清华大学 TSINGHUA UNIVERSITY 数学作业纸

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Problem 2.3.9.

(a) $M = P_{23}E_{21} = \begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}\begin{bmatrix}1&0&0\\-1&1&0\\0&0&1\end{bmatrix} = \begin{bmatrix}1&0&0\\0&0&1\\-1&1&0\end{bmatrix}$

(b) $M = E_{31}P_{23} = \begin{bmatrix}1&0&0\\0&1&0\\-1&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix} = \begin{bmatrix}1&0&0\\0&0&1\\-1&1&0\end{bmatrix}$

we need $E_{31}$ to exchange row 3 after the $P_{23}$.

Problem 2.3.12.

Sol. $\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}\begin{bmatrix}1&2&3\\4&5&6\\7&8&9\end{bmatrix}\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}$

$= \begin{bmatrix}7&8&9\\4&5&6\\1&2&3\end{bmatrix}\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}$

$= \begin{bmatrix}9&8&7\\6&5&4\\3&2&1\end{bmatrix}$

$\begin{bmatrix}1&0&0\\-1&1&0\\-1&0&1\end{bmatrix}\begin{bmatrix}1&2&3\\1&3&1\\1&4&0\end{bmatrix}$

$= \begin{bmatrix}1&2&3\\0&1&-2\\0&2&-3\end{bmatrix}$

Problem 2.3.17

Sol.

It gives $\begin{bmatrix}1&1&1\\1&2&4\\1&3&9\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix} = \begin{bmatrix}4\\8\\14\end{bmatrix}$

and $a=2, b=1, c=1$ is the solution.

Problem 2.3.28.

~~Sol~~. Since $AB = I$, $ABC = IC = C$

Proof.

$\Rightarrow A(BC) = C$

$\Rightarrow AI = C$ (Since $BC = I$)

$\Rightarrow A = C$ QED.

Problem 2.4.6.

Sol. When $A = \begin{bmatrix}1&2\\0&0\end{bmatrix}$, $B = \begin{bmatrix}1&0\\3&0\end{bmatrix}$

$(A+B)^2 = \left(\begin{bmatrix}1&2\\0&0\end{bmatrix} + \begin{bmatrix}1&0\\3&0\end{bmatrix}\right)^2 = \begin{bmatrix}2&2\\3&0\end{bmatrix}\begin{bmatrix}2&2\\3&0\end{bmatrix} = \begin{bmatrix}10&4\\6&6\end{bmatrix}$

$A^2 + 2AB + B^2 = \begin{bmatrix}1&2\\0&0\end{bmatrix}^2 + 2\begin{bmatrix}1&2\\0&0\end{bmatrix}\begin{bmatrix}1&0\\3&0\end{bmatrix} + \begin{bmatrix}1&0\\3&0\end{bmatrix}^2$

$= \begin{bmatrix}1&2\\0&0\end{bmatrix} + \begin{bmatrix}14&0\\0&0\end{bmatrix} + \begin{bmatrix}1&0\\3&0\end{bmatrix} = \begin{bmatrix}16&2\\3&0\end{bmatrix}$

So $(A+B)^2 \neq A^2 + 2AB + B^2$

$(A+B)^2 = A^2 + AB + BA + B^2$

Problem 2.4.15.

Sol. (a) True (let A be a m×n matrix, since $A^2$ is defined, $n=m$, A has to be square)

(b) False (Let A be a 2×3 matrix and B be a 3×2 matrix. Then AB and BA are both defined and A and B are not sequare)

(c) True (If AB and BA are both defined then A has to be a m×n matrix and B has to be a n×m matrix then AB has to be a m×m matrix, BA has to be a n×n matrix. AB and BA are both square).

(d) False (If $B=0$, A is unnessary to be $I$)

Problem 2.4.18.

Sol. (a) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 2 & 1 & \frac{2}{3} \\ 3 & \frac{3}{2} & 1 \end{bmatrix}$

Problem 2.4.21

Sol.

$$A^2 = \begin{bmatrix} 0&2&0&0 \\ 0&0&2&0 \\ 0&0&0&2 \\ 0&0&0&0 \end{bmatrix}\begin{bmatrix} 0&2&0&0 \\ 0&0&2&0 \\ 0&0&0&2 \\ 0&0&0&0 \end{bmatrix} = \begin{bmatrix} 0&0&4&0 \\ 0&0&0&4 \\ 0&0&0&0 \\ 0&0&0&0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 0&2&0&0 \\ 0&0&2&0 \\ 0&0&0&2 \\ 0&0&0&0 \end{bmatrix}\begin{bmatrix} 0&0&4&0 \\ 0&0&0&4 \\ 0&0&0&0 \\ 0&0&0&0 \end{bmatrix} = \begin{bmatrix} 0&0&0&8 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \end{bmatrix} \quad (\text{Since } A^3 = A(AA))$$

$$A^4 = \begin{bmatrix} 0&0&0&8 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \end{bmatrix}\begin{bmatrix} 0&2&0&0 \\ 0&0&2&0 \\ 0&0&0&2 \\ 0&0&0&0 \end{bmatrix} = \begin{bmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \end{bmatrix}$$

$$Av = \begin{bmatrix} 0&2&0&0 \\ 0&0&2&0 \\ 0&0&0&2 \\ 0&0&0&0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 2y \\ 2z \\ 2t \\ 0 \end{bmatrix} \qquad A^2v = \begin{bmatrix} 0&0&4&0 \\ 0&0&0&4 \\ 0&0&0&0 \\ 0&0&0&0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 4z \\ 4t \\ 0 \\ 0 \end{bmatrix}$$

$$A^3v = \begin{bmatrix} 0&0&0&8 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 8t \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad A^4v = 0.$$

Problem. 2.4.26.

$$AB = \begin{bmatrix} 1&0 \\ 2&4 \\ 2&1 \end{bmatrix}\begin{bmatrix} 3&3&0 \\ 1&2&1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}\begin{bmatrix} 3&3&0 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix}\begin{bmatrix} 1&2&1 \end{bmatrix} = \begin{bmatrix} 3&3&0 \\ 6&6&0 \\ 6&6&0 \end{bmatrix} + \begin{bmatrix} 0&0&0 \\ 4&8&4 \\ 1&2&1 \end{bmatrix}$$

$$= \begin{bmatrix} 3&3&0 \\ 10&14&4 \\ 7&8&1 \end{bmatrix}$$

Problem 2.6.32.

Sol. $AX = A\cdot[x_1\ x_2\ x_3] = [Ax_1\ Ax_2\ Ax_3] = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

Graded Problems.

Problem 1.

Sol. $\begin{bmatrix}a&b\\c&d\end{bmatrix}\begin{bmatrix}0&1\\0&0\end{bmatrix} = \begin{bmatrix}0&1\\0&0\end{bmatrix}\begin{bmatrix}a&b\\c&d\end{bmatrix}$

$$\begin{bmatrix}0&a\\0&c\end{bmatrix} = \begin{bmatrix}c&d\\0&0\end{bmatrix}$$

So we have $a=d$, $c=0$

$A = \begin{bmatrix}a&b\\0&a\end{bmatrix}$ and $\begin{bmatrix}0&1\\0&0\end{bmatrix}$ commute

$\begin{bmatrix}a&b\\c&d\end{bmatrix}\begin{bmatrix}0&0\\1&0\end{bmatrix} = \begin{bmatrix}0&0\\1&0\end{bmatrix}\begin{bmatrix}a&b\\c&d\end{bmatrix}$

$$\begin{bmatrix}b&0\\d&0\end{bmatrix} = \begin{bmatrix}0&0\\a&b\end{bmatrix}$$

so we have $b=0$, $a=d$

$A = \begin{bmatrix}a&0\\c&a\end{bmatrix}$ and $\begin{bmatrix}0&0\\1&0\end{bmatrix}$ commute.

Let $\begin{bmatrix}a&0\\c&a\end{bmatrix} = \begin{bmatrix}a&b\\0&a\end{bmatrix}$

we have $c=b=0$

so $\begin{bmatrix}a&0\\0&a\end{bmatrix}$ commute with both $\begin{bmatrix}0&0\\1&0\end{bmatrix}$ and $\begin{bmatrix}0&1\\0&0\end{bmatrix}$

Problem 2.

$$\begin{bmatrix}2&-1&0&0\\-1&2&-1&-1\\0&-1&2&0\\0&-1&0&2\end{bmatrix}\begin{bmatrix}-1&0&0&1\\0&1&-1&0\\0&-1&1&0\\1&0&0&-1\end{bmatrix} = \begin{bmatrix}-2&-1&1&2\\0&3&-3&0\\0&-3&3&0\\2&-1&1&-2\end{bmatrix}$$

$$\begin{bmatrix}1&1&1&1\\1&2&3&4\\1&4&9&16\end{bmatrix}\begin{bmatrix}1&-1\\-2&2\\1&1\\-1&0\end{bmatrix} = \begin{bmatrix}1\\1\\1\end{bmatrix}[1\ -1] + \begin{bmatrix}1\\2\\4\end{bmatrix}[-2\ 2] + \begin{bmatrix}1\\3\\9\end{bmatrix}[1\ 1] + \begin{bmatrix}1\\4\\16\end{bmatrix}[-1\ 0]$$

$$= \begin{bmatrix}1&-1\\1&-1\\1&-1\end{bmatrix} + \begin{bmatrix}-2&2\\-4&4\\-8&8\end{bmatrix} + \begin{bmatrix}1&1\\3&3\\9&9\end{bmatrix} + \begin{bmatrix}-1&0\\-4&0\\-16&0\end{bmatrix}$$

$$= \begin{bmatrix}-1&2\\-4&6\\-14&16\end{bmatrix}$$