清華大学 TSINGHUA UNIVERSITY 数学作业纸

班级：计27 姓名：郑东森 编号：2022010799 科目：Calculus (1) 第 1 页

§2.6.39.

Sol. When $x<3$, $f(x)=x^2-1$, $f(x)$ is C^0 at $(-\infty,-3)$

When $x>3$, $f(x)=2ax$, $f(x)$ is C^0 at $(3,+\infty)$

When $x=3$. $\lim\limits_{x\to3^-}f(x)=3^2-1=8$, $\lim\limits_{x\to3^+}f(x)=3\times2a=6a$

for $f(x)$ to be C^0 we must have $8=6a \Rightarrow a=\frac{4}{3}$.

§2.6.46.

Sol. $\cos x=x \Leftrightarrow \cos x-x=0$.

Let $f(x)=\cos x-x$.

$f(\frac{\pi}{2})=\cos\frac{\pi}{2}-\frac{\pi}{2}=-\frac{\pi}{2}<0$

$f(-\frac{\pi}{2})=\cos(-\frac{\pi}{2})+\frac{\pi}{2}=\frac{\pi}{2}>0$

Since $f(x)$ is C^0

then $\exists x_0\in(-\frac{\pi}{2},\frac{\pi}{2})$ s.t $f(x_0)=0$ according to the Intermediate Value Theorem.

Thus the equation $\cos x=x$ has at least 1 solution.

§2.6.60.

Sol. Since f is C^0 at $x=c$

then $\forall\varepsilon>0, \exists\delta>0$ s.t $|x-c|<\delta \Rightarrow |f(x)-f(c)|<\varepsilon$.

$\Rightarrow f(c)-\varepsilon<f(x)<f(c)+\varepsilon$.

If $f(c)>0$. Let $\varepsilon=\frac{1}{2}f(c)$ then

$\frac{1}{2}f(c)<f(x)<\frac{3}{2}f(c) \Rightarrow f(x)>0$ on $(c-\delta, c+\delta)$

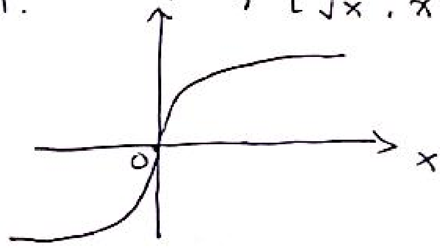
if $f(c)<0$, Let $\varepsilon=-\frac{1}{2}f(c)$ then

$\frac{3}{2}f(c)<f(x)<\frac{1}{2}f(c) \Rightarrow f(x)<0$ on $(c-\delta, c+\delta)$

Thus there is an interval $(c-\delta, c+\delta)$ about c where f has the same sign

§2.7.43.

Sol. $y=\begin{cases}-\sqrt{|x|}, & x\le0\\ \sqrt{x}, & x>0\end{cases}$



(a) The graph appears to have a vertical tangent at $x=0$

(b)

$$\lim_{\Delta x\to0^-}\frac{f(0+\Delta x)-f(0)}{\Delta x}=\lim_{\Delta x\to0^-}\frac{-\sqrt{|\Delta x|}}{-|\Delta x|}=\lim_{\Delta x\to0^-}\frac{1}{\sqrt{|\Delta x|}}=+\infty$$

$$\lim_{\Delta x\to0^+}\frac{f(0+\Delta x)-f(0)}{\Delta x}=\lim_{\Delta x\to0^+}\frac{\sqrt{\Delta x}}{\Delta x}=\lim_{x\to0^+}\frac{1}{\sqrt{\Delta x}}=+\infty$$

$\Rightarrow y$ has a vertical tangent at $x=0$

P145 . 19.

Sol. let Earth's equator be a ring

Let a which is a point in the ring ~~be the start point and also end point of the ring~~

so we can break the ring into a chain that starts with a and end with a.

Define the position of the start point is 0 and the position of the end point is 1.

Define $T(x)$ is the temperature at the point whose position is x.

then $T(0)=T(1)$

Since $T(x)$ is C^0 in $(0,1)$

then $\exists x_0 \in (0,1)$, $T(x_0)=T(0)=T(1)$ according to the Intermediate Value Theorem.

which means there exists a pair of antipodal points on Earth's equator where the temperature are the same.

§3.1.58.

(a) Sol. $|f(0)| \le 0^2 \Rightarrow f(0)=0$.

$$f'(0)=\lim_{\Delta x \to 0}\frac{f(0+\Delta x)-f(0)}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(\Delta x)}{\Delta x}$$

Since for $|x|\le 1$, $|f(x)|\le x^2$ then $-\Delta x \le \frac{f(\Delta x)}{\Delta x}\le \Delta x$

Therefore $f'(0)=\lim_{\Delta x\to 0}\frac{f(\Delta x)}{\Delta x}=0$ according to The Intermediate Value Theorem.

Thus f is differentiable at $x=0$ and $f'(0)=0$.

(b) Sol. when $x\neq 0$, $f(x)=x^2 \sin\frac{1}{x} \le x^2 \cdot 1 = x^2$

when $x=0$ $f(0)=0\le 0^2$

So $f(x)\le x^2$

By part (a), f is differentiable at $x=0$ and $f'(0)=0$

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§3.2.53.

Sol. a. $\frac{d}{dx}(uvw) = w\frac{d}{dx}(uv) + uv\frac{dw}{dx}$

$= w(u\cdot\frac{dv}{dx} + v\frac{du}{dx}) + uv\frac{dw}{dx}$

$= wu\cdot\frac{dv}{dx} + wv\frac{du}{dx} + uv\frac{dw}{dx}$

b. $\frac{d}{dx}(u_1u_2u_3u_4) = u_1u_2\frac{d}{dx}(u_3u_4) + u_1u_3u_4\frac{du_2}{dx} + u_2u_3u_4\frac{du_1}{dx}$

$= u_1u_2(u_3\frac{du_4}{dx} + u_4\frac{du_3}{dx}) + u_1u_3u_4\frac{du_2}{dx} + u_2u_3u_4\frac{du_1}{dx}$

$= u_1u_2u_3\frac{du_4}{dx} + u_1u_2u_4\frac{du_3}{dx} + u_1u_3u_4\frac{du_2}{dx} + u_2u_3u_4\frac{du_1}{dx}$

c. ① when $n=2$, $\frac{d}{dx}(u_1u_2) = u_1\frac{du_2}{dx} + u_2\frac{du_1}{dx}$

② Assume for $n=k$ $(k\geq 2, k\in N^*)$,

$\frac{d}{dx}(u_1\cdots u_k) = u_1u_2\cdots u_{k-1}u_k' + u_1u_2\cdots u_{k-2}u_{k-1}'u_k + \cdots + u_1'u_2\cdots u_k$

Then when $n=k+1$,

$\frac{d}{dx}(u_1\cdots u_{k+1}) = u_1u_2\cdots u_{k-2}(u_{k-1}u_k)' + u_1u_2\cdots u_{k-3}u_{k-3}'u_{k-2}(u_{k-1}u_k) + \cdots$

$+ u_1'u_2\cdots u_{k-2}(u_{k-1}u_k)$

$= u_1u_2\cdots u_{k-2}u_{k-1}u_k' + u_1u_2\cdots u_{k-2}u_{k-1}'u_k + \cdots$

$+ u_1'u_2\cdots u_{k-2}u_{k-1}u_k$.

By ①.②. we have $\frac{d}{dx}(u_1u_2\cdots u_n) = u_1u_2\cdots u_n' + u_1u_2\cdots u_{n-2}u_{n-1}'u_n + \cdots$

$+ u_1'u_2\cdots u_n$.

§3.4.48

Sol. $\lim_{x\to 0^-} g(x) = \lim_{x\to 0^-} x+b = b$, $\lim_{x\to 0^+} g(x) = \lim_{x\to 0^+}\cos x = 1$.

for $g(x)$ to be C^0 at $x=0$ we must have $b=1$.

left-hand derivative: $\frac{d}{dx}(x+b)\big|_{x=0} = 1$.

right-hand derivative: $\frac{d}{dx}(\cos x)\big|_{x=0} = -\sin 0 = 0$

Thus left-hand derivative is not equal to right-hand derivative which means $g(x)$ is not differentiable at $x=0$ for any b.

Bonus. P145. 17.

a. Pf. Assume $\lim_{x\to 0} f(x) = 0$

$\forall \varepsilon > 0$ choose $\delta = \frac{\varepsilon}{2}$ s.t.

if $|x-0| < \delta \Rightarrow -\frac{\varepsilon}{2} < x < \frac{\varepsilon}{2}$

Then $|f(x) - 0| \leq |x| < \frac{\varepsilon}{2} < \varepsilon$

Thus $\lim_{x\to 0} f(x) = 0$, f is continuous at $x=0$.

b. Pf. ① Assume $\exists c \neq 0$ s.t. $\lim_{x\to c} f(x) = L$, $L \neq 0$

choose $\varepsilon = \frac{|L|}{2}$, $\forall \delta > 0$ s.t.

$\exists x_0$, $|x_0 - c| < \delta$ s.t. x_0 is rational $\Rightarrow f(x_0) = 0$

$\Rightarrow |f(x_0) - L| = |-L| > \frac{|L|}{2} = \varepsilon$.

Thus lead to contradiction.

② Assume $\exists c \neq 0$ s.t. $\lim_{x\to c} f(x) = 0$

choose $\varepsilon = \frac{|c|}{2}$, $\forall \delta > 0$ s.t.

when $c > 0$, $\exists x_0$, $|x_0 - c| < \delta$ s.t. x is irrational and

$x_0 > c \Rightarrow f(x_0) = x_0 > c \Rightarrow |f(x_0) - 0| = |x_0| > |c| > \frac{|c|}{2} = \varepsilon$

when $c < 0$, $\exists x_0$, $|x_0 - c| < \delta$ s.t x is irrational and

$x_0 < c \Rightarrow f(x_0) = x_0 < c \Rightarrow |f(x_0) - 0| = |x_0| > |c| > \frac{|c|}{2} = \varepsilon$.

Thus lead to contradiction.

By ①②, f is not continuous at any nonzero value of x