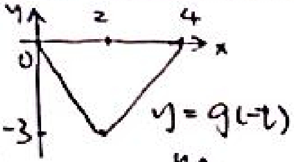
§1.5.50

(a) $g(-t)$

domain: $[0,4]$

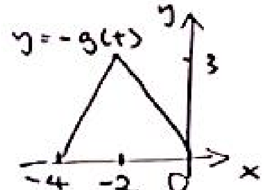
range: $[-3,0]$



(b) $-g(t)$

domain: $[-4,0]$

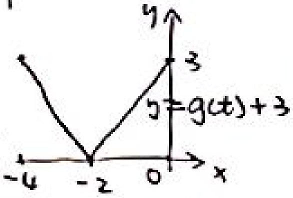
range: $[0,3]$



(c) $g(t)+3$

domain: $[-4,0]$

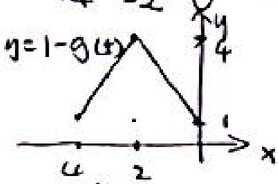
range: $[0,3]$



(d) $1-g(t)$

domain: $[-4,0]$

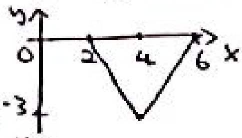
range: $[1,4]$



(e) $g(-t+2)$

domain: $[2,6]$

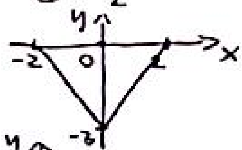
range: $[-3,0]$



(f) $g(t-2)$

domain: $[-2,2]$

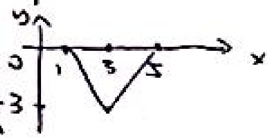
range: $[-3,0]$



(g) $g(1-t)$

domain: $[1,5]$

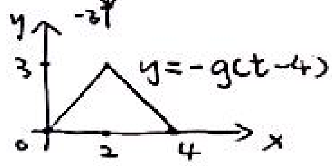
range: $[-3,0]$



(h) $-g(t-4)$

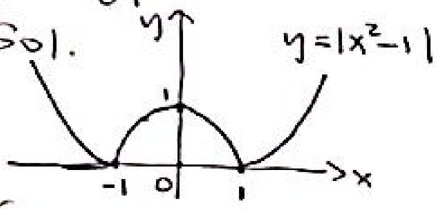
domain: $[0,4]$

range: $[0,3]$



§1.5.69

Sol.



§1.5.79

(a) $f(-x)g(-x)=-f(x)g(x)$

so fg is an ~~even~~ odd function.

(b) $\frac{f(-x)}{g(-x)}=-\frac{f(x)}{g(x)}$

so $\frac{f}{g}$ is an ~~odd function~~ odd function

(c) $\frac{g(-x)}{f(-x)}=-\frac{g(x)}{f(x)}$

so $\frac{g}{f}$ is an odd function

(d) $[f(-x)]^2=[f(x)]^2$

so f^2 is an even function

(e) $[g(-x)]^2=[-g(x)]^2=[g(x)]^2$

so g^2 is an even function

(f) $f(g(-x))=f(-g(x))=f(g(x))$

so $f\circ g$ is an even function.

(g) $(g\circ f)(-x)=g(f(-x))=g(f(x))$

so $g\circ f$ is an even function

(h) $(f\circ f)(-x)=f(f(-x))=f(f(x))$

so $f\circ f$ is an even function

(i) $(g\circ g)(-x)=g(g(-x))=g(-g(x))=-g(g(x))$

So $g\circ g$ is an ~~odd~~ odd function

§1.5.80.

Sol Let $f(x)=0$. $f(x)=f(-x)=0$, $f(-x)=-f(x)=-0=0$

So $f(x)=0$ is both even and odd.

清华大学 TSINGHUA UNIVERSITY　数学作业纸

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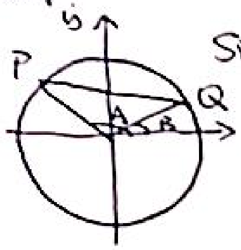
§1.6.49

Sol. $\sin^2\frac{\pi}{12} = \frac{1-\cos\frac{\pi}{6}}{2} = \frac{1-\frac{\sqrt{3}}{2}}{2} = \frac{2-\sqrt{3}}{4}$

§1.6.53

Proof

~~Point P locates~~



Since $P(\cos A, \sin A)$, $Q(\cos B, \sin B)$,

$$PQ = \sqrt{(\cos A-\cos B)^2+(\sin A-\sin B)^2} = \sqrt{(\cos(A-B)-1)^2+\sin^2(A-B)}$$

$$\Rightarrow (\cos A-\cos B)^2+(\sin A-\sin B)^2 = (\cos(A-B)-1)^2+1-\cos^2(A-B)$$

$$\Rightarrow 2-2(\cos A\cos B+\sin A\sin B) = 2-2\cos(A-B)$$

$$\Rightarrow \cos(A-B) = \cos A\cos B+\sin A\sin B \quad \text{QED}.$$

A1. Sol. $\cos(3x) = \sin(2x)$

$\sin(\frac{\pi}{2}-3x) = \sin(2x)$

So $\frac{\pi}{2}-3x = 2x-2k\pi$ or $\frac{\pi}{2}-3x = \pi-2x-2k\pi$, $k\in\mathbb{Z}$

$\Leftrightarrow 5x = \frac{\pi}{2}+2k\pi$ or $x = -\frac{\pi}{2}+2k\pi$, $k\in\mathbb{Z}$

$\Leftrightarrow x = \frac{\pi}{10}+\frac{2}{5}k\pi$ or $x = \frac{3\pi}{2}+2k\pi$, $k\in\mathbb{Z}$

A2. Sol. $\forall x\in[0,1]$, $f(x+1) = f(x)+2 \Leftrightarrow \forall x\in[1,2]$, $f(x) = f(x-1)+2$

~~is the as sam~~

Since $\forall x\in[1,2]$, $x-1\in[0,1]$

We have ~~f~~ $\forall x\in[1,2]$, $f(x) = 2(x-1)+2 = 2x$

So $f(x) = 2x$, $x\in[0,2]$

Bonus Exercises:

Sol. $\cos 3x = \cos 2x\cos x - \sin 2x\sin x$

$= (2\cos^2 x-1)\cos x - 2(1-\cos^2 x)\cos x$

$= 4\cos^3 x - 3\cos x$

Let $x = \frac{\pi}{10}$, $\cos\frac{3\pi}{10} = 4\cos^3\frac{\pi}{10} - 3\cos\frac{\pi}{10}$

Since A1, $\cos\frac{3\pi}{10} = \sin\frac{\pi}{5}$, we have $\sin\frac{\pi}{5} = 4\cos^3\frac{\pi}{10} - 3\cos\frac{\pi}{10}$

$\Rightarrow 2\sin\frac{\pi}{10}\cos\frac{\pi}{10} = 4\cos^3\frac{\pi}{10} - 3\cos\frac{\pi}{10}$

$\Rightarrow 2\sin\frac{\pi}{10} = 4(1-\sin^2\frac{\pi}{10}) - 3$

$\Rightarrow \sin\frac{\pi}{10} = \frac{\sqrt{5}-1}{4}$

$2\sin\frac{\pi}{10} = 4\cos^2\frac{\pi}{10} - 3 = 2\cos\frac{\pi}{5} - 1$

$\Rightarrow \frac{\sqrt{5}-1}{2} = 2\cos\frac{\pi}{5} - 1$

$\Rightarrow \cos\frac{\pi}{5} = \frac{\sqrt{5}+1}{4}$