清華大學 TSINGHUA UNIVERSITY 数学作业纸

班级：计22 姓名：郑[illegible] 编号：2022010799 科目：Linear Algebra 第 1 页

Problem 3.1.27.

(a) False zero vector is in C(A), so the space would not be a subspace.

(b) True

(c) True.

(d) False Let $A=[1]$. $A-I=[0]$.

C(A) is a line and C(A−I) is just origin.

Problem 3.1.28.

Sol. Let $A=\begin{bmatrix}1&1&2\\1&0&1\\0&1&1\end{bmatrix}$

C(A) are all the linear combination of $\begin{bmatrix}1\\1\\0\end{bmatrix}$ and $\begin{bmatrix}1\\0\\1\end{bmatrix}$

$\begin{bmatrix}1\\1\\1\end{bmatrix}$ is not a linear combination of them.

Problem 3.2.12

Sol. $A=[1\ \ -3\ \ -1]$ y, z are the free variables.

$\begin{bmatrix}3\\1\\0\end{bmatrix}$ and $\begin{bmatrix}1\\0\\1\end{bmatrix}$ are the two special solutions.

Problem 3.2.20.

Sol. $A=\begin{bmatrix}0&1\\0&0\end{bmatrix}$ s.t $N(A)=C(A)$.

Problem 3.2.21.

Sol. (a) $A=\begin{bmatrix}4&4&4&4\\4&4&4&4\\4&4&4&4\end{bmatrix}\to\begin{bmatrix}4&4&4&4\\0&0&0&0\\0&0&0&0\end{bmatrix}\to\begin{bmatrix}1&1&1&1\\0&0&0&0\\0&0&0&0\end{bmatrix}=R$.

rank = 1.

(b) $A=\begin{bmatrix}1&2&3&4\\2&3&4&5\\3&4&5&6\end{bmatrix}\to\begin{bmatrix}1&2&3&4\\0&-1&-2&-3\\0&-2&-4&-6\end{bmatrix}\to\begin{bmatrix}1&2&3&4\\0&-1&-2&-3\\0&0&0&0\end{bmatrix}$

$\to\begin{bmatrix}1&2&3&4\\0&1&2&3\\0&0&0&0\end{bmatrix}\to\begin{bmatrix}1&0&-1&-2\\0&1&2&3\\0&0&0&0\end{bmatrix}=R$

rank = 2.

(6) $A=\begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = R$

rank $= 1$.

Problem 3.2.32.

Sol. $A^T = \begin{bmatrix} -1 & 0 & 1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} -1 & 0 & 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & -1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \to$

$\begin{bmatrix} -1 & 0 & 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 & -1 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \to \begin{bmatrix} -1 & 0 & 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \to$

$\begin{bmatrix} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -1 & 0 & -1 & -1 \\ 0 & 1 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = R$

Let $Y = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and the three special solutions are

$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$

Problem 3.3.4.

Sol. $[A|b] = \left[\begin{array}{cccc|c} 1 & 3 & 1 & 2 & 1 \\ 2 & 6 & 4 & 8 & 3 \\ 0 & 0 & 2 & 4 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{cccc|c} 1 & 3 & 1 & 2 & 1 \\ 0 & 0 & 2 & 4 & 1 \\ 0 & 0 & 2 & 4 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{cccc|c} 1 & 3 & 1 & 2 & 1 \\ 0 & 0 & 2 & 4 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \to$

$\left[\begin{array}{cccc|c} 1 & 3 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{cccc|c} 1 & 3 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] = [R|d]$

$x_p = \begin{bmatrix} 1 \\ 0 \\ \frac{1}{2} \\ 0 \end{bmatrix}$, $x_n = c_1 \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 0 \\ -2 \\ 1 \end{bmatrix} \Rightarrow x = x_p + x_n$

$= \begin{bmatrix} 1 \\ 0 \\ \frac{1}{2} \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 0 \\ -2 \\ 1 \end{bmatrix}$

Problem 3.3.7.

Pf. Since $2r_1 + \frac{1}{2}r_3 = r_2$, then only if b satisfied

$b_3 - 2b_2 + 4b_1 = 0$, the system $Ax = b$ has solution.

the combination of the rows of A : $r_3 - 2r_2 + 4r_1 = 0$.

Problem 3.3.13.

(a) when b is not 0 : $A(2x_p) = 2b$.

$2x_p$ is not a solution of the system $Ax = b$.

(b) If R has free variable, you can set them into any value which means there has infinite particular solutions.

(c) Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$. $b = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$

The x_p with all free variables zero is $\begin{bmatrix} 10 \\ 0 \end{bmatrix}$.

But we have $x_p' = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$ is also a solution.

$\|x_p\| = 10$ $\|x_p'\| = 5\sqrt{2}$ and $10 > 5\sqrt{2}$

which means x_p is not the shortest solution.

(d) there always exists $x_n = 0$ in the nullspace.

Problem 3.3.34

(a) Sol. Since $Ax = 0$ just has one "special solution"

rank of A $= n - 1 = 4 - 1 = 3$.

(b) Sol. $R = \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(c) Sol. Since the rank of A is 3.

then C(A) is a 3-D subspace.

b is a vector has 3 coefficients

Therefore b is in C(A) which means $Ax = b$

is always solvable.

Problem 1.

Sol. $A=\begin{bmatrix}-3&-1&0&1\\-4&-1&1&1\\-1&1&4&-1\end{bmatrix}\to\begin{bmatrix}-1&1&4&-1\\-3&-1&0&1\\-4&-1&1&1\end{bmatrix}\to\begin{bmatrix}1&-1&-4&1\\-3&-1&0&1\\-4&-1&1&1\end{bmatrix}\to\begin{bmatrix}1&-1&-4&1\\0&-4&-12&4\\0&-5&-15&5\end{bmatrix}\to$

$\begin{bmatrix}1&-1&-4&1\\0&-4&-12&4\\0&0&0&0\end{bmatrix}\to\begin{bmatrix}1&-1&-4&1\\0&1&3&-1\\0&0&0&0\end{bmatrix}\to\begin{bmatrix}1&0&-1&0\\0&1&3&-1\\0&0&0&0\end{bmatrix}=R$

the special solutions are $\begin{bmatrix}1\\-3\\1\\0\end{bmatrix}$ and $\begin{bmatrix}0\\1\\0\\1\end{bmatrix}$

Problem 2.

Sol. $[A\ \ b]=\begin{bmatrix}2&-1&-1&b_1\\-1&2&-1&b_2\\-1&-1&2&b_3\end{bmatrix}\to\begin{bmatrix}2&-1&-1&b_1\\0&3/2&-3/2&b_2+\frac{1}{2}b_1\\0&-3/2&3/2&b_3+\frac{1}{2}b_1\end{bmatrix}\to\begin{bmatrix}2&-1&-1&b_1\\0&3/2&-3/2&b_2+\frac{1}{2}b_1\\0&0&0&b_1+b_2+b_3\end{bmatrix}$

$\to\begin{bmatrix}1&-1/2&-1/2&\frac{1}{2}b_1\\0&1&-1&\frac{2}{3}b_2+\frac{1}{3}b_1\\0&0&0&b_1+b_2+b_3\end{bmatrix}\to\begin{bmatrix}1&0&-1&\frac{2}{3}b_1+\frac{1}{3}b_2\\0&1&-1&\frac{2}{3}b_2+\frac{1}{3}b_1\\0&0&0&b_1+b_2+b_3\end{bmatrix}=[R\ \ d]$

When $b_1+b_2+b_3=0$, $b=\begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix}$ is in $C(A)$.

Let $b=\begin{bmatrix}1\\1\\-2\end{bmatrix}$, then

$[R\ \ d]=\begin{bmatrix}1&0&-1&1\\0&1&-1&1\\0&0&0&0\end{bmatrix}$ so $x_p=\begin{bmatrix}1\\1\\0\end{bmatrix}$, $x_n=x_3\begin{bmatrix}1\\1\\1\end{bmatrix}$

$x=x_p+x_n=\begin{bmatrix}1\\1\\0\end{bmatrix}+x_3\begin{bmatrix}1\\1\\1\end{bmatrix}$