



13. (3) $A = \{1, 2, 7, 8\}$

$$B = \{x | x^2 < 50\} = \{0, 1, 2, 3, 4, 5, 6, 7\}$$

$$C = \{x | 0 \leq x \leq 20 \wedge x \text{ 可被 3 整除}\}$$

$$= \{0, 3, 6, 9, 12, 15, 18\}$$

$$B - (A \cup C) = \{0, 1, 2, 3, 4, 5, 6, 7\}$$

$$- \{0, 1, 2, 3, 6, 7, 8, 9, 12, 15, 18\}$$

$$= \{4, 5\}$$

14. (1) $\cup \{ \{3, 4\}, \{\{3\}, \{4\}\}, \{3, \{4\}\}, \{\{3\}, 4\} \}$

$$= \{3, 4, \{3\}, \{4\}\}$$

(2) $\cap \{ \{1, 2, 3\}, \{2, 3, 4\}, \{3, 4, 5\} \}$

$$= \{3\}$$

15. $P(\emptyset) = \{\emptyset\}$, $PP(\emptyset) = \{\emptyset, \{\emptyset\}\}$

$$PPP(\emptyset) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$$

(1) $\cup \{ PPP(\emptyset), PP(\emptyset), P(\emptyset), \emptyset \}$

$$= \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$$

(2) $\cap \{ PPP(\emptyset), PP(\emptyset), P(\emptyset) \}$

$$= \{\emptyset\}$$

16. $A = \{\{\emptyset\}, \{\{\emptyset\}\}\}$

(1) $P(A) = \{\emptyset, \{\{\emptyset\}\}, \{\{\{\emptyset\}\}\}, \{\{\emptyset\}, \{\{\emptyset\}\}\}\}$

$$\cup P(A) = A = \{\{\emptyset\}, \{\{\emptyset\}\}\}$$

(2) $\cup A = \{\emptyset, \{\emptyset\}\}$

$$P(\cup A) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$$



$$\begin{aligned}
 17. (2) \text{ LHS} &= (A - B) - C \\
 &= (A \cap -B) \cap -C \\
 &= A \cap (-B \cap -C) \\
 &= A \cap ((-B \cap -C) \cup (-C \cap C)) \\
 &= A \cap (-C \cap (-B \cup C)) \\
 &= (A \cap -C) \cap -(B \cap -C) \\
 &= (A - C) - (B - C) = \text{RHS}.
 \end{aligned}$$

(4)

充分性:

$$A \subseteq C \wedge B \subseteq C \Rightarrow (A \cup B) \subseteq (C \cup C) \Rightarrow (A \cup B) \subseteq C$$

必要性:

$$\begin{aligned}
 A \cup B \subseteq C &\Leftrightarrow (\forall x)(x \in A \vee x \in B \rightarrow x \in C) \\
 &\Leftrightarrow (\forall x)((x \in A \rightarrow x \in C) \wedge (x \in B \rightarrow x \in C)) \\
 &\Leftrightarrow (\forall x)(x \in A \rightarrow x \in C) \wedge (\forall x)(x \in B \rightarrow x \in C) \\
 &\Leftrightarrow A \subseteq C \wedge B \subseteq C
 \end{aligned}$$

$$18. (1) A - B = B$$

$$\begin{aligned}
 B = A - B &= A \cap -B = (A \cap -B) \cap (A \cap -B) \\
 &= (A \cap -B) \cap B \\
 &= \emptyset
 \end{aligned}$$

$$A = A - \emptyset = A - B = B = \emptyset$$

故 $A = B = \emptyset$.

$$(3) A \cap B = A \cup B$$

$$B = B \cup (A \cap B) = B \cup (A \cup B) = A \cup B$$

$$A = A \cup (A \cap B) = A \cup (A \cup B) = A \cup B = B$$

故 $A = B$.



$$19. (2) (A-B) \cup (A-C) = \emptyset$$

$$\Leftrightarrow (A \cap -B) \cup (A \cap -C) = \emptyset$$

$$\Leftrightarrow A \cap (-B \cup -C) = \emptyset$$

$$\Leftrightarrow A \cap -(B \cap C) = \emptyset$$

$$\Leftrightarrow A \subseteq B \cap C.$$

$$(4) (A-B) \oplus (A-C) = \emptyset$$

$$\Leftrightarrow ((A-B) - (A-C)) \cup ((A-C) - (A-B)) = \emptyset$$

$$\Leftrightarrow (A-B) - (A-C) = \emptyset \wedge (A-C) - (A-B) = \emptyset$$

$$\Leftrightarrow (A-B) \subseteq (A-C) \wedge (A-C) \subseteq (A-B)$$

$$\Leftrightarrow A-B = A-C.$$

$$26. (1) A \times B = \emptyset$$

$$\Leftrightarrow \{ \langle x, y \rangle \mid x \in A, y \in B \} = \emptyset$$

$$\Leftrightarrow A = \emptyset \vee B = \emptyset.$$

$$(2) A = A \times A \Leftrightarrow A = \emptyset.$$

28. A, B, C 分别表示 $1 \sim 250$ 之间能被 $2, 3, 5$ 整除的数.

$$|A| = 250/2 = 125$$

$$|B| = 250/3 = 83.$$

$$|C| = 250/5 = 50.$$

$$|A \cap B| = 250/6 = 41.$$

$$|A \cap C| = 250/10 = 25$$

$$|B \cap C| = 250/15 = 16.$$

$$|A \cap B \cap C| = 250/30 = 8.$$

$$\begin{aligned} |A \cup B \cup C| &= |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C| \\ &= 125 + 83 + 50 - 41 - 25 - 16 + 8 = 184. \end{aligned}$$