NAME :

STUDENT ID:

NOTE: There are 3 Parts to this BLANK MIDTERM (total of 8 pages). For Part 1 (multiple choice) be sure to indicate your answer clearly as no partial credit will be awarded. Each question has a unique right answer.
Part 3 consists of a bonus exercise, which is not compulsory, and can give you some extra points. It is advised to finish all the compulsory questions before attempting the bonus ones. If your total is > 100 points, your assigned grade will be 100.
In order to receive full credit for Parts 2 and 3, you must show work to explain your reasoning. If you require additional sheets for Parts 2 or 3, be sure to put your name and ID to each additional page that you turn in with this exam. Use of calculators will NOT be permitted. You have 90 minutes complete this test.

Part 1 (25pts)

1. Let $f, g : \mathbb{R} \to \mathbb{R}$ be functions. When is it always true that the composite $f \circ g$ is an odd function?
   (A) f is even and g is odd.
   (B) f is even and g is even.
   (C) f is odd and g is odd.
   (D) f is odd and g is even.
   (E) None of the above

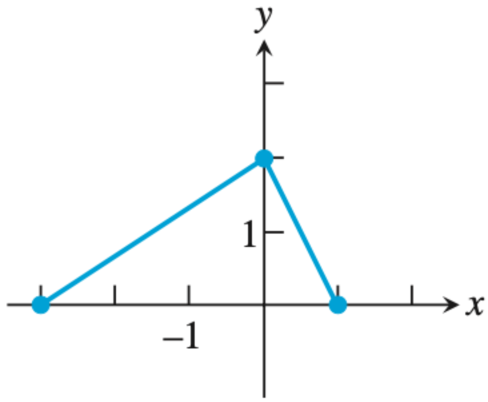
2. What is the least upper bound of the set $S = \{x \in \mathbb{Q}, x^3 < 2\}$?
   (A) $\sqrt{2}$
   (B) $2^{1/3}$
   (C) 8
   (D) The least upper bound does not exist.
   (E) None of the above.

3. Which of the assertions below is equivalent to "$\lim_{x\to+\infty} f(x) \neq L$" (for a function $f : \mathbb{R} \to \mathbb{R}$ and $L \in \mathbb{R}$)?
   (A) $\forall \epsilon > 0\ \exists M \geq 0$ such that $\forall x \geq M$, we have $\mid f(x) - L \mid > \epsilon$.
   (B) $\exists \epsilon > 0\ \exists M \geq 0$ such that $\forall x \geq M$, we have $\mid f(x) - L \mid > \epsilon$.
   (C) $\exists \epsilon > 0$ such that $\forall M \geq 0\ \forall x \geq M$, we have $\mid f(x) - L \mid > \epsilon$.
   (D) $\exists \epsilon > 0$ such that $\forall M \geq 0\ \exists x \geq M$ such that $\mid f(x) - L \mid > \epsilon$.
   (E) None of the above

4. How many asymptotes (vertical, horizontal and oblique) does the function $f(x) = \frac{x^3-x-1}{x^2+x-2}$ have?
   (A) 0
   (B) 1
   (C) 2
   (D) 3
   (E) None of the above

5. What is the (smallest) period of the function $f(x) = \mid \sin(x) \mid + \mid \cos(x) \mid$?
   (A) $\frac{\pi}{2}$
   (B) π
   (C) 2π
   (D) $\frac{3\pi}{2}$
   (E) f is not periodic.

Part 2a. (20 pts) Consider the function f whose graph is represented below.



Draw the graph of each function:

1. $y = f(-x)$.

2. $y = -2f(x+1) + 1$.

3. $y = -f(x)$.

4. $y = 3f(x-2) - 2$.

Part 2b. (15pts) Let $P(x) = a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$ be a degree 5 polynomial (where the coefficients a_i are real numbers and $a_5 \neq 0$). Prove that there exists $c \in \mathbb{R}$ such that $P(c) = 0$.

Part 2c. (10pts) Does the following limit exist? If yes, what is its value? (You need to justify your answer).

$$\lim_{x \to -1} \frac{\sqrt{x^2+8}-3}{x+1}$$

Part 2d. (15pts) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2 \cos(\frac{1}{x})$ for $x \neq 0$ and $f(0) = 0$. Prove that f is differentiable at $x = 0$, and compute $f'(0)$.

Part 2e. (15pts) Does the following limit exist? If yes, what is its value? (You need to justify your answer).

$$\lim_{x \to \frac{\pi}{4}} \frac{\sin(x) - \cos(x)}{x - \frac{\pi}{4}}$$

Part 3 (BONUS QUESTION). (10pts) Prove that for any $\epsilon > 0$, there exists $n \in \mathbb{N}$ such that $0 < \sin(\sqrt{n}) \leq \epsilon$.