Problem 1.2.7.

Sol. (b) $\cos\theta = \frac{V\cdot W}{\|V\|\cdot\|W\|} = \frac{2\times 2 - 2\times 1 - 1\times 2}{\sqrt{2^2+2^2+(-1)^2}\cdot\sqrt{2^2+(-1)^2+2^2}} = 0$

so $\theta = \frac{\pi}{2}$

(c) $\cos\theta = \frac{V\cdot W}{\|V\|\,\|W\|} = \frac{3\times(-1)+1\times(-2)}{\sqrt{3^2+1^2}\cdot\sqrt{(-1)^2+(-2)^2}} = \frac{-5}{\sqrt{10}\times\sqrt{5}} = -\frac{\sqrt{2}}{2}$

so $\theta = \frac{3}{4}\pi$.

Problem. 1.2.16.

Sol. Since $\|V\|^2 = 1^2+1^2+\cdots+1^2 = 9$, $\|V\| = 3$.

$u = \frac{V}{\|V\|} = (\frac{1}{3}, \frac{1}{3}, \cdots \frac{1}{3})$

~~Let w...~~

Let $w = \frac{(1,-1,0,\cdots,0)}{\sqrt{2}} = (\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 0, \cdots, 0)$

then since $w\cdot v = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = 0$

w is a unit vector in ~~the~~ 9D perpendicular to v.

Problem 1.2.22

Sol. $|V\cdot W|^2 - \|V\|^2\|W\|^2$

$= (v_1w_1 + v_2w_2)^2 - (v_1^2+v_2^2)(w_1^2+w_2^2)$

$= (v_1^2w_1^2 + 2v_1w_1v_2w_2 + v_2^2w_2^2) - (v_1^2w_1^2 + v_1^2w_2^2 + v_2^2w_1^2 + v_2^2w_2^2)$

$= 2v_1w_1v_2w_2 - v_1^2w_2^2 - v_2^2w_1^2$

$= -(v_1w_2 + v_2w_1)^2 \leq 0$

So $|V\cdot W|^2 \leq \|V\|^2\|W\|^2$

Problem. 1.2.27

Sol. $\|V+W\|^2 + \|V-W\|^2$

$= \|V\|^2 + 2V\cdot W + \|W\|^2 + \|V\|^2 - 2\cdot V\cdot W + \|W\|^2$

$= 2\|V\|^2 + 2\|W\|^2$

Problem. 1.2.33

Sol. Let $a_1 = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$
$a_2 = (\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2})$
$a_3 = (\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2})$
$a_4 = (\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2})$

It's easy to prove that every two of them are perpendicular. (It could be generated by Hadamard matrix).

Problem 1.3.4.

Sol. Since $w_2=(w_1+w_3)/2$, w_1, w_2, w_3 are ~~dependon~~ dependent

(~~There~~ The three vectors lie in a plane).

And $w_1-2w_2+w_3=0$.

Problem 1.3.5

Sol. $r_1-2r_2+r_3=2r_1-4r_2+2r_3=0$

so y's $=(1,-2,1)$ or $(2,-4,2)$.

Problem 1.3.12

Sol $\begin{bmatrix}0&1&0&0\\-1&0&1&0\\0&-1&0&1\\0&0&-1&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}b_1\\b_2\\b_3\\b_4\end{bmatrix} \Rightarrow \begin{cases}x_2=b_1\\-x_1+x_3=b_2\\-x_2+x_4=b_3\\-x_3=b_4\end{cases} \Rightarrow \begin{cases}x_1=-b_2-b_4\\x_2=b_1\\x_3=-b_4\\x_4=b_1+b_3\end{cases}$

$\Rightarrow \begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}0&-1&0&-1\\1&0&0&0\\0&0&0&-1\\1&0&1&0\end{bmatrix}\begin{bmatrix}b_1\\b_2\\b_3\\b_4\end{bmatrix}$ so $C^{-1}=\begin{bmatrix}0&-1&0&-1\\1&0&0&0\\0&0&0&-1\\1&0&1&0\end{bmatrix}$

Graded Problem.

Problem 1.

(a) $r=\frac{u}{\|u\|}=\frac{\begin{bmatrix}1\\2\\1\end{bmatrix}}{\sqrt{1^2+2^2+1^2}}=\begin{bmatrix}\frac{\sqrt6}{6}\\\frac{\sqrt6}{3}\\\frac{\sqrt6}{6}\end{bmatrix}$ is a unit vector pointing in the same direction as u.

(b) All vectors perpendicular to u could be ~~wrot~~ written as $\begin{bmatrix}a\\b\\-a-2b\end{bmatrix}$

Let $v=\begin{bmatrix}a\\b\\-a-2b\end{bmatrix}$ $w=\begin{bmatrix}w_1\\w_2\\-w_1-2w_2\end{bmatrix}$ we have

$aw_1+bw_2+aw_1+2aw_2+2bw_1+4bw_2=0$

$(2a+2b)w_1=-(2a+5b)w_2$

$\frac{w_1}{w_2}=\frac{-2a-5b}{2a+2b}$ so $w=\begin{bmatrix}-2a-5b\\2a+2b\\b-2a\end{bmatrix}$, v w u are perpendicular.

Problem 2.

Sol (a) $\begin{bmatrix}4&7\\1&2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}b_1\\b_2\end{bmatrix}$

$\Rightarrow \begin{cases}4x_1+7x_2=b_1\\x_1+2x_2=b_2\end{cases} \Rightarrow \begin{cases}x_1=2b_1-7b_2\\x_2=-b_1+4b_2\end{cases} \Rightarrow \begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}2&-7\\-1&4\end{bmatrix}\begin{bmatrix}b_1\\b_2\end{bmatrix}$

(b) So $A^{-1}=\begin{bmatrix}2&-7\\-1&4\end{bmatrix}$