4.(4) $(\neg(\exists x)P(x) \vee (\forall y)Q(y)) \rightarrow (\forall z)R(z)$

$= \neg(\neg(\exists x)P(x) \vee (\forall y)Q(y)) \vee (\forall z)R(z)$

$= ((\exists x)P(x) \wedge \neg(\forall y)Q(y)) \vee (\forall z)R(z)$

$= ((\exists x)P(x) \wedge (\exists y)\neg Q(y)) \vee (\forall z)R(z)$

$= (\exists x)(\exists y)(\forall z)((P(x) \wedge \neg Q(y)) \vee R(z))$

(8) $(\forall x)(P(x) \rightarrow Q(x)) \rightarrow ((\exists x)P(x) \rightarrow (\exists x)Q(x))$

$= \neg(\forall x)(\neg P(x) \vee Q(x)) \vee (\neg(\exists x)P(x) \vee (\exists x)Q(x))$

$= (\exists x)(P(x) \wedge \neg Q(x)) \vee (\forall x)\neg P(x) \vee (\exists x)Q(x)$

$= (\forall x)(P(x) \wedge \neg Q(x)) \vee (\forall y)\neg P(y) \vee (\exists z)Q(z)$

$= (\forall x)(\forall y)(\exists z)((P(x) \wedge \neg Q(x)) \vee \neg P(y) \vee Q(z))$

(9) $(\forall x)(P(x) \rightarrow (\exists y)Q(x,y)) \vee (\forall z)R(z)$

$= (\forall x)(\exists y)(P(x) \rightarrow Q(x,y)) \vee (\forall z)R(z)$

$= (\forall x)(\exists y)(\forall z)(\neg P(x) \vee Q(x,y) \vee R(z))$

SKOLEM 范式：

$(\forall x)(\forall z)(\neg P(x) \vee Q(x,f(x)) \vee R(z))$

(10) $(\exists y)(\forall x)(\forall z)(\exists u)(\forall v)P(x,y,z,u,v)$

SKOLEM 范式：

$(\forall x)(\forall z)(\forall v)P(x,a,z,f(x,z),v)$

5.(1) $(\forall x)(P(x) \vee Q(x)) \wedge (\forall x)(Q(x) \rightarrow \neg R(x)) \Rightarrow (\exists x)(R(x) \rightarrow P(x))$

推理规则：

① $(\forall x)(P(x) \vee Q(x)) \wedge (\forall x)(Q(x) \rightarrow \neg R(x))$	前提
② $(\forall x)(P(x) \vee Q(x))$	①
③ $P(y) \vee Q(y)$	全称量词消去.
④ $\neg P(y) \rightarrow Q(y)$	③置换.
⑤ $(\forall x)(Q(x) \rightarrow \neg R(x))$	①
⑥ $Q(y) \rightarrow \neg R(y)$	全称量词消去
⑦ $\neg P(y) \rightarrow \neg R(y)$	④⑥三段论.
⑧ $R(y) \rightarrow P(y)$	⑦置换.
⑨ $(\forall x)(R(x) \rightarrow P(x))$	全称量词引入
⑩ $R(c) \Rightarrow P(c)$	全称量词消去.
⑪ $(\exists x)(R(x) \rightarrow P(x))$	存在量词引入.

归结法：

原命题 $\Leftrightarrow (\forall x)(P(x)\vee Q(x))\wedge(\forall x)(Q(x)\to\neg R(x))\wedge\neg(\exists x)(R(x)\to P(x))$ 不可满足

$\Leftrightarrow (\forall x)(P(x)\vee Q(x))\wedge(\forall x)(\neg Q(x)\vee\neg R(x))\wedge(\forall x)(R(x)\wedge\neg P(x))$ 不可满足.

建立子句集：

$\{P(x)\vee Q(x),\ \neg Q(x)\vee\neg R(x),\ R(x),\ \neg P(x)\}$.

① $P(x)\vee Q(x)$
② $\neg Q(x)\vee\neg R(x)$
③ $R(x)$
④ $\neg P(x)$
⑤ $Q(x)$ ①④归结.
⑥ $\neg Q(x)$ ②③归结.
⑦ $\square$ ⑤⑥归结.

5.(4) $P(x)$：x是学生，$Q(x)$：x是研究生，$R(x)$：x是本科生，$S(x)$：x是高材生.

证明：

$(\forall x)(P(x)\to((Q(x)\vee R(x))\wedge(\neg Q(x)\vee\neg R(x))))$，$(\exists x)(P(x)\wedge S(x))$，$\neg Q(John)\wedge S(John)\Rightarrow P(John)\to R(John)$

① $(\forall x)(P(x)\to((Q(x)\vee R(x))\wedge(\neg Q(x)\vee\neg R(x))))$ 前提
② $P(John)$ 附加前提引入.
③ $(Q(John)\vee R(John))\wedge(\neg Q(John)\vee\neg R(John))$ ①②三段论.
④ $Q(John)\vee R(John)$ ③
⑤ $\neg Q(John)\to R(John)$ ④置换.
⑥ $\neg Q(John)\wedge S(John)$ 前提.
⑦ $\neg Q(John)$ ⑥
⑧ $R(John)$ ⑤⑦三段论.
⑨ $P(John)\to R(John)$ 条件证明规则.

归结法：建立子句集 $\{\neg P(x)\vee Q(x)\vee R(x),\ \neg P(x)\vee\neg Q(x)\vee\neg R(x),\ S(a),\ P(a),\ \neg Q(John),\ S(John),\ P(John),\ \neg R(John)\}$

① $\neg P(x) \vee Q(x) \vee R(x)$

② $\neg P(x) \vee \neg Q(x) \vee \neg R(x)$

③ $S(a)$

④ $P(a)$

⑤ $\neg Q(John)$

⑥ $S(John)$

⑦ $P(John)$

⑧ $\neg R(John)$

⑨ $Q(John) \vee R(John)$ ① ⑦ 归结.

⑩ $R(John)$ ⑨ ⑤ 归结

⑪ □ ⑧ ⑩ 归结