18 Reinforced Concrete Design

Design of Columns 2

- Combined Axial-Bending
- Interaction Diagram
- Biaxially Loaded Column

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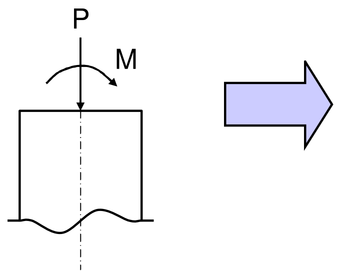
Combined Axial Load and Bending Moments

Bending moments can occur in columns because:

- Unbalance gravity loads
- Lateral loads: wind, earthquake

Equivalent Eccentricity $e = M/P$

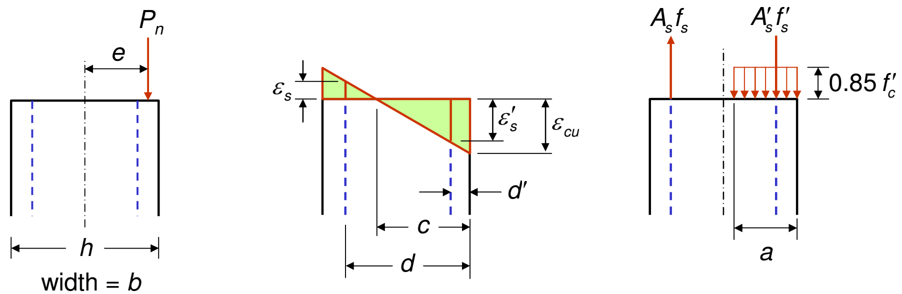


Minimum Eccentricity: $e_{min} = 0.6 + 0.03h$

Working Stress Design (WSD): $P_a \geq P\,,\;\; M_a \geq M$

Strength Design Method (SDM): $\phi P_n \geq P_u\,,\;\; \phi M_n \geq M_u$

Column subjected to eccentric compression



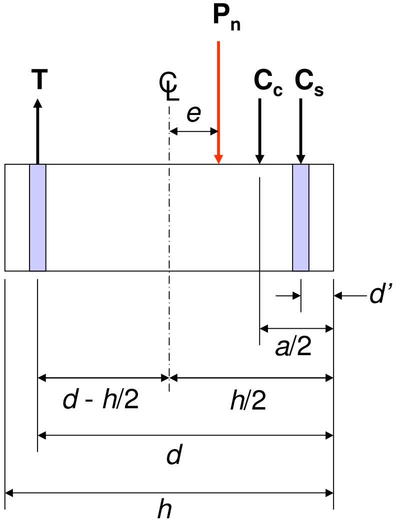
Equilibrium between external and internal axial forces requires that

$$[\Sigma F_y] \qquad P_n = 0.85 f'_c\, a\, b + A'_s f'_s - A_s f_s$$

Moment about centerline of the section of internal stresses and forces must be equal and opposite to the moment of external force P_n, so that

$$[\Sigma M_{\mathrm{CL}}] \qquad M_n = P_n e = 0.85 f'_c\, a b\left(\frac{h}{2}-\frac{a}{2}\right) + A'_s f'_s\left(\frac{h}{2}-d'\right) + A_s f_s\left(d-\frac{h}{2}\right)$$

Moment Strength of Column



Taking moment about centroid of section:

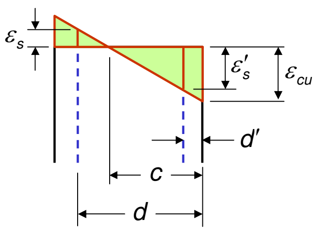
$$M_{n1} = C_c\left(\frac{h}{2}-\frac{a}{2}\right)+C_s\left(\frac{h}{2}-d'\right)+T\left(d-\frac{h}{2}\right)$$

$$= P_n \cdot e$$

Taking moment about tension steel:

$$M_{n2} = C_c\left(d-\frac{a}{2}\right)+C_s\left(d-d'\right)$$

$$= P_n \cdot \left(e+d-\frac{h}{2}\right)$$



Tension steel:

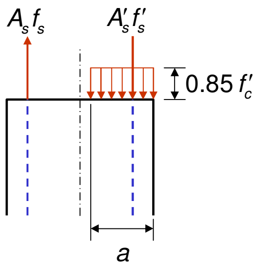
$$\varepsilon_s = \varepsilon_{cu}\frac{d-c}{c}$$

$$f_s = \varepsilon_s E_s = \varepsilon_{cu} E_s \frac{d-c}{c} \leq f_y$$

Compression steel:

$$\varepsilon_s' = \varepsilon_{cu}\frac{c-d'}{c}$$

$$f_s' = \varepsilon_s' E_s = \varepsilon_{cu} E_s \frac{c-d'}{c} \leq f_y$$



Concrete stress block:

$$a = \beta_1 c \leq h$$

$$C = 0.85\, f_c'\, a\, b$$

If we know

Neutral axis (c, a)

↓

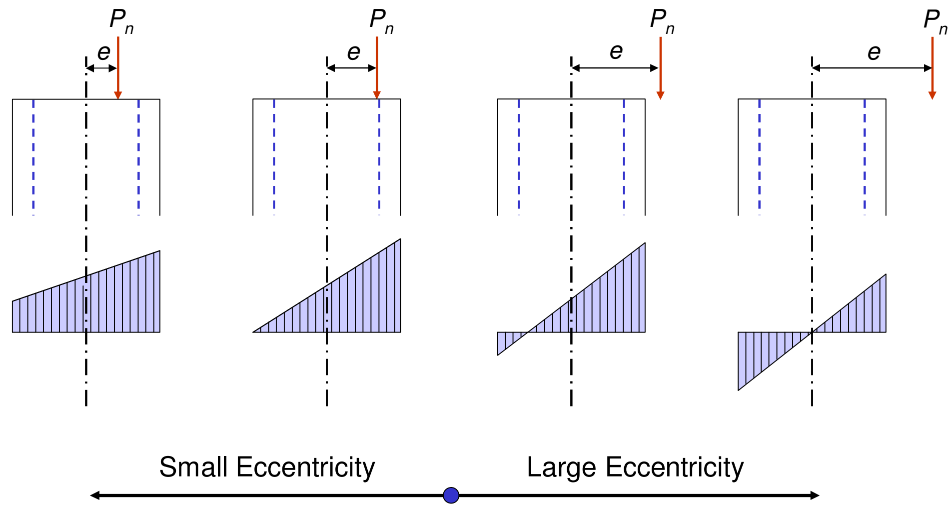
Strain condition
(ε_s , ε'_s)

↓

Stress condition
(f_s , f'_s)

↓

Column strength
(M_n, P_n)



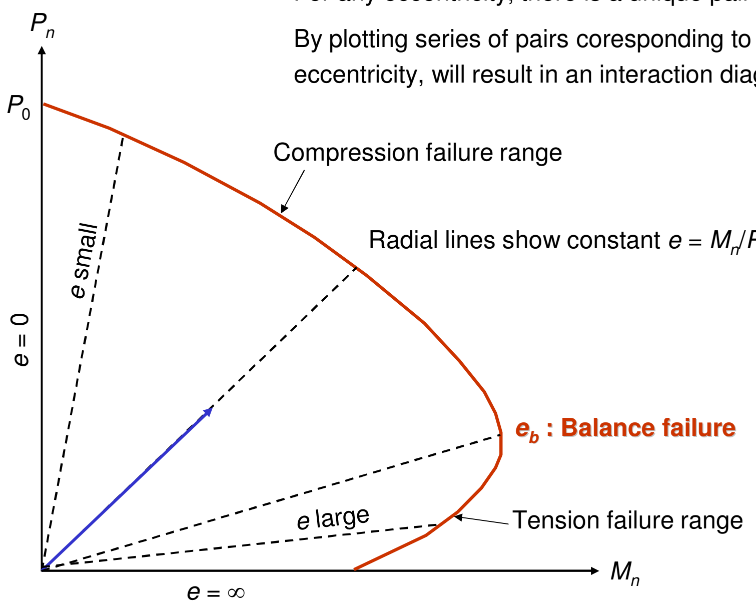
Large e $\rightarrow$ $f_s = f_y$ when $\varepsilon_c = \varepsilon_{cu} = 0.003$ (tension failure)

Small e $\rightarrow$ $f_s < f_y$ when $\varepsilon_c = \varepsilon_{cu} = 0.003$ (compression failure)

Interaction Diagram for Combined Bending and Axial Load

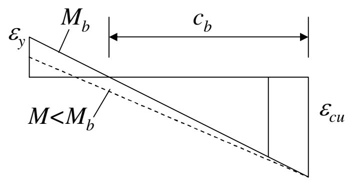
For any eccentricity, there is a unique pair of P_n and M_n.

By plotting series of pairs coresponding to a different eccentricity, will result in an interaction diagram.



Failure Mode Justification using e_b

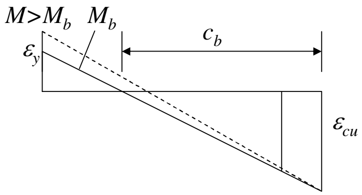
Case 1: $e < e_b$



$c > c_b \longrightarrow \varepsilon_s < \varepsilon_y \longrightarrow f_s < f_y$

Compression Failure

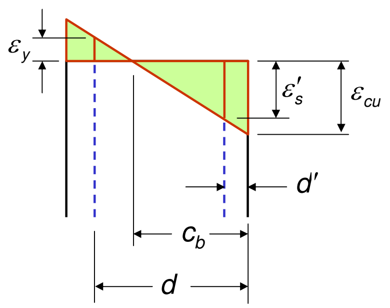
Case 2: $e > e_b$



$c < c_b \longrightarrow \varepsilon_s > \varepsilon_y \longrightarrow f_s > f_y$

Tension Failure

Balanced Failure, e_b



$M_b \;=\; P_b\, e_b$ Condition of failure that :

Concrete reaches the strain limit: ε_{cu} and

Tensile steel reaches the yield strain: ε_y

$$c_b \;=\; d\frac{\varepsilon_{cu}}{\varepsilon_{cu}+\varepsilon_y} \;=\; d\frac{6{,}120}{6{,}120+f_y}$$

$$a_b \;=\; \beta_1 c_b$$

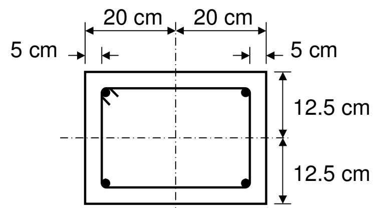
$$f_s' \;=\; \varepsilon_s' E_s \;=\; \varepsilon_{cu} E_s \frac{c_b - d'}{c_b} \;\le\; f_y$$

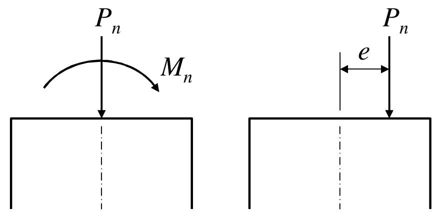
$$P_b \;=\; 0.85 f_c' a_b\, b \;+\; A_s' f_s' \;-\; A_s f_y$$

$$M_b \;=\; 0.85 f_c' a_b\, b\left(\frac{h}{2}-\frac{a_b}{2}\right) \;+\; A_s' f_s'\left(\frac{h}{2}-d'\right) \;+\; A_s f_y\left(d-\frac{h}{2}\right)$$

$$e_b \;=\; M_b / P_b$$

EXAMPLE 1 Column strength interaction diagram. A 25 x 40 cm column is reinforced with 4DB28. Concrete strength f'_c = 280 ksc and the steel yield strength f_y = 4,000 ksc





Balance condition:

$$c_b = \left(\frac{6{,}120}{6{,}120+4{,}000}\right)35 = 21.2 \text{ cm}, \qquad a_b = 0.85(21.2) = 18.0 \text{ cm}$$

$$C_c = 0.85(0.28)(18)(25) = 107 \text{ ton}$$

$$f_s' = E_s\varepsilon_{cu}\left(\frac{c-d'}{c}\right) = 6{,}120\left(\frac{21.2-5}{21.2}\right) = 4{,}677 \text{ ksc} \qquad \text{Yielding} \;\rightarrow\; f_s' = 4{,}000 \text{ ksc}$$

$$C_s = A_s' f_y = 12.32(4.0) = 49.3 \text{ ton}$$

$h = 40$ cm
$d' = 5$ cm
$d = 35$ cm
$a = 18$ cm

$$T = A_s f_y = 12.32(4.0) = 49.3 \text{ ton}$$

$$P_b = C_c + C_s - T = 107 + 49.3 - 49.3 = 107 \text{ ton}$$

$$M_b = C_c\left(\frac{h}{2} - \frac{a}{2}\right) + C_s\left(\frac{h}{2} - d'\right) + T\left(d - \frac{h}{2}\right)$$

$$= 107(20 - 18/2) + 49.3(20 - 5) + 49.3(35 - 20)$$

$$= 2{,}656 \text{ ton-cm} \quad = 26.6 \text{ ton-m}$$

$$e_b = \frac{M_b}{P_b} = \frac{2{,}656}{107} = 24.8 \text{ cm}$$

For c smaller than c_b = 21.2 cm will give e larger than e_b : tension failure

For example, choose c = 10 cm. By definition $f_s = f_y$

$$f_s' = 6{,}120\left(\frac{10-5}{10}\right) = 3{,}060 \text{ ksc}$$

$$a = 0.85 \times 10 = 8.5 \text{ cm}$$

$$C = 0.85(0.28)(8.5)(25) = 50.6 \text{ ton}$$

$$P_n = 50.6 + 12.32 \times 3.06 - 12.32 \times 4.0 = 39 \text{ ton}$$

$$M_n = 50.6(20 - 8.5/2) + 12.32 \times 3.06(20 - 5) + 12.32 \times 4.0(35 - 20)$$

$$= 2{,}102 \text{ ton-cm} = 21.0 \text{ t-m}$$

$$e = \frac{M_n}{P_n} = \frac{2{,}102}{39} = 53.9 \text{ cm}$$

For c larger than c_b = 21.2 cm will give e smaller than e_b : compression failure

For example, choose c = 30 cm.

$$a = 0.85 \times 30 = 25.5 \text{ cm}$$

$$C = 0.85(0.28)(25.5)(25) = 152 \text{ ton}$$

$$f_s = E_s \varepsilon_{cu}\left(\frac{d-c}{c}\right) = 6{,}120\left(\frac{35-30}{30}\right) = 1{,}020 \text{ ksc}$$

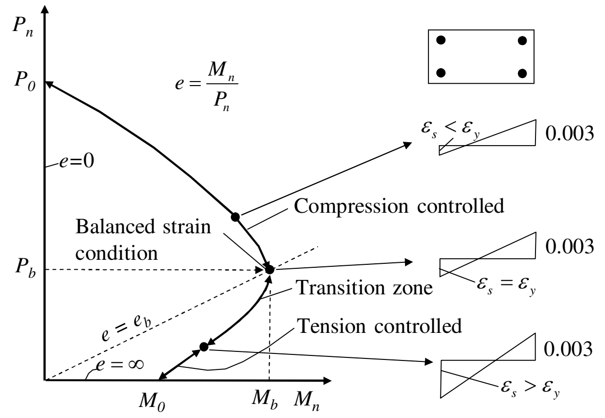
$$f_s' = 6{,}120\left(\frac{30-5}{30}\right) = 5{,}100 \text{ ksc} \qquad \text{Yielding} \rightarrow f_s' = 4{,}000 \text{ ksc}$$

$$P_n = 152 + 12.32 \times 4.0 - 12.32 \times 1.02 = 189 \text{ ton}$$

$$M_n = 152(20 - 25.5/2) + 12.32 \times 4.0(20 - 5) + 12.32 \times 1.02(35 - 20)$$

$$= 2{,}030 \text{ ton-cm} = 20.3 \text{ t-m} \qquad e = \frac{M_n}{P_n} = \frac{2{,}030}{189} = 10.7 \text{ cm}$$

Interaction Diagram

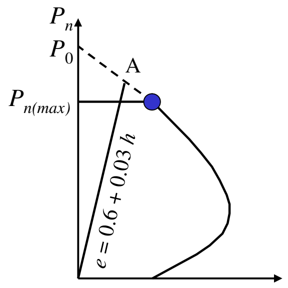


Maximum Strength in Axial Compression-ACI Code

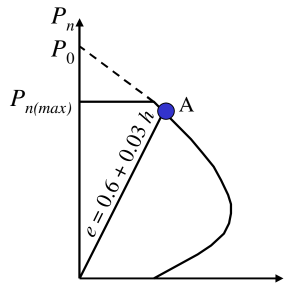
Tied column: $P_n = 0.80\left[0.85 f_c'\left(A_g - A_{st}\right) + f_y A_{st}\right]$

Spiral column: $P_n = 0.85\left[0.85 f_c'\left(A_g - A_{st}\right) + f_y A_{st}\right]$

Minimum eccentricity: $e = 0.6 + 0.03\ h$

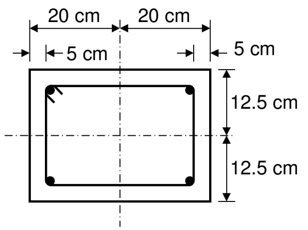


USE $P_{n(max)}$



USE P_n at point A

Column strength interaction diagram. A 25 x 40 cm column is reinforced with 4DB28. Concrete strength f'_c = 280 ksc and the steelyield strength f_y = 4,000 ksc



h = 40 cm
d' = 5 cm
d = 35 cm

Material Properties

f_c =	28.00	MPa	E_c =	25,045	MPa
f_y =	400.00	MPa	E_s =	200,000	MPa
ϕ_b =	1				
ϕ_c =	1				

Units: English / Metric

Compute Interaction Diagram

Rectangular Concrete Section

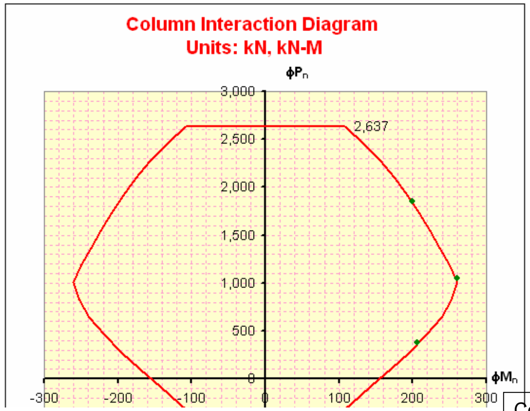
Height H =	400.00	mm
Width B =	250.00	mm
ρ_s =	2.46%	

RC18_InterDiagram.xls
Microsoft Excel Worksheet
83 KB

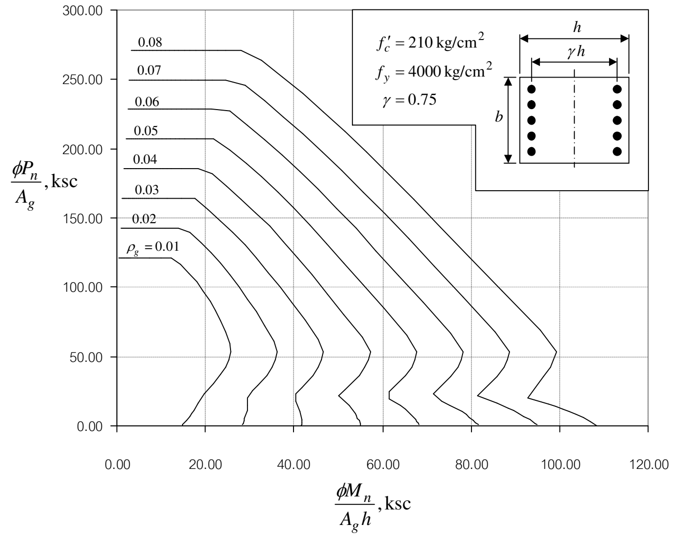
Steel Layer No.	Steel Area A_s mm^2	Dist. from bottom Y_s mm
1	1232.00	50.00
2	1232.00	350.00
3		

Factored Loads

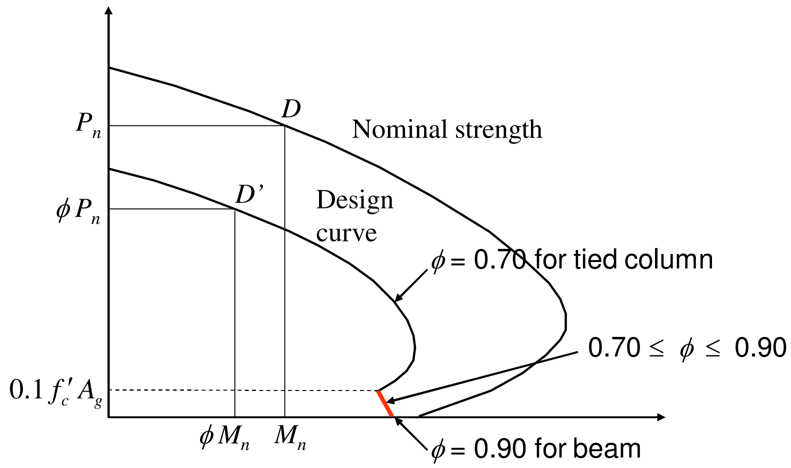
	Pu kN	Mu kN-M
1	1050	261
2	382	206
3	1852	199



Interaction Diagram of Rectangular Columns



Interaction Diagram for Design



Example: Design a rectangular reinforced concrete column with ties for service dead and live loads of 120 and 80 tons, respectively. Service dead and live load moments at the top about the strong axis are 15 and 8 t-m, respectively. Moments are negligible about weak axis. Assume moments at the bottom of the column as half those at the top. The column has unsupported height of 2.5 m and is bent in double curvature about strong axis and single curvature about the weak axis. Use f'_c = 210 ksc and f_y = 4,000 ksc.

1) Determine required strength

P_u = 1.4(120) + 1.7(80) = 304 ton

M_u = 1.4(15) + 1.7(8) = 34.6 t-m

2) Check column slenderness. Assume a 40 x 60 cm column size

a. Slenderness about weak axis (40 cm width)

k = 1.0 for braced compression member

r = 0.3x40 = 12 cm

kL_u/r = (1.0)(250)/12 = 20.8

With negligible moment about the weak axis, assume $M_1/M_2 = 1.0$

$kL_u/r < 34 - 12(M_1/M_2) = 22$ (single curvature)

Therefore, slenderness may be neglect about the weak axis.

b. Slenderness about strong axis (60 cm width)

$k = 1.0$ for braced compression member

$r = 0.3\text{x}60 = 18$ cm

$kL_u/r = (1.0)(250)/18 = 13.9$

$M_1/M_2 = -0.5$

$kL_u/r < 34 - 12(-0.5) = 40$

Therefore, slenderness may be neglect about the strong axis.

3) Design of column reinforcement

Compute $\dfrac{P_u}{A_g} = \dfrac{304(1{,}000)}{40 \times 60} = 126.7$

$$\frac{M_u}{A_g h} = \frac{34.6(1{,}000)(100)}{40 \times 60 \times 60} = 24.03$$

From interaction diagram, read $\rho_g = 0.025$

$A_{st} = \rho_g A_g = 0.025(40)(60) = 60.0\ \text{cm}^2$

USE 10DB28(A_s=61.58cm^2) Place 5DB28 bars on each 40 cm side.

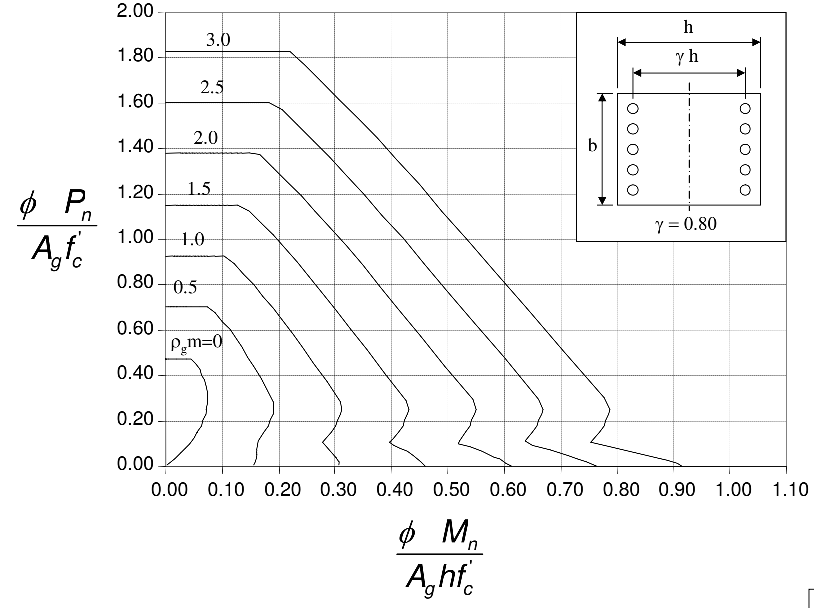
4) Select lateral reinforcement

Use DB10 ties with DB28 longitudinal bars

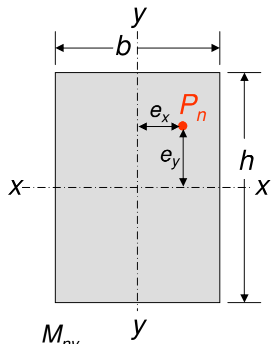
Spacing not greater than:	16(2.8)	= 44.8 cm
	48(1.0)	= 48.0 cm
	column size	= 40 cm **(control)**

USE DB10 @ 40 cm

Normalized Interaction Diagram



Biaxial Bending and Compression

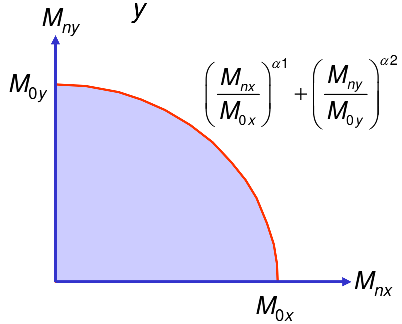


$M_{nx} = P_n e_{nx}$

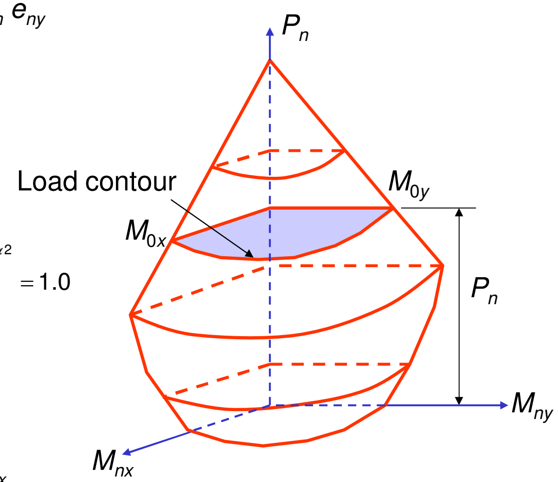
$M_{ny} = P_n e_{ny}$

Load Contour Method:

Cut failure surface at constant P_n



$$\left(\frac{M_{nx}}{M_{0x}}\right)^{\alpha 1} + \left(\frac{M_{ny}}{M_{0y}}\right)^{\alpha 2} = 1.0$$



Modified Load Contour Method

The interaction expression for the load and bending moments about the two axes is

$$\left(\frac{P_n - P_{nb}}{P_{no} - P_{nb}}\right) + \left(\frac{M_{nx}}{M_{nbx}}\right)^{1.5} + \left(\frac{M_{ny}}{M_{nby}}\right)^{1.5} = 1.0$$

where

P_n = nominal axial compression (positive), or tension (negative)

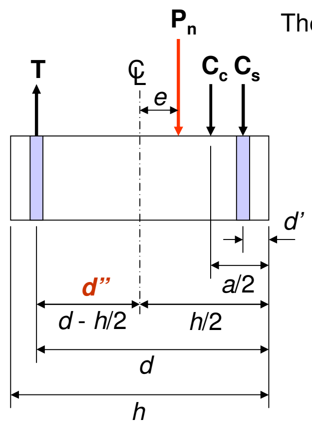
M_{nx}, M_{ny} = nominal bending moments about the *x*- and *y*-axis respectively

P_{no} = maximum nominal axial compression (positive) or axial tension (negative)

= $0.85\, f'_c\,(A_g - A_{st}) + f_y A_{st}$

P_{nb} = nominal axial compression at the limit strain states ($\varepsilon_t = 0.002$)

M_{nbx}, M_{nby} = nominal bending moment about the *x*- and *y*-axis respectively, at the limit strain state ($\varepsilon_t = 0.002$)



The value of P_{nb} and M_{nb} can be obtained from:

$$P_{nb} = 0.85 f'_c \beta_1 c_b b + A'_s f'_s - A_s f_y$$

$$M_{nb} = P_{nb} e_b = C_c\left(d - \frac{a}{2} - d''\right) + C_s(d - d' - d'') + T d''$$

where

a = depth of the equivalent block = $\beta_1 c_b$

$$c_b = \left(\frac{0.003}{f_y / E_s + 0.003}\right) d = \left(\frac{6{,}120}{6{,}120 + 4{,}000}\right) d$$

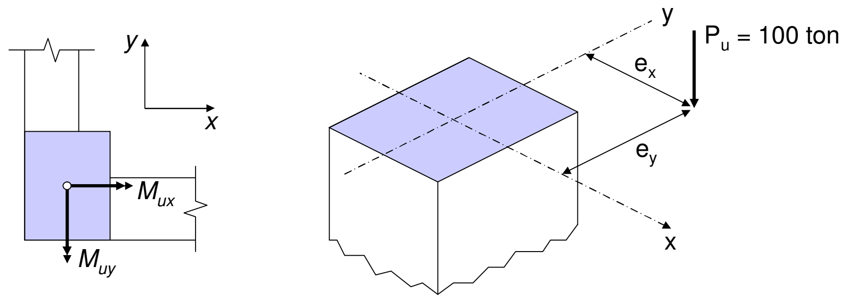
f'_s = stress in the compressive reinforcement

$= f_y$ if $f'_s \geq f_y$

T = Force in the tensile side reinforcement

EXAMPLE 18 – 3 : Design of a Biaxially Loaded Column by the Modified Load Contour Method

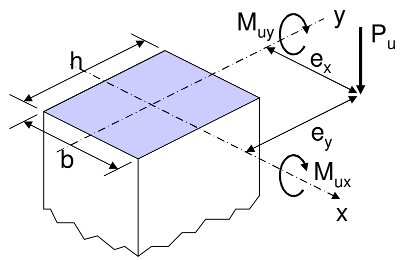
A nonslender corner column is subjected to a factored compressive load P_u = 100 ton, a factored bending moment M_{ux} = 18 t-m about the *x* axis, and a factored bending moment M_{uy} = 12 t-m about *y* axis. Given f'_c = 280 ksc, f_y = 4,000 ksc



Solution: Step 1: Calculate equivalent uniaxial bending moment assuming equal numbers of bars on all faces

Assume that ϕ = 0.70 for tied columns.

Required nominal P_n = 100/0.7 = 143 ton



Required nominal M_{nx} = 18/0.7 = 25.7 t-m

Required nominal M_{ny} = 12/0.7 = 17.1 t-m

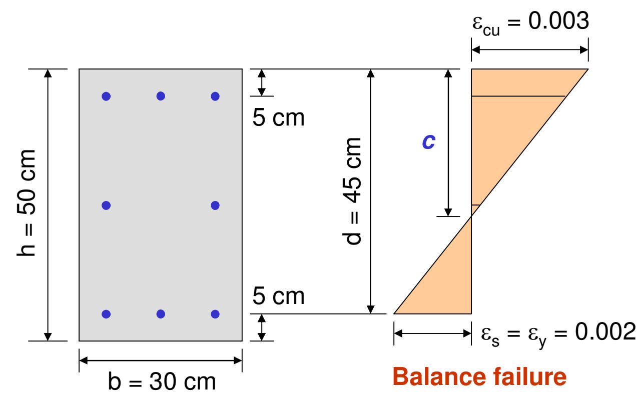
$e_y = M_{nx} / P_n$ = 25.7(100)/143 = 18.0 cm

$e_x = M_{ny} / P_n$ = 17.1(100)/143 = 12.0 cm

x : axis parallel to the shorter side *b*

y : axis parallel to the longer side *h*

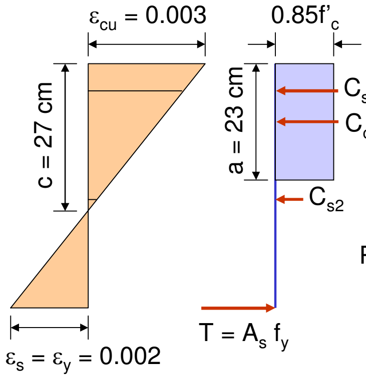
Assume column section : b = 30 cm, h = 50 cm, d' = 5 cm, and A_s = 8DB25



$$\frac{c}{d} = \frac{0.003}{0.003 + 0.002} = 0.6$$

c = 0.6(45) = 27 cm

a = β_1c = 0.85(27) = 23 cm



Compute P_{nb} :

f'_s = 6120(1-d'/c) = 6120(1-5/27)

= 4987 ksc USE f_y = 4000 ksc

f'_s = 6120(1-25/27) = 453 ksc

P_{nb} = nominal axial compression at the limit strain

= $C_c + C_{s1} + C_{s2} - T$

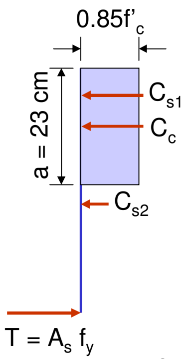
$C_c = 0.85f'_c$ b a = 0.85(0.28)(30)(23) = 164.2 ton

C_{s1} = (3DB25=14.73)(4.0) = 58.9 ton

C_{s2} = (2DB25=9.82)(0.453) = 4.5 ton

T = (3DB25=14.73)(4.0) = 58.9 ton

P_{nb} = 164.2 + 58.9 + 4.5 – 58.9 = 168.7 ton



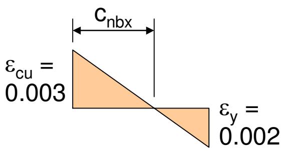
Compute M_{nbx} :

$$M_{nbx} = C_c\left(\frac{h}{2} - \frac{a}{2}\right) + C_s\left(\frac{h}{2} - d'\right) + T\left(d - \frac{h}{2}\right)$$

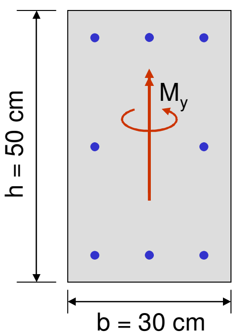
M_{nbx} = 164.2(25 – 23/2) + 58.9(25 – 5) + 58.9(45 – 25)

= 4573 t-cm = 45.7 t-m

$$e_{by} = \frac{M_{nbx}}{P_{nb}} = \frac{4573}{168.7} = 27.1 \text{ cm}$$



$e_{by} > e_y$ = 17.8 cm, hence compression failure



Compute M_{nby} :

$a_{nbx} = \beta_1 c_{nbx}$ = 0.85(0.6)(d=25) = 13 cm

M_{nby} = 0.85×0.28×50×12.75(15 – 13/2) + 58.9(15 – 5)

+ 58.9(25 – 15)

= 2468 t-cm = 24.7 t-m

$$P_{no} = 0.85 f'_c (A_g - A_{st}) + A_{st} f_y$$

= 0.85 x 0.24 (30 x 50 – 8 x 4.91) + 8 x 4.49 x 4.0

= 442 ton

Using the interaction surface expression for biaxial bending

$$\left(\frac{P_n - P_{nb}}{P_{no} - P_{nb}}\right) + \left(\frac{M_{nx}}{M_{nbx}}\right)^{1.5} + \left(\frac{M_{ny}}{M_{nby}}\right)^{1.5}$$

$$= \frac{143 - 168.7}{442 - 168.7} + \left(\frac{25.7}{45.7}\right)^{1.5} + \left(\frac{17.1}{24.7}\right)^{1.5}$$

= -0.094 + 0.422 + 0.576 = 0.904 < 1.00 **OK**

Hence, accept the design, namely,

b = 30 cm, h = 50 cm, d = 45 cm, and A_s = 8DB25