Problem 2.1.4.

Sol. If $z=2$ then $\begin{cases} x+y=0 \\ x-y=2 \end{cases} \Rightarrow \begin{cases} x=1 \\ y=-1 \end{cases}$ The first point is $(1,-1,2)$

If $z=0$ then $\begin{cases} x+y=6 \\ x-y=4 \end{cases} \Rightarrow \begin{cases} x=5 \\ y=1 \end{cases}$ The ~~sencond~~ second point is $(5,1,0)$

Halfway between those is $(3,0,1)$

Problem 2.1.17

Sol. Since $P\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y \\ z \\ x \end{bmatrix}$, $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

Since $Q\begin{bmatrix} y \\ z \\ x \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $Q = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

Problem 2.1.22

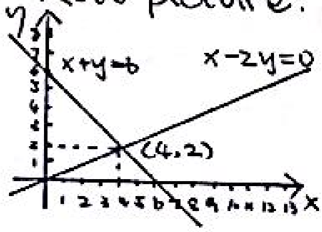
Sol. $Ax = [1 \ 4 \ 5]\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$Ax=0 \Rightarrow x$ lies on a 3D plane perpendicular to vector $(1,4,5)$.

The colums of A are vectors in only 1D space.

Problem 2.1.26.

Sol. Row picture:



Column Picture:

$x\begin{bmatrix} 1 \\ 1 \end{bmatrix} + y\begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \end{bmatrix}$

$\Rightarrow \begin{cases} x=4 \\ y=2 \end{cases}$

Problem 2.1.29

Sol. $u_2 = Au_1 = \begin{bmatrix} .8 & .3 \\ .2 & .7 \end{bmatrix}\begin{bmatrix} .8 \\ .2 \end{bmatrix} = \begin{bmatrix} .7 \\ .3 \end{bmatrix}$

$u_3 = Au_2 = \begin{bmatrix} .8 & .3 \\ .2 & .7 \end{bmatrix}\begin{bmatrix} .7 \\ .3 \end{bmatrix} = \begin{bmatrix} .65 \\ .35 \end{bmatrix}$

The sum of the components of u_0, u_1, u_2, u_3 is 1.

Problem 2.2.6.

Sol. Since $2x+4y$ times 2 is $4x+8y$, $b=4$.

let ①: $2x+4y=16$, ②: $4x+8y=g$

①×2: $4x+8y=32$, so $g=32$. and it makes ~~this system of~~ the equations. solvable.

Problem 2.2.13.

Sol.

$$\begin{aligned} 2x-3y\quad &= 3\\ 4x-5y+z &= 7\\ 2x-y-3z &= 5 \end{aligned} \xrightarrow[\text{Subtract 1 times row 1 from row 3}]{\text{Subtract 2 times row 1 from row 2}} \begin{aligned} 2x-3y\quad &= 3\\ y+z &= 1\\ 2y-3z &= 2 \end{aligned}$$

$$\xrightarrow{\text{Subtract 2 times row 2 from row 3}} \begin{aligned} 2x-3y\quad &= 3\\ y+z &= 1\\ -5z &= 0 \end{aligned} \Rightarrow \begin{cases} x=3\\ y=1\\ z=0 \end{cases}$$

Problem 2.2.18

Sol. $\begin{bmatrix}1&4&5\\2&8&10\\3&12&15\end{bmatrix}$ is one of the possible systems.

$\begin{bmatrix}1&4&5\\2&8&10\\3&12&15\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\10\\100\end{bmatrix} \Rightarrow x+4y+5z=1$ and $2x+8y+10z=10$

It is impossible, so no solution to this system with $b=(1,10,100)$.

when $b=(0,0,0)$, ~~the~~ the system has infinite solutions.

Problem 2.2.21.

Sol.

$$\begin{aligned} 2x+y\qquad &= 0\\ x+2y+z\quad &= 0\\ y+2z+t &= 0\\ z+2t &= 5 \end{aligned} \xrightarrow{r_2-\frac{1}{2}r_1} \begin{aligned} 2x+y\qquad &= 0\\ \tfrac{3}{2}y+z\quad &= 0\\ y+2z+t &= 0\\ z+2t &= 5 \end{aligned} \xrightarrow{r_3-\frac{2}{3}r_2} \begin{aligned} 2x+y\qquad &= 0\\ \tfrac{3}{2}y+z\quad &= 0\\ \tfrac{4}{3}z+t &= 0\\ z+2t &= 5 \end{aligned}$$

$$\xrightarrow{r_4-\frac{3}{4}r_3} \begin{aligned} 2x+y\qquad &= 0\\ \tfrac{3}{2}y+z\quad &= 0\\ \tfrac{4}{3}z+t &= 0\\ \tfrac{5}{4}t &= 5 \end{aligned} \Rightarrow \begin{cases} x=-1\\ y=2\\ z=-3\\ t=4 \end{cases}$$

$$\begin{aligned} 2x-y\qquad &= 0\\ -x+2y-z\quad &= 0\\ -y+2z-t &= 0\\ -z+2t &= 5 \end{aligned} \xrightarrow{r_2+\frac{1}{2}r_1} \begin{aligned} 2x-y\qquad &= 0\\ \tfrac{3}{2}y-z\quad &= 0\\ -y+2z-t &= 0\\ -z+2t &= 5 \end{aligned} \xrightarrow{r_3+\frac{2}{3}r_2} \begin{aligned} 2x-y\qquad &= 0\\ \tfrac{3}{2}y-z\quad &= 0\\ \tfrac{4}{3}z-t &= 0\\ -z+2t &= 5 \end{aligned}$$

$$\xrightarrow{r_4+\frac{3}{4}r_3} \begin{aligned} 2x-y\qquad &= 0\\ \tfrac{3}{2}y-z\quad &= 0\\ \tfrac{4}{3}z-t &= 0\\ \tfrac{5}{4}t &= 5 \end{aligned} \Rightarrow \begin{cases} x=1\\ y=2\\ z=3\\ t=4 \end{cases}$$

Graded Problem.

Sol (a) matrix-vector equation:

$$\begin{bmatrix} 2 & -2 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

vector equation:

$$x \cdot \begin{bmatrix} 2 \\ -1 \end{bmatrix} + y \begin{bmatrix} -2 \\ 2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

When $b = (1, 0)$.

$$\begin{array}{l} 2x - 2y = 1 \\ -x + 2y = 0 \end{array} \xrightarrow{r_1 + r_2} \begin{array}{l} x \qquad = 1 \\ -x + 2y = 0 \end{array} \Rightarrow \begin{cases} x = 1 \\ y = \frac{1}{2} \end{cases}$$

$$1 \cdot \begin{bmatrix} 2 \\ -1 \end{bmatrix} + \frac{1}{2} \cdot \begin{bmatrix} -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

When $b = (0, 1)$

$$\begin{array}{l} 2x - 2y = 0 \\ -x + 2y = 1 \end{array} \xrightarrow{r_1 + r_2} \begin{array}{l} x \qquad = 1 \\ -x + 2y = 1 \end{array} \Rightarrow \begin{cases} x = 1 \\ y = 1 \end{cases}$$

$$1 \cdot \begin{bmatrix} 2 \\ -1 \end{bmatrix} + 1 \cdot \begin{bmatrix} -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$