

Practice Final Exam

1. (a) Find the unique value of c such that the system of equations has at least one solution:

$$x_1 + x_2 + x_3 + x_4 = 3$$

$$x_1 + 2x_2 + 3x_3 + 4x_4 = c$$

$$x_1 - x_2 - 3x_3 - 5x_4 = 1$$

- (b) For the value of c you found, find *all* solutions to the system of equations.

2. Consider the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & -2 \\ 1 & -2 & 3 \end{bmatrix}.$$

- (a) Find all eigenvalues of A .
- (b) Find a basis of \mathbf{R}^3 consisting of eigenvectors for A .
- (c) Find the angles between the eigenvectors in the basis.

3. Determine whether the following sets of vectors are bases for \mathbb{R}^3 . In case a set of vectors is *not* linearly independent, show how to write one of the vectors as a linear combination of the others:

$$(a) \quad \left\{ \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} \right\} \qquad (b) \quad \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 9 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ 25 \end{bmatrix} \right\}$$

4. Find bases for the null space, column space, row space, and left null space of the matrix:

$$A = \begin{bmatrix} -3 & 1 & 4 & 4 \\ 1 & 2 & -1 & -2 \\ -3 & 8 & 5 & 2 \end{bmatrix}$$

5. (a) Diagonalize the matrix A : write $A = X\Lambda X^{-1}$ where X is invertible and Λ is diagonal.

$$A = \begin{bmatrix} 10 & 12 \\ -6 & -7 \end{bmatrix}$$

- (b) Show that the matrix B is *not* diagonalizable:

$$B = \begin{bmatrix} 8 & 9 \\ -4 & -4 \end{bmatrix}$$

6. Consider the (x, y) data points $(-1, 0)$, $(0, 1)$, $(1, 3)$, and $(2, 9)$.
- (a) Find the best least squares fit by a linear function $y = mx + b$ to the data.
 - (b) Plot your linear function from part (a) along the data on a coordinate system.
 - (c) Find the error of the least squares approximation.

7. (a) Find the inverse of

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 7 & 14 \end{bmatrix}$$

- (b) Use A^{-1} to solve the linear system of equations $A\mathbf{x} = (1, 2, 1)$.

8. Consider the matrix

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & -2 & 2 \end{bmatrix}.$$

- (a) Find the LU decomposition of A .
- (b) Find the volume of the box in \mathbf{R}^3 that is spanned by the columns of A .

9. (a) Find the determinants of the matrices

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 4 & 4 \\ 1 & -1 & 2 & -2 \\ 1 & -1 & 8 & -8 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ -3 & 0 & 3 \end{bmatrix}$$

- (b) Are the matrices A and B invertible?

10. (a) Find an orthonormal basis for the subspace V of \mathbf{R}^4 spanned by $(1, 1, 1, 1)$ and $(3, 2, 2, 1)$.
- (b) Find the projection matrix P for the orthogonal projection onto V , and compute the projection of $(0, 0, 1, 1)$ onto V .