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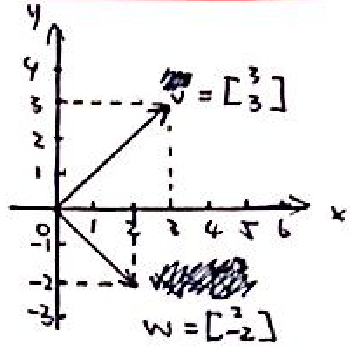
Promblem 1.1.3

Sol. As $2v = (v+w) + (v-w) = \begin{bmatrix}5\\1\end{bmatrix} + \begin{bmatrix}1\\5\end{bmatrix} = \begin{bmatrix}6\\6\end{bmatrix}$

we have $v = \frac{1}{2} \times (2v) = \frac{1}{2}\begin{bmatrix}6\\6\end{bmatrix} = \begin{bmatrix}3\\3\end{bmatrix}$

and $w = (v+w) - v = \begin{bmatrix}5\\1\end{bmatrix} - \begin{bmatrix}3\\3\end{bmatrix} = \begin{bmatrix}2\\-2\end{bmatrix}$



Promblem 1.1.6

Sol. (i) A linear combination of w and v is

$$cv + dw = c\begin{bmatrix}1\\-2\\1\end{bmatrix} + d\begin{bmatrix}0\\1\\-1\end{bmatrix} = \begin{bmatrix}c\\-2c\\c\end{bmatrix} + \begin{bmatrix}0\\d\\-d\end{bmatrix} = \begin{bmatrix}c\\d-2c\\c-d\end{bmatrix}$$

the sum of the components of $cv+dw$ is

$$c + d - 2c + c - d = 0$$

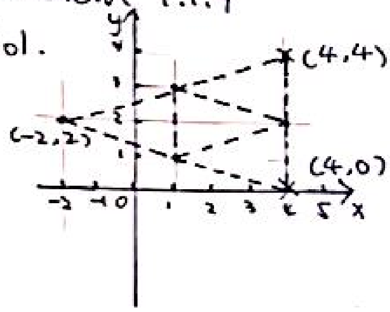
~~so the sum~~

(ii) We have $\begin{bmatrix}c\\d-2c\\c-d\end{bmatrix} = \begin{bmatrix}3\\3\\-6\end{bmatrix} \Rightarrow \begin{cases}c=3\\d-2c=3\\c-d=-6\end{cases} \Rightarrow \begin{cases}c=3\\d=9\end{cases}$

(iii) It's impossible because $3+3+6=12 \neq 0$ and the sum of the components of $cv+dw$ is 0 which we have proven in (i)

Promblem 1.1.9

Sol.



The forth corner can be (-2,2), (4,4) or (4,0).

Promblem 1.1.25

Sol. (i) Let $u = v = w = \begin{bmatrix}0\\0\\1\end{bmatrix}$ and their combinations fill only a line.

(ii) Let u, v become two vectors that have different directions, and $w = u + v$ so that ~~them~~ they are in the same plane ~~and~~

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Pramblem 1.1.26.

Sol. we have $c\begin{bmatrix}1\\2\end{bmatrix}+d\begin{bmatrix}3\\1\end{bmatrix}=\begin{bmatrix}14\\8\end{bmatrix}$

$$\Rightarrow \begin{bmatrix}c+3d\\2c+d\end{bmatrix}=\begin{bmatrix}14\\8\end{bmatrix}$$

$$\Rightarrow \begin{cases}c+3d=14\\2c+d=8\end{cases}$$

$$\Rightarrow \begin{cases}c=2\\d=4\end{cases}$$

Promblem 1.1.29.

Sol. (i) Let $xu+yv+zw=b$

$$\Rightarrow x\cdot\begin{bmatrix}1\\3\end{bmatrix}+y\begin{bmatrix}2\\7\end{bmatrix}+z\begin{bmatrix}1\\5\end{bmatrix}=\begin{bmatrix}0\\1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x+2y+z\\3x+7y+5z\end{bmatrix}=\begin{bmatrix}0\\1\end{bmatrix}$$

$$\Rightarrow \begin{cases}x+2y+z=0\\3x+7y+5z=1\end{cases}$$

$\begin{cases}x=1\\y=-1\\z=1\end{cases}$ and $\begin{cases}x=-2\\y=1\\z=0\end{cases}$ are two of the solutions.

(ii) No. if $u=v=w=\begin{bmatrix}1\\1\end{bmatrix}$ than it's impossible to have a combination of them that is equal to $\begin{bmatrix}0\\1\end{bmatrix}$

Graded Promblem.

(a) u and v span a plane in $\mathbb{R}^3$. $cu+dv=c\begin{bmatrix}2\\0\\0\end{bmatrix}+d\begin{bmatrix}0\\2\\2\end{bmatrix}=\begin{bmatrix}2c\\2d\\2d\end{bmatrix}$

(b) v and w span a plane in $\mathbb{R}^3$. $ev+fw=e\begin{bmatrix}0\\2\\2\end{bmatrix}+f\begin{bmatrix}2\\2\\3\end{bmatrix}=\begin{bmatrix}e+2f\\2e+2f\\2e+3f\end{bmatrix}$

(c) Let $\begin{bmatrix}2c\\2d\\2d\end{bmatrix}=\begin{bmatrix}2f\\2e+2f\\2e+3f\end{bmatrix}\Rightarrow\begin{cases}2c=2f\\2d=2e+2f\\2d=2e+3f\end{cases}\Rightarrow\begin{cases}c=f=0\\d=e\end{cases}$

so that the ans is $\begin{bmatrix}0\\a\\a\end{bmatrix}$, $a\in\mathbb{R}$. and it's a line.

(In other words, the intersection of two distinct plane in 3-dimensional space is a line).