



$$\begin{aligned}
 1. (1) & \neg(\exists x \wedge \exists y)(P(x) \wedge P(y) \wedge Q(x) \wedge Q(y) \wedge R(x, y)) \\
 & = (\forall x) \neg(\exists y)(P(x) \wedge P(y) \wedge Q(x) \wedge Q(y) \wedge R(x, y)) \\
 & = (\forall x)(\forall y) \neg(P(x) \wedge P(y) \wedge Q(x) \wedge Q(y) \wedge R(x, y)) \\
 & = (\forall x)(\forall y)(\neg(P(x) \wedge P(y) \wedge Q(x) \wedge Q(y)) \vee \neg R(x, y)) \\
 & = (\forall x)(\forall y)(\neg P(x) \vee \neg P(y) \vee \neg Q(x) \vee \neg Q(y) \vee \neg R(x, y))
 \end{aligned}$$

$$\begin{aligned}
 (4) & (\forall y)(\exists x)(P(x) \rightarrow q) \vee S(y) \\
 & = (\forall y)(\neg(\exists x)(P(x) \rightarrow q) \vee S(y)) \\
 & = (\exists x)(P(x) \rightarrow q) \vee (\forall y)S(y) \\
 & = (\forall x)(P(x) \rightarrow q) \vee (\forall y)S(y)
 \end{aligned}$$

$$\begin{aligned}
 (7) & (\exists x)P(x) \rightarrow (\forall x)Q(x) \\
 & = \neg(\exists x)P(x) \vee (\forall x)Q(x) \\
 & = (\forall x)(\neg P(x) \vee Q(x)) \\
 & \Rightarrow (\forall x)(\neg P(x) \vee Q(x)) \\
 & = (\forall x)(P(x) \rightarrow Q(x))
 \end{aligned}$$

2. (1) 在 1, 2 域上分析, 当 $P(1)=P(2)=Q(1)=F, Q(2)=T$ 时

$$\begin{aligned}
 & (\exists x)(P(x) \leftrightarrow Q(x)) \rightarrow ((\exists x)P(x) \leftrightarrow (\exists x)Q(x)) \\
 & = T \rightarrow (F \leftrightarrow T) = T \rightarrow F = F
 \end{aligned}$$

所以原公式不是普遍有效的

$$\begin{aligned}
 (3) & ((\exists x)P(x) \rightarrow (\forall x)Q(x)) \rightarrow (\forall x)(P(x) \rightarrow Q(x)) \\
 & = (\neg(\exists x)P(x) \vee (\forall x)Q(x)) \rightarrow (\forall x)(\neg P(x) \vee Q(x)) \\
 & = ((\forall x)\neg P(x) \vee (\forall x)Q(x)) \rightarrow (\forall x)(\neg P(x) \vee Q(x)) \\
 & = T
 \end{aligned}$$

故原公式是普遍有效的.

$$\begin{aligned}
 (5) & ((\exists x)P(x) \rightarrow (\exists x)Q(x)) \rightarrow (\exists x)(P(x) \rightarrow Q(x)) \\
 & = \neg(\neg(\exists x)P(x) \vee (\exists x)Q(x)) \vee (\exists x)(\neg P(x) \vee Q(x)) \\
 & = (\exists x)P(x) \vee \neg(\exists x)Q(x) \vee (\exists x)\neg P(x) \vee (\exists x)Q(x) \\
 & = T
 \end{aligned}$$

故原公式是普遍有效的.