# **Design of Slabs**

## 2 MODULE No. 2: DEFLECTION LIMIT STATES

## 2.1 Inputs

## 2.1.1 Material Properties

- ✓ Concrete compressive strength (f<sub>cu</sub>)
- ✓ Yield strength of longitudinal reinforcing steel bars (fy)
- ✓ Modulus of elasticity of reinforcing steel bars (E<sub>s</sub> = 200,000 N/mm²)

#### 2.1.2 Section Definition

- ✓ Section width (b=1000 mm for 1-meter-wide strip)
- ✓ Slab thickness (t<sub>s</sub>)
- ✓ Concrete cover to tension steel
- ✓ Area of reinforcing steel in tension (A<sub>s</sub>)
- ✓ Area of reinforcing steel in compression (As')
- ✓ Concrete cover to compression steel (d')
- ✓ CL to CL span (L)
- ✓ Slab type (One way, two way or cantilever)
- ✓ Slab continuity (Continuous form one side, Continuous from both sides, Simple)
- ✓ Non-structural elements condition
  - Slab connected to non-structural element not affected by deflection
  - Slab connected to non-structural element affected by deflection

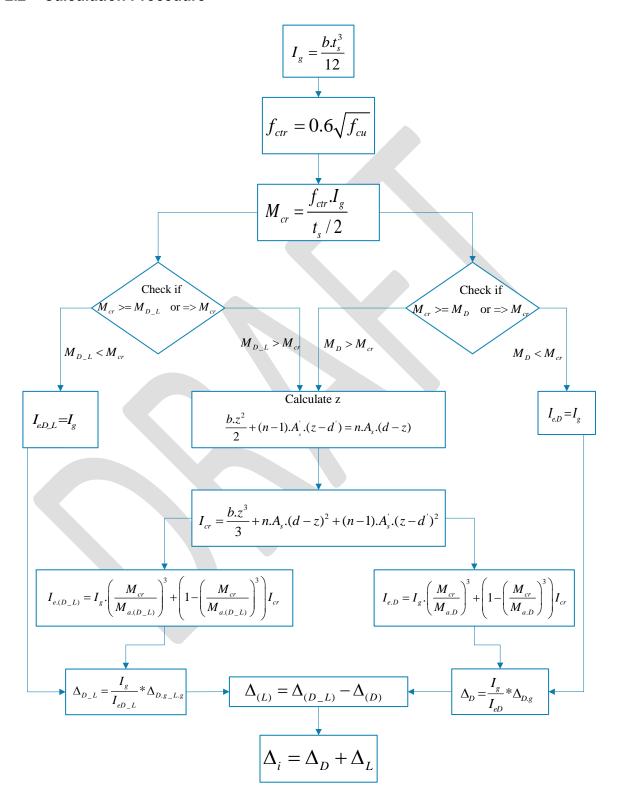
#### 2.1.3 Internal Forces

- ✓ Un-factored service Moment from analysis for dead loads (M<sub>D</sub>)
- ✓ Un-factored service Moment from analysis for dead and live loads (M<sub>D L</sub>)
- ✓ Deflection due to dead load ( $\Delta_{D.g}$ ) from gross moment of inertia analysis.
- ✓ Deflection due to dead and live loads  $(\Delta_{D,g\_L,g})$  from gross moment of inertia analysis.
- ✓ Percentage of sustainable live load.
- $\checkmark$  Time for long-term deflection calculation ( $T_1$ ) in months.

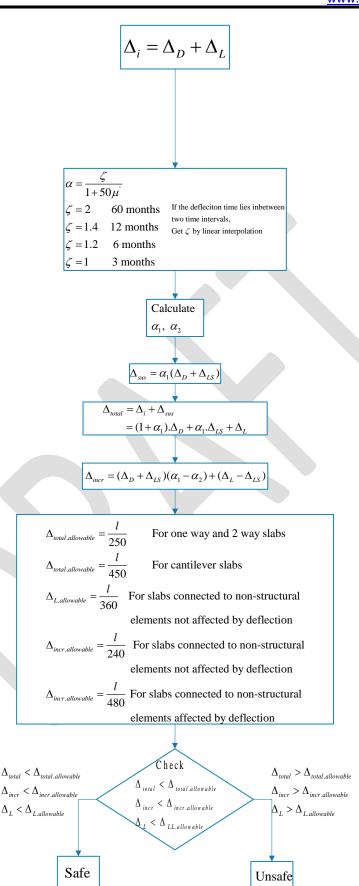


 $\checkmark$  Time of applying super-imposed dead loads (  $T_{\scriptscriptstyle 2}$  ) in months.

## 2.2 Calculation Procedure



www.gsconsultant.org



# 2.3 Design Outputs

- ✓ Check live load deflection.
- ✓ Check long-term deflection.
- ✓ Check additional deflection from applying super-imposed dead loads.



 $\Delta_{D.g} \coloneqq 3.38 \ \boldsymbol{mm}$ 



## 2.4 Example of Calculations using Mathcad

2.1 Input	
2.1.1 Material Properties	
$f_{cu} = 40 \frac{N}{mm^2}$	Concrete compressive strength
$f_{\mathbf{y}} = 360 \frac{\mathbf{N}}{\mathbf{mm}^2}$	Yield strength of reinforcing steel bar
$E_s = 200000 \frac{N}{mm^2}$	Modulus of elasticity of steel
2.1.2 Section Definition	
b := 1000 <b>mm</b>	Section width
$t_s\!\coloneqq\!250~m{mm}$	Slab thickness
conc.cover = 30 mm	Concrete Cover
$4_s = 393   \mathbf{mm}^2$	Area of reinforcing steel in tension
$4_s' = 393   \mathbf{mm}^2$	Area of reinforcing steel in compression
d'≔30 <b>mm</b>	Cover of compression steel.
L≔3.031 <b>m</b>	C.L to C.L span
$percent.sus.ll \coloneqq 0.25$	Percentage of sustained live load
$slab.type \coloneqq 1$	slab.type values 1= One way or two way 2= Cantilever
$slab.contin \coloneqq 3$	slab continuty values  1 = Continuous from two sides  2 = Continuous from one side  3 = simple
$non.structural \coloneqq 1$	Non-structural elements condition  1 = connected to non-structural elements  not affected by deflection.  2 = connected to non-structural elements  affected by deflection.
2.1.3 Internal Forces (From Ar	
$M_D \coloneqq 56.1 \; kN \cdot m$	Unfactored service Moment from dead load
$M_{D.g\_L.g}\!\coloneqq\!80.2$ kN·m	Unfactored service Moment from dead and
$\Delta_{D,g} \coloneqq 3.38   \mathbf{mm}$	live loads

Deflection due to dead loads



$$\Delta_{D,q\_L,q} \coloneqq 4.49 \ mm$$

$$T_1 \coloneqq 12$$

$$T_2 = 60$$

Deflection due to Dead and Live loads

Time of calculating long term deflection

Time of applying super imposed dead loads (months)

### 2.2 Calculation Procedure

## 2.2.1 General Calculations

$$E_{c} \coloneqq 4400 \; \frac{1000 \; \mathbf{kg}^{0.5}}{\mathbf{m}^{0.5} \cdot \mathbf{s}} \cdot \sqrt{f_{cu}} = \left\langle 2.783 \cdot 10^{4} \right\rangle \; \frac{\mathbf{N}}{\mathbf{mm}^{2}}$$

$$n \coloneqq \frac{E_s}{E_c} = 7.187$$

$$d := t_s - conc.cover = 220$$
 mm

$$L_e := \mathbf{if}(slab.contin = 1, 0.76 \cdot L, \mathbf{if}(slab.contin = 2, 0.87 \cdot L, L)) = \langle 3.031 \cdot 10^3 \rangle$$
 mm

$$I_g := \frac{b \cdot t_s^3}{12} = \langle 1.302 \cdot 10^9 \rangle \ \textit{mm}^4$$

## Gross-moment of inertia

$$f_{ctr} = 0.6 \ \frac{1000 \ kg^{0.5}}{m^{0.5} \cdot s} \cdot \sqrt[2]{f_{cu}} = 3.795 \ \frac{N}{mm^2}$$

$$M_{cr} \coloneqq \frac{f_{ctr} \cdot I_g}{\frac{t_s}{2}} = 39.528 \text{ kN} \cdot \text{m}$$

## Cracking Moment

$$z = 1 \, mm$$

$$z \coloneqq 1 \ \boldsymbol{m} \boldsymbol{m} \qquad z \coloneqq \operatorname{root} \left( \frac{b \cdot z^2}{2} - n \cdot A_s \cdot (d - z) + (n - 1) \cdot A_s' \cdot (z - d'), z \right) = 32.378 \ \boldsymbol{m} \boldsymbol{m}$$

$$I_{cr} = b \cdot \frac{z^3}{3} + n \cdot A_s \cdot (d-z)^2 + (n-1) \cdot A_s' \cdot (z-d')^2 = (1.108 \cdot 10^8) \, \,$$
mm<sup>4</sup>

#### 2.2.2 Check Dead Load

 $Check1 \coloneqq \mathbf{if} \langle M_{cr} > M_D, \text{``Ie}(\text{Dead load}) = \text{Ig"}, \text{``Calculate Ie"} \rangle = \text{``Calculate Ie"}$ 

$$I_{e.D} \coloneqq \left(\frac{M_{cr}}{M_D}\right)^3 \cdot I_g + \left(1 - \left(\frac{M_{cr}}{M_D}\right)^3\right) \cdot I_{cr} = \left(5.275 \cdot 10^8\right) \; \boldsymbol{mm}^4$$

$$I_{e.D}\!\coloneqq\!\mathbf{if}\big\langle Check1 = \text{``Calculate Ie"}, I_{e.D}, I_g\big\rangle \!=\! \big\langle 5.275 \cdot 10^8 \, \big\rangle \,\, \pmb{mm}^4$$

#### 2.2.2 Check Total Load

 $Check2 \coloneqq \mathbf{if} \langle M_{cr} > M_{D.g\_L.g}, \text{``Ie}(\text{Dead+Liave}) = \text{Ig''}, \text{``Calculate Ie''} \rangle = \text{``Calculate Ie''}$ 

$$I_{eD\_L} \coloneqq \left(\!\frac{M_{cr}}{M_{D.g\_L.g}}\!\right)^{\!3} \cdot I_g + \left(1 - \left(\!\frac{M_{cr}}{M_{D.g\_L.g}}\!\right)^{\!3}\right) \cdot I_{cr} \! = \! \left\langle2.534 \cdot 10^{\,8}\right\rangle \, \boldsymbol{mm}^{\,4}$$

$$I_{e.D\_L}\!\coloneqq\!\mathbf{if}\left\langle Check2=\text{``Calculate Ie''}\,,I_{eD\_L}\,,I_g\right\rangle\!=\!\left\langle 2.534\cdot10^8\right\rangle\;\pmb{mm}^4$$

### 2.2.3 Deflection Calculations

$$\Delta_D := \frac{I_g}{I_{s,D}} \cdot \Delta_{D,g} = 8.343 \text{ mm}$$

$$\Delta_{D\_L} \coloneqq \frac{I_g}{I_{e,D\_L}} \cdot \Delta_{D,g\_L,g} = 23.072 \ \textit{mm}$$

$$\Delta_L = \Delta_D L - \Delta_D = 14.729 \ mm$$

$$\Delta_{LS} \coloneqq percent.sus.ll \cdot \Delta_{L} = 3.682$$
 mm

$$\Delta_i \coloneqq \Delta_L + \Delta_D = 23.072 \ mm$$

Dead load deflection

Dead and Live load deflection

Live load deflection

sustained live load deflection

Instantinuous deflection

$$\zeta_1\!\coloneqq\!\mathbf{if}\left\langle T_1\!=\!3\,,1\,,\mathbf{if}\left\langle T_1\!=\!6\,,1.2\,,\mathbf{if}\left\langle T_1\!=\!12\,,1.4\,,\mathbf{if}\left\langle T_1\!\geq\!60\,,2\,,\text{``Use Linear Interpolation''}\right)\right\rangle\right\rangle$$

$$\zeta_1 \!=\! 1.4 \hspace{1cm} \alpha_1 \!:=\! \frac{\zeta_1}{1 \!+\! 50 \!\cdot\! \frac{A_s'}{b \!\cdot\! d}} \!=\! 1.285$$

$$\zeta_2\!\coloneqq\!\mathbf{if}\left\langle T_2\!=\!3\,,1\,,\mathbf{if}\left\langle T_2\!=\!6\,,1.2\,,\mathbf{if}\left\langle T_2\!=\!12\,,1.4\,,\mathbf{if}\left\langle T_2\!\geq\!60\,,2\,,\text{``Use Linear Interpolation''}\right)\right\rangle\right\rangle$$

$$\zeta_2 = 2 \qquad \qquad \alpha_2 \coloneqq \frac{\zeta_2}{1 + 50 \cdot \frac{A_s'}{h \cdot d}} = 1.836$$

$$\Delta_{total} \coloneqq \left(1 + \alpha_1\right) \cdot \Delta_D + \alpha_1 \cdot \Delta_{LS} + \Delta_L = 38.527 \ \textit{mm}$$

$$\Delta_{incr} := \langle \alpha_1 - \alpha_2 \rangle \cdot \langle \Delta_D + \Delta_{LS} \rangle + \langle \Delta_L - \Delta_{LS} \rangle = 4.423$$
 mm

#### 2.2.4 Allowable Deflection Calculations

$$\Delta_{\text{total.allow}} \coloneqq \text{if} \left( slab.type = 1, \frac{L_e}{250}, \frac{L_e}{450} \right) = 12.124 \text{ mm}$$

$$\Delta_{\text{L.allow}} = \frac{L_e}{360} = 8.419 \text{ mm}$$

$$\mathbf{\Delta_{incr.allow}} \coloneqq \mathbf{if} \left( non.structural = 1 \,, \frac{L_e}{240} \,, \frac{L_e}{480} \right) = 12.629 \,\, \mathbf{mm}$$

#### 2.2.4 Deflection Checks

$$\textbf{Check3} \coloneqq \textbf{if} \left( \triangle_{total} \negthinspace < \negthinspace \Delta_{\textbf{total.allow}} \text{, "Total Deflection Safe", "Total Deflection Unsafe"} \right)$$

$$\mathbf{Check4} \coloneqq \mathbf{if} \left( \Delta_L < \Delta_{\mathbf{L.allow}}, \text{``Live Load Deflection Safe''}, \text{``Live Load Deflection UnSafe''} \right)$$

$$\textbf{Check5} \coloneqq \textbf{if} \left( \triangle_{incr} < \Delta_{\textbf{incr.allow}}, \text{``Additional Deflection Safe"}, \text{``Additional Deflection UnSafe"} \right)$$

## 2.3 Output

Check3 = "Total Deflection Unsafe"

Check4 = "Live Load Deflection UnSafe"

Check5 = "Additional Deflection Safe"

