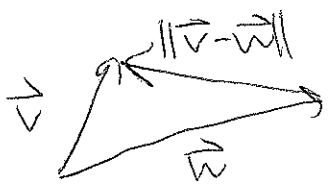
~~2. (10 points) Sketch the graph of the function $f(x) = \frac{x^2}{x^2-1}$, labeling all asymptotes, poles, maximum or minimum points, and inflection points.~~

Problem 3 $\|\vec{v}\| = 3$ and $\|\vec{w}\| = 5$

(a) Smallest and largest values of $\|\vec{v} - \vec{w}\|$:



Triangle inequality says

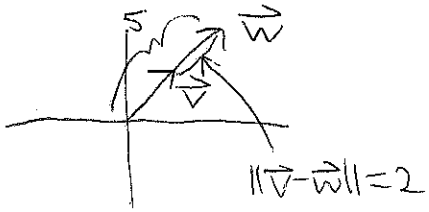
$\|\vec{v} - \vec{w}\| \leq \|\vec{v}\| + \|-\vec{w}\| = \|\vec{v}\| + \|\vec{w}\| = 3 + 5 = 8$

and $\|\vec{w}\| = \|(\vec{v} - \vec{w}) + (-\vec{v})\| \leq \|\vec{v} - \vec{w}\| + \|\vec{v}\|$

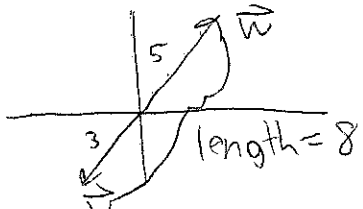
$\rightarrow \|\vec{v} - \vec{w}\| \geq \|\vec{w}\| - \|\vec{v}\| = 5 - 3 = 2$

So $2 \leq \|\vec{v} - \vec{w}\| \leq 8$

Smallest value when:



largest value when:

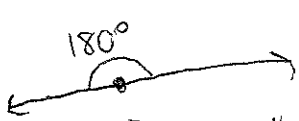


(b) Schwarz inequality:

$|\vec{v} \cdot \vec{w}| \leq \|\vec{v}\| \|\vec{w}\| = 3 \cdot 5 = 15$

$\rightarrow -15 \leq \vec{v} \cdot \vec{w} \leq 15$

smallest value when they point in opposite directions:



$\vec{v} \cdot \vec{w} = \|\vec{v}\| \|\vec{w}\| \cos 180°$

$= -15$

largest value when they point in same direction

$\vec{v} \cdot \vec{w} = \|\vec{v}\| \|\vec{w}\| \cos 0°$

$= 15$