limit deflections, the depth of a concrete slab for a floor required is generally more than what is required from consideration of strength only. In such situations, one has to incorporate only just enough steel that is required for strength requirements, and this will be less than the balanced percentage. The section thus chosen automatically becomes underreinforced.

5.5.1 PROCEDURE FOR ANALYSIS OF SECTIONS BY FORMULAE

The following step-by-step procedure can be used to analyse a given R.C.C. beam whereby one calculates the ultimate moment of resistance M_u of a given cross-section of concrete and steel having the given characteristic strengths.

Step 1: Calculate the p_t (lim) balanced steel required. For given grade of steel and concrete, find the p_t (lim) and A_{st} for balanced failure. If A_s provided is less than A_{st} balanced, only tension failure is possible. If A_s provided is more than A_{st} balanced, only compression failure is possible. Determine M_u from one of the formulae as shown in steps 2 and 3.

Step 2: (a) If failure occurs by compression in concrete, determine the value of M_u (lim) with respect to failure of concrete (compression failure) by Eq. (4.9).

$$M_u = 0.149 f_{ck} bd^2 \quad \text{for Fe 250}$$

$$M_u = 0.138 f_{ck} bd^2 \quad \text{for Fe 415}$$

(b) If the failure occurs by tension in steel, determine the value of M_u with respect to failure of steel (tension failure) by the expression from the equation

$$M_u = 0.87 f_y A_{st} \text{ (Lever arm)}$$

Therefore,

$$M_u = 0.87 f_y \frac{p}{100}\left(1 - \frac{1.005 f_y}{f_{ck}} \frac{p}{100}\right) bd^2$$

Alternatively, one can calculate the limiting moment with respect to concrete failure and steel failure separately without the calculation of balanced steel (Step 1) and the lesser of the two gives the moment of resistance.

5.5.2 PROCEDURE FOR DESIGN BY FORMULAE

1. Determination of minimum depth for given M_u and f_{ck}

To determine the minimum depth required for a beam or slab, one uses Eq. (4.9) which is

$$M_u = K f_{ck} b d^2$$

or

$$d = \sqrt{\frac{M_u}{K f_{ck} b}}$$

which becomes

$$d = \sqrt{\frac{6.6 M_u}{f_{ck} b}} \quad \text{for Fe 250}$$

$$= \sqrt{\frac{7.1 M_u}{f_{ck} b}} \quad \text{for Fe 415}$$

2. Determination of steel for balanced design

If the depth selected is equal to the depth obtained from the above formulae, the area of steel p_t (lim) to be provided will be that of a balanced section. Referring to Section 4.10, the values of p_t (lim) are given by

$$p_t \text{ (lim)} = 21.97\left(\frac{f_{ck}}{f_y}\right) \quad \text{for Fe 250}$$

$$p_t \text{ (lim)} = 19.82\left(\frac{f_{ck}}{f_y}\right) \quad \text{for Fe 415}$$

$$p_t \text{ (lim)} = 18.87\left(\frac{f_{ck}}{f_y}\right) \quad \text{for Fe 500}$$

where $p_t = \dfrac{100 A_{st}}{bd}$, see Eq. (4.11) and Table 4.4.

The various percentages for different grades of steel and concrete are given in Table 5.1.

TABLE 5.1 BALANCED PERCENTAGE OF STEEL p_t (lim)
(Ref. Table E, SP 16)

Type of steel	Strength of concrete			
	15	20	25	30
Fe 250	1.32	1.76	2.20	2.64
Fe 415	0.72	0.96	1.19	1.43
Fe 500	0.57	0.76	0.94	1.13

3. Determination of steel for a section with depth larger than for balanced section (underreinforced section)

As already pointed out in Section 4.13, this is the most common problem met with during design. One has to find the amount of steel required for a depth of concrete section larger than the minimum required to carry the moment with balanced steel ratio. Naturally the area of steel required will be less than that required for a balanced section. The required area should be obtained from the fundamental expression

$$A_{st} = \frac{M_u}{(0.87 f_y)\,(\text{Lever arm})}$$

Any one of the four procedures given in Section 4.13 may be used for calculating the area of steel for underreinforced section. This is illustrated in Example 5.5.

5.6 USE OF SP 16 FOR DESIGN OF BEAMS AND SLABS (METHOD 3)

The Indian Standards Institution in its Special Publication SP 16 Design Aids for Reinforced Concrete to IS 456 (1978) has given a number of charts and tables for design of R.C.C. members. One should be familiar with the derivation and use of these charts and tables to eliminate the drudgery of calculations in design offices.

The following are the data presented in SP 16 for design and analysis of beams and slabs of singly reinforced sections. The basis of these charts and tables was explained in Section 4.13.4.

1. Charts 1 to 18 give the moment of resistance per metre width for varying depths (5 to 80 cm) and varying percentage of steel, for various values of f_{ck} (15 and 20) using f_y = 250, 415 and 500. They are presented in different colour backgrounds for different grades of steel. Separate charts are presented for values of f_{ck} = 15 and 20. The use of these charts is therefore limited to these grades of concrete only.

2. Tables 1 to 4 of SP 16 give the reinforcement percentage needed for various values of M_u/bd^2 and f_y, for f_{ck} = 15, 20, 25 and 30. Tables 1 and 2 of SP 16 for f_{ck} = 15 and 20 are given as Tables 5.2 and 5.3 here.

3. Tables 5 to 44 of SP 16 give the moment of resistance per metre width for various thicknesses of slabs (t = 10 to 25 cm) for different bar diameters and spacing for various values of f_y and f_{ck}.

The data given in SP 16 can be used for quick and routine design of beams and slabs and its use is illustrated with the help of examples given at the end of this chapter.

TABLE 5.2 FLEXURE-REINFORCEMENT PERCENTAGE, p_t, FOR SINGLY REINFORCED SECTIONS

[Refer Table 1 of SP 16]

f_{ck} = 15 N/mm^2

M_u/bd^2 N/mm^2	f_y, N/mm^2			M_u/bd^2, N/mm^2	f_y, N/mm^2		
	250	415	500		250	415	500
0.30	0.141	0.085	0.071	1.50	0.796	0.480	0.398
0.35	0.166	0.100	0.083	1.52	0.809	0.487	0.404
0.40	0.190	0.114	0.095	1.54	0.821	0.495	0.411
0.45	0.215	0.129	0.107	1.56	0.834	0.503	0.417
0.50	0.240	0.144	0.120	1.58	0.847	0.510	0.423
0.55	0.265	0.159	0.132	1.60	0.860	0.518	0.430
0.60	0.290	0.175	0.145	1.62	0.873	0.526	0.436
0.65	0.316	0.190	0.158	1.64	0.886	0.534	0.443
0.70	0.342	0.206	0.171	1.66	0.899	0.542	0.449
0.75	0.368	0.221	0.184	1.68	0.912	0.550	0.456
0.80	0.394	0.237	0.197	1.70	0.925	0.558	0.463
0.82	0.405	0.244	0.202	1.72	0.939	0.566	0.469
0.84	0.415	0.250	0.208	1.74	0.952	0.574	0.476
0.86	0.426	0.257	0.213	1.76	0.966	0.582	0.483
0.88	0.437	0.263	0.218	1.78	0.980	0.590	0.490
0.90	0.448	0.270	0.224	1.80	0.993	0.598	0.497
0.92	0.458	0.276	0.229	1.82	1.007	0.607	0.504
0.94	0.469	0.283	0.235	1.84	1.021	0.615	0.511
0.96	0.480	0.289	0.240	1.86	1.035	0.624	0.518
0.98	0.491	0.296	0.246	1.88	1.049	0.632	0.525
1.00	0.502	0.303	0.251	1.90	1.063	0.641	0.532
1.02	0.513	0.309	0.257	1.92	1.078	0.649	0.539
1.04	0.524	0.316	0.262	1.94	1.092	0.658	0.546
1.06	0.536	0.323	0.268	1.96	1.107	0.667	0.553
1.08	0.547	0.329	0.273	1.98	1.121	0.676	0.561
1.10	0.558	0.336	0.279	2.00	1.136	0.685	
1.12	0.570	0.343	0.285	2.02	1.151	0.693	
1.14	0.581	0.350	0.290	2.04	1.166	0.703	
1.16	0.592	0.357	0.296	2.06	1.181	0.712	
1.18	0.604	0.364	0.302	2.08	1.197		
1.20	0.615	0.371	0.308	2.10	1.212		
1.22	0.627	0.378	0.314	2.12	1.228		
1.24	0.639	0.385	0.319	2.14	1.243		
1.26	0.650	0.392	0.325	2.16	1.259		
1.28	0.662	0.399	0.331	2.18	1.275		
1.30	0.674	0.406	0.337	2.20	1.291		
1.32	0.686	0.413	0.343	2.22	1.308		
1.34	0.698	0.420	0.349	2.24			
1.36	0.710	0.428	0.355				
1.38	0.722	0.435	0.361				
1.40	0.734	0.442	0.367				
1.42	0.747	0.450	0.373				
1.44	0.759	0.457	0.379				
1.46	0.771	0.465	0.386				
1.48	0.784	0.472	0.392				

Note: Blank space indicates inadmissible reinforcement percentage (see Table 5.1).

TABLE 5.3 FLEXURE-REINFORCEMENT PERCENTAGE, p_t, FOR SINGLY REINFORCED SECTIONS
[Refer Table 2 of SP 16]

f_{ck} = 20 N/mm²

M_u/bd^2 N/mm²	f_y, N/mm² 250	415	500	M_u/bd^2, N/mm²	f_y, N/mm² 250	415	500
0.30	0.140	0.085	0.070	2.22	1.203	0.725	0.602
0.35	0.164	0.099	0.082	2.24	1.216	0.733	0.608
0.40	0.188	0.114	0.094	2.26	1.230	0.741	0.615
0.45	0.213	0.128	0.106	2.28	1.243	0.749	0.621
0.50	0.237	0.143	0.119	2.30	1.256	0.757	0.628
0.55	0.262	0.158	0.131	2.32	1.270	0.765	0.635
0.60	0.286	0.172	0.143	2.34	1.283	0.773	0.642
0.65	0.311	0.187	0.156	2.36	1.297	0.781	0.648
0.70	0.336	0.203	0.168	2.38	1.311	0.790	0.655
0.75	0.361	0.218	0.181	2.40	1.324	0.798	0.662
0.80	0.387	0.233	0.193	2.42	1.338	0.806	0.669
0.85	0.412	0.248	0.206	2.44	1.352	0.814	0.676
0.90	0.438	0.264	0.219	2.46	1.366	0.823	0.683
0.95	0.464	0.280	0.232	2.48	1.380	0.831	0.690
1.00	0.490	0.295	0.245	2.50	1.394	0.840	0.697
1.05	0.517	0.311	0.258	2.52	1.408	0.848	0.704
1.10	0.543	0.327	0.272	2.54	1.423	0.857	0.711
1.15	0.570	0.343	0.285	2.56	1.437	0.866	0.719
1.20	0.597	0.359	0.298	2.58	1.451	0.874	0.726
1.25	0.624	0.376	0.312	2.60	1.466	0.883	0.733
1.30	0.651	0.392	0.326	2.62	1.481	0.892	0.740
1.35	0.679	0.409	0.339	2.64	1.495	0.901	0.748
1.40	0.707	0.426	0.353	2.66	1.510	0.910	0.755
1.45	0.735	0.443	0.367	2.68	1.525	0.919	
1.50	0.763	0.460	0.382	2.70	1.540	0.928	
1.55	0.792	0.477	0.396	2.72	1.555	0.937	
1.60	0.821	0.494	0.410	2.74	1.570	0.946	
1.65	0.850	0.512	0.425	2.76	1.585	0.955	
1.70	0.879	0.530	0.440	2.78	1.601		
1.75	0.909	0.547	0.454	2.80	1.616		
1.80	0.939	0.565	0.469	2.82	1.632		
1.85	0.969	0.584	0.484	2.84	1.647		
1.90	1.000	0.602	0.500	2.86	1.663		
1.95	1.030	0.621	0.515	2.88	1.679		
2.00	1.062	0.640	0.531	2.90	1.695		
2.02	1.074	0.647	0.537	2.92	1.711		
2.04	1.087	0.655	0.543	2.94	1.727		
2.06	1.099	0.662	0.550	2.96	1.743		
2.08	1.112	0.670	0.556	2.98	1.760		
2.10	1.125	0.678	0.562				
2.12	1.138	0.685	0.569				
2.14	1.151	0.693	0.575				
2.16	1.164	0.701	0.582				
2.18	1.177	0.709	0.588				
2.20	1.190	0.717	0.595				

Note: Blank space indicates inadmissible reinforcement percentage (see Table 5.1).

5.7 NECESSITY FOR SPECIFYING MAXIMUM AND MINIMUM TENSION STEEL IN BEAMS

The necessity for specifying the maximum and minimum tension steel in bending members can be explained as in the following sections.

5.7.1 MAXIMUM STEEL PERCENTAGE

Equation (4.7) for ultimate strength in terms of steel is given by

$$M_u = 0.87 f_y \frac{p}{100}\left(1 - \frac{f_y}{f_{ck}} \frac{p}{100}\right) bd^2$$

This equation suggests that the M_u increases with p in parabolic relation. Thus, with Fe = 415 and grade 20 concrete, the relationship can be expressed as

$$\frac{M_u}{bd^2} = 3.6p - 0.75p^2$$

Although an increase in steel raises the value of the moment of resistance, the introduction of large percentage of steel has an adverse effect of decreasing the ductility of the beam on overload. This can be shown from the following derivation. The effect of the percentage of steel on the depth of the neutral axis can be seen by writing the equilibrium equation as

$$0.36 f_{ck} b(x/d)d = f_{st} A_{st}$$

and reducing it to the form

$$\frac{x}{d} = \frac{f_{st} A_{st}}{0.36 f_{ck} bd} = \frac{f_{st}}{0.36 f_{ck}} \frac{p}{100}$$

For Fe 415 and $f_{ck} = 20$, the above expression reduces to $x/d = 0.58p$.

As will be shown in the later part of the text, ductility of the beam can be measured by the curvature (denoted by $1/R$) of the beam at failure. The equation for the curvature can also be written as

$$\frac{1}{R} = \frac{\text{strain in compression fibre}}{\text{depth of neutral axis}} = \frac{0.0035}{x}$$

and is derived from Fig. 5.2. Substituting for x/d, we get

$$\frac{1}{R} = \frac{0.0035}{0.58pd} = \frac{0.006}{pd}$$

This equation shows that there is a sharp fall in the ductility of the R.C.C. beam with increase in percentage of tension steel. Hence codes usually specify the maximum amount of steel to be allowed in bending members. IS 456 specifies this in clause 25.5.1 and has put the upper limit of tension steel as 4 per cent.

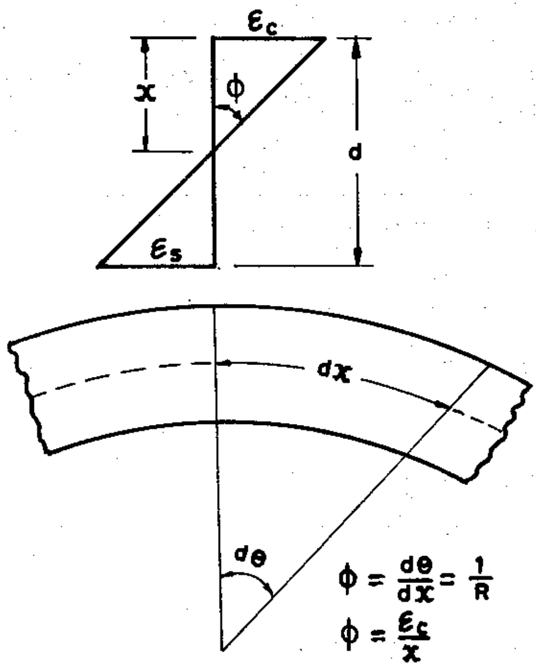


Fig. 5.2 Rotation of a section in bending.

As already indicated in Table 4.4, singly reinforced concrete beams should not normally have the area of tension steel more than 75 per cent of the steel for balanced ultimate failure.

5.7.2 MINIMUM STEEL PERCENTAGE

The minimum areas of longitudinal and transverse steel are also specified to take care of shrinkage and ductility of concrete. It is also necessary for giving some resistance to formation of diagonal tension cracks in shear and for ensuring minimum resistance to forces applied during construction. This is specified in IS 456: clause 25.5.1 as

$$\frac{A_s}{bd} = \frac{0.85}{f_y}$$

This works out to only 0.2 for Fe 415 and 0.35 for Fe 250. (For further discussion on this topic, refer Section 9.11.)

5.8 RECOMMENDED PROCEDURE FOR DESIGN AND ANALYSIS

All the three procedures mentioned in Section 5.3 and explained in Sections 5.6 and 5.7 will lead to the same results. However, the use of SP 16, considerably reduces the work of the designer and should be encouraged both in the classrooms and design offices. Working out design from fundamentals should be confined to classroom exercises for understanding the basis of design.

EXAMPLE 5.1 (Analysis of an underreinforced beam)

Calculate the ultimate moment carrying capacity of a rectangular beam with $b = 250$ mm, $d = 350$ mm, $A_{st} = 1800$ mm^2. Assume grade 30 concrete and Fe 250 steel. Work out the problem from fundamentals.

Ref.	Step	Calculations	Output
	1.	*Depth of* NA *for balanced failure*	
		$\dfrac{x}{d} = \dfrac{0.0035}{0.0055 + 0.87(f_y/E_s)}$	
		$f_y = 250$ N/mm^2, $E_s = 2 \times 10^5$ N/mm^2	
		$\dfrac{x}{d} = 0.53$	
		(Check with SP 16, Table B)	
	2.	*Balanced percentage of steel*	
		$0.87 f_y A_s = 0.36 f_{ck} bx$	
		$\left(\dfrac{p}{100} bd\right)(0.87 f_y) = 0.36 f_{ck} bx$	
		Substituting for x/d, we get	
		$\dfrac{p f_y}{f_{ck}} = \dfrac{0.36 \times 0.53}{0.87} \times 100 = 21.97$	
		(Check with SP 16 Table C)	
		For $f_y = 250$ N/mm^2, $f_{ck} = 30$ N/mm^2, we have	
		$p = \dfrac{21.97 \times 30}{250} = 2.64\%$	Balance $p = 2.64\%$
		(Check with SP 16 Table E)	
	3.	*Actual percentage of steel in beam*	
		Steel in beam $= \dfrac{1800 \times 100}{250 \times 350} = 2.05\%$	Steel failure controls
		Hence beam is underreinforced.	
	4.	*M_u steel failure*	
		$M_u = 0.87 f_y (p/100)[1 - (p/100)(f_y/f_{ck})]bd^2$	
		$= \dfrac{0.87 \times 250 \times 2.05}{100}\left(1 - \dfrac{2.05 \times 250}{100 \times 30}\right)bd^2$	
		$= 3.697 \times 250 \times 350^2$ Nmm $= 113$ kNm	$M_{us} = 113$ kNm

EXAMPLE 5.1 (cont.)

Ref.	Step	Calculations	Output
	5.	*M_u concrete (for confirmation only)*	
		M_u concrete = (compression) $(d - 0.416x)$	
		$M_u = (0.36 f_{ck} bx)(d - 0.416x)$	
		$x = 0.53d, \quad M_u = 0.149 f_{ck} bd^2$	
		(Alternatively, use SP 16, Table C.)	
		$M_u = 0.149 \times 30 \times 250 \times 350^2 = 137$ kNm	$M_{uc} = 137$ kNm
	6.	*Ultimate moment capacity*	
		Steel failure controls strength.	$M_u = 113$ kNm

EXAMPLE 5.2 **(Analysis of an overreinforced beam)**

Calculate the ultimate moment carrying capacity of a reinforced concrete section with $b = 250$ mm; $d = 400$ mm; $A_{st} = 3600$ mm^2. Assume grade 20 concrete and Fe 415 steel.

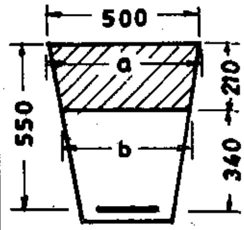
Ref.	Step	Calculations	Output
Method I		By comparing M_{us} for steel failure and M_{uc} for concrete failure.	
	1.	*M_{us} limit for steel failure*	
		$M_{us} = 0.87 f_y \dfrac{p}{100}\left(1 - \dfrac{f_y}{f_{ck}}\dfrac{p}{100}\right) bd^2$	
IS 456		$p = \dfrac{3600 \times 100}{250 \times 400} = 3.6$ per cent	
(Appendix E) cl. E.1.1b		$M_{us} = \dfrac{0.87 \times 415 \times 3.6}{100}\left(1 - \dfrac{415 \times 3.6}{20 \times 100}\right)$	
		$\times 250 \times 400^2 = 131.5$ kNm	$M_{us} = 131.5$
	2.	*M_{uc} limit for concrete failure (Fe 415)*	
SP 16 Table C		$M_{uc} = 0.138 f_{ck} bd^2$	
		$= 0.138 \times 20 \times 250 \times 400^2$	
		$= 110$ kNm	$M_{uc} = 110$
	3.	*Ultimate capacity*	
		$110 < 131.5$	
		Concrete failure controls $M_u = 110$ kNm	$M_u = 110$ kNm

EXAMPLE 5.2 (cont.)

Ref.	Step	Calculations	Output
Method II		*By determining neutral axis depth, x, for steel failure*	
	1.	*Depth of neutral axis*	
		$x = \dfrac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \dfrac{0.87 \times 415 \times 3600}{0.36 \times 20 \times 250}$	
		$= 722$ mm (Note $x \gg d$)	
		Limiting value of $x = 0.48d$ (Fe 415)	
		$= 0.48 \times 400 = 192$ mm	
		Hence the section is overreinforced. Concrete failure controls.	
	2.	*M_{uc} concrete*	
		$M_{uc} = 0.138 f_{ck} b d^2$	
		$= 0.138 \times 20 \times 250 \times 400^2$	
		$= 110$ kNm	$M_{uc} = 110$ kNm
Method III		*By use of Design Aid, SP 16*	
	1.	Percentage of steel, $p = \dfrac{A_{st} \times 100}{bd}$	
		$= \dfrac{3600 \times 100}{250 \times 400} = 3.6$	
SP 16	2.	For Fe 415 steel, $f_{ck} = 20$ N/mm^2	
Table E		Balanced steel percentage = 0.96	
		Beam is overreinforced.	M_{uc} controls
	3.	*M_{uc} concrete*	
		$M_{uc} = 0.138 f_{ck} b d^2$	
		$= 110$ kNm	$M_{uc} = 110$ kNm

EXAMPLE 5.3 **(Analysis of a trapezoidal beam)**

Find the ultimate (limiting state) moment capacity of a reinforced concrete trapezoidal section as shown in Fig. E.5.3 with the following dimensions: Top width = 500 mm; effective depth = 550 mm; width of beam at the level of the reinforcement = 300 mm; tension steel: 4 Nos. 25 mm rods. Assume $f_{ck} = 20$ N/mm^2 and $f_y = 415$ N/mm^2. Solve the problem from fundamentals using the rectangular stress block for compression in concrete.

Ref.	Step	Calculations	Output
	1.	*Assume a neutral axis* (NA) *and find strain in steel* Assume x = 210 mm Distance of steel from NA = 340 mm $\varepsilon_s = \dfrac{0.0035 \times 340}{210}$	 **Fig. E.5.3**
SP 16 Table A		= 0.0057 > 0.0038 (the yield strain of Fe 415 steel) Steel reaches yield at failure.	Steel yields
	2.	*Compression in concrete* Area of compression block = A_c A_c = (Average width) (210) mm^2 Average width $= \dfrac{1}{2}\left[500 + \left(300 + \dfrac{200 \times 340}{550}\right)\right]$ = 461.8 mm $A_c = 461.8 \times 210 = 96978$ mm^2 Average concrete stress = $0.36 f_{ck}$ $C = A_c\ (0.36 f_{ck})$ $C = (96978 \times 0.36 \times 20) \times 10^{-3}$ = 698 kN	C = 698 kN
	3.	*Tension in steel* A_s = 4 T 25 (1963 mm^2), i.e. 4 Nos. of 25 mm diameter high yield steel giving an area of 1963 mm^2. $T = 1963 \times 0.87 \times 415 \times 10^{-3} = 709$ kN	T = 709 kN
	4.	*Check T = C* C = 698 kN, T = 709 kN, $C = T$ (Approx.)	
	5.	*Find centre of compression and lever arm* Let CG be y from the top fibre. Then $y = \dfrac{h}{3}\left(\dfrac{a + 2b}{a + b}\right) = \dfrac{210}{3}\left[\dfrac{500 + 2(423)}{500 + 423}\right]$ = 102 mm LA = z = 550 − 102 = 448 mm	

EXAMPLE 5.3 (cont.)

Ref.	Step	Calculations	Output
	6.	*Find M_u*	
		$M_u = T \times z = 709 \times 448 \times 10^{-3} = 317$ kNm	$M_u = 317$ kNm
	7.	*Check using SP 16*	
		Average breadth $= b = \dfrac{500 + 300}{2} = 400$ mm	
		$d = 550, \quad \dfrac{A_s \times 100}{bd} = \dfrac{1963 \times 100}{400 \times 550} = 0.89\%$	
SP 16 Table 2		$\dfrac{M_u}{bd^2} = 2.62, \quad M_u = 2.62 \times 400 \times 550^2$	Check O.K.
		$= 317$ kNm	

V. Imp ***EXAMPLE 5.4*** **(Design of an overreinforced beam)**

A rectangular beam has $b = 200$ mm, $d = 400$ mm. If steel used is Fe 415 and grade of concrete is M20, find the steel required to carry a factored moment of 120 kNm.

Ref.	Step	Calculations	Output
		Solution by use of formulae	
	1.	*M_u limit of beam for concrete failure*	
		With Fe 415 steel, $M_u = 0.138 f_{ck} bd^2$	Section inadequate
		$M_u = 0.138 \times 20 \times 200 \times 400^2 = 88.32$ kNm	
IS 456 p. 109		According to Indian Standards, the above value is the capacity of the beam if the limiting value of $(x/d = 0.48)$ has to be kept. Hence the beam has to be designed as doubly reinforced (Chapter 6). However, a theoretical overreinforced beam can be designed to take the required moment.	
		Solution as an overreinforced beam	
	2.	*Determine the neutral axis required to carry a given M*	
Text Eq. (4.12)		$x/d = 1.20 - [(1.2)^2 - (6.6M)/(f_{ck}bd^2)]^{1/2}$	
		$= 1.20 - \left(1.44 - \dfrac{6.6 \times 120 \times 10^6}{20 \times 2 \times 4 \times 4 \times 10^6}\right)^{1/2}$	
		$= 0.75$	
		$0.75 >$ limiting value of 0.48	

EXAMPLE 5.4 (cont.)

Ref.	Step	Calculations	Output
SP 16 Table B		In practical designs the limiting value should not be exceeded.	
		Required x for the problem $= 0.75 \times 400$	
		$= 300$ mm	
	3.	*Strain in steel and consequent stress*	
		$\varepsilon_s = \dfrac{0.0035 \times 100}{300} = 0.0012$	
		Steel does not yield.	
		Stress $= f_s = 0.0012 \times E_s$	
		$= 0.0012\ (2 \times 10^5) = 240$ N/mm^2	
	4.	*Area of steel required*	
		$A_s f_s \quad = 0.36 f_{ck} bx$	
		$A_s\ 240 = 0.36 \times 20 \times 200 \times 300$	
		$A_s \quad = 1800$ mm^2	$A_s = 1800$ mm^2
		Note: This is not an acceptable design according to IS as $x/d > 0.48$. For solution as a doubly reinforced beam, refer Chapter 6.	
	5.	*Design by using SP 16*	
		This error will not creep in if the design is made by using SP 16.	
SP 16 Table 2		$M_u/bd^2 = \dfrac{120 \times 10^6}{200 \times 400^2} = 3.75$	
		In the table, the maximum value for $f_y = 415$ N/mm^2, $f_{ck} = 20$ N/mm^2 is 2.76 only. Hence beam should be doubly reinforced.	Beam to be doubly reinforced

EXAMPLE 5.5 **(Determination of A_s by various procedures)**

Determine the area of steel required for a beam $b = 300$ mm, $d = 675$ mm for carrying a factored moment of 185 kNm. Assume $f_y = 415$ N/mm^2 and $f_{ck} = 20$ N/mm^2. Solve the problem by various available methods.

Ref.	Step	Calculations	Output
	1.	*M_u limit in concrete failure* (Fe 415 steel)	
SP 16 Table C		$M_u = 0.138 f_{ck} bd^2 = 0.138 \times 20 \times 300 \times (675)^2$	
		$= 377$ kNm larger than $M = 185$ kNm	
		Hence design the beam as an underreinforced beam.	
Method I		*Determining steel by Eq. (4.12)*	
		Lever arm from neutral axis depth	
Text Eq. (4.12)	1.	$x/d = 1.2 - [(1.2)^2 - (6.6M_u)/f_{ck}bd^2]^{1/2}$	
		$= 1.2 - \left[1.44 - \dfrac{6.6 \times 185 \times 10^6}{20 \times 300 \times (675)^2}\right]^{1/2}$	
		$= 0.204$	
	2.	$z = d\left(1 - 0.42\dfrac{x}{d}\right) = 675\,[1 - (0.42 \times 0.204)]$	
		$= 617$ mm	
	3.	$A_s = \dfrac{M}{0.87 f_y z} = \dfrac{185 \times 10^6}{0.87 \times 415 \times 617} = 830\ \text{mm}^2$	$A_s = 830\ \text{mm}^2$
Method II		*Determining steel by Eq. (4.14)*	
		Lever arm from lever arm factor	
	1.	$f_1 = \dfrac{1.16 M_u}{f_{ck} bd^2} = \dfrac{1.16 \times 185 \times 10^6}{20 \times 300 \times (675)^2} = 0.0785$	
Text Eq. (4.14)	2.	$z/d = La = 0.5 + \sqrt{0.25 - f_1}$	
		$= 0.5 + \sqrt{0.25 - 0.0785} = 0.914$	
	3.	$z = 0.914 \times 675 = 617$ mm	
	4.	$A_s = \dfrac{M}{0.87 f_y z}$ as above $= 830\ \text{mm}^2$	$A_{st} = 830\ \text{mm}^2$
Method III		*Determining steel area from Eq. (4.15)*	
	1.	$q = 0.5 - \sqrt{0.25 - f_2},\ f_2 = \dfrac{1.149 M_u}{f_{ck} bd^2}$	
	2.	$f_2 = \dfrac{1.149 \times 185 \times 10^6}{20 \times 300 \times (675)^2} = 0.0777$	
		Hence, $q = 0.085$	

EXAMPLE 5.5 (cont.)

Ref.	Step	Calculations	Output
	3.	$A_{st} = \dfrac{qbdf_{ck}}{f_y} = \dfrac{0.085 \times 300 \times 675 \times 20}{415}$	
		$= 830 \text{ mm}^2$	$A_{st} = 830 \text{ mm}^2$
Method IV		*Solution by use of SP 16*	
	1.	*Design parameters*	
		$\dfrac{M}{bd^2} = \dfrac{185 \times 10^6}{300 \times (675)^2} = 1.35$	
		Fe 415 and $f_{ck} = 20 \text{ N/mm}^2$	
	2.	*Percentage of steel needed*	
SP 16 Table 2		$p = \dfrac{A_s}{bd} \times 100 = 0.409\%$	
		$A_s = \dfrac{0.409 \times 300 \times 675}{100} = 828 \text{ mm}^2$	$A_{st} = 828 \text{ mm}^2$

EXAMPLE 5.6 (Determination of A_s by SP 16)

A beam has $b = 260$ mm, $d = 460$ mm. Find the steel required to carry a factored moment of 165 kNm. Assume grade 25 concrete and Fe 415 steel. Solve the problem by use of formulae and by using SP 16.

Ref.	Step	Calculations	Output
Method I		*Solution by formulae*	
	1.	*Capacity of beam in concrete failure*	
		For Fe 415 steel, $M_u = 0.14 f_{ck} bd^2$ (approx.)	
		$M_u = 0.14 \times 25 \times 260 \times 460^2 = 193 \text{ kNm} > 165 \text{ kNm}$	Section O.K.
	2.	*Depth of* N.A. *required for given* B.M.	
		$x/d = 1.20 - \left(1.44 - \dfrac{6.6M_u}{f_{ck}bd^2}\right)^{1/2}$	
		$= 1.20 - \left(1.44 - \dfrac{6.6 \times 165 \times 10^6}{25 \times 260 \times (460)^2}\right)^{1/2}$	
		$= 0.40$	
IS 456 p. 109		0.40 < 0.48. (Hence the values of x/d are acceptable)	Steel yields

EXAMPLE 5.6 (cont.)

Ref.	Step	Calculations	Output
	3.	*Lever arm depth* $z = d\left(1 - 0.42\frac{x}{d}\right) = 460\,(1 - 0.42 \times 0.40)$ $= 382$ mm	
	4.	*Area of steel required* $A_s = \frac{M}{0.87 f_y z} = \frac{165 \times 10^6}{0.87 \times 415 \times 382} = 1196\,\text{mm}^2$	$A_s = 1196\,\text{mm}^2$
Method II		*Solution by use of SP 16*	
	1.	*Factor* $\frac{M}{bd^2} = \frac{165 \times 10^6}{260 \times (460)^2} = 3.00$	
SP 16 Table 3	2.	*Read-off percentage of steel* $p_t = 0.997\%$ $A_s = \frac{0.997}{100} \times 260 \times 460 = 1192\,\text{mm}^2$	$A_s = 1192\,\text{mm}^2$

EXAMPLE 5.7 **(Design of beam for given M_u)**

Design a beam to carry a factored moment of 145 kNm using grade 25 concrete and Fe 415 steel.

Ref.	Step	Calculations	Output
Method I		*Solution by formulae*	
	1.	*Determination of depth required*	
SP 16 Table C		For Fe 415 steel, $M_u = 0.138 f_{ck} bd^2$ Let $b = 230$ mm $d = \left(\frac{M_u}{0.138 f_{ck} b}\right)^{1/2} = \left(\frac{145 \times 10^6}{0.138 \times 25 \times 230}\right)^{1/2} = 427$ One can use this depth and find the balanced percentage of steel or use larger depth as under-reinforced beam. Adopt $d = 490$ mm.	
		Solution by formulae for $d = 490$	
	2.	*Determination of lever arm and area of steel* $\frac{x}{d} = 1.20 - \left[(1.2)^2 - \frac{6.6M}{f_{ck}bd^2}\right]^{1/2}$ $= 1.20 - \left(1.44 - \frac{6.6 \times 145 \times 10^6}{25 \times 230 \times 490^2}\right)^{1/2}$	

EXAMPLE 5.7 (cont.)

Ref.	Step	Calculations	Output
IS 456 p. 109		$= 1.20 - 0.864 = 0.336 < \frac{x}{d}$ limit $= 0.48$	
		$z = d\left(1 - 0.42\frac{x}{d}\right) = 490(1 - 0.42 \times 0.336)$	$z = 421$ mm
		$= 421$ mm	
		$A_{st} = \frac{M}{0.87 f_y z} = \frac{145 \times 10^6}{0.87 \times 415 \times 421} = 954 \text{ mm}^2$	$A_s = 954$ mm^2
Method II		*Solution by use of SP 16*	
	3.	*Parameters for SP 16*	
		$\frac{M}{bd^2} = \frac{145 \times 10^6}{230 \times (490)^2} = 2.62$	
		Fe $= 415$ N/mm^2	
SP 16 Table 3		Read off $p_t = 0.845\%$, $A_s = \frac{0.845 \times 230 \times 490}{100}$	
		$= 952$ mm^2	$A_s = 952$ mm^2

REVIEW QUESTIONS

5.1 How is 'analysis' different from 'design' of a reinforced member?

5.2 Which theory would you use for analysis of stresses for the serviceability condition of an R.C.C. beam?

5.3 Enumerate the three methods generally used for analysis and design of a reinforced concrete beam.

5.4 What is meant by strain compatibility? Enumerate the fundamental assumptions made.

5.5 Does the assumption that plane section before bending remains plane even after bending remain true till failure of the beam? Explain how this assumption can be verified in the laboratory.

5.6 Explain the term M_u (lim) and give the expression for this value for Fe 250 and Fe 415 steels.

5.7 Name the publications which give the design charts based on IS 456 (1978) and explain their merit for use in the design office. Can these charts be used for non-rectangular sections also?

5.8 Explain the necessity for specifying maximum and minimum tension steel in reinforced beams. What are their values?

5.9 What is meant by curvature of a beam? Draw the relationship between curvature and percentage of steel and show how large percentage of steel reduces the ductility of reinforced concrete beams.

5.10 Is it true that in most of the R.C.C. sections in bending, the depth adopted is more than that required from strength considerations only? What other factors are to be considered when determining the depth of a slab? What are the formulae to be used in determining the area of steel required? Will this area of steel be more or less than the balanced percentage?

PROBLEMS

5.1 A singly reinforced rectangular section is of breadth 150 mm and depth 350 mm. The tension steel consists of 3 Nos. high yield bars of 16 mm dia. bars (3T 16) and the stirrups are of mild steel 8 mm dia. Cover to steel is 25 mm. Assuming f_{ck} = 20 N/mm^2, determine the ultimate moment of resistance of the section.

5.2 A rectangular beam 200 × 400 mm is to be constructed in a place where the conditions of exposure can be classified as moderate as given in IS 456, Table 19. Find the reinforcement steel required if it has to resist a bending moment of 25 kNm due to working loads. The diameter of shear steel is 10 mm and the cover should be as per IS code. Assume f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2.

5.3 A rectangular beam is to be simply supported on two walls of 125 mm width with a 'clear span' of 6 m. The characteristic live load is 12 kN/m, f_{ck} = 20 N/mm^2, and f_y = 415 N/mm^2. What is the effective span of the beam? Design a suitable section for the beam and determine the necessary tension steel.

5.4 Explain the term '*limiting depth of neutral axis*' in R.C.C. beam design. Derive its value for a rectangular section using Fe 415 grade steel and M20 concrete. Explain the role of the strength of concrete, if any, on the limiting depth of neutral axis.

5.5 A beam of breadth 350 mm and depth of 400 mm has 4 rods of 32 mm as tension steel, with 50 mm cover to centre of steel. Assuming f_{ck} = 30 N/mm^2, and f_y = 415 N/mm^2, check whether the steel will reach yield when the strain in concrete reaches 0.0035. Comment on how this beam conforms to IS 456 conditions for limit state design for strength.

5.6 A singly reinforced beam of effective span 7.5 m has to carry a characteristic live load of 15 kN/m and dead load of 20 kN/m. M20 grade concrete and Fe 415 steel are used in the construction.

1. Determine the maximum and minimum effective depth of the beams that can be permitted according to IS 456 for the above load, assuming that deflection requirements need not be fulfilled and the breadth of the beam is 300 mm.
2. Design a practical beam for the above span and loading condition according to the usual design office practice (a) from fundamentals, and (b) by use of SP 16.

5.7 A rectangular beam has to carry a live load of 22 kN/m and an equal dead load over an effective span of 10 m. Design a beam for the following conditions with f_{ck} = 20 N/mm^2, and f_y = 415 N/mm^2.

1. Minimum steel prescribed by IS code is to be used in the beam.
2. One-third the balanced steel for the section is to be used in the beam.

5.8 IS 456 places a limit on the x/d ratio to ensure ductility of a beam section. Assuming f_{ck} = 20 N/mm^2, and f_y = 415 N/mm^2, find the ratio of steel for the limiting condition of neutral axis specified in the Indian code. Design a practical beam of b = 300 mm to carry a factored moment of 150 kNm.

5.9 A beam ABC has AB = 6 m and BC = 2.5 m. It is simply supported on A, continuous over B, and cantilevers over the span BC. It is subjected to a characteristic dead load of 15 kN/m, and a moving load of 20 kN/m, which can occupy the span fully or partially. The width and depth of the beam are to be as small as possible, but conforming to stability and serviceability conditions. Selecting the sections for maximum positive and negative moments, design the beam without using compression steel. Assume f_{ck} = 25 N/mm^2, and f_y = 415 N/mm^2. Sketch the arrangement of steel.

5.10 Find the ultimate moment capacity of a trapezoidal beam of top width 500 mm, bottom width 300 mm, and a depth of 800 mm. The reinforcement consists of 4 Nos. of 25 mm bars, placed with a clear cover of 35 mm to steel reinforcement. (Rectangular stress block with maximum stress of $0.4f_{ck}$ may be assumed instead of the theoretical rectangular parabolic block for ease of computation). Assume grade 20 concrete and Fe 415 steel.

5.11 A beam of trapezoidal cross-section is of width 300 mm at the top and 150 mm at the bottom. Its total depth is 400 mm. Design the section to withstand a factored moment of 40 kNm. Assume f_{ck} = 15 N/mm^2, f_y = 415 N/mm^2. Use 14 mm rods as main steel and 8 mm rods as stirrups. Environmental conditions can be classified as "moderate".

6

Design of Doubly Reinforced Beams

6.1 INTRODUCTION

Reinforced concrete beams provided with steel reinforcements in both the tension and compression zones are called *doubly reinforced beams*. It is essential to put steel in the compression zone also when the area of the concrete in the compression zone is inadequate to develop the full compression needed to resist the induced moments. This becomes necessary under the following states:

1. Where the construction depth is restricted and the moment the beam has to carry is greater than the moment capacity of the beam in concrete failure expressed by Eq. (4.9).

$$M_u = Kf_{ck}bd^2$$

This usually occurs at supports of continuous beams (Fig. 6.1) and in beams where the depth is controlled by architectural considerations.

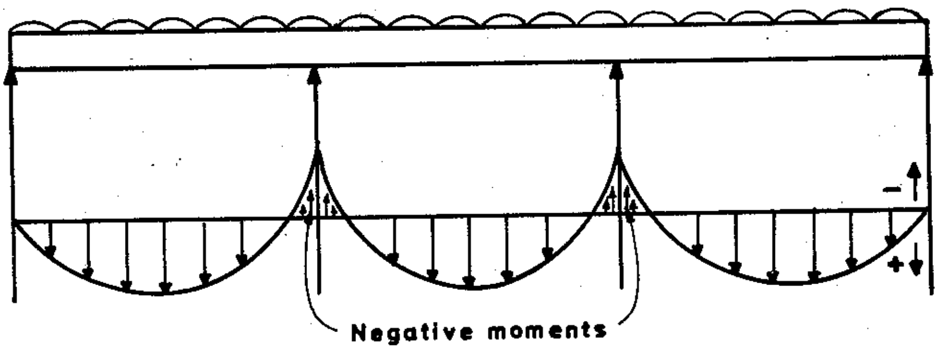


Fig. 6.1 Doubly reinforced sections at interior supports.

2. Where the bending moment at the section can change in sign (as may occur in a section in the span of a continuous beam with moving loads) so that the compression zone with one sign of the moment becomes the tension zone with the opposite sign of the moment, as in continuous bridge girders.

3. Where the compression steel can substantially improve the ductility of beams and its use is therefore advisable in members when large amount of tension steel becomes necessary for its strength. (In some codes like the Australian Code AS 1480, compression steel must be used even in normal beams if the percentage of tension steel exceeds three-fourths the balanced percentage.)

4. Compression steel is always used in structures in earthquake regions to increase their ductility.

5. Compression reinforcement will also aid significantly in reducing the long term deflections of beams.

6.2 BASIC CONSIDERATIONS

The failure theory as evolved for singly reinforced section holds good for doubly reinforced beams also. The main assumptions are the same as given in detail in Section 4.2.

1. R.C. sections in bending fail when the compressive strain in the concrete reaches the value of 0.0035.

2. Plane sections remain plane even after bending.

3. The stress at any point in steel and concrete can be taken as equal to the stress corresponding to the strain at that point of the stress-strain graph for the material (steel or concrete).

However, as the failure will not be brittle and sudden if there is steel in the compression zone also, the limitation specified for x/d ratios need not be strictly adhered to in doubly reinforced beams. It is also seen that even though shrinkage, creep and other properties of concrete will affect the actual state of stress of steel and concrete, these are not taken into consideration in estimating the collapse or ultimate strength. Laboratory tests have shown that the recommended method of design based on the above assumptions gives good results.

6.2.1 MINIMUM AND MAXIMUM STEEL AS COMPRESSION STEEL IN BEAMS

It should be noted that the small areas of reinforcement like the hanger bars provided in the compression zone should not be considered as compression steel. The minimum area of compression steel, which is required in a beam such that it can be considered as a doubly reinforced beam, is usually taken as 0.4 per cent of the area in compression (which may be taken as 0.2 per cent of the whole area including tension) in a beam. Steel below this area can reach yield stress in compression even without any loading just because of shrinkage and creep of concrete only.

The maximum amount of compression steel allowed in beams according to IS 456: clause 25.5.1.2 is 4 per cent of the total sectional area of the beam. The maximum tension and compression steel together should not be more than 8 per cent, which is also the allowed maximum percentage for a column section in many codes like the British Code. (However, IS 456: clause 25.5.3.1 limits the maximum steel in columns to 6 per cent only.)

6.2.2 YIELD STRESS IN COMPRESSION STEEL

It should also be clearly noted that IS 456 assumes that the stress-strain relationship for steel in tension and compression is the same. In both cases, design yield stress is $0.87f_y$. Codes like CP 110 used to assume that the yield stress in compression is less than the value in tension and equal to $0.72\,f_y$ only. However, the revised code BS 8110 (1985) assumes the same relationship in tension and compression for steel.

6.3 ACTION OF DOUBLY REINFORCED BEAMS

In doubly reinforced beams at ultimate moment, the resultant compressive force C (Fig. 6.2) consists

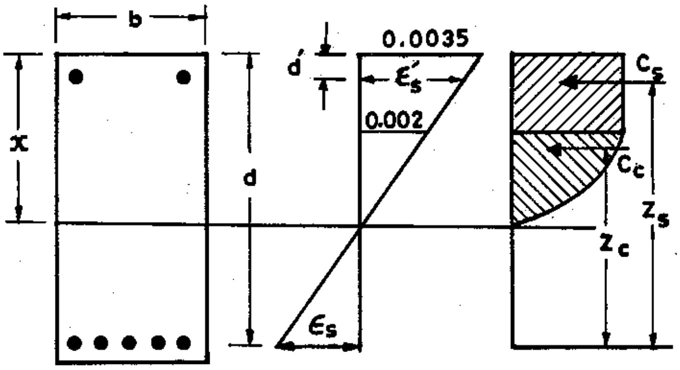


Fig. 6.2 Forces in doubly reinforced beam sections.

of two components, viz. C_c (due to concrete) and C_s (due to steel). In determining M_u, it is best to consider C_c and C_s separately with its separate lever arms z_c and z_s so that the equilibrium equations become

$$T = C_c + C_s$$

$$M_u = C_c z_c + C_s z_s$$

$$M_u = K f_{ck} b d^2 + C_s(d - d')$$

The value of C_s will depend on whether or not the compression steel will yield, and this in turn will depend on its position with respect to the outer concrete compression fibre which will reach a strain of 0.0035 at failure. The strain at the level of the compression steel is given in Fig. 6.2 as

$$\boxed{\varepsilon_s' = 0.0035(1 - d'/x)}$$

where d' is the distance of the centre of the compression steel from the extreme compression fibre.

The stress in the concrete will also depend on its position from the extreme fibre. If x_{max} is taken as $d/2$ approximately, and the strain at which concrete begins to yield ε_{co} in Fig. 2.3 is assumed as 0.002 (which is different from failure strain of 0.0035), then

$$0.0035\left(1 - \frac{d'}{0.5d}\right) \geq 0.002$$

which will be satisfied for d'/d up to 0.2. Hence the stress f_c in the concrete up to a depth $d' = 0.2d$ would reach the maximum value $0.45 f_{ck}$.

As regards state of stress in the compression steel, assuming depth of neutral axis to be at $0.5d$, the stress in steel for Fe 415 and Fe 500 for the various d'/d values, which can be computed from strain values, will be as given in Table 6.1. These can be calculated from the respective stress-strain curves of these steels as given in Table 6.2. Depending on the strain reached by the compression steel, the following two cases can arise regarding the stress in the reinforcement.

Case 1: Strain at level of compressive steel is at yield strain of steel. The stress in the compressive steel will be at yield stress equal to

$$\frac{f_y}{\gamma_m} = \frac{f_y}{1.15} = 0.87 f_y$$

TABLE 6.1 STRESS IN COMPRESSION REINFORCEMENT FOR d'/d RATIOS

Grade of Steel	d'/d 0.20	0.15	0.10	0.05	Strain at yield
Fe 250	218	218	218	218	0.00109
Fe 415	329	342	353	355	0.00380
Fe 500	370	395	412	424	0.00417

TABLE 6.2 SALIENT POINTS ON DESIGN STRESS-STRAIN CURVE
(Refer Table A of SP 16)

Stress level in terms of yield	Fe 415		Fe 500	
	Strain	Stress	Strain	Stress
$0.80 \times (0.87 f_y)$	0.00144	288	0.00174	347
$0.85 \times (0.87 f_y)$	0.00163	306	0.00195	369
$0.90 \times (0.87 f_y)$	0.00192	324	0.00226	391
$0.95 \times (0.87 f_y)$	0.00241	342	0.00277	413
$0.975 \times (0.87 f_y)$	0.00276	351	0.00312	423
$1.0 \times (0.87 f_y)$	0.00380	360	0.00417	434

Case 2: Strain at level of compressive steel is below yield strain. In this case the stress in the steel corresponds to that of the strain attained in the steel when the concrete reaches the ultimate strain of 0.0035. The stress will vary according to the d'/d value as indicated in Table 6.1, or it can be computed from fundamentals as given in Table 6.2.

6.4 STRESS-STRAIN RELATIONSHIP IN STEEL

The derivation of the design stress-strain curve from the laboratory curve for steel was already pointed out in Chapter 2. When designing doubly reinforced beams, it is necessary to know the values of the stress corresponding to any given strain. For this purpose the salient points on the design stress-strain curve can be taken from Table 6.2.

6.5 ANALYSIS AND DESIGN OF DOUBLY REINFORCED BEAMS

The following three methods, already indicated for singly reinforced beams, can be used for the analysis and design of doubly reinforced beams also:

1. Strain compatibility method using basic equations
2. Use of formulae
3. Use of design aids.

The procedures to be followed are now described.

6.6 STRAIN COMPATIBILITY METHOD (METHOD 1)

The procedure in this method is the same as in singly reinforced beams, explained in Chapters 4 and 5. Failure is assumed when the compression strain in concrete reaches 0.0035. The stresses in concrete and steel are computed from their resultant strains Rectangular stress blocks are easier for operation when dealing with irregular concrete sections. This method is generally used for analysis of beams; it involves the following procedure.

Step 1: Choose a value for x, the depth of the neutral axis. Assuming that the extreme compression fibre in concrete fails at a compressive strain of 0.0035, draw the strain distribution of the cross-section.

Step 2: Calculate the strain in the tension steel and the corresponding stress from Table 6.2. Determine the total tension in steel, i.e.

$$T = f_{st}A_{st} \tag{6.1}$$

Step 3: Calculate the strain in compression steel, the corresponding stress from the table, and the total compression in steel as

$$C_s = f_{sc}A_{sc} \tag{6.2}$$

Step 4: With the help of a suitable stress block, determine the compression in concrete. Using IS stress block, the compression in concrete is obtained as

$$C_c = 0.36f_{ck}bx \tag{6.3}$$

and its point of application is at $0.42x$ from the top.

Step 5: Determine the total compression

$$C = C_s + C_c \tag{6.4}$$

Step 6: Check whether total tension equals total compression. If $T = C$, the assumed neutral axis is satisfactory. Otherwise, choose another suitable depth of the neutral axis so that the tension becomes equal to compression.

Step 7: M_u, the moment of resistance of the section, is the moment of C_c and C_s about tension steel axis.

$$M_u = C_c(d - k_2x) + C_s(d - d') \tag{6.5}$$

6.7 USE OF FORMULAE STEEL BEAM THEORY—(METHOD 2)

The behaviour of R.C.C. beams with compression steel for ultimate load design is sometimes referred to as the *steel beam theory*. This is illustrated in Fig. 6.3.

The real beam is assumed to consist of two beams, namely, a singly reinforced beam which reaches its ultimate strength by failure of concrete in compression and a steel beam without any concrete but only compression and tension steel. The moment of resistance of the doubly reinforced beam will be the sum of the moment of resistance of the two beams. Let

M_{u1} = the maximum moment the concrete beam can carry = M_u (lim)

M_{u2} = the moment capacity the steel beam can carry

$M_u = M_{u1} + M_{u2}$

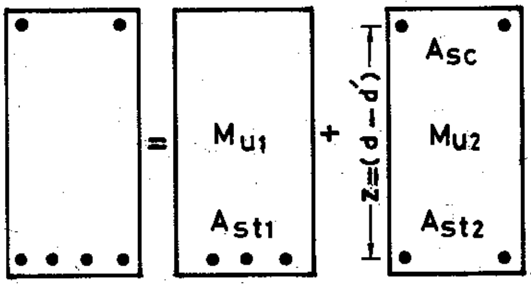


Fig. 6.3 Illustration of the steel beam theory.

Taking the effect of replacement of concrete in the compression zone by the compression steel also, we obtain

$$M_{u2} = (f_{sc} - f_{cc})\, A_{sc}(d - d')$$

where

f_{sc} = the stress in the compression steel corresponding to the strain reached by it when the extreme concrete fibre reaches a strain of 0.0035.

f_{cc} = stress in the concrete at the level of the compression steel

The value of f_{cc} is small and can be neglected for all practical purposes so that M_{u2} will be as given in IS 456: Appendix E.1-2.

$$M_{u2} = f_{sc}A_{sc}\,(d - d')$$

6.7.1 PROCEDURE FOR ANALYSIS USING FORMULAE

In analysis problems, data regarding A_{st}, A_{sc}, d, b and d' are given along with the grade of steel and concrete. Then it is necessary to determine the ultimate moment capacity of the beam. The procedure is to calculate the ultimate moment capacities separately: (a) in compression failure by failure of concrete, and (b) in tension failure by failure of tension steel and by taking lesser of the two capacities. The various steps are as follows:

Step 1: Calculate M_u for concrete failure as a singly reinforced beam

$$\begin{aligned} M_{u1} &= 0.149 f_{ck} bd^2 \quad \text{for Fe 250} \\ &= 0.138 f_{ck} bd^2 \quad \text{for Fe 415} \\ &= 0.133 f_{ck} bd^2 \quad \text{for Fe 500} \end{aligned} \tag{6.6}$$

Step 2: Determine the balanced steel required for the above moment. Per cent of A_{st1} = the percentage for balanced steel from Table 4.4. (see Table 4.4)

$$\begin{aligned} p_t &= \frac{21.97 f_{ck}}{f_y} \quad \text{for Fe 250} \\ &= \frac{19.82 f_{ck}}{f_y} \quad \text{for Fe 415} \\ &= \frac{18.87 f_{ck}}{f_y} \quad \text{for Fe 500} \end{aligned} \tag{6.7}$$

$$A_{st1} = \frac{p_t bd}{100}$$

Step 3: *Determine stress in compression steel.* Depending on d'/d ratio and using Table 6.1, determine stress in compression steel f_{sc}. This stress may be the yield value if the d'/d ratio is small; otherwise, it may be less than the yield value. If $d'/d < 0.2$, the stress (f_{cc}) of concrete at that level $= 0.45f_{ck}$. The total compression in steel is given by

$$C_s = (f_{sc} - f_{cc})A_{sc} \tag{6.8}$$

We may also neglect the effect of compression in the concrete (f_{cc}) as it is small compared to f_{sc}; taking moment about the tension steel, we obtain

$$C_s = f_{sc}A_{sc} \tag{6.8a}$$

Step 4: *Find M_{u2} additional moment in compression failure due to compression steel.*

$$M_{u2} = f_{sc}A_{sc}(d - d') \tag{6.9}$$

where

A_{sc} = area of compression steel

f_{sc} = stress in compression steel corresponding to its strain

Step 5: *Find total M_u from compression failure.* By considering the compressive stresses in concrete and steel, the limiting moment M_{uc} is given by

$$M_{uc} = M_{u1} + f_{sc}A_{sc}(d - d') \tag{6.10}$$

Step 6: *Find A_{st2} by tension failure.* To determine the capacity of the beam by failure in tension, let the additional steel available in tension in the 'steel beam' be A_{st2}. This is given by the equation

$$A_{st2} = (A_{st} - A_{st1}) \tag{6.11}$$

Step 7: *Find total M_u from tension failure.* Considering final failure to be due to tension steel reaching yield, the moment capacity M_{ut} is given by

$$M_{ut} = M_{u1} + A_{st2}(0.87f_y)(d - d') \tag{6.12}$$

Step 8: *Controlling value of M_u.* The lesser of the above two values M_{uc} (Eq. (6.10)) and M_{ut} (Eq. (6.12)) will be the actual moment of resistance of the beam.

6.7.2 PROCEDURE FOR DESIGN USING FORMULAE

In these problems, b and d and the grades of concrete and steel are given. We have to find A_{sc} and A_{st} for a given value M_u.

Step 1: As in analysis, find from Eq. (6.6),

$$M_{u1} = M_u \text{ (lim)}$$

$$M_{u1} = Kf_{ck}bd^2$$

Step 2: *Calculate A_{st1}*. Find A_{st1} from the expression for "balanced steel" assuming it reaches yield point. These are given by Eq. (6.7).

Step 3: Find M_{u2}:

$$M_{u2} = (M_u - M_{u1})$$

$$M_{u2} = 0.87 f_y A_{st2}(d - d') \tag{6.13}$$

Step 4: Calculate A_{st2}, the area of tension steel for the above M_{u2}:

$$A_{st2} = \frac{M_{u2}}{0.87 f_y (d - d')} \tag{6.14}$$

$$\text{Total steel required } A_s = A_{st1} + A_{st2} \tag{6.15}$$

Step 5: Find stress in compression steel, (f_{sc}) which will depend on d'/d. Find the stress from Table 6.1.

Step 6: Find the area of compression steel, A_{sc}, as

$$A_{sc} = \frac{A_{st2}(0.87 f_y)}{f_{sc} - f_{cc}} \tag{6.16}$$

$$A_{sc} = \frac{A_{st2}(0.87 f_y)}{f_{sc}} \text{ (approx.)} \tag{6.16a}$$

where f_{sc} = stress in compression steel corresponding to d'/d ratio at failure.

A_{st} and A_{sc} from Eqs. (6.15) and (6.16) respectively are the required tension and compression steel areas.

6.8 USE OF DESIGN AIDS SP 16 (METHOD 3)

Design Aid SP 16 contains a number of tables and charts for analysis and design of doubly reinforced beams.

The tables are easier to use than charts. Charts 19 and 20 give A_{st2}, the additional steel, for the steel beam for $(d - d')$ for Fe 250 only. If one has to use these charts for other grades of steels, it is to be modified by Table G on page 13 of SP 16.

The following tables of SP 16 give values for direct design of doubly reinforced beams. Tables 45–48 give data for Fe 250 steel and grades of steel with f_{ck} = 15, 20, 25, 30. Tables 49–52 give the data for Fe 415 steel and the above four grades of concrete. (Tables 49 and 50 for f_{ck} 15 and 20 are given as Tables 6.3 and 6.4 in this text.) Tables 53–56 are for Fe 500 and the above four grades of concrete.

The symbols used are A_{sc} = Area of compression steel and

$$p_c = \frac{A_{sc} 100}{bd}$$

A_{st} = area of tension steel

TABLE 6.3 FLEXURE-REINFORCEMENT PERCENTAGES FOR DOUBLY REINFORCED SECTIONS
(Table 49 of SP 16)

f_{ck} = 15 N/mm^2
f_y = 415 N/mm^2

M_u/bd^2, N/mm^2	d'/d = 0.05		d'/d = 0.10		d'/d = 0.15		d'/d = 0.20	
	p_t	p_c	p_t	p_c	p_t	p_c	p_t	p_c
2.08	0.719	0.003	0.720	0.003	0.720	0.003	0.720	0.003
2.10	0.725	0.009	0.726	0.009	0.726	0.010	0.727	0.011
2.20	0.754	0.039	0.757	0.041	0.759	0.045	0.761	0.050
2.30	0.784	0.069	0.787	0.073	0.791	0.080	0.796	0.089
2.40	0.813	0.099	0.818	0.106	0.824	0.115	0.831	0.127
2.50	0.842	0.129	0.849	0.138	0.857	0.150	0.865	0.166
2.60	0.871	0.160	0.880	0.170	0.889	0.185	0.900	0.205
2.70	0.900	0.190	0.910	0.202	0.922	0.220	0.935	0.244
2.80	0.929	0.220	0.941	0.234	0.954	0.255	0.969	0.282
2.90	0.959	0.250	0.972	0.267	0.987	0.290	1.004	0.321
3.00	0.988	0.280	1.003	0.299	1.020	0.325	1.039	0.360
3.10	1.017	0.311	1.034	0.331	1.052	0.360	1.073	0.399
3.20	1.046	0.341	1.064	0.363	1.085	0.395	1.108	0.438
3.30	1.075	0.371	1.095	0.395	1.117	0.430	1.142	0.476
3.40	1.104	0.401	1.126	0.427	1.150	0.465	1.177	0.515
3.50	1.134	0.432	1.157	0.460	1.183	0.500	1.212	0.554
3.60	1.163	0.462	1.188	0.492	1.215	0.535	1.246	0.593
3.70	1.192	0.492	1.218	0.524	1.248	0.571	1.281	0.631
3.80	1.221	0.522	1.249	0.556	1.280	0.606	1.316	0.670
3.90	1.250	0.552	1.280	0.588	1.313	0.641	1.350	0.709
4.00	1.279	0.583	1.311	0.621	1.346	0.676	1.385	0.748
4.10	1.309	0.613	1.342	0.653	1.378	0.711	1.420	0.787
4.20	1.338	0.643	1.372	0.685	1.411	0.746	1.454	0.825
4.30	1.367	0.673	1.403	0.717	1.443	0.781	1.489	0.864
4.40	1.396	0.703	1.434	0.749	1.476	0.816	1.524	0.903
4.50	1.425	0.734	1.465	0.781	1.509	0.851	1.558	0.942
4.60	1.455	0.764	1.495	0.814	1.541	0.886	1.593	0.980
4.70	1.484	0.794	1.526	0.846	1.574	0.921	1.627	1.019
4.80	1.513	0.824	1.557	0.878	1.606	0.956	1.662	1.058
4.90	1.542	0.855	1.588	0.910	1.639	0.991	1.697	1.097
5.00	1.571	0.885	1.619	0.942	1.672	1.026	1.731	1.136
5.10	1.600	0.915	1.649	0.975	1.704	1.061	1.766	1.174
5.20	1.630	0.945	1.680	1.007	1.737	1.096	1.801	1.213
5.30	1.659	0.975	1.711	1.039	1.769	1.131	1.835	1.252
5.40	1.688	1.006	1.742	1.071	1.802	1.166	1.870	1.291
5.50	1.717	1.036	1.773	1.103	1.835	1.201	1.905	1.329
5.60	1.746	1.066	1.803	1.136	1.867	1.236	1.939	1.368
5.70	1.775	1.096	1.834	1.168	1.900	1.271	1.974	1.407
5.80	1.805	1.126	1.865	1.200	1.932	1.306	2.008	1.446
5.90	1.834	1.157	1.896	1.232	1.965	1.341	2.043	1.485
6.00	1.863	1.187	1.927	1.264	1.998	1.376	2.078	1.523
6.10	1.892	1.217	1.957	1.296	2.030	1.411	2.112	1.562
6.20	1.921	1.247	1.988	1.329	2.063	1.446	2.147	1.601
6.30	1.950	1.278	2.019	1.361	2.095	1.481	2.182	1.640
6.40	1.980	1.308	2.050	1.393	2.128	1.517	2.216	1.678

SECTIONS

= 15 N/mm^2
= 415 N/mm^2

d = 0.20
p_c
0.003
0.011
0.050
0.089
0.127
0.166
0.205
0.244
0.282
0.321
0.360
0.399
0.438
0.476
0.515
0.554
0.593
0.631
0.670
0.709
0.748
0.787
0.825
0.864
0.903
0.942
0.980
1.019
1.058
1.097
1.136
1.174
1.213
1.252
1.291
1.329
1.368
1.407
1.446
1.485
1.523
1.562
1.601
1.640
1.678

TABLE 6.4 FLEXURE-REINFORCEMENT PERCENTAGES FOR DOUBLY REINFORCED SECTIONS

(Table 50 of SP 16)

f_{ck} = 20 N/mm^2
f_y = 415 N/mm^2

M_u/bd^2, N/mm^2	d'/d = 0.05		d'/d = 0.10		d'/d = 0.15		d'/d = 0.20	
	p_t	p_c	p_t	p_c	p_t	p_c	p_t	p_c
2.77	0.958	0.002	0.958	0.002	0.959	0.003	0.959	0.003
2.80	0.967	0.011	0.968	0.012	0.968	0.013	0.969	0.015
2.90	0.996	0.042	0.998	0.045	1.001	0.049	1.004	0.054
3.00	1.025	0.072	1.029	0.077	1.034	0.084	1.038	0.093
3.10	1.055	0.103	1.060	0.109	1.066	0.119	1.073	0.132
3.20	1.084	0.133	1.091	0.142	1.099	0.154	1.108	0.171
3.30	1.113	0.164	1.122	0.174	1.131	0.190	1.142	0.210
3.40	1.142	0.194	1.152	0.207	1.164	0.225	1.177	0.249
3.50	1.171	0.224	1.183	0.239	1.197	0.260	1.212	0.288
3.60	1.200	0.255	1.214	0.271	1.229	0.295	1.246	0.327
3.70	1.230	0.285	1.245	0.304	1.262	0.331	1.281	0.366
3.80	1.259	0.316	1.276	0.336	1.294	0.366	1.315	0.405
3.90	1.288	0.346	1.306	0.369	1.327	0.401	1.350	0.444
4.00	1.317	0.376	1.337	0.401	1.360	0.437	1.385	0.483
4.10	1.346	0.407	1.368	0.433	1.392	0.472	1.419	0.522
4.20	1.375	0.437	1.399	0.466	1.425	0.507	1.454	0.561
4.30	1.405	0.468	1.429	0.498	1.457	0.542	1.489	0.600
4.40	1.434	0.498	1.460	0.530	1.490	0.578	1.523	0.640
4.50	1.463	0.528	1.491	0.563	1.523	0.613	1.558	0.679
4.60	1.492	0.559	1.522	0.595	1.555	0.648	1.593	0.718
4.70	1.521	0.589	1.553	0.628	1.588	0.683	1.627	0.757
4.80	1.550	0.620	1.583	0.660	1.620	0.719	1.662	0.796
4.90	1.580	0.650	1.614	0.692	1.653	0.754	1.696	0.835
5.00	1.609	0.680	1.645	0.725	1.686	0.789	1.731	0.874
5.10	1.638	0.711	1.676	0.757	1.718	0.825	1.766	0.913
5.20	1.667	0.741	1.707	0.790	1.751	0.860	1.800	0.952
5.30	1.696	0.772	1.737	0.822	1.783	0.895	1.835	0.991
5.40	1.725	0.802	1.768	0.854	1.816	0.930	1.870	1.030
5.50	1.755	0.832	1.799	0.887	1.849	0.966	1.904	1.069
5.60	1.784	0.863	1.830	0.919	1.881	1.001	1.939	1.108
5.70	1.813	0.893	1.861	0.952	1.914	1.036	1.974	1.147
5.80	1.842	0.924	1.891	0.984	1.946	1.071	2.008	1.186
5.90	1.871	0.954	1.922	1.016	1.979	1.107	2.043	1.225
6.00	1.900	0.985	1.953	1.049	2.012	1.142	2.078	1.264
6.10	1.930	1.015	1.984	1.081	2.044	1.177	2.112	1.303
6.20	1.959	1.045	2.014	1.114	2.077	1.213	2.147	1.342
6.30	1.988	1.076	2.045	1.146	2.109	1.248	2.181	1.381
6.40	2.017	1.106	2.076	1.178	2.142	1.283	2.216	1.421
6.50	2.046	1.137	2.107	1.211	2.175	1.318	2.251	1.460
6.60	2.075	1.167	2.138	1.243	2.207	1.354	2.285	1.499
6.70	2.105	1.197	2.168	1.276	2.240	1.389	2.320	1.538
6.80	2.134	1.228	2.199	1.308	2.272	1.424	2.355	1.577
6.90	2.163	1.258	2.230	1.340	2.305	1.459	2.389	1.616
7.00	2.192	1.289	2.261	1.373	2.338	1.495	2.424	1.655
7.10	2.221	1.319	2.292	1.405	2.370	1.530	2.459	1.694

$$p_t = \frac{A_{st}\,100}{bd}$$

$$A_{st} = A_{st1} + A_{st2}$$

$$p_t = p_{t1} + p_{t2}$$

The tables give the relation between M_u/bd^2; the values of p_c and p_t are based on the following equations:

$$M_u = M_u\,(\text{lim}) + \frac{p_{t2}}{100}\,bd\,(0.87f_y)(d - d')$$

$$\frac{M_u}{bd^2} = \frac{M_u\,(\text{lim})}{bd^2} + \frac{p_{t2}}{100}\,0.87f_y\left(1 - \frac{d'}{d}\right) \qquad (6.17)$$

$$p_t = p_{t1}\,(\text{lim}) + p_{t2}$$

$$p_c = p_{t2}\left(\frac{0.87f_y}{f_{sc} - f_{cc}}\right)$$

so that p_t and p_c as also A_c and A_t can be determined.

6.9 SPECIFICATIONS REGARDING SPACING OF STIRRUPS IN DOUBLY REINFORCED BEAMS

Compression steel placed in doubly reinforced beams also has to be restrained against local buckling during its action like the compression steel in columns. The same rules regarding restraining of column reinforcements by lateral ties (IS 456: clause 25.5.3.2) apply to compression reinforcements in beams also. Accordingly, the minimum diameter of the stirrups (ties) should be 5 mm (usually taken as 6 mm) and the pitch should not be more than the least of the following:

1. The least lateral dimension
2. Sixteen times the diameter of the longitudinal steel
3. Forty-eight times the diameter of transverse reinforcement.

6.10 SUMMARY OF PROCEDURE FOR ANALYSIS AND DESIGN

The same methods used for design of singly reinforced beams can be used for doubly reinforced beams also: The steel beam theory is a convenient method for visualising the action of these beams and gives reliable results. The methods of analysis and design are illustrated with the help of examples given at the end of this chapter.

EXAMPLE 6.1 **(Analysis from fundamentals and formulae)**

Determine the ultimate moment capacity of a doubly reinforced beam with $b = 350$ mm, $d' = 60$ mm, $d = 550$ mm, $A_{sc} = 1690$ mm^2, $A_{st} = 4310$ mm^2, $f_{ck} = 30$ N/mm^2, and $f_y = 415$ N/mm^2.

Ref.
Method

SP 16
Table B

SP 16
Table A

SP 16
Fig. 3

following

(6.17)

l buckling
raining of
forcements
m (usually

reinforced
ese beams
ne help of

= 60 mm,

Ref.	Step	Calculations	Output
Method I		*Solution by strain compatibility*	
	1.	*Choose a suitable NA depth*	
SP 16 Table B		Limiting x/d for Fe 415 = 0.48, i.e. x = 264 mm	
		Adopt x = 260 mm	
	2.	*Check strain and stress in steel*	
SP 16 Table A		$\varepsilon_{st} = \dfrac{\varepsilon_c(d-x)}{x}$	
		$\varepsilon_{st} = \dfrac{0.0035 \times 290}{260} = 0.0039$	
		Yield strain of Fe 415 = 0.0038 – steel yields	
	3.	*Total tension in steel*	
		$T = 0.87 \times 415 \times 4310 \times 10^{-3} = 1556$ kN	
	4.	*Check strain in compression steel*	
		$\varepsilon_{sc} = \dfrac{0.0035 \times (260 - 60)}{260} = 0.0027$	
		Compression steel does not reach yield point.	
SP 16 Fig. 3		Corresponding stress in steel = 350 N/mm^2	
		$C_s = f_s A_s$ $\quad C_S = (\sigma_{sc} - \sigma_{cc}) A_{sc}$	
		$C_s = 350 \times 1690 \times 10^{-3} = 591.5$ kN	
		[*Note*: See also step 3.] $\quad \sigma_{sc} = .87\sigma_y \mid \sigma_{cc} = .446 \times f_{ck}$	
	5.	*Compression in concrete and its point of action*	
		$C_c = 0.36 f_{ck} bx = 0.36 \times 30 \times 350 \times 260 \times 10^{-3}$ $= 982.8$ kN	
		Centre of $C_c = 0.42x = 0.42 \times 260 = 109$ mm	
	6.	*Total compression*	
		$C = C_s + C_c = 591.5 + 982.8 = 1574$ kN	
	7.	*Check* $T = C$	
		T = 1556 kN, which is very near C = 1574 kN	
	8.	*Ultimate moment* (Moment of C_s and C_c about the tension steel (line of action of T))	
		$M_u = [591.5\,(550 - 60) + 982.8\,(550 - 109)] \times 10^{-3}$ $= 723.25$ kNm	M_u = 723 kNm

EXAMPLE 6.1 (cont.)

Ref.	Step	Calculations	Output
Method II		*Solution by formulae*	
	1.	*M_{u1} for concrete failure as singly reinforced* (Fe 415)	
		$M_{u1} = 0.138 f_{ck} bd^2 = 0.138 \times 30 \times 350 \times (550)^2$	
		$= 438$ kNm	
SP 16 Table E	2.	*Balanced Steel* (for Fe 415, $f_{ck} = 30$, $p = 1.43\%$)	
		$A_{st} = \frac{1.43}{100} \times 350 \times 550 = 2753\ \text{mm}^2$	
SP 16 Table F	3.	*Compression in steel*	
		$\frac{d'}{d} = \frac{60}{550} = 0.11, \quad f_{sc} = 351\ \text{N/mm}^2$	
		$C_s = 351 \times 1690 \times 10^{-3} = 593.2$ kN	
	4.	*Value of M_{u2} due to compression steel*	
		$M_{u2} = 593.2(550 - 60) \times 10^{-3} = 291$ kNm	
	5.	*Total M_u considering compression failure*	
		$M_u = M_{u1} + M_{u2} = 438 + 291 = 729$ kNm	M_u(comp) $= 729$ kNm
	6.	*Additional tension steel available*	
		$A_{st_2} = A_{st} - A_{st1} = 4310 - 2753 = 1557\ \text{mm}^2$	
	7.	*Moment capacity for steel failure (on steel beam theory)*	
		$M_{ut} = M_{u1} + A_{st_2}(0.87 f_y)(d - d')$	M_u (tension) $= 714$ kNm
		$= 438 + 1557 \times 0.87 \times 415(550 - 60) \times 10^{-6}$	
		$= 714$ kNm	
		The two methods give the same order of values.	

EXAMPLE 6.2 **(Analysis by formulae and SP 16)**

Determine the ultimate moment capacity of a beam $b = 280$ mm, $d = 510$ mm, $d' = 50$ mm, $A_{st} = 2455\ \text{mm}^2$, $A_{sc} = 402\ \text{mm}^2$, $f_{ck} = 30\ \text{N/mm}^2$, and $f_y = 415\ \text{N/mm}^2$.

Ref.	Step	Calculations	Output
Method I		*Solution by formulae*	
	1.	*M_{u1} for concrete failure as singly reinforced* (Fe 415)	
		$M_{u1} = 0.138 f_{ck} bd^2 = 0.138 \times 30 \times 280 \times (510)^2$	
		$= 301$ kNm	
SP 16 Table E	2.	*Balanced steel $p = 1.43\%$*	
		$A_{st} = \dfrac{1.43 \times 280 \times 510}{100} = 2042 \text{ mm}^2$	A_{st} > Balanced
	3.	*Stress in compression steel and compression in steel*	
SP 16 Table F		$\dfrac{d'}{d} = \dfrac{50}{510} = 0.10, \quad f_{sc} = 353 \text{ N/mm}^2$	
		$C_s = 353 \times 402 \times 10^{-3} = 141.9$ kN	
	4.	*Value of M_{u2} due to compression steel*	
		$M_{u2} = 141.9(510 - 50) \times 10^{-3} = 65.3$ kNm	
	5.	*Total M_{uc} for compression failure*	
		$M_{uc} = M_{u1} + M_{u2} = 301 + 65.3 = 366$ kNm	$M_{uc} = 366$ kNm
	6.	*Additional tension steel present as excess of balanced steel*	
		$A_{st2} = (2455 - 2042) = 413 \text{ mm}^2$	
		Percentage $= \dfrac{413 \times 100}{280 \times 510} = 0.3\% > 0.2\%$	More than nominal
	7.	*Ultimate moment capacity from steel beam theory* (tension steel failure)	
		$M_{ut} = M_{u1} + A_{st2}(0.87 f_y)(d - d')$	
		$= 301 + 413(0.87 \times 415)(510 - 50) \times 10^{-6}$	
		$= 370$ kNm	$M_{ut} = 370$ kNm
	8.	*Lesser of M_{uc} and M_{ut} controls*	
		Ultimate moments are more or less of equal capacity = 366 kNm	$M_u = 366$ kNm
Method II		*Solution by use of SP 16*	
	1.	*Parameters for use of tables*	
		$\dfrac{d'}{d} = \dfrac{50}{510} = 0.1, \quad p_c = \dfrac{402 \times 100}{280 \times 510} = 0.28\%$	
		$p_t = \dfrac{2455 \times 100}{280 \times 510} = 1.72\%$	

EXAMPLE 6.2 (cont.)

Ref.	Step	Calculations	Output
SP 16 Table 52	2.	*Use of Table in SP 16* Enter table with compression steel percentage (tension steel to be checked later) For $\frac{d'}{d} = 0.1, \quad p_c = 0.28\%, \quad \frac{M_u}{bd^2} = 5.00$ p_t required = 1.698 provided 1.72% Therefore, $M_u = 5.00 \times 280(510)^2 = 364$ kNm *Note*: A more exact method will be to determine the neutral axis and moments from strain compatibility by equations of equilibrium. However, the above simple methods give values safe for all practical purposes.	M_u = 364 kNm

EXAMPLE 6.3 **(Design of a doubly reinforced beam)**

Determine the reinforcements required for a beam b = 300 mm, D = 600 mm. Factored moment = 320 kNm. Assume f_{ck} = 15 N/mm^2, f_y = 415 N/mm^2.

Ref.	Step	Calculations	Output
Method I		*By formulae*	
	1.	*Calculate M_u for concrete failure* (Fe 415) $d = \left(D - \text{cover} - \frac{\phi}{2}\right) = \left(600 - 25 - \frac{25}{2}\right)$ $= 562.5$ mm $M_{u1} = 0.138 f_{ck} b d^2 = 0.138 \times 15 \times 300 \times (562.5)^2$ $= 196$ kNm $<$ required 320 kNm	Section doubly reinforced
SP 16 Table E	2.	*Balanced steel required p = 0.72%* $A_{st1} = \frac{0.72}{100}(300 \times 562.5) = 1215 \text{ mm}^2$	
	3.	*Moment to be taken by steel beam* $M_{u2} = (M_u - M_{u1}) = 320 - 196 = 124$ kNm	
	4.	*Calculate A_{st2} assuming tension steel yields* $A_{st2} = \frac{M_{u2}}{0.87 f_y (d - d')}, \quad d' = 25 + 12.5 = 37.5$ mm $A_{st2} = \frac{124 \times 10^6}{0.87 \times 415 \times (562.5 - 37.5)} = 654 \text{ mm}^2$	

EXAMPLE 6.3 (cont.)

Ref.	Step	Calculations	Output
	5.	*Total tension steel*	
		$A_{st} = A_{st1} + A_{st2} = 1215 + 654 = 1869$ mm^2	$A_{st} = 1869$ mm^2
	6.	*Stress in compression steel*	
		$\dfrac{d'}{d} = \dfrac{37.5}{562.5} = 0.067, \quad f_s = 353$ N/mm^2 (say)	
SP 16 Table F	7.	*Area of compression steel*	
		$A_{sc} = \dfrac{A_{st2} \times 0.87 f_y}{f_s} = \dfrac{654 \times 0.87 \times 415}{353}$	
		$= 669$ mm^2	$A_{sc} = 669$ mm^2
Method II		*Solution by SP 16*	
	1.	*Parameters for use of tables*	
		$\dfrac{d'}{d} = 0.07, \quad \dfrac{M_u}{bd^2} = \dfrac{320 \times 10^6}{300 \times (562.5)^2} = 3.37$	
SP 16 Table 49	2.	*Use of Tables*	
		Use $d'/d = 0.1$	
		$p_t = 1.117\%, \quad A_{st} = \dfrac{1.117}{100}(300 \times 562.5)$	
		$= 1885$ mm^2	
		$p_c = 0.418\%, \quad A_{sc} = \dfrac{0.418}{100}(300 \times 562.5)$	
		$= 705$ mm^2	

EXAMPLE 6.4 **(Design of a doubly reinforced beam)**

A *T* beam continuous over several supports has to carry a factored negative support moment of 1000 kNm. Determine the area of steel at supports if $b_w = 400$ mm, $b_{f_2} = 1600$ mm, $D_f = 100$ mm, $D = 610$ mm, $d' = 60$ mm, $f_{ck} = 30$ N/mm^2, $f = 415$ N/mm^2.

Ref.	Step	Calculations	Output
Method I		*Solution by formulae*	
		Note: As the *T* beam acts as a rectangular beam at support $b = 400$ mm, $d = (610 - 60) = 550$	
	1.	*Limiting moment for concrete failure* (Fe 415)	
		$M_{u1} = 0.138 f_{ck} bd^2 = 0.138 \times 30 \times 400 \times 550^2$	
		$= 501$ kNm	

EXAMPLE 6.4 (cont.)

Ref.	Step	Calculations	Output
SP 16 Table E	2.	*Balanced steel for above* (p_t = 1.43)	
		$A_{st1} = \dfrac{1.43 \times 400 \times 550}{100} = 3146 \text{ mm}^2$	
	3.	*Moment to be carried by steel beam*	
		$M_{u2} = 1000 - 501 = 499$ kNm	
	4.	*Additional tension steel*	
		$A_{st2} = \dfrac{499 \times 10^6}{0.87 \times 415(550 - 60)} = 2820 \text{ mm}^2$	
	5.	*Total tension steel*	
		$A_{st} = A_{st1} + A_{st2} = 3146 + 2820 = 5966 \text{ mm}^2$	$A_{st} = 5966 \text{ mm}^2$
	6.	*Stress in compression steel*	
SP 16 Table F		$\dfrac{d'}{d} = \dfrac{60}{550} = 0.1$, Stress in compression steel = 353 N/mm^2	
	7.	*Area of compression steel*	
		$A_{sc} = \dfrac{M_{u2}}{f_{sc}\,(d - d')} = \dfrac{499 \times 10^6}{353(550 - 60)} = 2884 \text{ mm}^2$	$A_{sc} = 2884 \text{ mm}^2$
		or	
		$A_{sc} = \dfrac{A_{st2} \times 0.87 \times 415}{f_{sc}} = \dfrac{2820 \times 0.87 \times 415}{353}$	
		$= 2884 \text{ mm}^2$	
Method II		*Solution by use of SP 16*	
	1.	*Parameters to be used for SP 16*	
		$\dfrac{M}{bd^2} = \dfrac{1000 \times 10^6}{400 \times (550)^2} = 8.26, \quad \dfrac{d'}{d} = 0.1$	
SP 16 Table 52	2.	*Results from Table 52*	
		$p_t = 2.70\%$, i.e. $A_s = \dfrac{2.7}{100}(400 \times 550) = 5940 \text{ mm}^2$	$A_{st} = 5940 \text{ mm}^2$
		$p_c = 1.35\%$, i.e. $A_s = \dfrac{1.35}{100}(400 \times 550) = 2970 \text{ mm}^2$	$A_{sc} = 2970 \text{ mm}^2$

REVIEW QUESTIONS

6.1 Enumerate at least three situations in which a doubly reinforced beam becomes necessary.

6.2 What are the assumptions in the theory of design of doubly reinforced beam regarding the role of compression steel?

6.3 What is the effect of creep and shrinkage on the state of compression steel stress in a doubly reinforced R.C.C. beam? Explain why a minimum steel area of 0.2 per cent of the cross-section should be present as compression steel if this is to be taken into account in the calculation of the strength of doubly reinforced beams.

6.4 What is the maximum and minimum percentage of tension reinforcement allowed in doubly reinforced beams?

6.5 What is the effect of the depth of cover to compression steel on its stress at ultimate load?

6.6 Give the formulae for analysis of a given doubly reinforced beam.

6.7 Derive the formulae for determination of the steel areas of a doubly reinforced beam of given dimensions to carry a given moment.

6.8 How will you check whether a beam of given dimension has to be designed as a doubly reinforced beam?

6.9 What are the rules regarding restraining of the compression steel from buckling in a doubly reinforced beam?

6.10 What are the specifications for spacing of transverse reinforcements in doubly reinforced beams?

PROBLEMS

6.1 Design a rectangular reinforced concrete beam of section 300 × 800 mm to carry a factored moment of 600 kNm. Assume f_{ck} = 20 N/mm^2, f_y = 415 N/mm^2, cover 25 mm, and size of stirrups 10 mm.

6.2 Design a rectangular reinforced concrete beam to resist service moments of 120 kNm from dead loads and 110 kNm from live loads. The beam dimensions should be 250 × 625 mm and cover 25 mm with 10 mm stirrups. f_y = 250 N/mm^2, and f_{ck} = 15 N/mm^2.

6.3 A reinforced concrete beam is 250 × 500 mm. The tension steel consists of 5 Nos. of 25 mm rods and the compression steel 5 Nos. of 12 mm rods. 10 mm stirrups are used for shear reinforcements and the clear cover is to be 25 mm. Determine the ultimate moment capacity of the section, assuming f_y = 415 N/mm^2 and f_{ck} = 20 N/mm^2.

6.4 A rectangular R.C. beam 300 × 600 mm is provided with 4 Nos. of 25 mm bars as compression steel. Determine the area of tension steel needed for the beam to attain its full moment capacity. Calculate also the corresponding ultimate moment of resistance. Assume f_{ck} = 20 N/mm^2, f_y = 415 N/mm^2. Cover to centre of main steel is 50 mm.

6.5 Determine the limiting moment of resistance of the R.C. beam sections shown in Fig. P. 6.5. It has been decided to increase the moment capacity of these beams by incorporating compression steel and adding tension steel if necessary. What will be the absolute maximum moments these beams can carry? Determine the areas of steel needed. Assume that f_{ck} = 25 N/mm^2, f_y = 415 N/mm^2, and cover to centre of steel is 50 mm.

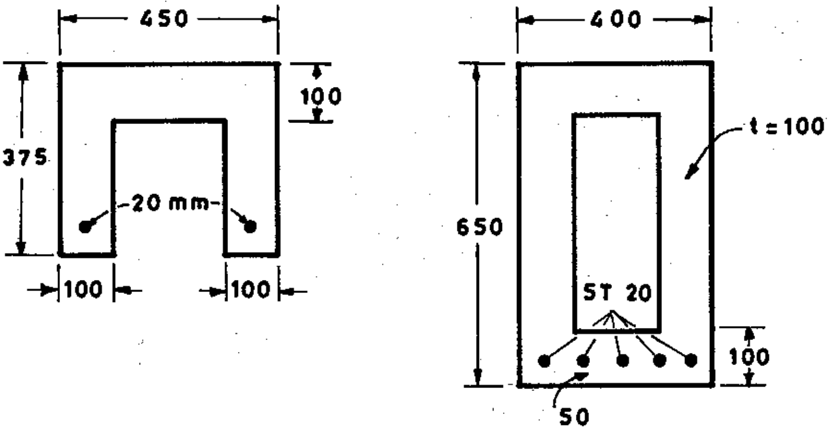


Fig. P.6.5 Reinforced concrete sections of beams.

6.6 A doubly reinforced beam 250 × 450 mm is reinforced with 4 bars of 25 mm diameter on the tension side and 4 bars of 18 mm on the compression side. Assuming an effective cover of 50 mm, M20 grade concrete, and Fe 415 steel, calculate the ultimate moment capacity of the section.

7

Limit State of Collapse in Shear (Design for Shear)

7.1 INTRODUCTION

Bending is usually accompanied by bending shear. Shear stress thus produced is accompanied by diagonal tension and compression as shown in Fig. 7.1.

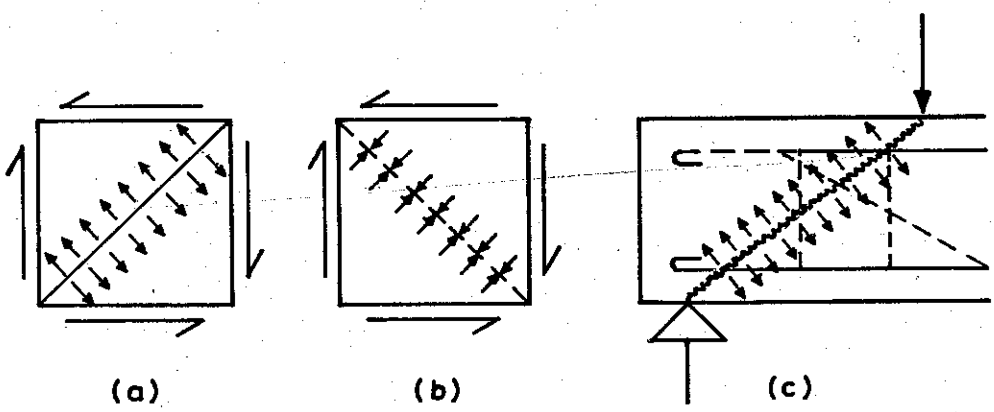


Fig. 7.1 Shear and diagonal stresses: (a) Diagonal tension; (b) Diagonal compression; (c) Tension in beams.

As concrete is weak in tension, large diagonal tension stresses can produce cracking and even failure of the concrete members. Hence beams should always be checked for safety against 'shear failure' in physical terms against diagonal tension and compression failures. If the shear stress is large, steel in the form of vertical stirrups or bent-up bars should be provided to take up the tensile stresses. Bending shear is sometimes referred to as 'one-way shear' and it should be distinguished from punching shear that occurs in slabs under concentrated load and sometimes referred to as 'two-way shear' (see also Chapter 22). This chapter gives the method of design of beams and slabs for bending shear.

7.2 TYPES OF SHEAR FAILURES

The various types of shear failures in beams are shown in Fig. 7.2. An indepth study of each type of failure will greatly help in understanding the interaction of shear with other forces. However, as it is not necessary for routine calculations and design, such a detailed treatment is not given in this text. Only the salient points necessary for routine design are presented.

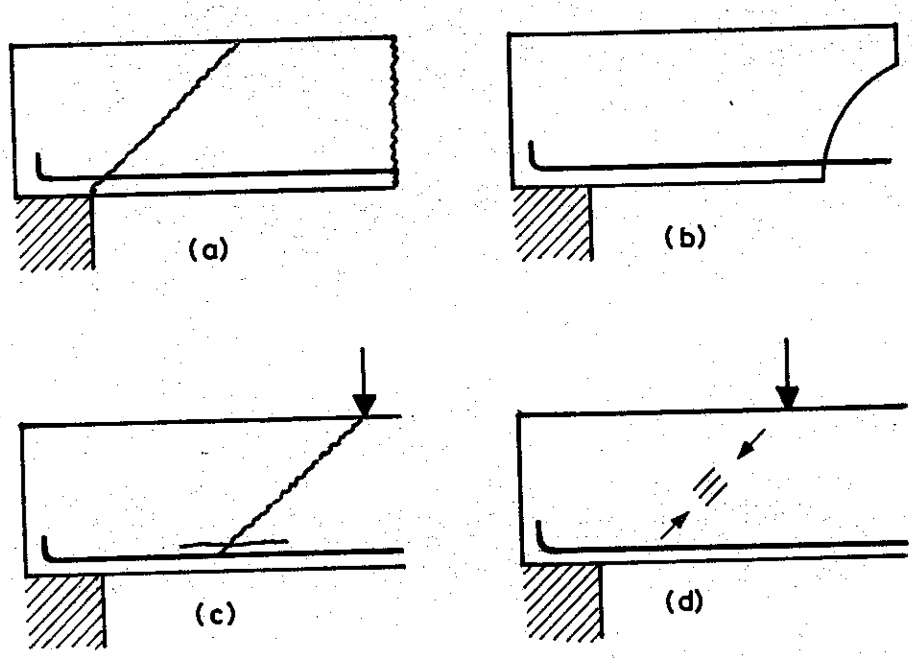


Fig. 7.2 Types of shear failures: (a) Shear-tension; (b) Shear-bending; (c) Shear-bond; (d) Shear-compression.

7.3 CALCULATION OF SHEAR STRESS

Theoretically the shear stress distribution in a rectangular reinforced section depends on the distribution of the normal stress. If we assume that concrete below the neutral axis gets cracked and does not absorb tension, then the theoretical distributions of shear stresses in the elastic and ultimate stages are as shown in Fig. 7.3(a) and Fig. 7.3(b), respectively. For the sake of simplicity, the value of normal shear stress across the cross-section of a beam is taken as the average shear on the section, i.e. the value obtained by dividing the shear force by the area of the section (IS 456: clause 39.1).

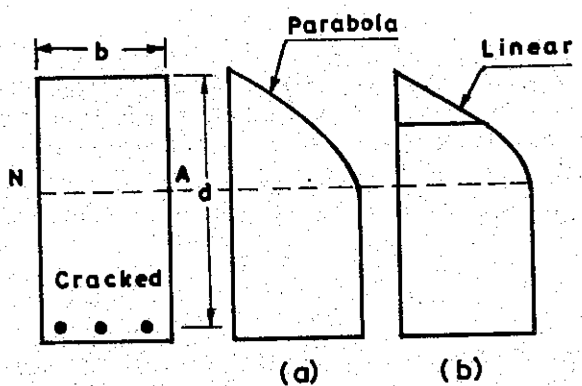


Fig. 7.3 Shear stress distribution in R.C.C. beams: (a) Elastic stage; (b) Ultimate stage.

Hence

$$\tau_v = \frac{V}{bd} \tag{7.1}$$

where

V = shear force
τ_v = nominal shear stress (sometimes it is denoted by the symbol v)
b = breadth (rib width for T and L beams)
d = effective depth of the member

For a T or L beam, the web (rib) only is assumed to resist the shear. Accordingly, the magnitude of the nominal shear stress is given by

$$v = \tau_v = \frac{V}{b_w d}$$

For solid circular section of diameter b_r,

$$\tau_v \simeq \frac{V}{b_r d}$$

As diagonal tension and compression produced by pure shear are equal in magnitude to the shear stress itself, the above value of the nominal shear is taken as a measure of the magnitude of diagonal tension which causes cracking of concrete. Since concrete is weak in tension, it is this diagonal tension that should be taken care of by proper design.

7.4 DESIGN SHEAR STRENGTH IN CONCRETE BEAMS

Recent laboratory tests have shown that the resistance of reinforced concrete beams to diagonal tension failure (as measured by the average shear) depends on two main factors: (a) the grade of concrete, and (b) the percentage of tension steel in the beam. Table 13 of IS 456 and Table 61 of SP 16 and Table 7.1 of the text give the ultimate allowable shear stress τ_c, called *design shear strength* of concrete in beams, as a function of both percentage of tension steel and grade of concrete.

Such a representation was first suggested by a study group of the Institution of Structural Engineers, U.K. in 1969. This has been adopted by IS 456 and BS 8110. It should be noted that the area of tension steel at a section to be taken into account is the area of steel which continues through the section (where the shear is being considered) for a distance which is at least equal to the effective depth beyond the section on either side. Thus, while considering shear at supports, one should take into account the fact that for continuous beams the top steel is the one that will be in tension, and for simply supported beams, the bottom steel is the one that will be in tension. When the 'tensile steel' at supports is less than 0.25 per cent, the lowest value given in Table 13 of IS 456 should be used.

Thus it should be clearly noted that the first value given in Table 13 of IS 456 is the design shear strength value for percentage of steel equal to or less than 0.25 per cent.

All beams where shear exceeds these allowable values of Table 13 of IS 456 should be provided with designed shear reinforcement. As shear failures are sudden and brittle, all important structures with shear stresses, even less than these safe values of the table, should also be provided

TABLE 7.1 DESIGN SHEAR STRENGTH OF CONCRETE, τ_c, N/mm²
(Table 61 of SP 16)

p_t	f_{ck}, N/mm²					
	15	20	25	30	35	40
≤ 0.20	0.32	0.33	0.33	0.33	0.34	0.34
0.30	0.38	0.39	0.39	0.40	0.40	0.41
0.40	0.43	0.44	0.45	0.45	0.46	0.46
0.50	0.46	0.48	0.49	0.50	0.50	0.51
0.60	0.50	0.51	0.53	0.54	0.54	0.55
0.70	0.53	0.55	0.56	0.57	0.58	0.59
0.80	0.55	0.57	0.59	0.60	0.61	0.62
0.90	0.57	0.60	0.62	0.63	0.64	0.65
1.00	0.60	0.62	0.64	0.66	0.67	0.68
1.10	0.62	0.64	0.66	0.68	0.69	0.70
1.20	0.63	0.66	0.69	0.70	0.72	0.73
1.30	0.65	0.68	0.71	0.72	0.74	0.75
1.40	0.67	0.70	0.72	0.74	0.76	0.77
1.50	0.68	0.72	0.74	0.76	0.78	0.79
1.60	0.69	0.73	0.76	0.78	0.80	0.81
1.70	0.71	0.75	0.77	0.80	0.81	0.83
1.80	0.71	0.76	0.79	0.81	0.83	0.85
1.90	0.71	0.77	0.80	0.83	0.85	0.86
2.00	0.71	0.79	0.82	0.84	0.86	0.88
2.10	0.71	0.80	0.83	0.86	0.88	0.90
2.20	0.71	0.81	0.84	0.87	0.89	0.91
2.30	0.71	0.82	0.86	0.88	0.91	0.93
2.40	0.71	0.82	0.87	0.90	0.92	0.94
2.50	0.71	0.82	0.88	0.91	0.93	0.95
2.60	0.71	0.82	0.89	0.92	0.94	0.97
2.70	0.71	0.82	0.90	0.93	0.96	0.98
2.80	0.71	0.82	0.91	0.94	0.97	0.99
2.90	0.71	0.82	0.92	0.95	0.98	1.00
3.00	0.71	0.82	0.92	0.96	0.99	1.01

with at least nominal minimum shear reinforcements. However, if the shear is only one-half the table value and in members of minor structural importance such as lintels, no shear reinforcement is deemed to be necessary (IS 456: clause 25.5.1.6).

Table 13 is difficult to use when design procedure has to be computerised. For this purpose, it is better to express the values by a formula. The semi-empirical formulae used to derive Table 13 is as follows (refer SP 24 (1983), Section 39.2): Denoting the design shear strength of concrete as τ_c, we get

$$\tau_c = \frac{0.85\sqrt{(0.8f_{ck})}\,\sqrt{(1+5\beta}-1)}{6\beta} \tag{7.2}$$

where

$$\beta = \frac{0.8 f_{ck}}{6.89 p_t}, \quad \text{but not less than 1}$$

$$p_t = \frac{100 A_s}{bd} \quad \text{(percentage of steel on rib width only)}$$

$0.8 f_{ck}$ = cylinder strength in terms of cube strength

0.85 = the reduction factor similar to $1/\gamma_m$

Table 13 of IS 456 is applicable for sections deeper than 300 mm or more. For lesser depths (as in slabs), the strength is larger as indicated in Section 7.12 and IS 456: clause 39.2.1.1.

The formula in BS 8110 for design shear strength of concrete is slightly different and is given by the expression

$$\tau_c = 0.79 (p_t)^{1/3} \left(\frac{400}{d}\right)^{1/4} \left(\frac{1}{\gamma_m}\right) \left(\frac{f_{ck}}{25}\right)^{1/3} \tag{7.3}$$

where

$\left(\dfrac{400}{d}\right)$ = the correction factor for depth and should not be less than 1

$\left(\dfrac{f_{ck}}{25}\right)$ = the correction factor for the strength of concrete and should not be greater than 40

γ_m = 1.25 is recommended

(*Note*: the value of p_t should not exceed 3.)

Any of the above formulae may be used as they give comparable values. There is a limit to the maximum shear stress value for which the beam can be strengthened by shear reinforcements. Beyond these values, diagonal compression can take over even if the diagonal tension is taken care of by steel reinforcements. Table 14 in IS 456 or Table J in SP 16, page 175 and Table 7.2 give the maximum values of allowable shear.

TABLE 7.2 MAXIMUM SHEAR STRESS IN CONCRETE, $\tau_{c\,\max}$

Concrete Grade	M_{15}	M_{20}	M_{25}	M_{30}	M_{35}	M_{40}
$\tau_{c\,\max}$ (N/mm^2)	2.5	2.8	3.1	3.5	3.7	4.0

As explained in SP 24, page 127, these values are obtained from the expression

$$\tau_{c\,\max} = 0.83 \sqrt{f_c} \quad \text{N/mm}^2 \tag{7.4}$$

where f_c is the cylinder strength. Converting the cylinder strength to cube strength by applying a factor of 0.85, and then applying a partial safety factor for material strength, $\gamma_m = 1.25$, we get

$$\tau_{c\,\max} = 0.62 \sqrt{f_{ck}} \text{ (approx.)} \tag{7.4a}$$

The values of $\tau_{c\,\max}$ recommended in BS 8110 (1985) are given by the expression

$$\tau_{c\,\max} = 0.8 \sqrt{f_{ck}} \text{ but} \ngtr 5 \quad \text{N/mm}^2 \tag{7.4b}$$

Thus the values recommended in IS can be considered as very safe. Under no condition should $\tau_v = V/bd$ exceed these values in beams. If it does, the section should be redesigned by changing the value of b and d. In slabs a maximum shear of only one-half of this value is allowed (see also Section 7.11).

It is well known that shear failure is a 'brittle failure' and should be avoided. Different codes ensure this in different ways. In the Indian as well as the British codes, the possibility of shear failure is excluded by giving enough safety factor for the combined action of concrete and tension steel.

7.5 TYPES OF SHEAR REINFORCEMENTS

As already explained, the shear steel is placed in reinforced concrete to counteract the cracking and shear failure. As shown in Fig. 7.4, the types of steel reinforcements that can be provided to resist diagonal tension are:

1. A system of vertical stirrups
2. A system of inclined stirrups placed at right angles to the diagonal tension cracks
3. Main tension steel bent up and placed as described in point 2.

Of these, the vertical stirrups and bent up main steel rods are most commonly used in practice. The mechanical actions of these two are different from each other to a certain extent. While bent-up bars are good in restricting crack-width, the stirrups help the longitudinal tension steel to maintain dowel action. From overall considerations, vertical stirrups are superior to inclined bars, but it may be more economical if some of the unwanted tension bars are also used for resisting shear. The combined use of stirrups and bent-up bars can lead to an economical and a technically superior solution. Both Indian and British Codes do not recommend that all the shear be taken by bent-up bars only. At least 50 per cent of the shear should be provided for by stirrups according to IS 456: clause 39.4. Thus, a combination of links and bent-up bars is much superior to using either links or bent-up bars only. Both dowel action and restriction of crack width to resist shear are present in such combinations.

7.6 DESIGN OF LINKS (STIRRUPS)

7.6.1 DESIGN PRINCIPLES

At ultimate shear failure of a beam, the shear forces are resisted by the combined action of concrete and shear steel. The resistance in concrete as shown in Fig. 7.5 is due to the combined effect of three factors, viz.

V_{cz} = shear carried by uncracked concrete (20 to 40 per cent of total shear in compression)
V_a = shear carried by aggregate interlock across the diagonal crack (33–50 per cent)
V_d = shear carried by dowel action of longitudinal tension steel (15–25 per cent)

Considering the maximum values that can be carried by each of the above effects, the expression for that portion of shear carried by concrete denoted by V_c can be written as

$$V_c = V_{cz} + V_a + V_d$$

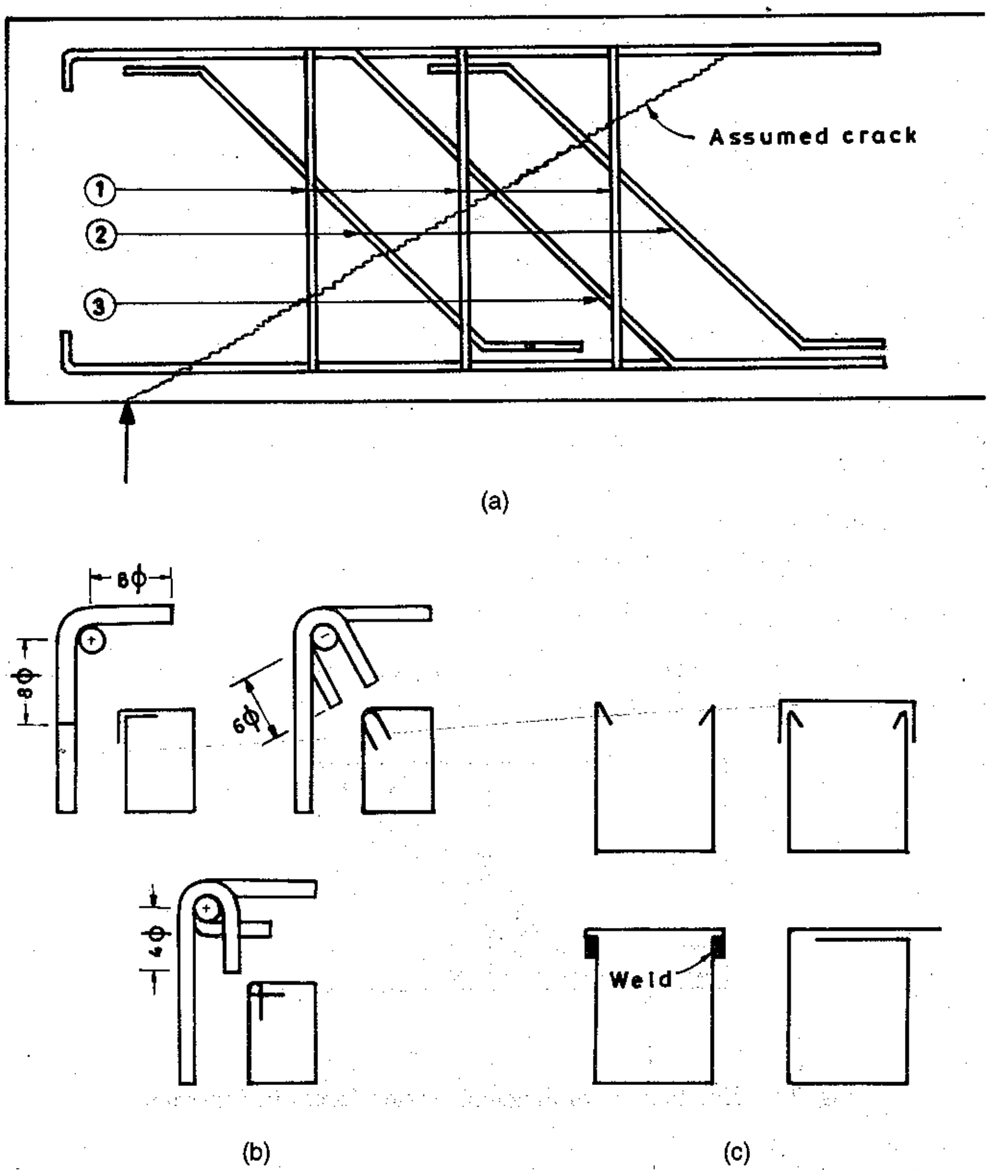


Fig. 7.4 **Types of shear reinforcements in beams: (a) Shear reinforcements—(1) System of vertical stirrups; (2) System of inclined stirrups; (3) Bent-up tension bar; (b) Detailing of ends of conventional types of stirrups; (c) Some special shapes of stirrups.**

Steel will then be required to carry only the balance of the shear. Let the shear to be carried by steel be V_s. Thus the value of total shear to be carried is given by

$$V = V_{\text{concrete}} + V_{\text{stirrup}} = V_c + V_s$$

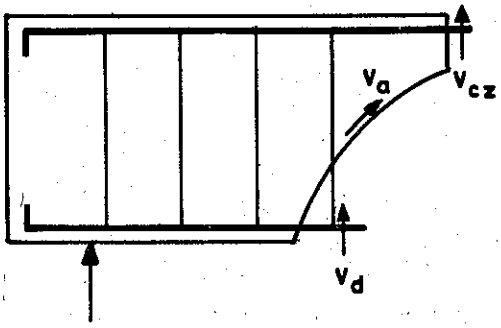


Fig. 7.5 Components of shear resistance in R.C.C. beams.

or the value of shear to be carried by stirrups is given by

$$V_s = V - V_c = (\tau_v - \tau_c)bd \tag{7.5}$$

where

τ_v = nominal shear stress
τ_c = design shear stress of concrete

7.6.2 SHEAR CAPACITY OF STIRRUPS

Consider the vertical equilibrium of forces across the diagonal crack, with the stress in the vertical stirrup reaching ultimate state of failure as in Fig. 7.6.

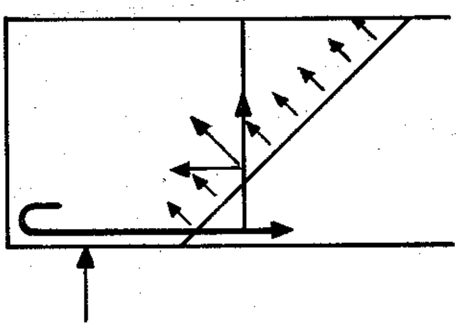


Fig. 7.6 Equilibrium of diagonal tension forces in concrete.

Assuming that the main steel is more than enough to balance the horizontal forces due to compression in the concrete and the diagonal tension force, the equilibrium of the vertical forces is given by the following expression: Let

A_{sv} = total area of the legs of shear links (for the usual stirrups there will be two legs)
s_v = spacing of links

The number of stirrups cut by a 45° crack line is given by

$$N = \frac{d}{s_v}$$

Therefore,
The s
Therefore,
Dividing b
Hence spa
or
This is the
reduced to
Table 62 o
the section
It is
tension ha
taken by t
always pa
detail in R
7.7 RUL
7.7.1 PR
IS 456: c
minimum
reinforcem

Therefore, the total shear resistance of the vertical stirrup system across the section is given by

$$V_s = 0.87(f_y\, A_{sv})\frac{d}{s_v} \tag{7.5a}$$

The shear to be carried by steel has already been derived as

$$V_s = (\tau_v - \tau_c)bd$$

Therefore,

$$(\tau_v - \tau_c)bd = \frac{0.87 f_y A_{sv} d}{s_v}$$

Dividing by bd, we get

$$(\tau_v - \tau_c) = \frac{0.87 f_y A_{sv}}{b s_v}$$

Hence spacing of stirrups is given by

$$s_v = \frac{0.87 f_y\, A_{sv}}{b(\tau_v - \tau_c)}$$

or

$$\frac{A_{sv}}{s_v} = \frac{b(\tau_v - \tau_c)d}{0.87 f_y\, d} = \frac{V_s}{0.87 f_y\, d} \tag{7.6}$$

This is the expression given in IS 456: clause 39.4. In SP 16 the above expression has been further reduced to the form

$$V_s/d = \left(\frac{\text{shear to be carried by steel}}{\text{depth in cm}}\right)$$

$$\frac{V_s}{d} = \frac{A_{sv}}{s_v}(0.87 f_y) \tag{7.6a}$$

Table 62 of SP 16 enables the direct design of two-legged stirrups from V_s/d in (kN/cm) values of the section. This table is given as Table 7.3 in the text.

It is very important to note that in equation (7.6a) only the vertical component of diagonal tension has been taken into account. The horizontal tension component of diagonal tension is to be taken by the main tension steel. Hence, for equilibrium of forces, the shear reinforcement should always pass around and enclose all the tension reinforcements of the beam. This is a construction detail in R.C.C. beams and should be carefully attended to.

7.7 RULES FOR MINIMUM SHEAR REINFORCEMENT

7.7.1 PRINCIPLES OF DESIGN

IS 456: clause 25.5.1.6 specifies that all beams should be provided with at least some minimum reinforcements called nominal steel even if calculations show that it is safe without these reinforcements.

TABLE 7.3 SHEAR-VERTICAL STIRRUP
(Table 62 of SP 16)
(VALUES OF V_s/d FOR TWO-LEGGED STIRRUPS, kN/cm

Stirrup Spacing, cm	f_y = 250 N/mm² Diameter, mm				f_y = 415 N/mm² Diameter, mm			
	6	8	10	12	6	8	10	12
5	2.460	4.373	6.833	9.839	4.083	7.259	11.342	16.334
6	2.050	3.644	5.694	8.200	3.403	6.049	9.452	13.611
7	1.757	3.124	4.881	7.028	2.917	5.185	8.102	11.667
8	1.537	2.733	4.271	6.150	2.552	4.537	7.089	10.208
9	1.367	2.429	3.796	5.466	2.269	4.033	6.302	9.074
10	1.230	2.186	3.416	4.920	2.042	3.630	5.671	8.167
11	1.118	1.988	3.106	4.472	1.856	3.299	5.156	7.424
12	1.025	1.822	2.847	4.100	1.701	3.025	4.726	6.806
13	0.946	1.682	2.628	3.784	1.571	2.792	4.363	6.286
14	0.879	1.562	2.440	3.514	1.458	2.593	4.051	5.833
15	0.820	1.458	2.278	3.280	1.361	2.420	3.781	5.445
16	0.769	1.366	2.135	3.075	1.276	2.269	3.545	5.104
17	0.723	1.286	2.010	2.894	1.201	2.135	3.336	4.804
18	0.683	1.215	1.898	2.733	1.134	2.016	3.151	4.537
19	0.647	1.151	1.798	2.589	1.075	1.910	2.985	4.298
20	0.615	1.093	1.708	2.460	1.020	1.815	2.836	4.083
25	0.492	0.875	1.367	1.968	0.817	1.452	2.269	3.267
30	0.410	0.729	1.139	1.640	0.681	1.210	1.890	2.722
35	0.351	0.625	0.976	1.406	0.583	1.037	1.620	2.333
40	0.307	0.547	0.854	1.230	0.510	0.907	1.418	2.042
45	0.273	0.486	0.759	1.093	0.454	0.807	1.260	1.815

Minimum steel is necessary to

1. guard against any sudden failure of a beam if concrete cover bursts and the bond to the tension steel is lost;
2. prevent brittle shear failure which can occur without shear steel;
3. prevent failure that can be caused by tension due to shrinkage and thermal stresses and internal cracking in the beams;
4. hold the reinforcements in place while pouring concrete; and
5. act as the necessary ties for the compression steel and make them effective.

The minimum quantity of shear reinforcement that should be provided for all beams except those of minor importance like lintels is given in IS 456: clause 25.5.1.6 by the equation

$$\frac{A_{sv}}{bs_v} = \frac{0.4}{f_y} \tag{7.7}$$

where

A_{sv} = total area of stirrup legs
s_v = spacing
b = breadth of web at level of tension steel
bs_v = cross-sectional area of the beam between two stirrups

(The formula in BS 8110 for minimum shear steel uses $0.87f_y$ instead of f_y as above). This works out to 0.16 per cent of the 'longitudinal section between stirrups' (bs_v) of the member with Fe 250 steel and 0.10 per cent for Fe 415 steel.

For a given value of A_{sv}, an expression for the spacing of nominal steel can be obtained from the rearrangement of equation (7.7). An expression for the spacing can be written as

$$s_v = \frac{f_y}{0.4}\left(\frac{A_{sv}}{b}\right) = 2.5f_y\left(\frac{A_{sv}}{b}\right)$$

Therefore,

$$s_v \simeq 1000\left(\frac{A_{sv}}{b}\right) \quad \text{for Fe 415} \tag{7.8}$$

Similarly,

$$s_v = 625\left(\frac{A_{sv}}{b}\right) \quad \text{for Fe 250} \tag{7.8a}$$

These are simple expressions that can readily give the nominal spacing of stirrups.

It should also be made sure that the spacing of the links does not exceed 0.75d or 450 mm (IS 456: clause 25.5.1.6) and these links satisfy the anchorage requirements. However, this provision of minimum steel is not applicable to design of slabs.

7.7.2 PROCEDURES FOR CALCULATION OF NOMINAL REINFORCEMENT FOR SHEAR IN BEAMS

In practice, one may use any one of the following two methods for determining this minimum steel for shear in beams.

Procedure 1 Calculate the spacing s_v from the code expression

$$s_v = 2.5f_y\left(\frac{A_{sv}}{b}\right)$$

which reduces to

$$s_v = 1000\left(\frac{A_{sv}}{b}\right) \quad \text{for Fe 415}$$

$$s_v = 625\left(\frac{A_{sv}}{b}\right) \quad \text{for Fe 250}$$

Procedure 2 Proceed as if the design is to be made for a value of $(\tau_v - \tau_c)$ equal to 0.35, i.e.

$$\frac{A_{sv}}{s_v} = \frac{0.4b}{f_y}$$

can be written as

$$\frac{A_{sv}}{s_v} = \frac{0.35bd}{0.87f_y\,d}$$

Comparing the above relation with equation (7.6), it can be seen that providing for nominal steel is equivalent to designing the shear reinforcement for

$$(\tau_v - \tau_c) = 0.35 \text{ N/mm}^2 \tag{7.9}$$

The corresponding shear reinforcement spacings can be read off from SP 16, Table 62 (Table 7.3 of text). This procedure is explained in Section 7.8.

7.8 GENERAL PROCEDURE FOR DESIGN OF BEAMS FOR SHEAR

The procedure for design of beams for shear is given in Table 7.4.

TABLE 7.4 DESIGN OF BEAMS FOR SHEAR

Case	Value of τ_v N/mm^2	Form of shear steel	Area of shear steel
1.	Less than $0.5\tau_c$ throughout the length of the beam	Minimum links as in case 2 for important structures. No links for structures of minor importance	
2.	$0.5\tau_c$ to $(0.5\tau_c + 0.35)$	Minimum links for whole length	$A_{sv} = \frac{0.35 b s_v}{0.87 f_y}$
3.	$(0.5\tau_c + 0.35)$ to $0.62\sqrt{f_{ck}}$*	Links or links with bent-up bars At least 50% resistance by stirrups	$A_{sv} = \frac{(\tau_v - \tau_c) b s_v}{0.87 f_y}$
4.	$>0.62\sqrt{f_{ck}}$*	Redesign section	

*$0.8\sqrt{f_{ck}}$ for BS 8110 as in equation (7.4b).

The calculation of steel area can be made by one of the following methods:

Method 1 (*Calculation by formula*) Choose a diameter ϕ for shear reinforcement and determine the spacing from

$$s_v = \frac{A_{sv}(0.87f_y)}{b(\tau_v - \tau_c)}$$

Method 2 From tabulated values of A_{sv}/s_v, calculate the value of $b(\tau_v - \tau_c)/(0.87f_y)$ and use tabulated values such as those given in Table 7.5 showing A_{sv}/s_v and diameter of bars to be chosen.

Method 3 (By use of Design Aid SP 16) Table 62 of SP 16 (Table 7.3 of the text) gives tables for design of vertical two-legged stirrups using formula (7.5a).

Expressing d in cm, equation (7.6) can be reduced to

$$\frac{V_s}{d} = 0.87f_y\left(\frac{A_{sv}}{s_v}\right)\left(\frac{1}{100}\right) \quad \text{[units will be kN/cm]}$$

TABLE 7.5 A_{sv}/s_v VALUES FOR DIFFERENT VALUES OF s_v AND ϕ

	Stirrup diameter (mm)			
s_v	8	10	12	16
100	1.00	1.57	2.26	4.02
150	0.67	1.05	1.51	2.68
200	0.50	0.79	1.13	2.01
250	0.63	0.63	0.90	1.61
300	0.33	0.52	0.75	1.34

where

V_s = the shear in excess of the shear capacity of concrete, which has to be carried by steel.

Table 7.3 gives the value of A_{sv} and s_v for various quantities of V_s/d for Fe 250 and Fe 415 steels.

7.9 STEP-BY-STEP PROCEDURE FOR DESIGN OF LINKS

The step-by-step procedure for design for vertical stirrups is now detailed:

Step 1: Calculate the maximum shear force V in the beam.

Step 2: Calculate the nominal shear stress $\tau_v = V/bd$ (for flanged beams use b = rib width).

Step 3: Check to see that τ_v is less than the maximum permissibie stress $0.62\sqrt{f_{ck}}$ or IS Table 14. Otherwise, change b or d to bring the value down. Read off design shear strength τ_c from Table 13 of IS 456, which depends on grade of concrete and the tensile steel ratio at the section considered.

Step 4: Design of the area and spacing of stirrups.

Case (a) Design for nominal steel. The diameter and spacing of nominal steel is determined by one of the two methods discussed already in Section 7.7. However, the spacing should not exceed $0.75d$ and the diameter of the stirrups should not be less than 6 mm.

Case (b) If τ_v is greater than τ_c, designed shear reinforcements are to be provided by using any of the three procedures given in Section 7.8.

Maximum spacing should be $0.75d$, and the diameter of stirrups should not be less than 6 mm.

Step 5: Check anchorage requirements and details.

(*Note*: At least 50 per cent of shear should be taken by vertical stirrups).

7.10 DESIGN OF BENT-UP BARS AS SHEAR REINFORCEMENTS

7.10.1 GENERAL PRINCIPLES

In addition to stirrups, a system of bent-up bars, bent usually at 45° and placed at specified spacing, can be used as shear reinforcement. However, such a system is not generally used nowadays in practice except in very heavy bridge girders. Usually, only one or two of the main longitudinal bars, which are not needed towards the end in the tension zone for resisting bending moment, are symmetrically bent-up and used as shear reinforcement. These will provide resistance against shear failure at the ends where the shear stresses are usually the largest.

The design of a system of bent-up bars is made by using the truss analogy in which the tension bars and the concrete are considered as the tension and compression members respectively of a truss (Fig. 7.7).

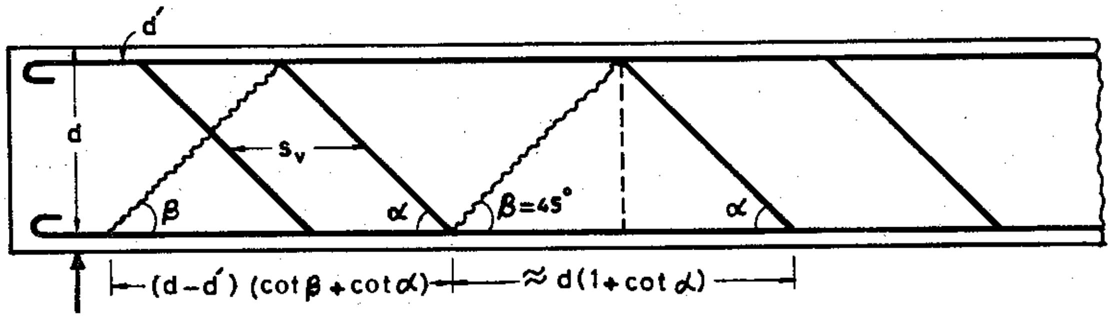


Fig. 7.7 Design of bent-up bar system for shear in RCC beams.

7.10.2 DESIGN FOR A SYSTEM OF BARS BENT AT AN ANGLE α AND AT A SPACING s_v

As already pointed out, this arrangement is only of theoretical interest since at present it is not practised in construction. The formulae for this system of reinforcement can be derived as follows: The horizontal length over which the bar is effective can be taken as equal to d (cot β + cot α), where α is the angle in which the bars are bent and β the direction of the shear compression (see Fig. 7.7).

Let the spacing = s_v. Hence the number of effective bars in one region is

$$N = \frac{(\cot\beta + \cot\alpha)(d - d')}{s_v}$$

Using truss analogy, the maximum shear that can be carried by one bar = A_{sv} (0.87f_y) sin α. Therefore, denoting shear carried by N bars as V_s, we get

$$V_s = A_{sv}(0.87 f_y)\sin\alpha\,(\cot\alpha + \cot\beta)\left(\frac{d - d'}{s_v}\right)$$

Again, when $\beta = 45°$ and $(d - d') = d$ (approx.), the above equation reduces to

$$V_s = \frac{A_{sv}(0.87 f_y)(\cos\alpha + \sin\alpha)d}{s_v} \tag{7.10}$$

as given in IS 456: clause 39.4(b). This can be rewritten as an expression for spacing, in the form

$$s_v = \frac{A_{sv}(0.87 f_y)(\cos\alpha + \sin\alpha)d}{V_s} \tag{7.10a}$$

where

d = effective depth
s_v = spacing of bent-up bars
A_{sv} = area of each bent-up bar
V_s = strength of shear reinforcement

BS 8110 specifies that a system of bent-up bars should be so spaced that both α and $\beta \geq 45°$, and the spacing does not exceed $1.5d$.

7.10.3 CALCULATION OF STRENGTH OF A SINGLE BENT-UP BAR

Of much more importance than a set of bent-up bars is the shear strength of one or two main tension bars bent to take shear near the supports.

The maximum shear that can be carried by one bent-up bar is given by IS 456: clause 39.4.

$$V_s = A_{sv}\,(0.87 f_y)\sin\alpha$$

This bar can be considered as effective over a horizontal distance of $d(1 + \cot\alpha)$, see Fig. 7.7. However, as the bar cannot be effective if the crack is at the bottom or top of the bar, one may limit the effectiveness of a single bar to $0.75d(1 + \cot\alpha)$ in usual cases and $0.5d(1 + \cot\alpha)$ in severe cases. The effect of one bar may be taken as effective half this distance on either side of the centre line of the bar (see Fig. 7.7). The shear resistance of single bent-up bars is given in Table 63 of SP 16 and Table 7.6 below.

TABLE 7.6 SHEAR—BENT-UP BARS
(Table 63 of SP 16)
Values of V_s for single bar, kN

Bar Diameter, mm	f_y = 250 N/mm^2		f_y = 415 N/mm^2	
	$\alpha = 45°$	$\alpha = 60°$	$\alpha = 45°$	$\alpha = 60°$
10	12.08	14.79	20.05	24.56
12	17.39	21.30	28.87	35.36
16	30.92	37.87	51.33	62.87
18	39.14	47.93	64.97	79.57
20	48.32	59.18	80.21	98.23
22	58.46	71.60	97.05	118.86
25	75.49	92.46	125.32	153.48
28	94.70	115.98	157.20	192.53
32	123.69	151.49	205.32	251.47
36	156.54	191.73	259.86	318.27

Note: α is the angle between the bent-up bar and the axis of the member.

7.11 ENHANCED SHEAR NEAR SUPPORTS

It has been observed from tests that shear failure at sections of beams and cantilevers without shear reinforcement will normally take place on a plane inclined at an angle 30° as shown in Fig. 7.8(a). Hence, if one considers failure at sections near the supports, it is customary to enhance the shear carrying capacity as is done in design of brackets, nibs, corbels etc., as explained in Chapter 21. Hence, when considering shears in beams which are supported on members which are in compression, Figs. 7.8(a) and 7.8(b), as different from case (c), where the support is in tension, one may proceed as follows (IS 456: clause 21.6.2):

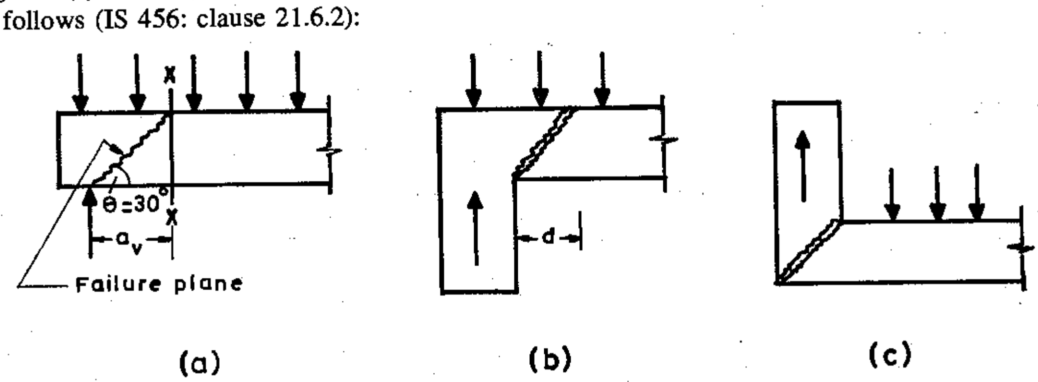


Fig. 7.8 Critical sections for shear: (a) Beams; (b) Face of support in compression; (c) Face of support in tension.

1. The critical section for shear is taken at a distance equal to the effective depth from the face of the support, when the beam is subjected to only uniformly distributed load. When it is subjected to a concentrated load, the critical section may be taken as twice the effective depth away from support or at the position of the concentrated load, whichever is critical.

2. For the "design shear" near the supports, the value τ_c of IS 456, Table 13 can be enhanced by a factor and is given by the equation

$$\text{Enhanced shear} = \tau_c 2d/a_v \qquad (7.11)$$

where a_v is the length of that part of a member transversed by a shear plane called the shear span as shown in Fig. 7.8(a). This is explained further in Chapter 21. However, in the case of members where the supports have to act in tension as in Fig. 7.8(c), the critical section for shear should be taken at the face of the support only. This difference is obvious when the nature of cracking that can happen is examined. It is also very important to note that for bottom loaded beams (where loads are hung from the underside of the beams), sufficient steel to carry the load should be given in addition to stirrups so that the beam is fully loaded as if from the top.

3. According to BS 8110, the total shear steel near supports for the distance a_v can be calculated from the following formula which takes into account the enhanced shear resistance also:

$$\Sigma A_{sv} = a_v b\left(\tau_v - \tau_c \frac{2d}{a_v}\right)\Big/0.87f_y$$

However,

$$\Sigma A_{sv} \geq 0.4a_v b/0.87f_y \text{ (nominal steel)}$$

This area is to be provided within the middle three quarters of a_v. If a_v is less than the effective depth, provision of horizontal shear steel is more effective than vertical steel as the action is similar to that in deep beams.

7.12 SHEAR IN SLABS

In practice, slabs less than 200 mm thick cannot be reinforced with shear reinforcements. Hence the shear stresses produced in slabs should be within the values allowable for concrete without shear steel. The design procedure for slabs is separately covered in Section 11.4.

7.13 DETAILING OF STEEL

For most beams, especially those subjected to UDL, it is sufficient to determine the points where no shear reinforcement (nominal steel only) is required as also to calculate the reinforcement required at points of maximum shear forces. The spacings between these points can be varied to suit the nature of the shear force diagram. This can be allocated in most cases by judgement rather than by further detailed calculations.

7.14 SHEAR IN MEMBERS SUBJECTED TO COMPRESSION AND BENDING

So far we have discussed only beams subjected to shear and bending. However, when shear is combined with other forces like torsion and compression, these should be treated differently.

The ACI code gives formulae applicable to the various combinations, and they can be made use of in such situations.

For bending, shear and compression as in prestressed concrete beams, IS 1343 (1980): clause 22.4 gives the necessary design formulae.

The case of shear combined with torsion is dealt with in Chapter 18.

7.15 SHEAR IN BEAMS OF VARYING DEPTH

Beams with constant width varying in depth are frequently used in R.C. construction. They are dealt with in IS 456 clause 39.1.1. The following two cases can occur in practice:

Case (a): The bending moment increases numerically in the same direction in which the effective depth increases.

Case (b): The bending moment decreases numerically in the direction in which the effective depth increases.

It can be deduced from Fig. 7.9 that the effective shear to be used for determining the shear stress is

$$V = V_\omega - \frac{M}{d} \tan \beta \quad \text{for case (a)}$$

$$V = V_\omega + \frac{M}{d} \tan \beta \quad \text{for case (b)}$$

where V_ω is the normal beam shear. This is dealt with in IS 456: clause 39.1.1.

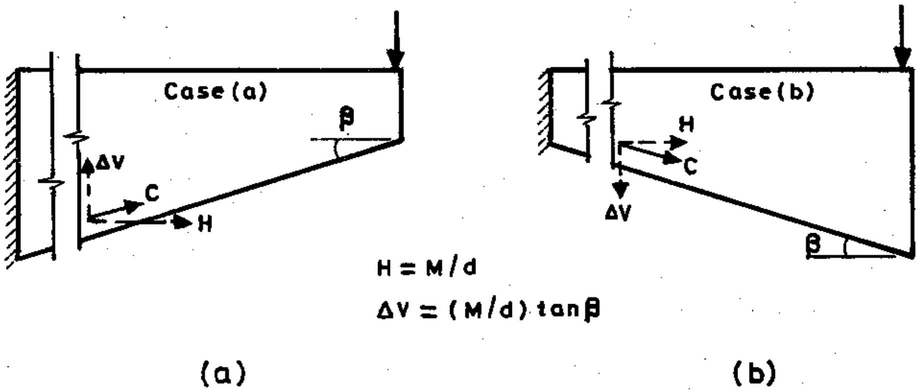


Fig. 7.9 Shear in beams of varying depth: (a) BM increases with increasing depth; (b) BM increases with decreasing depth.

Having determined V, the nominal shear stress is calculated by the expression

$$\tau_v = V/bd$$

and the shear steel is designed as already described in Table 7.4.

The application of this theory to haunched beams is illustrated in Fig. 7.10.

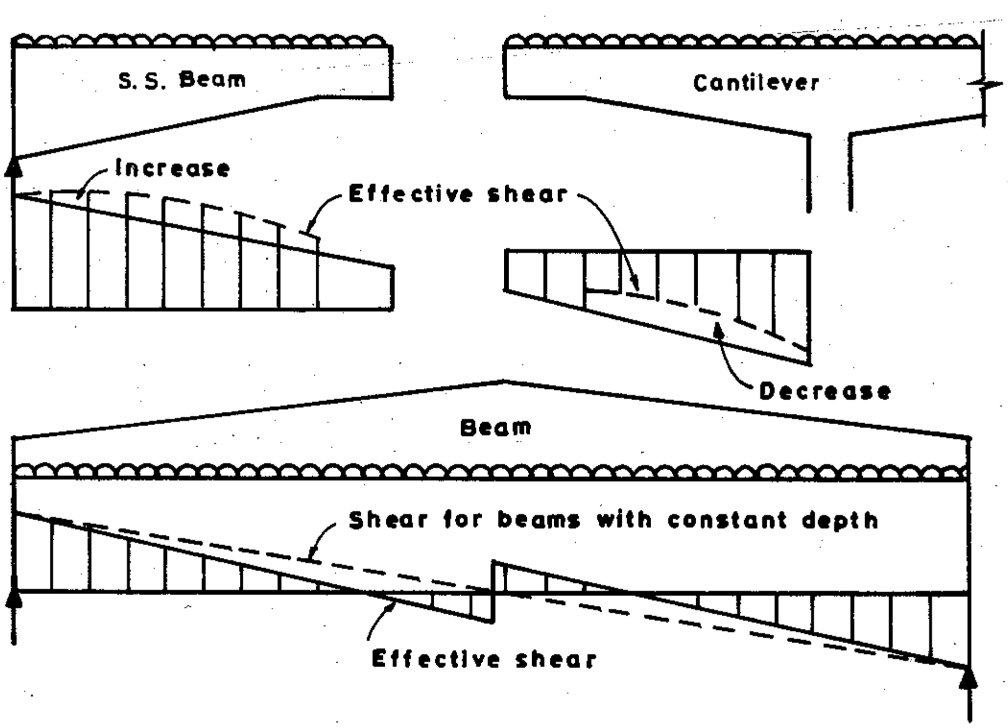


Fig. 7.10 Haunched beams and effective shears.

7.16 DETAILING OF VERTICAL STIRRUPS IN WIDE BEAMS

Even from the early days of the development of reinforced concrete, the usefulness of vertical stirrups and inclined bars in resisting shear was well known. This was explained by the truss analogy as shown in Figs. 7.11 and 7.12. In the case of vertical stirrups, the legs of the stirrups form the vertical members of a parallel chord truss, the diagonal compression members being the intact concrete between the verticals. The horizontal components of the tension are to be taken by the longitudinal steel and the horizontal compression by the uncracked concrete in the compression zone.

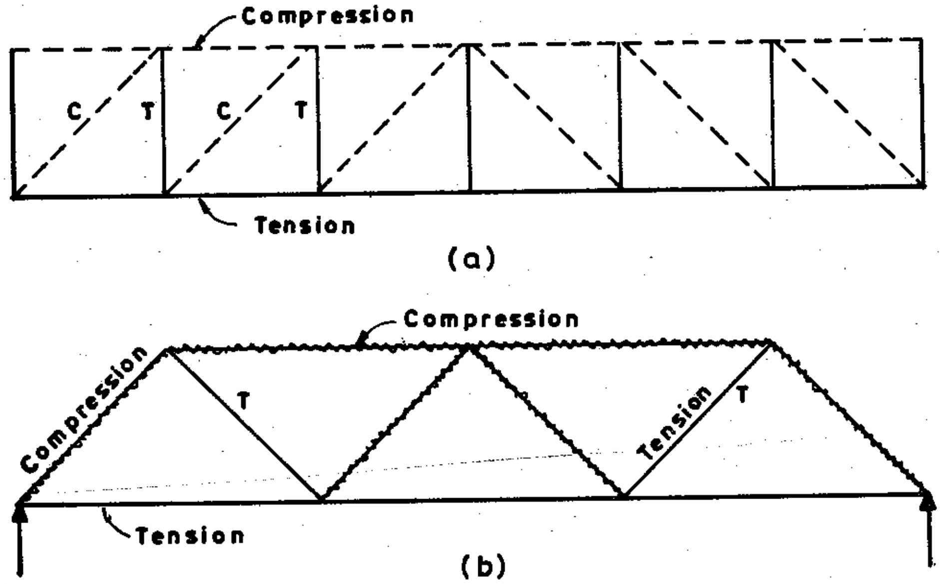


Fig. 7.11 Truss analogy of shear action: (a) Action of vertical stirrups; (b) Action of inclined bars.

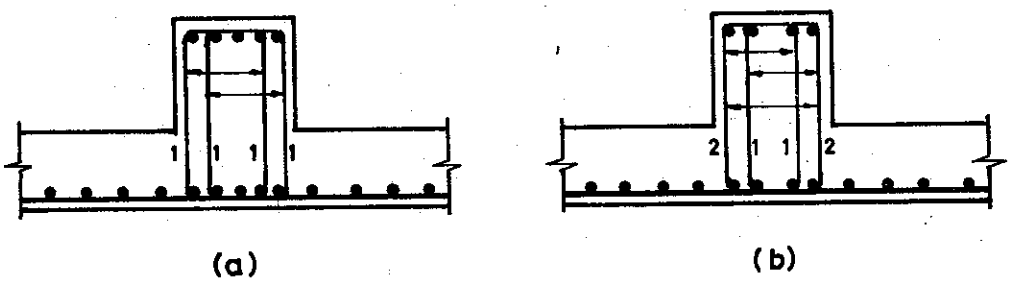


Fig. 7.12 Multilegged stirrups: (a) Four-legged stirrups; (b) Six-legged stirrups.

From the foregoing discussion, it is evident that in wide beams accommodating a large number of tension rods and carrying heavy shear forces (like beams of raft foundations in a ribbed raft), it is advisable to provide multilegged stirrups of smaller diameter going round all the longitudinal steel rods (rather than only the two outer bars) so that the longitudinal forces are evenly distributed among the tension bars provided in the beams. Even though there are no specific recommendations given in codes in this regard, the type of arrangement using multilegged stirrups shown in Fig. 7.12 is usually adopted in practice. It should also be noted that the truss model demands the full stress mobilisation of tension throughout the length of the stirrups, and not merely at its mid-height as is sometimes assumed.

7.17 DESIGN OF STIRRUPS AT STEEL CUT-OFF POINTS

According to IS 456: clause 25.2.3.2, when flexural reinforcements are terminated in the tension zone, it should satisfy two conditions regarding shear and stirrup area provided at the section as shown in Section 10.7. These conditions are:

1. The shear at the cut-off point does not exceed two-thirds the combined shear strength of concrete and steel.

Denoting the shear at the section as v equal to V/bd, the condition to be satisfied is

$$v \leq \frac{2}{3}(\tau_c + \tau_s)$$

This can be rewritten as

$$\tau_s \geq (1.5v - \tau_c)$$

where τ_s is the shear for which the stirrups at the section should be designed.

2. Stirrups area in excess of that normally required for shear and torsion should be provided along each terminated direction over a distance from the cut-off point equal to three-fourth the effective depth of the member. The area of this excess stirrup A'_{sv} is given by the relation

$$A'_{sv} = \frac{0.4bs}{f_y}$$

where

s = spacing which should not be more than $d/(8\beta_b)$

β_b = ratio of the area of bar cut off to the total area of bars

These are explained in Example 7.7.

EXAMPLE 7.1 (Design of stirrups for shear)

A rectangular beam with $b = 350$ mm and $d = 550$ mm has a factored shear of 400 kN at a section near the support. The steel at the tension side of the section consists of four 32 mm bars which are continued to support. Assuming $f_{ck} = 25$ and $f_y = 415$ (N/mm^2), design vertical stirrups for the section.

Ref.	Step	Calculations	Output
	1.	*Nominal shear stress* $V = 400$ kN $v = \frac{V}{bd} = \frac{400 \times 10^3}{350 \times 550} = 2.08\ \text{N/mm}^2$	
	2.	*Check for shear* $A_s = 3217$ mm^2	

EXAMPLE 7.1 (cont.)

Ref.	Step	Calculations	Output
SP 16 Table 61 Table J		$p = \dfrac{3217 \times 100}{350 \times 550} = 1.67\%$ $\tau_c = 0.76$ N/mm^2 and $\tau_{c\ max} = 3.1$ N/mm^2 Designed steel needed.	
	3.	*Design of stirrups* (a) *By calculating* from $\dfrac{A_{sv}}{s_v} = \dfrac{b(v - \tau_c)}{0.87 f_y}$ choose 10 mm two-legged links $A_{sv} = 157$ mm^2 $s_v = \dfrac{157 \times 0.87 \times 415}{350 \times (2.08 - 0.76)} = 123$ mm $s_{v\ max} = 0.75 \times 550 = 412.5$ mm	T 10 at 120 mm
		(b) *By use of SP 16* $V_s = (V - V_c) = 400 - \dfrac{0.76 \times 350 \times 550}{1000}$ $= 253.7$ kN	
SP 16 Table 62		$\dfrac{V_s}{d}$ (kN/cm) $= \dfrac{253.7}{55} = 4.61$ Choose 10 mm links – $s_v = 12$ cm	T 12 at 150 mm T 10 at 120 mm (4.726 kN/cm)

EXAMPLE 7.2 **(Design of stirrups for nominal shear)**

Design the shear reinforcement for a beam with $b = 350$ mm, $d = 550$ mm, $V_u = 125$ kN, $f_{ck} = 25$ N/mm^2, $f_y = 415$ N/mm^2. Percentage of steel is 1.67 per cent.

Ref.	Step	Calculations	Output
	1.	*Nominal shear stress* (v) $V = 125$ kN $v = \dfrac{V}{bd} = \dfrac{125 \times 10^3}{350 \times 550} = 0.65$ N/mm^2	
IS 456 Table 14 Table 13	2.	*Check for shear* $\tau_{c\ max} = 3.1$ N/mm^2, $\tau_c = 0.76$ N/mm^2 $v < \tau_c < \tau_{c\ max}$ but $v > \dfrac{1}{2}\tau_c$ Only nominal steel is needed.	

EXAMPLE 7.2 (cont.)

Ref.	Step	Calculations	Output
IS 456 Cl.25.5.1.6	3.	*Design of stirrups* (a) By the formula $\frac{A_{sv}}{bs_v} = \frac{0.4}{f_y}$ Choose 10 mm links. Then, $A_{sv} = 157\ \text{mm}^2$ $s_v = \frac{157 \times 415}{0.4 \times 350} = 465\ \text{mm}$ $s_{v\ \max} = 0.75 \times 550 = 412.5\ \text{mm}$	T 10 at 410 mm
Eq. (7.9) of text		(b) *By SP 16* (Design for nominal steel equivalent to designing section for an excess shear 0.35 N/mm^2.)	8 mm at 250 mm
		$\frac{V_{us}}{d} = \frac{0.35 \times 350 \times 550}{10^3 \times 55} = 1.23,\quad \text{kN/cm}$	
SP 16 Table 62		Choose 10 mm (spacing > 450 mm) (Max. spacing allowed = 410 mm)	T 10 at 410 mm

EXAMPLE 7.3 **(Use of bent-up bars for shear resistance)**

A beam with four 32 mm bars as main tension steel has two of its four main bars symmetrically bent at the ends of the beam at 45 degrees. Find the stirrups required for resistance against shear failure at the ends if the factored shear force at the section is 400 kN. Assume b = 350 mm, d = 550 mm, f_{ck} = 25, and f_y = 415 (N/mm^2).

Ref.	Step	Calculations	Output
	1.	*Nominal shear stress (v)* $V = 400\ \text{kN},\quad v = \frac{400 \times 10^3}{350 \times 550} = 2.08\ \text{N/mm}^2$	
SP 16 Table J Table 61	2.	*Check for shear* 2 bars only $\tau_{c\ \max} = 3.1\ \text{N/mm}^2,\quad p = \frac{A_s \times 100}{bd} = \frac{1608 \times 100}{350 \times 550}$ $= 0.83$ $\tau_c = 0.6\ \text{N/mm}^2,\quad v > \tau_c$ but $v < \tau_{c\ \max}$ Hence designed shear steel is needed.	
	3.	*Shear to be carried by steel V_s* $V_s = (2.08 - 0.6)350 \times 550 \times 10^{-3} = 284.9\ \text{kN}$	

EXAMPLE 7.3 (cont.)

Ref.	Step	Calculations	Output
	4.	*Shear resistance of bent-up bars*	
IS 456 p. 116		$V_{us} = 0.87 f_y A_{sv} \sin \alpha$ $= \dfrac{0.87 \times 415 \times 1608 \times 10^{-3}}{\sqrt{2}} = 411$ kN	
SP 16 Table 63		V_{us} for 2T 32 = 410.64 kN 2 T 32 can take more than the required shear. These bars are effective over $d(1 + \cot \alpha)$ = 550(2) = 1100 mm	
	5.	*Design for stirrups* At least $1/2V_s$ should be taken by stirrups. $V'_{us} = \dfrac{1}{2} \times 284.9 = 143$ kN	
SP 16 Table 62		$\dfrac{V'_{us}}{d} = \dfrac{143}{55} = 2.6$. Use 8 mm T 8 at 140 mm (Max. spacing allowed = 0.75 × 550 = 410)	T 8 at 140 mm

EXAMPLE 7.4 **(Design of T beam for shear)**

The T beam and slab system of a building are made of beams spaced at 2.4 m with clear span of 7.5 m between masonry walls of 300 mm thick. For the T beam D_f = 120 mm, b_w = 300 mm, D = 600 mm. If f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2, design the shear steel. Assume that 2 Nos. 28 mm bars of tension steel are continued to support and LL = 8 kN/m^2.

Ref.	Step	Calculations	Output
	1.	*Determine factored shear (spacing of beams = 2.4 m)*	
IS 456 Cl. 21.2		Effective span (7.5 + 0.3) or (7.5 + 0.6) = 7.8 m DL on beam = (2.4 × 0.12 × 25) + (0.3 × 0.48 × 25) = 10.8 kN/m LL on beam = 2.4 × 1 × 8 = 19.2 kN/m Factored load = 1.5 (10.8 + 19.2) = 45 kN/m Factored shear $= \dfrac{45 \times 7.8}{2} = 175.5$ kN	
IS 456 Cl. 21.6.2.1		(We can reduce the shear reinforcement by taking max. shear at (d) from supports.)	

EXAMPLE 7.4 (cont.)

Ref.	Step	Calculations	Output
	2.	*Nominal shear stress* $d = (D - c + \phi/2) = 600 - 50 = 550$ (say) $v = \dfrac{175.5 \times 10^3}{300 \times 550} = 1.06\ \text{N/mm}^2$	
	3.	*Check for shear* $A_s = 2\ \text{T}\ 28 = 1231\ \text{mm}^2$ $p = \dfrac{1231 \times 100}{300 \times 550} = 0.75\%$	
IS 456 Table 13 Table 14		$\tau_{c\ \max} = 2.8\ \text{N/mm}^2$, $\tau_c = 0.56\ \text{N/mm}^2$ $v > \tau_c < \tau_{c\ \max}$ Designed shear steel is needed.	
	4.	*Design of stirrups* Shear concrete can carry = V_c $= 0.56 \times 300 \times 550 \times 10^{-3}$ $= 92.4$ kN Shear to be taken by steel $V_s = 175.5 - 92.4 = 83.1$ kN $\dfrac{V_s}{d} = \dfrac{83.1}{55} = 1.51\ \text{kN/cm}$ Max. spacing $= 0.75 \times 550 = 412$	
SP 16 Table 62		Adopt 8 mm bars – Use T 8 at 200 $V_s/d = 1.815$).	T 8 at 200
	5.	*Region of nominal steel* Shear concrete can carry = 92.4 kN Distance from centre line of beam for this shear $= \dfrac{92.4}{45} = 2.05$ m Provide nominal steel for this region.	
Eq. (7.9) of text	6.	*Design for nominal steel* Design steel for excess shear = 0.35 N/mm^2 $\dfrac{V_s}{d} = \left(\dfrac{0.35 \times 300 \times 550}{1000}\right)\left(\dfrac{1}{55}\right) = 1.05\ \text{kN/cm}$	

EXAMPLE 7.4 (cont.)

Ref.	Step	Calculations	Output
SP 16 Table 62		Max. spacing = 0.75 × 550 = 412 mm 8 mm at 300 mm gives V_s/d = 1.21	T 8 at 300 mm
	7.	*Detailing of steel* 8 mm @ 200 mm from support to 1.85 m 8 mm @ 300 mm from 1.85 m to centre of beam Alternatively, 8 mm @ 200 throughout the span	

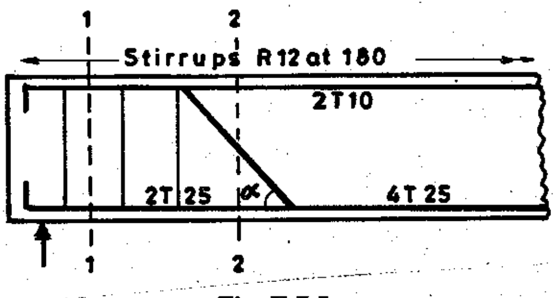


Fig. E.7.5.

EXAMPLE 7.5 **(Shear resistance of a given beam)**

Calculate the shear resistance at sections 1-1 and 2-2 of the beam, b = 300 mm, d = 600 mm, f_{ck} = 25, f_y = 250 (N/mm^2) for stirrups, as given in Fig. E.7.5.

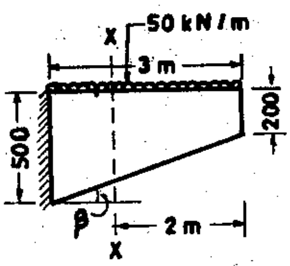
Ref.	Step	Calculations	Output
	1.	*Percentage of tension steel (at sections for shear) and τ_c* A_s of 2 Nos. 25 mm = 982 mm^2 $\frac{100A_s}{bd} = \frac{100 \times 982}{300 \times 600} = 0.54$	
IS 456 Table 13		Allowable τ_c = 0.5 N/mm^2 (approx.) Shear taken by concrete $V_c = 0.5 \times 300 \times 600 \times 10^{-3}$ = 90 kN	
	2.	*Shear taken by steel (R 12 at 180)* A_{sv} of 12 mm = 2 × 113 $V_s = \frac{A_{sv}}{s_v}(0.87 f_y)d$ $= \frac{2 \times 113}{180} \times 0.87 \times 250 \times 600 \times 10^{-3} = 164$ kN	

EXAMPLE 7.5 (cont.)

Ref.	Step	Calculations	Output
	3.	*Total shear resistance at section 1-1 near support* $V = V_c + V_s = 90 + 164 = 254$ kN	
	4.	*Shear strength of bent-up bars* $V_s = A_{sv}\,(0.87 f_y)\sin\alpha$ $= 982(0.87 \times 415)\sin 45 \times 10^{-3} = 250$ kN	
	5.	*Total shear resistance at section 2-2 through bent-up bar* $V = 254 + 250$ $= 504$ kN *Note*: An idea of the influence of bent-up bars can be obtained from the above numerical values.	

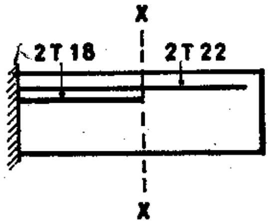
EXAMPLE 7.6 **(Analysis of shear in tapered beams)**

A tapered cantilever as shown in Fig. E.7.6 is of 3 metre span and has a constant width of 250 mm and tapers from 200 mm depth at the free end to 500 mm at the supports. Determine the nominal shear stress at a section 2 m from the free end assuming that the beam has to support a uniformly distributed load of 50 kN per metre length and cover to centre of steel is 50 mm.

Ref.	Step	Calculations	Output
	1.	*Shear and moment at section XX* $V_w = 2 \times 50 = 100$ kN $M = \dfrac{50 \times 2 \times 2}{2} = 100$ kN $d = D - \text{cover}$ $= 400 - 50 = 350$ mm $\tan\beta = \dfrac{500 - 200}{3000} = 0.10$	**Fig. E.7.6**
	2.	*Type of variation in section* The B.M. increases numerically in the same direction as the depth of the section increases.	
IS 456 Cl.39.1.1		$V = V_w - M/d \tan\beta$ $V = 100 - \dfrac{100 \times 0.1}{0.35} = 71.4$ kN Factored shear $V_u = 1.5 \times 71.4 = 107$ kN $v = \dfrac{V}{bd} = \dfrac{107 \times 10^3}{250 \times 350} = 1.23$ N/mm^2	

EXAMPLE 7.7 **(Design of shear steel at cut-off point of tension steel)**

A cantilever beam as given in Fig. E.7.7 is of 250 × 350 mm has 2 bars of 22 mm together with 2 bars of 18 mm as tension steel. At a section *XX* where the factored shear force is 110 kN, the 18 mm bars may be curtailed and the two 22 mm bars continued to the free end. Design the shear steel at the section *XX* (a) if the bars are not curtailed, and (b) if the bars are curtailed. Assume f_{ck} = 25 N/mm² and f_y = 415 N/mm².

Ref.	Step	Calculations	Output
	1.	*Design of shear steel without curtailment*	
		$d = 350 - 50 = 300$ mm	
		$v = \dfrac{V}{bd} = \dfrac{110 \times 10^3}{250 \times 300} \approx 1.50$ (say)	
		A_s = 2 T 22 (760 mm²) + 2T 18 (508)	
		= 1268 mm²	**Fig. E.7.7**
		$p = \dfrac{1268 \times 100}{250 \times 300} = 1.69\%$	
SP 16 Table 61		$\tau_c = 0.76$ N/mm², $\tau_{c\ max} = 3.1$ N/mm²	Designed shear steel needed
		$v > \tau_c$ but $< \tau_{c\ max}$	
Table J		$\dfrac{V_s}{d} = \dfrac{(v - \tau_c)bd}{d}$	
		$= \dfrac{(1.50 - 0.76)\ 250 \times 10}{1000} = 1.85$ kN/cm	
		Max. spacing of stirrups = 0.75 × 300	T 8 at 190 mm
		= 225 mm	(1.91 kN/cm)
SP 16 Table 62		Provide T8 at 190 mm (two-legged stirrups)	
	2.	*Design of shear steel with curtailment*	
		Condition (1)	
Text Section 10.7		Steel to be designed for $(1.5v - \tau_c)$	
IS 456 Cl.25.2.3.2(a)		τ_c depends on continued steel	
		$p = \dfrac{760 \times 100}{250 \times 300} = 1.01$	
		$\tau_c = 0.64$ N/mm², $\tau_{c\ max} = 3.1$ N/mm²	
		$v > \tau_c$ but $< \tau_{c\ max}$	
		$\dfrac{V_s}{d} = \dfrac{[(1.5 \times 1.5) - 0.64] \times 250 \times 10}{1000} = 4.02$	

EXAMPLE 7.7 (cont.)

Ref.	Step	Calculations	Output
SP 16 Table 62		Provide T 10 at 140 mm (4.05).	T 10 at 140 mm
		Condition (2)	
		Provide excess steel for distance 0.75d in the terminated direction.	
IS 456 Cl.25.2.3.2		$\beta = \dfrac{A_s'}{A_s} = \dfrac{508}{1268} = 0.4$	
		Max. $s = \dfrac{d}{8\beta} = \dfrac{300}{8 \times 0.4} = 93.7$ mm	
		$A_{sv}' = \dfrac{0.4bs}{f_y} = \dfrac{0.4 \times 250 \times 94}{415} = 22.6 \text{ mm}^2$	
		Provide this over 0.75d = 0.75 × 300 = 225 mm	
		Area of 2 legs of 6 mm = 56.5 mm^2	
		Hence provide 2 Nos. of 6 mm extra bars towards the free end of the cantilever over 225 mm distance.	2 Nos. 6 mm extra stirrups over 225 mm length.

REVIEW QUESTIONS

7.1 Explain how bending shear stresses produce tension cracks in concrete.

7.2 What is meant by punching shear and how is it different from bending shear?

7.3 Show how the expression for average shear on the section can be taken as a measure of diagonal tension in the design of shear in beams.

7.4 Enumerate the factors that affect the allowable shear in concrete in beams and write down the semi-empirical formula for shear strength of concrete in R.C.C. beams.

7.5 Why is it necessary to limit the allowable shear in concrete beams? Give an expression for this maximum value. Can this value be used for design of slabs also?

7.6 What are the types of reinforcements used to resist shear? Explain the action of different types of shear steel in resisting shear.

7.7 Why do specifications state that at least 50 per cent of the shear to be carried by steel should be in the form of stirrups?

7.8 Explain how the allowable shear strength of concrete in beams is greater near the supports of the beams? What is the formula used to calculate this allowable shear?

7.9 Explain why in beams the maximum shear to be checked is at a section at a distance equal to the depth of the beam?

7.10 What are the Indian specifications for allowable shear in slabs?

7.11 Explain how shear stress is calculated in beams of varying depth.

PROBLEMS

7.1 A reinforced concrete simply supported beam is of rectangular section 150 × 350 mm with 3 Nos. of 16 mm (3T 16) as tension steel continued to the ends. Assume f_{ck} = 25 N/mm^2, f_y = 415 N/mm^2, and that the member is to be used for an important structure. Determine

1. the maximum shear force for which no shear steel need be provided,
2. the size and spacing of the nominal stirrups that have to be provided,
3. the shear force up to which the above nominal spacing is applicable, and
4. the stirrup arrangement if the maximum shear expected at the end section of the beam is 40 kN under a uniformly distributed load over 6 m span.

7.2 Examine the following cases with respect to shear requirement and carry out one of the following options:

1. Provide designed shear steel.
2. Provide nominal steel.
3. Redesign the concrete section.

Assume f_{ck} = 20 N/mm^2, f_y = 415 N/mm^2 for main steel and f_y = 250 N/mm^2 for shear steel.

Case 1: Beam of breadth 200 mm, effective depth 450 mm, 6 Nos. of 20 mm bars at support as tension steel and maximum shear force 225 kN.

Case 2: Beam of breadth 100 mm, effective depth 140 mm, 3 Nos. of 12 mm bars at support as tension steel, maximum shear force 56 kN.

Case 3: Beam as in case 2 with maximum shear force 10 kN.

7.3 A rectangular section of a simply supported beam is 250 × 420 mm in section with effective cover of 40 mm to centre of reinforcement. It has 4 Nos. of 12 mm bars continued to the supports.

Find the shear capacity at the supports if the shear steel consists of double vertical stirrup of 8 mm dia at 200 mm spacing. Assume f_y = 250 N/mm^2 and f_{ck} = 20 N/mm^2.

7.4 Determine the ultimate shear resistance close to the support of the concrete section shown in Fig. P.7.4. Assume f_{ck} = 25 N/mm^2, f_y = 250 N/mm^2 for stirrups, and f_y = 415 N/mm^2 for bent-up bars.

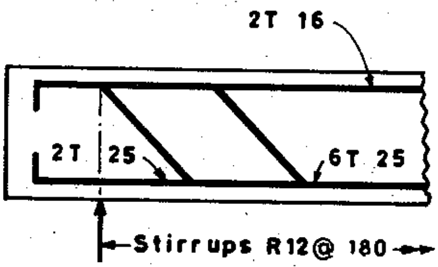


Fig. P.7.4.

7.5 The beam in Fig. P.7.4 is to be used for an effective span of 6.4 m with a total factored load of 800 kN as uniformly distributed over its length. Assuming f_{ck} = 15 and f_y = 415 N/mm^2 for main steel with f_y = 250 N/mm^2 for stirrups,

1. calculate the shear resistance of the bent-up bars and show with the help of a diagram over what distance it is effective, and

2. determine the additional links to be provided as shear reinforcement so that the beam is safe in shear according to IS 456.

7.6 A reinforced concrete simply supported beam is 300 × 500 mm in section. It has an effective span of 6 m and cover to centre of the main steel is 50 mm. It is reinforced at the centre of the beam with 5 Nos. of 20 mm, mild steel bars, of which 2 Nos. are curtailed at 0.15L from the supports. Mild steel bars are to be used as shear steel also.

If the shear force at the supports due to the total uniformly distributed characteristic load is 100 kN, design the shear reinforcement. Assume that grade 25 concrete is used for the beam.

7.7 1. A continuous beam ABC with spans $AB = BC = 8$ m is of rectangular section with breadth 350 mm and effective depth 800 mm. The characteristic dead load, inclusive of self weight, is 36 kN/m, and the characteristic imposed load is 45 kN/m. If $f_{ck} = 25$ N/mm^2, $f_y = 415$ N/mm^2, and the longitudinal steel arrangement is as shown in Fig. P.7.7(a), design the shear reinforcement at B.

2. If the above beam has flanges on either side as shown in Fig. P.7.7(b), what are the modifications, if any, that have to be done in the design for shear between A and B. Give reasons for your answer.

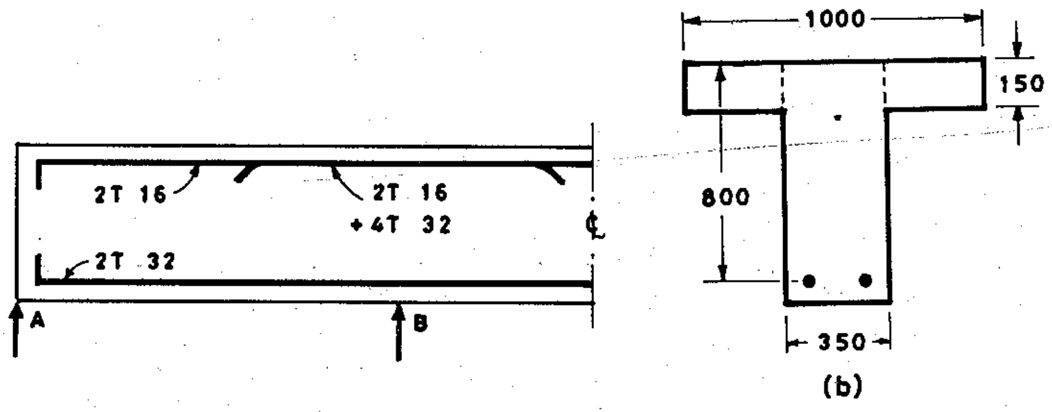


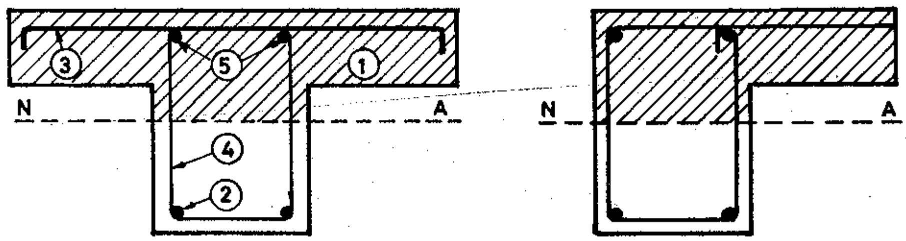
Fig. P.7.7.

8

Design of Flanged Beams

8.1 INTRODUCTION

When there is a reinforced concrete slab over a reinforced concrete beam, the slab and beam can be designed and constructed in such a way that they act together. The concrete in the slabs, which is on the compression side of the beam (in the middle portions of continuous beams), can be made to resist the compression forces, and the tension can be carried by the steel in the tension side of the beam. These combined beam and slab units are called *flanged beams.* They may be **T** or **L** beams, depending on whether the slab is on both or only on one side of the beam (see Fig. 8.1).



(a) (b)

Fig. 8.1 Flanged beams: (a) T beam—1–Compression in concrete; 2–Tension steel; 3–Transverse steel; 4–Stirrups for shear; 5–Anchorage of stirrups; (b) L beam.

One should be aware that continuous T or L beams act as flanged beams only between the supports where the bending moments are conventionally taken as positive and the slabs are on the compression side of the beam. Over the supports, where the bending moments are negative, the slabs are on the tension side and here the beam acts only as a rectangular beam, with the tension steel placed in the slab portion of the beam. Thus at places of negative moments these beams have to be designed as singly or doubly reinforced rectangular beams as shown in Fig. 8.2.

In order to make the slab and beam act together, transverse steel should be placed at the top of the slab with sufficient cover for the full effective width of slab. This steel is also useful to resist the shear stresses produced by the variation of compressive stress across the width of the slab, as shown in Fig. 8.3.

The four main components that constitute these flanged beams (in addition to the hanger bars) for which strength calculations are necessary (Fig. 8.1):

1. The compression flange
2. Tension steel

3. Transverse steel in the slab for integral action
4. Stirrups for shear.

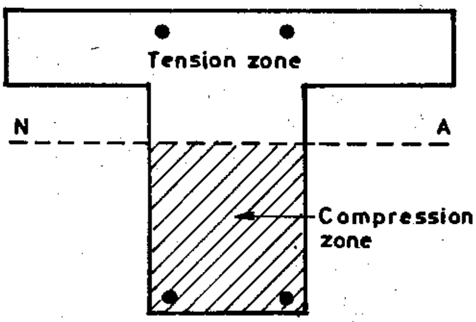


Fig. 8.2 Flanged beams over supports with negative moments.

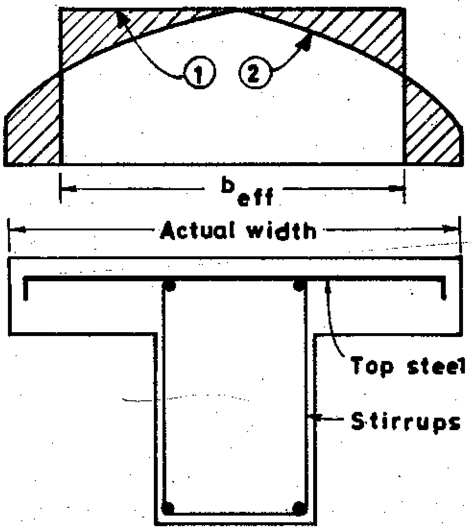


Fig. 8.3 Effective width of T beams: ① Actual stress distribution in compression flange, ② Assumed stress distribution in compression flange.

8.2 EFFECTIVE FLANGE WIDTH

An equivalent width of the slab with uniform stress distribution that can be assumed to act along with the beam for strength calculation is called *the effective width* of the flanged beam. The compressive stress in the flange just above the rib is higher than that at some distance away from the rib as in Fig. 8.3. The nature of this variation is very indeterminate, and the effective width concept that enables the use of an imaginary width of beam over which a uniform compressive stress is assumed to act is very useful.

The following recommendations given in IS 456: clause 22.1.2 regarding the effective width of flange can be considered as very conservative, and can be safely used in design.

1. The effective width of flange of T beams should be less than

Sagar Samaria

(i) $b_f = L_0/6 + b_w + 6D_f$ (8.1)

(ii) the actual width of the flange

2. For L beams, it should be less than

(i) $b_f = L_0/12 + b_w + 3D_f$ (8.2)

(ii) Actual width of the flange

3. For individual beams it may be taken as less than

(i) $b_f = \dfrac{L_0}{L_0/b + 4} + b_w$ or actual width for T beams (8.3)

(ii) $b_f = \dfrac{0.5L_0}{L_0/b + 4} + b_w$ or actual width for L beams (8.4)

where

b_f = the effective flange width

L_0 = the distance between the points of contraflexure, which can be considered to be the distance between supports in a simply supported span and 0.7 time the span distance between centre of supports (effective span) for continuous spans

b_w = the breadth of web

D_f = the thickness of flange

b = actual width of the flange

8.3 BASIS OF DESIGN AND ANALYSIS OF FLANGED BEAMS

The basic assumptions used for design of rectangular beams can be used for design of T beams also. The assumptions that plane section remains plane after bending and that failure takes place when the concrete strain reaches 0.0035 hold good for T beams also.

The most frequent problem in respect of T beams is the determination of the necessary steel required for a given section to carry a specified moment. Usually the full capacity of the T beam section in concrete failure will not be mobilised, and depending on the magnitude of the moment of resistance required, the neutral axis will occupy different positions with respect to the slab on top.

Three different cases (refer also IS 456, Appendix E.2) with respect to this position of the neutral axis and consequent design of T beams can be visualised (see Fig. 8.4).

Case 1 Neutral axis is within the flange. In this case the beam can be treated as a normal rectangular beam of width b_f and depth d.

Case 2 The neutral axis is below the flange, and the thickness of flange is small enough so that the stress in the concrete all over the slab is uniform and reaches the maximum value of the stress block $0.45f_{ck}$ (i.e. the minimum strain at the bottom of the slab reaches the value of 0.002 when the extreme fibre in compression reaches 0.0035).

Assuming that Fe 415 steel yields at a strain of 0.004 (Fig. 8.5), the following equation can be obtained:

$$\frac{0.004 + 0.0035}{d} = \frac{0.0035 - 0.002}{D_f}$$

$$\frac{D_f}{d} = 0.2$$

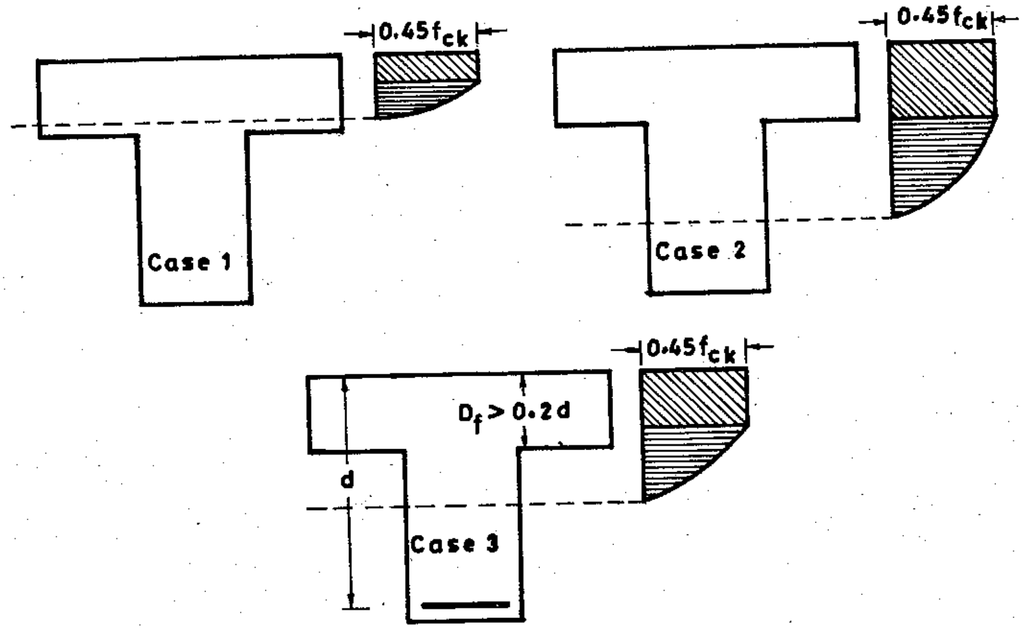


Fig. 8.4 Three possible positions of neutral axis in T beams.

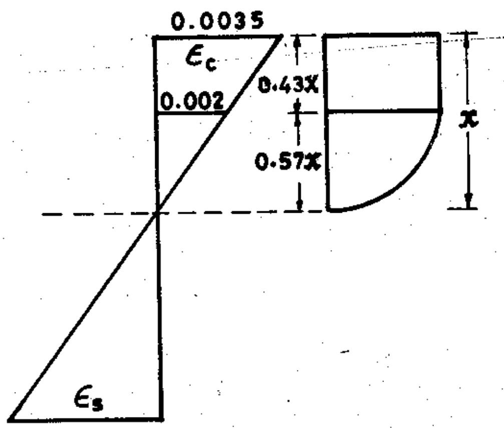


Fig. 8.5 Compression stress block in T beams.

Thus uniform stress distribution may be considered to occur in the flange when the flange thickness is less than $0.2d$.

This can also be explained in another way as follows: The limiting neutral axis depth for beams with steel Fe 250, Fe 415 and Fe 500 are $0.53d$, $0.48d$, and $0.46d$ respectively. As shown in Fig. 8.5 (explained in Section 2.6), the rectangular part of the stress-strain curve in concrete corresponding to a strain of 0.002 extends to $(0.43x)$. Hence the minimum depth for strain reaching the value of 0.002 will be

$$(0.43)\,(0.46d) = 0.198d \approx 0.2d$$

as obtained above.

Case 3 In this case the neutral axis is below the flange but the strain in the bottom of the slab is less than 0.002. This occurs when $D_f/d > 0.2$, so that the stress in the flange is also non-linear. To solve this case, no simple design method is available, but an approximation may be made, as explained later in this chapter.

8.4 T BEAM FORMULAE FOR ANALYSIS AND DESIGN

The procedure for design of a flanged beam consists in determining the value of x/d (the position of the neutral axis) and the value of A_s (the area of steel necessary to withstand the factored moment). The various cases that arise are examined in the following sections. The procedure is also summarised in Table 8.1.

8.4.1 CASE 1: NEUTRAL AXIS WITHIN THE FLANGE (DESIGN AS RECTANGULAR BEAM)

This is the most common case met within the design of buildings.

The formulae developed in Chapter 4 for design of rectangular beams hold good for this case also and can be used to find the position of the neutral axis. Thus, as shown in Section 4.11, an expression for x/d can be derived as

$$M_u = 0.36 f_{ck} \frac{x}{d}\left(1 - 0.416\frac{x}{d}\right) b_f d^2$$

Solving for x/d, we get

$$\frac{x}{d} = 1.2 \pm \sqrt{(1.2)^2 - \frac{6.68 M_u}{f_{ck} b_f d^2}} \tag{8.5}$$

where b_f = width of the flange.

One may use the following formulae also to check the depth of the neutral axis when the cross-section of the beam only is given:

$$x = \frac{0.87 f_y A_s}{0.36 f_{ck} b_f} \tag{8.6}$$

If $x/d < D_f/d$, then the neutral axis is inside the flange. x/d must be also restricted to the limiting values in the code as explained in Chapter 4. The procedure for analysis and design of flanged beams with neutral axis within the flange is the same as that for rectangular beams. The width of the beam is taken as the flange width.

8.4.2 CASE 2: NEUTRAL AXIS BELOW THE FLANGE ($D_f/d < 0.2$ FLANGE UNIFORMLY STRESSED)

As already explained in Section 8.3, when $D_f/d < 0.2$, the stress in the slab can be assumed to be uniform and equal to $0.446 f_{ck}$ with its centre of gravity $0.5 D_f$ below the top of the slab. As given in Appendix E2 of IS 456, the flanged section can be considered to be made of two elements (Fig. 8.6): a rectangular beam of width b_w and depth d and a slab of uniform compressive stress of width $(b_f - b_w)$.

Assuming b_f = breadth of flange, we have [$M_u = M_{u1}$ for rectangle [$(b_w \times d)$] + [M_{u2} for slab of width $(b_f - b_w) \times D_f$].

TABLE 8.1 PROCEDURE FOR T-BEAM DESIGN

T-Beam Design

$$X/d = 1.2 - \left[1.44 - 6.68M_u/(f_{ck}\,b_f\,d^2)\right]^{1/2} \quad \text{or } X = 0.87f_y A_s/(0.36 f_{ck} b_f)$$

(Case 1)	(Case 2) N.A. in web	(Case 3)
If $X/d < D_f/d$ (N.A. in flanges) Design as rectangular beams Find A_{st} by method (1) or (2) Method (1) $A_{st} = \dfrac{M}{(0.87f_y)\,Z}$ $Z = d(1 - 0.416\,\frac{x}{d})$ Method (2) Use SP 16 tables by using breadth $= b_f$ as for rectangular beams	If $X/d > D_f/d$ and $D_f < 0.2d$ Recalculate X/d from b_f/b_w, D_f/d, f_{ck} and b_w by $X/d = 1.2 - \left[1.44 - \frac{6.68M}{f_{ck}\,b_f\,d^2}\left(\frac{b_f}{b_w}\right) + 1.5\left(\frac{b_f}{b_w} - 1\right)\left(2 - \frac{D_f}{d}\right)\frac{D_f}{d}\right]^{1/2}$ $A_s = \dfrac{0.36 f_{ck} b_w X + 0.45 f_{ck}(b_f - b_w)D_f}{0.87 f_y}$	As in Case 2, but $D_f > 0.2d$ Substitute y_f for D_f $y_f = 0.15X + 0.65D_f$

Other useful formulae

(1) b_f required N. A. in flange $\qquad b_f = \dfrac{5.56M}{f_{ck}\,D_f(2d - D_f)}$

(2) Capacity in compression failure $\qquad M = k f_{ck} b_w d^2 + 0.45 f_{ck}(b_f - b_w)D_f\left(d - \frac{D_f}{2}\right)$

(3) Capacity in tension failure $\qquad M = 0.87 f_y A_s\left(d - \frac{D_f}{2}\right)$

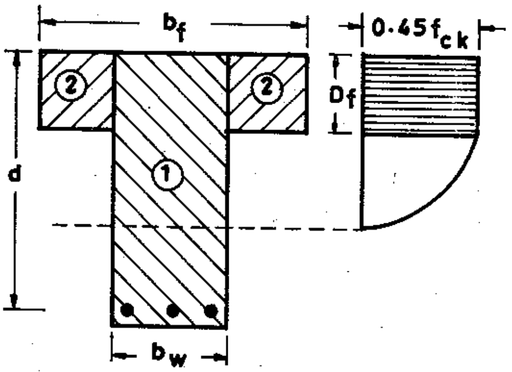


Fig. 8.6 Calculation of moment of resistance of T beams.

Taking moment of forces about the tension steel (Ref. IS 456, Appendix E2), we obtain

$$M_{u1} = 0.36 f_{ck} b_w x(d - 0.416x) \tag{8.7}$$

$$M_{u2} = 0.446 f_{ck}(b_f - b_w) D_f \left(d - \frac{D_f}{2}\right) \tag{8.8}$$

$$M_u = M_{u1} + M_{u2}$$

From equations (8.7) and (8.8), we have

$$M_u = 0.36 f_{ck} b_w x(d - 0.416x) + 0.45 f_{ck}(b_f - b_w) D_f \left(d - \frac{D_f}{2}\right)$$

Dividing throughout by (0.36) (0.42)$f_{ck} b_f d^2$, we get

$$\frac{6.68 M_u}{f_{ck} b_f d^2} = \frac{b_w}{b_f}\left(2.40 \frac{x}{d} - \left(\frac{x}{d}\right)^2\right) + 1.5\left(\frac{b_w}{b_f}\right)\left(\frac{b_f}{b_w} - 1\right)(2 - D_f)\left(\frac{D_f}{D}\right)$$

Rearranging the terms as a quadratic in x/d, we obtain

$$\left(\frac{x}{d}\right)^2 - 2.4\left(\frac{x}{d}\right) + \frac{6.68 M_u}{f_{ck} b_f d^2}\left(\frac{b_f}{b_w}\right) - 1.5\left(\frac{b_f}{b_w} - 1\right)\left(2 - \frac{D_f}{D}\right)\left(\frac{D_f}{D}\right) = 0$$

Let

$$k = \frac{6.68 M_u}{f_{ck} b_f d^2}\left(\frac{b_f}{b_w}\right) - 1.5\left(\frac{b_f}{b_w} - 1\right)\left(2 - \frac{D_f}{d}\right)\left(\frac{D_f}{d}\right)$$

Then solving for x/d, we obtain

$$\frac{x}{d} = 1.2 - \sqrt{1.44 - k} \tag{8.9}$$

Thus, if the equation for x/d derived for case 1 shows that the neutral axis is below the flange, then equation (8.9) derived for case 2 can be used to determine the position of x/d more accurately.

If x/d exceeds the limiting value of $(x/d)_{max}$, then only the limiting values should be used in further calculations.

Calculation of area of steel for case 2

The area of steel required is obtained by equating the compression to the tension, i.e.

$$C = T$$

where

$$C = 0.36 f_{ck} b_w x + 0.45 f_{ck} (b_f - b_w) D_f$$

$$T = f_{st} A_{st}$$

Assuming the yield of steel, we get

$$0.87 f_y A_{st} = 0.36 f_{ck} b_w x + 0.45 f_{ck} (b_f - b_w) D_f$$

Rearranging the expression for steel area for the moment, we obtain

$$A_{st} = \frac{C}{0.87 f_y}$$

where C is the quantity given above. Substituting for C, we have

$$A_{st} = \frac{0.36 f_{ck} b_w x + 0.45 f_{ck} (b_f - b_w) D_f}{0.87 f_y} \tag{8.10}$$

8.4.3 CASE 3: NEUTRAL AXIS BELOW THE FLANGE (D_f/d GREATER THAN 0.2-FLANGE NOT UNIFORMLY STRESSED)

When the flange thickness is greater than $0.2d$ and the neutral axis is below the flange, one cannot assume that the flange is uniformly stressed.

The expression for case 2 should be modified by substituting y_f for D_f in equations (8.8) for M_{u2} as suggested in IS 456, Appendix E2.2.1. Substitution of y_f for D_f is intended to bring case 3 similar to case 2 where most of the stress block is rectangular. The value of y_f is given by

$$y_f = (0.15x + 0.65 D_f) \tag{8.11}$$

but y_f should not be greater than D_f. (The derivation of the above expression is given in Section 8.4.4.) It should be noted that if x exceeds the limiting value of $(x/d)_{max}$, then only the limiting values should be used in the calculations.

Substituting y_f for D_f as in case 2 and taking b_f as the breadth of flange, the value of k in equation (8.9) is changed to k_1, i.e.

$$k_1 = \frac{6.68 M_u}{f_{ck} b_f d^2} \frac{b_f}{b_w} - 1.5\left(\frac{b_f}{b_w} - 1\right)\left(2 - \frac{y_f}{d}\right)\left(\frac{y_f}{d}\right)$$

$$\frac{x}{d} = 1.2 - \sqrt{1.44 - k_1} \tag{8.12}$$

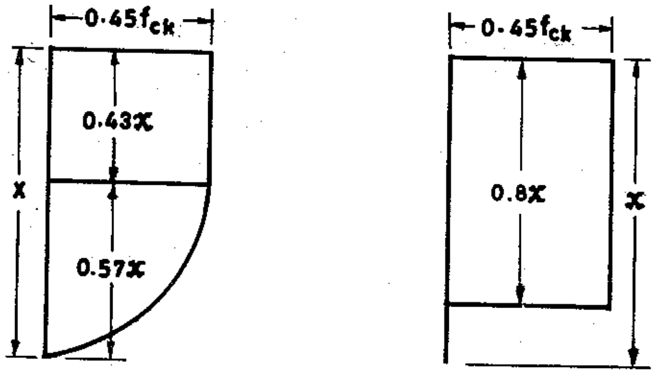
Equating tension to compression to find area of steel as givén in equation (8.10), one obtains

$$A_{st} = \frac{0.36 f_{ck} b_w (x) + 0.446 f_{ck} (b_f - b_w) y_f}{0.87 f_y}$$

giving the area of required steel.

8.4.4 DERIVATION OF EQUATION (8.11)

The equation $y_f = 0.15x + 0.65D_f$ has been derived on the basis of Whitney's stress block (Fig. 8.7) as follows: Let x be the depth of neutral axis and D_f the depth of the slab.



(a) (b)

Fig. 8.7 Equivalent stress block: (a) IS 456; (b) Whitney.

As y_f (to be substituted for D_f) will be a function of D_f and x, let $y_f = Ax + BD_f$. The constants A and B are solved by specifying the following two conditions to be satisfied by this equation:

(a) When $D_f = 0.43x$, $y_f = 0.43x$

(b) When $D_f = x$, $y_f = 0.80x$ (according to Fig. 8.7).

Solving for the two constants, one gets $A = 0.15$ and $B = 0.65$. Hence $y_f = 0.15x + 0.65D_f$. The moment of resistance is thus calculated as in case 2, using y_f instead of D_f.

The procedure for determining x/d and calculation of A_s for the various cases illustrated above is summarised in Table 8.1.

8.5 LIMITING CAPACITY OF T BEAMS BY USE OF DESIGN AIDS

An expression for the limiting capacity of a T beam section (by failure of concrete by compression) can be derived on the following assumptions for cases 1 and 2 above. The T beam is divided into rectangular beams of width b_w and effective depth d, and the remaining portion of the slab $(b_f - b_w)$ in width. The total limiting moment for cases 1 and 2 (see Sections 8.4.1 and 8.4.2) is given by

$$M_u = \text{(moment capacity of the rectangular beam)}$$
$$+ \text{(moment capacity of the remaining flange under uniform stress } 0.45f_{ck})$$

$$M_u = Kf_{ck}b_w d^2 + 0.45f_{ck}(b_f - b_w)D_f\left(d - \frac{D_f}{2}\right) \tag{8.13}$$

For Fe 415, this reduces to

$$M_u = 0.138f_{ck}b_w d^2 + 0.45f_{ck}(b_f - b_w)D_f\left(d - \frac{D_f}{2}\right) \tag{8.14}$$

For case 3 in Section 8.4.3, y_f should be substituted for D_f in equation (8.4). The values of the moment of resistance factor obtained by the above procedures have been tabulated in SP 16 in terms of $\frac{M_u\,(\text{lim})}{f_{ck}\,b_w d^2}$ for different values of b_f/b_w and D_f/d in Tables 57 to 59 of SP 16, for Fe 250, 415, and 500. The amount of steel necessary for these moments is not given in these tables. However, these tables are handy for the determination of the maximum moment the beam can take as singly reinforced beams in concrete failure, but are not useful for analysis and design when an arbitrary steel area (less than that necessary for the full utilisation of compression) has to be provided in the section. In most cases of practical design, the moment capacity of the T beam is much in excess of the applied moment and steel has to be provided in the beam only for this lower moment. Hence these tables are only of very limited practical use and serve to check the capacity of the 'concrete section' provided. Steel area to be provided in actual cases will be much less than that necessary for the full capacity. These calculations are illustrated through examples at the end of this chapter.

8.6 EXPRESSIONS FOR M_u AND A_s FOR PRELIMINARY DESIGN

A simple expression for moment of resistance of T beams can be obtained by assuming that the neutral axis is at the bottom of the slab and the lever arm length is $(d - D_f/2)$. The equation for preliminary design can be obtained as

$$M_u = 0.36 f_{ck} b_f D_f \left(d - \frac{D_f}{2}\right) \tag{8.15}$$

The area of flange width required for the neutral axis to be at the bottom of the flange can be obtained as

$$b_f = \frac{2M_u}{0.36 f_{ck} D_f (2d - D_f)}$$

or

$$b_f = \frac{5.56 M_u}{f_{ck} D_f (2d - D_f)} \tag{8.15a}$$

This required width may be compared with the effective width of the T beam to determine whether the neutral axis will be in the flange or below the flange.

An approximate and conservative value of the area of steel necessary for an applied factor moment M (which is generally less than the ultimate capacity) can be found by the simple expression

$$A_s = \frac{M_u}{0.87 f_y \left(d - \frac{D_f}{2}\right)} \tag{8.16}$$

Alternatively, if the area of steel is given (usually it will control the ultimate moment capacity), its magnitude is given by the simple equation

$$M_u = 0.87 f_y A_s \left(d - \frac{D_f}{2}\right) \tag{8.16a}$$

8.7 MINIMUM AND MAXIMUM STEEL IN FLANGED BEAMS

It should also be noted that the minimum percentage of steel to be provided in flanged beam as per IS 456: clause 25.5.1.1 is to be calculated only on the width of the web of T beam. The condition given in the above clause is that the percentage of minimum steel based on web width should be

$$\left(\frac{A_s}{b_w d}\right)100 = \frac{0.85}{f_y}$$

This works out to roughly 0.2 per cent with Fe 415 steel. In the same clause, IS stipulates that the maximum percentage of tension steel in T beams (based on web width) should not exceed 4 per cent. The comments made in Sections 9.10 and 9.11 regarding minimum areas of steel for rectangular beams are true for T beams also and should be studied carefully before adopting minimum steel for these beams.

8.8 TRANSVERSE REINFORCEMENT

As pointed out in the beginning of this chapter, transverse steel in the T beam, slab extending to full effective width of the slab is of utmost importance for the proper T beam action. IS 456: clause 22.1 specifies that this amount should be provided at the top of the slab and should not be less than 60 per cent of the main reinforcement of the slab at mid-span. BS 8110, however, specifies this area to be not less than 0.15 per cent of the longitudinal cross-sectional area of the flange. (The previous British Code CP 110 had specified this to be 0.3 per cent of the area.)

While the Indian specification is useful for slabs spanning in between the beams (where the main reinforcements are placed transverse to the beams), the British specification comes in handy for those beams where the main reinforcements of the slab are laid parallel to the beams and extra steel has to be provided as transverse steel if the beam has to be considered as T beams.

Thus, for example, the transverse reinforcement in a T beam with $D_f = 100$ mm and with the area of the main reinforcement of the slab at its middle as 450 mm^2/metre length, the transverse steel required according to IS and BS codes will be

1. As per IS 456, the required transverse steel = $0.6 \times 450 = 270$ mm^2/m
2. As per BS 8110, the required transverse steel = $\left(\frac{0.15}{100}\right) \times (100 \times 1000) = 150 \text{ mm}^2/\text{m}$

8.9 TABLES IN SP 24 FOR DESIGN OF T BEAMS

It has already been pointed out that in most cases of T beam design for buildings the neutral axis falls within the slab and the calculation of area of steel can be made as in the design of ordinary rectangular beams using Tables in SP 16, published in 1980. However, when the neutral axis falls below the bottom of the slab, these tables are not applicable. Hence design tables have been presented in the subsequent IS special publication SP 24 *Explanatory Hand Book on IS 456* published in 1983 for quick calculation of the area of steel for a given T beam to resist a specified bending moment.

8.10 DESIGN OF L BEAMS

An L beam is similar to a T beam except that the flange is available only or one side of the rib.

The same formulae derived for T beams apply to L beams also. The equivalent width formula for L beams is to be used to determine the width of the flange on these beams. However, as neither the areas nor the loadings of L beams are symmetrical about the centre of the beam, loading on these beams produces torsion. This torsion is resisted mainly by the rectangular beam part of the L beam. For this purpose, extra top steel and a fairly strong system of stirrups should be provided. These along with the bottom tension steel can resist the accidental torsion effectively. If these extra top steel and special stirrups are provided in L beams, they need not be specially designed for torsion under normal conditions of loading. Design for torsion is discussed in Chapter 18.

As L beams are located at the outside of buildings, provision of some extra top steel is also effective in resisting temperature stresses caused by exposure to the sun.

8.11 DESIGN OF FLANGED BEAMS IN SHEAR

As already pointed out in Section 7.3, shear in T and L beams is assumed to be taken up by the rectangular rib portion of the beam. Hence the design of these beams in shear is the same as that for rectangular beams already described in Chapter 7.

8.12 DETAILING OF REINFORCEMENTS

Detailing of the main reinforcements of rectangular and T beams carrying uniformly distributed load is shown in Figs. 8.8–8.10.

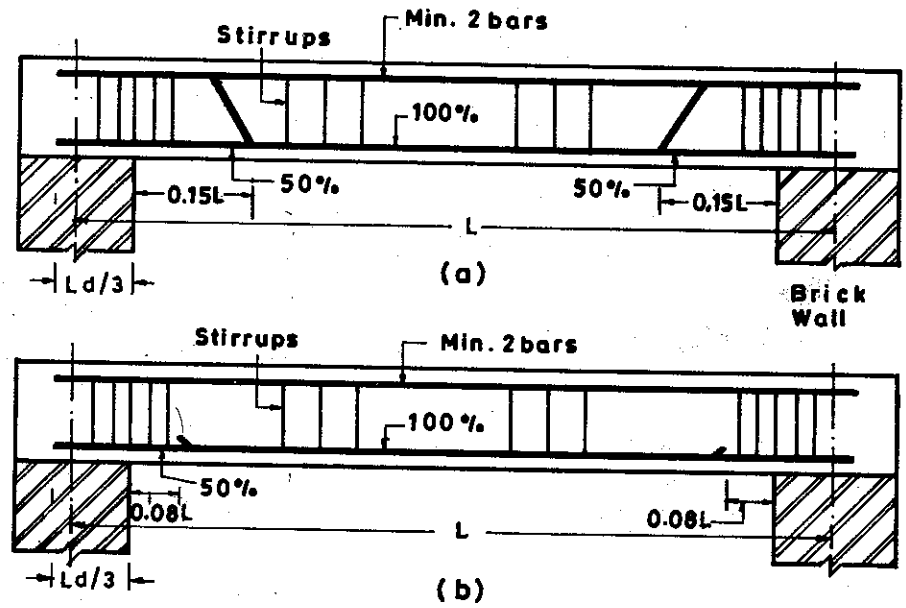


Fig. 8.8 Reinforcements for simply supported beams with UDL: (a) Using bent-up bars; (b) Using straight bars.

dth formula for
ever, as neither
am, loading on
r beam part of
rups should be
ion effectively.
/ need not be
on is discussed

op steel is also

aken up by the
ne same as that

mly distributed

bent-up bars;

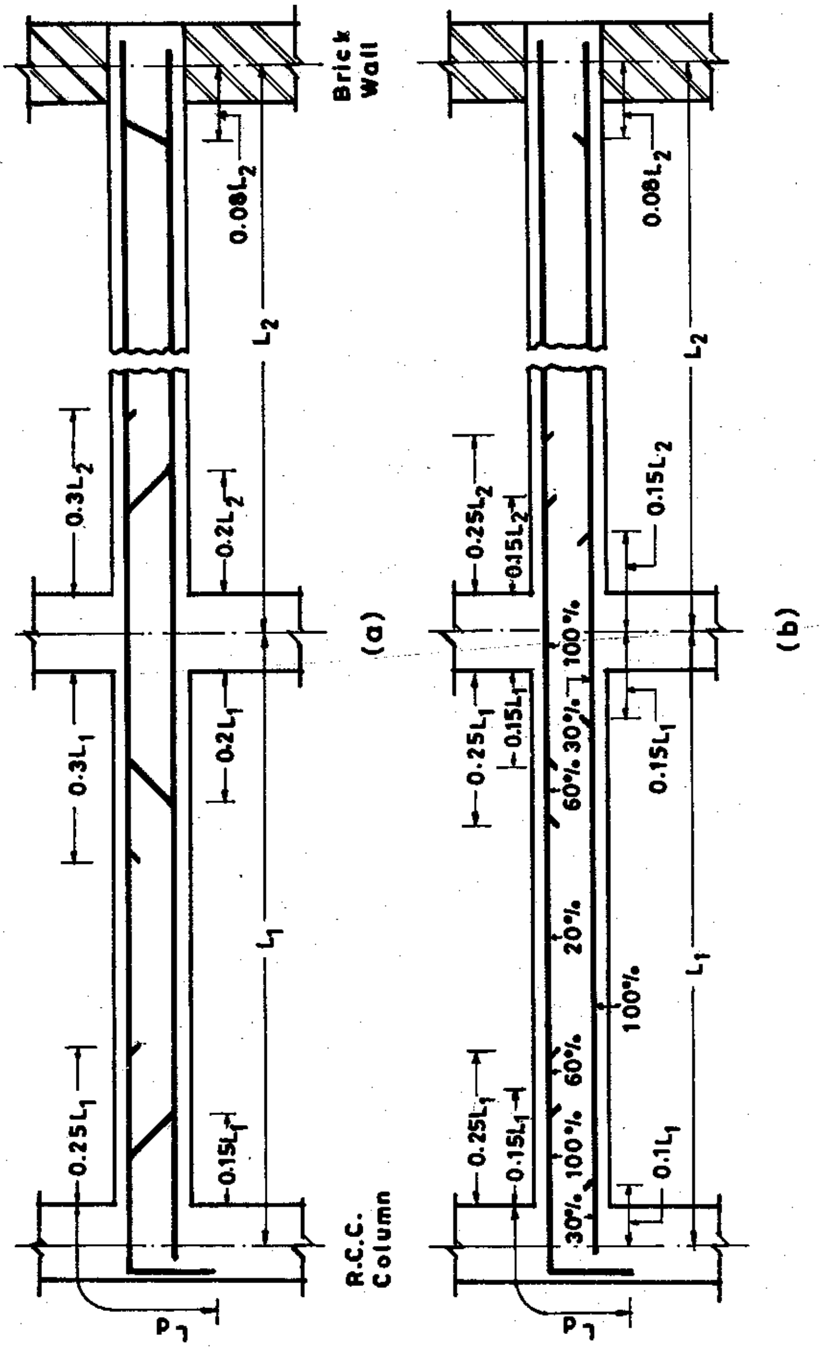


Fig. 8.9 Reinforcements drawing for continuous beams with UDL: (a) Using bent-up bars; (b) Using straight bars (Refer SP).

Sagar

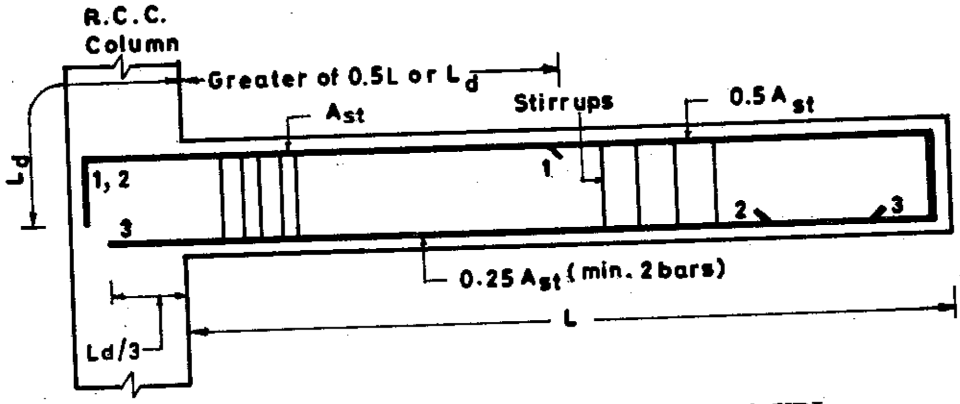


Fig. 8.10 Reinforcements for cantilever beams with UDL.

EXAMPLE 8.1 **(T-beam design)**

A series of beams placed at 2.5 m centres are supported on masonry walls and the effective span of the beam is 5 m. The slab thickness is 100 mm and ribs below the slab are 200 mm wide and 250 deep. If the slab and beams are so cast as to act together, determine the reinforcements at mid-span for the T beam to carry an imposed load of 5 kN/m² of the slab. Use Fe 415 steel and grade 20 concrete.

Ref.	Step	Calculations	Output
	1.	*Load on slab (characteristic loads)* DL = 0.100 × 25 = 2.5 kN/m² Finishes = 1.72 4.22 kN/m² LL = 5 kN/m²	
	2.	*Load on beams (characteristic loads)* DL from slab = 4.22 × 2.5 = 10.55 kN/m Ribs and finishes = (0.25 × 0.2 × 25) + 5% = 1.31 kN/m Total DL = 11.86 kN/m Total LL = 5 × 2.5 = 12.50 kN/m	w = 12.5 kN/m
	3.	*Factored load and moment* w_f = 1.5(12.50 + 11.86) = 36.54 kN/m $M = \frac{wl^2}{8} = \frac{36.54 \times 5 \times 5}{8} = 114.2$ kNm	M_u = 114.2 kNm

EXAMPL
Ref.
IS 456
Cl. 22.1.2
SP 16
Table
IS45
Cl.

Sagar

EXAMPLE 8.1 (cont.)

Ref.	Step	Calculations	Output
IS 456 Cl. 22.1.2	4.	*Effective width and depth* b_f = lesser of (a) $\frac{L}{6} + b_w + 6D_f = \frac{5}{6} + 0.20 + 6(0.1) = 1.63$ m (b) Centre to centre distance of beam = 2.50 m $b_f = 1630$ mm $d = 350 - \left(\text{cover} + \frac{\phi}{2}\right) = 300$ mm (approx.)	d = 300 mm
	5.	*Neutral axis depth* $\frac{x}{d} = 1.2 - \left(1.44 - \frac{6.68 M_u}{f_{ck} b_f d^2}\right)^{1/2}$ $= 1.2 - \left(1.44 - \frac{6.68 \times 114.20 \times 10^6}{20 \times 1630 \times 300^2}\right)^{1/2} = 0.12$ $x = 300 \times 0.12 = 36 \text{ mm} < D_f$	x inside flange
	6.	*Steel to be provided (design as rectangular beam)* $A_s = \frac{M_u}{0.87 f_y (d - 0.42x)} = \frac{114.20 \times 10^6}{0.87 \times 415\,(300 - 15)}$ $= 1109 \text{ mm}^2$	$A_s = 1109 \text{ mm}^2$
SP 16 Table 2	7.	*By SP 16* $\frac{M}{bd^2} = \frac{114.20 \times 10^6}{1630 \times 300 \times 300} = 0.78$ $p_t = \frac{100 A_s}{bd} = 0.227\%$ $A_s = \frac{0.227 \times 1630 \times 300}{100} = 1110 \text{ mm}^2$	$A_s = 1110 \text{ mm}^2$
IS456 Cl. 25.5.1	8.	*Check for minimum steel* Steel, based on breadth of web $= \frac{1110 \times 100}{200 \times 300}$ $p = 1.85\%$ Min. steel specified $= \left(\frac{0.85}{f_y}\right) \times 100 = 0.2\%$	As > Min

EXAMPLE 8.1 (cont.)

Ref.	Step	Calculations	Output
	9.	*Check for transverse steel* (a) IS: 60% of slab steel at mid-span (b) $BS = \frac{0.15bD}{100}$ $= \frac{0.15 \times 1000 \times 100}{100}$ $= 150\ \text{mm}^2/\text{m}$	$A_s = 150\ \text{mm}^2/\text{m}$

EXAMPLE 8.2 (Calculation of M_u limit for T beams)

Determine by approximate calculations the maximum moment of resistance by concrete failure of a T beam with the following dimensions: $b_w = 300$ mm; $b_f = 1500$ mm; $d = 600$ mm; $D_f = 150$ mm. Assume Fe 415 steel and $f_{ck} = 30\ \text{N/mm}^2$.

Ref.	Step	Calculations	Output
Method I	1.	*Approximate formula*	
		$M_u = 0.45 f_{ck} b_f D_f \left(d - \frac{D_f}{2}\right) = 0.45 \times 30 \times 1500$ $\times 150(600 - 75) = 1594$ kNm	$M_u = 1594$ kNm
Method II		*Use of formula in IS 456, Appendix E (cutting T beam as rectangular beam and rest of slab)*	
IS 456 E. 2.2.1	1.	$\frac{D_f}{d} = \frac{150}{600} = 0.25 > 0.2$ (Appendix E)	
	2.	$M_{u1} = 0.36 \frac{x_u}{d}\left(1 - 0.42 \frac{x_u}{d}\right) f_{ck} b_w d^2$ for rectangle	
IS 456 p. 109		$\frac{x_{u\,\max}}{d} = 0.48$ which reduces to $M_{u1} = 0.138 f_{ck} b_w d^2$ $M_{u1} = 0.138 \times 30 \times 300 \times (600)^2 = 447$ kNm	
	3.	$M_{u2} = 0.45 f_{ck} (b_f - b_w)\, y_f \left(d - \frac{y_f}{2}\right)$ for rest of slab $y_f = (0.15 x_u + 0.65 D_f)$ $= (0.15 \times 0.48 \times 600) + (0.65 \times 150)$ $= 43.2 + 97.5 = 140.7$ mm	

EXAMPLE 8.2 (cont.)

Ref.	Step	Calculations	Output
		$M_{u2} = 0.45 \times 30 \times 1200 \times 140.7 \times \left(600 - \frac{140.7}{2}\right)$	
		$= 1207$ kNm	
		$M_u = M_{u1} + M_{u2} = 447 + 1207 = 1654$ kNm	$M_u = 1654$ kNm
Method III		*Use of SP 16, Table 58*	
	1.	$\frac{D_f}{d} = \frac{150}{600} = 0.25, \quad \frac{b_f}{b_w} = \frac{1500}{300} = 5.00$	
SP 16 Table 58	2.	From the table,	
		$\frac{M_u}{f_{ck} b_w d^2} = 0.507$	
		$M_u = 0.507 \times 30 \times 300 \times (600)^2 = 1642$ kNm	$M_u = 1642$ kNm

EXAMPLE 8.3 **(Analysis of T beams)**

Determine the ultimate moment of resistance of the following T beam: $b_f = 450$ mm; $D_f = 150$ mm, $b_w = 300$ mm; $d = 440$ mm, $A_{st} = 2100$ mm^2. Assume $f_y = 415$ and $f_{ck} = 25$ (N/mm^2).

Ref.	Step	Calculations	Output
Method I		*By formulae*	
	1.	*Depth of N.A.*	
		$\frac{D_f}{d} = \frac{150}{440} = 0.34 > 0.2$	
		Use y_f instead of D_f	
	2.	*Calculate x for first approximation* (assuming x is within flange)	$x_m = .48d$
		$x = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f} = \frac{0.87 \times 415 \times 2100}{0.36 \times 25 \times 450}$	$= .48 \times 440$ $x_m = 211$ mm
		$= 187.2$ mm $> D_f$ (150)	
	3.	*Assume $x = 200$ and calculate y_f (equivalent depth of flange)*	$\frac{D_f}{x} = .75$
		$y_f = 0.15x + 0.65D_f = (0.15 \times 200 + 0.65 \times 150)$	
		$= 127.5$ mm	
	4.	*Recalculate depth of N.A. with y_f*	
		$x(0.36 f_{ck} b_w) + 0.45 f_{ck} (b_f - b_w) y_f = 0.87 f_y A_{st}$	

EXAMPLE 8.3 (cont.)

Ref.	Step	Calculations	Output
		$x(0.36 \times 25 \times 300) + 0.45 \times 25 \times 150 \times 127.5$ $= 0.87 \times 415 \times 2100$	
		$x = \dfrac{758205 - 215156}{2700} = 201$ mm	
		(Very near the value given in step 2)	
	5.	*Moment capacity by taking moments about steel*	
		M_{c1} = due to beam part $= 0.36 f_{ck} b_w \times (d - 0.42x)$	
		$= 0.36 \times 25 \times 300 \times 201\ (440 - 0.42 \times 201)$	
		$= 193$ kNm	
		M_{c2} = due to balance of the slab	
		$= 0.45 f_{ck}(b_f - b_w) y_f \left(d - \dfrac{y_f}{2}\right)$	
		$= 0.45 \times 25 \times 150 \times 127.5 \left(440 - \dfrac{127.5}{2}\right)$	
		$= 81$ kNm	
		$M_u = M_{c1} + M_{c2} = 193 + 81 = 274$ kNm	$M_u = 274$ kNm
Method II		The approximate formula for M_u is	
		$M_u = 0.87 f_y A_{st} \left(d - \dfrac{D_f}{2}\right)$	
		$= 0.87 \times 415 \times 2100 \left(440 - \dfrac{150}{2}\right) = 276$ kNm	$M_u = 276$ kNm
		(This method is normally used for all preliminary designs.)	
Method III		*By SP 16 tables for limiting capacity for T beams*	
SP 16 Table 58		$\dfrac{D_f}{d} = 0.34, \quad \dfrac{b_f}{b_w} = \dfrac{450}{300} = 1.5$	
		$M_u = 0.194 f_{ck} b_w d^2 = 0.194 \times 25 \times 300 \times 440^2$	
		$= 282$ kNm	$M_u = 282$ kNm

EXAMPLE 8.4 **(Determination of steel area for T beam)**

Find the area of steel required for the T beam with $b_f = 1500$ mm, $b_w = 300$ mm, $d = 600$ mm, $D_f = 150$ mm to carry an applied factored moment of 1080 kNm. (Note this moment is less than the limiting moment.) $f_{ck} = 30$ N/mm² and $f_y = 415$ N/mm².

Ref.	Step	Calculations	Output
Method I		*By formulae*	$x = \frac{.87f_y A_t}{.36f_{ck} b_f}$
	1.	*Determination of NA depth*	$.87 \times 415 \times A_t [d - .42x] = M$
		$$\frac{x}{d} = 1.2 - \left(1.44 - \frac{6.68M}{f_{ck}b_f d^2}\right)^{1/2}$$	
		$$\frac{6.68M}{f_{ck}b_f d^2} = \frac{6.68 \times 1080 \times 10^6}{30 \times 1500 \times (600)^2} = 0.44$$	
		Substituting and solving $x/d = 0.2$, we get	
		$x = 0.2 \times 600 = 120$ mm – NA in flange	NA in flange
	2.	*Determination of LA depth*	
		$$z = d\left(1 - 0.416\frac{x}{d}\right) = 600(1 - 0.416 \times 0.20)$$	
		$= 550$ mm	
	3.	*Determination of steel area*	
		$$A_s = \frac{M}{0.87f_y z} = \frac{1080 \times 10^6}{0.87 \times 415 \times 550} = 5439 \text{ mm}^2$$	$A_s = 5439$ mm²
Method II		*By SP 16* (applicable only for NA inside flange)	
	1.	x as above = 120 mm. NA inside flange.	
	2.	*Steel required is the same as that for a rectangular beam*	
		$$\frac{M}{b_f d^2} = \frac{1080 \times 10^6}{1500(600)^2} = 2.0$$	
SP 16 Table 4		$$p = \frac{100 \times A_s}{b_f d} = 0.605 \text{ per cent}$$	
		$$A_s = \frac{0.605 \times 1500 \times 600}{100} = 5445 \text{ mm}^2$$	$A_s = 5445$ mm²
Method III		*By approximate formula*	
		$$A_s = \frac{M}{0.87f_y(d - D_f/2)} = \frac{1080 \times 10^6}{0.87 \times 415 \times 525}$$	
		$= 5697$ mm²	$A_s = 5697$ mm²

EXAMPLE 8.5 **(Analysis of T beams)**

Determine the ultimate moment capacity of the following T beam: $b_f = 800$ mm, $D_f = 150$ mm, $b_w = 300$ mm, $d = 420$ mm, $A_{st} = 1470$ mm^2. Assume $f_y = 415$ N/mm^2 and $f_{ck} = 25$ N/mm^2.

Ref.	Step	Calculations	Output
Method I		*By use of formulae*	
	1.	*Assuming tension steel yields, find depth of NA*	
		$x = \dfrac{0.87 f_y A_s}{0.36 f_{ck}\, b} = \dfrac{0.87 \times 415 \times 1470}{0.36 \times 25 \times 800} = 73.7 \text{ mm} < D_f$	
		NA inside the flange	NA in flange
	2.	*Lever arm depth*	
		$z = (d - 0.42x) = [420 - (0.42 \times 73.7)] = 389$ mm	
	3.	*Moment of resistance*	
		$M_u = (0.36 f_{ck} bx)z = 0.36 \times 25 \times 800 \times 73.7 \times 389$	$M_u = 206$ kNm
		$= 206$ kNm	
Method II		*By use of SP 16 Tables*	
		NA inside slab: design as rectangle beam	
	1.	*Steel percentage*	
		$p = \dfrac{100 A_{st}}{b_f d} = \dfrac{100 \times 1470}{800 \times 420} = 0.44\%$	
	2.	*Moment of resistance*	
SP 16		For $p = 0.44\%$ and Fe 415,	
Table 3		$\dfrac{M_u}{bd^2} = 1.47$	
		$M = 1.47 \times 800 \times 420^2 = 207$ kNm	$M_u = 207$ kNm
Check		*Limiting capacity for concrete failure by SP 16 (M_{uc})*	
		$\dfrac{D_f}{d} = \dfrac{150}{420} = 0.36, \quad \dfrac{b_f}{b_w} = \dfrac{800}{300} = 2.70$ (say)	
SP 16 Table 58		$\dfrac{M_{uc}}{f_{ck}\, b_w d^2} = 0.334, \quad M_{uc} = 0.334 \times 25 \times 300\, (420)^2$	
		$= 442$ kNm	$M_{uc} = 442$ kNm
		M_{uc} is much more than the moment applied: It gives only the maximum capacity of the beam.	

EXAMPLE 8.6 **(Design of reinforcement for T beam)**

In a flanged beam, $b_f = 960$ mm, $b_w = 200$ mm, $D_f = 125$ mm, $d = 315$ mm, and factored moment = 240 kNm. Check the capacity of the beam to carry the load and if it is safe, design the steel required. Assume Fe 415 steel and grade 20 concrete.

Ref.	Step	Calculations	Output
	1.	*Check capacity as in Example 8.1*	
SP 16		$\frac{D_f}{d} = \frac{125}{315} = 0.4,\ \frac{b_f}{b_w} = \frac{960}{200} = 4.8,\ \frac{M_u}{f_{ck} b_w d^2} = 0.608$	
Method I	2.	*Design of steel by formulae*	
		(a) *Depth of NA*	
		$\frac{x}{d}$ (approx.) as rectangular beam $= 1.2 - \left(1.44 - \frac{6.6 M_u}{f_{ck} bd^2}\right)^{1/2}$	$M_{lim} = 213 \times 10^6$ N mm
		$\frac{x}{d} = 1.2 - \left(1.44 - \frac{6.6 \times 240 \times 10^6}{20 \times 960 \times (315)^2}\right)^{1/2} = 0.42$	
		$x = 0.42 \times 315 = 132$ mm > 125 – NA in web	NA in web
		(b) *Calculate y_f (Assume $x = 140$)*	
IS 456 E–2.2.1		$y_f = 0.15x + 0.65D_f = (0.15 \times 140) + (0.65 \times 125)$ $= 102.25$ mm	
		$\frac{y_f}{d} = \frac{102.25}{315} = 0.325$	$y_f = 102$ mm
		(c) *Find x/d with y_f/d for D_f/d in formula for NA in web*	
Eq. 8.12		$\frac{x}{d} = 1.2 - \left[1.44 - \frac{6.6 M_u}{f_{ck} bd^2}\frac{b_f}{b_w} + 1.5\left(\frac{b_f}{b_w} - 1\right)\left(2 - \frac{y_f}{d}\right)\left(\frac{y_f}{d}\right)\right]^{1/2}$	
		$= 1.2 - [1.44 - (0.83 \times 4.8)$ $+ (1.5 \times 3.8)(2 - 0.325) \times (0.325)]^{1/2}$	
		$= 1.2 - (1.44 - 4.00 + 3.10)^{1/2} = 0.465$	
		$x = 0.465 \times 315 = 146$ mm	$x = 146$ mm
		(d) *Steel area required*	
		$0.87 f_y A_{st} = 0.36 f_{ck} b_w x + 0.45(b_f - b_w) y_f f_{ck}$	
		$A_{st} = \frac{20(0.36 \times 200 \times 146 + 0.45 \times 760 \times 102)}{0.87 \times 415}$	
		$= 2515$ mm^2	$A_s = 2515$ mm^2

EXAMPLE 8.6 (cont.)

Ref.	Step	Calculations	Output
Method II		*Design of steel by approximate formula*	
	2.	$A_s = \dfrac{M_u}{0.87 f_y \left(d - \dfrac{D_f}{2}\right)} = \dfrac{240 \times 10^6}{0.87 \times 415 \left(315 - \dfrac{125}{2}\right)}$	
		$= 2632\ \text{mm}^2$	$A_s = 2632\ \text{mm}^2$

EXAMPLE 8.7 **(Analysis of a T beam)**

Calculate the moment carrying capacity of a T beam of the following dimensions: $b_f = 1100$ mm, $b_w = 400$ mm, $D_f = 200$ mm, $d = 650$ mm, $A_{st} = 8000\ \text{mm}^2$, $f_{ck} = 25\ \text{N/mm}^2$ for steel with Fe 415 and for steel with Fe 250.

Ref.	Step	Calculations	Output
		Case 1: Fe 415 steel	
	1.	*Find NA depth*	
		$x = \dfrac{0.87 f_y A_{st}}{0.36 f_{ck} b_f} = \dfrac{0.87 \times 415 \times 8000}{0.36 \times 25 \times 1100} = 291\ \text{mm}$	
		Neutral axis is in the web. Proceed as in Example 8.3:	NA in web
		Case 2: Fe 250 steel *Find NA depth*	
		$x = \dfrac{0.87 f_y A_{st}}{0.36 f_{ck} b_f} = \dfrac{0.87 \times 250 \times 8000}{0.36 \times 25 \times 1100} = 175\ \text{mm}$	
		Neutral axis is inside the flange. Proceed as in Example 8.4.	NA in flange

EXAMPLE 8.8 **(Analysis of T beam with compression steel)**

Calculate the ultimate moment of resistance of a T beam with the following dimensions: $b_f = 650$ mm, $b_w = 150$ mm, $d = 450$ mm, $D_f = 150$ mm, A_{st} (in tension) $= 2827\ \text{mm}^2$, A_{sc} (in compression) $= 314\ \text{mm}^2$, cover to compression steel $= 25$ mm, $f_{ck} = 30\ \text{N/mm}^2$; Fe = 415 grade steel.

Ref.	Step	Calculations	Output
		Solution from fundamentals	
	1.	*Position of NA* (Assume both A_{st} and A_{sc} yields as the area of compression steel is small)	
		$0.36 f_{ck} b x = 0.87 f_y (A_{st} - A_{sc})$	

EXAMPLE 8.8 (cont.)

Ref.	Step	Calculations	Output
		$x = \dfrac{0.87 \times 415 \times (2827 - 314)}{0.36 \times 30 \times 650} = 129 \text{ mm} < D_f$	NA in flange
	2.	Check strains in steels $\varepsilon_{st} = \dfrac{0.0035(450 - 129)}{129} = 0.0087$ (steel yields) $\varepsilon_{sc} = \dfrac{0.0035 \times 104}{129} = 0.0028$ (steel yields)	
	3.	*Calculation of lever arms and moments* Failure by yielding of tension steel Total M_u is given by $M_{u1} = 0.87 f_y (A_{st} - A_{sc})(d - 0.42x)$ $M_{u2} = 0.87 f_y (A_{sc})(d - d')$ $M_u = [(0.87 \times 415 \times 2513 \times 396) + (0.87 \times 415 \times 314 \times 425)] \times 10^{-6}$ $= 407.5$ kNm	$M_u = 407.5$ kNm

V. Imp **EXAMPLE 8.9** **(Design of T beam with compression steel)**

A T beam has to develop an ultimate moment of resistance of 450 kNm. Its web width is 200 mm, flange width 750 mm, slab thickness 100 mm, and total depth 550 mm. Determine the necessary steel for the section using M20 concrete and Fe 415 steel. Assume cover to centre of steel to be 50 mm.

Ref.	Step	Calculations	Output
Method I	1.	*Ultimate capacity of T beam:* *From fundamentals* Considering T beam with central part as a rectangular beam and the balance flanges under uniform stress, we bave $M = 0.138 f_{ck} b_w d^2 + 0.45 f_{ck}(b_f - b_w) D_f \left(d - \dfrac{D_f}{2}\right)$ $= 0.138 \times 20 \times 200(500)^2 + 0.45 \times 20 \times 550 \times 100\ (500 - 50)$ $= 138 + 222 = 360$ kNm	$M = 360$ kNm

EXAMPLE 8.9 (cont.)

Ref.	Step	Calculations	Output
Method II SP 16	1.	*By use of SP 16* Moment capacity can be calculated from SP 16	
Table 58		$\dfrac{M}{f_{ck}bd^2} = 0.359$ gives $M = 360$ kNm	
	2.	*Type of beam and reinforcement* 360 < 450 (Given) Beam should be a doubly reinforced beam Tension and compression steel are required	Beam to be doubly reinforced
	3.	*Calculation of tension steel for M = 360 kNm*	
Step 1 above		Tension steel required = (steel for rectangular beam for 138 kNm) + (steel for 222 kNm from flanges) LA for rectangular beam = $(d - 0.416x)$ x for Fe 415 = $0.476d$ $\text{LA} = [d - (0.416)(0.476)d]$ $= 0.8d$ $= (0.8 \times 500) = 400$ mm	$z_1 = 400$ mm
		LA for flanges $= \left(d - \dfrac{D_f}{2}\right)$ $= (500 - 50) = 450$ mm	$z_2 = 450$ mm
		Tension steel as singly reinforced T beam $A_{st1} = \dfrac{138 \times 10^6}{0.87 \times 415 \times 400} + \dfrac{222 \times 10^6}{0.87 \times 415 \times 450}$ $= 2322 \text{ mm}^2$	$A_{st1} = 2322 \text{ mm}^2$
	4.	*Compression steel by steel beam theory* $M = (450 - 360) = 90$ kNm $\dfrac{d'}{d} = \dfrac{50}{500} = 0.1, \quad x = 0.476d$ Compression steel strain $= \dfrac{0.0035(0.376)}{0.476} = 0.00276$	
SP 16 Table A		Stress in steel = 352 N/mm^2 $A_{sc} = \dfrac{90 \times 10^6}{352 \times 450} = 568 \text{ mm}^2$	$A_{sc} = 568 \text{ mm}^2$

EXAMPLE 8.9 (cont.)

Ref.	Step	Calculations	Output
	5.	*Tension steel by steel beam theory*	
		Tension steel stress = $0.87 \times 415 = 361$ N/mm^2	
		$A_{st2} = \dfrac{568 \times 352}{361} = 554$ mm^2	$A_{st2} = 554$ mm^2
	6.	*Total steel areas required*	
		Tension steel = $2322 + 554 = 2876$ mm^2	5T28 (3076 mm^2)
		Compression steel = 568 mm^2	3T16 (603 mm^2)
	7.	*Check percentages of steel*	
		Percentage of tension steel = $\dfrac{3076 \times 100}{200 \times 500} = 3.1$	
		Percentage of compression steel = $\dfrac{603 \times 100}{200 \times 500} = 0.6$	Design is satisfactory
		This is > 0.2 and < 4.0 per cent.	

EXAMPLE 8.10 **(Analysis: T beam with large compression steel)**

A T beam has flange width 1600 mm, web width 250 mm, thickness of slab 120 mm, effective depth 450 mm, area of tension steel (A_{st}) 4600 mm^2, area of compression steel (A_{sc}) 2938 mm^2, and cover to centre of steel 60 mm. Find the ultimate moment of resistance of the beam. Assume $f_{ck} = 20$ N/mm^2 and $f_y = 415$ N/mm^2.

Ref.	Step	Calculations	Output
		A_{sc} large and hence may not reach yield.	
		Use "Strain compatibility method"	
	1.	*Assume NA at bottom of slab x = 120*	
Text		Strain in compression steel	
Fig. 4.1 SP 16 Fig. 3		$\varepsilon_{sc} = \dfrac{0.0035 \times 60}{120} = 0.00175$	
		Steel stress (f_{sc}) = 315 N/mm^2	
		Compression in steel = $C_1 = (A_{sc})$ (Stress)	
		$C_1 = 2938 \times 315 = 925 \times 10^3$ N	
		Compression in concrete $C_2 = 0.36 f_{ck} bx$	
		$C_2 = 0.36 \times 20 \times 1600 \times 120 = 1382 \times 10^3$ N	
		Total $C = 925 + 1382 = 2307$ kN	$C = 2307$ kN
		Strain in tension steel = $\dfrac{0.0035 \times 330}{120} = 0096$	
		Steel reaches yield.	

EXAMPLE 8.10 (cont.)

Ref.	Step	Calculations	Output
		$T = 0.87 \times 415 \times 4600 = 1660 \times 10^3$ N	$T = 1660$ kN
		$C > T$. NA is within flange < 120 mm	
	2.	*Actual position of NA*	
		This has to be found by trial and error.	
		Assume $x = 90$ mm	
		$\varepsilon_{sc} = \dfrac{0.0035 \times 30}{90} = 0.0011,\ f_{sc} = 220$	
		$C_1 = 2938 \times 220 = 646 \times 10^3$ N	
		$C_2 = 0.36 \times 20 \times 1600 \times 90 = 1036 \times 10^3$ N	
		$C = 646 + 1036 = 1682$ kN $\approx T = 1660$	$C \approx T$
Text Fig. 4.2	3.	*Ultimate moment capacity*	
		CG of $C = \dfrac{(646 \times 60) + (1036 \times 0.42 \times 90)}{1682}$	C_2 acts at $0.42x$
		$= 46.33$ mm	(As T is known accurately, take moment of T.)
		$M_u = T$ (LA) $= 1660 \times 10^3\ (450 - 46.33)$	
		$= 670 \times 10^6$ N mm	$M_u = 670$ kNm

REVIEW QUESTIONS

8.1 What are T beams and L beams? What are the principal components of these beams?

8.2 Why do continuous T beams at supports have to be designed as rectangular beams?

8.3 What are the four main components for which designs are to be made in T beams?

8.4 Explain the concept of effective width of T beams and write down the formulae for the effective width of T beams.

8.5 What is the distance usually assumed for the distance between points of zero moments in a continuous T beam?

8.6 Indicate the three cases regarding position of neutral axis in the design of T beams.

8.7 How does one calculate the value of M_u (lim) for T beams? Under normal conditions will the factored moment in beams in buildings due to characteristic loads be greater or less than the M_u limit?

8.8 Can we provide steel for M_u (lim) or for the factored moment in a T beam?

8.9 What is the difference between analysis and design of T beams?

8.10 What are the functions of transverse steel in the slab portion of flanged beams? Write down the specifications with reference to IS and BS codes regarding the provision of transverse steel in T beams.

8.11 What are the assumptions made in the design of T and L beams for shear? Sketch the shape of stirrups to be used for L beams and T beams.

8.12 What is the minimum percentage of tension steel that should be provided in T beams? Is it based on the flange width or web width?

8.13 What is the maximum percentage of steel that is allowed with T beams?

8.14 The edge beams of a building are to be provided as E11 beams. What special considerations are to be taken in its design?

8.15 Give a simple and approximate formula that can be used to determine the area of steel for T beams for a given factored moment.

PROBLEMS

8.1 Determine the approximate position of the NA at ultimate load and the ultimate moment of resistance of a T beam with the following dimensions: Breadth of flange = 750 mm, breadth of web = 250 mm, depth of slab = 120 mm, effective depth of T beam = 450 mm, area of tension steel = 3500 mm^2. Grade 15 concrete and Fe 415 steel are used in the construction.

8.2 Determine the area of steel required in a T beam with the following dimensions for an ultimate moment of resistance of 460 kNm: Depth of slab = 100 mm, breadth of flange 700 mm, breadth of web 300 mm, total depth 550 mm. Assume f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2. Use 10 mm steel for stirrups and cover to steel 25 mm.

8.3 A floor system consists of a slab 100 mm thick cast integral on beams spaced at 2.5 m centre and spanning 6 m. The beam part has a width 300 mm and total depth of 550 mm. The floor is to be designed for a characteristic live load of 3.0 kN/m^2 and a load of 1.0 kN/m^2 for partitions and 0.72 kN/m^2 for finishes, excluding the dead load of the floor system. Calculate the main steel required (diameter and number of rods) at the mid-section of the beam assuming f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2.

8.4 Determine the moment of resistance of an isolated T beam with the following data: Width of flange = 1000 mm, width of web = 200 mm, depth of slab = 100 mm, effective depth = 500 mm, tension steel 5 Nos. of 28 mm dia (5T 28), compression steel 3 Nos. of 16 mm dia (3T 16). State clearly the assumption, if any, made in the calculations. Assume that grade 20 concrete is used.

8.5 An L beam has the flange width of 900 mm, with the thickness of slab 100 mm. The web below is 250 × 500 mm. Determine the areas of steel required for it to carry a limiting moment of 600 kNm. Assume f_{ck} = 20 N/mm^2 and grade 415 steel.

8.6 A floor system for an office consists of slabs over continuous beams *ABC* of constant cross-section with *AB* = 8 m and *BC* = 6 m spaced at 5 m intervals and supported at *A*, *B* and *C* on columns 300 mm × 300 mm in size. If the slab depth is chosen as 175 mm, design the beam at mid-span of *AB* and at support *B*. Assume a practical width for the beam: f_y = 415 N/mm^2 f_{ck} = 20 N/mm^2.

9

Design of Bending Members for Serviceability Requirements of Deflection and Cracking

9.1 INTRODUCTION

In addition to the two limit state conditions already dealt with (i.e. durability on exposure to the environments and ultimate strength at overloads), reinforced concrete structures must also satisfy the *serviceability conditions* under the action of the dead and live loads that act normally on the structure. Two of the important serviceability conditions are:

1. The member should not undergo excessive deformation (i.e. limit state of deflection).
2. The crack width at the surface in the reinforced concrete member should not be more than that which is normally allowed by codes of practice (i.e. the limit state of cracking).

Even though other limit states like limit state of vibration can be specified and are applicable to special structures such as bridges, the two conditions above are generally accepted as very important conditions to be satisfied by every structure under service loads. Codes also specify the partial safety factors for load combinations under which these are to be checked. According to IS 456, Table 12 (Table 2.1 of the text), the combinations of loads for serviceability conditions should be the largest of the following:

1. 1.0 DL + 1.0 LL
2. 1.0 DL + 1.0 WL
3. 1.0 DL + 0.8 LL + 0.8 WL (EL)

For control of deflection, two methods are usually described in codes of practices:

1. The empirical method of keeping the span-effective depth ratios of the members not more than those specified in the codes.
2. The theoretical method of calculating the actual deflection and checking it with the allowable deflection in codes of practice.

Similarly, for control of crack width, two methods are recommended:

1. The empirical method of detailing the reinforcements according to the provisions of the code regarding spacing of bars, minimum steel ratios, curtailment and anchorage of bars, lapping of bars, etc.
2. The theoretical method of calculating the actual width of cracks and checking whether they satisfy the requirements in the codes for the given environmental conditions.

Greater attention to deflection and cracking of concrete structures has to be given with the aid of modern methods of R.C. design structures, as these methods allow higher stresses than the conventional method, both in concrete and steel. During the past few decades, the maximum allowable stresses have nearly been doubled for steel and increased considerably for concrete. Thus, whereas most of the steel used in older R.C.C. members were only of grade Fe 250, in modern construction, steel of grades of Fe 415 and Fe 500 are very commonly used. This necessitates better control of deflection and cracking conditions.

9.2 DESIGN FOR LIMIT STATE OF DEFLECTION

Excessive deflection of beams and slabs is not only an eyesore in itself, but it can also cause cracking of partitions. As given in IS 456: clause 22.2, the commonly accepted limits of allowable deflections are:

1. A final deflection of span/250 for the deflection of horizontal bending members like slabs and beams due to all loads so as not to be noticed by the eye and thus is not an eyesore.

2. A deflection of span/350 or 20 mm, whichever is less, for these members, after the construction of the partitions and finishes etc., to prevent damage to finishes and partitions.

We shall discuss later the methods for estimating deflection by calculation, but now describe the empirical method to limit deflection.

9.3 EMPIRICAL METHOD OF DEFLECTION CONTROL IN BEAMS

One can roughly express the allowable deflection/span ratio of beams in terms of length/depth ratio as can be shown by the following derivation. Let the deflection of a simply supported beam under UDL be expressed by the formula

$$a = \frac{5}{384}\frac{wL^4}{EI} = \left(\frac{5}{48}\right)\left(\frac{wL^2}{8}\right)\left(\frac{L^2}{EI}\right) = \frac{5}{48}\left(\frac{M_{\max}}{I}\right)\frac{L^2}{E}$$

Putting $M/I = f/y$ and $y = d/2$, we get

$$\frac{a}{L} = \frac{5}{48}\left(\frac{2f}{d}\right)\left(\frac{L}{E}\right) = \frac{5}{24}\left(\frac{f}{E}\right)\left(\frac{L}{d}\right)$$

Thus deflection/span is a function of L/d, taking f and E as constant values.

If one were to assume allowable values of

$$f = 5 \text{ N/mm}^2, \qquad E = 10 \text{ kN/mm}^2, \qquad \frac{a}{L} = \frac{1}{350}$$

then the span/depth ratio can be obtained as

$$\frac{L}{d} = \frac{1 \times 24(10 \times 10^3)}{350 \times 5 \times 5} = 27$$

This means that assuming certain terms as constants, the allowable deflection/span ratio can be controlled by the span/depth ratio. This principle is used for specifying the span/depth ratio for control of deflection in beams.

9.3.1 BASIC SPAN-DEPTH RATIOS

The empirical procedure for control of deflection is to control the span to effective depth ratio. It assumes that the deflection of beams and slabs will depend on the following factors:

1. The span/effective depth ratios
2. Type of supports, as to whether simply supported, fixed or continuous
3. Percentage of tension steel or the stress level in the steel at service loads if more than the necessary steel is provided at the section
4. Percentage of compression steel provided
5. Type of beam (whether it is flanged or rectangular).

IS 456: clause 22.2 gives the values of the basic span/effective depth ratios to be used for beams and slabs with spans less than 10 m. Separate values have been given for cantilevered, simply supported and continuous beams and slabs. It may be noted that for ordinary two-way slabs supported on the four sides, the shorter span controls the deflection. The recommended values of the basic span/depth ratios in IS code are given in Table 9.1 and Fig. 9.1.

TABLE 9.1 BASIC VALUES OF SPAN-EFFECTIVE DEPTH RATIOS FOR DEFLECTION CONTROL OF BEAMS

Type of support	Rectangular sections	Flange beams
Cantilever	7	Multiply values for rectangular
Simply supported	20	by factor F_3
Continuous	26	(see Fig. 9.4)

For simply supported and continuous spans over 10 m, these ratios are to be multiplied by a factor F

$$F = \frac{10}{\text{span in metres}}$$

For cantilevers over 10 m in length, the actual deflection should be estimated by calculations, and the requirements for limit state of deflection checked.

9.3.2 MODIFICATION FACTORS OF BASIC SPAN/DEPTH RATIOS

For any given case, modifying factors have to be applied to the basic span/effective depth ratios to obtain the following effects:

1. Effect of percentage of tension steel and service stress in the steel (Modification factor F_1)
2. Effect of compression steel (Modification factor F_2)
3. Type of beam—flanged or rectangular (Modification or reduction factor F_3)
4. Effect of redistribution of moments, which affects steel stress at a section and consequently the value of F_1 as already calculated by formulae.

Flanged beams are first treated as rectangular beams with width equal to effective flange width and then the modification factor F_3 is applied since there is no concrete below the flange. The final allowable span effective depth ratio will be

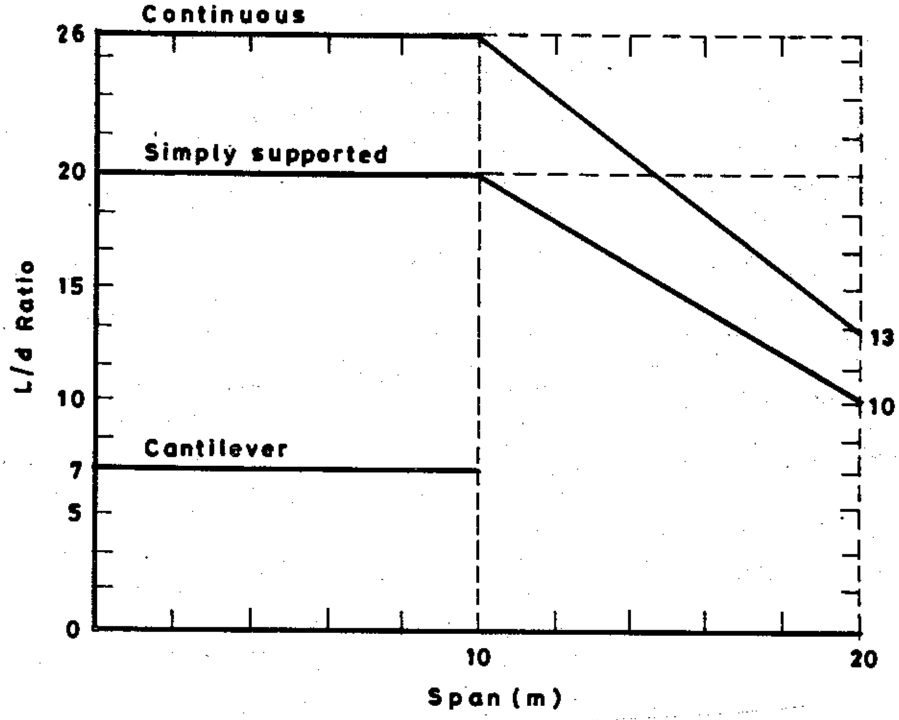


Fig. 9.1 Basic span-depth ratio for beams and slabs.

$$\frac{L}{d} \text{ ratio} = (\text{basic ratio})(F_1)\,(F_2)\,(F_3)$$

The steps for incorporating these corrections are now described.

9.3.3 PROCEDURE FOR CHECKING DEFLECTION

Step 1: Depending on condition of supports, choose the basic span/effective depth ratio from Table 9.1 if the span is 10 m or less. If it is greater than 10 m, reduce the values as indicated in Table 9.1 and Fig. 9.1.

Step 2: Determine modification factor F_1 which depends on the type of steel used (corresponding to the service stress in steel) and the percentage of steel required in the beam at the point of maximum deflection. The modification factor F_1 is obtained from IS 456 Fig. 3 which is given as Fig. 9.2 here.

A separate chart is given in IS for each grade of steel (corresponding to the service stresses used). It may be assumed that the service stress in a beam designed by limit state method will be approximately $5/8f_y$ (i.e. $0.625f_y$). In these charts the percentage of steel to be used is the theoretical percentage required at the point of maximum moment in the span, and not the actual percentage provided. In fact, using more steel than that required reduces the steel stress at service load and hence the deflection. Therefore, the factor F_1 should increase with the amount of steel provided over and above that required at the section.

For computerised design, the original equation, from which the values of F_1 given in Table 9.1

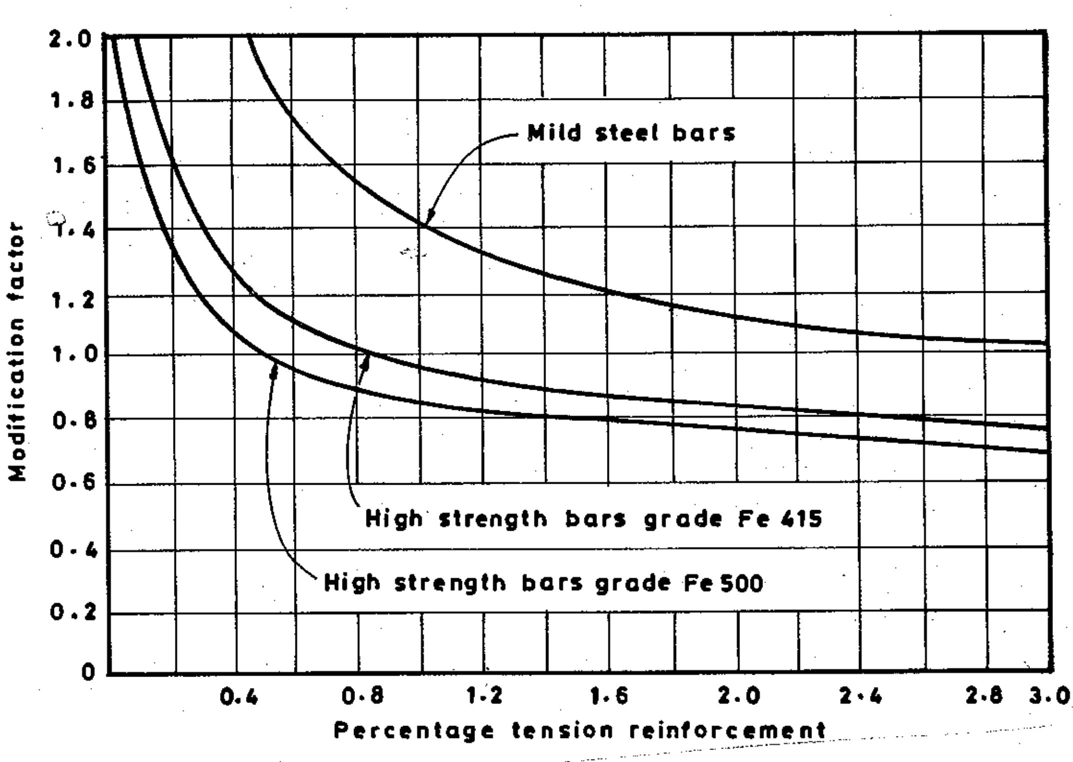


Fig. 9.2 Modification factor for tension reinforcement (IS 456 Fig. 3)

of IS 456 have been derived, should be used. This equation as given in SP 24, para 22.2.1 (page 55) is

$$F_1 = \frac{1}{(0.225 + 0.003 f_s + 0.625 \log_{10} p_t)} \tag{9.1}$$

where

$$p_t = \frac{100 A_s}{bd}$$

This equation is very useful for determining the value of F_1 more accurately than the one given in Fig. 3 of IS or when computer procedures are used for checking the deflection. The service stress f_s is obtained from the equation

$$f_s = \frac{5}{8} f_y \frac{A_s \text{ (required)}}{A_s \text{ (provided)}} \frac{1}{\beta} \tag{9.2}$$

where

$$\beta = \frac{\text{moment at section after redistribution}}{\text{moment at section before redistribution}}$$

Note: The equation corresponding to BS 8110 (1985) is much simpler and is given by

$$F_1 = 0.55 + \frac{477 - f_s}{120\left(0.9 + \dfrac{M}{bd^2}\right)} \le 2.0 \tag{9.3}$$

where f_s is as defined above and M is the maximum design moment.

In T beam the ratio of tensile steel to be used for determination of F_1 is to be based on the depth and the breadth of the flange and corrections of this is made by the special factor of T beams, viz. F_3, as given in step 4.

It should be remembered that with higher grades of steel used (i.e. higher service stresses) or with larger theoretical percentage of steel needed for the beam, the value of the multiplying factor F_1 becomes smaller, i.e. the necessary depth for the same span increases.

Step 3: Determine the modification factor F_2 corresponding to the percentage of compression reinforcement provided at the point of maximum moment. This is given in IS 456, Fig. 4 (page 58) and Fig. 9.3 of the text. The larger the percentage of compression reinforcement, the larger will be

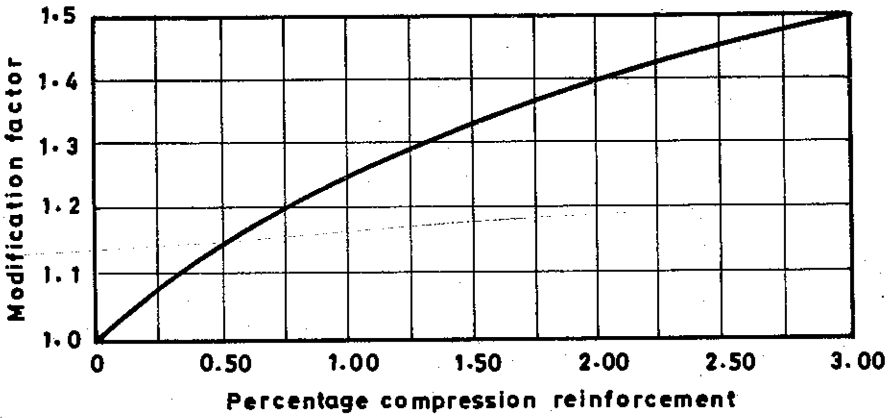


Fig. 9.3 Modification factor for compression reinforcement. (IS 456 Fig. 4)

the factor F_2. As in step 1, for T beams the width to be considered is the effective flange width. The compression steel can include all bars in the compression zone. It may be noted that increasing the percentage of compression steel is the best method to control deflection in critical cases as it can be done without decreasing the strain in tension steel required by limit state design.

The curve for F_2 is based on the formula given in SP 24, page 55, the formula being

$$F_2 = \frac{1.6p_c}{p_c + 0.275} \le 1.5$$

The corresponding expression in BS 8110 (1985) is

$$F_2 = 1 + \left(\frac{p_c}{3 + p_c}\right) \le 1.5$$

where p_c is the percentage of compression steel.

Step 4: As the factors F_1 and F_2 for flanged beams are calculated with the effective flange width (b_f), a reduction factor F_3 should be used to allow for the reduced area in the tension zone.

In normal rectangular beams the concrete in the tension zone also contributes to the stiffness of the member. The reduction factor depends on the ratio of web width (b_w) to effective flange width (b_f) as follows:

1. For $b_w/b_f = 0.3$ and below, the value of $F_3 = 0.8$
2. For $b_w/b_f = 1.0$, the value of $F_3 = 1.0$
3. For intermediate values, the value of F_3 is obtained by linear interpolation from Fig. 5 of IS page 58 (Fig. 9.4 in this text).

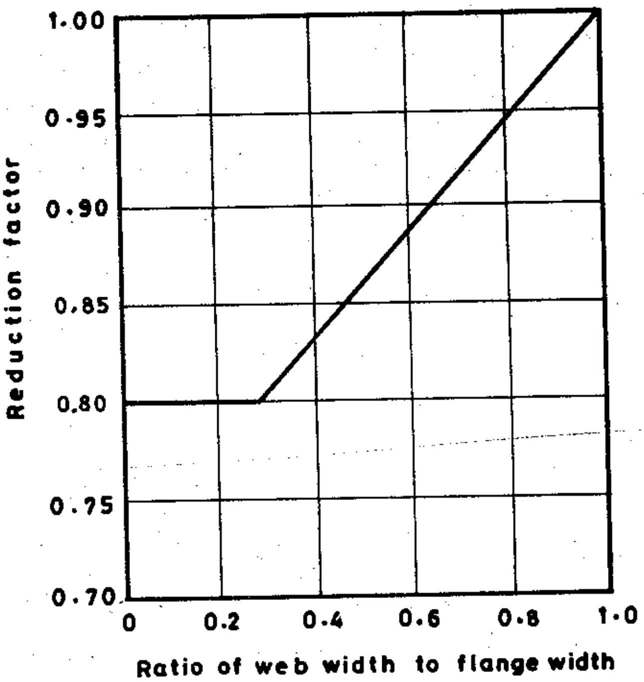


Fig. 9.4 Reduction factor for flanged beams. (IS 456 Fig. 5)

The equation for F_3 according to both IS and BS can be written as

$$F_3 = 0.8 + \frac{2}{7}\left(\frac{b_w}{b_f} - 0.3\right) \nless 0.8$$

Step 5: The final span/depth ratio allowed is

(Basic ratio) (F_1) (F_2) (F_3).

The ratio should not exceed this in the designed structure.

9.3.4 DEFLECTION CONTROL IN SLABS

Today, there are no accurate methods for estimating deflection of slabs. The empirical procedure recommended for control of deflection for slabs is the same as in beams, i.e. to limit the span/depth ratio as indicated. The same modifying factors as given above are used. In two-way slabs supported on all four sides, the shorter span of the two-way slab is taken for calculation with the amount of reinforcements in that direction at the centre of this span taken as the percentage of tension reinforcement.

As the ratio of percentage of steel in slabs is usually much smaller than in beams, the factor F_1 will be generally greater than unity. Usually, a percentage of steel equal to 0.4 may by assumed for slabs, so that a value of $F_1 = 1.4$ can be used for preliminary design of slabs using Fe 415 steel. Thus for slabs, one may assume a larger l/d ratio of $20 \times 1.4 = 28$ for simply supported and $26 \times 1.4 = 36$ for continuous slabs (see also IS 456: clause 23.1 and Section 11.1 in this text).

9.4 EMPIRICAL METHOD OF CONTROL OF CRACKING IN BEAMS

The allowable crack width in concrete depends on the environmental condition to which it is exposed. The usual values adopted in design have been given in Table 9.2.

Three aspects of cracking are of importance, namely the effect of cracking on the appearance, corrosion, and stiffness of the beam. Structural cracking can be classified according to the cause of cracking. These cracks may belong to any one of the following categories:

1. Flexural cracks
2. Shear or diagonal tension cracks
3. Splitting cracks along with reinforcement due to bond and anchorage failure
4. Temperature and shrinkage cracks.

The general specifications in codes regarding detailing of reinforcements are meant to reduce such cracking to allowable limits.

9.4.1 CRACKING OF R.C.C. SECTIONS UNDER LOADING

Reinforced concrete sections, when subjected to a bending moment, can be considered to undergo the following different phases in elastic behaviour.

Phase 1: Initially the sections remain uncracked as assumed in the theory of design of prestressed concrete section.

Phase 2: Subsequently the sections get partially cracked as assumed in the currently used theories for calculations of deflections under service loads. It is generally assumed that by limiting the tensile stress in concrete below the neutral axis, the width of cracking can also be controlled. Thus with grade 30 concrete, in the design of water tanks by CP 2007 by the Alternate Method of Design of calculating stresses in composite sections with $m = 15$, the maximum value of direct tension in concrete, is limited to 1.44 N/mm^2 and the value of bending tension to 2.02 N/mm^2, to control the width of cracks.

Phase 3: Finally the sections undergo extensive cracking, deflection and failure.

9.4.2 METHODS OF LIMITING CRACK WIDTHS

Under service loads the crack width in concrete should not be excessive. In general, under normal conditions, crack width at the surface of concrete should not exceed 0.3 mm for the sake of appearance. However, in 'aggressive' conditions of exposure for control of corrosion, the cracks at points nearest to the main reinforcement are generally specified not to be more than 0.004 that of the nominal cover to the reinforcement. In this connection it should be remembered that according to modern codes (IS 1343 (1980): clause 19.3.2), prestressed concrete is classified as type 1 to type 3 structures, depending on the allowable width of cracks as shown in Table 9.2. Reinforced concrete structures are a class by themselves and, with normal design, are assumed to develop crack widths up to 0.3 mm under service loads.

TABLE 9.2 CLASSIFICATION OF STRUCTURES ACCORDING TO CRACK WIDTH
(IS 1343 (1980))

Type of concrete structure	Allowable crack width
Prestressed: Type 1 (No tension in the section)	0
Prestressed: Type 2 (No visible cracking)	0 to 0.1 mm
Prestressed: Type 3	0.1 to 0.2 mm
Reinforced concrete normal design	0.3 mm

The 0.3 mm limit of crack width (appearance requirement) can be generally met by good practice of detailing reinforcements. Requirement of safety against environment (corrosion requirement) is to be satisfied by specifying the cover. This empirical method of crack control is applicable only to mild or moderate environments. For aggressive environments the crack width should be less than 0.3 mm and the actual values should be calculated by theory and checked against specifications.

9.4.3 PROCEDURE IN EMPIRICAL METHOD OF CRACK CONTROL

The empirical method of crack control assumes that cracks will be within the allowable limits, by detailing of steel in the structure according to the rules laid down in codes of practice. The important factors to be considered are the following:

1. Maximum and minimum spacing of reinforcements (bar spacing rules)
2. Maximum and minimum areas of steel in the member
3. Curtailment of reinforcement bars
4. Anchorage of reinforcement bars
5. Lapping of steel
6. Stress level in steel
7. Conformity with the general layout of steel reinforcement in the given structure according to accepted practice
8. Cover to reinforcement
9. Maximum and minimum sizes of steel to be used for the various types of steel in the member.

These are dealt with in the following sections.

9.5 BAR SPACING RULES FOR BEAMS

Considering factor 1, namely bar spacing rules, the major parameters that affect the crack width in concrete beams are as follows:

1. The distance of crack from the nearest reinforcement bar spanning the crack.
2. Distance from neutral axis of the cross-section.
3. Mean strain at the level of the section considered.

These parameters can be related to the distribution of the reinforcements in the beam in terms of the following factors:

1. Maximum horizontal bar spacing (the closer the bar spacing, the finer will be the cracks)
2. Minimum vertical and horizontal bar spacings
3. Arrangement of side reinforcements for members whose depth is larger than 750 mm
4. Corner distance to the nearest steel.

Modern codes give elaborate rules for each of the above. The usual recommendations can be summarised as in the following section.

9.5.1 MINIMUM BAR SPACING RULE FOR BEAMS

The bar spacing (horizontally and vertically) is given in IS 456: clause 25.3.1. It should not be less than the diameter of the largest bar and also not less than the maximum size of aggregate plus 5 mm, i.e. (h_{agg} + 5 mm) as shown in Fig. 9.5, for proper placement of concrete.

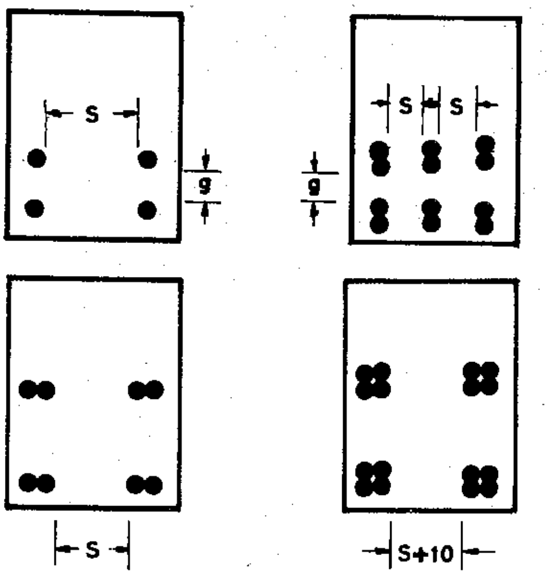


Fig. 9.5 Minimum spacing between group of bars (BS 8110).

9.5.2 MAXIMUM HORIZONTAL BAR SPACING RULES FOR BEAMS

The distance allowed between the main reinforcement at a section in the tension zone is a function of the stress level in the steel and the redistribution of moments to and from that section. It should not be more than those specified in Table 10, of IS 456: clause 25.3.2 (Table 9.3).

TABLE 9.3 CLEAR DISTANCES BETWEEN BARS (mm)
(Table 10 of IS 456)

f_y N/mm^2	Percentage redistribution to or from section considered				
	− 30	− 15	0	+ 15	+ 30
250	215	260	300	300	300
415	125	155	180	210	235
500	105	130	150	175	195

Note: BS 8110 (1985) uses the following expression:

$$\text{Clear spacing} = \frac{75{,}000\beta}{f_y} \quad \text{or} \quad \frac{47{,}000\beta}{f_s} \le 300$$

$$\beta = \frac{\text{Moment after redistribution}}{\text{Moment before redistribution}}$$

f_s = Estimated service stress in the reinforcement

This table gives the spacing for the given yield strength of steel and percentage of redistribution of moments. It should be remembered that with negative redistribution (i.e. the moment at the section is distributed to another section), the actual working stress in steel is bound to be higher than that with no redistribution, and hence the difference in spacings shown in the above table. For applying this rule, only bars whose diameters are at least 0.45 times the diameter of the largest bar in the sections are considered. This is called the *0.45 rule*.

9.5.3 CORNER DISTANCE RULE (BRITISH PRACTICE)

As shown in Fig. 9.6, the clear distance a_c from the corner of the beam to the surface of the nearest longitudinal bar should not be more than one-half the clear distance given in IS 456, Table 10 (Table 9.3).

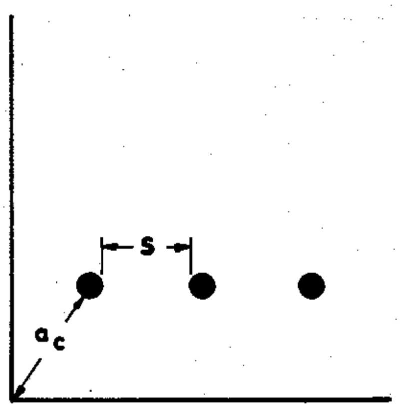


Fig. 9.6 Corner distance and spacing of bars.

9.5.4 SIDE REINFORCEMENT RULE

According to IS 456: clause 25.5.1.3, if the total depth of the beam is greater than 750 mm, side reinforcements are to be provided along the two faces as shown in Fig. 9.7 at a spacing s_b which should not be greater than 300 mm (or web thickness, whichever is less). The total area of such reinforcement shall also be not less than 0.1 per cent of web area. As can be visualised, this steel is a must on the tension part of the beam below the neutral axis.

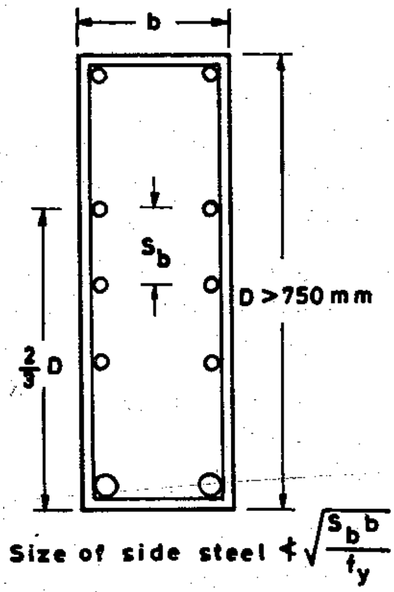


Fig. 9.7 Spacing of side reinforcement.

To guard against the bar yielding locally at a crack, in BS 8110, the spacing of the side reinforcement is limited to 250 mm and the diameter of the side reinforcement too should not be less than the value in mm given by the expression

$$\phi = \sqrt{\frac{s_b b}{f_y}}$$

where

s_b = vertical spacing in mm

b = breadth of the beam in mm

f_y = characteristic strength in N/mm^2

9.5.5 RULE FOR AGGRESSIVE ENVIRONMENTS

In aggressive environments, the rules given in 1 to 4 in the beginning of the sections are applicable only if f_y is limited to 300 N/mm^2. For higher values of f_y, allowable values of crack width are to be calculated and checked for safety.

9.6 BAR SPACING RULES FOR SLABS

9.6.1 SPACING OF MAIN REINFORCEMENTS

According to IS 456: clause 25.3.2, the only rule to be followed for spacing of main bars in slabs is that the spacing should not be more than three times the effective depth or 450 mm, whichever is smaller. However, BS 8110: clause 3.12.11.2.7 gives more elaborate guidelines for fixing the clear distances of the main reinforcements in slabs. Accordingly, the first rule is that the clear spacing to be adopted for the main bars in slabs should not exceed three times the effective depth or 750 mm, whichever is less.

No further checks are considered necessary for normal external and internal environments if

1. the thickness of the slab does not exceed 250 mm using Fe 250 steel,
2. the thickness of the slab does not exceed 200 mm using Fe 415 steel, and
3. the percentage of steel is less than 0.3 per cent.

Further, when none of the above conditions apply, the bar spacing should comply with IS Table 10 if the reinforcement percentage exceeds 1.0. However, for slabs with reinforcement percentage less than 1.0, the bar spacing should be equal to that given in Table 10 divided by the value of the reinforcement percentage.

For slabs exposed to aggressive environment, either adopt spacings as for normal conditions with f_y limited to 300 N/mm^2, or calculate theoretically the actual crack width that will occur and compare it with the allowable values for the environment.

9.6.2 SPACING OF SECONDARY REINFORCEMENT

According to IS, the spacing of secondary reinforcement in slabs should not exceed five times the effective depth or 450 mm, whichever is smaller (IS 456: clause 25.3.2).

9.7 MINIMUM PERCENTAGES OF STEEL IN BEAMS AND SLABS FOR CRACK CONTROL

The second requirement to be fulfilled for empirical crack control is the minimum percentage of steel to be provided.

1. The minimum amount of steel to be provided in beams is based on effective depth and is given in IS 456: clause 25.5.1 as

$$\frac{A_s}{bd} = \frac{0.85}{f_y}$$

(This works out to 0.20 per cent for Fe 415 steel and 0.34 per cent for Fe 250 steel. It is rather low as discussed in Section 9.11 and the amounts given in that section are recommended for practical designs.)

2. In slabs the minimum steel in the main direction should be 0.12 per cent of the total cross-sectional area when using high yield steel and 0.15 per cent when using mild steel (IS clause 25.5.2.1). This percentage is based on the total depth and gross area, and not on the effective depth.

3. The minimum percentage of secondary steel in slabs should be 0.12 per cent and 0.15 per cent on gross cross-section as above (IS clause 25.5.2.1), depending on the type of steel used.

4. Secondary steel has to be provided across the full effective width of flanges of flanged beams for integral action of slab and beam. This has already been dealt with under T beams. The amount of steel, according to British Code, should not be less than 0.15 per cent of the longitudinal cross-section of flange, and according IS, it should not be less than 60 per cent of the main steel at mid-span (IS 456: clause 22.1.1).

5. Minimum links and stirrups have to be provided in beams as given in Chapter 7 on shear reinforcement. Also, when compression steel has been provided in beams, it should be restrained by links of at least one quarter diameter of the largest compression bar and these links are to be provided at a space not greater than 12 times the diameter of the smallest compression bar.

IS 456: clause 25.5.1.6 gives the minimum quantity of shear reinforcement in beams as

$$\frac{A_s}{bs_v} \geq \frac{0.4}{f_y}$$

6. It should also be noted that according to IS clause: 25.5.1.1, the maximum percentage of steel allowed in beams is 4 per cent in tension and 4 per cent in compression, the percentage of steel in T beams being based on the rib width.

7. The maximum diameter of steel in slabs should not exceed one-eighth the total thickness of the slab (see IS clause 25.5.2.2).

9.8 CURTAILMENT, ANCHORAGE AND LAPPING OF STEEL

Requirements 3 to 5 for empirical crack control are meant to satisfy the (a) curtailment, (b) anchorage, and (c) lapping of reinforcement bars. Every bar should satisfy these requirements, and these are dealt with at greater length in Chapter 10.

9.9 STRESS LEVEL IN STEEL

The sixth criterion to control the size of cracks in reinforced members is the stress level (and the strain) in the tension reinforcement. The 0.3 mm crack width that is currently allowed in R.C. members goes with the presently allowed state of stress ($5/8f_y$) in the tension steel. If the crack width in R.C. members is to be less, then the stress level in the steel should be reduced or prestressing with "suitable levels of prestressing" should be adopted for the structure. Thus it is reasonable to expect lesser crack width with Fe 250 steel, and hence some authorities recommend the use of lower grade steel for thin structures like shells exposed to weather, water retaining structures, and structures subjected to adverse atmospheric conditions.

9.10 OTHER REQUIREMENTS

The three other requirements, namely, conformity with recommended layout of steel, cover to steel and size of reinforcements are described in the chapters dealing with the design of the various reinforced concrete members.

9.11 COMMENTS ON MINIMUM PERCENTAGES OF STEEL TO BE PROVIDED IN BEAMS AND SLABS

9.11.1 MINIMUM REINFORCEMENT IN BEAMS

IS 456: clause 25.5.1.1 stipulates that for isolated rectangular beams the area of tension reinforcement should not be less than that given by the formula

$$\frac{A_s}{bd} = \frac{0.85}{f_y}$$

which works out to 0.34 per cent for Fe 250 and 0.20 per cent for Fe 415 steel.

Some authorities, however, consider the above minimum quantities as inadequate for beams and recommend higher minimum percentages; this is now explained.

The minimum percentage of steel in beams should be arrived at from two considerations: First, from shrinkage and creep point of view, and second, from the probability of cracking of the beams on the tension side when the tension in concrete reaches a stress equal to modulus of rupture. The first criterion controls the minimum percentage of steel required in slabs.

The second criterion is meant to control the minimum steel required for beams of very large depths, which may be required for architectural reasons. The amount of steel required by theory of R.C.C. beams can be very small. The theoretical strength calculated on the basis of cracked section theory for such a beam with low reinforcement can be less than the strength calculated on the theory of a homogeneous concrete beam failing with the extreme fibre stress reaching modulus of rupture. Hence the required condition for minimum percentage of steel should be

$$\begin{bmatrix}\text{Strength as reinforced}\\ \text{concrete beam}\end{bmatrix} > \begin{bmatrix}\text{Strength as plain}\\ \text{concrete beam}\end{bmatrix}$$

According to IS 1343 (1980): clause 5.2.2, the value of modulus of rupture is given by

$$f_{cr} = 0.7\sqrt{f_{ck}}, \qquad M_{cr} = f_{cr}\left(\frac{I_g}{y_t}\right)$$

Putting

$$\frac{I_g}{y_t} = \frac{bd^2}{6}$$

$$M_{cr} = 0.7\left(\frac{bd^2}{6}\right)(\sqrt{f_{ck}})$$

The resistance moment of a reinforced rectangular beam (on cracked section theory) is given approximately by

$$M_u = A_s f_y \text{ (lever arm)} = A_s f_y\left(d - \frac{x}{2}\right)$$

Even though very small reinforcement ratio may give a lever arm as high as 0.95d, one may safely assume that

$$z = 0.70d$$

In rectangular beams the ratio of total depth to effective depth is very nearly unity so that the total depth can be approximated to effective depth, i.e.

$$A_s\,(0.87\,f_y)(0.70d) = 0.7\,\frac{bd^2}{6}\,\sqrt{f_{ck}}$$

$$\frac{A_s}{bd} = \frac{0.17\sqrt{f_{ck}}}{f_y}$$

For $f_{ck} = 25$,

$$\frac{A_s}{bd} = \frac{0.85}{f_y}$$

This value has been obtained by considering DL and LL only without taking into account accidental overloads. It may be noted that both BS and IS specify only the above-mentioned minimum steel ratios. But ACI code specifies the minimum steel as

$$\frac{A_s}{bd} = \frac{200}{f_y} \quad \text{in lb/m}^2$$

$$= \frac{138}{f_y} \quad \text{in SI units}$$

The minimum steel for the IS and ACI requirements works out to the following values:

Fe 250, IS: 0.34%; ACI: 0.55%
Fe 415, IS: 0.20%; ACI: 0.33%

In order to cater for any overloads due to vibration, settlement etc, the values recommended in the ACI code is found to be more acceptable in practice. In large beams designed with minimum steel as recommended by IS code, extensive cracking (but not failure) has been observed in some situations.

9.11.2 MINIMUM REINFORCEMENT IN SLABS

In slabs, because of its large cover/effective depth ratio the value of the total depth to effective depth when compared to beams can be as large as 1.25 so that the above coefficients will work out much smaller. Also, any overload will be better distributed laterally in slabs than in beams. Thus the minimum steel specified in IS code (0.15 per cent for mild steel and 0.12 per cent for high strength bars) based on shrinkage and temperature effects, and not on strength, can be safely accepted in practice for slabs.

9.11.3 MINIMUM STEEL FOR T BEAMS

Beams in monolithic slab construction act as T beams rather than as rectangular beams. Hence the minimum steel expression for these cases will be slightly different. For example, the ratio can be easily worked out for a typical T beam which is usually defined as one with the flange width four times the web width and flange thickness 20 per cent of the total depth and effective depth 0.9 the total depth. For such a beam, the I_{gr} works out to about 1.75 times that of the rectangle with the width equal to the web width and with the same depth as the T beam. Two cases can be identified.

Case 1: T beams with slab in compression. The value for minimum percentage will be of the same order as in the rectangular section (b_w and d).

Case 2: T beam at supports, with slab in tension. The value works out approximately to 50 per cent more than in case 1. However, in most T beams, the support sections are designed as doubly reinforced beam and the conditions for minimum steel are automatically taken care of in these sections.

9.12 RECOMMENDATIONS FOR CHOOSING DEPTH OF R.C.C. BEAMS

The following recommendations may be used in choosing the depth of R.C.C. beams to satisfy the rules regarding minimum and maximum steels in the section.

For beams with tension steel only, the depth selected should be such that the percentage of tension steel is not more than 75 per cent of the balanced steel.

In order to avoid cracking of beams under accidental overloads, the minimum steel provided should not be less than 45 per cent of the balanced steel. With Fe 415 steel and grade 15 concrete, the balanced percentage of steel is 0.72 and the minimum recommended steel works out to 0.33 per cent, as already stated. Other factors to be considered in choosing depth of beams are dealt with in Section 12.8.

9.13 SLENDERNESS LIMITS FOR BEAMS FOR STABILITY

It should also be noted that, according to IS 456; clause 22.3 in the conditions to be satisfied for stability, the compression side of the beam may be likened to a slender column that can buckle sidewards and horizontally. The conditions are:

1. In a simply supported or continuous beam, the clear distance between the lateral restraints should not exceed $60b$ or $250b^2/d$, whichever is less.

2. For a cantilever, the distance between the free end of the cantilever and the lateral restraint should not exceed $25b$ or $100b^2/d$, whichever is less.

EXAMPLE 9.1 **(Deflection control of T beam by empirical method)**

Check the deflection requirement for the following T beam continuous over 12 m spans. Flange width (b_f) = 1200 mm, web width (b_w) = 250 mm, effective depth 400 mm area of tension steel 1260 sq. mm, area of compression steel 402 sq. mm. Assume that (a) Fe 415 steel, and (b) Fe 250 steel are used in construction.

Ref.	Step	Calculations	Output
	1.	*Basic and actual span depth ratios* Continuous beam 12 m span	
		Actual $\dfrac{L}{d} = \dfrac{12000}{400} = 30$	
IS 456 Cl. 22.2.1		Theoretical L/d for L = 12 m = $\dfrac{26 \times 10}{12} = 21.6 < 30$	

EXAMPLE 9.1 (cont.)

Ref.	Step	Calculations	Output
	2.	*Modification factor F_1 (steel stress and percentage)*	
		Assuming steel provided is equal to that necessary for yield stress of Fe 415, steel,	
		Percentage of tension steel $= \dfrac{100A_s}{bd}$	
		$= \dfrac{100 \times 1260}{1200 \times 400} = 0.26$	
SP 24 page 55		$F_1 = \dfrac{1}{0.225 + 0.003f_s + 0.625 \log_{10}(p_t)} \le 2.0$	
		Using Fe 415, we obtain	
		$f_s = (5/8)f_y = 0.625 \times 415 = 259$ N/mm^2 (Fe 415)	
IS 456 Fig. 3		$F_1 = 1.56$	$F_1 = 1.56$ (a)
		Using Fe 250, we get	
		$f_s = 0.625 \times 250 = 156$ N/mm^2 (Fe 250)	
		$F_1 = 2.0$ (Check Fig. 3)	$F_1 = 2.0$ (b)
	3.	*Modification factor F_2 for compression steel*	
		Percentage $A_{sc} = \dfrac{100 \times 402}{1200 \times 400} = 0.08$	
		Even though this is less than 0.2%, the minimum required for consideration as compression steel, one may consider the above steel for reduction of deflection. By using the formula	
		$F_2 = 1 + \dfrac{p_c}{3 + p_c}$	
		given in Step 3, Section 9.3.3, we get	
		$F_2 = 1 + \left(\dfrac{0.08}{3 + 0.08}\right) = 1.026$	
IS 456 Fig. 4		Also, from the figure, $F_2 = 1.02$	$F_2 = 1.02$
	4.	*Reduction factor F_3 for T beam*	
IS 456 Fig. 5		$\dfrac{b_w}{b_f} = \dfrac{250}{1200} = 0.21 < 0.3$,	$F_3 = 0.80$
		$F_3 = 0.80$	

EXAMPLE 9.1 (cont.)

Ref.	Step	Calculations	Output
	5.	*Allowable span depth ratio* Allowable span depth ratio = (basic) (F_1) (F_2) (F_3) (a) Fe 415 = 21.6 × 1.56 × 1.026 × 0.80 = 27.6 (b) Fe 250 = 21.6 × 2.00 × 1.026 × 0.80 = 35.5 The span depth ratio of 30 is allowable for Fe 250 but not quite satisfactory for Fe 415 steel.	Not O.K. Satisfactory

EXAMPLE 9.2 **(Deflection control of beam by empirical method)**

A simply supported beam of 12 m span is provided with 1.6 per cent tension reinforcement (with no compression steel). The area of tension steel required by calculation is only 1.25 per cent. Determine the depths of beam for (a) Fe 500, and (b) Fe 415 steels if the deflection is to be within the safe limits.

Ref.	Step	Calculations	Output
	1.	*Basic span depth ratio* Simply supported beam = 20	
	2.	*Modification factor for tension steel* Service stress = $\frac{5}{8} f_y \left(\frac{A_s \text{ required}}{A_s \text{ provided}}\right)$ $f_s = \frac{5}{8} \times 500 \times \frac{1.25}{1.60} = 244 \text{ N/mm}^2$ for Fe 500 $= \frac{5}{8} \times 415 \times \frac{1.25}{1.60} = 202 \text{ N/mm}^2$ for Fe 415	
Eq. (9.1)		(a) For Fe 500, $F_1 = \dfrac{1}{0.225 + (0.003 f_s) + 0.625 \log_{10}\left(\dfrac{100 A_s}{bd}\right)(\text{prov.})}$ $F_1 = \dfrac{1}{0.225 + (0.003)(244) + 0.625 \log_{10} 1.6} = 0.922$	
IS 456 Fig. 3		From Fig. 3 for $p = 1.25\%$, $F_1 = 0.82$. (*Note*: The formula uses actual stress which may be less than $\frac{5}{8} f_s$. Hence, when steel provided is greater than that needed, it is preferable to use the formula.) (b) For Fe 415, using $f_s = 202$, N/mm², we get $F_1 = 1.04$	

EXAMPLE 9.2 (cont.)

Ref.	Step	Calculations	Output
	3.	*Modification factor for compression steel*	
		$F_2 = 1.0$	
	4.	*Allowable span depth ratios for 10 m*	
		For Fe 500, $(s/d) = 20 \times 0.922 \times 1 = 18.5$ (approx.)	
		For Fe 415, $(s/d) = 20 \times 1.04 \times 1 = 20.80$	
	5.	*Depths required (for over 12 m span > 10 m)*	
		Using Fe 500 steel, we obtain	
		$d = \dfrac{\text{span}}{18.5} \times \dfrac{12}{10} = \left(\dfrac{12000}{18.5}\right) 1.2 = 779$ say 780 mm	$d = 780$ mm
		Using Fe 415 steel, $d = 1.2\left(\dfrac{12000}{20.80}\right) = 693$ mm	$d = 693$ mm

EXAMPLE 9.3 (Crack control in beams—The empirical method)

Detail the steel for the following beam to conform to the empirical rules in IS 456 for crack control in beams. $b = 450$ mm, $A_{st} = 6$ Nos. of 25 mm Fe 415 [2950 mm^2), total depth of beam = 950 mm.

Ref.	Step	Calculations	Output
IS 456	1.	*Minimum tension reinforcement for beams*	
		$d = 950 - 25 - 12.5 \approx 910$ mm	
		$p = \dfrac{(A_s)100}{bd} = \dfrac{2950 \times 100}{450 \times 910} = 0.72\%$	
Cl. 25.5.1.1		Min. steel $= \dfrac{0.85 \times 100}{415} = 0.20\%$	
		$p > 0.20$ and $< 0.04bD$ (max : ten. steel)	Steel % O.K.
	2.	*Compression steel*	
Cl. 25.5.1.2		Nil	
	3.	*Side steel*	
Cl. 25.5.1.3		$D = 950 > 750$ side steel has to be provided.	
		Spacing not to exceed 300 mm and steel to be provided in $(2/3)D$ from tension face.	
		$(2/3)D = (2/3)\ 950 = 633$	
		Net distance = 633 − 40 = 593	
		Provide two rods on each face at 295 mm centre-to-centre from tension steel.	

EXAMPLE 9.3 (cont.)

Ref.	Step	Calculations	Output
		Diameter of steel $= \left(\dfrac{s_b b}{f_y}\right)^{1/2}$	
		$= \left(\dfrac{295 \times 450}{415}\right)^{1/2}$ 17.8 mm	
		Use 20 mm bars – 4 bars of 20 mm = 1260 mm^2	2T20 at 295 mm
		Total area to be not less than 0.1 per cent	
		Web area $= \dfrac{0.1 \times 450 \times 950}{100} = 427.5$ sq. mm	Area O.K.
	4.	*Spacing of tension reinforcement*	
Cl. 25.3.1		$[b - 2 \times \text{cover} - \phi] = 450 - 50 - 25 = 375$ mm	
		6 rods needs 5 spacings $s = \dfrac{375}{5} = 75$ mm	
		Clear distance = 75 – 25 = 50 mm	
		(a) Min. spacing = 25 mm < 50 mm	
		(b) Max. spacing – (415 steel) = 180 > 50 mm	Spacing O.K.
Table 10		Hence spacing is acceptable.	
		Notes: (i) Tension steel should also be checked for (a) anchorage and (b) curtailment.	
		(ii) Shear steel should be checked for (a) minimum quantity, and (b) spacing.	
		(iii) For BS 8110, the corner distance should also be checked.	

EXAMPLE 9.4 **(Crack control in slabs)**

A slab of total depth 150 mm is designed for two-way action over 5 m × 6 m. The reinforcements in short span consist of 12 mm at 240 mm spacing and that in the long span is 12 mm at 320 mm spacing. Check whether these satisfy IS 456 empirical rules for crack control.

Ref.	Step	Calculations	Output
IS 456	1.	*Minimum steel requirements (based on total depth)*	
		Area provided: 12 mm at 320 = 353 mm^2/m	
		Percentage $= \dfrac{353 \times 100}{1000 \times 150} = 0.24$	
		Percentage of 12 mm at 240 $= \dfrac{471 \times 100}{1000 \times 150} = 0.31$	
Cl. 25.5.2.1		Percentage > 0.12 (min) specified for Fe 415	Percentage O.K.

EXAMPLE 9.4 (cont.)

Ref.	Step	Calculations	Output
	2.	*Max. diameter of bar*	
Cl. 25.5.2.2		Max. diameter = $\frac{150}{8} = 18.75 > 12$ mm	Diameter O.K.
	3.	*Spacing of steel*	
		Mean depth (d) = 150 − 15 − 12 = 123 mm	
Cl. 25.3.2b		Max. spacing = 3 (effective depth) or 450 mm	
		= 3 × 123 = 369 mm < 450 mm	
		Hence spacing of 240 and 320 is acceptable.	Spacing O.K.

REVIEW QUESTIONS

9.1 Why is limitation of deflection in a structure called a serviceability condition? Name another serviceability condition commonly used in limit state design.

9.2 Write down the loading conditions for checking for serviceability.

9.3 Indicate the allowable limits of deflection in slabs and beams. Does allowable depth depend on span length?

9.4 Explain with derivation how span/depth ratio can be used to control deflection in beams.

9.5 What is 'basic span/depth ratio'? Would the initial depth you assume in your design of a T beam be greater or less than that for a rectangular beam from allowable deflection in beam?

9.6 What are the major factors that affect deflection and how do these affect deflection in beams?

9.7 What are the methods available in IS 456 to ensure that the deflection of the beam one has designed is within allowable limits?

9.8 How does one use the curves given in IS 456 for modification factors to check deflection in computer-aided design of beams?

9.9 In two-way slabs which of the spans (the shorter or larger) is taken for checking deflection?

9.10 What is the magnitude of crack width allowed in concrete structures?

9.11 What methods are available in IS for control of crack width in R.C. members? Are the methods and formulae for calculation of crack width given in IS code?

9.12 Discuss briefly the rules given in IS 456 regarding the following:
(a) Maximum and minimum steel in R.C.C. beams
(b) Minimum and maximum spacing of steel in R.C.C. slabs
(c) Stress level in steel at working load in structures designed by IS 456.

9.13 When will one provide side reinforcement in beams? What are the specifications regarding its position?

9.14 Comment on the formula given in IS 456 for minimum area of reinforcement of beams.

9.15 What broad practical rules would you use in choosing a depth for an R.C.C. beam?

PROBLEMS

9.1 A three span continuous T beam section with equal spans of 6.5 m has the following dimensions: Breadth of flange 1150 mm, breadth of web 250 mm, effective depth 390 mm, area of tension steel required 1234 mm^2, area provided 1260 mm^2, and area of compression steel provided 402 mm^2. Check the beam for deflection requirement using empirical rules assuming grade 20 concrete and Fe 415 steel.

9.2 A simply supported rectangular beam of 14 m span is of breadth 300 mm and effective depth 750 mm. Tension steel required is 3082 mm^2 and that provided is 3220 mm^2, 2 Nos. of 16 mm bars are provided as compression steel. Check the beam for deflection consideration according to IS 456 assuming grade 20 concrete and Fe 415 steel.

If the depth cannot be increased in a beam, suggest two methods by which deflection requirements for the beam can be satisfied.

9.3 A rectangular beam has a breadth of 250 mm and effective depth of 700 mm. The area of the tension steel provided is 3927 mm^2 and that of the compression steel is 981 mm^2. Check the deflection requirements for the beam according to IS 456, if it has a simply supported span of 12 metres. f_y = 415 N/mm^2.

9.4 A beam is 300 × 800 mm in cross-section. It has 4 Nos. of 20 mm bars as tension steel placed with the centre of the steel 50 mm from the bottom. The maximum bending moment at mid-span has been reduced by 10% by redistribution. Shear steel consists of 12 mm bars placed at 700 mm and the hanger bars consist of two 15 mm bars. Check whether the distance between the bars and the corner distances are satisfied for safety against cracking. Determine the size and spacing of any other steel that has to be provided for the section according to IS 456. Comment on the size and spacing of shear reinforcement.

9.5 A beam is 650 × 1500 mm. Draw a section of the beam and sketch the different types of steel that you will provide for the section, indicating the minimum diameter and spacing admissible according to IS 456. Assume that the concrete is made with 20 mm coarse aggregates.

9.6 Comment on the formula for minimum area of tensile steel as provided by IS 456 and compare it with the amount recommended by ACI code. Should the minimum percentage of area of tension steel to be provided for a beam be more or less than that should be provided for a slab? Give reasons.

A continuous beam of ABC of depth 500 mm and width 300 mm has been designed and detailed with the following arrangement of Fe 415 reinforcements:

Point	Redistribution of moments	Spacing of steel
Support B	– 20%	80 mm
Mid-span AB	+ 3	90 mm
Mid-span BC	0	160 mm

Check whether the above distribution of steel satisfies the provisions for cracking in beams according to IS 456.

9.7 A T beam is as shown in Fig. P.9.7. Calculate the minimum and maximum areas of Fe 415 steel that is allowed according to IS 456 (1978).

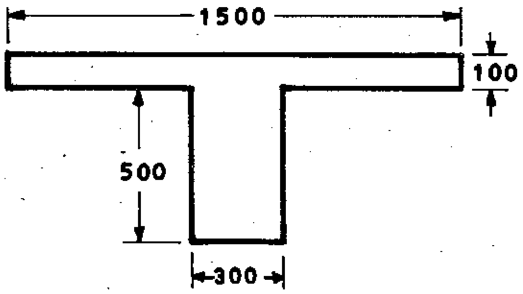


Fig. P.9.7.

9.8 Determine the maximum area of tension steel allowed in a beam 300 mm wide and 550 mm deep.

9.9 The maximum size of aggregates to be used for a reinforced concrete beam is 20 mm. Determine the minimum horizontal and vertical distances between (a) individual bars, (b) horizontal pairs, and (c) bundles of four bars.

9.10 Check whether the following arrangements of reinforcement satisfy the bars spacing rules in beams shown below. Assume that cover is 40 mm.

Indicate any modification that should be made.

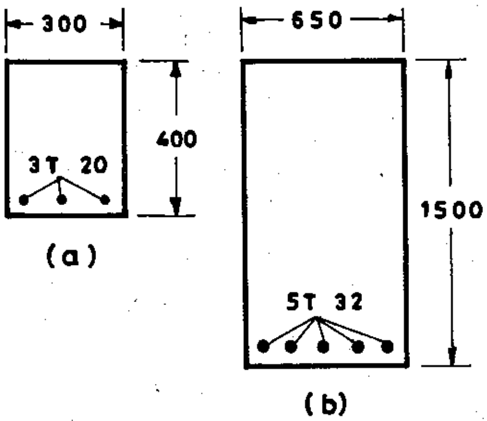


Fig. P.9.10.

9.11 Determine the minimum amounts of main and secondary reinforcements that should be provided for a slab of 150 mm total depth, clear cover to steel 20 mm. Assume that Fe 250 steel of 10 mm dia (R 10) is used as reinforcement.

9.12 Examine the following cases and give reasons for your decisions.

Case 1: The cross-section of a simply supported beam is 150 × 500 mm and it has a span of 10 m. Does it satisfy stability conditions?

Case 2: A cantilever beam is of 2.5 m in length and 110 × 600 mm in cross-section. Can it be allowed to be constructed?

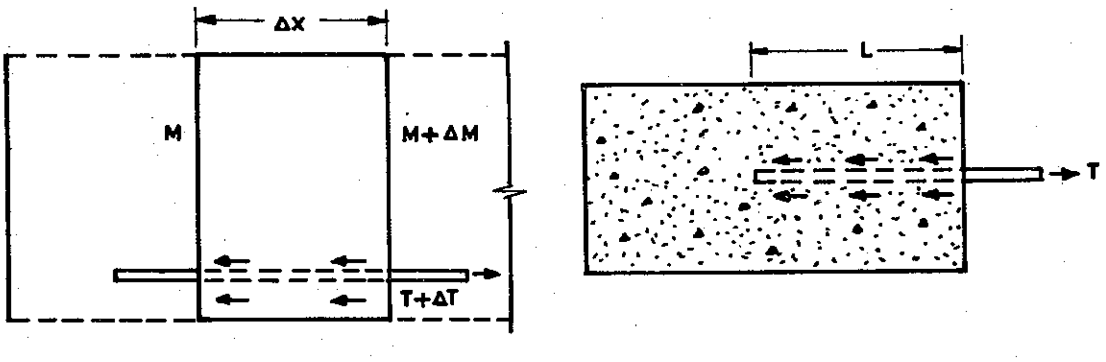
10

Bond, Anchorage, Development Lengths and Splicing

10.1 INTRODUCTION

The term 'bond' in reinforced concrete design refers to the adhesion or the shear stress that occurs between concrete and steel in a loaded member. It is the bond between steel and concrete that enables the two materials to act together without slip. The assumption that in a R.C.C. beam plane sections remain plane even after bending will be valid only if there is perfect bond or no slip between concrete and steel.

The magnitude of this bond stress at a point is called *local bond.* It varies along a member depending on the variation of bending moment as shown in Fig. 10.1. Similarly, in order to develop the full tension in the steel placed at the mid-section of a beam, it should be properly anchored on both sides of the section so that the full tension capacity of the steel reinforcement is developed.



(a) (b)

Fig. 10.1 Bond stresses: (a) Local (flexural) ; (b) Anchorage (average).

The average stress that acts along this anchorage length is called the *average anchorage bond.* Even though local bond varies along the length of the anchorage, its 'average value' is taken for design. The length or extension that should be provided on either side of the point of maximum tension in the steel so that the average bond stress is not exceeded, is called the *development length in tension.* Development length should be ensured in compression steel also. This length for development of compressive stress in steel is called *development length in compression.*

Till recently, since smooth mild steel bars were used as reinforcements, both local bond and development lengths were important and they had to be checked separately in routine designs. However, with the wide use of high bond bars (where the mechanics of bond is more complex and the action is not only adhesion of steel with concrete, but also mechanical locking by the projections on the steel bars as shown in Fig. 10.2), more emphasis is laid on development length requirements than on local bond. Thus, IS 456: clause 25.2.2 deals with requirements for proper anchorage of reinforcements in terms of development length, L_d only.

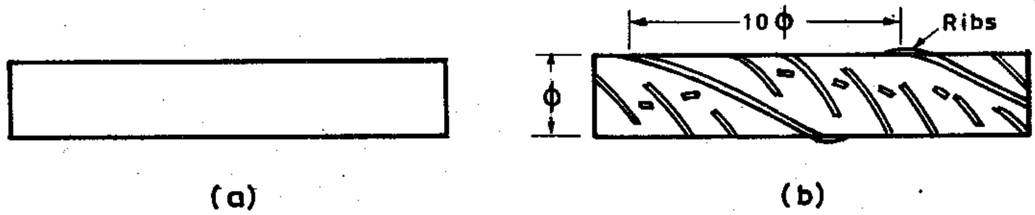


Fig. 10.2 Nature of bond in reinforcement bars: (a) Smooth bars; (b) Deformed bars.

Hooks, bends, extensions etc. provided at the ends of bars are sometimes referred to as *end anchorages*, and their anchorage length is denoted by the symbol L_a.

10.2 LOCAL (OR FLEXURAL) BOND

Local bond (also known as flexural bond) at a point is the rate of change of tension in the steel at a given location in a R.C.C. member. With mild steel smooth bars where adhesion and friction are the main components of local bond, this is important, and an expression for the magnitude of flexural bond at a point can be derived as follows: In a distance dx over the length of the beam, let the increase in tension be equal to T as given by Fig. 10.1. Then we get

$$T = \frac{dM}{jd}$$

Let u be the local bond stress and ΣO the perimeter of the steel area provided. Equating the forces, we obtain

$$u\,(\Sigma O)(dx) = T = \frac{dM}{jd}$$

$$u = \frac{dM}{dx}\left(\frac{1}{\Sigma Ojd}\right)$$

Therefore,

$$\text{Local bond stress} = \frac{V}{\Sigma Ojd}.$$

However, as already mentioned, the mechanism of bond between ribbed steel (which has projections along its length) and concrete is different from that in smooth bars and the above expression for local bond stress is not strictly valid for ribbed steel. The projections shown in Fig. 10.2 are so designed that bond failures will not normally occur, and hence checking of local bond stresses (which is obligatory for designs with mild steel smooth bars) is not required when using high bond bars.

10.3 AVERAGE (ANCHORAGE) BOND STRESS

With modern high bond bars the mechanism of reinforcement anchorage is due to

1. adhesion of concrete and steel,
2. shear strength of concrete, and
3. interlocking of ribs with concrete.

Codes specify that, with high bond bars, the condition to be satisfied is that the average resistance called the *average bond stress,* developed along the full length of the bar surface embedded in the concrete, should be safe at ultimate loads. Ultimate average anchorage bond stress for plain bars in tension according to IS 456 is given in clause 25.2.1.1 and Table 10.1.

TABLE 10.1 DESIGN ANCHORAGE BOND STRENGTH OF DEFORMED BARS (τ_{bd})
(Fe 415 Bars in Tension: IS Clause 25.2.1.1)

Concrete grade	15	20	25	30	35	40
Bond strength N/mm^2	1.60	1.92	2.24	2.40	2.72	3.04

Notes: 1. Design ultimate average bond stress for

$$\text{Fe 250 steel} = \left(\frac{\tau_{bd} \text{ for Fe 415}}{1.6}\right)$$

2. BS 8110 uses the expression

$$\tau_{bd} = \beta\sqrt{f_{ck}} \tag{10.1}$$

In tension,

$\beta = 0.28$ for plain bars, 0.50 for deformed bars

For bars in compression, $\beta' = 1.25\beta$

It may be noted that the value of design ultimate anchorage bond stress in compression is larger because

1. the compression tends to increase the diameter and tension tends to decrease the diameter of the bar,
2. the end of compression bar also contributes to the transfer of load, and
3. the adverse effects of flexural cracks are absent in the compression zone.

10.4 DEVELOPMENT LENGTH

The length of bar necessary to develop the full strength of the bar is called the *development length* L_d, see Fig. 10.3.

The expression of L_d can be derived as taking design yield strength in tension as $0.87f_y$,

$$L_d(\pi\phi)\,\tau_{bd} = \frac{\pi\phi^2(0.87f_y)}{4}$$

$$L_d = \frac{(0.87f_y)\phi}{4\,\tau_{bd}} \quad \text{for tension} \tag{10.2}$$

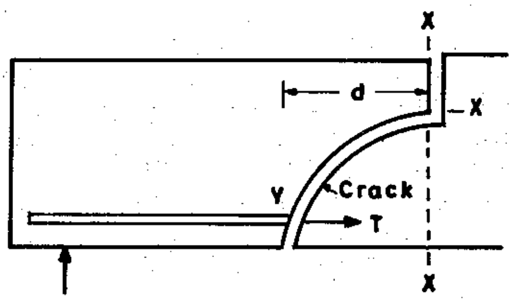


Fig. 10.3 Necessity of extending bar by d beyond theoretical cut-off point.

Taking design strength of steel in compression also as $0.87f_y$ for Fe 415 steel, we get

$$L_d = \frac{(0.87f_y)\phi}{4\tau_{bd}(1.25)} \quad \text{for compression} \tag{10.2a}$$

For $f_{ck} = 25$ and $f_y = 415$, $L_d = 41\phi$ in tension and 32ϕ in compression. Hence as a rough rule full anchorage of steel stressed to ultimate strength may be taken as 40ϕ for tension and 30ϕ in compression. The provisions in BS and ACI for development length are more complex than these simple rules in IS 456.

It should also be noted that when the actual reinforcement provided is more than that theoretically required, so that the actual stress in steel will be less than the full stress, the development length required may be reduced according to the relation

$$L'_d = (L_d)\left(\frac{A_s \text{ required}}{A_s \text{ provided}}\right) \tag{10.3}$$

This principle is used in design of footings and other short bending members where bond is critical. By providing more steel than required by theory, the bond requirement can be satisfied.

10.5 END ANCHORAGE OF BARS

It is the practice in detailing of steel to continue the reinforcement (both tension and compression steel) beyond the point where it is theoretically required for a distance equal to the effective depth of the beam or 12 times the diameter of the bar. This length is called end anchorage, L_d. The need for extension of reinforcement is evident from Fig. 10.3. If there is a diagonal crack, the force in steel will correspond to force at X and not Y (Refer SP 24 and IS 456: clause 25.2.3.1). The condition to be satisfied is

$L_a = d$ or 12ϕ, whichever is greater

The value of the L_a will form part of the total anchorage length L_d in Section 10.6.

10.6 CHECKING DEVELOPMENT LENGTHS OF TENSION BARS

The development length should be checked (in theory) at

1. sections of maximum bending moments,
2. supports,

3. points of inflection, and at
4. points of cut-off of reinforcement.

10.6.1 CHECKING AT POINTS OF MAXIMUM MOMENTS

At the section where the bending moment is maximum, the development length L_d of the reinforcement should be checked on both sides of the section. As this length depends on the diameter of the bars used, its diameter should be chosen such that the anchorage length available is equal to or greater than the development length required for that diameter. This is particularly true for short bending members like footings and lintels, and for such members this check becomes obligatory.

10.6.2 CHECKING OF DEVELOPMENT LENGTHS AT SUPPORTS

The fact that the development length is satisfied at the point of maximum bending moment does not guarantee that the development lengths at other points along with the beam are also automatically ensured. For example, in a beam with uniformly distributed load (UDL), the provision of full development length required for resistance in tension at the centre of the beam does not guarantee that at the quarter span from the supports the required development length will be ensured. This is because at quarter points the bending moment being parabolic, its value is three quarters the maximum value. Hence the development length required will also be 0.75 times the full development length value at the centre. This requires checking of the development lengths at supports as also at points of inflection. At supports the value of L_a provided should be safe and is determined as follows: Let L_a be the end anchorage provided and L_d the development length required at distance x from the support. Let the bending moment capacity of the bars continuing into the support be M_1 and shear at support V. For all shapes of bending moments it is evident from Fig. 10.4 that M_1 will be less than Vx. Hence a safe anchorage length will be given by the expression

$$L_a \geq L_d - \frac{M_1}{V}.$$

Therefore, the condition to be satisfied for full development length will be as given in IS 456: clause 25.2.3.3:

$$L_a + \frac{M_1}{V} \geq L_d \tag{10.4}$$

In IS 456 the value of M_1 is taken as the maximum moment of resistance of the section assuming that all the reinforcements at the section are stressed to $0.87f_y$.

L_a = end anchorage including hooks and bends beyond the centre of the support.

In some codes including IS 456, because of the compression exerted in the concrete by simple supports, the formula allows a 30 per cent increase in bond stress at these points. Hence the condition to be fulfilled is modified to have less value of L_a given by the expression

$$L_a + \frac{1.3M_1}{V} \geq L_d \tag{10.4a}$$

The diameter of the main bars should be so chosen as to satisfy these requirements.

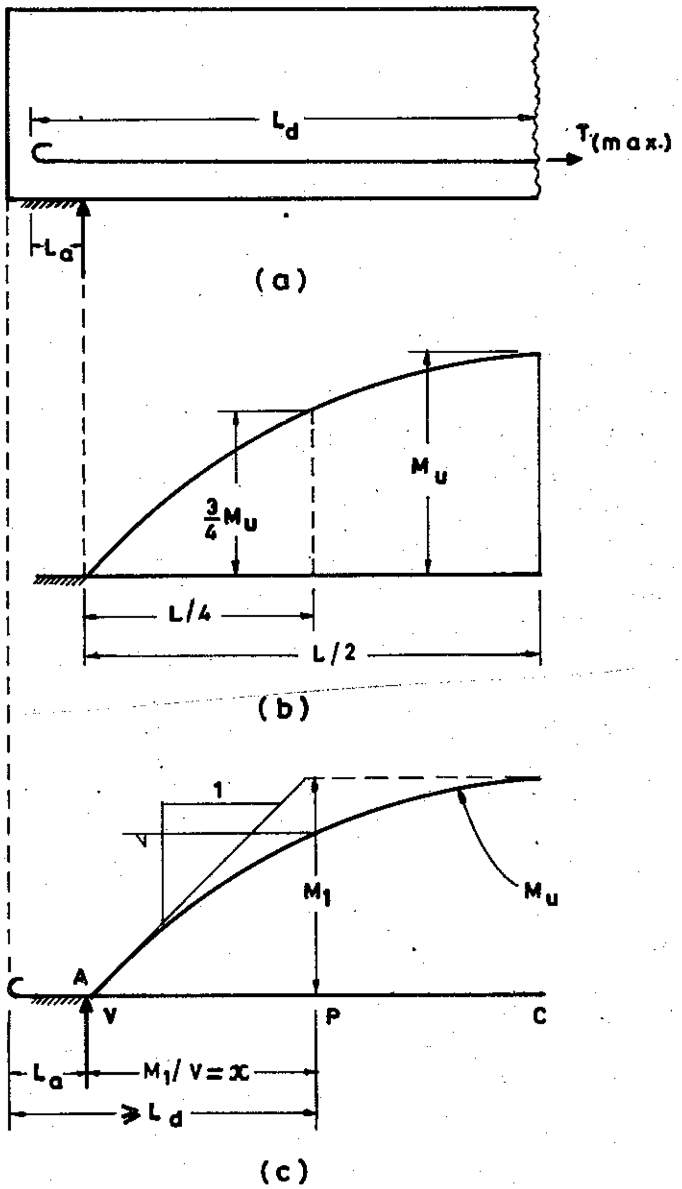


Fig. 10.4 **Checking development lengths: (a) Checking L_d from point of max: B.M.; (b) and (c) Checking L_d for points near the support.**

10.6.3 CHECKING AT POINTS OF INFLECTION

The condition at the point of inflection is shown in Fig. 10.5.

The same derivation as for simply supported end yields the formula

$$\left[\frac{M_1}{V} + (\text{Actual } L_a, \text{ but not greater than } d \text{ or } 12\phi)\right] \geq L_d$$

where V is the shear at the point of inflection.

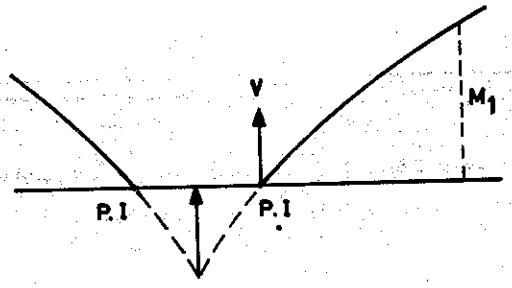


Fig. 10.5 Development length at point of inflection.

10.6.4 CHECKING AT INFLECTION POINTS NEAR INTERIOR SUPPORTS

IS 456 does not require the above formula to be applied to negative steel. This is due to the fact that the rate of decrease of bending moment at these supports is very large so that the condition that the total length of reinforcement from the point of maximum stress should be more than the development length will satisfy other requirements also.

10.6.5 CHECKING AT POINTS OF CUT-OFF

The above formula can be used for checking the development lengths between cut-off points also.

Let

$M_{ic} = (M_{i+1} - M_i)$ between the i and $(i + 1)$ theoretical cut-off points.

V_i = shear force at ith cut-off point

The condition to be satisfied reduces to

$$L_a + \frac{M_{ic}}{V_i} \geq L_d$$

This checking can be quite tedious. A simpler procedure is to extend all bars for a distance equal to the full anchorage length beyond the point where the bars are to be cut-off (TCP) as given in Section 10.7.

10.7 CONDITIONS FOR TERMINATION OF TENSION REINFORCEMENT IN FLEXURAL MEMBERS

All steel (whether in tension or in compression) should extend 12ϕ (or effective depth) beyond the theoretical cut-off point (TCP), i.e. the point at which the bar can be cut-off. However, termination of tension steel in tension zone should satisfy at least one of the following conditions (IS 456: clause 25.2.3):

1. It should extend the full anchorage length beyond TCP.

2. The actual shear capacity at the physical cut-off point (PCP) should be 1.5 times the "design shear" at that section (BS 8110 requires it to be twice).

3. Excess stirrup area not less than $(0.4s_v/f_y)$ is provided along each terminated bar over a distance from the point of cutoff for $(3/4d)$ (IS 456: clause 25.2.3.2).

4. At PCP, the continuing bars should provide double the area required for flexure.

10.8 DEVELOPMENT LENGTH OF COMPRESSION BARS

The real performance of bond in compression bars is not fully known. However, the present practice is to consider it similar to that in tension; only the projection lengths (and not equivalent length as in tension) of hooks, bends and extensions beyond bends are taken as effective in adding to compression bond (IS 456: clause 25.2.2.2). In the design of compression steel for beams, the sizes of bars are generally so selected as to satisfy the available development length. Alternatively, the area of steel may be increased and thus the stress in steel decreased to suit the development length available. The reduced stress in steel will decrease the value of L_d required. As already stated, the design bond strength in compression is usually taken as 25 per cent higher than the value in tension so that one may use the simple rule that development length required in compression is 25 per cent less than that required in tension.

10.9 EQUIVALENT DEVELOPMENT LENGTH OF HOOKS AND BENDS

Part of the development of a tension bar can be provided for by means of mechanical devices such as bends, hooks, etc. which are classified as *anchorage length.* But hooks and bends are not considered fully effective as anchorage in compression. Only their projected length may be considered in design.

Plain bars in tension should always be hooked at the ends. The maximum dimensions of the hooks will depend on the dimensions of the concrete member in which it is to be accommodated. However, there are some specifications in the codes concerning the minimum radius permitted in bending of bars. This ensures that bars do not crack on bending and the concrete inside the hook is not stressed too much when the bar is in tension. The minimum radius specified for a hook is 2ϕ for an M.S. bar and 4ϕ for a high yield bar, see Fig. 10.6.

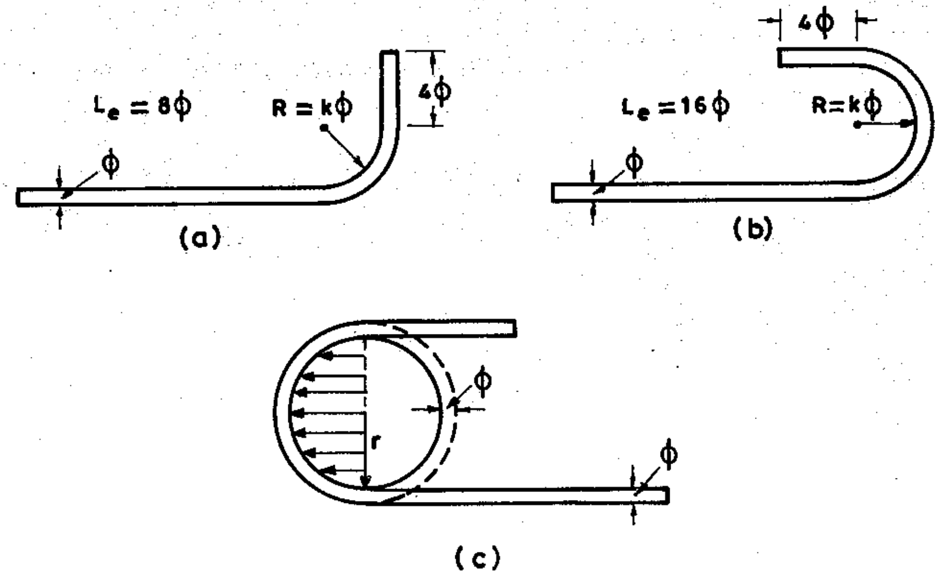


Fig. 10.6 Hooks and bends in reinforcement bars: (a) Standard 90° bend; (b) Standard hook (k = 2 for M.S. and 4 for HYD bars); (c) Stresses inside bends.

Bars bent in the specified way are said to have been provided with standard hooks and bends. The equivalent length of anchorage of these standard bends, according to IS 456: clause 25.2.2, is 4 times the diameter of the bar for each 45° bend subject to a maximum of 16 times the diameter of the bar. Thus the equivalent length of standard hooks and bends according to IS is as given in Table 10.2 or the actual length. It should be noted that the required development length for stirrups is provided by standard bends and hooks.

TABLE 10.2 SPECIFICATIONS AND EQUIVALENT LENGTH OF BENDS AND HOOKS
(IS 456: clause 25.2.2)

Type of bar	Bend		Hook		Equivalent length	
	Angle	Extension	Angle	Extension	90°	180°
Straight or inclined bars	90°	4ϕ	180°	4ϕ	$2(4\phi) = 8\phi$	$4(4\phi) = 16\phi$
Stirrups	90°	8ϕ	135° 180°	6ϕ 4ϕ	Assume full anchorage	

U-type bends are preferred for mild steel and L-type bends for deformed bars. When detailing stirrups with ends having 90° bend only, it is usually recommended that a cover of at least twice the size of the link at the bend shall be available:

10.10 BEARING STRESSES INSIDE HOOKS (MINIMUM RADIUS OF BENDS)

When a tension bar can be assumed not to be stressed beyond a distance of four times the size of the bar from the end of a bend, it is not necessary to check the bearing stresses. Otherwise, in highly stressed bars the bearing stresses inside the bend are to be checked by the following formula as obtained from IS 456: clause 25.2.2.5. (Ref. SP 24 (1983) page 60)

$$2T = f_b 2r\phi$$

$$f_b = \text{bearing stress} = \frac{T}{r\phi}$$

where

T = tensile force due to design load in the bar

r = internal radius of the bend

ϕ = diameter of bar

This bearing stress should not exceed the value given by the following formula (IS 456: clause 25.2.2.5):

$$\text{Bearing stress} = \frac{1.5 f_{ck}}{1 + 2\,(\phi/a)}$$

where a is the centre-to-centre distance between bars or group of bars perpendicular to the plane of the bend. For bars adjacent to the face of the member, a is taken as the cover plus the size of the bar. (Thus for a 10 mm bar with 15 mm cover, a will be 25 mm.)

10.10.1 MINIMUM RADIUS OF BENDS

Equating the above two expressions for bearing stress, one gets

$$\frac{0.87 f_y \pi\phi}{4r} = \frac{1.5 f_{ck}}{1 + 2\,\phi/a}$$

$$r \geq \frac{0.87 \times \pi}{1.5 \times 4}\left(\frac{f_y}{f_{ck}}\right)\left(1 + \frac{2\phi}{a}\right)\phi$$

$$r \geq 0.456\,\frac{f_y}{f_{ck}}\left(1 + \frac{2\phi}{a}\right)\phi \tag{10.5}$$

This equation controls the diameter of bend. Thus, for example, if

$\phi = 10$ mm, $f_y = 415$, $f_{ck} = 20$; $a = 25$ mm

Substituting these values into equation (10.5), we get

$$r \geq (0.456)(10)\left(\frac{415}{20}\right)\left(1 + \frac{2 \times 10}{25}\right) \geq 170 \text{ mm}$$

10.10.2 CHANGE OF DIRECTIONS

When detailing of bars is carried out, special care should be given at points where there is a change of direction of steel so that they do not tend to break away the concrete cover provided for the steel (see also Chapter 20 for staircase) or produce very high compression in the concrete at the bend. The reinforcement should also be restrained by stirrup and other devices. For example, when column bars are bent at top of floors and spliced with steel from the next storey, laterals at close spacings should be provided for the horizontal component of the forces in these bent column bars, see Fig. 10.7.

10.11 ANCHORAGE OF A GROUP OF BARS

As already pointed out, anchorage of steel bars is accomplished by fixing of the tension or compression bars in concrete and providing the required development lengths. In conventional practice, placing of bars, one touching the other, was not allowed. A space equal to the diameter of bar was to be left in between the bars to develop the bond stresses. In modern practice up to four bars can be bundled together to avoid congestion in heavily reinforced sections such as over the supports of continuous T beams. However, groups of bars, when used in compression, should be carefully examined for the additional provision of links for containment of the compression bars.

According to IS 456: clause 25.2.1.2, the development length required by each bar of the bundled bars is that of the individual bar increased by 10 per cent for the two bars, 20 per cent for three bars and 33 per cent for four bars which are in contact. More than four bars in contact are not allowed to be used. Care should also be taken not to stop all the bars of the group together. A spacing of at least 40ϕ (which is equivalent to full development of a bar) should be maintained between the cutoff of each alternate bar at least till the bars are reduced to two.

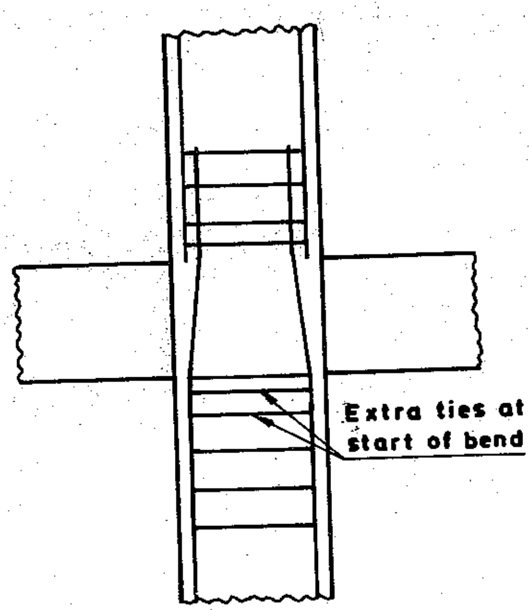


Fig. 10.7 Laterals at change in direction of bars.

10.12 SPLICING OF BARS

When it is necessary to transfer the force from one bar to another in the same line of action (since it becomes necessary due to non-availability of long bars and when the length of the bar is to be extended), the force can be transmitted by splicing as shown in Fig. 10.8. The requirements for splicing of bars are covered in IS 456: clause 25.2.5. Splicing is carried out by (a) lapping of bars, (b) a mechanical joint, and (c) a welded joint.

In lapped bars the forces are transferred by bond from one bar to the concrete surrounding it and simultaneously by bond to the other bar forming the splice. Thus the concrete is subjected to high shear and splitting stresses which may cause cracking unless adequate precautions are taken to avoid it. Thus normally bars larger than 36 mm are not spliced by lapping; but if they have to be lapped, 6 mm spirals at 100 mm pitch should be provided around the lap as specified for splices in "direct tension" members in IS 456: clause 25.2.5.1(e).

IS specifies that splicing in flexural members should not be at sections where the bending moment is more than 50 per cent of the moment of resistance, and not more than one-half of the bars should be spliced at a section.

10.13 LAP SPLICES

Because of the splitting action in the concrete, the amount of cover at the point of splicing by lap splicing is very important. If the cover is inadequate, special reinforcements in the form of stirrups or spirals should be provided at the lap splice.

10.13.1 IS PROVISIONS FOR LAPS

IS 456: clause 25.2.5 specifies elaborate provisions for lap splicing of bars in tension and compression.

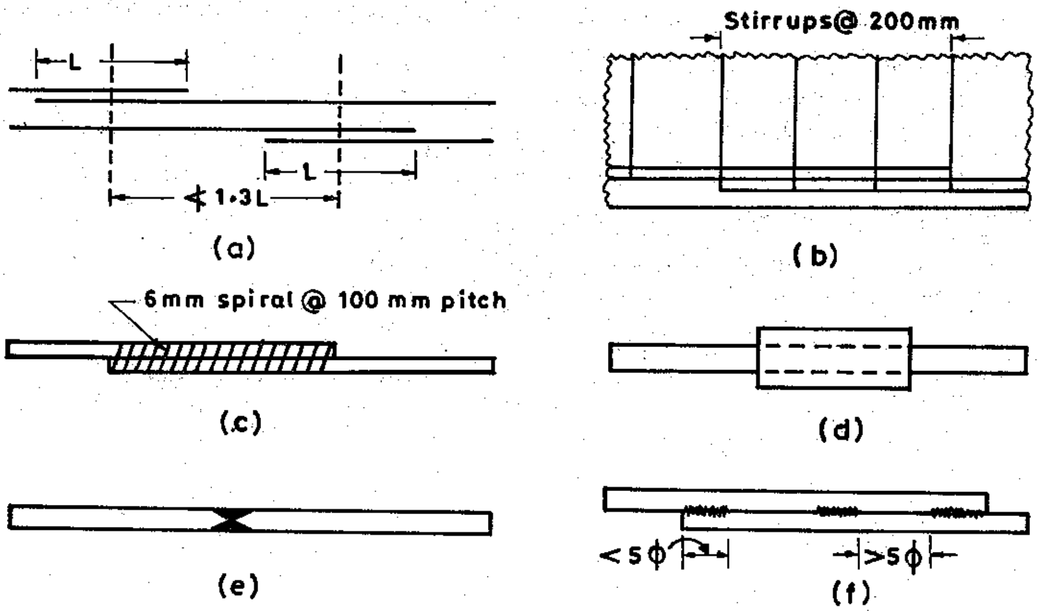


Fig. 10.8 Splicing of reinforcement bars: (a) Splicing of bars; (b) Stirrups at splice points; (c) Splicing of bars $\phi > 36$ mm; (d) Mechanical joints $\phi > 36$ mm; (e) Butt welding; (f) Lap welding of bars.

These should be carefully studied and followed in designs. The important points to be remembered are:

1. Lap splices are not usually allowed for bars more than 36 mm, but if such rods are to be spliced they should be properly welded. When welding of cold worked bars is allowed, the special instructions applicable to these bars should be followed. Where lapping has to be done under unusual circumstances, such as splicing in areas of large moments or more than one-half of the bars is spliced, additional closely spaced spirals should be provided around the bars and the length of lap should be increased.

2. Laps should be always staggered. Their centre-to-centre distances should not be less than 1.3 times the required lap length. Bars to be lapped should be placed either vertically one above the other, or horizontally one beside the other.

3. The total lap length (including bends, hooks etc.) in flexural tension should not be less than 30ϕ or the full development length, L_d, as calculated, whichever is greater.

4. Lap lengths in 'direct tension' should be 30ϕ or $2L_d$, whichever is greater. Tension splices should be enclosed in spirals made from 6 mm bars with pitch not more than 100 mm. Hooks are also to be provided at the end of the tension bars.

5. Lap length in compression should be more than 24ϕ or L_d in compression. When the columns are subjected to bending, the lap length may have to be increased to the value in bending tension if the bar is found to act in tension. In columns where longitudinal bars are offset at a splice, detailing should be done according to standard practice. The lower end of the upper (new) bar is supported on hardened concrete. The main bars may extend more than one floor if it is convenient for construction. Extra ties should be given at the laps to take up the horizontal forces due to change

of directions of the lapped bars (see Chapter 13). All bars may be spliced at the same floor level or, alternatively, the splices may be staggered.

6. With two different diameters of bars, lap length should be calculated on the diameter of the smaller bar.

7. When bundled bars are to be spliced by lapping, one bar at a time is to be spliced and the splicing should be staggered.

8. If the general rules regarding laps cannot be obeyed in a construction, special welded splices or mechanical connections should be provided (IS 456: clause 25.2.5.2).

10.13.2 BS PROVISIONS FOR LAPS

BS 8110 considers the positions in which the splices occur and stipulates special provisions for sections where splicing has to be made. The principal provisions in BS 8110 for tension laps can be summarised as follows:

1. The minimum lap length for bar reinforcement (for very lightly stressed bars) should not be less than 15ϕ or 300 mm, whichever is greater.

2. Where the bars lapped exceed size 20 mm and the cover is less than 1.5 times the size of the smaller bar, transverse links of at least one quarter the size of the smallest bars spliced at spacing not exceeding 200 mm should be provided throughout the lap length.

3. In normal condition, the tension anchorage length required is the one necessary to develop the required stress in the reinforcement. Where the lap occurs at the top of a section as cast or at a corner, and the minimum cover is less than double the size of the bar, the lap length should be increased by a factor 1.4. In very unfavourable situation it should be increased by a factor of 2.

4. The maximum amount of reinforcement in a layer, including tension laps, should be such that the sum of the rod sizes should not exceed 40 per cent of the breadth at that level.

For compression laps the British code requires that the lap length should be more than 25 per cent of the compression anchorage length required to develop the stress in the steel. Thus for columns with steel subjected to the full design stress compression, lap length should be equal to the tension anchorage length required.

10.14 DESIGN OF BUTT JOINTS IN BARS

Lapping of bars, especially with large diameter bars as in columns and foundation structures, tends to increase consumption of steel. Hence butt joints for compression bars are commonly used instead of laps.

For bars in compression, the load may be transferred by end bearing of square 'saw cut ends held in concentric contact' by suitable sleeve or other coupler with the concrete cover specified as for normal reinforcement.

For bars in tension only approved standard mechanical couplers are allowed. When tested on standard gauge length, they should have such strengths that the elongation at a stress level of $0.6f_y$ is not more than 0.1 mm and the ultimate strength of the coupled joint should be more than that of the solid bar.

10.15 WELDED LAP JOINTS

Welded lap joints are also used to reduce the length of laps. The length of weld in a single pass should not normally exceed 5ϕ, and if a larger length of weld is to be required, it should be divided into sections with space not less than 5ϕ in between. The strengths of welded bars should preferably be 100 per cent the design strength of the bar.

10.16 CURTAILMENT OF BARS AND THEIR ANCHORAGE

Bending moment at mid-spans of beams requires maximum area of steel. Towards the ends of the beam some of these steel may be stopped by curtailing them (see Fig. 10.9).

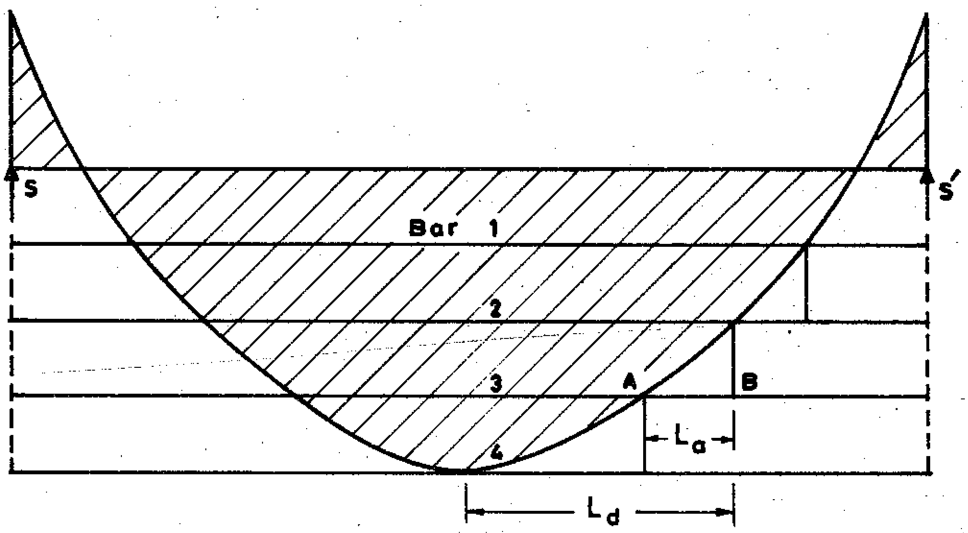


Fig. 10.9 Cut-off point of reinforcements: (A, theoretical cut-off point (TCP); B, physical cut-off point (PCP)).

In practice, the theoretical cut-off point (TCP) and the actual or physical cut-off point (PCP) differ. The distance at which the PCP occurs should not be less than either the effective depth of the member or twelve times the bar size. In addition, the bar as a whole should satisfy the requirement of development length.

Simplified empirical rules have been laid down for detailing of reinforcements for slabs and beams of nearly equal spans with UDL to comply with anchorage and other requirements. They are discussed in Chapters 11 and 12.

10.17 USE OF SP 16 FOR CHECKING DEVELOPMENT LENGTH

Tables 64 to 66, SP 16 give the tension and compression development lengths (L_d) required for a design strength of $0.87f_y$ (the same in tension and compression).

$$L_d \text{ (in compression)} = \frac{L_d \text{ (in tension)}}{1.25}$$

The length required (L'_d) for any other stress level that exists in the structure can be determined from these values by the expression

$$L'_d = \frac{f_s}{0.87 f_y} L_d$$

Table 67 of SP 16 gives the anchorage value of hooks and bends for tension reinforcement. In tension anchorage, the effect of hooks, bends and straight lengths beyond bends, if provided, can be considered as development length. In compression bars, only the projected length of hooks, bends, etc. are generally considered as effective towards development length.

10.18 IMPORTANCE OF LAPS AND ANCHORAGE LENGTH

Laps in steel increase cost as well as difficulty in placing of concrete. When using large diameter bars in columns and foundations such as rafts, correct estimation of laps is required. Savings in lap length can considerably reduce cost and consumption of steel in the structure.

Recent codes for footings and other foundation structures have relaxed the provision for anchorage length of starter bars for foundation structures as explained in Chapter 22. These can be utilised to effect economy of steel in construction.

EXAMPLE 10.1 (Bond and anchorage in beams)

A cantilever carrying a uniformly distributed load has a breadth of 150 mm and effective depth of 260 mm. The reinforcement consists of four 16 mm bars. If the factored total load is 75 kN, calculate (a) the maximum local bond stress, and (b) the anchorage length required. Assume $f_{ck} = 30$ N/mm^2 and $f_y = 415$ N/mm^2.

Ref.	Step	Calculations	Output
IS 456	1.	*Local bond stress* (*Note*: Local bond is not generally checked for ribbed bars but only for smooth bars.) $u = \frac{V}{\Sigma Ojd} \approx \frac{V}{\Sigma Od}$, $\Sigma O = 4(\pi \times 16) = 201$ $u = \frac{75 \times 10^3}{201 \times 260} = 1.44$ N/mm^2	
Cl. 25.2.1.1	2.	*Anchorage length required* Bond stress (τ_{bd}) for grade 30 concrete and Fe 415 steel = 1.5 × 1.6 = 2.4 N/mm^2 Anchorage length = full development length	
Cl. 25.2.1		$L_d = \frac{(0.87 f_y)\phi}{4\tau_{bd}} = 37.6\phi = 602$ mm *Note*: SP 16: Table 65 checks anchorage length for 16 mm bars, the length being 602 mm. The steel should extend an equivalent length (including hooks, bends, etc.) of 602 mm from the face of the support into the support.	

EXAMPLE 10.2 **(Compression laps)**

A tied column of a multistoreyed building has 32 mm rods for the longitudinal steel. Assuming $f_{ck} = 25$ N/mm^2 and $f_y = 415$ (N/mm^2), (i) calculate the lap length required, and (ii) state how this length can be reduced to make savings in steel consumption.

Ref.	Step	Calculations	Output
IS 456 Cl. 25.2.2.2	1.	*Required lap length (L)* L = development length of bars (only projected length of hooks and bends is considered)	
IS 456 Cl. 25.2.1.1		Bond stress = τ_{bd} in compression = (τ_{bd} in tension) × 1.25	
IS 456 Cl. 25.2.1		$L_d = \dfrac{(0.87 \times 415)\phi}{4 \times 1.25\ (1.6 \times 1.4)}$ $= 32.2\ (32) = 1030$ mm	
SP 16 Table 65		*Note*: SP 16 Table 65 gives the value directly, which is 1032 mm	
		As 32 mm bars weigh 6.3 kg per metre length, the weight of steel involved in these splices will be large.	
IS 456 Cl. 25.2.2.3	2.	*Methods of reducing lap length* Use a shorter lap length with welding. Adopt a lap length of 15ϕ together with lap welding at 5ϕ gaps. The welds will be designed to carry the equivalent force (F) for a lap of $(32.2 - 15)\phi = 17.2\phi$. $F = (0.87 \times 415)\left(\dfrac{\pi \times 32^2}{4}\right)\left(\dfrac{17.2}{32.2}\right)$ N $= 124.7$ kN	

EXAMPLE 10.3 **(Curtailment of tension bars)**

A reinforced concrete beam of 4 m span requires 7 Fe 415 bars of 16 mm, as tension bars. Find the distance from the centre of the beam where the central bar can be curtailed. Assume $f_{ck} = 15$ N/mm^2 and $d = 300$ mm. (*Note*: All curtailments should be such that the arrangement of steel can be kept symmetrical.)

Ref.	Step	Calculations	Output
IS 456	1.	*Theoretical point of cutoff from BM consideration* $\dfrac{(wL^2/8) - (wx^2/2)}{wL^2/8} = (6/7)$ bars where x is measured from midpoint $7(L^2 - 4x^2) = 6L^2$ i.e. ($x = 760$ mm)	

EXAMPLE 10.3 (cont.)

Ref.	Step	Calculations	Output
	2.	*Development length for max. tension at centre in Fe 415 steel*	
Cl. 25.2.1.1		τ_{bd} for M15 concrete = 1.6 for Fe 415 bars	
Cl. 25.2.1		$L_d = \dfrac{\phi\sigma_s}{4\tau_{bd}} = \dfrac{16(0.87 \times 415)}{4 \times 1.6} = 902$ mm	
	3.	*Physical cut-off point (PCP)* The theoretical cut-off (TCP) is larger of 1 or 2 above	
Cl. 25.2.3.1		PCP = TCP + d or 12ϕ = 902 + 450(or 12 × 16) = 1350 mm from centre line of beam	
		Note: Even though from BM consideration steel can be terminated 760 mm from centre, from other considerations it has to be extended to 1350 mm.	

EXAMPLE 10.4 **(Checking development length at supports)**

A simply supported beam is 6 m in span and carries a characteristic load of 60 kN/m. If 6 Nos. of 20 mm bars are provided at the centre of the span and 4 Nos. of these bars are continued into the supports, check the development at the supports assuming grade 15 concrete and Fe 415 steel.

Ref.	Step	Calculations	Output
	1.	*BM and SF in the beam*	
		Design load = 1.5 × 60 = 90 kN/m SF = V_0 = 90 × 3 = 270 kN	
		$M_{max} = \dfrac{wl^2}{8} = \dfrac{90 \times 6 \times 6}{8} = 405$ kNm	
	2.	*Moment of resistance of bars continued into support (4 bars)*	
		One may calculate the exact value if beam dimensions are given. As an estimate,	
		$M_1 = \dfrac{4}{6}(405) = 270$ kNm	
	3.	*L_d for 20 mm bars*	
SP 16		M15 concrete Fe 415 steel	
Table 65		L_d = 1128 mm	

EXAMPLE 10.4 (cont.)

Ref.	Step	Calculations	Output
IS 456 Cl. 25.2.3.3	4.	*Check development length at support* Using 30 per cent increase as in IS, we get $\frac{1.3M_1}{V_0} = \frac{1.3 \times 270 \times 1000}{270} = 1300$ mm Condition to be satisfied: $L_0 + \frac{1.3M_1}{V_0} > L_d$ where L_0 = anchorage beyond support line $L_0 + 1300 > 1128$	L_d satisfied (with no L_a)

REVIEW QUESTIONS

10.1 Explain the terms (a) bond and anchorage, (b) development length, lap length, and anchorage lengths.

10.2 What are high bond bars? Why is it necessary to specify projections on bars made from Fe 415 and Fe 500 steel, whereas such projections are not obligatory for Fe 250 steel?

10.3 Explain the action of bond in high bond-bars.

10.4 Explain the terms average bond stress and local bond stress. Derive expressions for these. Write down the expression for checking these.

10.5 Explain why, according to modern specifications, local bond need not be checked for high bond bars.

10.6 What is meant by 'full development length'? What is its approximate value for tension and compression in terms of the diameter of the bar?

10.7 If the theoretical steel needed is A_s and if much more steel than necessary has been provided, can the development length for the bars be of reduced value than the theoretical?

10.8 Why can the bond stress in compression bars be assumed to be more than that in tension bars?

10.9 What is meant by 'equivalent length' of hooks and bends? Can they be used to increase the theoretical development length of (a) tension bars, and (b) compression bars?

10.10 What is meant by 'anchorage' of steel bars in R.C. construction? What are the IS Code provisions for providing anchorage of shear reinforcement? (IS 456: clause 25.2.2.4).

10.11 What is meant by bundling of bars? What is the maximum number of bars that can contribute to a bundle? What are the specifications regarding development length of bundled bars?

10.12 When do steel reinforcements require splicing? What is the length of bars that are usually available in the market? Enumerate the methods used for splicing.

10.13 Up to what diameter of steel bars are allowed to be extended by splicing? What method would you use to splice large bars in a bridge girder?

10.14 Enumerate the rules for splicing of tension bars and the extra precautions to be taken to reduce cracking at these points.

10.15 Enumerate the rules for splicing of compression bars.

PROBLEMS

10.1 A simply supported beam of 5 m span with uniformly distributed load is 250 × 500 mm in section. One-third of its steel bars is extended into the support. The factored shear force at the centre of support is 100 kN. Determine the maximum diameter of bars that can be allowed according to IS 456: clause 25.2.3.3, "regarding tension steel continued into simple supports". Assume f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2.

10.2 A simply supported beam of 200 × 600 mm is designed to support a uniformly distributed load of 50 kN/m over a span of 5 m. If it is proposed to use 20 mm bars as main bars, check its adequacy for development length. Grade 20 concrete and Fe 415 steel are used for the beam.

10.3 Explain why IS codes do not insist on the condition $L_d \ngtr M_1/V + L_0$ for tension steel at the interior support of a continuous beam in clause 25.2.3.3. How does one check the anchorage length of bars in the interior support?

A continuous beam ABCD is simply supported on supports A and D and is continuous over B and C. Over support B the design needs 4 bars of 32 mm at top and 3 bars of 32 mm at the bottom for a support moment of 768 kNm. Check the anchorage length that should be provided for the tension steel. What are the requirements to be fulfilled by the negative moment reinforcement?

10.4 A continuous beam ABCD carrying a UDL is 300 × 500 mm in section. It is simply supported at A and D and continuous on B and C. In span BC at the point of inflection the shear force is 175 kN. Determine the sizes of bar to be continued beyond the point of inflection so that the anchorage length conditions are satisfied. Assume f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2. The moment capacity of the section at the region of the point of inflection is 130 kNm. There is also enough space for the standard anchorage length between the support and the point of inflection.

10.5 An R.C. cantilever beam has a span of 3 m and free breadth of 450 mm with the depth varying from 200 mm at the end to 500 mm at the fixed end. It carries a UDL of 15 kN/m. Calculate the maximum size of bars that can be used as reinforcement for the member, if one-half of the steel is to be cut off at midspan of the member. The bond requirements should be fully satisfied. Assume f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2.

10.6 (a) Determine the allowable bearing stress inside a hooked bar of 20 mm dia, if f_{ck} = 40 N/mm^2, f_y = 250 N/mm^2, with the distances between the bars of the group being 40 mm.

(b) Determine the mimimum allowable radius of bend of a 20 mm bar if f_y = 415 N/mm^2, f_{ck} = 30 N/mm^2, and centre-to-centre distances between the bars of the group is 40 mm.

10.7 A 25 mm bar in a tension member is to be lapped with a 16 mm bar. The joint has to transmit the full tension allowed by the code. Determine the lap length required if f_y = 415 N/mm^2 and f_{ck} = 20 N/mm^2. What is the special provision that should be made for splicing of tension members?

11

Design of One-way Slabs

11.1 INTRODUCTION

Reinforced concrete solid slabs are constructed in one of the following ways (see Fig. 11.1):

1. One-way slabs
2. Two-way slabs
3. Flat slabs
4. Flat plates.

One-way slabs are those supported continuously on the two opposite sides so that the loads are carried along one direction only. The direction in which the load is carried in one-way slabs is called the *span*. It may be in the long or short direction. One-way slabs are usually made to span in the shorter direction since the corresponding bending moments and shear forces are the least. The main reinforcements are provided in the span direction. Steel is also provided in the transverse direction, to distribute any unevenness that may occur in loading and for temperature and shrinkage effects in that direction. This steel is called distribution steel or secondary reinforcement. The main steel is calculated from the bending moment consideration and under no circumstances should it be less than the minimum specified by the code. The secondary reinforcement provided is usually the minimum specified by the code for such reinforcement.

Two-way slabs are those slabs that are supported continuously on all four sides and are of such dimensions that the loads are carried to the supports along both directions. They are discussed in more detail in Chapter 12.

Flat slabs and flat plates are those multispan slabs which directly rest on columns without beams. Flat slabs differ from flat plates in that they have either drop panels (increased thickness of slab) or column capitals in the regions of the columns. Flat plates have uniform slab thickness, and the high shear resistance around the columns are obtained usually by the provision of special reinforcements called 'shear-head reinforcements' placed in the slab around the columns.

11.2 LIVE LOAD ON SLABS IN BUILDINGS

Dead load of slabs consists of its own weight and in addition, the weight of finishes, fixtures and partitions. Live load or imposed load is specified as per IS code. This live load varies according to the use for which the building is to be put after construction. It is important to note that for design of buildings these live loads are considered as either acting on the full span or assumed to be absent altogether in the span. In continuous slabs, they are to be so placed as to get the maximum bending moment and shear effect in the structure. In design of slabs for other structures

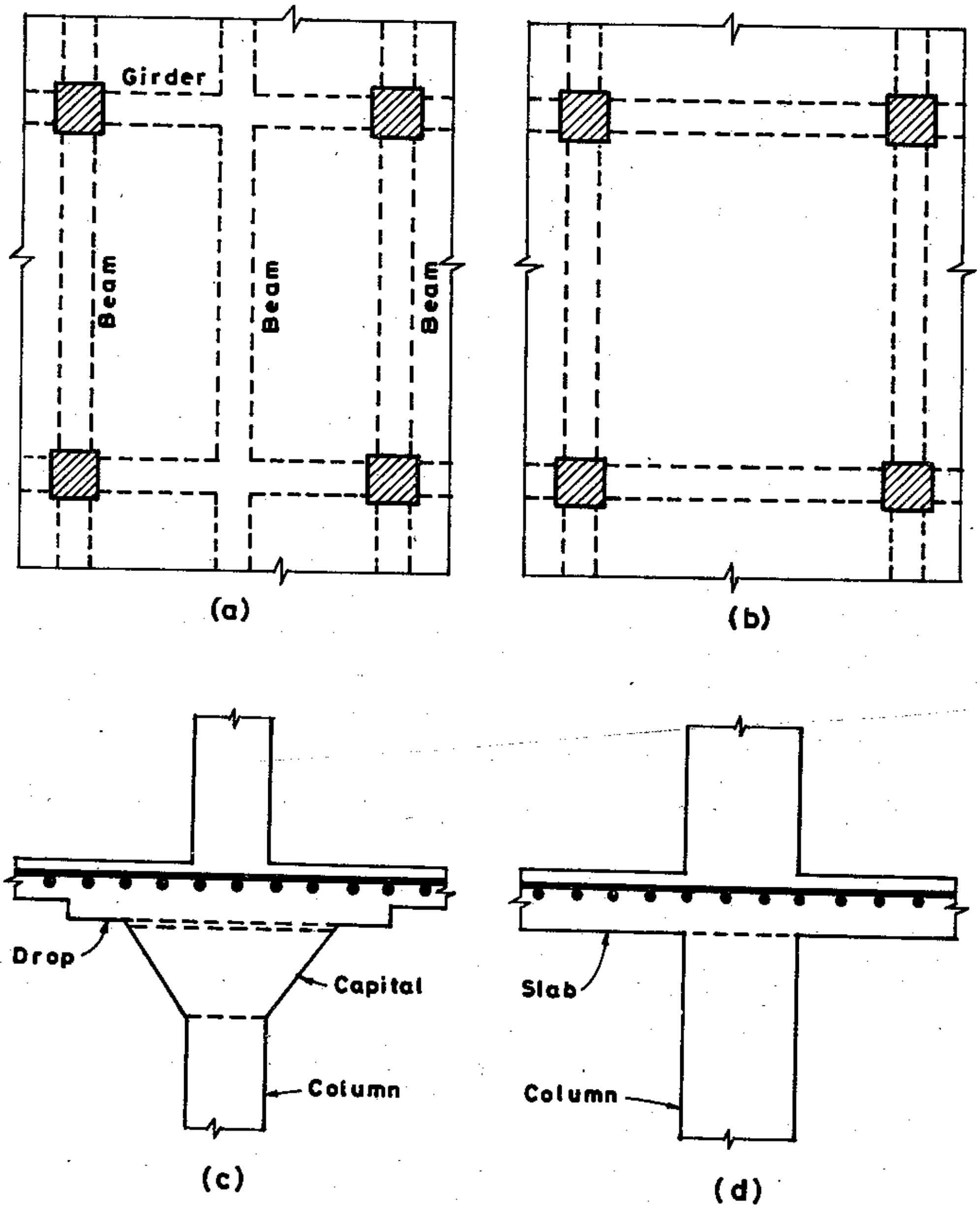


Fig. 11.1 Types of reinforced concrete slabs: (a) One-way slab; (b) Two-way slab; (c) Flat slab; (d) Flat plate.

like bridges, the effects of partial loading of the slab may have to be considered.

According to IS 875, the loading on slabs for buildings are calculated as follows:

1. Self-weight at 25 kN/m^3 for reinforced concrete.
2. Finishes and partitions generally at 1.5 kN/m^2.

3. Characteristic imposed loads as specified depending on the use of the building. For example, for residential and office buildings, these loads are

(a) For roofs $= 1.5$ kN/m^2 with access and 0.75 kN/m^2 without access

(b) For floors $= 2.0$ kN/m^2 for residential floors
$= 3.0$ kN/m^2 for office floors

In buildings the dead load is generally equal to or greater than the live load so that the dead load to live load ratio is usually unity or more than unity. When this ratio is 1.33 or more, the maximum moments obtained by considering all loads as dead loads are allowed by the Indian Code for arriving at the design moments, shear etc. (IS 456: clause 21.4.1).

11.3 STRUCTURAL ANALYSIS OF ONE-WAY SLABS WITH UDL USING COEFFICIENTS

One-way slabs, because of their one-way action, are analysed as beams of unit width. However, codes allow the use of simple coefficients for calculation of moments and forces in continuous beams and slabs of uniform loading with more or less equal spans and continuous on at least three *spans*. Spans are considered as equal, if the differences in span are not more than 15 per cent of the larger of the spans. These coefficients for ultimate bending moments and shears are given in Tables 7 and 8 of IS 456 and Table 11.1 of the text. It should be noted that when using this table, redistribution of moments between sections is not allowed.

TABLE 11.1 BENDING MOMENT AND SHEAR FORCE COEFFICIENTS FOR BEAMS AND ONE-WAY SLABS
(IS 456, Tables 7 and 8)

	End support	Near middle of end span	At first interior support	At middle of interior span	At all other interior supports
Moment					
DL		+ 1/12	− 1/10	+ 1/24	− 1/12
LL		+ 1/10	− 1/9	+ 1/12	− 1/9
Shear					
DL	0.4		0.6 (outer) 0.55 (inner)		0.5
LL	0.45		0.6 (outer) 0.6 (inner)		0.6

Notes: (i) For bending moment the coefficient is multiplied by total design load and effective span.
(ii) According to IS 456: clause 21.5.2, beams with partially restrained supports, develop restraining moment which may be taken as WL/24 and the corresponding increase in the shear coefficients as 0.05.
(iii) For SF the coefficient is multiplied by total design load.

11.4 DESIGN FOR SHEAR IN SLABS

Normally the thickness of slabs is so chosen that the shear can be resisted by concrete itself and the slab does not need extra shear reinforcements. It is only in extreme situations where the thickness tends to be very large that shear reinforcements are allowed to be used for slabs.

Shear tests on solid slabs have shown that when these shallow members are less than 300 mm thick, they have an increased shear resistance compared to members such as beams which are 300 mm or more in depth. This shear enhancement factor for shallow depths is given in IS 456: clause 39.2 and Table 11.2 of the text.

TABLE 11.2 SHEAR MULTIPLYING FACTOR FOR SLABS
($k\tau_c$ for slabs: IS 456: clause 39.2)

Overall depth of slab (mm)	k
300 or more	1.00
275	1.05
250	1.10
225	1.15
200	1.20
175	1.25
150 or less	1.30

Note: The formula suggested in BS 8110:

$$k = \left(\frac{400}{d}\right)^{1/4}$$ k < 1.

where $\left(\frac{400}{d}\right)$ should not be taken as less than 1.

The enhanced shear is ($k\tau_c$), where k is called the shear enhancement factor for slabs. According to the above table, k varies from 1.30 for slabs of 150 mm or less to 1.00 for slabs of 300 mm or more. BS 8110 also allows the use of the shear enhancement factor and its value is given by the expression

$$k = \left(\frac{400}{d}\right)^{1/4}, \qquad \left(\frac{400}{d}\right) \nless 1$$

According to IS 456, the rules to be followed in the design of slabs for shear are:

1. No shear reinforcement should be provided for slabs less than 200 mm thick. However, the increased value of shear resistance in slabs can be taken into account in design.

2. It is preferable to design slabs without any extra shear reinforcements.

3. However, if found necessary, shear reinforcements may be allowed to be provided in slabs which are 200 mm or more in thickness. The spacing of these reinforcements can be increased to "d" instead of "$0.75d$" as in beams.

4. In no case, even with provision of shear reinforcements, according to IS 456: clause 30.2.3.1, should the maximum shear stress in slabs due to ultimate load exceed one-half that allowed in beams as given in Table 14 of IS 456 (Table 7.2 of the text).

11.5 CONSIDERATIONS FOR DESIGN OF SLABS

11.5.1 CHOOSING SPAN EFFECTIVE DEPTH RATIO FOR SLABS

For a given type of support condition, the same span/depth basic ratio as given for beams in IS 456 are applicable for slabs also. However, as the percentage of reinforcements in slabs is generally low, the effective span/depth ratios can be much larger than the basic ratios as already indicated in Chapter 9. It should also be remembered that with heavy loadings when the percentage of steel in slabs increases, this ratio will tend to be the same as in beams. For the first trial, a convenient percentage of steel may be assumed for the slabs and the span/effective depth ratio calculated. Thus, assuming 0.3 per cent of steel, the correction factor F_1 from Fig. 3 of IS 456 will be 1.4, and the span/effective depth ratio for a continuous slab will be of the order of $1.4 \times 26 = 36$. Because of these considerations. IS 456: clause 23.1 recommends the following span-overall depth factor for two-way slabs using Fe 415 steel:

Simply supported two-way slabs	28 (beams 20)
Continuous two-way slabs	32 (beams 26)

For one-way slabs, a ratio of 25 and 30 may be more appropriate.

11.5.2 CONCRETE COVER

In IS 456 the normal cover specified for slabs is a fixed amount of 15 mm, independent of the grade of concrete. Increase in cover is to be provided under special conditions of exposure as explained in Chapter 3. Also, a minimum cement content and maximum water cement ratio are specified for different environment conditions. As already pointed out in Chapter 3, this practice is different from the British practice where cover is related to the grade of concrete. The specified cover of 15 mm for mild conditions of exposure, assumed in IS 456: clause 25.4 should be suitably increased for other conditions of exposure. However, as increased cover raises the dead load, the slab has to carry, one should be judicious in the choice of cover. Strict maintenance of the chosen cover during construction and using a good grade of concrete for the construction will go a long way to ensure durability of slabs and reduce the dead load due to cover.

11.5.3 CALCULATION OF STEEL AREA

It should be noted that the depth of slab chosen from deflection requirements will be usually greater than the depth required for balanced design. Hence the area of steel required will be less than the balanced amount. The fundamental formula used for this purpose is

$$M_u = (f_{st} A_{st}) \times (\text{lever arm})$$

Any one of the procedures explained in Section 4.13 for determining steel area for an underreinforced section may be used for the calculations.

11.5.4 DESIGN PROCEDURE

The procedure of design of one-way slabs is to consider then as 'beams of one metre width' in the short direction. The various steps in design are as follows:

Step 1: Assuming a suitable overall thickness for the slab, calculate the factored loads (dead and live load) for design. This initial guess for thickness of slab may be made from empirical relations between depth and span. The allowable span/overall depth ratio of slabs may be taken as given in Section 11.5.1. The minimum depth for ease of construction is 100 mm. The factored load is (1.5 DL + 1.5 LL). A suitable cover depending on exposure condition should be assumed.

Step 2: Considering the slab as beam of one metre width and using effective span, determine the maximum bending moments M for the ultimate factored load. For continuous slabs, coefficients of Table 11.1 (IS 456 Table 7) may be used for this purpose. Otherwise, any established elastic analysis may be used. In the latter case redistribution of moments is also allowed.

Step 3: Using the formula $M_u = Kf_{ck}bd^2$ and $b = 1000$ mm, find the minimum effective depth required as in beams.

Add cover and find the total depth of slab from strength considerations. Check the depth with the depth assumed in step 1. Generally, the depth from Step 1 will be more than that obtained from the strength formula.

Step 4: Check the depth used for shear. As the actual percentage of steel at supports is not known, the check is only approximate. A value of τ_c corresponding to the lowest percentage of steel in Table 13 of IS 456 may be used for this purpose. This value can be increased by a factor k. The depth used should be such that, in the final analysis, the slab is safe without any shear reinforcements.

Step 5: As the depth selected is usually greater than the minimum depth d, the tension steel required will be less than the balanced amount for the section. Determine the steel required by a suitable formula or by use of SP 16 charts and tables (see Section 11.6).

Step 6: Check whether this steel is less than the minimum percentage of the gross section specified for slabs, namely, 0.12 per cent with high yield steel and 0.15 per cent with mild steel bars. If so, provide at least the specified minimum. Table 11.3 may be used for this purpose. (IS 456: clause 25.5.2.1)

TABLE 11.3 SPACING OF DISTRIBUTION STEEL FOR SLABS (cm)

Thickness of slab (cm)	Fe 250 (0.15%) Diameter of bar (mm)				Fe (0.12%) Diameter of bar (mm)			
	6	8	10	12	6	8	10	12
10	18	30			23			
11	17	30			21	35		
12	15	27	40		19	30		
13	14	25	40		18	30		
14	13	23	35		16	29		
15	12	22	30		15	27	40	
17.5	10	19	29		13	23	35	
20	9	16	26	35	11	20	30	
22.5	8	14	23	30	10	18	29	40
25	7	13	20	30	9	16	26	35

Step 7: Choose a suitable diameter for reinforcement and determine the spacing of steel. For crack control, this spacing should suit the bar spacing rules for slabs. In general, the spacing of main steel should not exceed three times effective depth or 450 mm, whichever is smaller, but for slabs less than 300 mm it is better to limit the spacing to 200 mm [IS 456: clause 25.3.2b].

Step 8: Recheck for shear stresses, using the actual percentage of steel available.

Step 9: Check the adopted depth for deflection using the empirical method given in Chapter 9.

Step 10: Provide necessary distribution (secondary) reinforcement. This too should not be less than the specified per cent of the cross-sectional area of the slab, namely, 0.12 per cent for high yield bars and 0.15 per cent for rolled mild steel bars. This steel is usually placed over the main reinforcement bars to maximise the effective depth for the main reinforcements and facilitate the order of placing of steel. Check also the spacing of the secondary steel, so that it does not exceed five times the effective depth.

Step 11: For the slab forming the top flange of a T or L beam, the transverse reinforcement provided on the top surface should extend across the full effective width of the flange. According to IS 456: clause 22.1.1, this transverse steel should not be less than 60 per cent of the main steel at mid-span of the slab and according to BS 8110 the amount should not be less than 0.15 per cent of the longitudinal cross-sectional area of the flange (both for high yield steel and mild steel).

These steels are only for tying up the slabs with the beams, and not for absorbing any stresses in the compression zone of T beams. When the slab is spanning across T beams, the negative steel may be used for this purpose. When the slab is spanning in the direction of the T beams, separate steel has to be employed for this transverse steel to make the beam act as a T beam.

Detailing of steel in slabs subjected to uniformly distributed load is shown in Figs. 11.2 and 11.3. (In Fig. 11.3, the bar marks indicate the following: 1, Main positive tension steel; 2, secondary steel for 1; 3, main negative tension steel; 4, secondary steel for 3; 5, reinforcement for flanges; 6, secondary steel for 5; and 7, lap bars (optional).)

11.6 USE OF DESIGN AIDS SP 16

IS Publication SP 16 gives various tables and charts for rapid design of slabs. One may use either tables 1 to 4 of SP 16, which are usually used for design of singly reinforced beam sections by assuming $b = 1000$, or tables 5 to 44 giving the moment of resistance of slabs of specific thicknesses 10 to 25 cm for different values of f_{ck} and f_y. The tables are made for the standard 15 mm clear cover to slab reinforcement.

The minimum steel ratio of 0.12 per cent for Fe 415 and 0.15 per cent for Fe 250 with respect to gross area to be provided as distribution steel can also be read off for the various thicknesses of these slabs, from Table 11.3.

11.7 CONCENTRATED LOAD ON ONE-WAY SLABS

The bending effects due to concentrated loads on one-way slabs are usually analysed by the effective width method discussed in Sections 11.7.1 and 11.7.2. The corresponding effects on two-way slabs are analysed by Pigeaud's method which is given in Chapter 12. It may be noted that even though one-way slabs can also the analysed by Pigeaud's method, effective width method is more commonly used for such slabs. For these, both bending moment and beam shear produced by

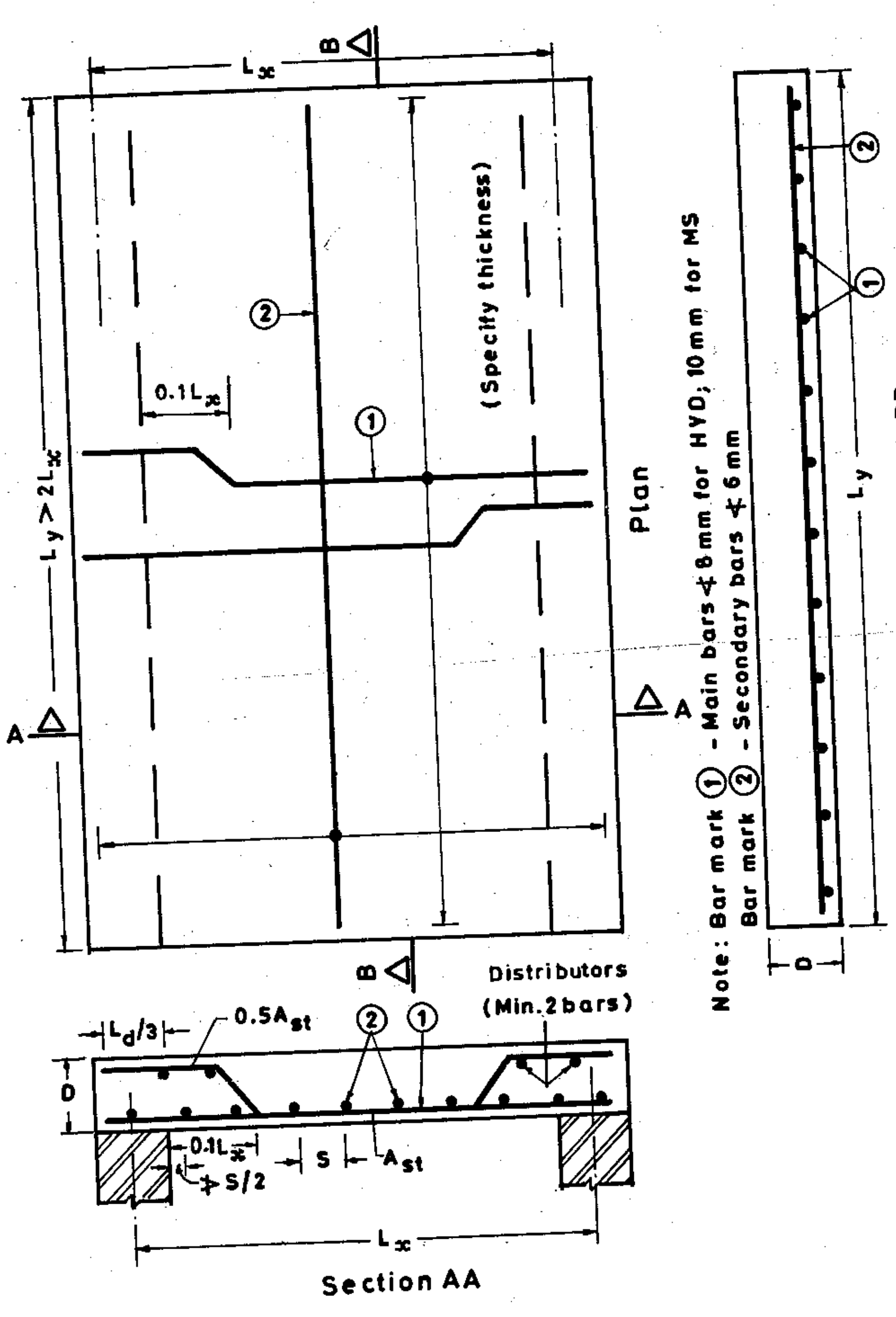


Fig. 11.2a Layout of steel in one-way simply supported slabs with UDL.

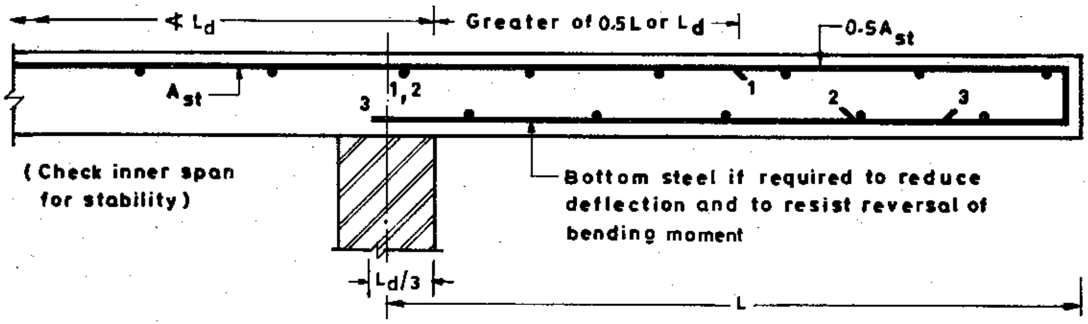


Fig. 11.2b Layout of reinforcement in cantilever slabs.

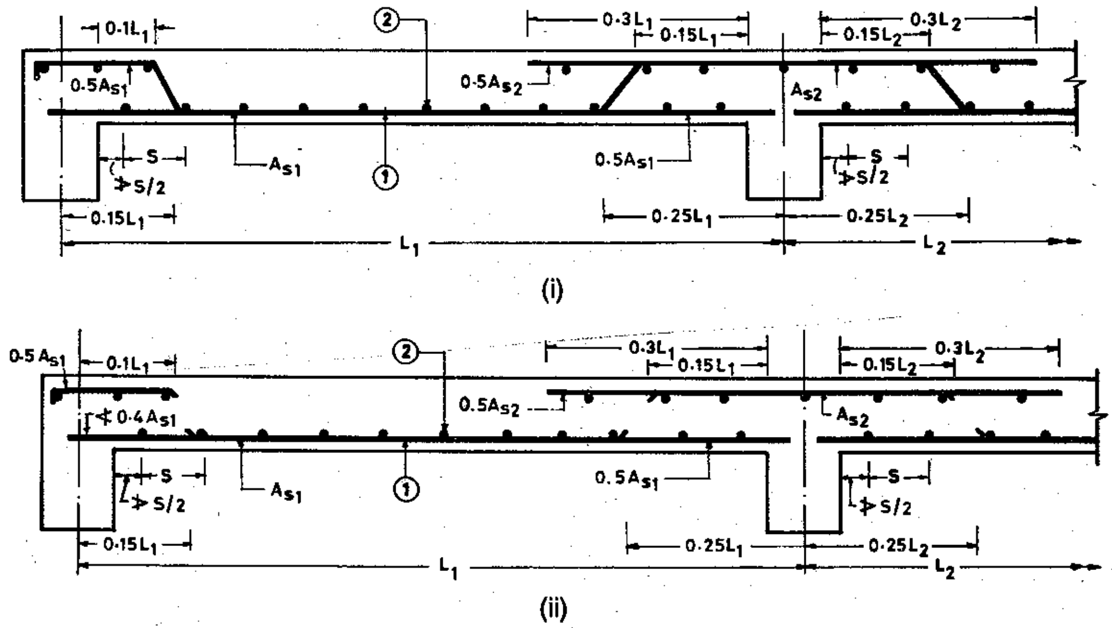


Fig. 11.3a Layout of steel in one-way continuous slabs under UDL: (i) Using bent-up bars; (ii) Using straight bars only.

concentrated loads have to be considered. It may be noted that the position of loads to produce the maximum bending moment will be different from that for producing maximum shear.

11.7.1 THEORY OF EFFECTIVE WIDTH METHOD

Let a concentrated load of length y_1 (along the span) and width x_1 (parallel to the supports) be placed on a one-way slab of span L as shown in Fig. 11.4. (Usually, in the wheeled vehicle the dimension of the contact area along the road way will be smaller than the dimensions perpendicular to it, while in the tracked vehicle the dimensions along the road will be larger than that perpendicular to it.)

The load is first considered as dispersed along the width (x_1) through the wearing coat only at 45° so that the width of the load after dispersion is given by

$$a = x_1 + 2\ (\text{thickness of wearing coat})\ \text{(see Fig. 11.4a)}.$$

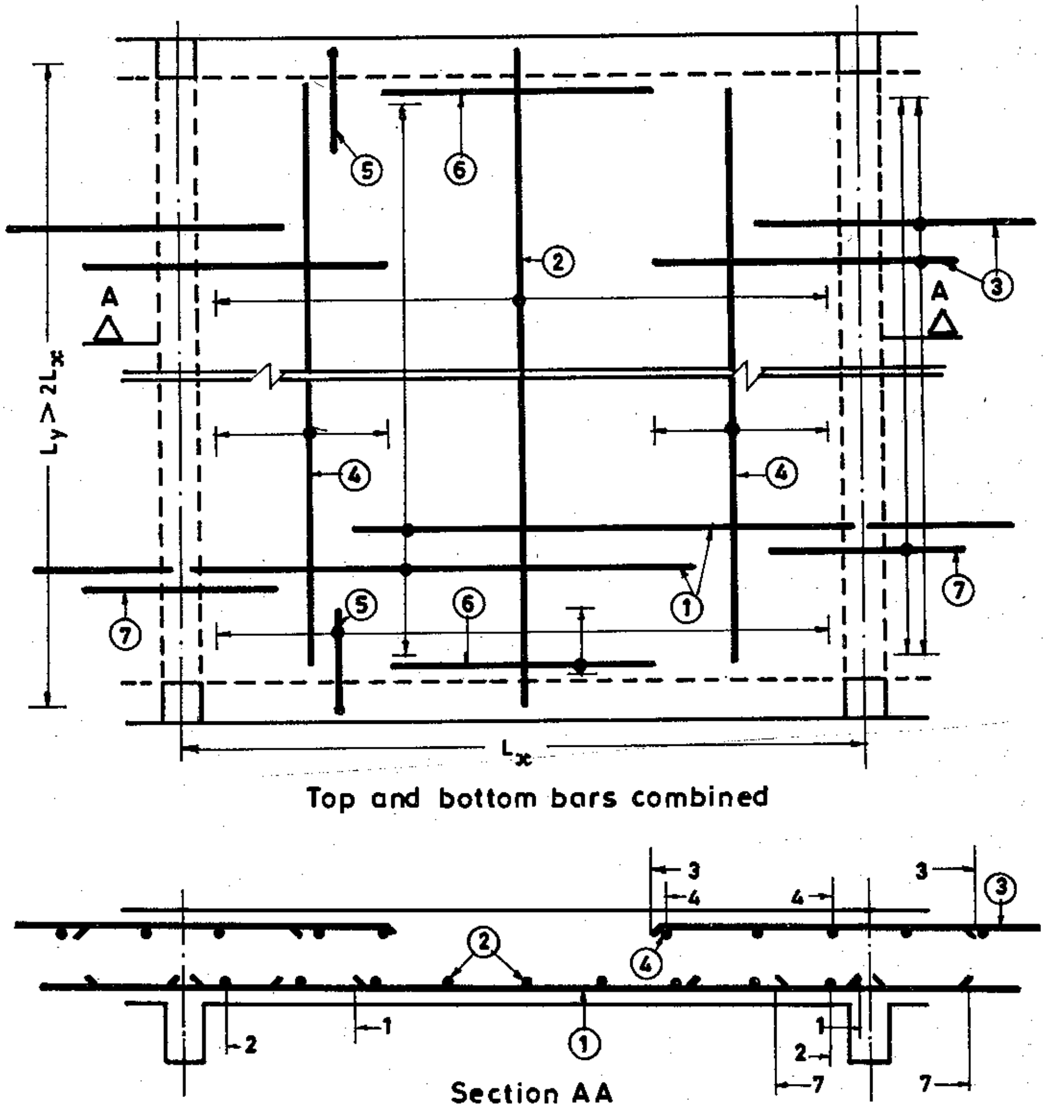


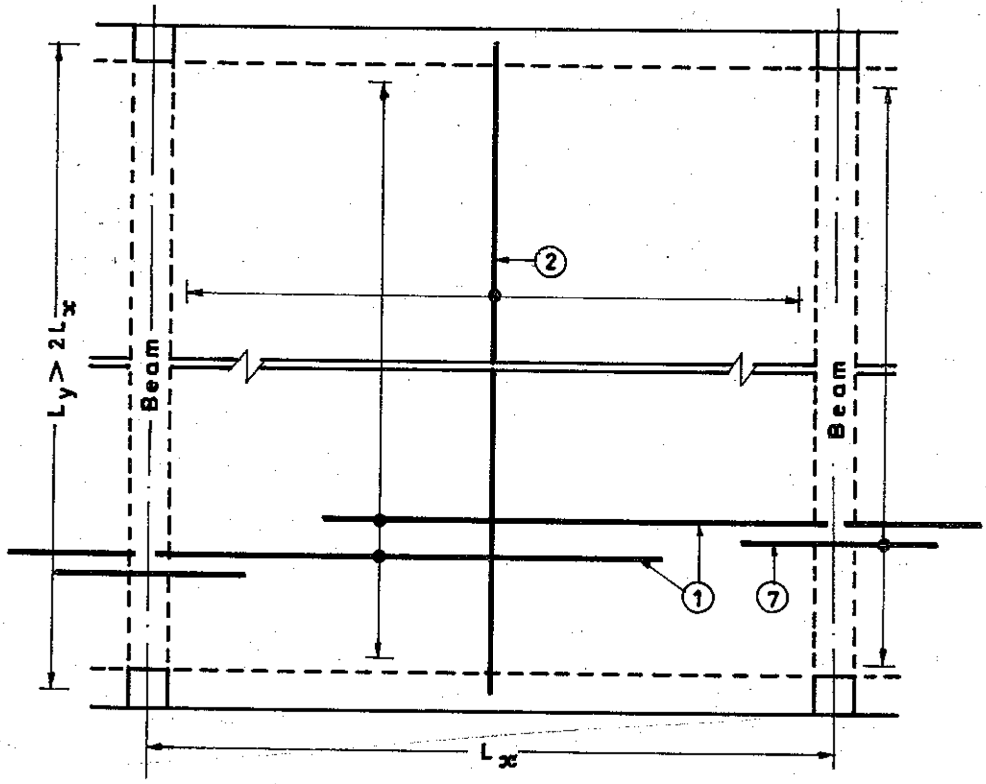
Fig. 11.3b Reinforcement drawings for one-way continuous slab using straight bars.

Some designers take a as the distributed width through the depth of the slab also, in which case

$$a = x_1 + 2 \text{ (wearing coat + effective depth)}$$

Similarly, let $e = y_1 + 2$ (wearing coat + effective depth). The first method is more conservative and generally used for highway slabs. The second method may be used in other safer situations. The effective width b_e is then calculated by the expression

$$b_e = kx\left(1 - \frac{x}{L_e}\right) + a$$



Bottom bars

Fig. 11.3c Layout of bottom bars in Fig. 11.3b.

given in IS 456: clause 23.3.2.1 (a to c), Fig. 11.4.

k = constant given in Table 9 of IS 456 (Table 11.4 of the text) depending on the ratio of the width of the slab (B) to its effective span (L), and the nature of the slab whether it is simply supported or continuous. (A value of $k = 1.2$ on either side of load width as shown in Fig. 11.4c may be assumed for all one-way slabs.)

x = distance of the centroid of load from the near support

L_e = effective span

a = dispersed width of the contact area of the concentrated load parallel to the supported edge. (Dispersion is taken at 45° through wearing coat only.)

Under no circumstances should the effective width exceed the actual width of the slab, and when the concentrated load is close to an unsupported edge of the slab, effective width should not exceed the above value or one-half the above value plus the distance of the load from the unsupported edge, whichever is less.

For determining the effective length of the load in the direction of the span, the dispersion of the load along the bridge through full effective depth of the slab is usually taken into account,

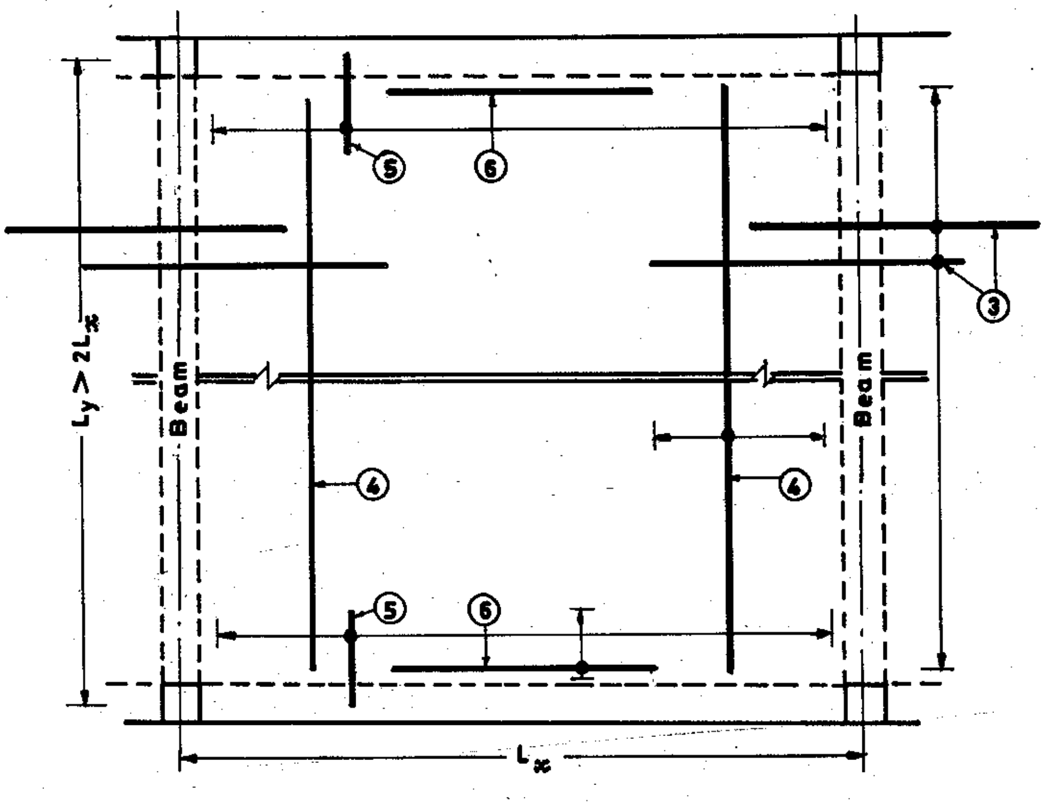


Fig. 11.3d Layout of top bars in Fig. 11.3b.

thus assuming at 45° through both the wearing coat and the effective depth. Such dispersion is taken into account for each load when the loads are placed one behind the other on the slab along the direction of the span.

When the effective widths and the effective lengths of loads as calculated above overlap, for two or more loads, the effective widths for each load should be considered separately and marked. If the effects overlap, the slab should be designed for the combined effects of the two loads on the overlapped portion.

11.7.2 EFFECTIVE WIDTH OF CANTILEVER SLABS

For cantilever slabs the effective width is given by the expression in IS 456: clause 23.3.2.1(d):

$$b_e = 1.2a_1 + a$$

where

a_1 = distance of the concentrated load from the face of the cantilever

a = width of the contact area of concentrated load parallel to the support (as in the case of the one-way slab)

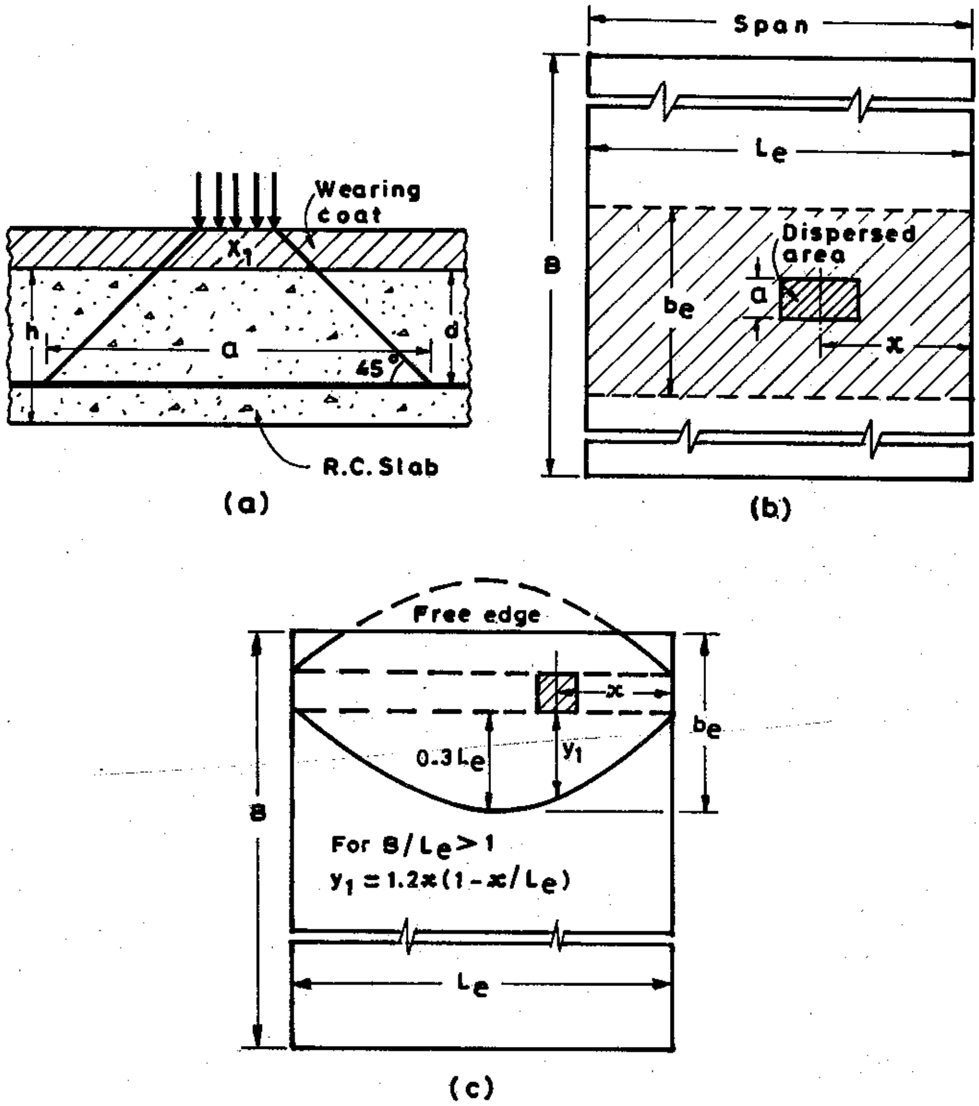


Fig. 11.4 Concentrated load on one-way slabs: (a) Dispersion of load; (b) load width; and (c) variation of load width with position of load.

11.7.3 DETERMINATION OF MAXIMUM BENDING MOMENT AND SHEAR

The maximum bending moment is determined by placing the load in such a position as to produce the maximum moments and then determining the corresponding effective width.

Similarly, for determining the maximum bending shear the load is so placed as to produce maximum shear. The corresponding effective width and shear are calculated. The procedure is shown in Example 11.3.

TABLE 11.4 VALUES OF *k* FOR CONCENTRATED LOADS ON SLABS (ONE-WAY SLABS) FOR EQUIVALENT WIDTH METHOD
(IS 456, Table 9)

B/L	Types of slabs	
	Simply supported	Continuous
0.1	0.4	0.4
0.2	0.8	0.8
0.3	1.16	1.16
0.4	1.48	1.44
0.5	1.72	1.68
0.6	1.96	1.84
0.7	2.12	1.96
0.8	2.24	2.08
0.9	2.36	2.16
1.0	2.48	2.24

EXAMPLE 11.1 **(Design of one-way slab)**

Design a simply supported R.C.C. slab for a roof of a hall 4 m × 10 m (inside dimensions) with 230 mm walls all around. Assume a live load of 4 kN/m^2 and finish 1 kN/m^2. Use grade 25 concrete and Fe 415 steel.

Ref.	Step	Calculations	Output
	1.	*Calculate factored loads*	ϕ = 10 mm
IS 456 23.1		Assume $d = \frac{\text{span}}{25} = \frac{4000}{25} = 160$ mm	d = 160 mm
		(For slabs, a larger *L/d* than for beam can be assumed)	
		Total depth (h) = 160 + 5 + 15 (cover) = 180 mm	h = 180 mm
		Dead load = 0.180 × 25 = 4.5 kN/m^2	
		Finish = 1.00 kN/m^2	
		Total DL = 5.50 kN/m^2	
		LL = 4.0 kN/m^2	
		Factored (design load) = 1.5 (5.50 + 4.0) = 14.25 kN/m^2	w = 14.25 kN/m^2
		Span length of slab	
		Span = effective span + d = 4.0 + 0.16 = 4.16 m	
		Total load on span per metre width	
		W = 4.16 (14.25) = 59.28 kN	

Sagar Samaria.

EXAMPLE 11.1 (cont.)

Ref.	Step	Calculations	Output
	2.	*Ultimate moments and shears* $M = \frac{WL}{8} = \frac{59.28 \times 4.16}{8} = 30.83$ kNm $V = \frac{W}{2} = \frac{59.28}{2} = 29.64$ kN	
SP 16 Table C	3.	*Check depth for bending* For Fe 415 steel, $M = 0.138 f_{ck} b d^2$ $d = \left(\frac{M}{0.138 f_{ck}\, b}\right)^{1/2} = \left(\frac{30.83 \times 10^6}{0.138 \times 25 \times 1000}\right)^{1/2}$ $= 94.5$ mm	$d = 160$. O.K.
IS 456 Table 13	4.	*Rough calculation for shear* $v = \frac{V}{bd} = \frac{29.64 \times 10^3}{1000 \times 160} = 0.185\ \mathrm{N/mm^2}$ τ_c for grade 25 concrete = 0.36 (min) $v < \tau_c$	Shear O.K.
	5.	*Calculation of steel areas* (Depth is greater than minimum required; section is underreinforced. Any of the four methods described in Chapter 4 can be used.) *Procedure (i) from formula for NA depth* $\frac{x}{d} = 1.2 - \left(1.44 - \frac{6.6M}{f_{ck} b d^2}\right)^{1/2}$ $\frac{6.6M}{f_{ck} b d^2} = \frac{6.6 \times 30.83 \times 10^6}{25 \times 1000 \times (160)^2} = 0.318$ $\frac{x}{d} = 1.2 - 1.059 = 0.141$ $z = d\left(1 - 0.416\,\frac{x}{d}\right) = 150.6$ mm	$\frac{x}{d} < 0.5$ (O.K.)
	•	$A_s = \frac{30.83 \times 10^6}{0.87 \times 415 \times 150.6} = 567\ \mathrm{mm^2}$ *Procedure (2) By SP 16 Table (3)* $\frac{M}{bd^2} = \frac{30.83 \times 10^6}{1000 \times (160)^2} = 1.204$	

EXAMPLE 11.1 (cont.)

Ref.	Step	Calculations	Output
SP 16 Table 3		$p_t = 0.354$	
		$A_s = \dfrac{0.354 \times 1000 \times 160}{100} = 567\ \text{mm}^2$	
	6.	*Main steel*	
		Provide T10 at 130 mm. Then, $A_s = 604\ \text{mm}^2/\text{m}$	T10 at 130 mm ($604\ \text{mm}^2$)
	7.	*Check for control of cracks*	
		Min $p_t = 0.12$, $A_s = \dfrac{0.12 \times 1000 \times 180}{100} = 216\ \text{mm}^2$	p O.K.
IS 456 25.5.2.2		$d = 10 < \dfrac{180}{8}$ mm	Diameter O.K.
		Max spacing = $3d$ = 480 mm	Spacing O.K.
	8.	*Recheck for shear* (see step 4)	
		$p = \dfrac{604 \times 100}{1000 \times 160} = 0.38\%$, $f_{ck} = 25\ \text{N/mm}^2$	
		Assume support steel as 0.19 per cent.	
IS 456 Table 13		$\tau_c = 0.36$, $v = 0.185 < \tau_c$ Slab is safe in shear.	
	9.	*Check for deflection*	
IS 456 22.2.1		Basic span depth ratio = 20	
		Factor F_1 for $p_t = 0.38 = 1.35$	
		Allowable $L/d = 1.35 \times 20 = 27$	
IS 456 Fig. 3		Assumed $L/d = 25$	
		Hence safe in deflection.	Deflection O.K.
	10.	*Provide secondary steel*	
		$A_s = \dfrac{0.12\,(b \times h)}{100} = \dfrac{0.12 \times 1000 \times 180}{100} = 216\ \text{mm}^2$	
		Provide 8 mm at 200 mm spacing = $251\ \text{mm}^2$	T8 at 200 mm ($251\ \text{mm}^2$)
		Spacing less than $5d = 5 \times 160 = 800$ mm	Spacing O.K.
	11.	*Detail steel*	
		Detail steel according to standard practice.	

EXAMPLE 11.2 **(Design of continuous slabs)**

Design the interior span of a continuous one-way slab for an office floor continuous over T beams spaced at 4 m centres. Assume f_{ck} = 25 N/mm^2 and Fe 415 steel.

Ref.	Step	Calculations	Output
	1.	*Calculate factored loads*	
		Assume $d = \frac{\text{span}}{30} = \frac{4000}{30} = 135$ mm (approx.)	$\phi = 16$ mm
		Total depth $(h) = 135 + 8 + 15 = 158$ mm	
		Assume $h = 160$ mm, $d = 137$ mm	$d = 137$ mm $h = 160$ mm
		Dead load = $0.160 \times 25 = 4.0$ kN/m^2	
		Finish = 1.0 kN/m^2	
		Total DL = 5.0 kN/m^2	
		LL for office floor = 3.0 kN/m^2	
IS 456 21.4.1.b		Ratio of $\frac{\text{LL}}{\text{DL}} = 0.6$ less than 0.75	Separate analysis of DL + LL not needed
		Design (factored load) $= 1.5(5.0 + 3.0) = 12$ kN/m^2	
		Span load = $W = 12 \times 4 = 48$ kN/m width	
	2.	*Ultimate moments*	
IS 456 Table 7		At interior support $= \frac{Wl}{12} = \frac{48 \times 4}{12} = 16$ kNm	
		At interior span $= \frac{Wl}{24} = 8.0$ kNm	
	3.	*Check depth for moment*	
		(For Fe 415 steel), $d = \left(\frac{M}{0.138 f_{ck} b}\right)^{1/2}$	
		$d = \left(\frac{16 \times 10^6}{0.138 \times 25 \times 1000}\right)^{1/2} = 68$ mm	Adopt $d = 137$ mm
	4.	*Rough check for shear*	
IS 456 Table 13		$v = \frac{48}{2} = 24$ kN, $\tau_{c\,(\min)}$ for $M25 = 0.36$ N/mm^2	
		$v = \frac{24 \times 10^3}{1000 \times 137} = 0.175 < 0.36$	Shear O.K.

EXAMPLE 11.2 (cont.)

Ref.	Step	Calculations	Output
	5.	*Calculation of steel areas*	
		Adopted depth is greater than required. Hence section is underreinforced. Any of the four methods in Chapter 4 can be used.	
	6.	*Procedure by using SP 16 Tables*	
		(a) *Support steel*	
SP 16 Table 3		$\frac{M}{bd^2} = \frac{16 \times 10^6}{1000 \times (137)^2} = 0.852$	
		$p_t = 0.246$ per cent	
		$A_s = \frac{p_t bd}{100} = \frac{0.246 \times 1000 \times 137}{100}$	
		$= 337 \text{ mm}^2$	
		Minimum steel area $= \frac{0.12bh}{100}$	
		$= \frac{0.12 \times 1000 \times 160}{100} = 192 \text{ mm}^2$	
		(b) Span steel need be only one-half at the support	
		$A_s = 169 \text{ mm}^2 < 192$	
		Provision of main steel	
		For moments in span = 169 mm^2	
		Provide minimum steel = 192 mm^2	
SP 16 Table 96		Hence provide T10 at 380 mm in span	T10 at 380 mm (207 mm^2)
		This gives support steel $A_s = 414$ mm^2	T10 at 190 mm (414 mm^2)
	7.	*Check for cracking*	
		Steel area greater than 0.12%bh (192 mm^2)	p O.K.
		Spacing less than $3d = 3 \times 16 = 48$ cm	Spacing O.K.
		Diameter of rod $< \frac{h}{8}\left(\frac{160}{8} = 20 \text{ mm}\right)$	Diameter O.K.
	8.	*Recheck for shear*	
		v is very much less than τ_c (min for $M25$)	

EXAMPLE 11.2 (cont.)

Ref.	Step	Calculations	Output
	9.	*Check for deflection (at middle of slab)*	
		Basic L/d ratio = 20	
IS 456		p_t (required) = 0.12 per cent, $F_1 > 2$	
Fig. 3		(Alternatively, calculate F_1 from formula)	
		$F_1 = \dfrac{1}{0.225 + 0.003 f_s - 0.625 \log\left(\dfrac{1}{p_t}\right)}$	
		$f_s = \dfrac{5}{8} \times f_y \dfrac{A_s \text{ (required)}}{A_s \text{ (provided)}} = \dfrac{5 \times 415 \times 169}{8 \times 207}$	
		$= 211.76 \text{ N/mm}^2$	
		Hence, $F_1 = 3.51$	
		Max. value allowed = 2	
		Span depth ratio allowed = 2 × 20 = 40	
		Adopted L/d = 30	Deflection O.K.
	10.	*Provide secondary steel (see Steps 5 and 6)*	
		Provide 10 mm at 38 cm	T10 at 380 mm
	11.	*Detail steel as per standard practice*	
	12.	*Check for top steel for T-beam action*	
IS 456 22.1.1(b)		The detailing arrangements provide more than 60 per cent of main steel in mid-span of the slab as transverse steel.	T-beam action O.K.

EXAMPLE 11.3 **(Effective width for one concentrated load)**

A bridge slab spans between two longitudinal girders placed 2 m apart. Assuming an effective depth of 125 mm for the slab, determine the design bending moment and shear for a wheel load of 25 kN on a contact area of 300 × 75 (mm), the lesser dimension being in the direction of the wheel. Use the effective width method and assume that the slab is simply supported.

Ref.	Step	Calculations	Output
	1.	*Determine a and c assuming 45° dispersion through effective depth*	
		$a = 75 + (2 \times 125) = 325$ mm	
		$c = 300 + (2 \times 125) = 550$ mm (along span)	
		The load acts through 325 × 550 mm area	

EXAMPLE 11.3 (cont.)

Ref.	Step	Calculations	Output
	2.	*Maximum BM* is produced when load is placed at centre of span = M_0 $M_0 = \frac{W}{2}\left(\frac{L}{2} - \frac{c}{4}\right)$ $= \frac{25}{2}\left(\frac{2}{2} - \frac{0.55}{4}\right)$ $= 10.78$ kNm	
IS 456 Table 9 Fig. 11.4(e)	3.	*Effective width normal to the span over which M_0 acts.* $b_e = kx\left(1 - \frac{x}{L}\right) + a$ k depends on $\frac{\text{width of slab}}{\text{effective span}} = \frac{B}{L}$ $\frac{B}{L} = \frac{\text{very large}}{2} > 1$ $k = 2.48$ (or 1.24 on either side) $x = 1$ m $b_e = 2.48\ (1)\ (1 - 1/2) + 0.325$ $= 1.565$ m	
	4.	*Moment per unit width* $M = \frac{M_0}{b_e} = \frac{10.78}{1.565}$ $= 6.90$ kNm/m	
	5.	*Design Moment* $M_u = 1.5M = 1.6 \times 6.9$ $= 10.35$ kNm per metre width (*Note*: If the slab is continuous, reduce span moment by 20 per cent for continuity)	$M_u = 10.35$ kNm
	6.	*Maximum SF* V_{max} when load is near support. Assume wheel load touches the support $c = 300 + 125$ (dispersion to one side only) $= 0.425$ m	

EXAMPLE 11.3 (cont.)

Ref.	Step	Calculations	Output
		$x = \dfrac{0.425}{2} = 0.213$ m	
		$V_0 = \dfrac{25(2 - 0.213)}{2} = 22.34$ kN	
	7.	*Effective width over which V_0 acts*	
		$b_e = kx\left(1 - \dfrac{x}{L}\right) + a$	
		k is only a function of B/L and has a value of 2.48 as before, $x = 0.213$ m	
		$b_e = 2.48 \times 0.213\left(1 - \dfrac{0.213}{2}\right) + 0.325$	
		$= 0.797$ m	
	8.	*Shear per unit width*	
		$V = \dfrac{V_0}{b_e} = \dfrac{22.34}{0.797} = 28$ kNm	
		$V_u = 1.5 \times 28 = 42$ kN per metre width	$V_u = 42$ kN

EXAMPLE 11.4 **(Effective width of two concentrated loads)**

A slab for a culvert has a two-lane roadway of 7.5 m with 600 mm kerb on the sides. The slab has an effective span of 6.4 between the abutments. Assuming an effective depth of 500 mm for the slab and that its top wearing coat is 80 mm thick, find the maximum design BM and SF due to IRC class AA tracked vehicle loading, each of 350 kN on 850 × 3600 mm area at 2050 centres.

Ref.	Step	Calculations	Output
		(*Note*: With two loads, it is better to work with dispersion lengths and effective width components on each side of the load separately for each load.)	
	1.	*Dispersion lengths assuming 45° dispersion*	
		On each side of load = 500 + 80 = 580 mm	
		$a = 850 + (2 \times 580) = 2010$ mm	
		$c = 3600 + (2 \times 580) = 4760$ mm	
	2.	*Maximum BM (When load is centre of span)*	
		$M_0 = \dfrac{W}{2}\left(\dfrac{L}{2} - \dfrac{c}{4}\right)$	

EXAMPLE 11.4 (cont.)

Ref.	Step	Calculations	Output
		Load at $x = \frac{L}{2} = \frac{6.4}{2} = 3.2$ $M_0 = \frac{2 \times 350}{2}\left(3.2 - \frac{4.76}{4}\right)$ $= 703.5$ kNm	
	3.	*Effective width of the tracked wheel load* Effective width components (a) width of load on road surface = 850 mm (b) dispersion = 580 mm on each side (c) Distribution = $kx(1 - x/L)$	
IS 456 Table 9		$\frac{B}{L} = \frac{7.5 + 1.2}{6.4} = 1.36 > 1$ $k = 2.48, \quad x = \frac{6.4}{2} = 3.2$ m	
Text Fig. 11.4(c)		It can be taken as $k' = 1.24$ on each side with $x = 3.2$ m $k'x(1 - x/L) = 1.24 \times 3.2\ (1 - 1/2)$ $= 1.984$ m on each side Length of dispersion between the two track loads $= 1984 + 580 + \frac{850}{2} = 2989$ mm As 2989 mm is greater than 2050, the distance between centres of loads, effective width of the two loads at centre of load overlaps.	
	4.	*Minimum effective width* This is obtained not when the load is at the centre of the width of the slab but when load is near the curb as shown now.	
	Case (a)	*Load at centre* Effective width $= 2\left(1984 + 580 + \frac{850}{2}\right) + 2050$ $= 2(2989) + 2050$ mm $= 8025$ mm $= 8.025$ m	

Output

EXAMPLE 11.4 (cont.)

Ref.	Step	Calculations	Output
	Case (b)	*Load near curb* Minimum clearance of wheel from kerb (IRC) $= 1200$ mm Effective width $= \left(600 + 1200 + \frac{850}{2}\right) + 2050 + 2989$ mm $= 7.264$ m Least value of the width $= 7.264$ m	
	5.	*BM per unit width* $M = \frac{M_0}{7.26} = \frac{703.5}{7.26} = 96.9$ kNm/m $M_u = 1.5M = 145.4$ kNm per metre width	$M_u = 145.4$ kNm/m
	6.	*Maximum shear V* Maximum shear obtained when track loads touch the support. Distance of C.G. of loads from support $x = 1.8$ m $V_{max} = \frac{2 \times 350(6.4 - 1.8)}{6.4} = 503$ kN	
	7.	*Effective width component when* $x = 1.8$ m As B/L is same as in step 3 $k = 2.48$ $k' = 1.24$ on either side With $x = 1.8$ m from support $k'x(1 - x/L) = 1.24(1.8)\left(1 - \frac{1.8}{6.4}\right)$ $= 1600$ mm on either side. Length of load dispersion from centre line of one load in the direction of the other load. $= 1600 + 580 + 425 = 2605$ mm 2605 mm is greater than 2050; the two distributions overlap at centre of loads.	
	8.	*Minimum effective width for shear* Load at 1200 from curb.	

EXAMPLE 11.4 (cont.)

Ref.	Step	Calculations	Output
		Effective width from load to curb side	
		$= 580 + 1600 > 1200 + 600$	
		Hence limit to 1800	
		Effective width	
		$= \left(600 + 1200 + \frac{850}{2}\right) + 2050 + (2605)$	
		$= 6880$ mm	
	9.	*Shear per unit width*	
		$V = \frac{V_0}{6.88} = \frac{503}{6.88}$	
		$= 73.1$ kN/m	
		$V_u = 1.5 \times 73.1 = 109.7$ kN per metre width	$V_u = 109.7$ kN/m

REVIEW QUESTIONS

11.1 Distinguish between one-way and two-way slabs. How are they analysed for determining BM and SF under a UDL?

11.2 What is meant by class 300 loading in IS 875?

11.3 What are the limitations in using the BM and SF coefficients for analysing a one-way slab continuous over supports?

11.4 What are the rules for designing slabs for shear? Why is shear magnification factor applicable for slabs less than 300 mm thick?

11.5 Why is the span/effective depth ratio of slabs larger than that for beams?

11.6 What type of slabs are usually used in practice, underreinforced or overreinforced?

11.7 What are the specifications according to Indian Standards with respect to the following:

(a) Minimum reinforcement to be provided as main and secondary reinforcements in slabs
(b) Maximum shear allowed in slabs

11.8 Show how the distribution steel is placed with respect to the main reinforcements (above or below the main steel):

(a) at the centre part of the slab,
(b) at supports portion of the slab.

Explain the reasons for your choice of placement.

11.9 What are the factors that affect deflection consideration of one-way slabs? How can the crack width in slabs be controlled?

11.10 What value of cover to the main steel is assumed in Tables 5 to 44 given in SP 16? How does one read off the distribution steel needed for various thicknesses of the slab from these tables?

PROBLEMS

11.1 A reinforced concrete slab is to span a series of rooms 3 m × 8 m. The slab is subjected to a live load of 4 kN/m^2. Design an interior R.C.C. slab using M20 concrete and torsteel bars. Sketch the layout of the reinforcements with (a) system of straight bars and (b) system of bent-up bars.

11.2 A verandah slab cantilevers 2 m over a fixed wall. The slab has to take a live load of 4 kN/m^2. Design the slab using grade 20 concrete and high yield deformed bars. Sketch the details of placement of the steel.

11.3 A simply supported one-way slab is reinforced with Fe 415 high yield deformed bars. The reinforcement consists of 10 mm bars at 150 mm, along the span and 10 mm bars at 325 mm, perpendicular to the span of the slab. If the effective span is 4.5 m and the slab thickness is 200 mm, determine the maximum safe load it can carry. Check also the deflection requirements of the slab and adequacy of the secondary steel.

11.4 A simply supported floor slab 6.5 m × 3.0 m has to carry a half-brick partition wall of reinforced brickwork 3 m in height built along the full 3 metre span at the centre of the slab in addition to an imposed characteristic load of 2.5 kN/m^2. Design the floor slab assuming f_{ck} = 20 N/mm^2, f_y = 415 N/mm^2.

11.5 Design the slab given in Problem 11.4 assuming that the wall is built along the 6.5 metre length along the centre line of the slab.

11.6 A one-way slab is continuous over a series of walls 300 mm thick spaced at 4.0 m centres. It has to carry a characteristic live load of 4 kN/m^2 and supports, in addition, a strip load of 6 kN/m due to a light weight partition wall placed along the span of the slab, the width of partition being 100 mm. Assuming f_{ck} = 25 N/mm^2 and f_y = 415 N/mm^2, design the slab and draw the details of reinforcement using (a) straight bars and (b) bent-up bars.

12

Ly is always the greater side.

Design of Two-way Slabs

12.1 INTRODUCTION

Slabs which are supported on unyielding supports like walls on all four sides are called *two-way slabs*. The span in the larger direction is denoted by l_y and that in the shorter direction by l_x. The distribution of loads in the l_y and l_x directions will depend on the ratio l_y/l_x. When $l_y/l_x > 2$, it can be shown that most of the loads are transmitted along the shorter l_x directions and the slab acts as a one-way slab.

Beam supports which are sufficiently stiff can be considered as unyielding and slabs on these beams also act as two-way slabs. Beam supports which deflect significantly under the loading from slabs, come under *slabs on flexible beams*, and cannot be strictly classified as conventional two-way slabs. In these slabs the load distribution and bending moments produced are different from slabs on unyielding supports.

The boundaries of a two-way slab can be fully restrained (continuous), simply supported, or partially restrained at the edges as shown in Fig. 12.1. Discontinuous edges of two-way slabs,

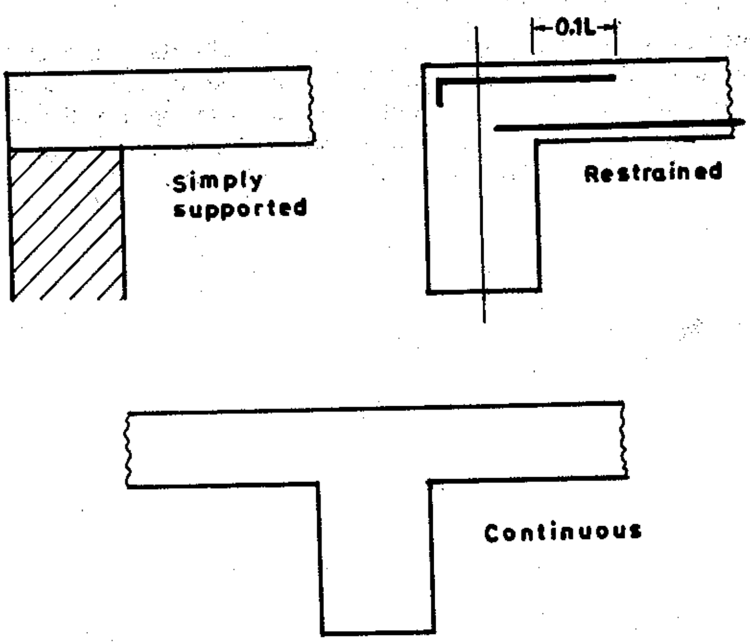


Fig. 12.1 Conditions of support of two-way slabs.

which are supported on beams are sometimes taken as simply supported or partially restrained. However, an arbitrary value of one-half the span moment or $wL^2/24$ is usually provided for the end moment of these discontinuous edges to take care of the moments due to partial fixity that may develop there. It is important that detailing of reinforcements of these slabs too should correspond to these assumptions of nil, partial or full fixity. The method of design of these two-way slabs is discussed in this chapter.

12.2 ACTION OF TWO-WAY SLABS

When a slab simply supported on all the four sides is loaded, the corners tend to curl and lift up. This is to compensate for the non-uniform distribution of pressure exerted by the slab on the supporting walls. This behaviour can be easily demonstrated by supporting a rigid cardboard with L_y less than $2L_x$ on its four sides and pressing it downwards (see Fig. 12.2).

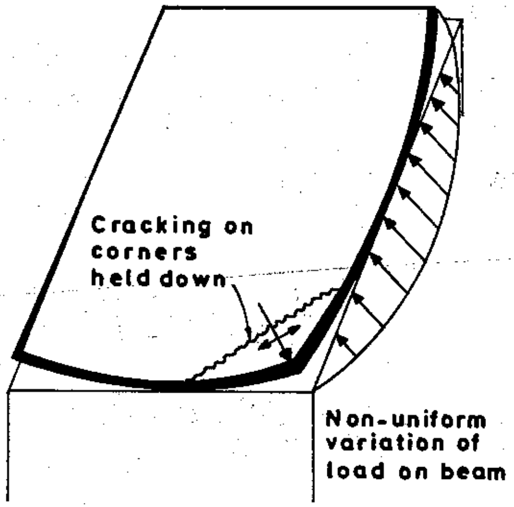


Fig. 12.2 Action of two-way slabs.

In practice, the upward movement described above may be prevented by a wall or such other construction at the edge of the slab. In such circumstances it is said that the corner is 'torsionally restrained'. Unless top steel is provided for such slabs, these will crack at the corners. These slabs also tend to carry loads by spanning diagonally across the corners. Hence, in addition to the top steel along the diagonal, bottom steel is also needed at the corners. Even though for both these purposes (i.e. the curling effect and diagonal transfer of load at corners) steel is needed diagonally, it is more convenient to provide two-way steel in the x and y directions at the top and bottom surfaces of the slab. This steel is known as *corner steel.*

12.3 MOMENTS IN TWO-WAY SLABS SIMPLY SUPPORTED ON ALL SUPPORTS

For computation of moments in simply supported cases of two-way slabs, Table 23 in Appendix C of IS 456 can be used. These are derived from the Rankine-Grashoff formula (see Fig. 12.3) which we now discuss.

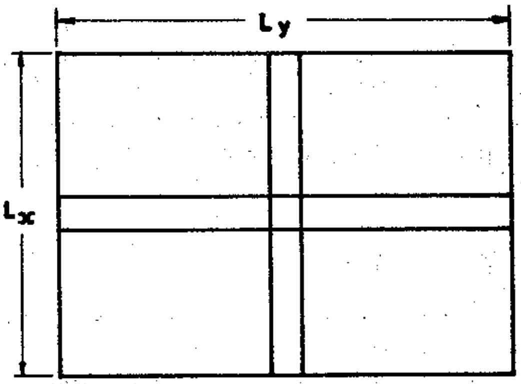


Fig. 12.3 Basis of Rankine-Grashoff formula.

12.3.1 RANKINE-GRASHOFF FORMULA

This formula is obtained by equating the deflection of middle strips in the two perpendicular directions. Let q_x and q_y be the loads in the respective directions due to a load q. Then

$$\frac{5}{384}\frac{q_x L_x^4}{EI} = \frac{5}{384}\frac{q_y L_y^4}{EI}$$

$$q = q_x + q_y$$

$$q_x = q\left(\frac{L_y^4}{L_x^4 + L_y^4}\right) = q\frac{(L_y/L_x)^4}{1 + (L_y/L_x)^4}$$

$$M_x = \frac{q}{8}\frac{(L_y/L_x)^4}{1 + (L_y/L_x)^4}L_x^2 = q\beta_x L_x^2$$

where

$$\beta_x = \frac{(L_y/L_x)^4}{8[1 + (L_y/L_x)^4]} \tag{12.1}$$

Similarly,

$$M_y = \frac{q}{8}\left(\frac{L_x^4}{L_x^4 + L_y^4}\right)L_y^2 = \frac{q}{8}\left(\frac{(L_y/L_x)^2}{[1 + (L_y/L_x)^4]}\right)L_x^2 = q\beta_y L_x^2$$

where

$$\beta_y = \frac{(L_y/L_x)^2}{8[1 + (L_y/L_x)^4]} \tag{12.1a}$$

(Note the difference between β_x and β_y in the numerator.) Values of the above expressions are given in Table 12.1. They are given as α_x and α_y in IS 456, Table 23. For computer application, the above formula itself is preferable to the values from the table. For example, when $L_y/L_x = 1.4$,

TABLE 12.1 BENDING MOMENT COEFFICIENTS FOR SIMPLY SUPPORTED TWO-WAY SLABS [EQ. (12.1)]

[Table 23 of IS 456 (1978)]

L_y/L_x	1.0	1.1	1.2	1.3	1.4	1.5	2.0	3.0
β_x	0.062	0.074	0.084	0.093	0.099	0.104	0.118	0.124
β_y	0.062	0.061	0.059	0.055	0.051	0.037	0.029	0.014

$$\beta_y = \frac{(1.4)^2}{8[1 + (1.4)^4]} = \frac{1.96}{38.73} = 0.051$$

$$\beta_x = \frac{(1.4)^4}{8[1 + (1.4)^4]} = \frac{3.84}{38.73} = 0.099$$

Thus the maximum moments per unit width for two-way simply supported slabs which do not have adequate provision for preventing corners from lifting up are calculated from Table 23 of IS 456 and from the following equations:

$$M_x = \beta_x w L_x^2, \qquad M_y = \beta_y w L_x^2 \qquad (12.1b)$$

where M_x and M_y are the moments at mid-span on strips of unit width and spans L_x and L_y, respectively. (Note that L_x, the shorter span, is used for calculation of M_x and M_y.)

12.3.2 MARCUS CORRECTION

In practice, the cutting of the slab into simple strips to calculate moments is a very rough approximation. Loads are carried also by torsion of the slab, and H. Marcus proposed a simple method for the correction for the load carried by torsion. Correspondingly, the bending moment for simply supported slabs as calculated by Rankine-Grashoff method is reduced. For slabs with the corners held down, the carrying of load by torsion in the slab is arbitrarily increased by a further amount in the Marcus correction for such slabs, so that the load carried by bending is further reduced.

It can be shown that for a simply supported slab there is approximately 40 per cent reduction in the bending ~~moments for a two-way~~ square slab due to the twin effects of torsional restraint and corners being ~~held down.~~ It is reduced to 9 per cent for a simply supported one-way slab. For a slab fixed at the ends, the corresponding values are only 14 per cent and 3 per cent, respectively. However, as the use of Rankine-Grashoff formula and Marcus correction for analysis of restrained slabs is too cumbersome for design office use, more convenient data as presented in Table 22 of IS code, which can be expressed by simple equations and thus suitable for computer aided design, are nowadays recommended by IS and BS codes for calculation of moments in restrained two-way slabs.

12.4 MOMENTS IN TWO-WAY RESTRAINED SLABS WITH CORNERS HELD DOWN

12.4.1 ANALYSIS BY COEFFICIENTS

Restrained slabs are defined in codes as those slabs which are cast integral with R.C.C. frames and which are not free to lift up at the corners. They may be continuous or discontinuous at the edges.

Those which are discontinuous at edges are also referred to as simply supported. Coefficients given in IS 456, Table 22 in Appendix C (Table 12.2 here) can be used for analysis of such slabs.

The conditions to be satisfied for use of these coefficients are:

1. The loading on the adjacent spans should be the same.
2. The spans in each direction should be approximately equal.

The span moments per unit width (which are considered as positive in sign) and the negative moments at continuous edges for these slabs are calculated from the equations

$$M_x = \alpha_x w L_x^2 \quad \text{from span } l_x$$
$$M_y = \alpha_y w L_x^2 \quad \text{from span } l_y \qquad (12.2)$$

where α_x and α_y are coefficients (see Table 12.2). The slab is also divided into the middle strip and the edge strips (column strip) as in Fig. 12.4, the middle strip forming three-fourths of the slab width. As explained in Section 12.2, torsion steel must be provided at the discontinuous edges as specified in the code.

The coefficients in Table 22 have been derived from yield line analysis made by Tayler and others and corrected for non-uniform distribution of steel*. Till recently, similar tables based on Westergard's elastic analysis of plates (but corrected for redistribution of moments as observed from tests) were used. These were presented in IS 456 (1964), CP 114 (1965), and ACI 318 (1963) as tables. These have now been replaced in IS 456 (1978) and BS 8110 by Table 22 of IS 456. ACI has discontinued the method of analysis of two-way slabs by method of coefficients and has adopted a general method of analysis called the *equivalent frame method* which is discussed under analysis of flat slabs.

For computer aided designs or in the absence of the tables, the values of the coefficients of the tables can be derived as follows (see also Fig. 12.5 and BS 8110): Let

N_d = No. of discontinuities ($0 \le N_d \le 4$)
α_y and α_x = coefficient for span moments (positive) per unit width
α_1 and α_2 = coefficients for the negative moments over the shorter edges
α_3 and α_4 = coefficients for the negative moments over the longer edges (the coefficient for the continuous edge is taken as 4/3 times the coefficient for the span moment).

The values of α_y and α_x and the support moments are determined as follows:

1. Determination of α_y

Let α_y be the coefficient for the span moment for span l_y and α_1 and α_2 the support moments in the L_y direction. Then, α_y can be found from the equation

$$\alpha_y = \frac{24 + 2N_d + 1.5N_d^2}{1000} \qquad (12.3)$$

The values of α_1 and α_2 can be obtained from α_y from the relations

$$\alpha_1, \alpha_2 = 0 \quad \text{for discontinuous edges}$$
$$\alpha_1, \alpha_2 = 4/3\ \alpha_y \quad \text{for continuous edges}$$

(*Note*: The above expression does not depend on L_y/L_x ratio.)

*R. Tayler, B. Hayes and M. Bhai, Coefficients for design of slabs by yield line theory, *Concrete*, **13**, 5, 1969.

TABLE 12.2 BENDING MOMENT COEFFICIENTS FOR RECTANGULAR PANELS SUPPORTED ON FOUR SIDES WITH PROVISION FOR TORSION AT CORNERS

[Table 22 of IS 456 (1978)]

TABLE 12.2 BENDING MOMENT COEFFICIENTS FOR RECTANGULAR PANELS SUPPORTED ON FOUR SIDES WITH PROVISION FOR TORSION AT CORNERS

[Table 22 of IS 456 (1978)]

Case no.	Type of panel and moments considered	Short span coefficients α_x (Values of L_y/L_x)								Long span coefficients α_y for all values of L_y/L_x
		1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0	
1.	*Interior Panels*									
	Negative moment at continuous edge	0.032	0.037	0.043	0.047	0.051	0.053	0.060	0.065	0.032
	Positive moment at mid-span	0.024	0.028	0.032	0.036	0.039	0.041	0.045	0.049	0.024
2.	*One Short Edge Discontinuous*									
	Negative moment at continuous edge	0.037	0.043	0.048	0.051	0.055	0.057	0.064	0.068	0.037
	Positive moment at mid-span	0.028	0.032	0.036	0.039	0.041	0.044	0.048	0.052	0.028
3.	*One Long Edge Discontinuous*									
	Negative moment at continuous edge	0.037	0.044	0.052	0.057	0.063	0.067	0.077	0.085	0.037
	Positive moment at mid-span	0.028	0.033	0.039	0.044	0.047	0.051	0.059	0.065	0.028
4.	*Two Adjacent Edges Discontinuous*									
	Negative moment at continuous edge	0.047	0.053	0.060	0.065	0.071	0.075	0.084	0.091	0.047
	Positive moment at mid-span	0.035	0.040	0.045	0.049	0.053	0.056	0.063	0.069	0.035
5.	*Two Short Edges Discontinuous*									
	Negative moment at continuous edge	0.045	0.049	0.052	0.056	0.059	0.060	0.065	0.069	
	Positive moment at mid-span	0.035	0.037	0.040	0.043	0.044	0.045	0.049	0.052	0.035
6.	*Two Long Edges Discontinuous*									
	Negative moment at continuous edge									0.045
	Positive moment at mid-span	0.035	0.043	0.051	0.057	0.063	0.068	0.080	0.088	0.035
7.	*Three Edges Discontinuous (One Long Edge Continuous)*									
	Negative moment at continuous edge	0.057	0.064	0.071	0.076	0.080	0.084	0.091	0.097	
	Positive moment at mid-span	0.043	0.048	0.053	0.057	0.060	0.064	0.069	0.073	0.043
8.	*Three Edges Discontinuous (One Short Edge Continuous)*									
	Negative moment at continuous edge									0.057
	Positive moment at mid-span	0.043	0.051	0.059	0.065	0.071	0.076	0.087	0.096	0.043
9.	*Four Edges Discontinuous*									
	Positive moment at mid-span	0.056	0.064	0.072	0.079	0.085	0.089	0.100	0.107	0.056

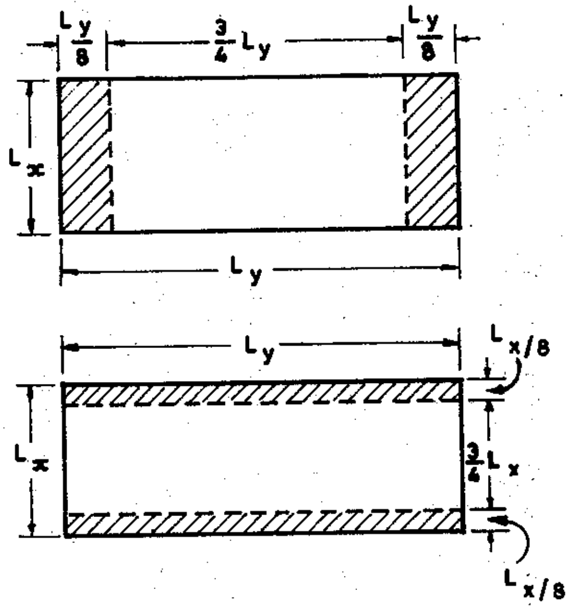


Fig. 12.4 Edge and middle strips of two-way slabs.

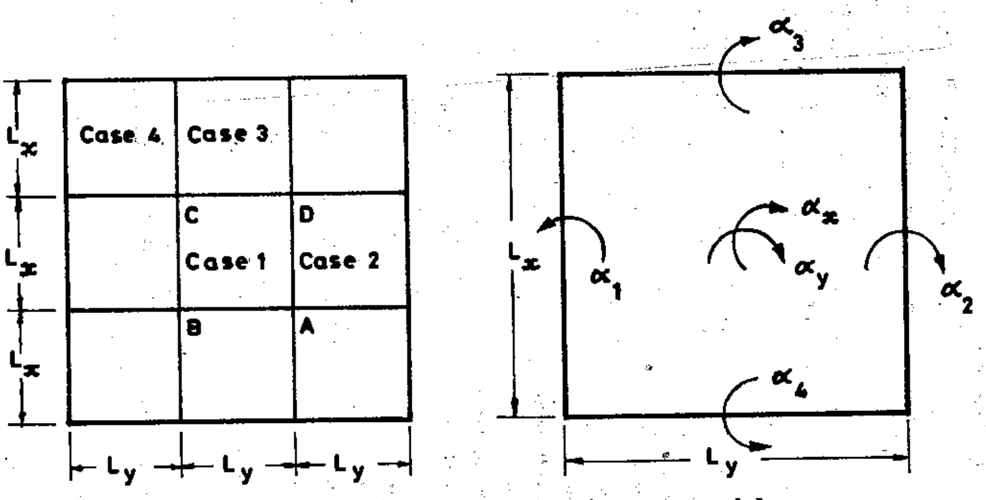


Fig. 12.5 Coefficients for moments in one-way slabs.

2. Determination of α_x

Having determined α_y, the value of α_x can also be determined from the relations

$$r = \frac{2}{9}\left[3 - \sqrt{18}\,\frac{L_x}{L_y}\left(\sqrt{\alpha_y + \alpha_1} + \sqrt{\alpha_y + \alpha_2}\right)\right] \tag{12.4}$$

$$\sqrt{r} = \sqrt{\alpha_x + \alpha_3} + \sqrt{\alpha_x + \alpha_4} \tag{12.5}$$

The values of α_3 and α_4 are obtained from the assumptions

$$\alpha_3,\ \alpha_4 = 0 \qquad \text{for discontinuous edges}$$

$$\alpha_3,\ \alpha_4 = 4/3\ \alpha_x \qquad \text{for continuous edges}$$

The solution of these equations can be further simplified by putting

$$\alpha_1 = K_1\alpha_y, \qquad \alpha_2 = K_2\alpha_y, \qquad \alpha_3 = K_3\alpha_x, \qquad \alpha_4 = K_4\alpha_x$$

Substituting these values, Eq. (12.4) becomes

$$r = \frac{2}{9}\left[3 - \sqrt{18}\,\frac{L_x}{L_y}\left(\sqrt{\alpha_y + K_1\alpha_y} + \sqrt{\alpha_y + K_2\alpha_y}\right)\right] \tag{12.5a}$$

Similarly, squaring Eq. (12.5), we obtain

$$r = \alpha_x\left(\sqrt{1 + K_3} + \sqrt{1 + K_4}\right)^2 \tag{12.5b}$$

Equating the two and solving for α_x, we get

$$\alpha_x = \frac{r}{\left(\sqrt{1 + K_3} + \sqrt{1 + K_4}\right)^2} \tag{12.5c}$$

Example. For a slab with $L_y/L_x = 1.5$ with all the four edges continuous, we can proceed as follows:

$$N_d = 0$$

Substituting this value in Eq. (12.3), we get

$$\alpha_y = \frac{24 + 0 + 0}{1000} = 0.024$$

For continuous edges, K_1 and $K_2 = 4/3$. Hence,

$$K_1\alpha_y = K_2\alpha_y = 0.032$$

$$\alpha_y + K_1\alpha_y = 0.024 + 0.032 = 0.056$$

$$\alpha_y + K_2\alpha_y = 0.056$$

Putting these values in Eq. (12.5a), we obtain

$$r = \frac{2}{9}\left[3 - \sqrt{18}\left(\frac{1.0}{1.5}\right)2\sqrt{0.056}\right] = 0.37$$

Also, as both edges in the L_x direction are continuous, we have

$$K_3,\ K_4 = 4/3$$

$$1 + K_3 = 2.33, \qquad 1 + K_4 = 2.33$$

Substituting in Eq. (12.5c), we get

$$\alpha_x = \frac{0.37}{(2\sqrt{2.33})^2} = 0.0397$$

Thus the values of the moment coefficients for the slab are as follows: In the L_x direction,

Positive at mid-span = 0.0397

Negative at edges = 0.0397 (4/3) = 0.053

In the L_y direction,

Positive at mid-span = 0.024

Negative at edges = 0.024 (4/3) = 0.032

These are the same as those in Table 12.2. It may be clearly noted that these tables refer to analysis of slabs under uniformly distributed loads only and do not apply to concentrated loads. The following points are to be remembered when using the table:

1. The negative (or support) moments of continuous edges are 4/3 times the span moments.

2. The long span moment coefficient α_y is a constant for a given condition of the slab, irrespective of span ratios. This is strictly not true, but can be taken as approximately true since the variation is not much. The value of α_y is given by Eq. (12.3).

3. The short span coefficient varies sharply with the variation of the ratio of the spans.

4. While using this table for a series of slabs, the moments calculated at an interior support will sometimes be different on the two sides of that support because of difference in the continuity condition of the slabs on the opposite sides of the support. One way to take care of this difference is to follow the empirical rule that if the ratio of the lesser to the larger of these moments is less than 0.8, the difference may be distributed in proportion to the relative stiffness of the slabs. Torsion steel is to be provided for the slab as detailed in Section 12.16.

12.4.2 EQUIVALENT FRAME ANALYSIS

It may be noted that the analysis of two-way slabs by use of the bending moment coefficients is as simple as in the case of analysis of one-way slabs and beams by means of coefficients (see Tables 7 and 8 of IS 456). However, they are approximate and hence redistribution of moments is not allowed in the resulting moments. As already pointed out, for analysis of two-way slabs, the American code ACI uses a general method called the *equivalent frame method* which considers the slab system as a series of frames interlocking at right angles in the long and short directions. Redistribution of moments is also allowed in the results obtained by this theoretical method.

12.5 ARRANGEMENT OF REINFORCEMENTS

While using design of two-way slabs with the help of coefficients, restrained slabs are considered to be divided in each direction into middle and edge strips as in Fig. 12.4. The moments given in Table 12.2 (IS Table 22) apply only to the middle strips, and no further redistribution is allowed for these moments. The edge strips have to be reinforced only with nominal minimum steel for crack control (Table 11.3).

The middle strip should have steel (negative and positive) calculated by using Table 12.2 for the various sections. In the edge strips the steel is placed as positive steel at the bottom of the slab.

12.6 NEGATIVE MOMENTS AT DISCONTINUOUS EDGES

Negative moments may be experienced at discontinuous edges since, in practice, they are not supported on rollers but partially restrained at their ends. The magnitude of this moment depends on the degree of fixity at the edge of the slab and is indeterminate. The usual practice is to provide at these edges top reinfocement for negative moment equal to (0.042 wl^2), i.e. $wl^2/24$. However, IS 456: clause 01.6 of Appendix C recommends provision of 50 per cent of steel provided at mid-span along these edges, and this negative steel has to extend into the span 0.1 times the span length, as indicated in Fig. 12.6.

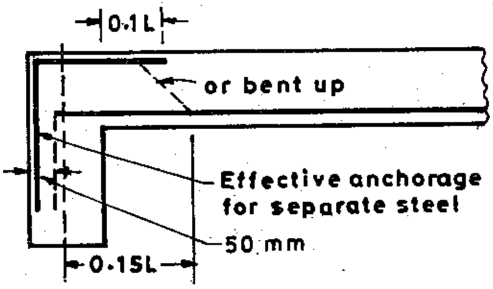


Fig. 12.6 Details of steel at discontinuous edge.

12.7 CHOOSING SLAB THICKNESS

The following three equally important conditions regarding the minimum thickness should be satisfied by the slab.

Condition 1: The minimum should be such that it is safe in compression. Thus the depth is calculated for the greater value of the negative moment on the short span denoted by M_u. Hence

$$M_u = Kf_{ck}bd^2$$

Therefore,

$$d = \sqrt{\frac{M_u}{Kf_{ck}b}}$$

Total thickness = d (short) + 0.5ϕ + cover. The steel for the short span is placed at the lowest layer so that for the long strip the total thickness = d (long) + 0.5ϕ + ϕ + cover. An average of ($d + \phi$) may also be used for practical design purposes.

Condition 2: The slab should satisfy the span/effective depth ratio to control deflection. For this purpose the short span is considered in the calculation of L/d ratios. As explained in Section 11.5, span/effective depth ratios of 28 for simply supported slabs and 32 for continuous slabs may be adopted for initial trials.

Condition 3: The slab should be safe in shear without shear reinforcements as in the case of one-way slabs.

12.8 SELECTING DEPTH AND BREADTH OF SUPPORTING BEAMS

As already pointed out, the depth of beams used for supporting the slab should be sufficient to justify the assumptions of unyielding supports. The empirical relation used in Swedish Regulations between depth of beams and depth of slabs as given in Fig. 12.7 may be used for arriving at the

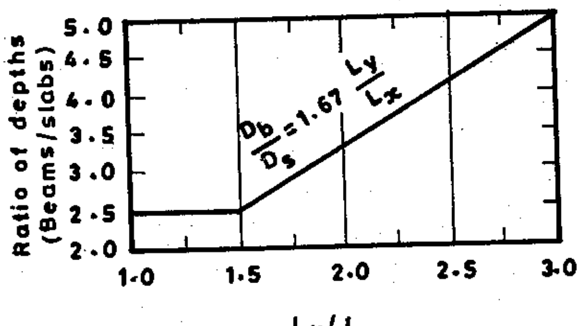


Fig. 12.7 Depths of beams for rigidity.

preliminary depth of these beams. It can be seen that the depth of beam necessary from this consideration lies between 2.5 to 5.0 times the depth of slab, depending on the ratio of L_y to L_x. It can be represented by the equation

$$\frac{D_b}{D_s} = 1.67\frac{L_y}{L_x} \text{ but } \nless 2.5 \tag{12.6}$$

where

D_b = depth of the beam

D_s = depth of the slab

Again the breadth b of the beam is to be selected to ensure adequate torsional rigidity. An empirical relationship giving breadth as a function of the length of short span can be derived as

$$b = 3.24(L_x)^{1/3} \tag{12.7}$$

where b and L_x are expressed in centimetres. Thus for $L_x = 300$ cm, b works out to 22 cm.

Having selected these preliminary values for depth and breadth, they can be later checked by other methods for deflection control and torsional strength. Other factors that affect the choice of depth and breadth of beams have already been discussed in Section 9.12.

12.9 CALCULATION OF AREAS OF STEEL

As already explained in Chapter 11, the depth of slab selected from deflection criterion will be generally greater than the minimum required from strength considerations. The areas of steel are calculated on the assumption that the short span steel will be placed below the long span steel.

12.10 DETAILING OF REINFORCEMENTS

12.10.1 SIMPLY SUPPORTED TWO-WAY SLABS DESIGNED BY RANKINE-GRASHOFF FORMULAE

The moments obtained by the formulae apply to unit width in the long and short directions. The reinforcements are distributed uniformly in each direction will be seen in Fig. 12.13.

12.10.2 DETAILING OF RESTRAINED TWO-WAY SLABS

Restrained slab moments obtained from Table 22 are the moments for the middle strips. These middle strips consist of three quarters of the appropriate panel width in each direction. The steel calculated for the respective moment is distributed uniformly in the middle strip as will be shown in Figs. 12.14–12.16. Each direction is to be provided only with the minimum (nominal) reinforcements placed at the bottom of the slabs. In addition, corner steel to resist the torsional stresses produced in these slabs are provided at discontinuous edge as described in the next Section.

12.10.3 CORNER REINFORCEMENTS FOR TWO-WAY SLABS

Three types of corners, viz. C_1, C_2, C_3 as given in Fig. 12.8, can be identified. At corners such as C_3 where there is no discontinuity, no corner steel is required. At corners like C_1 where the slab is discontinuous on both edges or at C_2 where it is discontinuous only on one edge, extra reinforcements are to be provided. These reinforcements will consist of two mats, one placed on the top and the other at the bottom of the slab, with each mat having reinforcements in both x and y directions.

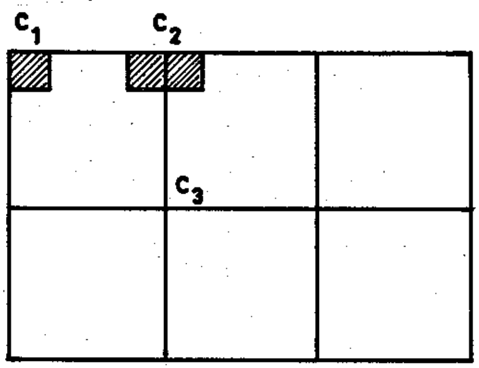


Fig. 12.8 Types of corners for corner steel.

Where the slab is discontinuous on both sides of a corner as in C_1, full torsion steel has to be provided. The area of the full torsion reinforcement per unit width in each of the four layers (directions) should be as follows:

$$\begin{bmatrix}\text{Area of full corner steel}\\ \text{per unit width in each}\\ \text{of the four layers}\end{bmatrix} = \begin{bmatrix}\text{3/4 (area required for the maxi-}\\ \text{mum span moment in the}\\ \text{slab per unit width)}\end{bmatrix} \tag{12.8}$$

These steels are to be provided for a distance of one-fifth the short span ($L_x/5$), as will be shown in Fig. 12.17.

At corners like C_2 where the slab is discontinuous only on one side, half of the above area is to be provided as corner steel in each of the layers. Corner steel need not be provided in corners such as C_3 which is continuous on all sides.

12.11 LOADS ON SUPPORTING BEAMS

According to IS 456: clause 23.5, the total loads that act on the support beams for two-way slabs may be assumed as the load within the respective area of the slab bounded by the intersection of 45° line from the corners with the median line of the panel parallel to the long side (Fig. 12.9). As is well known from yield line theory of slabs, this is a good approximation if all the sides are similarly supported, either as discontinuous or as continuous.

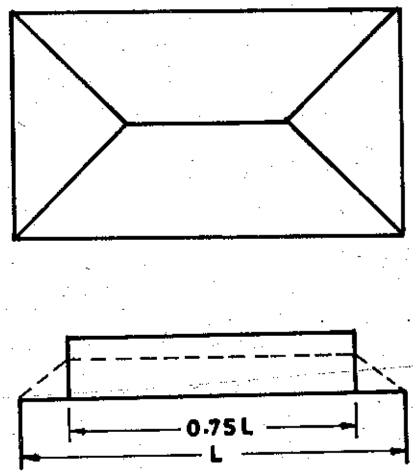


Fig. 12.9 Transfer of load from slab to beam.

The total load so obtained can be converted to an equivalent distributed load for design of these beams. As the intensity of the loads on the beam is non-uniform along its length, being higher at the central portion than at the ends (Fig. 12.2), some codes like BS 8110 make a further assumption and calculate the equivalent load as if it is acting only on 3/4 of the length L_x and L_y (Fig. 12.9).

12.11.1 EXPRESSIONS FOR LOAD ON THE BEAM IN THE SHORT DIRECTION (SHEAR FROM SLAB IN THE LONG DIRECTION)

Considering a slab simply supported on all sides and assuming 45° dispersion of loads as in Fig. 12.9, the total load acting on the beam in the short direction is equal to

$$\frac{1}{2}wL_x\,\frac{L_x}{2} = w\frac{(L_x)^2}{4}$$

If this is assumed to be uniformly distributed load w_x acting over the middle $\frac{3}{4}L_x$, then

$$w_x \text{ on short beam} = \left[\frac{w(L_x)^2}{4}\right]\left[\frac{4}{3(L_x)}\right] = \frac{1}{3}wL_x \qquad (12.9)$$

This load on the beam in the short direction can be considered as shear per unit length from the slab in the long direction. Thus,

$$w_x = (r_y w)L_x$$

where r_y will be the shear coefficient for the slab in the long direction. The value of r_y for the slab simply supported on all edges or fixed on all edges will be 1/3 as shown above.

Thus, r_y can be considered as a coefficient for calculation of shear per unit length of the slab in the long direction.

12.11.2 EXPRESSION FOR LOAD ON THE BEAM IN THE LONG DIRECTION (SHEAR FROM THE SLAB IN THE SHORT DIRECTION)

The load acting on one of the beams in the long direction is given by

$$\frac{wL_xL_y}{2} - \frac{wL_x^2}{4} = \frac{wL_x}{4}(2L_y - L_x)$$

Assuming this load to be uniformly distributed over the middle $\frac{3}{4}L_y$, the equivalent load on L_y is

$$w_y = \left[\frac{wL_x}{4}(2L_y - L_x)\right] \Big/ \left(\frac{3}{4}L_y\right) = \frac{1}{3}\left(2 - \frac{1}{k}\right)wL_x \tag{12.10}$$

where $k = L_y/L_x$. This load on the long beam can be taken as due to shear from unit length of the slab in the short direction. Thus the load expressed in terms of the short span (similar to the expression for bending moment) is given by

$$w_y = (\gamma_x w)L_x \tag{12.10a}$$

where γ_x will be the shear coefficient for the slab spanning in the short direction. For the two-way slab simply supported or fixed at all the edges (with symmetrical end conditions), we have

$$\gamma_x = \frac{1}{3}\left(2 - \frac{1}{k}\right)$$

This expression will be modified when the fixing moments at the ends are not equal. The values of γ_x and γ_y for various values of k for the nine cases given in Table 22 of IS 456 for two-way slabs can be tabulated as in Table 12.3. A similar table is also presented in BS 8110.

As an example for a slab with two adjacent discontinuous edges, let $L_x = 4$ m, $L_y = 6$ m, and $k = 1.5$.

The shears per unit length of the slab will be given by referring to Table 12.3, see case 4, and substituting the values obtained in the Eq. (12.10a) with $L_x = 4$ m, we obtain

Shear per unit length on continuous edge in the short direction = $0.54w(4) = 2.16w$
Shear per unit length on discontinuous edge in the short direction = $0.35w(4) = 1.40w$
Shear per unit length on continuous edge beam in the long direction = $0.40w(4) = 1.60w$
Shear per unit length on discontinuous beam in the long direction = $0.26w(4) = 1.04w$

Thus the maximum shear for the slab is $2.16w$ at the continuous edge in the short direction. These loads from slabs may be also assumed to be uniformly distributed on 3/4 length of the

TABLE 12.3 SHEAR FORCE COEFFICIENTS FOR UNIFORMLY LOADED TWO-WAY RECTANGULAR SLABS GIVEN IN TABLE 12.2
(Shear in span $L_x = \gamma_x L_x$ per unit width)

Case No. of Table 12.2		γ_x (for shear in slab) in the short direction for L_y/L_x								γ_y (in the long direction)
		1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0	
1 and 9		0.33	0.36	0.39	0.41	0.43	0.45	0.48	0.50	0.33
2	C*	0.36	0.39	0.42	0.44	0.45	0.47	0.50	0.52	0.36
	D†									0.24
3	C	0.36	0.40	0.44	0.47	0.49	0.51	0.55	0.59	0.36
	D	0.24	0.27	0.29	0.31	0.32	0.34	0.36	0.38	
4	C	0.40	0.44	0.47	0.50	0.52	0.54	0.57	0.60	0.40
	D	0.26	0.29	0.31	0.33	0.34	0.35	0.38	0.40	0.26
5	C	0.40	0.43	0.45	0.47	0.48	0.49	0.52	0.54	
	D									0.26
6	C									0.40
	D	0.26	0.30	0.33	0.36	0.38	0.40	0.44	0.47	
7	C	0.45	0.48	0.51	0.53	0.55	0.57	0.60	0.63	
	D	0.30	0.32	0.34	0.35	0.36	0.37	0.39	0.41	0.29
8	C									0.45
	D	0.29	0.33	0.36	0.38	0.40	0.42	0.45	0.48	0.30

*C—continuous edge; †D—discontinuous edge

beams for calculation of B.M. and shears in the beams. However, more exact expressions for the calculation of equivalent loads for bending moment and shear forces in the beams are used when dealing with frame analysis of buildings for gravity loads.

12.12 CRITICAL SECTION FOR SHEAR IN SLABS

The maximum shear calculated above is at the end of the slab along the centre line of the beam, and this can be assumed as the design shear for all practical purposes. If a more refined calculation is called for, the critical section for shear according to IS 456: clause 21.6, can be taken to be at a distance equal to the depth of the slab away from the edge of the beam (Fig. 12.10). Thus, the shear for design will be

$$V = V_0 - w'\left(d + \frac{b}{2}\right) \text{ per unit length} \tag{12.11}$$

where

V_0 = shear as calculated by Section 12.11

w' = average load on the slab in the given direction

d = effective thickness of slab

b = breadth of the beam

The slab is designed so that it is safe in shear without shear steel as indicated in Section 7.10.

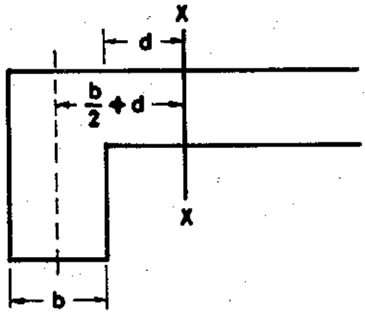


Fig. 12.10 Critical section for shear in slabs.

12.13 PROCEDURE FOR SAFETY AGAINST EXCESSIVE DEFLECTION

As already explained, deflection of slabs is controlled by span/effective depth ratio similar to the case of beams and one-way slabs. For two-way slabs, the shorter span and the percentage of steel in that direction have to be considered for this purpose. In Chapter 9 the correction factors to be used in basic span depth ratios for slabs for deflection control have already been explained. Methods for computation of deflection of slabs are generally used only under special circumstances.

12.14 PROCEDURE FOR CONTROL OF CRACK-WIDTH

Under normal circumstances, crack control of these slabs can be assumed to be satisfactory if the empirical rules for detailing of reinforcements for slabs as given in Chapter 9 are followed.

12.15 PROCEDURE FOR DESIGN OF TWO-WAY SIMPLY SUPPORTED SLABS

Step 1: Assume a slab thickness with proper cover to steel. This is to be based on (span/effective depth) ratio of shorter span.

The minimum practical depth of slab is 90 to 100 mm. As the percentage of steel in slabs is low compared to that in beams, a larger value of *L/d* ratio than that of beams will be found to be acceptable for slabs. (The correction factor is larger in slabs due to low percentage of steel used in them.)

Usually, the following span/effective depth ratios may be assumed for preliminary design (IS 456: clause 23.1):

Cantilever slabs = 12

Simply supported slabs = 30 to 35

Continuous slabs = 35 to 40

Assume suitable concrete cover of at least 15 mm, depending on the environmental conditions.

Step 2: Calculate the design load and the value of dead and live loads (LL) for the slab:

$$w = (1.5\ \text{LL} + 1.5\ \text{DL})$$

Step 3: *Calculate design moment.* Determine l_y/l_x, where l_x is the shorter span, find the moment coefficients from Table 23 of IS 456, and calculate the moments M_x and M_y.

Step 4: Calculate maximum shear and check shear stress in the slab. For calculating the shear, assume shear based on load distribution of the beams as explained in Section 12.11 or use Table 12.3. For refined analysis, critical section may be assumed as in Section 12.12.

Step 5: Calculate steel required in both directions. Check the value of M_x/bd^2. It should not be greater than that allowed for compression failure in concrete. (d is the centre of steel in the x-direction.) Calculate the area of reinforcement required. Choose diameter and find the spacing. Check for maximum spacing allowed: $3d$ or 450 mm. For slabs less than 300 mm, limit spacing to 200 mm. These steps should be carried out for both directions.

Step 6: Check for deflection. The span depth ratio for deflection is based on l_x and A_{s1}.

Step 7: Check for cracking minimum steel in both directions and spacings.

Step 8: Detail the steel preferably as given in simplified detailing of reinforcement in slabs.

12.16 PROCEDURE FOR DESIGN OF TWO-WAY RESTRAINED SLABS (WITH TORSION AT CORNERS)

Step 1: Assume a slab thickness with proper cover to steel.

Step 2: Calculate the design load from dead and live loads:

$$w = (1.5 \text{ DL} + 1.5 \text{ LL})$$

Step 3: Draw the slab pattern of each slab in plan and show which case in Table 22 of IS 456 each of the slabs refers to. Find L_y/L_x for the slab.

Step 4: Write down the shear force coefficients on the assumptions based on distribution of slab loads to supporting beams or Table 12.3. Make approximate check for shear.

Step 4 (a): Write down the moment coefficients M_x and M_y from Table 22 or using formulae 12.3 to 12.5. Determine the maximum moment for the middle strips in the long and short directions. Make adjustment for the negative moments at continuous edges of common spans, as explained in Section 12.4, if necessary.

(b): Take each panel, check depth for bending moment, ensure that $M/(bd^2)$ is not greater than that allowed for both l_x and l_y directions, and calculate the steel required in each direction at mid-span and supports of the middle strip. One may assume the average effective depth for both the directions for calculation of steel area.

$$d = (h - \text{cover} - \text{diameter of reinforcement})$$

Find the required spacings of steel.

(c): Calculate the nominal steel for the edge strips.

(d): Identify the corners to be provided with corner steel and calculate the corner steel required. It is equal to 0.75 times the area of the maximum positive steel and is to be provided over $0.2L_x$ the width at the corners.

(e): If the shear check in step 4 is critical, make a final check for shear, taking the value of design shear strength as that allowed for the area of steel provided at the edges and using the enhancement factor explained in Chapter 11.

Step 5: Check the deflection (the span/depth ratio in short direction with the corresponding percentage of steel).

Step 6: Check for cracking (the minimum steel in both directions). Check the rule for spacing of steel.

Step 7: Detail the main steels (edge strip steels and corner steels) preferably according to standard practice.

12.17 CONCENTRATED LOAD ON TWO-WAY SLABS

As slabs are two-dimensional structures, concentrated load produces saucer-shaped deformation. It is difficult to analyse this deformation. Hence an equivalent plane structure analysis is used, which will always be approximate. We have seen in Chapter 11 that the bending moment and shear force due to concentrated loads on one-way slabs are analysed by the equivalent width method: Two-way slabs under concentrated loads are analysed by theory of plates as indicated in Section 12.18.

12.18 METHODS BASED ON THEORY OF PLATES FOR CONCENTRATED LOADS ON TWO-WAY SLABS (PIGEAUD'S METHOD)

The two well-known methods based on theory of elasticity for determination of bending moments in slabs due to concentrated loads, e.g. wheel loads, are Pigeaud's and Westergard's methods. Of these, the former method is more popular than the latter and is commonly used for design of bridge slabs. The procedure to apply this method is given now. However, it should be remembered that the standard design curves available for Pigeaud's method are for loads placed at the centre of two-way simply supported slabs as shown in Fig. 12.11, and for other cases of loading and support conditions the values from the curves have to be suitably modified.

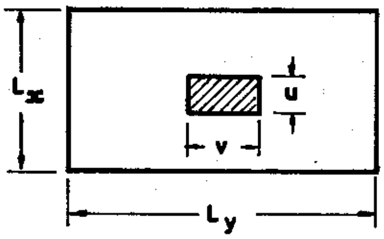


Fig. 12.11 Position of concentrated load for Pigeaud's curves.

12.18.1 DISPERSION OF LOADS

For using Pigeaud's curves, the wheel load is first assumed to be applied through a contact area and dispersed along the contact breadth and length as already described in Chapter 11.

12.18.2 DESIGN PROCEDURE BY PIGEAUD'S CURVES

The following notations are used for the slab and the load. K = ratio of long span (L_y) to short span (L_x).(*Note*: In some texts, Pigeaud's curves are presented for the ratios L_x/L_y instead of L_y/L_x).

u = loaded breadth corresponding to L_x after dispersion

v = loaded length corresponding to L_y after dispersion

m_x = coefficient of moment along the short span

m_y = coefficient of moment along the long span

M_x = moment along the short span for unit width

M_y = moment along the long span for unit width

μ = Poisson's ratio is generally taken as 0.2 for reinforced concrete

P = wheel load

The curves which can be directly used for design are given in Charts 12.1–12.7. The values of m_x and m_y for a particular set of u/L_x and v/L_y values are read off from these curves for the particular

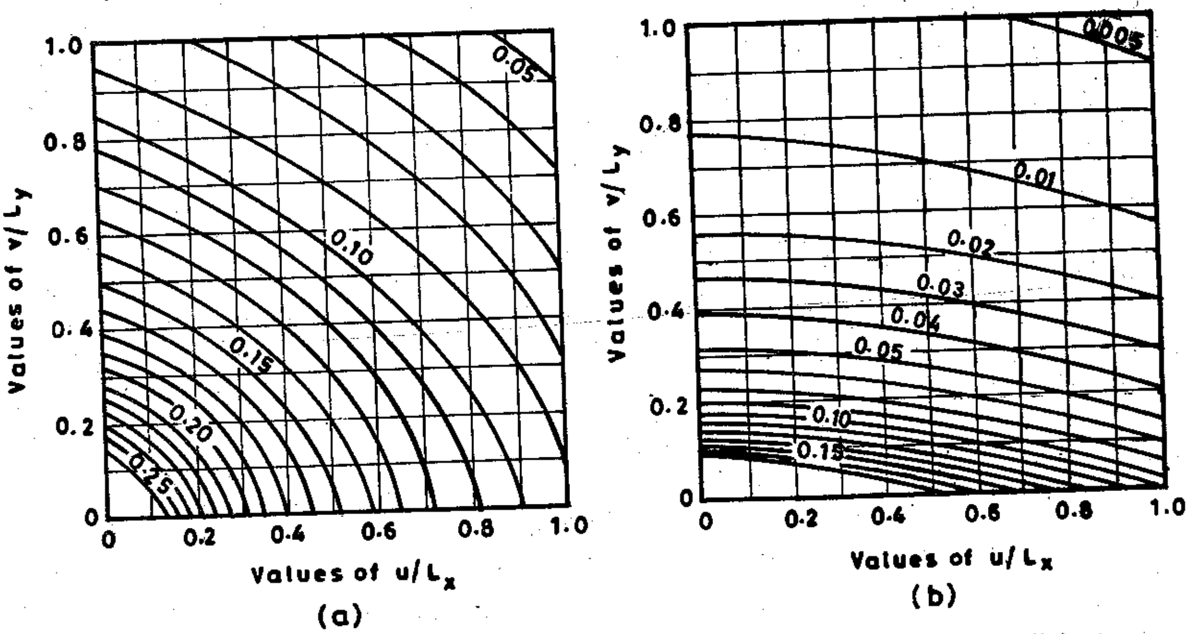


Chart 12.1 Moment coefficients by Pigeaud's Method L_y/L_x = 2.5: (a) Coefficient m_x, (b) Coefficient m_y.

values of K. The magnitudes of M_x and M_y are obtained from the relations:

$$M_x = P(m_x + \mu m_y), \qquad M_y = P(m_y + \mu m_x)$$

(*Note*: The spans of the slab do not enter directly into these expressions.)

For one-way slab the curve with K equal to infinity (with the usual ratios u/L_x and v/L_y may be used for design. For uniformly distributed load the curves may be made use of by putting u/L_x and v/L_y as both equal to unity. Pigeaud's methods are most accurate when K is greater than 1.8 and values of v/L_y are not very small.

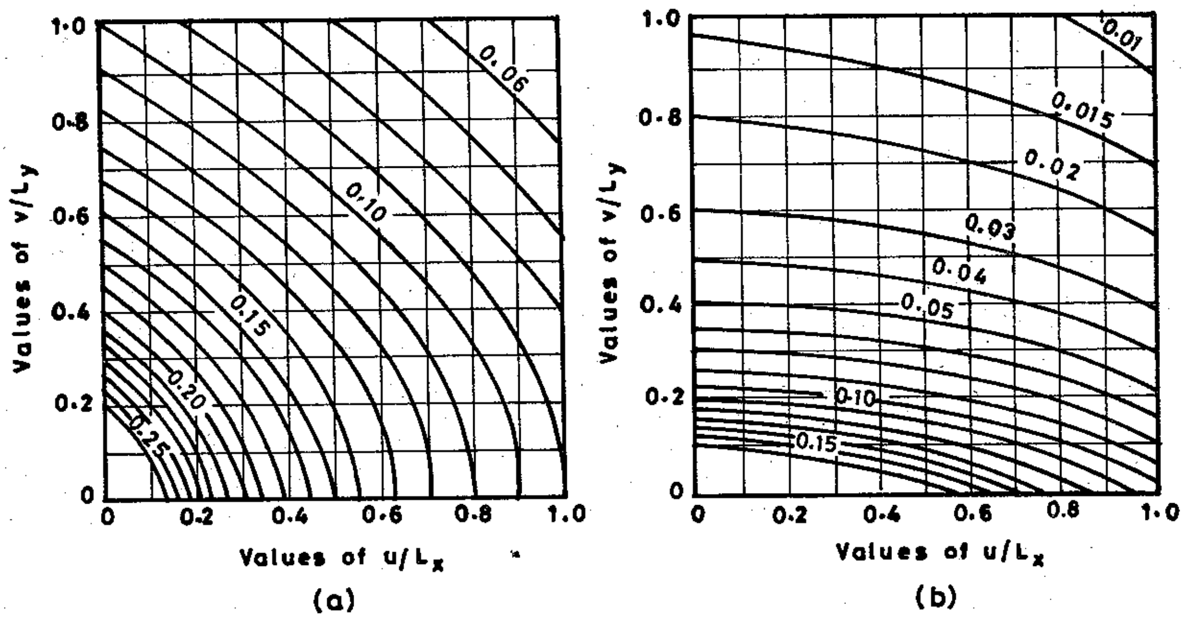


Chart 12.2 Moment coefficients by Pigeaud's Method $L_y/L_x = 2.0$: (a) Coefficient m_x, (b) Coefficient m_y.

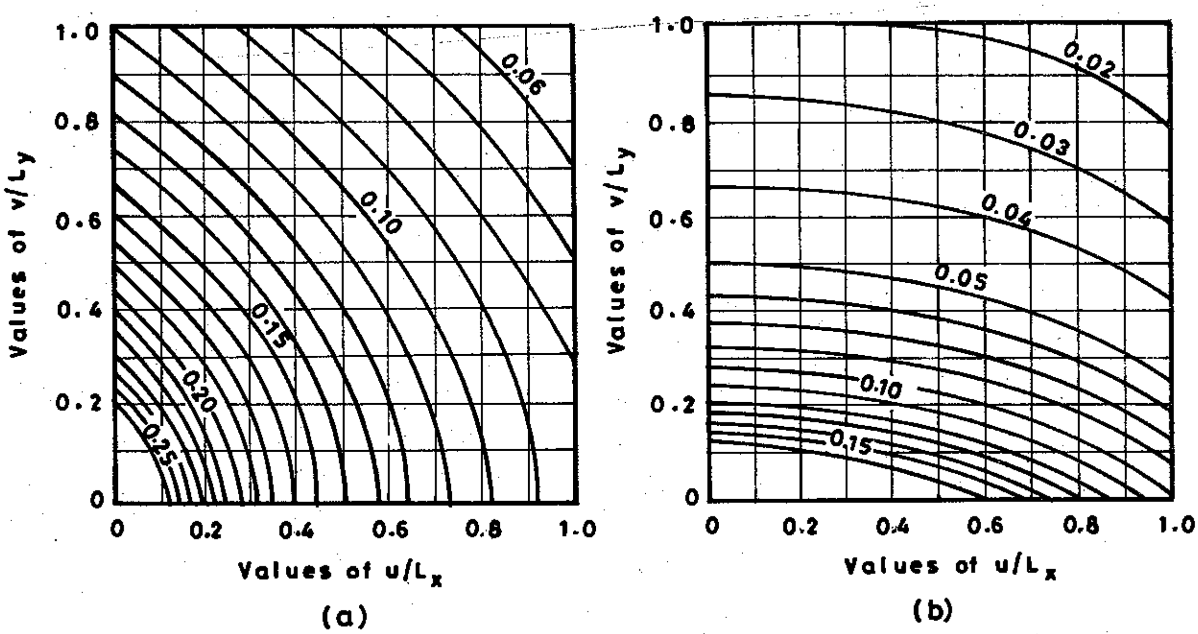


Chart 12.3 Moment coefficients by Pigeaud's Method $L_y/L_x = 1.67$: (a) Coefficient m_x, (b) Coefficient m_y.

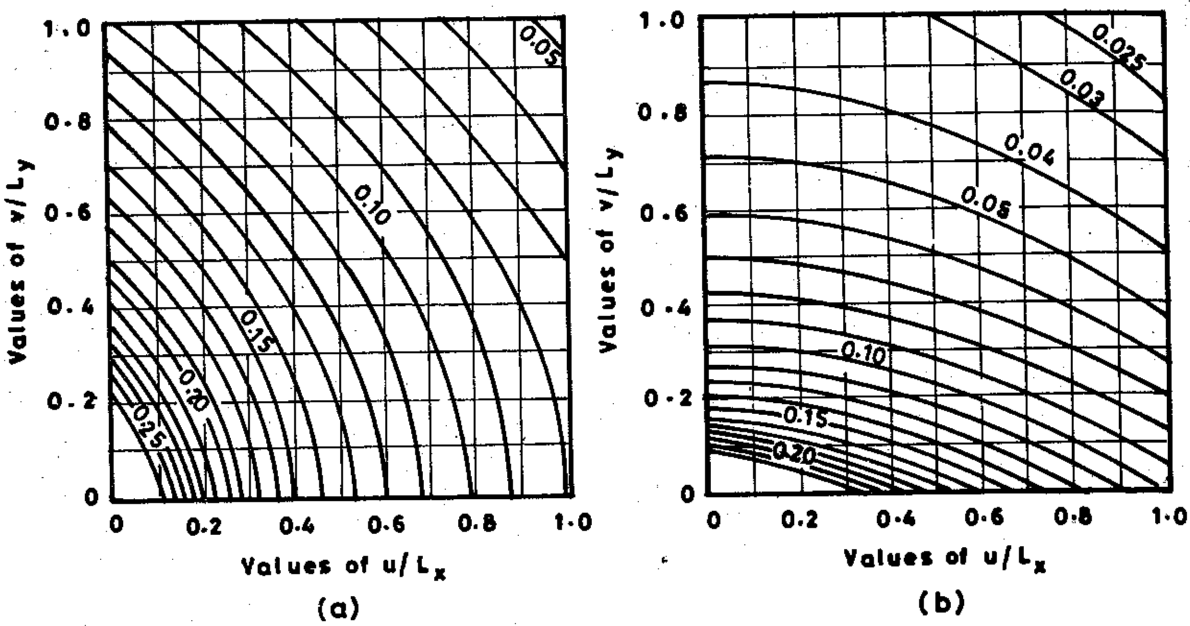


Chart 12.4 Moment coefficients by Pigeaud's Method $L_y/L_x = 1.41$: (a) Coefficient m_x, (b) Coefficient m_y.

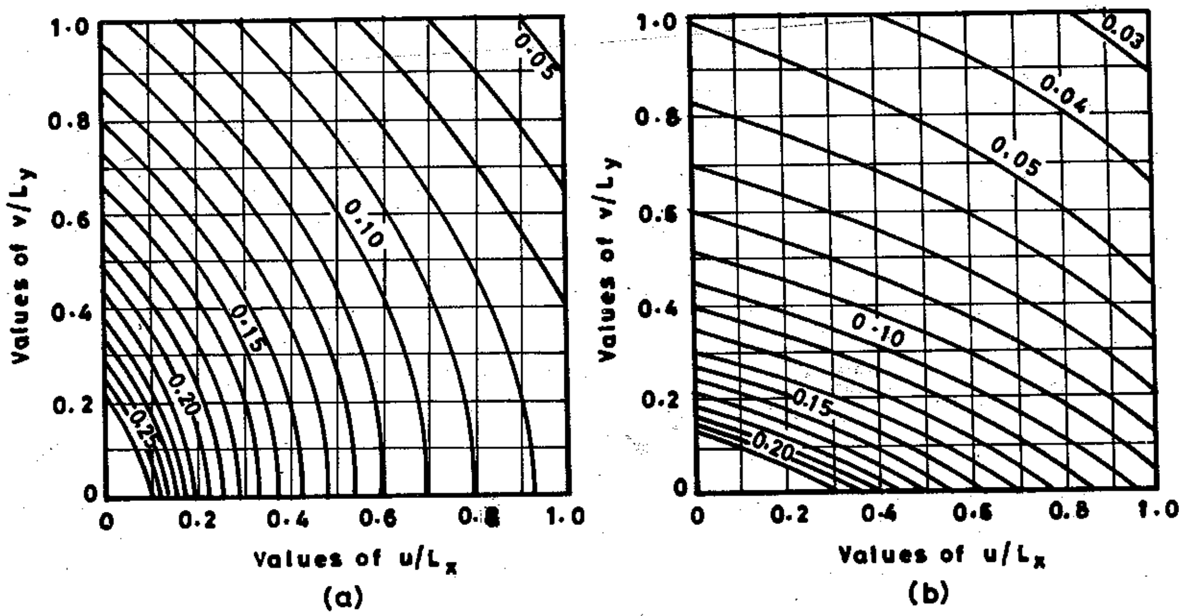


Chart 12.5 Moment coefficients by Pigeaud's Method $L_y/L_x = 1.25$: (a) Coefficient m_x, (b) Coefficient m_y.

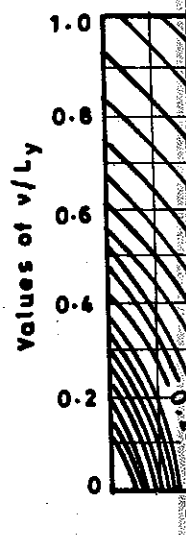


Chart 12.

Chart

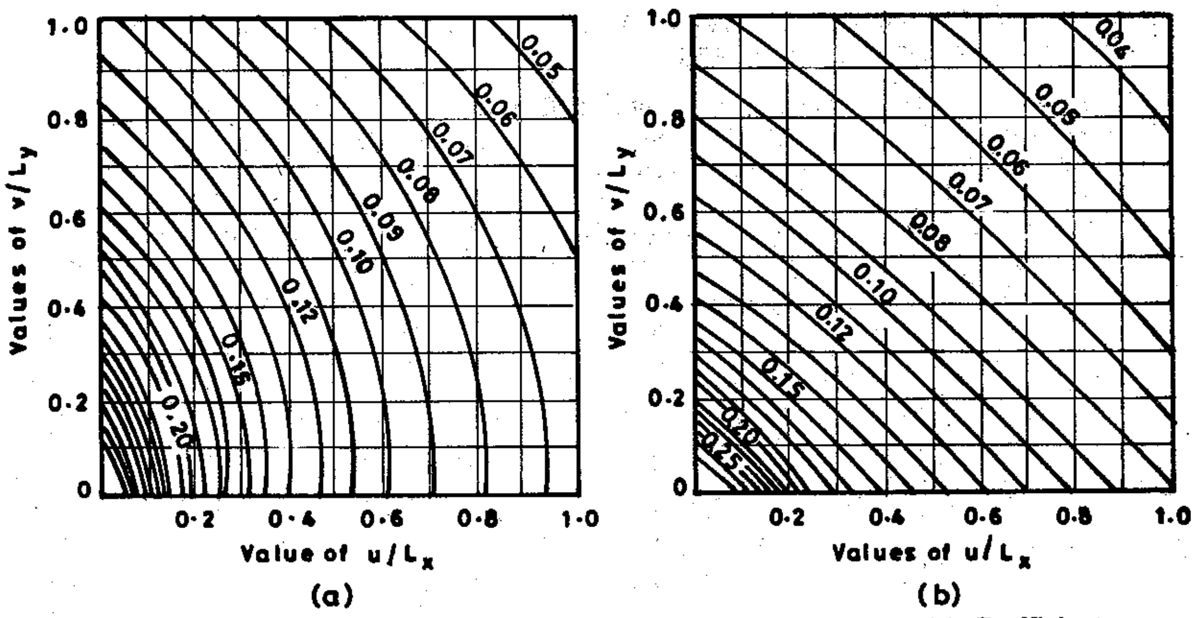


Chart 12.6 Moment coefficients by Pigeaud's Method $L_y/L_x = 1.11$: (a) Coefficient m_x, (b) Coefficient m_y.

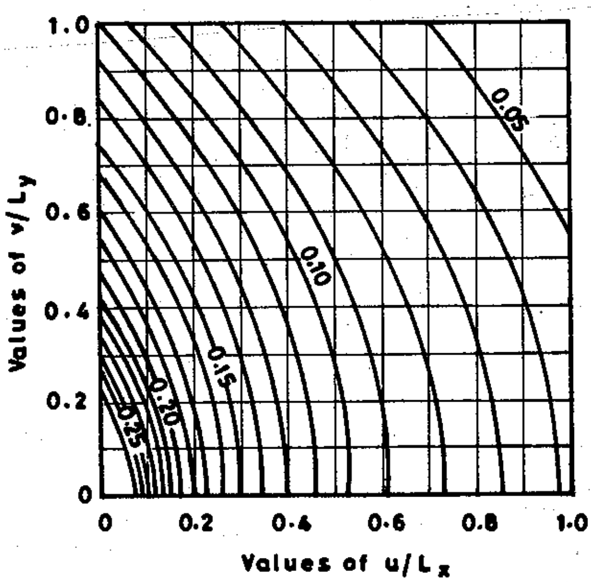
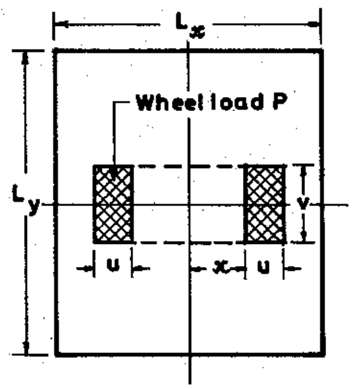
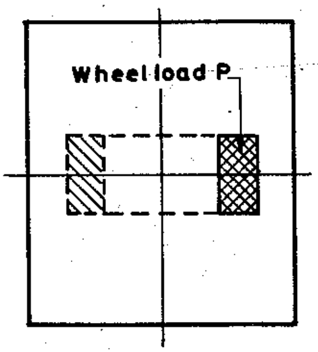
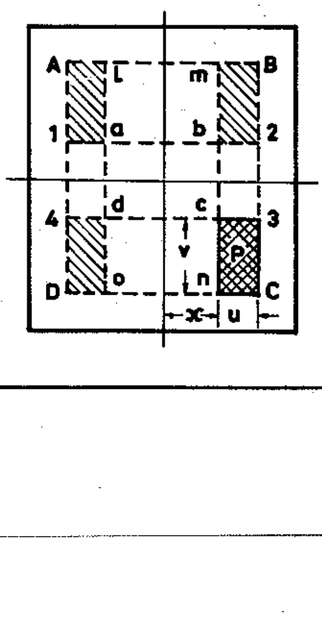


Chart 12.7 (Coefficient $m_x = m_y$). Moment coefficients by Pigeaud's Method $L_y/L_x = 1.0$.

12.18.3 USE OF PIGEAUD'S CURVES FOR NON-CENTRAL LOADS AND SLABS RESTRAINED AT SUPPORTS

When the concentrated load on the slab is not centrally placed or when two concentrated loads are symmetrically placed on the slab, the bending moments can be calculated with the help of the standard Pigeaud's curves using the procedure shown in Table 12.4.

TABLE 12.4 MODIFICATION OF PIGEAUD'S METHOD FOR ECCENTRIC LOADS

Case 1 	Find intensity of load P/u per m. (I) Find m_{1x} and m_{1y} for $2(u+x)/L_x$ and v/L_y and multiply by $2(u+x)$ (II) Find m_{2x} and m_{2y} for $2x/L_x$ and v/L_y and multiply by $2x$ (III) Design B.M = deduct (II) from (I) and multiply by P/u
Case 2 	The B.M. at the centre line will be one-half the value in Case 1
Case 3 	Intensity of load P/uv per m² (I) Find moment coeff. for ABCD (II) Find moment coeff. for abcd (III) Find moment coeff. for 1234 (IV) Find moment coeff. for lmno (V) Substract (III) – (IV) from (I) + (II) As the remaining loading will be 4 times the given load multiply by P/4 uv

Similarly, when using Pigeaud's curves, which are basically derived for simply supported slabs, for continuous or fixed slabs, it is customary to estimate the moment by first making a "span-ratio adjustment" shown in Fig. 12.12 and then reducing the bending moment so obtained for the span by using a reduction factor for continuity as shown in Table 12.5.

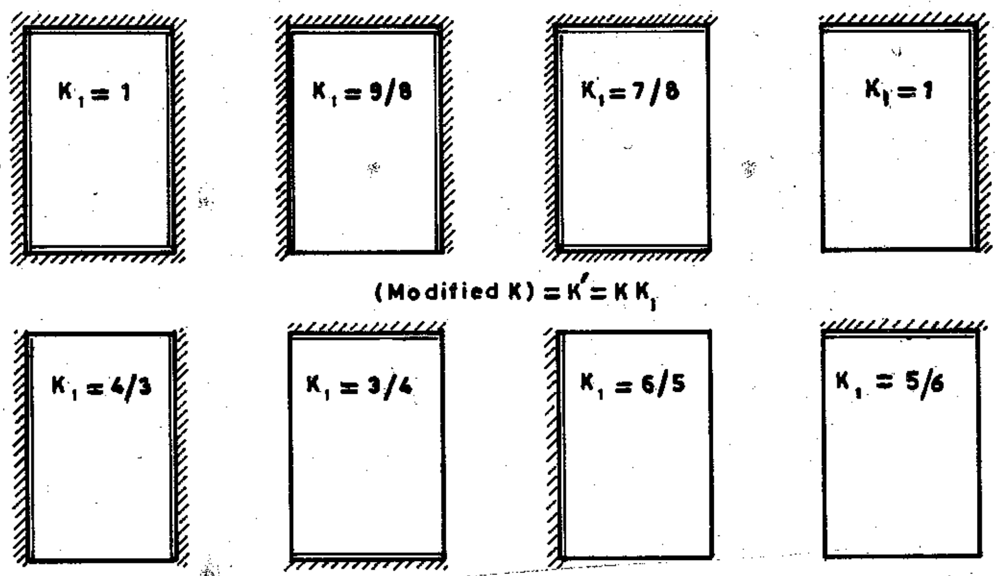


Fig. 12.12 Span ratio adjustment for continuous and fixed slabs for use of Pigeaud's curves.

For rough designs one may assume that in fully continuous slabs the moment at the support and the span are the same and equal to 80 per cent of that of a simply supported slab*. The analysis of two-way slabs under concentrated loads is done with the help of Examples 12.4 and 12.5.

TABLE 12.5 BENDING MOMENT REDUCTION FACTORS FOR CONTINUITY IN PIGEAUD'S METHOD

Span moments		Support moments	
Position	Factor	Position	Factor
Interior	0.70	Interior	0.90
End	0.85	End	0.25
		Penultimate	0.95

*For more details, refer to C.E. Reynolds and J.C. Steedman, R.C. Designers' Handbook, Viewpoint Publications, London, 1989.

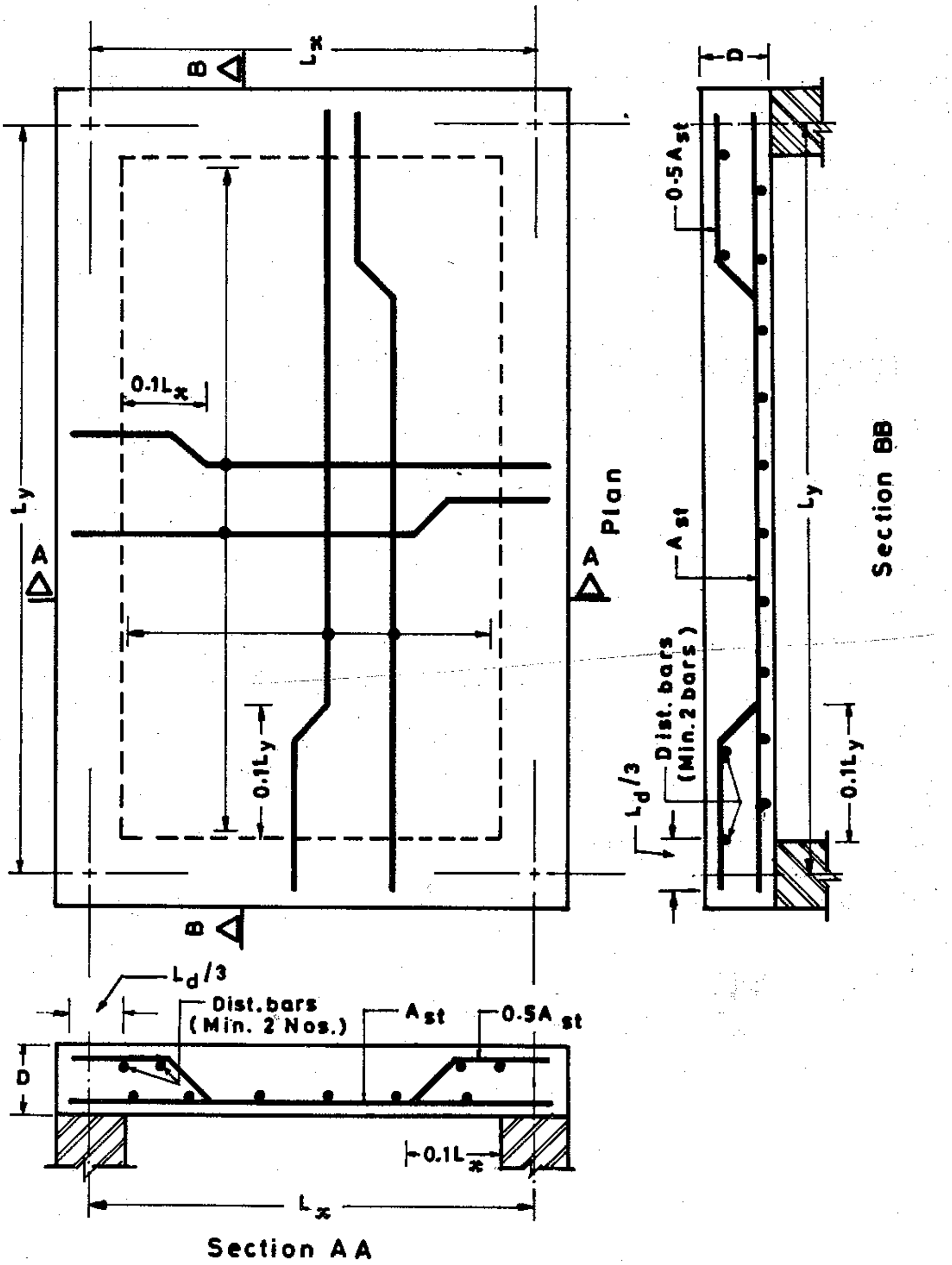
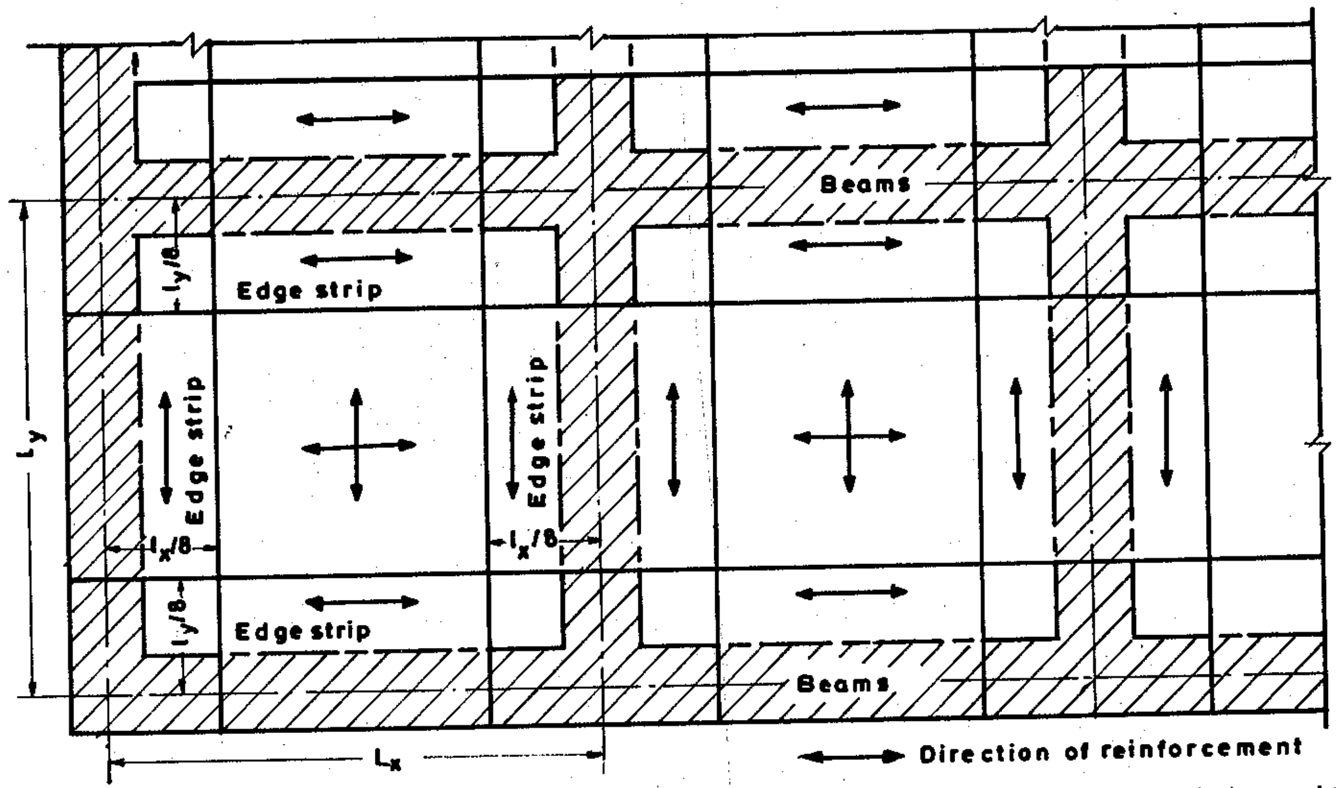


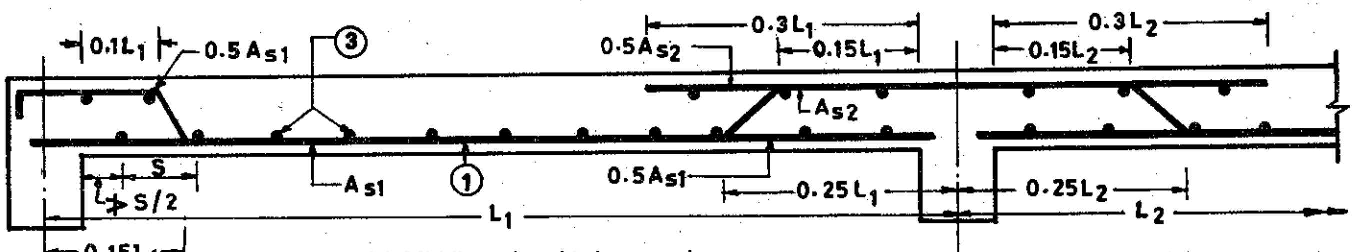
Fig. 12.13 Layout of steel in two-way simply supported slabs.

Section BB

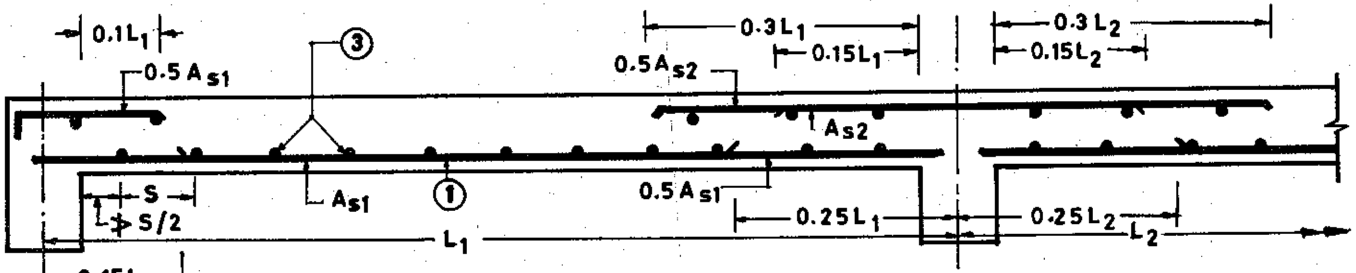


Note: Bars in the edge strip should be of the same length and diameter as those in middle strip but the pitch may be increased to give the minimum reinforcement permitted.

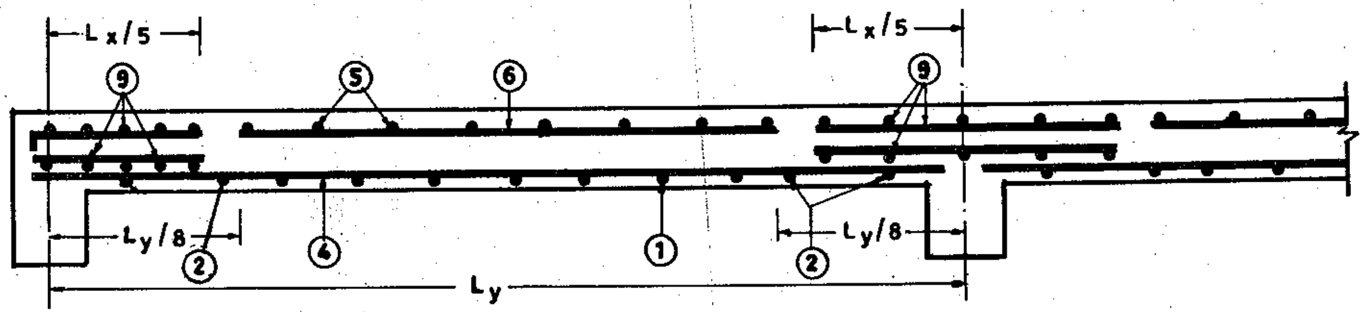
Fig. 12.14 Two-way restrained slabs: arrangement of strips and directions of reinforcement.



(a) Middle strip with bent-up bars



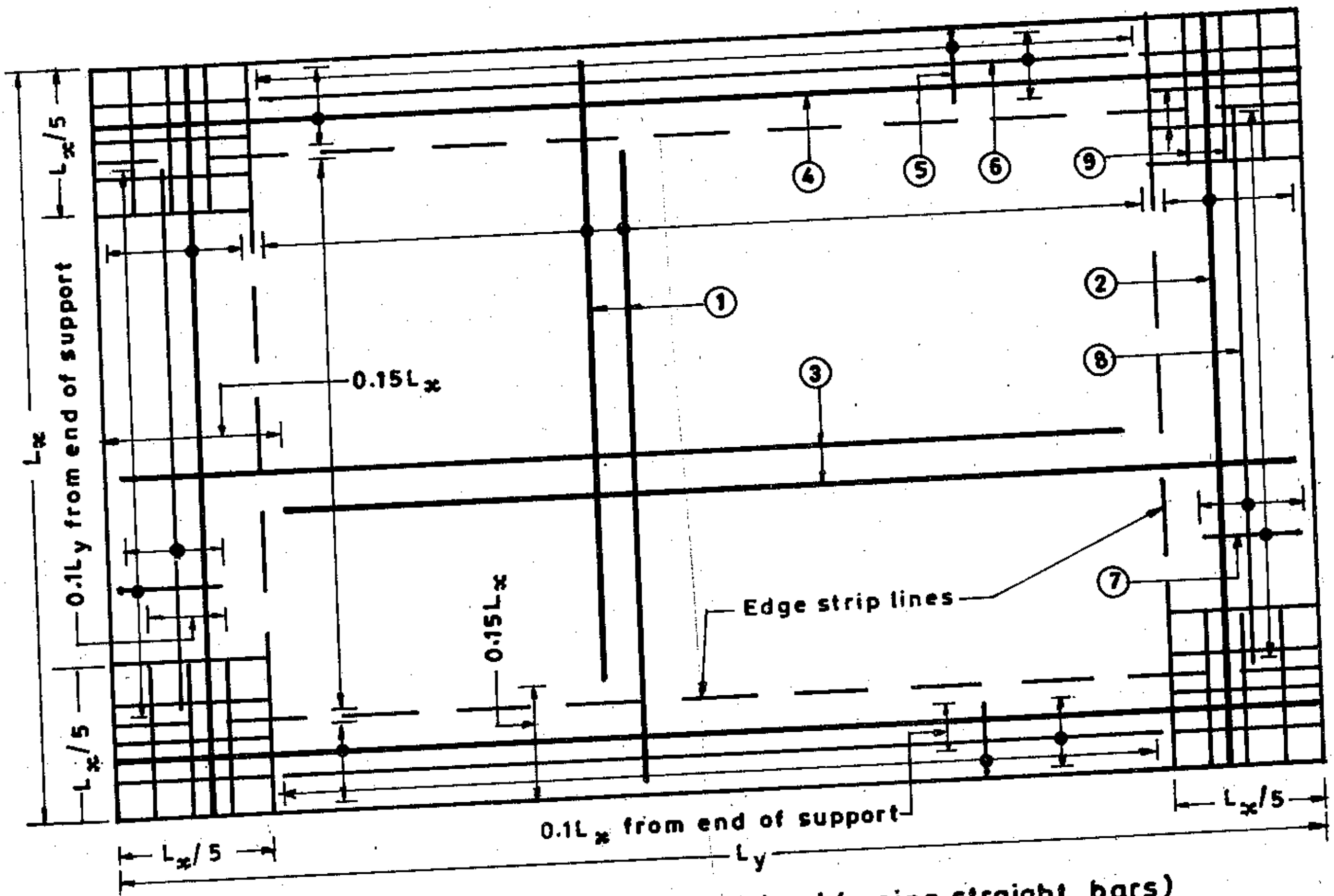
(b) Middle strip with straight bars



(c) Edge strip with straight bars (L_y direction)

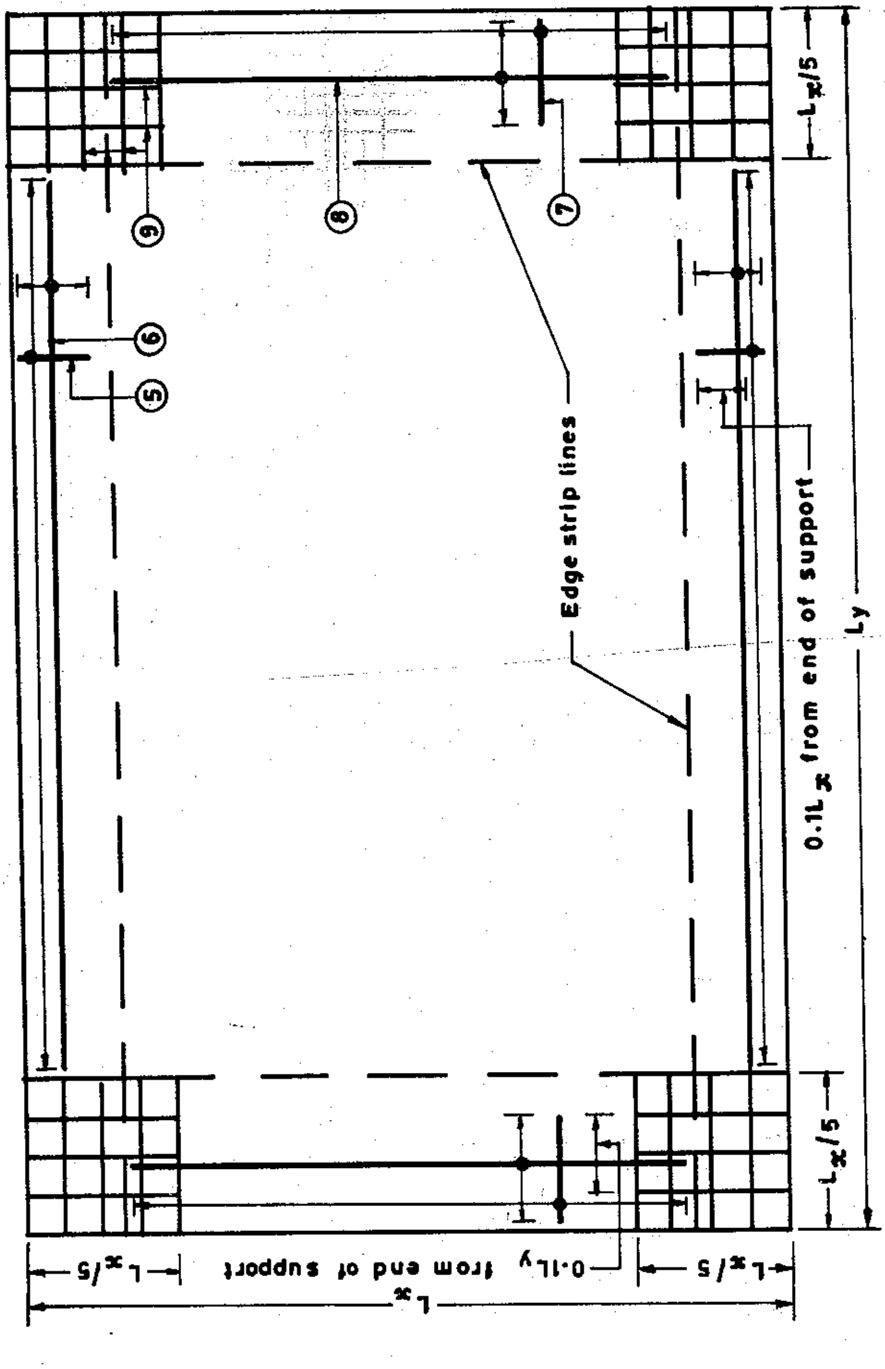
Fig. 12.15 **Layout of steel in restrained two-way slabs: (1, Main steel along L_x direction in middle strip; 2, Main steel along L_y direction edge strip; 3, Main steel along L_y direction in middle strip; 4, Main steel along L_y direction in edge strip; 5, Main steel over supports along L_x direction; 6, Secondary steel for 5; 9, Torsional steel mesh at corners, top and bottom.)**

Fig. 12.15 Layout of steel in restrained two-way slabs: (1, Main steel along L_x direction in middle strip; 2, Main steel along L_y direction edge strip; 3, Main steel along L_y direction in middle strip; 4, Main steel along L_y direction in edge strip; 5, Main steel over supports along L_x direction; 6, Secondary steel for 5; 9, Torsional steel mesh at corners, top and bottom.)



Top and bottom plan combined (using straight bars)

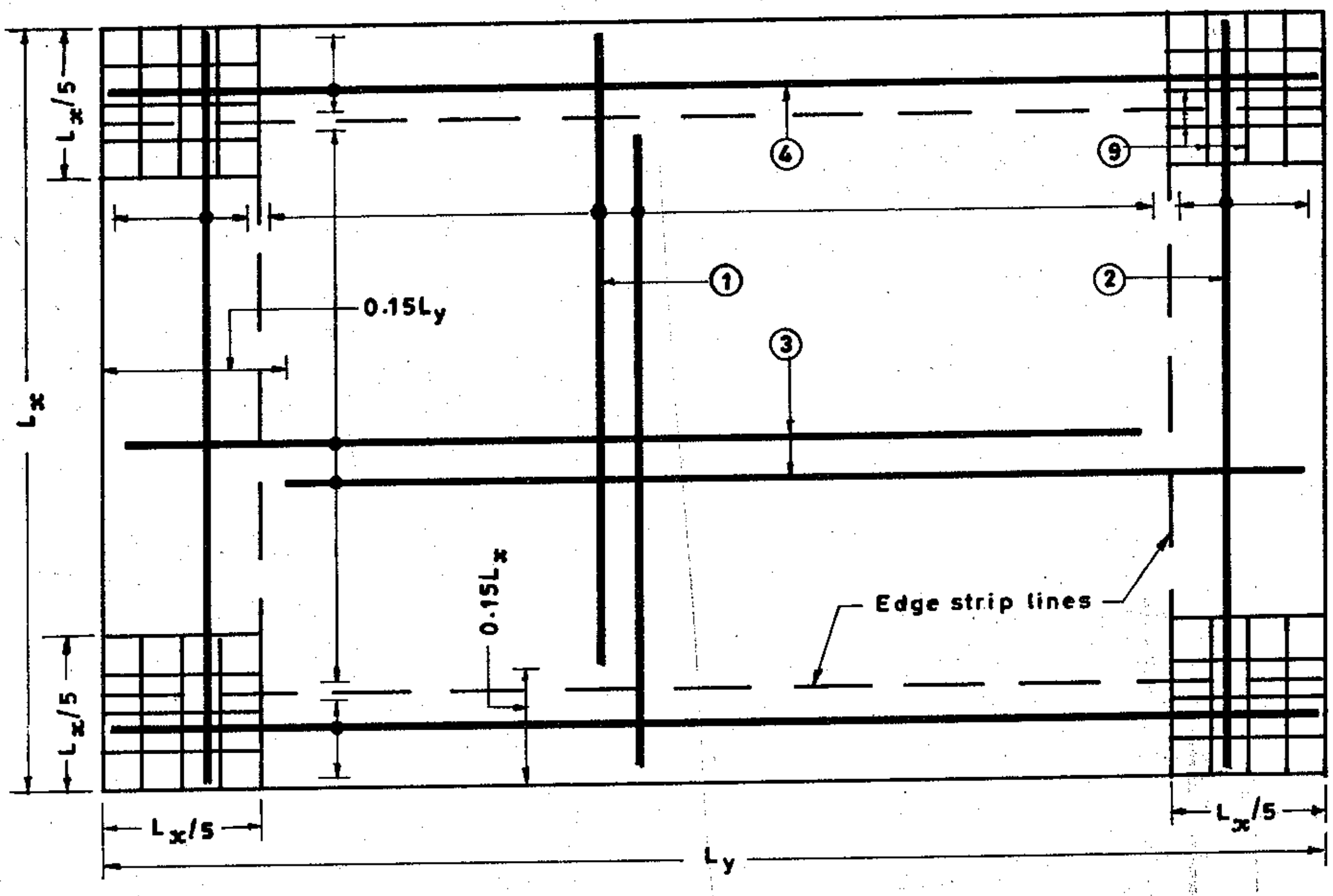
Fig. 12.16a Reinforcement drawing for two-way restrained slabs. (1, Main steel along L_x direction in middle strip; 2, main steel along L_x direction in edge strip; 3, main steel along L_y direction in middle strip; 4, main steel along L_y direction in edge strip; 5, main steel over supports along L_x direction; 6, secondary steel for 5; 7, main steel over supports along L_x direction; 8, secondary steel for 7; 9, torsional steel mesh at corners top and bottom.)



Top plan

Fig. 12.16b Layout of top bars in Fig. 12.16a. (5, Main steel over supports along L_x direction; 6, secondary steel for 5; 7, main steel over supports along L_x direction; 8, secondary steel for 7; 9, torsional steel mesh at corners top and bottom.)

Fig. 12.16b Layout of top bars in Fig. 12.16a. (5, Main steel over supports along L_x direction; 6, secondary steel for 5; 7, main steel over supports along L_x direction; 8, secondary steel for 7; 9, torsional steel mesh at corners top and bottom.)



Bottom plan

Fig. 12.16c Layout of bottom bars in Fig. 12.16a. (1, Main steel along L_x direction in middle strip; 2, main steel along L_y direction in edge strip; 3. main steel along L_y direction in middle strip; 4, main steel along L_y direction in edge strip; 9, torsional steel-mesh at corners top and bottom.)

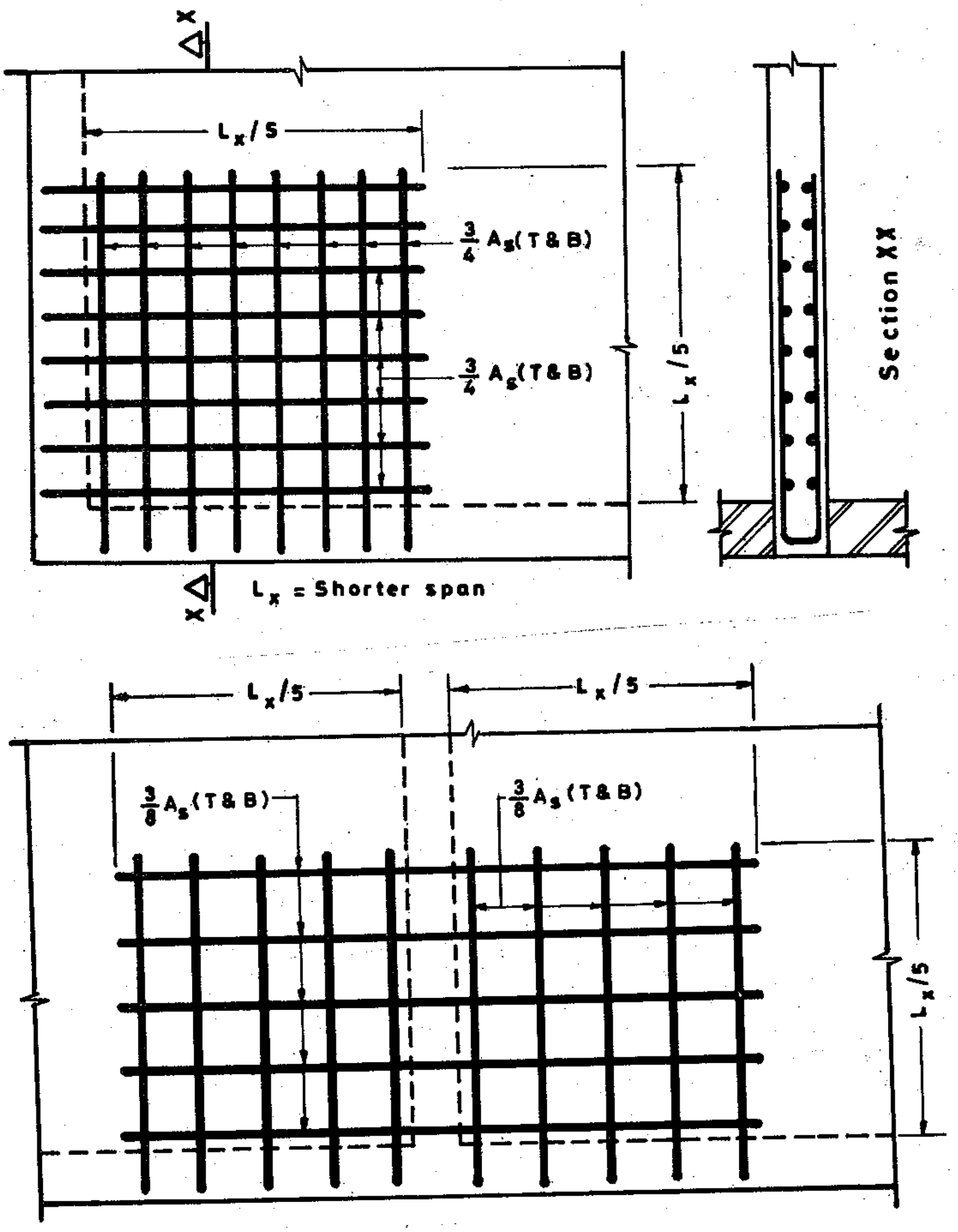


Fig. 12.17 Layout of corner steel in restrained two-way slabs: (a) Corner with two discontinuous ends; (b) Corner with one discontinuous end.

12.19 DESIGN OF CIRCULAR SLABS

Circular slabs when loaded act as two-way slabs. For convenience, they are usually analysed in polar coordinates so that the bending moments are expressed as radial moments (M_r) and tangential moment (M_θ), see Fig. 12.18. The theory of analysis of circular slabs and the formulae for these

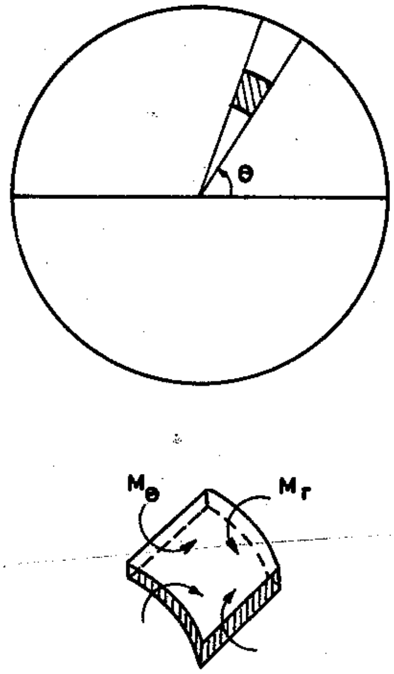


Fig. 12.18 Radial and tangential moments in circular slabs.

moments and shears for the commonly occurring cases of loading are available in standard text books on Advanced Mechanics of Materials or on Theory of Elasticity. Some of these formulae are given in Table 12.6, and the distribution of moments is shown in Figs. 12.19 to 12.21. Many other cases of loadings can be solved from these standard values by method of superposition as shown in Fig. 12.22.

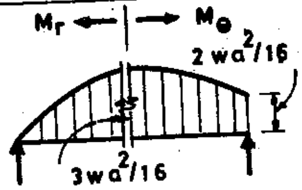
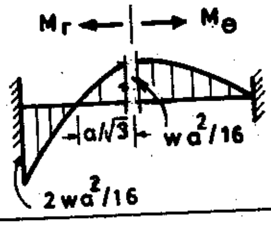
Positive radial and tangential moments are taken as those which need tension steel at the bottom in those respective directions.

12.19.1 DESIGN OF UNIFORMLY LOADED CIRCULAR SLABS

The formulae for the radial and tangential moments and shears for a circular slab simply supported and fixed at the edges are given in Table 12.6. Usually, the value of Poisson's ratio for reinforced concrete is taken as zero and with this value the pattern of distribution of moments is as shown in the table. The radial moment for a simply supported slab at the edges is zero but for a slab fixed at the edges it is quite large, being twice the radial moment at the centre. For partially fixed slab, the value will be somewhere between the simply supported and fixed cases. The distribution of these moments is parabolic. Similarly, the distribution of the circumferential moment is also parabolic, being maximum at the centre and equal to the radial moment at the centre. But at the edges it has 2/3 its value at the centre for the simply supported case. Even though it reduces to zero at the ends

TABLE 12.6 MOMENTS AND SHEARS IN CIRCULAR SLABS WITH UNIFORM LOAD = w/m^2.

[Radius of slab = a]

Case No.	Moments and shears at r from centre	$\mu = 0$
1. Simply supported (S.S.) at ends	$M_r = w/16\,[(3 + \mu)(a^2 - r^2)]$ $M_\theta = w/16\,[a^2(3 + \mu) - r^2(1 + 3\mu)]$ $V = 0.5\,wr$	
2. Fully fixed at ends	$M_r = w/16\,[a^2(1 + \mu) - r^2(3 + \mu)]$ $M_\theta = w/16\,[a^2(1 + \mu) - r^2(1 + 3\mu)]$ $V = 0.5\,wr$	

Note: The central deflection of simply supported slab = $5/64wa^4/EI_c$ and that of fixed slab is only 1/5 the above value.

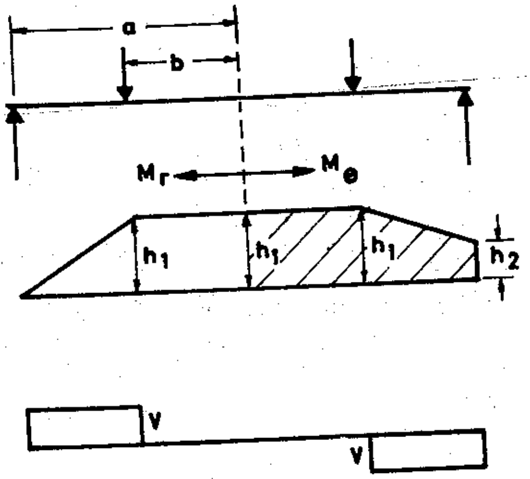


Fig. 12.19 Moments and shears in S.S. slabs with load P along a circle.

for a fully fixed case, full fixity is difficult, and a minimum value of this circumferential moment should always be assumed in designs.

12.19.2 PROVISION OF REINFORCEMENTS

The logical form of steel reinforcement for the radial moment is steel along the radial direction. Similarly, the best form of steel for circumferential moment is the one placed as concentric circles as illustrated in Fig. 12.23a. However, it is easily seen that such placement is difficult. Each circumference has its own length, and the length of all radial steels is not the same. There will be congestion of steel at the centre. Hence, it is much simpler to adopt a rectangular grid of steel placed in the X- and Y-direction in the slab, as illustrated in Fig. 12.23b.

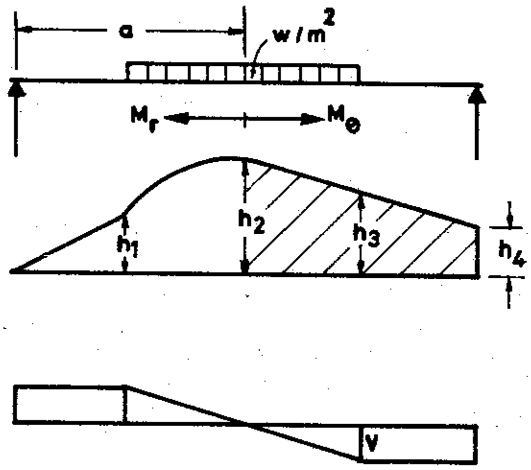


Fig. 12.20 **Moments and shears in S.S. slabs loaded at the centre with UDL.**

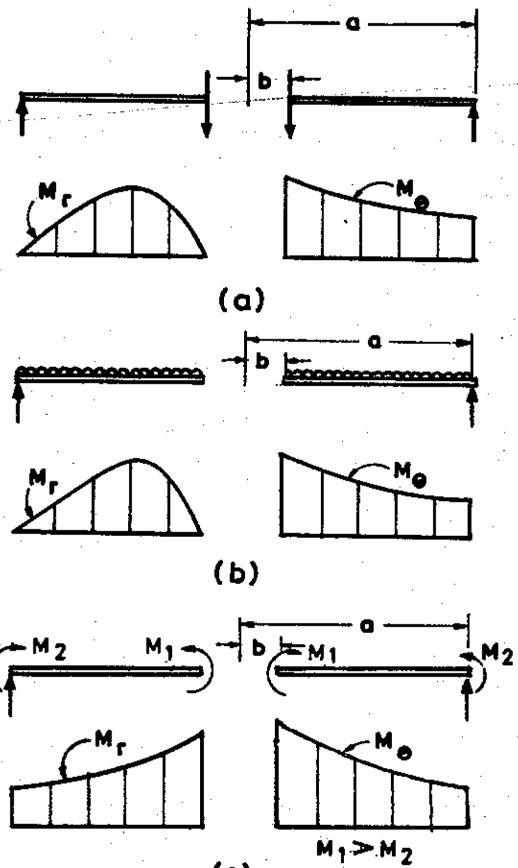


Fig. 12.21 **Moments in simply supported annular slabs: (a) Loaded along inner circumference by line load, (b) Loaded with UDL, (c) Loaded with moments along inner and outer circumferences.**

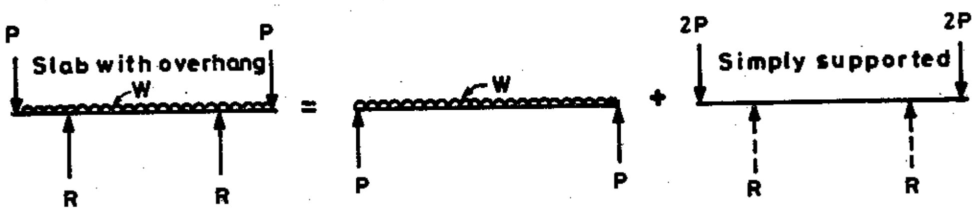


Fig. 12.22 Analysis of circular slabs by superposition.

Such an alternate arrangement of reinforcement in the form of rectangular mesh is possible by using the theory described in the yield theory of slabs that the ultimate moment of resistance in any direction of an isotropically reinforced slab is the same and the torsional moment is zero. Using this concept in all practical designs, the arrangement of reinforcement usually adopted for the central portion of the slab is in the form of a mesh, the intensity of reinforcement in either direction of the mesh being that required for the bigger of the radial and circumferential stress, thus forming an isotropically reinforced slab. It is, however, evident that such a mesh will not be able to take the tension produced by circumferential moments near the edges of the slab since there is not enough development length available for these bars at these places. Special bars should therefore be provided near the edges for circumferential and for negative radial moments.

For positive circumferential moments near the edges, separate steel in the form of circular rings has to be placed at the bottom for some distance from the edges. For a simply supported slab, the circumferential moment at edges is 2/3 that of the moment at centre. Hence the ring steel with some spacing as at the centre has to be provided for a distance which is at least equal to 2/3 of the development length of the bars. For negative radial moments at the edges, separate radial steel with enough anchorages (by provision of bends at the far ends) is provided on top of the slab. These bars are then extended well into the slab to a distance of at least 12ϕ or the effective depth beyond the point of inflection. The arrangement of steel at the bottom and top are shown in Fig. 12.23c. The above theory of placement of steel for circular slabs should be clearly understood while detailing steel for these slabs. It is also necessary to select the diameter of the reinforcement so that the necessary development length is ensured in the slab.

It is thus obvious that there is no difficulty in placing radial and circumferential steel in circular slab with hole at the centre (annular slabs). For such slabs, placing of steel in the radial and circumferential direction is recommended.

12.19.3 CHECKING DEPTH FOR SHEAR AND MOMENTS

It is necessary to check the depth of the slab not only for strength to withstand the moment but also to withstand the maximum shear without additional shear reinforcements.

12.19.4 CIRCULAR SLABS WITH LOADS OTHER THAN UDL

The distribution of M_r and M_θ in simply supported slabs with a circular line load and partially distributed loads are shown in Figs. 12.20 and 12.21. It can be seen from Fig. 12.21 that cut-outs introduce considerable circumferential moments under uniformly distributed loads and line loads around the inner hole.

The design and detailing of circular plates are illustrated by Examples 12.7 and 12.8.

2P
ted

is possible
f resistance
ent is zero.
adopted for
nt in either
umferential
a mesh will
of the slab
es. Special
ative radial

of circular
ported slab,
g steel with
al to 2/3 of
radial steel
slab. These
pth beyond
Fig. 12.23c.
tood while
nent so that

ial steel in
n the radial

ent but also

nd partially
hat cut-outs
d line loads

12.8.

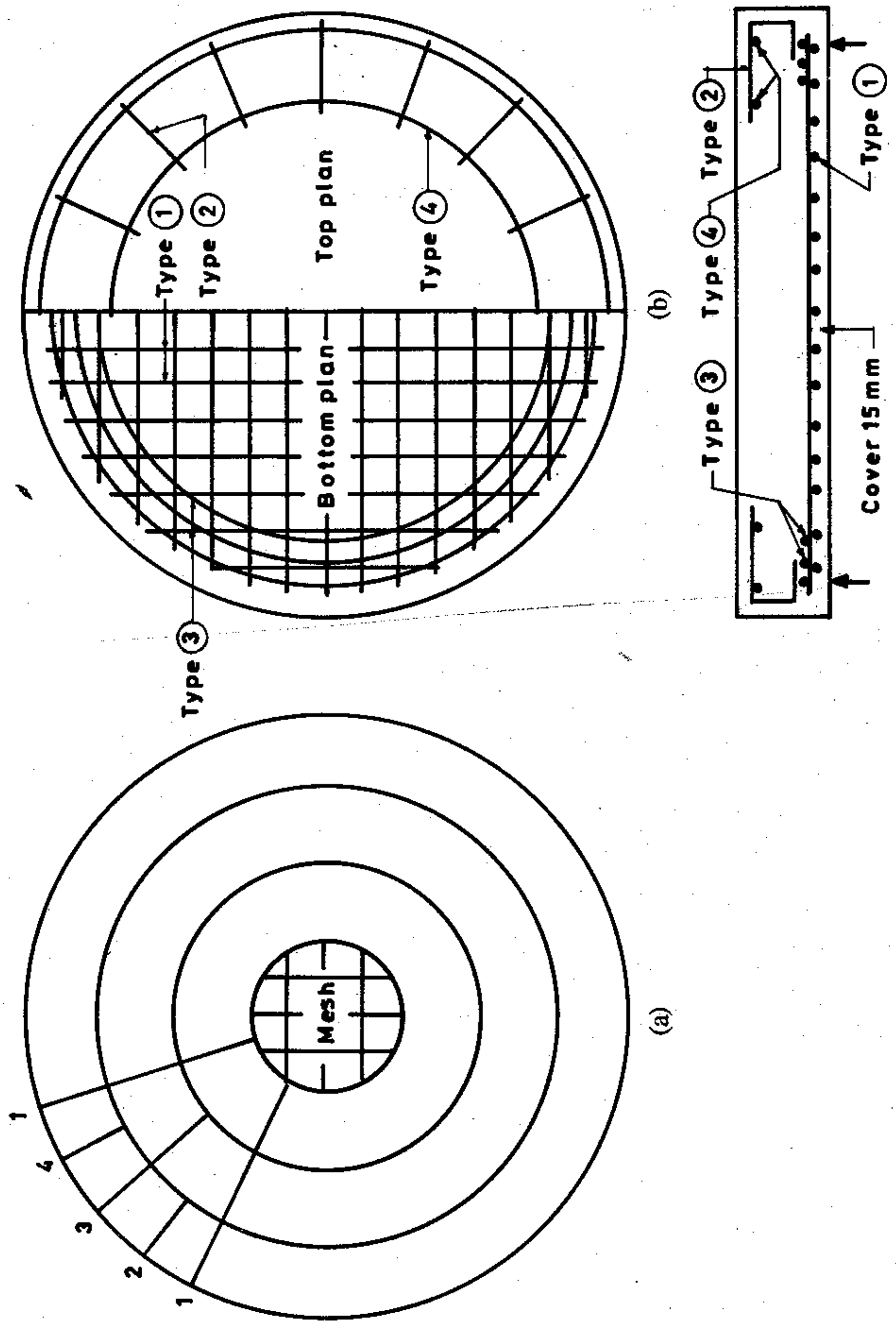


Fig. 12.23 Arrangement of steel in circular slabs: (a) Radial arrangement, (b) Rectangular grid arrangement, (c) Grid arrangement (b)—in section.

EXAMPLE 12.1 **(Design of simply supported two-way slab with UDL)**

Design a reinforced concrete slab 6.3 × 4.5 m simply supported on all the four sides. It has to carry a characteristic live load of 10 kN/m^2 in addition to its dead weight. Assume M25 concrete and Fe 415 steel; also assume that the exposure condition to environment can be classified as mild.

Ref.	Step	Calculations	Output
	1.	*Thickness of slab and durability consideration* $L_x = 4.5$ m Cover for mild exposure = 15 mm	
IS 456 23.1		As the span is large and loading heavy, assume a span depth ratio of 25 instead of the usual 35. $d = \frac{4500}{25} = 180$ mm $h = 180 + 5 + 15 = 200$ mm	$d = 180$ mm $h = 200$ mm
	2.	*Design load* DL = 0.200 × 25 × 1 = 5.00 kN/m^2 LL = 10.00 kN/m^2	
IS 456 Table 12		Design load = (Factored load) = 1.5 (DL + LL) = 22.5 kN/m^2	
	3.	*Maximum factored moment and checking for depth* $\frac{L_y}{L_x} = \frac{6.3}{4.5} = 1.4$	
IS 456 Table 23		From these tables, $\alpha_x = 0.099$, $\alpha_y = 0.051$ $M_x = \alpha_x w L_x^2 = 0.099 \times 22.5 \times 4.5^2$ $= 45.10$ kNm $M_y = \alpha_y w L_x^2 = 0.051 \times 22.5 \times 4.5^2$ $= 23.20$ kNm	$M_x = 45.10$ kNm $M_y = 23.20$ kNm
		Check depth for maximum bending moment $M_{\max} = 0.138 f_{ck} bd$ $d = \frac{45.10 \times 10^6}{0.138 \times 25 \times 1000} = 114 < 180$ mm	d is enough
	4.	*Check for shear* Referring to Table 12.3 of text for case No. 9 and $L_y/L_x = 1.40$, Coefficient for shear = γ_x in L_y direction = 0.33 = γ_x in L_x direction = 0.43	

EXAMPLE 12.1 (cont.)

Ref.	Step	Calculations	Output
		Max. design shear $= \gamma_x w L_x$	
		$= 0.43 \times 22.5 \times 4.5$	
		$= 43.5$ kN	$V_u = 43.5$ kN
		$\dfrac{V}{bd} = \dfrac{43.5 \times 10^3}{1000 \times 179} = 0.24$	$v = 0.24$ N/mm^2
IS 456 Table 13		Safe min: shear for M25 concrete is greater than 0.36 and hence slab is safe in shear. (One may also take the approximate value of shear force per metre width as 0.5 wL_x; the use of Table 12.3 as above is a refinement which may not be necessary in most cases.)	
	5.	*Areas of steel*	
		For steel in short direction, $d = 179$ mm	
		$\dfrac{M}{bd^2} = \dfrac{45.10 \times 10^6}{1000 \times 179 \times 179} = 1.40$	
SP 16 Table 3		Percentage steel (p_t) = 0.417	
		$A_s = \dfrac{0.417 \times 1000 \times 179}{100} = 746$ sq.mm/m	
SP 16 Table 96		Use 12 mm at 130 mm centres giving $A_s = 870$ mm^2	T12 at 130 mm (870 mm^2)
		Spacing less than $3d$,	
		For steel in the long direction, $d = 179 - 12 = 167$	
		$\dfrac{M}{bd^2} = \dfrac{23.2 \times 10^6}{1000 \times 167 \times 167} = 0.83$	
SP 16 Table 3		$p_t = 0.240\%$ (more than min. steel)	
		$A_{st} = \dfrac{0.240 \times 1000 \times 167}{100} = 400$ mm^2	
		Use 12 mm at 280 mm, giving 404 mm^2/m Spacing less than $3d$.	T12 at 280 mm (404 mm^2)
	6.	*Check deflection*	
IS 456 22.2.1		Basic span depth ratio = 20	
		Per cent steel along $L_x = 0.417$ (needed)	
IS 456 Fig. 3		$F_1 = 1.28$	
		Allowable $1/d$ ratio = (basic) (F_1)	
		$= 20 \times 1.28$	
		$= 25.6$	

EXAMPLE 12.1 (cont.)

Ref.	Step	Calculations	Output
		Actual span/depth ratio = 4.5/0.179 = 25.14 Therefore, the assumed span depth ratio is enough to control deflection.	Deflection O.K.
IS 456 25.5.2.1	7.	*Cracking* Steel more than min. 0.12 per cent Spacing of steel < 3d Diameter of steel $< \frac{200}{8} = 25$ mm	Cracking O.K.

EXAMPLE 12.2 **(Design of restrained two-way slabs with UDL)**

A room 17.5 m × 10 m has brick walls all around and it is to be covered with a reinforced concrete slab supported on the walls and on the central beams in the East-West and North-South directions along the middle of the room. The slab has to carry a live load of 4 kN/m². Assume mild exposure condition and that M20 concrete with Fe 415 steel is used for the construction. The slabs are restrained on top of the walls by the brick masonry built above it. Design the slab.

Ref.	Step	Calculations	Output
	1.	*Thickness of slab and durability consideration* The roof consists of four slabs of $L_x = 5$ m, $L_y = 8.75$ m	
IS 456 25.4 and 23.1		Cover for mild exposure = 15 mm Assume $\frac{\text{span}}{\text{depth}}$ ratio = 32 $d = \frac{5000}{32} = 156$ mm Assume $h = 180$ mm Actual $d = 180 - 15 - 12 = 153$ mm	Cover = 15 mm $h = 180$ mm $d = 153$ mm
	2.	*Design load* Dead load = (0.180) (1) (25) = 4.5 kN/m²	
IS 456 Table 12		Live load = 4.0 kN/m² Design load = 1.5 (DL + LL) = 12.75 kN/m²	$w_d = 12.75$ kN/m²
	3.	*Type of slab*	
IS 456 Table 22		Two-way slab: Case 4. Two adjacent edges are discontinuous $\frac{L_y}{L_x} = \frac{8.75}{5} = 1.75$	

EXAMPLE 12.2 (cont.)

Ref.	Step	Calculations	Output
	4.	*Check for shear*	
		Referring to Table 12.3 of text for case 4,	
		Max. shear coefficient = 0.57	
		Max. shear $V = \gamma_s w l_x$	
		$= (0.57)\ (12.75)\ (5) = 36.3$ kN	
		Shear stress = v	
		$v = \dfrac{36.3 \times 1000}{1000 \times 153} = 0.237\ \text{N/mm}^2$	
IS 456 Table 13		Min. for M20 concrete = 0.36	Shear O.K.
	5.	*Bending moment coefficients—middle strip*	
		For $L_y/L_x = 1.75$: Case 4.	
IS 456 Table 22		(a) *Short direction*	
		$M = \beta_x w L_x^2$	
		Negative on continuous edge $= 0.084 \times 12.75 \times 5^2$	
		$= 26.78$ kN/m	
		Positive at mid-span $= 0.063 \times 12.75 \times 5^2$	
		$= 20.08$ kN/m	
IS 456		Negative on discontinuous edge = One-half span moment	
		= 10.04 kN/m	
		[*Note*: That the negative moments 4/3 (M of positive) are at continuous edge and 1/2 (M of positive) are at discontinuous edge.]	
		(b) *Long direction*	
		Positive at mid-span $= 0.035 \times 12.75 \times 5^2$	
		$= 11.16$ kNm	
		Negative on continuous edge $= \dfrac{4}{3} \times 11.16 = 14.88$ kNm	
		Negative on discontinuous edge $= \dfrac{1}{2} \times 11.16 = 5.58$ kNm	
	6.	*Reinforcements in edge strip*	
		Total depth $= h = 180$	
IS 456 25.5.2.1		Min. $A_s = \dfrac{0.12}{100} \times 1000 \times 180 = 216\ \text{mm}^2/\text{m}$	A_s (min) = 216 mm^2/m
		Max. spacing $= 3d = 459$ mm	s (max) = 459 mm

EXAMPLE 12.2 (cont.)

Ref.	Step	Calculations	Output
		From steps 5(a) and (b), tabulate the bending moments and steel required.	

$(M) \times 10^6$	$\frac{M}{bd^2}$*	p	$A_s^\dagger = (\text{mm}^2)$	Output
(a) Short span				
− 26.78	1.14	0.340	520	T12 at 200 (566)
+ 20.08	0.86	0.251	384	T12 at 275 (411)
− 10.04	0.43	0.123	216	T12 at 400 (283)
(b) Long span				
− 14.88	0.64	0.184	282	T12 at 400 (283)
+ 11.16	0.48	0.137	216	T12 at 400 (283)
− 5.58	0.24	< 0.12	216	T12 at 400 (283)

Ref.: SP 16 Table

$*bd^2 = 1000 \times 153 \times 153 = 23.4 \times 10^6$

$^\dagger \quad A_s = \frac{p}{100} \times 1000 \times d = p \times 10 \times d$

Note: As shown in Chapter 4, the value of p increases more than proportionally with M. Hence the area of steel at the continuous supports is slightly larger than 4/3, and at the discontinuous edges slightly less than one-half the steel at mid-span. Selection of mid-span steel should be made by taking note of this fact.

Ref.	Step	Calculations	Output
		(c) *Reinforcement at edge strip*	
		Nominal steel = 216 mm²/m	T10 at 300 mm / (262 mm²/m)
		(d) *Full corner steel*	
		In both directions X and Y	At corner B,
		Each layer = 0.75 maximum positive steel	T10 at 270 mm / (291 mm²/m)
		= 0.75 × 384 = 288 mm²/m	2 lays: T and B
Fig. 12.5		Distance to be covered = $0.2L_x$ = 1 m	
		Provide for 1.0 m × 1.0 m	
		Full at corner B (both edges are discontinuous)	
		one-half at corners C and A (one edge discontinuous)	
		None at corner D (both edges continuous)	
IS 456 22.2.1	7.	*Deflection*	
		Deflection to be based on short span	
		Basic span depth ratio = 23 (say)	

EXAMPLE 12.2 (cont.)

Ref.	Step	Calculations	Output
		Steel at mid-span = 0.251	Limit state of
IS 456		Factor F_1 for Fe 415 = 1.45	deflection
Fig. 3		Allowable ratio = 23 × 1.45 = 33 Actual ratio = 32 (step 1) Hence ratio of 32 is acceptable.	OK
	8.	*Cracking* All steels at more than minimum spacing and less than $3d$	Limit state of cracking OK
	9.	*Arrangement of steel* Detail steel according to standard practice.	

EXAMPLE 12.3 **(Restrained two-way slab with UDL)**

Determine the values of the bending moment coefficients for a rectangular panel with L_y/L_x ratio of 1.2 with two adjacent sides discontinuous by using the formulae in BS 8110 instead of Table 22 of IS 456. (*case 4*)

Ref.	Step	Calculations	Output
	1.	*Determination of α_y, α_1 and α_2 in L_y directions*	
		N_d = No. of discontinuous edges = 2	
		$\alpha_y = \dfrac{24 + 2N_d + 1.5N_d^2}{1000} = \dfrac{24 + 4 + 6}{1000}$	
		= 0.034 (max. positive)	$\alpha_y = 0.034$
		$\alpha_2 = 0$ (provide steel for $\alpha_y/2$)	$\alpha_2 = 0$
		$\alpha_1 = \dfrac{4}{3} \times 0.034 \left(\dfrac{4}{3}\text{ max. positive}\right)$	
		= 0.045 (negative)	$\alpha_1 = -0.045$
		These values are applicable irrespective of L_y/L_x ratio.	
	2.	*Determination of α_x, α_3, α_4 in L_x directions*	
		$\gamma = \dfrac{2}{9}\left[3 - \sqrt{18}\,\dfrac{L_x}{L_y}\left(\sqrt{\alpha_y + \alpha_1}\right) + \sqrt{(\alpha_y + \alpha_2)}\right]$	
		$\sqrt{\gamma} = \sqrt{\alpha_x + \alpha_3} + \sqrt{\alpha_x + \alpha_4}$	
		Substituting, we get $L_x/L_y = \left(\dfrac{1}{1.2}\right)$	

EXAMPLE 12.3 (cont.)

Ref.	Step	Calculations	Output
		Remembering that max. negative = $\frac{4}{3}$ max. positive, we have	
		$\alpha_x + \alpha_3 = \alpha_x + \frac{4}{3}\alpha_x = 2.33\alpha_x$	
		$\alpha_x + \alpha_4 = \alpha_x + 0 = \alpha_x$	
		$\gamma = \frac{2}{9}\left[3 - \sqrt{18}\left(\frac{1}{1.2}\right)(\sqrt{0.034 + 0.045} + \sqrt{0.034 + 0})\right]$	
		$= 0.302$	
		$\sqrt{\gamma} = \sqrt{2.33\alpha_x} + \sqrt{\alpha_x} = 2.526\sqrt{\alpha_x}$	
		Substituting the values, we get	
		$\sqrt{0.302} = 2.526\sqrt{\alpha_x}$	
		$\alpha_x = \frac{0.302}{(2.526)^2} = 0.047$ (positive)	$\alpha_x = 0.047$
		$\alpha_3 = \frac{4}{3} \times 0.047 = 0.063$ (negative)	$\alpha_3 = -0.063$
		$\alpha_4 = 0$ (provide steel for $a_x/2$)	$\alpha_4 = 0$
		These are very nearly the same values as in Table 22 of IS 456, and the procedure is useful for computer aided design.	

EXAMPLE 12.4 **(Concentrated load on two-way slabs)—Pigeaud's curves**

A bridge of 16 m span consists of three longitudinal girders spaced at 2.5 m with five cross girders spaced at 4 m centres. Assuming a wearing coat thickness of 80 mm, determine by Pigeaud's method the moments and shears produced by the IRC class AA tracked vehicle (350 kN on 850 mm × 3600 mm contact area spaced at 2050 mm centres). Assume that the slab is simply supported.

Ref.	Step	Calculations	Output
	1.	*Slab depth*	
		$d = \frac{2500}{12} \approx 200$ mm	
	2.	*Calculate u, v*	
		$u = [0.85 + 2(0.20 + 0.08)] = 1.41$ m	
		$v = [3.6 + (2 \times 0.28)] = 4.16$ m	
	3.	*Calculate coefficients*	
		$\frac{u}{L_x} = \frac{1.41}{2.5} = 0.56, \quad \frac{v}{L_y} = 1$	

EXAMPLE 12.4 (cont.)

Ref.	Step	Calculations	Output
Chart 12.3		$K = \dfrac{L_y}{L_x} = \dfrac{4.0}{2.5} = 1.6$ (assuming simply supported condition)	
		$m_x = 7.3 \times 10^{-2}$, $m_y = 2.0 \times 10^{-2}$ (approx.)	
	4.	*Calculate moments*	
		$M_y = P(m_y + \mu m_x)$	
		$= 350(2 + 0.15 \times 7.3) \times 10^{-2} = 10.8$ kNm	$M_y = 10.8$ kNm
		$M_x = P(m_x + \mu m_y)$	
		$= 350(7.3 + 0.15 \times 2) \times 10^{-2} = 26.6$ kNm	$M_x = 26.6$ kNm
	5.	*Calculate max. shear force*	
		Position load near supports; shear can be calculated by equivalent width as in Example 12.3 or by directly using dispersed width only. Using the second method, we get (a) the shear in the X-direction as	
		$V_1 = \dfrac{350(4 - 2.08)}{4(1.41)} = 119$ kN/m length	
		and (b) the shear in the Y-direction as	
		$V_2 = \dfrac{350(2.5 - 0.705)}{2.5(4.16)} = 60.4$ kN/m	$V_{\max} = \dfrac{119 \text{ kN}}{\text{metre length}}$

EXAMPLE 12.5 **(Analysis for moments for concentrated load on restrained rectangular slabs)**

Applying the corrections for fixity and continuity of the edges, calculate the moments in one of the inner two-way slabs of the bridge given in Example 12.4 due to a concentrated load of 350 kN with $u/L_x = 0.56$ and $v/L_y = 1$.

Ref.	Step	Calculations	Output
	1.	*Span-ratio adjustment for restrained slab*	
Fig.12.12		Case 3: $K_1 = \dfrac{7}{8}$, $K = K_1 \dfrac{L_y}{L_x}$	
		$K = \dfrac{7}{8} \times 1.6 = 1.4$ (Use Chart 12.4)	
	2.	*Values of coefficient of moments and moments*	
		$\dfrac{u}{L_x} = 0.56$, $\dfrac{v}{L_y} = 1$, $K = 1.41$	
Chart 12.4		$m_x = 6.8 \times 10^{-2}$, $m_y = 2.8 \times 10^{-2}$	
		$M_y = P(m_y + \mu m_x)$, $M_x = P(m_x + \mu m_y)$	

EXAMPLE 12.5 (cont.)

Ref.	Step	Calculations	Output
		$M_y = 350[(2.8 + (0.15 \times 6.8)] \times 10^{-2} = 13.37$ kNm	
		$M_x = 350[6.8 + (0.15 \times 2.8)] \times 10^{-2} = 25.27$ kNm	
	3.	*Effect of continuity of edges along L_y*	
Table 12.5		Modified M_y = (Reduction factor) (M_y on simply supported case)	
		M_y' (+ve) mid-span = 0.70 × 13.37 = 9.36 kNm	
		M_y' (−ve) supports = 0.90 × 13.37 = 12.03 kNm	
	4.	*Effect of continuity of edges along L_x*	
		Slab continuous on one end and restrained on the other.	
		M_x (+ve) mid-span = 0.85 × 25.27 = 21.48 kNm	
Table 12.5		M_x (−ve) end support = 0.25 × 25.27 = 6.32 kNm	
		M_x (−ve) penultimate end = 0.95 × 25.27 = 24.0 kNm	
	5.	*Alternative approximate analysis*	
		M = 0.8 (M for simply supported case)	
Thumb rule		M_y = 0.8 (10.8 from Example 12.4) = 8.64 kNm	
Text 12.18.3		M_x = 0.8 (26.6 from Example 12.4) = 21.28 kNm	
		(*Note*: In bridges the effect of impact factor should also be taken into account.)	

EXAMPLE 12.6 **(Analysis for moments in bridge slab under tracked vehicle)**

A reinforced concrete T beam bridge has three beams spaced at 2.5 m each and cross girders at 4 m. The slab thickness is 200 mm and the wearing coat thickness is 80 mm. Calculate the bending moments due to I.R.C. class AA tracked vehicle (850 mm × 3600 mm track and 350 T load) placed at the centre of the slab.

Ref.	Step	Calculations	Output
	1.	*Values of coefficients for moments*	
		$u = [0.85 + (2 \times 0.08)] = 1.01$ m	
		$v = [3.60 + (2 \times 0.08)] = 3.76$ m	
		$\frac{u}{L_x} = \frac{1.01}{2.5} = 0.404, \quad \frac{v}{L_y} = \frac{3.76}{4.0} = 0.94$	
		$K = \frac{L_y}{L_x} = \frac{4.0}{2.5} = 1.6$	
Chart		Use chart $K = 1.67$	
$K = 1.67$		$m_x = 0.085, \quad m_y = 0.024$	

EXAMPLE 12.6 (cont.)

Ref.	Step	Calculations	Output
	2.	*Short span design moment due to LL* $M_x = P(m_x + \mu m_y) = 350[0.085 + (0.15 \times 0.024)]$ $= 31.01$ kNm Continuity factor = 0.8 Impact factor = 1.25 Design moment $M_{ux} = 1.5 \times 1.25 \times 0.8 \times 31.01 = 46.52$ kNm	
	3.	*Long span design moment due to LL* $M_y = P(m_y + \mu m_x)$ $= 350[0.024 + (0.15 \times 0.085)] = 12.85$ kNm Design moment M_{uy} $M_{uy} = 1.5 \times 1.25 \times 0.8 \times 12.85 = 19.27$ kNm	

EXAMPLE 12.7 **(Design of partially fixed circular slab)**

A circular slab is 5 m in effective diameter and is partially fixed at the edges. It is loaded with a characteristic live load of 5 kN/m^2. Using rectangular mesh as main reinforcement at the centre and suitable type of steel at the edges, design the reinforcements for the slab. Assume f_{ck} = 15 and f_y = 415 (N/mm^2).

Ref.	Step	Calculations	Output
	1.	*Calculation of design load* $\left(\text{Usual } \dfrac{\text{span}}{\text{total depth}} \text{ ratio is 25 to 40}\right)$. Adopt 40.	
		Thickness of slab $t = \dfrac{5000}{40} = 125$ mm	Cover 15 mm
		$d = t - c - \phi = 125 - 15 - 5 = 105$ mm	d = 100 (assume)
IS 456: Table 12		Design load = 1.5 (DL + LL) = w $w = 1.5[(0.125 \times 25) + 5.0] = 12.2$ kN/m^2	Use 10 mm bars
	2.	*Moments and shears at centre and edges* Assuming $\mu = 0$, moments for circular slab $M = kwa^2$. The values of k are:	

EXAMPLE 12.7 (cont.)

Ref.	Step	*End condition*	*Moment*	*μ at edges*	*μ at centre*	Output
		Simply supported	M_r M_θ	0 2/16	3/16 3/16	
Table 12.6		Fixed	M_r M_θ	– 2/16 0	1/16 1/16	At centre
		Partially fixed	M_r M_θ	– 1/16 (50 to 80 per cent at centre)	2/16 2/16	$M_r = (2/16)wa^2$ $M_\theta = (2/16)wa^2$

Ref.	Step	Calculations	Output
		Max. shear = $0.5wa$	
	3.	*Calculation of moments and shears*	
		$M_r = M_\theta$ at centre	
		Max. moment = $2/16(wa^2)$, $a = 2.5$ m	
		Substituting for wa, we get	
		Max. moment = $(2/16)\ 12.2(2.5)^2 = 9.53$ kNm width	$M_r = M_\theta = 9.53$ kNm
		Max. shear = $0.5 \times 12.2 \times 2.5 = 15.25$ kN/m	$V = 15.25$ kNm
	4.	*Design of depth of slab (M15 concrete)*	
		$d = \left(\dfrac{9.53 \times 10^6}{0.138 \times 15 \times 1000}\right)^{1/2} = 67.9 < 100$ mm	
		Rough check for shear	
		$v = \dfrac{V}{bd} = \dfrac{15.25 \times 10^3}{1000 \times 100} = 0.153\ \text{N/mm}^2$	
		τ_c (min) = $0.35 > v$	Safe in shear
	5.	*Variation of M_r along the radius*	
Text 12.19.1		Equation for $M_r = \dfrac{3w}{16}(k_1a^2 - k_2r^2)$ (parabolic)	
		Evaluating k_1 and k_2 from the boundary conditions, we get	
		$r = 0, \quad M_r = \dfrac{2}{16}wa^2$ (assumed), $\quad k_1 = 2/3$	
		$r = a, \quad M_r = -\dfrac{1}{16}wa^2$ (assumed), $\quad k_2 = 1$	
		$M_r = \dfrac{3w}{16}\left(\dfrac{2}{3}a^2 - r^2\right)$	
		For $M_r = 0$,	
		$r = \left(\dfrac{2}{3}\right)^{1/2} a = 0.816 \times 2500 = 2041$	

EXAMPLE 12.7 (cont.)

Ref.	Step	Calculations	Output
		Distance of point of inflection from edge $= 2500 - 2041 = 459$ mm (say 460 mm)	Contraflexure at 460 mm
	6.	*Variation of M_θ along the radius* [For a fixed slab M_θ varies from maximum at the centre to zero at the edges. For a simply supported slab, M_θ at end is 2/3 of that at centre and is of the same sign.] Assume M_θ = 50 to 80% at the centre.	
	7.	*Types of steel to be provided in the slab* *Type 1*: Rectangular mesh for positive B.M. (M_θ and M_r) in the central portion at the bottom. *Type 2*: Radial steel for negative B.M. (M_r) at the ends placed on top of the slab. *Type 3*: Circumferential steel at the edges for M_θ at the bottom of the slab. *Type 4*: Circumferential steel at the edges for fixing the radial steel at top of the slab.	
	8.	*Type 1 steel for M_r and M_θ at central portion* $\dfrac{M}{bd^2} = \dfrac{9.53 \times 10^6}{1000(100)^2} = 0.953$	
SP 16: Table 1		$f_{ck} = 15$ N/mm², $f_y = 415$ N/mm², $p = 0.286 > 0.12$ $A_s = \dfrac{0.286}{100} \times 100 \times 1000 = 286\ \text{mm}^2$ T10 at 250 gives 314 (mm²). (Max. allowed spacing $3d = 300$ mm) Use mesh reinforcement (isotropic slab) with T10 at 250 mm both ways. (The spacing may be increased beyond the points of contraflexure.)	Use mesh T10 at 250 both ways
	9.	*Type 2 steel for negative M_r at supports* This steel is provided as radial steel Max. negative moment = 1/2 max. at centre	$M_r = -4.77$ kNm
SP 16 Table 1		$\dfrac{M}{bd^2} = 0.477, \quad p = 0.137$ $A_s = \dfrac{0.137}{100} \times 100 \times 1000 = 137\ \text{mm}^2$ Max. spacing = 300 mm	T10 at 300 (262 mm²)

EXAMPLE 12.7 (cont.)

Ref.	Step	Calculations	Output
IS 456: Cl.25.2.3.1		Steel should extend beyond point of contraflexure by 12ϕ $= 460 + 12\phi = 460 + 120 = 580$ (600 mm)	$L = 600$ mm
	10.	*Type 3 steel for M_θ at edges*	
		The mesh steel will not be effective due to lack of development length. Provide circumferential steel for development length.	
SP 16: Table 65		$L_d = 564$ mm	
		Spacing required = 80% at centre (approx.) $= \frac{250}{0.8} = 313$ (max. = 300 mm)	Three rings at 300
		Use three rings spaced 300 mm in 600 mm length.	
	11.	*Type 4 steel for fixing radial steel*	
		Provide 3 rings for 600 mm length as in Type 3.	Three rings at 300

EXAMPLE 12.8 **(Detailing of radial steel in circular slab)**

A simply supported circular slab is 6 m in diameter and 150 mm thick. For maximum moments at the centre, it has to be reinforced with steel of 12 mm bars at 200 mm spacing. Work out a radial and circumferential arrangement of steel if the slab is to be reinforced uniformly over its surface.

Ref.	Step	Calculations	Output
	1.	*Circumferential steel*	
		Provide circular rings at 200 mm spacing, starting at the outer perimeter towards the centre.	T12 at 200 mm
	2.	*Radial steel*	
		(a) Number of spacings (N) at 200 mm on the outer circumference	
		$N = \frac{2\pi(3000)}{200} = 94.2$ (assume 96 spaces)	
		Place radially 96 bars of 12 mm diameter at outer circumference by dividing it into 24 sectors and 4 bars in each sector.	24 sectors of 4 bars (96 T12 at the outer perimeter)
		(b) The alternate bars (No. 2 and 4 bars of each sector) can be theoretically cut off at radius R_2.	
		$R_2 = \frac{48 \times 200}{2\pi} = 1520$ mm (say at 1500 mm)	$R_2 = 1500$ mm

EXAMPLE 12.8 (cont.)

Ref.	Step	Calculations	Output
		(c) The third bar in each sector can be discontinued theoretically at radius R_3. Then $$R_3 = \frac{24 \times 200}{2\pi} = 760 \text{ mm}$$	$R_3 = 760$ mm
	3.	*Rectangular arrangement at the centre* M_r and M_θ are maximum at the centre and it is difficult to lay continuous steel through the centre of the circular slab. It is better to lay the centre portion as rectangular grid with 12 mm bars at 200 mm spacing. *Note*: The radial distribution is more suitable for annular slabs, where the reinforcements need not be continued through the centre.	Mesh at centre

***EXAMPLE 12.9* (Approximate analysis of centrally supported circular slab by superposition)**

A circular slab is supported by a central reinforced concrete column. Assuming the radius of the slab to be a, that of the post to be b, and the factored total load on the slab to be w per unit area, indicate how the design moments can be calculated.

Ref.	Step	Calculations	Output
	1.	*Method of analysis* The combined effects of cases (a) and (b) below give the required answer for the problem.	
Table 12.6	2.	*Case (a): Simply supported case with UDL* The worst design moment occurs at the edge of the column. The distribution of M_r and M_θ is parabolic. $$M = \frac{3}{16} w(k_1 a^2 - k_2 r^2)$$ From boundary conditions, $k_1 = k_2 = 1$. Hence, $$M_r = \frac{3}{16} wa^2[1 - (r/a)^2]$$ $$\text{At } r = b, \quad M_r = \frac{3}{16} wa^2 \left[1 - \left(\frac{b}{a}\right)^2\right]$$ $$M_\theta = \frac{2}{16} wa^2 + \frac{wa^2}{16}[1 - (b/a)^2]$$	
Fig. 12.20	3.	*Case (b): Simply supported case with load inside the circle* Radius of loaded circle $b < a$. For values of $r \le b$, the moments per unit lengths are	$w_1 = (a/b)^2 w$

EXAMPLE 12.9 (cont.)

Ref.	Step	Calculations	Output
		$M_r = -\frac{3}{16} w_1 r^2 + \frac{1}{4} w_1 b^2 \left(1 - \log_e \frac{b}{a} - \frac{b^2}{4a^2}\right)$	
		$M_\theta = -\frac{1}{16} w_1 r^2 + \frac{1}{4} w_1 b^2 \left(1 - \log_e \frac{b}{a} - \frac{b^2}{4a^2}\right)$	
	4.	*Resultant moments*	
		Combining these moments with proper signs, the maximum design moments can be calculated.	

REVIEW QUESTIONS

12.1 What type of construction will enable two-way slabs to be classified as (a) simply supported on four sides, and (b) restrained on all the sides? What is the differences in the action of these slabs?

12.2 Distinguish the action of slabs continuous over their supports from those that are cast with ends monolithically with their supporting beams.

12.3 Show (with the help of a sketch) the nature of moments set up at the corners of the slab restrained at the edges. Do these moments produce tension or compression at the bottom of the slab?

12.4 How does one (a) check for deflections of two-way slabs, and (b) control crack width in two-way slabs?

12.5 How is the maximum shear in two-way slabs calculated? Which is the critical section to be considered for checking for shear in a slab supported on beams?

12.6 Is there any difference between the action of shear in slabs and shear in beams? Compare the allowable shear strength in slabs to that in beams. Would you consider providing shear steel in an ordinary two-way slab?

12.7 Enumerate the methods used for determining the bending moments in (a) one-way slab, and (b) two-way slabs due to concentrated loads.

12.8 Sketch the nature of the 'contact area' of a wheel load on a bridge slab.

12.9 Explain the basis of Pigeaud's and Westergards' method of design of two-way slabs. What are the limitation of Pigeaud's curves? Which of these methods would one use for design of bridge slabs?

12.10 Mark the edge and middle strip portions of a two-way slab and explain the differences in their action. Give IS rules for reinforcing these strips.

PROBLEMS

12.1 A room is 3 × 4.5 m, and the walls are built with 250 mm thick brick work. It is covered with a simply supported slab which has to take an imposed characteristic load of 2.0 kN/m². Design the slab and sketch the layout of the reinforcements. Explain whether this slab needs any corner steel as reinforcement. Assume f_{ck} = 15 N/mm^2 and f_y = 415 N/mm^2.

12.2 A framed building (slabs on beams and columns) with columns spaced at 4m in the North-South direction and columns spaced at 5 m in the East-West direction has R.C.C. slabs over the beams. The slab has to carry a characteristic live load of 3 kN/m^2 in addition to a floor finish of 1 kN/m^2, and the dead load. Using concrete of grade 20 and Fe 415 steel design a suitable corner slab. Sketch the details of placing of the main and corner steel.

12.3 For the same layout as in Problem 12.2, design an interior slab and sketch the layout of steel using grade 20 concrete and high yield deformed bars as reinforcement.

12.4 A simply supported two-way reinforced slab 3 × 4.5 m is supported on all the four sides, and supports a concentrated load at the centre of the slab. This load is 57 kN in magnitude and is applied over an area 50 × 25 cm with 50 cm along the shorter span, as shown in Fig. P.12.4 position (o). If the slab has to carry an additional characteristic live load of 3 kN/m^2, design the slab and sketch the reinforcements. Assume f_{ck} = 25 N/mm^2 and f_y = 415 N/mm^2.

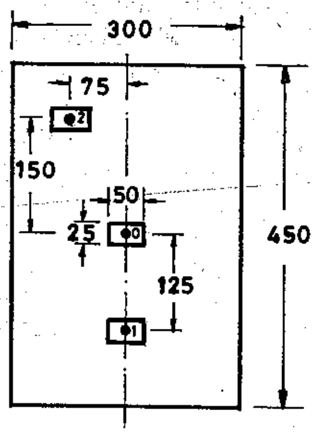


Fig. P.12.4.

12.5 Design the slab in Problem 12.4 if the slab is continuous over all supports. Draw the arrangement of steel.

12.6 Design the slab given in Problem 12.4 if the load is placed (a) in position 1, (b) in position 2, as shown in Fig. P.12.4.

12.7 Design a roof for a circular room 5 m inside diameter. The thickness of the wall is 250 mm and the slab projects all round by 1.25 m. Assume an imposed load of 1.5 kN/m^2. Assume f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2.

12.8 Design a circular slab having 3 m in diameter and supported on a central column 40 cm in diameter. The slab has to take a superimposed characteristic load of 1.5 kN/m^2. Assume f_{ck} = 25 N/mm^2 and f_y = 415 N/mm^2.

Sagar

13

Limit State of Collapse in Compression Design of Axially Loaded Short Columns

13.1 INTRODUCTION

Members in compression are called columns and struts. The term 'column' is reserved for members which transfer loads to the ground and the term 'strut' is applied to a compression member in any direction, as those on a truss. Column members whose height is not more than three times its lateral dimension are called *pedestals* while the term 'wall' is used to compression members whose breadth is more than four times the thickness of the wall.

It is well known from the theory of structures that the modes of failure of a column depend on its slenderness ratio. This ratio is expressed in IS and BS practice for reinforced concrete rectangular columns as the ratio of the effective length L_e to its least lateral dimension (d), (L_e/d) ratio. In steel columns the slenderness ratio is generally expressed as the effective length to its least radius of gyration (L_e/γ) ratio. This practice is continued for R.C.C. columns in ACI code. Effective length L_e of a column is different from its unsupported length L_0 as explained in Chapter 15 of the text.

Columns, when centrally loaded, fail in one of the three following modes, depending on the slenderness ratio (see Fig. 13.1).

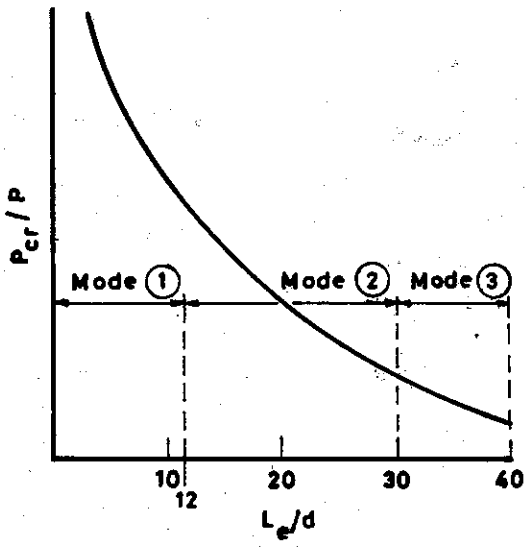


Fig. 13.1 Modes of failure of columns.

Mode 1: P

The colum
reach the

Mode 2: C

Short colu
loaded axi
additional
of these di

Mode 3: F

Very long
yield stres
Failu
members
beyond a
explained
This

13.2 SH

IS 456 cla
dimension
is of dime

If any of
are less th
10 for un
L/r ratio i
and L/r =

13.3 BR

Columns
like wind
the wind
shear wal
braced co
loads by t
be in one

Mode 1: Pure compression failure

The column fails under axial loads without undergoing any lateral deformation. Steel and concrete reach the yield stress values at failure. The collapse of the column is due to material failure.

Mode 2: Combined compression and bending failure

Short columns can be subjected to direct load (*P*) and moment (*M*). Slender columns even when loaded axially undergo deflection along their length as beam columns, and these deflections produce additional moments in the columns. When material failure is reached under the combined action of these direct loads and bending moment, it is called combined compression and bending failure.

Mode 3: Failure by elastic instability

Very long columns can become unstable even under small loads well before the material reaches yield stresses. Under such cases the member fails by lateral 'elastic buckling'.

Failure by the third mode is unacceptable in practical construction (see Fig. 13.1). R.C.C. members that may fail by this type of failure is prevented by the codal provision that columns beyond a specified slenderness (30 for unbraced columns) should not be allowed in structures as explained in Section 13.5.

This chapter gives the method of calculation of the carrying capacity of short columns.

13.2 SHORT COLUMNS

IS 456 classifies rectangular columns as short when the ratio of the effective length (L_e) to the least dimension is less than 12. This ratio is called the *slenderness ratio* of the column. If the column is of dimension $b \times D$, then there are two slenderness ratios namely,

$$\text{Slenderness ratio about major axis} = \frac{L_{ex}}{D}$$

$$\text{Slenderness ratio about minor axis} = \frac{L_{ey}}{b}$$

If any of these two ratios is equal to or more than 12, it is called a *slender column*. If both ratios are less than 12, it is a *short column*. In BS the dividing ratio is taken as 15 for braced column and 10 for unbraced columns. (The term 'braced column' is explained in Section 13.3.) In ACI the L/r ratio is used instead of the L/b ratio. The dividing line is taken as $L/r = 34$ for braced columns and $L/r = 22$ for unbraced columns.

13.3 BRACED AND UNBRACED COLUMNS

Columns can be planned in a structure so that they do not have to withstand any horizontal load like wind and earthquake loads. Thus, for example, when the columns of water-tower are braced, the wind load is taken by the interaction of column bracings. In tall buildings, lateral supports like shear walls can be provided so that the lateral loads are taken by them. Such columns are called *braced columns*. Other columns, where the lateral loads have to be resisted in addition to vertical loads by the strength of the columns themselves, are considered as unbraced columns. Bracings can be in one direction or in more than one direction, depending on the likelihood of the direction of the lateral loads. For example, for the plan arrangement shown in Fig. 13.2, the lateral forces in the *YY*-direction have to be taken by the columns themselves; hence they are unbraced in that

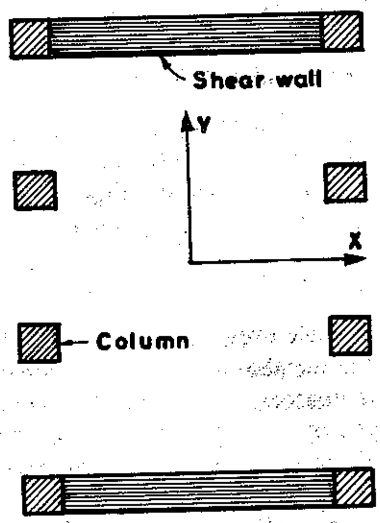


Fig. 13.2 Bracing of columns.

direction. However, in the *XX*-direction the lateral loads can be taken by the shear walls so that the columns can be considered as braced in the *XX*-direction only. The following chart gives the classification of columns according to IS.

Given column of free length, L_0
(Effective length, L_e)

$\frac{L_e}{b} < 12$ (Short) $\qquad$ $\frac{L_e}{b} > 12$ (Long)

It should be noted that in IS code the effect of bracing is reflected in the calculation of the effective length of the columns and consequent classification as a short or long column. This is explained in Chapter 15. In this chapter, only the design of short columns is dealt with.

13.4 UNSUPPORTED AND EFFECTIVE LENGTH (HEIGHT) OF COLUMNS

The unsupported length or height of a column (L_0) is generally taken as the clear height of the columns. It is defined in IS 456, clause 24.1.3 for various cases of constructions. However, as is well known in the theory of structures, the effective length of the column is different from the unsupported length. Effective length (L_e) is dependent on the bracing and end conditions. It should be noted that for braced columns the effective column height is less than the clear height between restraints, whereas for unbraced and partially braced columns the effective height is greater than the clear height. The effect of fixity of ends on the equivalent length of isolated columns is as shown by Fig. 13.3. More accurate methods available for determination of effective length of columns in frames are given in Chapter 15.

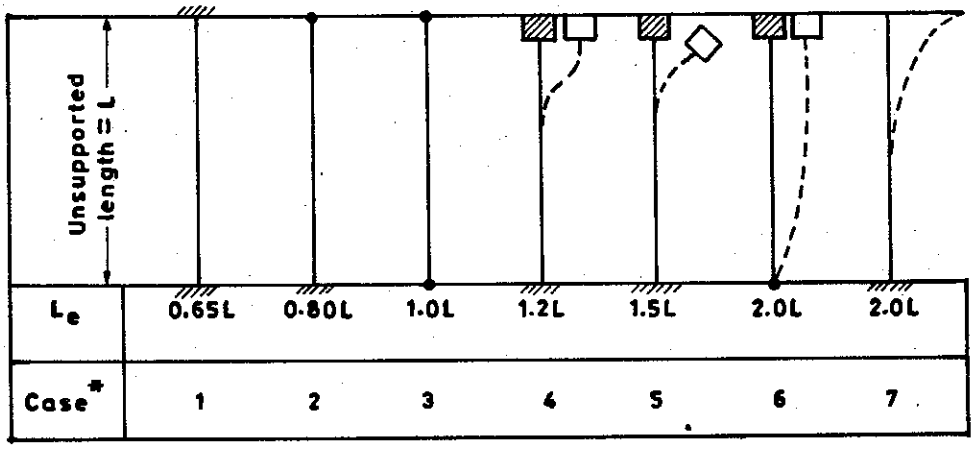


Fig. 13.3 Equivalent length of isolated columns (Ref. IS 456 Table 24).

13.5 SLENDERNESS LIMITS FOR COLUMNS

As already stated, columns should never be of such dimensions that they fail by buckling. All columns should fail by material failure only. For this purpose, the clear distance between restraints (L_0) should never exceed 60 times the minimum dimensions of the column. (IS 456: clause 24.3.1). For unbraced columns it is recommended to keep this value as 30. In cantilever columns in addition to the above restriction ($L_0 \leq 60b$), the clear height should also not exceed (according to IS 456: clause 24.3.2) the value

$$L_0 = \frac{100b^2}{D}$$

where D is the depth of cross-section measured in the plane under consideration and b is the width of the cross-section.

13.6 DERIVATION OF DESIGN FORMULA FOR SHORT COLUMNS

13.6.1 ASSUMPTIONS

The main assumptions made for limit state design of columns failing in pure compression, as given in IS 456: clause 38.1 are:

1. Plane sections remain plane in compression.

2. The maximum compressive strain in concrete in axial compression is 0.002 (the general criteria for failure are given in Chapter 14).

3. The design stress-strain curve of steel in compression is taken to be the same as in tension (Fig. 13.4). The stress-strain curve of steels like Fe 250 which are hot rolled is bi-linear and the maximum design stress at yield is a definite point taken as $f_y/1.15 = 0.87f_y$.

However, the design stress-strain curve for Fe 415 steel which is cold twisted is taken in IS 456 as non-linear beyond the stress value of $(0.8f_y)/1.15 = 0.7f_y$. It may be noted that at a strain of 0.002, the stress will be approximately $0.75f_y$ with these types of steel.

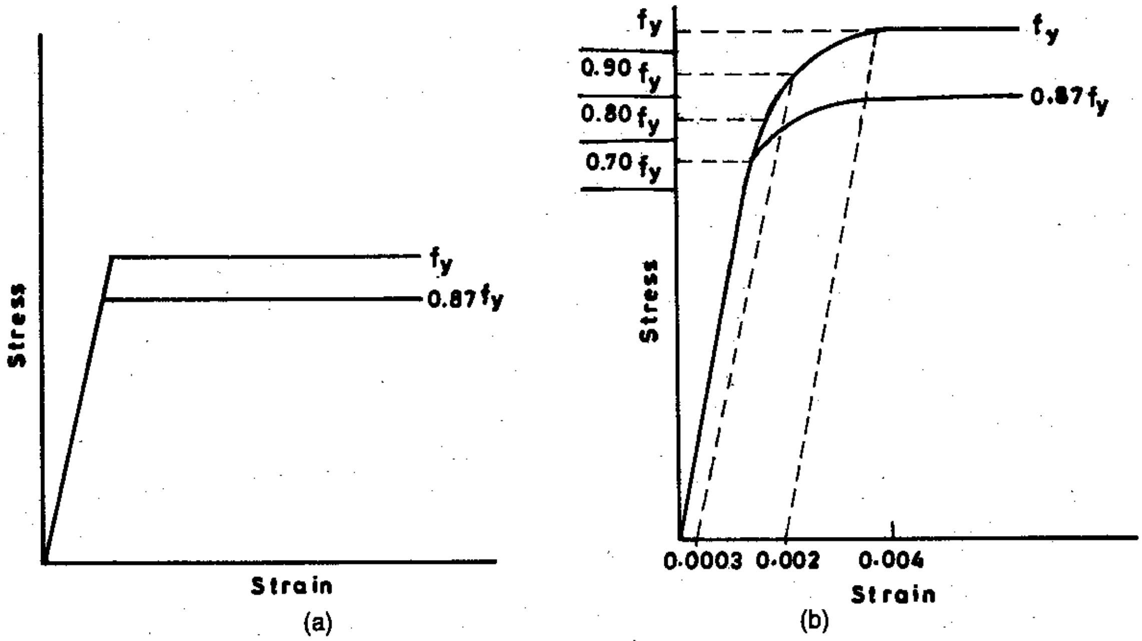


Fig. 13.4 Stress-strain curve for steel: (a) Mild steel bars, (b) Cold worked deformed bars.

4. The stress-strain curve of concrete (Fig. 13.5) as given in IS 456 is such that the maximum strength attained in concrete is $0.446f_{ck}$. This was explained in Chapter 4. The above value is usually approximated to $0.45f_{ck}$, and this is reached at a strain in concrete equal to 0.002.

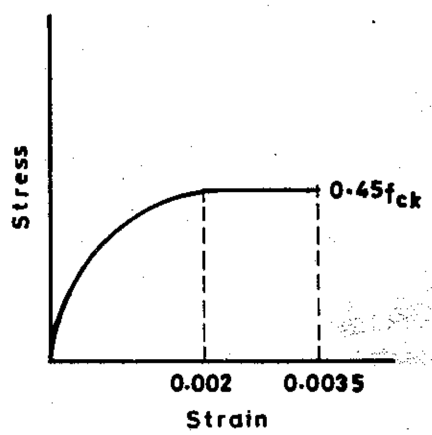


Fig. 13.5 Stress-strain curve for concrete.

13.6.2 DERIVATION OF FORMULA

The ultimate failure is assumed to be reached when the section reaches a uniform compression strain of 0.002. Ultimate load is given by the expression

Ultimate load = load carried by concrete + load carried by steel

$$P = A_c f_c + A_s f_s$$

where f_c and f_s are the stresses in the material at a uniform strain of 0.002.

The compression in concrete f_c at failure ($\varepsilon_c = 0.002$) is given by

$$f_c = 0.45 f_{ck}$$

The compression in steel f_s at failure ($\varepsilon_c = 0.002$) will be

$$f_s = 0.87 f_y \text{ for steel with bilinear stress-strain curve as in Fe 250 steel}$$

$$= 0.75 f_y \text{ for steel with stress-strain curve as in Fe 415 steel}$$

Hence, the ultimate carrying capacity of the column P_u is given by the expressions

1. For Fe 415 steel,

$$P = A_c(0.45 f_{ck}) + A_s(0.75 f_y - 0.45 f_{ck})$$

2. For Fe 250 steel,

$$P = A_c(0.45 f_{ck}) + A_s(0.87 f_y - 0.45 f_{ck})$$

However, it is never possible to apply the load centrally on a column. Accidental eccentricities are bound to happen. Indian and the British codes allow an accidental eccentricity of 5 per cent of the lateral dimension of the column in the plane of bending ($0.05D$) in the strength formula itself. For this purpose the ultimate load P_u derived above is reduced by 10 per cent.

Neglecting the last negative term in the above expressions, the equation for P_u for Fe 415 steel reduces to

$$P_u = 0.9(0.45 f_{ck} A_c + 0.75 f_y A_s)$$

i.e.

$$P_w = 0.4 f_{ck} A_c + 0.67 f_y A_s \tag{13.1}$$

as given in IS 456: clause 38.3.

With Fe 250 steel the corresponding expression will be

$$P_u = 0.4 f_{ck} A_c + 0.75 f_y A_s \tag{13.2}$$

as given in BS 8110 which assumes a bilinear stress-strain curve for all grades of steel.

It should be clearly noted that these formulae already take into account a maximum accidental eccentricity of ($0.05D$) or ($0.05b$) in these columns.

13.7 CHECKING ACCIDENTAL ECCENTRICITY

Accidental eccentricities are caused by imperfections in construction, inaccuracy in loading etc. The BS Code assumes that its value will be equal to $0.05D$, but not more than 20 mm.

IS 456: clause 24.4 gives an expression for the possible minimum eccentricity as

$$e_{min} = \frac{L_0}{500} + \frac{D}{30} \text{ but not less than 20 mm} \tag{13.3}$$

where

L_0 = the unsupported length

D = lateral dimensions in the plane of bending

Note: In IS 456, fourth revision, e_{min} to be not more than 20 mm as in BS 8110.

For sections other than rectangular, the Explanatory Handbook SP 24 recommends a value of $L_e/300$, where L_e is the effective length of the column. (This expression is similar to the provision in DIN 1045.)

Thus, for example, for a column 600 × 450 of unsupported height 3 m, considering the long direction according to IS formula,

$$e_{\min} = \frac{L}{500} + \frac{D}{30} = \frac{3000}{500} + \frac{600}{30} = 26 \text{ mm}$$

As 26 mm is greater than the minimum specified 20 mm, use $e_{\min}$ = 26 mm. Then

$$\frac{e_{\min}}{D} = \frac{26}{600} = 0.043$$

Considering the short direction, we have

$$e_{\min} = \frac{3000}{500} + \frac{450}{30} = 21 \text{ mm} > 20 \text{ mm}$$

Hence,

$$\frac{e}{b} = \frac{21}{450} = 0.047$$

Both these values are less than the specified ratio of 0.05, and hence the simple column formula is applicable to the above column. If the eccentricities are more, then the column has to be designed as subjected to direct load P and moment Pe (see Chapter 14).

13.8 DESIGN OF LONGITUDINAL STEEL

The area of the longitudinal steel is to be calculated by the strength formula given above. However, there are a number of other factors that should be taken into account in choosing the longitudinal steel and they are given in IS 456: clause 25.5.3.1. The important provisions for design of longitudinal steel are:

1. The minimum diameter of longitudinal steel should be 12 mm.

2. There should be at least four longitudinal bars in a rectangular column and at least six of them in a circular column.

3. The percentage of longitudinal steel should not be less than 0.8, nor should it be greater than 6 per cent of gross cross-sectional area of the section. With very little steel, shrinkage effects will cause high stress in steel. With large amount of steel, there will be congestion which will prevent proper placement of concrete, since usually lapping of all column bars is done at the base of each column height so that the percentage of steel at the lapping point will be double the percentage at the centre of the column.

Hence, even though a maximum of 6 per cent is allowed, normally the percentage of steel in columns should not exceed 4. In columns of cross-sections larger than those required to carry the load, these minimum areas of steel are to be based on the concrete area required to resist the direct stress, and not on the actual area of concrete in the column.

4. Proper cover should be provided for the longitudinal steel. The minimum cover should be 40 mm. For columns of 200 mm or less, it may be 25 mm if the diameter of the bar is 12 mm (IS clause 25.4.1).

5. The spacing of the longitudinal bars measured along the periphery of the column should not exceed 300 mm (IS clause 25.5.3.1g).

13.9 DESIGN OF LATERAL TIES (LINKS)

All longitudinal bars (as these are in compression) should be properly restrained by ties, tied at proper intervals, so that the steel bars do not in their turn act as long columns. Detailed rules are given in IS 456: clause 25.5.3.2 regarding provision of transverse steel. Briefly stated, these rules are:

1. Links should be so arranged that every corner and alternate longitudinal bar, if spaced not more than 75 mm, should have lateral support provided by the corner of a link having an internal angle of not more than 135 degrees. For circular columns the support is calculated adequate if they are provided by circular ties touching the longitudinal steel. Recommended arrangements of column bars and ties are given in SP 34, Section 7 as shown in Fig. 13.6.

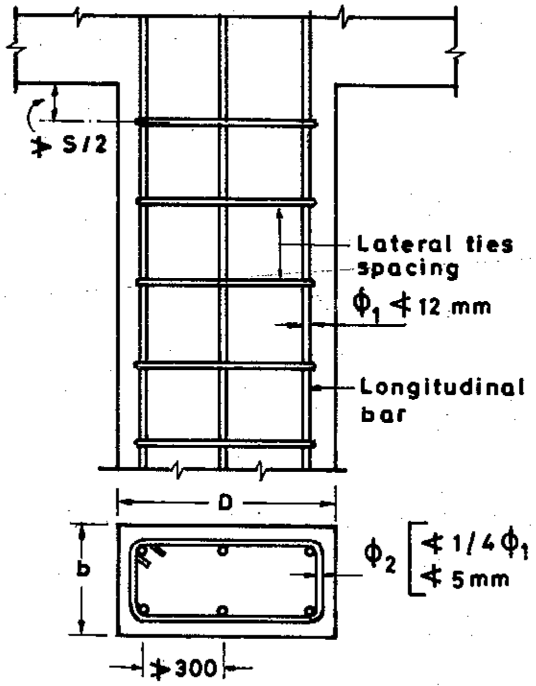


Fig. 13.6 Arrangement of steel in tied columns.

2. The diameter of the links should be at least one quarter of the largest diameter of the longitudinal steel. In any case, the links should not be less than 5 mm in diameter.

3. The spacing of the links should not exceed the least of the following:
   (a) The least lateral dimension of column
   (b) Sixteen times the diameter of the smallest longitudinal bar
   (c) Fortyeight times the diameter of the link.

4. Proper cover should be provided for the links.

(*Note*: The provision for restraining compression steel in columns is also similar to the provisions for restraining the longitudinal compression steel in beams.)

13.10 DESIGN OF SHORT COLUMN BY SP 16

Charts 24 to 26 of the IS publication Design Aids SP 16 can be used for routine office design of short columns. These charts are made from the column formula

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_s$$

If

$$A_g = \text{area of cross-section}$$

$$p = \text{percentage of steel} = 100 A_s / A_g$$

The areas of steel and concrete are given by

$$A_s = \frac{p A_g}{100}$$

$$A_c = A_g - A_s = A_g \left(1 - \frac{p}{100}\right)$$

Rewriting Eq. (13.1) with the above quantities, we obtain

$$\frac{P_u}{A_g} = 0.4 f_{ck} \left(1 - \frac{p}{100}\right) + 0.67 f_y \left(\frac{p}{100}\right)$$

or

$$P_u = \left[0.4 f_{ck} + \frac{p}{100} (0.67 f_y - 0.4 f_{ck})\right] A_g \qquad (13.4)$$

Charts 24 to 26 of SP 16 have been prepared from these formulae for Fe 250, Fe 415 and Fe 500 and f_{ck} = 15, 20, 30, 35 and 40. (Chart 25 for Fe 415 steel is given as chart 13.1 in this book.)

To use the design chart, choose the value of the factored design load P_u, and proceed horizontally till the A_g corresponding to the size of the column selected is reached. The value of the percentage of steel $(100A_s)/A_g$ required for the adopted value of f_{ck} is read off from the lower half of Chart 13.1.

13.11 PROCEDURE FOR DESIGN OF CENTRALLY LOADED SHORT COLUMN

The step-by-step procedure for design of a centrally loaded short column can be arranged as follows:

Step 1: Compute the factored load on the column.

Step 2: Choose a suitable size for the column, depending on the size of the beam that has to be placed on it and the architectural requirements. Usually, the beams are accommodated inside the column. Check also the minimum eccentricity.

Step 3: Determine the effective length and slenderness of the column about the principal axes. If it is less than 12, it can be considered as a short column. If it is 12 or more, it is to be designed as a long column.

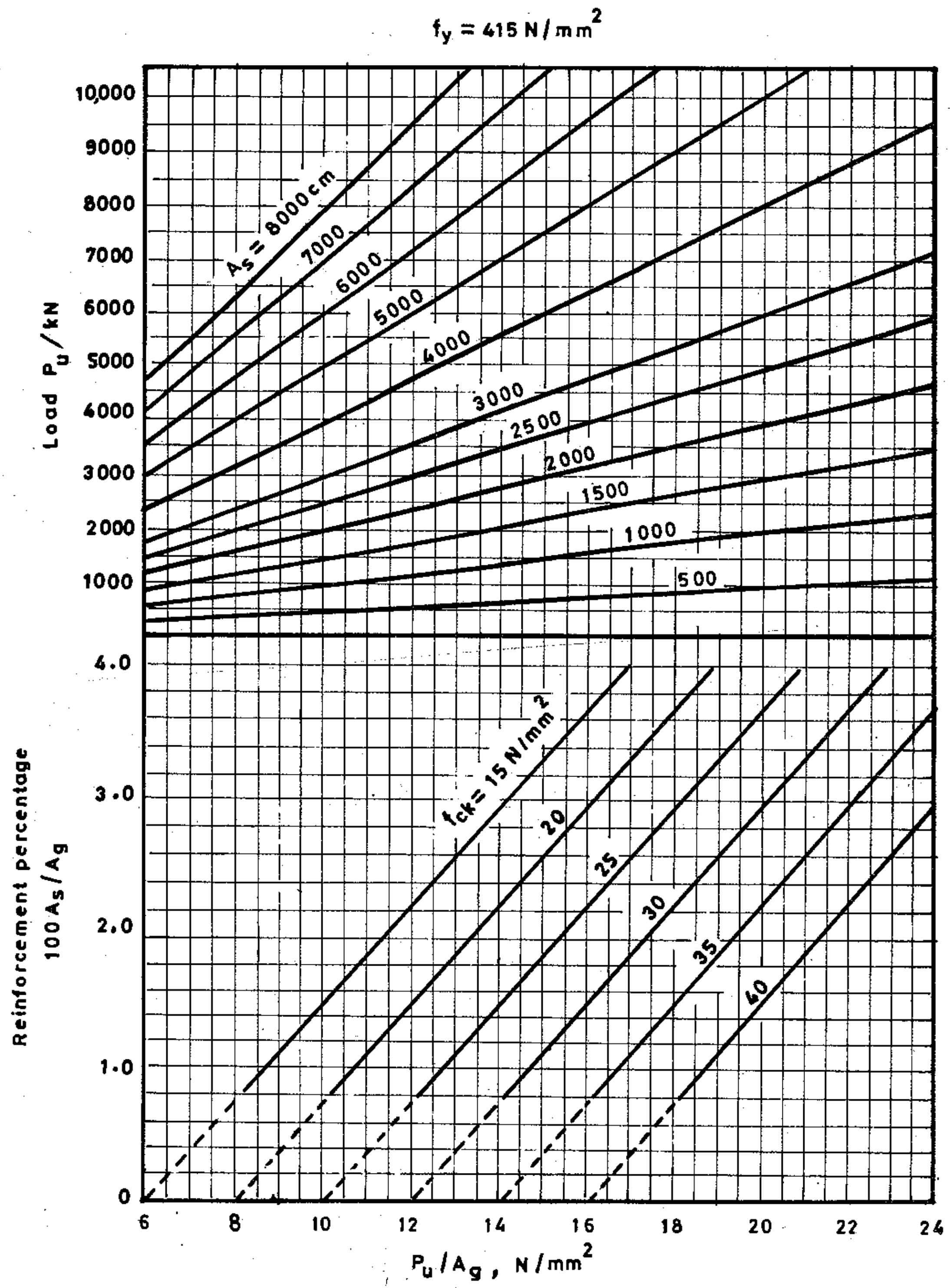


Chart 13.1 Design of Axially Loaded columns f_y = 415 (Chart 25 of SP 16).

Step 4: Compute the area of the longitudinal steel required by either (a) by using the formulae or (b) by using SP 16 in the following manner:

(a) $$P = 0.4f_{ck}A_c + 0.67f_yA_{sc}$$

or

$$P = \left[0.4f_{ck} + \frac{p}{100}(0.67f_y - 0.4f_{ck})\right]A_g$$

(b) By use of SP 16 as already indicated in Section 13.10. The minimum percentage of steel adopted should be greater than 0.8. As regards the maximum percentage, it should be less than 4 in normal designs where lapping becomes essential; where lapping is not adopted, one may accept a percentage of up to 6.

Step 5: Detail the steel by choosing a suitable size and number (size not to be less than 12 mm and for a symmetrical arrangement with at least four bars for a rectangular column and six bars for a circular column). Adopt a suitable cover to the steel (clause 25.4.1) and check perimeter spacing of bars (IS clause 25.5.3.1) is not more than 300 mm as shown in Fig. 13.6.

Step 6: Detail the transverse steel. Adopt a suitable size, determine spacings, etc. as explained in Section 13.8. The procedure is illustrated by Example 13.2.

13.12 STRENGTH OF HELICALLY REINFORCED SHORT COLUMN

IS 456: clause 25.5.3.2(d) deals with design of helically reinforced column. In working stress design, it was the practice to consider the strength of spirals also in contributing to the strength of the column. Tests on spirally reinforced columns show that the additional strength due to spirals in working stress design can be estimated by considering the volume of spiral steel per unit height of the column is approximately twice as effective as the same volume were put as longitudinal steel. Hence, the equation for strength of spiral columns in working load (P_c) is usually written as

$$P_c = \text{(load taken by core)} + \text{(load taken by longitudinal steel)} + 2(V_{sh})\text{(stress in spiral)}$$

where V_{sh} is the volume of the spiral per unit length of the column which is also termed as the equivalent area of helical steel per unit height of the column. However, when dealing with ultimate loads and limit state design, it has been observed that

1. The containing effect of spirals is useful only in the elastic stage and it is lost when the spirals also reach yield point.

2. The spirals become fully effective only after the concrete cover over the spirals spalls off after excessive deformation.

Hence in ultimate load estimation, the strength of spirally reinforced columns is expressed by

1. P_u (spirally reinforced column) = $1.05P_u$ (tied column)

2. the volume of spirals to be provided which is calculated on the principle that it should be adequate to offset the loss of strength of the cover which cracks up at ultimate stage.

13.13 CALCULATION OF SPACING OF SPIRALS

From point 2 given above, one can derive an expression to calculate the area and spacing of spirals used for these columns as follows: As shown in Fig. 13.7, let

s = pitch or spacing of spirals used

a = area of spiral steel

D = diameter of the column

D_k = diameter of the core

The condition is that the loss of strength due to spalling of cover should be equal to the contribution due to spirals.

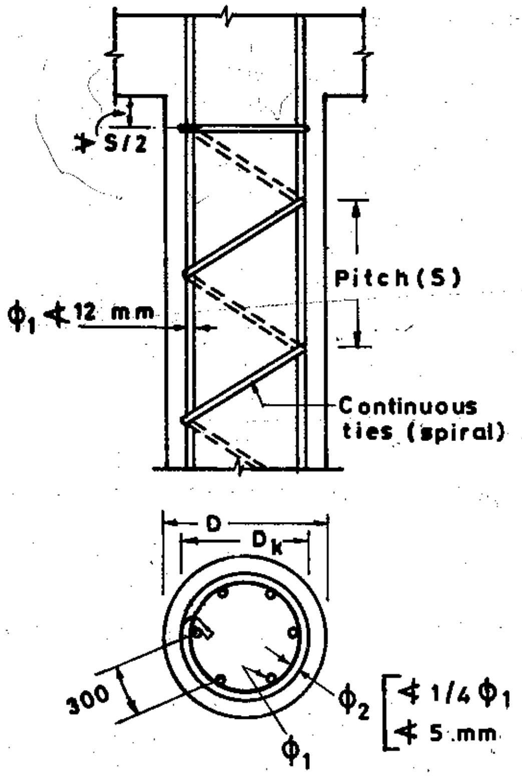


Fig. 13.7 Arrangement of steel in spirally reinforced columns.

Taking A_k as the area of the core and A_g as the area of cross-section and using the same assumptions about the action of the spiral as is used in the elastic design, the relationship at failure is given by

$$2V_{sh}(0.87f_y) = 0.63f_{ck}(A_g - A_k)$$

$$V_{sh} = 0.36(A_g - A_k)\left(\frac{f_{ck}}{f_y}\right)$$

which can be reduced to the form given in IS 456: clause 38.4.1 as

$$\left(\frac{V_{sh}}{A_k}\right) = 0.36\left(\frac{A_g}{A_k} - 1\right)\left(\frac{f_{ck}}{f_y}\right)$$

where

A_g = gross area of section

A_k = area of core

This expression gives the ratio of the volume of the helical reinforcement required for the volume of the core per unit height of the column.

Simplifying this expression further, one can write

V_{sh} = (volume of the spiral in one ring) × (No. of rings per unit length)

Taking D_k as the diameter of the core, we get

$$V_{sh} = \frac{(\text{area of spiral})\pi D_k}{\text{spacing (pitch)}} = \frac{a\pi D_k}{s}$$

Rewriting the IS equation by using the above value for V_{sh}, we obtain

$$\frac{aD_k}{s} = \frac{0.36(D^2 - D_k^2)}{4}\frac{(f_{ck})}{f_y}$$

$$s = \frac{4aD_k}{0.36(D^2 - D_k^2)}\left(\frac{f_y}{f_{ck}}\right)$$

$$s = \frac{11.1aD_kf_y}{(D^2 - D_k^2)f_{ck}} \tag{13.5}$$

which can be used as the expression for calculating the pitch of the spirals for a given steel of cross-sectional area a.

The rules regarding detailing of helical steel are given in IS 456: clause 25.5.3.2. The main considerations are:

1. The diameter of the helicals shall be at least 5 mm or one-fourth the diameter of longitudinal steel.

2. The pitch shall be (a) as derived from formula given in Eq. (13.5), (b) not more than 75 mm, (c) not more than 1/6th core diameter, (d) not less than 25 mm, (e) not less than three times the diameter of the steel bar forming the helix.

If the diameter and the pitch of the spirals do not comply with the above rules, the strength is to be taken as only that of a tied column of similar dimensions.

13.14 PLACEMENT OF STEEL IN CIRCULAR COLUMNS

Circular columns can be tied or spirally reinforced. The minimum number of longitudinal rods should be six and the diameter of the rods should not be less than 12 mm. In tied columns the laterals are individual ties whereas in helically reinforced columns a continuous helix of steel of the required diameter goes around the longitudinal steel at a given pitch.

13.15 COMPARISON OF TIED AND SPIRALLY REINFORCED COLUMNS

Even though there is not much difference in the ultimate load carrying capacity of ties and spirally reinforced columns, as discussed above, there is a marked difference between them in the type of failure that takes place, i.e. the load deformation curve obtained from tests. Whereas tied columns fail suddenly, spirally reinforced columns fail gradually, exhibiting considerable amount of ductility and deformation. Consequently, there will be a lot of warning before final failure of such columns.

13.16 DESIGN OF NON-RECTANGULAR COLUMNS

The use of non-rectangular columns like *I* or *T* sections is quite common in buildings, bridges, etc. (see Fig. 13.8). The *L/b* rule cannot be directly applied to these sections for their classification into short and long columns.

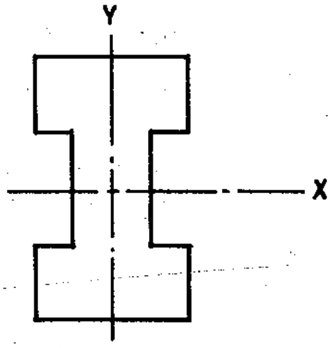


Fig. 13.8 Columns of non-rectangular sections.

One method to overcome this difficulty is to use the ACI code where the length effects are considered by the ratio of the effective length to radius of gyration and not the ratio of effective length to breadth as in BS and IS codes. This is the method used for steel columns and can as well be used for R.C. columns also. For a rectangular column the radius of gyration is given by

$$r^2 = \frac{I}{A} = \frac{Db^3}{12(Db)} = \frac{b^2}{12}$$

$$r = \frac{b}{\sqrt{12}} = 0.289b$$

$$= 0.3b \text{ (as given by ACI code)}$$

Hence, the condition $L/b = 12$ will reduce to

$$\frac{L}{b} = \frac{0.3L}{r} = 12$$

i.e.

$$\frac{L}{r} = 40$$

ACI 318 (1983) takes L/r as the criterion for column design and its clause 10.11.4 gives the following conditions for short and long columns:

1. For compression members braced against side sway, the effects of slenderness may be neglected when

$$\frac{L_e}{r} < \left[34 - 12\,\frac{M_1}{M_2}\right]$$

where M_1 and M_2 are the numerically smaller and larger bending moments respectively, at the ends of the column and the ratio M_1/M_2 is taken as positive for single curvature cases and negative for double curvature. Also, when the member is subjected to large transverse loading other than the end moments, the ratio is taken as +1.

2. For compression members not braced against side sway, the effects of slenderness may be neglected when

$$\frac{L_e}{r} < 22$$

These principles may be used in the design of non-rectangular reinforced concrete columns.

13.17 DETAILING OF COLUMNS

Columns are to be detailed according to standard practice shown schematically in Figs. 13.9 to Fig. 13.11. The line sketches of reinforcements of columns in buildings are indicated as in Fig. 13.11.

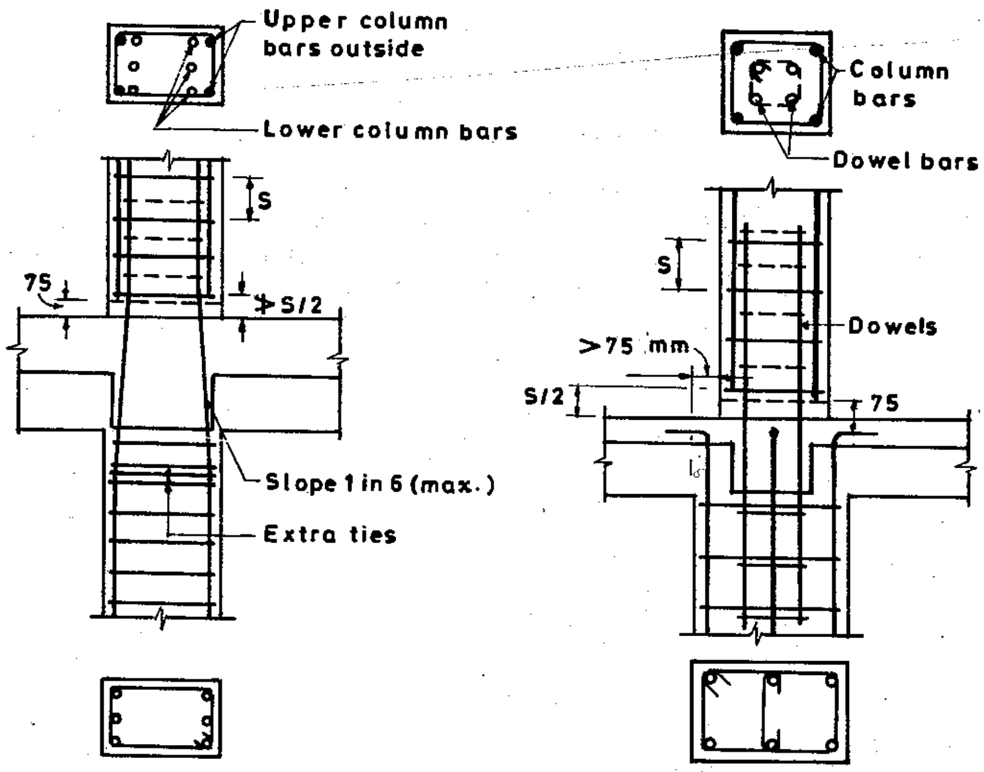


Fig. 13.9 Splicing of columns.

Fig. 13.10

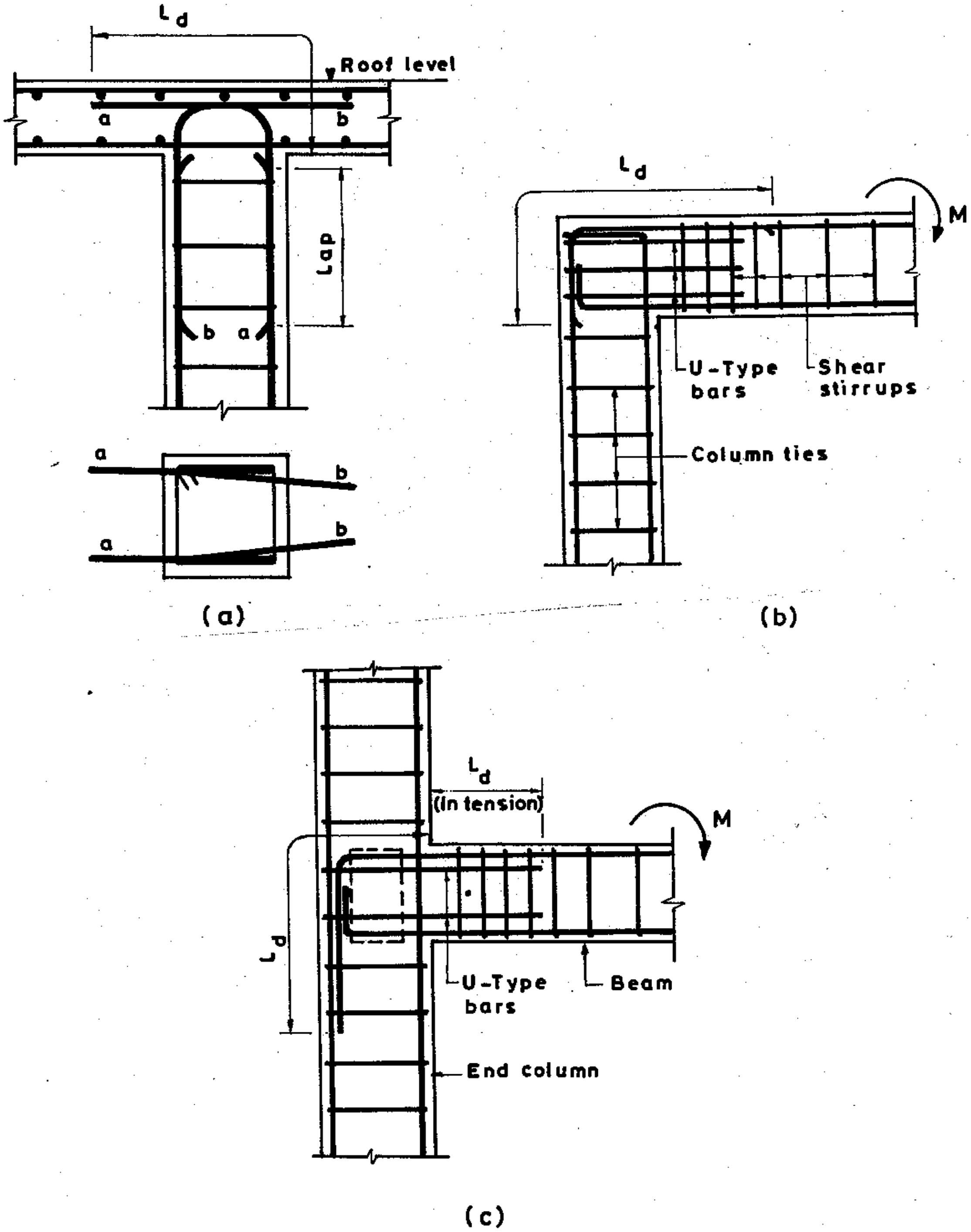
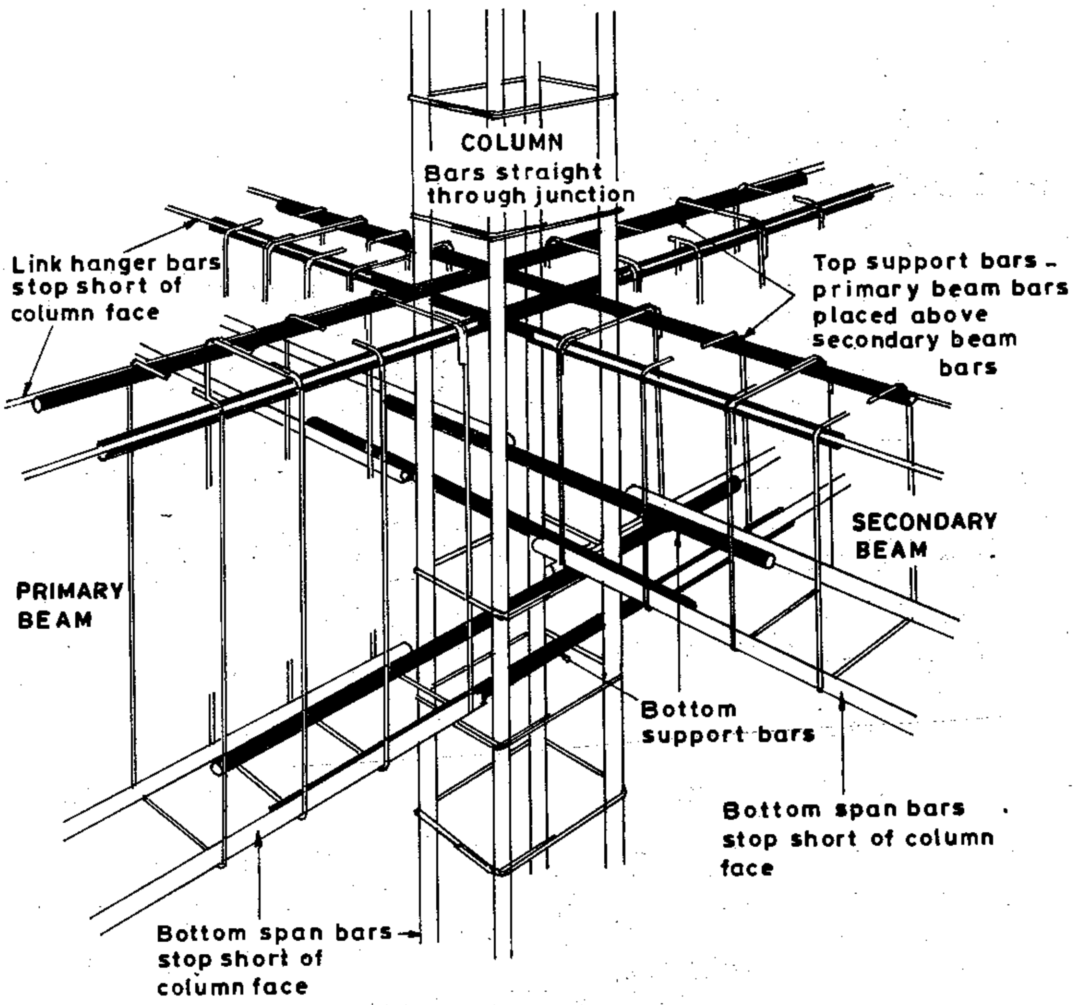


Fig. 13.10 Details of reinforcement at column junctions: (a) Termination of column bars inside slab, (b) Fixed-end joint in a column, and (c) Typical detail of beam-column junction at external column.

(Cont.)



(d).

Fig. 13.10 **Details of reinforcement at column junctions: (d) Detail of beam-column junction at internal column.**

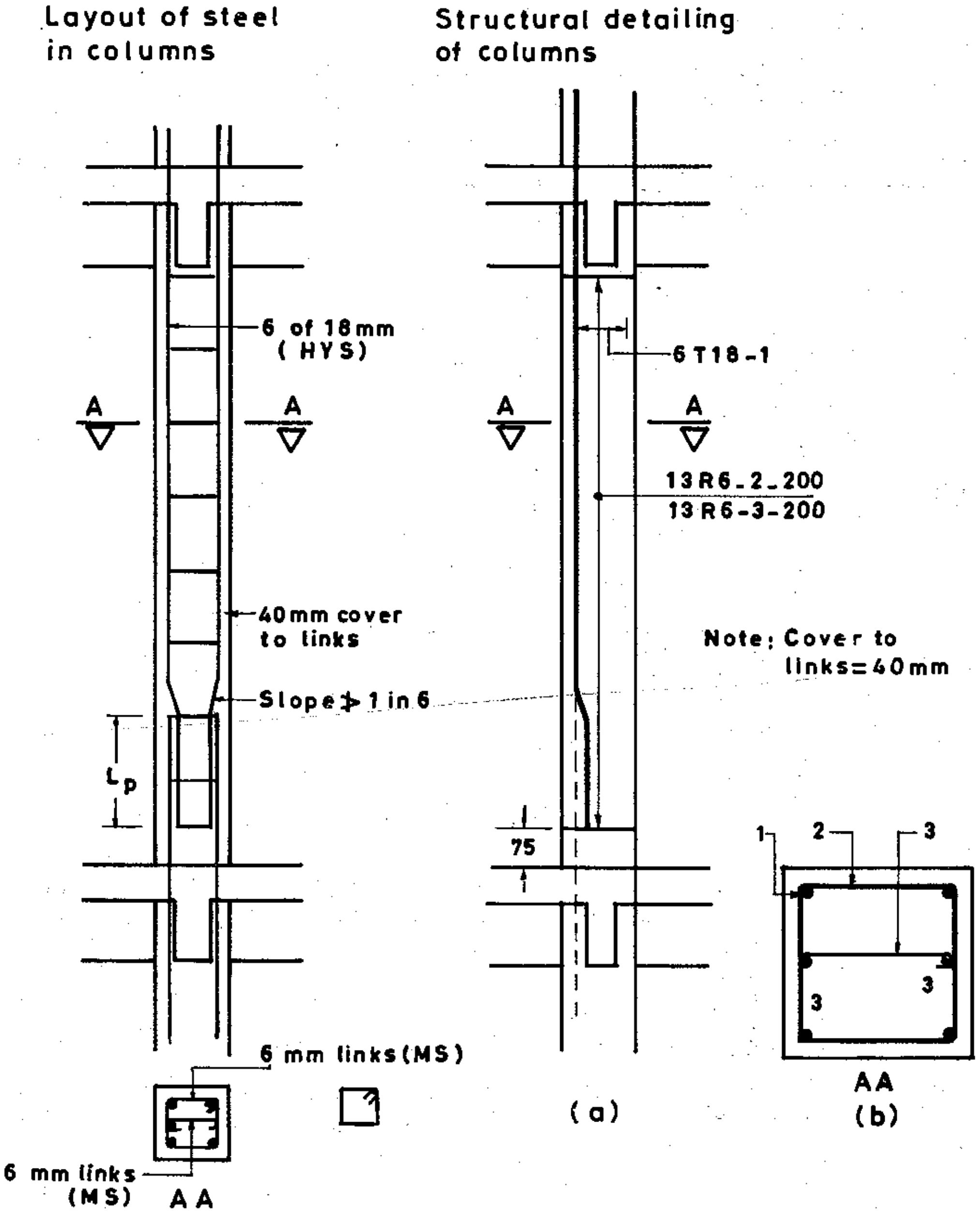


Fig. 13.11 Detailing of R.C.C. Columns.

EXAMPLE 13.1 **(Definitions)**

A column 400 × 400 mm has an unsupported length of 7 m and effective length of 4.5 m. Can it be designed as a short column under axial compression, if the load is placed centrally on it?

Ref.	Step	Calculations	Output
	1.	*Slenderness ratio consideration*	
		$\frac{L_e}{D} = \frac{4500}{400} = 11.25$	
IS 456 Cl. 24.1.2		As columns with slenderness less than 12 can be considered as short, the column is short.	Column is short
	2.	*Eccentricity considerations*	
IS 456 Cl. 24.4		$e_{\min} = \frac{L_0}{500} + \frac{D}{30} \not< 20$ mm $= \frac{7000}{500} + \frac{400}{30}$ $= 14 + 13.3$ $= 27.3$ mm is greater than 20 mm Adopt 27.3 mm	
		The eccentricity for which short column formula is applicable is $D/20$.	
IS 456 Cl. 38.3		$D/20 = \frac{400}{20} = 20$ mm $e_{\min} > D/20$	
		Hence formula for axial load is not applicable. Column should be designed as subject to axial load and moment due to $e_{\min}$. $(M = Pe_{\min})$.	Design as short column subjected to P and M.

EXAMPLE 13.2 **(Design of axially loaded R.C. Short column)**

Design an axially loaded tied column 400 × 400 mm pinned at both ends with an unsupported length of 3 m for carrying a factored load of 2300 kN. Use grade 20 concrete and Fe 415 steel.

Ref.	Step	Calculations	Output
	1.	*Factored load on column* $P_u = 2300$ kN	
	2.	*Size of column and check* $e_{\min}$	
IS 456 Cl. 24.4		Size of column = 400 × 400, $\frac{D}{20} = 20$ mm $e_{\min} = \frac{L}{500} + \frac{D}{30} = \frac{3000}{500} + \frac{400}{30} = 19.33 < 20$ mm	

EXAMPLE 13.2 (cont.)

Ref.	Step	Calculations	Output
		e_{min} less than $D/20$ is assumed in the formula. Hence short column formula for axial load can be used.	
	3.	*Calculation of slenderness*	
IS 456 Cl. 24.1.2		$L_e = 1.0L = 3000$ mm	
		$\frac{L_e}{b} = \frac{3000}{400} = 7.5 < 12$	Short column
	4.	*Find area of steel and check percentage*	
IS 456 Cl. 38.3		(a) *By formula,* $P_u = 0.4 f_{ck} A_c + 0.67 f_y A_s$	
		$2300 \times 10^3 = 0.4 \times 20 \times (400^2 - A_s) + 0.67 \times 415 \times A_s$	
		$A_s = 3777\ \text{mm}^2, \quad p = \frac{3777}{400^2} \times 100 = 2.36\%$	
		This is more than 0.8% and less than 6%. Hence O.K.	
		(b) *By SP 16,* $A_g = 1600\ \text{cm}^2, \quad P = 2300$ kN,	
SP 16 Chart 25		$p = 2.4\%$	
		$A_s = \frac{2.4 \times 400 \times 400}{100} = 3840\ \text{mm}^2$	$\left(\frac{\text{Use 8T 25}}{3927\ \text{mm}^2}\right)$
	5.	*Detail the longitudinal steel*	
Cl. 25.4.1		Use cover = 40 mm	
		Steel spacing = $\frac{(400 - 40 - 40 - 25)}{2} = 147.5$	
Cl. 25.5.3.1(g)		Clear spacing between bars = 147.5 − 25 = 122.5 < 300	
	6.	*Design transverse steel*	
		Diameter of links: not less than $\frac{25}{4}$ or 5 mm;	
		Use 10 mm	
Cl. 25.5.3.2(c)		Spacing not less than	
		(a) dimension of column = 400 mm	
		(b) 16 times ϕ of long steel = 16 × 25 = 400 mm	
		(c) 48 times ϕ of trans. steel = 48 × 10 = 480 mm	
		Adopt 400 mm.	
		Use Fe 250 steel (R) for ties.	R10 at 400 mm

EXAMPLE 13.3 **(Design of helically reinforced columns)**

Design a circular pin-ended column 400 mm dia and helically reinforced, with an unsupported length of 4.5 m to carry a factored load of 900 kN. Assume M 30 concrete and Fe 415 steel.

Ref.	Step	Calculations	Output
	1.	*Factored load* $P_u = 900$ kN	
	2.	*Size of column* $D = 400$ mm, cover = 40 mm	
		$D_{core} = 320$ mm	
		$D/20 = \frac{400}{20} = 20$ mm	
		$e_{min} = \frac{L_0}{500} + \frac{400}{30} = \frac{4500}{500} + \frac{400}{30} = 22.3 \text{ mm} > 20 \text{ mm}$	
		As $e_{min} > D/20$, theoretically short column formula for centrally loaded column is not applicable. However, the column is designed as centrally loaded, as the moment to be considered is small.	
	3.	*Slenderness of column*	
		$\frac{L_e}{D} = \frac{4500}{400} = 11.25 < 12.$ Short column	Centrally loaded short column
IS 456	4.	*Area of longitudinal steel*	
Cl. 38.4		$P_u = 1.05(0.4 f_{ck} A_c + 0.67 f_y A_s)$	
		$A_c = \frac{\pi \times 400^2}{4} = 125.6 \times 10^3 \text{ mm}^2, \quad \frac{P}{1.05} = \frac{900}{1.05} = 857 \text{ kN}$	
		$857 \times 10^3 = [0.4 \times 30 \times (125600 - A_s) + 0.67 \times 415 A_s]$	
		$= 1507 \times 10^3 + A_s\,(278 - 12)$	
		Concrete itself can carry more than the required load.	
IS 456		Hence provide minimum steel	
Cl. 25.5.3.1(b)		$A_{s(min)} = 0.8\%$ (of area required to carry P)	
		A_c to resist given $P = \frac{900 \times 10^3}{1.05 \times 0.4 \times 30} = 71428 \text{ mm}^2$	
		$A_{s(min)} = \frac{0.8}{100} \times (71428) = 571.4 \text{ mm}^2$	$\left(\frac{6\text{T}12}{678 \text{ mm}^2}\right)$
Cl. 25.5.3.1(c)		Provide 6 Nos. of 12 mm bars giving area 678 mm^2 as minimum number of bars allowed is 6 Nos.	
	5.	*Design spirals*	
		Choose 6 mm; $a = 28$ mm^2 (area); s = pitch	

EXAMPLE 13.3 (cont.)

Ref.	Step	Calculations	Output
Cl. 38.4.1 Text Eq. (13.5)		(a) $s = \dfrac{11.1aDf_y}{(D^2 - D_c^2)f_{ck}} = \dfrac{11.1 \times 28 \times 400 \times 415}{57600 \times 30}$ $= 30$ mm	
Cl. 25.5.3.2(d)		(b) Spacing not more than 75 mm (c) Spacing not more than $\frac{320}{6} = 53.3$ mm (d) Spacing not less than 25 (e) Spacing not less than $6 \times 3 = 18$ mm Choose 30 mm spacing.	(R6 at 30 mm)

REVIEW QUESTIONS

13.1 What are braced and unbraced columns? What parameter in column design is affected by bracing?

13.2 What is meant by equivalent length of a column? Explain how column behaviour is affected by the effective length.

13.3 Write down the formula for the strength of a centrally loaded R.C. column in limit state design. How much accidental eccentricity is assumed in this formula?

13.4 What is the minimum percentage of steel allowed in an R.C. column? Explain why it is necessary to specify the minimum percentage.

13.5 What is the maximum percentage of steel allowed in R.C. column? Explain why it is necessary to specify this maximum percentage.

13.6 Why is it necessary to have lateral ties in a column? What are the specifications for these laterals?

13.7 What is the difference between 'lap length' and 'anchorage length' in columns? How are the column bars of a multistoreyed building continued from column to column? Can one lap all the reinforcements at the same level? How is lapping in columns different from that in beams?

13.8 How does helically reinforced columns differ from tied columns in their behaviour when tested to destruction?

13.9 In what situations would one recommend the use of helically reinforced column?

13.10 In construction practice a much stronger mix than used in beams is recommended for columns. Explain how this practice can lead to considerable economy in construction.

PROBLEMS

13.1 A circular column is of unsupported height 4 m and is effectively held in position at both ends and restrained against rotation at one end. If the diameter of the column is restricted to 400 mm, determine the effective length of the column as well as the main steel and laterals required for it to carry a factored axial load of 1500 kN: f_{ck} = 20 and f_y = 415 N/mm^2.

13.2 A short circular column 6 m long is to carry a characteristic load of 250 kN. Assuming both the ends of the columns are fully restrained, design the column (column size, main steel and laterals) if the column is to be made as a spirally reinforced column: f_{ck} = 15 and f_y = 415 N/mm^2.

13.3 A column of unsupported height 5 m is subjected to an axial load of 800 kN. If it is assumed as hinged at both ends, design the section with minimum possible size using M20 concrete and grade 415 steel. Detail the lateral ties for this column.

13.4 An I-shaped section as shown in Fig. P.13.4 is to be used as a column for architectural purposes. If the column is hinged in position at both ends, determine the maximum height of the column so that it can be considered as a short column. Sketch the type of main reinforcements and laterals you will provide for this column.

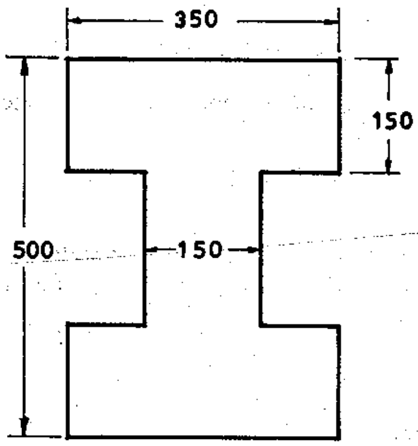


Fig. P.13.4.

13.5 Determine the carrying capacity of the column in Problem 13.4 assuming that the column is short and f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2, and minimum area of steel is provided in the column.

13.6 A tied column is 500 mm diameter with a central hole of 200 mm. It is reinforced with 8 nos. of 25 mm diameter and laterals of 10 mm diameter bars. Assuming f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2 for the main steel and f_y = 250 N/mm^2 for the laterals, determine the maximum axial load it can take as a short column. Detail the laterals.

Calculate the maximum height up to which it can act as a short column, if both ends are fully restrained in position and rotation.

13.7 (a) A reinforced T section column is as shown in Fig. P.13.7. It is to support a load of 600 kN as a short column at the centre of gravity of the section. Determine the reinforcements required and indicate how it will be arranged, to meet the requirements for a centrally loaded column: f_{ck} = 20 N/mm^2, f_y = 415 N/mm^2.

(b) Ho
the section

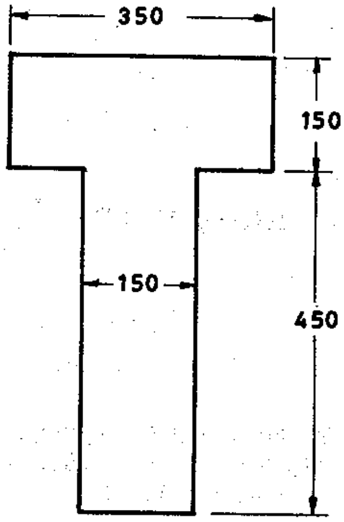


Fig. P.13.7.

(b) How will the problem be solved if the point of application of the load and the C.G. of the section do not coincide?

14

Design of Short Columns with Moments

14.1 INTRODUCTION

Columns, such as the external columns of framed buildings, or columns carrying crane loads through corbels of a column, are subjected not only to direct loads (P), but also to moments (M) due to the eccentricity in application of the load (Fig. 14.1). In the above columns, the eccentricity is with respect to one axis only and these columns are said to be under uniaxial bending. On the other hand, a corner column of a building is subjected to eccentric load along both the X and Y axes. Such columns are said to be under biaxial bending.

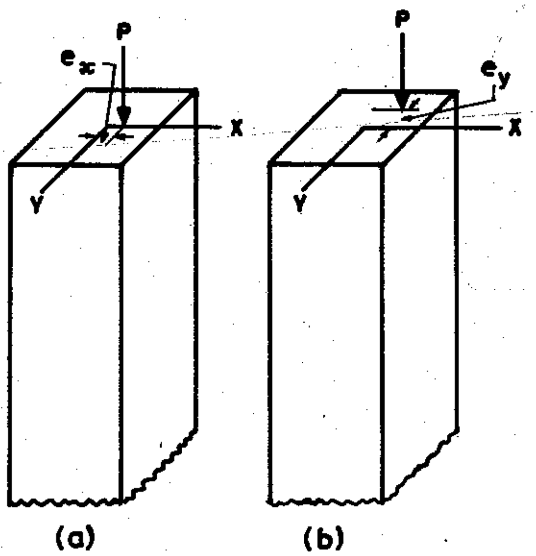


Fig. 14.1 Columns under direct load and moments: (a) Uniaxial bending, (b) Biaxial bending.

In Chapter 13 it was pointed out that the short column formula takes into account accidental eccentricity to a certain extent. However, if the accidental eccentricity given by the expression

$$\frac{L}{500} + \frac{D}{30} \text{ or } 20 \text{ mm}$$

(whichever is more) is greater than $0.05D$ already provided in the formula for short column, then the short column formula cannot be used for design of such columns. The theory of short columns subjected to axial load and moments should be used for their strength calculations.

Sagar Saman

It will be seen in Chapter 16 that in slender columns, even when they are subjected to central load, bending moments are produced as they undergo deflection along its length as a beam column. Such columns also have to be designed as eccentrically loaded columns.

The theory of design of these columns subjected to central load P and bending moment M is discussed in this chapter.

14.2 METHODS OF DESIGN

The three methods that are commonly used to design these columns are:

1. Use of design charts (interaction diagrams)
2. Use of equilibrium equation to determine the minimum steel required
3. A simplified approximate method considering the section as a doubly reinforced beam.

Of these, the interaction diagram is extensively used for design of rectangular or circular columns with symmetric arrangement of steel. The equilibrium method is based on fundamental concepts and is applicable to any cross-section and any arrangement of steel.

The simplified method is found useful for columns with large eccentricities where the column acts more like a beam.

14.3 UNIAXIAL BENDING (DESIGN ASSUMPTIONS)

In eccentrically loaded columns the strain distribution across the section will not be uniform as in the case of centrally loaded columns. As there is bending in addition to direct load, the strain distribution will vary linearly across the section as in the case of beams. The following assumptions as given in IS 456: clause 38.1(b) are used to calculate the value of P_u and M_u of a given section:

1. Plane sections remain plane even after bending.

2. The strain at different points in the section will be different. The maximum compressive strain in concrete at failure is the governing criterion for ultimate failure. The magnitude of this failure strain is given by the expression (see also Fig. 14.2)

$$\varepsilon_c = 0.0035 - 0.75\,\varepsilon_c' \tag{14.1}$$

where ε_c is the 'maximum' strain in compression in the section at failure and ε_c' is the 'minimum' strain in compression in the section at failure. The minimum compression strain in a beam ε_c' is equal to zero so that $\varepsilon_c = 0.0035$. In an axially loaded column, $\varepsilon_c = \varepsilon_c'$ so that ε_c at failure = 0.002. It is interesting to note that the above expression is also equivalent to assuming that the strain distribution diagram rotates around a fulcrum at a distance $3D/7$ from the highly compressed edge. This fulcrum point is the same point where the concrete strain will be 0.002 at failure with the neutral axis at the far edge of the section, as shown in Fig. 14.2.

3. The design compression stress block under the varying strain is the same as assumed for beams. It is rectangular parabolic with the maximum stress value $0.446f_{ck}$ (approximately equal to $0.45f_{ck}$) at failure.

4. The design stress-strain curve for steel in compression is the same as in tension. The same design curve as for beams is assumed for columns also.

5. The tensile strength of concrete is ignored.

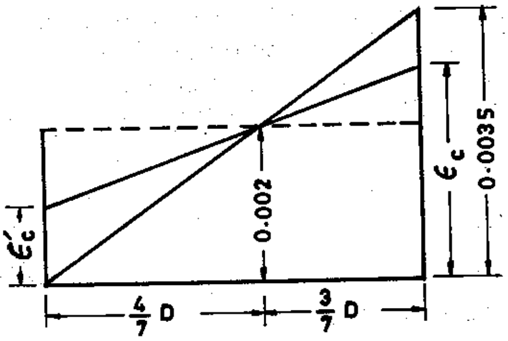


Fig. 14.2 Failure strain in concrete under direct load and moments.

14.4 STRESS-STRAIN CURVE FOR STEEL

One should have a good idea of the stress-strain curves recommended by IS for the different types of steels to correctly assess the stresses corresponding to the strain in the steel. According to IS 456, mild steel (Fe 250) has a bilinear stress-strain curve, and failure strain on the design stress-strain curve is given by the expression

$$\varepsilon_s = \frac{0.87 \times 250}{200 \times 10^3} = 0.00109$$

However, the design stress-strain curves of cold drawn bars, like Fe 415 steel, is not bilinear. There is a linear and a non-linear strain, for the stress levels beyond $0.80 f_y$ so that the strains on the design stress-strain curve corresponding to the various design stress levels will be as in Table 14.1, see Fig. 14.3 (see also Chapter 4).

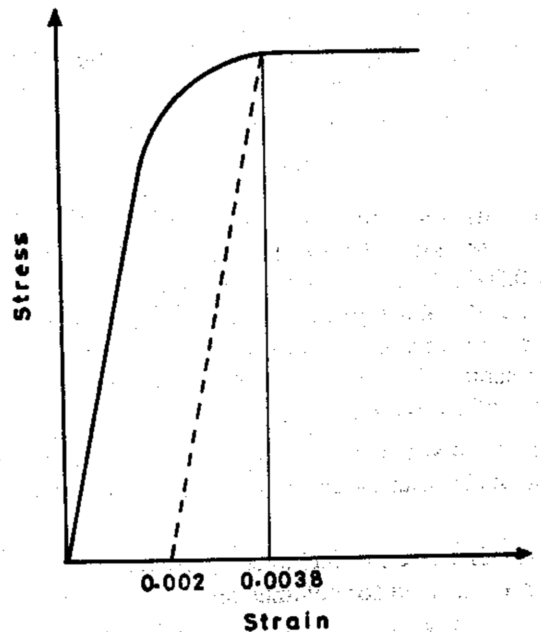


Fig. 14.3 Stress-strain curve for Fe 415 steel.

TABLE 14.1 VALUES OF DESIGN STRESS-STRAIN CURVE FOR Fe 415 STEEL
(Ref: SP 16, Table A)

Stress level	Strain		
	Elastic	Inelastic	Total
$0.8 \times (0.87f_y) = 0.7f_y$	0.00144	0	0.00144
$0.85 \times (0.87f_y) = 0.74f_y$	0.00153	0.0001	0.00163
$0.90 \times (0.87f_y) = 0.78f_y$	0.00162	0.0003	0.00192
$0.95 \times (0.87f_y) = 0.83f_y$	0.00171	0.0007	0.00241
$0.975 \times (0.87f_y) = 0.85f_y$	0.00176	0.0010	0.00276
$1.00 \times (0.87f_y) = 0.87f_y$	0.00180	0.0020	0.00380

14.5 COLUMN INTERACTION DIAGRAM

A column subjected to varying magnitudes of P and M will act with its neutral axis at varying points as described now.

Case 1: When $P = P_u$ and $M = 0$ (Axial load only)

In this case it acts as a column. The strain distribution across the section is uniform (Fig. 14.4b). Its ultimate load P_u is reached when the compression strain reaches the failure strain of $\varepsilon_c = 0.002$. The corresponding stresses in concrete and steel can be calculated and, by using the equilibrium equation, P_u can be calculated as

$$P_u = (A_c) \times (\text{Stress in concrete}) + (A_s) \times (\text{Stress in steel})$$

See Chapter 13.

Case 2: When $P = 0$, $M = M_u$ (Moment only)

In this case the member acts as a pure beam. The failure moment is the ultimate moment in pure bending (Fig. 14.4). The ultimate moment M_u was calculated on the basis of the assumption that ultimate failure is reached when $\varepsilon_c = 0.0035$.

In this case the section is subjected to a pure couple and the neutral axis is determined from the two equilibrium equations

$$C = T$$

$$M_u = \text{moment of the couple}$$

Case 3: When both P and M act on the member

When an eccentrically loaded column is subjected to P and M, for every load P there is a particular value of M which will cause failure. Thus there will be infinite combinations of P_u and M_u, which can safely act together for a given R.C. section. The particular value of M_u for a given value of P_u can be found only by trial and error and the work is quite tedious. It will be more convenient especially for routine design to construct a curve showing the P_u, M_u combination and the read off value of M_u for a given value of P_u or vice versa. Such a curve showing the limiting values of P_u

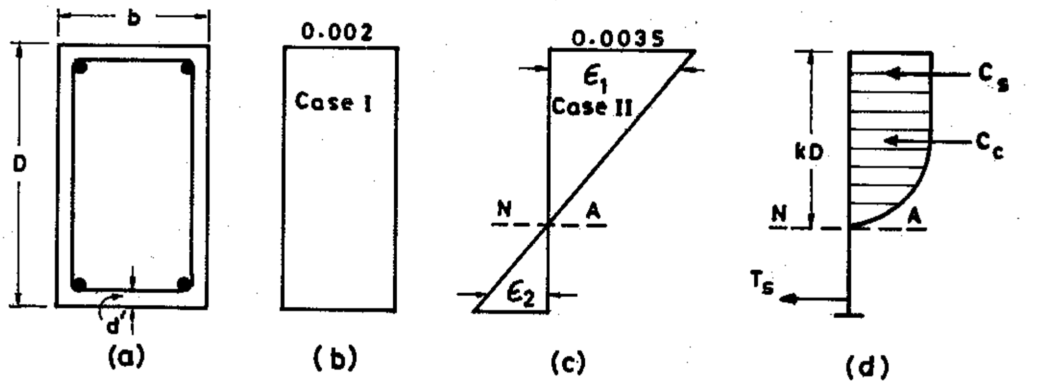


Fig. 14.4 **Strain distribution in column section with P and M: (a) Section, (b) Failure with axial load only (N.A. at infinity), (c) Failure with N.A. inside section, (d) Forces under ultimate load with N.A. inside the section.**

and M_u is called a P-M interaction curve. This can be made non-dimensional by using a diagram such as Fig. 14.5.

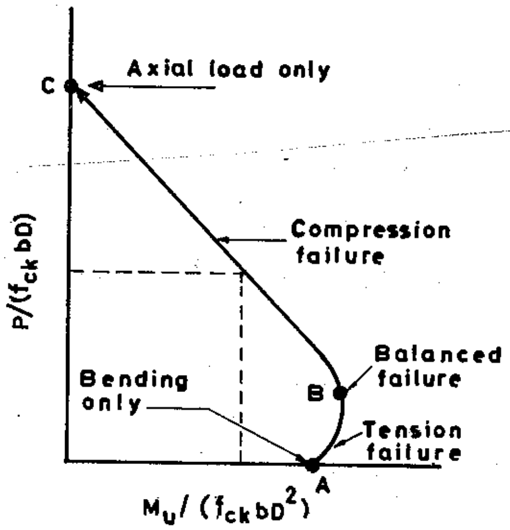


Fig. 14.5 **Column interaction diagram.**

$\dfrac{M}{f_{ck}bD^2}$, (non-dimensional quantity along x-axis)

$\dfrac{P}{f_{ck}bD}$, (non-dimensional quantity along y-axis)

Thus, the interaction curves give the strength envelopes for the reinforced concrete section subjected to combinations of direct load and bending moment (P and M). Points outside these diagrams represent failure of the column. Combinations on or inside the diagram are safe.

14.6 USE OF EQUILIBRIUM EQUATIONS TO CONSTRUCT THE INTERACTION DIAGRAM FOR RECTANGULAR SECTIONS

14.6.1 INTERCEPT ON y-AXIS (AXIAL LOAD ONLY)

The intercept of the interaction diagram on the y-axis represents the strength of the section as a short column without corrections made for accidental eccentricity. Considering a rectangular section (Fig. 14.4), with symmetrical arrangement of steel (with one-half the percentage of steel, on either side) the equation for the ultimate load can be written as

$$P_u = 0.446 f_{ck} bD + \frac{pbD}{100}(f_{sc} - 0.446 f_{ck})$$

Rearranging the terms, we get

$$\frac{P_u}{f_{ck} bD} = 0.446 + \frac{p}{100 f_{ck}}(f_{sc} - 0.446 f_{ck}) \tag{14.2}$$

where p represents the total steel in the section.

The second term within the brackets can be either neglected or in all cases an average value of $f_{ck} = 20$ (corresponding to M 20 concrete) can be used for calculations. The above value gives the y-intercept for the interaction diagram. It may be noted that as the strain distribution at failure is uniform, equal to 0.002, one may assume that the neutral axis for this case lies at infinity. It can also be seen from the above equation that it is more convenient to represent the non-dimensional curves for various p/f_{ck} values rather than for the percentage of steel.

14.6.2 INTERCEPT ON x-AXIS (MOMENT ONLY)

The intercept on the x-axis represents the strength of the section as a beam in pure bending. Considering the same R.C. section as in the previous case, but with symmetric steel, the value of M_u can be calculated from the equilibrium equation as obtained for theory of beams in bending.

First, the position of the neutral ~~axis can be determined~~ from the condition that tension will be equal to the compression. The equilibrium equation for this condition is

$$\text{Compression in concrete} + \text{Compression in steel} = \text{Tension in steel}$$

$$C_c + C_s = T_s$$

Assuming that the neutral axis in pure bending is at $k_1 D$, we get

$$C_c = 0.36 f_{ck} b k_1 D$$

$$C_s = f_1\left(\frac{p_1}{100}\right)bD \quad \text{(where } f_1 \text{ is the stress in compression steel)}$$

$$T_s = f_2\left(\frac{p_2}{100}\right)bD \quad \text{(where } f_2 \text{ is the stress in tension steel)}$$

The values of f_1 and f_2 will correspond to the strain ε_1 and ε_2 of these steels assuming that the compressive strain in concrete at failure $\varepsilon_c = 0.0035$ and that steel is placed with cover equal to

d' so that $d'/D = m$. Assuming that steel is placed symmetrically and p represents the percentage of total steel in the section, we have

$$0.36 f_{ck} b k_1 D + f_1 \frac{(0.5p)}{100 f_{ck}} f_{ck} bD = f_2 \frac{(0.5p)}{100 f_{ck}} f_{ck} bD$$

$$\varepsilon_1 = \frac{\varepsilon_c}{k_1 D}(k_1 - m)D$$

$$\varepsilon_2 = \frac{\varepsilon_c}{k_1 D}(D - mD - k_1 D) = \frac{\varepsilon_c (1 - m - k_1) D}{k_1 D}$$

From the above equations, $k_1 D$ can be obtained. The ultimate moment capacity of the section is the moment of the couple which can be easily calculated by taking the moment of C_c and C_s about the tension steel. This will give the moment M_u for $P = 0$.

It may be noted that the neutral axis for a section subjected to pure bending will be inside the section as the tension and compression should balance.

14.6.3 INTERMEDIATE POINTS (COMPRESSION, BALANCED AND TENSION FAILURES)

For all intermediate points the neutral axis can be between the above two extreme cases, as represented by the axially loaded column (with the neutral axis at infinity) and the section in pure bending (with the neutral axis at $k_1 D$).

The values of P_u and M_u corresponding to any given position of neutral axis can be calculated from the equilibrium equations

$$P_u = C_c + C_{s1} \pm C_{s2}$$

$$M_u = \text{moment of the forces at the centre of the column about which eccentricities } (Pe = M) \text{ are calculated} \qquad (14.3)$$

14.6.4 BALANCED FAILURE CONDITION

One of the special cases that can be investigated is the one where concrete reaches failure strain and the steel the yield strain in tension simultaneously. This is called the balanced figure condition (point B in Fig. 14.5). In this case the neutral axis will be within the section and its position can be determined by the condition $\varepsilon_c = 0.0035$ and ε_s = yield strain. The value of C_c and its position of application can be easily determined from standard formulae. The corresponding axial load and moments can also be calculated. The procedure is shown in Example 14.3.

It can be noted that points from A to B on the interaction diagram Fig. 14.5 represent tension failure condition, where the steel on the tension face reaches yield point before failure. Points B to C represent compression failure condition where the failure is initiated by concrete reaching its ultimate strain first. In the first case, the neutral axis will lie between pure bending and balanced failure conditions. In the second case it will lie between the balanced and axial load conditions.

When the neutral axis is outside the section (Fig. 14.6), part of the parabola will be outside the concrete area. Then the values of kD will be greater than D (i.e. $k >$ unity). Table 14.2 (as given in SP 16) provides the area of the stress block and the position of centroid for various values of k. These values can be used to simplify the calculations for case c.

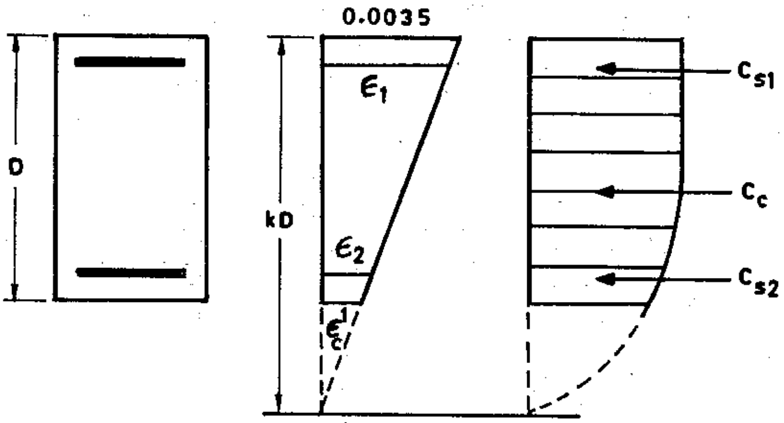


Fig. 14.6 Columns with neutral axis outside the section.

TABLE 14.2 STRESS BLOCK PARAMETERS WHEN THE NEUTRAL AXIS LIES OUTSIDE THE SECTION

(Refer Table H, SP 16)

k	Area of stress block	Distance of centroid from highly compressed edge
1.00	$0.361 f_{ck} D$	$0.416D$
1.05	$0.374 f_{ck} D$	$0.432D$
1.10	$0.384 f_{ck} D$	$0.443D$
1.20	$0.399 f_{ck} D$	$0.458D$
1.30	$0.409 f_{ck} D$	$0.468D$
1.40	$0.417 f_{ck} D$	$0.475D$
1.50	$0.422 f_{ck} D$	$0.480D$
2.00	$0.435 f_{ck} D$	$0.491D$
2.50	$0.440 f_{ck} D$	$0.495D$
3.00	$0.442 f_{ck} D$	$0.497D$
4.00	$0.444 f_{ck} D$	$0.499D$

A slightly different value of P_b obtained on the assumptions that $\varepsilon_{cu} = 0.0035$ and $\varepsilon_{su} = 0.002$ as described in Section 16.5.2 is used for calculating the reduction factor that corrects the deflection in long columns.

14.7 APPLICATION TO CIRCULAR SECTIONS

The stress block parameters for rectangular sections are not applicable to circular sections. Hence circular sections are usually divided into strips and the forces on these strips are summed up for determining the total forces and moments due to stresses in concrete. This procedure has been followed in SP 16 to obtain the design curves.

The extreme fibre strain for circular section is also taken as 0.0035, even though some authorities feel that the failure strains in compression of circular sections will be less than that for rectangular sections. Alternatively, a simple rectangular stress distribution of $0.4f_{ck}$ may be assumed for the compression block for simplified calculation (Fig. 14.7).

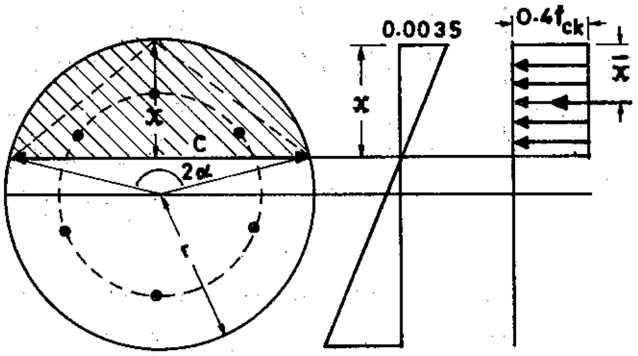


Fig. 14.7 Circular column under direct load and moments.

The following relationships for circular sections are useful in computations of such sections. Let c be the length of the chord at a distance x from the extreme fibre (depth of neutral axis), see Fig. 14.7, given by

$$\left(\frac{c}{2}\right)^2 = x(2r - x)$$

$$\cot\left(90 - \frac{\alpha}{2}\right) = \tan\frac{\alpha}{2} = \frac{2x}{c}$$

The area of the circular segment (shaded in Fig. 14.7) is

$$A = r^2\left(\frac{2\alpha - \sin 2\alpha}{2}\right)$$

where α is in radians. The depth of the centre of gravity is given by

$$\bar{x} = \left(r - \frac{c^3}{12A}\right)$$

14.8 INTERACTION CURVES IN SP 16

Interaction curves for rectangular and circular columns with symmetrical arrangement of steel for Fe 250, Fe 415 and Fe 500, and for various cover ratio d'/D values are given in Charts 27 to 62 in SP 16. Typical curves are shown in Charts 14.1 and 14.2. The steel ratios are represented by p/f_{ck} in the diagram and it varies from 0 to 0.26. It should be remembered that p is the percentage of the 'total steel' with respect of bD and it is distributed symmetrically on the two faces or on all the four faces as shown in the diagrams given for the corresponding set of interaction diagrams in SP 16. (Charts 14.1 and 14.2 presented here are from SP 16.) The values of the stress level in the steel on the 'tension side' are represented in the interaction diagrams by the dashed line so that the type of failure can be easily identified.

ough some
han that for
be assumed

ch sections.
l axis), see

of steel for
ts 27 to 62
esented by
percentage
es or on all
iagrams in
evel in the
so that the

Sagar Samaria. Sagar

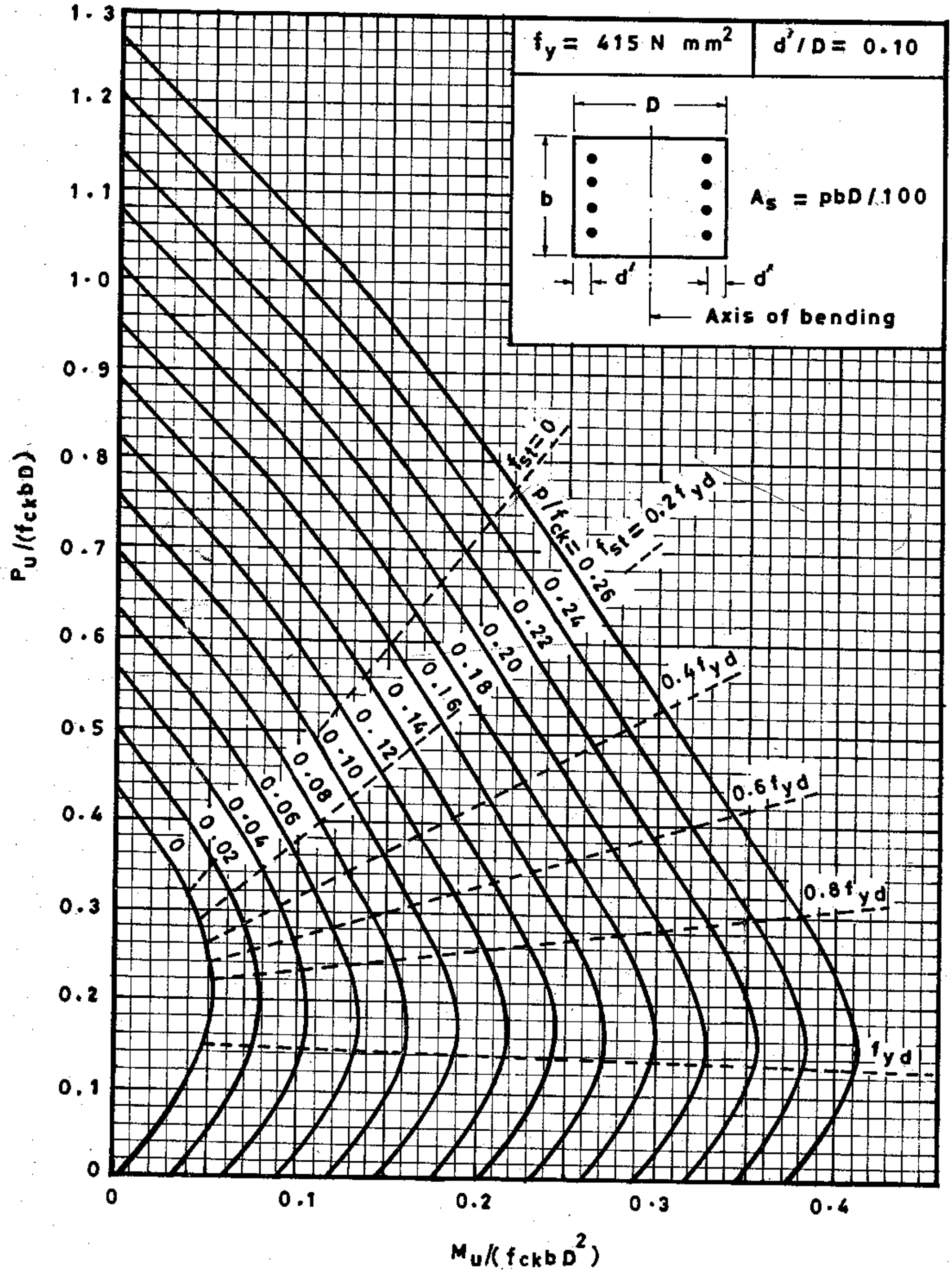


Chart 14.1 Column Interaction Diagram (Chart 32 of SP 16).

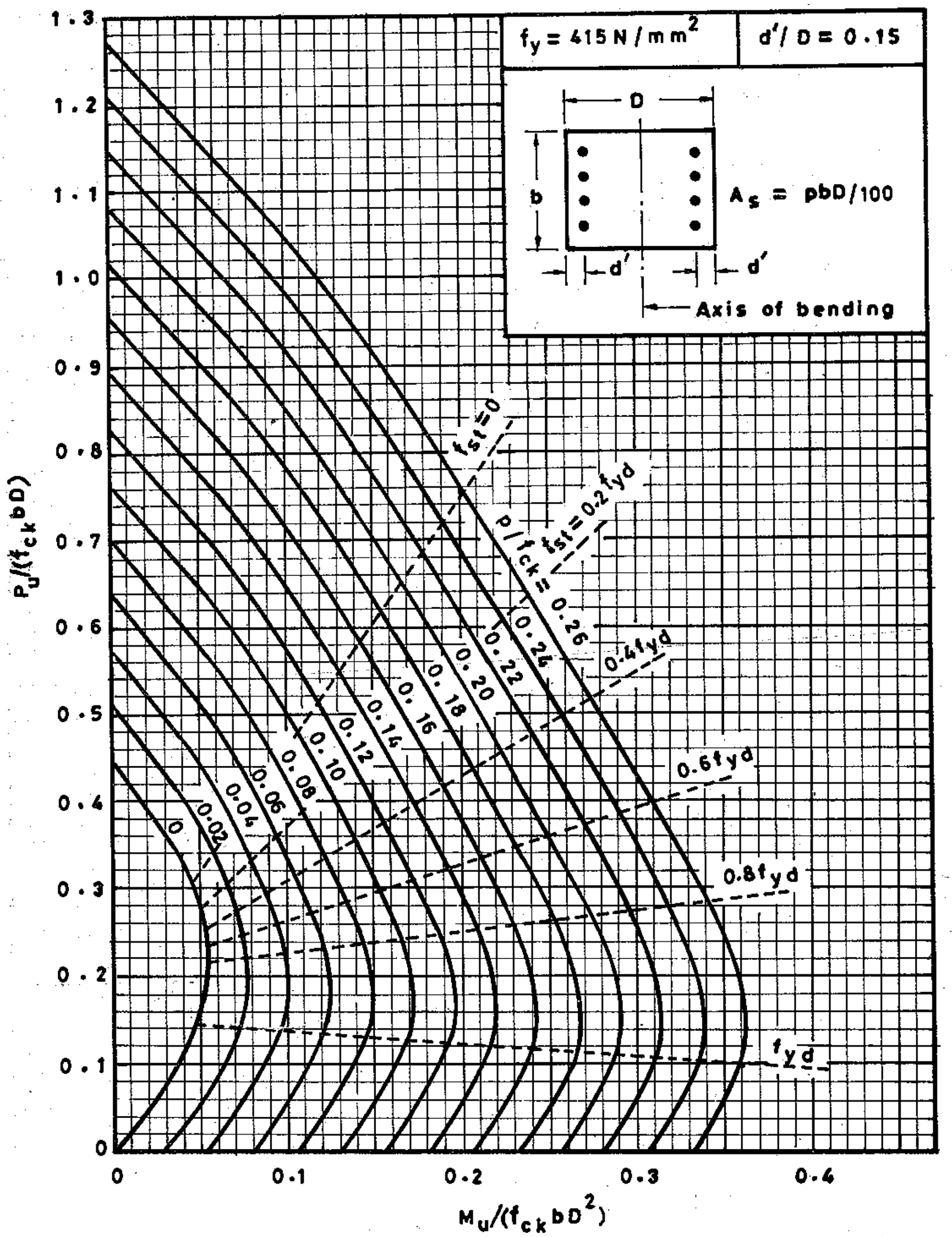


Chart 14.2 Column Interaction Diagram (Chart 33 of SP 16).

14.9 INTERACTION DIAGRAM FOR $P = 0$

The interaction curve for $p/f_{ck} = 0$ (for a rectangular section with no steel as shown in Fig. 14.8)

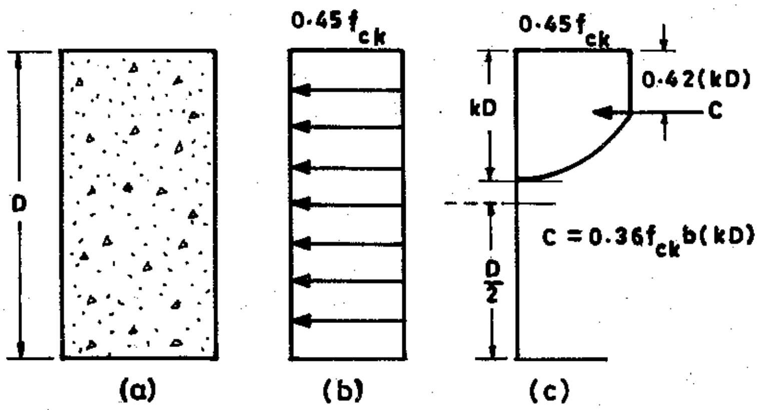


Fig. 14.8 Basis of drawing of interaction diagram for columns with no steel: (a) Section, (b) Forces with central load, (c) Forces with eccentric load.

can be derived as follows:

Case 1: Neutral axis at infinity: Column centrally loaded

$$P = P_u, \qquad M = 0$$

The concrete strain at failure is 0.002 and stress is uniform equal to $0.446 f_{ck}$. Therefore,

$$P = 0.446 f_{ck} bD$$

Rearranging the terms, we get

$$\frac{P}{f_{ck} bD} = 0.446$$

Case 2: Let the neutral axis be inside the section at kD from the compression face. Using the equilibrium equation for bending, we obtain

$$P_u = 0.36 f_{ck} b(kD)$$

Points on the Y-ordinate will be obtained by rearranging the terms as

$$\frac{P_u}{f_{ck} bD} = 0.36k \tag{14.4a}$$

Taking moments about the centre of the section about which eccentricities are measured as shown in Fig. 14.8(c), we get

$$M_u = 0.36 f_{ck} bkD(D/2 - 0.42kD)$$

Points on the X-ordinate will be given by rearranging the terms as

$$\frac{M_u}{f_{ck} bD^2} = 0.36k(0.5 - 0.42k) \tag{14.4b}$$

Putting different values of k in Eqs. (14.4a) and (14.4b), the Y and X ordinates are obtained as

1. For $k = 1$, $Y = 0.36$ and $X = 0.029$
2. For $k = 0.5$, $Y = 0.18$ and $X = 0.12$
3. For $k = 0$, $Y = 0$ and $X = 0$

These are the values plotted in the interaction diagrams given in SP 16.

14.10 SHAPE OF INTERACTION CURVES

The shape of the interaction diagram will vary with the shape of the stress-strain diagram of steel, with kinks corresponding to the kinks in the stress-strain for steel. This can be noted from the differences in the shapes of the curves given in SP 16 for Fe 250, 415 and 500 steels.

14.11 ACCIDENTAL ECCENTRICITY IN COLUMNS

It was shown in Chapter 13 that the accidental eccentricity that should be assumed in a R.C. column according to IS 456 (1978) is

$$e_{\min} = \frac{L}{500} + \frac{D}{30}, \text{ but not less than 20 mm (see note on page 271)}$$

With small values of M and P, the consequent eccentricity $e = M/P$ may be small. In such cases one should note that the design moment should be always larger of the following:

$$M = M \text{ (given)}$$

$$M = Pe_{\min} \text{ (due to accidental eccentricity)}$$

14.12 USE OF INTERACTION DIAGRAMS FOR DESIGN AND ANALYSIS (METHOD 1)

Case 1: Use of interaction curves to determine the area of steel required for a given column size for specified P and M

Step 1: Check whether the column is short or long. If it is long, proceed as in Chapter 15. If short, proceed as follows:

Step 2: Find the following design parameters:

$$e = \frac{M}{P} > e_{\min} = \frac{L}{500} + \frac{D}{30} \text{ or 20 mm}$$

Find $\dfrac{d'}{D}, \dfrac{P_u}{f_{ck}bD}$ and $\dfrac{M_u}{f_{ck}bD^2}$ for column design.

Step 3: Determination of areas of steel from interaction curve is as follows: Choosing proper curve for grades of steel and d'/D, find p/f_{ck}. Calculate $A_s = p/100(bD)$. Distribute this total area A_s as distributed in the sketch given in SP 16 for the interaction diagram.

Case 2: Use of interaction curve to analyse the safety of the given column for given P and M, with symmetrical distribution of steel.

Step 1: Find the following parameters:

$$\frac{d'}{D}, \quad \frac{p}{f_{ck}}, \quad \frac{P_u}{f_{ck}bD}, \quad \frac{M_u}{f_{ck}bD^2}$$

Step 2: Determine safety of column as follows: Determine the point corresponding to the given

$$\frac{P_u}{f_{ck}bD}, \quad \frac{M_u}{f_{ck}bD^2}$$

on the corresponding interaction curve for p/f_{ck}. If the point is on or inside the interaction curve, the column is safe. If the point is outside the curve, it is unsafe.

14.13 DESIGN OF ECCENTRIC COLUMNS BY EQUILIBRIUM EQUATION (METHOD 2)

Another method that can be used for design of eccentrically loaded column is to work from fundamentals by using equilibrium equation and to arrive at the necessary steel for a given section of breadth b, depth D, with given P and M, as follows:

Step 1: Assume the arbitrary depth of neutral axis. Let the extreme fibre in concrete reach failure strain as explained in the assumptions in Section 14.3 above.

Step 2: Determine the strains in the steels.

Step 3: Determine the compression force in concrete by using Table 14.1 or by other means. Find also the stress f_s in compression steel that will be provided near the compression face from the strain at the level of steel.

Step 4: Determine the area of steel to be provided at the compression face (A_{s1}) by taking moments of all the forces about the position of the steel at the tension face. The moment equilibrium equation will be

$$P[e + D/2 - d'] = \begin{pmatrix} \text{Moment of compression} \\ \text{in concrete about tension} \\ \text{steel} + f_{sc}A_{s1}(d - d') \end{pmatrix}$$

Step 5: Determine A_{s2}, the steel required on the tension face, from the second equation of equilibrium of forces

$$P = C_c + f_{s1}A_{s1} \pm f_{s2}A_{s2}$$

Step 6: From these, determine the total area of steel

$$A_s = A_{s1} + A_{s2}$$

Plot the value of A_s against the value of the depth of the neutral axis assumed (Fig. 14.9).

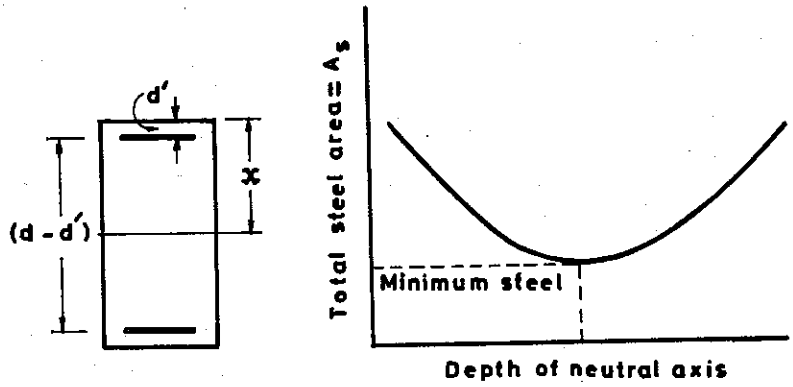


Fig. 14.9 Plot to determine minimum steel in section under direct load and moment.

Step 7: Assume other suitable values of the depth of neutral axis and plot the values of A_s (total steel) needed, in the above diagram. The values of the neutral axis corresponding to minimum value of A_s can be taken as the optimum solution of the problem.

Step 8: Provide the steel A_{s1} and A_{s2} as obtained in step 7 for optimum value of x. The procedure is similar to that given in Examples 14.1 and 14.2.

14.14 SIMPLIFIED METHOD (METHOD 3)

Columns under large eccentricity of load may be solved by considering them as equivalent to a doubly reinforced beam with a concentrated load acting on it (for equilibrium of forces) as shown in Fig. 14.10.

The effect of P and M may be regarded as equivalent to a modified moment ($M + M_a$) and a force P applied along the steel on the tension side, where

$$M + M_a = M + P\left(\frac{D}{2} - d_2\right) \tag{14.5}$$

In addition, P acts through the tension steel.

As tables for doubly reinforced beams for these large eccentricities will not be available, calculation has to be made from the basic equations. The simplified stress block with $0.4f_{ck}$ as uniform compression can be used for satisfying the equilibrium of forces. Taking moments about the tension steel, we get

$$\begin{aligned} M_a &= (0.4f_{ck}b)\,(0.5d)(0.75d) + 0.72f_yA_{sc}(d - d') \\ &= 0.15f_{ck}bd^2 + 0.72f_yA_{sc}(d - d') \end{aligned} \tag{14.6}$$

from which the value of A_{sc}, the area of compression steel, can be calculated.

For equilibrium of forces, the area of tension steel A_{st} is given by the equation

$$\begin{aligned} 0.87f_yA_{st} &= (0.4f_{ck}b)(0.5\,d) + 0.72f_yA_{sc} \\ &= 0.2f_{ck}bd + 0.72f_yA_{sc} \end{aligned} \tag{14.7}$$

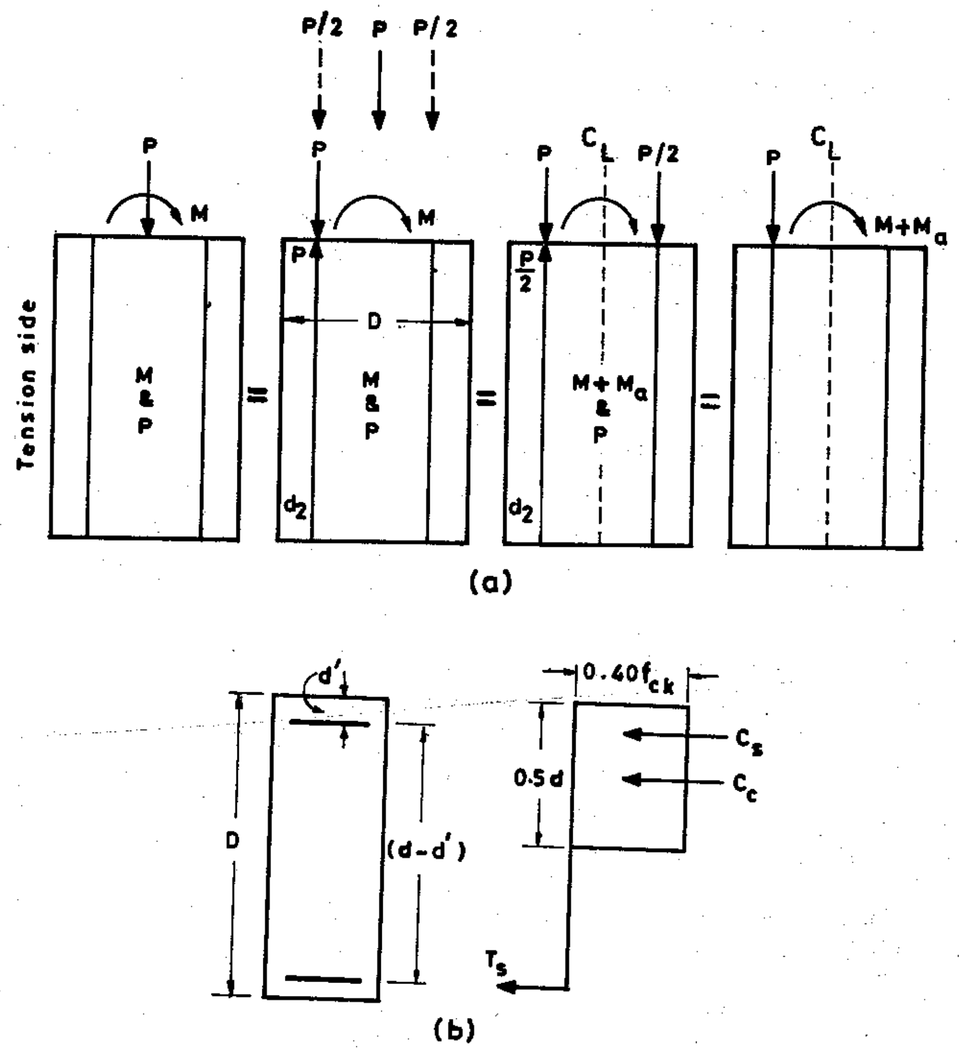


Fig. 14.10 **Approximate method for design of eccentrically loaded columns with large eccentricity: (a) Equivalent moment and force, (b) Simplified stress block.**

However, a correction has to be made for the value of P acting as compression P on tension steel. This reduces the tension steel required and the area of the reduced tension steel is given by

$$A_{st1} = A_{st} - \left(\frac{P}{0.87 f_y}\right) \tag{14.8}$$

It is evident from Eq. (14.3) that the equations are valid only when the beam theory as explained above is valid for the situation, i.e. the eccentricity is larger than $(D/2 - d_2)$. The above procedure is illustrated by Example 14.5.

14.15 MEMBER SUBJECTED TO BIAXIAL BENDING

IS 456: clause 38.6 deals with biaxial bending. The analysis may be used by one of the following two methods:

1. By choosing the neutral axis which is in the XY-plane. Calculations are made from fundamentals to satisfy the equilibrium of load and moments about both the axes. (This method is quite tedious and is not generally recommended for routine design.)

2. By the use of the formula recommended for use of IS 456. The above code recommends the use of the following relation:

$$\left(\frac{M_x}{M_{x1}}\right)^{\alpha_n} + \left(\frac{M_y}{M_{y1}}\right)^{\alpha_n} \leq 1.0 \tag{14.9}$$

where M_x and M_y are the applied moments about the X and Y axes and M_{x1} and M_{y1} are the maximum uniaxial moments the column can take under the actual load P by bending along the XX- and YY-axis, respectively.

α_n is related to the value of P/P_z

where

P = design load on the column
$P_z = 0.45 f_{ck} + 0.75 f_y A_{sc}$ (i.e. value of P_u when $M = 0$)
α_n = exponent whose value is to be taken as follows:

P/P_z	α_n
0.2	1.0
0.8	2.0

The intermediate values are to be obtained by interpolation, as shown in Fig. 14.11. The value of α_n can be also determined by the equation

$$\alpha_n = \frac{2}{3}\left(1 + \frac{5}{2}\frac{P}{P_z}\right)$$

which should be within 1.0 and 2.0 as above.

The IS code formula follows the "Bresler Load Contour Method". It is based on the concept of a failure surface which is the envelope of a number of interaction curves for different axes of bending of a column, as shown in Fig. 14.12.

Any point of the failure surface corresponds to failure condition in a column about a neutral axis and any point inside the surface can be considered safe. A horizontal section at each level defines a load contour corresponding to a particular P/P_z value. The general form of the contour is defined by relation (14.9).

It may be noted that the limiting value $\alpha_n = 1.0$ for $P/P_z = 0.2$ represents the equation of a straight line and the value of $\alpha_n = 2$ for $P/P_z = 0.8$ represents a circle. These values are meant to represent the shape of the interaction diagrams at these points. Application of this formula is greatly facilitated by Chart 64 given in SP 16 (Chart 14.3 of the text). Its use is illustrated by Example 14.6.

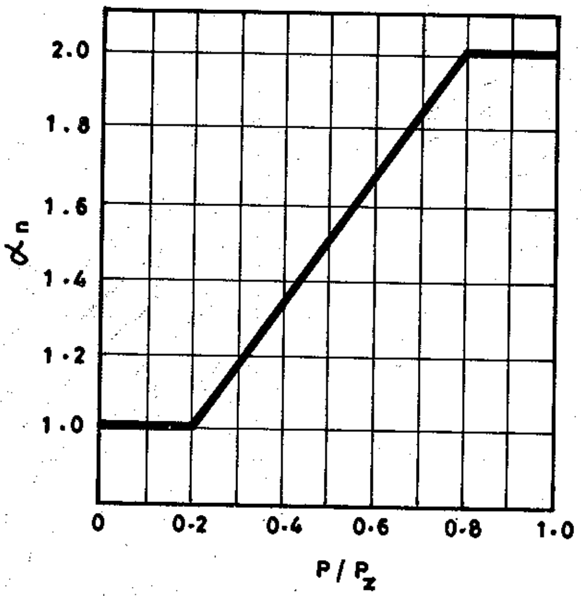


Fig. 14.11 **Illustration of coefficient α_n for biaxial bending of columns.**

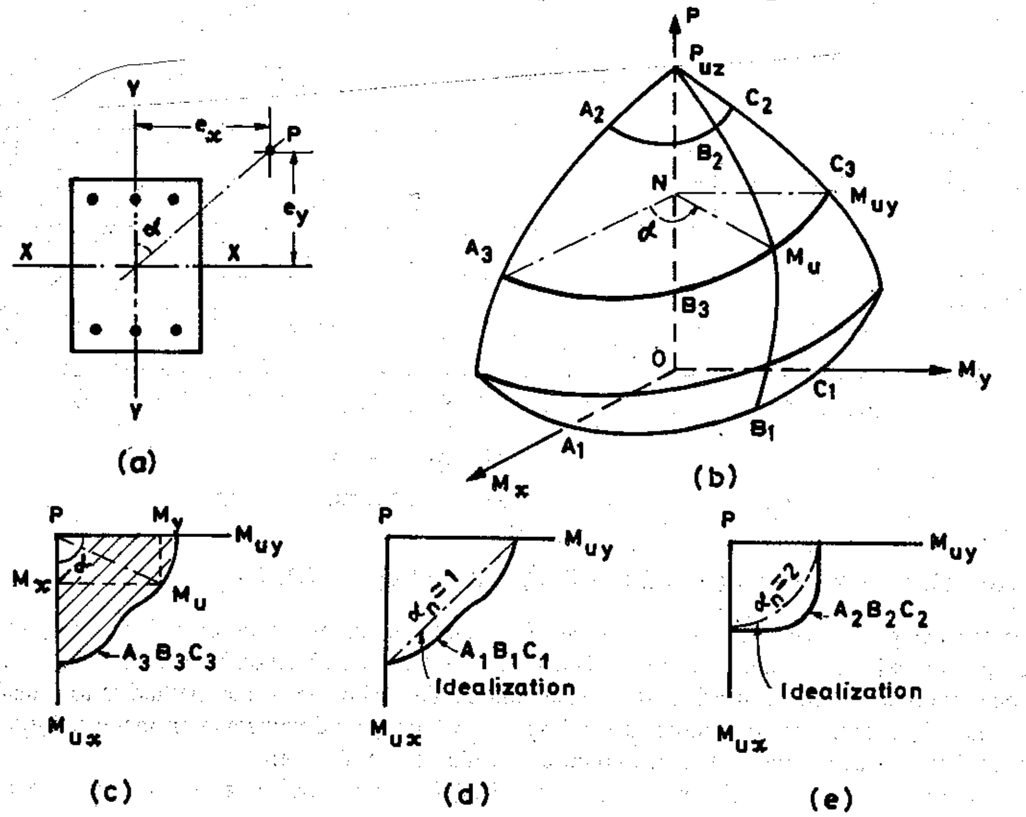


Fig. 14.12 **Interaction curve for biaxial bending of columns: (a) Section, (b) Interaction surface, (c) Section at A_3C_3, (d) Section at A_1C_1, (e) Section at A_2C_2.**

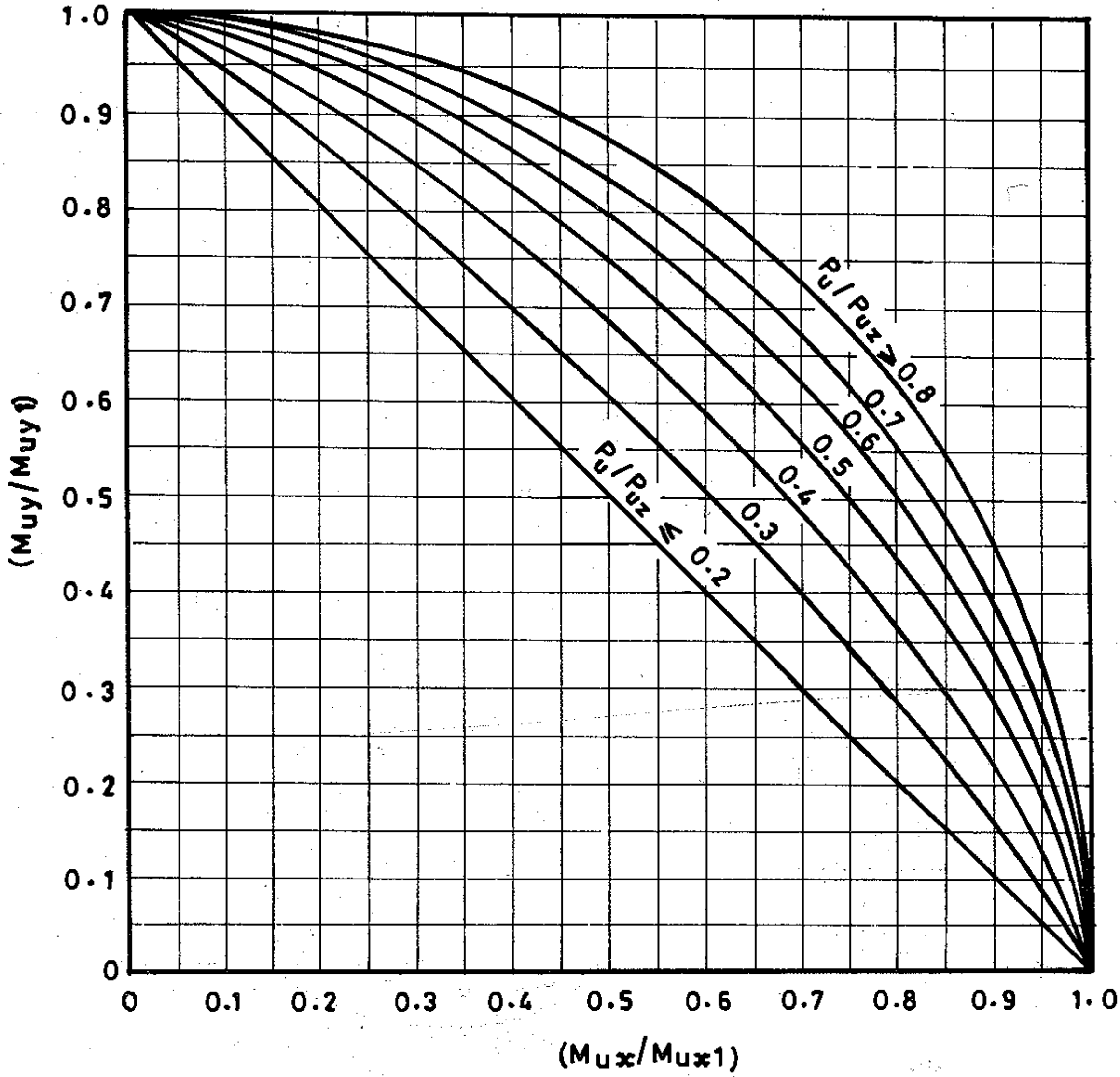


Chart 14.3 Check for Safety in Biaxial Bending.

14.16 SIMPLIFIED BS 8110 METHOD FOR BIAXIAL BENDING

For the design of symmetrically reinforced rectangular columns under biaxial bending, results comparable with those obtained by the Bresler method can be obtained by the simplified design procedure recommended in BS 8110: clause 3.8.4.5. The principle of the method is to transform the biaxial bending case to a uniaxial bending case which should withstand an increased moment about that axis according to the two conditions which are now given.

Let the column be subjected to (P, M_x and M_y). Then it can be designed for uniaxial bending of (P, M_x') or (P, M_y'), depending on the following conditions:

Condition 1: When $M_x/d \geq M_y/b'$, M_x controls the design and the column is to be designed for P and M'_x, where

$$M'_x = M_x + \alpha \frac{d}{b'} M_y \tag{14.10}$$

Condition 2: When $M_x/d < M_y/b'$, M_y controls the design and the column is to be designed for P and M'_y, where

$$M'_y = M_y + \alpha \frac{b'}{d} M_x \tag{14.11}$$

In the above expression,

d = effective depth with respect to major axis and total depth D

b' = effective depth with respect to minor axis and total depth b

$$\alpha = \text{coefficient} = \left(1 - \frac{7}{6}\frac{P}{f_{ck}bD^2}\right)$$

14.17 SHEAR IN COLUMNS SUBJECTED TO MOMENTS

BS 8110 requires that columns which are subjected to axial load and bending are checked for shear also. If there are no imposed load along the height of the column, the shears at top and bottom of the column are equal and given by V so that (with proper signs)

$$V = \frac{M_{\text{top}} + M_{\text{bottom}}}{\text{storey height}}$$

This shear will be acting over the entire height of the column. The shear stress is given by the formula

$$v = V/bd$$

As the concrete is under compression and shear, its shear strength is larger than the value for pure shear. The enhanced shear recommended in BS 8110 is given by the formula

$$\tau'_c = \tau_c + 0.75 \frac{P}{A_c} \frac{Vd}{M} \leq 0.8\sqrt{f_{ck}} \text{ or } 5 \text{ N/mm}^2$$

whichever is less. The critical section for shear on the column is usually taken as $2d$ from the bottom of the beam. Normally columns should be safe without extra shear reinforcement. This is illustrated by Example 14.9.

14.18 REPRESENTATION OF COLUMN DESIGN CHARTS

There are many types of representation of the column interaction diagrams. The commonly used ones are:

1. $\dfrac{M_u}{f_{ck}bD^2}$ on the x-axis and $\dfrac{P_u}{f_{ck}bD}$ on the y-axis for various values of p/f_{ck}. It also gives the value of f_{st}, the stress in steel. Plots are made for different values of f_y and d'/D. This is used in IS publication SP 16.

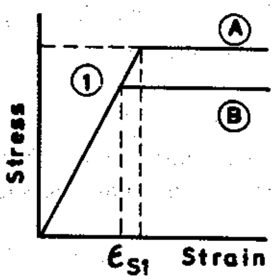
2. M_u/bd^2 on the x-axis and P_u/bD on the y-axis for various values of steel percentages 0 to 8. It also gives the reduction factor k given by the Eq. (16.2). Plots are made for different values of f_{ck}, f_y and d'/D. This representation is used in BS 8110 charts. Some American publications also use this type of plots.

3. $\dfrac{M_u}{f_{ck}bD^2}$ on the x-axis and $\dfrac{P_u}{f_{ck}bD}$ on the y-axis for various values of $\dfrac{pf_y}{f_{ck}} = 0$ to 1.4. Charts are plotted for different values of d'/D. This is used in the Institution of Structural Engineers Manual and is non-dimensional in its representation. This has the advantage that a smaller number of charts will cover all the combinations required for the designer.

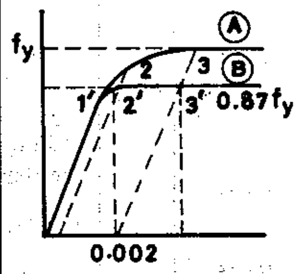
All these representations can be used in design. The first representation as given in SP 16 is commonly used in India.

EXAMPLE 14.1 **(Design stress-strain curve of steel)**

Draw the design stress-strain curves for Fe 250 (mild steel bars) and Fe 415 (cold worked bars) given that the residual strain for Fe 415 steel is nil at $0.8f_y$ and 0.0003 at $0.9f_y$ (as given on p. 4 of SP 16).

Ref.	Step	Calculations	Output
	1.	*Case 1: Fe 250 steel (Mild Steel)*	
		Mild steel has a bilinear stress-strain curve.	
		Strain at yield $= \dfrac{f_y}{E_s} = \dfrac{250}{200 \times 10^3} = 0.00125$	
SP 16 page 4		Yield stress on design curve $= 0.87f_y$	
		$\varepsilon_{s1} = \dfrac{0.87 \times 250}{200 \times 10^3} = 0.00109$	
		The design stress-strain curve is as shown. Curve A is the curve obtained from laboratory test; curve B is the design curve for Fe 250 steel.	**Fig. E.14.1a**
	2.	*Case 2: Fe 415 steel (Cold worked bars)*	
		When it is specified that the yield stress of a steel (which has a non-linear stress-strain curve) is 415 N/mm^2, it refers to the proof stress, i.e. the residual strain on unloading is 0.002 or the proof stress is 415 N/mm^2. It is also known that the slope of the unloading	

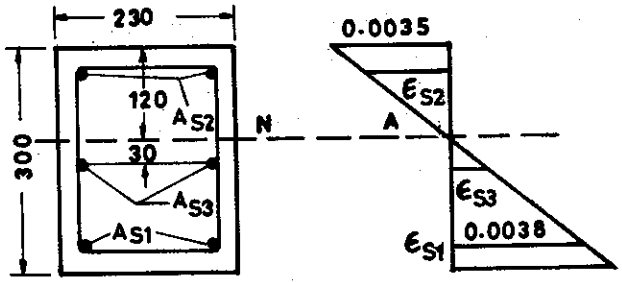
EXAMPLE 14.1 (cont.)

Ref.	Step	Calculations	Output
		curve is the same as the initial loading curve obtained from laboratory tests (A).	
		Stress at which the design stress curve starts off from curve $A = \dfrac{0.8f_y}{1.15}$	
		ε_{s1} (strain at 1) $= \left(\dfrac{0.8f_y}{1.15}\right)\left(\dfrac{1}{E_s}\right)$	
		$= \dfrac{0.8[0.87f_y]}{E_s} = \left(\dfrac{0.7f_y}{E_s}\right)$	
		$\varepsilon_{s1} = \dfrac{0.8 \times 0.87 \times 415}{200 \times 10^3} = 0.00144$	**Fig. E.14.1b**
		As there is no residual strain, $\varepsilon_{s1} = 0.00144$ Let strain at point 3 = ε_{s3}. ε_{s3} be the yield strain. Residual strain = 0.002	
SP 16 page 4		$\varepsilon_{s3} = 0.002 + \dfrac{0.87 \times 415}{200 \times 10^3} = 0.00380$	
		Let ε_{s2} be the strain corresponding to 0.9 times the yield. Residual strain = 0.0003.	
		ε_{s2} = (residual strain) + $\dfrac{0.9(0.87 \times 415)}{200 \times 10^3}$	
		$= 0.0003 + 0.00162$	
		$= 0.00192.$	

EXAMPLE 14.2 **(Calculation of P and M for teusiou failure of steel)**

A short column is 230 × 300 mm and is reinforced with four rods of 20 mm, one at each of the corners and two rods of 16 mm, one each at the middle of the longer sides. Calculate the value of P and M for tension failure of steel by bending on the major axis. Assume cover = 40 mm, f_{ck} = 20 N/mm^2, Fe 415 steel. Assume cover to centre of steel as 50 mm.

EXAMPLE 14.2 (cont.)

Ref.	Step	Calculations	Output
		 **Fig. E.14.2**	
		Taking A_{s1} and A_{s2} as steels in tension and compression, the depth of neutral axis is calculated as follows:	
	1.	*Strains when tension steel reaches yield*	
IS 456 Cl. 37.1		$\varepsilon_c = 0.0035$ $\varepsilon_{s1} = 0.002 + \dfrac{f_y}{1.15E_s} = 0.0038$ $\dfrac{x}{d} = \dfrac{0.0035}{0.0073} = 0.48$ $x = 0.48 \times 250 = 120$ mm Stress at $\varepsilon_{s1} = f_{s1} = (0.87)\ (415) = 361$ N/mm^2 $\varepsilon_{s3} = \dfrac{0.0035}{120} \times 30 = 0.000875$ (less than yield) Stress at $\varepsilon_{s3} = f_{s3} = (0.000875)(E_s) = 175$ N/mm^2 $\varepsilon_{s2} = \dfrac{0.0035}{120} \times 70 = 0.00204$ (less than yield)	
SP 16 Fig. 3		For $\varepsilon_{s3} = 0.00204$, we have $f_{s2} = 330$ N/mm^2	
	2.	*Direct load P under the above condition* As strain in steel is greater than 0.002, the stress in concrete at level of steel is $0.45f_{ck} = 0.45 \times 20 = 9$ N/mm^2 P = compression in concrete + compression in steel − tension in steel $= (0.36f_{ck} \times b)_x + (330 - 9)628 - (361 \times 628)$ $- (175 \times 402)$	

EXAMPLE 14.2 (cont.)

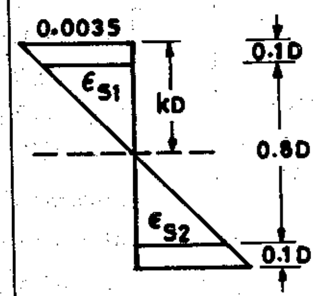
Ref.	Step	Calculations	Output
		($2T\ 20$ mm = 628 mm^2, $2T\ 16$ mm = 402 mm^2) $= [(0.36 \times 20 \times 120 \times 230) + (201.6) - (226.7)$ $- (70)] \times 10^{-3}$ $= [198.7 - 95.1] = 103.6$ kN	$P = 103.6$ kN
	3.	*Moment M under the condition in step 1* M = moment of all forces about the C.G. of column $M = [(198.7)(150 - 0.42x) + (201.6 \times 100)$ $+ (226.7 \times 100)] \times 10^{-3}$ $= 19.8 + 20.2 + 22.6 = 62.6$ kNm	$M = 62.6$ kNm

EXAMPLE 14.3 **(Construction of interaction diagram)**

Construct the interaction diagram for a column of size $b \times D$ with steel ratio $p/f_{ck} = 0.1$ for $d'/D = 0.1$ and Fe 250 steel. Assume steel is placed equally on the two sides and moment is applied about the major axis.

Ref.	Step	Calculations	Output
	1.	*Intercept on the y-axis (acting as column N.A. at infinity)* $$P_u = 0.45 f_{ck} bD + \frac{p}{100} bd (f_{sc} - 0.45 f_{ck})$$ The second $0.45f_{ck}$ is the deduction for concrete area occupied by steel. It may be neglected, or an average value corresponding to $f_{ck} = 20$ may be used as in SP 16. Neglecting the term for simplicity, we get	$k = \infty$
		$$\frac{P_u}{f_{ck} bD} = 0.45 + \frac{p}{100 f_{ck}} (f_{sc})$$ Failure strain of column = 0.002. Fe 250 steel reaches yield $f_{sc} = 0.87 f_y$. $$\frac{P_u}{f_{ck} bD} = 0.45 + \left(\frac{0.1}{100}\right)(0.87 \times 250) = 0.67$$	$x = 0$, $y = 0.67$
	2.	*Intercept on the x-axis (acting as a pure beam N.A. within section)* (A_{s1} and A_{s2} are steels in compression and tension respectively) Tension = compression, i.e. $P = 0$ $$0.36 f_{ck} bkD + f_{s1}\left(\frac{0.5p}{100 f_{ck}}\right) f_{ck} bD - f_{s2}\left(\frac{0.5p}{100 f_{ck}}\right) f_{ck} bD = 0$$	

EXAMPLE 14.3 (cont.)

Ref.	Step	Calculations	Output
		where f_{s1} and f_{s2} are stresses in steel, depending on the strain in these steels at failure. Taking A_{s1} as steel on the more compressed face, we obtain $\varepsilon_{s1} = \frac{0.0035(kD - 0.1D)}{kD} = \frac{0.0035(k-0.1)}{k}$ $f_{s1} = \varepsilon_{s1} E_s$ $\varepsilon_{s2} = \frac{0.0035(0.9-k)}{k}$	 **Fig. E.14.3**
		Assuming ε_{s2} reaches yield at failure and ε_{s1} may not have reached yield and simplifying the equation above, we get	
IS 456 Cl.4.6.2		$E_s = 200 \times 10^3, \quad \frac{p}{f_{ck}} = 0.1$ $0.36k + \frac{0.0035(k-0.1)}{k} \times 200 \times 10^3 \left(\frac{0.5}{100}\right)(0.1)$ $- 0.87 \times 250 \times \frac{0.5}{100}(0.1) = 0$	
		Solving $0.36k^2 + 0.241k - 0.035 = 0$ for k, we obtain $k = 0.125$ and $f_{s1} = 140$ N/mm^2	$k = 0.125$
		As $P = 0$, M is obtained by taking moment of compression about tension steel. $M = 0.36 f_{ck} b(0.125D)(0.9D - 0.42 \times 0.125D)$ $+ 140\left(\frac{0.05}{100}\right) f_{ck} bD(D - 0.2D)$ This reduces to $\frac{M}{f_{ck} bD^2} = 0.36 \times 0.125(0.848) + 0.056 = 0.094$	$x = 0.094$ $y = 0$
		Check strain in $\varepsilon_{s2} = \frac{0.0035(0.9 - 0.125)}{0.125} = 0.0217$ Yield strain of Fe 250 $= \frac{0.87 \times 250}{200 \times 10^3} = 0.0011$ Hence steel yields.	
	3.	*Taking balanced failure as the one in which tension steel yields, we get* ε_{s2} = yield strain of steel	

EXAMPLE 14.3 (cont.)

Ref.	Step	Calculations	Output
		Steel yields at $\varepsilon_{s2} = 0.0011$	
		$\varepsilon_c = 0.0035$	
		$\dfrac{\varepsilon_c}{\varepsilon_{s2} + \varepsilon_c} = \dfrac{0.0035}{0.0046} = \dfrac{kD}{0.9D}$	
		Solving $k = 0.685$, we obtain	$k = 0.685$
		$\varepsilon_{s1} = \dfrac{0.0035(0.685 - 0.1)}{0.685}$	
		$= 0.003$ (steel yields)	
		$\varepsilon_{s2} = \dfrac{0.0035(0.9 - 0.685)}{0.685}$	
		$= 0.0011$ (steel yields)	
		$P = 0.36 f_{ck} bkD$ + (compression in steel − tension in steel)	
		$\dfrac{P}{f_{ck} bD} = (0.36)(k) = (0.36)(0.685) = 0.25$	
		Moment of the compression forces and the tension about the centroid of section at $D/2$ from the edge. Taking both tension and compression steel together, we get	
		$M = 0.36 f_{ck} bkD(0.5D - 0.42kD)$	
		$+ \left(\left(\dfrac{0.05}{100}\right) f_{ck} bD\right)(0.87 f_y)(0.8D)$	
		$\dfrac{M}{f_{ck} bD^2} = 0.36k(0.5 - 0.42k) + 0.087$	
		$= 0.36 \times 0.685(0.5 - 0.288) + 0.087$	
		$= 0.052 + 0.087$	
		Hence,	
		$\dfrac{M}{f_{ck} bD^2} = 0.139, \quad \dfrac{P}{f_{ck} bD} = 0.25$	$x = 0.139$ $y = 0.25$ (Check with SP 16 Chart 28)
		Note: (i) For pure bending, N.A. is at $0.125D$ (ii) For balanced failure, N.A. is at $0.685D$ (iii) For pure compression, N.A. is at infinity. Other positions for N.A. can be assumed and the curve plotted. For N.A. more than $1.0D$ the stress block parameters in Table H of SP 16 can be used with advantage.	

EXAMPLE 14.3 (cont.)

Ref.	Step	Calculations	Output
	4.	*Neutral axis at $k = 1.00$*	$k = 1.0$
		$\varepsilon_{s1} = \dfrac{0.0035 \times 0.9}{1.0} = 0.0031$ (yields)	
		$f_{s1} = 0.87 \times 250 = 217.5$ N/mm^2	
		$\varepsilon_{s2} = \dfrac{0.0035 \times 0.1}{1.0} = 0.00035$	
		$f_{s2} = \varepsilon_{s1} E_s = 70$ N/mm^2	
		$P = 0.36 f_{ck} bD + \dfrac{0.05}{100} f_{ck} bD\,(217.5 + 70)$	
		$\dfrac{P}{f_{ck} bD} = 0.36 + 0.109 + 0.035 = 0.506$	
		By taking moments about centre line of column, we obtain	
		$\dfrac{M}{f_{ck} bD^2} = 0.36\,(0.5 - 0.42) + (0.109 \times 0.4) - (0.035 \times 0.4)$	
		$= 0.0288 + 0.0436 - 0.014$	
		$= 0.058$	
		Hence the ordinates are	$x = 0.06$
		$\dfrac{P}{f_{ck} bD} = 0.5, \quad \dfrac{M}{f_{ck} bD^2} = 0.06$	$y = 0.5$
	5.	Check the curve with Chart 28 of SP 16.	

EXAMPLE 14.4 **(Calculation of steel for given P_u and M_u)**

A column 300 × 400 mm has an unsupported length of 3 m and effective length of 3.6 m. If it is subjected to $P_u = 1100$ kN and $M_u = 230$ kNm about the major axis, determine the longitudinal steel using $f_{ck} = 25$ N/mm^2 and $f_y = 415$ N/mm^2. Assume $d' = 60$ mm.

Ref.	Step	Calculations	Output
	1.	*Calculation of slenderness*	
IS 456 Cl. 24.4		$L_e = 3600, \quad L = 3000, \quad \dfrac{L_e}{400} = \dfrac{3600}{400} = 9.0$	
		$e_{\min} = \dfrac{L}{500} + \dfrac{D}{30} = \dfrac{3000}{500} + \dfrac{400}{30} = 6 + 13.3 = 19.3$ mm	
BS 8110 Sec. 13.7		$\dfrac{D}{20} = \dfrac{400}{20} = 20$ mm	
	2.	*Calculation of parameters*	
		$\dfrac{M}{P} = e = \dfrac{230 \times 10^6}{1100 \times 10^3} = 209$ mm > 20 mm (design for M)	

EXAMPLE 14.4 (cont.)

Ref.	Step	Calculations	Output
		$\dfrac{P_u}{f_{ck}bD} = \dfrac{1100 \times 10^3}{25 \times 300 \times 400} = 0.37$	
		$\dfrac{M_u}{f_{ck}bD^2} = \dfrac{230 \times 10^6}{25 \times 300 \times 400^2} = 0.19$	
		$\dfrac{d'}{D} = \dfrac{60}{400} = 0.15$	
	3.	*Amount of steel required*	
SP 16 Chart 33		$\dfrac{p}{f_{ck}} = 0.15, \quad p = 0.15 \times 25 = 3.75\%$	
		$A_s = \dfrac{3.75 \times 300 \times 400}{100} = 4500$ – (8 Nos. 28 mm = 4926)	(4T 28 on each side)
IS 456 Cl.25.3.1		Spacing of steel $= \dfrac{300 - 108}{3} = 64$ mm < 300 mm	
		> (aggregate size + 5 mm)	
	4.	*Detail ties as in Example 13.2 (axially loaded column)*	

EXAMPLE 14.5 **(Use of approximate method)**

Calculate the area of steel required for a column 300 × 400 mm for $P_u = 1100$ kN and $M_u = 230$ kNm, by the approximate method. Assume $d' = 60$ mm, $f_{ck} = 25$ N/mm^2, and Fe 415 steel.

Ref.	Step	Calculations	Output
	1.	*Eccentricity of M*	
		$\dfrac{M}{P} = e = \dfrac{230 \times 10^3}{1100} = 209$ mm	
		$\left(\dfrac{D}{2} - d'\right) = 200 - 60 = 140$ mm	
		As $e = 209 > 140$, the approx. method is applicable.	
	2.	*Equivalent increased moment*	
		Let $(D - d') = d = 340$ mm	
Eq. 14.5		$M_a = M + P\left(\dfrac{D}{2} - d'\right) = 230 + (1100 \times 140) \times 10^{-3} = 384$ kNm	
	3.	*Compression and steel areas required for* M_a	
Eq. 14.6		$384 \times 10^6 = 0.15 f_{ck} b d^2 + 0.72 f_y A_{sc}(d - d')$	
		$= 0.15 \times 25 \times 300 \times (340)^2$	
		$+ 0.72 \times 415 \times A_{sc}(340 - 60)$	

EXAMPLE 14.5 (cont.)

Ref.	Step	Calculations	Output
		$A_{sc} = \dfrac{254 \times 10^6}{0.72 \times 415 \times 280} = 3036 \text{ mm}^2$	
		Using the second equation here, we get	
Eq. 14.7		$0.87 f_y A_{st} = 0.2 f_{ck} bD + 0.72 f_y A_{sc}$	
		$= (0.2 \times 25 \times 300 \times 340) + (0.72 \times 415 \times 3036)$	
		$A_{st} = \dfrac{1417 \times 10^3}{0.87 \times 415} = 3925 \text{ mm}^2$	
	4.	*Reduction in tension steel for P*	
Eq. 14.8		$A_{st1} = 3925 - \dfrac{1100 \times 10^3}{0.87 f_y} = 3925 - 3047 = 878 \text{ mm}^2$	
		Total steel = 3036 + 878 = 3914 mm²	$A_s = 3914$ (mm²)

EXAMPLE 14.6 **(Biaxial bending—Bresler method)**

Determine the longitudinal steel required for a column 400 × 600 mm carrying $P_u = 1600$ kN, factored M (major axis) = 120 kNm, and factored M (minor axis) = 90 kNm. Assume $f_{ck} = 15$ N/mm², $f_y = 415$ N/mm², $d' = 60$ mm.

Ref.	Step	Calculations	Output
	1.	*Check for accidental eccentricity*	
		Equivalent eccentricity of loads is given by	
		$\dfrac{M_x}{P} = \dfrac{120 \times 10^3}{1600} = 75 \text{ mm}$	
		$\dfrac{M_y}{P} = \dfrac{90 \times 10^3}{1600} = 56 \text{ mm}$	
IS 456 Cl. 24.4		Both are more than 20 mm minimum.	
	2.	*Assume percentage of steel*	
		(Assume steel larger than required by P and M_x)	
		$\dfrac{d'}{D} = \dfrac{60}{600} = 0.1$	
		$\dfrac{M_x}{f_{ck} bD^2} = \dfrac{120 \times 10^6}{15 \times 400 \times 600 \times 600} = 0.06$	
		$\dfrac{P_u}{f_{ck} bD} = \dfrac{1600 \times 10^3}{15 \times 400 \times 600} = 0.44$	

EXAMPLE 14.6 (cont.)

Ref.	Step	Calculations	Output
SP 16 Chart 44		$\frac{p}{f_{ck}} = 0.05$	
		Assume a higher value $\frac{p}{f_{ck}} = 0.085$	
		Assumed $p = 0.085 \times 15 = 1.27$ per cent	Column as in Chart reinforced all around
		$A_s = \frac{1.27 \times 400 \times 600}{100} = 3048 \text{ mm}^2$	Use $\frac{12T\,18}{3053 \text{ mm}^2}$
		(Use 12 Nos. 18 mm – 3053 mm^2)	
	3.	*Find the moment capacities M_{x1} and M_{y1}*	
		About the x-axis,	
		$\frac{d'}{D} = 0.1, \quad \frac{P}{f_{ck}bD} = 0.44, \quad \frac{p}{f_{ck}} = 0.08$	
		$\frac{M_{x1}}{f_{ck}bD^2} = 0.095$	
SP 16 Chart 44		$M_{x1} = 0.095 \times 15 \times 400 \times 600^2 = 205.2$ kNm	
		About the y-axis,	
		$\frac{d'}{D} = \frac{60}{400} = 0.15, \quad \frac{P}{f_{ck}bD} = 0.44$	
		$\frac{p}{f_{ck}} = 0.08$	
SP 16 Chart 45		$\frac{M_{y1}}{f_{ck}Db^2} = 0.085$	
		$M_{y1} = 0.085 \times 15 \times 600 \times 400^2 = 122.4$ kNm	
	4.	*Calculate α_n*	
		$P_z = 0.45 f_{ck}A_c + 0.75 f_y A_s$	
		$= [0.45 \times 15 \times 400 \times 600 + 0.75 \times 415$	
		$\times 3053] \times 10^{-3}$	
		$= 2570$ kN	
IS 456 Cl. 38.6		$\frac{P}{P_z} = \frac{1600}{2570} = 0.62, \quad \alpha_n = 1.7$	
		$\left[\text{By formula } \alpha_n = \frac{2}{3}\left(1 + \frac{5}{2}\frac{P}{P_z}\right) = 1.7\right]$	

EXAMPLE 14.6 (cont.)

Ref.	Step	Calculations	Output
	5.	*Criteria for biaxial bending* $$\left(\frac{M_x}{M_{x1}}\right)^{\alpha_n} + \left(\frac{M_y}{M_{y1}}\right)^{\alpha_n} \le 1.0$$ $\left(\frac{120}{205}\right)^{1.7} + \left(\frac{90}{122}\right)^{1.7} = 0.4 + 0.6 = 1.0$ Hence column is just safe.	
	6.	*Alternative method*	
SP 16 Chart 63		(a) Calculate P_z $p = 1.27\%$, $f_y = 415$ N/mm², $f_{ck} = 15$ N/mm² $\frac{P_z}{A_g} = 10.6$, $P_z = 10.6 \times 600 \times 400 \times 10^{-3}$ $= 2544$ kN	
		(b) Check capacity by SP 16, Chart 64. Then we get $\frac{P}{P_z} = \frac{1600}{2544} = 0.63$, $\frac{M_x}{M_{x1}} = \frac{120}{205} = 0.58$ $\frac{M_y}{M_{y1}} = \frac{90}{122} = 0.74$	
SP 16 Chart 64		For $\frac{M_x}{M_{x1}} = 0.58$, $\frac{M_y}{M_{y1}}$ allowed $= 0.745 > 0.74$	Column is safe.

EXAMPLE 14.7 **(Biaxial bending—BS 8110 method)**

Design a symmetrically reinforced short column 450 × 450 mm under biaxial bending with the following factored loads: $P = 1000$ kN, $M_x = 75$ kNm, $M_y = 60$ kNm. Assume $f_{ck} = 15$ N/mm², $f_y = 415$ N/mm².

Ref.	Step	Calculations	Output
	1.	*Solution by enhanced moment method (BS 8110)* *Find condition of loading* $d = 450 - 40 - \frac{\phi}{2}$. Assumed $d = 400$ mm b' also equals 400 mm.	

EXAMPLE 14.7 (cont.)

Ref.	Step	Calculations	Output
		$\dfrac{M_x}{d} = \dfrac{75 \times 10^6}{400} = 187.5 \times 10^3$	
		$\dfrac{M_y}{b'} = \dfrac{60 \times 10^6}{400} = 150.0 \times 10^3$	
		$\dfrac{M_x}{d} > \dfrac{M_y}{b'}$	$\dfrac{M_x}{d}$ controls the design
	2.	*Find enhanced moment about M_x*	
		$M'_x = M_x + \alpha \dfrac{d}{b'} M_y$	
		The value of α depends on $\dfrac{P}{f_{ck} bD}$	
		$\dfrac{P}{f_{ck} bD} = \dfrac{1000 \times 10^3}{15 \times 450 \times 450} = 0.329$	
		$\alpha = \left(1 - \dfrac{7}{6} \dfrac{P}{f_{ck} bD}\right) = 1 - \dfrac{7}{6}(0.329) = 0.62$	
		$M'_x = 75 + 0.62 \times 1 \times 60 = 112$ kNm	$M'_x = 112$ kNm
	3.	*Design for uniaxial bending for M'_x*	
		$\dfrac{d'}{D} = \dfrac{50}{450} = 0.11$	
SP 16 Chart 44		$\dfrac{M'_x}{f_{ck} bD^2} = \dfrac{112 \times 10^6}{15 \times 450 \times 450 \times 450} = 0.082$	
		$P/f_{ck}bD = 0.33$	
		$\dfrac{p}{f_{ck}} = 0.05$ or $p = 0.05 \times 15 = 0.75\%$	$p = 0.75\%$
		CHECKING BY BRESLER METHOD	
	1.	*Assume the percentage of steel*	
		$p = 0.75\%$	Assume 0.75%
	2.	*Find moment capacity M_{ux1} and M_{uy1}*	
SP 16 Chart 44		For $\dfrac{P}{f_{ck} bD} = 0.33$,	
		$\dfrac{M_u}{f_{ck} bD^2} = 0.082$	
		$M_{ux1} = M_{uy1} = 0.082 f_{ck} bD^2 = 112$ kNm	$M_u = 112$ kNm

EXAMPLE 14.7 (cont.)

Ref.	Step	Calculations	Output
	3.	*Find P_z*	
		$P_z = 450 \times 450 \left[0.45 \times 15 + 0.75 \times 415 \times \frac{0.75}{100}\right] \times 10^{-3}$	
		$= 1840$ kN	
	4.	*Calculate load ratios*	
		$\frac{P}{P_z} = \frac{1000}{1840} = 0.54$	
		$\frac{M_x}{M_{ux}} = \frac{75}{112} = 0.67$	
		$\frac{M_y}{M_{uy}} = \frac{60}{112} = 0.54$	
	5.	*Determine safety of column*	
SP 16 Chart 64		For $\frac{P}{P_z} = 0.54$ and $\frac{M_x}{M_{ux}} = 0.67$,	
		$\frac{M_y}{M_{uy}} = 0.62$	
		This value is greater than 0.54 in column. Hence column design with 0.75 per cent steel is very safe.	Design is safe.

EXAMPLE 14.8 **(Shear in column)**

A square column 300 × 300 mm reinforced with 4T 20 rods (1256 mm^2) is subjected to the following ultimate loads:

$P_u = 70$ kN, M_{1x} (moment at top of the column) = 55 kNm

M_{2x} (moment at bottom of the column) = 65 kNm

If the column height is 3 m, check its safety in shear. Assume a cover of 50 mm to the centre of steel and $f_{ck} = 30$ N/mm^2.

Ref.	Step	Calculations	Output
	1.	*Calculation of shear stress*	
		$V = \frac{65 + 55}{3} = 40$ kN	
		$v = \frac{40 \times 10^3}{300 \times 250} = 0.53\ \text{N/mm}^2$	

EXAMPLE 14.8 (cont.)

Ref.	Step	Calculations	Output
	2.	*Find τ_c' value*	
		$b = 300, \quad d = 250$	
IS 456		$p_t = \left(\dfrac{1256 \times 100}{2 \times 300 \times 250}\right) = 0.83\%$	
Table 13		τ_c for $f_{ck} = 30$ N/mm^2 = 0.62 N/mm^2	
	3.	*Increased shear due to compression*	
		$\tau_c' = \tau_c + 0.75\left(\dfrac{P}{A_c}\right)\left(\dfrac{Vd}{M}\right)$	
		$\dfrac{Vd}{M} = \dfrac{40 \times 10^3 \times 250}{65 \times 10^6} = 0.154 < 1$	
		$\tau_c' = 0.62 + \left(\dfrac{0.75 \times 70 \times 10^3}{300 \times 300}\right)(0.154)$	
		$= 0.72$ (approx.)	
	4.	*Check against maximum shear*	
		$\tau_{c\max} = 0.8(f_{ck})^{1/2} = 0.8\sqrt{30}$	
		$= 4.38 < 5$ N/mm^2	
		$v < \tau_c' \le \tau_{c\max}$	
		Hence section is safe in shear. Only nominal shear steel is required.	Provide nominal steel for shear.

REVIEW QUESTIONS

14.1 Explain the assumption regarding strain distribution in a section under eccentric loading at ultimate failure.

14.2 Indicate how the interaction diagram for columns under combined axial load and bending can be drawn. What are the advantages in representing it in a non-dimensional form?

14.3 What are the three methods available for design of columns subjected to P and M? Which of these is readily useful for use in design office?

14.4 What is meant by balanced failure of a column section subjected to P and M? How will you determine the combination of P and M that will cause balanced failure of a given section?

14.5 Explain how a section under moderate axial load can take more bending moment than its ultimate bending moment capacity in pure bending.

14.6 It can be noted that the interaction diagrams are convex outwards (always from the origin). Is there any significance for this property of the interaction curve?

14.7 Give examples of columns that are in practice subjected to biaxial bending.

14.8 What are the methods determining the strength of columns under biaxial bending? What is meant by interaction surface for these columns?

14.9 What are the rules to be followed for placement of steel when using the charts in SP 16 for biaxial bending. What does the percentage of steel (p) in these charts signify?

14.10 Explain the difference in shapes that one can notice in the interaction diagrams using Fe 250 and Fe 415 steels.

PROBLEMS

14.1 A concrete column 500 × 700 mm is of effective height 6 m and is provided with 6 Nos. of 25 mm bars as longitudinal steel. Determine its ultimate bending moment capacity about the major axis when it will be subjected to an ultimate axial load of 2.5 MN. Assume f_{ck} = 25 N/mm^2 and f_y = 415 N/mm^2, and a clear cover according to IS 456 for normal conditions of exposure. Place the steel for maximum moment capacity about major axis.

14.2 A rectangular column of effective height of 4 m is subjected to a characteristic axial load of 800 kN and bending moment of 100 kNm about the major axis of the column. Design a suitable section for the column so that the width should not exceed 400 mm. Use the minimum percentage of longitudinal steel. Assume f_y = 415 N/mm^2 and f_{ck} = 20 N/mm^2.

14.3 A trapezoidal section as shown in Fig. P.14.3 is used as a column. Check the moment capacity of the column if a characteristic load of 500 kN is applied at the point P shown in the figure. Assume that the column is reinforced with 4 Nos. high yield deformed bars of 20 mm diameter at the four corners. Assume f_y = 415 N/mm^2, f_{ck} = 20 N/mm^2.

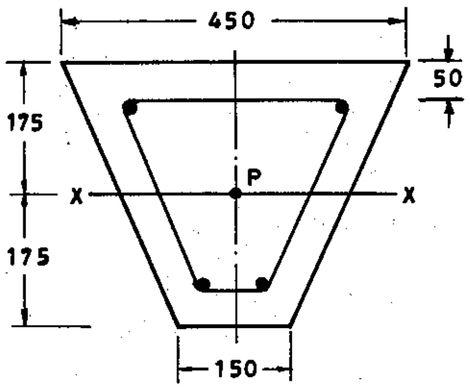


Fig. P.14.3.

14.4 A short column 250 mm square has to carry an ultimate axial load of 600 kN along with ultimate moments of 60 kNm about one axis and 40 kNm about the other axis. Assuming f_{ck} = 30 N/mm^2, f_y = 415 N/mm^2, and cover to be the minimum as per IS 456; design the longitudinal steel.

14.5 Design the longitudinal steel for a 500 × 300 mm column with ultimate loads P_u = 2300 kN, M_{ux} = 300 kNm and M_{uy} = 120 kNm. Assume that f_{ck} = 30, f_y = 415 N/mm^2 and that the column is short.

15

Effective Length of Columns

15.1 INTRODUCTION

The effective length or effective height of the column (L_e) is different from its unsupported length L_0. The effective length can be defined as that length which corresponds to the length of a pin jointed column that can carry the same axial load as the given column. Alternatively, effective length can also be defined as the length between the points of contraflexure, real or imaginary, of a buckled column. The difference between braced and unbraced columns was explained in Chapter 13. Figure 15.1 shows the mode of failures of braced and unbraced individual columns and Fig. 15.2 their mode of failure in a framework. It is seen that the effective length of braced columns can range from $0.5L_0$ to L_0, and that of the unbraced column from L_0 to a very large value. This effective length has to be obtained from the actual or 'free' length by considering the degree of end-restraint of the compression member. The following three methods are generally used to find the effective length of column:

1. Use of table of coefficients for isolated columns
2. Wood's charts given in IS and More Land alignments charts in ACI for columns in frames
3. By formulae (BS and ACI) for columns in frames.

These are explained in this chapter.

15.2 TABLE OF COEFFICIENTS

The degree of end restraint of a column is expressed with respect to the following two end conditions:

1. How the end is held in position, the ends being effectively held or not held in position or movement.

2. How the end is restrained against rotation: effectively restrained or not effectively restrained against rotation.

With the two ends having combinations of the above conditions of end restraints, several cases of stable compression members can be obtained. Recommended values of effective length for these various cases have been traditionally used in R.C. design to determine the effective length of columns.

Table 24 of IS 456, which is simple and straightforward to use, gives the conventional values of effective length of compression members with different end conditions. The values are as given in Fig. 13.3. These are to be used for design of isolated columns.

The British Code BS 8110 has presented these more systematically with the help of the following data shown in Table 15.1.

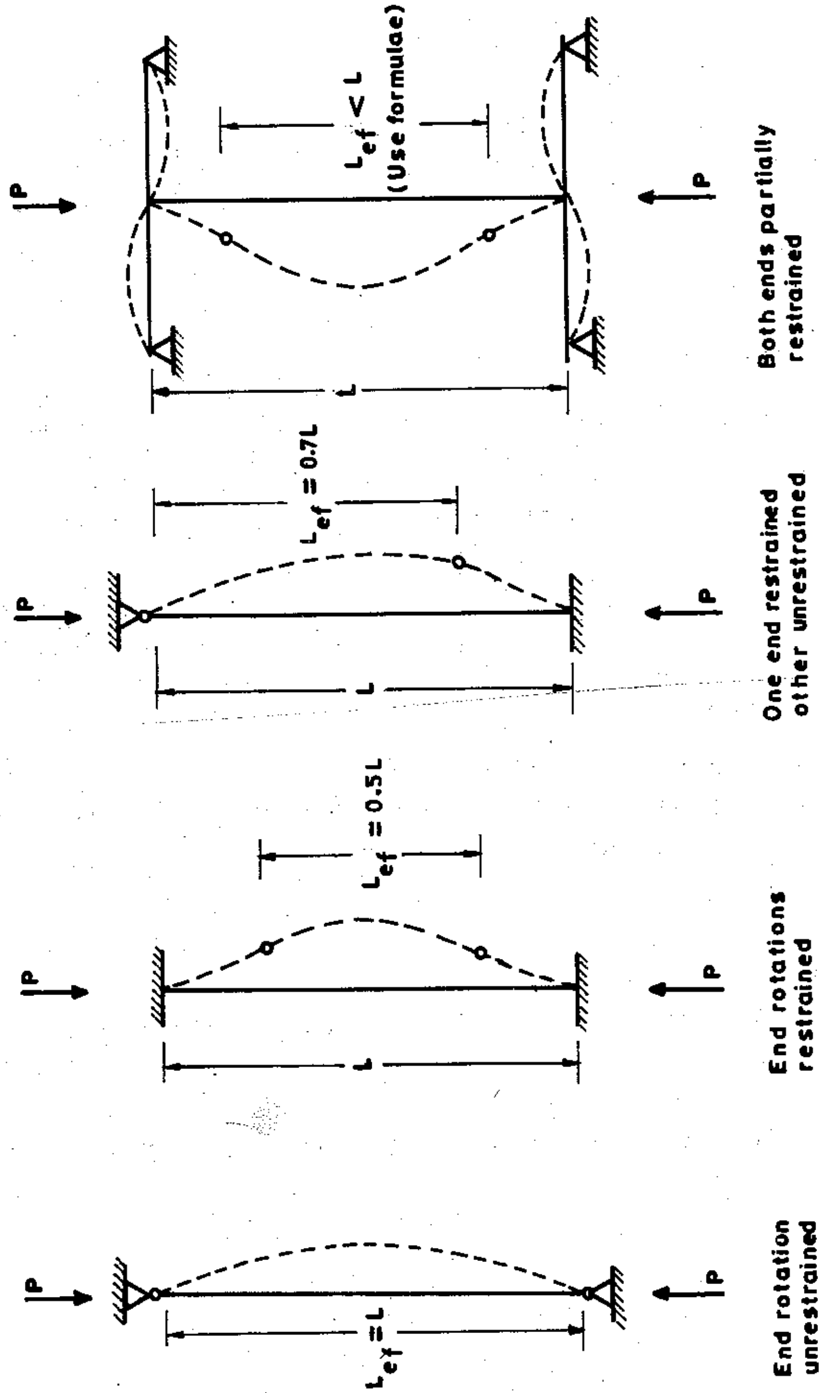


Fig. 15.1a Effective length of columns without joint translation.

Fig. 15.1a Effective length of columns without joint translation.

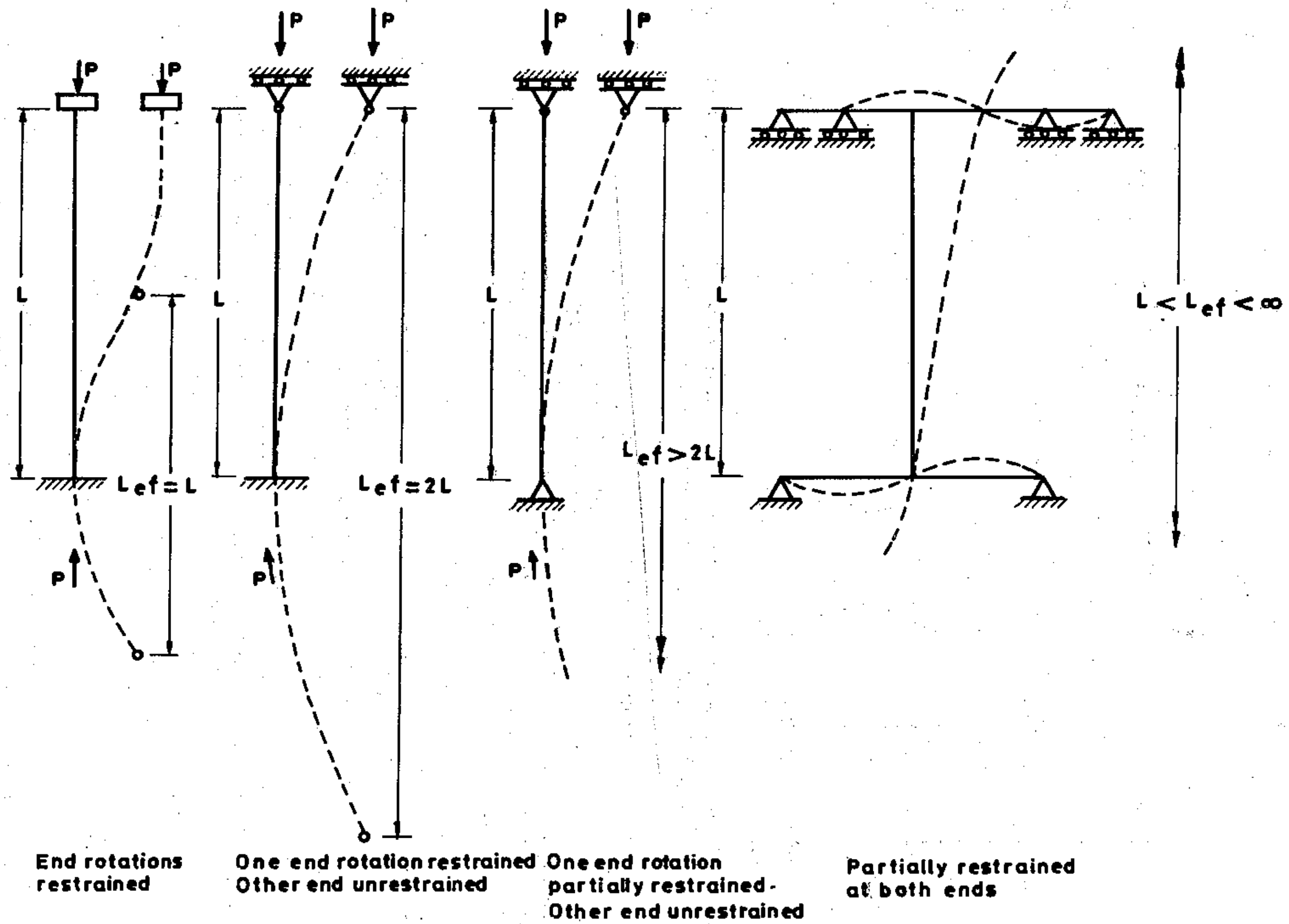
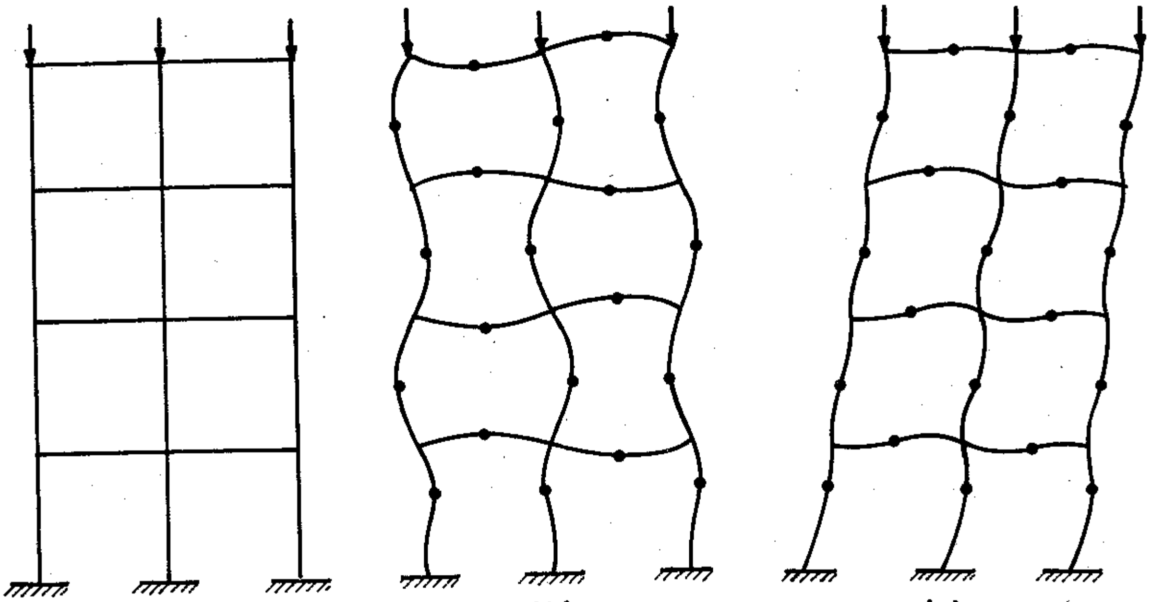


Fig. 15.1b Effective length of columns with joint translation.



(a) (b) (c)

Fig. 15.2 Failure modes of columns in frames: (a) Frame, (b) Braced frame (single curvature bending), (c) Unbraced frame (double curvature bending).

TABLE 15.1 EFFECTIVE LENGTH COEFFICIENTS
(Based on BS 8110)
(Br = Braced, U.Br. = Unbraced)

End condition at top	End condition at bottom					
	1		2		3	
	Br.	U.Br.	Br.	U.Br.	Br.	U.Br.
1	0.75	1.2	0.80	1.3	0.90	1.6
2	0.80	1.3	0.85	1.5	0.95	1.8
3	0.90	1.6	0.95	1.8	1.00	
4		2.2				

End condition 1: End of column fixed (i.e. connected with beams of depth as much as the size of the column or connected to foundations and designed to carry moments).
End condition 2: End of column connected to beams shallower than the size of column.
End condition 3: End of column provides nominal restraint only against rotation.
End condition 4: End of column is unrestrained against movement and rotation, like end of a cantilever column in an unbraced structure.

15.3 WOOD'S CHARTS FOR COLUMNS IN BUILDING FRAMES

For estimating the effective length of columns in frames, IS 456 uses the results of research conducted by R.H. Wood (presented as graphs in IS 456, Appendix D, Figs. 24 and 25). These are

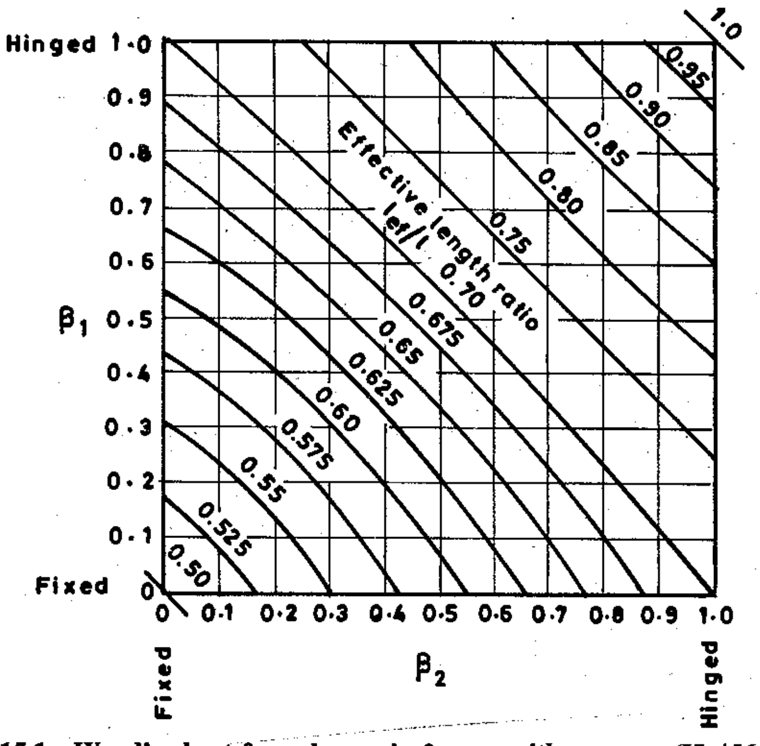


Chart 15.1 **Wood's chart for columns in frames with no sway** (IS 456 Fig. 24).

reproduced as Charts 15.1 and 15.2 of this text*. The basis of the Vanishing Stiffness Method used by R.H. Wood to obtain these curves is explained in SP 24 and may be referred for a better understanding of the derivation of the diagrams. Chart 15.1 gives the effective length ratios for columns in a frame with no sway (braced columns) and Chart 15.2 the values for columns in frames without restraint against sway (unbraced columns). In this connection, it may be noted that even though what can be considered as braced and unbraced is not defined in IS 456, one may use the concept as given in the ACI-318 Code for this purpose. Thus, a compression member located in any storey may be assumed as braced if the bracing elements (like shear walls or lateral support of the storey) has a stiffness of at least six times the sum of the stiffness of all the columns in that storey. In Figs. 24 and 25 of IS 456, values of β_1 and β_2 correspond to the stiffness at the top and bottom ends of the column respectively (Fig. 15.3).

The value of β is given by the expression

$$\beta = \frac{\Sigma K_c}{\Sigma K_c + \Sigma K_b}$$

where

K_c = stiffness of column

K_b = stiffness of beam

*Effective length of columns in multistoreyed buildings, *Structural Engineer*, July 1974.

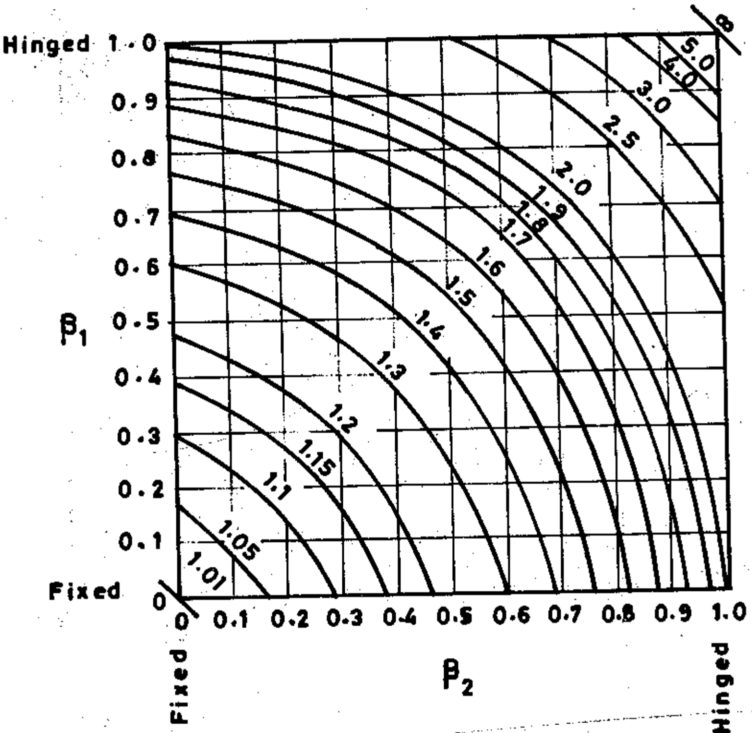


Chart 15.2 **Wood's chart for columns in frames not restrained against sway** (IS 456 Fig. 25).

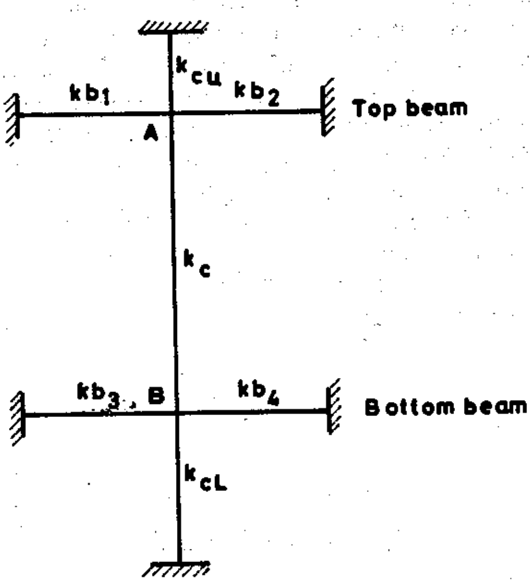


Fig. 15.3 **Column stiffness in Wood's chart.**

Thus, at the top joint (Fig. 15.3),

$$\beta_1 = \frac{K_c + K_{cu}}{K_c + K_{cu} + \Sigma K_{bt}}$$

where

$$\Sigma K_{bt} = \text{sum of stiffness of beams at top.}$$

At the bottom joint,

$$\beta_2 = \frac{K_c + K_{cl}}{K_c + K_{cl} + \Sigma K_{bb}}$$

where ΣK_{bb} = sum of stiffness of beams at bottom. The summation is to be made for all the members framing into the joint in the bending direction considered. It is designated at top as (β_1) and bottom as (β_2).

It may be noted here that whereas the maximum effective length of a column which is in a 'no-sway' (braced) mode is limited to L_0, that of unbraced column (subjected to sway) can be very large, theoretically, as much as infinity. This can be explained by the fact that in a frame without bracing, its stability depends on the rotational restraint provided, and the yielding of the restraint can lead to instability of the column. Thus, if $K_b = 0$ at both top and bottom of the column, then β_1 and $\beta_2 = 1$, and the column is unstable.

If the foundation is not to be designed to resist moments, then K of foundation $K_f = 0$ and $\beta_2 = K_c/K_c = 1$ for use in Wood's charts.

If the foundation is designed to resist moments,

$$K_f = K_c$$

so that $\beta_2 = K_c/(K_c + K_f) = 0.50$ which is applicable in the charts.

15.4 MODIFICATION FOR BEAM STIFFNESS

In SP 24 (The Explanatory Handbook on IS 456, Appendix D) it is shows as to how the stiffness of the beams should be modified while using Wood's charts. As given in SP 24, the modifications are:

1. To determine β_1 and β_2 use the substitute frame method.
2. For braced frames, the beam stiffness should be taken as

$$K_b = \frac{I}{2L} = 0.5\,\frac{I}{L}$$

3. For unbraced frames, the beam stiffness should be taken as

$$K_b = \frac{3}{4}\,\frac{I}{L/2} = 1.5\,\frac{I}{L}$$

as the beams bend in double curvature (see Fig. 15.2).

The method of computation to be used in these cases is illustrated in Example 15.3.

15.5 ACI CHARTS

For preliminary design, the ACI Code recommends another chart known as the *Jackson and Moreland Alignment Chart* for estimating effective length. As in Wood's chart, separate charts are available for braced and unbraced columns. They may be considered as similar to Wood's charts.

15.6 USE OF FORMULAE FOR COLUMNS IN BUILDING FRAMES

The charts recommended by IS or ACI are not easily amenable to computer aided design. Hence BS 8110 recommends the use of formulae for estimation of effective lengths. The 'Commentary on ACI 318' (1983) has also adopted the same formula for use in the American Code.

In BS 8110, the effective height of columns in framed structures is calculated by taking the lower values obtained from the equations for braced columns or unbraced columns, as the case may be.

1. *Braced columns*

$$L_e = L_0\,[0.7 + 0.05(\alpha_{c1} + \alpha_{c2})] < L_0$$

$$L_e = L_0\,(0.85 + 0.05\alpha_{c\,\min}) < L_0$$

2. *Unbraced columns*

$$L_e = L_0\,[1.0 + 0.15(\alpha_{c1} + \alpha_{c2})]$$

$$L_e = L_0\,(2.0 + 0.3\alpha_{c\,\min})$$

where

L_0 = clear height between end restraints

L_e = effective height

α = the ratio of sum of column stiffness to the sum of beam stiffness in the appropriate plane of bending given by $\Sigma K_c/\Sigma K_b$

α_{c1} = ratio at lower end

α_{c2} = ratio at upper end

$\alpha_{c\,\min}$ = lesser of α_{c1} and α_{c2}

These equations are simple and can be readily used for design.

For calculation of stiffness of beams the centre to centre distance of restraints is taken for the value of length L.

15.7 α_c FOR SPECIFIC CONDITIONS AS GIVEN IN BS

When foundation is designed to resist moment such as in a normal pad footing, α_c is taken as 1.0.

When the mass of the concrete base is of width and depth greater than 4 times the depth of the column, it can be assumed as fixed i.e. $\alpha_c = 1.0$ (as in pile caps).

When columns and bases are designed to resist only nominal moments, one may take $\alpha_c = 10.0$.

When the column junction is with beams designed only as simply supported, $\alpha_c = 10.0$.

This method using formulae is at present considered as the best method to determine the effective length of columns in building frames. It is to be expected that IS will adopt this procedure in the next revision of the code.

EXAMPLE 15.1

An R.C. column 500 × 400 mm is 3.2 m long between two floor levels. At its top, beams of 500 mm and 350 mm frame into the column in the direction of the major axis of the column while beams of 300 mm frame in the direction of the minor axis. Calculate the unsupported length of the column in the two axes.

Ref.	Step	Calculations	Output
	1.	*Calculations for buckling on the XX-axis*	
		The depth of beams along the *YY*-axis is effective Beams size (which are of equal size) = 300 mm L_{0x} = 3200 – 300 = 2900 mm	L_{0x} = 2900 mm
	2.	*Calculations on the YY-axis*	
		The depth of beam along the *XX*-axis is effective Lesser depth of beam = 350 mm L_{0y} = 3200 – 350 = 2850 mm	L_{0y} = 2850 mm

EXAMPLE 15.2

A braced R.C. column of effective length 3 m is 400 × 400 mm in size. Determine the L_e/L_0 coefficients under the following cases as per BS 8110:

1. The ground level column as a base not designed to resist moment and the top of the column ends in a beam 450 mm in depth with nominal restraint between the beam and column.
2. The second floor level column is fixed to a beam 500 mm deep at the top and to a beam 300 mm deep at the bottom.

Ref.	Step	Calculations	Output
	1.	*Ground level column*	
Table 15.1		Condition at bottom—nominal restraint against rotation (end condition 3) Condition at top—nominal restraint against rotation (end condition 3) L_e/L_0 ratio = 1.0	
Table 15.1	2.	*Second Floor Column*	
		At bottom, it is fixed to beam shallower than column (end condition 2) At top, it is fixed to beam deeper than column (end condition 1) L_e/L_0 ratio = 0.80	

EXAMPLE 15.3

An unbraced portal frame ABCD is composed of columns AB and CD and beam BC. It is hinged at the base. The columns are 300 × 500 mm and the beam 300 × 800 mm. The heights of the columns are 8 m and the span of the beam 12 m. (a) Using Wood's chart determine the effective length; (b) calculate the value by BS 8110 formulae.

Ref.	Step	Calculations
		Method A: *By use of Wood's Chart*
	1.	Sectional properties

Member	$I \times 10^6$	$L \times 10^3$	$K_c \times 10^3$
Column	$\frac{300 \times 500^3}{12} = 3125$	8	$\frac{3}{4}\frac{I}{L} = 293$
Beam	$\frac{300 \times 800^3}{12} = 12{,}800$	12	$\frac{3}{2}\frac{I}{L} = 1600$

Ref.	Step	Calculations
	2.	Ratio of stiffnesses $\beta = \dfrac{\Sigma K_c}{\Sigma K_c + \Sigma K_{b''}}$

Position	β-value
Top	$\beta_1 = \dfrac{293}{293 + 1600} = 0.155$
Bottom	β_2 = Hinged = 1

Ref.	Step	Calculations
IS 456 Chart 15.2	3.	Calculation of L_e/L_0 Refer chart for $\beta_1 = 0.155$ and $\beta_2 = 1.0$. $L_e/L_0 = 2.15, \quad L_e = 2.15 \times 8 = 17.2$ m
		Method B: *By BS 8110 formula*
	1.	*Calculate the α values* $\alpha = \dfrac{\Sigma K_c}{\Sigma K_b}$ Column $K = \dfrac{I}{L} = 391 \times 10^3$ Beam $K = \dfrac{I}{L} = 1067 \times 10^3$ $\alpha_{c2} = \dfrac{391}{1067} = 0.366$ $\alpha_{c1} = 10.0$
	2.	*Calculate L_e* (taking the lesser of the two values) $L_e/L_0 = [1.0 + 0.15\,(\alpha_{c1} + \alpha_{c2})]$

EXAMPLE 15.3 (cont.)

Ref.	Step	Calculations
		or
		$L_e/L_0 = [2.0 + 0.3\alpha_{c\ \min}]$
		i.e. $= [1.0 + 0.15\ (10.366)] = 2.55$
		or $= [2.0 + 0.3 \times 0.366] = 2.11$
		Hence
		$L_e/L_0 = 2.11$
		$L_e = 2.11L_0 = 2.11 \times 8 = 16.9$ m

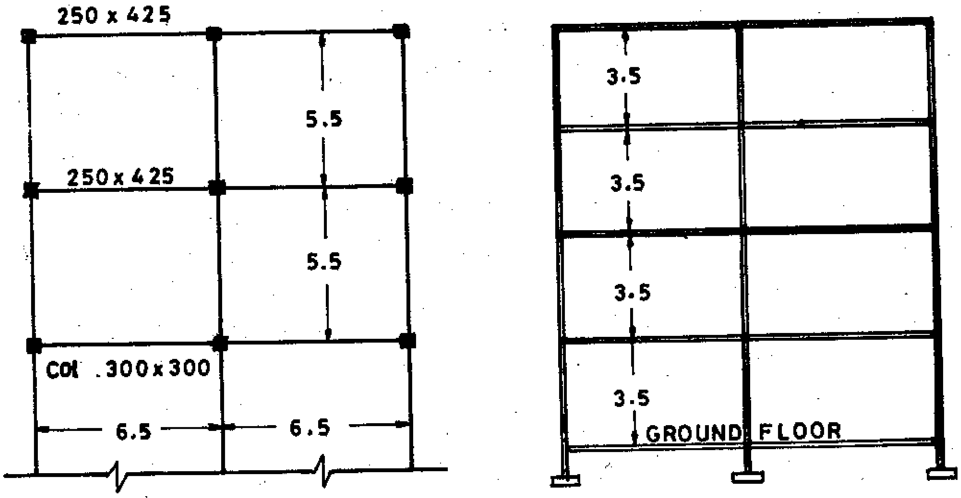
EXAMPLE 15.4

An internal column 300 × 300 mm has beams 250 × 500 mm framing into it from either side. If the storey height is 4 m, determine the equivalent height of the column if the column is braced. Assume that beam lengths are 6 m and 8 m on either side and the column foundation is designed to resist moment.

Ref.	Step	Calculations	Output
	1.	*Sectional properties (centre to centre lengths are taken)*	
		Member $\quad I \times 10^6 \quad L \times 10^3$	$K = (I/L)10^3$
		Column $\quad \dfrac{300 \times 300^3}{12} = 675 \quad 4$	168.7
		Beam 1 $\quad \dfrac{250 \times 500^3}{12} = 2604 \quad 6$	434.0
		Beam 2 $\quad = 2604 \quad 8$	325.5
	2.	*Calculation of* α *values* $\quad \alpha = \dfrac{\Sigma K_c}{\Sigma K_b}$	
		Position $\quad \alpha$ value	
		Top $\quad \alpha_{c2} = \dfrac{168.7 + 168.7}{434 + 325.5} = 0.44$	
		Bottom $\quad \alpha_{c1} = 1.0$ (braced)	
	3.	*L_e/L_0 by formula for braced column*	
		Take lesser of the following two values:	
		$L_e/L_0 = [0.7 + 0.05\ (\alpha_{c1} + \alpha_{c2})]$	
		or $[0.85 + 0.05\alpha_{c\ \min}]$	
		$[0.7 + 0.05\ (1 + 0.44)] = 0.772$	
		$= [0.85 + 0.05 \times 0.44] = 0.872$	
		Therefore, $L_e/L_0 = 0.77$	

EXAMPLE 15.5

The column and beam layout of a four storey building is as shown in Fig. E.15.5. The frames are braced so that they do not resist lateral loads. Find the equivalent heights of the central columns and check whether they should be designed as short or long columns.



(a) Plan (b) Section

Fig. E.15.5.

Method A: Effective Length by Wood's Chart (IS Method) (Assume beam 300 × 525 in *YY*-direction)

Ref.	Step	Calculations				
IS 456 24.1.3	1.	Unsupported lengths (L_0) = (Full length – beam depth)				
			XX-direction	*YY-direction*		
		Roof to 3rd floor	= 3500 – 425 = 3075 mm	3500 – 525 = 2975 mm		
		3rd to 2nd floor	= 3500 – 425 = 3075 mm	3500 – 525 = 2975 mm		
		2nd to 1st floor	= 3500 – 425 = 3075 mm	3500 – 525 = 2975 mm		
		1st to ground floor	= 3500 – 425 = 3075 mm	3500 – 525 = 2975 mm		
SP 24 p. 152	2.	*Section properties*				
		Member	$I \times 10^6$	L	$(I/L)\ 10^3$	$K \times 10^3$
		(a) Column	$\frac{(300 \times 300^3)}{12} = 675$	3500	193	193
		(b) Beam along *XX*	$\frac{250\ (425)^3}{12} = 1600$	6500	246	123*
		(c) Beam along *YY*	$\frac{300\ (525)^3}{12} = 3620$	5500	658	329*

*K for beams for braced frame = 0.5 I/L

EXAMPLE 15.5 (cont.)

Ref.	Step	Calculations
SP 24	3.	*Ratios of stiffness*

$$\beta_1 = \frac{\Sigma K_c}{\Sigma K_c + \Sigma K_b}$$

Joint	β (*XX*-direction)	β (*YY*-direction)
R	$\frac{193}{193 + (2 \times 123)} = 0.44$	$\frac{193}{193 + 2(329)} = 0.23$
3	$\frac{2 \times 193}{(2 \times 193) + (246)} = 0.61$	$\frac{(2 \times 193)}{386 + 658} = 0.37$
2	$\frac{2 \times 193}{(2 \times 193) + (246)} = 0.61$	$\frac{386}{386 + 658} = 0.37$
1	$\frac{2 \times 193}{(2 \times 193) + (246)} = 0.61$	$\frac{386}{386 + 658} = 0.37$
B	$\beta = 1$	$\beta = 1$

(Foundation not designed for bending)

Step 4. *Effective heights* (Read off values from Fig. 24; IS 456, p. 142) Example $\beta_1 = 0.44$, $\beta_2 = 0.61$, Ratio = 0.70

Ref.: IS 456 Fig. 24

	XX-direction			*YY*-direction		
Storey	Ratio	l_0	l_e	Ratio	l_0	l_e
R – 3	0.70	3075	2152	0.59	2975	1755
3 – 2	0.74	3075	2275	0.62	2975	1844
2 – 1	0.74	3075	2275	0.62	2975	1844
1–base	0.85	3075	2614	0.79	2975	2350

Step 5. *Classification*

	XX-direction		*YY*-direction		Classification
Storey	*D*	$l_{e/D}$	*b*	$l_{e/b}$	
R – 3	300	7.17	300	5.85	Short column
3 – 2	300	7.58	300	6.15	Short column
2 – 1	300	7.58	300	6.15	Short column
1 – B	300	8.71	300	7.83	Short column

All columns can be designed as short column.

Method B: Effective Length by Formulae (BS 8110)

Step 1&2. Steps 1 and 2 as in IS method

EXAMPLE 15.5 (cont.)

Ref.	Step	Calculations
	3.	Ratio of stiffness $\alpha = \dfrac{\Sigma I_c}{\Sigma I_B}$

Joint	XX-direction	YY-direction
R	$\dfrac{193}{2 \times 246} = 0.39$	$\dfrac{193}{2 \times 658} = 0.15$
3	$\dfrac{2 \times 193}{2 \times 246} = 0.78$	$\dfrac{2 \times 193}{2 \times 658} = 0.30$
2	$\dfrac{2 \times 193}{2 \times 246} = 0.78$	$\dfrac{2 \times 193}{2 \times 658} = 0.30$
1	$\dfrac{2 \times 193}{2 \times 246} = 0.78$	$\dfrac{2 \times 193}{2 \times 658} = 0.30$
B	1.0	1.0

Note: Stiffness of base taken as 1.0

4. Effective heights – braced column. Lesser of (a) and (b) below.

(a) $l_e/l_0 = 0.7 + 0.05(\alpha_{c1} + \alpha_{c2}) < L_0$

or (b) $l_e/l_0 = 0.85 + 0.05\alpha_{c\,\min} < L_0$

where $\alpha = \dfrac{\Sigma K_{\text{col}}}{\Sigma K_{\text{beam}}}$.

Storey	XX-direction	YY-direction
R – 3	0.7 + 0.05(0.39 + 0.78) = 0.76	0.7 + 0.05(0.15 + 0.30) = 0.72
	0.85 + 0.05(0.39) = 0.87	0.85 + 0.05(0.15) = 0.86
3 – 2	0.7 + 0.05(0.78 + 0.78) = 0.78	0.7 + 0.05(0.30 + 0.3) = 0.73
	0.85 + 0.05(0.78) = 0.89	0.85 + 0.05(0.30) = 0.87
2 – 1	0.7 + 0.05(0.78 + 0.78) = 0.78	0.7 + 0.05(0.3 + 0.3) = 0.73
	0.85 + 0.05(0.78) = 0.89	0.85 + 0.05(0.30) = 0.87
1 – B	0.7 + 0.05(0.78 + 1.0) = 0.79	0.7 + 0.05(0.3 + 1.0) = 0.77
	0.85 + 0.05(0.78) = 0.89	0.85 + 0.05(0.30) = 0.87

Tabulate these values in step 5.

5. *Calculation of effective lengths*

	XX-direction			YY-direction		
Storey	Ratio	l_0	l_e	Ratio	l_0	l_e
R – 3	0.76	3075	2337	0.72	2975	2142
3 – 2	0.78	3075	2399	0.73	2975	2172
2 – 1	0.78	3075	2399	0.73	2975	2172
1 – B	0.79	3075	2429	0.77	2975	2291

EXAMPLE 15.5 (cont.)

Ref.	Step	Calculations
	6.	*Classification (L/D)*

	XX-direction		*YY*-direction		Classification
Storey	*D*	$l_{e/D}$	*b*	$l_{e/b}$	
R – 3	300	7.79	300	7.14	Short column
3 – 2	300	8.00	300	7.24	Short column
2 – 1	300	8.00	300	7.24	Short column
1 – B	300	8.10	300	7.64	Short column

Ref.	Step	Calculations
		All columns can be designed as short column.
		Method C: Effective Length by Coefficients
		1. *BS 8110 method*
Table 15.1		End condition (Beams depth > depth of column) No. 1 at top—No. 1 at bottom l_e/l_0 coefficient = 0.75
IS 456 Table 24		2. *IS 456 method* End conditions—Effectively held in position and restrained against rotation at both ends. l_e/l_0 coefficient = 0.65

REVIEW QUESTIONS

15.1 Define effective length of column.

15.2 What are the methods available to determine effective length of columns in IS 456 ?

15.3 Outline R.F. Wood's method to determine effective length of R.C.C. columns in building frames.

15.4 Outline the BS and ACI formulae to determine effective length of columns. Why is the formula superior to tables and charts for design purposes ?

15.5 What is meant by moment of inertia of a concrete section? What methods are specified by IS code ? (Refer IS 456: clause 21.3).

15.6 What is the difference between "Frames with restraint against-sway" (braced) and "Frames without restraint against sway" (unbraced) ? How does bracing affect effective length of beams and columns ?

15.7 Under what condition will the effective length be infinity and what is the meaning of this condition ?

15.8 Can the effective lengths of the same column be different in the *XX*- and *YY*-axes ?

15.9 When will the effective length be greater than its actual length ?

15.10 Between the different methods available to determine effective length which would you prefer to use and why ?

PROBLEMS

15.1 A portal frame ABCD is composed of columns AB and CD of 7.5 m height and beam BC 11 m in span. It is hinged at A and D. The columns are 300 × 600 mm on which the beam 300 × 850 mm is fixed monolithically. Determine the effective height of the column by means of Wood's chart give in IS 456 assuming the frame to be braced adequately to prevent sway. Calculate the effective length by BS formula also.

15.2 The layout of the framework of an unbraced building is as shown in Fig. P.15.1. Determine the effective length of the middle column above the first floor levels by the BS code formula. Also determine the value using Wood's chart.

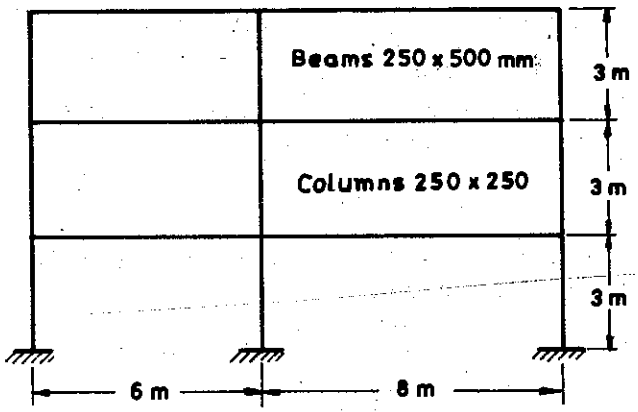


Fig. P.15.1.

15.3 An unbraced framed building consists of three bays of 9 m span. It is three storeys in height, each of 3.6 m. The columns are 250 × 250 mm size and the beams may be assumed as 250 × 600 mm in section. Determine the effective length of the exterior and interior columns of the middle storey

1. By IS 456 Table 24
2. By use of Wood's chart
3. By use of BS 8110 formulae.

15.4 If the frame in Problem 15.3 is braced adequately to prevent relative translation of its joints, determine the modified effective lengths of the column.

16

Design of R.C. Slender Columns

16.1 INTRODUCTION

It was seen in Chapter 14 that in rectangular columns when the ratio of the effective length of the columns to its lateral dimension exceeds 12, it is called a *slender* or *long column* according to IS code. In BS 8110, the limits for a short column are put as 15 for braced and 10 for unbraced columns. The difference between the behaviour of short and slender columns is that, when slender columns are loaded even with axial loads, the lateral deflection (measured from their original centre lines along its length) becomes appreciable, whereas in short columns this lateral deflection is very small and can be neglected as shown in Fig. 16.1. The magnitude of this deflection is denoted by the symbol *e*.

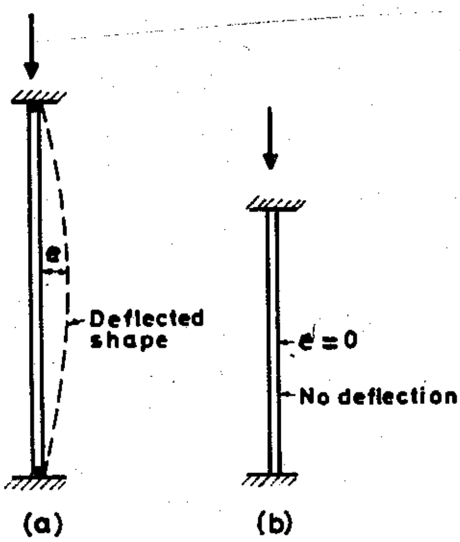


Fig. 16.1 Behaviour of long and short columns: (a) Long column, (b) Short column.

In slender columns the moment produced by this deflection is large and has to be taken into account in design. Hence slender columns, even if centrally loaded, have to be designed for not only the external axial forces acting on them but also for the secondary moment produced by the lateral deflection.

There are three major methods that are used to take into account the slenderness effect of these columns. They are:

1. The reduction coefficient method
2. The additional moment method
3. The moment magnification method.

The reduction coefficient method as given in IS 456 is generally recommended for working stress design where designs are made for service loads (not factored loads) using allowable stresses in steel and concrete. The procedure is to reduce the allowable load by a reduction factor that depends on the slenderness of the column as given in IS 456: clause 45.3.

In the limit state method, however, where one works with factored loads and ultimate strengths of steel and concrete, one of the other two methods is generally recommended. The British and the Indian codes specify the use of the additional moment method as given in IS 456: clause 38.7.1, whereas ACI recommends the use of moment-magnification method. In this Chapter the design of slender columns by the additional moment method is discussed in detail and the basic principles of the moment magnification method are briefly explained.

16.2 MAXIMUM PERMITTED LENGTH OF COLUMNS

It has already been pointed out in Chapter 13 that in order to avoid buckling failures, IS 456: clause 24.3.1 limits the unsupported length between restraints to 60 times the least lateral dimension. In practice, the unsupported length to breadth ratio of column is restricted to 60 in braced columns and 30 in unbraced columns. This restriction will ensure that the final failure will be due to material failure only and the classical buckling failure will be avoided.

16.3 BASIS OF ADDITIONAL MOMENT METHOD

As already pointed out, slender columns, even when loaded axially, produce moments along their length due to lateral deflection. If e_{max} is the maximum deflection, along the axis of the column, the moment produced by this deflection is given by the expression

$$M_{max} = Pe_{max}$$

This is the additional moment that has to be added to the external moments to obtain the design moment. This additional moment is usually designated as M_{add} or M_a so that

$$M_{add} = Pe_{add}$$

where

$$e_{add} = e_{max}$$

The analysis for e_{add}, which is a second order analysis, is also called P-Δ analysis. In unbraced frames such an analysis should also include the effect of sway deflections. However, this aspect is not dealt with in this text and is found in literature on structural analysis of frames.

16.4 EXPRESSION FOR LATERAL DEFLECTION

In theory of structures the elastic deflection of a bending member is represented by the formula

$$\frac{d^2y}{dx^2} = \frac{M}{EI} = \frac{I}{R}$$

$(1/R)$ is called the curvature, which may be defined as the change in slope over unit length of the member, assuming it to be constant along this length. Integrating the above expression twice, we get

$$y = \iint \left(\frac{1}{R}\,dx\right) dx = \iint \left(\frac{M}{EI}\,dx\right) dx$$

This means that the deflection depends on the distribution of curvature or M/EI diagram, along the length of the member.

Assuming that the moment curvature relationship is linear and is not influenced by axial load, one can express the maximum deflection in terms of $(1/R)_{\max}$. Thus for a UDL on a simply supported beam with a parabolic distribution of bending moment, the maximum deflection can be expressed as

$$e_{\max} = \frac{5}{384}\left(\frac{wL^4}{EI}\right) = \frac{(5)8}{384}\left(\frac{wL^2}{8}\right)\frac{L^2}{EI}$$

$$= \frac{(M_{\max})}{EI}\,\frac{L^2}{9.6}$$

$$= \frac{L^2}{9.6}\,(1/R)_{\max}$$

Taking a very conservative estimate of a uniform rectangular distribution of the bending moment Fig. 16.2 and working out the deflection as above, we get

$$e_{\max} = \frac{L^2}{8}\,(1/R)_{\max}$$

Taking a very unconservative estimate of a triangular distribution of the bending moment diagram, we obtain

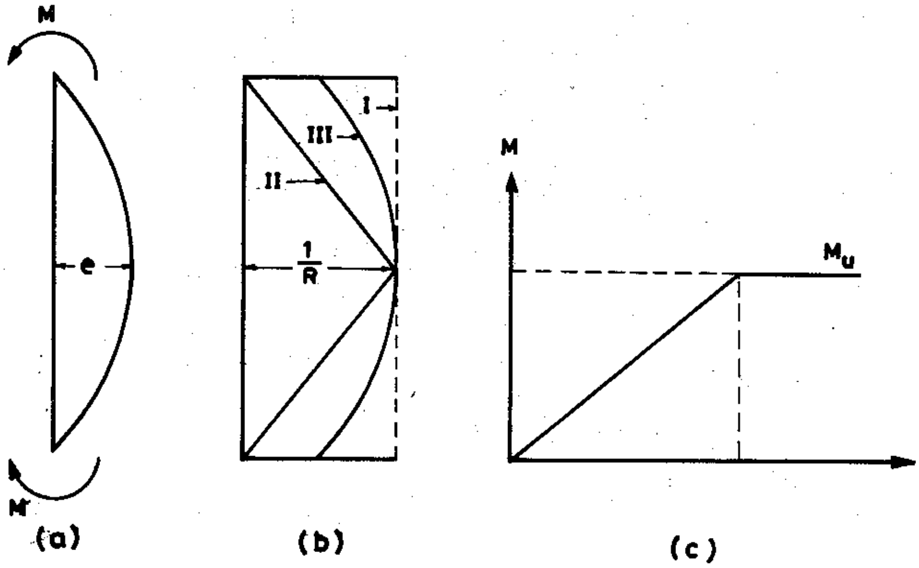


Fig. 16.2 Curvature of bent columns: (a) Deflected shape at ultimate load; (b) Curvature (M/EI) diagrams showing rectangular, triangular, and assumed probable distribution; (c) Linear moment-curvature relationship.

$$e_{\max} = \frac{L^2}{12}(1/R)_{\max}$$

Hence, one may assume that a reasonable estimate of $e_{\max}$ can be obtained from the expression

$$e_{\max} = \frac{L^2}{11}(1/R)_{\max}$$

Thus the eccentricity to be taken into account for calculation of the additional moment is equal to

$$e_{\text{add}} = \frac{L_e^2}{11}(1/R)_{\max}$$

where L_e is the effective length of the column.

Assuming that the bending strains at failure (ultimate conditions) are as given in Fig. 16.3 (balanced failure), we have

$$(1/R)_{\max} = \frac{\varepsilon_c + \varepsilon_s}{D} = \frac{0.0035 + 0.002}{D} = \frac{1}{182D}$$

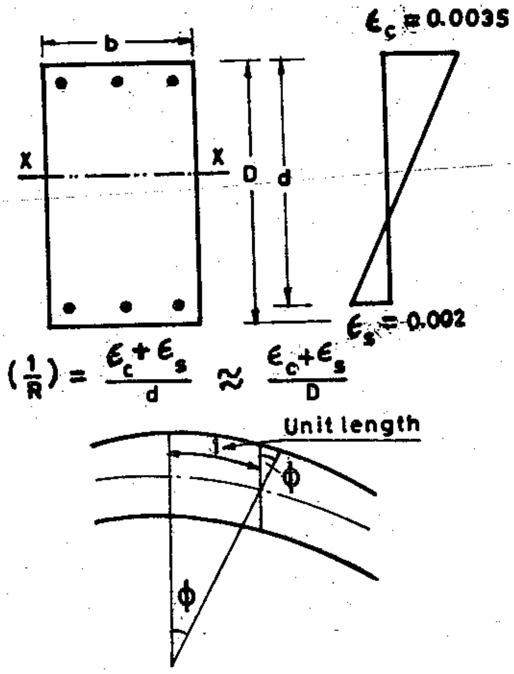


Fig. 16.3 Curvature-strain relationship.

Substituting this value, we get

$$e_{\text{add}} = \frac{L_e^2}{11}\left(\frac{1}{182D}\right)$$

This can be approximated to the value given in IS 456: clause 38.7.1.

$$e_{\text{add}} = \frac{D}{2000}\left(\frac{L_e}{D}\right)^2$$

Both IS 456 and BS 8110 use the reduced form of this equation, viz.

$$\frac{e_{\text{add}}}{D} = \frac{1}{2000}\left(\frac{L_e}{D}\right)^2$$

Putting the right-hand side expression as β, we have

$$e_{\text{add}} = \beta D$$

Therefore,

$$M_{\text{add}} = M_a = Pe_{\text{add}}$$

The value of β for (L_e/D) ratios can be read off from Table I of SP 16 or calculated from the above formula. If the value of additional moment on the XX-axis is designated by M_{ax} and that on YY-axis as M_{ay}, they can be calculated by the formulae

$$M_{ax} = \frac{PD}{2000}\left(\frac{L_{ex}}{D}\right)^2$$
$$M_{ay} = \frac{Pb}{2000}\left(\frac{L_{ey}}{b}\right)^2 \qquad (16.1)$$

where

P = axial load on the member
L_{ex} = effective length in respect to major axis
L_{ey} = effective length in respect to minor axis
D = depth at right angles to the X-axis
b = breadth at right angles to the Y-axis

16.5 REDUCTION FACTOR FOR ADDITIONAL MOMENT

It is quite obvious from physical considerations that the lateral deflection of a column must be less when a large portion of the column section is in compression. The expression for $e_{\max}$ derived above was on the assumption that the curvature is the one corresponding to the balanced failure where the maximum strain in tension steel $\varepsilon_s = 0.002$ and the maximum compression strain in concrete $\varepsilon_c = 0.0035$ at failure. For any value of P for which the strain ε_s is less than that of balanced failure, the deflection and hence the additional moment should also be less. Even though IS has made this modification optional, this reduction in many cases can be substantial, and should be taken into account from economy point view. The reduced deflection can be expressed by the reduction factor k given by IS 456: clause 38.7.1.1 as

$$k = \frac{P_z - P}{P_z - P_b} \le 1$$
$$k = \frac{1 - P/P_z}{1 - P_b/P_z} \le 1 \qquad (16.2)$$

where

k = the reduction factor that corrects the deflection in column due to axial load
P = axial load on the member
P_b = axial load corresponding to balanced failure
P_z = ultimate axial load the column can carry

The value of P_z is given by

$$P_z = 0.45 f_{ck} A_c + 0.75 f_y A_s$$

By putting $A_c = A_g$, we get

$$\frac{P_z}{A_g} = \left(0.45 f_{ck} + 0.75 f_y \frac{p}{100}\right)$$

SP 16 Chart 63 gives the value of P_z based on this equation and is given as Chart 16.1 in this text.

This chart is slightly different from the design charts 24 to 26 of SP 16 for ultimate axial loads P_u in columns where a reduction in value of P_z was introduced to account for accidental eccentricity.

The derivation of this formula for k is clear from the shape of the interaction diagram shown in Fig. 16.4. The value of k will depend on P/P_z and P_b/P_z ratios.

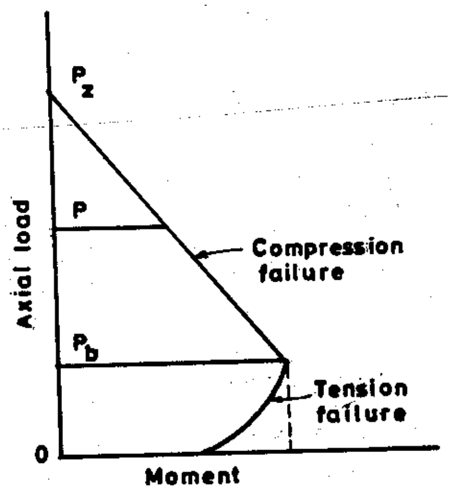


Fig. 16.4 Interaction diagram for columns.

By applying the reduction factor, the value of the additional moment is given by

$$M_{add} = Pke_{add} \tag{16.3}$$

16.5.1 CALCULATION OF P_b

An approximate expression for P_b, the load corresponding to balanced failure for rectangular sections with steel on the two opposite sides, can also be derived by assuming the bending stress distribution for balanced failure. If A_{s1} and A_{s2} are the tension and compression steel areas and f_{s1} and f_{s2} the stresses in them (assuming d_c as the depth of neutral axis and d as effective depth), we get

$$P_b = 0.4 f_{ck} b d_c + f_{s2} A_{s2} - f_{s1} A_{s1}$$

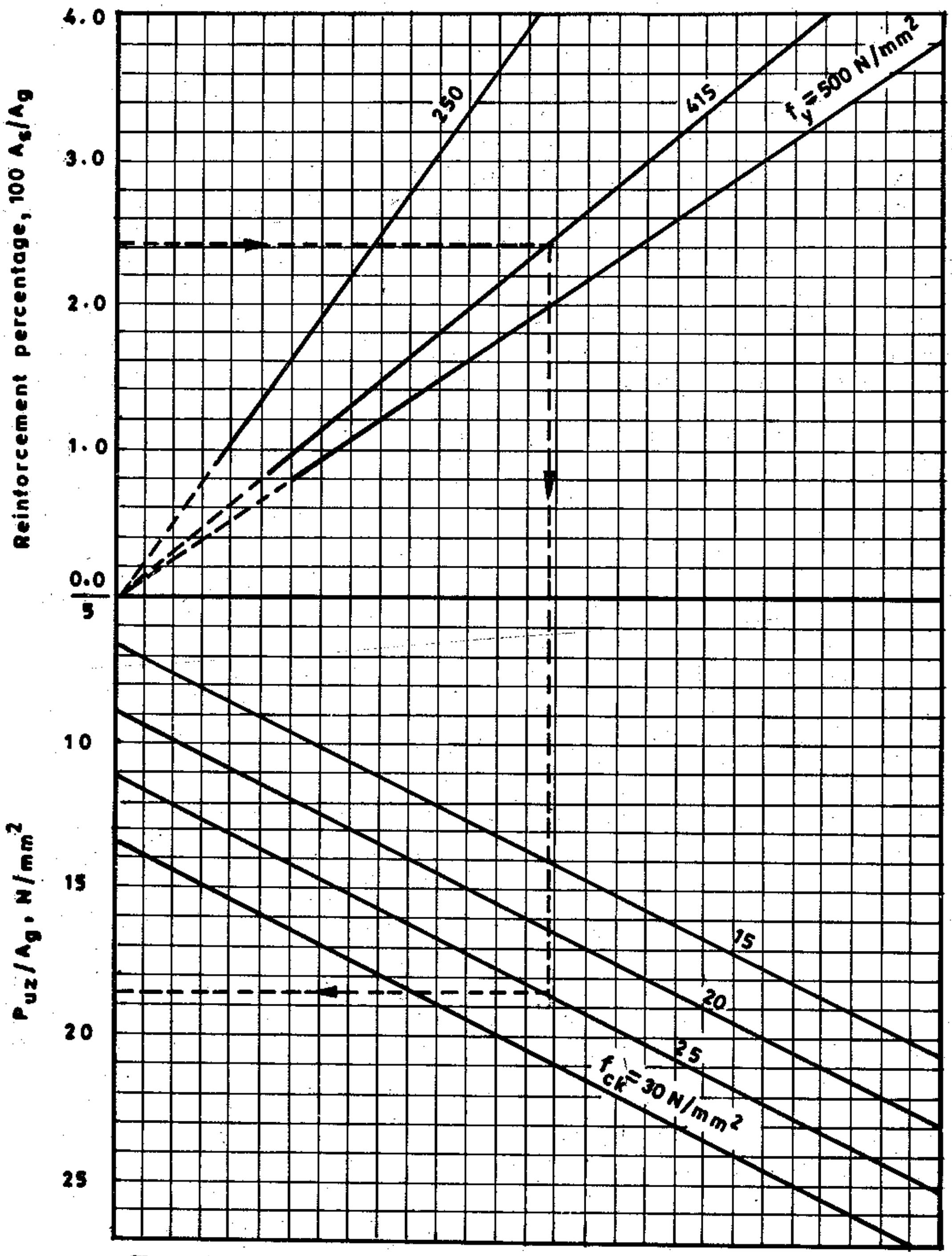


Chart. 16.1 Theoretical value of P_z in columns (Chart 63 of SP 16).

where

$$\frac{d_c}{d} = \frac{0.0035}{0.0035 + 0.002}$$

$$d_c = 0.636d$$

Assuming tension steel and compression steels are equal and are equally stressed so that $f_{s2}A_{s2} = f_{s1}A_{s1}$ for a section where equal steel is put on the two opposite sides, we have

$$P_b = (0.4f_{ck}b)\ (0.636d) = 0.254f_{ck}bd$$

16.5.2 USE OF SP 16 FOR DETERMINATION OF P_b

Table 60 on page 171 of SP 16 (Table 16.2 of the text) gives a more accurate method to calculate P_{bal}, especially when steel is distributed on all faces. It uses an expression in the following forms:

1. *For rectangular section*

$$\frac{P_b}{f_{ck}bD} = \left(k_1 + k_2\frac{p}{f_{ck}}\right) \tag{16.4}$$

2. *For circular section*

$$\frac{P_b}{f_{ck}D^2} = \left(k_1 + k_2\frac{p}{f_{ck}}\right) \tag{16.4a}$$

k_1 and k_2 can be read off from the Table 16.1, where p is the percentage of the total steel in the column.

TABLE 16.1 VALUES OF k_1 AND k_2 FOR VALUES OF P_b
(SP 16, Table 60)

	Section	d'/D			
		0.05	0.10	0.15	0.20
Value of k_1	Rectangular	0.219	0.207	0.196	0.184
	Circular	0.172	0.160	0.149	0.138
	Rectangular				
	Equal steel on two sides	0.096	0.082	0.046	– 0.022
Value of k_2 for Fe 415 steel	Equal steel on four sides	0.424	0.328	0.203	0.028
	Circular	0.410	0.323	0.201	0.036

The value of P_b as obtained above is slightly different from that obtained by the method described in Section 14.6.4.

It may however be noted that the value of k cannot be estimated until the area of the steel in the section is known. Hence, one has to initially assume a percentage of steel and determine the value of k and check for safety by successive approximation.

Having obtained P_b as above, Chart 65 on page 150 of SP 16 (Chart 16.2 here) gives a quick method of obtaining k, the reduction factor from P/P_z and P_b/P_z values.

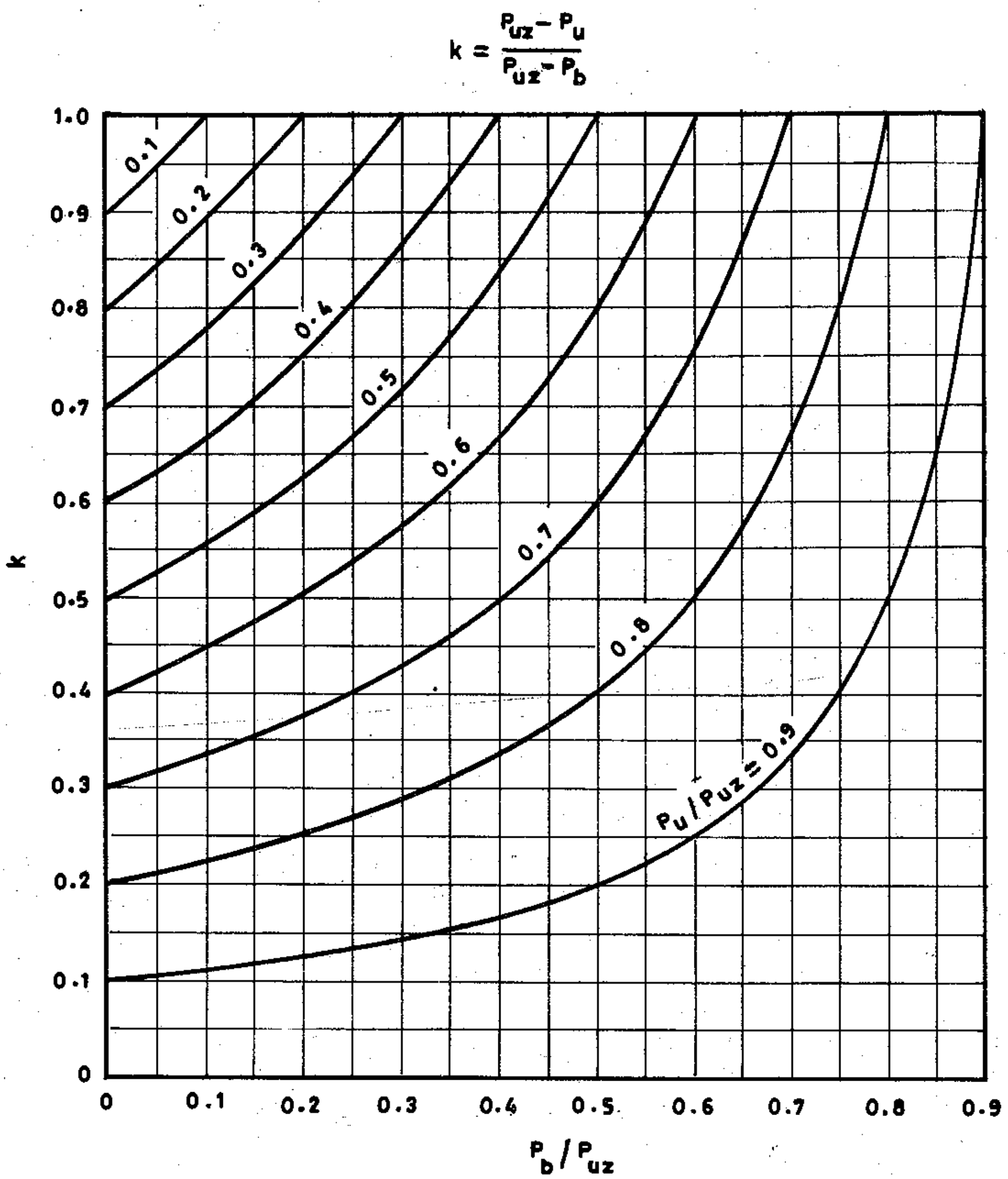


Chart 16.2 Reduction factor *k* for additional moment in columns (Chart 65 of SP 16).

16.6 FACTORS AFFECTING BEHAVIOUR OF SLENDER COLUMNS

As already discussed, columns are usually classified as one of the following types:

1. Pin-ended columns
2. Braced columns
3. Unbraced columns.

They can also bend in single curvature or double curvature. The various types of bending that can occur in column is shown in Figs. 16.5–16.10. Analysis of structures like building frames gives the value of the axial load and the moments at top and bottom of the columns. The resulting bending moment diagram will indicate whether the column is bent in single or double curvature. If the moments on the two ends are opposite in sign, the column bends in single curvature.

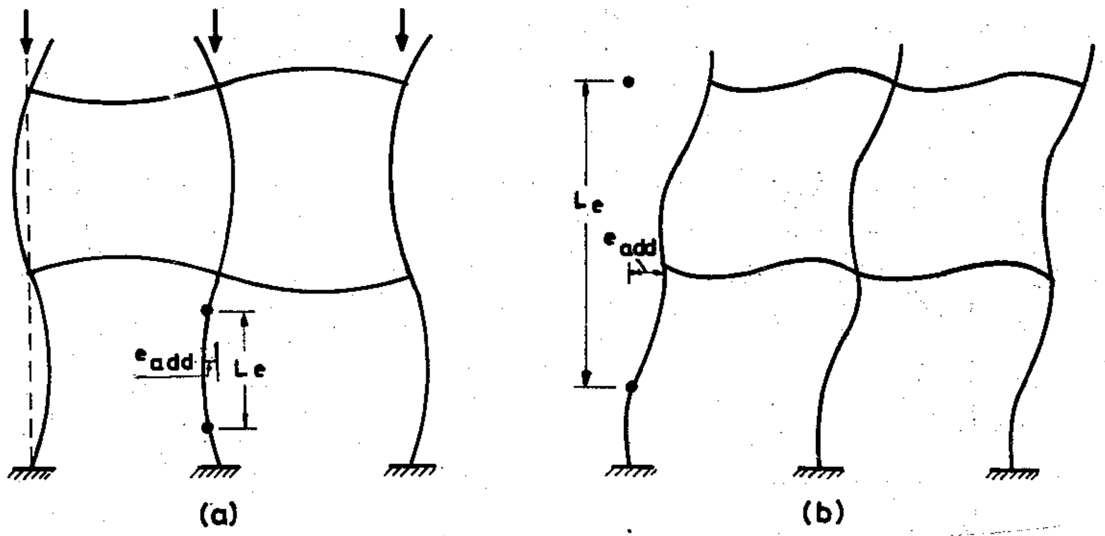


Fig. 16.5 Bending of columns in frames: (a) Braced, (b) Unbraced.

In designing slender columns to get the total moment M_t for final design, one has to determine the combined effects of all the following three factors:

1. The initial moment M_i caused by the end moments, M_1 and M_2. (The larger value is taken as positive and designated as M_2. If the bending is in double curvature, M_1 is taken as negative.)
2. Moment due to accidental eccentricity usually designated by the term M_{min} or M_m.
3. Additional moment = M_{add} or M_a.

The magnitude of M_i will depend on whether the columns are in single or double curvature and whether they are braced or unbraced as explained in notes 1 and 2 of IS 456: clause 38.7.1. The Explanatory Handbook on IS 456, SP 24, Section 38.7 may also be referred to for more details. Some of the salient features of behaviour of slender columns are now explained.

16.7 DESIGN MOMENT IN BRACED COLUMNS WITH INITIAL MOMENTS

The distribution of moments over the height of the column for the different cases of applied moments in typical braced columns are shown in Figs. 16.6–16.8. Let

M_2 = the larger end moment obtained from structural analysis, and

M_1 = the smaller end moment obtained from structural analysis, and is taken as negative if the column is bent in double curvature.

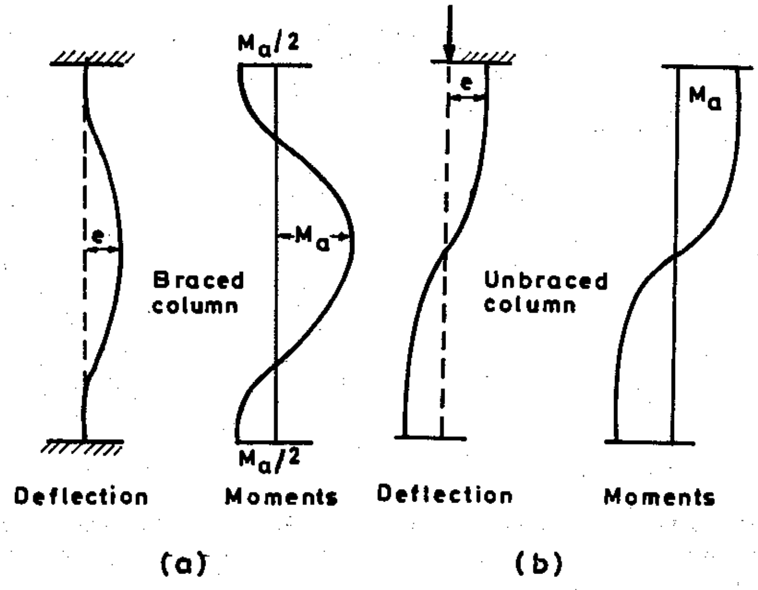


Fig. 16.6 Column bracing affecting position of e_{max}: (a) Braced (e_{add} max at middle), (b) Unbraced (e_{max} at ends).

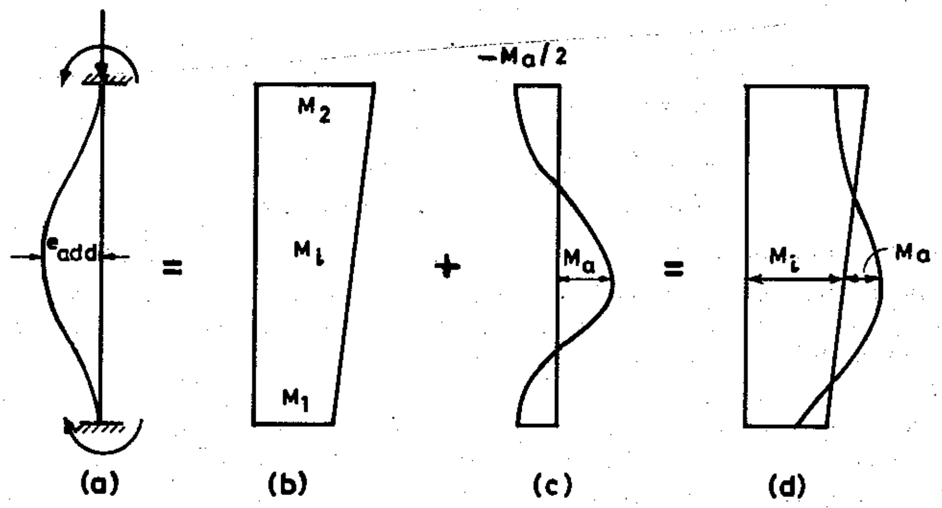


Fig. 16.7 Distribution of moments in braced columns with end moments bent in single curvature: (a) End condition, (b) Initial moment M_i, (c) Additional moment, (d) Design moment.

Then the expression for the value of the initial moment M_i for braced column due to M_2 and M_1 according to IS 456: clause 38.7.1, note 1 is

$$M_i = 0.4M_1 + 0.6M_2 \text{ (taking care of the signs)}$$

but not less than $0.4M_2$.

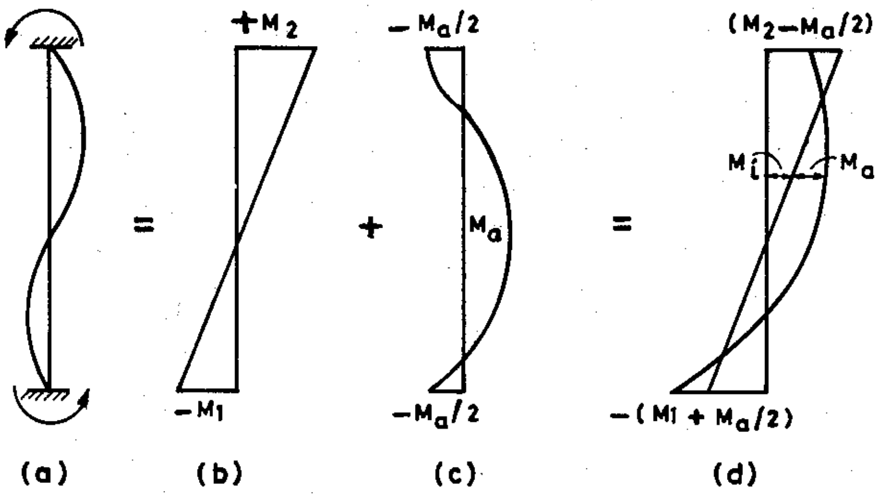


Fig. 16.8 Distribution of moments in braced columns with end moments bent in double curvature: (a) End condition, (b) Initial moments M_i, (c) Additional moment, (d) Design moment.

Thus the design moment M_t should be larger of the four following values as can be seen from the bending moment envelopes

$$M_t = M_2 \qquad \text{(at } M_2 \text{ end)}$$

$$M_t = M_1 + \frac{1}{2} M_{add} \qquad \text{(at } M_1 \text{ end)}$$

$$M_t = M_i + M_{add} \qquad \text{(at intermediate points)}$$

$$M_t = M_{min} \qquad \text{(at intermediate points)}$$

The following cases can occur.

1. Braced columns bent in single curvature The conditions that exist in a braced column that bend in single curvature are shown in Fig. 16.7. The moments to be considered are also indicated in the figure. M_1 and M_2 are both taken as positive as the column is bent in single curvature.

2. Braced column bent in double curvature The behaviour of braced columns bent in double curvature and the corresponding moments along the length of the column are shown in Fig. 16.8. M_1 and M_2 are opposite in signs as the column is bent in double curvature, the numerical larger moment being taken as M_2.

16.8 DESIGN MOMENTS IN UNBRACED COLUMNS

The distribution of additional moment over the height of unbraced columns is shown in Figs. 16.9 and 16.10. The maximum lateral deflection occurs at the middle of the 'equivalent column', which will be at the end of the real column. Hence, the maximum additional moment also occurs at the ends. The end which has the stiffer joint will have the maximum M_i value.

Unbraced columns at any given level subject to lateral loads usually deflect equally so that

Sagar

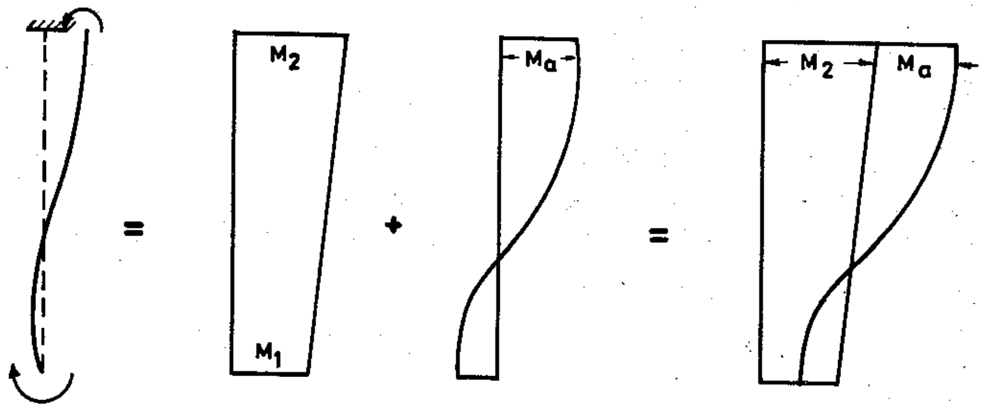


Fig. 16.9 **Distribution of moments in unbraced columns with end moments bent in single curvature.**

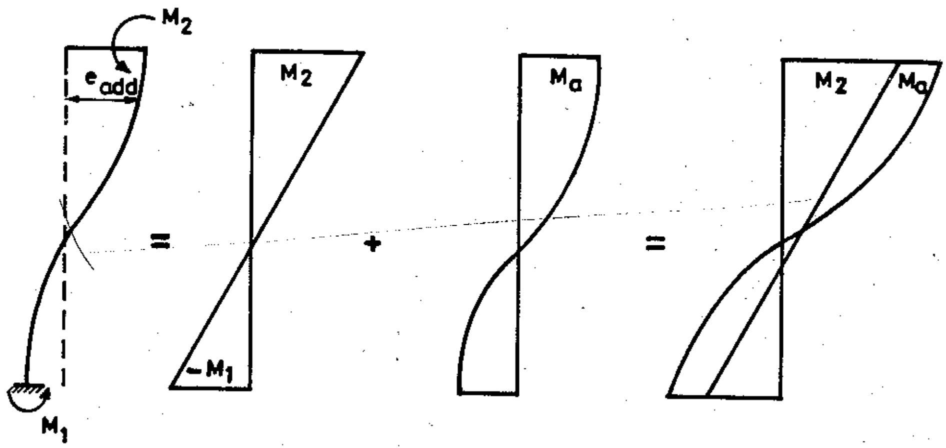


Fig. 16.10 **Distribution of moments in unbraced columns with end moments bent in double curvature.**

the average deformation may be taken as (denoting deformation by e)

$$e_{av} = \frac{\Sigma e_1}{n}$$

$$e_1 = e_{\text{add}} \text{ calculated for each column}$$

where

$$\Sigma e_1 = \text{sum of deflection of all the bays}$$

$$n = \text{number of bays}$$

The total design moment is given by

$$M_t = M_2 + M_{\text{add}}$$

The conditions to be considered in the case of unbraced column bent in single curvature and double curvature respectively are shown in Figs. 16.9 and 16.10.

16.9 SLENDER COLUMNS BENT ABOUT BOTH AXES

When slender columns are subjected to significant bending moments about both the axes, additional moments are to be calculated for both directions of bending. These additional moments are to be combined with the initial moments to obtain the total design moments in the principal directions. However, the 'minimum eccentricity' has to be assumed to act only about one axis at a time. With moments acting on both the axes, the column should be designed for biaxial bending.

16.10 DESIGN PROCEDURE

After obtaining the values of P and M_t in the principal planes of bending (XX and YY directions), the column is designed for the effect of these direct load and moment (as columns subjected to P and M) as explained in Chapter 15. This is illustrated by examples given at the end of this chapter. The various steps to be followed are now given:

Step 1: Determine the effective length and the slenderness ratio in the XX and YY axes. If the slenderness is 12 or above, about any of the axes, it is to be designed as a long column about that axis. If it is slender on both axes, the moments about both the axes should be considered.

Step 2: Calculate the additional moment M_{add}. Then

$$M_{ax} = Pe_{ax} = \frac{PD}{2000}\left(\frac{L_{ex}}{D}\right)^2$$

$$M_{ay} = Pe_{ay} = \frac{PB}{2000}\left(\frac{L_{ey}}{b}\right)^2$$

Step 3: From structural analysis determine end moments M_1 and M_2.

Step 4: Determine the value of M_i, taking due care whether it is braced or unbraced and bent in single or double curvature.

Step 5: Determine the moments caused by accidental eccentricity $M_{\min}$.

Step 6: Choose the design moment as the greater of the following values at the ends and the intermediate points. For a braced column:

$$\begin{aligned} M_t &= M_2 \\ &= M_1 + \frac{1}{2} M_{\text{add}} \\ &= M_i + M_{\text{add}} \\ &= M_{\min} \end{aligned}$$

Step 7: Make a preliminary design on P and M_t and find the area of steel, as required.

Step 8: Determine the reduction factor k and calculate the modified design value

$$M_{t2} = M_i + kM_{\text{add}}$$

Step 9: If the column is slender on both the axes, determine M_t on both the axes and design the column for biaxial bending for P and M_{xt} about the X-axis with P and M_{yt} about the Y-axis by using the interaction diagrams in SP 16.

16.11 DESIGN PROCEDURE TO DETERMINE k VALUES

As seen from the above procedure, an economical design of long column can be carried out only by trial and error. However, it will be found easier to arrive at a solution if one provides initial dimensions of the concrete section in such a way that the additional moment can be provided by adjusting the steel areas only. It was shown in Section 16.5 that P_b is not very sensitive to the percentage of steel if it is distributed on the two opposite faces only and its value depends mostly on the concrete section selected. Thus, the value of P_b/P_z for 4 per cent of steel (which is usually the steel used in columns) will be of the order of 0.25 only. In this region slight increase in P/P_z considerably decreases the value of k.

In design calculations, it has been suggested to decrease the value of k obtained by the first trial itself by 20 to 30 per cent so that the checking can be done for a lesser value of M_{add}. This procedure tends to reduce the number of trials needed to arrive at the final solution.

16.12 PRINCIPLE OF MOMENT MAGNIFICATION METHOD

The ACI Code uses the moment magnification method for design of slender columns. This method automatically takes into account the additional moment caused by the deflection of a slender elastic (pin ended) beam column in the presence of an axial load. The formulae can be derived as follows: Consider a braced column as shown in Fig. 16.11. Let

δ_0 = maximum 'primary deflection' due to primary 'applied bending moment'

δ_1 = 'maximum additional deflection' due to long column effect

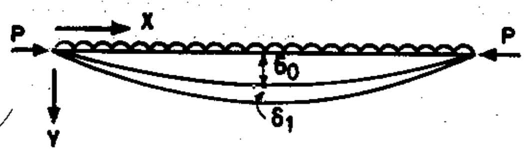


Fig. 16.11 Primary and secondary bending in long columns.

Assuming that the deflections are in the shape of sine curves, the equation to the total deflection curve for bending in single curvature can be represented as follows: y is the deflection and x is the distance along the length of the beam:

$$y = (\delta_0 + \delta_1) \sin \frac{\pi x}{L}$$

As

$$M = Py,$$

we have

$$M = P(\delta_0 + \delta_1) \sin \frac{\pi x}{L}$$

Since the deflection δ_1 is due to the above bending moment (by making use of the moment area or conjugate beam principle), this deflection will be equal to the moment of the $M/(EI)$ diagram between the support and mid-span taken about the support. Hence,

$$\delta_1 = \left[\int_0^{l/2} \frac{P}{EI} (\delta_0 + \delta_1) \sin \frac{\pi x}{L}\, dx\right] \left(\frac{L}{\pi}\right)$$

where L/π is the distance of the C.G. of the sine curve from 0 to $L/2$.

$$\delta_1 = (\delta_0 + \delta_1) \frac{PL^2}{\pi^2 EI}$$

Putting

$$\frac{\pi^2 EI}{L^2} = P_c \text{ (the Euler load)}$$

we obtain

$$\delta_1 = (\delta_0 + \delta_1) \frac{P}{P_c}$$

Again, putting $P/P_c = \alpha$, we get

$$\delta_1 = \frac{\delta_0 \alpha}{1 - \alpha}$$

$$\alpha_{\max} = \delta_0 + \frac{\delta_0 \alpha}{1 - \alpha} = \frac{\delta_0}{1 - \alpha} \qquad (16.5)$$

Putting

M_t = maximum design moment due to all effects

M_m = applied moment on the column from elastic analysis

M_t = (applied moment) + (additional moment due to the deflection of the beam column)

$$M_t = M_m + \frac{P\delta_0}{1 - \alpha}$$

$$= \frac{M_m}{1 - \alpha}\left(1 + \frac{P_c \alpha}{M_a} \delta_0 - \alpha\right)$$

$$= \frac{M_m}{1 - \alpha}\left[1 + \left(\frac{P_c \delta_0}{M_a} - 1\right)\alpha\right]$$

The expression within the brackets can be denoted as C_m.

$$M_{\max} = M_m \left(\frac{C_m}{1 - \alpha}\right) = M_m \delta_m \qquad (16.6)$$

The factor $[C_m/(1 - \alpha)] = \delta_m$.

δ_m is called the moment magnifier which amplifies the column moment to account for the effect of the axial load on these moments.

In the revised ACI 318 (83) code the moments due to gravity loads and lateral loads have been clearly separated for braced frames and the moment magnifier has been applied separately so that

$$M_{total} = M_t = M_g\delta_b + M_s\delta_s \qquad (16.7)$$

where δ_b for gravity loads and δ_s for sway moment are taken separately. The total magnified moment due to P is directly given by Eq. (16.7).

Thus, while the ACI method directly gives the total magnified design moment M_t, in the IS method the total moment has to be obtained by adding the additional moment to the initial moment as shown in Fig. 16.12. The additional moment method is more fundamental in its approach than the moment magnification method.

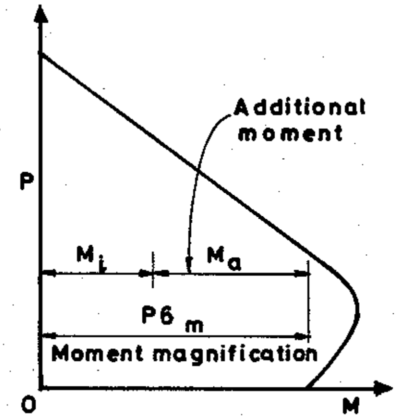


Fig. 16.12 Moment magnification method.

EXAMPLE 16.1 **(Column slender about the minor axis)**

An R.C. column 500 × 400 mm is subjected to an axial ultimate load of 2500 kN and bent in single curvature about the minor axis with $M_{y\,(top)} = 90$ kNm and $M_{y\,(bottom)} = 120$ kNm as ultimate moments. If $L_0 = 7.2$ m and $L_e = 5.75$ m on both axes, calculate the design moments for the column.

Step	Bending about *XX*	Bending about *YY*
1.	*Calculation of slenderness (IS 456: Cl. 24.1.2)*	
	$\frac{L_e}{D} = \frac{5750}{500} = 11.5 < 12$	$\frac{L_e}{b} = \frac{5750}{400} = 14.3 > 12$
	Column is short on the *XX*-axis	Column is long on the *YY*-axis
2.	*Calculation of e_{min} and M_{min} (IS 456: Cl. 24.4)*	
	$e_{min} = \frac{L_0}{500} + \frac{D}{30} \not< 20$ mm	
	$e_{min} = \frac{7200}{500} + \frac{500}{30}$ = 31.1 mm > 20 mm	$e_{min} = \frac{7200}{500} + \frac{400}{30}$ = 27.7 mm > 20 mm

EXAMPLE 16.1 (cont.)

Step	Bending about *XX*	Bending about *YY*
	$M_{\min} = \dfrac{2500 \times 31.1}{1000} = 77.8$ kNm	$M_{\min} = \dfrac{2500 \times 27.7}{1000} = 69$ kNm
3.	*Find initial moment M_i (IS 456: Cl. 38.7.1 Note 1)*	
	$M_i = 0$	$M_i = (0.6M_2 + 0.4M_1) \nless 0.4M_2$ $M_i = (0.6 \times 120 + 0.4 \times 90)$ $= 108$ kNm
4.	*Calculation of e_a and M_a (IS 456: Cl. 38.7.1)*	
	$e_a = 0$	$\dfrac{e_a}{b} = \left(\dfrac{L_e}{b}\right)^2 \Big/ 2000 = 0.10$ $= (14.3)^2/2000 = 0.10$ $e_a = 0.1 \times 400 = 40$ mm
	Additional moment M_a	
	$M_a = 0$	$M_a = Pe_aK$ (assume $K = 1$) $M_a = \dfrac{2500 \times 40}{1000} = 100$ kNm
5.	*Find design moment M_t* (as greater of the following loads according to BS 8110):	
	(a) $M_t = M_2 = 0$	(a) $M_t = 120$ kNm
	(b) $M_t = M_1 + \dfrac{M_a}{2} = 0$	(b) $M_t = 90 + \dfrac{100}{2} = 140$ kNm
	(c) $M_t = M_i + M_a = 0$	(c) $M_t = 108 + 100 = 208$ kN
	(d) $M_t = M_{\min} = 77.8$ kNm	(d) $M_t = 69$ kNm
	Design the slender column for the following loads as shown in Example 16.3.	
	$M_{tx} = 77.8$ kNm $P = 2500$ kN	$M_{ty} = 208$ kNm $P = 2500$ kN

EXAMPLE 16.2 **(Column slender on both axes)**

An R.C. braced column 300 × 500 mm with L_0 = 9 m, L_e = 6.75 m has $M_{y\,(\text{top})}$ 70 kNm and $M_{y\,(\text{bottom})}$ 10 kNm as ultimate moments. P_u = 1700 kN. If the column is bent in double curvature, determine the design moments (*YY* is the minor axis).

Step	Bending about *XX*	Bending about *YY*
1.	*Calculation of slenderness (IS 456: Cl. 24.1.2)* $\frac{L_e}{D} = \frac{6750}{500} = 13.5 > 12$ Slender about *XX*	$\frac{L_e}{b} = \frac{6750}{300} = 22.5 > 12$ Slender about *YY*
2.	*Calculation of $e_{\min}$ and $M_{\min}$ (IS 456: Cl. 24.4)* $e_{\min} = \frac{L_0}{500} + \frac{D}{30}$ $= \frac{9000}{500} + \frac{500}{30} = 34.7 \text{ mm} > 20$ $M_{\min} = \frac{1700 \times 34.7}{1000} = 59.0 \text{ kNm}$	$e_{\min} = \frac{9000}{500} + \frac{300}{30} = 28 \text{ mm} > 20$ $M_{\min} = \frac{1700 \times 28}{1000} = 47.6 \text{ kNm}$
3.	*Find initial moment M_i (IS 456: Cl. 38.7.1, Note 1)* $M_i = (0.6M_2 + 0.4M_1) \nless 0.4M_2 = 0$	$M_2 = 70, \quad M_1 = -10$ $M_i = (0.6 \times 70 - 0.4 \times 10) = 38 \text{ kNm}$ Also, $M_i = 0.4 \times 70 = 28 \text{ kNm}$ Use $M_i = 38$ kNm
4.	*Find additional eccentricity and additional moment (IS 456: Cl. 38.7.1)* For $L_e/D = 13.5$, $\frac{e}{D} = (L_{e/D})^2/2000 = 0.091$ Additional moment = Pe $M_a = \frac{(0.091 \times 500)1700}{1000}$ $= 77.4 \text{ kNm}$	$\frac{e}{b} = (22.5)^2/2000 = 0.253$ $M_a = \frac{1700 \times 0.253 \times 300}{1000} = 129 \text{ kNm}$

EXAMPLE 16.2 (cont.)

Step	Bending about XX	Bending about YY
5.	*Calculate design moment M_t (largest of the following values as per BS 8110)*	
	(a) $M_t = M_2 = 0$	(a) $M_t = 70$
	(b) $M_t = M_1 + \frac{M_a}{2} = \frac{77.4}{2} = 38.7$	(b) $M_t = 10 + \frac{129}{2} = 74.5$
	(c) $M_t = M_i + M_a = 77.4$	(c) $M_t = 38 + 129 = 167$
	(d) $M_t = M_{min} = 59.0$	(d) $M_t = 47.6$
6.	Design column for $p = 1700$, $M_x = 77.4$ and $M_y = 167$ kNm, as in Example 16.3.	

EXAMPLE 16.3

Design the longitudinal steel for a braced column 500 × 400 mm bent in single curvature with $f_{ck} = 40$ N/mm² and $f_y = 415$ N/mm² and the following data: Effective length about both axes = 6.5 m, unsupported length = 7 m. Factored load = 2500 kN, factored moments on major axis are 250 kNm at the top and 200 kNm at bottom. The factored moments about minor axis are 120 kNm at top and 100 kNm at the bottom.

Step	Bending about XX	Bending about YY
1.	*Calculate slenderness of column*	
	$L_e/D = \frac{6500}{500} = 13 > 12$ – long	$L_e/b = \frac{6500}{400} = 16.25$ – long
2.	*Calculate e_{min} and M_{min} (IS 456: Cl. 24.4)*	
	$e_{min} = \frac{L_0}{500} + \frac{D}{30}, \frac{D}{20}$ or 20 mm, whichever is greater.	
	$\frac{7000}{500} + \frac{500}{30} = 30.7$	$\frac{7000}{500} + \frac{400}{30} = 27.3$
	$\frac{D}{20} = \frac{500}{20} = 25.0$	$\frac{b}{20} = \frac{400}{20} = 20$
	Adopt $e_{min} = 30.7$ mm	Adopt $e_{min} = 27.3$ mm
	$M_{min} = \frac{2500 \times 30.7}{1000} = 76.8$ kNm	$M_{min} = \frac{2500 \times 27.3}{1000} = 68.3$ kNm

EXAMPLE 16.3 (cont.)

Step	Bending about XX	Bending about YY
3.	*Find initial moment*	
	$(M_i = 0.4M_1 + 0.6M_2) \nless 0.4M_2$	
	$M_{ix} = 0.4 \times 200 + 0.6 \times 250$ $= 230$ kNm	$M_{iy} = 0.4 \times 100 + 0.6 \times 120$ $= 112$ kNm
4.	*Find e_a and M_a (SP 16, p. 106 or Table 16.1)*	
	For $\frac{L_e}{D} = 13$, we have $\frac{e_a}{D} = 0.085$	$\frac{L_e}{b} = 16.25$, $\frac{e_a}{b} = 0.132$
	By formula $e_a = \frac{D}{2000}\left(\frac{l_e}{D}\right)^2$,	
	$e_a = 0.085 \times 500 = 42.5$ mm	$e_a = 0.132 \times 400 = 52.8$ mm
	$M_{ax} = \frac{2500 \times 42.5}{1000} = 106.25$ kNm	$M_{ay} = \frac{2500 \times 52.8}{1000} = 132$ kNm
5.	*Find design moment M_t, taking greater of the following values ($K = 1$):*	
	(a) $M_t = M_2 = 250$ kNm	(a) $M_t = 120$ kNm
	(b) $M_t = M_1 + \frac{M_a}{2} = 200 + \frac{106}{2} = 253$	(b) $M_t = 100 + \frac{132}{2} = 166$ kNm
	(c) $M_t = M_i + M_a = 230 + 106 = 336$	(c) $M_t = 112 + 132 = 244$ kNm
	(d) $M_t = M_{min} = 76.8$ kNm	(d) $M_t = 68.3$ kNm
	Adopt $M_{tx} = 336$ kNm	Adopt $M_{ty} = 244$ kNm
	$P = 2500$ kN, $M_x = 336$ kNm, $M_y = 244$ kNm	
6.	*Design for biaxial bending with $K = 1$ (Use BS 8110 method)*	
	$\frac{P}{f_{ck}bD} = \frac{2500 \times (10)^3}{40 \times 500 \times 400} = 0.31$	
	$\alpha = \left[1 - \frac{7}{6}\left(\frac{P}{f_{ck}bD}\right)\right] = 0.64$	
	$\frac{M_x}{d} = \frac{336 \times 10^6}{450} = 746 \times 10^3$	
	$\frac{M_y}{b'} = \frac{244 \times 10^6}{350} = 697 \times 10^3$	

EXAMPLE 16.3 (cont.)

Step	Bending about *XX*	Bending about *YY*
	M_x controls the design.	
	$M'_x = 336 + 0.64\left(\dfrac{244}{350}\right) \times 450$	
	$= 536$ kNm	
7.	*Using SP 16, find the steel required*	
	$\dfrac{d'}{D} = \dfrac{50}{500} = 0.10, \quad \dfrac{P}{f_{ck}bD} = 0.31$	
	$\dfrac{M'_x}{f_{ck}bD^2} = \dfrac{536 \times 10^6}{40 \times 400 \times 500 \times 500} = 0.13$	(Use chart 44)
	$\dfrac{P}{f_{ck}} = 0.09$ or $p = 0.09 \times 40$	
	$= 3.6$ per cent	
8.	*Assuming 3.6 per cent steel, work out the reduction factor K*	
	$K = \dfrac{P_z - P}{P_z - P_b}$	$P_{uz} = 0.45 f_{ck} A_c + 0.75 f_y A_s$
	$P_z = 400 \times 500 \left[0.45 \times 40 + \left(\dfrac{3.6}{100}\right) \times 0.75 \times 415\right] \times 10^{-3}$	
	$= 5841$ kN	$P_z = 5841$ kN
	$P_b = 0.254 f_{ck} bd$ (as in a beam)	
	$= 0.254 \times 40 \times 400 \times 450 \times 10^{-3}$	$P_b = 0.254 \times 40 \times 500 \times 350 \times 10^{-3}$
	$= 1829$ kN	$= 1778$ kN.
	(Alternatively, calculate P_b from Table 60 of SP 16)	
	$K = \dfrac{5841 - 2500}{5841 - 1829} = 0.83$	$K = \dfrac{5841 - 2500}{5841 - 1778} = 0.82$
9.	*Calculate the revised additional moment*	
	$M_a = 0.83 \times 106.25 = 88.19$ kNm	$M_a = 0.83 \times 132 = 109.56$ kNm
10.	*And the revised M_t from Step 6*	
	$M_{tx} = 230 + 88.19 = 318.19$ kNm	$M_{ty} = 112 + 109.56 = 221.56$ kNm
	$P = 2500$ kN, $M_x = 318.2$ kNm	$M_y = 221.6$ kNm

EXAMPLE 16.3 (cont.)

Step	Bending about *XX*	Bending about *YY*
11.	*Design for revised M_{tx} and M_{ty} by BS 8110* $\frac{M_x}{d} = \frac{318.2 \times 10^6}{450 \times 10^3} = 706$ kN $\frac{M_y}{b'} = \frac{221.6 \times 10^6}{350 \times 10^3} = 633$ kN $\frac{M_x}{d}$ controls $- \frac{P}{f_{ck} bD} = 0.31$ (as before), $\alpha = 0.64$ $M'_x = 318.19 + 0.64 \times \frac{450}{350} \times 221.6$ $= 500.5$ kNm Using Chart 44 of SP 24 (Steel all around) find P_t. $\frac{M}{f_{ck} bD^2} = \frac{500.5 \times 10^6}{40 \times 400 \times 500 \times 500} = 0.125,$ $\frac{P}{f_{ck} bD} = 0.31$ Then we get $\frac{P}{f_{ck}} = 0.085.$ Hence, $P = 0.085 \times 40 = 3.4$ per cent steel $A_s = \frac{3.4 \times 400 \times 500}{100} = 6800$ mm^2 [12T 28 giving 7389 mm^2] Provide steel all around as in Chart 44.	

EXAMPLE 16.4

Determine the longitudinal steel for a braced column 400 × 300 mm bent in double curvature with the following values: $f_{ck} = 30$ N/mm^2 and $f_y = 415$ N/mm^2. $L_0 = 7$ m, L_{ex} (on major axis = 6 m, L_{ey} (on minor axis) = 5 m. The ultimate moments at top $M_x = 40$ kNm; $M_y = 30$ kNm; at bottom $M_x = 22.5$ kNm; $M_y = 20$ kNm. $P_u = 1500$ kN.

Step	Calculations about the *XX*-axis	Calculations about the *YY*-axis
1.	*Calculation of slenderness of column* $\frac{L_e}{D} = \frac{6000}{400} = 15 > 12$	 $\frac{L_e}{b} = \frac{5000}{300} = 16.7 > 12$

EXAMPLE 16.4 (cont.)

Step	Bending about XX	Bending about YY
2.	*Calculation of $e_{\min}$ and $M_{\min}$ (IS 456: Cl. 24.4)* $e_{\min} = \frac{L}{500} + \frac{D}{30} \nless 20$ mm $\left(\text{BS code} \nless \frac{1}{20} D \text{ or } b\right)$ $e_x = \frac{7000}{500} + \frac{400}{30} = 27.3 \text{ mm} > 20$ $M_{\min} = \frac{1500 \times 27.3}{1000} = 41.0$ kNm	 $e_y = \frac{7000}{500} + \frac{300}{30} = 24 \text{ mm} > 20$ $M_{\min} = \frac{1500 \times 24}{1000} = 36.0$ kNm
3.	*Find M_i (IS 456: Cl. 38.7.1, note (i))* $M_i = (0.6M_2 + 0.4M_1)$ $M_2 = 40, \quad M_1 = -22.5$ $M_i = (0.6 \times 40 - 0.4 \times 22.5)$ $= 15.0$ kNm $M_i \nless 0.4M_2 = 16$ kNm	 $M_2 = 30, \quad M_1 = -20$ $M_1 = (0.6 \times 30 - 0.4 \times 20)$ $M_i = 10$ kNm $(M_i \nless 0.4 \times 30 = 12 \text{ kNm})$
4.	*Find e_a and M_a (SP 16 or Table 16.1)* For $\frac{L_e}{D} = 15, \quad \beta = 0.113$ $e_a = 0.113 \times 400 = 45.2$ mm $M_{ax} = \frac{1500 \times 45.2}{1000} = 67.8$ kNm	 $\frac{L_e}{b} = 16.7, \quad \beta = 0.140$ $e_a = 0.14 \times 300 = 42$ mm $M_{ay} = \frac{1500 \times 42}{100} = 63$ kNm
5.	*Find design moment according to BS 8110* (a) $M_t = M_2 = 40$ kNm (b) $M_t = M_1 + \frac{M_a}{2}$ $= 22.5 + 33.9 = 56.4$ kNm (c) $M_t = M_i + M_a$ $= 16 + 67.8 = 83.8$ kNm (d) $M_t = M_{\min} = 41$ kNm Adopt $M_{tx} = 83$ kNm (say) $P = 1500$ kN, $\quad M_x = 83$ kNm, $M_y = 75$ kNm	 (a) $M_t = 30$ kNm (b) $M_t = 20 + 31.5 = 51.5$ kNm (c) $M_t = 12 + 63 = 75$ kNm (d) $M_t = 36.0$ kNm $M_{ty} = 75$ kNm

EXAMPLE 16.4 (cont.)

Step	Bending about XX	Bending about YY
6.	*Design for biaxial bending with $K = 1$ (Use BS 8110 method)* $d = 400 - 50 = 350$ mm, $b' = 300 - 50 = 250$ mm $\frac{P}{f_{ck}bD} = \frac{1500 \times 10^3}{30 \times 300 \times 400} = 0.42$ $\alpha = \left(1 - \frac{7}{6}\frac{P}{f_{ck}bD}\right) = 0.51$ $\frac{M_x}{d} = \frac{83 \times 10^6}{350} = 237 \times 10^3$ $\frac{M_y}{b'} = \frac{75 \times 10^6}{250} = 300 \times 10^3.$ Hence M_y controls the design $M'_y = 75 + 0.51(83)\left(\frac{250}{350}\right) = 105$ kNm	
7.	*Using SP 16, find the steel required (Chart 46)* $\frac{d'}{D} = \frac{50}{300} = 0.17$ $\frac{M_y}{f_{ck}Db^2} = \frac{105 \times 10^6}{30 \times 400 \times 300^2} = 0.097,$ $\frac{P}{f_{ck}bD} = 0.42$ $\frac{p}{f_{ck}} = 0.10, \quad p = 0.10 \times 30 = 3$ per cent	
8.	*Assuming 3.0 per cent steel determine the reduction factor K* $K = \frac{P_z - P}{P_z - P_b}$ $P_z = 400 \times 300\left(0.45 \times 30 + \frac{3.0}{100} \times 0.75 \times 415\right) = 2740$ kN	$P_z = A_c\left(0.45f_{ck} + \frac{p}{100} \times 0.75 \times f_y\right)$

EXAMPLE 16.4 (cont.)

Step	Bending about XX	Bending about YY
	$P_z = 2740$ kN	$P_z = 2740$ kN
	$P_b = \left(k_1 + k_2 \dfrac{p}{f_{ck}}\right) f_{ck} bD$	
	From Table 60 of SP 16,	
	$\dfrac{d'}{D} = \dfrac{50}{400} = 0.13, \quad A_c = 300 \times 400$	
	$P_b = (0.201 + 0.253 \times 0.10) \times 30 \times A_c \times 10^{-3} = 815$ kN	
	$P_b = 0.254 f_{ck} bd$ (as beam) $= 0.254 \times 30 \times 300 \times 350 \times 10^{-3} = 800$ kN	$P_b = 0.254 \times 30 \times 400 \times 250 = 762$ kN
	$K = \dfrac{2740 - 1500}{2740 - 800} = 0.64$	$K = \dfrac{2740 - 1500}{2740 - 762} = 0.63$
9.	*Calculate revised additional moment*	
	$M_a = 0.64 \times 67.8 = 43.0$	$M_a = 0.63 \times 63 = 39.7$
10.	*Revised design moment (in kNm)*	
	(a) $M_t = 40$	(a) $M_t = 30$
	(b) $M_t = 22.5 + \dfrac{43.0}{2} = 44$	(b) $M_t = 20 + \dfrac{39.7}{2} = 39.9$
	(c) $M_t = 16 + 43 = 59$	(c) $M_t = 12 + 39.7 = 51.7$
	(d) $M_t = 41$	(d) $M_t = 36.0$
	Adopt $M_{tx} = 59$ kNm	Adopt $M_{ty} = 51.7$ kNm
11.	*Design for revised M_x and M_y*	
	$\dfrac{M_x}{d} = \dfrac{59 \times 10^6}{350} = 1.69 \times 10^5,$	
	$\dfrac{M_y}{b'} = \dfrac{51.7 \times 10^6}{250} = 2 \times 10^5$	
	$\dfrac{M_y}{b'}$ controls the design.	
	Find α for $\dfrac{P}{f_{ck} bD} = 0.42, \quad \alpha = 0.51$	

EXAMPLE 16.4 (cont.)

Step	Bending about XX	Bending about YY
	$M'_y = M_y + \alpha M_x \left(\frac{b'}{d}\right)$	
	$= 51.7 + \left(0.51 \times 59 \times \frac{250}{350}\right)$	
	$= 73.2$ kNm	
	Using SP 16 Chart 46, we get	
	$\frac{d'}{D} = \frac{50}{300} = 0.17$	
	$\frac{M_y}{f_{ck} D b^2} = \frac{73.2 \times 10^6}{30 \times 400 \times 300 \times 300} = 0.068$	
	$\frac{P}{f_{ck} bD} = 0.42$	
	$\frac{P}{f_{ck}} = 0.06, \; P = 0.06 \times 30 = 1.8$	
	$A_{st} = \frac{1.8}{100} \times 400 \times 300 = 2160 \text{ mm}^2$	
	[8T 20 giving 2513 mm^2]	
	Provide steel all around the column as shown in SP 16, Chart 46.	

EXAMPLE 16.5

An unbraced column 400 mm square is subjected to the following: Factored loads $P = 3200$ kN. At the top, $M_x = 76$ kNm and $M_y = 68$ kNm. At the bottom, $M_x = 38$ kNm and $M_y = 34$ kNm, $L_0 = 5$ m; $L_e = 6.0$ m at both the axes. Assuming $f_{ck} = 40$ and $f_y = 415$ N/mm^2, design the longitudinal steel.

Step	Calculations about the XX-axis	Calculations about the YY-axis
1.	*Find slenderness of column*	
	$\frac{L_e}{D} = \frac{6000}{400} = 15 > 12$	
	Column is slender about XX	Column is slender about YY also.
2.	*Find M_{min}* (IS 456: Cl. 24.4).	
	$e_{min} = \frac{L_0}{500} + \frac{D}{30}$ or 20 mm	
	$= \frac{5000}{500} + \frac{400}{30} = 23.3$ mm	$e_{min} = 23.3$ mm

EXAMPLE 16.5 (cont.)

Step	Bending about *XX*	Bending about *YY*
3.	*Obtain initial moment*	
	As given in the given data above, M is larger at the top of the column	
4.	*Find additional moment*	
	For $\frac{L_e}{D} = 15$, SP 16, Table I,	
	$\beta = 0.113$	
	$e_a = 0.113 \times 400 = 45.2$ mm	$e_a = 45.2$ mm
	$M_{ax} = \frac{3200 \times 45.2}{1000} = 145$ kNm	$M_{ay} = 145$ kNm
5.	*Find design moment* (column is unbraced and top moments control).	
	$M_{tx} = 76 + 145 = 221$ kNm	$M_{ty} = 68 + 145 = 213$ kNm
6.	*Make preliminary design for biaxial bending* $K = 1$.	
	Assume $d = b' = 350 (d' = 50)$	
	$\frac{M_x}{d} = \frac{221 \times 10^6}{350} = 631 \times 10^3$	$\frac{M_y}{b'} = \frac{213 \times 10^6}{350} = 608 \times 10^3$
	$\frac{P}{f_{ck} bD} = \frac{3200 \times 10^3}{400 \times 400 \times 40} = 0.50$	
	$\alpha = \left[1 - \frac{7}{6} \frac{P}{f_{ck} bD}\right] = 0.42$	
	$\frac{M_x}{d}$ controls the design.	
	$M'_x = 221 + 0.42 \times 213 \times \frac{350}{350} = 310.5$ kNm	
7.	*Using SP 16, determine the steel required*	
	$\frac{d'}{D} = \frac{50}{400} = 0.125, \quad \frac{P}{f_{ck} bD} = 0.50$	(Use Chart 45)
	$\frac{M'_x}{f_{ck} bD^2} = \frac{310.5 \times 10^6}{40 \times 400 \times 400 \times 400} = 0.121$	

EXAMPLE 16.5 (cont.)

Step	Bending about XX	Bending about YY
	$\dfrac{p}{f_{ck}} = 0.14,$	
	$p = 0.14 \times 40 = 5.6$ per cent, all around	
8.	*Work out reduction factor K*	
	$P_b = 0.254 f_{ck} bd$	
	$= 0.254 \times 40 \times 400 \times 350 \times 10^{-3}$	
	$= 1422$ kN (approx.)	
	$P_z = A_c \left[0.45 f_{ck} + \left(\dfrac{p}{100}\right) \times 0.75 f_y\right]$	
	$= 400 \times 400 \left(0.45 \times 40 + \dfrac{5.6}{100} \times 0.75 \times 415\right) \times 10^{-3}$	
	$= 5669$ kN	
	$P = 3200$ kN	
	$K = \dfrac{P_z - P}{P_z - P_b}$	
	$K = \dfrac{5669 - 3200}{5669 - 1422} = \dfrac{2469}{4247} = 0.58$ for both axes.	
9.	*Calculate the revised additional moment*	
	$M_a = 0.58 \times 145 = 84.1$ kNm,	$M_a = 0.58 \times 145 = 84.1$ kNm
10.	*Revised additional moment*	
	$M_{tx} = 76 + 84.1 = 160.1$ kNm,	$M_{ty} = 68 + 84.1 = 152.1$ kNm
11.	*Redesign for biaxial bending (M_x controls)*	
	$M'_x = 160.1 + 0.42 \times 152.1 = 224$ kNm	
	Using SP 16 Chart 45, we get	
	$\dfrac{M'_x}{f_{ck} bD^2} = \dfrac{224 \times 10^6}{40 \times 400 \times 400 \times 400} = 0.88,$	
	$\dfrac{P}{f_{ck} bD} = 0.50$	

EXAMPLE 16.5 (cont.)

Step	Bending about XX	Bending about YY
	$\frac{p}{f_{ck}} = 0.11$ Hence $p = 0.11 \times 40 = 4.4$ per cent Area of steel $= \frac{4.4}{100} \times 400 \times 400 = 7040\ \text{mm}^2$ Provide 12T 28 giving 7389 mm^2 (4 rods on each face).	

REVIEW QUESTIONS

16.1 What is the maximum length of columns allowed by IS for R.C. columns ? Give reason for specifying their limits.

16.2 What are the factors that affect behaviour of slender columns ?

16.3 How does bracing affect the behaviour of slender columns ? Explain how bracing can be provided for columns in multistoreyed buildings.

16.4 Distinguish between the failure patterns of reinforced concrete short and slender columns.

16.5 Define curvature and show how columns can bend in single and double curvatures.

16.6 Derive the IS code expression for the additional eccentricity in slender columns.

16.7 Explain the 'reduction factor' method for design of slender columns. Under what conditions is the method specified to be used in IS code ?

16.8 How does one design a slender column which is slender about both the axes ?

16.9 What is meant by *P-D* effect ? State under what conditions one has to consider this effect in design of buildings.

16.10 Explain the methods used to determine the design moment in slender columns by ACI code. Derive the expression for meant magnification factor for braced columns according to ACI code.

PROBLEMS

16.1 A slender rectangular column 350 × 450 mm has an unsupported length of 6 m. It is braced with the ends of the column providing only nominal restraint against rotation. If it has to carry a characteristic imposed load of 250 kN and moments of 30 kNm about the major axis and moment of 15 kNm about the minor axis, design the column assuming $f_{ck} = 25\ \text{N/mm}^2$, $f_y = 415\ \text{N/mm}^2$.

16.2 A slender braced circular column bent in single curvature has an unsupported length of 8 m and effective length of 5 m. It is to be 350 mm in diameter and is to carry a factored axial load of 50 T with factored moments at the top of 40 kNm and at the bottom of 25 kNm. Design the reinforcement required assuming $f_{ck} = 25\ \text{N/mm}^2$ and $f_y = 415\ \text{N/mm}^2$.

16.3 A column 400 × 600 mm in size bent in single curvature is of unsupported length 8 m, and the effective lengths about the major axis is 6 m and about the minor axis 7 m. It is subjected to an axial factored load of 1500 kN alongwith the following factored moments:

At the top M_{xx} = 175 kNm, M_{yy} = 75 kNm
At the bottom, M_{xx} = 225 kNm, M_{yy} = 125 kNm

Design the reinforcement using M_{25} concrete and grade 415 steel.

16.4 A column 350 × 350 mm has an unsupported length of 8 m and equivalent length of 5 m about both the axes. It is loaded with characteristic loads P = 50 T, M_{xx} (top) = 40 kNm, M_{xx} (bottom) = – 25 kNm. Assuming the column is bent in double curvature, design the longitudinal steel if f_{ck} = 30 N/mm^2 and f_y = 415 N/mm^2.

16.5 Design the longitudinal reinforcement for a braced slender column 500 × 400 mm in section for a factored axial load of 2000 kN. The factored moments at the top are M_{xx} = 150 kNm. M_{yy} = 100 kNm and the moments at the bottom M_{xx} = 250 kNm and M_{yy} = 110 kNm. Assume f_{ck} = 30 N/mm^2 and f_y = 415 N/mm^2.

17

Design of Concrete Walls Carrying Vertical Loads

17.1 INTRODUCTION

A vertical load-bearing member whose breadth is more than four times its thickness is called a *wall*. A wall is considered to be a reinforced wall if the percentage of steel is not less than 0.4 so that the strength of reinforcement can also be taken into consideration when calculating its load carrying capacity. When the steel percentage is low (< 0.4 per cent), the wall is assumed to carry the whole load without the help of steel reinforcement, and it is called a *plain concrete wall.* Vertical as well as horizontal loads, as in earthquake or wind loads, may be arranged to be carried by properly designed reinforced and plain walls.

IS 456: clause 31, Section 4 (Special Design Requirements for Structural Members and Systems) deals very briefly with the design of ordinary reinforced concrete walls which have to carry vertical loads only. Special walls, e.g. shear walls or cantilever retaining walls which have to carry horizontal loads, require more detailed analysis and are not considered as ordinary walls. According to Indian practice, ordinary plain walls are to be designed by IS 1905 (1969) Code of Practice for Structural Safety of Buildings—Masonry walls and R.C. walls should be designed by IS 456 (1978).

Detailed rules for design of walls are laid down in BS 8110 where the design of R.C. walls and plain walls is dealt with separately. This chapter covers the design of concrete walls according to the provisions in IS and BS codes. Wall elements of retaining walls, bunkers, and silos which act as slabs are designed mostly to resist bending. They are similar to reinforced slabs in their action. When considering ordinary walls, it should also be borne in mind that even though they are primarily designed for vertical loads, they should also have the capacity to transmit the usual horizontal loads like wind loads to their peripheral members.

17.2 SLENDERNESS RATIO OF WALLS

The carrying capacity of walls depends on their slenderness. The slenderness of plain walls is to be calculated as in the case of masonry walls. Thus, according to the Indian Code of Masonry Walls, IS 1905 (1969), the slenderness ratio of a masonry wall is defined as the effective height divided by the effective thickness or its effective length divided by the effective thickness, whichever is less. It is important to note that the lesser of the two values is taken as the slenderness ratio. This slenderness controls the stress factors in the masonry. The effective height (along the vertical direction) is to be taken, for calculation depends, as in columns, on the 'free height' as well as the conditions of support of its base and top. Similarly, the effective length (along the horizontal direction) depends on the free length and conditions of support along its length such as crosswalls.

The slenderness of R.C. concrete walls, on the other hand, is to be taken according to its method of construction. If it is constructed monolithically with adjacent side constructions, it is considered as short or slender with reference to its height only, without reference to its length. when the ratio of the effective height to its thickness does not exceed 12, it is considered as short. If it is equal or more than 12, it is considered as slender. The slenderness effect in R.C. walls is thus similar to that already considered under R.C. columns. These walls are classified as braced or unbraced, similar to columns, according to the provisions made for carrying the lateral loads. However, where the construction of R.C. walls with adjacent construction is such that the loads can be assumed to be transmitted to the reinforced wall as in simply supported condition, the effective height is to be assessed as for plain walls with respect to both effective height and effective length measured horizontally.

17.2.1 BRACED AND UNBRACED WALLS

The effect of cross walls in R.C. concrete walls is, sometimes, taken into account for considering it as braced or unbraced, depending on the capacity of the cross wall to resist lateral forces, as in R.C. columns. The wall can be considered as braced if all the lateral forces on it are borne by the walls constructed at right angles to the wall being considered. Otherwise, it is considered as unbraced. The overall stability of a multistorey building should not depend on unbraced walls alone. Careful bracing of walls should be planned for these structures. Even when walls are considered as unbraced, it is recommended to design the columns and walls which are provided at right angles to it, to assist the unbraced wall to carry at least 25 per cent of the lateral loads to ensure a stable design.

17.2.2 EFFECTIVE HEIGHT OF PLAIN WALLS IN BS 8110

It has already been mentioned that the IS code considers plain concrete wall design as similar to masonry wall design. But in BS 8110, plain walls are also classified as braced or unbraced, and their effective height should be calculated according to Tables 17.1 and 17.2, where L_0 is the unsupported height of the wall.

TABLE 17.1 EFFECTIVE HEIGHT OF UNBRACED PLAIN CONCRETE WALLS (BS Practice)

Nature of wall	Unbraced
With a roof of floor spanning at right angles on top of the wall	$1.5L_0$
With no roof or floor on top of the wall	$2L_0$

Note: L_0 = clear height of wall between lateral supports.

TABLE 17.2 EFFECTIVE HEIGHT OF BRACED PLAIN CONCRETE WALLS (BS Practice)

Nature of wall	Lateral support resists rotation and movement	Lateral support resists movement only
With roof or floor on top at right angles	$0.75L_0$	L_0
With no roof or floor on top	$2L_0$	$2.5L_0$

Note: L_0 is the clear distance between lateral supports.

We now give the classification of concrete walls into eight categories as per BS 8110: For design purposes, a given wall is first classified into one of these categories and designed as indicated in Sections 17.4 and 17.5.

CLASSIFICATION OF CONCRETE WALLS

- Concrete walls
  - R.C. walls
    - Braced
      - Short (1)
      - Slender (2)
    - Unbraced
      - Short (3)
      - Slender (4)
  - Plain walls
    - Braced
      - Short (5)
      - Slender (6)
    - Unbraced
      - Short (7)
      - Slender (8)

17.3 LIMITS OF SLENDERNESS

IS 456, Section 31.3 and the British Code give guidelines for the dimensions of R.C. walls. These may be summarised as follows:

Normally, the thickness of an R.C. wall should not be less than 100 mm. If the slenderness ratio of the wall exceeds 12, slenderness effects should be considered for the walls as in the case of columns. The limits of slenderness allowed are given in the classification in Section 17.2.2.

TABLE 17.3 SLENDERNESS LIMITS OF CONCRETE WALLS

Steel (Percentage)	Type of wall	Braced or Unbraced	Max L_e/h
0.4 or less	Plain	Braced	30
0.4 or less	Plain	Unbraced	30
0.4–1.0	Reinforced	Braced	40
1–4	Reinforced	Braced	45
0.4–1.0	Reinforced	Unbraced	30
1–4	Reinforced	Unbraced	30

17.4 DESIGN OF R.C. WALLS (ACCORDING TO IS 456 AND BS 8110)

According to IS 456: clause 31.2, the load carrying capacity of R.C. walls is to be calculated as in the case of columns. It further recommends that while designing R.C. walls the strength of walls may be increased by the values given in Table 17.4. This means that the longer the wall compared to its height, the larger is the percentage increase in strength. The length is defined as the overall length; where large size openings occur, the length is to be taken as that between adjacent openings. When the larger dimension of the opening exceeds about one-quarter the height of the wall, it may be considered as large (as in the case of openings in beams).

TABLE 17.4 INFLUENCE OF HEIGHT-LENGTH RATIO ON STRENGTH OF R.C. WALLS
(Ref: IS 456: clause 31.2)

Ratio of storey height to length of wall	0.5 or less	1.0	1.5 or more
Percentage increase	20	10	0

BS 8110: clause 3.9.3 recommends R.C. walls to be designed according to the principles of design of columns. The axial load in a wall is calculated by assuming that the beams and slabs transmit the load as in a simple hinged support. Moments are to be calculated by structural analysis, depending on the degree of fixity. However, a minimum eccentricity of load of 20 mm or 1/20th (0.5 per cent) the thickness of the wall is to be taken for the design for all walls also as in columns. An R.C. wall can be classified into one of the four types shown in Section 17.2.2. The design of each type is made, as described in Section 17.5.

17.5 DESIGN OF R.C. WALLS (ACCORDING TO BS 8110)

Cases 1 to 4 of the classification given in Section 17.2.2 are considered in this section.

17.5.1 CASE 1. DESIGN OF SHORT BRACED R.C. WALLS

Short braced RC walls not subjected to any moments may be designed by the formula (as in column)

$$P = 0.4f_{ck}A_c + 0.75A_sf_y \tag{17.1}$$

where

A_c = area of concrete

A_s = area of compression reinforcement

Short unbraced R.C. walls supporting approximately symmetrical arrangement of slabs (spans on either side not differing by more than 15 per cent) may be designed by the formula

$$P = 0.35f_{ck}A_c + 0.67A_sf_y \tag{17.2}$$

With short R.C. walls, which have to resist moments in the plane of the wall as also moments at right angles to it, special consideration should be given first to bending in the plane of the wall to establish a distribution of tension and compression along the length of the walls, and then to the bending at right angles to the plane of the walls.

17.5.2 CASE 2: DESIGN OF SLENDER BRACED R.C. WALLS

Let the thickness of wall be h. The slenderness ratio Le/h should be limited to 40 to 45 as given in Table 17.3. The design is made as slender compression members, according to IS Code: clause 38.7.1 with additional moment

$$M_{ax} = \frac{P_uh}{2000}\left(\frac{L_e}{h}\right)^2 \tag{17.3}$$

This is equivalent to taking the additional eccentricity e_a as in slender columns given by

$$e_a = \frac{h}{2000}\left(\frac{L_e}{h}\right)^2$$

A trial and error method may have to be used for the design. Where the wall is reinforced with only one central layer of steel, the additional moments are doubled.

17.5.3 CASE 3: DESIGN OF SHORT UNBRACED R.C. WALL

The section should be designed as columns to withstand the combination of P and M, the minimum moment being that due to an eccentricity of $h/20$ or 20 mm.

17.5.4 CASE 4: DESIGN OF SLENDER UNBRACED R.C. WALL

As mentioned in Table 17.3, the slenderness should not exceed 30. The design is as in case 2 above.

17.6 DESIGN OF PLAIN WALLS (ACCORDING TO BS 8110) (CASES 5 TO 8)

Plain walls are also classified into one of the four types (cases 5 to 8) in Section 17.2.2 and are designed as follows according to B.S. 8110: clause 3.9.4.

17.6.1 CASE 5: DESIGN OF SHORT BRACED PLAIN CONCRETE WALLS

Plain short braced walls are designed by using the following formulae:

$$P = (h - 2e_x)\,\alpha f_{ck} \tag{17.4}$$

where

P = maximum axial load per unit length

h = thickness of wall

e_x = resultant eccentricity at right angles to the plane of the wall (minimum value $h/20$)

α = stress-reduction coefficient or factor depending on the type of concrete and dimension of walls as given in Table 17.5 (an average value of 0.3 may be used for all practical purposes as recommended in BS 8110).

TABLE 17.5 STRESS-REDUCTION FACTOR FOR PLAIN CONCRETE WALLS (α)

(Height/Length) ratio	Grade 25 and above	Below grade 25
Less than 0.5	0.5	0.4
Greater than 1.5	0.4	0.35

Note: (i) An average value of 0.3 may be used for all cases as recommended in BS 8110.
(ii) Interpolation is allowed for height-length ratio between 1.5 and 0.5.

In addition, when the length of the wall is less than four times its thickness and the vertical steel is less than 0.4 per cent (in which case the member is to be considered as plain wall), a reduction coefficient varying from 1.0 for length of walls four times the thickness to 0.8 for the length of wall equal to the thickness has to be used.

17.6.2 CASE 6: DESIGN OF SLENDER BRACED PLAIN CONCRETE WALL

The slenderness ratio should not exceed 30 and the load should be taken as the lesser of the following:

$$P = (h - 2e_x)\ \alpha f_{ck}$$

(which is the same as 17.4)

$$P = (h - 1.2e_x - 2e_a)\alpha f_{ck} \tag{17.5}$$

where e_a is the additional eccentricity due to deflection equal to $\frac{1}{2500h}(L_e)^2$, and is recommended in BS 8110 (this expression being similar to IS code: clause 38.7.1 for slender columns), e_x is as explained in case 1, and α is taken as 0.30 or as given in Table 17.5.

17.6.3 CASE 7: SHORT UNBRACED PLAIN CONCRETE WALL

The carrying capacity P of these walls should be lesser of the following (similar to slender columns):

$$P = (h - 2e_{x1})\alpha f_{ck} \tag{17.6}$$

$$P = (h - 2e_{x2} - e_a)\alpha f_{ck} \tag{17.7}$$

where e_{x1} and e_{x2} are the eccentricities at top and bottom and e_a the additional eccentricity. α is taken as 0.30 or as given in Table 17.5.

17.6.4 CASE 8: SLENDER UNBRACED PLAIN CONCRETE WALL

The slenderness ratio of unbraced plain concrete wall should not exceed 30. For walls within the above limits the design is carried out as in case 3.

17.7 DESIGN OF TRANSVERSE STEEL IN CONCRETE WALLS

In plain walls the provision of transverse reinforcement to retain vertical bars against buckling as in columns need not be applied as the vertical bars are not called upon to carry the load. However, in R.C. walls where the vertical steel has to take loads, the transverse steel should be properly designed to restrain the vertical steel against buckling. Transverse reinforcement is provided by horizontal steel. The minimum amount of horizontal steel, needed is specified in various codes such as the IS and BS codes, and the rules in this respect are now given.

17.8 RULES FOR DETAILING OF STEEL IN CONCRETE WALLS

The minimum steel to be provided in walls is expressed in terms of the gross area of concrete. The provisions for steel are as follows:

1. Indian Standards—IS 456, clause 31.4

(a) Minimum vertical steel for plain walls should be 0.12% for high yield bars and welded fabric and 0.15% for mild steel bars.

The maximum spacing should be 450 mm or three times the wall thickness, whichever is less.

(*Note*: For R.C. walls the vertical steel should not be less than 0.4 per cent, irrespective of the type and grade of steel.)

(b) Minimum horizontal steel for all types of walls (plain or reinforced) should be 0.20% for high yield bars with diameter not larger than 16 mm and 0.25% for mild steel bars.

The maximum spacing of bars should be 450 mm or three times the wall thickness, whichever is less.

(It should be noted that the amount of minimum horizontal steel specified in IS 456 is higher than that for the vertical steel.)

2. British Standards

(a) *Plain walls.* Only nominal horizontal and vertical steel should be provided in plain walls. The function of steel is to control cracking due to shrinkage, temperature, etc. The minimum vertical and horizontal steel should be 0.25 per cent for high yield and 0.30 per cent for mild steel. All the steel should be placed at the exposed side for external walls. For internal walls, half the steel is placed near each side of these walls.

(b) *R.C. walls.* Steel is provided in the form of vertical steel, horizontal steel, and links (if found necessary) as follows:

(i) *Vertical steel.* Minimum vertical steel should be 0.4 per cent in one or two layers and the maximum vertical steel should be 4 per cent.

(ii) *Horizontal steel.* Minimum steel should be as in the Indian code, but when the compression steel is more than 2 per cent of the area of the section, horizontal steel should be at least 0.25 per cent for high yield steel and 0.30 per cent for mild steel. The diameter should not be less than one-quarter that of vertical steel, and not less than 6 mm.

(iii) *Placing of steel in thick walls.* In walls greater than 220 to 250 mm thick, the steel is placed in two layers. Not less than one-half and not more than two-third steel is placed at the exterior face and the balance at the interior face. The vertical and horizontal steel are not placed further apart than three times the wall thickness or 450 mm.

(iv) *Links through thickness of walls.* With large amount of vertical compression steel (more than 2 per cent), it is a good practice to provide horizontal steel through the thickness of walls, in the form of links. The diameter of the links should not be less than one-quarter the diameter of the main rods or 6 mm. The spacing of links too should be less than twice the thickness of wall in horizontal or vertical direction. In the vertical direction the spacing should not be more than 16 times the diameter of the main steel. No un-restrained vertical bar should be farther than 200 mm of a restrained bar Fig. 17.1.

Table 17.6 gives the minimum steel required for different percentages of steel for varying thicknesss of walls. It can be used as a ready reckoner for the selection of the necessary minimum steel for design of R.C. walls.

17.9 GENERAL CONSIDERATIONS IN DESIGN OF WALLS

It has already been stated that the overall stability of a multistoreyed building should never depend on its unbraced walls alone. In all cases of walls (even when walls are considered as unbraced) at least 25 per cent of the lateral load should be capable of being resisted by columns, crosswalls, or both.

TABLE 17.6 MINIMUM REINFORCEMENT IN WALLS

Wall thickness	Steel spacings for given percentage of steel in two layers		
(mm)	0.20	0.25	0.4
100	6 mm at 280 mm	6 mm at 200 mm	8 mm at 250 mm
125	6 mm at 220 mm	6 mm at 175 mm	8 mm at 200 mm
150	6 mm at 175 mm	6 mm at 150 mm	10 mm at 250 mm
175	6 mm at 150 mm	8 mm at 225 mm	10 mm at 200 mm
200	6 mm at 140 mm	8 mm at 200 mm	12 mm at 275 mm
225	6 mm at 125 mm	10 mm at 275 mm	12 mm at 250 mm
250	8 mm at 200 mm	10 mm at 250 mm	12 mm at 225 mm
275	10 mm at 275 mm	10 mm at 225 mm	12 mm at 200 mm
300	10 mm at 250 mm	10 mm at 200 mm	12 mm at 175 mm

Note: Steel should be provided on both faces at the given spacing; if a single layer is used, half the spacing.

A panel wall which is constructed as an infilling for a structural frame may be considered as non-load bearing, but it should be sufficiently strong to resist the wind pressure to the frame. For this purpose the panels should be given enough bearing by setting them in rebates in the members of the frame or by means of steel dowels.

Vertical loads may be carried by plain or R.C. walls. With heavy loads or when the load is applied with considerable eccentricity or when moments are to be transferred to the wall in addition to vertical loads, it is better to use a reinforced concrete wall than a plain concrete wall. Loads with small eccentricities can as well be carried by plain walls as this eccentricity can be taken into account in the design formula for plain walls also. It is essential that, while detailing steel, attention should be devoted to ease of construction. For economy in steel, it is preferable in R.C. walls to provide the maximum effective depth to steel in the direction of moments, but from the steel fabricators point of view, it is more convenient to have horizontal bars placed on the side of the vertical steel. On the other hand, with smaller cover, horizontal bars next to the formwork may tend to seggregate the concrete and make the aggregate to wedge in between the formwork and longitudinal bars. All these factors should be considered before finalising the detailing of steel.

17.10 PROCEDURE FOR DESIGN OF CONCRETE WALLS

Step 1: Arrive at a layout of the wall and its dimensions.

Step 2: Select type of wall—plain or R.C. (Plain walls are used when the factored load to be carried is light, and is of the order of 0.2 to 0.3 $(f_{ck}h)$ per metre length of wall.)

Step 3: Determine the effective length and slenderness ratio of the concrete wall and classify the wall into one of the eight types given in Section 17.2.2.

Step 4: Design the wall using the appropriate formulae given in Sections 17.4 and 17.5.

Step 5: Detail the steel according to the rules given in Section 17.6.

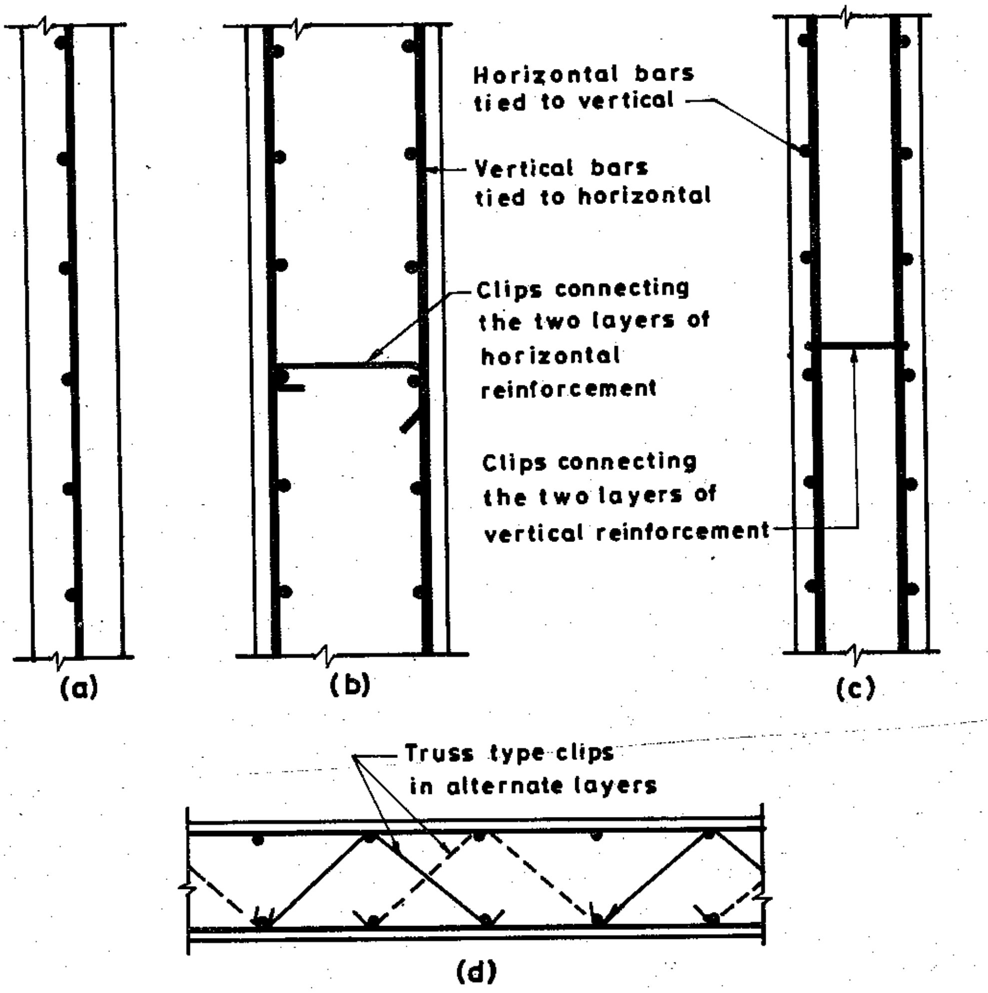


Fig. 17.1 Detailing of steel in reinforced concrete walls: (a) Wall thickness ≤ 170 mm, (b) Wall thickness > 220 mm, (c) Wall thickness > 170 mm but < 220 mm, (d) Truss type clips (links) in alternate layers in a wall.

17.11 DETAILING OF STEEL

Standard detailing of steel in walls is given in IS publication SP 34 *Handbook on Concrete Reinforcement and Detailing*. These recommendations for vertical steel and splices on top of wall are given in Figs. 17.1 to 17.3.

17.12 CONCENTRATED LOADS ON WALLS

The following approximation, as can be obtained from the definition of a wall, may be used for design of concentrated loads on walls. The horizontal length of wall to be considered as effective in carrying concentrated loads on a wall should not exceed the width of bearing of the load plus four times the wall thickness or the centre-to-centre distance between the loads, whichever is less.

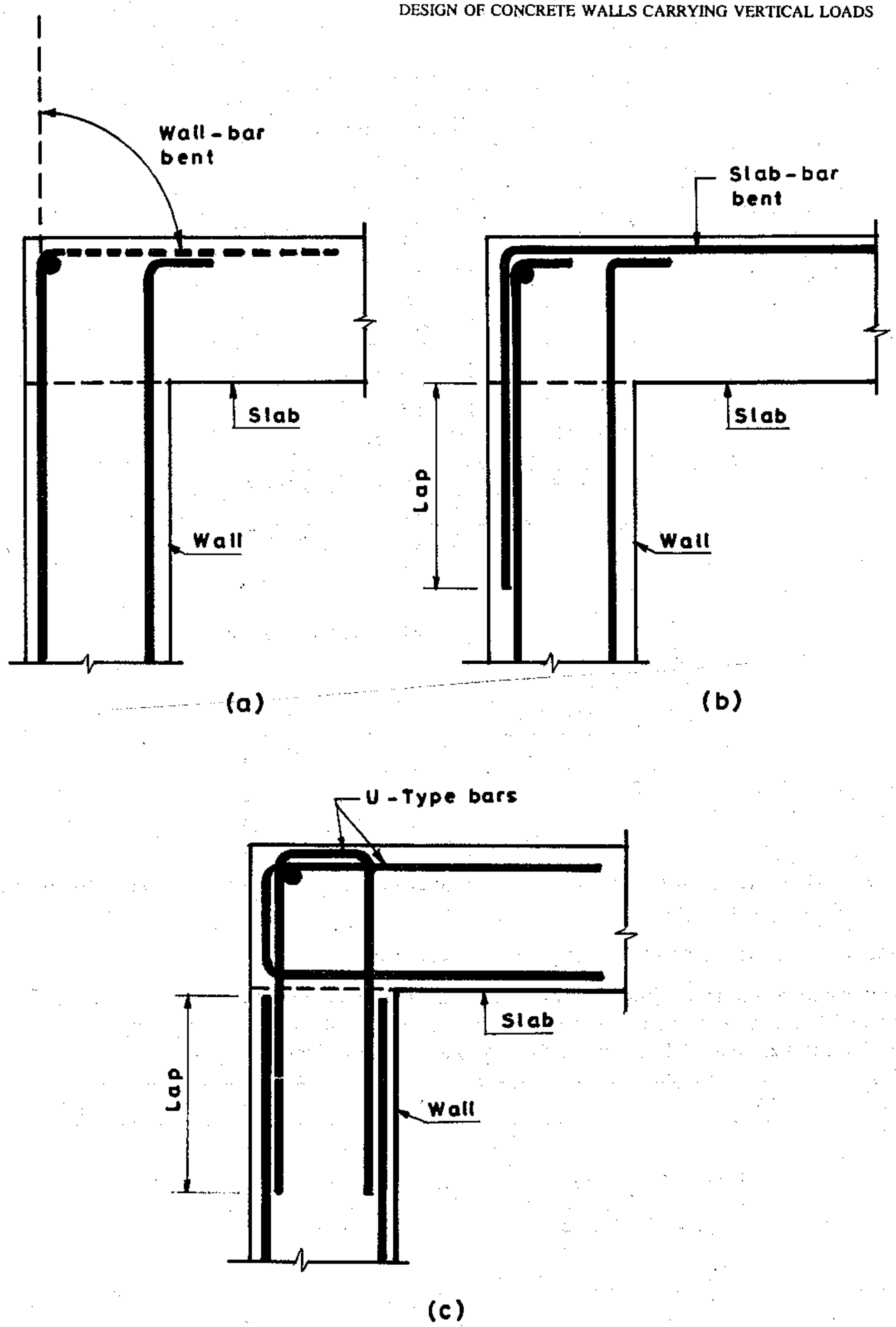


Fig. 17.2 **Splices at top of walls: (a) Deformed bars < 10 mm, (b) Deformed bars > 10 mm, (c) Deformed bars > 10 mm or mild steel of any diameter.**

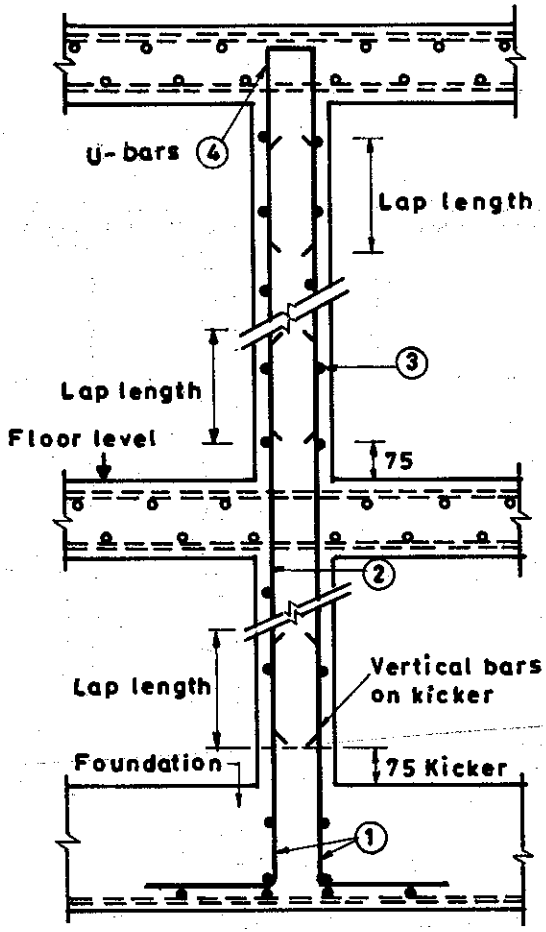


Fig. 17.3 Reinforcement layout in walls (1. Foundation bars; 2. Vertical bars; 3. Horizontal bars; 4. U-bars).

EXAMPLE 17.1 **(Design of braced plain concrete wall)**

A plain concrete wall is 3 m high, 100 mm thick, and 4 m in length between cross walls. The loads are carried to the wall through a floor at the top. Assuming that there are no openings on the wall, determine the load the wall can carry. $f_{ck} = 20$ and $f_y = 415$ N/mm^2.

Ref.	Step	Calculations	Output
	1.	*Determine slenderness of wall*	
		$L_0 = 3$ m. Assume $L_e = 0.75L_0$	
Table 17.2		$\frac{L_e}{h} = \frac{(0.75)(3000)}{100} = 22.5 > 12 < 30$ (vertical)	
Table 17.4		$\frac{L_d}{h} = \frac{(0.75 \times 4000)}{100} = 30$ (horizontal)	Wall is slender
		Hence slenderness ratio = 22.5	Case 6

EXAMPLE 17.1 (cont.)

Ref.	Step	Calculations	Output
	2.	*Calculation of load capacity* Lesser value of	
Eq. (17.4)		$P = (h - 2e_x)\alpha f_{ck}$	
Eq. (17.5)		$P = (h - 1.2e_x - 2e_a)\alpha f_{ck}$ e_x = Resultant eccentricity of load $\nless h/20$ e_a = additional eccentricity due to slenderness $e_x = \dfrac{h}{20} = \dfrac{100}{20} = 5$ mm $e_a = \dfrac{h}{2000}(L_e/h)^2 = \dfrac{(22.50)^2 \times 100}{2000}$ $= 25.3$ mm	
Table 17.5		α from Table 17.5 for height/length = 3/4 $\alpha = 0.38$ $P_u = [100 - (2)(5)]0.38 \times 20 = 684$ N/mm $P_u = [100 - (1.2)(5) - 2(25.3)]0.38 \times 20$ $= 330$ N/mm For one metre length, $P_u = 330$ kN Allowable load $= \dfrac{330}{1.5} = 220$ kN/metre	Allowable load 220 kN/m
	3.	*Steel to be provided*	
Table 17.7		Minimum steel 0.25 per cent for Fe 415 Provide 6 mm at 200 mm vertically and horizontally.	Use T6 at 200

EXAMPLE 17.2 **(Design of braced R.C. wall)**

Design the wall in Example 17.1 (with the same thickness) if it has to carry a factored load of 600 kN per metre length. (Note: As the capacity of the plain wall is only 330 kN, the wall has to be reinforced.)

Ref.	Step	Calculations	Output
	1.	*Determine slenderness* $L_0 = 3$ m, $L_e = 0.75L_0$	
Table 17.4		$\dfrac{L_e}{h} = \dfrac{(0.75)(3000)}{100} = 22.50 > 12 < 40$	Wall is slender Case (2)

EXAMPLE 17.2 (cont.)

Ref.	Step	Calculations	Output
	2.	*Calculate minimum accidental eccentricity* $e_x = e_{\min} = \dfrac{h}{20}$ or 20 mm $\dfrac{h}{20} = \dfrac{100}{20} = 5$ mm Adopt $e_x = 20$ mm	Design as slender column
	3.	*Additional eccentricity* (e_a as in R.C. column) $e_a = \dfrac{(L_e)^2}{2000h} = \dfrac{(2250)^2}{2000 \times 100} = 25.3$ mm	
	4.	*Calculate M and P for design* Assume total eccentricity = $(e_x + e_a) = 45.3$ $M = Pe = \dfrac{600 \times 45.3}{1000} = 27.18$ kNm $P = 600$ kN	
SP 16 Chart 34	5.	*Determine steel from interaction diagram* $\dfrac{d'}{D} = \dfrac{20}{100} = 0.2,\quad \dfrac{P}{f_{ck}bD} = \dfrac{600 \times 10^3}{20 \times 1000 \times 100} = 0.30$ $\dfrac{M}{f_{ck}bD^2} = \dfrac{27.18 \times 10^6}{20 \times 1000(100)^2} = 0.14$ $\dfrac{p}{f_{ck}} = 0.12,\quad p = 0.12 \times 20 = 2.4\%$	
	6.	*Detail the steel* (a) *Vertical steel* = $\dfrac{2.4 \times 1000 \times 100}{100} = 2400$ mm^2 Provide 16 mm at 16 cm spacing on each face.	Use T16 at 160 on both faces (2513 mm^2)
Table 17.7		(b) *Horizontal steel*—As vertical steel is more than 2 per cent, provide 0.25 per cent horizontal steel for both faces (6 mm at 200 mm spacing). Maximum allowed spacing = $3h$ = 300	T6 at 200 on both faces
		(c) *Links*— As vertical steel is more than 2 per cent, provide links—6 mm at 200 mm. [Min. diameter = 6 mm, spacing $2h$ = 200 in either direction: Vertical spacing $\ngtr 16 \times$ diameter of main steel]	T6 at 200 vertically and horizontally

EXAMPLE 17.3 **(Design of R.C. with end moments)**

(Design a reinforced concrete wall of height 5 m, which is restrained in position and direction at both ends and has to carry at its top a factored load $P_u = 600$ kN and factored moment $M_u = 25$ kNm, at right angles to the plane of the wall.

Ref.	Step	Calculations	Output
	1.	*Determine thickness of wall* Assume a moderate height to thickness ratio say 22. $h = \frac{5000}{22} = 225$ mm (approx.) Adopt an R.C. wall as the load is large. $L_e = 0.65L_0 = 3250$ mm	
	2.	*Determine slenderness of wall* $\frac{L_e}{h} = \frac{0.65L_0}{h} = \frac{0.65 \times 5000}{225} = 14.4 > 12 < 40$	Slender R.C. wall Case 2.
	3.	*Calculate minimum accidental eccentricity* $e_{\min} = \frac{h}{20}$ or 20 mm $\frac{h}{20} = \frac{225}{20} = 11.25$. Adopt 20 mm	
	4.	*Calculate e_x* Eccentricity due to $M = \frac{M}{P} = \frac{25 \times 10^6}{600 \times 10^3}$ $= 41.6$ mm $e_{\min} < e$. Hence adopt $e_x = 41.6$ mm	$e_x = 41.6$ mm
	5.	*Additional eccentricity as in R.C. columns* $e_a = \frac{(L_e)^2}{2000h} = \frac{(3250)^2}{2000 \times 225} = 23.5$ mm	$e_a = 23.5$ mm
	6.	*Calculate design P and M* Total eccentricity $= e_x + e_a = 41.6 + 23.5 = 65.1$ $M_t = \frac{600 \times 65.1}{1000} = 39.1$ kNm	
	7.	*Determine percentage of steel from SP 16* $P = 600$, $M = 39.1$ $d' = 40$(cover) + 6 (for 12 mm rods) = 46 mm $\frac{d'}{D} = \frac{46}{225} = 0.20$	

EXAMPLE 17.3 (cont.)

Ref.	Step	Calculations	Output
		$\dfrac{P}{f_{ck}bD} = \dfrac{600 \times 10^3}{20 \times 1000 \times 225} = 0.13$	
		$\dfrac{M}{f_{ck}bD^2} = \dfrac{39.1 \times 10^6}{20 \times 1000 \times (225)^2} = 0.04$	
SP 16 Chart 34		$\dfrac{p}{f_{ck}} = 0.$ Provide minimum 0.4 per cent	
	8.	*Detail the reinforcement*	
Table 17.7		(a) *Vertical steel* 6 mm at 250 centres on both faces satisfies maximum spacing of $3h$ or 450 mm as vertical steel.	T6 at 250 on both faces
		(b) *Horizontal steel* Minimum 0.2 per cent. Providing 6 mm diameter steel at 450 mm spacing on both faces satisfies conditions for maximum spacing.	T6 at 450 on both faces
		(c) *Links* No links required as vertical steel is less than 2 per cent.	No links

REVIEW QUESTIONS

17.1 State the differences between
(a) column and wall,
(b) plain walls and R.C. walls, and
(c) braced and unbraced walls.

17.2 State why the provisions in IS 456 under walls are not applicable to design of shear walls.

17.3 Indicate how the effective height and slenderness ratio of (a) a plain wall and (b) an R.C. wall member are calculated.

17.4 What are the limits of slenderness laid down for plain and R.C. walls ?

17.5 What is the basic philosophy of design of
(a) plain walls, and
(b) R.C. walls according to IS code ?

17.6 What is the influence of the length-height ratio on R.C. walls in Indian code, IS 456 ?

17.7 What is the minimum reinforcement to be placed in concrete walls to control shrinkage and temperature cracking ? Does this apply to plain walls also ?

17.8 What are the rules regarding detailing of transverse steel in R.C. walls ? When does one use links through the thickness in R.C. walls and what functions do they perform ?

17.9 What general considerations should be taken into account in planning a multi-storey building on R.C. walls ?

17.10 Explain the principles of design of a slender reinforced concrete wall to carry vertical load.

PROBLEMS

17.1 The layout of a system of R.C.C. walls for a building is as shown in Fig. P.17.1. The walls are 20 cm thick, the height between floors being 3 m. The walls support an R.C.C. slab 12 cm thick on top. Determine the effective height, effective length, and slenderness ratio of the walls marked A and B. Would you consider C as a wall or a column ?

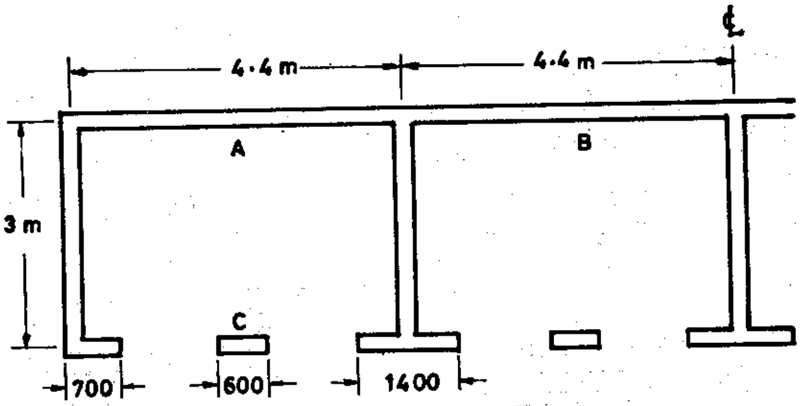


Fig. P.17.1.

17.2 A plain concrete wall is 3.5 m high and 5 m in length. It has to carry a characteristic load of 250 kN/m. Determine the required thickness of the wall and detail the steel required. Assume f_y = 415 N/mm^2 and f_{ck} = 25 N/mm^2.

17.3 Design a braced reinforced concrete wall of height 3.5 m and length 4 m to carry 400 kN per metre length of wall. Assume grade 20 concrete and Fe 415 steel.

17.4 A braced reinforced concrete wall is 30 cm in thickness and 3 m in height. It is restrained at the bottom by the foundation and at the top by a reinforced concrete slab 15 cm thick. If the wall is to carry a load of 500 tons per metre length, determine the vertical and horizontal steel required. Sketch the arrangement of steel. Assume f_{ck} = 20 N/mm^2, f_y = 415 N/mm^2.

18

Design For Torsion

18.1 INTRODUCTION

Many types of loadings produce torsion in reinforced concrete members. The resultant torsion may be classified into two types (see Fig. 18.1):

1. Primary or equilibrium torsion
2. Secondary or compatibility torsion

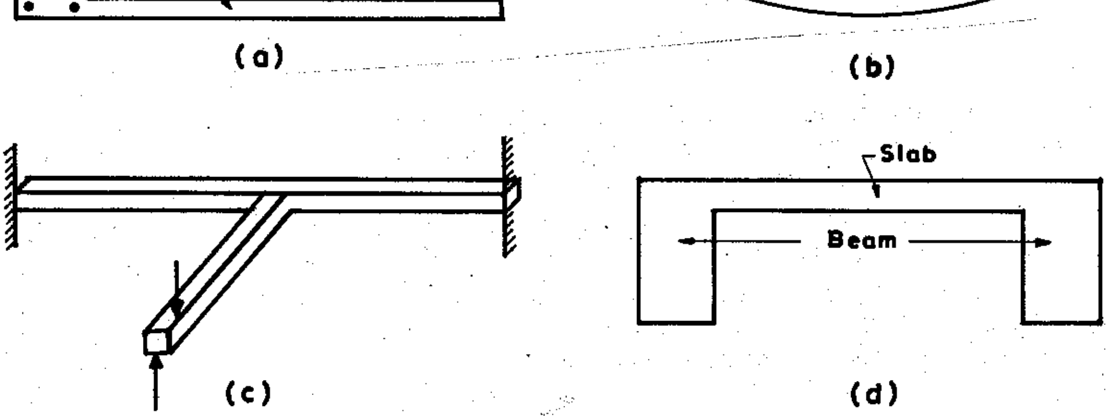


Fig. 18.1 Primary and secondary torsion: (a) Portico slab, (b) Bow girder, (c) Indeterminate frame, (d) Edge beams.

The first type is that which is required to maintain basic static equilibrium, and the second is the one required to maintain only compatibility condition between members. In general, one may say that torsion in statically determinate structure is of the equilibrium type and torsion in statically indeterminate structures may be either of the equilibrium or the compatibility type. In statically indeterminate structures, there are more than one load path along which loads can be distributed and equilibrium maintained, so that the structure can be made safe without taking minor torsional effects into account. Such neglect, at most, will produce some cracking, but not failure. However, in structures in which a large part of the load is applied unsymmetrically, torsion will have to be considered carefully.

Torsion is a major component if it is of the equilibrium type as also in situations where the torsional stiffness of the members has been taken into account in the structural analysis. In other cases of secondary torsion, provision of nominal shear reinforcements according to codes of practice may be assumed to take care of the incidental effects. Thus the small amount of unintentional torsion in most of the conventional beams and slabs can be ignored in design and supplied by proper detailing of reinforcements.

18.2 ANALYSIS FOR TORSIONAL MOMENT IN A MEMBER

Moments provided in a structure on application of loads are taken by some members in bending and other members by torsion, depending on their disposition. Just as bending moments are distributed among the members sharing the moments in proportion to their bending stiffness, i.e. (EI/L) values, the factor that determines the transfer of torsional moment is the torsional stiffness (GC/L), where G is the elastic shear modulus and C the torsion constant. This principle is used to determine torsional moments carried by members in structural analysis as illustrated in Examples 18.1 to 18.3. C is an important property of the section in shear and is similar to the property I of a beam in bending.

18.2.1 CALCULATION OF TORSION CONSTANT C

The value C for a rectangle $b \times D$ (where b is the smaller dimension) is given by

$$C = KDb^3$$

where K is St. Venant's torsional constant which varies with the ratio of D/b. The value of K for various ratios of D/b is given in Table 18.1.

TABLE 18.1 VALUES OF K AND α IN TORSION OF RECTANGLES

$$\left(C = KDb^3, \quad \tau_{\max} = \frac{T}{\alpha b^2 D}\right)$$

D/b	Value of		D/b	Value of	
	K	α		K	α
1.0	0.141	0.208	3.0	0.263	0.267
1.2	0.166	0.219	4.0	0.281	0.282
1.5	0.196	0.231	5.0	0.291	0.291
2.0	0.229	0.246	10.0	0.312	0.312
2.5	0.249	0.258	α	0.333	0.333

A more convenient expression for C, giving values close to those obtained from the above table, for value of $D/b < 10$, has been derived by Timoshenko as

$$C = \left(1 - 0.63\frac{b}{D}\right)\left(\frac{b^3 D}{3}\right) \tag{18.1}$$

The following approximation can be made when dealing with flanged beams and other sections which can be assumed to be rectangular (see Fig. 18.2).

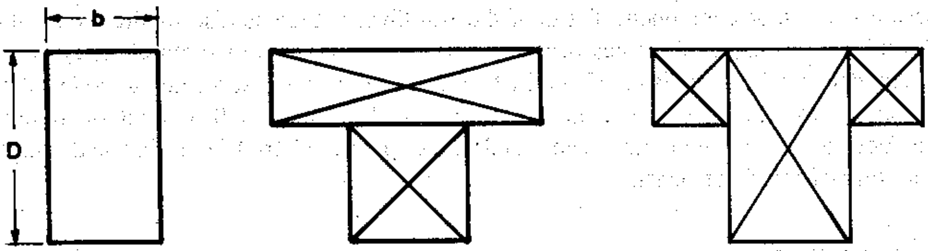


Fig. 18.2 Torsion of flanged sections.

1. For analysis, T, L or I sections are divided into component rectangles and the C value is the combined (added up) value of the component rectangles. The division should be such that the value of C obtained for the whole section, i.e.

$$C = \Sigma\left(1 - 0.63\frac{b}{D}\right)\left(\frac{b^3 D}{3}\right)$$

should be the largest possible value.

2. Hollow box sections can be treated as solid when the wall thickness is more than $D/4$ (one-fourth the depth). Otherwise, it can be divided into its component rectangle, and the value of the total torsional stiffness determined. It should, however, be remembered that sections of thickness less than $b/10$ are not suitable for reinforced concrete in torsion due to its large flexibility.

18.2.2 BENDING AND TORSIONAL STIFFNESS OF R.C. MEMBERS

That the magnitude of distribution of moments as torsion to adjoining members is small can be seen from the following argument: *First,*

$$\text{Value of } G = \frac{E}{2(1+\mu)}$$

If $\mu = 0.15$, then

$$G = \frac{E}{2.3}, \quad \mu = 0, \quad G = \frac{E}{2}$$

which shows that G is very low as compared to E.

Secondly, for a rectangle $D = 2b$, the value of I is about three times that of C.

The ratio of stiffness in bending to stiffness in torsion for adjoining members can therefore be obtained as

$$\frac{EI}{L} : \frac{GC}{L} = 6.9 \quad \text{(approx. 7)}$$

Hence beams are several times more stiff in bending than in torsion. Thus very little of the moment is transferred as torsion to members while the major part is transferred as bending moments.

18.2.3 TORSIONAL RIGIDITY OF R.C. MEMBERS

On the basis of laboratory tests, BS 8110 (part 2): clause 2.4.3 states that for structural analysis or design, the torsional rigidity may be calculated by assuming $G = 0.42$ times the modulus of elasticity of concrete and C equal to one-half of the St. Venant value calculated for the plain concrete section.

18.3 TORSIONAL SHEAR STRESS ANALYSIS OF RECTANGULAR SECTIONS

Having determined the torsional moment acting on a section by elastic analysis, the next step is to determine the maximum shear stress that will be produced in the section. It can be calculated by elastic or plastic analysis.

Elastic analysis gives a distribution as shown in Fig. 18.3. The value of maximum shear stress occurs at mid-point of the larger side and its magnitude is given by

$$\tau_{\max} = \frac{T}{\alpha b^2 D} \tag{18.2}$$

The value of α is given in Table 18.1 for different D/b ratios.

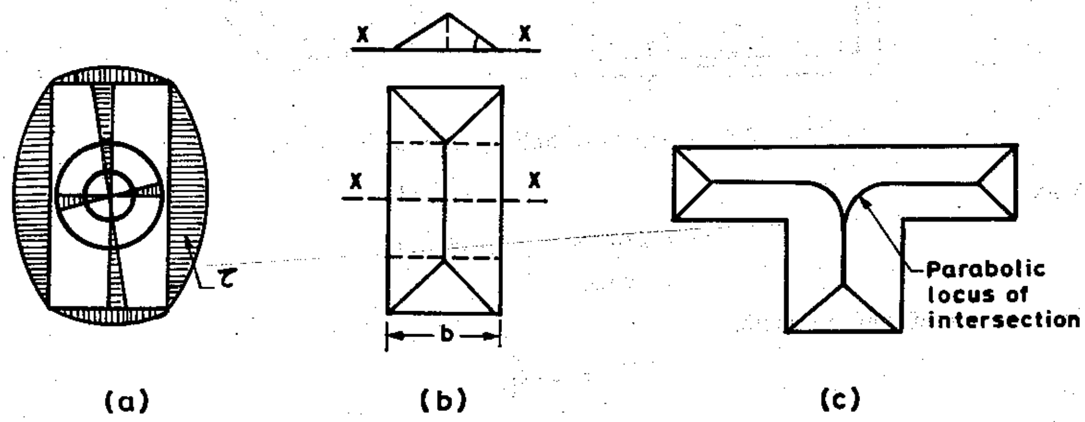


Fig. 18.3 Analysis of rectangular sections in torsion: (a) Elastic stress, (b) and (c) Sand heap analogy.

However, the value of the stress to be used for limit state design should be the value obtained by plastic analysis. This can be easily obtained by the sand heap analogy which gives the following principles:

1. Plastic torsion = volume of sand heap
2. Slope of sand heap = twice the constant plastic shear stress

From Fig. 18.3(b) which represents the sand heap for a rectangle, the height of the heap (one-half breadth × slope) is

$$\text{Height of sand heap} = (1/2b)\,(2\tau_t) = b\tau_t$$

$$\text{Volume of sand heap} = 1/2\,(D-b)\,b\,(b\tau_t) + \frac{1}{3}\,b^2 b\tau_t$$

$$T = 1/2 b^2\left(D - \frac{b}{3}\right)\tau_t$$

Hence the maximum shear by plastic analysis is given by

$$\tau_t = \frac{2T}{b^2(D - b/3)} \tag{18.3}$$

For the hollow section the constant shear stress on the walls can be approximated as follows (see Fig. 18.4):

$$\tau_1 t_1 = \tau_2 t_2 \text{ (for shear flow)}$$

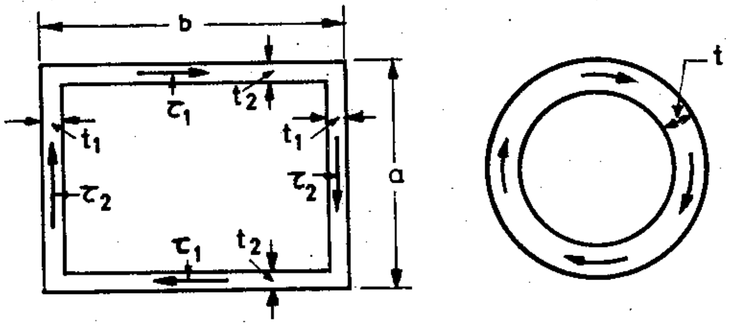


Fig. 18.4 Analysis of hollow sections in torsion.

Also,

$$T = 2\tau_1 t_1 \times (a - t_2)\,(b - t_1)$$
$$= 2\tau_1 t_1 A$$

For a section of uniform width,

$$\tau_t = \frac{T}{2At}$$

where

A = area enclosed by the centre line of the walls

t = wall thickness

It should be noted that the combined effect of bending shear and torsional shear is to increase the shear on one side and decrease it on the opposite side (refer also to torsion and shear in flat plates).

18.3.1 TORSIONAL STRENGTH OF CONCRETE BEAMS (ACCORDING TO BS 8110)

According to BS 8110, one has to assess the shear produced in bending shear and torsional shear separately.

As already pointed out in Chapter 7, the stresses can be evaluated by the equation

$$\tau_v = V/bd$$

The safe strength of concrete in bending shear without shear reinforcement is given by τ_c (Table 13, IS 456). Similarly, the shear produced by torsion can be evaluated by plastic theory by Eq. (18.3), as

$$\tau_t = \frac{2T}{b^2(D - b/3)}$$

BS 8110 stipulates that without torsion reinforcement the above value should not exceed τ_t given by

$$\tau_{tc} = 0.067\sqrt{f_{ck}} \quad \text{or} \quad 0.4\ \text{N/mm}^2$$

Also, even if the section is fully reinforced for shear and torsion, the maximum value of shear $(\tau_v + \tau_t)$ should not exceed $\tau_{c\,(\max)} = 0.8\sqrt{f_{ck}}$ or $5\ \text{N/mm}^2$. (It may be noted that, Table 14 of the IS 456 is derived from the expression approximated by $\tau_{c\,(\max)} = 0.63\sqrt{f_{ck}}$.)

In the case of small sections ($y_1 < 550$ mm), (Fig. 18.6) the above value can be modified as

$$\tau'_{c\,(\max)} = (y_1/550)\,(\tau_{c\max})$$

18.4 TORSIONAL STRESS IN FLANGED SECTIONS

To determine maximum shear stress in sections *I*, *T* or *L*, and other rectangular sections, the section may be assumed to be divided into component rectangles with the largest possible rectangle as one of its components. The torsion can be assumed to be distributed to each rectangle in proportion to its torsional stiffness (KDb^3). Then by elastic analysis, we get

$$T_n = T\frac{K_n D_n b_n^3}{\Sigma K_n D_n b_n^3} \tag{18.4}$$

By plastic analysis the value is given by

$$T_n = T\frac{D_n b_n^3}{\Sigma D_n b_n^3} \tag{18.5}$$

The maximum stress in each rectangle is then found by Eq. (18.2) or (18.3) for elastic and plastic analysis, respectively.

18.5 REINFORCEMENTS FOR TORSION IN R.C. BEAMS

Ultimate failure of a beam in torsion, according to Hsu's skew bending theory, is by rotation about a skew axis as shown in Fig. 18.5. (This can be easily demonstrated by taking a brittle material like a piece of chalk and giving it a twist at each of its ends by hand in the opposite directions.) Because of the skewness of the surface of failure (unlike in simple bending where it is planar), reinforcements for torsion should consist not only of transverse links (loop reinforcement) but also of longitudinal steel provided at all the four corners of the beam (Fig. 18.6). The requirement can be explained both by the skew bending theory as also by the simple space truss model (Fig. 18.7) similar to the plane truss model proposed as early as in 1929, for bending shear.

18.6 INTERACTION CURVES FOR COMBINED SHEAR AND TORSION

The strength of rectangular section in combined shear and torsion has been a subject of much research. Theories have been built on the basis of a large number of laboratory tests. It is generally represented by an interaction curve and different people have proposed different shapes based on their observations from laboratory test results.

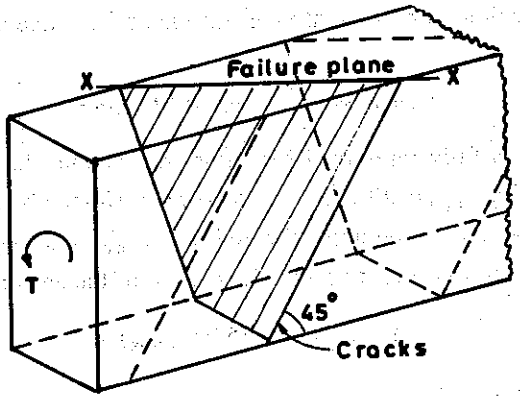


Fig. 18.5 Failure of beam section in torsion.

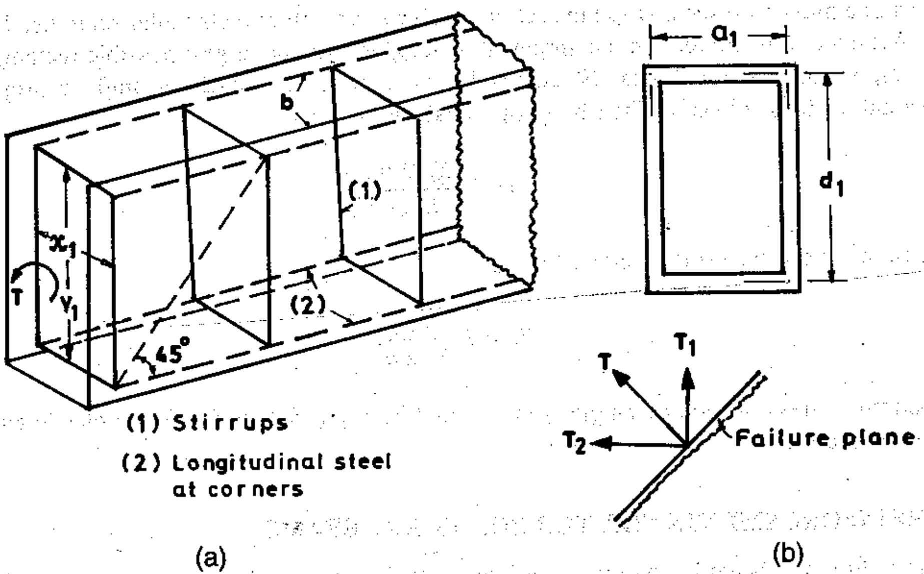


Fig. 18.6 Reinforcement of beams for torsion: (a) Stirrups, (b) Longitudinal steel at corners.

18.6.1 INTERACTION CURVE FOR CONCRETE WITHOUT WEB STEEL UNDER V AND T

If

V_{u0} = shear strength of beam without torsion

T_{u0} = torsional strength without shear

T_u, V_u = the design torsion and shear respectively

It has been found that for beams without web reinforcement the interaction curve can be represented by a quarter circle, as shown in Fig. (18.8a).

$$\left(\frac{T_u}{T_{u0}}\right)^2 + \left(\frac{V_u}{V_{u0}}\right)^2 = 1 \tag{18.6}$$

Equation (18.6) has been assumed by ACI in its design.

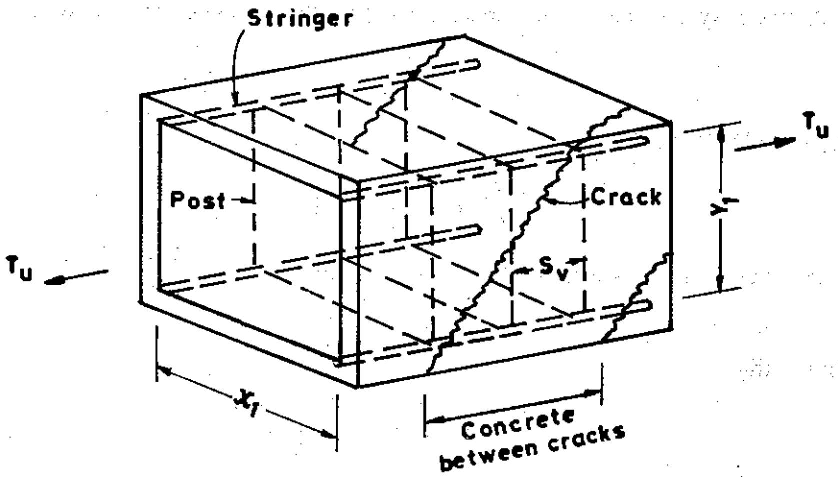


Fig. 18.7 Space-truss model for torsion in RCC beams.

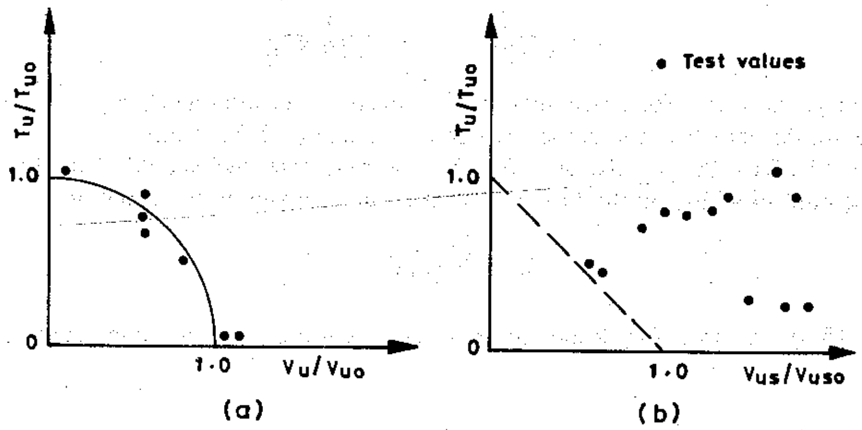


Fig. 18.8 Interaction curves for torsion and shear: (a) without web steel, (b) with web steel.

18.6.2 INTERACTION CURVE FOR CONCRETE WITH WEB STEEL UNDER V AND T

For beams with web reinforcement it has been found that the interaction curve can be assumed to follow the conservative linear relation represented by the equation (Fig. 18.8b)

$$\left(\frac{T_u}{T_{u0}} + \frac{V_{us}}{V_{us0}}\right) = 1 \tag{18.7}$$

where

T_u = torsional moment in the section

V_{us} = shear force carried by the web steel out of the total shear force in the section (V_u)

V_{us0} = shear strength of reinforcement assuming no torsion is present

T_{u0} = torsional strength of reinforcement assuming no shear is present

Based on tests one may assume that the shear carried by the web steel (V_{us}) is 40 per cent of V_u, i.e.

$$V_{us} = 0.4V_u$$

Accordingly, Eq. (18.7) reduces to

$$\left(\frac{T_u}{T_{u0}} + \frac{V_u}{2.5V_{us0}}\right) = 1 \tag{18.8}$$

From Eq. (7.5a) denoting s = spacing of stirrups, we have

$$V_{us0} = 0.87f_y A_{sv}(d/s)$$

It can be shown that

$$T_{u0} = 0.87f_y A_{sv}\left(\frac{x_1 y_1}{s}\right)$$

Substituting these values in equation (18.8), we get

$$A_{sv} = \frac{T_u s}{x_1 y_1 (0.87f_y)} + \frac{V_u s}{2.5d\,(0.87f_y)} \tag{18.8a}$$

The equation given in IS 456: clause 40.4.3 is obtained by substituting $d_1 = y_1$, where d_1 is the centre-to-centre distance between corner bars in the direction of the depth.

Clause 40.4.3 (of IS 456) also specifies that the transverse steel should naturally be not less than that required to withstand the full equivalent shear V_e given by the equation

$$A_{sv} = \frac{(\tau_{ve} - \tau_c)\,bs}{0.87f_y} \tag{18.8b}$$

Equations (18.8) are used for design of transverse reinforcement for combined torsion and shear.

18.7 PRINCIPLES OF DESIGN OF SECTIONS FOR TORSION BY DIFFERENT CODES

The design procedure to be adopted when torsion is present in R.C. members depends on the code to be used. The IS, BS and ACI propose different methods for torsion design, even though the resultant design is considered equally safe. When torsion is present along with 'bending shear', IS recommends the use of an equivalent shear for which the shear steels are calculated. Again in IS when torsion is present as combined with bending, an equivalent bending moment is calculated and reinforcement for this equivalent bending moment is provided as longitudinal steel.

In the British practice, the section is separately analysed for maximum torsional stresses, and depending on the magnitude of the resultant stress, the torsional reinforcements are calculated. Steel is also calculated separately for shear and bending moments. The values of reinforcements thus calculated individually are combined and provided as stirrups and longitudinal steel.

The ACI procedure for design for torsion is to accommodate torsional shear in the same way as in flexural shear, i.e. part of the torsional moment may be considered as carried by concrete without web steel and the remainder by stirrups. The part carried by concrete is obtained from a quarter-circle interaction curve as explained in Section 18.6.1.

18.8 DESIGN FOR TORSION BY BS 8110

It has already been pointed out that, according to BS 8110, torsion is treated separately and provided for separately. This method is explained first as it gives an insight into the fundamentals of design of R.C. beams for torsion.

1. *Area of stirrups*

A simple expression for the area of the stirrups to withstand torsion can be obtained by assuming the cracking pattern in torsion (which is in the form of a helix) to be inclined at 45° to the horizontal as shown in Fig. 18.5. The torsion is withstood by the moment of forces in the stirrups in the x and y directions about the centre. Denoting the centre-to-centre distances between the links as x_1 and y_1, horizontally and vertically (see Fig. 18.6), we get

$$\text{Torsion force in each link} = \frac{A_{sv}}{2}(0.87 f_y) = F$$

where A_{sv} is the area of both legs of the stirrups; moment of vertical legs in one link $= Fx_1$

$$\text{No. of links cut by 45° line} = \frac{y_1}{s_v}$$

$$\text{Moment of the vertical links in the crack} = \frac{Fx_1y_1}{s_v}$$

Similarly,

$$\text{Moment of horizontal legs in one link} = Fy_1$$

$$\text{No. of links cut by 45° line} = \frac{x_1}{s_v}$$

$$\text{Moment of horizontal links in the crack} = \frac{Fy_1x_1}{s_v}$$

$$T_u = \frac{2Fx_1y_1}{s_v} = \frac{2A_{sv}}{2s_v}(0.87 f_y)x_1y_1$$

$$T_u = (0.87 f_y A_{sv} x_1 y_1)/s_v \qquad (18.9)$$

Using an efficiency factor 0.8, we get

$$T_u = \frac{0.8 A_{sv}(0.87 f_y)x_1y_1}{s_v}$$

$$\frac{A_{sv}}{s_v} = \frac{T_u}{0.8\,x_1y_1(0.87 f_y)}$$

or

$$\frac{T_u}{0.8x_1y_1} = \frac{A_{sv}(0.87 f_y)}{s_v} \qquad (18.9a)$$

which is a simple expression for design using Table 62 of SP 16 or Table 7.3 of the text. It should be remembered that this steel is to be provided in addition to the steel to be supplied for shear V_u. The procedure is shown in Example 18.4.

2. *Area of additional longitudinal steel* (A_{s1})

At least four numbers of steel bars should be placed symmetrically inside the four corners of the links for the links to be effective. It is usually specified that the clear distances between these bars should not exceed 300 mm. These bars are meant to take care of the component of the tensile force in the longitudinal direction (Fig. 18.6). According to this concept, the volume of the longitudinal steel required will be the same as the volume of the transverse hoops. Taking a distance equal to spacing of stirrup and equating the forces, we get the area of additional longitudinal steel as

$$A_{s1}(f_{y1})s_v = \frac{A_{sv}}{2}(f_y)2(x_1 + y_1)$$

$$A_{s1}f_{y1}s_v = A_{sv}f_y(x_1 + y_1)$$

where

A_{s1} = total area of the additional longitudinal steel

A_{sv} = area of the two legs of stirrups

f_{y1} = yield stress of the longitudinal steel

f_y = yield stress of links

x_1, y_1 = centre-to-centre distance of links.

The formula for design is

$$A_{s1} = \frac{A_{sv}}{s_v}\frac{f_y}{f_{y1}}(x_1 + y_1) \tag{18.10}$$

This area of steel has to be provided in addition to the bending steel to take care of torsion.

18.9 PRINCIPLES OF DESIGN FOR COMBINED BENDING, SHEAR AND TORSION BY IS 456

As already explained, in the design procedure for torsion according to IS 456, it is not necessary to calculate the shear stresses produced by torsion separately as in BS 8110. The former gives the analysis for combined effects of torsion shear and bending shear. Bending shear and torsion are combined to an equivalent shear V_e. Similarly, the bending moment and torsional moment are combined to an equivalent bending moment M_e. The R.C. section is then designed for V_e and M_e.

18.9.1 CALCULATION OF EQUIVALENT SHEAR AND DESIGN FOR STIRRUPS

An empirical relation for equivalent shear due to the combined effects of torsion and shear has been suggested in IS 456: clause 40.3 as

$$V_e = V_u + 1.6\,T_u/b \tag{18.11}$$

where

V_e = equivalent shear

V_u = design shear

T_u = design torsion

b = breadth of section

The equivalent nominal shear stress will be given by the expression

$$\tau_{ve} = \frac{V_e}{bD} \tag{18.12}$$

Under no condition should the above value exceed τ_{max} given in Table 14 of IS 456 (Table 7.2 of the text). The area of shear steel should satisfy two conditions: First, the area of reinforcement A_{sv} should satisfy the same equation as used for bending shear. (IS code: clause 40.4.3). Hence

$$\frac{A_{sv}}{s_v} = b\left(\frac{\tau_{ve} - \tau_c}{0.87 f_y}\right) \tag{18.13}$$

where s_v = spacing of the stirrups.

Secondly, a linear interaction curve as shown by Eq. (18.7) is assumed, and the steel should satisfy the condition given in IS 456, clause 40.4.3, already derived as Eq. (18.8a).

$$\frac{T_u s_v}{A_{sv} b_1 d_1 (0.87 f_y)} + \frac{V_u s_v}{2.5 d_1 (0.87 f_y) A_{sv}} = 1$$

Rewriting this equation in the usual form for design of shear steel, it becomes

$$\frac{A_{sv}}{s_v}(0.87 f_y) = \frac{T_u}{b_1 d_1} + \frac{V_u}{2.5 d_1}$$

18.9.2 CALCULATION OF EQUIVALENT BENDING MOMENT AND DESIGN FOR LONGITUDINAL STEEL

In IS 456 the effect of bending moment and torsion is converted into an equivalent total bending (M_e) given in IS 456: clause 40.4.2 by the equation

$$M_e = M_u \text{ (bending)} \pm M_t \text{ (equivalent torsion)}$$

The equivalent bending moment (M_t) due to torsion T_u is given by

$$M_t = \frac{T_u\,(1 + D/b)}{1.7} \tag{18.14}$$

so that the equivalent total bending moment is given by

$$M_e = M_u \pm \frac{T_u\,(1 + D/b)}{1.7}$$

where

T_u = design torsional moment

M_u = design bending moment

D = overall depth

b = breadth of beam

The above equation for bending moment is derived from the interaction curve between bending moment and torsion and the three possible modes of failure, the theory of which is beyond the scope of this text. Thus in combined bending and torsion the longitudinal steel should be designed for this equivalent total moment given by

$$M_{e1} = M_u + \frac{T_u(1 + D/b)}{1.7} \tag{18.15}$$

18.9.3 DESIGN FOR COMPRESSION STEEL

If $M_t > M_u$, then there can be reversal of moment, and longitudinal steel has to be provided on the flexural compression face also, so that the beam can withstand the equivalent moment

$$M_{e2} = M_t - M_u \tag{18.16}$$

Additional steel is provided for this moment on the compression side of the beam.

18.10 DETAILING OF TORSION STEEL

IS 456: clause 25.5.1.7 gives the rules regarding detailing of torsion steel, which are to be read along with clause 40.4.3. These rules can be summarised as follows:

1. The spacings of stirrups should not exceed x_1, or

$$\frac{x_1 + y_1}{4} \text{ or } 300 \text{ mm}$$

2. There should be at least one longitudinal bar placed at each corner of the stirrups. When the cross-sectional spacing exceeds 450 mm, additional longitudinal bars should be provided to satisfy the minimum reinforcement and spacing rules regarding side face reinforcement. That is, there should be a minimum of 0.1 per cent longitudinal steel spaced at not more than 300 mm or thickness of web—IS 456: clause 25.5.1.3.

18.11 DESIGN RULES ACCORDING TO IS 456

The following is the procedure for design of beam for torsion according to IS 456, clause 40.

18.11.1 DESIGN FROM FUNDAMENTALS

Step 1: Determine design moments shear and torsion M_u, V_u and T_u.

Step 2: Determine equivalent moments and longitudinal steels. Calculate

$$M_t = \frac{T_u\,(1 + D/b)}{1.7}$$

(a) $M_{e1} = M_u + M_t$

Design tension steel for M_{e1}

(b) If $M_t > M_u$, then

$$M_{e2} = M_t - M_u$$

Design steel on compression face for reversal of moment = M_{e2}

Step 3: Determine the equivalent shear given by the equation

$$V_e = V_u + 1.6\,\frac{T_u}{b}$$

Step 4: Find shear stress and check for maximum shear. Determine the shear stress as

$$\tau_{ve} = \frac{V_e}{bd}$$

Check that this should be less than the maximum allowed.

Step 5: Calculate area of shear links from the formula

$$\frac{A_{sv}}{s_v} = \frac{(\tau_{ve} - \tau_c)b}{0.87f_y}$$

Step 6: Check for interaction of shear and torsion given by the relation

$$A_{sv} = \frac{T_u}{(0.87f_y)b_1d_1} + \frac{V_u}{(0.87f_y)2.5d_1}$$

Adopt the larger value of steps 5 and 6 for the shear steel. The spacing should not exceed x_1 or $(x_1 + y_1)/4$ or 300 mm (IS 456: clause 25.5.1.7).

18.11.2 PROCEDURE FOR DESIGN WITH SP 16

The calculation in steps 5 and 6 for shear steel can be facilitated by use of SP 16, Table 62 (Table 7.3 of the text) which gives $\dfrac{A_{sv}(0.87f_y)}{s_v}$ values for different sizes of steel and spacings. Steps 5 and 6 above can be modified respectively as

$$(\tau_{ve} - \tau_c)b = \frac{A_{sv}}{s_v}(0.87f_y)$$

$$\frac{T_u}{b_1d_1} + \frac{V_u}{2.5d_1} = \frac{A_{sv}}{s_v}(0.87f_y)$$

Calculate

$$[(\tau_{ve} - \tau_c)b] + \left(\frac{T_u}{b_1d_1} + \frac{V_u}{2.5d_1}\right)$$

and read off the diameter of stirrups and their spacings from table 62 of SP 16 or Table 7.3 of the text.

18.12 DESIGN PROCEDURE ACCORDING TO BS 8110

18.12.1 RULES FOR DESIGN

Let the section be subjected to bending moment M, shear V, and torsion T. It is necessary to design the transverse and longitudinal steel.

First, the section is designed for longitudinal steel for the bending moment M. Then it is designed for the shear produced by the bending shear and torsion as described now.

18.12.2 CHECKING OF SHEAR CAUSED BY V AND T

As has been explained in Section 18.3.1,

$$\tau_v = \frac{V}{bd}$$

$$\tau_t = \frac{2T}{b^2(D - b/3)}$$

$$\tau_{tc} = 0.067\sqrt{f_{ck}} \ngtr 0.4 \text{ N/mm}^2$$

$$\tau_v + \tau_t \ngtr \tau_{c\,(\max)}$$

$$\tau_{c\,(\max)} = 0.8\sqrt{f_{ck}} \ngtr 5.0 \text{ N/mm}^2$$

The rules for design are given in Table 18.2.

TABLE 18.2 DESIGN FOR SHEAR AND TORSION BS 8110 (1985)

Bending shear stress	Torsion shear stress	
	$\tau_t < \tau_{tc}$	$\tau_t > \tau_{tc}$
Less than safe in concrete ($\tau_v < \tau_c$)	Nominal shear steel, no torsion steel	Designed torsion steel
Greater than the safe value in concrete ($\tau_v > \tau_c$)	Designed shear steel, no torsion steel	Designed shear and torsion steel

18.12.3 DESIGN FORMULA FOR TORSION

It has already been shown that for design of stirrups for torsion, one has to calculate the areas of links and the longitudinal steel. These are given by the following formulae:

1. The area of links is given by Eq. (18.9) as

$$\frac{A_{sv}}{s_v} = \frac{T_u}{0.8x_1y_1(0.87f_y)}$$

2. The area of longitudinal bars is given by Eq. (18.10) as

$$A_{s1} = \frac{A_{sv}}{s_v}\left(\frac{f_y}{f_{y1}}\right)(x_1 + y_1)$$

18.12.4 MAXIMUM SPACING ALLOWED FOR LINKS AND LONGITUDINAL STEELS IN BS 8110

According to BS 8110, the spacing of links should not exceed x_1 or $y_1/2$ or 200 mm to control cracking. The distance between the longitudinal steels should not exceed 300 mm.

18.12.5 PROCEDURE FOR DESIGN FOR TORSION BY BS 8110

Step 1: Find the area of tension steel for M.

Step 2: Calculate $\tau_v = V/bd$.

Step 3: Calculate τ_t due to torsion.

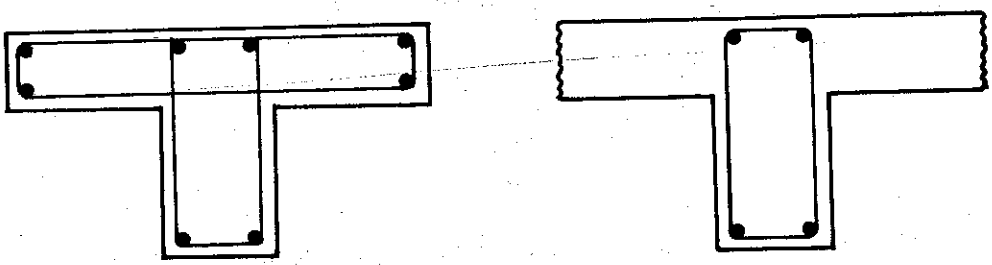
Step 4: Design shear and torsion steel as per Table 18.2.

Step 5: Calculate additional longitudinal steel by Eq. (18.10). Place this area as rods around the periphery of the beam, as already specified in Section 18.5.

18.13 ARRANGEMENT OF LINKS FOR TORSION IN FLANGED BEAMS

It is easy to arrange the stirrup and longitudinal steel for rectangular section. However, when treating flanged beams like T or I beams, it is not to be treated as one whole. It should be split into its constituent rectangles as explained in Section 18.4, and each rectangle detailed for torsion. Studies have shown that treating flanged beams with the largest rectangle, by taking the torsion gives conservative values, and this procedure is recommended in IS 456: clause 40.1.1.

In those cases where different rectangles are taken as resisting torsion, each rectangle must be suitably reinforced with the necessary links (Fig. 18.9).



(a) (b)

Fig. 18.9 Reinforcing flanged sections in torsion: (a) Torsion shared by two rectangles, (b) Torsion taken hy one rectangle.

18.14 TORSION IN BEAMS CURVED IN PLAN

Circular beams (e.g. the ring beam under a circular water tank supported on columns) are subjected to bending and torsion moments. Their distribution along the circumference depends on the number of column supports. The magnitude of these moments can be expressed as a function of the uniformly distributed load w and the radius of the ring beam r as follows: Let α and β be taken as given in Fig. 18.10.

The general formulae for torsion and bending moment at angle β are given by

1. $T_\beta = wr^2(\beta - \alpha + \alpha \cos \beta - \alpha \sin \beta \cot \alpha)$
2. $M_\beta = wr^2(\alpha \sin \beta - 1 + \alpha \cot \alpha \cos \beta)$
3. T_β is maximum when $M_\beta = 0$

Ref.

The maximum values of moments and torsion can be calculated as

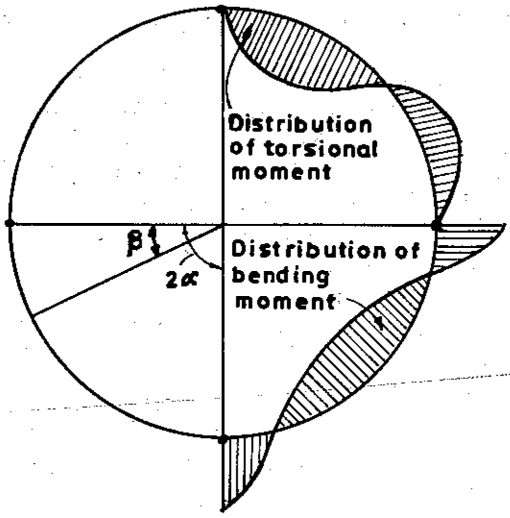
Sagging moment at mid-span = $2wr^2\alpha\lambda_1$ (18.19)

Hogging moment at supports = $2wr^2\alpha\lambda_2$ (18.20)

Maximum torsional moment = $2wr^2\alpha\lambda_3$ (18.21)

The angle at which the torsional moment is a maximum (as measured from the centre line of supports is taken as β_0. The values of α, λ_1, λ_2, λ_3 and β_0 for various conditions of support are given in Table 18.3.

Text Table 18.1



Text Eq. (18.1), Section 18.2

Text Eq. (18.1a)

Fig. 18.10 Distribution of torsional moment in ring beams (n = 4).

TABLE 18.3 MOMENT COEFFICIENT FOR TORSION IN RING BEAMS

No. of supports	2α (degrees)	λ_1	λ_2	λ_3	β (degrees)
4	90	0.070	0.137	0.021	19.25
6	60	0.045	0.089	0.009	12.75
8	45	0.033	0.066	0.005	9.5
10	36	0.027	0.054	0.003	7.5
12	30	0.023	0.045	0.002	6.25

EXAMPLE 18.1 **(Analysis of structure for torsion)**

The beam XY is rigidly connected to AB and $A'B'$ and loaded with a factored load of 200 kN at its centre. Beams AB and $A'B'$ are rigidly fixed at their ends. All beams are R.C. T-section as shown in Fig. E.18.1. Determine the torsional moment in the member AB.

(18.19)

(18.20)

(18.21)

re line of
upport are

degrees)
19.25
12.75
9.5
7.5
6.25

00 kN at
as shown

Ref.	Step	Calculations
		Note: The problem is solved by moment distribution
	1.	*Calculate C* [Elastic method]
		Cut the T-section into rectangles
		(a) 2-3-5-6, and (b) 2 times 1-2-2′-1′
Text Table 18.1		$\frac{D}{b}$ for part (a) $= \frac{600}{300} = 2.0, \quad K = 0.23$
		Assuming continuity with 2-2′ and 3-3′, we have
		$\frac{D}{b}$ for part (b) $= \frac{300}{200} = 1.5, \quad K = 0.195$
Text Eq. (18.1), Section 18.2.3		$C = \Sigma \frac{1}{2} KDb^3$ $= \frac{1}{2}[0.23 \times 60 \times 30^3 + 0.195 \times 30 \times 20^3]$ $= 2.1 \times 10^5 \text{ cm}^4$
		Calculation of C by Formula [Plastic method]
Text Eq. (18.1a)		$C = \frac{1}{2} \Sigma \left(1 - 0.63 \frac{b}{D}\right)\left(\frac{b^3 D}{3}\right)$
		For Fig. E.18.1(a),
		$C_1 = \left(1 - 0.63 \frac{30}{60}\right)\left(\frac{30^3 \times 60}{3}\right)$ $= 3.7 \times 10^5 \text{ cm}^4$

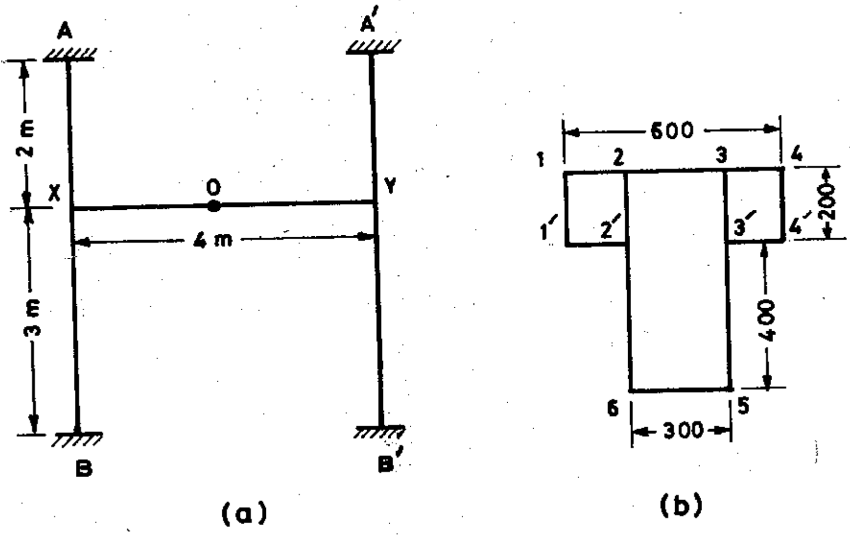


Fig. E.18.1.

EXAMPLE 18.1 (cont.)

Ref.	Step	Calculations
		For Fig. E.18.1(b),
		$C_2 = \left(1 - 0.63\dfrac{20}{30}\right)\left(\dfrac{20^3 \times 30}{3}\right)$
		$= 0.46 \times 10^5 \text{ cm}^4$
		$C = \dfrac{1}{2}(3.7 + 0.46)10^5$
		$= 2.08 \times 10^5 \text{ cm}^4$
	2.	*Calculate I*
		$\text{C.G. } \bar{y} = \dfrac{(600 \times 200 \times 100) + (300 \times 400 \times 400)}{(600 \times 200) + (300 \times 400)} = 250 \text{ mm}$
		$I = \dfrac{60 \times (20)^3}{12} + 60 \times 20 \times (15)^2 + \dfrac{30(40)^3}{12} + 30 \times 40 \times 15^2$
		$= 7.4 \times 10^5 \text{ cm}^4$, i.e. $I = 3.56C$
	3.	*Value of G*
		Assume $\mu = 0.15$
		$G = \dfrac{E}{2(1+\mu)} = \dfrac{E}{2(1+0.15)} = 0.4E$ (approx.)
	4.	*Stiffness coefficients of members meeting at X*
		$K_{AX} = \dfrac{GC}{L} = \dfrac{0.4EC}{2} = 0.2EC$
		$K_{BX} = \dfrac{GC}{L} = \dfrac{0.4EC}{3} = 0.133EC$
		$K_{XY} = \dfrac{4EI}{L} = \dfrac{4E \times 3.56C}{4} = 3.56EC$
		$\dfrac{1}{2}K_{XY} = 1.76EC$
		(Only one-half stiffness is considered because of symmetry)
	5.	*Distribution factors*
		$F_{XA} = \dfrac{0.2}{0.2 + 0.13 + 1.76} = \dfrac{0.2}{2.09} = 0.10$
		$F_{XB} = \dfrac{0.13}{2.09} = 0.06$
		$F_{XO} = \dfrac{1.76}{2.09} = 0.84$

EXAMPLE 18.1 (cont.)

Ref.	Step	Calculations
	6.	*Fixing Moment* $\frac{Wl}{8} = \frac{200 \times 4}{8} = 100$ kNm
	7.	*Distribution*

Joint:	A	X			B
Member:	AX	XA	XO	XB	BX
Distribution Factor:		0.1 (Torsion)	0.84 (Bending)	0.06 (Torsion)	
		– 10	100 – 84	– 6	
	– 10 – 10(T)	– 10(T)	– 84(M)	– 6(T)	– 6 – 6(T)

Note: In contrast to bending moment, full torsion is carried over from one end to the other end of a fixed beam.

EXAMPLE 18.2 **(Analysis of structure for torsion)**

The car porch of a building is shown in Fig. E.18.2, the beams and the slab being cast monolithic. The live load on the beam due to U.D.L. on the slab is 2 kN/m. Determine the torsion and shear due to live load for which the beam should be designed.

Ref.	Step	Calculations
		(*Note*: Torsion is taken about the centre line of the beam.)
	1.	*Torsion per metre length due to* LL
		Factored LL = 1.5 × LL = 1.5 × 2 = 3 kN/m
		Torsion per unit length of beam
		$= 3 \times \frac{3}{2} = 4.5$ kNm/m
		The torsion is zero at the centre and maximum at the two fixed ends of the beam.
		Torsion at ends = $4.5 \times \frac{6}{2} = 13.5$ kNm
		Torsion at critical section for shear
		= 4.5(3 – effective depth) = 4.5(3 – 0.65)
		= 10.57 kNm

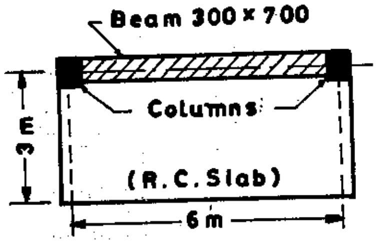


Fig. E.18.2

EXAMPLE 18.2 (cont.)

Ref.	Step	Calculations
	2.	*Design shear* LL shear at critical section (*d* from end of the beam) $= G(3 - 0.65) = 21.15$ kN
	3.	*Equivalent shear due to live load* $V_e = V + 1.6\dfrac{T}{b} = 21.15 + \dfrac{1.6(10.57)}{0.30}$ kNm $= 77.52$ kN *Note*: The beam is to be designed for the combined effects of bending shear and torsion due to dead and live loads.

EXAMPLE 18.3 **(Analysis of T beam in torsion)**

A T beam is as shown in Fig. E.18.3. If the section is subjected to a torsion of 150 kNm, calculate the torsion carried by the two main rectangular portions of the T beam, assuming (a) elastic theory and (b) plastic theory.

Ref.	Step	Calculations
	1.	*Proportioning of torsion by elastic theory* $T_1 = T\dfrac{K_1 D_1 b_1^3}{\Sigma KDb^3}$
Text Table 18.1		For flange, $\dfrac{D}{b} = \dfrac{350}{120} = 2.91, \quad K_1 = 0.26$ For web, $\dfrac{D}{b} = \dfrac{800}{350} = 2.30, \quad K_2 = 0.24$ $K_1D_1b_1^3 + K_2D_2b_2^3 = 0.26 \times 350 \times 120^3$ $+ 0.24 \times 800 \times 350^3$ $= (1.57 + 82.32)10^8$ mm^4 $T_1 = \dfrac{150 \times 82.32}{83.89} = 147.2$ kNm $T_2 = \dfrac{150 \times 1.57}{83.89} = 2.8$ kNm (The major part is carried by the web only)
	2.	*Calculation by plastic theory* $T_1 = T\dfrac{D_1b_1^3}{D_1b_1^3 + D_2b_2^3}$ $= 350 \times 120^3 + 800 \times 350^3$ $= (6.05 \times 10^8 + 343 \times 10^8) = 349.05 \times 10^8$

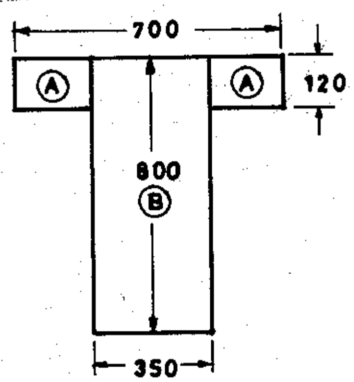


Fig. E.18.3

EXAMPLE 18.3 (cont.)

Ref.	Step	Calculations
		$T_1 = 150\left(\dfrac{343}{349.05}\right) = 147.4$ kNm
		$T_2 = 150\left(\dfrac{6.05}{349.05}\right) = 2.6$ kNm
		Note: (a) The provision in IS 456: clause 40.1.1 to design flanged beams, assuming torsion is taken fully by the beam, is fully justified.
		(b) Proportioning by plastic theory is simpler than proportioning by elastic theory and gives reasonably good results.

EXAMPLE 18.4 **(Design of beam in torsion by IS 456)**

The T beam given in Example 18.3 is subjected to a bending moment of 215 kNm, shear of 150 kN, and torsion of 105 kNm. Assuming $f_{ck} = 30$ and $f_y = 415$ (N/mm^2), design the reinforcements according to IS 456. Cover to centre of steel is 50 mm.

Ref.	Step	Calculations	Output
IS 456: Cl. 40.1.1	1.	*Note:* Assume that torsion is fully taken by web.	
Cl. 40.4.2	2.	*Equivalent BM*	
		$M_e = M_u + M_t = M_u + \dfrac{T}{1.7}\left(1 + \dfrac{D}{b}\right)$	
		$= 215 + \dfrac{105}{1.7}\left(1 + \dfrac{800}{350}\right) = 215 + 202.9$	
		$= 417.9$ kN	
	3.	*Calculation of longitudinal steel*	
		$M_t \ngtr M_u$ (If M_t exceeds M_u, provide compression steel for $M_t - M_u$)	
		Hence design for M_e only.	
		$\dfrac{M_e}{bd^2} = \dfrac{417.9 \times 10^6}{350 \times 750^2} = 2.12$	
SP 16 Table 4		$p = 0.646,\ A_s = \dfrac{0.646}{100}(350 \times 750) = 1696$ mm^2	Provide 6 Nos. 20 mm $A_s = 1885$ mm^2 (allowed for beams) greater than min. and less than max.
		Percentage provided $= \dfrac{1885 \times 100}{350 \times 750} = 0.72$ per cent	
	4.	*Determine equivalent shear*	
		$V_e = V + 1.6\dfrac{T}{b}$,	

EXAMPLE 18.4 (cont.)

Ref.	Step	Calculations	Output
		Using kNm/m units, we obtain	
		$V_e = 150 + 1.6\frac{105}{0.35} = 150 + 480 = 630$ kN	
IS	5.	*Find shear stress*	
Table 13		$\tau_e = \frac{V_e}{bd} = \frac{630 \times 10^3}{350 \times 750} = 2.4\ \text{N/mm}^2$	
Table 14 $f_{ck} = 30$		This is more than $\tau_c = 0.58$ for 0.72% steel (assuming full extension of steel) and less than $\tau_{max} = 3.5\ \text{N/mm}^2$.	
	6.	*Design of stirrups*	
IS 456 Cl. 40.4.3		Two conditions should be satisfied:	
		Condition 1	
		$\frac{A_{sv}(0.87 f_y)}{s_v} = \frac{T}{b_1 d_1} + \frac{V}{2.5 d_1}$	
		$= \frac{105 \times 10^8}{250 \times 700} + \frac{150 \times 10^3}{2.5 \times 700} = 685.7\ \text{N/mm}$	
		Condition 2	
		$\frac{A_{sv}(0.87 f_y)}{s_v} = (\tau_e - \tau_c)b = (2.4 - 0.58) \times 350$	Design for
		$= 637$ N/mm	685.7 N/mm
	7.	*Design of stirrups*	
		$\frac{A_{sv}(0.87 f_y)}{s_v} = \frac{V_{US}}{d}, = \left(\frac{685.7}{1000}\right) \times 10 = 6.9$ kN/cm	Provide T10
SP 16 Table 62,		Adopt 10 mm at 8 cm.	two-legged stirrups 8 cm spacing.
		$\frac{V_{us}}{d} = 7.09$	
Text, Table 7.3		Spacing should not exceed $x_1 = 250$ mm or	Cover to centre of steel = 50 mm
IS Cl. 25.5.1.7		$\frac{x_1 + y_1}{4} = \frac{250 + 700}{4} = 237.5$ or 300 mm	
	8.	*Provision of longitudinal steel*	
		Longitudinal steel consists of 6 Nos. of 20 mm diameter at bottom and nominal hangers, 12 mm at top. As the depth of beam is more than 750 mm, provide side reinforcement 0.05 per cent on both faces.	

EXAMPLE 18.4 (cont.)

Ref.	Step	Calculations	Output
Cl. 25.5.1.3		$A_s = \frac{0.05}{100}(800 \times 350) = 140\ \text{mm}^2$ 2 Nos. 10 mm bars, $A_s = 157\ \text{mm}^2$ Spacing $= \frac{750}{3} = 250$ mm	Provide two bars of 10 mm on each face.

EXAMPLE 18.5 **(Design of beam in torsion by BS 8110)**

Design the T beam in Example 18.4 according to the provision of BS 8110.

Ref.	Step	Calculations	Output
		(*Note*: According to BS 8110, the bending moment, shear and torsion are calculated separately and provided for.)	
	1.	*Calculation of steel for bending moment 215 kNm* Capacity with N.A. at bottom of slab $M_1 = (0.36 \times 30 \times 700 \times 120)(750 - 0.42 \times 120)$ $= 634$ kNm As 215 kNm is very much less than 634 kNm, NA is inside the slab. Calculating the steel necessary for 215 kNm, we get $\frac{M_U}{bd^2} = \frac{215 \times 10^6}{700 \times 750 \times 750} = 0.55$	
SP 16 Table 4		$p = 0.156,\ A_s = \frac{0.156 \times 700 \times 750}{100} = 819$ sq.mm Percentage of steel on web area $= \frac{942 \times 100}{350 \times 750} = 0.36\%$	Provide 3 T20 $A_s = 942\ \text{mm}^2$ (> min. required)
	2.	*Calculation of steel for shear $V = 150$ kN* $\tau_v = \frac{V}{bd} = \frac{150 \times 10^3}{350 \times 750} = 0.57$ τ_c for 0.3% steel = 0.40 N/mm² Design the shear steel required.	
	3.	*Calculation of steel for torsion $T = 105$ kNm* Assume that torsion is taken fully by the beam part $\tau_t = \frac{2T}{b^2\left(D - \frac{b}{3}\right)} = \frac{2 \times 105 \times 10^6}{350^2\left(800 - \frac{350}{3}\right)}$ $= 2.51\ \text{N/mm}^2$	

EXAMPLE 18.5 (cont.)

Ref.	Step	Calculations	Output
		$\tau_t > \tau_{tc} = 0.067\sqrt{f_{ck}} = 0.067\sqrt{30} = 0.37\ \text{N/mm}^2$	
		Hence, τ_t requires designed reinforcement.	
	4.	*Determine* $A_{sv}/s_v\ (0.87f_y)$ *for shear and torsion*	
		$A_{sv}/s_v\ (0.87f_y)$ for shear $= (\tau_v - \tau_c)b$	
		$= (0.57 - 0.40)350$	
		$= 59.5$ N/mm	
		$\dfrac{A_{sv}}{s_v}(0.87f_y)$ for torsion $= \dfrac{T}{0.8x_1y_1}$	
		$x_1 = 350 - (2 \times 30) - 10 = 280$	Assume 30 mm cover.
		$y_1 = 800 - (2 \times 30) - 10 = 730$	
		For torsion,	
		$\dfrac{A_{sv}}{s_v}(0.87f_y) = \dfrac{105 \times 10^6}{0.8 \times 280 \times 730} \approx 650\ \text{N/mm}$	
		Total $= 59.5 + 650 = 709.5$	
	5.	*Design of shear steel by SP 16* (Using Fe 415 steel)	
		$\dfrac{V_{us}}{d} = \dfrac{A_{sv}}{s_v}(0.87f_y)$ in kN/cm	
SP 16 Table 62		$= \dfrac{709.5 \times 10}{1000} = 7.09$; (Use 8.167)	T 12 at 100
	6.	*Design of shear steel by formula*	
		$\dfrac{A_{sv}}{s_v} = \dfrac{59.5}{0.87f_y} + \dfrac{650}{0.87f_y} = 0.165 + 1.80 = 1.96$	
		Choosing 12 mm rods, we get	
		$A_{sv} = 226\ \text{mm}^2$ for two legs	
		Therefore,	
		$s_v = \dfrac{226}{1.96} = 115$ mm	T 12 at 115
	7.	*Check spacing for max. specified*	
		s_v should not be greater than	
		$x_1 = 280$ mm	
		$\dfrac{x_1 + y_1}{4} = \dfrac{280 + 730}{4} = 252$ mm or 300 mm	Satisfactory

EXAMPLE 18.5 (cont.)

Ref.	Step	Calculations	Output
	8.	*Extra longitudinal steel for torsion at corners* $A_{st} = \frac{A_{sv}}{s_v}\left(\frac{f_{yv}}{f_{yl}}\right)(x_1 + y_1) = 1.80 \times 1(280 + 730)$ $= 1818\ \text{mm}^2$. Use 4 bars 25 mm dia. Provide one bar in each corner. $A_s = 1968$ mm.	Use 4T 25

EXAMPLE 18.6 **(Torsion in channel sections)**

A precast edge beam of a building is of channel section shown in Fig. E.18.6. It is of span 12 m and is restrained at the ends. Precast floor slabs are placed on the lower flange as shown. Assuming that the load from the slabs is 30 kN per metre length, determine the twisting moment for which the edge beam should be designed.

Step	Calculations
1.	*Calculation of shear centre** $e = \dfrac{b/2}{1 + \frac{1}{6}(wh/bt)}$ from centre of web $= \dfrac{435}{2\left(1 + \dfrac{250 \times 1300}{6 \times 435 \times 200}\right)} = 134$ mm $= 134 - 250/2 = 9$ mm from outer edge
2.	*Position of centre of gravity of channel* Taking moments about the outer edge, we get $\bar{x} = \dfrac{(1100 \times 250 \times 125) + (560 \times 200 \times 2 \times 280)}{(1100 \times 250) + (560 \times 200 \times 2)}$ $= \dfrac{97095 \times 10^3}{499 \times 10^3} = 195$ mm inside
3.	*Loads on channel* Factored DL = 1.5(0.499 × 25) = 18.75 kN Factored load on flange = 1.5 × 30 = 45 kN Total load = 63.75 kN
4.	*Design torsional moment per unit length on channel* Due to DL of channel = $18.75\,\dfrac{(195 + 9)}{1000} = 3.83$ kNm/m

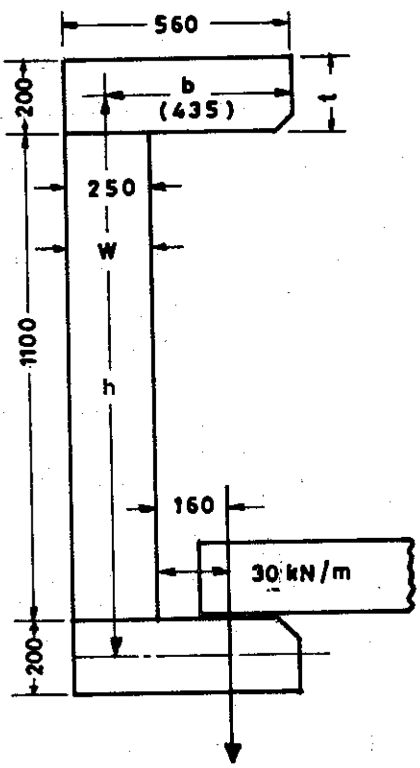


Fig. E.18.6

*Refer any book on Advanced Mechanics of Materials.

EXAMPLE 18.6 (cont.)

Step	Calculations
	Due to load from slab $= 45.0\,\dfrac{(9 + 250 + 160)}{1000} = 18.86$ kNm/m
	Total moment = 22.69 kNm
5.	*Maximum torsion at the ends*
	$T = \dfrac{22.69 \times 12}{2} = 136$ kNm

EXAMPLE 18.7 **(Design of channel section)**

Determine the torsional steel for the reinforced concrete channel section in Example 18.6. $f_{ck} = 25$, $f_y = 415$ (N/mm^2).

Ref.	Step	Calculations	Output
	1.	*Torsion in main rectangle (web)*	
		$T_1 = \dfrac{T(t^3h)}{\Sigma t^3 h}$	
		Web $(t^3h) = 250^3 \times 1500 = 23.4 \times 10^9$	
		Flanges $(t^3h) = 2 \times 200^3 \times 310 = 5.0 \times 10^9$	
		$T_1 = \dfrac{136 \times 23.4}{28.4} = 112$ kNm	$T = 112$ kNm
	2.	*Design shear* (τ_v)	
		$V = \dfrac{wl}{2} = \dfrac{63.75 \times 12}{2} = 382.5$ kN	
		$\tau_v = \dfrac{V}{bd} = \dfrac{382.5 \times 10^3}{250 \times 1450} = 1.06$ N/mm^2	$V = 382.5$ kN
SP 16 Table 61		Assume τ_c for grade 25 concrete = 0.40 $(\tau_v - \tau_c) = 1.06 - 0.40 = 0.66$ N/mm^2	
	3.	*Torsional shear* (τ_t)	
		$\tau_t = \dfrac{2T}{t^2(h - t/3)} = \dfrac{2 \times 112 \times 10^6}{250 \times 250(1500 - 250/3)}$	
		$= 2.53$ N/mm^2	
		Total shear $= 1.06 + 2.53 = 3.59$ N/mm^2	

EXAMPLE 18.7 (cont.)

Ref.	Step	Calculations	Output
BS 8110		(This is less than $\tau_{max} = 0.8\sqrt{f_{ck}} = 4$ N/mm^2	
		But greater than IS value = 3.1 N/mm^2)	
		$\tau_{t\,min} = 0.067\sqrt{f_{ck}} = 0.335$	
	4.	*Design of shear by BS 8110*	
		$\tau_t > \tau_{t\,min}$ $\tau_v > \tau_c$ Design for shear and torsion	
		$\dfrac{A_{sv}(0.87f_y)}{s_v}$ for shear $= (\tau_v - \tau_c)b = \dfrac{V}{d}$	
		$= 0.66 \times 250 = 165$ N/mm	
		$\dfrac{A_{sv}(0.87f_y)}{s_v}$ for tension $= \dfrac{T}{0.8x_1y_1}$	$x_1 = 250 - (2 \times 35)$ $= 180$ mm $y_1 = 1500 - 70$ $= 1430$ mm
		$= \dfrac{112 \times 10^6}{0.8 \times 180 \times 1430}$ $= 543.9$ N/mm	
		Total $\dfrac{V}{d} = \dfrac{(543.9 + 165) \times 10}{1000}$ in kN/cm	
SP 16 Table 62		$= 7.08$ kN/cm (708 N/mm)	
		Provide 12 mm at 100 mm$\left(\text{this gives } \dfrac{V_{us}}{d} = 8.16\right)$	T 12 at 100
	5.	*Design shear by IS 456*	
		$V_e = V + 1.6\left(\dfrac{T}{b}\right)$	$d = 1500 - 50$
		$= 382.5 + \dfrac{1.6(112)}{0.250}$	$= 1450$ mm
		$= 1099$ kN	
		Capacity for concrete $= \tau_c bd$	
		$= \dfrac{0.4 \times 250 \times 1500}{1000} = 150$ kN	
		$\dfrac{V_s}{d} = \left(\dfrac{1099 - 150}{145}\right) = 6.5$ kN/cm	
		Provide 12 mm at 120 mm gives $V_{us}/d = 6.8$)	T 12 at 120

EXAMPLE 18.7 (cont.)

Ref.	Step	Calculations	Output
Text Eq. (18.10)	6.	*Longitudinal steel for torsion by BS 8110* According to BS, $A_{st} = \dfrac{A_{sv}}{s_v}\left(\dfrac{f_y}{f_{y1}}\right)(x_1 + y_1)$ $\dfrac{A_{sv}}{s_v} = \dfrac{543.9}{0.87 \times 415} = 1.50$ $A_{st} = (1.50)(180 + 1430) = 2415\ \text{mm}^2$ 8 T 20(2513 mm^2 – two on each corner) In addition, provide steel for bending also.	8 T 20 (two on each corner)
		$M_{\max} = \dfrac{wl^2}{10} = \dfrac{63.75(12)^2}{10} = 918\ \text{kNm}$ $\dfrac{M}{bd^2} = \dfrac{918 \times 10^6}{250(1450)^2} = 1.75$ $p = 0.532\%$ $A_s = \dfrac{0.532 \times 250 \times 1450}{100} = 1928\ \text{mm}^2$ Use 7 T 20 (2199 mm^2)	Steel for M and T 11 T 20 on tension side 4 T 20 on compression side.
	7.	*Longitudinal steel by IS 456* $M_e = M_u + \dfrac{T_u(1 + D/b)}{1.7}$ $= 918 + \dfrac{112(1 + 1500/250)}{1.7}$ $= 918 + 461 = 1379\ \text{kNm} \quad (M_t < M_u)$ $\dfrac{M_e}{bd^2} = \dfrac{1379 \times 10^6}{250(1450)^2} = 2.62$	
SP 16 Table 3		$p = 0.845\%$ $A_s = \dfrac{0.845 \times 250 \times 1450}{100} = 3063\ \text{mm}^2$ Use 10 T 20 (3141 mm^2). As $M_t < M_u$, compression steel is not required.	Use 10 T 20 on tension side
	8.	Detail steel according to standard practice. *Notes*: (i) The lower flange should be designed to take the weight from the floor steel. (ii) The stirrups should be so designed that they should be able to carry also the load from the lower flange to the top of the beam in addition to taking care of shear and torsion.	

EXAMPLE 18.7 (cont.)

Ref.	Step	Calculations	Output
		A_s required to carry load $= \dfrac{\text{load/m}}{0.87 f_y}$	
		$= \dfrac{45 \times 10^3}{0.87 \times 415} = 125\ \text{mm}^2/\text{m}$	
		Hence,	
		$\dfrac{A_{sv}}{s_v}(0.87 f_y)$ for shear and torsion = 708.9 N/mm	
Step 4		$A_{sv} = \dfrac{708.9 \times 1000}{0.87 \times 415} = 1963\ \text{mm}^2$/metre length	
		Total steel required = 1963 + 125 = 2088 mm^2/m	
SP 16 Table 96		Area provided by the stirrups (12 mm at 100 mm) $= 2 \times 1131 = 2262\ \text{mm}^2/\text{m} >$ required (2088).	

EXAMPLE 18.8 **(Torsional strength of a rectangular section)**

A reinforced concrete rectangular beam has a breadth of 350 mm and effective depth of 800 mm. It has a factored shear force of 105 kN at section *XX*. Assuming that $f_{ck} = 25$, $f_y = 415$ (N/mm^2), and percentage of tensile steel at that section is 0.5 per cent, determine the torsional moment the section can resist (a) if no additional reinforcement for torsion is provided, and (b) if the maximum steel for torsion is provided in the section.

Work out the problem according to BS 8110 and IS 456 principles of design for torsion.

Ref.	Step	Calculations	Output
	A.	SOLUTION (ACCORDING TO BS 8110)	
Eq. (18.17)	1.	*Torsional shear capacity without torsion steel*	
		$\tau_{t\,(\text{min})} = 0.067\sqrt{f_{ck}} \not> 0.4\ \text{N/mm}^2$	
		$= 0.067\sqrt{25} = 0.33\ \text{N/mm}^2$	
	2.	*Torsion to produce the above stress*	
Eq. (18.3)		$T = (1/2)\tau_t b^2 (D - b/3)$	
		$= 0.5 \times 0.33 \times 350^2 \times (800 - 350/3) \times 10^{-6}$	
		$= 13.8$ kNm	$T_0 = 13.8$ kNm
	3.	*Maximum shear capacity* c (max)	
Eq. (18.18)		$\tau_{c\,(\text{max})} = 0.8\sqrt{f_{ck}} \not> 5\ \text{N/mm}^2$	
		$\tau_{c\,(\text{max})}$ = (due to shear) + (due to torsion)	
		$= 0.8\sqrt{25} = 4\ \text{N/mm}^2$	

EXAMPLE 18.8 (cont.)

Ref.	Step	Calculations	Output
	4.	*Shear capacity from torsion*	
		$\tau_v = \dfrac{V}{bd} = \dfrac{105 \times 10^3}{350 \times 800} = 0.375$	
		$\tau_t = (4 - 0.375) = 3.625\ \text{N/mm}^2$	
	5.	*Torsion to produce* $\tau_t = 3.625\ \text{N/mm}^2$	
		$T = (1/2)\tau_t b^2(D - b/3)$	
		$= 0.5 \times 3.625 \times 350^2(800 - 350/3) \times 10^{-6}$	
		$= 151.7$ kNm	$T_{max} = 151.7$ kNm
	B.	SOLUTION (ACCORDING TO IS 456)	
IS 456 Table 13	1.	$\tau_{c\,(min)}$ *for p = 0.5%*	
		$\tau_{c\,(min)} = 0.49\ \text{N/mm}^2$	
	2.	*Allowable torsion from formula for equivalent shear produced by torsion*	
		$\dfrac{(105 + 1.6T/b)}{bd} = 0.49$	
		$T = \left[\dfrac{(0.49 \times 350 \times 800) - (105)}{1000}\right](0.35/1.6)$	
		$= 7.0$ kNm	$T_0 = 7.0$ kNm
IS 456 Table 14	3.	*Max. shear capacity (torsion + shear)*	
		$\tau_{c\,(max)} = 0.63\sqrt{f_{ck}} = 3.1\ \text{N/mm}^2$	
	4.	*Torsion to produce the above shear along with*	
		$V = 105$ kN $\dfrac{(105 + 1.6T/b)}{bd} = 3.1$	
		$T_{max} = \left[\dfrac{(3.1 \times 350 \times 800) - (105)}{1000}\right](0.35/1.6)$	
		$= 166.90$ kNm	$T_{max} = 166.9$ kNm

REVIEW QUESTIONS

18.1 Sketch the pattern of cracking in a beam under torsional moment.

18.2 Explain the difference between primary and secondary torsion. Give two examples each.

18.3 What stresses are produced by torsion in (a) RC rectangular section, and (b) RCT section ? How does one detail steel for these sections ?

18.4 Explain the nature of the interaction curves that have been obtained by tests for combined shear and torsion in RC members (a) without shear steel, and (b) with shear steel.

18.5 Explain the BS and IS methods of design of RC members for torsional moment.

18.6 Explain the action of longitudinal steel along with stirrups in resisting torsional stresses in RC beams.

18.7 What is 'equivalent shear' as applied to torsion and shear in IS 456 ?

18.8 Explain the term 'torsional stiffness' of a beam. Show that the magnitude of moment transferred to members perpendicular to the plane of bending of building frame is small in magnitude and can be neglected in practice.

18.9 Explain the principles of the sand-heap analogy for determining the stresses in plastic analysis of a section in torsion.

18.10 Show how torsion is present in the bottom ring beam of a water tank supported by columns under the ring beam. What is its magnitude when the columns are six in number ? Sketch the distribution of the bending moment and torsional moment along the ring beam.

PROBLEMS

18.1 Calculate the stresses produced in a rectangular beam 350 × 800 mm due to torsion of 30 kNm. What is the nature of these stresses and what type of reinforcements should be provided to resist these stresses ?

18.2 A rectangular R.C.C. beam is 400 × 900 mm in size. Assuming the use of grade 25 concrete and Fe 415 steel, determine the maximum ultimate torsional moment the section can take if (a) no torsion reinforcement is provided, and (b) maximum torsion reinforcement is provided.

18.3 An R.C.C. section 200 × 400 mm is subjected to a characteristic load twisting moment of 2.5 kNm and a transverse shear of 600 kN. Assuming the use of grade 25 concrete Fe 415 steel, determine the reinforcements required according to (a) IS 456, and (b) BS 8110.

18.4 An L beam has a flange width of 700 mm, web width 350 mm, and thickness of slab 120 mm. Its total depth is 800 mm. If the factored design moment is 200 kNm, design shear 150 kN, and torsional moment 110 kNm, design the steel for the beam section. Assume $f_{ck} = 30$ N/mm^2 and $f_y = 415$ N/mm^2.

18.5 A spandrel beam is continuous over columns spaced at 8 m. It is rectangular in cross-section 300 × 600 mm. The beam has a slab 120 mm thick cantilevering 1.8 m from the face of support and cast level with the top of the beam acting as a portico. The slab is to be designed for a characteristic live load of 2.0 kN/m^2. Assuming $f_{ck} = 20$ and $f_y = 415$ N/mm^2, design the beam reinforcements.

19

Design of R.C. Members in Tension

19.1 INTRODUCTION

Reinforced concrete is not an ideal material for tension members. However, in members like hangers and ties of bow girder bridges or in structures like circular water tanks, the main stresses are direct tension. In such cases, steel is to be used principally to take all the tension, and the concrete is considered only as a protective cover for the steel reinforcements. In these structures, care should be taken to see that the cracks that may appear in the concrete, while the steel is stressed in tension, are only of small sizes so that they are not harmful and should not lead to corrosion of the steel. This condition will essentially require limiting of the values of allowable stresses in the steel and concrete so that the strains in the steel and the concrete around the steel are not high. In structures like water tanks with fixed bases, the walls are subjected to bending tension.

In this chapter we discuss only the methods of design of members under direct tension and those under bending tension (where cracking should be limited). It will be shown that, at present, such members in direct tension are designed by elastic theory alone, but members subjected to bending tension can be designed by either elastic or limit state theory.

19.2 DESIGN METHODS FOR MEMBERS IN DIRECT TENSION

Theoretically, both elastic and the limit state methods of approach can be used in this problem. The aim is to limit the crack-width, and the procedure is essentially to arrive at the size of the concrete member which will be uncracked at service loads. In the elastic method the amount of reinforcement to be provided is determined on the assumption that the concrete does not carry any load and all the loads are carried by the steel. The allowable stress in steel and concrete is also so limited that the composite section remains practically uncracked. The design procedure recommended has been in use for the past several years. Hence, even though the design is not theoretically very sound, the resulting design is found to give satisfactory results. With the improvement in materials of high bond bars, which tend to distribute cracking as small cracks along the length of the reinforcement, higher stresses in steel than those used before for smooth bars have been found to be acceptable for this design procedure also. Structures designed on this elastic theory tend to be larger in size and have higher margins of safety than those designed on the more recently developed ultimate load theory.

In the limit state of the ultimate load method, the procedure is to calculate the crack-width and check whether it will be less than the specified value. However, as an accepted theoretical method for estimating crack-width has not yet been evolved for members in direct tension, such members generally continue to be designed by elastic theory, as we now discuss.

19.2.1 COVER TO REINFORCEMENT

The cover to be provided should comply with the code requirements. As regards environmental conditions, BS 5337 the British code on water retaining structures, classifies exposure into the following three categories:

Class A—Exposed to wetting and drying, such as underside of roof of liquid retaining structures (allowed crack-width, 0.1 mm)

Class B—Exposed to continuous contact with water, e.g. walls of liquid retaining structures (allowed crack-width, 0.2 mm)

Class C—Not so exposed, for instance, members exposed only to outside air (allowed crack-width, 0.3 mm)

Cover of surfaces not exposed to water can be controlled by the general rules governing concrete cover given for various environmental conditions, but the minimum cover should be 40 mm for surfaces in contact with water.

19.2.2 SPACING OF STEEL

In practice, for members up to 200 mm thick, the reinforcements can be provided in one layer and in members over 200 mm thick, the steel should be equally divided on both faces. In both cases the spacing of steel is to be kept less than 300 mm.

19.2.3 MINIMUM STEEL (SECONDARY REINFORCEMENTS)

A specified minimum of steel should at least be provided in the two principal directions perpendicular to each other to offset the effects of shrinkage, temperature changes etc. The amount of steel for this purpose is generally taken as 0.3 per cent of the total sectional area of concrete for deformed bars and 0.5 per cent for plain bars.

19.3 ELASTIC METHOD OF DESIGN OF TENSION MEMBERS

As already pointed out, with the present state of knowledge, it is not possible to calculate accurately the maximum widths of the tensile cracks due to direct tension. The only possible solution therefore is to limit the allowable stresses. This design procedure assumes 'no cracked section' at working loads. The design makes use of working (characteristic) loads and the principles of elastic theory and modular ratio. Sometimes lower working stresses are adopted, depending on the exposure condition.

Both steel and concrete are assumed to be elastic. The value of modular ratio is taken as 15. The whole section, including the concrete cover to reinforcement, is assumed to be effective in direct tension and the area of reinforcement is calculated assuming that the whole tension is taken by steel only.

19.4 DESIGN PROCEDURE FOR DIRECT TENSION

Based on the above theory, the step-by-step procedure for design of R.C. section in direct tension can be summarised as in the following steps.

TABLE 19.1 ALLOWABLE STRESSES IN STEEL FOR DIRECT TENSION
(British Standards 5337–1976)
(see also Table 19.5)

Exposure*	Permissible → Stress (N/mm^2)	
	Plain bars	Deformed bars
A	85	100
B	115	130
C	125	140

*Refer to Section 19.2.1 for classification of exposure.

Step 1: *Calculate area of steel required.* Choosing a suitable value of f_s (Table 19.1), depending on exposure and type of steel bar to be used, determine the area of steel A_{s1} given by

$$A_{s1} = \frac{T}{f_s} \tag{19.1}$$

(Also, area of steel A_{s1} should be greater than the minimum A_s of 0.3 to 0.5 per cent required in all members, as given in step 3.)

Provide this area of steel A_{s1} (greater than A_s) in the direction of T.

Determine the size and spacing of bars. The spacing should be less than 300 mm.

Step 2: *Check concrete stress f_c.* The equivalent area of concrete A_e is given by

$$A_e = A_c + (m - 1)A_{s1} \tag{19.2}$$

where A_c = area of concrete.

$$\text{Concrete stress} = f_c = \frac{T}{A_e} \tag{19.3}$$

The value of f_c should be equal or less than the allowable tensile stress in concrete (Table 19.2) if cracking of concrete is not allowed. However, if cracking of concrete is allowed, stresses as in Table 19.2a (IS 456: clause 44.1.1) may be used for design.

TABLE 19.2 ALLOWABLE STRESSES IN CONCRETE IN DIRECT TENSION WITHOUT CRACKING OF CONCRETE

Concrete grade	Permissible direct tension (N/mm^2)
30	1.44
25	1.31

It is also necessary to give special attention to lap length of bars in tension.

Step 3: *Area of secondary reinforcement (A_{s2}).* The minimum secondary steel to be provided, which should be based on the concrete area, is 0.3 per cent for deformed bars and 0.5 per cent for plain bars. Thus A_s is given by

TABLE 19.2a ALLOWABLE STRESSES IN CONCRETE IN DIRECT TENSION ALLOWING CRACKING OF CONCRETE
(Refer IS 456: clause 44.1.1)

Grade of concrete	15	20	25	30	35	40
Tensile stress (N/mm^2)	2.0	2.8	3.2	3.6	4.0	4.4

$$A_s = \frac{A_c}{100}(0.3) \quad \text{for deformed bars}$$

$$A_s = \frac{A_c}{100}(0.5) \quad \text{for plain bars} \tag{19.4}$$

Step 4: *Check cover to reinforcements.* The rule for minimum cover should be satisfied. In addition to these calculations, the clear distance between bars should be 300 mm, to limit the crack-width in tension members.

If $A_{s1} > A_s$, no further steel is required. Provide the minimum steel in the perpendicular direction also for structures such as concrete walls. The lap length recommended in tension members is given in Table 19.3.

TABLE 19.3 TENSION LAP LENGTHS
(British Practice)

Allowable stress / Grade of Concrete	Plain bars		Deformed bars	
	85	115	100	130
25	24d	32d	20d	26d
30	22d	29d	18d	24d

19.5 DESIGN OF MEMBERS IN BENDING TENSION AS IN WATER TANKS

In design of members like water retaining structures, which are subjected to bending, care should be taken to satisfy the two conditions: (a) strength, and (b) limiting the width of tensile cracks.

The design procedure recommended in BS 5337 (1976): Structural Use of Concrete for Retaining Aqueous Liquids which has replaced the earlier British Code CP 2007, can be adopted for this purpose.

Two methods of approach are recommended by BS 5337. The first method is the traditional elastic approach. The second method is the limit state method in which the size of the crack-width is calculated and is limited to a value which depends on the type of exposure of the member to environmental conditions. The elastic method is much simpler in approach than the limit state method.

Comparing the resulting member obtained by the two methods, as already stated, it will generally be found that the second limit state method is likely to result in a thinner section and the use of lesser reinforcement. While the margin of safety under service load conditions is likely to be smaller for the structures designed by the two methods, the margin of safety under ultimate condition will be more for structures designed by the elastic method. This higher factor of safety depends on such parameters as the thickness of the section and the strength of the materials used in construction.

When large bending tension is combined with large direct tension, it is essential to have a thorough analysis combining the two methods. However, in most of the practical cases this is not necessary, since large bending (as in rectangular water tanks) is associated with only very small direct tension, and very large direct tensions (as in circular tanks) are accompanied only by very small bending forces.

19.5.1 LIMIT STATE METHOD OF CRACKING

The first factor to be identified in the limit state (as well as the elastic) method is the environmental or exposure condition. As already stated in Section 19.2.1, BS 5337 (1976) defines three classes of exposure, and the maximum width of cracks permitted under the three classes of exposure conditions A, B and C are 0.1, 0.2 and 0.3 mm, respectively.

As cracking, is a semi-random phenomenon, the aim is to calculate the crack-widths which in all probability will not be exceeded. In the limit state method of design for bending tension, these crack-widths are estimated for conditions of bending (flexural) tension in mature concrete by the formula given in Appendix C of BS 5337 (1976). As this method of design is still in the developmental stage, it is not dealt with further in this chapter.

19.5.2 'DEEMED TO BE SATISFIED' METHOD (ELASTIC METHOD)

An alternative method is to consider the above limit state to be 'deemed to be satisfied' if the steel and concrete stress under service conditions or elastic conditions are not allowed to exceed certain values. This is similar to the elastic method for direct tension described earlier. Thus calculations are made to satisfy the two conditions of strength and resistance to cracking as indicated now. The lap-lengths recommended in R.C. members in tension are given in Table 19.3.

1. Strength calculations

Strength is determined on the assumption that the section is cracked on the tension side. Elastic designs with modular ratio and allowed stresses in concrete and steel are used. The allowable stresses recommended in BS 5337 are given in Tables 19.4 and 19.5.

TABLE 19.4 PERMISSIBLE CONCRETE STRESSES FOR STRENGTH CALCULATION BY ELASTIC METHOD (N/mm²)

Grade	Compression		Shear	Bond	
	Direct	Bending		Average	Local
30	8.37	11.0	0.87	1.0	1.49
25	6.95	9.15	0.77	0.9	1.36

The formulae used in bending strength calculations are as follows: Let d be the effective depth and x the depth of the neutral axis. Then

$$x = \frac{d}{1 + f_s/mf_c} \tag{19.5}$$

The moment of resistance is obtained as

$$M = 1/2b \times f_c\left(d - \frac{x}{3}\right) \tag{19.6}$$

TABLE 19.5 PERMISSIBLE STEEL STRESSES FOR STRENGTH CALCULATION
(By Elastic Method)

Type of stress	Exposure*	Permissible stresses (N/mm^2)	
		plain bars	deformed bars
Flexural tension and shear	A	85	100
	B	115	130
	C	125	140
Compression	A to C	125	140

*See Section 19.2.1

The area of tension steel for bending is

$$A_s = \frac{M}{f_s\,(d - x/3)} \tag{19.7}$$

Shear stress (denoted by v) is given by

$$v = \frac{V}{bz} \tag{19.8}$$

where

$$z = (d - x/3) \text{ the lever arm} \tag{19.9}$$

The same curves and tables presented in SP 16 for working stress design of reinforced concrete described in Appendix A of the text with appropriate values for f_s and f_c given in Tables 19.4 to 19.6 can be used for this purpose.

2. Calculations for cracking

To check cracking conditions, calculations can be made on the assumption that concrete can take some tension. This allowable tension in concrete is to be limited to such values as to ensure the section to be uncracked or within allowable crack-widths. For bending strength conditions, this is done with the help of the following steps using the values given in Table 19.6. While employing this elastic method and the above values, it may be assumed that the maximum crack-width will not be larger than 0.3 mm.

TABLE 19.6 PERMISSIBLE CONCRETE TENSION IN BENDING
(Calculation by Elastic Method for Cracking)

Grade	Tension in bending (N/mm^2)	Shear ($v = V/bz$) (N/mm^2)
30	2.02	2.19
25	1.84	1.94

Step 1: *Calculate the neutral axis of uncracked section.* The depth of neutral axis will be different from that in which the section is assumed as cracked and will be given by the following

expression (taking h as the total depth of the section):

$$x = \frac{h + 2(m-1)\dfrac{A_s}{bh}d}{2(m-1)\dfrac{A_s}{bh} + 2}$$

Step 2: Calculate the maximum tensile stress in concrete f_t from the expression for moment.

$$M = f_t bh \left[\left(\frac{h-x}{3}\right) + (m-1)\frac{A_s}{bh}\,\frac{d-x}{h-x}\,(d - x/3)\right] \tag{19.10}$$

If this stress is greater than the allowable stress, the depth of the section or the area of steel provided should be increased.

Step 3: Check the shear stresses by the usual shear formula for R.C. beams.

19.6 MINIMUM STEEL AREAS AND COVER

As already stated in Section 19.2.3, the minimum steel percentage to be provided in each of the two directions at right angles is 0.5 per cent for plain and 0.3 per cent for deformed bars. Minimum cover for concrete exposed to water should be 40 mm.

19.7 INTERACTION CURVES FOR BENDING WITH TENSION

Interaction diagrams for rectangular sections under combined bending and tension are given in Design Aid SP 16—Charts 66 to 85. These charts give the ultimate strength without taking cracking into account. They are constructed as follows:

Strength under pure axial tension in given by

$$P_u = \frac{pbD}{100}(0.87 f_y)$$

Rearranging the terms, we get

$$\frac{P_u}{f_{ck}bD} = \frac{p}{100 f_{ck}}(0.87 f_y)$$

In the charts, $P_u/(f_{ck}bD)$ is plotted on the y-axis and $M_u/(f_{ck}bD^2)$ on the x-axis as in column interaction diagrams.

The necessary steel ratio p/f_{ck} can be read off from these charts. The required steel A_s is given by

$$A_s = \frac{pbD}{100}$$

Charts 66 to 75 of (SP 16) are for sections with reinforcements on two sides and Charts 76 to 85 are for sections with reinforcements on all four sides. The limitation of these charts is that they give the ultimate strength without any reference to allowable cracking.

EXAMPLE 19.1 **(Design for direct tension)**

A concrete ring 160 mm thick is subjected to a hoop tension of 115 kN per metre length. Using plain bars determine the area of steel required. Assume exposure condition class B, M 30 concrete, and m = 15.

Ref. (Text)	Step	Calculations	Output
	1.	*Steel area for strength*	
Section 19.2		Exposure condition class B f_s = 115 N/mm^2	
Eq. (19.1)		$A_s = \dfrac{115 \times 10^3}{115} = 1000$ mm^2	
		Provide R 12 at 200 mm on each face	
		(A_s = 1130 mm^2)	R 12 at 200 / 565 mm^2
		(Spacing to be less than 300 mm)	on each face.
	2.	*Check concrete stresses*	
Eq. (19.2)		$A_e = A_c + (m - 1)A_{s1}$ = (160 × 1000) + (14) (1130) = 175,820 mm^2	
Eq. (19.3)		$f_{ct} = \dfrac{T}{A_e} = \dfrac{115{,}000}{175{,}820} = 0.65$ N/mm^2	f_{ct}, O.K.
Table 19.2		Less than 1.44 N/mm^2 is allowed.	Concrete tension O.K.
	3.	*Area of secondary steel*	
		Minimum 0.5 per cent, i.e. 0.25 on each face	
Eq. (19.4)		$A_{s2} = \dfrac{0.25}{100} \times 160 \times 1000 = 400$ mm^2	
		Provide 8 mm at 125 mm, giving 402 mm^2	R 8 at 125 / 402 mm^2
	4.	*Cover to steel*	on each face
		Adopt minimum cover 40 mm.	

EXAMPLE 19.2 **(Design of member in bending and tension)**

The thickness of the cantilever wall of a water tank is 200 mm. Assuming that the max. bending moment is 14 kNm, find the steel area required using grade 30 concrete, 12 mm dia Fe 250 steel, m = 15 and cover = 40 mm.

Ref. (Text)	Step	Calculations	Output
		SOLUTION	
	1.	*Exposure conditions and permissible stresses*	
Section 19.2.1		Exposure condition, class B.	
		f_{st} = 115 N/mm^2, f_{cb} = 11.0 N/mm^2	

EXAMPLE 19.2 (cont)

Ref. (Text)	Step	Calculations	Output
Tables 19.4 to 19.6		$f_{ct} = 2.02$ N/mm^2 $d = 200 - 40 - 6 = 154$	Min. cover 40 mm
	2.	*Strength calculation*	
Eq. (19.5)		$x = \dfrac{d}{1 + \dfrac{f_{st}}{m f_{cc}}} = \dfrac{154}{1 + \dfrac{115}{15 \times 11}} = 90.8$ mm	
Eq. (19.7)		$A_s = \dfrac{M}{f_{st}\left(d - \dfrac{x}{3}\right)} = \dfrac{14 \times 10^6}{115(154 - 30.2)} = 983$ mm^2	$\dfrac{\text{R12 at 100}}{(1130)}$
	3.	*Check min. steel*	
		$\dfrac{A_s(100)}{bh} = \dfrac{1130 \times 100}{1000 \times 200} = 0.565\% > 0.5\%$ (min)	Min. steel O.K.
	4.	*Check cracking*	
Eq. (19.9)		$x = \dfrac{h + 2(m-1)\dfrac{A_s}{bh}d}{2(m-1)\dfrac{A_s}{bh} + 2}$ $= \dfrac{200 + 2(14)\,0.00565 \times 154}{2(14)(0.00565) + 2}$ $= 104.0$ mm Substitute in the equation for moment $x = 104$ mm	
Eq. (19.10)		$M = f_t bh\left[\left(\dfrac{h-x}{3}\right) + (m-1)\dfrac{A_s}{bh}\left(\dfrac{d-x}{h-x}\right)\left(d - \dfrac{x}{3}\right)\right]$ $14 \times 10^6 = f_t \times 1000 \times 200\left[(32.0) + (14 \times 0.56 \times 10^{-2}) \times \dfrac{50.0}{96} \times 119.3\right]$ $f_t = 1.9$ N/mm$^2 < 2.02$ allowed.	Concrete tension O.K.

REVIEW QUESTIONS

19.1 Enumerate two situations where the reinforced concrete member will be (a) in direct tension, and (b) in bending combined with direct tension.

19.2 What is the basis of design of R.C. members in direct tension ?

19.3 Indicate the elastic procedure for design of R.C. members in the bending moment and direct tension. Why is it called the "Deemed to be satisfied" method ?

19.4 Explain the classifications of exposure conditions used in BS 5337 (1976) and give the crack-widths allowed in each of these conditions for durability.

19.5 How can one reduce crack-width in R.C. members subjected to tension ?

19.6 What are the requirements for cover in water tanks ?

19.7 In an R.C. member subjected to direct tension, how will one detail the reinforcement required ?

19.8 Explain why the SP 16 curves Charts 66 to 85 for combined bending and tension cannot be directly used in design of water tanks.

19.9 What are the rules regarding distribution of secondary steel in (a) members in direct tension, and (b) members in direct tension and bending ?

19.10 Between smooth bars and ribbed steel, which would you prefer for water tank construction ? Is there any economy in using ribbed bars for these structures ?

19.11 Discuss the merits and demerits of using high strength concrete in water tank construction. What are the main points to be considered in specifying a concrete mix for the construction of a water tank ?

PROBLEMS

19.1 A reinforced concrete hanger of a bowstring girder is square in section and has to resist an axial tension of 350 kN due to characteristic loads. If the concrete used is of grade 20, design the member using (a) Fe 415 steel, and (b) Fe 250 steel.

19.2 The bending moment diagram in the walls and the floor of a long rectangular water tank supported on beams A and B are shown in Fig. P.19.2. Assuming a wall thickness of 220 mm and slab thickness of 320 mm, check the adequacy of the concrete sections. Determine also the amount of steel for the maximum moment section of the floor and the walls. f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2.

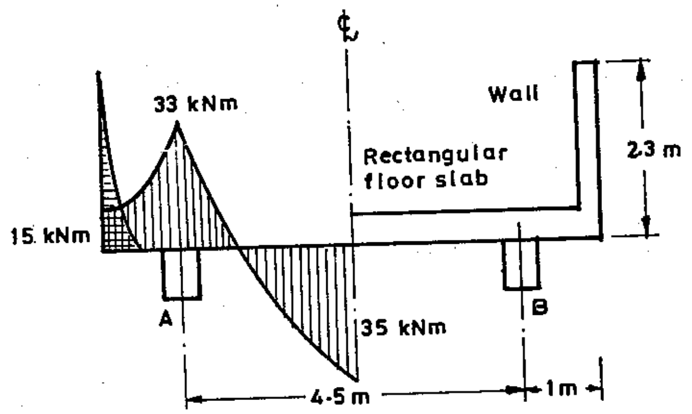


Fig. P.19.2.

19.3 The wall of a reinforced concrete water tank is subjected to a bending moment due to characteristic load of 30 kNm per metre length and a tension of 50 kN/m. Design a suitable section for the wall, using grade 20 concrete and Fe 415 steel.

19.4 A reinforced circular water tank wall resting on bitumen filling at its base (flexible base) is 10 m in diameter and is to have 2.5 m of water when it is full. Assuming 20 grade concrete and Fe 415 steel, find a suitable thickness for the wall and the circumferential as well as vertical reinforcements required. Sketch the layout of the reinforcements along the height of the wall.

19.5 An R.C.C. rectangular section is 400 × 600 mm and reinforced with 6 Nos. 20 mm bars as shown in Fig. P.19.5. Determine the maximum tensile force that can be applied on the section at an eccentricity of 500 mm, assuming $f_y = 415$ and $f_{ck} = 25$ N/mm^2.

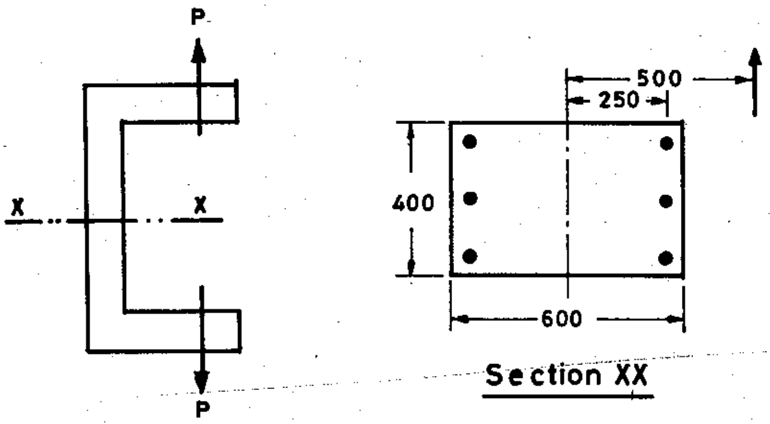


Fig. P.19.5.

20

Design of Staircases

20.1 INTRODUCTION

There are many types of staircases like the slab, the spiral, the free standing, etc. Each of these has to be designed in a prescribed way. In this chapter only the slab type staircases are dealt with. The different parts of staircases used for normal buildings are shown in Fig. 20.1. These parts are 'going', 'rise', tread', 'nosing' and 'waist'.

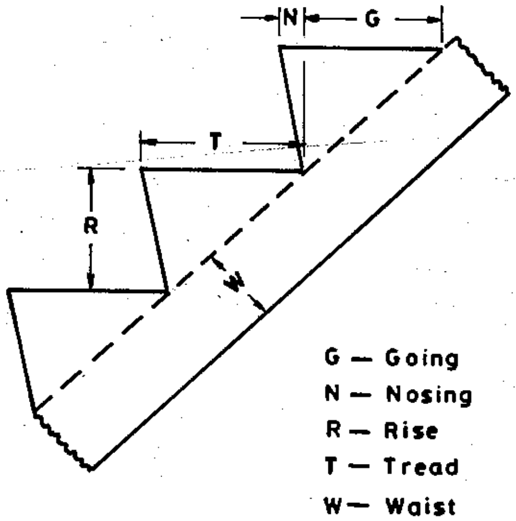


Fig. 20.1 Parts of a staircase.

The magnitude of the rise and tread to be adopted will depend on the type and use of the building. In public buildings the treads have to be larger than in residential buildings or factories. In dwelling houses and factories, the tread can be smaller, say of the order of 250 mm, but in public buildings where large number of persons use the staircase, it should be larger, i.e. it should be between 270 mm and 300 mm. Similarly, the maximum rise allowed in public buildings will have to be smaller than that allowed in residential buildings, where the rise may be about 160 mm, but in public buildings it has to be only around 150 mm, and in factories it can be as much as 190 mm. Usually the number of risers in one flight should not be more than 16.

The empirical formula commonly used for establishing the relation between the 'rise' and 'going' of the finished staircases is that twice the rise (R) plus the going (G) shall be between 550 and 700 (expressed in millimetres)

$$(2R + G) > 550 \text{ mm but} < 700 \text{ mm or} = 600 \text{ mm (approx.)}$$

In all cases the stairs should have their rise and treads equal for every step in each flight and preferably the same in all the flights of the buildings:

20.2 PRINCIPLES OF DESIGN

IS 456 (1978): clause 32 deals with the design of slab type of staircases. They are classified into two types as illustrated in Figs. 20.2 and 20.3:

1. Those spanning horizontally (transversely)
2. Those spanning longitudinally.

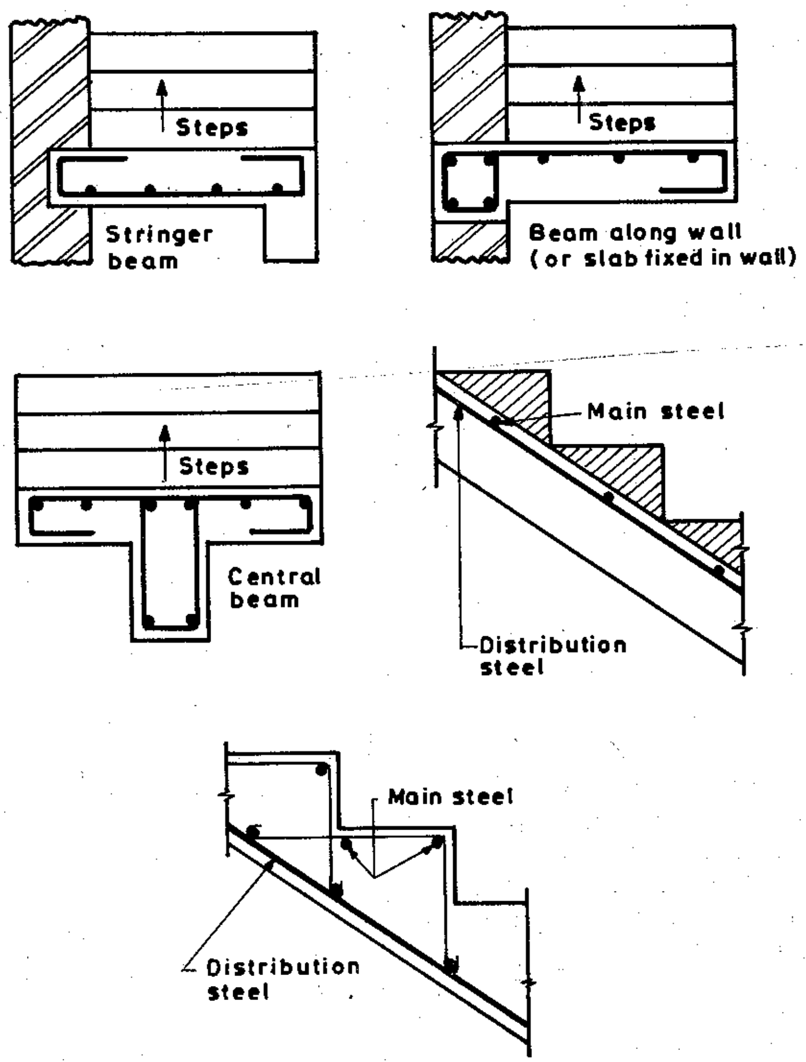


Fig. 20.2 Types of stairs spanning transversely and detailing of steel in them.

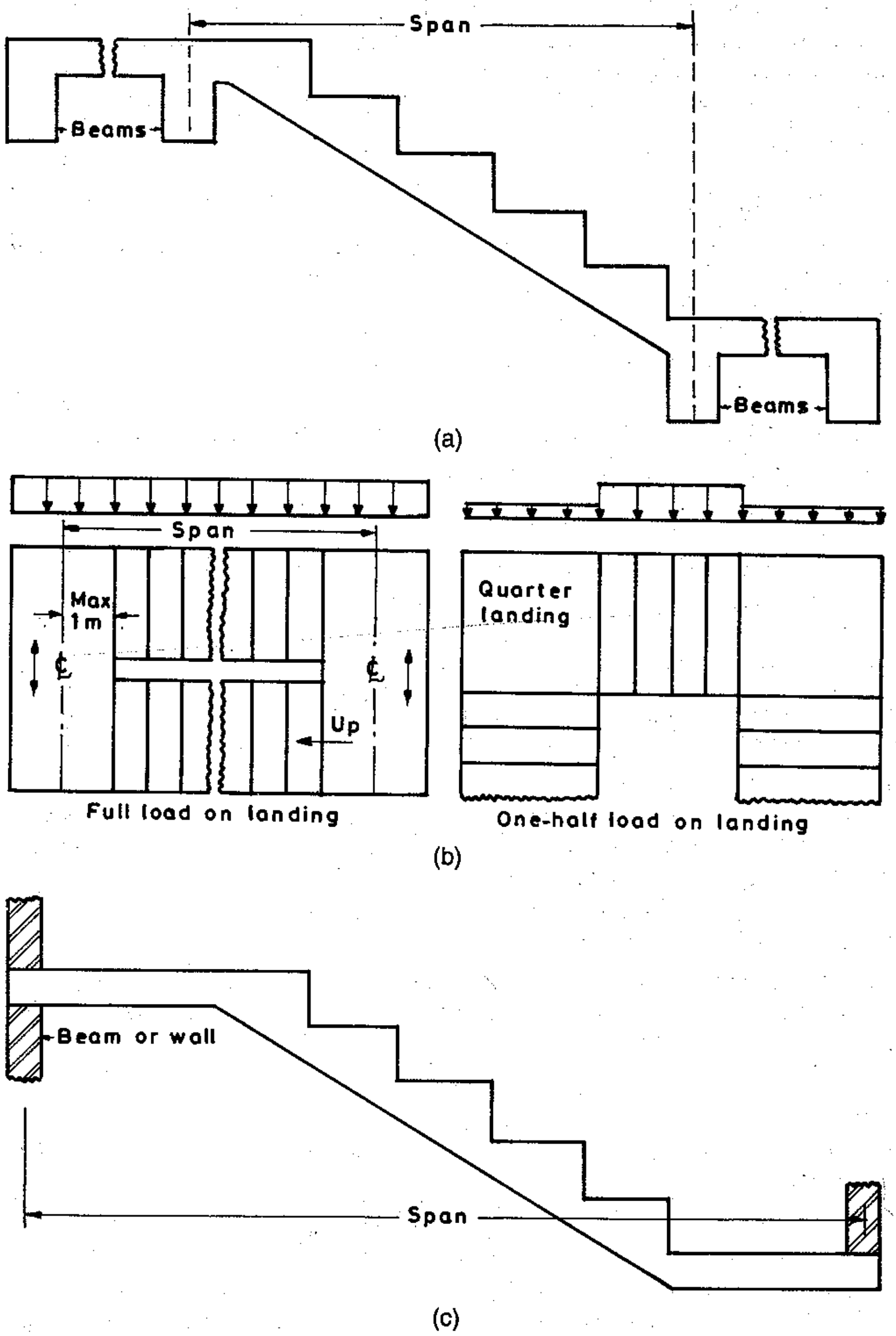


Fig. 20.3 Types of stairs spanning longitudinally: (a) Slab between beams, (b) Slab between landings, (c) Slab between walls.

Detailed recommendations regarding calculations of effective span and the method of determining the distribution of loads on staircases are given in IS 456: clause 32. They are shown in Fig. 20.3. Once these are determined, the bending moments at critical section are calculated on the basis of principles of structural analysis, and the staircase is designed for all practical purposes as an R.C. slab.

20.3 APPLIED LOADS

The live load for design of staircases is to be taken from IS 875 (1964) on Loading Standards. The normal values used are:

for *residential buildings* not liable to overcrowding 2 kN/m^2

for *public buildings* liable to overcrowding 5 kN/m^2

20.4 DESIGN OF STAIRS SPANNING TRANSVERSELY (HORIZONTALLY)

The following categories shown in Fig. 20.2 come under this type:

1. Steps cantilevering from wall on one side.
2. Steps spanning between supports, e.g. walls, or stringer beam at each side
3. Steps cantilevering to both sides from a central 'spine' beam.

20.5 STAIRS SPANNING LONGITUDINALLY

Longitudinally spanning stairs are supported at the top and bottom of the flights and may be unsupported or supported on walls along the sides as shown in Fig. 20.3. At top and bottom they can be supported by one of the following methods:

1. By beams spanning at right angles (transverse) to the staircase slab and cast monolithic with the stairs, the stair slab being continued by landing slabs between beams (Fig. 20.3a). This generally happens in framed structures.

2. By landing slabs spanning at right angles (transverse) to the staircase slab (Fig. 20.3b), so that the loading on the landing slab is distributed in both directions.

3. With the landing slab itself spanning in the same direction as the stairs and ending on beams or walls (Fig. 20.3c), so that full loading on the landing slab is carried in one direction only. This is the usual layout in residential houses.

20.6 EFFECTIVE SPAN

IS 456: clause 32.1 gives the following rules for calculation of the effective spans, depending on the way the slabs are supported:

1. When the stairs span longitudinally and are supported at top and bottom by beams, the effective span is the distance between the respective centres of the beams.

2. When the stairs span longitudinally and are supported by the landings, on top and bottom, which span in the transverse direction (perpendicular to the stairs), the effective span is to be taken as the total going plus half the width of landing on each end or one metre, whichever is smaller.

3. When the stairs span longitudinally with the landing slab also spanning in the same direction as the stairs, the effective span is the centre-to-centre distance between the supporting beams or walls.

4. In the case of stairs spanning transversely (horizontally in the transverse direction), the effective width is taken as the effective span.

20.7 DISTRIBUTION OF LIVE LOADING

The two principles specified in IS 456: clause 32.2 regarding distribution of load are the following (see also figures in IS 456):

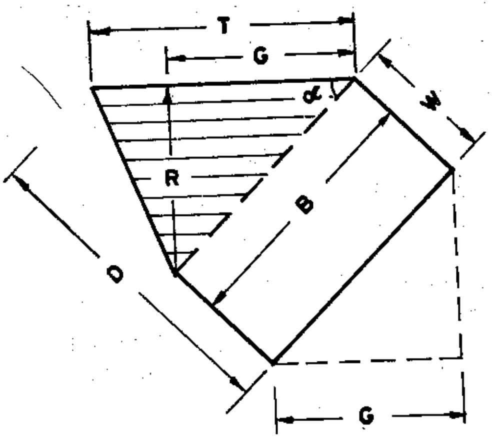
1. The load in areas common to any two spans (such as in stairs with open walls) should be taken as one-half in each direction.

2. When longitudinal flights (or landing) are built with their sides at least 110 mm into the walls, the load from 150 mm strip adjacent to the wall may be deducted from the loaded area, and the effective breadth of the staircase for design can be increased by 75 mm, taking it as the clear distance plus 75 mm.

20.8 CALCULATION OF DEAD LOADS

The general practice is to calculate the dead load acting along the horizontal plan projection. The dead load due to the waist is calculated along the slope of the staircase, and it is then converted along the horizontal plane as shown in Fig. 20.4. The live load given in codes assumes that it is acting along the horizontal span and this is directly used in calculations.

The method of calculation of these loads is indicated in Section 20.11 and Example 20.1.



$$B = \sqrt{G^2 + R^2}$$

Dead load per m^2 of horizontal span = $\left[WB + \frac{RT}{2}\right]\frac{25}{G}$

with dimensions in metres

Fig. 20.4 Calculation of dead load of staircase slab.

20.9 DEPTH OF SECTION FOR CALCULATION OF AREA OF STEEL

The staircase slab is designed as a one-way reinforced slab. The depth of the section for design purposes of longitudinally spanning slabs is taken as the minimum thickness perpendicular to the soffit of the staircase, i.e. the waist (W).

For transverse staircases, there is no universal method to determine the effective depth. The usual practice is to base the calculation, considering each step and to use the average depth of the step for the calculations of the area of steel. Alternatively, the effective lever arm is taken as approximately equal to $D/2$ (see Fig. 20.4).

20.10 DETAILING OF STEEL IN LONGITUDINALLY SPANNING STAIRS

After calculating the necessary steel, it should be placed in the slab according to accepted field practice. Figures 20.5 to 20.8 illustrate the practice recommended in the Bureau of Indian Standards Publication SP 34.

The principles generally followed in placing steel in longitudinally spanning stairs are as follows:

1. The maximum positive B.M. in the span for a simply supported case is taken as $wl^2/8$. One-half of the tension steel is taken over the supports as negative steel according to the usual practice in slabs.

2. If the slab is built into beams at the supports, as in Fig. 20.3(c), the maximum positive moment at the centre of the span is taken as 90 per cent of the simply supported case and at least two-third of the quantity of steel at the centre is carried over to the supports as negative steel.

3. The loading and effective width of slabs built into the side walls is shown in Fig. 18 of IS 456 and the design of such cases is dealt with in Example 20.3.

4. If the slab is continuous over the beams, as in Fig. 20.3(a), the span moment is taken as $wl^2/10$, and the area of steel placed at the supports is the same as that of the positive reinforcement in the span, so that both supports and the span are designed for $wl^2/10$.

5. Distribution steel as in the case of slabs (0.12 per cent for Fe 415 steel) is also placed on top of the main steel in the transverse direction. Details of placing steel are shown in Figs. 20.5 to 20.7.

6. Care should be taken while detailing of corners such that the steel when pulled in tension does not tend to pull out the concrete over the reinforcement as indicated in Fig. 20.8.

The procedure for design of slab type staircases is shown by Examples 20.1 to 20.4.

20.11 CALCULATION OF DEAD LOADS AND EFFECTIVE DEPTHS

The live load for staircases given in codes is the loading on the horizontal projected area. Hence, for designs it is necessary to express dead loads also on the horizontal projection. The following procedure is generally used in design calculations.

1. Slabs spanning transversely

These slabs are designed as individual steps of width equal to the going G and the effective span depending on the width of the staircase, as explained in Section 20.6. The dead loads are calculated for each step. Referring to Fig. 20.4, and considering one metre length along the span, the dead load on width G per metre length can be expressed as

$$[(1/2TR + BW)\ 25]\ \text{kN/m}$$

Additional dead load due to finish of thickness F is equal to

$$= [(FT)23.5] \text{ kN/m}$$

$$\text{Total DL} = [(1/2TR + BW) 25 + (FT) 23.5] \text{ kN/m} \tag{20.1}$$

If the effective depth d is taken as the average depth, it can be expressed as follows: In Fig. 20.1, put

$$H = (W/\cos \alpha)$$

$$d = 1/2[H + (R + H)] - \text{cover} - \phi/2$$

$$= H + \frac{R}{2} - \text{cover} - \frac{\phi}{2}$$

$$d = \left(\frac{W}{\cos \alpha} + \frac{R}{2}\right) - \text{cover} - \frac{\phi}{2} \tag{20.2}$$

If the effective depth d is taken as $D/2$ in Fig. 20.4, then

$$D = (T \sin \alpha + W)$$

$$d = 1/2(T \sin \alpha + W) \tag{20.3}$$

2. Slabs spanning longitudinally

Dead load on each step of length G on horizontal projection is given by

$$\text{Weight of each step} = 1/2RT \times 25 \text{ kN}$$

$$\text{Weight of waist} = WB \times 25 \text{ kN}$$

$$\text{Weight of finish} = FT \times 23.5 \text{ kN}$$

$$\text{Dead load/m}^2 = \left(\frac{\text{total load}}{G}\right)$$

$$\text{DL} = \left(\frac{WB}{G} + \frac{RT}{2G}\right)25 + \left(\frac{FT}{G}\right)23.5 \text{ kN/m}^2 \tag{20.4}$$

The stairs are designed as a slab of depth equal to the waist. The effective depth d is given by

$$d = \left(W - \text{cover} - \frac{\phi}{2}\right) \tag{20.5}$$

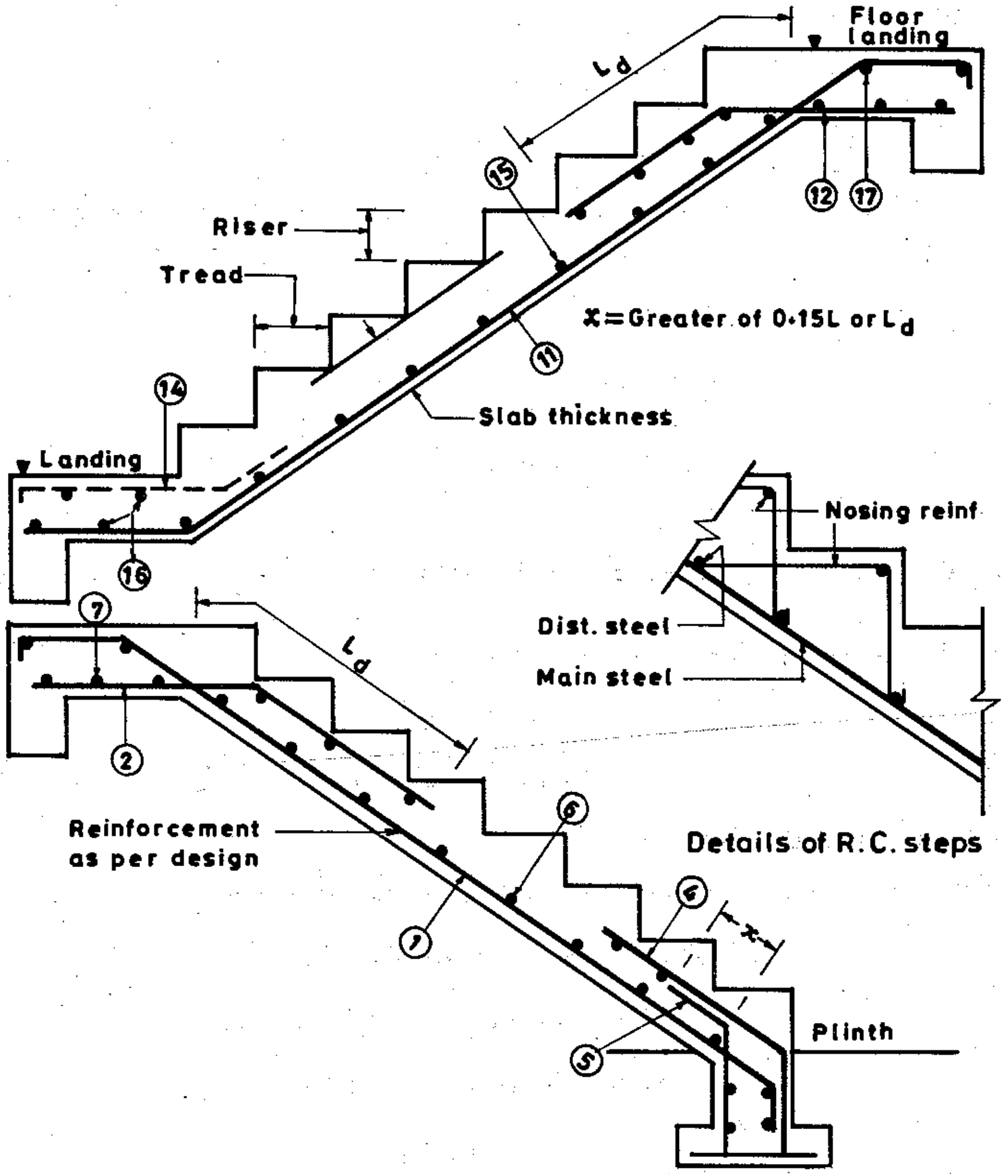


Fig. 20.5 **Reinforcement drawing of staircase slabs continuous with landings (1, 2, 11, 12, main tension steel; 14, main tension steel over support; 4, 5, anchorage steel; 6, 7, 15, 16, 17 secondary steel).**

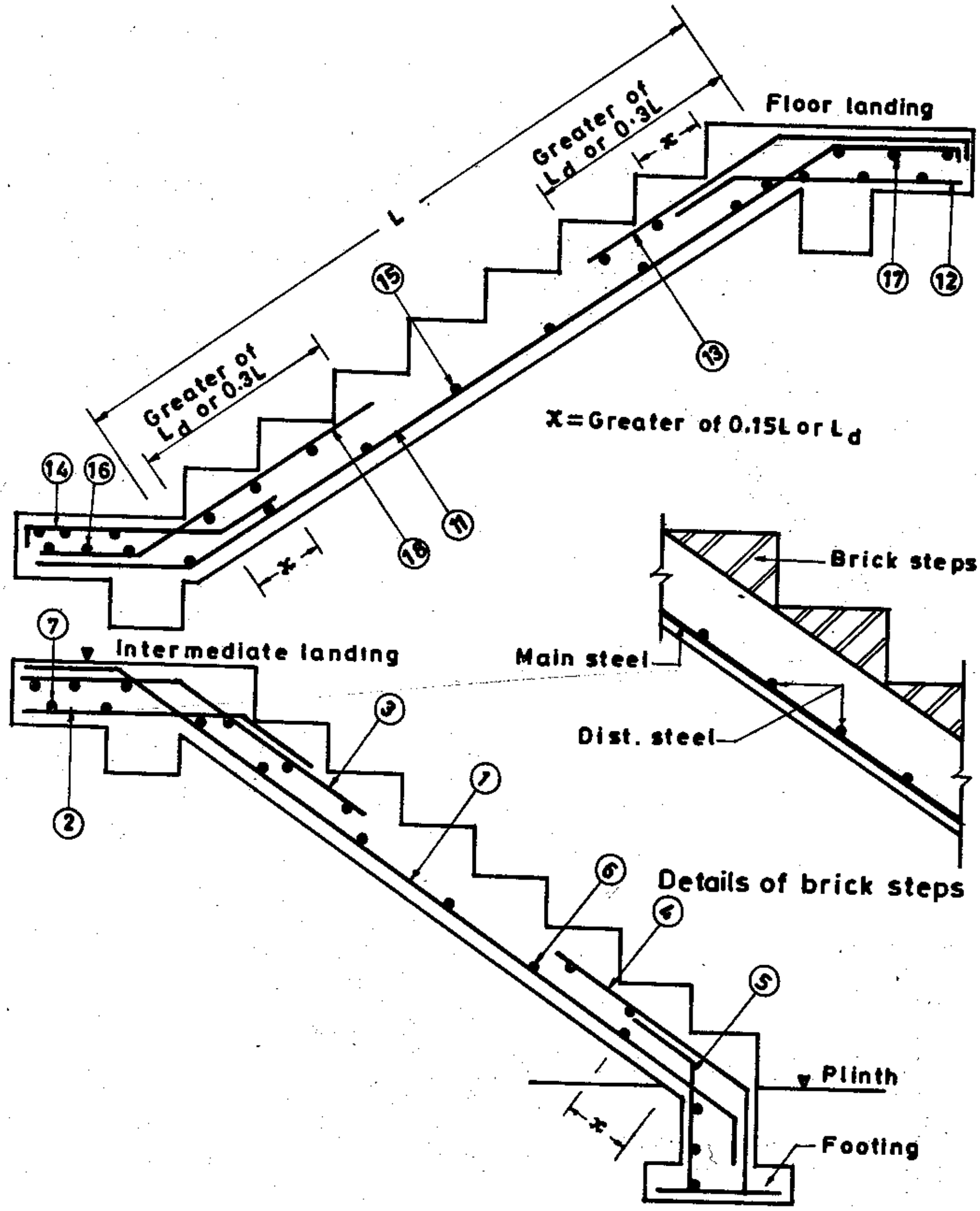


Fig. 20.6 Reinforcement drawing of staircase slabs supported on beams before landing (1, 2, 11, 12, main tension steel; 3, 13, 19, main tension steel over support; 4, 5, anchorage steel; 6, 7, 15, 16, 17, secondary steel; 18, optional tension steel).

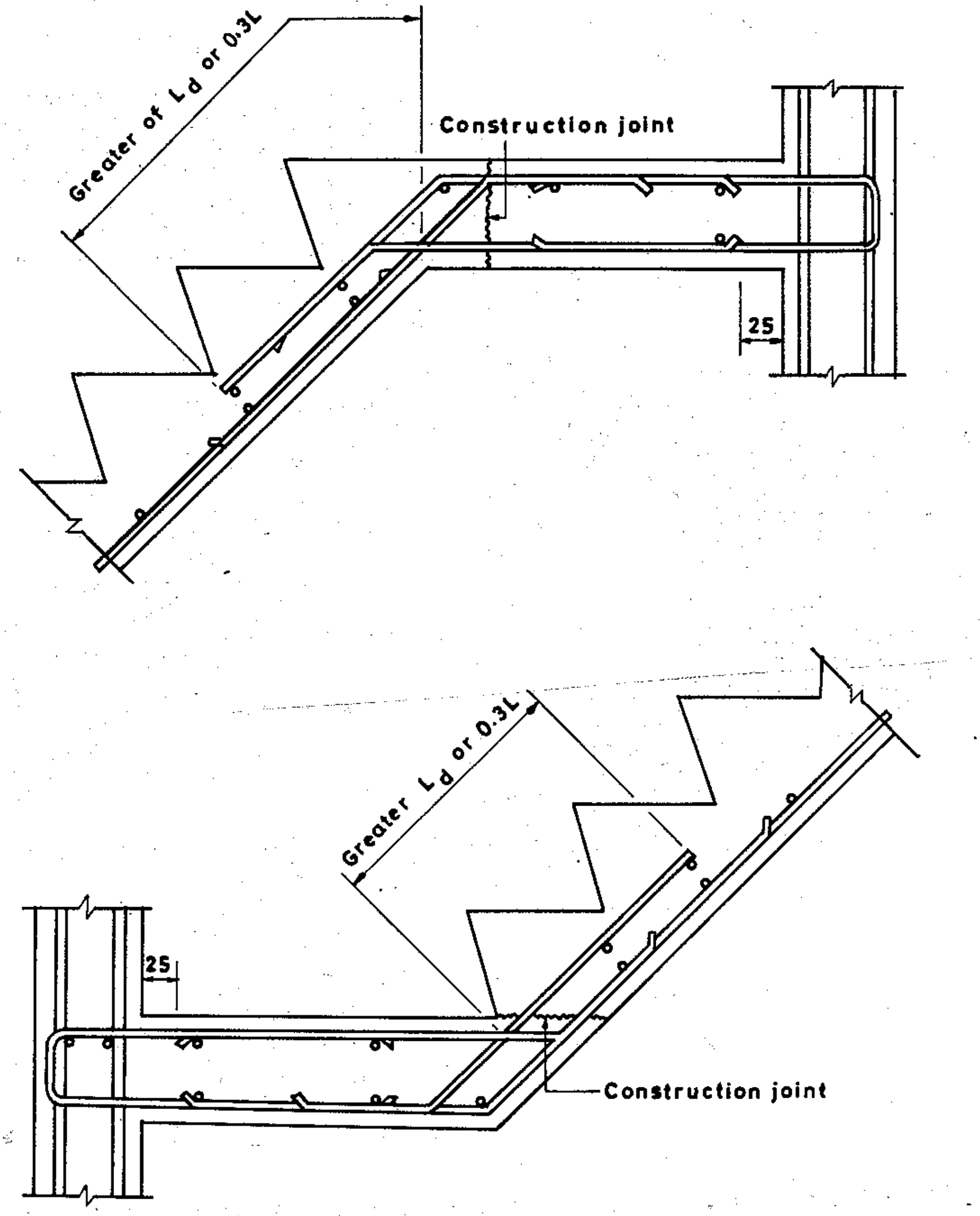
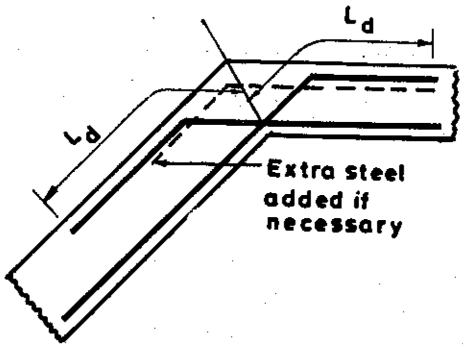


Fig. 20.7 Details of landings with construction joints.



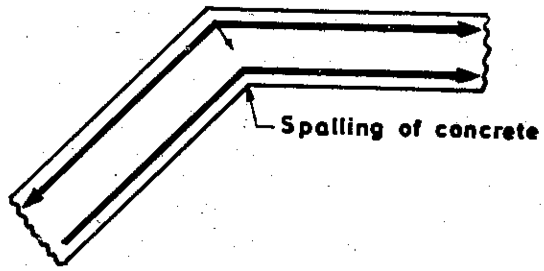


Fig. 20.8 Detailing of staircase slab at corners: (a) Correct method, (b) Incorrect method.

EXAMPLE 20.1 **(Design of staircases)**

A staircase has the following dimensions: waist W = 75 mm; nosing N = 25 mm; rise R = 175 mm; going G = 225 mm. The live load expected is 2 kN/m^2. Calculate the factored loads and effective depths for design if the staircase is spanning (a) transversely (horizontally), and (b) longitudinally.

Ref.	Step	Calculations	Output
Text Fig. 20.4	1.	The breadth B of a step along the slope is $B = \sqrt{G^2 + R^2}$ $= \sqrt{225^2 + 175^2}$ $= 285$ mm	
	2.	*Case (1): Slab spanning transversely* Dead load for width $G = 225$ mm	
Eq. (20.1)	2.1	$\text{DL} = \left(\frac{TR}{2} + BW\right)\frac{25}{1}$ $= \left(\frac{0.250 \times 0.175}{2} + 0.285 \times 0.075\right)25$ $= 1.08$ kN/m transversely.	
	2.2	LL on 225 mm width $= 0.225 \times 2.0$ $= 0.45$ kN/m transversely.	
IS 456 Cl. 35.4	2.3	Design load $= 1.5(1.08 + 0.45)$ $= 2.295$ kN/m transversely.	Factored design load = 2.3 kN/m

EXAMPLE 20.1 (cont.)

Ref.	Step	Calculations	Output
Text Eq. (20.2)	2.4	Effective depth from mean depth $d = \dfrac{W}{\cos\alpha} + \dfrac{R}{2} - \text{cover} - \dfrac{\phi}{2}$ $= \dfrac{75 \times 285}{225} + \dfrac{175}{2} - 15 - 6$ $= 161.5$ mm	$d = 161.5$ mm
Text Eq. (20.3) Fig. 20.4	2.5	Assuming lever arm depth as $D/2$, we have $jd = \dfrac{1}{2}(T \sin\alpha + W)$ $= \dfrac{1}{2}\left(\dfrac{250 \times 175}{285} + 75\right)$ $= 114$ mm	$d = 114$ mm
	3.	*Case 2: Slab spanning longitudinally*	
	3.1	Dead load per sq.m on horizontal span:	
Eq. (20.4)		$\text{DL} = \dfrac{1}{G}\left(WB + \dfrac{RT}{2}\right)25$ $= \left(0.075 \times 285 + \dfrac{0.175 \times 250}{2}\right)\dfrac{25}{225}$ $= 4.81$ kN/m^2 Live load on horizontal span:	
	3.2	LL = 2.0 kN/m^2	
	3.3	Design load = 1.5(4.81 + 2.0) = 10.22 kN/m^2	Factored Design Load = 10.22 kN/m^2
Eq. (20.5)	3.4	Effective depth $= W - \text{cover} - \dfrac{\phi}{2}$ $= 75 - 15 - 6$ $= 54$ mm *Note*: (i) If only the slab is built of R.C. and the steps are built with brickwork, suitable changes should be made in calculating the dead weights. (ii) For the above type of staircases spanning transversely, the steel area is calculated by considering the effective depth as the thickness of the slab.	$d = 54$ mm

EXAMPLE 20.2 **(Design of longitudinal staircase)**

A longitudinal type of staircase spans a distance of 3.75 m centre-to-centre of beams. The rise R = 175 mm, going G = 250 mm, tread T = 270 mm. The treads have 15 mm granolithic finish and consist of 15 steps. Assuming grade 25 concrete and Fe 415 steel, design the staircase for a live load of 5 kN/m². Assume breadth of staircase as 1.5 m.

Ref.	Step	Calculations	Output
	1.	*Thickness of waist W*	Design longitudinal type staircase
		Assume span/effective depth ratio = 30	
		$d = \frac{3750}{30} = 125$	
IS 456		Total $W = 125 + \text{cover} + \frac{\phi}{2}$	
Cl. 25.4		$= 125 + 15 + 6 = 146$ mm	Cover = 15 mm
		Assume 150 mm thickness of waist	W = 150 mm
		$d = 150 - 15 - 6 = 129$ mm	d = 129 mm
	2.	*Dead load from staircase slab (on plan)*	
		$B = \sqrt{G^2 + R^2} = \sqrt{250^2 + 175^2} = 305$ mm	
Eq. (20.4)		$\text{DL} = \frac{1}{G}\left(WB + \frac{RT}{2}\right)25 + \left(\frac{FT}{G}\right)23.5$	
		$= \frac{1}{0.250}\left[(0.150 \times 0.305) + \frac{0.175 \times 0.270}{2}\right]25$	
		$+ \frac{1}{0.250}(0.015 \times 0.270)23.5$	
		$= 7.32\ \text{kN/m}^2$ on plan	
	3.	*Design load*	
IS 456 Cl. 35.4		$w = 1.5(\text{DL} + \text{LL}) = 1.5(7.32 + 5.0)$	Factored design load = 18.48 kN/m²
		$= 18.48\ \text{kN/m}^2$	
	4.	*Bending moment and moment capacity of slab*	
		$M = \frac{wl^2}{10} = \frac{18.48 \times 3.75^2}{10} = 26.0$ kNm	
		Capacity of slab (d = 129, f_{ck} = 25, f_y = 415)	

EXAMPLE 20.2 (cont.)

Ref.	Step	Calculations	Output
SP 16 Table D		$M_u = 3.45bd^2$ $= 3.45 \times 1000 \times 129^2 = 57.41$ kNm Depth provided is satisfactory. [*Note: The BM at mid-point calculated with w and L on horizontal projection of slab is equal to the BM at that section of the inclined slab normal to the section.]	
	5.	*Area of main steel* $\dfrac{M}{bd^2} = \dfrac{26.0 \times 10^6}{1000(129)^2} = 1.56$	
SP 16 Table 3		$p = 0.470 >$ min $A_s = \dfrac{pbd}{100} = \dfrac{0.47 \times 1000 \times 129}{100}$ $= 606\ \text{mm}^2/\text{m}$	
SP 16 Table 95		Total steel in 1.5 m width = $1.5 \times 606 = 909\ \text{mm}^2$	Provide 8 Nos. of 12 mm (904 mm^2)
IS 456 E. 1.1		*Alternative method* Calculate A_{st} from the equation $M_u = 0.87 f_y A_{st} \left(d - \dfrac{A_{st} f_y}{b f_{ck}} \right)$	
IS 456 Cl. 25.5.2.1	6.	*Distribution steel* $A_s = \dfrac{0.12}{100} \times 1000 \times 150 = 180\ \text{mm}^2/\text{m}$	Use 8 m at 270 mm (186 mm^2
	7.	*Check for shear* $V = \dfrac{wl}{2} = \dfrac{18.48 \times 3.75}{2} = 34.65$ kN	
IS 456 Table 13		$v = \dfrac{34.65 \times 10^3}{1000 \times 129} = 0.27\ \text{N/mm}^2$ τ_c for $f_{ck} = 25 = 0.36\ \text{N/mm}^2$	No shear steel required
	8.	*Check for deflection*	
IS 456 Fig. 3		% tension steel > 0.470 Multiplying factor $F_2 = 1.2$ Allowable $L/d = 1.2 \times 26 = 31.2$ Assumed $L/d = 30 < 31.2$	L/d ratio O.K.

EXAMPLE 20.2 (cont.)

Ref.	Step	Calculations	Output
	9.	*Check for cracking*	Cover = 15 mm
IS 456 Cl. 25.3.2		Spacing of main bars = 208 – 12 = 196 mm	Less than $3d$
		Spacing of second steel = 270 – 8 = 262 mm	Less than $5d$
	10.	*Detailing and check for stability*	
		Detail as per standard practice.	
		Extend steel to supports as internal ties.	

EXAMPLE 20.3 **(Design of longitudinal staircase built into wall)**

Assuming that the staircase in Example 20.2 is built into the adjacent wall by 110 mm, design the necessary steel.

Ref.	Step	Calculations	Output
	1.	*Modified widths*	
IS 456 Cl. 32.2		Decreased loaded breadth = 1.5 – 0.15 = 1.35 m	Design longitudinal type staircase
Fig. 18 IS 456		Increased effective breadth = 1.5 + 0.075 = 1.575 m	
	2.	*Design load*	
		As in Example 20.3, w = 18.48 kN/m^2	
	3.	*Design load from loaded breadth*	
		Total design load = 18.48 × 1.35 = 24.95 kN/m length	Total design load = 24.95 kN/m
	4.	*Bending moment and moment capacity*	
		$M = \frac{wl^2}{10} = \frac{24.95 \times 3.75^2}{10}$ = 35.08 kNm	
SP 16 Table D		$M_u = 3.45bd^2$(effective breadth = 1575 mm) = 3.45 × 1575 × 129^2 = 90.4 kNm	Depth of slab O.K.
		Depth is satisfactory.	

EXAMPLE 20.3 (cont.)

Ref.	Step	Calculations	Output
	5.	*Area of steel required*	
		$\frac{M}{bd^2} = \frac{35.08 \times 10^6}{1575 \times (129)^2} = 1.33$	
SP 16 Table 3		Percentage of steel = 0.395	Area of main steel A_s = 802 mm^2
		$A_s = \frac{0.395 \times 1575 \times 129}{100}$	
		$= 802\ mm^2$	
		Note: By fixing the staircase to the wall, the theoretical steel area has been reduced from 909 mm^2 to 802 mm^2.	
	6–10	Steps 6 to 10 as in Example 20.2.	

EXAMPLE 20.4 **(Design of cantilever staircase)**

A staircase slab has waist W = 75 mm, rise R = 175 mm, tread T = 250 mm, and going G = 225 mm. It is built into an adjoining wall and cantilevers for a clear width of 1.5 m. The longitudinal span is 3.5 m. Assuming a live load of 3.0 kN/m^2 and use of M 15 concrete and Fe 415 steel, calculate the necessary reinforcements.

Ref.	Step	Calculations	Output
		Each tread is designed as a cantilever beam	Design a transverse staircase
	1.	*Calculation of B*	
		$B = \sqrt{G^2 + R^2} = \sqrt{225^2 + 175^2}$	
		= 285 mm	
	2.	*Loads on each step for one metre transversely*	
		DL = weight of (waist + steps)	
		$= \left(BW + \frac{TR}{2}\right)25$	
		$= \left(0.285 \times 0.075 + \frac{0.250 \times 0.175}{2}\right)25$	
		= 1.07 kN/m	
		LL on 0.225 m width along the step transversely = 0.225 × 3 = 0.68 kN/m	
IS 456 Cl. 32.2		(One may neglect the live load on 0.15 m width adjacent to wall because of fixity.)	

EXAMPLE 20.4 (cont.)

Ref.	Step	Calculations	Output
	3.	*Design loads*	
IS 456 CL. 35.4		$w = 1.5(\text{DL} + \text{LL}) = 1.5(1.07 + 0.68)$ $= 2.63$ kN/m	Design load = 2.63 kN/m
	4.	*Bending moment* as a cantilever: $M = \dfrac{2.63 \times 1.5^2}{2} = 2.95$ kNm *Moment capacity* from effective depth: Effective depth = mean depth − cover − $\phi/2$ Mean depth $= \dfrac{R}{2} + W\dfrac{B}{G} = \dfrac{175}{2} + 75\left(\dfrac{285}{225}\right)$ $= 182$ mm $d = 182 - 25 - 6 = 151$ mm $M_u = Kbd^2 = 2.07bd^2$ $= 2.07 \times 225(151)^2$ $= 10.6$ kNm > 2.95 kNm	Assume cover = 25 mm (top of slab) $d = 151$ mm
	5.	*Area of main steel* (for each step of width) 225 mm and mean effective depth 151 mm $\dfrac{M}{bd^2} = \dfrac{2.95 \times 10^6}{225(151)^2} = 0.575$	
SP 16 Table 1		Percentage of steel = 0.167 $A_s = \dfrac{0.167 \times 225 \times 151}{100} = 56.7$ mm^2 Provide one 10 mm rod on top of each step (79 mm^2). Spacing 285 mm is less than $3d$. *Alternative method*: Assume lever arm = $D/2$ $D = (W + T \sin \alpha)$ $= 75 + \dfrac{250 \times 175}{285} = 229$ mm LA = 114.5 mm $A_s = \dfrac{2.95 \times 10^6}{114.5 \times 0.87 \times 415} = 71$ mm^2 Provide one rod of 10 mm (79 mm^2) for each step.	

EXAMPLE 20.4 (cont.)

Ref.	Step	Calculations	Output
	6.	*Distribution steel*	
Table 11.3 of text		Provides secondary steel for an equivalent thickness of slab of 180 mm (6 mm at 100 mm)	
	7.	*Check for deflection*	
		L/d ratio $= \frac{1500}{151} = 9.9$	
IS 456 Fig. 3		Percentage of steel = 0.357	
		$F_1 = 1.4$	
IS 456 Cl. 22.2.1		Permissible L/d = basic × 1.4	
		$= 7 \times 1.4 = 9.8$	L/d Satisfactory
	8.	*Check for shear*	
		$V = wl = 2.63 \times 1.5$	
		$= 3.95$ kN	
IS 456 Table 13		$v = \frac{V}{bd} = \frac{3.95 \times 10^3}{225 \times 151} = 0.12\ \text{N/mm}^2$	
		Min v for M 15 concrete = 0.35 N/mm²	
		Hence section is safe without shear steel.	
	9.	*Check for crack-width*	
IS 456 Cl. 25.3.2		Spacing of main steel < $3d$	
		Spacing of secondary steel < $5d$	
	10.	*Detailing*	
		Detail steel according to standard practice.	

REVIEW QUESTIONS

20.1 Illustrate, with the help of diagrams, the following terms: rise, tread and going.

20.2 Explain the two types into which the slab types of staircase can be broadly divided into.

20.3 The landing slabs of a staircase can be arranged to span in the direction of the staircase or transverse to it. What is the effect of each of this case on the effective span of the staircase slab ?

20.4 Is the live load specified in IS code in kN/m² taken along the length of the slab or the projected horizontal length of the slab ?

20.5 In the construction of staircases, the step can be cast along with the slab in concrete or it can be constructed after the slab is cast in brickwork. Discuss the advantages and disadvantages of each of the above construction procedures.

20.6 Sketch the layout of a slab staircase cantilevering from a wall, and detail the typical reinforcements used.

20.7 Sketch the layout of a slab staircase spanning longitudinally and supported on beam at its ends. Detail the typical reinforcement if they are (a) simply supported on the beams, and (b) continuous over the beams.

20.8 What is meant by a "dog-legged staircase ?" Sketch a layout of such a staircase and indicate the spans for design.

20.9 What is a free standing staircase ? Sketch such a staircase and explain its stability and design principles.

20.10 Sketch the effective span and the loading to be used to calculate the BM and SF in a step of a staircase contilevering from the wall.

PROBLEMS

20.1 A staircase slab spanning longitudinally is set into pockets of two supporting beams, one on either end, in successive landings between floors. If the effective span of the slab is 3 m horizontally and the rise of the stairs is 1.5 m with 250 mm treads and 150 risers, design and detail the staircase slab. Assume a live load of 3 kN/m^2, finishes 0.5 kN/m^2, f_{ck} = 20 N/mm^2, and f_y = 415 N/mm^2.

20.2 Lay out a longitudinal staircase 1.25 m wide for a residential flat to climb 3.2 m between floors in a staircase room 4.5 × 5.5 m. Design the slab and detail the steel, assuming f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2. Assume that the steps are built on the slab in brickwork.

20.3 A staircase of 1.2 m width for an office building consists of each step built into the wall with a bearing of 110 mm along the flight with the tread = 250 mm and rise = 200 mm. Design the staircase and sketch the layout of reinforcements, assuming f_{ck} = 15 N/mm^2 and f_y = 415 N/mm^2.

20.4 A dog-legged staircase for a residential flat consists of 18 steps, each of 300 mm tread and 180 mm rise, with an intermediate landing 1.2 m in width at the middle. The width of staircase is also 1.2 m. If the flights are of equal number of steps, design the staircase and detail the steel. f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2.

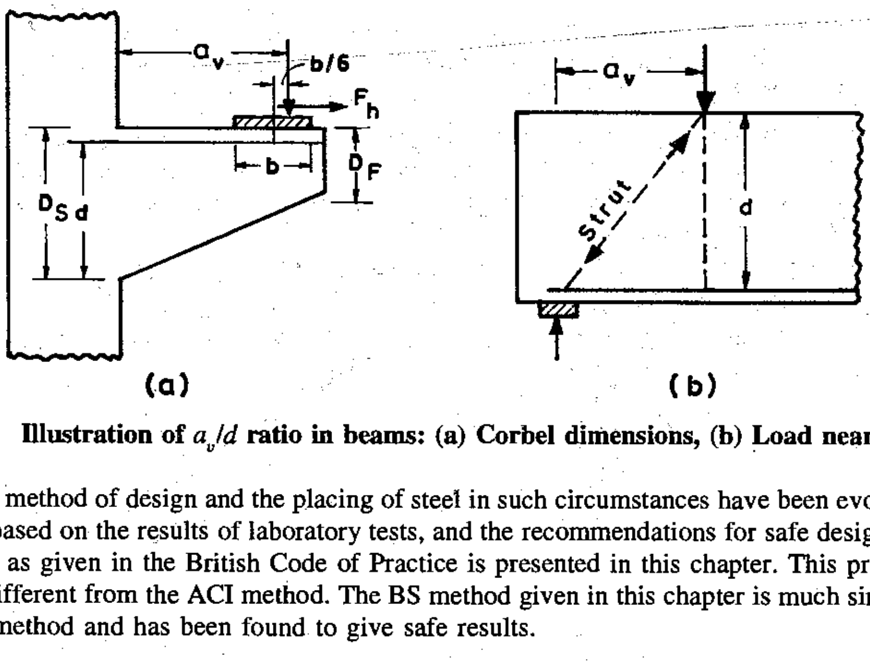
20.5 Sketch the layout of a two-flight staircase for a residential flat whose floor to floor height is 3.2 m. The landing is to be placed such that there will be a head room of at least 2 m clear under the first landing slab, for an entrance door. Design the staircase slab assuming its width as 1.1 m, rise 160 mm, and tread 250 mm, f_{ck} = 20 N/mm^2, f_y = 415 N/mm^2.

21

Design of Corbels, Brackets and Nibs

21.1 INTRODUCTION

The case of shear resistance of R.C. members in which the loads are applied very near the supports is different from that in which the loads are applied far away from the supports. The ratio of the distance of the load application from the support a_v to the effective depth of the member supporting the load d is called a_v/d ratio or the (shear span/depth) ratio. Results of laboratory tests show that, when this ratio is low (less than 2 as in structural members like brackets, pile caps or in beams with the load applied near the supports), the load transfer to the support can be assumed to take place more by 'strut' action than by simple bending (see Fig. 21.1). Under these conditions of low a_v/d ratio, the shear strength as obtained from laboratory test results is shown in Fig. 21.2.



(a) (b)

Fig. 21.1 Illustration of a_v/d ratio in beams: (a) Corbel dimensions, (b) Load near support.

The method of design and the placing of steel in such circumstances have been evolved from a theory based on the results of laboratory tests, and the recommendations for safe design of these structures as given in the British Code of Practice is presented in this chapter. This procedure is slightly different from the ACI method. The BS method given in this chapter is much simpler than the ACI method and has been found to give safe results.

21.2 ALLOWABLE SHEAR IN BEAMS FOR LOWER a_v/d VALUES ACCORDING TO BS 8110

As already stated, tests show that there is a sharp increase in shear strength when a_v/d is less than 2.

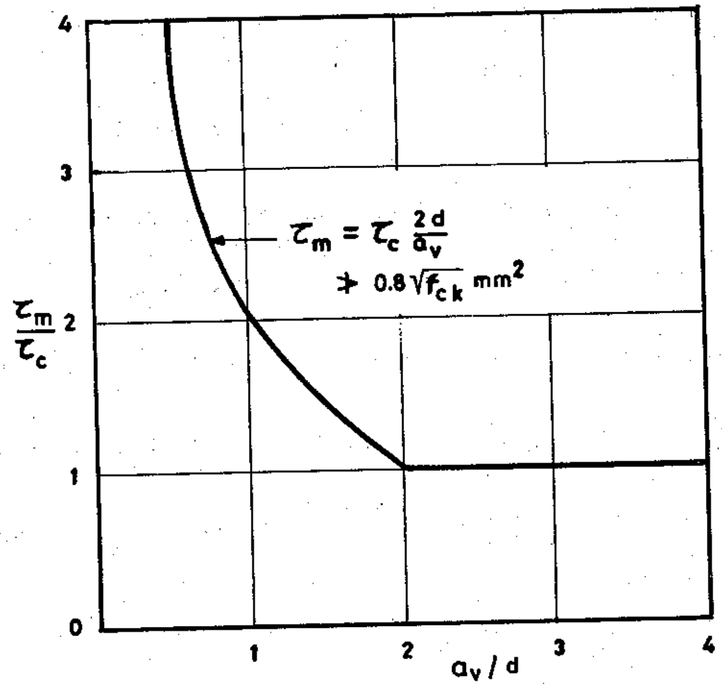


Fig. 21.2 Illustration of the influence of a_v/d ratio on shear resistance.

Hence the enhanced shear strength (τ_m) according to BS code 8110 is given by the modified expression

$$\tau_m = \tau_c\left(\frac{2d}{a_v}\right)$$

where τ_c is the design shear strength for normal beams as given in IS 456, Table 13 (Table 7.1 of the text). In any case, $(2d/a_v)$, the shear enhancement factor, should not yield values greater than the value τ_m specified in Table 14 of IS 456. This maximum value in BS 8110 is given by the expression $0.8\sqrt{f_{ck}}$ subject to a maximum value of 5 N/mm².

21.3 DESIGN OF CORBELS OR BRACKETS

Corbels or brackets are short cantilevers whose a_v/d value is less than 1.0, and the depth (D_f) at the end face of these members is not less than one-half of the depth D_s at the support. These corbles carry the load by strut action as shown in Fig. 21.3 and, therefore, the layout of reinforcement for corbels should be as shown in Fig. 21.4.

21.4 INITIAL DIMENSIONING OF CORBELS (CRITERIA FOR CORBEL ACTION)

The initial dimensioning of the bracket is carried out from the following principles:

1. The ultimate bearing pressure on concrete should not be exceeded. The following are the values generally recommended:

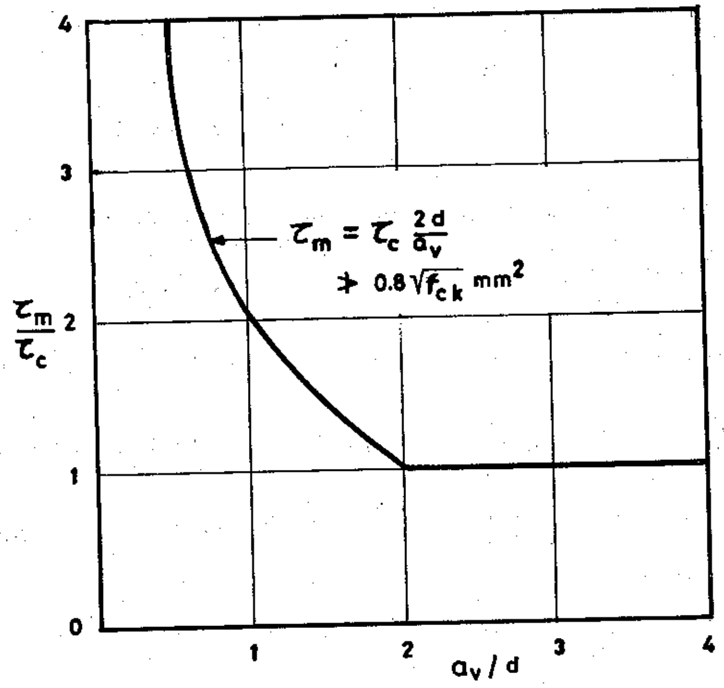


Fig. 21.2 Illustration of the influence of a_v/d ratio on shear resistance.

Hence the enhanced shear strength (τ_m) according to BS code 8110 is given by the modified expression

$$\tau_m = \tau_c\left(\frac{2d}{a_v}\right)$$

where τ_c is the design shear strength for normal beams as given in IS 456, Table 13 (Table 7.1 of the text). In any case, ($2d/a_v$), the shear enhancement factor, should not yield values greater than the value τ_m specified in Table 14 of IS 456. This maximum value in BS 8110 is given by the expression $0.8\sqrt{f_{ck}}$ subject to a maximum value of 5 N/mm^2.

21.3 DESIGN OF CORBELS OR BRACKETS

Corbels or brackets are short cantilevers whose a_v/d value is less than 1.0, and the depth (D_f) at the end face of these members is not less than one-half of the depth D_s at the support. These corbles carry the load by strut action as shown in Fig. 21.3 and, therefore, the layout of reinforcement for corbels should be as shown in Fig. 21.4.

21.4 INITIAL DIMENSIONING OF CORBELS (CRITERIA FOR CORBEL ACTION)

The initial dimensioning of the bracket is carried out from the following principles:

1. The ultimate bearing pressure on concrete should not be exceeded. The following are the values generally recommended:

(a) Bearing with no padding material $0.4f_{ck}$

(b) Bearing in cement mortar $0.6f_{ck}$

(c) Bearing on steel plate cast into member $0.8f_{ck}$ (BS)
$0.9f_{ck}$ (IS)

The width b of the member should be determined from the size of the bearing required. If a plate is used, the size of bearing plate is to be calculated by assuming that the bearing stress should not be greater than $0.8f_{ck}$ (provided the horizontal force acting in the problem is less than 10 per cent of the vertical load).

2. Generally, the value of a_v/d should not be greater than 0.6, and in any case, it should not exceed 1.0.

3. From the design shear aspect, for low values of a_v/d, the depth at the support should be large enough.

4. The depth at the far end should be at least one-half the depth at the support.

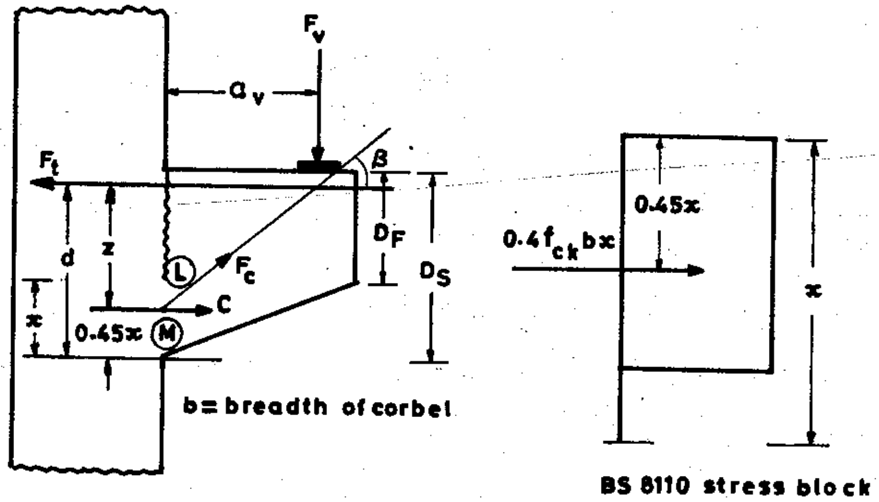


Fig. 21.3 **Forces in a corbel.**

21.5 EQUILIBRIUM OF FORCES IN A CORBEL

The design of corbels, according to BS 8110, is based on the assumption shown in Fig. 21.3. The vertical load denoted by F_v is resisted by strut action. It is in equilibrium under the action of the horizontal steel force F_t and the inclined force F_c from concrete compression with a potential crack along the face of the supporting member. The forces F_c and F_t can be calculated from the assumptions that F_c acts at $0.45x$ in the compressive zone of concrete as assumed in BS 8110 compression block. Its line of action at the face of the support is at a distance equal to the lever arm from the centre of the tensile force. The compressive force F_c should be mobilised by concrete and the tensile force F_t by the main steel provided in the direction of F_t.

21.6 ANALYSIS OF FORCES IN CORBELS

As shown in Fig. 21.3, let

a_v = a = distance of load from the face of the support
b = breadth of the corbel
d = effective depth of the corbel at the face of the support
z = lever arm distance (distance of the centre of compression from the centre of the tension steel)
F_v = applied vertical load
F_t = tension in the horizontal direction due to F_v
F_c = compression developed in concrete as strut action to support F_v

Using triangle of forces, we get

$$F_t = F_v(a/z) \tag{21.1}$$

$$F_c = \frac{F_v(a^2 + z^2)^{1/2}}{z} \tag{21.2}$$

An expression for the compressive force F_c can also be obtained from the strength of the concrete which is in compression. Let

x = height of the compression concrete at the root of the corbel from the bottom side

Assuming that the resultant compression passes through the depth $0.45x$, the expression for the lever arm can be written as

$$z = (d - 0.45x)$$

Hence

$$x = 2.2(d - z)$$

As the area perpendicular to the force F_c over which the compression is built up is $x \cos \beta$, an expression for F_c can be written as

$$F_c = 0.4 f_{ck} b(x \cos \beta)$$

Substituting for x and $\cos \beta$, we obtain

$$F_c = 0.88 f_{ck} b(d - z)\frac{a}{(a^2 + z^2)^{1/2}} \tag{21.3}$$

Equating Eqs. (21.2) and (21.3), we obtain

$$0.88 f_{ck} bd(1 - z/d)\, az = F_v\,(a^2 + z^2)$$

By putting

$$\frac{F_v}{0.88 f_{ck} bd} = k, \quad a/d = \gamma \tag{21.4}$$

from relations (21.2) to (21.4), we obtain the equation

$$\left(\frac{z}{d}\right)^2 - \left(\frac{\gamma}{\gamma + k}\right)\left(\frac{z}{d}\right) + \left(\frac{k}{k + \gamma}\right)\left(\frac{a}{d}\right)^2 = 0 \tag{21.5}$$

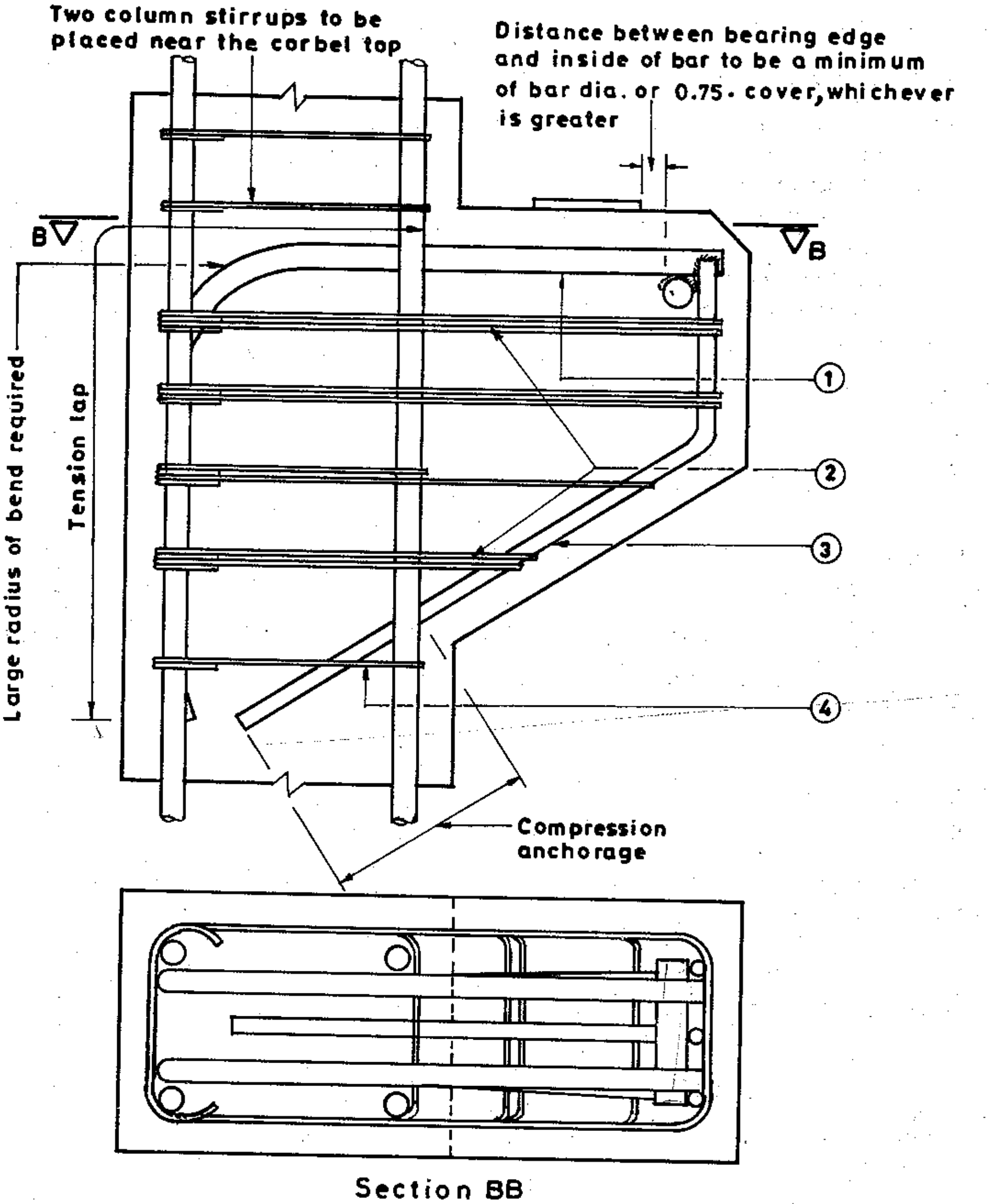


Fig. 21.4a Reinforcement drawing of corbel when using 18 mm diameter or more as main tensile reinforcement (1. Main tensile bars; 2. Horizontal links, total area of which should not be less than 0.25 of area of main tensile reinforcement; 3. Compression bars, total area of which should not be less than 1000 mm²/metre width of corbel; 4. Extra binders).

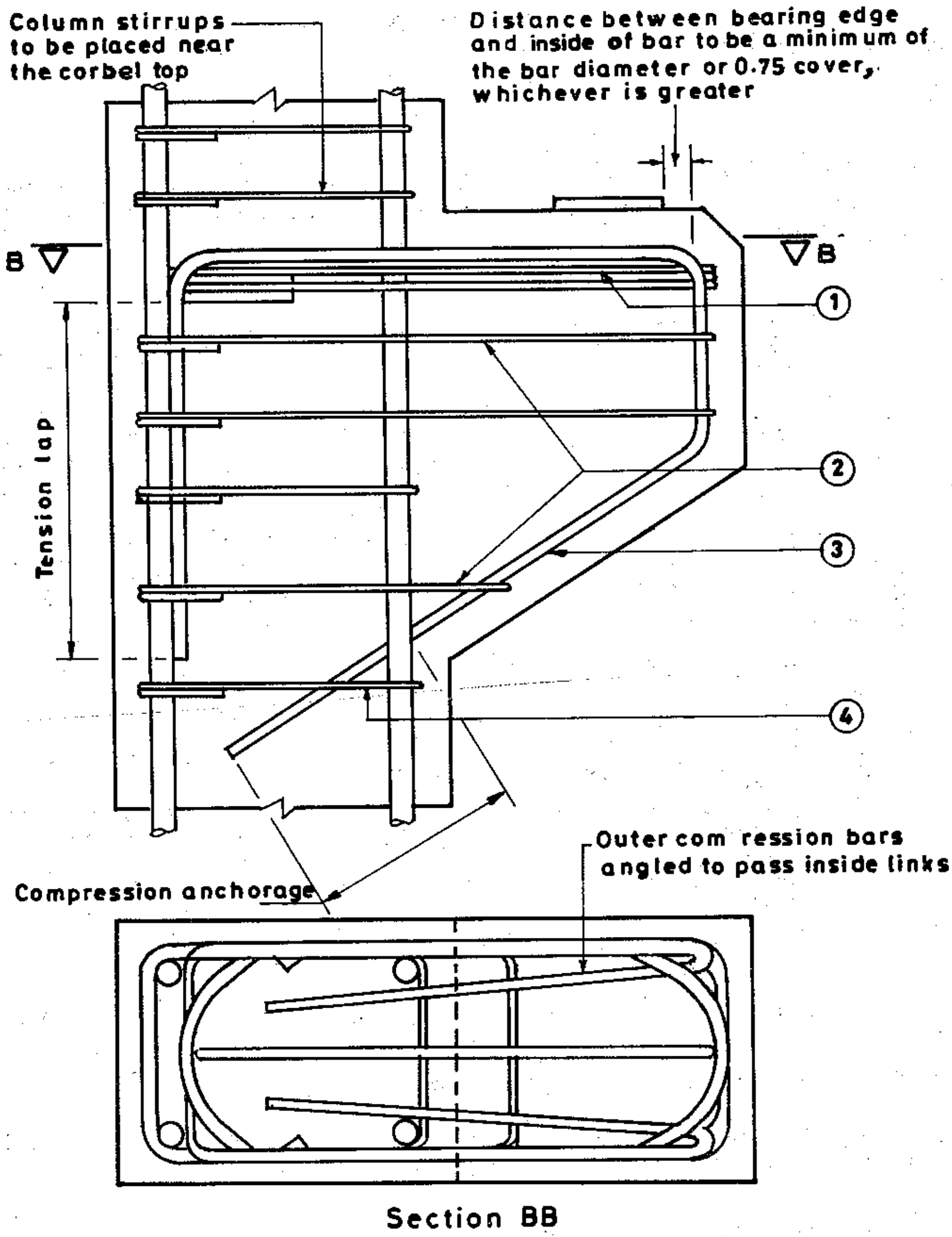


Fig. 21.4b Reinforcement drawing of corbel when using 16 mm diameter or less as main tensile reinforcement (1. Main tensile bars; 2. Horizontal links, total area should not be less than 0.25 of area of main tensile reinforcement; 3. Compression bars, total area of which should not be less than 1000 mm²/metre width of corbel; 4. Extra binders).

The values of z/d for given values of a/d and $F_v/(f_{ck}bd)$ can be calculated from Eq. (21.5). Alternatively, the value of z/d can be obtained from Chart 21.1 for given values of a/d and

$$\frac{F_v}{f_{ck}bd} = \frac{v}{f_{ck}}$$

Once z is known, the value of x, F_t, ε_s can be easily calculated as shown in Examples 21.1 and 21.2. Alternatively, a graphical solution based on the above principles can be used to give the necessary values (Chart 21.1).

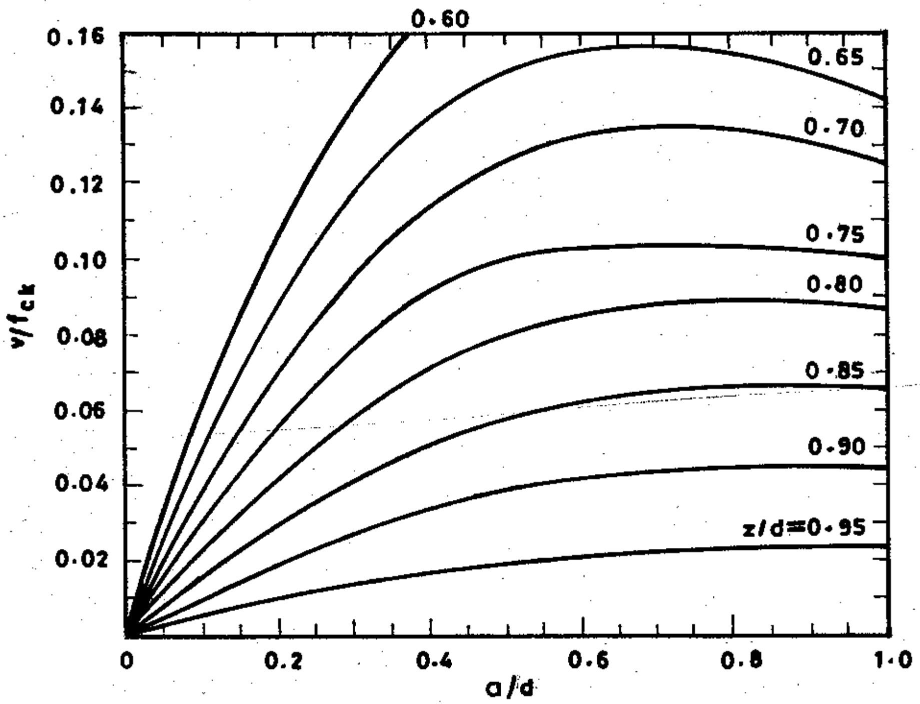


Chart 21.1 Illustration of z/d values for design of corbels.

21.7 DESIGN CALCULATION FOR STEEL AREAS

As already stated, the standard detailing of steel for a corbel consists of the main horizontal steel A_{st}, the horizontal shear steel A_{su}, and the compression steel A_{sc} as shown in Fig. 21.4.

21.7.1 CALCULATION FOR MAIN TENSION STEEL A_{st}

The area of main steel A_{st} should satisfy the following three conditions:

1. The area of tension steel is to be calculated from the expression

$$A_{st} = F_t/f_s$$

and the value of f_s is to be obtained from the ultimate load condition, by using strain compatibility equation with the depth of neutral axis as x . As the failure strain of concrete is 0.0035,

$$\varepsilon_s = \frac{0.0035(d-x)}{x}$$

and the values of f_s should correspond to ε_s in the stress-strain curve of the type of steel used.

In most cases it will be found that ε_s is very low so that it will be better to use mild steel in preference to high strength bars for reinforcement for this purpose. As mild steel reinforcements can also be bent and welded more easily than cold rolled bars, they are preferred as reinforcements for corbels. It may be necessary to weld bars onto the longitudinal steel (as shown in Fig. 21.4) to satisfy anchorage requirements.

2. The second condition to be satisfied is that x should be within the limit specified; otherwise, compression steel of at least 0.1 per cent should be provided to satisfy the limit conditions.

3. The third condition is that F_t should not be less than $1/2F_v$. If there are any horizontal forces acting on the bearing, these too should be added to the F_t for calculation of main steel.

21.7.2 CALCULATION FOR SHEAR STEEL A_{su}

It has been found by tests that premature diagonal tension failure will not occur in corbels if closed stirrups or ties parallel to the main tension steel are provided as specified below (see Fig. 21.4):

1. A_{su} should not be less than half the main tension steel required to resist F_t (Fig. 21.4).

2. The shear steel should be placed in the upper two-third effective depth of the corbel at the support.

21.7.3 COMPRESSION STEEL A_{sc}

Even though theoretically, the compression reinforcement is not required, the practice as illustrated in SP 34 is followed:

1. In the arrangement where the main tension bars at the front face of the corbel are welded to a transverse bar, only nominal compression steel to anchor the stirrups is provided (Fig. 21.4).

2. Alternatively, all the main tension bars are bent back to form loops to act as compression steel (Fig. 21.4b).

21.7.4 HORIZONTAL FORCES ON BEARING

If any horizontal forces can develop at the point of application of the load, they should also be properly taken care of. For this purpose, the bearing plate is generally welded to the main steel. With no live load the value of F_h is taken as not less than $0.2F_v$.

21.8 PROCEDURE FOR DESIGN OF CORBELS

Step 1: *Dimensioning the corbel*

(a) *Breadth of bearing plate.* Assuming a design bearing pressure of $0.8f_{ck}$, determine the length of bearing plate.

(b) *Depth of bracket at support.* As the design value for shear near supports can be much larger than the normal value in Table 13 of IS 456 but not more than $\tau_{c\,\max}$ of Table 14, assume a suitable value for shear nearer to $\tau_{c\,\max}$. Calculate the effective depth d at the root and at the face of the bracket:

$$d = F_v/\tau_c b, \qquad D = d + \text{cover} + \phi$$

(c) *Check the dimensions for criterion for corbel* (as in Section 21.4). The value of a/d should be preferably less than 0.6, but always not more than 1.0.

As a conservative estimate, one may assume that the load is acting at the outer edge of the plate.

Step 2: Determination of lever arm depth. Determine z, the lever arm depth, from Eq. (21.5) or by using design chart 21.1. Also calculate $x = 2.22d(1 - z/d)$ and check if it is greater than the limiting value of x.

Step 3: Resolution of forces. The values of F_t and F_c can be calculated from

$$F_t = F_v\ (a/z)$$

The BS code also requires that F_t should be at least $1/2F_v$ obtained from equation (21.1).

Step 4: Calculation area of main steel. Stress in steel f_s should correspond to the strain ε_s. From the value of x obtained in step 2, find

$$\varepsilon_s = \varepsilon_c\left(\frac{d-x}{x}\right)$$

Here, assuming $\varepsilon_c = 0.0035$, find f_s corresponding to the value of ε_s. Then,

$$A_{st} = F_t/f_s$$

If there is any additional horizontal force, then

$$A_{st} = \frac{F_t + F_h}{f_s}$$

Step 5: Check minimum and maximum percentage of steel. A_{st} should not be greater than 1.3 per cent and not less than 0.4 per cent of bd. If it exceeds the maximum allowed, increase the depth and redesign.

Step 6: Area of horizontal shear steel A_{su} should be at least one-half the area of main tension steel, A_{st}, i.e.

$$A_{su} = 1/2A_{st}$$

Provide it as loops in the upper two-third part of the total depth of the corbel at the support.

Step 7: Check the section for shear. Knowing the percentage of steel, the exact value of allowable shear $\tau_m = \tau_c\ (2d/a_v)$ is known, and the section can be checked for safety.

Step 8: Detail the steel. According to standard practice as given in SP 34: clause 7.7 (see Fig. 21.4).

21.9 DESIGN OF CONTINUOUS NIBS (BEAM SHELVES)

Nibs or 'beam shelves' are used to carry floor units in a prefabricated system. Usually, it is 100 mm or more in depth and the width of bearing should be enough to carry the prefabricated members. It may also be continuous, depending on the prefabrication arrangements. Continuous nibs less than 300 mm in depth are designed in accordance with the rules for design of cantilever slabs. In addition, the points to be remembered in the detailing of the members are:

1. The distance a_v representing the line of action of the load for calculating BM and shear enhancement factor for design purposes should be taken as the distance from the centre line of the nearest vertical leg of the stirrup in the beam to the outer face of the main horizontal steel of the nib, as shown in Fig. 21.5.

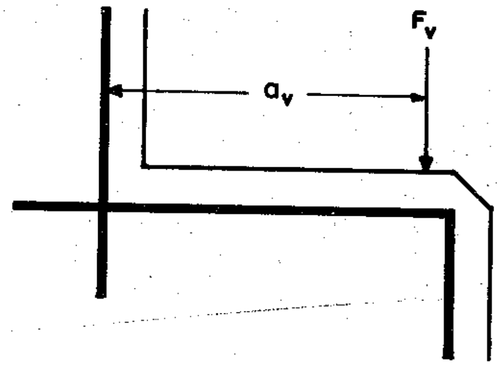


Fig. 21.5 Distance a_v for bending moment and shear.

2. Additional ties or links should be provided as hangers in the structure (beam) connected with the nib. The load on the nib has to be carried to the compression zone of the supporting beam. These hangers are used not only to resist shear in the beam but also to transfer the load from the nib to the compression side of the beam.

The additional steel area (in addition to the area necessary to carry the shear) for the hangers is given by

$$A_{s(\text{add})} = \frac{F_v}{0.87 f_y}$$

3. The Cement and Concrete Association, U.K. recommends the cantilever part of the nibs to be reinforced both horizontally and vertically with two systems of reinforcements. Accordingly, the best way is to provide steel in normal nibs in the form of horizontal loops, as shown in Fig. 21.6. Their areas are given by

$$\text{Horizontal nib steel } A_s = \frac{F_v a}{0.87 f_y z}$$

where z is the lever arm and a is the distance of the load F_v from the nearest hanger bar.

$$\text{Inclined nib steel } A_s = \frac{F_v}{0.87 f_y \sin\theta}$$

where θ is the angle of the inclined loop with the horizontal.

These steels are to be secured in position by additional fixing rods running parallel to the nib, as shown in Fig. 21.6.

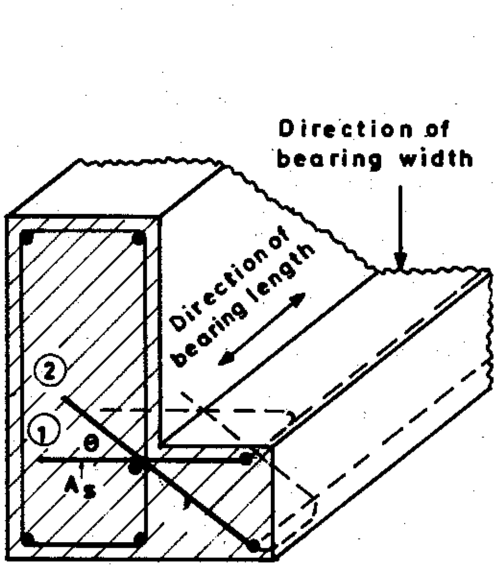


Fig. 21.6 Reinforcements in nibs with large loads (1. Horizontal loops; 2. Inclined loops).

For lightly loaded nibs the horizontal steel may be bent as a loop with the vertical hanger nearest to the load as centre of the bearing with a radius not less than 3ϕ inside the loop, as shown in Fig. 21.7.

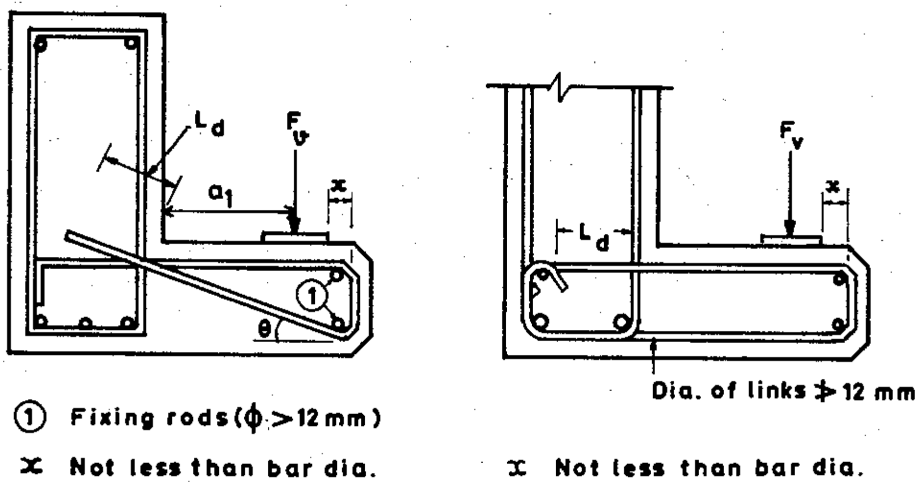


Fig. 21.7 Reinforcements in nibs with light loads.

EXAMPL
Design a
300 × 30
Ref.
Text
Section 2
IS 456
Table 14
Text
Eq. (21.5
Text
Chart 21

EXAMPLE 21.1 **(Design of a corbel)**

Design a corbel to carry a factored load of 500 kN at a distance of 200 mm from the face of a 300 × 300 mm column. Assume that grade 30 concrete is used for construction.

Ref.	Step	Calculations	Output
	1.	*Dimensioning of corbel*	
		Bearing length = size of column = 300 mm	
Text Section 21.4		Assuming bearing plate,	
		Bearing strength = $0.8f_{ck} = 0.8 \times 30 = 24$ N/mm^2	
		Width of plate $= \dfrac{500 \times 10^3}{300 \times 24} = 69$ mm	
		As corbel is an isolated member, increase the calculated width by 20 mm (69 + 20) = 89	Adopt 90 × 300 plate
		Estimation of depth d	
IS 456 Table 14		$\tau_{c\,max} = 3.5$. Assume 3 N/mm^2.	
		$d = \dfrac{500 \times 10^3}{300 \times 3} = 555$ mm	
		$D = d + \text{cover} + \phi/2 = 555 + 40 + 10$ $= 605$ mm	
		Depth at the face = $D/2 \approx 300$ mm	
		Check for strut action	
		$a/d = 200/555 = 0.36 < 0.6$	Acts as corbel
	2.	*Determination of lever arm*	
Text Eq. (21.5)		$(z/d)^2 - \left(\dfrac{r}{r+k}\right)(z/d) + \left(\dfrac{k}{r+k}\right)(a/d)^2 = 0$	
		$k = F_v/0.88f_{ck}bd = \dfrac{500 \times 10^3}{0.88 \times 30 \times 300 \times 555} = 0.114$	
		$a/d = r = 0.36$	
		Substituting, we get	
		$(z/d)^2 - 0.76(z/d) + 0.0312 = 0$	
		Hence, $(z/d) = 0.72$	
Text Chart 21.1		To check by chart, it is necessary that	
		$v/f_{ck} = F_v/(f_{ck}bd) = 0.88k = 0.1$	z/d checked
		For $a/d = 0.36$, $z/d = 0.72$	
		$z = 0.72 \times 555 = 400$ mm	
		$(d - z) = 0.45x$	

EXAMPLE 21.1 (cont.)

Ref.	Step	Calculations	Output
		Hence $x = \dfrac{(555 - 400)}{0.45} = 345$ mm $\dfrac{x}{d} = \dfrac{345}{555} = 0.62 > \dfrac{x}{d}$ limit for steel (x/d limit for Fe 250 = 0.531 and Fe 450 = 0.479) Adequate steel should be used in compression also. The support steel for the main steel and shear steel can satisfy this condition.	
	3.	*Resolution of forces* $F_t = \dfrac{500 \times a}{z} = \dfrac{500 \times 200}{400} = 250$ kN $F_t \not< \dfrac{1}{2} F_v = \dfrac{500}{2} = 250$ kN	
	4.	*Area of tension steel* $A_{st} = \dfrac{F_t + F_h}{f_s}$, $(F_h = 0)$ $\varepsilon_s = \dfrac{0.0035(d - x)}{x} = \dfrac{0.0035 \times 210}{345} = 0.0021$ Fe 250 reaches yield. A_s for Fe 250 $= \dfrac{F_t}{0.87 f_y} = \dfrac{250 \times 10^3}{0.87 \times 250}$ $= 1149$ mm^2 Fe 415 does not reach yield strain 0.0038.	Tension steel 4 T 16
SP 16 Fig. 3		$f_s = 330$ N/mm^2 at $\varepsilon_s = 0.0021$ $A_{st} = \dfrac{250 \times 10^3}{330} = 758$ mm^2. Use 4 rods of 16 mm dia.	(804 mm^2)
	5.	*Check for minimum and maximum steel* $\dfrac{100 A_{st}}{bd} = \dfrac{804 \times 100}{300 \times 555} = 0.48 > 0.4$, but $< 1.3\%$	
	6.	*Area of shear steel* (Minimum) $A_{sv} = \dfrac{1}{2} A_{st} = \dfrac{804}{2} = 402$ mm^2 Provide 4 Nos. 10 mm links (each of 2 legs) in the upper two-third depth. Spacing $= \dfrac{2 \times 555}{3 \times 4} = 92.5$	4 T 10 links (628 mm^2)

EXAMPLE 21.1 (cont.)

Ref.	Step	Calculations	Output
	7.	*Shear capacity of section*	
IS 456 Table 13		τ_c for 0.48 per cent steel = 0.49 N/mm^2	
		$a/d = 0.36$	
		Increased shear strength $= \dfrac{0.49 \times 2}{0.36} = 2.72\ \text{N/mm}^2$	
		The shear capacity of concrete is	
		$\dfrac{2.72 \times 300 \times 555}{1000} = 453\ \text{kN}$	
		Shear capacity of steel $= \dfrac{0.87 f_y A_s d}{s_v}$	
		$= \left(\dfrac{0.87 \times 415 \times 157 \times 555}{92.5 \times 1000}\right)$	
		$= 340\ \text{kN}$	
		Total shear capacity = (340 + 453) kN	
		= 793 kN > 500 kN	Design is safe
	8.	Detail steel according to standard practice.	

REVIEW QUESTIONS

21.1 Explain the term 'shear span' and indicate what is its effect on strength of R.C. members.

21.2 Tests show that the value of shear strength of beams decreases with increase in (shear-span depth) ratio. Draw the nature of the experimental curve obtained, and write down the expression for enhanced shear as given by British practice.

21.3 When will you classify a cantilever projection from a column, as a corbel ? Explain how the load is carried by a corbel.

21.4 Sketch the dimension of a corbel to take a load of 300 kN at a distance of 250 mm from the face of a column 250 × 250.

21.5 What is the design compressive strength for bearing in limit state design ? How does it compare with the average design compressive strength in bending compression in beams and direct compression in columns ?

21.6 Explain the influence of horizontal ties in a bracket and discuss why shear steel in the form of vertical ties or links is not very useful in R.C. brackets.

21.7 Sketch the equilibrium of forces in a corbel assuming the depth of neutral axis at one-half the depth.

21.8 Sketch the typical layouts of the steel in a corbel.

21.9 What is a nib and in what situations would you use them ? What is the difference in design procedure between a corbel and a nib ?

21.10 Sketch the typical layout of steel in a nib.

PROBLEMS

21.1 Design a corbel for a 300 mm square column to support a vertical ultimate load of 500 kN, with its line of action 200 mm from the face of the column. Assume that grade 20 concrete and mild steel are used for the construction.

21.2 Design a suitable bracket (corbel) on a square column of 350 mm to carry a beam, which exerts a reaction of 270 kN due to characteristic loads. A clearance of at least 25 mm is to be provided between the face of the column and the face of the beam to accommodate inaccuracies in construction, and an allowance of 20 mm is to be provided for spalling. Assuming f_{ck} = 30 N/mm^2, f_y = 415 N/mm^2, design the bracket and sketch the details of the reinforcement.

21.3 Design a corbel to support a reaction due to a characteristic dead load of 80 kN and live load of 120 kN. This reaction acts at 200 mm from the face of the column which is 350 mm square in section. There is also a horizontal reaction of 30 kN due to shrinkage restraint of beams etc. Design the corbel and sketch the reinforcements. Show the detailing to counteract the horizontal reaction on the corbel. Assume f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2.

21.4 A continuous concrete nib is to be provided to a reinforced concrete beam, cast *in situ*. The nib is to support a series of precast floor units 450 mm wide and 150 mm deep. These floor units have a clear span of 3.5 m and exert an ultimate total reaction of 25 kN per metre length on the nib. The dry bearing of the floor units on the beam can exert a pressure of $0.4f_{ck}$. Assuming that an allowance of 20 mm has to be provided for spalling and an allowance of 25 mm has to be made for the face of the columns for inaccurate dimensions, design a suitable nib and sketch the layout of reinforcement. Assume f_{ck} = 30 N/mm^2 and f_y = 415 N/mm^2.

22

Design of Footings, Pedestals and Pile Caps

22.1 INTRODUCTION

The function of a foundation or substructure is to safely transfer the loads from the superstructure to the ground. Different types of foundation structures like isolated footings, continuous footings, combined footings, slab rafts, piles, piled rafts, caissons are used for this purpose. Design of foundation structure is a subject in its own right and only the design of elementary foundation structures, such as reinforced concrete individual footings, plain footings, pedestals and pile caps, is covered in this chapter. The aim is to illustrate the method of approach of limit state design to foundation problems.

22.2 DESIGN LOADS FOR FOUNDATION DESIGN

The condition to be satisfied by the subsoil in design of foundations is that its safe bearing capacity (which is based on both strength and settlement) should not be exceeded by the loads from the structure. As the safe bearing capacity is obtained from principles of soil mechanics by dividing the ultimate capacity of the soil by a suitable factor of safety, its value represents the serviceability condition, and not the limit state condition. Accordingly, the loads to be used to determine the size of the foundation should be the service loads and not the factored loads. The loads to be used are

1. Dead plus imposed	1.0 DL + 1.0 LL
2. Dead plus wind	1.0 DL + 1.0 WL
3. Dead plus imposed + wind (or earthquake)	1.0 DL + 0.8 LL + 0.8 WL (EL)

In multistoreyed buildings, one should take advantage of the allowable reduction in live load for residential and office buildings.

22.2.1 EFFECT OF HORIZONTAL LOADS

While considering the resistance of the structure to horizontal loads, it should be noted that horizontal loads are to be resisted by the friction and cohesion of the ground, together with the passive resistance of the soil in contact with the vertical faces of the foundation. Limit state design may conveniently be used for this purpose. However, for mobilising friction, only 0.9 times the dead load should be considered as vertical loads.

If the structure is to be planned to take the horizontal load by moments at its base, it should be properly fixed to the ground so that a fixing moment opposite to the applied moment will be produced by the reactions. Alternatively, the foundation pressure should be considered as non-uniform (refer Section 22.4).

22.3 BASIS OF DESIGN OF FOOTINGS

Footings under walls are called one-way footings and those under columns, two-way footings. The first step in design of footings is to calculate the necessary area from the formula

$$\text{Area of footings} = \frac{\text{service load on column or wall above}}{\text{safe bearing capacity of soil below}}$$

Having thus determined the size of the footings, its structural design is carried out by using factored loads and principles of limit state design as already discussed in the case of other R.C. members. The main items to be designed are the thickness of the footing and its reinforcement. The thickness should be sufficient to

1. resist the shear force without shear steel and the bending moment without compression steel;

2. give the structure the required structural rigidity so that the foundation reaction below can be assumed (see Sections 22.4 and 22.5);

3. withstand the corrosion that can be caused from the ground. (This minimum cover required is not less than 75 mm when the concrete is cast against the ground, and not less than 40 mm when it is cast against a layer of blinding concrete of 75 to 80 mm thickness.)

It is also important to remember that the percentage of steel provided should not be less than 0.15 for Fe 250 and 0.12 for Fe 415 steels as specified for slabs in IS 456: clause 22.5.2.1.

22.4 SOIL PRESSURE ON FOUNDATIONS

In most designs of foundations and especially in individual footing design, the soil is considered as elastic and the R.C. foundation structure as infinitely rigid. Hence, if foundation pressures on these rigid structures are assumed as uniformly distributed on the base, it is necessary that the centre of gravity of the external load system always coincide with the C.G. of the loaded area. Otherwise, there will be a variation of pressure on the base of the foundation which, for rigid foundations, may be assumed as linearly varying. In all layout of foundations, this basic principle should always be borne in mind.

22.5 CONVENTIONAL ANALYSIS OF FOUNDATIONS SUBJECTED TO VERTICAL LOAD AND MOMENTS

With a vertical load P and a moment M acting on a column, the base pressure under the rigid footing will be non-uniform but will be linearly varying from a maximum value of q_1 to a minimum value of q_2 as shown in Fig. 22.1.

If the centre of gravity of the foundation and the line of application of P coincide, the values of q_1 and q_2 are given by Fig. 22.1 as check footing for pressure.

$$q_1 = \frac{P}{BL} + \frac{6M}{BL^2}, \qquad q_2 = \frac{P}{BL} - \frac{6M}{BL^2} \tag{22.1}$$

$q_1 = \frac{P}{A} + \frac{M}{Z}$

no tension should develop.

Z

where

L = the dimensions along the plane of action of the moment

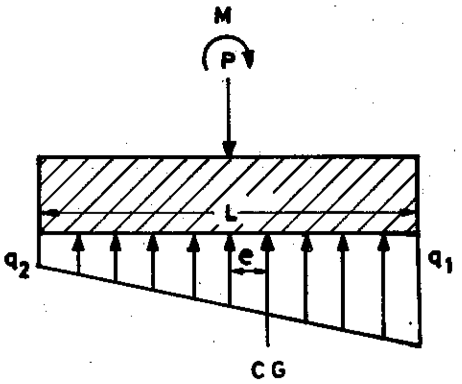


Fig. 22.1 Non-uniform pressure under eccentrically loaded footings.

B = the breadth

q_1 and q_2 = the foundation pressures

The eccentricity e produced by the resultant base reaction will produce the necessary moments to counteract the applied moment so that

$$e = M/P$$

In soils such as clays, large non-uniform base pressures can lead to differential settlements and consequent tilt in the column. Hence the ratio of (q_1/q_2) should not exceed 2 to 4, depending on whether it is caused by permanent or transient loads. In the case of small eccentricities due to P and M, the centre of gravity of the foundation itself may be offset from the line of action of the load P to produce uniform resultant distribution on the base as shown in Fig. 22.2.

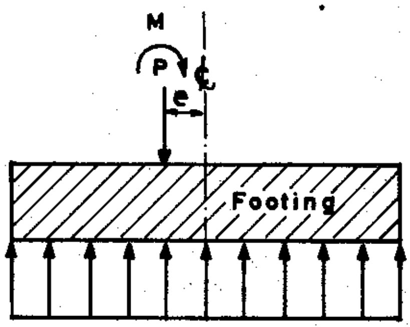


Fig. 22.2 Eccentric loading to produce uniform soil reaction.

The value of $e = M/P$ gives indication of the probable variation of q_1 and q_2. If $e = L/6$, according to the middle third rule, $q_2 = 0$ and q_1 will be twice the average pressures. If e is in the range $L/10 - L/12$, the eccentricity can be considered to be small. When e is greater than $L/6$, part of the base will not be in contact. The maximum soil pressures q on the foundation can be calculated from the following equation (see Fig. 22.3)

$$P = \frac{1}{2} qsB$$

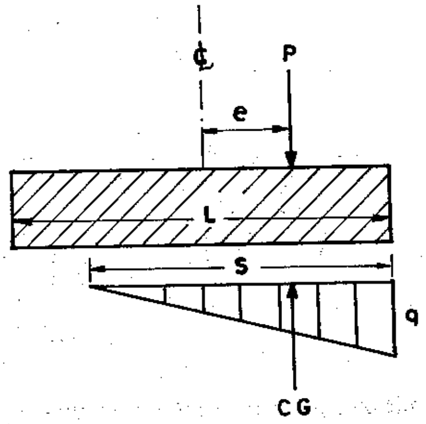


Fig. 22.3 Effect of large eccentricity of load on foundation pressures.

where B is the breadth of the footing s is the length of foundation in contact with the ground. The value of the eccentricity e is given by the equation

$$\frac{M}{P} = e = \left(\frac{L}{2} - \frac{s}{3}\right)$$

In footings subjected to biaxial bending, the effect of variation of base pressures in both planes should be considered and combined to get the final values.

22.6 DESIGN OF INDEPENDENT FOOTINGS

IS 456: clause 33 deals with the design of independent footings. The main recommendations can be summarised as in the following section:

22.6.1 GENERAL PLANNING

1. There can be three types of individual footings: rectangular, sloped or stepped. They may rest on soil, rock or on piles. The minimum thickness of edge of the footing on soil and rock should be 150 mm and that on top of piles should not be less than 300 mm.

2. The column transfers the load to top of footing by bearing. In limit state design, the value of the pressure allowed under direct compression on an unreinforced loaded area of same size is to be limited to $0.45f_{ck}$ as given by IS 456: clause 33.4. However, when the supporting area is larger than the loading area on all sides, it may be increased by the factor A_1/A_2 (where in the case of a footing A_1 = area of the footing and A_2 = area of the column base), but the factor should not be greater than 2.

3. If the above permissible stresses are exceeded, the transfer of forces should be with steel reinforcement, by extending the reinforcement into the footing or by providing dowel (starter) bars. According to IS 456, if dowel bars are provided, they should extend into the column a distance equal to the development length of the column base (IS 456: clause 33.4.4). However, this requirement for development length has been relaxed in BS 8110 (1985): clause 3.12.8.8. According to BS code, compression bond stress that develops on starter bars within bases need not be checked provided

the starter bars extend down to the level of the bottom reinforcement. Application of this clause can reduce the depth required in footings and save on steel reinforcement.

According to IS, the depth required for anchorage is given by

$$L_d = \frac{\phi f_s}{4\tau_{bd}}$$

where

ϕ = diameter of bar

f_s = stress in the bar

τ_{bd} = design bond stress (IS 456: clause 25.2.1.1) and Chapter 10)

For grade 20 concrete and Fe 415, this works out to

$$L_d = \frac{\phi \times 0.87 \times 415}{4 \times 2.4} = 37.6\phi$$

usually it is rounded off as 40ϕ.

When the depth required for the above development length or other causes is very large, it is more economical to adopt a stepped or sloped footing so as to reduce the amount of concrete that should go into the footing.

4. According to IS 456: clause 33.4.3, the extended longitudinal bars or dowels should be at least 0.5 per cent of the cross-sectional area of the supported columns or pedestal, and a minimum of four bars should be provided. The diameter of the dowels should not exceed the diameter of the column bars by 3 mm.

As seen from the above discussions, the depth of the footings can be generally taken as that obtained from the point of shear and bending moment only. In all cases, the depth should be such that extra shear reinforcement or compression steel for bending should be avoided.

5. Rectangular footings are generally used with rectangular columns. The most economical proportions of the footing are given if the rectangular base projects the same distance beyond all the column faces, so that the footing requires the minimum amount of materials.

22.6.2 CALCULATION OF SHEAR FOR DESIGN

In many cases the thickness of the footing will be determined by requirements for shear. Both one-way shear (wide-beam shear) and two-way shear (punching shear) requirements are prevalent as discussed now.

1. One-way shear (wide beam shear)

One-way shear is similar to bending shear in slabs. Considering the footing as a wide beam, the shear is taken along a vertical plane extending the full width of the base located at a distance equal to the effective depth of the footing (i.e. a dispersion of 45°) in the case of footings on soil and a distance equal to half the effective depth of footings for footings on piles (see Fig. 22.4a).

The allowable shear stress is the same as in beams. The tension reinforcement to be considered for estimating the allowable shear should continue for a distance equal to full effective depth beyond the section. For routine design, the lowest value of allowable shear in Table 13 of IS 456, i.e. 0.35 N/mm^2, is recommended. In one-way shear, the shear force to be resisted is the sum of

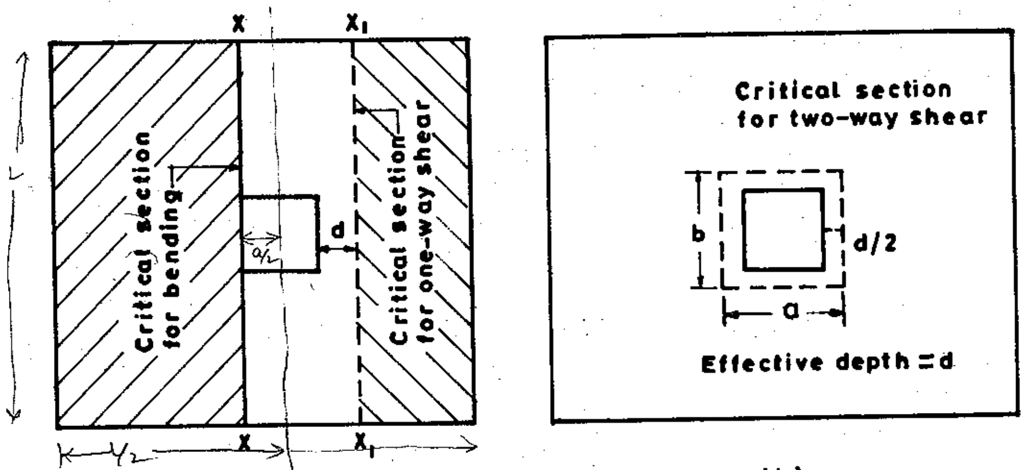


Fig. 22.4 Critical sections for bending and shear for footings on soil: (a) Bending and one-way shear, (b) Punching sheat.

the upward forces in the foundation area from the critical section to the edge of the footing. The consequent shear per unit area is given by

$$\tau_v = \frac{V}{bd} \tag{22.2}$$

This τ_v should not be more than the τ_c specified.

2. Two-way or punching shear

Punching shear indicates the tendency of the column to punch through the footing slab, as illustrated in Fig. 22.4(b). As it has two-way action, it is also called two-way shear. This type of shear occurs also in flat slabs around the supporting columns.

According to IS 456, which follows ACI in general principles, punching shear has to be checked along the surface of a truncated cone around the column load called *critical perimeter* which should be at a distance $d/2$ from the column face. The ultimate shear strength in two-way shear is similar to the two-way shear in flat slabs without moment transfer. The maximum value of shear stress in limit state design according to IS 456: clause 30.6.3.1 is taken as $0.25\sqrt{f_{ck}}$ (0.5 + ratio of (short side to long side of column)), but $\not> 0.25\sqrt{f_{ck}}$.

Let S represent the total upward reaction from the area between the critical section for two-way shear and the edge of the footing. This should be resisted by the footing surface around the perimeter of the critical section

$$S = 2(b + a)d\tau_p \tag{22.3}$$

The value of τ_p should not be more than the maximum specified value.

It should be noted, however, that the situation in punching shear in footings is much safer than that in flat slabs since, unlike the latter, the footings are supported fully on the soil below.

The British practice is to use the same design values for two-way shear as those for one-way shear. These values are much lower than those recommended for punching shear in IS and ACI codes. In order to adjust such low values of BS code, the critical perimeter is taken at a distance equal to effective depth d (instead of $d/2$) from the column face. This will result in larger perimeter length and reduced requirement for shear strength.

Both these approaches are found to give safe values for design, especially in footings.

22.6.3 BENDING MOMENT FOR DESIGN

IS 456: clause 33.2.3 states that the bending moment to be taken for design of footings is the moment of the reaction forces due to the applied load (excluding the weight of the foundation itself) at the following sections:

1. At the face of the column for footings supporting a reinforced concrete column as shown in Fig. 22.4a.

2. Half-way between the centre line and edge of the wall for footings under masonry walls.

3. Half-way between the face of the column and edge of the gusseted base for footings under gusseted bases.

Moments should be considered both in the X and Y directions and the necessary areas of steel provided in both directions.

The steel for the above bending moment is placed as detailed under placement of reinforcement (Section 22.7). The footing is to be considered as a slab and the rules for minimum reinforcement for slabs should apply to those in flat slabs. This recommendation is very important as in many cases of design of footings the reinforcement calculated from bending moment consideration can be less than the minimum required as a slab by IS 456: clause 25.5.2.1.

22.7 MINIMUM DEPTH AND STEEL REQUIREMENTS

As already pointed out, the depths of footings are governed by the following considerations:

1. The depth should be safe in one-way and two-way shear without shear reinforcements.

2. The depth should be safe for the bending moment without compression reinforcement.

3. The depth, according to IS 456, should develop the necessary transfer bond length by the main bars or dowel bars. (This condition is relaxed in BS 8110).

The minimum steel should be as in slabs: 0.15 per cent for Fe 250 and 0.12 per cent for Fe 415. The steel required in the X and Y directions should be distributed across the cross-section of the footing as follows (see also Fig. 22.5):

1. In one-way reinforced footings as in wall footings, the main steel is distributed uniformly across the full width of the footing.

2. In two-way square reinforced footings also, the steel is distributed uniformly across the full width of the footing.

3. In two-way rectangular footings, the steel in the long direction is distributed uniformly over the width of the footing. But as regards the steel in the short direction, more of this steel is placed on the column portion than in the outer portion. For this purpose, a column band equal to the width of the footing is marked beside the column, along the length of the footing, as shown in Fig. 22.5. The portion of the reinforcement to be placed at equal spacing in this band is determined by the equation

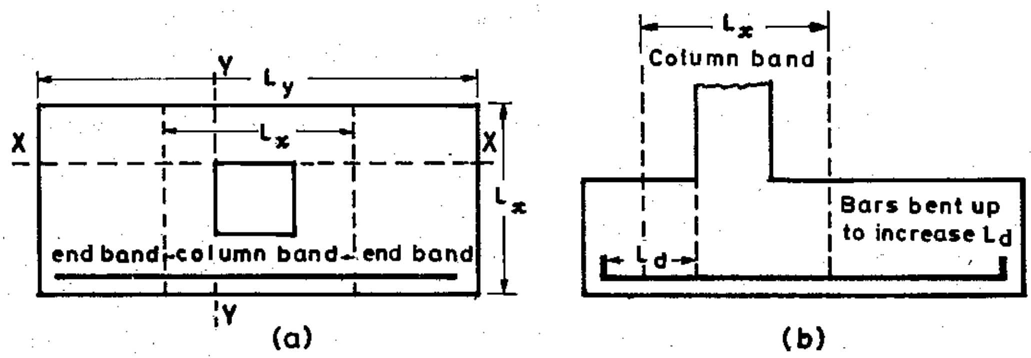


Fig. 22.5 Placement of steel in isolated footings: (a) Central column (plan), (b) Non-central column (elevation).

$$\text{Reinforcement in column band} = \frac{2}{(y/x + 1)} A_t$$

where

A_t = total area of reinforcement in the short direction

y/x = ratio of the length to breadth of footing

The remaining steel is placed at uniform spacing outside the column band.

22.8 CHECKING FOR DEVELOPMENT LENGTHS OF REINFORCEMENTS IN FOOTINGS

Individual footings are small sized members with heavy moments. The sizes of the bars selected should be small enough to develop the full development length in the available dimension of the footing. Thus, in Fig. 22.5(a), the anchorage length L_d required, for the sizes of the bars selected for the long and short direction, should be less than one-half of the lengths for the footing in the respective directions. One way of increasing the development of bars in footings is to bend them up at 90° as shown in Fig. 22.5(b), but this is not an economical solution.

22.9 PROCEDURE FOR DESIGN

The major steps in the design of a footing square or rectangular can be summarised as follows:

Step 1: Determine the plan area from the allowable bearing capacity and service loads from the column, assuming a reasonable weight for the footing.

Step 2: Taking the dead and live load, determine the ultimate soil reaction for factored design load.

Step 3: Determine the depth for one-way shear, assuming a design shear strength value τ_c. Theoretically, this value depends on the percentage of steel in the slab. For preliminary design, a value of $\tau_c = 0.35$ N/mm^2 may be assumed.

Step 4: Determine the depth from bending considerations and adopt the larger depth obtained from steps 3 and 4.

Step 5: Check the depth adopted for safety against punching shear. If it is not sufficient, increase the depth so that the footing is safe in punching shear.

Step 6: Choose the largest depth required considering steps 3 to 5.

Step 7: Calculate the reinforcement required in the X and Y directions from bending moment considerations. The steel provided at the section for maximum moment should not be less than the minimum specified for slabs.

Step 8: Check the development length required and choose the proper diameter of bars.

Step 9: Distribute the steel as specified in the code (see Section 22.7).

Step 10: Provide necessary cover to reinforcement and find the total depth of footing required.

22.10 DESIGN OF SQUARE FOOTINGS OF UNIFORM DEPTH

Square footings are often met with in practice and it is worthwhile to derive the expression for their design. Let the data for design be as follows:

$$\text{Size of footing} = L \times L$$

$$\text{Column size} = a \times a$$

$$\text{Factored load} = P$$

$$\text{Service load + wt. of footing} = P_1$$

$$\text{Bearing capacity} = q_a$$

$$\text{One-way shear value} = \tau_c$$

$$\text{Two-way shear value} = \tau_p$$

Step 1: Plan size of footing

$$A = \frac{\text{service load}}{\text{allowable bearing capacity}}$$

Step 2: Soil reaction for limit state design

$$q = \frac{\text{factored load}}{\text{area}} = \frac{P}{L^2}$$

Step 3: By considering one-way shear, the depth is obtained from the shear at section at X_1X_1 at d from the face of the column (Fig. 22.4) for a square footing ($L \times L$).

$$\text{Shear } V = \frac{P}{L^2} L \left(\frac{L-a}{2} - d \right)$$

$$= \frac{P}{2L} (L - a - 2d)$$

If τ_c is the design shear strength, then

$$\tau_c L d = V$$

Substituting for V and simplifying the expression, we get

$$d = \frac{P(L-a)}{2(P + \tau_c L^2)} \tag{22.4}$$

For calculation purposes, it is convenient to work in metres with τ_c expressed in N/m^2. As the actual value of τ_c will depend on the percentage of steel present at the section continued for a distance d on both sides of the section, it is sometimes recommended to adopt the lowest value of τ_c (namely, the one corresponding to a percentage of steel equal to 0.15 per cent) for routine calculations.

Step 4: The depth from bending moment consideration is obtained by taking moments at face of column XX.

$$M = \frac{P}{L^2}\left[\frac{L(L-a)^2}{8}\right] = \frac{P}{8L}(L-a)^2 \;=\; \frac{P}{L^2} \times \underbrace{L\left(\frac{L-a}{2}\right)}_{\text{area}} \times \frac{(L-a)}{4} \tag{22.5}$$

at a distance from XX.

$M = KLd^2$ (as singly reinforced beam)

$$d = \left(\frac{M}{KL}\right)^{1/2} \tag{22.5a}$$

where K is obtained from Table 4.3.

Step 5: The depth should satisfy two-way shear or punching shear at section $d/2$ from the column face.

Critical perimeter = $4(a + d)$. Considering equilibrium of forces, we get

$$(P/L^2)[L^2 - (a + d)^2] = 4(a + d)\, d\tau_p \tag{22.6}$$

where $\tau_p = 0.25\sqrt{f_{ck}}$. It is easier to check the value of τ_p in this expression for the value of d obtained from Eqs. (22.4) and (22.5) than to solve the equation for d.

Step 6: Take the larger of the depths as obtained from steps 3 to 5.

Step 7: Find the area of steel required from the equation

$$M = A_s f_s jd$$

(Use SP 16 for easy determination of area of steel.)

Step 8: Check the development length. Select the size of bar whose development length is less than $1/2(L - a)$; otherwise, provide the development length by 90° bend at the ends.

Step 9: Detail steel as in Section 22.7 (see Fig. 22.6).

22.11 DESIGN OF SLOPED SQUARE FOOTINGS

In India thick footings are designed and constructed as sloped footing, sloping from column face

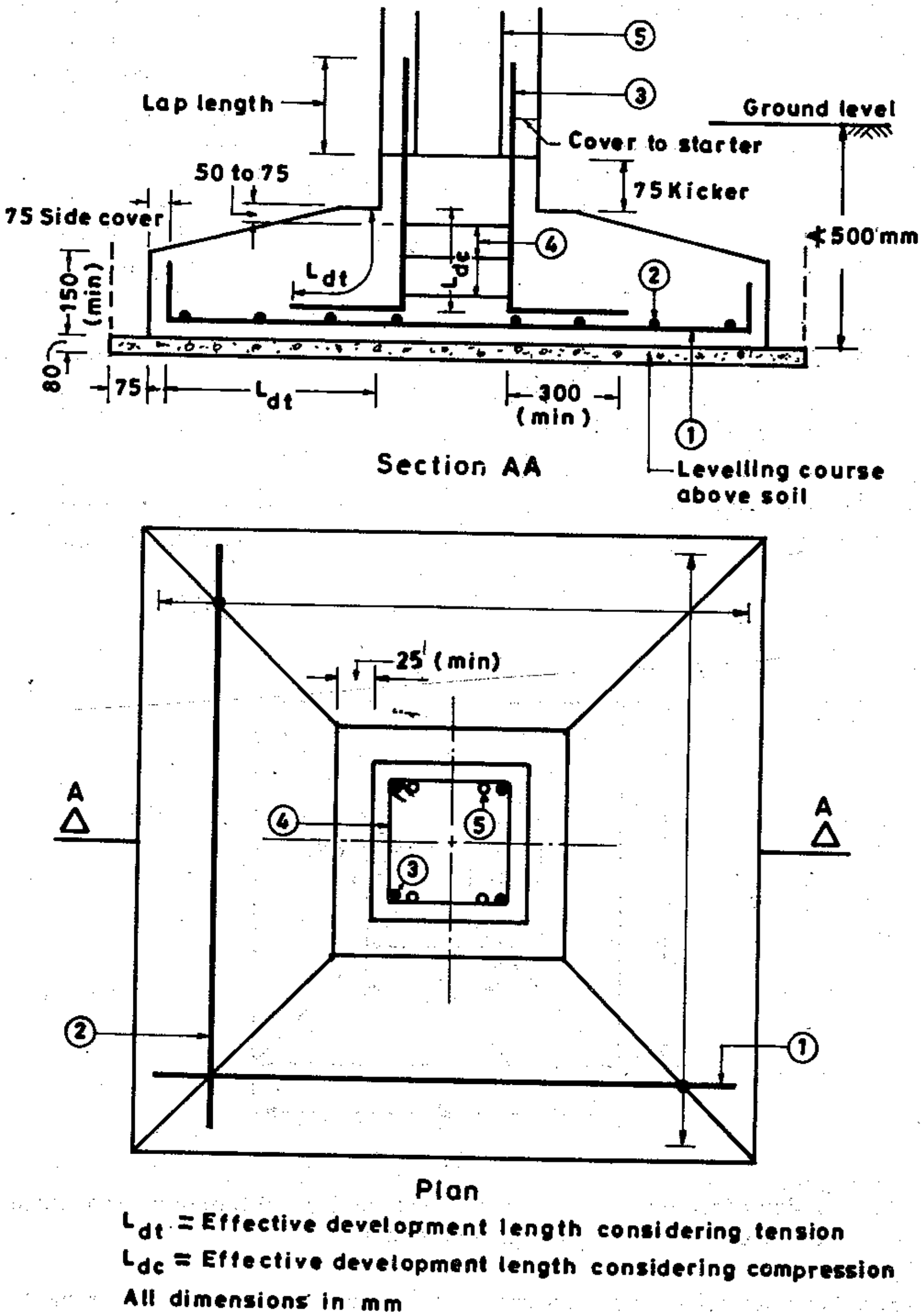


Fig. 22.6 Reinforcement drawing of a footing. (1–5: Bar marks. 1. Main reinforcement bent upwards if required; 2. main reinforcement in other direction; 3. starter bars; 4. stirrup, unless specified use T8 @ 300, minimum 3; 5. column bars.)

to the edge. This saves the amount of concrete required. Sloped footings generally require more depth but less steel than block footings. Figure 22.7(a) shows such a sloped footing. According to IS: 456: clause 23.1.2, the edge thickness should not be less than 150 mm. The slope should

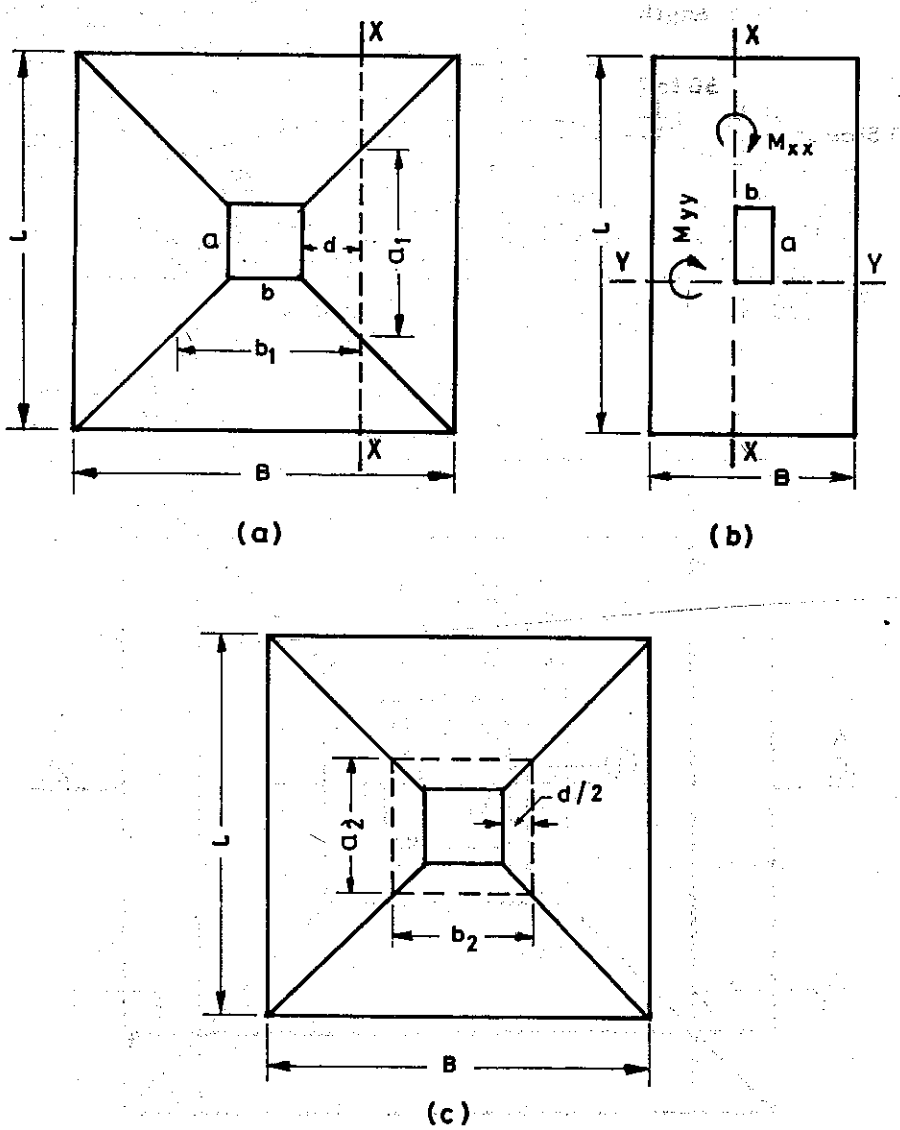


Fig. 22.7 Design of footings: (a) Design of sloped footing, (b) Design of rectangular footing, (c) Checking of two-way shear in footings.

not exceed one vertical to three horizontal if top forms are to be avoided. In most cases the concrete to be placed on the slope has to be relatively dry so that it does not slide down along the slope. As the strength of the footing depends on the compressive strength of this concrete along the slope, special care should be taken in the placement, compaction and curing to get it free from voids.

The procedure for design of sloped footings varies among designers. This is due to the fact that there have not been many large scale tests on sloped footings and an accepted yield line pattern at failure of centrally loaded and eccentrically loaded sloped footings is not yet available.

22.11.1 DESIGN PROCEDURE

A sloped footing can be designed by using the following steps:

Step 1: *Determination of required depth.* The aim here is find a reasonable value which will be larger than that required for block footings for the depth of the footing. This depth is to be checked for shear and used for calculation of steel area. The bending moment is taken at the face of the column and any of the following three procedures can be used for this purpose:

(a) The first method is to assume that the bending moment is the same as that due to a block footing in the XX and YY planes. The expressions are simple and can be derived as follows:

$$\text{Footing size} = (L \times B) \text{ with } L > B$$

$$\text{Column size} = (a \times b) \text{ with } a > b$$

Taking M_{XX} as moment on section normal to shorter span and M_{YY} as moment on section normal to larger span, we get

$$M_{XX} = \left(\frac{P}{LB}\right) L \left(\frac{B-b}{2}\right)\left(\frac{B-b}{4}\right)$$

$$M_{XX} = \frac{P}{8B}(B-b)^2 \tag{22.7}$$

Similarly,

$$M_{YY} = \frac{P}{8L}(L-a)^2 \tag{22.7a}$$

(b) A less conservative method which will give a lesser depth of footing is to assume that the failure plane will be along the diagonals and the moment to be resisted is that due to the loads in the trapezoidal area only, as shown in Fig. 22.7a. Accordingly,

$$M_{XX} = \frac{P}{LB}\left[\left(\frac{L+a}{2}\right)\left(\frac{B-b}{2}\right)\right]\left[\left(\frac{2L+a}{L+a}\right)\frac{1}{3}\left(\frac{B-b}{2}\right)\right]$$

$$M_{XX} = \frac{P}{24LB}(2L+a)(B-b)^2 \tag{22.7b}$$

Similarly,

$$M_{YY} = \frac{P}{24LB}(2B+b)(L-a)^2 \tag{22.7c}$$

In both cases the moment is to be resisted by the dimensions (a) and (b) respectively at the base of the column. Thus,

$$M_{XX} = Ka(d_{XX})^2, \qquad M_{YY} = Kb(d_{YY})^2$$

so that

$$d_{XX} = \left(\frac{M_{XX}}{Ka}\right)^{1/2}, \qquad d_{YY} = \left(\frac{M_{YY}}{Kb}\right)^{1/2} \tag{22.8}$$

The larger of the d values is taken as the required depth.

(c) Some designers use method (a) for calculation of moments but assume that the moment is resisted by an effective breadth larger than that of the dimension at the column base. One such approximation made is to use the formula

$$b_{\text{eff}} = b + \frac{1}{8}(B - b), \qquad a_{\text{eff}} = a + \frac{1}{8}(L - a) \tag{22.9}$$

However, as this step is for an estimate of the depth (which can as well be assumed by an experienced designer) procedure (a) is preferred as it gives a larger value than procedures (b) and (c). However, for calculation of moment for steel area (as shown in step 4), procedure (b) gives more economical results.

Step 2: Checking for one-way shear. One-way shear is checked by taking a section at a distance equal to the effective depth as obtained from step 1 from the face of the column (not from the edge on top of the footing).

The shear force V_1 to be resisted is taken as that acting in the corresponding quadrant into which the slope footing is divided as shown in Fig. 22.7(a). The area of concrete resisting the shear is taken as the breadth of the quadrant b_1 at the section multiplied by the depth of the concrete at that section d_1. Accordingly, for equilibrium at failure,

$$V_1 = b_1 d_1 \tau_c$$

Step 3: Checking for two-way shear. The depth of the column should also be enough to resist two-way (punching) shear. The section for two-way shear is $d/2$ from the face of the column (where d is the effective depth at the face of the column).

The lengths a_2, b_2 as well as the depth d_2 resisting the perimeter shear are calculated. The shear force to be resisted, i.e. V_2 is given by

$$V_2 = \frac{P}{LB}(LB - a_2 b_2)$$

The condition to be satisfied is

$$V_2 = 2(a_2 + b_2)\tau_p d_2$$

Step 4: Calculation of the area of steel required. The area of the steel required is to be calculated from moment considerations. The conventional method is to use procedure (a) explained in step 1. However, as already point out, procedure (b) gives a more economical solution.

The area of the steel is obtained from the calculated moment by the equation

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{b d f_{ck}}\right)$$

by using SP 16 or any other method.

The step-by-step procedure for design is illustrated by Example 22.3.

22.12 DETAILING OF STEEL

Typical detailing of steel in footings is shown in Fig. 22.7. The IS specifications have been already discussed in Section 22.7.

22.13 DESIGN OF RECTANGULAR FOOTINGS

The considerations are the same as already derived above. The corresponding moments are

$$M_{XX} = \frac{P}{8B}(B - b)^2, \quad M_{YY} = \frac{P}{8L}(L - a)^2$$

where M_{XX} is the moment on section normal to span L_x and M_{YY} the moment on section normal to span L_y.

22.14 PLAIN CONCRETE FOOTINGS

Plain concrete footings are used under brick walls, brick pillars etc. where the pressures transmitted at foundation level are small.

The angle at which the load is assumed to be transferred to the ground through a hard stratum like concrete is called the *angle of dispersion*. According to IS 456: clause 33.1.3, this angle of dispersion is taken as a function of the ratio f_{ck}/q_0. The concrete strength to the bearing capacity of material is given by the relation

$$\tan \alpha \not< 0.9\left(\frac{100q_0}{f_{ck}} + 1\right)^{1/2} \tag{22.10}$$

However, some authorities take its value as a constant equal to 45°.

The quantity to be found out is the depth D of the footing necessary to disperse the load to the base through the angle as specified above. Under this condition there will be no tension at the base and no steel reinforcements are necessary in these plain footings.

The solution of the problem is illustrated by Example 22.1

22.15 DESIGN OF PEDESTALS

Pedestal is a short compression member (the height being usually less than three times its least lateral dimension) placed at the base of columns to transfer the load of the column to a footing pile cap, mat, etc. as shown in Fig. 22.8. Pedestals become essential in the layout of steel columns. It is also provided for steel columns in factory buildings as a precaution against possible corrosion of steel, from foundations and wet floors. Thus in industrial buildings where the floors are washed regularly, the column bases should be 50 to 100 mm above the floor level. Otherwise, special precautions should be taken to encase the bottom part of the column in concrete.

Another factor that makes the introduction of pedestals under steel columns compulsory is the large tolerances that have to be provided in civil engineering constructions. Steel columns are fabricated in workshops to exact sizes. The use of a pedestal makes it convenient to make adjustments for the variation of the foundation levels in construction.

In reinforced column construction the enlargement of its base as a pedestal is practised by many engineers to makeup levels, supply larger bearing areas to foundations, and provide enough development length for reinforcements. In such cases, the pedestals and the base footing together act as a stepped footing.

If these pedestals are reinforced as in columns with longitudinal and lateral steels, they should be treated as enlargement of the column. If they are not provided with longitudinal steel, then they should be treated as footings.

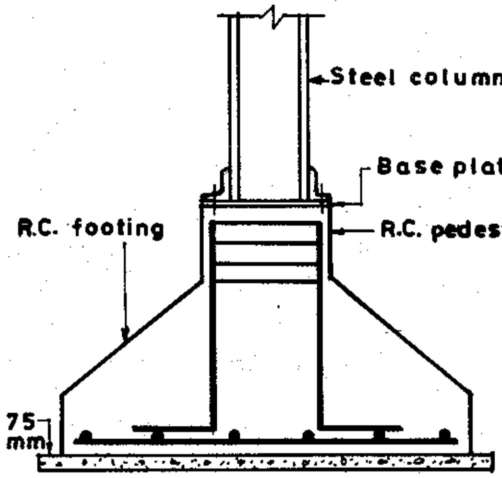


Fig. 22.8 Layout of steel for R.C. pedestal.

22.15.1 DESIGN CALCULATION

Based on the theory of bearing capacities explained in Section 22.5, one can arrive at a pedestal which may be designed as a plain or reinforced pedestal.

When the stress on the top of the pedestals is lesser than $0.45 f_{ck}(A_2/A_1)$, theoretically no steel is required in the pedestal. However, in practice, for pedestals used for R.C. and steel columns, it is advisable to have at least 0.4 per cent of the area of the pedestal as nominal longitudinal steel with 12 mm laterals binding them together as in columns.

When the stress on the base is greater than $(0.45 f_{ck}(A_2/A_1))$, the reinforcement shall be provided for developing the excess force either by extending the longitudinal bars from the column into the base or by means of dowels from base to columns (IS 456: clause 33.4.1).

An interpretation of this rule obviously means that it is sufficient that steel in the base be less than in the column, i.e. the total area of the dowels need not be the same as the area of longitudinal steel in the columns. Accordingly, dowels can be of the same number but of less area. However, the Indian construction practice is to continue all the longitudinal bars from the column to the base of the foundation structure and provide laterals for these bars as in the top column. This is a very safe and conservative practice.

Example 22.5 illustrates the design of pedestals.

22.16 DESIGN OF PILE CAPS

Pile caps are used to transmit column loads to the pile foundation. The plan dimension of the pile cap is based on the fact that the actual final position of piles can be in construction up to 10 cm out of line from the theoretical centrelines. Pile caps should therefore be made very large to accommodate these deviations. In practice, pile caps are extended as much as 15 cm beyond the outerface of the piles.

The important parameters in design of pile caps are:

1. Shape of pile cap
2. Depth of pile cap

3. Amount of steel to be provided
4. Arrangement of reinforcement.

To standardise pile cap design, R.T. Whittle and D. Beattie gave the following recommendations which are commonly used in British practice.

22.16.1 SHAPE OF PILE CAP TO BE ADOPTED

The standard shapes and types of arrangements to be used in layout of piles should depend on the number of piles in the foundation. In this context, the following requirements should be taken into account:

1. The minimum spacing of piles permitted from soil mechanics depends on the type of pile and the soil conditions. CP 2004 requires a minimum centre-to-centre spacing of twice the diameter of the piles for end-bearing and three times the diameter for friction piles. (IS 2911: part I, Sections 1 and 2 recommend a minimum spacing of two and a half times the diameter of the pile for both driven cast-in-situ and bored cast-in-situ piles.) For accommodating deviations in driving of piles, the size of the pile cap is made 300 mm more than the outer-to-outer distance of the exterior piles. A cover of 75 mm is also usually provided for the pile cap surfaces in contact with earth and 60 mm against blinding concrete of 75 to 80 mm thick. In marine situations the cover should be increased to a minimum of 80 mm.

2. Another requirement in arriving at the shape of the pile cap is that the centre of gravity of the piles and the pile cap should coincide so that all the piles are equally loaded.

3. In arriving at the final layout, the need to provide suitable reinforcement is a major consideration.

Based on these three requirements, the recommended shapes of the pile cap for two to nine piles are as shown in Figs. 22.9 and 22.10. For example, as a pile cap for a three-piles group, any one of the shapes A, B and C as in Fig. 22.11 can be considered. However, A was rejected because of doubtful shear capacity, and B was rejected because of difficulty in setting out of steel. But shape C has been adopted as it can have more practical layout of reinforcement and will be also strong in shear.

22.16.2 CHOOSING APPROXIMATE DEPTH OF PILE CAP

Study of the cost of pile caps for pile diameters of 40 to 60 cm showed that the cost per kN of load varied with the depth of pile cap. The trend in variation of cost for 40 cm and 60 mm diameter pile is shown in Fig. 22.12 (p. 484). Accordingly, one may generalise the relationship for an economical depth and arrive at the following relation: Assuming (h_p) as the diameter of pile and (h) as the total thickness of the pile cap, we get

$$\begin{aligned} h &= (2h_p + 100)\ \text{mm} \quad \text{for } h_p \ngtr 550\ \text{mm} \\ h &= 1/3\,(8h_p + 600)\ \text{mm} \quad \text{for } h_p \geq 550\ \text{mm} \end{aligned} \tag{22.11}$$

22.16.3 DESIGN OF PILE CAP REINFORCEMENT AND CHECKING THE DEPTH FOR SHEAR

There are two alternative theories on which pile caps can be assumed to transfer the loads from the columns to the pile foundation. They are (a) the truss theory, and (b) the beam theory.

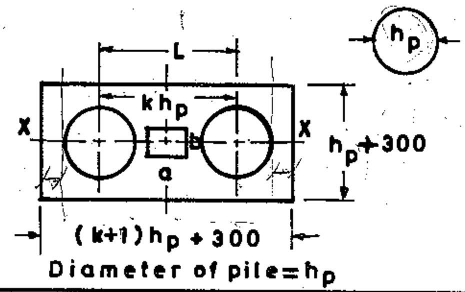
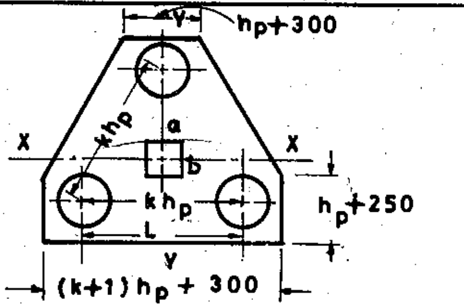
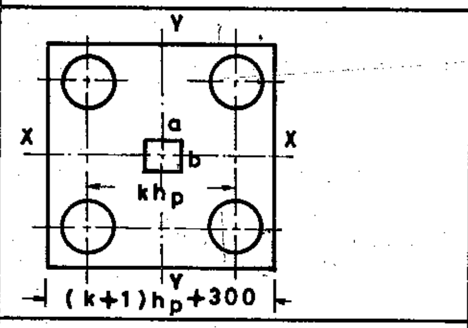
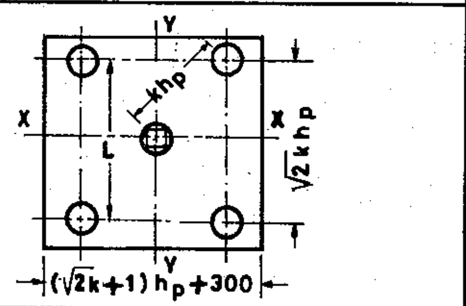
No. of piles	Shape of pile cap	Tension by truss theory
2		Along XX $= \frac{P}{12Ld}(3L^2 - a^2)$
3		Along XX $= \frac{P}{36Ld}(4L^2+b^2-3a^2)$ Along YY $= \frac{P}{18Ld}(2L^2 - b^2)$
4		Along XX $= \frac{P}{24Ld}(3L^2 - a^2)$ Along YY $= \frac{P}{24Ld}(3L^2 - b^2)$
5		Along XX $= \frac{P}{30Ld}(3L^2 - a^2)$ Along YY $= \frac{P}{30Ld}(3L^2 - b^2)$

Fig. 22.9 Shapes of pile caps and value of tensile force for pile caps.

Even in conventional designs when the angle of dispersion of load θ is less than 30° (tan 30° = 0.58), i.e. the value of (a_v/d) ratio as shown in Fig. 22.13 is less than 0.6, one is allowed to assume the load to be transferred to the pile by strut action, AB being in compression and BC in

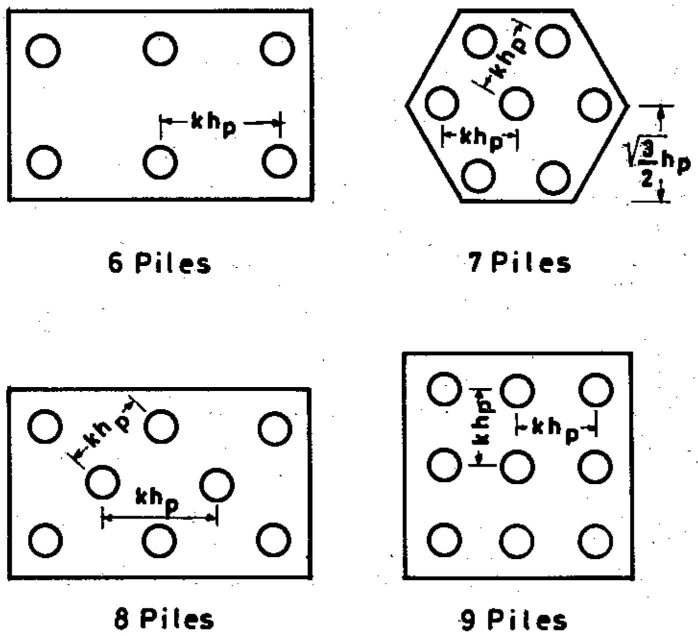


Fig. 22.10 Shapes of pile caps (6–9 piles).

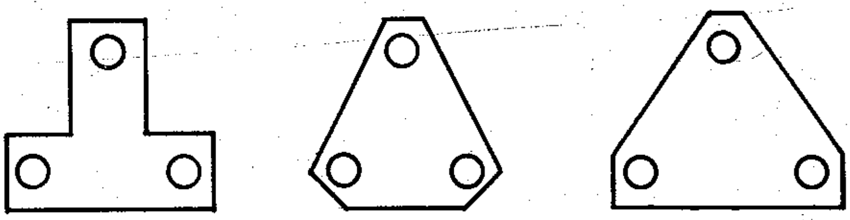


Fig. 22.11 Possible layout of pile caps for three piles.

tension. This is called the 'truss action' (Fig. 22.14). Experiments show that this action (as in deep beams and corbels) can be predominant even up to (a_v/d) ratio equal to 2. In this truss action the tensile force between pile heads is assumed to be taken by the reinforcement as in an arch, and so great care should be taken to tie the ends of the reinforcements at its ends as needed in the case of an arch.

When the shear span/depth ratio of (a_v/d) is 2 or more, bending action is more predominant than truss action and the tensile reinforcement at the bottom acts like the tension reinforcement in an ordinary beam. Then the pile caps are to be designed by the beam theory.

In some cases the design by beam theory may require far less steel than that by the truss theory. It should, however, be remembered that the realisation of these different actions is not so much for the determination of the amount of steel and the savings that can be done in steel quantity as for the appreciation of the real behaviour of the pile cap and for the method of detailing of steel in pile caps. The necessity of anchoring the main steel at their ends should be fully appreciated when conditions are favourable for truss action. With truss action, the ends of the steel should be given full anchorage by providing the full development length.

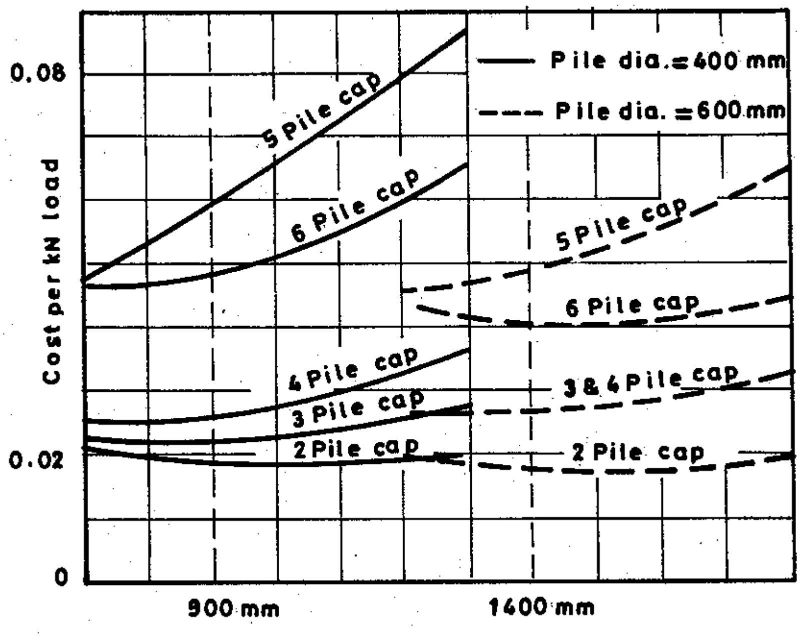


Fig. 22.12 Variation of cost of pile cap with pile diameters.

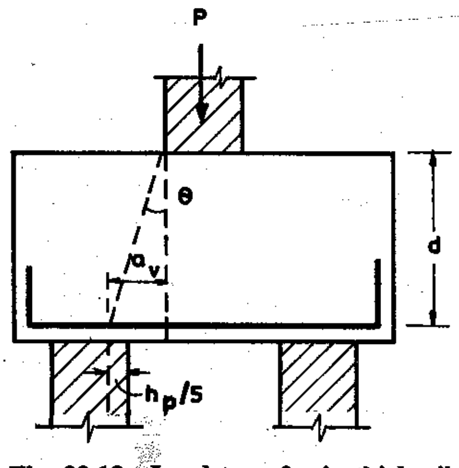


Fig. 22.13 Load transfer in thick pile caps.

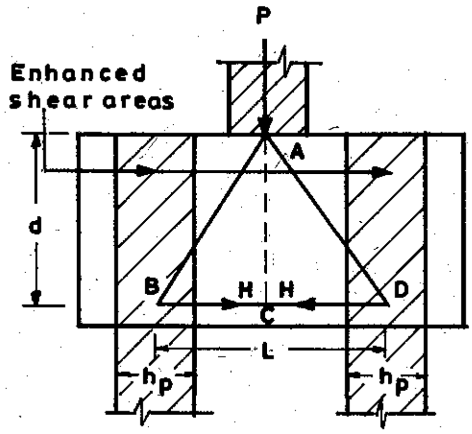


Fig. 22.14 Truss action in pile caps.

Theoretically, one can assume higher anchorage bond stresses on top of the pile due to the effect of the compression present at the junction of the pile head and the pile cap.

1. Design by truss action (Method 1)

(a) *Calculation of tensile force.* The truss action can be visualised in groups of piles up to five piles. For a pile cap over two piles, taking H as the tensile force and P as load from the column, from the triangle of forces in Fig. 22.14, we have

$$\frac{H}{P/2} = \frac{L/2}{d}$$

The tensile force is thus given by the expression

$$H = \frac{PL}{4d} \tag{22.12}$$

The area of steel required is given by

$$A_s = \frac{H}{0.87 f_y} = \frac{PL}{4(0.87 f_y)d}$$

If the size of the column ($a \times a$) is also taken into account, the magnitude of the tensile force will be given by

$$H = \frac{P}{12Ld}(3L^2 - a^2) \tag{22.12a}$$

These values of H for pile caps for piles up to 5 in number are tabulated in Fig. 22.9.

The ideal arrangement of reinforcement obtained from truss theory is to place the steel in bands joining the top of piles with enough extension of the steel for full anchorage. In practice, this concentration of steel over piles is rarely possible and an even distribution across the section with some concentration along the piles is resorted to, as will be seen in Section 22.15.4.

(b) *Design for shear.* It has already been pointed out that truss action can be visualised for pile caps up to five piles. In the area where truss action is prevalent, the shear is as in deep beams and corbels. The shear resistance is large as already explained in Chapter 21. With a pile cap for two piles as shown in Fig. 22.14, considering the downward force, it can be seen that the safe shear capacity of the section is given by the increased shear capacity in the shaded area plus the shear capacity of the unshaded area. Hence the equation to be used for checking the allowable shear resistance of the section can be written as

$$P = 2(dh_p)2\tau_c\left(\frac{d}{a_v}\right) + (b - 2h_p)\tau_c d \tag{22.13}$$

where τ_c is the design shear strength of concrete in IS 456, Table 13. The section should be safe without extra shear reinforcement.

2. Design for bending action (Method 2)

When (a_v/d) ratio is more than 2 as in shallow pile caps or with the arrangement of 6 or more piles, bending action is more predominant than truss action. In this case the pile cap is designed as a normal beam for bending moment and shear. The reinforcement is evenly distributed over the section.

For bending action the load on the pile cap may be considered as uniformly distributed or concentrated. In IS 2911, the load from the column is dispersed at 45° angle from the top of the pile cap to the middle of the pile cap. The reaction from the pile is also taken as distributed at 45° from the edge of the pile up to the mid-depth of pile cap. The maximum bending moment and shear force are calculated on this basis. However, it is much easier to consider the loads as concentrated loads and calculate the BM and SF. The depth should be such that no extra shear reinforcement is necessary for the section.

22.16.4 ARRANGEMENT OF REINFORCEMENTS

The detailing of reinforcement in pile caps depends on their shape. In general seven types of bars are used as pile cap reinforcements (Fig. 22.15). They are:

1. Type (1) steel main bars placed at the bottom in the *XX*-direction bent up at their ends to increase anchorage.

2. Type (2) steel main bars placed at the bottom in the *YY*-direction also bent up at their ends.

3. Type (3) steel consisting of two or three layers of 16 mm diameter horizontal ties (lacer bars) fixed to the upstands of the main bars as secondary steel to resist bursting.

4. Type (4) bars, the column starter bars, which are *L* shaped and turned back at the level of the bottom reinforcement. They are held together by links at two or more levels.

5. Type (5) bars which are the reinforcements from the pile which are extended into the pile cap for its full development length in compression.

6. Type (6) bars which are the top steel provided as compression steel in the slab if required by calculation. These reinforcements are laid out as shown in Fig. 22.15 and tied together to form a cage before casting the pile cap.

7. Type (7) bars are the links to the column bars and 7A links to the pile reinforcements.

The arrangement of reinforcements for various types of pile caps is shown in Fig. 22.16.

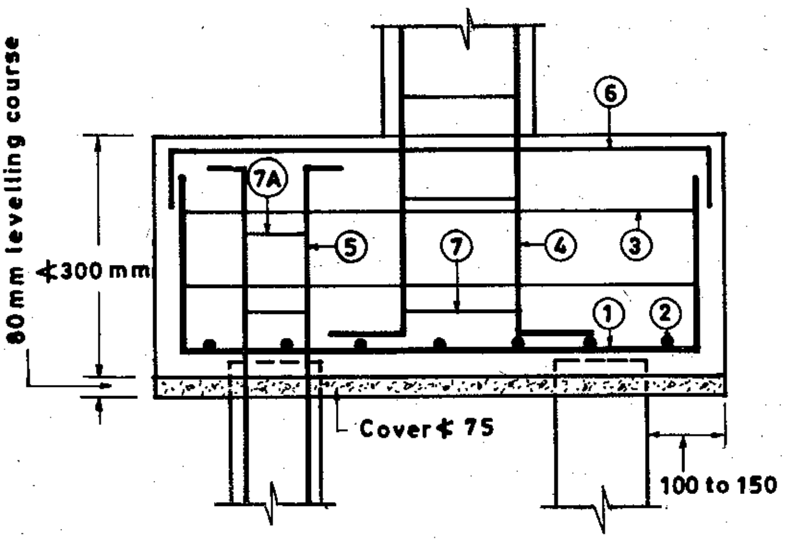


Fig. 22.15 Reinforcement drawing of pile caps. (1–7: Bar marks. 1, 2, Main bars; 3, horizontal ties to resist bursting (T12–150); 4, starter bars; 5, pile bars; 6, top bars; 7, 7A links.)

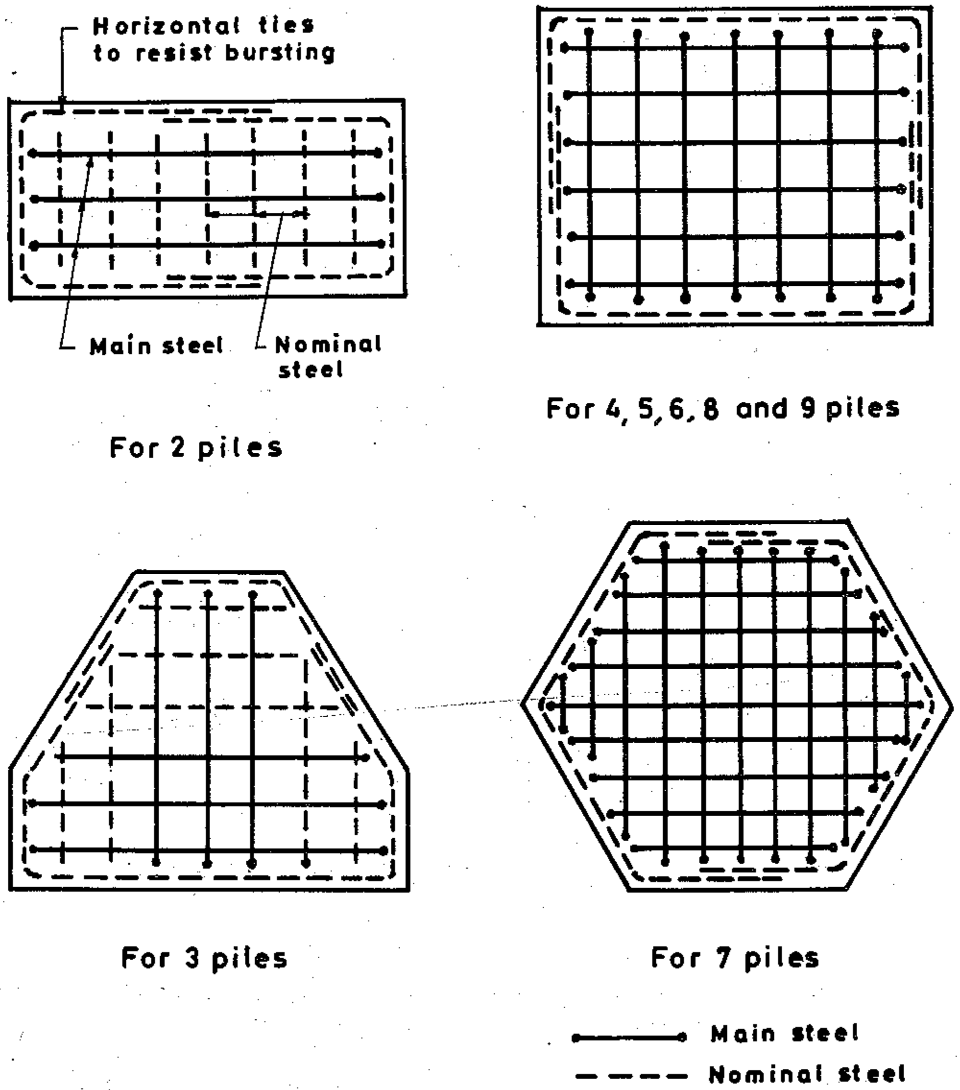
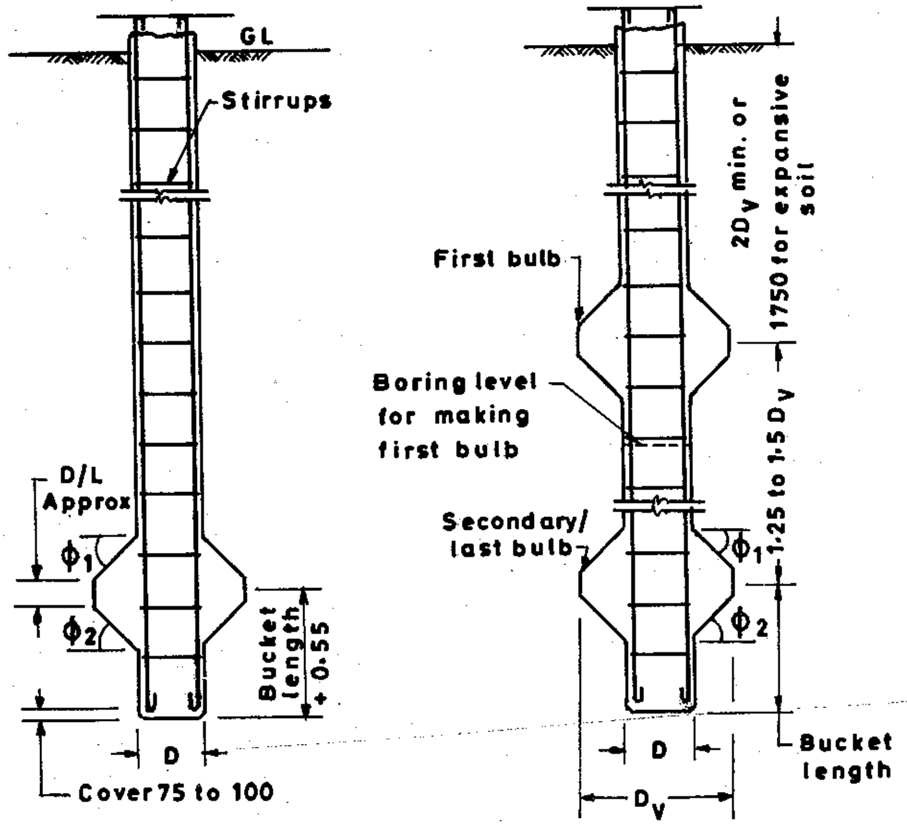


Fig. 22.16 Arrangement of reinforcement for pile caps.

22.17 UNDER-REAMED PILE FOUNDATIONS

Shallow under-reamed piles with pile cap, e.g. conventional piles, are used to carry light column loads. They may also be used with grade beams for wall foundations. The latter type is extensively used in India for construction of residential buildings of one or two storeys, especially in expansive soils. It is also used in other soils where excavation for conventional foundation should be avoided due to restricted space or other reasons.

IS 2911, Part III (1980) deals with the use of under-reamed piles. They can be single or double under-reamed as shown in Fig. 22.17. The diameter of the bulb (D_u) is usually 2.5 times the shaft diameter (D), and the bulbs are separated $1.25D_u$ to $1.5D_u$. The first bulb is usually at a



$\phi_1 = 45°$ approx., $\phi_2 = 30°–45°$ approx., D_v = Normally 2.5 D

(a) (b)

Fig. 22.17 Typical details of bored cast *in situ* under-reamed pile foundations: (a) Section of single under-reamed, (b) Section of multi under-reamed.

minimum depth of 1.75 m from the ground level. The bulbs are installed by manual labour by special boring tools. For carrying light column loads, they are detailed in the same way as conventional piles. The type of layout for carrying wall loads of residential houses is shown in Fig. 22.18 and is now described. As it is not possible to produce bulbs in sandy soil under water level, they should not be used in such soils.

22.17.1 DESIGN OF UNDER-REAMED PILE FOUNDATION FOR WALLS OF BUILDINGS

Under-reamed foundations for walls consist of a series of under-reamed piles spaced at suitable intervals, and a continuous grade beam is laid over these piles; the walls are built on these grade beam and piles. For the design of such a foundation, the following factors should be taken into account:

1. The spacing of the piles should depend on the capacity of the pile and the load it has to carry; it is usually between 1 m and 3 m. Generally, it should not exceed 3 m to limit the sizes

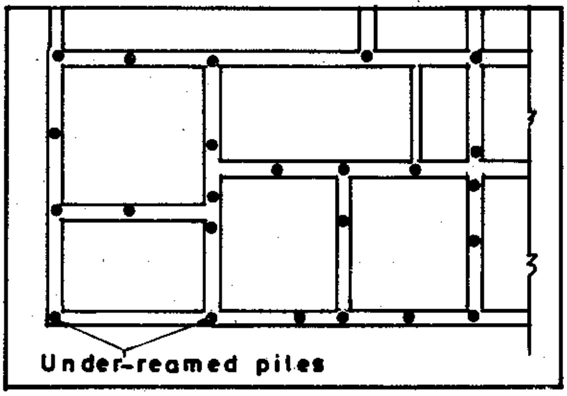


Fig. 22.18 Layout for carrying wall loads.

of the grade beams. The minimum spacing allowed for the under-reamed piles is $2D_u$. In critical cases it can be reduced to $1.5D_u$, in which case the capacity of the pile should be reduced by 10 per cent. For a building these piles are so planned that a pile is placed at the junctions or very near the junctions of the walls, as shown in Fig. 22.18.

2. The carrying capacity of these piles is based on the field tests conducted by the Central Building Research Station, India. They have standardised the pile diameters and the reinforcements to be provided (not less than 0.4 per cent for high yield deformed steel). The safe load capacity uplift and lateral loads for these under-reamed piles in medium compact sandy soils (N values, 10 to 30) and clayey soils of medium consistency (N value, 4 to 8) and with bulb diameters equal to 2.5 times the shaft diameter and 3.5 m to 4.5 m deep are given in Table of IS 2911 (Part III). The safe vertical load capacities for single under-reamed piles are reproduced in Table 22.1. It can be

TABLE 22.1 SAFE LOAD FOR VERTICAL UNDER-REAMED PILES [(L = 3.5 M) IN SANDY AND CLAYEY SOILS INCLUDING BLACK COTTON SOILS]

(Refer IS 2911, part 3 (1980))

Diameter of pile (mm)	Longitudinal steel—HYDS (mm.)	6 mm stirrups; measurement (mm)	Single under-reamed (tons)	Strength increase in 300 mm length (tons)	Strength decrease in 30 mm length (tons)
200	3–10	180	8	0.9	0.7
250	4–10	220	12	1.15	0.9
300	4–12	250	16	1.4	1.1
375	5–12	300	24	1.8	1.4
400	6–12	300	28	1.9	1.9
450	7–12	300	35	2.15	1.7
500	9–12	300	42	2.4	1.9

Notes: (i) N value for sand (10 to 30) and clay (4 to 8).
(ii) Diameter of bulb is 2.5 times diameter of pile.
(iii) Capacity of double under-reamed is 1.5 times that of single under-reamed pile.
(iv) Spacing of piles should be such that there is undisturbed soil for a distance of not less than 0.5 times the diameter of the bulb between the bulbs.

seen that this table assumes a nominal bearing capacity of about 40 T/m^2 and about 2 m depth below the ground level calculated on the nominal area of the bulbs. For softer soils the capacity of the pile should be reduced, and for harder soils it may be increased correspondingly. The safe load on double under-reamed piles is taken as 1.5 times that assumed for single under-reamed piles.

3. Grade beams are meant to transfer the wall load to the ground. The width of the beam is made slightly larger than the pile diameter and not less than the width of the wall it has to carry. In expansive soils these grade beams are isolated (not supported) on the soil below. In non-expansive soils, it is better that the grade beams are placed on an 8 cm sand filling and a levelling course of 8 cm thick so that it can also transfer part of the load to the ground below. A span depth ratio of 7 to 10 with a minimum depth of 15 cm is used in practical construction.

4. For the design of these beams, according to IS 2911 (Part 3): clause 5.3, the maximum bending moment in the case of beams supported on the ground during construction is taken as $wL^2/50$, where w is the uniformly distributed load per metre run (with a maximum height of two storeys) and L is the spacing of the piles. With unsupported beams the bending moment is increased to $wL^2/30$. However, for concentrated loads on the beam the full bending moment should be considered.

Alternatively, even though not suggested in the code, the grade beams may be designed as in the case of lintels so that only the wall load inside a 60° triangle is transferred onto the grade beam. As the height of the triangle of load will be $\sqrt{3}\,(L/2)$, the weight of the triangle of masonry becomes

$$W = \frac{\sqrt{3}\,(L^2)}{4}(t \times 19)\ \text{kN} \tag{22.14}$$

The bending moment due to this load will work out to be

$$M = \frac{WL}{6}\ \text{kNm} \tag{22.15}$$

In either case the depth and reinforcement calculated will be small. A minimum depth of 15 cm is always adopted for the beams.

5. For detailing of steel in these beams, IS 2911 (Part 3) recommends that equal amount of steel should be provided at the top and bottom of the grade beam. It should also be not less than three bars of 8 mm high yield steel at top and bottom. The stirrups should consist of 6 mm M.S. bars at 300 mm spacing, which may be reduced to 100 mm at the door openings near the wall up to a distance of three times the depth of the beam.

A better method of detailing would be to bend two of the four bars from the under-reamed piles to each side of the pile into the grade beams as top steel and extend it to a length up to quarter span of the beam on both sides of the pile. In addition, two top bars are provided continuous at top of the grade beam as hanger bars. The reinforcements required at the bottom (minimum of 3 Nos. of 8 mm bars) are provided as continuous bottom reinforcement. The recommended sizes of grade beams are given in Table 22.2.

The design of an under-reamed pile-grade beam system is given in Example 22.7.

TABLE 22.2 RECOMMENDED SIZE OF BEAMS OVER UNDER-REAMED PILES
(Table for Fe 250; correspondingly reduce for Fe 415)

Load on beam (kN/m)	Effective span of beams							
	1.5 m		1.8 m		2.1 m		2.4 m	
	D (mm)	A_{st} (Nos. dia)	D (mm)	A_{st} (Nos. dia)	D (mm)	A_{st} (Nos. dia)	D (mm)	A_{st} (Nos. dia)
1. *Beam width* 230 mm								
15	150	4 T 10	150	4 T 10	150	4 T 10	150	4 T 10
30	150	4 T 10	150	4 T 10	150	4 T 10	180	4 T 10
45	150	4 T 10	150	4 T 10	180	4 T 10	200	4 T 10
60	150	4 T 10	180	4 T 10	200	4 T 10	200	4 T 10
75	180	4 T 10	200	4 T 10	200	4 T 12	200	4 T 16
2. *Beam width* 345 mm								
15	150	4 T 10	150	4 T 10	150	4 T 10	150	4 T 10
30	150	4 T 10	150	4 T 10	180	4 T 10	180	4 T 10
45	150	4 T 10	150	4 T 10	180	4 T 10	180	4 T 10
60	150	4 T 10	150	4 T 10	180	4 T 10	200	4 T 10
75	150	4 T 10	150	4 T 10	200	4 T 12	200	4 T 12
90	180	4 T 10	150	4 T 12	200	4 T 12	200	5 T 12
105	180	4 T 10	150	4 T 12	200	4 T 12	200	6 T 12
3. *Beam width* 460 mm								
60	150	4 T 12	150	4 T 12	150	4 T 12	180	4 T 12
75	150	4 T 12	150	4 T 12	180	4 T 12	200	4 T 12
90	150	4 T 12	180	4 T 12	200	4 T 12	200	4 T 16
105	150	4 T 12	180	4 T 12	200	4 T 16	200	4 T 16
120	180	4 T 12	200	4 T 12	200	4 T 16	200	4 T 16
135	180	4 T 12	200	4 T 12	200	4 T 16	200	4 T 16
150	180	4 T 12	200	4 T 16	200	4 T 16	200	4 T 16

Note: (i) A_{st} = Bottom reinforcement; stirrups 6 mm at 300 mm centres; concrete M 20.
(ii) Top steel at pile supports of cast *in-situ* beams too should be equal to A_{st}.

22.18 COMBINED FOOTINGS

There are many types of combined footings that are used in practice (see Fig. 22.19). As the length of these combined footings increases, they can no longer be considered as rigid so that the distribution of foundation pressures assumed in Section 22.4 is no longer valid. Accurate determination of the foundation reactions (or loading of the footings from the ground) can be made only by the various theories of beams and slabs on elastic foundations. Many methods of design which assume full rigidity of the foundation use the theory explained in this Chapter for the approximate design of these structures.

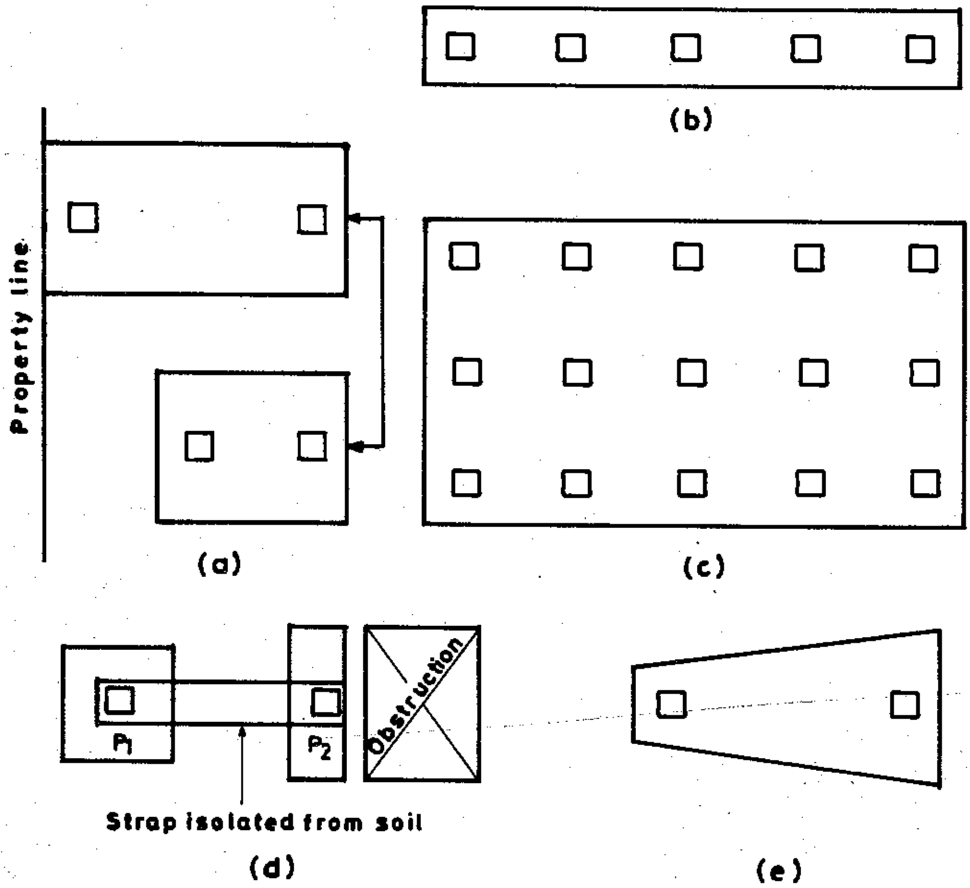


Fig. 22.19 **Types of combined footings: (a) Combined footings, (b) Strip footing, (c) Mat foundation, (d) Strap footing ($P_1 > P_2$), (e) Trapezoidal footing.**

EXAMPLE 22.1 **(Design of plain concrete footing)**

Design a plain concrete footing for a 450 mm wall carrying 300 kN per metre length. Assume grade 20 concrete and the bearing capacity of the soil to be 200 kN/m^2.

Ref.	Step	Calculations	Output
	1.	*Plan area required* $P = 300 +$ self wt. at 10 per cent $= 330$ kN Base $= \dfrac{330}{200} = 1.65$ m	

EXAMPLE 22.1 (cont.)

Ref.	Step	Calculations	Output
	2.	*Angle of dispersion*	
IS 456 33.1.3		Bearing capacity = 20 T/m^2 = 0.2 N/mm^2	
		For $\frac{f_{ck}}{q} = \frac{20}{0.2} = 100$, we have	
		$\tan\alpha = 0.9\sqrt{\frac{100q}{f_{ck}} + 1} = 1.27$	
		$\alpha = 51.8°$	$\alpha = 51.8°$
	3.	*Depth of footing*	
		Length of projection = $\frac{1}{2}(1650 - 450) = 600$ mm	
		$\frac{D}{600} = 1.27$	
		$D = 762$ mm	
		(*Note*: According to IS 456, the angle of dispersion is taken as a function of the ratio f_{ck}/q. Alternatively, some designers assume that the angle has a constant value of 45°.)	
	4.	*Checking of transfer stress*	
IS 456 33.4		Transfer stress = $\frac{300 \times 10^3}{1000 \times 450} = 0.66$ N/mm^2	Width of 1650 mm is safe.
		which is less than	
		$0.45 f_{ck} = 0.45 \times 20 = 9$ N/mm^2	
		Hence it is safe.	

EXAMPLE 22.2 **(Design of square footing)**

A solid footing has to transfer a dead load of 1000 kN and an imposed load of 400 kN from a square column 400 × 400 mm (with 16 mm bars). Assuming $f_y = 415$, and $f_{ck} = 20$ N/mm^2, and safe bearing capacity to be 200 kN/m^2, design the footing.

Ref.	Step	Calculations	Output
	1.	*Required plan area*	
IS 456 35.4		Load = 1.0 DL + 1.0 LL + wt. of footing	
		= 1000 + 400 + 100 (say) = 1500 kN	

EXAMPLE 22.2 (cont.)

Ref.	Step	Calculations	Output
		Area $= \dfrac{1500}{\text{BC}} = \dfrac{1500}{200} = 7.5\ \text{m}^2$	
		Adopt 2.8 m square base of constant depth.	Use 2.8 m square
	2.	*Ultimate soil reaction (only* DL + LL *to be taken)*	
		Design load = 1.5 DL + 1.5 LL	
		= 1.5(1400) = 2100 kN	
		Reaction $= 2100/(2.8)^2 = 268 \approx 270\ \text{kN/m}^2$	
Text Eq. (22.4)	3.	*Depth for one-way shear* (Assuming min. shear = 0.35 N/mm^2)	
		$d = \dfrac{P(L-b)}{2P + 700L^2}$ (in metres)	
		$= \dfrac{2100(2.8 - 0.4)}{2 \times 2100 + 700(2.8)^2} = 0.520\ \text{m} = 520\ \text{mm}$	
	4.	*Depth for two-way shear*	
		IS critical section at $d/2$ from face as in flat slabs.	
		Perimeter $= 4(b + d) = 4(0.4 + 0.52) = 3.68$ m	
		Shear $= 270(2.8^2 - 0.92^2) = 1890$ kN	
		Shear value $= 0.25\sqrt{f_{ck}} = 0.25\sqrt{20} = 1.12\ \text{N/mm}^2$	
		$d = \dfrac{1890 \times 10^3}{1.12 \times 3680} = 459\ \text{mm}$	d = 520 O.K.
Text Eq. (22.5)	5.	*Depth from bending*	
		$M = \dfrac{P(L-b)^2}{8L} = \dfrac{2100(2.8 - 0.4)^2}{8 \times 2.8} = 540\ \text{kNm}$	
		$d = \sqrt{\dfrac{M}{2.76L}} = \sqrt{\dfrac{540 \times 10^6}{2.76 \times 2800}} = 264\ \text{mm}$	d = 520 mm O.K.
		(*Note*: According to BS the critical perimeter is at $1.5d$ from the column face, and the allowable shear is, as in slabs, a function of percentage of steel present.)	

EXAMPLE 22.2 (cont.)

Ref.	Step	Calculations	Output
	6.	*Reinforcement required*	
		$\frac{M}{bd^2} = \frac{540 \times 10^6}{2800 \times (520)^2} = 0.71$	
SP 16 Table 2		Steel percentage = 0.206 (> min. 0.15)	
		$A_s = \frac{0.206 \times 2800 \times 520}{100} = 3000 \text{ mm}^2$	Use 10 of 20 mm (A_s = 3141 mm^2)
	7.	*Check development length (for bond) of main steel*	
SP 16 Table 65		Length from face of column $= \frac{1}{2}(2800 - 400) = 1200$ mm	
		Development length for 20 mm ϕ = 940 (less than 1200)	
	8.	*Distribution of steel* (Cover = 75 mm)	
IS 456 Cl. 25.3.2		Square footing: steel uniformly distributed	
		Spacing $= \frac{2800 - 150\,(\text{cover}) - 20}{9} = 290$ mm	$\frac{(10\text{ T }20)}{290 \text{ mm}^2}$
		Spacing < 3d and 450 mm	
	9.	*Anchorage of compression bars (According to IS)*	
SP 16 Table 65		Anchorage for 16 mm bars = 602 mm	
		Available d = 520 (satisfactory)	
		(This condition is not satisfied in BS 8110)	
	10.	*Total depth* (Concrete cast on ground)	
Text See point 3, Sec. 22.3		Assume cover to steel = (25 + 50) = 75 mm	
		With two layers of steel, D = 520 + 20 + 75 = 615	D = 615 mm
IS 456 Cl. 25.4.2		[*Note*: According to IS 456, with large diameter column bars, the depth for compression anchorage will be large. However, in BS 8110 the compression bond does not require checking, if the starter bars extend to the bottom of the footing.]	

EXAMPLE 22.3 **(Design of dowels in column)**

A square footing 3 × 3 m and 60 cm thickness supports a 450 × 450 mm column; f_y = 415 and f_{ck} = 25 N/mm^2.

Design the dowels (starter bars) between the column and the footings if the characteristic loads at the base of column are 780 kN live load and 1000 kN DL

Ref.	Step	Calculations	Output
	1.	*Max. load on base of column*	
		Design load = 1.5(DL + LL)	
		= 1.5(780 + 1000) = 2670 kN	
	2.	*Max. allowable stress on top of the footing*	
IS 456 Cl. 33.4		Permissible bearing = $0.45f_{ck}$	
		$= 0.45 \times 25 = 11.25$ N/mm^2	
		Increase the value due to increased area	
		$\sqrt{\dfrac{A_1}{A_2}} = \left(\dfrac{3000}{450}\right) > 2.0$	
		Limiting value = $2 \times 11.25 = 22.5$ N/mm^2	
	3.	*Load taken by concrete*	
		$P_c = 22.5 \times 450 \times 450 = 4556$ kN	
	4.	*Area of dowel steel*	
IS 456 Cl. 33.4.3		The entire column load can be transferred by the concrete above. However, codes and field practice require a minimum dowel area of 0.5% of area of column.	
		$A_s = 0.005(450)^2 = 1012$ mm^2	4 T 20 / 1257 mm^2
		Provide 4 Nos. of 20 mm dia.	
		Notes: (i) The general practice is to provide dowel bars of the same diameter as the column bars. (ii) IS and ACI require that the development length of dowels into the footing be checked. BS 8110 does not require this checking if the other requirements for the thickness of footing (BM and shear) are satisfied. However, the development length of bars inside the column should be satisfied in all cases.	

EXAMPLE 22.4 **(Design of rectangular footing)**

Design a footing for a 500 × 350 mm column using 20 mm bars as dowels to transmit characteristic loads of 600 kN as dead load and 400 kN as live load to a foundation with safe bearing capacity of 120 kN/m^2. Assume grade 20 concrete and Fe 415 steel.

Ref.	Step	Calculations	Output
	1.	*Plan area of footing*	
		A = characteristic load/safe BC	$a = 500$
		$= \dfrac{600 + 400 + 100\,(\text{wt. of footing})}{120}$	$b = 350$
		$= 9.17 \text{ m}^2$	
		$\dfrac{a}{b}$ ratio of column $= \dfrac{500}{350} = 1.42$	
		$1.42B^2 = 9.17, \quad B = 2.55, \quad L = 3.60$	$L = 3600$
		Area provided = 9.18 m^2	$B = 2550$
	2.	*Ultimate soil reaction*	
		$p = \dfrac{1.5(\text{DL} + \text{LL})}{\text{area}}$	
		$= \dfrac{1.5(600 + 400)}{9.18} = 163 \text{ kN/m}^2$	Assume block footing
	3.	*Depth from one-way shear*	
Text Fig. 22.6(b)		For max. V, take section along the breadth in the YY-direction at a distance d from the column.	
		$V = 163 \times B\left(\dfrac{L - a - 2d}{2}\right)$	
		$= 163 \times 2.55\left(\dfrac{3.60 - 0.5 - 2d}{2}\right)$	
		$= \tau_c \times 2.8 \times d$	
IS 456 Table 13		Assume $\tau_{c\,(\min)}$ (for $f_{ck} = 20$) = 0.36 N/mm^2	$\tau_c = 360$ kN/m^2
		$d(720 \times 2.8 + 163 \times 2.55 \times 2) = 163 \times 2.55 \times 3.1$	
		$d = 0.45$ m	$d = 450$ mm
		(Check section in the XX-direction also)	
IS 456 Cl. 30.6.3.1	4.	*Check depth for two-way shear*	
		Shear strength $= 0.25\sqrt{f_{ck}}$	
		$= 0.25\sqrt{20} = 1.12 \text{ N/mm}^2$	
		As $b > 1/2a$, no correction is needed.	

EXAMPLE 22.4 (cont.)

Ref.	Step	Calculations	Output
		Taking section at $d/2$ around column, we get	
		$V = 163[9.18 - (a + d)(b + d)]$	
		$= 163[9.18 - 0.95 \times 0.80] = 1372$ kN	
		$\tau_p = \dfrac{1372}{2(a + d + b + d)d}$	
		$= \dfrac{1372 \times 10^3}{2(1750) \times 450} = 0.87 < 1.11\ \text{N/mm}^2$	τ_p O.K.
Text Fig. 22.6(b)	5.	*Depth from bending*	
		Section YY	
		$M_{\text{long}} = \dfrac{P}{LB} B \left(\dfrac{L-a}{2}\right)\left(\dfrac{L-a}{4}\right) = \dfrac{P(L-a)^2}{8L}$	
		$= \dfrac{1500(3.6 - 0.5)^2}{8 \times 3.6} = 500$ kNm	
Text Eq. (22.9)		*Section XX*	
		$M_{\text{short}} = \dfrac{P(B-b)^2}{8B} = \dfrac{1500(2.55 - 0.35)^2}{8 \times 2.55}$	
		$= 355$ kNm	
	6.	*Reinforcement required*	
		Longitudinal direction	
		For A_{sl},	
SP 16 Table 2		$\dfrac{M}{bd^2} = \dfrac{500 \times 10^6}{2550 \times (450)^2} = 0.96, \quad p = 0.282,$	
		$A_s = \dfrac{0.282 \times 2550 \times 450}{100} = 3236\ \text{mm}^2$	$\dfrac{16\ \text{T}\ 16}{3217\ \text{mm}^2}$
		Short direction	
		For A_{sb},	
		$\dfrac{M}{bd^2} = \dfrac{355 \times 10^6}{3600(450)^2} = 0.49, \quad p = 0.143,$	
		$A_s = \dfrac{0.143 \times 3600 \times 450}{100} = 2317\ \text{mm}^2$	$\dfrac{12\ \text{T}\ 16}{2412\ \text{mm}^2}$

EXAMPLE 22.4 (cont.)

Ref.	Step	Calculations	Output
	7.	*Development length in short direction*	
SP 16 Table 65		16 mm rods = 752 mm	
		$1/2(B-b) = \frac{1}{2}(2550-350) = 1100$ mm	
		$1100 - 2(\text{cover}) = 1020 \text{ mm} > L_d$	L_d O.K.
	8.	*Placing of steel*	
IS 456 Cl. 33.3.1		(a) Reinforcement in long direction is placed at uniform spacing.	
		$\text{Spacing} = \dfrac{2550 - 2(40) - 16}{15} = 163 \text{ mm}$	
		(b) For reinforcement in short direction,	
		$\beta = \dfrac{3.6}{2.55} = 1.41$	
		$\dfrac{2}{\beta + 1} = \dfrac{2}{2.41} = 0.83$	
		As this percentage is high, place the steel uniformly on the shorter side also.	
	9.	*Transfer of load to base of column*	
		Capacity = $(0.45 f_{ck})$ 2(area)	
		$= 0.45 \times 20 \times 2 \times 500 \times 350$	
		$= 3150 \text{ kN} > 1500 \text{ kN}$	
IS 456 Cl. 33.4.3		Hence dowels are not theoretically needed, but at least four rods (equal to 0.5 per cent area of column) are extended to the footing.	
IS 456 Cl. 33.4.2		*Note*: If 20 mm bars are needed as dowels to transfer the load, then according to IS, the depth of footing should be equal to the development length.	
		L_d of 20 mm bars = 752 mm	
		This depth is too large for a footing of constant depth. In such cases use a stepped footing or a sloped footing to reduce the amount of concrete. Hence use a pedestal 350 mm high around the column with an offset of 200 mm around the column. Pedestal will be $(350 + 400) \times (550 + 400)$ in plan. The rest of the footing will be stepped and will be of constant depth of $750 - 350 = 400$ mm. However, the provision that the depth of footing should satisfy the development is not specified in BS 8110.	

EXAMPLE 22.5 **(Design of pedestals)**

Design a concrete pedestal for supporting a steel column carrying a total factored load of 1700 kN. The size of the base plate is 300 mm square. Assume grade 25 concrete and Fe 415 steel.

Ref.	Step	Calculations	Output
		Case (1): Design for unreinforced pedestal	
	1.	*Size of pedestal*	
		Bearing strength $f_{cb} = 0.45 f_{ck}$	
IS 456 Cl. 33.4		$= 0.45 \times 25 = 11.25$ N/mm^2	
		Max. allowed strength $= 2 \times 11.25 = 22.5$ N/mm^2	
		Pressure on base plate $= \dfrac{1700 \times 10^3}{300 \times 300} = 19$ N/mm^2	
		Min. size ($L \times L$) to carry this pressure is given by the condition	
		$11.25\left(\dfrac{L}{300}\right) = 19, \quad L = 507$ mm	Pedestal size 510 × 510 mm
		Choose pedestal size 510 × 510 mm.	
	2.	*Provision of steel*	
		Theoretically, the above pedestal need not be reinforced. To avoid brittle failure, 0.4 per cent is usually provided.	
		$A_s = \dfrac{0.4 \times 510 \times 510}{100} = 1040$ mm^2	$\dfrac{4\text{ T }20}{1256\text{ mm}^2}$
		Provide 4 Nos. of 20 mm rods and the usual laterals.	
		Case (2): Design as a reinforced pedestal	
		Adopt minimum size for pedestal (10 mm clearance) = 310 × 310 mm	
IS 456 Cl. 33.4		Safe pressure = 11.25(3.1/3.0) = 11.63	
		Load carried by pedestal is equal to	
		$11.63 \times 310 \times 310 \times 10^{-3} = 1117$ kN	
		Balance load = 1700 − 1117 = 583 kN	
		A_s required $= \dfrac{583 \times 10^3}{0.87 \times 415} = 1614$ mm^2.	$\dfrac{4\text{ T }25}{1963\text{ mm}^2}$
		Percentage of steel $= \dfrac{1614 \times 100}{310 \times 310} = 1.68\%$	
		This is greater than the minimum, i.e. 0.4%.	

EXAMPLE 22.6 **(Design of sloped footing)**

Design a sloped square footing for a circular column 500 mm in diameter and intended to carry a characteristic load of 1000 kN. The safe bearing capacity of the soil is 200 kN/m^2. Assume that grade 15 concrete and Fe 415 steel are used for the construction.

Ref.	Step	Calculations	Output
	1.	*Size of footing required* (self wt. 10%) $\sqrt{A} = \left(\dfrac{1000 + 100}{200}\right)^{1/2} = 2.34$ m Adopt 2.4 × 2.4 m footing.	
IS 456 Cl. 33.2.2	2.	*Size of equivalent square* $\dfrac{500}{\sqrt{2}} = 354$ mm Provide a square ledge 600 × 600 around the column. Calculations are made on column size 354 × 354 with a ledge 600 × 600.	
	3.	*Ultimate loads from column* Factored load = 1.5 × 1000 = 1500 kN Upward pressure $= \dfrac{1500 \times 10^3}{2400 \times 2400} = 0.26$ N/mm^2	
Text Sec. 22.11 SP 16 Table C IS 456 Cl. 31.1.2	4.	*Depth from moment consideration* Taking moment about face of column For a conservative estimate of (d), $M_{xx} = \dfrac{P}{8L}(B - b)^2$ $= \dfrac{1500 \times 10^3}{8 \times 2400}(2400 - 354)^2 = 327 \times 10^6$ N/mm^2 Breadth resisting moment = 600 mm $M_u = 0.138 f_{ck} b d^2$ for Fe 415 $d = \sqrt{\dfrac{327 \times 10^6}{0.138 \times 15 \times 600}} = 513$ mm Provide effective depth of 600 mm at column face and 250 mm at the end. Provide cover to steel of 75 mm. This is a conservative estimate of d.	$d = 513$ mm

 SAGAR SAMARIA.

EXAMPLE 22.6 (cont.)

Ref.	Step	Calculations	Output
IS 456 Cl. 33.2.4.1(a)	5.	*Check depth from one-way shear* Take section at effective depth from the face of column (d = 600). Distance of section from edge of footing is equal to $\left(\frac{2400-354}{2}\right) - 600 = 1023 - 600 = 423$ mm Breadth of column face at this section with 45° diagonal $b_1 = 2400 - 2 \times 423 = 1554$ mm^2 $V_1 = \left(\frac{2400+1554}{2}\right)(423)(0.26)$ N $= 217$ kN Effective depth of section, d_1 $d_1 = 250 + \frac{(600-250)\times 423}{(2400-600)\times 0.5} = 414$ mm	
IS 456 Table 13		$\tau_{c(\text{reqd.})} = \frac{217\times 10^3}{1554 \times 414} = 0.33$ N/mm^2 Less than τ_c for grade 15 concrete with nominal steel.	Safe in one-way shear
IS 456 Cl. 33.2.4.1(b)	6.	*Check for two-way shear* Section at $d/2$ from face of column $d/2 = \left(\frac{600}{2}\right) = 300$ mm from column face Distance of section from edge of footing is equal to $\left(\frac{2400-354}{2}\right) - 300 = 723$ mm $b_2 = 2400 - 2 \times 723 = 954$ mm $d_2 = 250 + \frac{(600-250)\times 723}{900} = 531$ $V_2 = 0.26[(2400)^2 - (954)^2] = 1261$ kN	
IS 456 Cl. 30.6.3.2		$\tau_p = \frac{1261 \times 10^3}{4 \times 954 \times 531} = 0.62$ N/mm^2 $0.25\sqrt{f_{ck}} = 0.25\sqrt{15} = 0.97$ N/mm^2	Safe in two-way shear

EXAMPLE 22.6 (cont.)

Ref.	Step	Calculations	Output
	7.	*Area of steel required*	
		The effective depth d was calculated on b = 600. The steel area is calculated as follows:	
Section 22.11.1		(a) According to conventional method, A_s is calculated from M obtained from step 4 above.	
		$\frac{M}{bd^2} = \frac{327 \times 10^6}{600 \times (600)^2} = 1.51$	
		$p = 0.484$	
		$A_s = \frac{0.484}{100}(600 \times 600) = 1742$	$\frac{16\text{ T }12}{1809}$ (BW)
		(b) A more economical solution is to take moments of the pressures inside the diagonals	
		$M_x = \frac{P}{24LB}(2L + a)(B - b)^2$	
		$= \frac{0.26}{24}(4800 + 354)(2400 - 354)^2$	
		$= 234$ kNm	
SP 16 Table 1		$\frac{M}{bd^2} = \frac{234 \times 10^6}{600(600)^2} = 1.08$	
		$p = 0.329$ (more than minimum)	
		$A_s = \frac{0.329}{100} \times 600 \times 600 = 1185\ \text{mm}^2$	
		Alternative method	
Text Table 4.5		$\frac{M}{f_{ck}bd^2} = \frac{234 \times 10^6}{15 \times 600(600)^2} = 0.07$	
		LA factor = 0.9 (approx.)	
		$A_s = \frac{234 \times 10^6}{0.87 \times 415 \times 0.9 \times 600} = 1200\ \text{mm}^2$	$\frac{11\text{ T }12}{1244}$ (BW)
	8.	*Distribution of steel—Check for spacing*	
		Distribute total steel uniformly as per IS. Hence,	
IS 456 Cl. 25.3.2		Spacing $= \frac{[2400 - (75 \times 2)]}{10} = 225$ mm The spacing is less than 450 and 3d.	75 mm side cover Spacing O.K.

EXAMPLE 22.6 (cont.)

Ref.	Step	Calculations	Output
	9.	*Check for development length*	
SP 16 Table 65		L_d of 12 mm bars = 677 mm	
		Length available $= \dfrac{2400 - 600}{2} = 900$ mm	
		This is just sufficient and, if necessary, the bars can be given a standard bend upwards at the ends.	L_d O.K.
	10.	*Overall dimensions of footing*	
		Provide cover to effective depth of 75 mm	
		$L = B = 2400$ mm	
		Footing depth varies from (250 + 75) at edge to (600 + 75) at the column face.	
		[*Note*: Step 7 may be modified by calculating A_s on full breadth and mean depth. Thus, mean depth considering the whole width is equal to	
		$\left[600(600) + (2400 - 600)\dfrac{(600+250)}{2}\Big/2400\right] = 469$ mm]	
SP 16 Table 1		$\dfrac{M}{bd^2} = \dfrac{234 \times 10^6}{2400(469)^2} = 0.443, \quad p = 0.13\%$	
		$A_s = \dfrac{0.13 \times 2400 \times 469}{100} = 1460$	$\dfrac{13\text{ T }12}{(1470\text{ mm}^2)}$
		Note: The value is between that obtained by procedures (a) and (b) in step 7.	

EXAMPLE 22.7 **(Design of under-reamed pile foundation)**

The main brickwall of a room of a residential building is 225 mm thick and has a loading of 40 kN/m at foundation level. Another crosswall of the same thickness joins it and transmits a concentrated load of 35 kN. Design a layout of under-reamed piles and grade beam for the foundation of the main wall.

Ref.	Step	Calculations	Output
	1.	*Layout of foundation*	
		Place one pile (P_1) at junction of walls. Piles (P_2) at 2 m spacing along the walls.	
	2.	*Loads on piles and grade beams*	
		Load on P_1 = (40 × 2) + 35 = 115 kN	
		Load on P_2 = 40 × 2 = 80 kN	
		Load on grade beam = 40 kN/m	

EXAMPLE 22.7 (cont.)

Ref.	Step	Calculations	Output
Text Table 21.1	3.	*Design of piles* Piles of 200 mm, single under-reamed has a capacity of 80 kN. Adopt for P_2. Piles of 200 mm, double under-reamed has a capacity of 120 kN. Adopt for P_1 (pile at the junction).	From Table 22.1, d = 200 mm D = 500 mm A_s = 3 R 10 Rings R_6 at 180
Text Table 22.2	4.	*Design of grade beam* Load on grade beam = 40 kN/m Effective span = 2000 – 200 = 1800 mm Adopt 150 mm deep beam with 4 Nos. of 10 mm at bottom and the same at the top.	From Table 22.2 Depth = 150 mm 4 R 10 at T + B links 6 mm at 300
	5.	*Always check bearing capacity of soil* Diameter of under-ream = 2.5d $= 2.5 \times 200 = 500$ mm Bearing area of pile $= \dfrac{\pi (0.5)^2}{4} = 0.196\ \text{m}^2$ Expected bearing capacity at 3 m depth is equal to $\dfrac{P}{A} = \dfrac{80}{0.196} = 408\ \text{kN/m}^2$	Bearing capacity of soil at 3 m depth to be 40 tons/m^2

EXAMPLE 22.8 **(Design of pile caps using truss theory)**

A column 550 mm square has to carry a factored (design) load of 2600 kN to be supported on 4 piles each of 450 mm diameter and spaced at 1350 mm centres. Design a suitable pile cap assuming f_{ck} = 25 N/mm^2 and f_y = 415 N/mm^2.

Ref.	Step	Calculations	Output
Text Fig. 22.9	1.	*Arrangement of pile cap* With extension of pile cap equal to 150 mm on all sides from outer side of piles, Size of pile cap = 1350 + 450 + 300 = 2100 mm	
Text Sec. 22.16.2	2.	*Depth of pile cap* Empirical formula = $2h_p$ + 100 = 1000 mm (too high)	

EXAMPLE 22.8 (cont.)

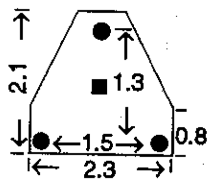
Ref.	Step	Calculations	Output
		Adopt effective depth d = 1/2 spacing of pile $= \dfrac{1350}{2}$ = 675 mm and 75 mm cover. Assuming 20 mm rods, $D = 675 + 75 + 10 = 760$ mm	
Text Fig. 22.13	3.	*Check for truss action* Shear span = a_v $a_v = 675 - 275 - 225 + \dfrac{450}{5} = 265$ mm $\dfrac{d}{a_v} = \dfrac{675}{265} = 2.55$ Truss action exists.	
Text Fig. 22.9	4.	*Tension steel* $T = \dfrac{P}{24Ld}(3L^2 - b^2)$ $= \dfrac{2600}{24 \times 1350 \times 675}(3(1350)^2 - 550^2)$ = 614 kN $A_s = \dfrac{614 \times 10^3}{0.87 \times 415} = 1701 \text{ mm}^2$ Provide 16 T 12, giving 1809 mm^2	Provide steel ~~16~~ 12 T ~~12~~ each way. 16
	5.	*Percentage of steel provided*	
IS 456 Cl. 25.5.1.1		$p = \dfrac{1809 \times 100}{2100 \times 675} = 0.127\% > 0.12\%$ (This is less than the 0.33% steel usually provided according to ACI for beams in which bending action is predominant.)	
	6.	*Check for shear* (f_{ck} = 25)	
Text page 101		$\tau_c' = \tau_c\left(\dfrac{2d}{a_v}\right) = \dfrac{0.36 \times 2 \times 675}{265}$ $= 1.83 < 3.1 \text{ N/mm}^2$	
IS 456 Table 13, 14		Shear $= \left(\dfrac{V}{2100 \times 675}\right) = \dfrac{2600 \times 1000}{2(2100 \times 675)}$ $= 0.92 \text{ N/mm}^2 < \tau_c' = 1.83 \text{ N/mm}^2$	

EXAMPLE 22.8 (cont.)

Ref.	Step	Calculations	Output
	7.	*Detail steel* Detail steel according to standard practice. [*Note*: If the depth of pile cap provided is less than that given for truss action, then the pile cap is designed as a beam and checked for bending and shear.]	

EXAMPLE 22.9 **(Design of pile cap using bending theory)**

Design a pile cap for a system of 3 piles supporting a column 500 mm square and carrying an axial load of 600 kN. Assume that the diameter of the pile is 400 mm f_{ck} = 20 N/mm^2, f_y = 415 N/mm^2.

Ref.	Step	Calculations	Output
Fig. 22.9	1.	*Layout of pile cap* Assume standard layout with tolerance of 200 mm for cap. Spacing of pile $\nless 2h_p = 3 \times 400 = 1200$ mm Adopt 1.5 m; Assume D = 600 mm Bending theory becomes applicable when the depth to be provided is small, $d = 600 - 75 - \frac{16}{2} = 517$ mm	s = 1.5 m d = 517 mm
	2.	*Loads on pile cap* Volume of pile cap = plan area × depth Base length = 1.5 + 0.4 + 0.2 + 0.2 = 2.3 m $\text{Height} = \left(\frac{1.5 \times \sqrt{3}}{2}\right) + 0.4 + 0.4$ $= 1.3 + 0.8 = 2.1$ m $\text{Plan area} = (2.3 \times 2.1) - (2.1 - 0.8)\left(\frac{2.3 - 0.8}{2}\right)$ $= 3.86$ sq.m Wt. of pile cap = 3.86 × 0.6 × 24 = 55.5 kN Assume wt. as concentrated load at CG. Load on each pile $= \frac{1}{3}(600 + 56) = 219$ kN Factored load = 1.5 × 219 ≈ 328 kN	 **Fig. E.22.9**

Sagar Samanta

EXAMPLE 22.9 (cont.)

Ref.	Step	Calculations	Output
	3.	*Transfer of load*	
		Let the load be first transferred in the *YY*-direction and then the *XX*-direction.	
Fig. 22.9		According to IS 2911, part I: clause 5.12.1, loads and reactions can be dispersed to mid-depth of pile cap. Taking loads as concentrated is a conservative estimate.	
	4.	*Bending in the YY-direction*	Assumed simply supported
		$M_1 = 328 \times \frac{2}{3} \times 1.3 = 284$ kNm	$b = 800$ mm $f_{ck} = 20$
		$d_1 = \sqrt{\frac{284 \times 10^6}{0.138 \times 800 \times 20}} = 359 < 517$	
SP 16 Table 2		$\frac{M_1}{bd^2} = \frac{284 \times 10^6}{800 \times 517 \times 517} = 1.33, \quad p = 0.4$	
		$A_s = \frac{0.4 \times 800 \times 517}{100} = 1654 \text{ mm}^2$	Use 8 T 16 1608 mm²
	5.	*Shear in the YY-direction*	
		$v = \frac{328 \times 10^3}{800 \times 517} = 0.79 \text{ N/mm}^2$	
IS 456 Table 13		τ_c for 0.4% steel = 0.39	
		$\tau_c' = \tau_c \left(\frac{2d}{a_v}\right)$	
Fig. 22.13		$a_v = \frac{2}{3}(1.3) - 0.2 - 0.25 + \frac{0.40}{5} = 0.5$	Safe in shear without steel
		$\tau_c' = \frac{0.39 \times 2 \times 0.57}{0.42} = 0.8 \text{ N/mm}$	
	6.	*Bending in the XX-direction*	
		$M_2 = 328 \times \frac{1.5}{2} = 246 \text{ kNm} < M_1$	Use 8 T 16
	7.	*Shear in the XX-direction*	
Fig. 22.13		$a_v = \frac{1.5}{3} - 0.2 - 0.25 + 0.08 = 0.38$	
		$\tau_c' = 0.39 \left(\frac{2 \times 0.517}{0.38}\right) = 1.06 \text{ N/mm}^2$	Shear O.K.

EXAMPLE 22.9 (cont.)

Ref.	Step	Calculations	Output
SP 34 Fig. 6.12	8.	*Arrangement of steel* (a) Arrange main steel as beams in the *YY* and *XX* directions. (b) Provide 5 Nos. of 10 mm rods as circumferential steel around the projected pile reinforcements which extend to top of pile cap. [*Note*: Two-way shear also may have to be considered at $d/2$ from column face where such action can develop, as in the case of a pile cap over a large number of piles. For this example, such action may not develop and hence it is not considered.]	

EXAMPLE 22.10 **(Design of pile cap with truss and bending action)**

Design a pile cap for a group of piles consisting of 6 piles of 350 mm diameter to support a 450 mm square column carrying a factored load of 280 tons. Assume f_{ck} = 25 N/mm^2 and f_y = 415 N/mm^2.

Ref.	Step	Calculations	Output
	1.	*Layout of pile* (6 pile group)	
Fig. 22.10		Spacing $\nless 3h_p = 3 \times 350 = 1050$	s = 1.1 m
		Using a close tolerance of 100 mm all around	
		L of cap = 2.2 + 0.35 + 0.20 = 2.75 m	
		B of cap = 1.1 + 0.35 + 0.20 = 1.65 m	d = 750 mm
		$D \nless 2h_p + 100 = (2 \times 350) + 100 = 800$ mm	Cover = 40 mm
		$d = 800 - 40 - \frac{20}{2} = 750$ mm	Over base concrete
	2.	*Load on pile*	
		(γ_m) DL = 1.5(2.75 × 1.65 × 0.8 × 24) = 130 kN	
		Load on each pile = (2800 + 130)/6 = 488 kN	
	3.	*Transfer of load* Assume the following: Transfer to middle pile by truss action Transfer to end pile by cantilever bending	
	4.	*Bending in the XX-direction as cantilever*	
		$M = 2 \times 488 \times 1.1 = 1074$ kNm	
SP 16 Table 3		$d = \left(\frac{1074 \times 10^6}{0.138 \times 1650 \times 25}\right)^{1/2} = 434$ mm	Depth O.K.
		$\frac{M}{bd^2} = \frac{1074 \times 10^6}{1650 \times 750 \times 750} = 1.15, \; p = 0.39$	

EXAMPLE 22.10 (cont.)

Ref.	Step	Calculations	Output
		$A_s = (0.39 \times 1650 \times 750)/100 = 4826\ \text{mm}^2$	Use 16 T 20 $5026\ \text{mm}^2$
		Actual $p = \dfrac{5026 \times 100}{1650 \times 750} = 0.40$	
	5.	*Shear along xx (Section yy)*	
SP 16 Table 61		$v = \dfrac{2 \times 488 \times 10^3}{1650 \times 750} = 0.79\ \text{N/mm}^2$ τ_c for 0.4 per cent steel $= 0.45\ \text{N/mm}^2$	
Fig. 22.13		$a_v = 1100 - \dfrac{1}{2}(450 + 350) + \dfrac{350}{5} = 770\ \text{mm}$ $\tau_c' = \tau_c\, 2\left(\dfrac{d}{a_v}\right) = 0.45 \times 2 \times \dfrac{750}{770} = 0.88\ \text{N/mm}^2$	Shear O.K. without steel
	6.	*Steel in the YY-direction*	
Fig. 22.9		Tension for truss action over central two piles $T = \dfrac{P(3L^2 - a^2)}{12Ld}$ $L = 2.2, \quad a = 0.45, \quad d = 0.75$ $T = \dfrac{(2 \times 488)(14.32)}{12 \times 2.2 \times 0.75} = 705.9\ \text{kN}$	
		$A_s = \dfrac{706 \times 10^3}{0.87 \times 415} = 1955\ \text{mm}^2$	Use 4 T 25 $1963\ \text{mm}^2$
		Provide (a) 4 T 25 over central piles 2 T 20 over outer piles 2 T 20 in space in-between. (Total number 10 rods. Maximum spacings less than that allowed for distribution steel)	
	7.	*Arrangement of steel*	
Fig. 22.15		(a) Main steel (16 Nos. 20 mm—ends bent up to full depth) in the *XX*-direction along the length	
Fig. 22.16		(b) Steel in the *YY*-direction along the breadth, as in step 6. (c) Horizontal ties T 12 at 300 mm (d) Starter bars, pile bars, links etc. as per standard specifications.	
	8.	*Punching shear.* This may be checked at $d/2$ from column edge.	

REVIEW QUESTIONS

22.1 In limit state design, to determine the area of an isolated footing for a given column load and safe bearing capacity of soil, should we use the factored load or the service load as the basis of calculation ?

22.2 Having determined the area of the footing, should be use the factored load or the service load for design of the reinforced concrete details of the footing ? Then how will the soil reaction from below compare with the safe bearing capacity ?

22.3 When can a foundation be considered as rigid ? What will be the difference in the action if the footing is flexible as in the case of a thin concrete footing on rock foundation ?

22.4 What are one-way and two-way shears in footings ? What is the difference between the methods of checking for two-way shear or punching shear in IS (ACI) and BS codes ?

22.5 Sketch the placement of steel in (a) square footing, and (b) rectangular footing with a non-central load. What is the amount of minimum steel prescribed for footings according to IS 456 ? What consideration should be given in choosing the diameter of the reinforcements used in isolated footings ? How can the development length of these bars be increased ?

22.6 When will you use a plain concrete footing ? Discuss the formula given in IS 456 for the dispersion of load in plain footings.

22.7 Theoretically is it necessary to continue all the steel in a reinforced concrete column into a footing also ? What are the criteria to be considered, and what is the prescribed minimum necessary ?

22.8 Explain why pedestals are essential in the layout of a building with steel columns. What are the considerations for determining the dimensions of a pedestal ?

22.9 Explain the two theories usually assumed, showing as to how the column load is carried to the piles through pile caps. Under what conditions would you use each of the theories ? If it is prescribed that the depth of the pile cap should be minimum, which of the theories will be applicable ?

22.10 Sketch the details of reinforcements for a pile cap over a group of four piles, indicating the different shapes of reinforcements that will be incorporated in the pile cap.

PROBLEMS

22.1 A brick wall 300 mm thick is used for a double storey (ground and first floor) building 4 m high from the foundation to the ground floor and 3 m high from the first floor to the roof. Assuming the rooms to be 4 m square, calculate the load on the interior wall and sketch a suitable layout of a stepped footing of bricks and plain cement concrete (PCC) for the foundation. Assume a safe bearing capacity of 100 kN/m^2. (The interior walls carry loads from both sides of the wall.)

22.2 Design a suitable reinforced concrete continuous footing for the above wall assuming that the wall starts from the foundation with a thickness of 300 mm without any stepping. Sketch the details of the steel to be provided. Use grade 15 concrete and Fe 415 steel.

22.3 A square footing 3.5 m × 3.5 m is used for a square column 300 × 300 mm carrying a total ultimate load of 1500 kN (150 t). Safe bearing capacity of the soil is 100 kN/m^2. Using grade 20 concrete and Fe 415 steel, design the footing (a) as a pad footing (constant depth); (b) as a sloped footing.

Give reasons and indicate which of these you would recommend for construction.

22.4 Design a pad footing (footing of constant depth) for a rectangular column 300 × 450 mm carrying an axial factored load of 1500 kN. The safe bearing capacity of the soil is 120 kN/m^2. Use M 20 concrete and grade 415 steel.

22.5 A 400 mm square column is to carry an axial characteristic load of 1000 kN resting on 2 piles of 20 cm diameter, each having more than 500 kN capacity. Design a suitable pile cap using M 20 concrete and, Fe 415 steel (a) employing the truss theory, and (b) employing the beam theory.
Explain which of the two theories is more applicable to this design.

22.6 The outer brick walls of a double storey (ground and first floor) residential building with 4 m square rooms is 300 mm thick. The height of the lower storey wall is 4 m from foundation level and that of the first storey wall is 3 m. The walls have to carry the self weight as well as the weight of the loads from the first floor and the roof.

1. Estimate the loads on the outer wall.

2. Using double under-reamed piles of 250 mm diameter with safe load capacity of 9 tons and grade beams, design a suitable foundation system for the outer walls.

22.7 Design a suitable under-reamed pile and grade beam for the inner walls of Problem 22.6 using 250 mm piles with safe load capacity of 9 tons.

Appen

Workin

A.1 INTR

In olden day
experience.
structural ele
only after th
beam by ela
the followin

where

For de
the above e

This formul

Hence, by s
a specified
used for ela
value of M
$M = fZ$. A s
tables.

Durin
reinforced c
used for hon
of the conc
concrete wa

The th
name "mod
referred als

Appendix A

Working Stress Method of Design

A.1 INTRODUCTION

In olden days, sizes of structural members were adopted by master builders from thumb rules and experience. The practice of 'design of structures' by which one arrives at the dimensions of the structural elements after theoretical calculation is of recent origin. It was adopted by civil engineers only after the elastic theories of stress analysis were developed. The analysis of a homogeneous beam by elastic theory of bending was first proposed by Bernoulli at about 1750 A.D. He derived the following classical formula for bending of beams:

$$M = f(I/y) = fZ$$

where

$$f = \text{maximum allowable working stress}$$

$$Z = \text{modulus of the section}$$

For design of a rectangular homogeneous beam like a wooden beam of breadth b and depth d, the above expression for M becomes

$$M = f\frac{bd^2}{6} = 0.167fbd^2$$

This formula can also be expressed by the general formula

$$M = Qbd^2$$

Hence, by specifying the maximum allowable stress in a material, the dimensions required to carry a specified bending moment can be easily calculated by assuming the b/d ratio. This method is still used for elastic design of timber and steel sections. Thus, in the design of steel beams for a given value of M, a safe value of f is first fixed and the value of Z is calculated from the formula $M = fZ$. A suitable section with the required value of Z can then be chosen from the standard steel tables.

During the evolution of the theory of design of beams of non-homogeneous sections like reinforced concrete also, the early attempts were directed at evolving a formula similar to the one used for homogeneous sections like wood or steel. This was accomplished in 1890 by the introduction of the concept of modular ratio. Complete elastic behaviour without creep, shrinkage, etc. of concrete was assumed in these methods.

The theory of elastic design of R.C. member by using the above theory is also known by the name "modular ratio method". As the design is based on safe working stress, it is sometimes referred also as the working stress method to differentiate it from the limit state method.

IS 456 (1978): Section 6 deals with this method. The present Indian code allows R.C. members to be continued to be designed by this method even though the codes of most developed countries have adopted the ultimate limit state method. It is still, however, the only method available to estimate the state of stress of concrete and steel at working loads. The elastic theory with some corrections is always useful whenever calculations are made for serviceability limit states, as for example, the calculation of the neutral axis for estimation of crack-width. Hence, familiarity with the elastic method is needed even when dealing with limit state design of beams. The basis of the working stress method for simple beams is, therefore, spelt out in the following Sections.

A.2 DESIGN FOR BENDING

The two fundamental concepts established from early days for the elastic theory of design of R.C. beams were those of modular ratio $m = E_s/E_c$, and the importance of perfect bond between steel and concrete. Bond ensures that there is no slip between steel and concrete during bending so that Bernoulli's assumption that plane sections remain plane even after bending can be assumed to be true in R.C.C. beams also.

As already stated, the elastic theory also gives a method of estimating the concrete and steel stresses in the section. The steel stress is taken as equal to modular ratio times the theoretical stress of concrete at the level of the steel. It becomes $\sigma_{st} = m\sigma_c$ in tension steel and $\sigma_{sc} = 1.5m\sigma_c$ in the compression steel as shown in Section A3.1.

The value of E_s for steel in the elastic stage is a constant equal to $(200)10^3$ N/mm^2 as obtained by laboratory tests. But the value of E_c is assumed to vary with the strength of concrete. In order to take indirectly into account the effect of shrinkage and creep in concrete, the value of E_c used is not the value obtained from short term loading tests (which can be taken as $E_c = 5700\sqrt{f_{ck}}$), but a value entirely different from these test values. Hence, the value of the modular ratio m is to be calculated (according to IS 456: clause 43.3), not from (E_s/E_c) obtained from tests, but from the expression

$$m = (280/f_{ck}) = (280/(3\sigma_{cb})) \tag{A.1}$$

where

f_{ck} = the characteristic strength of concrete

σ_{cb} = maximum permissible compressive stress in concrete

The assumptions made in the working stress method are given in IS 456: clause 43.3.

A.3 DESIGN PROCEDURE

There are two procedures for design of bending members by working stress method. They are (a) the transformed area method, (b) the formulae method.

The first method is a general method applicable to all shapes of sections, and the second method is usually applicable only in rectangular or other regular sections.

A.3.1 TRANSFORMED AREA METHOD

Following the classical theory of elasticity, when dealing with two materials of different moduli of elasticity, in this method, the steel area is transformed into its equivalent concrete area.

Thus, assuming that concrete does not absorb tension, the equivalent area of tension steel = mA_s, where A_s = area of tension steel.

As concrete takes compression and is subjected to creep and shrinkage, the equivalent area of steel in compression is given by the expression $(1.5m - 1)\ A_s'$, where A_s' is the area of compression steel as shown in Fig. A.1. In IS 456: Table 16, the stress in compression steel is taken as (1.5 m) times the stress in the surrounding concrete.

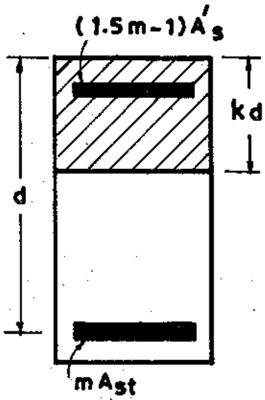


Fig. A.1 Illustration of transformed area method.

From these data the position of the neutral axis (which is the centre of gravity of the transformed section) and the moment of inertia (I) of the equivalent area of concrete about the neutral axis can be easily calculated.

The stresses in concrete (f_c) at any level are given by

$$f_c = \frac{M}{I} y$$

The stress in steel at the corresponding level of the concrete is given by

$$f_{st} = mf_c \quad \text{in the tension area}$$

$$f_{sc} = 1.5\ mf_c \quad \text{in the compression area}$$

The above procedure can be used to analyse the stresses in a given reinforced concrete beam section for given service bending moment. Designs can also be made by formulae derived from this theory for regular sections like rectangles as described in Section A.3.2.

A.3.2 FORMULAE FOR DESIGN OF A SINGLY REINFORCED SECTION

Let

b = breadth of the rectangular beam

d = effective depth

σ_{cb} = maximum allowable bending compressive stress in concrete given in IS 456 Table 45 (see Table A.1).

The values given in the table are approximately equal to $(f_{ck}/3)$.

Taking σ_{st} = maximum allowable tensile stress in steel given in IS 456: Table 16 (Table A.2), the values are approximately equal to $(f_y/1.8)$.

TABLE A.1 PERMISSIBLE STRESSES IN CONCRETE
(Refer IS 456, Table 15)

Grade of concrete	Bending stress in compression σ_{cb} (N/mm^2)	Direct stress in compression, σ_{cc} (N/mm^2)	Permissible bond strees for plain tension bars, τ_{bd} (N/mm^2)
M 10	3.0	2.5	—
M 15	5.0	4.0	0.6
M 20	7.0	5.0	0.8
M 25	8.5	6.0	0.9
M 30	10.0	8.0	1.0
M 35	11.5	9.0	1.1
M 40	13.0	10.0	1.2

Note: (i) Bond stress given above can be increased by 25 per cent for bars in compression.
(ii) Bond stress given above can be increased by 40 per cent for deformed bars.

TABLE A.2 PERMISSIBLE STRESSES IN STEEL REINFORCEMENT
(Refer IS 456, Table 16)

Type of steel	Tension (N/mm^2)	Compression (N/mm^2)	Shear (N/mm^2)
Fe 250			
Up to 20 mm	140	130	140
Over 20 mm	130	130	130
Fe 415			
Up to 20 mm	230	190	230
Over 20 mm	230	190	230

Let

A_s = area of tension steel

kd = depth of neutral axis from the extreme compression fibre

k = neutral axis factor

As both steel and concrete are assumed to behave elastically, the concrete stress block will be triangular and the total compression will be the area of the stress block as shown in Fig. A.2 and given by the equation

$$C = 1/2\sigma_{cb}(kd)b \tag{A.2}$$

This compression acts at the C.G. of the triangle. The total tension

$$T = \sigma_{st}A_s \tag{A.3}$$

and acts at the effective depth.

As total tension is equal to total compression in pure bending, we have

$$C = T$$

The resisting moment is the moment of the couple due to C and T.

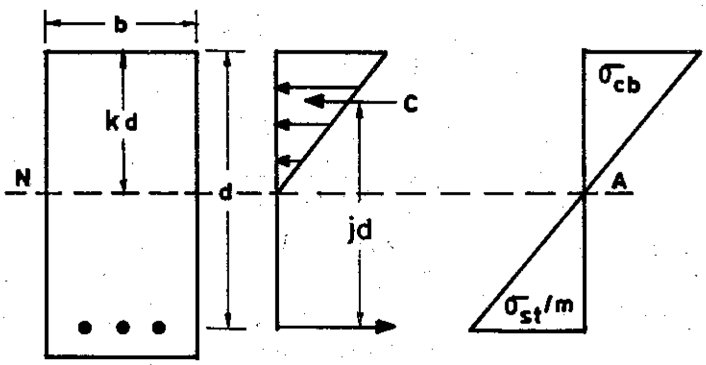


Fig. A.2 Forces in a singly reinforced section.

For maximum compressive stress which is allowed in the extreme concrete fibre during bending as given in Table A.1, one can derive an expression for the limiting moment of resistance of a reinforced concrete beam as follows: Let the lever arm of the couple be denoted by jd:

$$jd = (d - kd/3) = d(1 - k/3)$$

Taking moment of the compression about the C.G. of the steel area, we get

$$M = (\sigma_{cb}/2)\,(kd)\,(b)d\,(1 - k/3)$$
$$= \frac{bd^2}{2}\,\sigma_{cb}\,k(1 - k/3)$$
$$= \sigma_{cb}(kj/2)bd^2$$

Putting $(kj/2) = K$, we obtain

$$M = K\sigma_{cb}bd^2 \tag{A.4}$$

Denoting $(K\sigma_{cb}) = Q$, we get

$$M = Qbd^2 \tag{A.4a}$$

Again, considering the maximum stress allowed in steel (f_s) as given in Table A.2, the moment of resistance should be limited to

$$M = T(jd) = A_s f_s(jd) \tag{A.5}$$

The actual moment of resistance will be lesser of the values given by equations (A.4) and (A.5).

A.4 BALANCED SECTIONS

A reinforced concrete section in bending, in which the maximum allowable stresses in concrete and steel are reached, simultaneously, is called a *balanced section.* In working stress theory, the distribution of strain and stress is assumed to be linear as shown in Fig. A.2(b).

The following formulae for the balanced condition can be easily derived from this assumption:

$$\frac{kd}{d(1-k)} = \frac{\sigma_{cb}}{\sigma_{st}/m} = \frac{m\sigma_{cb}}{\sigma_{st}}$$

Therefore,

$$k = \frac{m\sigma_{cb}}{\sigma_{st} + m\sigma_{cb}} \tag{A.6}$$

$$j = (1 - k/3) \tag{A.7}$$

As already pointed out, according to IS 456: clause 43.3, the value of m is assumed to be a function of cube strength, and is given by

$$m = \frac{280}{3\sigma_{cb}} = \frac{93.3}{\sigma_{cb}}$$

Substituting for m in equation (A.6), we obtain

$$k = \frac{93.3}{\sigma_{st} + 93.3} \tag{A.6a}$$

It can be seen from this equations that, unlike in other codes, in IS 456 the values of the neutral axis factors and lever arm factors become constants for a given value of σ_{cb} and σ_{st}. The values of n, j and k obtained according to IS 456 will be as shown in Table A.3.

TABLE A.3 BEAM FACTORS BY W.S. METHOD: IS 456

Grade of steel	Grade of concrete	IS 456		
		k	j	K
Fe 250	15	0.40	0.87	0.174
	20	0.40	0.87	0.174
	30	0.40	0.87	0.174
Fe 415	15	0.29	0.90	0.131
	20	0.29	0.90	0.131
	30	0.29	0.90	0.131

As already shown in equations (A.4) and (A.4a), the value of the resisting moment can be expressed as

$$M = \sigma_{cb} K b d^2$$

This is of the same form as the equation derived for traditional elastic design of homogeneous section derived in Section A.1.

As shown in Section A.1 for a homogeneous rectangular beam (like a timber beam), M is obtained as

$$M = 0.167 f b d^2$$

For a reinforced concrete beam the general equation becomes

$$M = K\sigma_{cb} b d^2$$

Substituting for the value of K for different grades of steel, this equation reduces to

$$M = 0.174\sigma_{cb}bd^2 \quad \text{for Fe 250} \tag{A.8}$$

$$M = 0.131\sigma_{cb}bd^2 \quad \text{for Fe 415} \tag{A.9}$$

$$M = 0.116\sigma_{cb}bd^2 \quad \text{for Fe 500} \tag{A.9a}$$

(This may be compared with the values of 0.149, 0.138 and 0.133 $f_{ck}bd^2$ derived for ultimate moments for the above steels by the limit state method).

A.4.1 BALANCED STEEL PERCENTAGES

The necessary steel for a singly reinforced balanced section is known as balanced steel percentage, and it can be obtained by equating the maximum allowable compression and tension. Putting

$$p_t = \frac{100A_s}{bd}$$

we get

$$A_s = \frac{p_t bd}{100}$$

Equating the forces, we obtain

$$1/2\sigma_{bc}(kd)b = \frac{p_t}{100}bd\sigma_{st}$$

$$p_t = 50k(\sigma_{cb}/\sigma_{st}) \tag{A.10}$$

The values of the balanced percentage of steel for a singly reinforced balanced section for various grades of concrete and steel can be tabulated as given in Table A.4. Sections in which the steel provided is less than that necessary for the balanced section are called *under-reinforced sections.* Sections in which the steel provided is more than the balanced sections are called *over-reinforced sections.* The following general observations can be made regarding the design of singly reinforced rectangular sections:

1. For resisting a given bending moment with a specified grade of steel, a higher strength of concrete will result in smaller section ($b \times d$) required to resist the given moment. The corresponding percentage of steel based on the ($b \times d$) values will also be larger.

2. For a given section with a specified strength of concrete, the use of higher grade steel will obviously require less percentage of steel to resist a given bending moment.

TABLE A.4 PERCENTAGE OF TENSILE REINFORCEMENT FOR BALANCED SECTION
(Refer SP 16, Table L)

σ_{cb}	σ_{st} (N/mm^2)		
(N/mm^2)	140	230	275
5.0	0.71	0.31	0.23
7.0	1.00	0.44	0.32
8.5	1.21	0.53	0.39
10.0	1.43	0.63	0.46

A.5 ANALYSIS OF A GIVEN SECTION IN BENDING

For analysing a reinforced section in bending, the position of the neutral axis is determined by finding out the centre of gravity of the transformed area. For a rectangular singly reinforced section (putting $p = p_t$, since there is no compression steel) by taking moment of areas about the neutral axis at depth (kd), we have

$$b(kd)\,(kd/2) = (p/100)\,(bd)(m)d\,(1 - k)$$

Simplifying, we get

$$k^2 + (p/50)mk - (p/50)m = 0$$

Solving the equation for the positive value of k, we obtain

$$k = -\,(p/100)m + [(p/100)^2m^2 + 2(p/100)m]^{1/2}$$

$$= -\,mp' + (m^2p'^2 + 2mp')^{1/2}, \quad p' = p/100 = \frac{A_s}{bd} \tag{A.11}$$

The value of the resisting moment can be expressed as

$$M = (p/100)bd\sigma_{st}d(1 - k/3)$$

Hence,

$$\frac{M}{bd^2} = (p/100)(1 - k/3)\sigma_{st} \tag{A.12}$$

For specific values of k and σ_{st}, the values of M/bd^2 for varying values of p can be determined. In SP 16, Tables 68 to 71, the values of (M/bd^2) for various steel ratios have been tabulated for the different grades of steel and concrete commonly used in practice. These can be used for calculating the necessary steel for a beam of given section ($b \times d$) for resisting a given bending moment.

A.6 DEPTH OF NEUTRAL AXIS WITH COMPRESSION STEEL

The above expression gives the moment of resistance with only tension steel in the section. By assuming equivalent area for compression steel, as explained in Section A.3.1, similar expressions can be obtained for doubly reinforced beams also. Values of (M/bd^2) for various steel ratios are given in Tables 72 to 79 of SP 16, and they can be used for routine design of doubly reinforced beams.

SP 16, Tables 91 to 94 give the depths of the neutral axis calculated on the theory using $(m - 1)A_{sc}$ as the equivalent area of compression steel for different amounts of tension steels, ratios of compression steel to tension steel, and d'/d ratios, where d' is the depth of compression steel as shown in Fig. A.3. This depth of neutral axis is important in estimating crack-width in beams for assuming the serviceability limit state. This neutral axis is different from the neutral axis for stress analysis using $(1.5m - 1)A_{sc}$ for compression steel.

A.7 OTHER DESIGN PROBLEMS BY WORKING STRESS METHOD

It is possible to use working stress method based on elastic theory for design of doubly reinforced

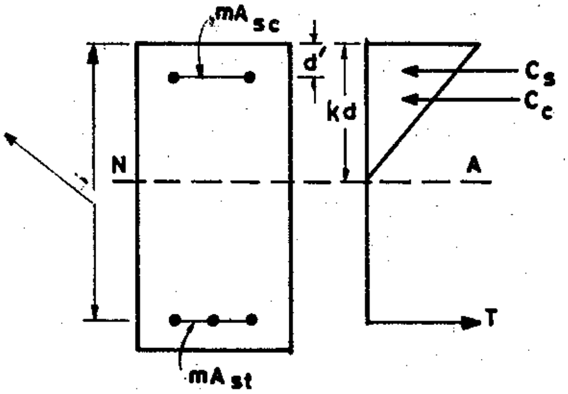


Fig. A.3 Illustration of Forces in a doubly reinforced section.

beams, *T*-beams, compression members, and members in shear. These are alternative methods of design that are allowed by the IS code, but discontinued by many other codes after the introduction of the limit state theory.

However, as the elastic theory of bending is an important concept for estimating stresses at working loads and for analysing reinforced concrete members in the limit state of serviceability, a knowledge of the elastic theory as presented above becomes a necessary tool for limit state design also. These concepts are necessary in the calculations of deflection and crack-width in reinforced concrete members.

EXAMPLE A.1 **(Analysis of rectangular beam)**

A singly reinforced concrete beam is of width 450 mm and effective depth 715 mm. It is reinforced with 8 Nos. 20 mm mild steel bars. Assuming grade 15 concrete, determine its moment of resistance according to the working stress method. Determine also the stress in steel when the beam is subjected to the above moment.

Ref.	Step	Calculations	Output
	1.	*Determination of NA depth x*	
		$(bx^2/2) = mA_s(d - x)$	$A_s = 2513$ mm^2
IS 456 43.3		$m = 280/3\sigma_{cb} = 280/15 = 18.67$	
		$\dfrac{450x^2}{2} = (18.67)(2513)(715 - x)$	
		$x^2 + 208.6x - 149024 = 0$	
		$x = 295.7$ mm	
	2.	*M_1 due to concrete failure* $\sigma_{cb} = 5$ N/mm^2	
IS 456 Table 15		$M_1 = (bx\sigma_{cb}/2)(d - x/3)$	
		LA = (715 – 295.7/3) = 616.4 mm	

EXAMPLE A.1 (cont.)

Ref.	Step	Calculations	Output
		$M_1 = (450 \times 295.7 \times 5 \times 616.4)/2$ $= 205$ kNm	
IS 456 Table 16	3.	*M_2 due to steel failure* $\sigma_{st} = 140$ N/mm^2 $M_2 = A_s\sigma_{st}$ (LA) $= 2513 \times 140 \times 616.4$ $= 216.9$ kNm	M_1 controls beam overreinforced. Moment of resistance $= 205$ kNm
	4.	*Stress in steel when $M = M_1$* $\sigma_{st} = m$ (stress in concrete) $= 18.67 \times 5(715 - 295.7)/295.7$ $= 132$ N/mm$^2 < 140$ N/mm^2	$\sigma_{st} = 132$ N/mm^2

EXAMPLE A.2 **(Analysis of moment capacity of T beams)**

Determine the moment of resistance of a T beam with flange width 1200 mm, web thickness 300 m, slab thickness 100 mm, effective depth 360 mm, and area of tension steel 2945 mm^2. Assume M 15 concrete and Fe 250 steel. Use the working stress method.

Ref.	Step	Calculations	Output
IS 456 43.3	1.	*Determination of* NA *depth* (a) Taking moment of areas neglecting web, we get $1200 \times 100\,(x - 50) = 18.67 \times 2945(360 - x)$ $x = 147.4$ mm (b) Taking moments with web also, we obtain $300(x)\,(x/2) + 900 \times 100(x - 50)$ $= 18.67(2945)\,(360 - x)$ $x^2 + 967x - 161960 = 0$ $x \approx 144$ mm	$m = \dfrac{280}{3\sigma_{cb}} = 18.67$ Contribution of web can be neglected
IS 456 Table 15	2.	*Moment capacity with* $\sigma_{cb} = 5$ N/mm^2 Stress at top of slab $= 5$ N/mm^2 Stress at bottom $= \dfrac{5 \times 44}{144} = 1.53$ N/mm^2 Average stress $= (5 + 1.53)/2 = 3.26$ N/mm^2	

EXAMPLE A.2 (cont.)

Ref.	Step	Calculations	Output
		$\text{C.G.} = \left(\dfrac{5+3.06}{5+1.53}\right)\left(\dfrac{100}{3}\right) = 41.14$ from top	
		$M_1 = (3.26 \times 100 \times 1200)\ (360 - 41.14)$	
		$= 124.7$ kNm	
	3.	*Moment capacity with* $\sigma_{st} = 140$ N/mm^2	
IS 456		$M_2 = \sigma_{st} A_s (\text{LA})$	
Table 16		$= 140 \times 2945(360 - 41.14)$	
		$= 131.5$ kNm	Overreinforced beam
	4.	*Moment of resistance*	
		M_1 controls $= 124.7$ kNm	$M_1 = 124.7$ kNm

EXAMPLE A.3 **(Calculation of stresses in T beams)**

A T beam has its flange width 1200 mm, web width 300 mm, slab depth 100 mm, effective depth 410 mm, area of tension steel 3927 mm^2, area of compression steel 1963 mm^2. If it is subjected to a bending moment of 200 kNm, determine the maximum stresses in concrete, tension and compression steels. Assume cover to centre of steel as 35 mm.

Ref.	Step	Calculations	Output
IS 456 43.3	1.	*Position of* NA *(neglecting compression in web)*	
IS 456 Table 16 (iii)		$1200 \times 100\ (x - 50) + 1963(1.5m - 1)(x - 35)$	$m = \dfrac{280}{3 \times 5} = 18.67$
		$= 3927 \times m(410 - x)$	$(1.5m - 1) = 27.7$
		$x = 154$ mm	
		2. *Moment of Inertia I*	
		$1200 \times 100\ (154 - 50)^2 + 1963 \times 27.7(154 - 35)^2$	
		$+ 3927 \times 18.67(410 - 154)^2$	
		$= 6873 \times 10^6$ mm^4	
	3.	*Calculation of stresses* $M = 200 \times 10^6$	
		Compression stress in concrete $= \sigma_c$	
		$\sigma_c = \dfrac{200 \times (10)^6 \times 154}{6873 \times (10)^6} = 4.49$ N/mm^2	$\sigma_c = 4.49$ N/mm^2
		σ_{sc} = compression in steel $= (M/I)y \times 1.5m$	
IS 456		$= \dfrac{200(154 - 35)(1.5 \times 18.67)}{6873}$	
Table 16		$= 97$ N/mm^2	$\sigma_{sc} = 97$ N/mm^2

EXAMPLE A.3 (cont.)

Ref.	Step	Calculations	Output
		σ_{st} = Tension in steel = $(M/I)y \times m$ $= \dfrac{200(410 - 154) \times 18.67}{6873}$ $= 139$ N/mm^2	$\sigma_{st} = 139$ N/mm^2

EXAMPLE A.4 **(Design of singly reinforced beam)**

Determine the reinforcement for a T beam with flange width = 1500 mm, web width = 300 mm, thickness of slab = 100 mm, effective depth 735 mm, to carry a moment of 380 kNm due to characteristic loads. Use M 15 concrete and Fe 415 steel. Use working stress design.

Ref.	Step	Calculations	Output
	1.	*Approx. capacity of* T *beam* $x = kd$ $k = \dfrac{93.3}{\sigma_{st} + 93.3} = 0.29$ $x = 0.29 \times 735 = 212$ mm N.A. below the slab. σ_c at top of slab = 5 N/mm^2 σ_c at bottom of slab $= \dfrac{5 \times 112}{212} = 2.64$ N/mm^2 C.G. of area $= \left(\dfrac{A + 2B}{A + B}\right)\dfrac{L}{3}$ $= \dfrac{100}{3}\left(\dfrac{5 + 5.28}{7.64}\right) = 44.8$ mm LA = (735 − 44.8) = 690 mm $M = \left(\dfrac{5 + 2.64}{2}\right)(1500 \times 100)(690)$ = 395 kNm > M given	$\sigma_{st} = 230$ Singly reinforced beam
	2.	*Tension steel required* $A_{st} = \dfrac{M}{f_s jd} = \dfrac{380 \times 10^6}{230 \times 690}$ $= 2395$ mm^2	$A_{st} = 2395$ mm^2
	3.	*Using approx. method* LA = $(d - D_f/2)$ = (735 − 50) $A_s = \dfrac{380 \times 10^6}{230(735 - 50)} = 2412$ mm^2	$A_{st} = 2412$ mm^2

EXAMPLE A.5 **(Design of a doubly reinforced T beam)**

A T beam has a flange width of 1500 mm, web width of 400 mm, slab thickness of 120 mm, and an effective depth of 550 mm with cover to centre of steel of 35 mm. (a) Determine, using the working stress method, the steel required to carry a moment of 375 kNm due to characteristic loads. Assume M 15 concrete and mild steel rods over 20 mm as reinforcement. (b) Also determine the steel area required by the limit state method.

Ref.	Step	Calculations	Output
IS 456	1.	*Approx. moment capacity (no compression steel)*	
Table 16(a)		$\sigma_{cb} = 5\ \text{N/mm}^2$	
		$\sigma_{st} = 130\ \text{N/mm}^2$ (diameter > 20 mm)	
		$m = \dfrac{280}{3 \times 5} = 18.7$	
		$k = \dfrac{93.3}{\sigma_{st} + 93.3} = 0.4$ (for rectangular beam)	
		$x = kd = 0.4 \times 550 = 220$ mm	
		σ_{cb} top (of slab) $= 5\ \text{N/mm}^2$	
		σ_{cb} bottom (of slab) $= \dfrac{5 \times 100}{220} = 2.27\ \text{N/mm}^2$	
		Depth of compression $= \dfrac{120}{3}\left[\dfrac{5 + 2(2.27)}{5 + 2.27}\right]$	
		$= 40\left[\dfrac{5 + 4.54}{5 + 2.27}\right]$	
		= 52 mm from top	
		$M = \left(\dfrac{5 + 2.27}{2}\right)(120 \times 1500)(550 - 52)$	
		= 326 kNm (moment of resistance)	Doubly reinforced
		M given (375 kNm) > M.R. (Use compression steel)	beam
	2.	*Find area of tension steel (steel beam theory)*	
		$A_{st} = A_{st_1}$ (for moment of resistance) $+ A_{st_2}$ (for ΔM)	
		$A_{st_1} = \dfrac{326 \times 10^6}{130(550 - 52)} = 5036\ \text{mm}^2$	
		$A_{st_2} = \dfrac{(375 - 326) \times 10^6}{130(550 - 35)} = 732\ \text{mm}^2$	
		$A_{st} = 5036 + 732 = 5768\ \text{mm}^2$	$A_{st} = 5768\ \text{mm}^2$

EXAMPLE A.5 (cont.)

Ref.	Step	Calculations	Output
	3.	*Area of compression steel*	
		$A_{sc} = \dfrac{\text{Force in compression steel}}{\sigma_{cb}\,(1.5m - 1)}$	
		σ_{cb} at compression steel level (NA as in step 1)	
		$= \dfrac{5(220 - 35)}{220} = 4.20\ \text{N/mm}^2$	
		$A_{sc} = \dfrac{732 \times 130}{4.2\,(1.5 \times 18.7 - 1)} = 837\ \text{mm}^2$	$A_{sc} = 837\ \text{mm}^2$
Table 16(b)		*Design by limit state method*	
	1.	*Design moment*	
		$M_u = 1.5 \times 375 = 562.5$ kNm	
	2.	T *beam parameters*	
		$b_f/b_w = 1500/400 = 3.75$	
		$D_f/d = 120/550 = 0.22$	
SP 16		$M_u/f_{ck}bd^2$ from table $= 0.389$	
Table 57		$M_u = 0.389 \times 400 \times (550)^2 \times 15$ $= 706$ kNm	
		$M_{lim} < M_u$ (no compression steel)	A_{sc} not needed
	3.	*Approx. area of tension steel*	
		$A_{st} = M/0.87f_y(d - D_f/2)$	
		$= 562.5 \times 10^6/0.87 \times 250(550 - 60)$	
		$= 5278\ \text{mm}^2$	$A_{st} = 5278\ \text{mm}^2$

EXAMPLE A.6 **(Use of SP 16 for design)**

A rectangular beam of breadth 250 mm and effective depth 800 mm with cover of 40 mm to centre of steel is to be designed for M 20 grade concrete and Fe 250 grade steel by the working stress method. Determine the area of steel required if the moment due to characteristic load it has to carry is (a) 150 kNm, and (b) 200 kNm.

Ref.	Step	Calculations	Output
		Case (a): $M = 150$ kNm	
	1.	*Design parameters*	$\sigma_{cb} = 7\ \text{N/mm}^2$
		$M/bd^2 = \dfrac{150 \times 10^6}{250\,(800)^2} = 0.937$	$\sigma_{cb} = 140\ \text{N/mm}^2$

EXAMPLE A.6 (cont.)

Ref.	Step	Calculations	Output
SP 16 Table 69	2.	*Area of steel required from SP 16*	
		$p = 0.76$ per cent	
		$A_s = \dfrac{0.76 \times 250 \times 800}{100} = 1520\ \text{mm}^2$	$A_{st} = 1520\ \text{mm}^2$
		Case (b): $M = 200$ kNm	
	1.	*Design parameters*	
SP 16 Table 69		$M/bd^2 = \dfrac{200 \times 10^6}{250 \times (800)^2} = 1.25$	
		Beam has to be doubly reinforced.	
	2.	*Areas of steel from SP 16*	
SP 16 Table 73		$p_t = 1.028\%,\quad p_c = 0.033\%$	
		$A_{st} = \dfrac{1.028 \times 250 \times 800}{100} = 2056\ \text{mm}^2$	$A_{st} = 2056\ \text{mm}^2$
		$A_{sc} = \dfrac{0.033 \times 250 \times 800}{100} = 66\ \text{mm}^2$	$A_{sc} = 66\ \text{mm}^2$

EXAMPLE A.7 **(Comparison of L.S.M. and W.S.M.)**

Using SP 16, determine the steel by the limit state method for (b) in Example A.6.

Ref.	Step	Calculations	Output
	1.	*Design moment*	$f_{ck} = 20$
		$M_u = 1.5 \times 200 = 300$ kNm	$f_y = 250$
	2.	*Design parameters*	
SP 16 Table D		$\dfrac{M_u}{bd^2} = \dfrac{300 \times 10^6}{250\,(800)^2} = 1.88$	
		$\dfrac{M_u\ (\text{lim})}{bd^2}$ for singly reinforced $= 2.98$	
	3.	*Area of tension steel*	
SP 16 Table 2		Steel $= 0.988$ per cent	
		$A_{st} = \dfrac{0.988 \times 250 \times 800}{100} = 1976\ \text{mm}^2$	$A_{st} = 1976\ \text{mm}^2$
		No compression steel is required.	

REVIEW QUESTIONS

A.1 When was the method of calculation of strength of reinforced beams first proposed ? What were the basic assumptions made in the proposed method ?

A.2 What are the advantages and disadvantages of the design of R.C. members by working stress method ?

A.3 How will one estimate the stresses attained by steel and concrete at the working load of a beam ? Explain the assumptions made.

A.4 Is the value of modular ratio used in working stress design the same as that obtained by the results of short-term laboratory tests on steel and concrete ? Explain what value is used in design by working stress method.

A.5 Write down the formula for modular ratio proposed in IS 456. What is the advantage of using IS value as different from a constant value $m = 15$ as is used in some other codes ?

A.6 How does one estimate the stresses in the compression and tension steels by working stress method ? Why is there difference in the procedures between the two types of steels ? Explain why the permissible stress in bending compression is more than that in direction compression.

A.7 What is a balanced section according to the working stress theory ? Will such a section be balanced according to the limit state theory also ?

A.8 Will the depth of the neutral axis as calculated by working stress theory be the same as that calculated by the limit state theory ? Explain how both can really exist in a beam and indicate how it can be demonstrated by laboratory tests.

A.9 What were the limitations of the working stress method that led to the development of the limit state method of design of concrete members ?

A.10 What will generally be the difference between the results obtained between two beams of the same size but one designed by the working stress method and the other by the limit state method ?

PROBLEMS

A.1 Determine the position of the neutral axis and the moment of resistance of a beam 300 mm wide and 550 mm depth reinforced with 4 Nos. 18 mm bars in tension and 4 Nos. 14 mm bars in compression with a cover to the centre of steel of 50 mm. Assume grade 15 concrete and Fe 415 steel. Estimate also the depth of the neutral axis by use of SP 16, Tables 91 to 94. In which situation would you use each of the above values of the neutral axis?

A.2 A rectangular beam is 300 mm wide and of 550 mm effective depth. Determine the area of tension reinforcement required in the section is to be a balanced section. Assume $\sigma_{cb} = 7$ N/mm^2, $\sigma_{st} = 140$ N/mm^2 and $m = 13$. Determine also the elastic moment of resistance of the section.

A.3 A rectangular beam is 300 mm wide and of effective depth 550 mm. It is provided with 4 Nos. 20 mm HYD bars as tension steel. Determine the elastic moment of resistance of the beam assuming grade 15 concrete.

A.4 Reinforced beams of span 6 m and spaced at 3 m centres carry a 100 mm floor slab of a hall 6 × 18 m. The rib width of the beams is restricted to 300 mm and the beams rest on walls 450 mm thick. The live load on the floor is 4 kN/m^2. Assuming that $\sigma_{cb} = 5$ N/mm^2, $\sigma_{st} = 140$

N/mm^2, $m = 19$ and that the slab and beams are cast as an integral unit, design the beam for flexure by elastic theory. Detail the reinforcements in the beam.

A.5 Design a reinforced rectangular beam of breadth 300 mm to carry a characteristic live load of 12 kN/m over an effective span 8 m, using grade 15 concrete and Fe 415 steel by the working stress method. Determine the steel requirement of the above beam by the limit state method.

A.6 A rectangular beam 300 × 550 mm with a cover of 50 mm to the centre of steel has 4 rods of 16 mm as tension steel. Determine the maximum stresses in concrete and in steel when the beam is subjected to a moment of 60 kNm.

A.7 A T beam section with a width of flange of 1500 mm, width of web 300 mm, depth of slab 100 mm, and effective depth 500 mm is subjected to a moment of 250 kNm. Comment on the safety of the beam if the concrete used is of grade 20 and the steel is of grade Fe 415.

Estimate also the maximum moment the beam can carry according to the Limit State Method.

Appendix B

General Data for Designs

TABLE B.1 DEAD LOADS

(Refer IS 876 (1986), Part I: Structural Safety of Buildings—Loading Standards—Dead Loads)

A. Weight of Some Building Materials

Material	Unit weight (kN/m^3)
Concrete—plain	24
Concrete—reinforced	25
Brick work	19
Stone work	21–27
Cement plaster	20.5
Lime plaster	17.5
Timber	6–10
Brick jelly concrete	19.5

B. Weight of Some Building Parts on Components

Component	Unit weight (N/m^2)
Roofing AC sheets	160 (on plan area)
Roofing GI sheets	150 (on plan area)
Mangalore tiles with battens	650
Ceiling plaster	250
Floor finish	600–1000
Floor—mortar screed (10 mm)	200
Floor—Stone tiles (25 mm)	600
Floor—Ceramic (20 mm)	500
Floor—Terrazo (20 mm)	500
Partition wall	1000
Steel truss (10 to 20 m spans)	85–120
Purlins for trusses	60–90
Bracing for trusses	12–15
Roof finish—Felt (10 mm)	8
Bitumen macadam (10 mm)	220

TABLE B.2 IMPOSED LOADS

(Refer IS 876 (1986): Part II—Loading Standards)

A. Loads on Floors

S. No.	Type of buildings	Use of floor	UDL (kN/m^2)
1.	*Residential premises*		
	(a) Dwelling houses	All rooms	2.0
			1.5 (BS)
	(b) Boarding houses, hostels	All rooms	2.0
			1.5 (BS)
	(c) Hotels and hospitals	Bedrooms and hospital wards	2.0
2.	*Places of public assembly or access*		
	(a) Public assembly	With fixed seats	4.0
	Theatres		
	Display centres		
	Waiting halls		
	Sports halls		
	Dance halls	Without fixed seats	5.0
	(b) Dance halls		5.0
	(c) Churches, class rooms		4.0
			3.0 (BS)
	(d) Banking halls		3.0
	(e) Library	Reading rooms	3.0
			2.5 (BS)
		Book shelves	10.0
			4.0 (BS)
	(f) Shops	Display and sale	4.0
3.	*Commercial and Industrial premises*		
	(a) Offices	With separate storage	2.5
		Without separate storage	4.0
		Storage space	5.0
		Computer room	3.5
	(b) Factories (BS)	Small work rooms	2.5
		Small workshops	5.0
		Medium workshop	7.5
		Large workshop	10.0
		Machinery sheds	4.0
	(c) Warehouses	Small	5.0
		Medium	7.5
		Large	10.0
	(*Note*: The minimum load for slabs should be on an area of 2.5 m^2, and 6 m^2 for beams)		
	(d) Garrages	Light vehicles less than 25 kN in weight	4.0
		Heavy vehicles less than 40 kN in weight	7.5
4.	*Ancillary areas*		
	Stairs and landings	Dwellings (not liable to overcrowding)	2.0
			1.5 (BS)
	Balconies	Liable to overcrowding	5.0

(*Note*: A minimum of 1.3 kN concentrated load should be supported by cantilever steps.)

TABLE B.2 (cont.)

B. Loads on Roofs

Type	UDL (kN/m^2)
Roof with access	1.50
Roof without access	0.75

C. Horizontal Loads on Parapets and Balustrades

Type	UDL (kN/m)
In places of assembly	2.25
Domestic stairways	0.35
Others	0.75

TABLE B.3 AREAS OF BARS (mm)
(Ref: SP 16 Table 95)

No. of bars \ Diameter of bars (mm)	6	8	10	12	16	20	25	32
1	28.3	50.3	78.5	113.1	201.1	314.2	490.0	804.2
2	56.5	100.5	157.1	226.2	402.1	628.3	981.7	1,608
3	84.8	150.8	235.6	339.3	603.2	942.5	1,473	2,413
4	113.1	201.1	314.2	452.4	804.2	1,257	1,963	3,217
5	141.4	251.3	392.7	565.5	1,005	1,571	2,454	4,021
6	169.6	301.6	471.2	678.6	1,206	1,885	2,945	4,825
7	197.9	351.9	549.8	791.7	1,407	2,199	3,436	5,630
8	226.2	402.1	628.3	904.8	1,608	2,513	3,927	6,434
9	254.5	452.4	706.9	1,018	1,810	2,827	4,418	7,238
10	282.7	502.7	785.4	1,131	2,011	3,142	4,909	8,042
Perimeter of one bar (mm)	18.8	25.1	31.4	37.6	50.2	62.8	78.5	100.5

TABLE B.4 AREAS OF BARS AT GIVEN SPACINGS (mm^2)
(Refer SP 16 Table 96)

Spacing (mm) \ Diameter of bars (mm)	6	8	10	12	16	20	25	32
50	565	1,005	1,571	2,262	4,021			
75	377	670	1,047	1,508	2,680	4,188	6,545	
100	282	503	785	1,131	2,010	3,141	4,908	8,042
125	226	402	628	904	1,608	2,513	3,926	6,434
150	188	335	523	754	1,340	2,094	3,272	5,361
175	161	287	448	646	1,149	1,795	2,805	4,595
200	141	251	392	565	1,005	1,570	2,454	4,021
225	125	223	349	502	893	1,396	2,181	3,574
250	113	201	314	452	804	1,256	1,963	3,217
275	102	182	285	411	731	1,142	1,785	2,924
300	94	167	261	377	670	1,047	1,636	2,680

TABLE B.5 UNIT WEIGHTS AND WEIGHTS AT SPECIFIED SPACING OF BARS

Unit Weights			Weights of bars in kg per m²								
Size (mm)	Weight/m (kg)	Length/ tonne (m)	Spacing of bars (mm)								
			75	100	125	150	175	200	225	250	275
6	0.222	4,505	2.960	2.220	1.776	1.480	1.269	1.110	0.987	0.888	0.807
8	0.395	2,532	5.267	3.950	3.160	2.633	2.257	1.975	1.756	1.580	1.436
10	0.616	1,623	8.213	6.160	4.928	4.107	3.520	3.080	2.738	2.464	2.240
12	0.888	1,126	11.84	8.880	7.104	5.920	5.074	4.440	3.947	3.552	3.229
16	1.579	633	21.05	15.79	12.63	10.53	9.023	7.895	7.018	6.316	5.742
20	2.466	406	32.88	24.66	19.73	16.44	14.09	12.33	10.96	9.864	8.967
25	3.854	259	51.39	38.54	30.83	25.69	22.02	19.27	17.13	15.42	14.01
32	6.313	159		63.13	50.50	42.09	36.07	31.57	28.06	25.25	22.96
40	9.864	101			78.91	65.76	56.37	49.32	43.84	39.46	35.87

*Basic weight = 0.00785 kg/mm²/m

TABLE B.6 CONVERSION FACTORS

Quantity	To convert from	Convert to	Multiply by
UDL	kips/ft	kN/m	14.59
	kN/m	lb/ft	68.52
Forces	kips	kN	4.448
	N	lb	0.2248
Moments	ft-kips	kN-m	1.356
	kN-m	ft-kips	0.7376
Stresses	ksi	MPa	6.895
	MPa	ksi	0.1450

TABLE B.7 SI UNITS

1. *Prefixes*
   - k kilo 10^3
   - M mega 10^6
   - m milli 10^{-3}
   - μ micro 10^{-6}

2. *Some equivalents*

(a) Newton-force unit (kg m/s²). Assuming acceleration due to gravity as 10 m/s², 1 kg force is taken as 10 N
1 kg force = 1 (acceleration due to gravity) = 9.81 N

(b) 1 bar = 1 kg/cm² = 0.1 N/mm²
(Strength of concrete 250 bar = 25 N/mm²)

(c) Pascal (Pa) = N/m²

$$\text{MPa} = \frac{\text{N} \times 10^6}{1000 \times 1000} = \text{N/mm}^2$$

(d) 10 kN = 1 ton

(e) 1 quintal = 100 kg

(f) 1 kg/cm² = 10 tons/m² = 100 kN/m² ≈ 1 ton/sq. ft.

(g) 1 ton/m² = 0.1 kg/cm² = 0.01 N/mm².

TABLE B.8 PRELIMINARY ESTIMATION OF QUANTITIES OF MATERIALS

A. Materials for 1 m^3 of Finished Concrete

Minimum mix (Traditional)	Grade without site control	Grade with site control	Cement (kg/m^3)	Coarse sand (m^3)	Coarse aggregate. (m^3)
1 : 1$^1/_2$: 3	20	30	405	0.41	0.82
1 : 2 : 4	15	25	310	0.42	0.84
1 : 3 : 6	10	20	215	0.44	0.88
1 : 4 : 8	used only as PCC		160	0.45	0.90
1 : 5 : 10	for base concrete		130	0.46	0.92

Notes:
(i) Average volume of dry sand = 40% of volume of concrete
(ii) For moist sand, allow 15 to 20% bulking
(iii) One bag of cement = 0.035 m^3 (30 bags approx. = 1 m^3)
(iv) 1 m^3 = 35.315 ft^3
(v) Sum of volume of ingredients = 1.5 (volume of mixed concrete)
(vi) Concrete used for R.C. work should have a minimum cement content of 220 kg/m^3 as specified in IS 456, Table 19.

B. Approximate Steel (HYD bars) in Structural Members (kg/m^3)

(i) Foundation	40–100
(ii) Columns	80–320
(iii) Beams	80–160
(iv) Lintels and slabs	30–80
(v) Cast *in situ* piles	30–50

C. Overall Consumption of Steel (HYD bars) in Buildings (kg/m^2)

(i) Residences with load bearing walls	9.0–20
(ii) Flats and hostels	30–50
(iii) Residences and office: multistoried buildings	40–70

Appendix C

Formulae for Some Charts and Tables in IS 456

1. *Modification factor for tension steel (F_1)*

   IS 456, Fig. 3: (p. 150)*

   $F_1 = 1/(0.225 + 0.003f_s + 0.625 \log p_t) \leq 2.0$

   BS 8110: (p. 150)

   $$F_1 = 0.55 + \frac{477 - f_s}{120\left(0.9 + \dfrac{M}{bd^2}\right)} \leq 2.0$$

2. *Modification factor for compression steel (F_2)*

   IS 456, Fig. 4: (p. 151)

   $$F_2 = \frac{1.6p_c}{p_c + 0.275} \leq 1.5$$

   BS 8110: (p. 151)

   $F_2 = 1 + p_c/(3 + p_c) \leq 1.5$

3. *Modification for flanged beams (F_3)*

   IS 456, Fig. 5: (p. 152)

   $$F_3 = 0.8 + 0.3\left(\frac{b_w}{b_f} - 0.3\right) \not< 0.8$$

4. *Effective width in one-way solid slabs with concentrated loads*

   IS 456, Table 9: (p. 199)

   $$b_e = kx\left(1 - \frac{x}{L_e}\right) + a$$

   Use $k = 2.48$ for values $B/L > 1$ (One-way slabs)

   BS 8110 formula for one-way slabs: (p. 199)

   $$b_e = a + 1.2x\left(1 - \frac{x}{L_e}\right) \text{ on either side of load strip.}$$

*Page numbers given in brackets in this Appendix refer to pages in the text.

5. *Design ultimate bond stress*

IS 456: clause 25.2.1.1, BS 8110: (p. 172)

$$\tau_{bd} = \beta\sqrt{f_{ck}}$$

β varies with type of steel

$$\beta = 0.28 \quad \text{for plain bars}$$
$$= 0.50 \quad \text{for deformed bars}$$
$$\beta' = 1.25\beta \quad \text{for compression.}$$

6. *Maximum clear spacing of reinforcement in beams*

IS 456, Table 10:

BS 8110: (p. 156)

$$s \le \frac{47{,}000}{\text{estimated service stress}} \le 300 \text{ mm}$$

7. *Design shear strength of concrete τ_c*

IS 456, Table 13 and clause 39.2: (p. 92)

$$\tau_c = \frac{0.85\sqrt{0.8f_{ck}}\,(\sqrt{1+5\beta}-1)}{6\beta}\,(300/d)^{1/4}$$

where

$$\beta = 0.8f_{ck}/6.89p_t \nless 1.0$$

BS 8110: (p. 93)

$$\tau_c = 0.79(p_t)^{1/3}\,(400/d)^{1/4}\,(f_{ck}/25)^{1/3}\,(1/\gamma_m)$$

$$\gamma_m = 1.25$$

8. *Maximum shear stress $\tau_{c\,max}$*

IS 456, Table 14: (p. 93)

$$\tau_{c\max} = 0.83\sqrt{\text{cyl. strength}} = 0.63\sqrt{f_{ck}} \ngtr 4 \text{ N/mm}^2$$

BS 8110: (p. 93)

$$\tau_{c\max} = 0.8\sqrt{f_{ck}} \ngtr 5 \text{ N/mm}^2$$

9. *Two-way restrained slabs*

IS 456, Table 22 and BS 8110: (p. 217)

$$\alpha_y = (24 + 2N_d + 1.5N_d^2)/1000$$

$$\gamma = \frac{2}{9}\left[3 - \sqrt{18}\left(\frac{L_x}{L_y}\right)(\sqrt{\alpha_y + \alpha_1} + \sqrt{\alpha_y + \alpha_2})\right]$$

$$\sqrt{\gamma} = \sqrt{\alpha_x + \alpha_3} + \sqrt{\alpha_x + \alpha_4}$$

(See Section 12.4.1 for details)

10. *Two-way slabs simply supported on four sides*

IS 456, Table 23 and BS 8110: (p. 216)

$$\beta_x = \frac{1}{8}\frac{(L_y/L_x)^4}{1 + (L_y/L_x)^4}$$

$$\beta_y = \frac{1}{8}\frac{(L_y/L_x)^2}{1 + (L_y/L_x)^4}$$

11. *Effective height of columns in frames*

IS 456:

Fig. 24 and Fig. 25

BS 8110: (p. 332)

Braced columns—values less than

$L_e = L_0[0.7 + 0.05(\alpha_{c1} + \alpha_{c2})] < L_0$

$L_e = L_0[0.85 + 0.05\alpha_{c\,min}] < L_0$

Unbraced column—values less than

$L_e = L_0[1.0 + 0.15(\alpha_{c1} + \alpha_{c2})]$

$L_e = L_0[2.0 + 0.3\alpha_{c\,min}]$

Index

Limit State Design of Reinforced Concrete

by

P.C. VARGHESE

Designed as a text for the undergraduate students of Civil Engineering, covering the first course on Reinforced Concrete Design and as a reference for the practising engineers, this comprehensive and well-organized book deals with the internationally accepted Limit State Design. The Indian Code, IS 456 (1978), "Code of Practice for Plain and Reinforced Concrete" which covers mostly the Limit State Method, has been discussed in rich detail. The book also deals with the British and American Codes to familiarize the reader with the other major codes. What distinguishes the text is that it has a *large number of worked-out examples to illustrate the theory* and numerous tables and diagrams reproduced from the publications of BIS which would be of enormous help to the professional as well as the students. Besides, separate sections have been devoted to Detailing of Reinforcement in the field. Both the student and the professional should find this volume a useful companion and an invaluable reference.

P.C. VARGHESE (M.S., M. Engg. (Harvard), Ph.D.) is Honorary Professor of Civil Engineering at Anna University (College of Engineering, Guindy), Madras. Earlier, he taught at IIT Kharagpur and IIT Madras. He was also associated for a decade with the development of the University of Moratuwa, Colombo, as the UNESECO Chief Technical Advisor. Besides, he has worked as a concrete and soil specialist at Hirakud Dam Project, Construction Engineer at IIT Kharagpur, UN Advisor to the Ministry of Works and Housing, Sri Lanka, and as Consultant to a number of projects in India and abroad.

ISBN 81-203-0860-3

9 788120 308602