Design of Beams

3 Module No. 2: Ultimate Flexure Strength (R_DOUBLE-SECTION)

3.1 Inputs

3.1.1 Material Properties

- ✓ Concrete compressive strength (f_{cu})
- ✓ Reduction factor of concrete compressive strength (γ_c=1.50)
- ✓ Yield strength of longitudinal reinforcing steel bars (f_y)
- ✓ Reduction factor of reinforcing steel yield strength(γ_s=1.15)
- ✓ Modulus of elasticity of reinforcing steel bars (E_s = 200,000 N/mm^2)

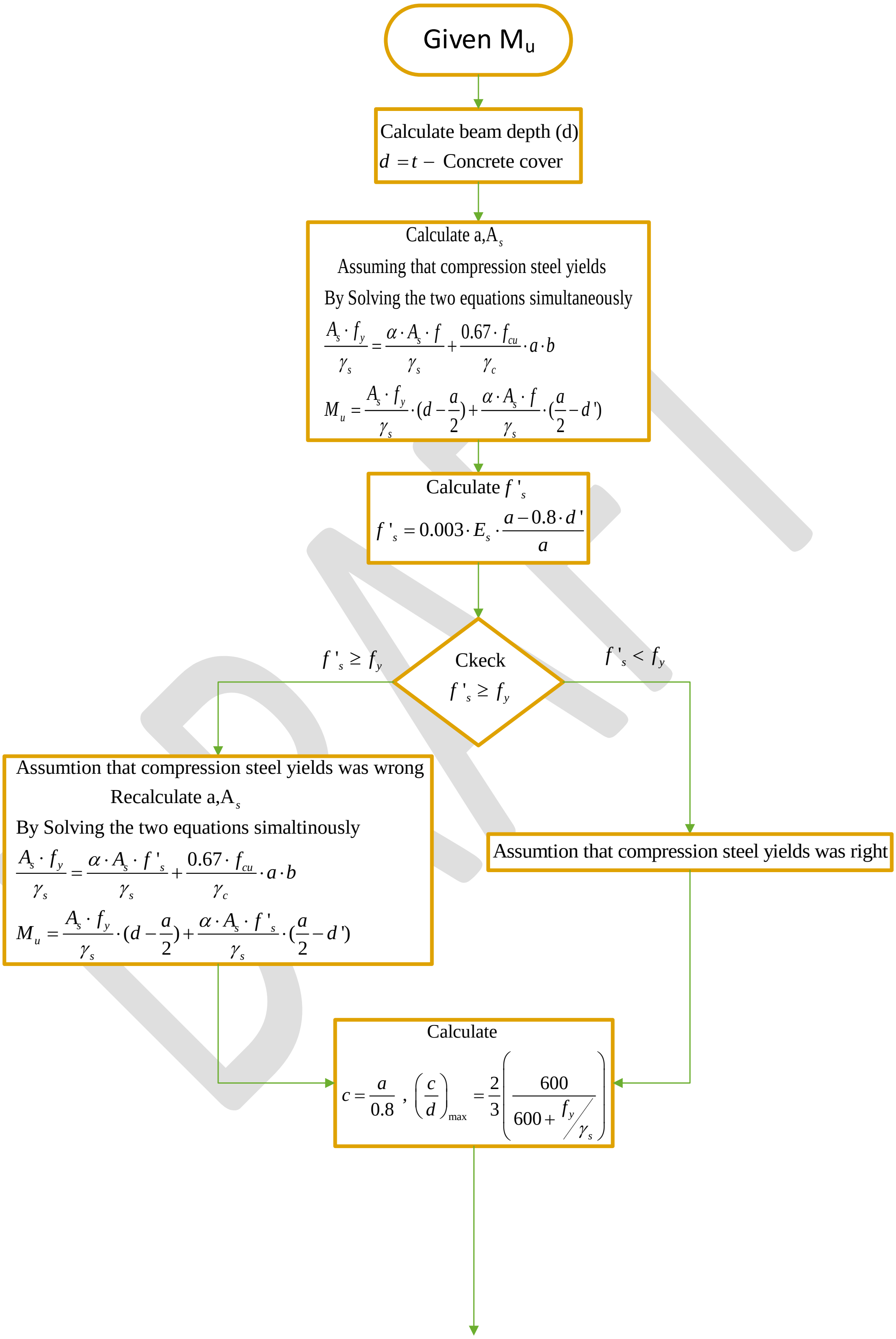
3.1.2 Section Definition

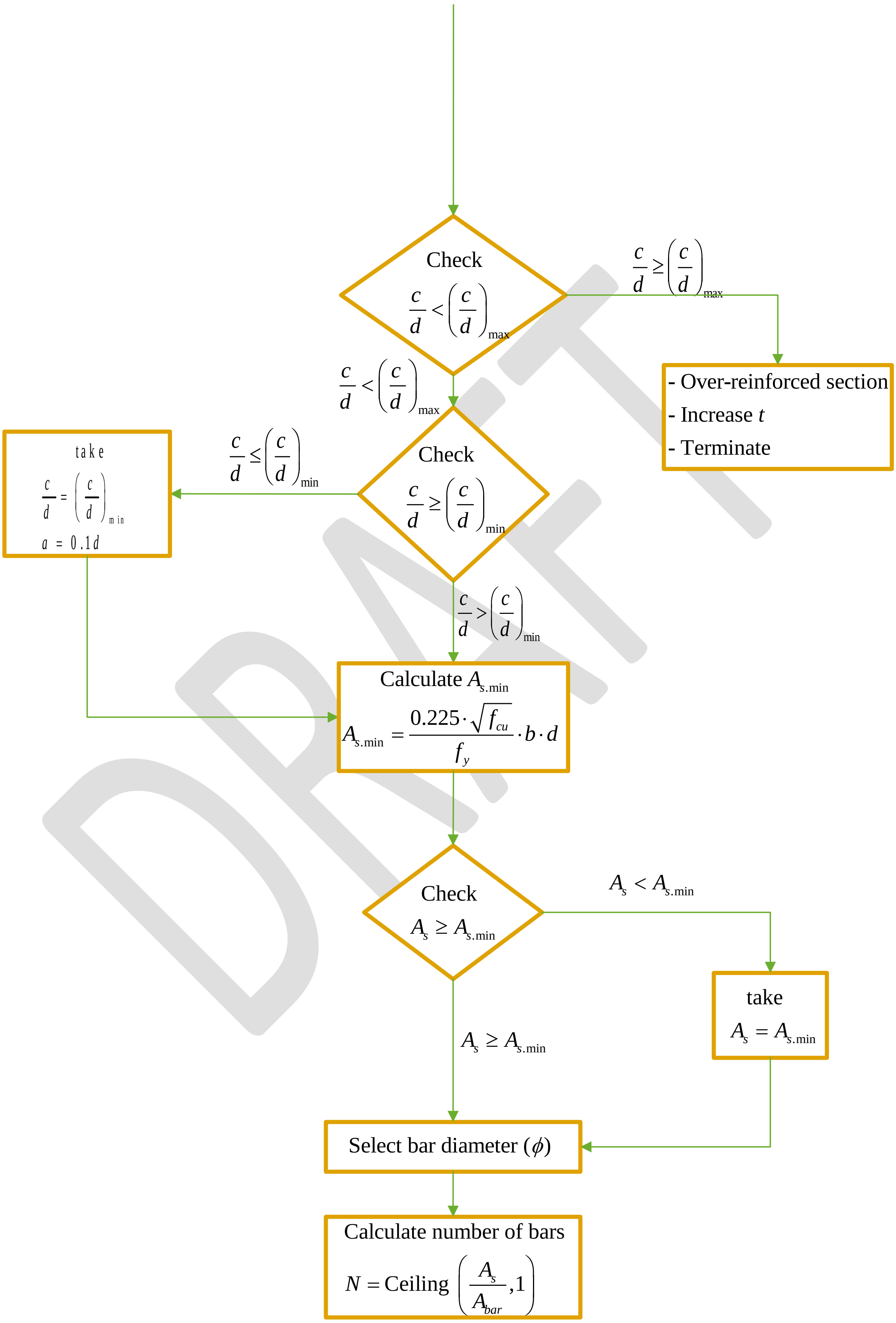
- ✓ Beam width (b)
- ✓ Total beam depth (t)
- ✓ Concrete cover
- ✓ Beam span (L)

3.1.3 Internal Forces

- ✓ Ultimate Factored Moment from analysis (M_u)

3.2 Calculation Procedure




Check
$\frac{c}{d} < \left(\frac{c}{d}\right)_{max}$
$\frac{c}{d} \geq \left(\frac{c}{d}\right)_{max}$
- Over-reinforced section
- Increase t
- Terminate
$\frac{c}{d} < \left(\frac{c}{d}\right)_{max}$
Check
$\frac{c}{d} \geq \left(\frac{c}{d}\right)_{min}$
$\frac{c}{d} \leq \left(\frac{c}{d}\right)_{min}$
take
$\frac{c}{d} = \left(\frac{c}{d}\right)_{min}$
$a = 0.1d$
$\frac{c}{d} > \left(\frac{c}{d}\right)_{min}$
Calculate $A_{s.min}$
$A_{s.min} = \frac{0.225 \cdot \sqrt{f_{cu}}}{f_y} \cdot b \cdot d$
Check
$A_s \geq A_{s.min}$
$A_s < A_{s.min}$
take
$A_s = A_{s.min}$
$A_s \geq A_{s.min}$
Select bar diameter (ϕ)
Calculate number of bars
$N = \text{Ceiling}\left(\frac{A_s}{A_{bar}}, 1\right)$

3.3 Design Outputs

- ✓ Beam total depth (t).
- ✓ Diameter of bars.
- ✓ Number of bars.

3.4 Example of Calculations using Mathcad

3.1 Input

3.1.1 Material Properties

$f_{cu} := 30\ MPa$	Concrete compressive strength
$f_y := 360\ MPa$	Yield strength of reinforcing steel bar
$E_s := 200000\ MPa$	Modulus of easticity of steel
$\gamma_c := 1.5$	Concrete strength reduction factor
$\gamma_s := 1.15$	Steel strength reduction factor

3.1.2 Section Definition

$b := 250\ mm$	Beam width
$t := 700\ mm$	Total beam depth
$conc.cover := 50\ mm$	Concrete Cover
$d' := 100\ mm$	Cover for compression steel
$\alpha := 0.3$	Percentage of compression steel to tension steel

3.1.3 Internal Forces

$M_u := 400\ kN \cdot m$	Ultimate Factored Moment from analysis

3.2 Calculation Procedure

$d := t - conc.cover = 650\ mm$

$a := 1\ mm$

$A_s := 1\ mm^2$

$$\frac{A_s \cdot f_y}{\gamma_s} = \frac{\alpha \cdot A_s \cdot f_y}{\gamma_s} + \frac{0.67 \cdot a \cdot b \cdot f_{cu}}{\gamma_c}$$

$$\frac{A_s \cdot f_y}{\gamma_s} \cdot \left(d - \frac{a}{2}\right) + \frac{\alpha \cdot A_s \cdot f_y}{\gamma_s} \cdot \left(\frac{a}{2} - d'\right) = M_u$$

$$\begin{bmatrix} a_1 \\ A_{s1} \end{bmatrix} := \mathbf{find}\left(a, A_s\right) = \begin{bmatrix} 0.147\ m \\ 0.002\ m^2 \end{bmatrix}$$

$a_1 = 147.01\ mm$

$A_{s1} = 2247.447\ mm^2$

$a := 1\ mm$

$A_s := 1\ mm^2$

$$\frac{A_s \cdot f_y}{\gamma_s} = \frac{\alpha \cdot A_s \cdot 0.003 \cdot E_s \cdot \frac{a - 0.8 \cdot d'}{a}}{\gamma_s} + \frac{0.67 \cdot a \cdot b \cdot f_{cu}}{\gamma_c}$$

$$\frac{A_s \cdot f_y}{\gamma_s} \cdot \left(d - \frac{a}{2}\right) + \frac{\alpha \cdot A_s \cdot 0.003 \cdot E_s \cdot \frac{a - 0.8 \cdot d'}{a}}{\gamma_s} \cdot \left(\frac{a}{2} - d'\right) = M_u$$

$$\begin{bmatrix} a_2 \\ A_{s2} \end{bmatrix} := \mathbf{find}\left(a, A_s\right) = \begin{bmatrix} 0.159\ m \\ 0.002\ m^2 \end{bmatrix}$$

$a_2 = 158.768\ mm$

$A_{s2} = 2259.547\ mm^2$

$$\begin{bmatrix} a \\ A_s \end{bmatrix} := \text{if } 0.003 \cdot E_s \cdot \frac{a_1 - 0.8 \cdot d'}{a_1} \geq f_y \ \left\| \begin{bmatrix} a_1 \\ A_{s1} \end{bmatrix} \right. \ \text{else} \ \left\| \begin{bmatrix} a_2 \\ A_{s2} \end{bmatrix} \right.$$

$A_s = 2259.547\ mm^2$

$A'_s := \alpha \cdot A_s = 677.864\ mm^2$

$a = 158.768\ mm$

$$c := \frac{a}{0.8} = 198.46\ mm$$

$$Check_1 := \text{if } \frac{c}{d} \ge \frac{2}{3} \cdot \frac{0.003}{0.003 + \frac{f_y}{\gamma_s \cdot E_s}}$$

$$\quad \text{“Over reinforced section, increase t”}$$

$$\text{else}$$

$$\quad \text{“No need to increase t”}$$

$$Check_1 = \text{“No need to increase t”}$$

$$c := \text{if}(c < 0.125 \cdot d, 0.125 \cdot d, c)$$

$$c = 198.46\ mm$$

$$a := 0.8 \cdot c = 158.768\ mm$$

$$A_{smin} := \frac{0.225\ \sqrt{MPa} \cdot \sqrt{f_{cu}} \cdot b \cdot d}{f_y} = 556.281\ mm^2$$

$$A_s := \text{if}(A_s < A_{smin}, A_{smin}, A_s)$$

$$A_s = 2260\ mm^2$$

$$\phi 6 := 28.3\ mm^2$$

$$\phi 8 := 50.3\ mm^2$$

$$\phi 10 := 78.5\ mm^2$$

$$\phi 12 := 113\ mm^2$$

$$\phi 14 := 154\ mm^2$$

$$\phi 16 := 201\ mm^2$$

$$\phi 18 := 254\ mm^2$$

$$\phi 20 := 314\ mm^2$$

$$\phi 22 := 380\ mm^2$$

$$\phi 25 := 491\ mm^2$$

$$\phi 28 := 616\ mm^2$$

$$\phi 32 := 804\ mm^2$$

$$\phi 38 := 1134\ mm^2$$

$$N_1 := \text{Ceil}\left(\frac{A_s}{\phi 16}, 1\right) = 12$$

$$N_2 := \text{Ceil}\left(\frac{A'_s}{\phi 16}, 1\right) = 4$$

3.3 Output

$t = 700\ mm$

$b = 250\ mm$

Use $N_1 = 12\ \ \phi 16$ Bottom Reinforcement

Use $N_2 = 4\ \ \phi 16$ Top Reinforcement