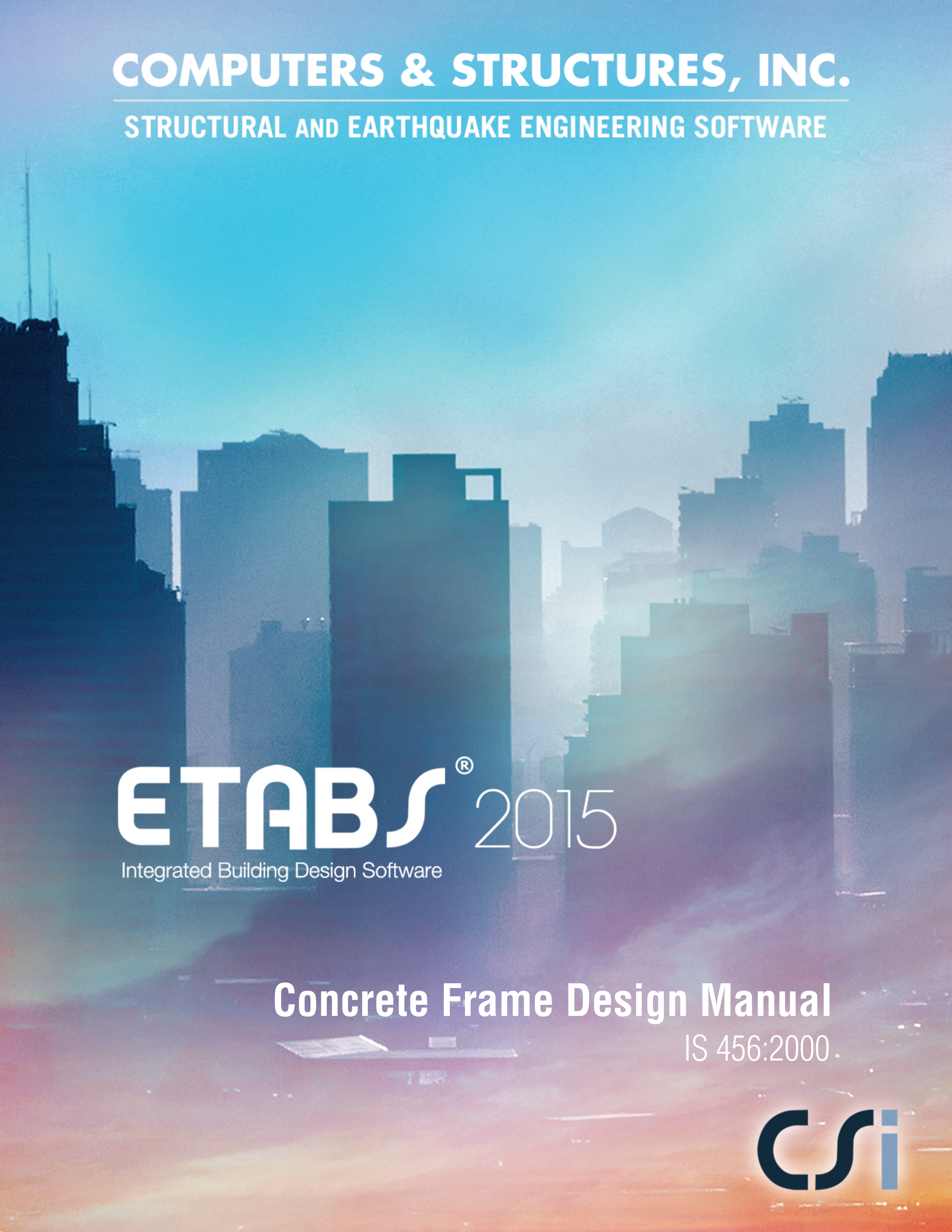

COMPUTERS & STRUCTURES, INC.

STRUCTURAL AND EARTHQUAKE ENGINEERING SOFTWARE

ETABS® 2015

Integrated Building Design Software

Concrete Frame Design Manual

IS 456:2000

Concrete Frame Design Manual

IS 456:2000

For ETABS® 2015

ISO ETA082914M26 Rev.0
Proudly developed in the United States of America December 2014

Copyright

Further information and copies of this documentation may be obtained from:

Computers & Structures, Inc.
http://www.csiamerica.com/

info@csiamerica.com (for general information)
support@csiamerica.com (for technical support)

DISCLAIMER

CONSIDERABLE TIME, EFFORT AND EXPENSE HAVE GONE INTO THE DEVELOPMENT AND TESTING OF THIS SOFTWARE. HOWEVER, THE USER ACCEPTS AND UNDERSTANDS THAT NO WARRANTY IS EXPRESSED OR IMPLIED BY THE DEVELOPERS OR THE DISTRIBUTORS ON THE ACCURACY OR THE RELIABILITY OF THIS PRODUCT.

THIS PRODUCT IS A PRACTICAL AND POWERFUL TOOL FOR STRUCTURAL DESIGN. HOWEVER, THE USER MUST EXPLICITLY UNDERSTAND THE BASIC ASSUMPTIONS OF THE SOFTWARE MODELING, ANALYSIS, AND DESIGN ALGORITHMS AND COMPENSATE FOR THE ASPECTS THAT ARE NOT ADDRESSED.

THE INFORMATION PRODUCED BY THE SOFTWARE MUST BE CHECKED BY A QUALIFIED AND EXPERIENCED ENGINEER. THE ENGINEER MUST INDEPENDENTLY VERIFY THE RESULTS AND TAKE PROFESSIONAL RESPONSIBILITY FOR THE INFORMATION THAT IS USED.

Contents

Chapter 1
Introduction

The design of concrete frames is seamlessly integrated within the program. Initiation of the design process, along with control of various design parameters, is accomplished using the **Design** menu.

Automated design at the object level is available for any one of a number of user-selected design codes as long as the structures have first been modeled and analyzed by the program. Model and analysis data, such as material properties and member forces, are recovered directly from the model database, and no additional user input is required if the design defaults are acceptable.

The design is based on a set of user-specified loading combinations. However, the program provides default load combinations for each supported design code. If the default load combinations are acceptable, no definition of additional load combinations is required.

In the design of columns, the program calculates the required longitudinal and shear reinforcement. However, the user may specify the longitudinal steel, in which case a column capacity ratio is reported. The column capacity ratio gives an indication of the stress condition with respect to the capacity of the column.

The biaxial column capacity check is based on the generation of consistent three-dimensional interaction surfaces. It does not use any empirical formulations that extrapolate uniaxial interaction curves to approximate biaxial action.

Interaction surfaces are generated for user-specified column reinforcing configurations. The column configurations may be rectangular, square, or circular, with similar reinforcing patterns. The calculation of moment magnification factors, unsupported lengths, and strength reduction factors is automated in the algorithm.

Every beam member is designed for flexure and shear at output stations along the beam span. All beam-column joints are investigated for existing shear conditions. For Ductile Moment Resisting frames, the shear design of the columns, beams, and joints is based on the moment capacities of the members. Also, the program will produce ratios of the beam moment capacities with respect to the column moment capacities, to investigate weak beam/strong beam aspects, including the effect of axial force.

Output data can be presented graphically on the model, in tables for both input and output data, or on a calculation sheet prepared for each member. For each presentation method, the output is in a format that allows the engineer to quickly study the stress conditions that exist in the structure and, in the event the member reinforcing is not adequate, aids the engineer in taking appropriate remedial measures, including altering the design member without rerunning the entire analysis.

1.1 Organization

This manual is designed to help you quickly become productive using the concrete frame design options of “Indian IS 456-2000.” Chapter 2 provides detailed descriptions of the Design Prerequisites used for the Indian IS-456 2000 code. Chapter 3 provides detailed descriptions of the process used when the Indian IS 456-2000 code is selected. The appendices provide details on certain topics referenced in this manual.

1.2 Recommended Reading/Practice

It is strongly recommended that you read this manual and review any applicable "Watch & Learn" Series™ tutorials, which are found on our web site, http://www.csiamerica.com, before attempting to design a concrete frame. Additional information can be found in the online Help facility available from within the program.

Chapter 2
Design Prerequisites

This chapter provides an overview of the basic assumptions, design preconditions, and some of the design parameters that affect the design of concrete frames.

In writing this manual it has been assumed that the user has an engineering background in the general area of structural reinforced concrete design and familiarity with the IS 456:2000 code.

2.1 Design Load Combinations

The design load combinations are used for determining the various combinations of the load cases for which the structure needs to be designed/checked. The load combination factors to be used vary with the selected design code. The load combination factors are applied to the forces and moments obtained from the associated load cases and are then summed to obtain the factored design forces and moments for the load combination.

For multi-valued load combinations involving response spectrum, time history, and multi-valued combinations (of type enveloping, square-root of the sum of the squares or absolute) where any correspondence between interacting quantities is lost, the program automatically produces multiple sub combinations using maxima/minima permutations of interacting quantities. Separate combinations with negative factors for response spectrum cases are not required because the program automatically takes the minima to be the negative of the maxima for response spectrum cases and the previously described permutations generate the required sub combinations.

When a design combination involves only a single multi-valued case of time history, further options are available. The program has an option to request that time history combinations produce sub combinations for each time step of the time history. Also an option is available to request that moving load combinations produce sub combinations using maxima and minima of each design quantity but with corresponding values of interacting quantities.

For normal loading conditions involving static dead load, live load, wind load, and earthquake load, and/or dynamic response spectrum earthquake load, the program has built-in default loading combinations for each design code. The combinations are based on the code recommendations and are documented for each code in the corresponding manuals.

For other loading conditions involving moving load, time history, pattern live loads, separate consideration of roof live load, snow load, and so on, the user must define design loading combinations either in lieu of or in addition to the default design loading combinations.

The default load combinations assume all static load cases declared as dead load to be additive. Similarly, all cases declared as live load are assumed additive. However, each static load case declared as wind or earthquake, or response spectrum cases, is assumed to be non-additive with each other and produces multiple lateral load combinations. Also wind and static earthquake cases produce separate loading combinations with the sense (positive or negative) reversed. If these conditions are not correct, the user must provide the appropriate design combinations.

The default load combinations are included in design if the user requests them to be included or if no other user-defined combination is available for concrete design. If any default combination is included in design, then all default combinations will automatically be updated by the program any time the design code is changed or if static or response spectrum load cases are modified.

Live load reduction factors can be applied to the member forces of the live load case on an element-by-element basis to reduce the contribution of the live load to the factored loading.

The user is cautioned that if time history results are not requested to be recovered in the analysis for some or all of the frame members, the effects of those loads will be assumed to be zero in any combination that includes them.

2.2 Design and Check Stations

For each load combination, each element is designed or checked at a number of locations along the length of the element. The locations are based on equally spaced segments along the clear length of the element. The number of segments in an element is requested by the user before the analysis is made. The user can refine the design along the length of an element by requesting more segments.

When using the IS 456:2000 design code, the requirements for joint design at the beam to column connections are evaluated at the top most station of each column. The program also performs a joint shear analysis at the same station to determine if special conditions are required in any of the joint panel zones. The ratio of the beam flexural capacities with respect to column flexural capacities considering axial force effects associated with the weak beam/strong column aspect of any beam-column intersection is reported.

2.3 Identifying Beams and Columns

In the program all beams and columns are represented as frame elements. But design of beams and columns requires separate treatment. Identification for a concrete element is accomplished by specifying the frame section assigned to the element to be of type beam or column. If any brace element exists in the frame, the brace element also would be identified as either a beam or a column element, depending on the section assigned to the brace element.

2.4 Design of Beams

In the design of concrete beams, in general, the program calculates and reports the required areas of steel for flexure and shear based on the beam moments, shears, load combination factors, and other criteria, which are described in detail in the next chapter. The reinforcement requirements are calculated at a user-defined number of stations along the beam span.

All of the beams are designed for major direction flexure, major shear and torsion only. Effects due to any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

In designing the flexural reinforcement for the major moment at a particular section of a particular beam, the steps involve the determination of the maximum factored moments and the determination of the reinforcing steel. The beam section is designed for the maximum positive and maximum negative factored moment envelopes obtained from all of the load combinations. If torsion is present, each design moment is modified by two fictitious moments that are the actual factored moment plus an equivalent moment due to torsion and the actual factored moment minus an equivalent moment due to torsion. The equivalent moment due to torsion is proportional to the torsion and the associated scale factor is a function of the beam section aspect ratio. Negative beam moments produce top steel. In such cases, the beam is always designed as a Rectangular section. Positive beam moments produce bottom steel. In such cases, the beam may be designed as a Rectangular beam or a T-Beam. For the design of flexural reinforcement, the beam is first designed as a singly reinforced beam. If the beam section is not adequate, the required compression reinforcement is calculated.

In designing the shear reinforcement for a particular beam for a particular set of loading combinations at a particular station due to the beam major shear, the steps involve the determination of the factored shear force, the determination of the shear force that can be resisted by concrete, and the determination of the reinforcement steel required to carry the balance. If there is any torsion, the design shear is modified by a fictitious shear that is the actual shear force plus an equivalent shear due to torsion. If the nominal shear stress due to the modified shear force is less than the nominal allowable shear stress, no closed hoop torsion rebar is needed. In that case, only minimum shear link is needed. However, if torsion is present and if the nominal shear stress due to the modified shear force is more than the nominal allowable shear stress, closed hoop torsion rebar is calculated.

Special considerations for seismic design as required in the seismic code “IS 13920:1993” are incorporated into the program for IS 456:2000.

2.5 Design of Columns

In the design of columns, the program calculates the required longitudinal steel, or if the longitudinal steel is specified, the column stress condition is reported in terms of a column capacity ratio, which is a factor that gives an indication of the stress condition of the column with respect to the capacity of the column. The design procedure for the reinforced concrete columns of the structure involves the following steps:

- Generate axial force-biaxial moment interaction surfaces for all of the different concrete section types of the model.
- Check the capacity of each column for the factored axial force and bending moments obtained from each loading combination at each end of the column. This step also is used to calculate the required reinforcement (if none was specified) that will produce a capacity ratio of 1.0.

The generation of the interaction surface is based on the assumed strain and stress distributions and some other simplifying assumptions. Those stress and strain distributions and the assumptions are documented in Chapter 3.

The shear reinforcement design procedure for columns is very similar to that for beams, except that the effect of the axial force on the concrete shear capacity needs to be considered.

For certain special seismic cases, the design of columns for shear is based on the capacity shear. The capacity shear force in a particular direction is calculated from the moment capacities of the beams framing into the column.

2.6 Design of Joints

To ensure that the beam-column joint of special moment resisting frames possesses adequate shear strength, the program performs a rational analysis of the beam-column panel zone to determine the shear forces that are generated in the joint. The program then checks this against design shear strength.

Only joints having a column below the joint are checked. The material properties of the joint are assumed to be the same as those of the column below the joint. The joint analysis is done in the major and the minor directions of the column. The joint design procedure involves the following steps:

- Determine the panel zone design shear force
- Determine the effective area of the joint
- Check panel zone shear stress

The joint design details are documented in Chapter 3.

2.7 P-Delta Effects

The program design process requires that the analysis results include the P-Delta effects. For the individual member stability effects (local effects), the moments are magnified with additional moments, as documented in Chapter 3 of this manual.

For lateral drift effects (global effect), the program assumes that the P-Delta analysis is performed and that the amplification is already included in the results. The moments and forces obtained from P-Delta analysis are further amplified for individual column stability effect as required by the code.

The users of the program should be aware that the default analysis option in the program is that P-Delta effects are not included. The user can include the P-Delta analysis and set the maximum number of iterations for the analysis. The default number of iteration for P-Delta analysis is 1. Further details on P-Delta analysis are provided in Appendix A of this design manual.

2.8 Element Unsupported Lengths

To account for column slenderness effect, the column unsupported lengths are required. The two unsupported lengths are l_{33} and l_{22}. These are the lengths between support points of the element in the corresponding directions. The length l_{33} corresponds to instability about the 3-3 axis (major axis), and l_{22} corresponds to instability about the 2-2 axis (minor axis).

Normally, the unsupported element length is equal to the length of the element, i.e., the distance between END-I and END-J of the element. The program, however, allows users to assign several elements to be treated as a single member for design. This can be done differently for major and minor bending, as documented in Appendix B of this design manual.

The user has options to specify the unsupported lengths of the elements on an element-by-element basis.

2.9 Choice of Input Units

English as well as SI and MKS metric units can be used for input. But the codes are based on a specific system of units. All equations and descriptions presented in this manual correspond to that specific system of units unless otherwise noted. For example, the IS 4546:2000 code is published in Millimeter-Newton-Second units. By default, all equations and descriptions presented in the "Design Process" chapter correspond to Millimeter-Newton- Second units. However, any system of units can be used to define and design the structure in the program.

Chapter 3
Design Process

This chapter describes in detail the various aspects of the concrete design procedure that is used by the program when the user selects the Indian IS 456-2000 code option in the program. This covers the basic design code "IS 456:2000 Indian Standard Plain and Reinforced Concrete Code of Practice (IS 2000), the seismic code "IS 13920:1993 (Reaffirmed 1998, Edition 1.2, 2002-2003), Indian Standard – Ductile Detailing of Reinforced Concrete Structures Subjected to Seismic Forces – Code of Practice" (IS 2002), and a part of the draft seismic code (Jain and Murty 2008). Various notations used in this chapter are listed in Section 3.1. For referencing to the pertinent sections of the Indian codes in this chapter, a prefix "IS" followed by the section number is used. The relevant prefixes are "IS," "IS 13920," and "IS 13920 Draft" for the basic code IS 456:2000, the seismic code IS 13920:1993, and the draft seismic code, respectively.

English as well as SI and MKS metric units can be used for input. The code is based on Millimeter-Newton-Second units. For simplicity, all equations and descriptions presented in this chapter correspond to Millimeter-Newton-Second units unless otherwise noted.

3.1 Notation

The various notations used in this chapter are described in this section:

A_c Area of concrete, mm^2

A_{cv} Area of section for shear resistance, mm^2
Area of concrete used to determine shear stress, mm^2

A_g Gross cross-sectional area of a frame member, mm^2
Gross area of concrete, mm^2

A_s Area of tension reinforcement, mm^2

A'_s Area of compression reinforcement, mm^2

$A_{s(\text{required})}$ Area of steel required for tension reinforcement, mm^2

A_{st} Total area of column longitudinal reinforcement, mm^2

A_{sv} Total cross-sectional area of links at the neutral axis, mm^2
Area of shear reinforcement, mm^2

A_{sv}/s_v Area of shear reinforcement per unit length of the member, mm^2/mm
Area of transverse torsion reinforcement (closed stirrups) per unit length of the member, mm^2/mm

D Overall depth of a beam or slab, mm

D_f Flange thickness in a T-beam, mm

E_c Modulus of elasticity of concrete, N/mm^2

E_s Modulus of elasticity of reinforcement, assumed as 200,000 N/mm^2

I_g Moment inertia of gross concrete section about the centroidal axis neglecting reinforcement, mm^4

I_{se} Moment of inertia of reinforcement about the centroidal axis of member cross-section, mm^4

K_{22} Effective length factor in local axis 2

K_{33}	Effective length factor in local axis 3
L	Clear unsupported length, mm
M_{ns}	Non-sway component of factored end moment, N-mm
M_s	Sway component of factored end moment, N-mm
M_{single}	Design moment resistance of a section as a singly reinforced section, N-mm
M_u	Ultimate factored design moment at a section, N-mm
M_{u2}	Factored moment at a section about 2-axis, N-mm
M_{u3}	Factored moment at a section about 3-axis, N-mm
P_b	Axial load capacity at balanced strain conditions, N
P_c	Critical buckling strength of column, N
$P_{\max}$	Maximum axial load strength allowed, N
P_o	Axial load capacity at zero eccentricity, N
P_u	Factored axial load at a section, N
V_c	Shear force resisted by concrete, N
V_E	Shear force caused by earthquake loads, N
V_{D+L}	Shear force from span loading, N
$V_{\max}$	Maximum permitted total factored shear force at a section, N
V_p	Shear force computed from probable moment capacity, N
V_s	Shear force resisted by steel, N
V_u	Shear force of ultimate design load, N
Z	Lever arm, mm
a	Depth of compression block, mm
a_b	Depth of compression block at balanced condition, mm

a_{max}	Maximum allowed depth of compression block, mm
b_1	Center-to-center distance between corner bars in the direction of the width = $b - 2c$
b	Width or effective width of the section in the compression zone, mm
b_f	Width or effective width of flange, mm
b_w	Average web width of a flanged beam, mm
c	Depth to neutral axis, mm
c_b	Depth to neutral axis at balanced condition, mm
d_1	Center-to-center distance between corner bars in the direction of the depth = $d - 2c$
d	Effective depth of tension reinforcement, mm Distance from compression face to tension reinforcement, mm
d'	Effective depth of compression reinforcement, mm Concrete cover-to-center of reinforcing, mm
$d_{compression}$	Depth of center of compression block from most compressed face, mm
$f_c^{'}$	Specified compressive strength of concrete, N/mm^2
f_{cd}	Design concrete strength = f_{ck} / γ_c, N/mm^2
f_{ck}	Characteristic compressive strength of concrete, N/mm^2
$f_s^{'}$	Compressive stress in beam compression steel, N/mm^2
f_{yd}	Design yield strength of reinforcing steel = f_y / γ_s, N/mm^2
f_y	Characteristic strength of reinforcement, N/mm^2
f_{ys}	Characteristic strength of shear reinforcement, N/mm^2
h	Overall depth of a column section, mm

k	Enhancement factor of shear strength for depth of the beam Effective length factor
l_{22}	Length of column about the local axis 2
l_{33}	Length of column about the local axis 3
m	Normalized design moment, $M/\alpha f_{ck}\, db^2$
r	Radium of gyration of column section, mm
s_v	Spacing of the shear reinforcement along the length of the beam, mm
v_c	Allowable shear stress in punching shear mode, N
x_u	Depth of neutral axis, mm
$x_{u,\max}$	Maximum permitted depth of neutral axis, mm
α	Concrete strength reduction factor for sustained loading Reinforcing steel overstrength factor
β	Factor for the depth of compressive force resultant of the concrete stress block
β_c	Ratio of the maximum to minimum dimensions of the punching critical section
γ_c	Partial safety factor for concrete strength
γ_f	Partial safety factor for load
γ_m	Partial safety factor for material strength
γ_s	Partial safety factor for steel strength
γ_v	Fraction of unbalanced moment transferred by eccentricity of shear
δ	Enhancement factor of shear strength for compression
δ_{ns}	Moment magnification factor for non-sway moments
δ_s	Moment magnification factor for sway moments

ε_c	Strain in concrete
$\varepsilon_{c,\max}$	Maximum usable compression strain allowed in extreme concrete fiber (0.0035 mm/mm)
ε_s	Strain in tension steel
ε_s'	Strain in compression steel
ρ	Tension reinforcement ratio, A_s/bd
τ_c	Basic design shear stress resisted by concrete, N/mm^2
τ_v	Average design shear stress resisted by the section, N/mm^2
$\tau_{c,\max}$	Maximum possible design shear stress permitted at a section, N/mm^2
τ_{cd}	Design shear stress resisted by concrete, N/mm^2
τ_c	Basic design shear stress resisted by concrete, N/mm^2

3.2 Design Load Combinations

The design loading combinations are the various combinations of the prescribed response cases for which the structure is to be checked/designed. The program creates a number of default design load combinations for a concrete frame design. Users can add their own design load combinations as well as modify or delete the program default design load combinations. An unlimited number of design load combinations can be specified.

To define a design load combination, simply specify one or more response cases, each with its own scale factor. The scale factors are applied to the forces and moments from the load cases to form the factored design forces and moments for each design load combination. There is one exception to the preceding. For spectral analysis model combinations, any correspondence between the signs of the moments and axial loads is lost. The program uses eight design load combinations for each such loading combination specified, reversing the sign of axial loads and moments in major and minor directions.

As an example, if a structure is subjected to dead load, D, and live load, L, only, the IS 456:2000 design check may need one design load combination only, namely, 1.5D + 1.5L. However, if the structure is subjected to wind, earthquake, or other loads, numerous additional design load combinations may be required.

For the IS 456:2000 code, if a structure is subjected to dead (D), live (L), pattern live (PL), wind (W), and earthquake (E) load, and considering that wind and earthquake forces are reversible, the following load combinations may need to be defined (IS 36.4.1, Table 18):

1.5D (IS 36.4.1)

1.5D + 1.5L (IS 36.4.1)

1.5D ± 1.5W
0.9D ± 1.5W (IS 36.4.1)
1.2D ± 1.2L ± 1.2W

1.5D ± 1.5E
0.9D ± 1.5E (IS 36.4.1)
1.2D + 1.2L ± 1.2E

These are also the default load combinations in the program whenever the Indian IS 456:2000 code is used. The user should use other appropriate design load combinations if roof live load is separately treated, or if other types of loads are present. The pattern loading is approximately, but conservatively, performed in the program automatically. Here PL is the approximate pattern load that is the live load multiplied by the Pattern Live Load Factor. The Pattern Live Load Factor can be specified in the Preferences. While calculating forces for the specified pattern load combination, the program adds forces for the dead load, assuming that the member geometry and continuity are unchanged from the model, and the forces for the pattern live load, assuming the beam is simply supported at the two ends. The Pattern Live Load Factor should normally be taken as 0.75 (IS 31.5.2.3). If the Pattern Live Load Factor is specified to be zero, the program does not generate pattern loading.

Live load reduction factors can be applied to the member forces of the live load case on a member-by-member basis to reduce the contribution of the live load to the factored loading. However, such a live load case must be specified as type Reducible Live Load.

For slender compression members, the code recommends the use of a second order frame analysis, also called a P-Δ analysis, which includes the effect of sway deflections on the axial loads and moments in a frame. For an adequate and rational analysis, realistic moment curvature or moment rotation relationships should be used to provide accurate values of deflections and forces. The analysis also should include the effect of foundation rotation and sustained loads. Because of the complexity in the general second order analysis of frames, the code provides an approximate design method that takes into account the "additional moments" due to lateral deflections in columns (IS 39.7). See also Clause 38.7 of SP-24 1983 (IS 1993) for details.

Hence, when using the Indian IS 456:2000 code, it is recommended that the user include P-Delta analysis. With this option, the program can capture the lateral drift effect, i.e., the global effect or P-Δ effect, very nicely. But the program does not capture the local effect (P-δ effect) to its entirety because most often the column members are not meshed. To capture the local effects in columns, the program uses the approximate formula for additional moments as specified in the code (IS 39.7.1). Two major parameters in calculating the additional moments are the effective length factors for major and minor axis bending. The effective length factors for columns are computed using a code-specified procedure (IS 25.2, Annex E). If P-Δ analysis is not included, the program calculates effective length factors, *K*, assuming the frame is a sway frame (sway unrestrained) (IS Annex E, Figure 27). However, if the P-Δ analysis is included, the program assumes the member is prevented from further sway and assumes that the frame can be considered non-sway where $K < 1$ (IS Annex E, Figure 26). In that case, the program takes *K* equal to 1 conservatively. For more information on P-Δ and P-δ effects, please refer to Appendix A. See Appendix C concerning the determination of *K* factors.

3.3 Design Strength

The design strength for concrete and steel is obtained by dividing the characteristic strength of the material by a partial factor of safety, γ_m. The values of γ_m used in the program are as follows:

Partial safety factor for steel, γ_s = 1.15, and (IS 35.4.2.1)

Partial safety factor for concrete, γ_c = 1.50. (IS 35.4.2.1)

These factors are already incorporated in the design equations and tables in the code. Although not recommended, the program allows them to be overwritten. If they are overwritten, the program uses them consistently by modifying the code-mandated equations in every relevant place.

3.4 Column Design

The program can be used to check column capacity or to design columns. If the geometry of the reinforcing bar configuration of each concrete column section has been defined, the program will check the column capacity. Alternatively the program can calculate the amount of reinforcing required to design the column based on provided reinforcing bar configuration. The reinforcement requirements are calculated or checked at a user-defined number of design/check stations along the column height. The design procedure for the reinforced concrete columns of the structure involves the following steps:

- Generate axial force/biaxial moment interaction surfaces for all of the different concrete section types of the model. A typical biaxial interaction surface is shown in Figure 3-1. For reinforcement to be designed, the program generates the interaction surface for the range of allowable reinforcement, which by default is 0.8 to 6 percent for all types of concrete frames (IS 26.5.3.1).

- Calculate the capacity ratio or the required reinforcing area for the factored axial force and biaxial (or uniaxial) bending moments obtained from each loading combination at each station of the column. The target capacity ratio is taken as the Utilization Factor Limit when calculating the required reinforcing area.

- Design the column shear reinforcement.

The following three subsections describe in detail the algorithms associated with these steps.

3.4.1 Generate Biaxial Interaction Surfaces

The column capacity interaction volume is numerically described by a series of discrete points that are generated on the three-dimensional interaction failure surface. In addition to axial compression and biaxial bending, the formulation allows for axial tension and biaxial bending considerations (CP 6.2.1.4a). A typical interaction diagram is shown in Figure 3-1.

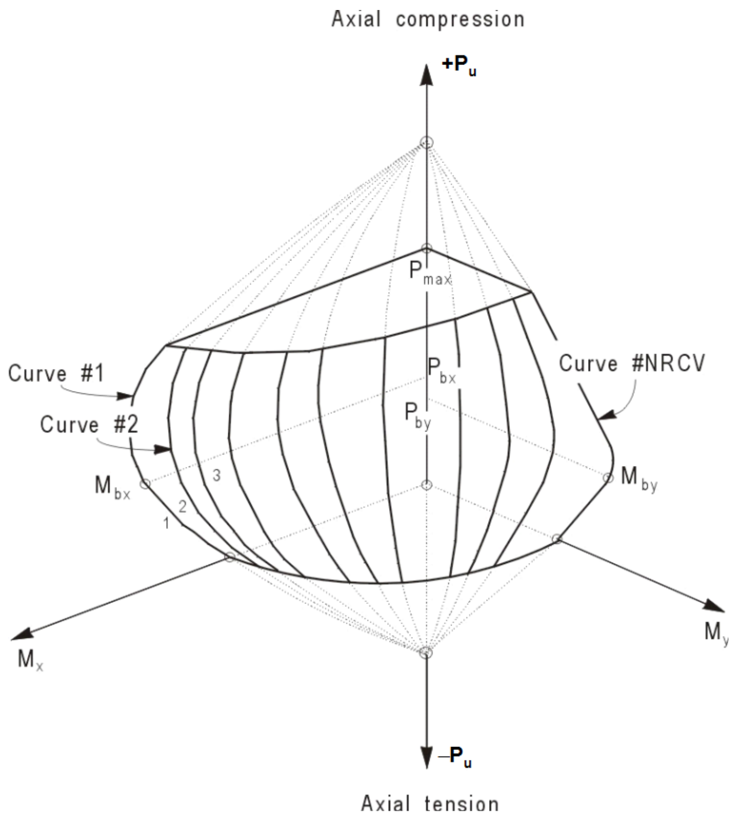

Figure 3-1 A typical column interaction surface

The coordinates of these points are determined by rotating a plane of linear strain in three dimensions on the section of the column, as shown in Figure 3-2.

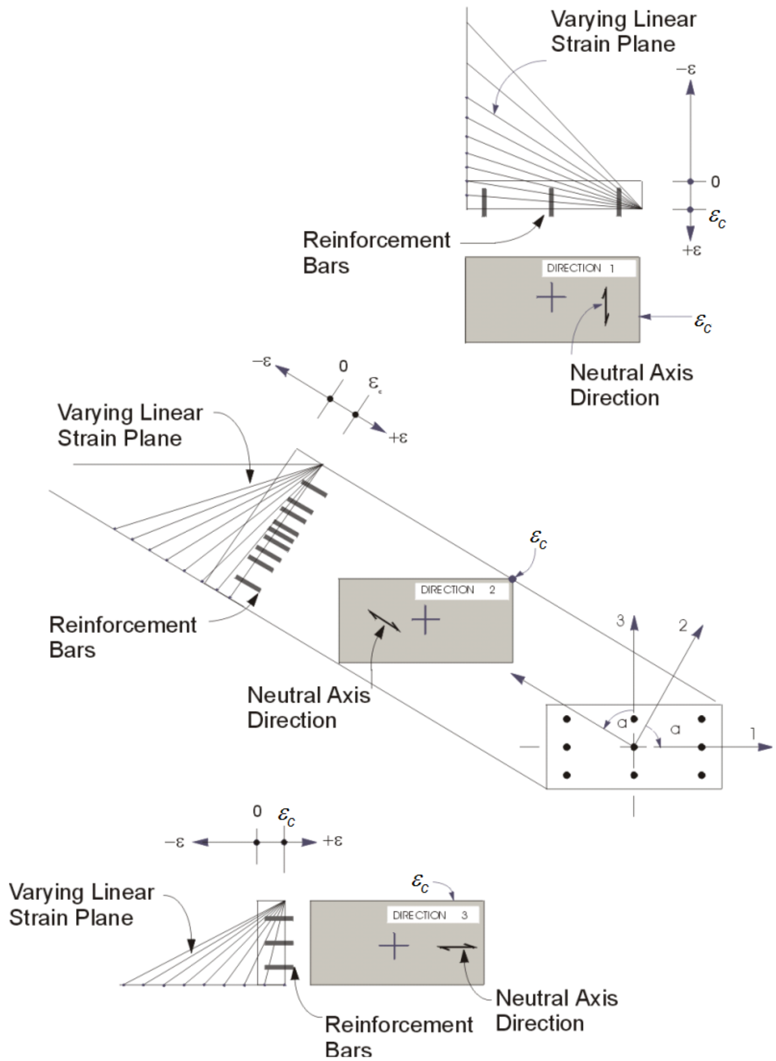

Figure 3-2 Idealized strain distribution for generation of interaction surface

The linear strain diagram limits the maximum concrete strain, ε_c, at the extremity of the section, as given by the following equations:

(a) When there is any tensile strain in the section, the maximum strain in concrete at the outermost compression fiber is taken as 0.0035 (IS 38.1(b)).

ε_c = 0.0035, when tensile strain is present (IS 38.1(b))

(b) When the section is uniformly compressed, the maximum compression strain in concrete is taken as 0.002 (IS 39.1(a)).

$\varepsilon_c = 0.002,$ when the section is uniformly compressed (IS 39.1(a))

(c) When the entire section is under non-uniform compression, the maximum compressive strain at the highly compressed extreme fiber is taken as 0.0035 minus 0.75 times the strain at the least compressed extreme fiber (IS 39.1(b)).

$\varepsilon_{c,max} = 0.0035 - 0.75\ \varepsilon_{c,min},$ when the section is non-uniformly compressed (IS 39.1(b))

The formulation is based consistently on the basic principles of limit state of collapse under compression and bending (IS 38, 39).

The stress in the steel is given by the product of the steel strain and the steel modulus of elasticity, $\varepsilon_s E_s$, and is limited to the design strength of the steel, f_y / γ_s (IS 38.1(e)). The area associated with each reinforcing bar is assumed to be placed at the actual location of the center of the bar, and the algorithm does not assume any further simplifications with respect to distributing the area of steel over the cross-section of the column, as shown in Figure 3-2.

The concrete compression stress block is assumed to be parabolic, with a stress value of $0.67 f_{ck}/\gamma_m$ (IS 38.1.c). See Figure 3-3. The interaction algorithm provides corrections to account for the concrete area that is displaced by the reinforcement in the compression zone.

The equivalent concrete compression stress block is assumed to be rectangular, with a stress value of 0.36 f_{ck} (IS 38.1(c), Figure 22), as shown in Figure 3-3. The depth of the equivalent rectangular block, a, is taken as:

$$a = \beta_1 x_u \qquad \text{(IS 38.1(c), Figure 22)}$$

where x_u is the depth of the stress block in compression strain and,

$$\beta_1 = 2 \times 0.42 = 0.84 \qquad \text{(IS 38.1(c), Figure 22)}$$

The maximum compressive axial strength is limited to P_u where,

$$P_u = [0.4\, f_{ck} A_c + 0.67\, f_y A_{sc}], \text{ tied} \qquad \text{(IS 39.3)}$$

$$P_u = 1.05[0.4\, f_{ck} A_c + 0.67\, f_y A_{sc}], \text{ spiral} \qquad \text{(IS 39.4)}$$

However, the preceding limit is not normally reached unless the section is heavily reinforced.

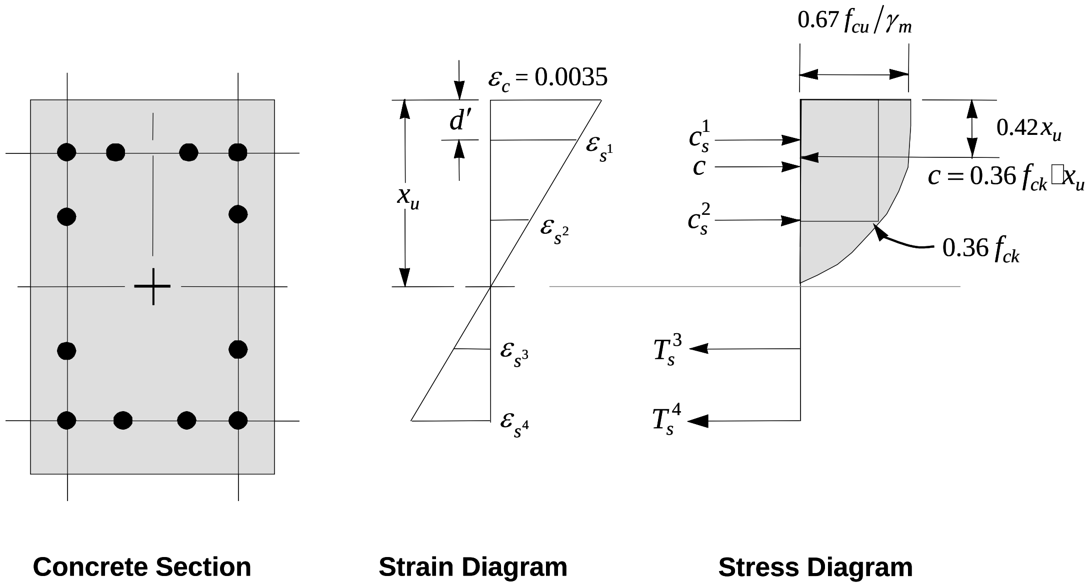

Figure 3-3 Idealization of stress and strain distribution in a column section

3.4.2 Calculate Column Capacity Ratio

The column capacity ratio is calculated for each design loading combination at each output station of each column. The following steps are involved in calculating the capacity ratio of a column for a particular design load combination at a particular location:

- Determine the factored moments and forces from the analysis load cases and the specified load combination factors to give P_u, M_{u2}, and M_{u3}.
- Determine the additional moments to account for column slenderness and compute final design column moments.

- Determine if the point, defined by the resulting axial load and biaxial moment set, lies within the interaction volume.

The final design moments depend on the identification of the individual columns as Ductile, Ordinary, or Nonsway.

The following three subsections describe in detail the algorithms associated with that process.

3.4.2.1 Determine Factored Moments and Forces

The loads for a particular design load combination are obtained by applying the corresponding factors to all of the load cases, giving P_u, M_{u2}, and M_{u3}.

Columns are designed for minimum eccentricity moment, $Pe_{,\min}$, when $M < Pe_{,\min}$, where

$$e_{\min} = \begin{cases} L/500 + \dfrac{D}{30} \\ L/500 + \dfrac{b}{30} \end{cases} \qquad \text{(IS 25.4)}$$

where D and b are dimensions of the column perpendicular to the local 3 and 2 axes. The minimum eccentricity is applied in only one direction at a time (IS 25.4)

3.4.2.2 Determine Additional Moments for Column Slenderness

The program computes the slenderness ratios as l_{33} / D and l_{22} / b, where l_{33} and l_{22} are effective lengths of the column about local axis 3 and 2, while D and b are dimensions of the column perpendicular to the local 3 and 2 axes. If the slenderness ratio is greater than 12, the column is considered as slender in that plane (IS 25.1.2). Effectively, the column may be slender in one or both planes.

If the column is slender in a plane, additional slenderness moments, M_{a2} and M_{a3}, are computed using the following formula:

$$M_{a3} = k\frac{P_u D}{2000}\left\{\frac{l_{33}}{D}\right\}^2 \qquad \text{(IS 39.7.1, 39.7.1.1)}$$

$$M_{a2} = k\frac{P_u b}{2000}\left\{\frac{l_{22}}{b}\right\}^2 \qquad \text{(IS 39.7.1, 39.7.1.1)}$$

where,

$$k = \frac{P_{uz} - P_u}{P_{uz} - P_b} \le 1 \qquad \text{(IS 39.7.1.1)}$$

P_{uz} = $0.45\, f_{ck}A_c + 0.75 f_y A_{sc}$ (IS 39.6)

M_{a2}, M_{a3} = Additional moment to account for column slenderness about column local axes 2 and 3 respectively

P_u = Factored axial force in column for a particular load combination

P_{uz} = Theoretical axial capacity of the section

P_b = Axial load corresponding to the condition of maximum compressive strain of 0.0035 in concrete and tensile strain of 0.002 in the outermost layer of tensile steel

l_{33}, l_{22} = Effective length of column about the local axes 2 and 3 respectively

$l_{22} = k_{22}L_{22}$

$l_{33} = k_{33}L_{33}$`

D, b = Lateral column dimension perpendicular to local axes 3 and 2 respectively

k = Reduction factor for reducing additional moments

Other variables are described in *Section 3.1 Notation* in this chapter.

When designing a column, A_{sc} is not known in advance, and so P_{uz} and P_b are not known. In such cases, k is conservatively taken as 1.

$k = 1$

The program calculates the effective length factors based on Annex E of the code (IS 25.2, 39.7, Annex E) with the assumption that the frame is a sway-frame and using the chart given in IS Figure 27. However, if P-Δ analysis is included, the program assumes that the member is prevented from further sway and takes $k = 1$ conservatively,

The use of code-specified additional moment (IS 39.7) is an approximate procedure (IS SP-24, 1983, Clause 38.7). It is recommended that the user included P-Delta analysis. With this option, the program can capture the lateral drift effect (i.e., global effect or P-Δ effect), but the program does not capture local effect (i.e., P-δ effect) to its entirety through analysis. To capture the local effect correctly, the program uses the approximate design formula for additional moments, with the assumption that $k = 1$.

See Appendix A for more information on P-Δ and P-δ effects. Also see *Section 3.2 Design Load Combination* for additional information. See Appendix C on the determination of K factors.

3.4.2.3 Determine Capacity Ratio

As a measure of the stress condition of the column, a capacity ratio is calculated. The capacity ratio is basically a factor that gives an indication of the stress condition of the column with respect to the capacity of the column.

Before entering the interaction diagram to check the column capacity, the final moments are evaluated as outlined previously. The point (P, M_2, M_3) is then placed in the interaction space shown as point L in Figure 3-4. If the point lies within the interaction volume, the column capacity is adequate; however, if the point lies outside the interaction volume, the column is overstressed.

This capacity ratio is achieved by plotting the point L and determining the location of point C. Point C is defined as the point where the line OL (if extended outwards) will intersect the failure surface. This point is determined by three-dimensional linear interpolation between the points that define the failure surface. See Figure 3-4. The capacity ratio, CR, is given by the ratio $\frac{OL}{OC}$.

- If $OL = OC$ (or $CR = 1$), the point lies on the interaction surface and the column is stressed to capacity.
- If $OL < OC$ (or $CR < 1$), the point lies within the interaction volume and the column capacity is adequate.
- If $OL > OC$ (or $CR > 1$), the point lies outside the interaction volume and the column is overstressed.

The maximum of all of the values of CR calculated from each load combination is reported for each check station of the column along with the controlling P_u, M_{u2}, M_{u3} set and associated load combination number.

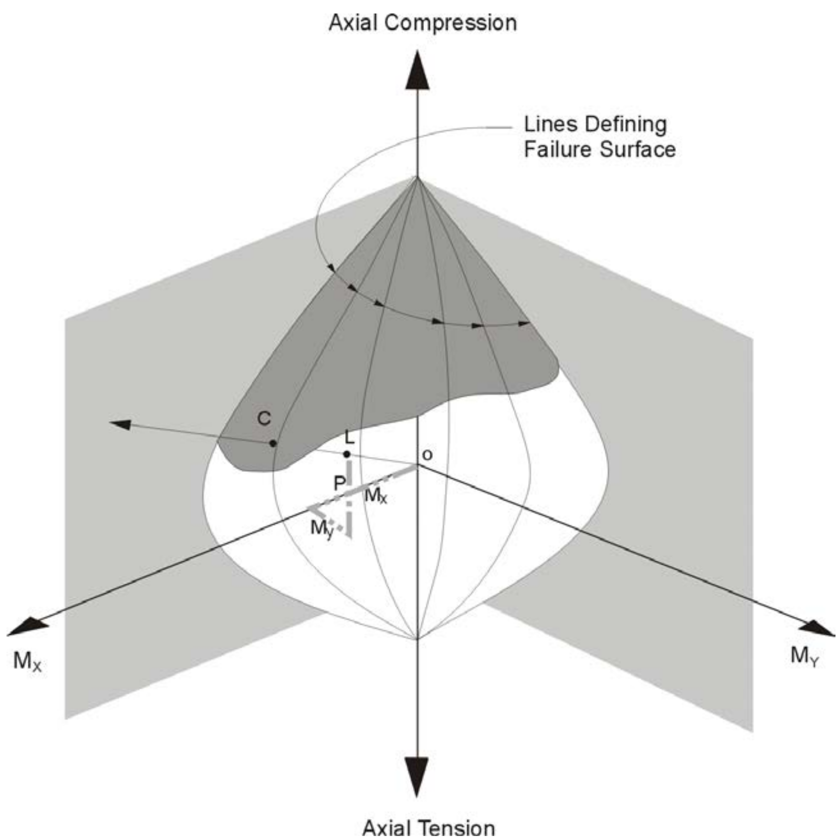

Figure 3-4 Geometric representation of column capacity ratio

3.4.3 Required Reinforcing Area

If the reinforcing area is not defined, the program computes the reinforcement that will give a column capacity ratio equal to the Utilization Factor Limit, which is set to 0.95 by default.

3.4.4 Design Column Shear Reinforcement

The shear reinforcement is designed for each design combination in the major and minor directions of the column. The following steps are involved in designing the shear reinforcement for a particular column for a particular design load combination resulting from shear forces in a particular direction:

- Determine the factor forces acting on the section, P_u and V_u. Note that P_u is needed for the calculation of τ_{cd}.

- Determine the shear force, τ_{cd}, which can be resisted by concrete alone.

- Calculate the reinforcement, A_{sv}/s_v, required to carry the balance.

For Ductile frames, shear design of columns is also based on the capacity shear that a column must resist without overstress to ensure a ductile behavior (IS 13920 7.3.4). The capacity shear in a column is computed from the beam capacities framing into the column.

The following three sections describe in detail and algorithms associated with this process.

3.4.4.1 Determine Section Forces

- In the design of column shear reinforcement of an ordinary moment resisting concrete frame, the forces for a particular design load combination, namely, the column axial force, P_u, and the column shear force, V_u, in a particular direction are obtained by factoring the load cases with the corresponding design load combination factors. Column shear ends are computed at the beam face.

- In the shear design of ductile moment resisting frames (i.e., seismic design), the shear capacity of the column is checked for capacity shear in addition to the requirement of the ordinary moment resisting frame. The capacity shear force in the column, V_u, is determined from consideration of the maximum shear forces that can be generated in the column when beams connected with the column reach their ultimate moment capacity. Two different capacity shears are calculated for each direction (major and minor). The beam object connectivity with the column along the column major and minor axes is retrieved from the model database, with information regarding beam size, steel, and so on. The probably capacity is computed assuming appropriate partial material safety factors, γ_c and γ_s; capacity is the same as described in IS 13920 Clause 6.3.3. The moment capacities of beams are then related to column shear using the following equation.

$$V_u = 1.4\left[\frac{M_u^{\;bL} + M_u^{\;bR}}{H}\right] \qquad \text{(IS 13920 7.3.4)}$$

The column shear in both directions (column major and minor axes) is computed using the preceding equation.

It should be noted that the points of inflection shown in Figure 3-5 are taken at midway between actual lateral support points for the columns, and *H* is taken as the mean of the two column heights. If there is no column at the top of the joint, *H* is taken to be equal to one-half of the height of the column below the joint.

When beams are not oriented along the major and minor axes of the column, appropriate components of the flexural capacity are used. If the beam is oriented at an angle θ with the column major axis, the appropriate component, $M_u^b \cos\theta$ or $M_u^b \sin\theta$, of the beam flexural capacity is used in calculating capacity shear in the column for joint rotations in the clockwise and counterclockwise directions. Also, the positive and negative moment capacities are used appropriately based on the orientation of the beam with respect to the column local axis.

- The column is designed for maximum capacity shear but never less than the factored shear obtained from the design load combination.

$$V_u = 1.4\left[\frac{M_u^{\;bL} + M_u^{\;bR}}{H}\right] \geq V_{u,\text{factored}} \qquad \text{(IS 7.3.4)}$$

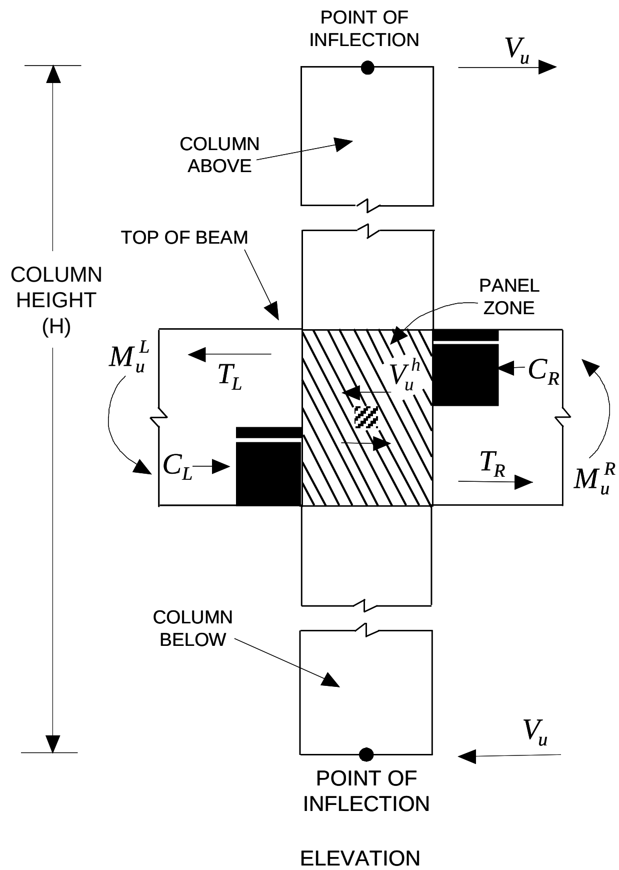

Figure 3-5 Column shear force V_u

3.4.4.2 Determine Concrete Shear Capacity

Given the design set force P_u and V_u, the shear force carried by the concrete, V_c, is calculated as follows:

$$V_c = \tau_{cd} A_{cv}$$

where,

A_{cv} = Effective area under shear as shown in Figure 3-6. For column shapes other than rectangular or circular, it is taken as 5/6 times the gross axial area.

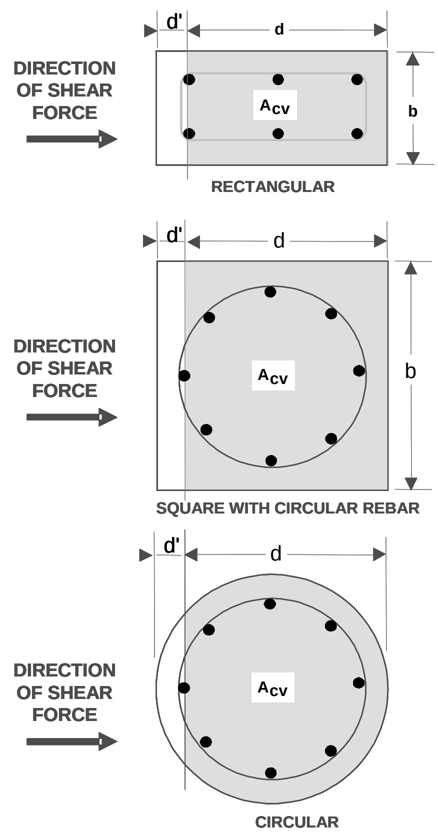

Figure 3-6 Shear stress area, A_{cv}

$$\tau_{cd} = k\delta\tau_c \qquad \text{(IS 40.2)}$$

$$\delta = \begin{cases} 1 + 3\dfrac{P_u}{A_g f_{ck}} \le 1.5 & \text{if } P_u > 0, \text{ Under Compression} \\ 1 & \text{if } P_u \le 0, \text{ Under Tension} \end{cases} \qquad \text{(IS 40.2.2)}$$

k is always taken as 1, and (IS 40.2.1.1)

τ_c is the basic design shear strength for concrete, which is given by IS Table 19. It should be mentioned that the value of γ_c has already been incorporated in IS Table 19 (see Note in IS 36.4.2.1).

The following limitations are enforced in the determination of the design shear strength as is done in the Table.

$$0.15 \le \frac{100 A_s}{bd} \le 3$$

$f_{ck} \le 40$ N/mm^2 (for calculation purposes only)

The determination of τ_c from Table 19 of the code requires two parameters: $100\frac{A_s}{bd}$ and f_{ck}. If $100\frac{A_s}{bd}$ becomes more than 3.0, τ_c is calculated based on $100\frac{A_s}{bd}$ = 3.0. If $100\frac{A_s}{bd}$ becomes less than 0.15, τ_c is calculated based on $100\frac{A_s}{bd}$ = 0.15. Similarly, if f_{ck} is larger than 40 N/mm^2, τ_c is calculated based on f_{ck} = 40 N/mm^2. However, if f_{ck} is less than 15 N/mm^2, τ_c is reduced by a factor of $\left(\frac{f_{ck}}{0.15}\right)^{1/4}$. If γ_c is chosen to be different from 1.5, τ_c is adjusted with a factor of $\left(1.5/\gamma_c\right)$. The absolute maximum limit on nominal shear stress, $\tau_{c,\max}$ is calculated in accordance with IS Table 20, which is reproduced in the table that follows (IS 40.2.3, Table 20):

Maximum Shear Stress, $\tau_{c,\max}$ (N/mm^2)
(IS 40.2.3, IS Table 20)

Concrete Grade	M15	M20	M25	M30	M35	M40
$\tau_{c,\max}$ (N/mm^2)	2.5	2.8	3.1	3.5	3.7	4.0

If f_{ck} is between the limits, linear interpolation is used.

$$\tau_{c,\max} = \begin{cases} 2.5 & \text{if} \quad f_{ck} < 15 \\ 2.5 + 0.3\dfrac{f_{ck} - 15}{5} & \text{if} \quad 15 \le f_{ck} < 20 \\ 2.8 + 0.3\dfrac{f_{ck} - 20}{5} & \text{if} \quad 20 \le f_{ck} < 25 \\ 3.1 + 0.4\dfrac{f_{ck} - 25}{5} & \text{if} \quad 25 \le f_{ck} < 30 \\ 3.5 + 0.2\dfrac{f_{ck} - 30}{5} & \text{if} \quad 30 \le f_{ck} < 35 \\ 3.7 + 0.3\dfrac{f_{ck} - 35}{5} & \text{if} \quad 35 \le f_{ck} < 40 \\ 4.0 & \text{if} \quad f_{ck} > 40 \end{cases} \qquad \text{(IS 40.2.3)}$$

3.4.4.3 Determine Required Shear Reinforcement

Given V_u and V_c, the required shear reinforcement in the form of stirrups or ties within a spacing, *s*, is given for rectangular and circular columns by the following:

- Compute nominal shear stress using the following equation:

$$\tau_v = \frac{V_u}{A_{cv}} \qquad \text{(IS 40.1)}$$

  where V_u is design shear force and A_{cv} is the column section area resisting shear.

- Calculate the basic permissible nominal shear stress, τ_c, and the design permissible nominal shear stress, τ_{cd} (IS 40.2.1, Table 19).

- Calculate the absolute maximum permissible nominal shear stress, $\tau_{c,\max}$, using the procedures described in the previous section (IS 40.2.3, Table 20).

- Compute required shear reinforcement as follows:

  If $\tau_v \le \tau_{cd}$, provide minimum links defined by

$$\frac{A_{sv}}{s_v} = \frac{0.4 b_w}{0.87 f_{ys}}, \qquad \text{(IS 40.3, 26.5.1.6)}$$

else if $\tau_{cd} < \tau_v \le \tau_{c,\max}$, provide links given by

$$\frac{A_{sv}}{s_v} = \frac{(\tau_v - \tau_{cd}) b_w}{0.87 f_{ys}} \ge \frac{0.4 b_w}{0.87 f_{ys}}, \qquad \text{(IS 40.4)}$$

else if $\tau_v > \tau_{c,\max}$, a failure condition is declared. (IS 40.2.3)

In calculating the shear reinforcement, a limit is imposed on the f_{yv} as

$$f_{yv} \le 415 \text{ N/mm}^2. \qquad \text{(IS 40.4)}$$

In the preceding expressions, for a rectangular section, b_w is the width of the column, d is the effective depth of the column, and A_{cv} is the effective shear area, which is equal to $b_w d$. For circular sections, b_w is replaced with D, which is the external diameter of the column and d is the effective depth of the column, and A_{cv} is replaced with the grayed area shown in Figure 3-6.

The maximum of all the calculated A_v/s values, obtained from each design load combination, is reported for the major and minor directions of the column, along with the controlling combination name.

The column shear reinforcement requirements reported by the program are based purely on shear strength consideration. Any other requirements to satisfy spacing considerations or transverse reinforcement volumetric considerations must be investigated independently of the program by the user.

3.5 Beam Design

In the design of concrete beams, the program calculates and reports the required areas of steel for flexure and shear based on the beam moments, shear forces, torsions, design load combination factors, and other criteria described in the text that follows. The reinforcement requirements are calculated at a user-defined number of check/design stations along the beam span.

All beams are designed for major direction flexure, shear, and torsion only. Effects resulting from any axial forces and minor direction bending that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design longitudinal reinforcement
- Design shear reinforcement
- Design torsion reinforcement

3.5.1 Design Beam Longitudinal Reinforcement

The beam top and bottom flexural steel is designed at check/design stations along the beam span. The following steps are involved in designing the flexural reinforcement for the major moment for a particular beam for a particular section:

- Determine the maximum factored moments
- Determine the reinforcing steel

3.5.1.1 Determine Factored Moments

In the design of flexural reinforcement of any concrete beam, the factored moments for each design load combination at a particular beam section are obtained by factoring the corresponding moments for different load cases with the corresponding design load combination factors.

If torsion is present, the factored moment, M_u, is modified with the equivalent torsion moment, M_t, as follows:

$$M_{e1} = M_u + M_t \qquad \text{(IS 41.4.2)}$$

$$M_{e2} = M_u - M_t \qquad \text{(IS 41.4.2)}$$

where,

M_{e1} = equivalent resultant moment

M_{e2} = equivalent resultant moment

M_t = equivalent torsional moment

$$= \quad T_u = \left(\frac{1+D/b}{1.7}\right) \qquad \text{(IS 41.4.2)}$$

T_u = factored torsion

D = overall depth of the beam, and

b = width of the beam or beam web (if T beam)

The beam is designed for both M_{e1} and M_{e2}.

The beam section is then designed for the factored moments or equivalent resultant moments obtained from all of the design load combinations. Positive moments produce bottom steel. In such cases, the beam may be designed as a Rectangular beam or a T-beam. Negative moments produce top steel. In such cases, the beam is always designed as a Rectangular beam.

3.5.1.2 Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the grade of concrete.

The design procedure is based on the simplified parabolic stress block, as shown in Figure 3-7 (IS 38.1, Figures 21 and 22). The area of the stress block, C, and the depth of the center of the compressive force from the most compressed fiber, $d_{\text{compression}}$, are taken as:

$$C = a f_{ck} x_u \text{ and} \qquad \text{(IS 38.1.b)}$$

$$d_{\text{compression}} = \beta x_u, \qquad \text{(IS 38.1.c)}$$

where x_u is the depth of the compression block, and a and β are taken respectively as:

$$a = 0.36\text{, and} \qquad \text{(IS 38.1.c)}$$

$$\beta = 0.42. \qquad \text{(IS 38.1.c)}$$

a is the reduction factor to account for sustained compression and the partial safety factor for concrete. *a* is taken as 0.36 for the assumed parabolic stress block (IS 38.1). The β factor establishes the location of resultant compressive force in concrete in terms of the neutral axis depth.

Furthermore, it is assumed that moment redistribution in the member does not exceed the code-specified limiting value. To safeguard against non-ductile failures, the code also places a limit on the neutral axis depth, as follows (IS 38.1):

f_y	$x_{u,\max}/d$
250	0.53
415	04.8
500	0.46

The program uses interpolation between the three discrete points given in the code.

$$\frac{x_{u,\max}}{d} = \begin{cases} 0.53 & \text{if} \quad f_y \le 250 \\ 0.53 - 0.05\dfrac{f_y - 250}{165} & \text{if} \quad 250 < f_y \le 415 \\ 0.48 - 0.02\dfrac{f_y - 415}{85} & \text{if} \quad 415 < f_y \le 500 \\ 0.46 & \text{if} \quad f_y \ge 500 \end{cases} \qquad \text{(IS 38.1.f)}$$

When the applied moment exceeds the capacity of the beam as a singly reinforced beam, the area of compression reinforcement is calculated on the assumption that the neutral axis depth remains at the maximum permitted value. The maximum fiber compression is taken as:

$$\varepsilon_{c,\max} = 0.0035 \qquad \text{(IS 38.1.b)}$$

and the modulus of elasticity of steel is taken to be

$$E_s = 200{,}000 \text{ N/mm}^2. \qquad \text{(IS 5.6.3, 38.I.e, Figure 23)}$$

The design procedure used by the program, for both rectangular and flanged sections (L-beams and T-beams) is summarized in the subsections that follow. It is assumed that the design ultimate axial force can be neglected; hence all the beams are designed for major direction flexure, torsion, and shear only. Effects of torsion are considered by adjusting the factored moment with $\pm M_t$, as explained earlier.

3.5.1.2.1 Design of a Rectangular Beam

For rectangular beams, the limiting depth of neutral axis, $x_{u,\max}$, and the moment capacity as a singly reinforced beam, M_{single}, are obtained first for a section.

The reinforcing steel area is determined based on whether M is greater than, less than, or equal to M_{single}. (See Figure 3-7.)

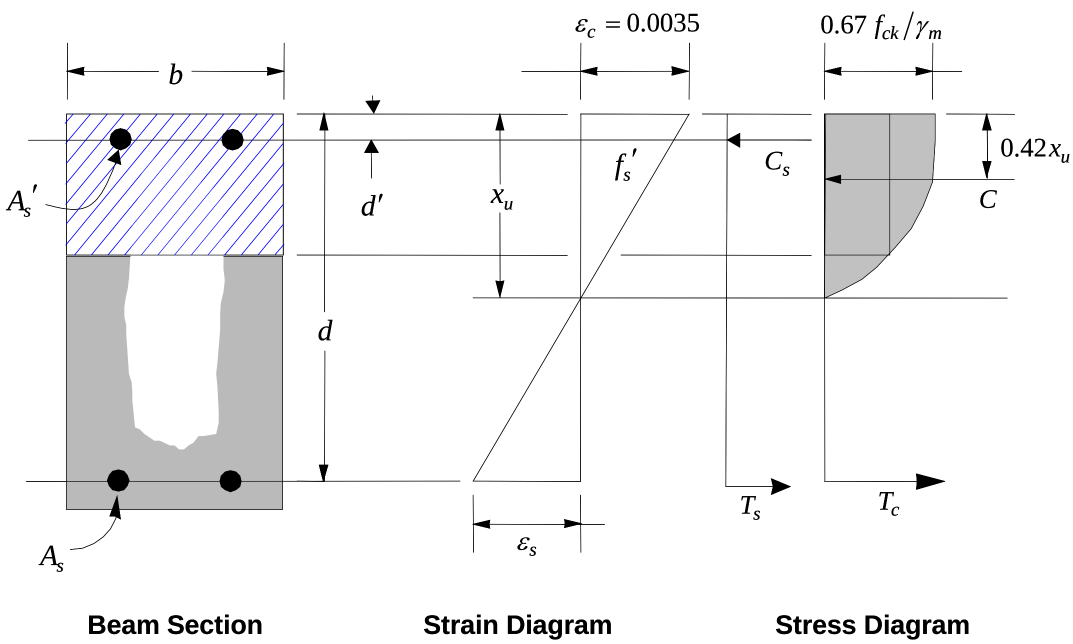

Figure 3-7 Rectangular beam design

- Calculate the limiting depth of the neutral axis.

$$\frac{x_{u,\max}}{d} = \begin{cases} 0.53 & \text{if} \quad f_y \le 250 \\ 0.53 - 0.05\dfrac{f_y - 250}{165} & \text{if} \quad 250 < f_y \le 415 \\ 0.48 - 0.02\dfrac{f_y - 415}{85} & \text{if} \quad 415 < f_y \le 500 \\ 0.46 & \text{if} \quad f_y \ge 500 \end{cases} \qquad \text{(IS 38.1.f)}$$

- Calculate the limiting ultimate moment of resistance of the section as a singly reinforced beam.

$$M_{\text{single}} = \alpha f_{ck} b d^2 \frac{x_{u,\max}}{d}\left[1 - \beta \frac{x_{u,\max}}{d}\right], \text{ where} \qquad \text{(IS G-1.1.c)}$$

α = 0.36, and (IS G-1.1.c, 38.1.c)

β = 0.42.

- Calculate the depth of neutral axis x_u as

$$\frac{x_u}{d} = \frac{1 - \sqrt{1 - 4\beta m}}{2\beta},$$

where the normalized design moment, m, is given by

$$m = \frac{M_u}{bd^2 \alpha f_{ck}}.$$

- If $M_u \le M_{single}$, the area of tension reinforcement, A_s, is obtained from

$$A_s = \frac{M_u}{\left(f_y / \gamma_s\right) z}, \text{ where} \qquad \text{(IS G-1.1)}$$

$$z = d\left\{1 - \beta \frac{x_u}{d}\right\}. \qquad \text{(IS 38.1)}$$

This is the top steel if the section is under negative moment and the bottom steel if the section is under positive moment.

- If $M_u > M_{\text{single}}$, the area of compression reinforcement, A'_s, is given by

$$A_s' = \frac{M_u - M_{\text{single}}}{f_s'\left(d - d'\right)}, \qquad \text{(IS G-1.2)}$$

where d' is the depth of the compression steel from the concrete compression face, and

$$f_s' = \varepsilon_{c,\max} E_s \left[1 - \frac{d'}{x_{u,\max}}\right] \le \frac{f_y}{\gamma_s}. \qquad \text{(IS G-1.2)}$$

This is the bottom steel if the section is under negative moment and top steel if the section is under positive moment. From equilibrium, the area of tension reinforcement is calculated as

$$A_s = \frac{M_{\text{single}}}{\left(f_y / \gamma_s\right) z} + \frac{M_u - M_{\text{single}}}{\left(f_y / \gamma_s\right)(d - d')}, \quad \text{where} \qquad \text{(IS G-1.2)}$$

$$z = d\left\{1 - \beta \frac{x_{u,\max}}{d}\right\}. \qquad \text{(IS 38.1)}$$

A_s is to be placed at the bottom and A'_s is to be placed at the top if M_u is positive, and A'_s is to be placed at the bottom and A_s is to be placed at the top if M_u is negative.

3.5.1.2.2 Design as a T-Beam

In designing a T-beam, a simplified stress block, as shown in Figure 3-8, is assumed if the flange is under compression, i.e., if the moment is positive. If the moment is negative, the flange comes under tension, and the flange is ignored. In that case, a simplified stress block similar to that shown in Figure 3-7 is assumed on the compression side.

3.5.1.2.2.1 Flanged Beam Under Negative Moment

In designing for a factored negative moment. M_u (i.e., designing top steel), the calculation of the steel area is exactly the same as described for a rectangular beam, i.e., no T-beam data is used. The width of the web, b_w, is used as the width of the beam.

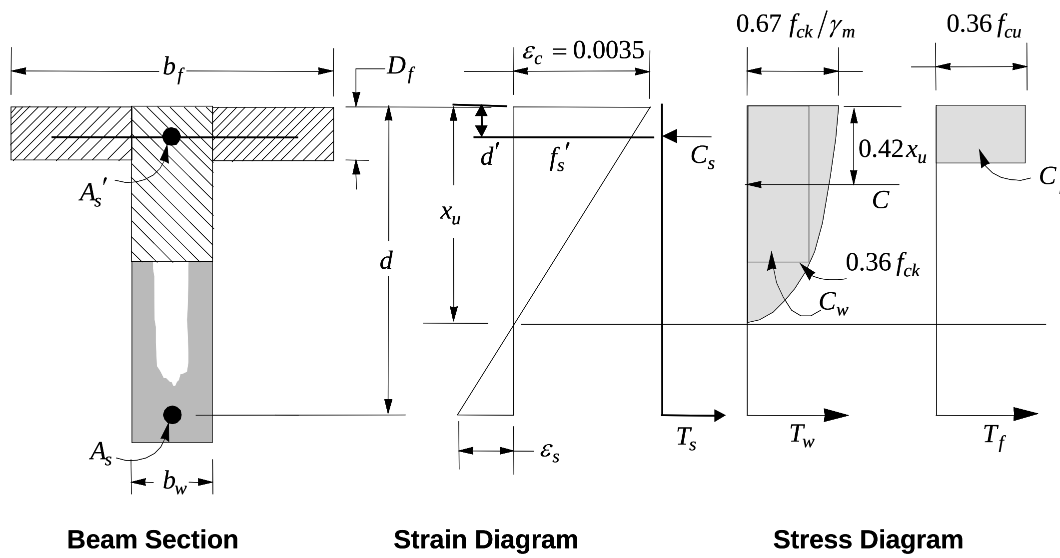

Figure 3-8 T beam design

3.5.1.2.2.2 Flanged Beam Under Positive Moment

With the flange in compression, the program analyzes the section by considering alternative locations of the neutral axis. Initially, the neutral axis is assumed to be located in the flange. Based on this assumption, the program calculates the depth of the neutral axis. If the stress block does not extend beyond the flange thickness, the section is designed as a rectangular beam of width b_f. If the stress block extends beyond the flange, additional calculation is required. (See Figure 3-8.)

- Assuming the neutral axis to lie in the flange, calculate the depth of the neutral axis, x_u, as

  $$\frac{x_u}{d} = \frac{1 - \sqrt{1 - 4\beta m}}{2\beta},$$

  where the normalized design moment, m, is given by

  $$m = \frac{M_u}{b_f d^2 \alpha f_{ck}}.$$

- If $\left(\frac{x_u}{d}\right) \le \left(\frac{D_f}{d}\right)$, the neutral axis lies within the flange. The subsequent calculations for A_s are exactly the same as previously defined for Rectangular section design (IS G-2.1). However, in this case, the width of the compression flange, b_f, is taken as the width of the beam, b. Compression reinforcement is required when $M_u > M_{single}$.

- If $\left(\frac{x_u}{d}\right) > \left(\frac{D_f}{d}\right)$, the neutral axis lies below the flange. Then calculation for A_s has to parts. The first part is for balancing the compressive force from the flange, C_f, and the second part is for balancing the compressive force from the web, C_w, as shown in Figure 3-8.

  – Calculate the ultimate resistance moment of the flange as:

$$M_f = 0.45\, f_{ck}\, (b_f - b_w) y_f\, (d - 0.5\, y_f), \qquad \text{(IS G-2.2)}$$

  where y_f is taken as follows:

$$y_f = \begin{cases} D_f & \text{if} \quad D_f \le 0.2d \\ 0.15x_u + 0.65D_f & \text{if} \quad D_f > 0.2d \end{cases} \qquad \text{(IS G-2.2)}$$

  – Calculate the moment taken by the web as:

$$M_w = M_u - M_f.$$

  – Calculate the limiting ultimate moment resistance of the web for only tension reinforcement.

$$M_{w,\text{single}} = \alpha f_{ck} b_w d^2 \frac{x_{u.\max}}{d}\left[1 - \beta \frac{x_{u,\max}}{d}\right] \text{ where,} \qquad \text{(IS G-1.1)}$$

$$\frac{x_{u,\max}}{d} = \begin{cases} 0.53 & \text{if} \quad f_y \le 250 \\ 0.53 - 0.05\dfrac{f_y - 250}{165} & \text{if} \quad 250 < f_y \le 415 \\ 0.48 - 0.02\dfrac{f_y - 415}{85} & \text{if} \quad 415 < f_y \le 500 \\ 0.46 & \text{if} \quad f_y \ge 500 \end{cases} \qquad \text{(IS 38.1.f)}$$

$$\alpha = 0.36, \text{ and} \qquad \text{(IS G-1.1.c, 38.1.c)}$$

= 0.42. (IS G-1.1.c, 38.1.c)

– If $M_w \le M_{w,\text{single}}$, the beam is designed as a singly reinforced concrete beam. The area of steel is calculated as the sum of two parts: one to balance compression in the flange and one to balance compression in the web.

$$A_s = \frac{M_f}{\left(f_y/\gamma_s\right)\left(d - 0.5 y_f\right)} + \frac{M_w}{\left(f_y/\gamma_s\right) z}, \text{ where}$$

$$z = d\left\{1 - \beta\frac{x_u}{d}\right\},$$

$$\frac{x_u}{d} = \frac{1 - \sqrt{1 - 4\beta m}}{2\beta}, \text{ and}$$

$$m = \frac{M_w}{b_w d^2 \alpha f_{ck}}.$$

– If $M_w > M_{w,\text{single}}$, the area of compression reinforcement, A_s', is given by:

$$A_s' = \frac{M_w - M_{w,\text{single}}}{f_s'\left(d - d'\right)},$$

where d' is the depth of the compression steel from the concrete compression face, and

$$f_s' = \varepsilon_{c,\max} E_s \cdot \left[1 - \frac{d'}{x_{u,\max}}\right] \le \frac{f_y}{\gamma_s}, \qquad \text{(IS G-1.2)}$$

From equilibrium, the area of tension reinforcement is calculated as:

$$A_s = \frac{M_f}{\left(f_y/\gamma_s\right)\left(d - 0.5 y_f\right)} + \frac{M_{w,\text{single}}}{\left(f_y/\gamma_s\right) z} + \frac{M_w - M_{w,\text{single}}}{\left(f_y/\gamma_s\right)\left(d - d'\right)},$$

where

$$z = d\left\{1 - \beta\frac{x_{u,\max}}{d}\right\}.$$

A_s is to be placed at the bottom, and A_s' is to be placed at the top for positive moment.

3.5.1.3 Minimum and Maximum Tensile Reinforcement

The minimum flexural tensile steel required for a beam section is given by the following equation (IS 26.5.1.1):

$$A_s \geq \begin{cases} \dfrac{0.85}{f_y} bd & \text{Rectangular beam} \\ \dfrac{0.85}{f_y} b_w d & \text{T-beam} \end{cases} \qquad \text{(IS 26.5.1.1)}$$

An upper limit on the tension reinforcement for beams (IS 26.5.1.1) and compression reinforcement (IS 26.5.1.2) has been imposed to be 0.04 times the gross web area.

$$A_s \leq \begin{cases} 0.4bd & \text{Rectangular beam} \\ 0.4b_w d & \text{T-beam} \end{cases} \qquad \text{(IS 26.5.1.1)}$$

$$A_s' \leq \begin{cases} 0.4bd & \text{Rectangular beam} \\ 0.4b_w d & \text{T-beam} \end{cases} \qquad \text{(IS 26.5.1.2)}$$

3.5.1.4 Special Consideration for Seismic Design

For ductile moment resisting concrete frames (seismic design), the beam design satisfies the following additional conditions (see also Table 3-1):

Table 3-1: Design Criteria

Type of Check/Design	Ordinary/Non-Sway Moment Resisting Frames (Non-Seismic)	Ductile Moment Resisting Frames (Seismic)
Column Check (PMM interaction)	Specified Combinations	Specified Combinations
Column Design (PMM interaction)	Specified Combinations 0.8% < ρ < 6%	Specified Combinations 0.8% < ρ < 6%
Column Shears	Specified Combinations	Specified Combinations Column Capacity Shear
Beam Design Flexure	Specified Combinations $\rho \leq 0.04$ $\rho \geq \frac{0.85}{f_y}$	Specified Combinations $\rho \leq 0.025$ $\rho \geq \frac{0.85}{f_y}$ $\rho \geq \frac{0.24\sqrt{f_{ck}}}{f_y}$
Beam Minimum Rebar Override Check	No Requirement	$A_s{}^{bot}, \text{end} \geq \frac{1}{2}\max\left\{A_s^{top}, \text{left } A_s^{top}, \text{right}\right\} \text{end}$ $A_s{}^{bot}, \text{span} \geq \frac{1}{4}\max\left\{A_s^{top}, \text{left}, A_s^{top}, \text{right}\right\} \text{end}$ $A_s{}^{top}, \text{span} \geq \frac{1}{4}\max\left\{A_s^{top}, \text{left}, A_s^{top}, \text{right}\right\} \text{end}$ $M_{u,\text{end}}^{+} \geq \frac{1}{2} M_{u,\text{end}}^{-}$ $M_{u,\text{span}}^{+} \geq \frac{1}{4}\max\left\{M_u^{+}, M_u^{-}\right\} \text{end}$ $M_{u,\text{span}}^{-} \geq \frac{1}{4}\max\left\{M_u^{+}, M_u^{-}\right\} \text{end}$

Table 3-1: Design Criteria

Type of Check/Design	Ordinary/Non-Sway Moment Resisting Frames (Non-Seismic)	Ductile Moment Resisting Frames (Seismic)
Beam Design Shear		
	Specified Combinations	Specified Combinations
		Beam Capacity Shear (V_e)
Joint Design		
	No Requirement	Checked for shear (Informative)
Beam/Column Capacity Ratio		
	No Requirement	Checked (Informative)

- The minimum longitudinal reinforcement shall be provided at both the top and bottom of the section. Any of the top and bottom reinforcement shall not be less than $A_{s(\min)}$ (IS 13920 6.2.1, IS 26.5.1.1).

$$A_{s(\min)} \geq \max\left\{\frac{0.24\sqrt{f_{ck}}}{f_y} b_w d, \text{ and } \frac{0.85}{f_y} b_w d\right\}$$

(IS 13920 6.2.1; IS 26.5.1.1)

- The beam flexural steel is limited to a maximum given by

$$A_s \leq 0.025\, b_w d. \qquad \text{(IS 13920 6.2.2)}$$

- At any end (support or joint force) of the beam, the beam bottom reinforcement area (i.e., associated with the positive moment) would not be less than 1/2 of the beam top reinforcement area (i.e., associated with the negative moment) at that end (IS 13920 6.2.3).

- Neither the top nor bottom reinforcement area at any of the sections within the beam would be less than 1/4 of the maximum top reinforcement area of any of the beam end (support or joint force) stations (IS 6.2.4).

3.5.2 Design Beam Shear Reinforcement

The shear reinforcement is designed for each design load combination at each check/design station of the beam element along its span. The assumptions in designing the shear reinforcement are as follows:

- The beam sections can be prismatic or non-prismatic. The program keeps track of any variation in beam width and depth along the length of the beam object, and the actual width and depth of the beam at the design station are considered. However, web width is assumed to remain uniform at a section, and the web is assumed to always be vertical.

- The effect on the concrete shear capacity of any concentrated or distributed load in the span of the beam between two columns is ignored. Also, the effect of the direct support on the beams provided by the columns is ignored.

- All shear reinforcement is assumed to be perpendicular to the longitudinal reinforcement.

- The effect of torsion is considered for the design of shear reinforcement when design for torsion is included in the preferences.

- For Ductile beams (seismic design), capacity shear resulting from moment capacities at the ends along with factored gravity loads also is considered in design.

The shear reinforcement is designed for each loading combination in the major direction of the beam. In designing the shear reinforcement for a particular beam for a particular loading combination, the following steps are involved (IS 40.1).

The shear reinforcement is designed for each design load combination at a user-defined number of stations along the beam span. The following steps are involved in designing the shear reinforcement for a particular station because of beam major shear:

- Determine the factored shear force, V_u, T_u, and M_u.

- Determine the shear stress, τ_{cd}, that can be resisted by the concrete.

- Determine the reinforcement steel A_{sv}/s_v, required to carry the balance.

The following three sections describe in detail the algorithms associated with this process.

3.5.2.1 Determine Shear Force and Moment

In the design of beam shear reinforcement of an Ordinary moment resisting concrete frame, the shear forces and moments for a particular design load combination at a particular beam section are obtained by factoring the associated shear forces and moments with the corresponding design load combination factors.

In the design of Ductile beams (i.e., seismic design), the shear capacity of the beam also is checked for the capacity shear resulting from the maximum moment capacities at the ends along with the factored gravity load. This check is performed only for Ductile beams in addition to the design check required for Ordinary beams. The capacity shear force, V_p, is calculated from the maximum moment capacities of each end of the beam and the gravity shear forces. The procedure for calculating the design shear force in a beam from the maximum moment capacity is the same as described in the code (IS 13920 6.3.3).

The design shear force is then give by (IS 13920 6.3.3)

$$V_u = \max\left\{V_{u,a}, V_{u,b}\right\} \qquad \text{(IS 13920 6.3.3, Figure 4)}$$

$$V_{u,a} = V_{p1} + V_{D+L} \qquad \text{(IS 13920 6.3.3, Figure 4)}$$

$$V_{u,a} = V_{p2} + V_{D+L} \qquad \text{(IS 13920 6.3.3, Figure 4)}$$

where V_p is the capacity shear force obtained by applying the calculated maximum ultimate moment capacities at the two ends of the beams acting in two opposite directions. Therefore, V_p is the maximum of V_{p1} and V_{p2} where,

$$V_{p1} = \frac{M_I^- + M_J^+}{L}, \text{ and}$$

$$V_{p2} = \frac{M_I^+ + M_J^-}{L},$$

where,

M_I^- = Moment capacity at end I, with top steel in tension

M_J^+ = Moment capacity at end J, with bottom steel in tension

M_I^+ = Moment capacity at end I, with bottom steel in tension

M_J^- = Moment capacity at end J, with top steel in tension

L = Clean span of beam.

The moment strengths are determined using a strength reduction factor of 1.0 and the reinforcing steel stress equal to αf_y, where α is equal to 1.25. If the reinforcement area has not been overwritten for Ductile beams, the value of the reinforcing area envelope is calculated after completing the flexural design of the beam for all the design load combinations. Then this enveloping reinforcing area is used in calculating the moment capacity of the beam. If the reinforcing area has been overwritten for Ductile beams, this area is used in calculating the moment capacity of the beam. The reinforcing area can be overwritten. If the beam section is a variable cross-section, the cross-sections at the two ends are used along with the user-specified reinforcing of the envelope of reinforcing, as appropriate. V_{D+L} is the contribution of factored shear force from the in-span distribution of gravity loads with the assumption that the ends are simply supported.

If the user overwrites the major direction length factor, the full span length is used. However, if the length factor is not overwritten, the clear length will be used. In the latter case, the maximum of negative and positive moment capacities will be used for both the negative and positive moment capacities in determining the capacity shear.

- For non-prismatic sections, the calculated V_u is modified with V_e, where V_e is given by:

$$V_e = V_u \pm \frac{M_u}{d}\tan\beta \qquad \text{(IS 40.1.1)}$$

where

V_u = factored shear (or capacity shear with gravity load),

M_u = factored bending moment at the section,

d = effective depth,

β = angle between the top and bottom edge of the beam.

The negative sign in the formula applies when the bending moment, M_u, increases numerically in the same direction as the effective depth increases, and the positive sign applies when the moment decreases numerically in the same direction.

- If torsion is present, the factored shear force, V_u, is modified with the equivalent shear force, V_e, as follows:

$$V_e = V_u + 1.6\frac{T_u}{b} \qquad \text{(IS 41.3.1)}$$

where

V_e = equivalent shear,

V_u = factored shear (or capacity shear with gravity load),

T_u = factored torsion,

b = width of the beam or T-beam web.

Then the beam is designed for V_e.

3.5.2.2 Determine Concrete Shear Capacity

Given the design set force P_u and V_u, the shear force carried by the concrete, V_c, is calculated as follows:

$$V_c = \tau_{cd} A_{cv}$$

where,

A_{cv} = Effective area under shear, as shown in Figure 3-6. For column shapes other than rectangular or circular, it is taken as 5/6 times the gross axial area.

$$\tau_{cd} = k\delta\tau_c \qquad \text{(IS 40.2)}$$

$$\delta = \begin{cases} 1+3\dfrac{P_u}{A_g f_{ck}} \le 1.5 & \text{if } P_u > 0, \text{ Under Compression} \\ 1 & \text{if } P_u \le 0, \text{ Under Tension} \end{cases} \qquad \text{(IS 40.2.2)}$$

k is always taken as 1, and (IS 40.2.1.1)

τ_c is the basic design shear strength for concrete, which is given by IS Table 19. It should be mentioned that the value of γ_c has already been incorporated in IS Table 19 (see Note in IS 36.4.2.1). The following limitations are enforced in the determination of the design shear strength as is done in the Table.

$$0.15 \le \frac{100 A_s}{bd} \le 3$$

$f_{ck} \le 40$ N/mm^2 (for calculation purposes only)

The determination of τ_c from Table 19 of the code requires two parameters: $100\dfrac{A_s}{bd}$ and f_{ck}. If $100\dfrac{A_s}{bd}$ becomes more than 3.0, τ_c is calculated based on $100\dfrac{A_s}{bd} = 3.0$. If $100\dfrac{A_s}{bd}$ becomes less than 0.15, τ_c is calculated based on $100\dfrac{A_s}{bd} = 0.15$. Similarly, if f_{ck} is larger than 40 N/mm^2, τ_c is calculated based on f_{ck} = 40 N/mm^2. However, if f_{ck} is less than 15 N/mm^2, τ_c is reduced by a factor of $\left(\dfrac{f_{ck}}{0.15}\right)^{1/4}$. If γ_c is chosen to be different from 1.5, τ_c is adjusted with a factor of $(1.5/\gamma_c)$. The absolute maximum limit on nominal shear stress, $\tau_{c,\max}$ is calculated in accordance with IS Table 20, which is reproduced in the table that follows (IS 40.2.3, Table 20):

Maximum Shear Stress, $\tau_{c,max}$ (N/mm^2) (IS 40.2.3, IS Table 20)						
Concrete Grade	M15	M20	M25	M30	M35	M40
$\tau_{c,max}$ (N/mm^2)	2.5	2.8	3.1	3.5	3.7	4.0

If f_{ck} is between the limits, linear interpolation is used.

$$\tau_{c,\max} = \begin{cases} 2.5 & \text{if} \quad f_{ck} < 15 \\ 2.5 + 0.3\dfrac{f_{ck} - 15}{5} & \text{if} \quad 15 \le f_{ck} < 20 \\ 2.8 + 0.3\dfrac{f_{ck} - 20}{5} & \text{if} \quad 20 \le f_{ck} < 25 \\ 3.1 + 0.4\dfrac{f_{ck} - 25}{5} & \text{if} \quad 25 \le f_{ck} < 30 \\ 3.5 + 0.2\dfrac{f_{ck} - 30}{5} & \text{if} \quad 30 \le f_{ck} < 35 \\ 3.7 + 0.3\dfrac{f_{ck} - 35}{5} & \text{if} \quad 35 \le f_{ck} < 40 \\ 4.0 & \text{if} \quad f_{ck} > 40 \end{cases} \qquad \text{(IS 40.2.3)}$$

3.5.2.3 Determine Required Shear Reinforcement

Given V_u, T_u, and M_u, the required shear reinforcement in the form of stirrups within a spacing, s_v, is given for Rectangular beams or T-beams by the following:

- Calculate the design nominal shear stress as

$$T_u = \frac{V_u}{bd} \qquad \text{(IS 40.1)}$$

when b is the width of the rectangular beam or the width of the T-beam web, i.e., ($b = b_u$),

– If the beam is non-prismatic, V_u is replaced by V_e, as explained earlier

$$V_e = V_u \pm \frac{M_u}{d}\tan\beta \qquad \text{(IS 40.1.1)}$$

– If the beam is subjected to torsion, T_u, in addition to shear V_u, the V_u is replaced by V_e, as explained earlier.

$$V_e = V_u + 1.6\frac{T_u}{b} \qquad \text{(IS 41.3)}$$

– If the beam is non-prismatic and is subjected to torsion, both the effects are considered in design:

$$V_e = V_u + 1.6\frac{T_u}{b} \pm \frac{M_u}{d}\tan\beta \qquad \text{(IS 41.3, 40.1.1)}$$

- Calculate the basic permissible nominal shear stress, τ_c, and the design permissible nominal shear stress, τ_{cd}, following the procedure described in the previous section (IS 40.2.1, Table 19, 40.2).

- Calculate the absolute maximum permissible nominal shear stress, $\tau_{c,\max}$, following the procedure described in the previous section (IS 40.2.3, Table 20).

The shear reinforcement is computed as follows:

- If $\tau_v \leq \tau_{cd}$, provide minimum links defined by

$$\frac{A_{sv}}{s_v} = \frac{0.4 b_w}{0.87 f_{ys}}, \qquad \text{(IS 40.3, 26.5.1.6)}$$

else if $\tau_{cd} < \tau_v \leq \tau_{c,\max}$, provide links given by

$$\frac{A_{sv}}{s_v} = \frac{\left(\tau_v - \tau_{cd}\right) b_w}{0.87 f_{ys}} \geq \frac{0.4 b_w}{0.87 f_{ys}}, \qquad \text{(IS 40.4)}$$

else if $\tau_v > \tau_{c,\max}$,

a failure condition is declared (IS 40.2.3)

In calculating the shear reinforcement, a limit was imposed on the f_{ys} of

$$f_{ys} \leq 415 \text{ N/mm}^2. \qquad \text{(IS 39.4)}$$

The maximum of all calculated A_{sv}/s_v values, obtained from each load combination, is reported along with the controlling shear force and associated load combination number.

The beam shear reinforcement requirements displayed by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

If torsion is present and if $\tau_u > \tau_{cd}$, the shear links calculated in the section are replaced by two-legged closed hoops enclosing the corners of the longitudinal bars. The calculation of those closed hoops is given in the following section.

3.5.3 Design Beam Torsion Reinforcement

The torsion reinforcement is designed for each design load combination at a user-defined number of stations along the beam span. The following steps are involved in designing the reinforcement for a particular station because of beam torsion:

- Determine the factored forces.
- Determine the reinforcement steel required.

Note that the torsion design can be turned off by choosing not to consider torsion in the Preferences.

3.5.3.1 Determine Factored Forces

In the design of torsion reinforcement of any beam, the factored torsions, shear, and bending moment for each design load combination at a particular design station are obtained by factoring the corresponding torsion, shear, and bending moment for different load cases with the corresponding design load combination factors.

In a statistically indeterminate structure where redistribution of the torsional moment in a member can occur due to redistribution of internal forces upon cracking, the design T_u is permitted to be reduced in accordance with code (IS 41.1). However, the program does not try to redistribute the internal forces and to reduce T_u. If redistribution is desired, the user should *release* the torsional degree of freedom in the structural model, or should specify torsional stiffness modifiers to reduce torsion (IS 41.1).

3.5.3.2 Determine Torsion Reinforcement

Normally the torsional resistance of a beam is provided by the closed hoops, longitudinal rebar, and concrete compression diagonals. The code uses a simplified approach in designing torsion of beams. Torsional reinforcement is determined for a fictitious bending moment, M_e, which is a function of actual bending moment, M_u, and torsion, T_u (IS 41.1). Similarly, web reinforcement is determined for a fictitious shear, V_e, which is a function of actual shear, V_u, and torsion, T_u (IS 41.1). If V_e is small ($\tau_{ve} \leq \tau_{cd}$), stirrup links suffice and if V_e is large ($\tau_{ve} > \tau_{cd}$), closed hoops are calculated to resist the torsion T_u and the interactive V_u.

If torsion is present, the factored moment, M_u, is modified with the equivalent torsion moment, M_t, as follows:

$$M_{e1} = M_u + M_t \qquad \text{(IS 41.4.2)}$$

$$M_{e2} = M_u - M_t \qquad \text{(IS 41.4.2)}$$

where,

M_{e1} = equivalent resistant moment

M_{e2} = equivalent resistant moment

M_t = equivalent torsional moment

$$= T_u \left(\frac{1 + D/b}{1.7} \right) \qquad \text{(IS 41.4.2)}$$

T_u = factored torsion,

D = overall depth of the beam, and

b = width of the beam or brace web (if T-beam)

The beam is designed for both M_{e1} and M_{e2}, as discussed in *Section 3.5.1 Design Beam Longitudinal Reinforcement*. The longitudinal rebar calculated this way includes flexural rebar for M_u and additional rebar needed to resist T_u torsion (IS 41.4.2.1). No further consideration is needed for equivalent moment.

Given T_u, V_u, and M_u, the required torsion reinforcement in the form of two-legged closed hoops within a spacing, s_v, is given for Rectangular beams or T-beams by the following:

- Calculate the design nominal shear stress as

$$T_{ue} = \frac{V_e}{bd}, \text{ where} \qquad \text{(IS 40.1, 41.3.1)}$$

$$V_e = V_u + 1.6\frac{T_u}{b},$$

where b is the width of the rectangular beam or the width of the t beam web (i.e., $b = b_w$).

– If the beam is non-prismatic and is subjected to torsion, V_e is taken as

$$V_e = V_u + 1.6\frac{T_u}{b} \pm \frac{M_u}{d}\tan\beta \qquad \text{(IS 4.13, 40.1.1)}$$

– All parameters in the preceding expressions have been explained thoroughly in *Section 3.5.2 Design Beam Shear Reinforcement*.

- Calculate the basic permissible nominal shear stress, τ_c, and the design permissible nominal shear stress, τ_{cd}, following the procedure described in the previous section (IS 40.2.1, Table 19, 40.2).

- Calculate the absolute maximum permissible nominal shear stress, $\tau_{c,\max}$, using the procedure described in the previous section (IS 40.2.3, Table 20).

- The two-legged closed hoop reinforcement is computed as follows:

– If $T_u = 0$ or $\tau_{ve} \le \tau_{cd}$,

$$\frac{A_t}{s_v} = 0 \qquad \text{(IS 41.3.2)}$$

else if $\tau_{cd} < \tau_u \le \tau_{c,\max}$,

$$\frac{A_t}{s_v} = \frac{T_u}{b_1 d_1\left(0.87 f_{ys}\right)} + \frac{V_u}{2.5 d_1\left(0.87 f_{ys}\right)} \qquad \text{(IS 41.4.3)}$$

else if $\tau_u > \tau_{c,\max}$,

a failure condition is declared (IS 41.3.1)

In calculating the hoop reinforcement, a limit is imposed on the f_{ys} as

$f_{ys} \leq 415$ N/mm^2 (IS 39.4)

The dimensions b_1 and d_1 are taken as follows (Figure 3-9):

b_1 = center-to-center distance between corner bars in the width direction
= $b - 2c$,

d_1 = center-to-center distance between corner bars in the depth direction
= $d - 2c$,

c = cover-to-center, it is taken as 50 mm.

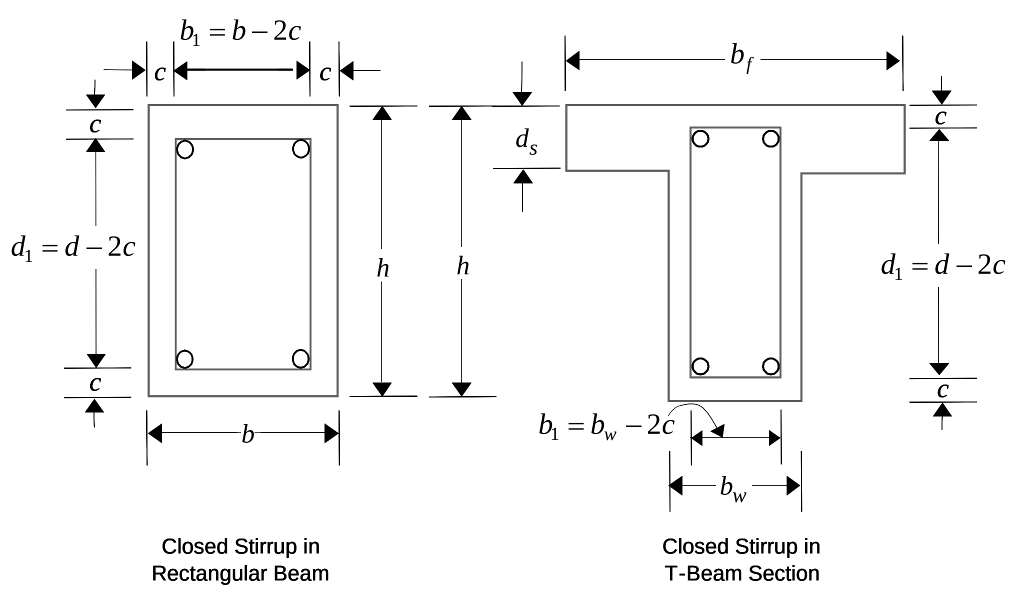

Figure 3-9 Closed stirrup and section dimensions for torsion design

The maximum of all the calculated A_t and A_t /s values obtained from each design load combination is reported along with the controlling combination names.

The beam torsion reinforcement requirements reported by the program are based purely on strength considerations. Any minimum stirrup requirements and longitudinal rebar requirements to satisfy spacing considerations must be investigated independently of the program by the user.

3.5.4 Design Joints for Ductile Frames

Current seismic code (IS 13920 1993) does not cover joint design, and hence as an interim arrangement, a joint design procedure as outlined in the IS 13920 draft code has been adopted for the program and is described in this section. This joint shear check should be taken as informational only.

To ensure that the beam-column joint of ductile frames possesses adequate shear strength, the program performs a rational analysis of the beam-column panel zone to determine the shear forces that are generated in the joint. The program then checks this against design shear strength.

Only joints having a column below the joint are checked. The material properties of the joint are assumed to be the same as those of the column below the joint.

The joint analysis is completed in the major and minor directions of the column. The joint design procedure involves the following steps:

- Determine the panel zone design shear force, V_u^h
- Determine the effective area of the joint
- Check panel zone shear stress

The algorithms associated with these three steps are described in detail in the following three sections.

3.5.4.1 Determine the Panel Zone Shear Force

Figure 3-10 illustrates the free body stress condition of a typical beam-column intersection for a column direction, major or minor.

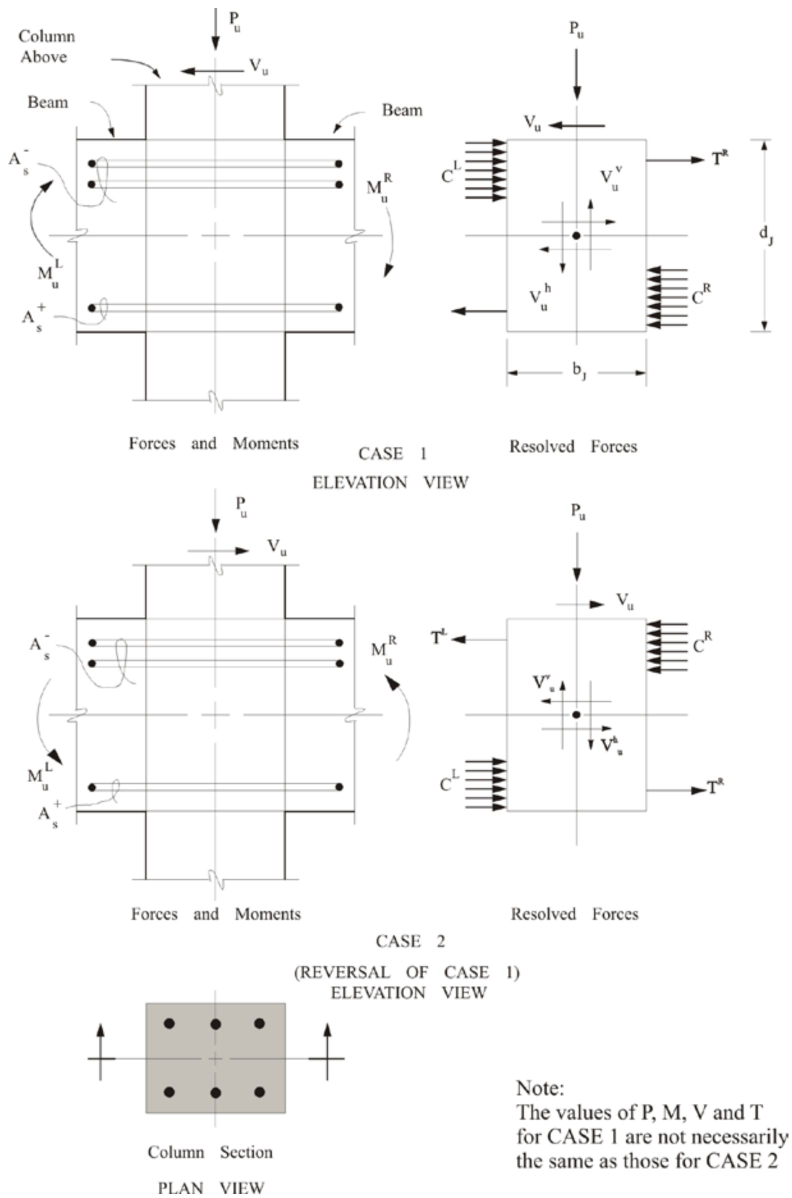

Figure 3-10 Beam-column joint analysis

The force V_u^h is the horizontal panel zone shear force that is to be calculated. The forces that act on the joint are P_u, V_u, M_u^L, and M_u^R. The force P_u and V_u are axial force and shear force, respectively, from the column framing into the top of the joint. The moments M_u^L and M M_u^R are obtained from the beams framing into the joint. The program calculates the joint shear force V_u^h by resolving the moments into *C* and *T* forces. Noting that $T_L = C_l$ and $T_R = C_R$,

$$V_u^h = T_L + T_R - V_u$$

The location of *C* and *T* forces is determined by the direction of the moment. The magnitude of *C* or *T* forces is conservatively determined using basic principles of limit state design theory (IS C8.2.1, ACI 318-02 10.2).

The program resolves the moments and the *C* and *T* forces from beams that frame into the joint in a direction that is not parallel to the major or minor direction of the column along the direction that is being investigated, thereby contributing force components to the analysis. Also, the program calculates the *C* and *T* for the positive and negative moments, considering the fact that the concrete cover may be different for the direction of moment.

In the design of Ductile moment resisting concrete frames, the evaluation of the design shear force is based on the moment capacities (with reinforcing steel stress, αf_y [IS 13920 Draft 8.2.4], where α = 1.25 and no material partial safety factors $\gamma_c = \gamma_s = 1$) of the beams framing into the joint (IS 13920 Draft 8.2.4). The *C* and *T* forces are based on these moment capacities. The program calculates the column shear force V_u from the beam moment capacities, as follows (see Figure 3-5):

$$V_u = \left[\frac{M_u^L + M_u^R}{H} \right]$$

It should be noted that the points of inflection shown in Figure 3-5 are taken as midway between actual lateral support points for the columns. If no column exists at the top of the joint, the shear force from the top of the column is taken as zero.

The effects of load reversals, as illustrated in Case 1 and Case 2 of Figure 3-10, are investigated and the design is based on the maximum of the joint shears obtained from the two cases.

3.5.4.2 Determine the Effective Area of Joint

The joint area, b_j, h_j, that resists the shear forces is assumed always to be rectangular in plan view. The dimensions of the rectangle correspond to the major and minor dimensions of the column below the joint, except if the beam framing into the joint is very narrow or very wide. The effective width of the joint area to be used in the calculation is limited to the width of the beam plus half the depth of the column (IS 13920 Draft 8.2.2). The joint area for joint shear along the major and minor directions is calculated separately.

Effective point width is taken from the following two equations:

$$b_j = \begin{cases} \min\left[b_c; b_b + 0.5D_c\right] & \text{if} \quad b_c > b_b \\ \min\left[b_b; b_c + 0.5D_c\right] & \text{if} \quad b_c < b_b \end{cases} \qquad \text{(IS 13920 Draft 8.2.2)}$$

Effective depth of joint is taken as the depth of column h_c; see Figure 3.11.

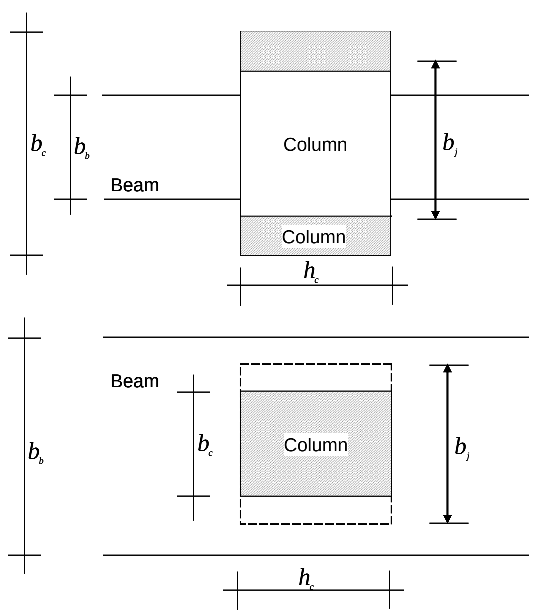

Figure 3-11 Effective Width of Joint (Plan View) from IS 13920 Draft

$h_j = h_c$

It should be noted that if the beam frames into the joint eccentrically, the preceding assumptions may not be conservative and the user should investigate the acceptability of the particular joint.

3.5.4.3 Check Panel Zone Shear Stress

The panel zone shear stress, τ, is evaluated by dividing the shear force by the effective area of the joint and comparing it with the following design shear strength, τ_c (IS 13920 Draft 8.2.1).

Panel zone stress, τ, is computed using the following equation

$$\tau = \frac{V_j}{b_j h_j} \qquad \text{(IS 13920 Draft 8.2.1)}$$

where

V_j = Design shear force at the joint

b_j = Effective width of joint

h_j = Effective depth of joint

The allowable nominal shear stress is taken as follows:

$$\tau_c = \begin{cases} 1.5\sqrt{f_{ck}} & \text{for joints confined on all four sides,} \\ 1.2\sqrt{f_{ck}} & \text{for joints confined on three faces or on two opposite faces,} \\ 1.0\sqrt{f_{ck}} & \text{for all other joints.} \end{cases}$$

(IS 13920 Draft 8.2.1)

A beam that frames into a face of a column at the joint is considered in this program to provide confinement to the joint if at least three-quarters of the face of the joint is covered by the framing member (IS 13920 Draft 1.3). This has been adopted from ACI Section 21.5.3.1.

The joint shear stress ratio is calculated as follows:

$$R = \frac{\tau}{\tau_c}$$

For joint design, the program reports the joint shear, V_j, the joint shear stress, τ, the allowable joint shear stress, τ_c, and a capacity ratio.

3.5.4.4 Beam-Column Flexural Capacity Ratios

Current seismic code (IS 13920:1993) does not cover beam-column flexural capacity ratio. However, as an interim arrangement, beam-column capacity ratio checks as outlined in the IS 13920 Draft code have been adopted as described in this section. This check information is taken as informational only.

The program calculates the ratio of the sum of the beam moment capacities to the sum of the column moment capacities for Ductile frames only. For Ductile frames, at a particular joint for a particular column direction, major or minor (IS 13920 Draft 7.2.1):

$$\sum M_c \geq 1.1 \sum M_g \qquad \text{(SI 13920 Draft 7.2.1)}$$

$\sum M_c$ = Sum of flexural strengths of columns framing into the joint, evaluated at the faces of the joint. Individual column flexural strength is calculated for the associated factored axial force.

$\sum M_g$ = Sum of flexural strengths of the beams framing into the joint, evaluated at the faces of the joint.

The capacities are calculated with the appropriate material partial safety factors, γ_c and γ_s. The beam capacities are calculated for reversed situations (Cases 1 and 2) as illustrated in Figure 3-10 and the maximum summation obtained is used.

The moment capacities of beams that frame into the joint in a direction that is not parallel to the major or minor direction of the column are resolved along the direction that is being investigated and the resolved components are added to the summation.

The column capacity summation includes the column above and the column below the joint. For each load combination, the axial force, P_u, in each of the columns is calculated from the program design load combinations. For each design load combination, the moment capacity of each column under the influence of the corresponding axial load is then determined separately for the major and minor directions of the column, using the uniaxial column interaction diagram (see Figure 3-12). The moment capacities of the two columns are added to give the capacity summation for the corresponding design load combination. The maximum capacity summations obtained from all of the design load combinations is used for the beam-column capacity ratio.

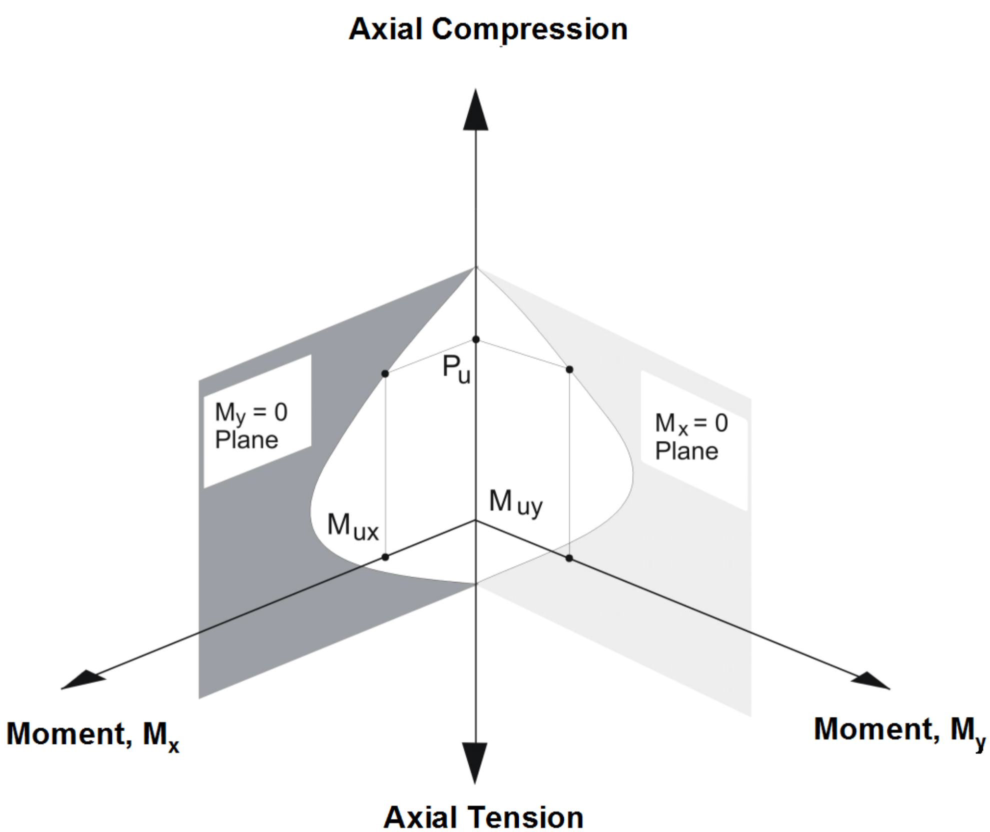

Figure 12 Moment capacity M_u at a given axial load P_u

The beam-column capacity ratio is determined for a beam-column joint only when the following conditions are met:

- the frame is ductile moment resisting
- a column of concrete material exists above the beam-column joint
- all of the beams framing into the column are concrete frame
- the connecting member design results are available
- the load combo involves seismic load

The amplified beam-column flexural capacity ratio $\left(1.1\sum M_g / \sum M_c\right)$ are reported only for ductile frames involving seismic design load combinations. If this ratio is greater than 1.0, a warning message is printed in the output.

APPENDICES

Appendix A
Second Order P-Delta Effects

Typically, design codes require that second order P-Delta effects be considered when designing concrete frames. They are the global lateral translation of the frame and the local deformation of members within the frame.

Consider the frame object shown in Figure A-1, which is extracted from a story level of a larger structure. The overall global translation of this frame object is indicated by Δ. The local deformation of the member is shown as δ. The total second order P-Delta effects on this frame object are those caused by both Δ and δ.

The program has an option to consider P-Delta effects in the analysis. When P-Delta effects are considered in the analysis, the program does a good job of capturing the effect due to the Δ deformation shown in Figure A-1, but it does not typically capture the effect of the δ deformation (unless, in the model, the frame object is broken into multiple elements over its length).

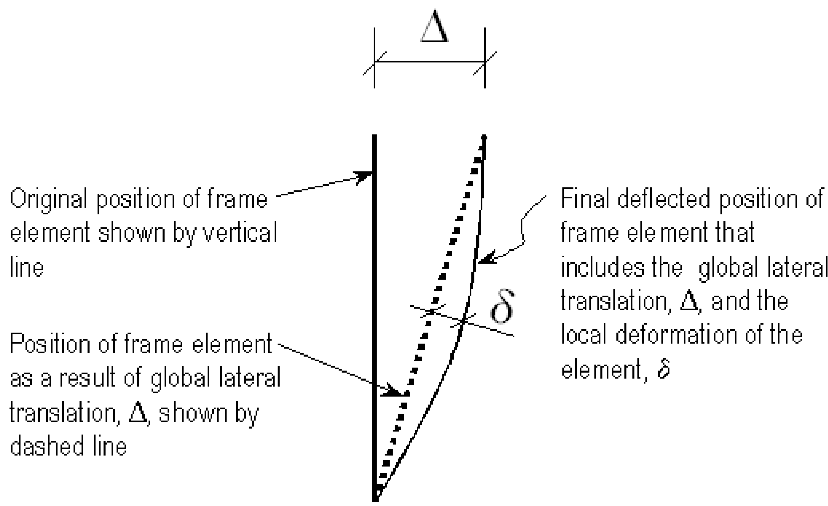

Figure A-1 The total second order P-delta effects on a frame element caused by both Δ and δ

Consideration of the second order P-Delta effects is generally achieved by computing the flexural design capacity using a formula similar to that shown in the following equation.

$$M_{CAP} = a(M_{nt} + bM_{lt}) \quad \text{where,}$$

M_{CAP} = Flexural design capacity required

M_{nt} = Required flexural capacity of the member assuming there is no joint translation of the frame (i.e., associated with the δ deformation in Figure A-1)

M_{lt} = Required flexural capacity of the member as a result of lateral translation of the frame only (i.e., associated with the Δ deformation in Figure A-1)

a = Unitless factor multiplying M_{nt}

b = Unitless factor multiplying M_{lt} (assumed equal to 1 by the program; see the following text)

When the program performs concrete frame design, it assumes that the factor b is equal to 1 and calculates the factor a. That b = 1 assumes that P-Delta effects have been considered in the analysis, as previously described. Thus, in general, when performing concrete frame design in this program, **consider P-Delta effects in the analysis before running the design**.

Appendix B
Member Unsupported Lengths

The column unsupported lengths are required to account for column slenderness effects. The program automatically determines the unsupported length ratios, which are specified as a fraction of the frame object length. Those ratios times the frame object length give the unbraced lengths for the members. Those ratios can also be overwritten by the user on a member-by-member basis, if desired, using the overwrite option.

There are two unsupported lengths to consider. They are L_{33} and L_{22}, as shown in Figure B-1. These are the lengths between support points of the member in the corresponding directions. The length L_{33} corresponds to instability about the 3-3 axis (major axis), and L_{22} corresponds to instability about the 2-2 axis (minor axis).

In determining the values for L_{22} and L_{33} of the members, the program recognizes various aspects of the structure that have an effect on those lengths, such as member connectivity, diaphragm constraints and support points. The program automatically locates the member support points and evaluates the corresponding unsupported length.

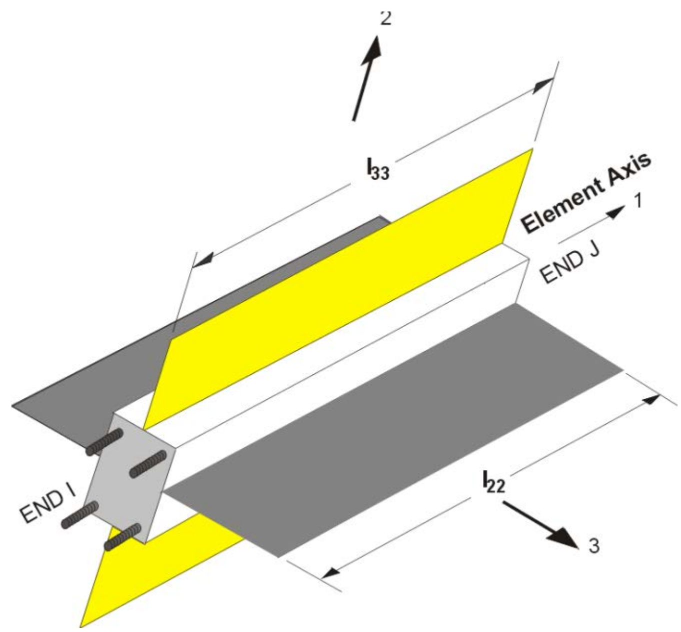

Figure B-1 Axis of bending and unsupported length

It is possible for the unsupported length of a frame object to be evaluated by the program as greater than the corresponding member length. For example, assume a column has a beam framing into it in one direction, but not the other, at a floor level. In that case, the column is assumed to be supported in one direction only at that story level, and its unsupported length in the other direction will exceed the story height.

Appendix C
Effective Length Factor of Columns

The effective length factor, K, is the ratio of the effective length of the column, l_e, to the unsupported length, l, of the column ($K = l_e/l$). The program calculates the effective length factors of concrete columns in frame structures based on Annex E of the code (IS 25.2, 39.7, Annex E).

If the column is identified as Ordinary or Ductile, it is taken as a sway frame member. For such column members, the program calculates K using the chart given in IS Figure 27 (IS Annex E-1, Figure 27). In such cases, K is always greater than 1. Also the program enforces a lower limit of 1.2 on the K values:

$K \geq 1$ (non-sway frame)

If the P-Δ analysis is turned on, the program assumes that the joints are prevented from further sway and takes $K = 1$ for all framing types.

$K = 1$ (if P-Δ has been completed)

There are two K-factors associated with each column. One for major direction bending (K_{33}) and the other is for minor direction bending (K_{22}). The K-factors are used in calculating the code-specified additional moment (IS 39.7), which tries to capture the P-δ effect.

The *K*-factor algorithm has been developed for building-type structures, where the columns are vertical and the beams are horizontal, and the behavior is basically that of a moment-resisting frame for which the *K*-factor calculation is relatively complex. For the purpose of calculating *K*-factors, the objects are identified as columns, beam and braces. All frame objects parallel to the Z-axis are classified as columns. All objects parallel to the X-Y plane are classified as beams. The remainders are considered to be braces.

The beams and braces are assigned *K*-factors of unity. In the calculation of the *K*-factors for a column object, the program first makes the following four stiffness summations for each joint in the structural model:

$$S_{cx} = \sum\left(\frac{E_c I_c}{L_c}\right)_x \qquad S_{bx} = \sum\left(\frac{E_b I_b}{L_b}\right)_x$$

$$S_{cy} = \sum\left(\frac{E_c I_c}{L_c}\right)_y \qquad S_{by} = \sum\left(\frac{E_b I_b}{L_b}\right)_y$$

where the *x* and *y* subscripts correspond to the global *X* and *Y* directions and the *c* and *b* subscripts refer to column and beam. The local 2-2 and 3-3 terms EI_{22}/L_{22} and EI_{33}/L_{33} are rotated to give components along the global *X* and *Y* directions to form the $(EI/L)_x$ and $(EI/L)_y$ values. Then for each column, the joint summations at END-I and the END-J of the member are transformed back to the column local 1-2-3 coordinate system, and the β-values for END-I and the END-J of the member are calculated about the 2-2 and 3-3 directions as follows (IS Annex E, E-1 Note 2):

$$\beta^I{}_{22} = \frac{S^I{}_{c22}}{S^I{}_{c22} + S^I{}_{b22}} \qquad \beta^J{}_{22} = \frac{S^J{}_{c22}}{S^J{}_{c22} + S^J{}_{b22}}$$

$$\beta^I{}_{33} = \frac{S^I{}_{c33}}{S^I{}_{c33} + S^I{}_{b33}} \qquad \beta^J{}_{33} = \frac{S^J{}_{c33}}{S^J{}_{c33} + S^J{}_{b33}}$$

If a rotational release exists at a particular end (and direction) of an object, the corresponding value of β is set to 0.9. If all degrees of freedom for a particular joint are deleted, the β-values for all members connecting to that joint will be set to 0.5 for the end of the member connecting to that joint. Finally, if β^I and β^J are known for a particular direction, the column *K*-factors for the corresponding direction are calculated as follows:

$$K = \frac{l_e}{l} = \left[\frac{1 - 0.2(\beta_1 + \beta_2) - 0.12\beta_1\beta_2}{1 - 0.8(\beta_1 + \beta_2) + 0.60\beta_1\beta_2} \right]^{1/2}$$

The preceding expression closely and slightly conservatively approximates the Woods' chart (Wood 1974), which is reproduced in Indian codes as Figure 27 (IS Annex E-1, Note 1). The preceding expression has appeared in Figure E2 of Annex E of British code BS 5950-1:2000, and in Section E.2(12) of Annex E of Eurocode ENV 1993-1-1:1992. This expression approximates the Woods chart, which is reproduced in British code as Figure E.2 in Annex E and reproduced in Eurocode as Figure E.2.2 in Annex E.

The following are some important aspects associated with the column *K*-factor algorithm:

- An object that has a pin at the joint under consideration will not enter the stiffness summations calculated above. An object that has a pin at the far end from the joint under consideration will contribute only 50% of the calculated *EI* value. Also, beam members that have no column member at the far end from the joint under consideration, such as cantilevers, will not enter the stiffness summation.

- If there are no beams framing into a particular direction of a column member, the associated β value will be 1. If the β value at any one end of a column for a particular direction is 1, the *K*-factor corresponding to that direction is set equal to unity.

- If rotational releases exist at both ends of an object for a particular direction, the corresponding *K*-factor is set to unity.

- The automated *K*-factor calculation procedure can occasionally generate artificially high *K*-factors, specifically under circumstances involving skewed beams, fixed support conditions, and under other conditions where the program may have difficulty recognizing that the members are laterally supported and *K*-factors of unity are to be used.

- All *K*-factors produced by the program can be overwritten by the user. These values should be reviewed and any unacceptable values should be replaced. Consult the program Help for information about applying overwrites.

- The beams and braces are assigned *K*-factors of unity.

Appendix D
Concrete Frame Design Preferences

The Concrete Frame Design Preferences are basic assignments that apply to all of the concrete frame members. Table D-1 lists the Concrete Frame Design Preferences for the IS 456 2000 code. Default values are provided for all preference items. Thus, it is not necessary to specify or change any of the preferences. However, at least review the default values to ensure they are acceptable. Some of the preference items also are available as member specific overwrite items. The Overwrites are described in Appendix E. Overwritten values take precedence over the preferences.

Table D-1 Preferences

Item	Possible Values	Default Value	Description
Number Interaction Curves	Multiple of 4 ≥ 4	24	Number of equally spaced interaction curves used to create a full 360 *deg* interaction surface (this item should be a multiple of four). We recommend 24 for this item.

Item	Possible Values	Default Value	Description
Number of Interaction Points	Any odd value ≥ 5	11	Number of points used for defining a single curve in a concrete frame should be odd.
Consider Minimum Eccentricity	No, Yes	Yes	Toggle to consider if minimum eccentricity is considered in design.
Gamma (Steel)	> 0	1,15	Strength reduction factor for rebar
Gamma (Concrete)	> 0	1.5	The strength reduction factor for concrete.
Pattern Live Load Factor	≥ 0	0	The scale factor for performing pattern loading for live loads. Zero means no pattern loading.
Utilization Factor Limit	> 0	0.95	Stress ratios that are less than or equal to this value are considered acceptable.

Appendix E
Concrete Frame Overwrites

The concrete frame design overwrites are basic assignments that apply only to those elements to which they are assigned. Table E-1 lists concrete frame design overwrites for the IS 456 2000 code. Default values are provided for all overwrite items. Thus, it is not necessary to specify or change any of the overwrites. However, at least review the default values to ensure they are acceptable. When changes are made to overwrite items, the program applies the changes only to the elements to which they are specifically assigned.

Table E-1 Overwrites

Item	Possible Values	Default Value	Description
Current Design Section	Any defined concrete section	Analysis section	The design section for the selected frame objects. When this overwrite is applied, any previous auto select section assigned to the frame object is removed.

Item	Possible Values	Default Value	Description
Element Type	Ductile, Ordinary, Non-sway	Ductile	Frame type per moment frame definition. The program default is Ductile, which the user can overwrite if needed.
Live Load Reduction Factor	≥ 0	Calculated	The reduced live load factor. A reducible live load is multiplied by this factor to obtain the reduced live load for the frame object. Specifying 0 means the value is program determined.
Unbraced Length Ratio (Major)	≥ 0	Calculated	Unbraced length factor for buckling about the frame object major axis. This item is specified as a fraction of the frame object length. Multiplying this factor times the frame object length gives the unbraced length for the object. Specifying 0 means the value is program determined.
Unbraced Length Ratio (Minor)	≥ 0	Calculated	Unbraced length factor for buckling about the frame object minor axis. Multiplying this factor times the frame object length gives the unbraced length for the object. Specifying 0 means the value is program determined. This factor is also used in determining the length for lateral-torsional buckling.
Effective Length Factor (*K* Major)	> 0	Calculated	Effective length factor for buckling about the frame major axis. This item is specified as a fraction of the frame object length.
Effective Length Factor (*K* Minor)	> 0	Calculated	Effective length factor for buckling about the frame minor axis. This item is specified as a fraction of the frame object length.
Top Left Rebar Area for Capacity	≥ 0	0	The beam top rebar area at the left end. If this is overwritten, it is used to calculate beam end moment, which is used to calculate beam capacity shear, column capacity shear, beam/column capacity ratio, and partial zone shear force. Specifying zero means the value is program determined.
Bottom Left Rebar Area for Capacity	≥ 0	0	The beam bottom rebar area at the left end. If this is overwritten, it is used to calculate beam end moment, which is used to calculate beam capacity shear, column capacity shear, beam/column capacity ratio, and partial zone shear force. Specifying zero means the value is program determined.

Item	Possible Values	Default Value	Description
Top Right Rebar Area for Capacity	≥ 0	0	The beam top rebar area at the right end. If this is overwritten, it is used to calculate beam end moment, which is used to calculate beam capacity shear, column capacity shear, beam/column capacity ratio, and partial zone shear force. Specifying zero means the value is program determined.
Bottom Right Rebar Area for Capacity	≥ 0	0	The beam bottom rebar area at the right end. If this is overwritten, it is used to calculate beam end moment, which is used to calculate beam capacity shear, column capacity shear, beam/column capacity ratio, and partial zone shear force. Specifying zero means the value is program determined.

References

BSI, 2001, BS 5950-1:2000 Structural Use of Steelwork in Building – Part 1: Code of Practice for Design – Rolled and Welded Sections, British Standard Institution.

CEN, 1993, ENV 1993-1-1 Eurocode 3: Design of Steel Structures, Part 1.1 – General Rules and Rules for Building, European Committee for Standardization, Brussels, Belgium.

CSI, 2008. CSI Analysis Reference Manual, Computers and Structures, Inc., Berkeley, California.

IS, 1983, Explanatory Handbook on Indian Standard Code of Practice for Plain and Reinforced Concrete IS:456-1978, SP:24-1983, Bureau of Indian Standards, New Delhi, India.

IS, 2000. IS 456:2000 – Indian Standard – Plain – Code of Practice, Reinforced Concrete, Fourth Edition, Second Reprint, October 2000, Bureau of Indian Standards, Manak Bhavan, 9 Bahadur Shah Zafar Marg, New Delhi 110002, India.

IS, 2002. IS 13920:1993 (Reaffirmed 1998, Edition 12, 2002-03) Indian Standard Ductile Detailing of Reinforced Concrete Structures Subject to Seismic Forces – Code of Practice, Incorporating Amendment Nos. 1 & 2, Bureau of Indian Standards, Manak Bhavan, 9 Bahadur Shah Zafar Marg, New Delhi 11002, India.

Jain, S.K. and C.V.R Murty, 2008, Proposed Draft Provision, and Commentary on Ductile Detailings of RC Structures Subjected to Seismic Forces; Document IITK-GSDMA-EQII-V4.0, IITK-GSDMA-EQ16-V3.0, Indian Institute of Technology Kanput Kanpur, India.

White, D. W. and J. F. Hajjar, 1991. "Application of Second-Order Elastic Analysis in LRFD: Research to Practice," Engineering Journal, American Institute of Steel Construction, Inc., Vol. 28, No. 4.