15.09.2020

Digital Image Processing (CSE/ECE 478)

Lecture-10: Frequency Domain Processing

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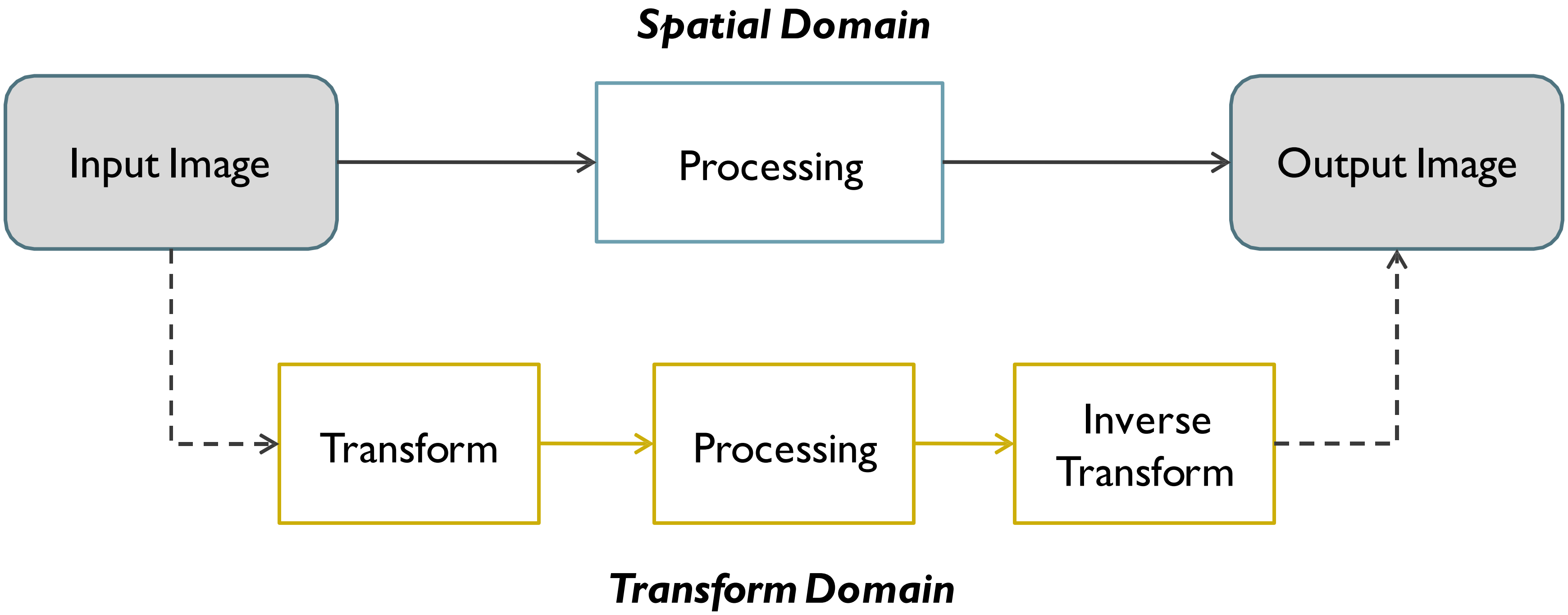
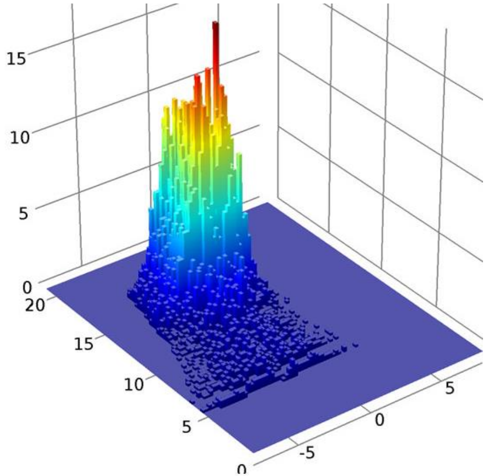
CVIT

Spatial vs. Transform Domain Processing





$$F[m,n] = \sum_{y=0}^{y=(N-1)} \sum_{x=0}^{x=(M-1)} f[x,y]\, e^{-2\pi j\left(\frac{mx}{M} + \frac{ny}{N}\right)}$$

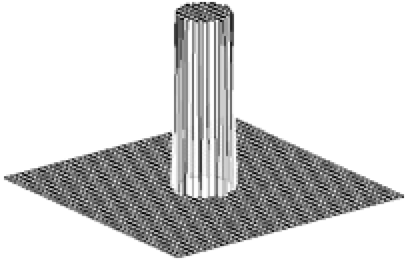
$$f[x,y] = \frac{1}{MN} \sum_{n=0}^{n=(N-1)} \sum_{m=0}^{m=(M-1)} F[m,n]\, e^{2\pi j\left(\frac{mx}{M} + \frac{ny}{N}\right)}$$

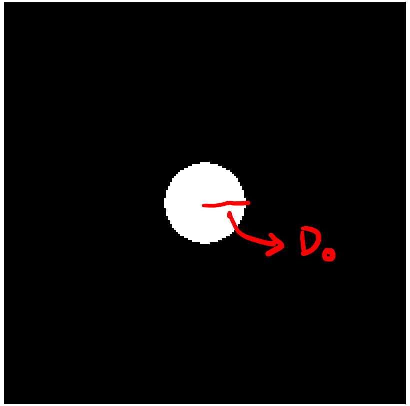
Image Enhancement and Filtering in Frequency Domain

Ideal Low Pass Filters

$$I \xrightarrow{DFT} \underbrace{F(u,v)\,H(u,v)} \xrightarrow{IDFT} I_{LPF}$$

$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le \underline{D_0} \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$$





D_0

$$H(u,v) \xrightarrow{IDFT} \underbrace{h(x,y)}$$

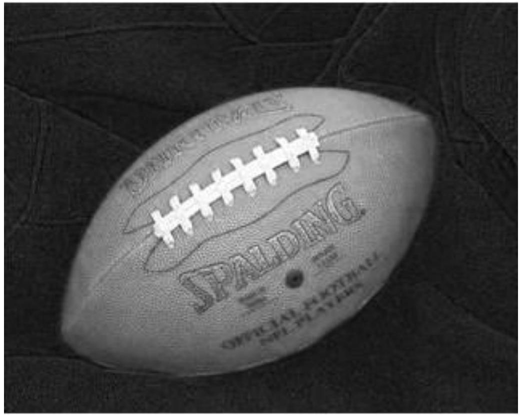
$$\underbrace{I * h(x,y)}$$

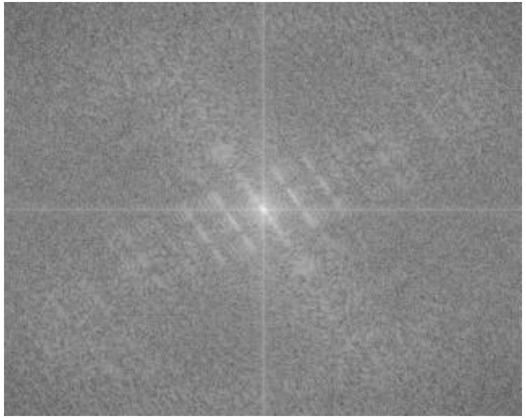
where $D(u,v) = [(u - M/2)^2 + (v - N/2)^2]^{1/2}$

$D_0 \rightarrow$ cut off frequency

Image courtesy: Gonzalez and Woods

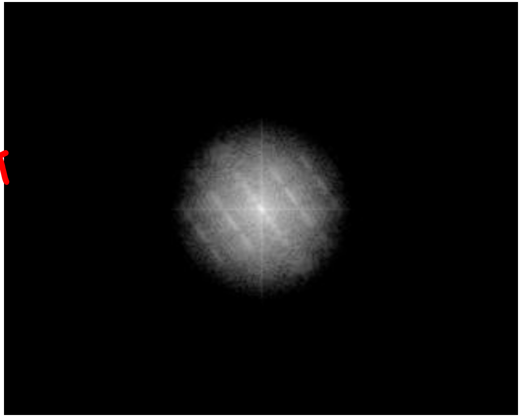
Ideal Low Pass Filters





$F(u,v)$

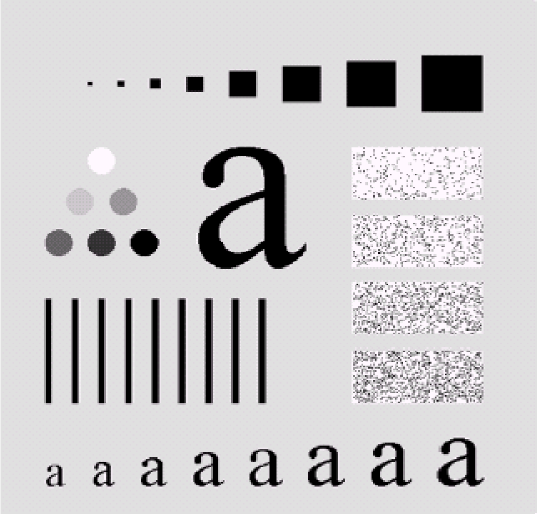
IDFT

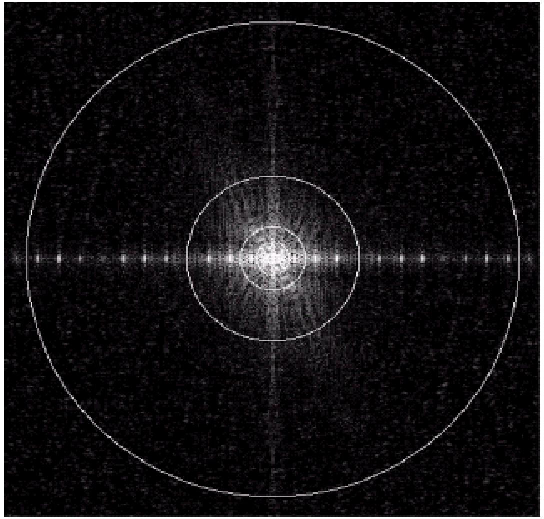


$F(u,v)H(u,v)$

Image courtesy: cs.uregina.ca

Ideal Low Pass Filters

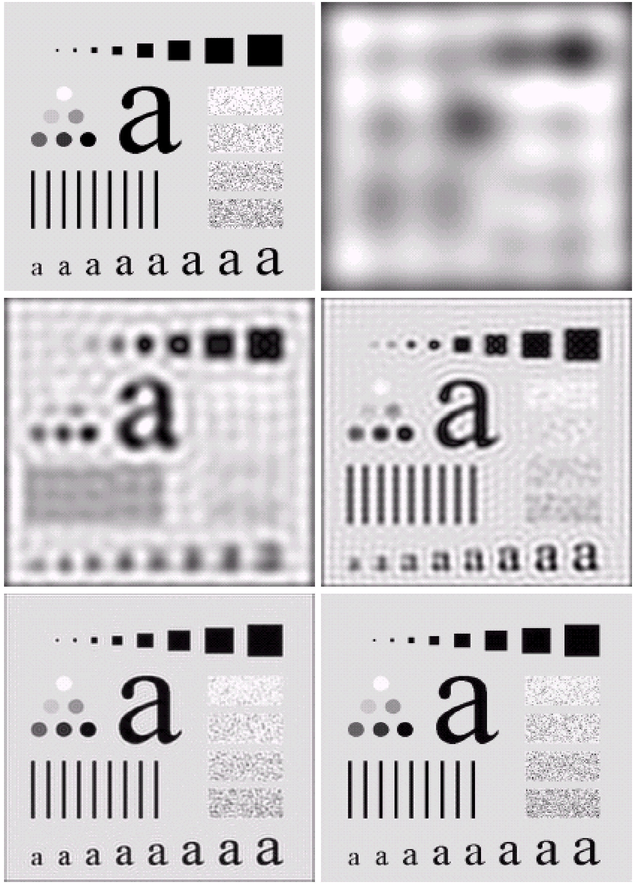




Radii 10,30,60,160 and 460 → power 87, 93.1, 95.7, 97.8 and 99..2

Image courtesy: Gonzalez and Woods

Ideal Low Pass Filters



ILPF radius 30

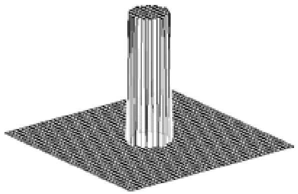
ILPF radius 60

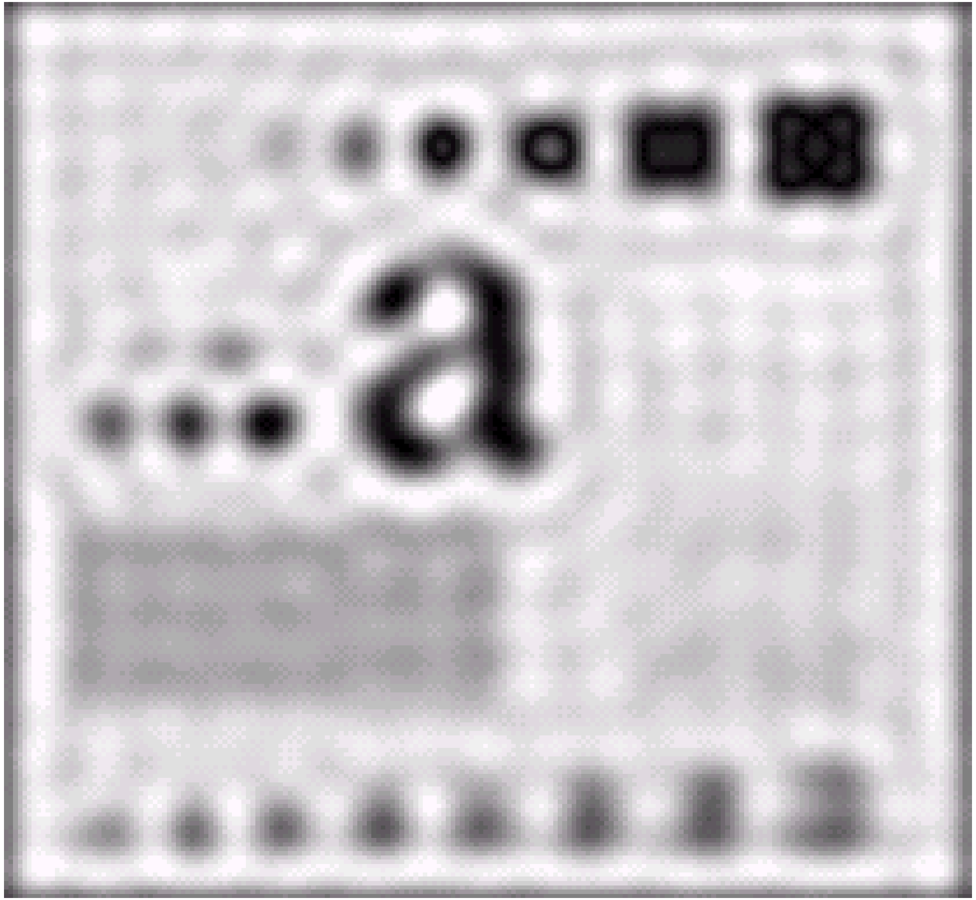
ILPF radius 160

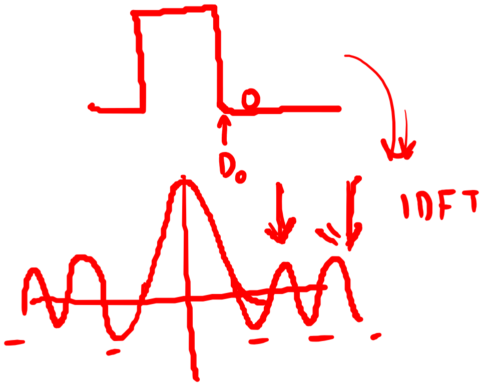
ILPF radius 460

Image courtesy: Gonzalez and Woods

Ideal Low Pass Filters



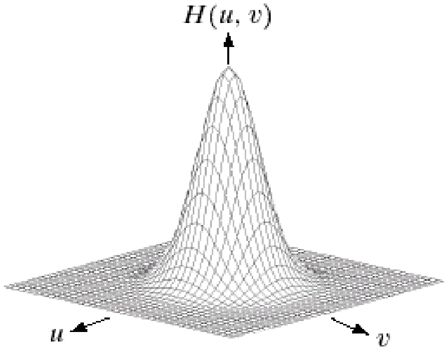


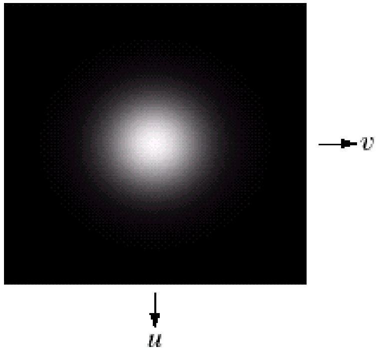


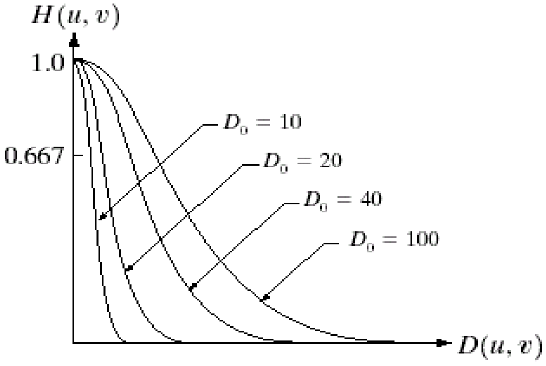
ILPF radius 30

Image courtesy: Gonzalez and Woods

Gaussian Low Pass Filters



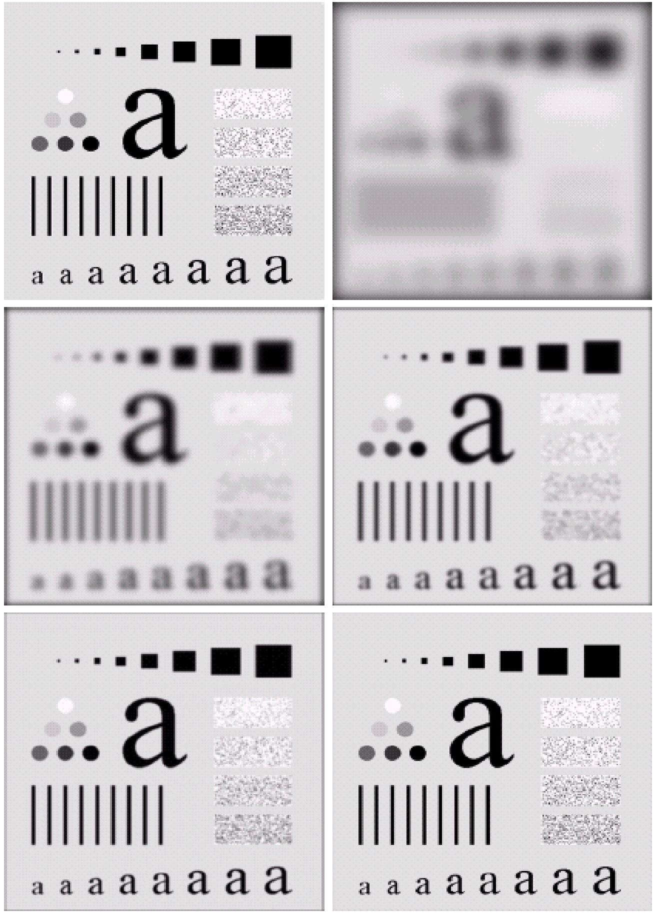




$$H(u,v) = e^{-D^2(u,v)/2D_0^{\,2}}$$

Image courtesy: Gonzalez and Woods

Gaussian Low Pass Filters (GLPF)



GLPF cut off frequency 10

GLPF cut off frequency 30

GLPF cut off frequency 60

GLPF cut off frequency 160

GLPF cut off frequency 460

Image courtesy: Gonzalez and Woods

Comparison (ILPF, BLPF, GLPF)

Butterworth

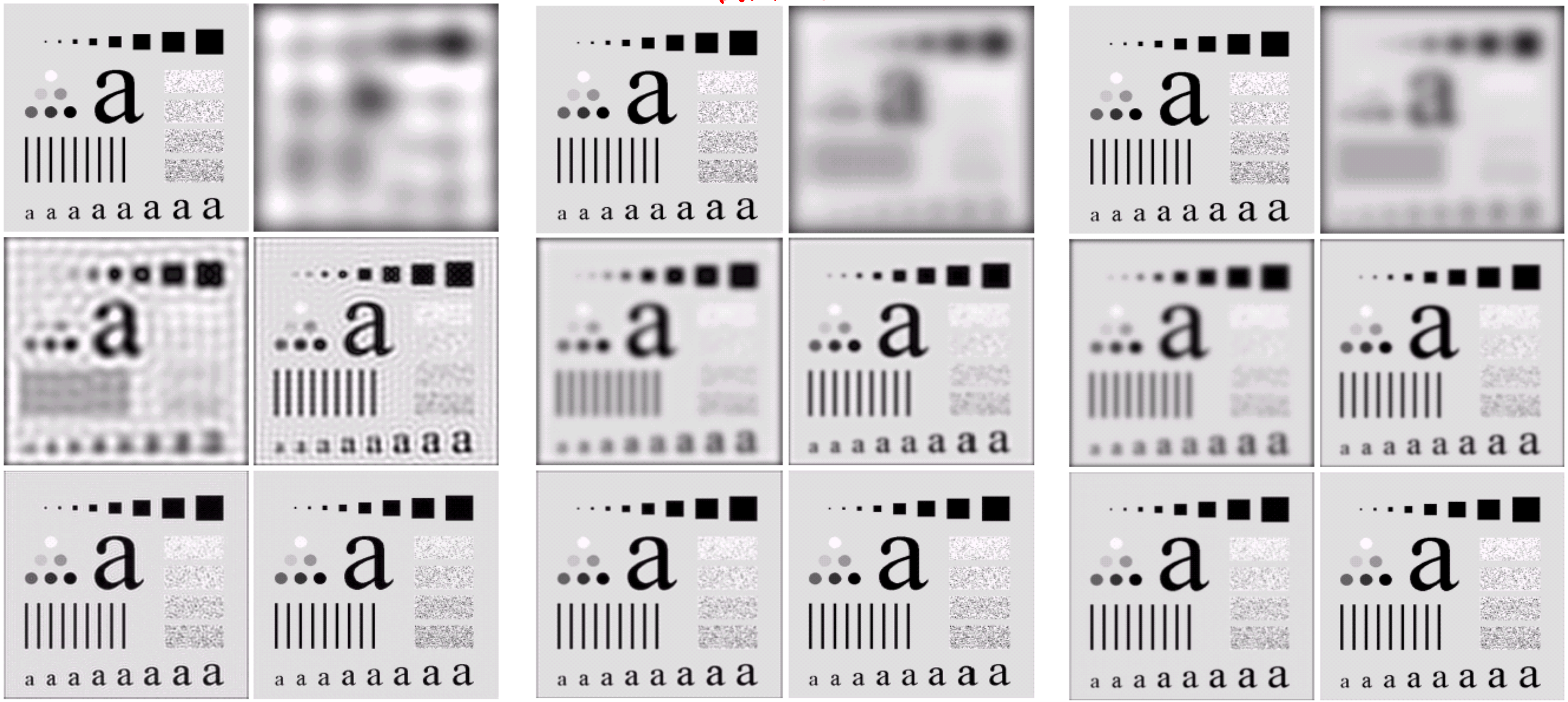


Image courtesy: Gonzalez and Woods

Low pass filtering application

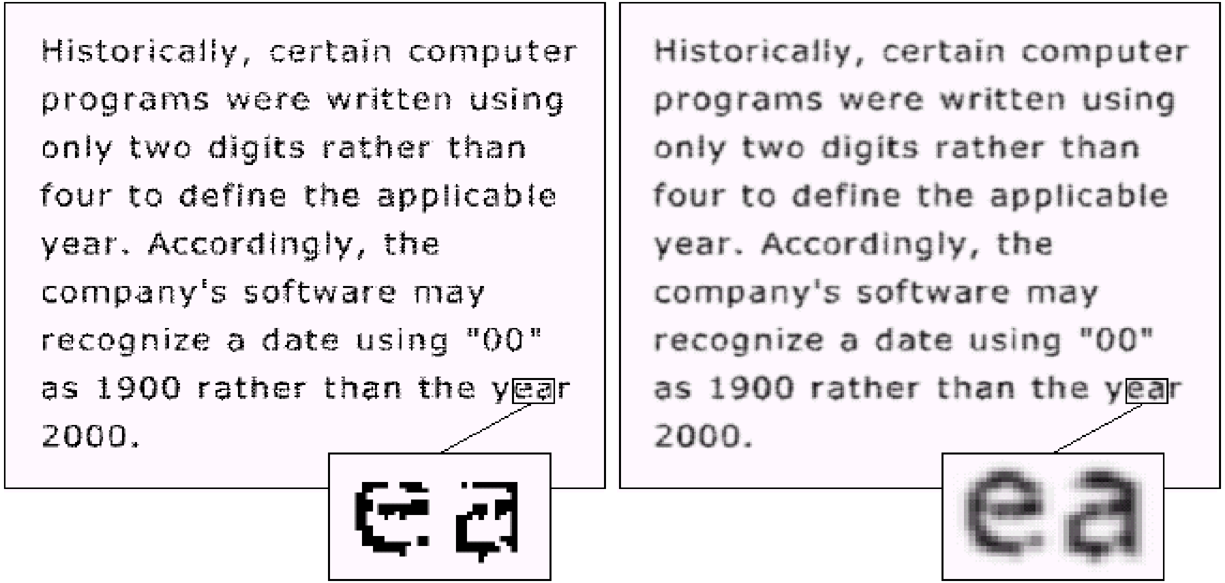
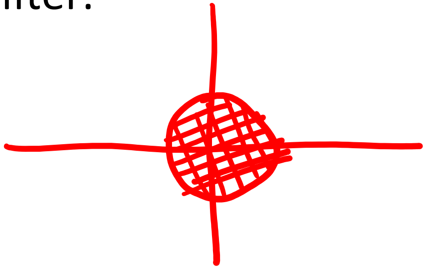


Image courtesy: Gonzalez and Woods

Image Sharpening in Frequency Domain

High Pass filter can be obtained from a given low pass filter:

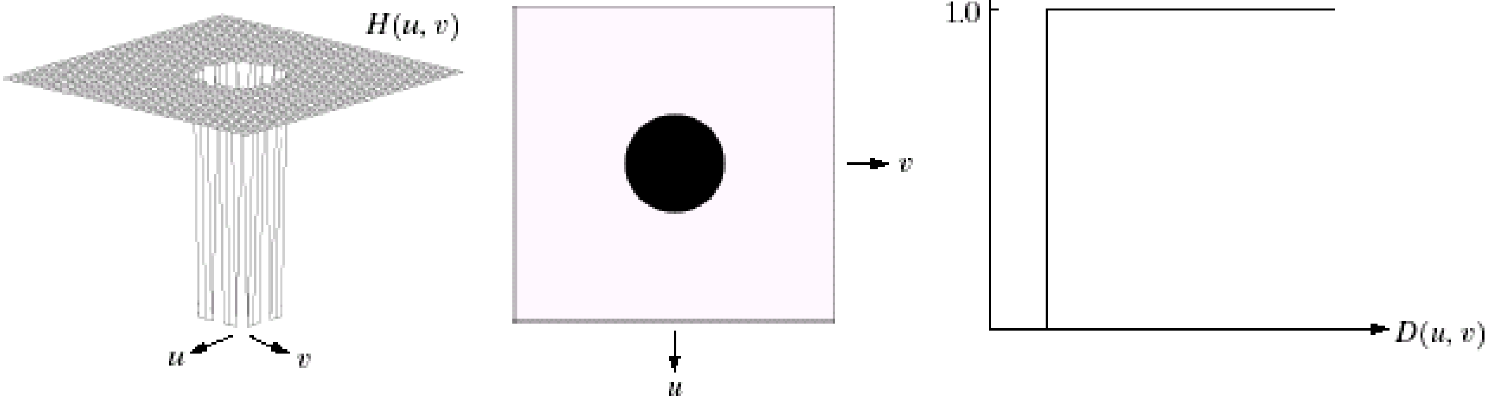
$$\underline{H_{hp}(u, v)} = 1 - \underbrace{H_{lp}(u, v)}$$



Ideal High Pass Filters

$$H_{hp}(u, v) = 1 - H_{lp}(u, v)$$

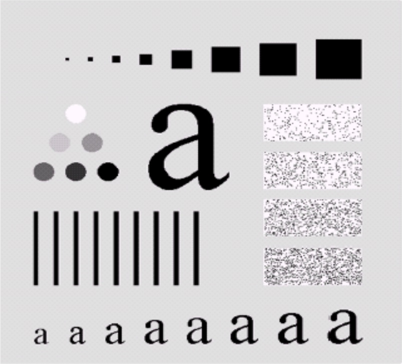
$1 - H_{lp}$

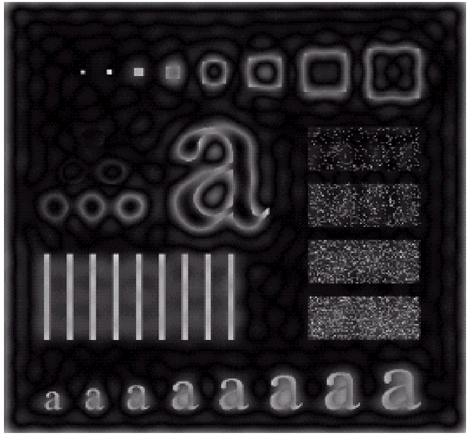


$$H(u,v) = \begin{cases} 0 \text{ if } D(u,v) \le D_0 \\ 1 \text{ if } D(u,v) > D_0 \end{cases}$$

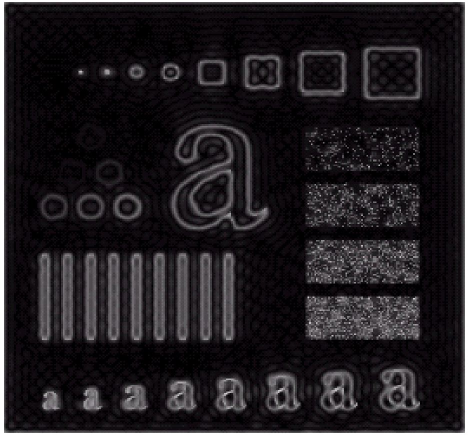
Image courtesy: Gonzalez and Woods

Ideal High Pass Filters

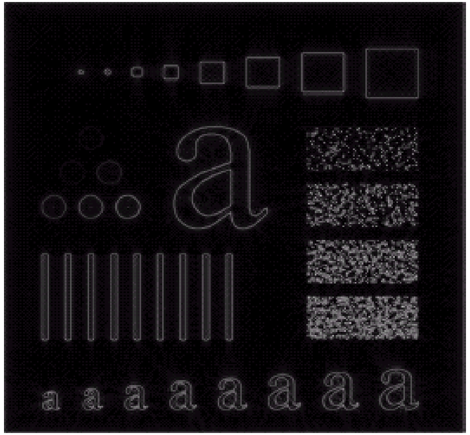




IHPL with $D_0 = 30$



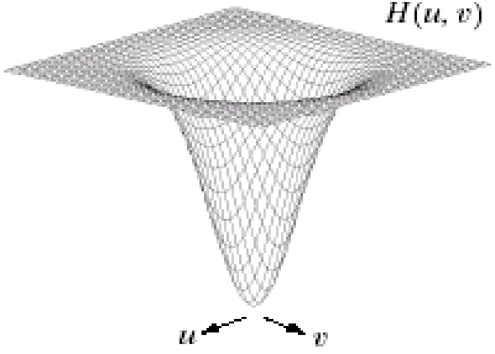
IHPF with $D_0 = 60$

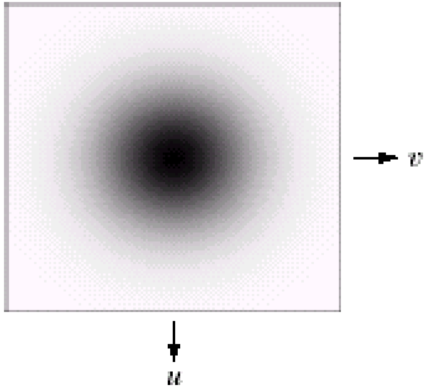


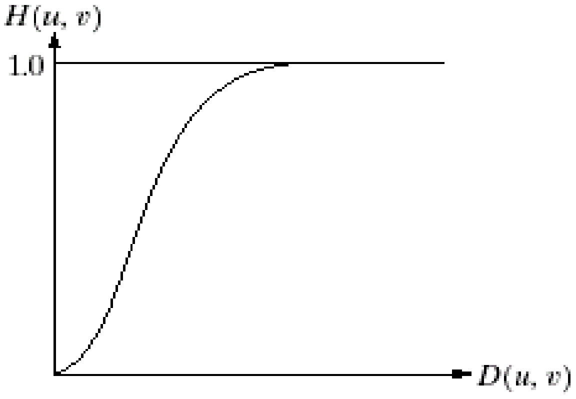
IHPF with $D_0 = 160$

Image courtesy: Gonzalez and Woods

Gaussian High Pass Filters



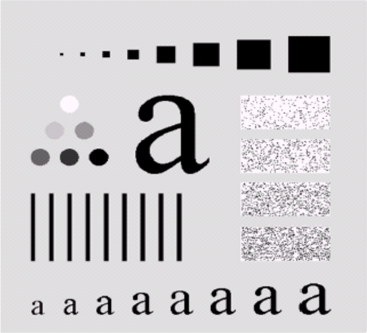


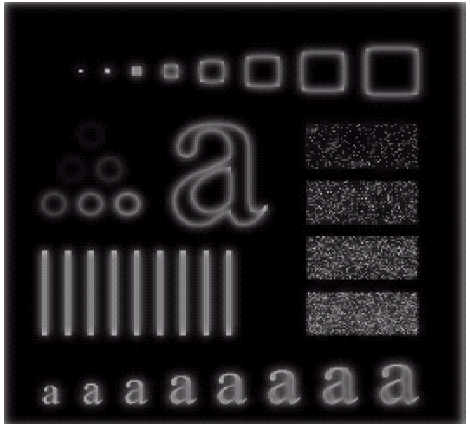


$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0{}^2}$$

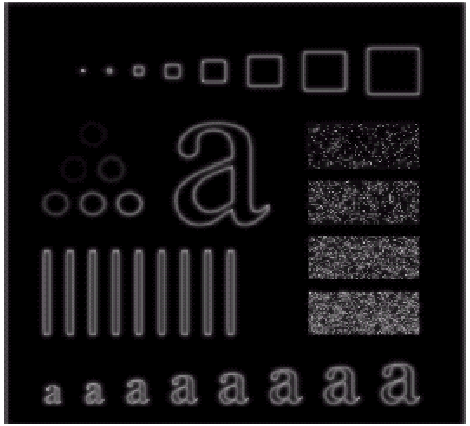
Image courtesy: Gonzalez and Woods

Gaussian High Pass Filters

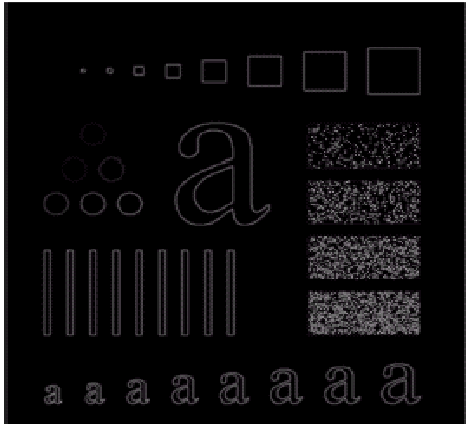




GHPL with $D_0 = 30$



GHPF with $D_0 = 60$



GHPF with $D_0 = 160$

Image courtesy: Gonzalez and Woods

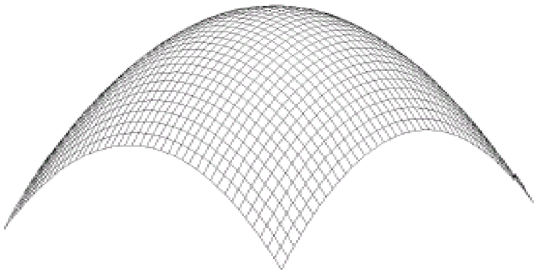
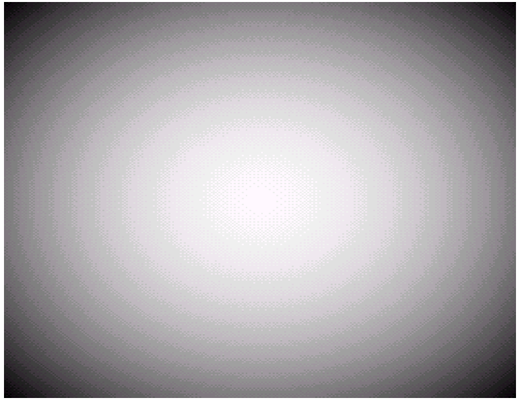
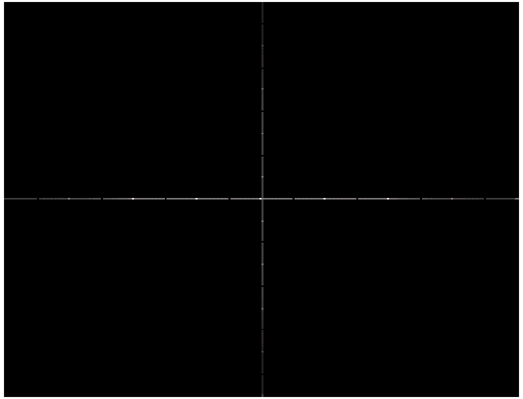
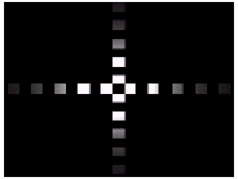
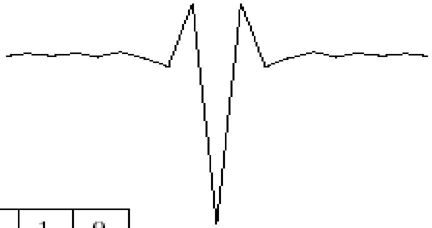
Laplacian in frequency domain

$$\Im[\frac{d^n f(x)}{dx^n}] = (ju)^n F(u)$$

$$\Im[\frac{\partial^2(f(x,y))}{\partial x^2} + \frac{\partial^2(f(x,y))}{\partial y^2}] = (ju)^2 F(u,v) + (jv)^2 F(u,v)$$

$$= -(u^2 + v^2) F(u,v)$$

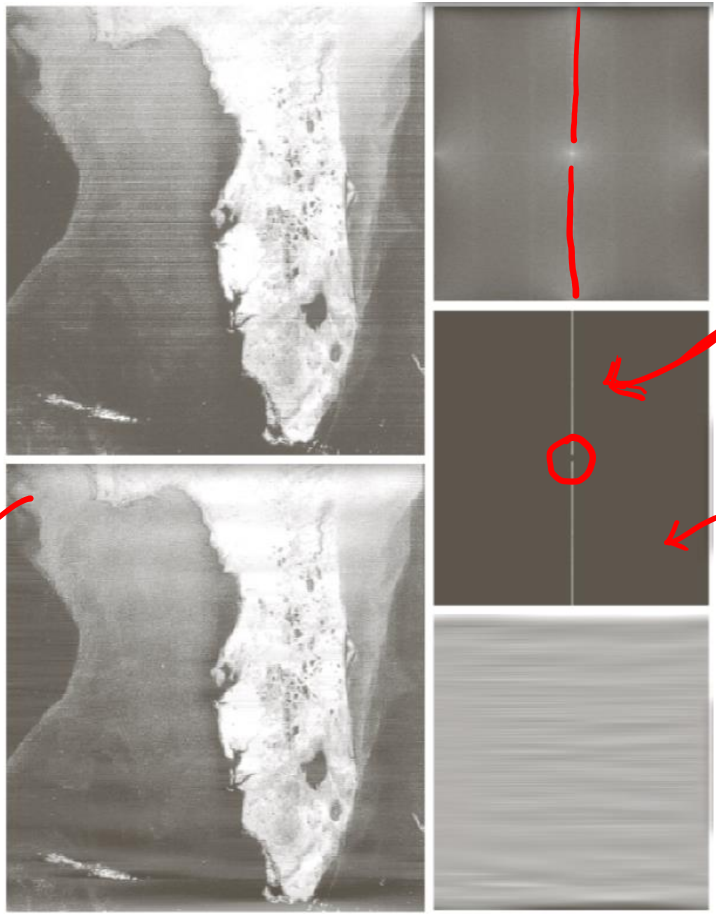
Laplacian in frequency domain

0	1	0
1	−4	1
0	1	0

Image courtesy: Gonzalez and Woods

Notch Reject filter (Notch pass filter)

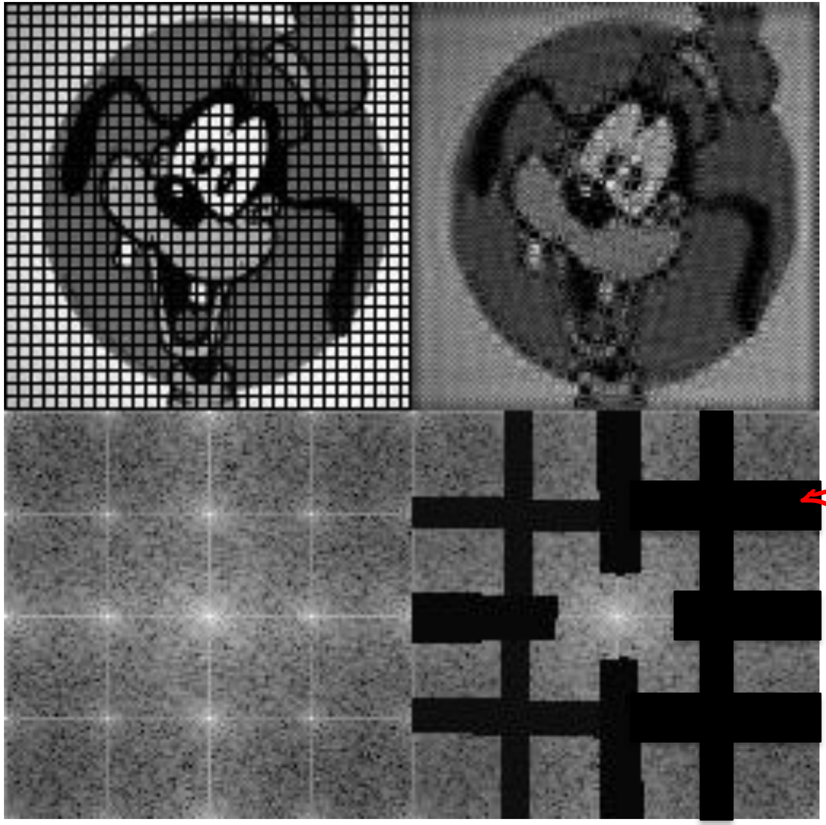


(1 - m)

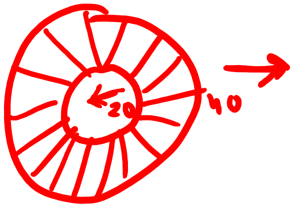
noise profile

Image courtesy: Gonzalez and Woods

Artifact removal



Filtering in frequency domain



- Band reject (Band pass filters) ✓
- Unsharp Masking and High boost filtering ✓
- Homomorphic filtering

$$I - I_{LPF}$$

$$I + c\nabla^2 I$$

$$I(x,y) = \underline{M}(x,y)\,\underline{d}(x,y)$$

$$\log I(x,y) = \underbrace{\log M(x,y)}_{} + \underbrace{\log d(x,y)}_{}$$

e

$\mathcal{F}(M(x,y))$

e

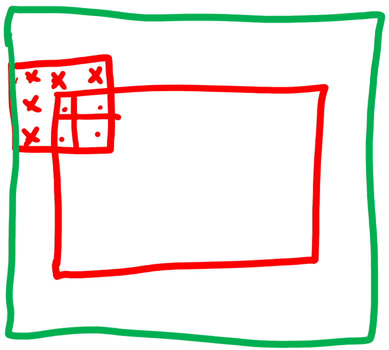
$\mathcal{F}($

Additional considerations

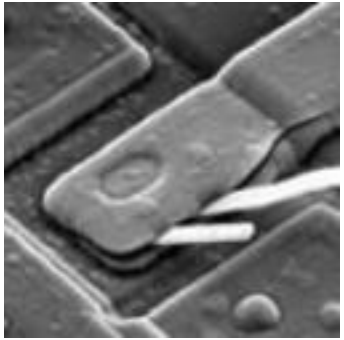
- Circular convolution → Wraparound error
  - Zero padding

$I \xrightarrow{DFT} F$

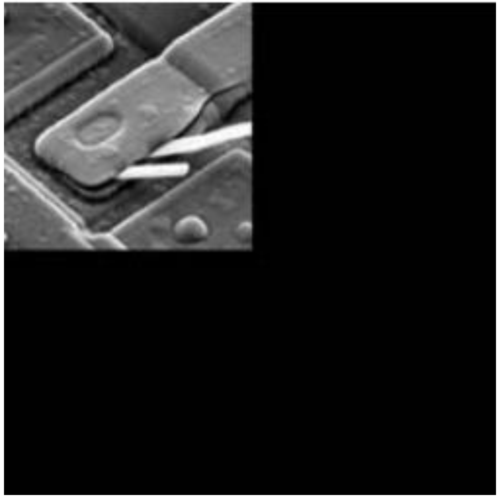
FH



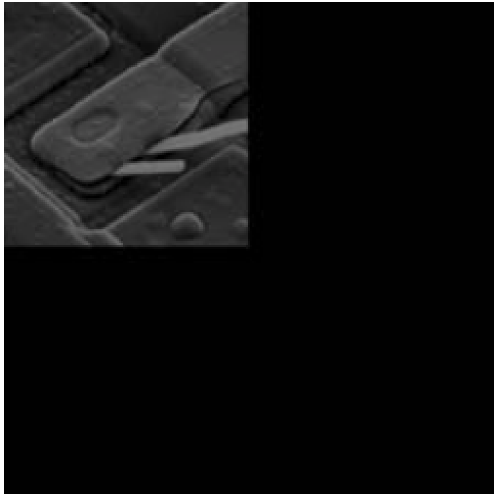
Recipe for transform domain processing



Given: M x N image f



1: pad f_p to size P x Q
where P = 2M, Q = 2N



2: Multiply f_p by $(-1)^{(x+y)}$

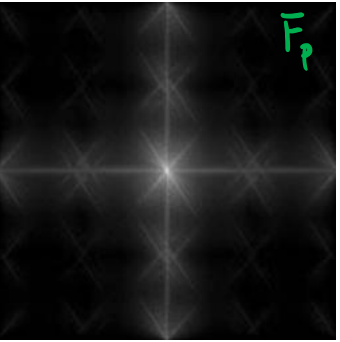
3: Compute $F_p = DFT(\tilde{f}_p)$

$\tilde{f}_p$

m+a

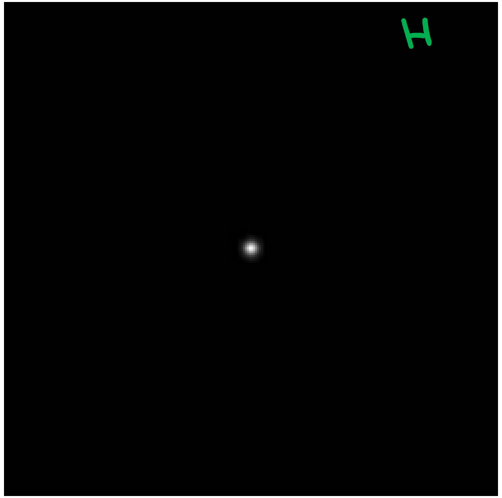
$F[u,v] =$

Recipe for transform domain processing



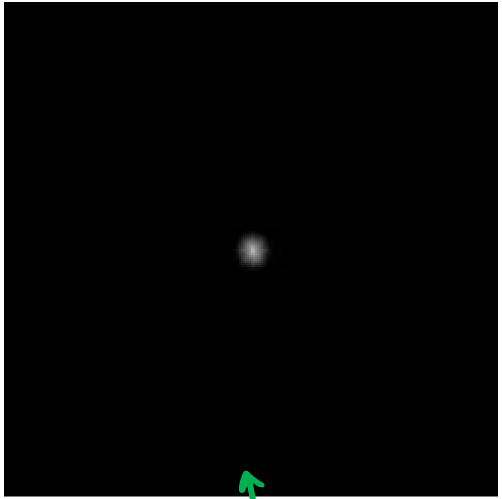
F_p - Fourier Spectrum of f_p

P x Q

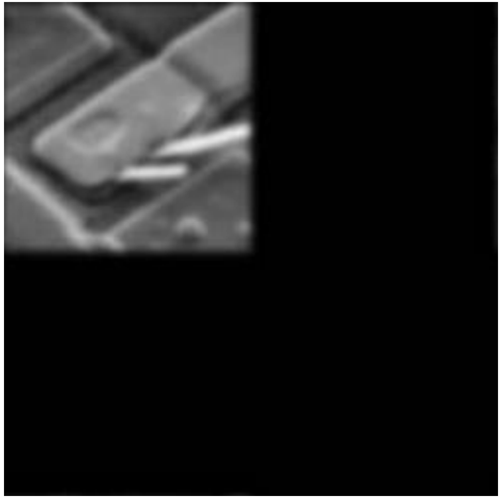


4: Centered Gaussian low pass spectral filter H

P x Q



5: Compute $G_p = \mathrm{H}F_p$



6: Compute $\mathrm{Re}[IDFT\{G_p\}](-1)^{(x+y)}$

3x3

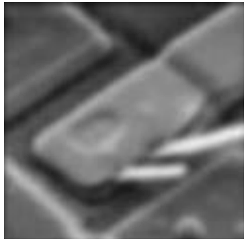
f

f → FFT → F

0 → FFT

Re(F) → IFFT → A

Im(F) → IFFT → B



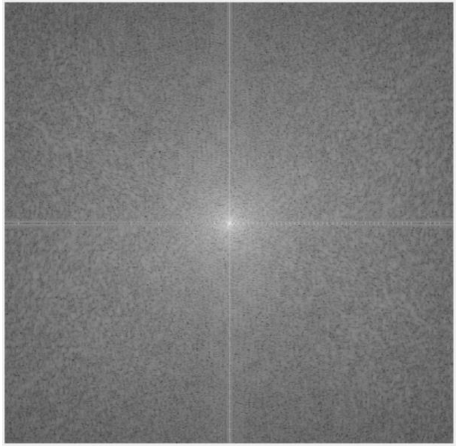
7: Filtered result

Correspondence to spatial filtering

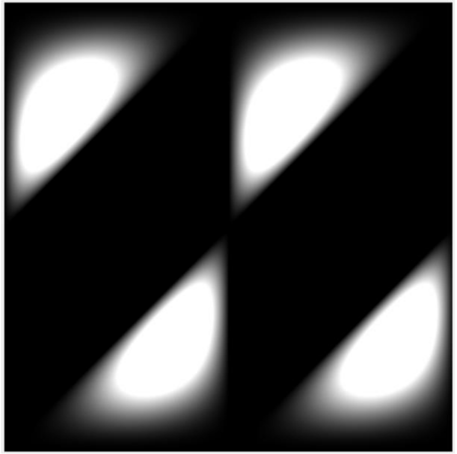
```
f = rgb2gray(imread('boy.jpg'));
```

-1	0	1
-2	0	2
-1	0	1

```
h = [-1 0 1; -2 0 2; -1 0 1];
```



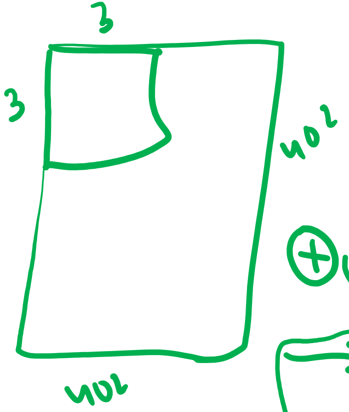
```
F = fft2(double(f), 402, 402);
```

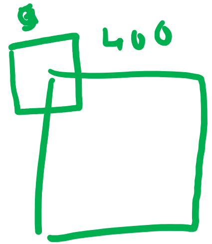


```
H = fft2(double(h), 402, 402);
```

```
F_fH = fftshift(H).*fftshift(F);
ffi = ifft2(ifftshift(F_fH));
```

Correspondence to spatial filtering





400 + 2

```
%Sobel filter in frequency domain
f = rgb2gray(imread('boy.jpg'));
h = [-1 0 1; -2 0 2; -1 0 1];
F = fft2(double(f), 402, 402);
H = fft2(double(h), 402, 402);
F_fH = fftshift(H).*fftshift(F);
ffi = ifft2(ifftshift(F_fH));
```

400x400

exact padding

$F \cdot H$

$(-1)^{x+y}$

imshow(ffi)

$(-1)^{x+y}$

J = imfilter(f,h)

f

fftshift(f)

$\forall u,v$

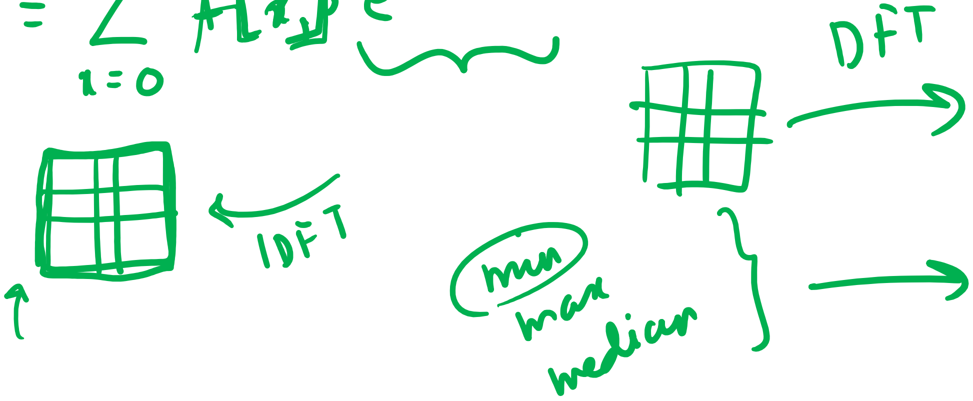
$F(u,v) \times H(u,v)$

$(a_1 + i b_1) \cdot (a_2 + i b_2)$

Frequency Domain vs Spatial Domain Filtering

- Any linear spatial filter
- Guide the process of spatial filter design

$$F[u] = \sum_{x=0}^{M-1} f[x] \underbrace{e^{-j2\pi ux}}$$



Related Topics

- Gabor filters
- Wavelets
- Shape descriptors

References

- G & W (4.5.1, 4.5.2, 4.5.5, 4.6 – 4.11)
- http://mstrzel.eletel.p.lodz.pl/mstrzel/pattern_rec/fft_ang.pdf

Digital Image Processing (CSE/ECE 478)

Morphological Processing

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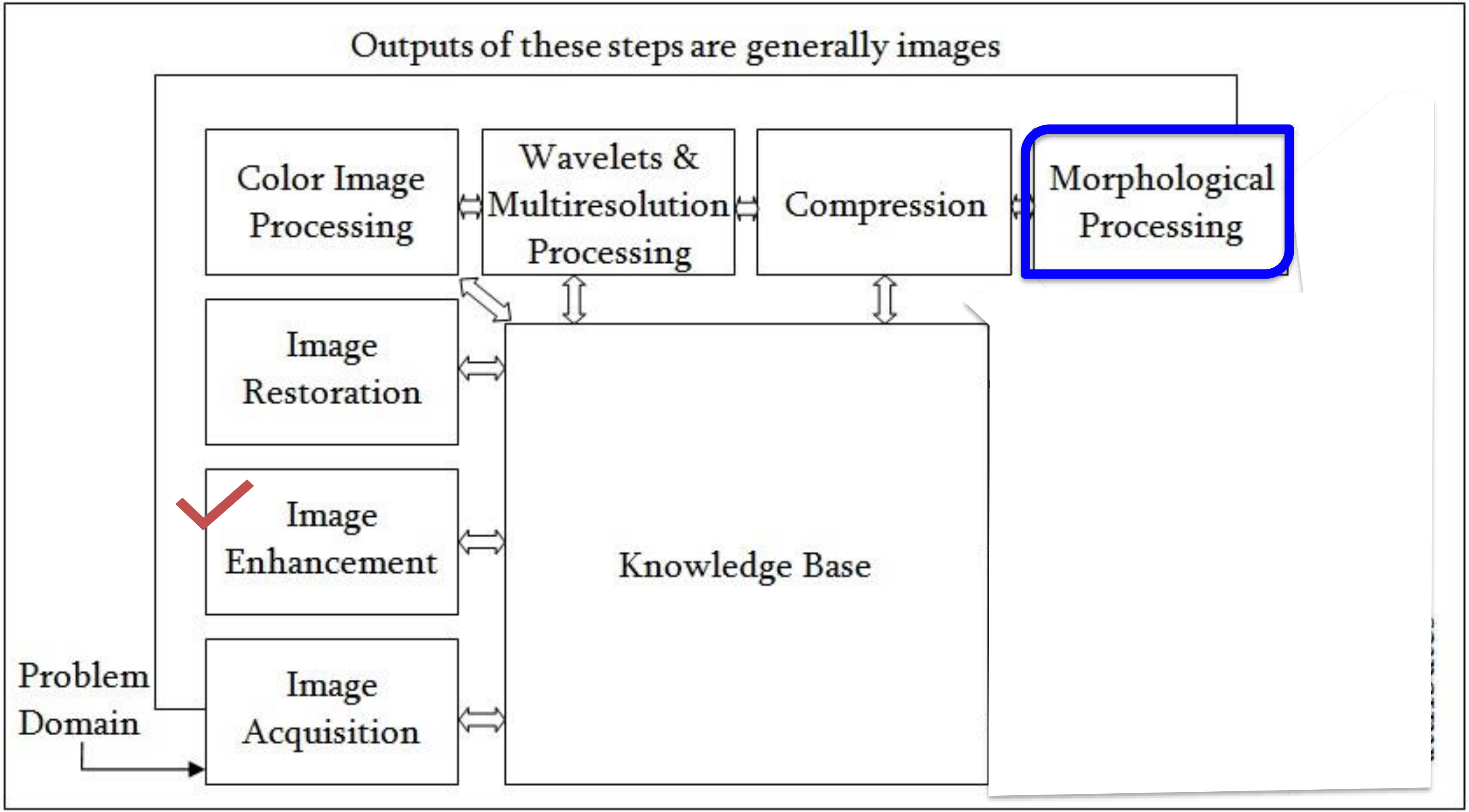
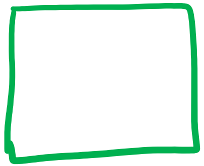


Outputs of these steps are generally images
Color Image Processing
Wavelets & Multiresolution Processing
Compression
Morphological Processing
Image Restoration
Image Enhancement
Image Acquisition
Knowledge Base
Problem Domain

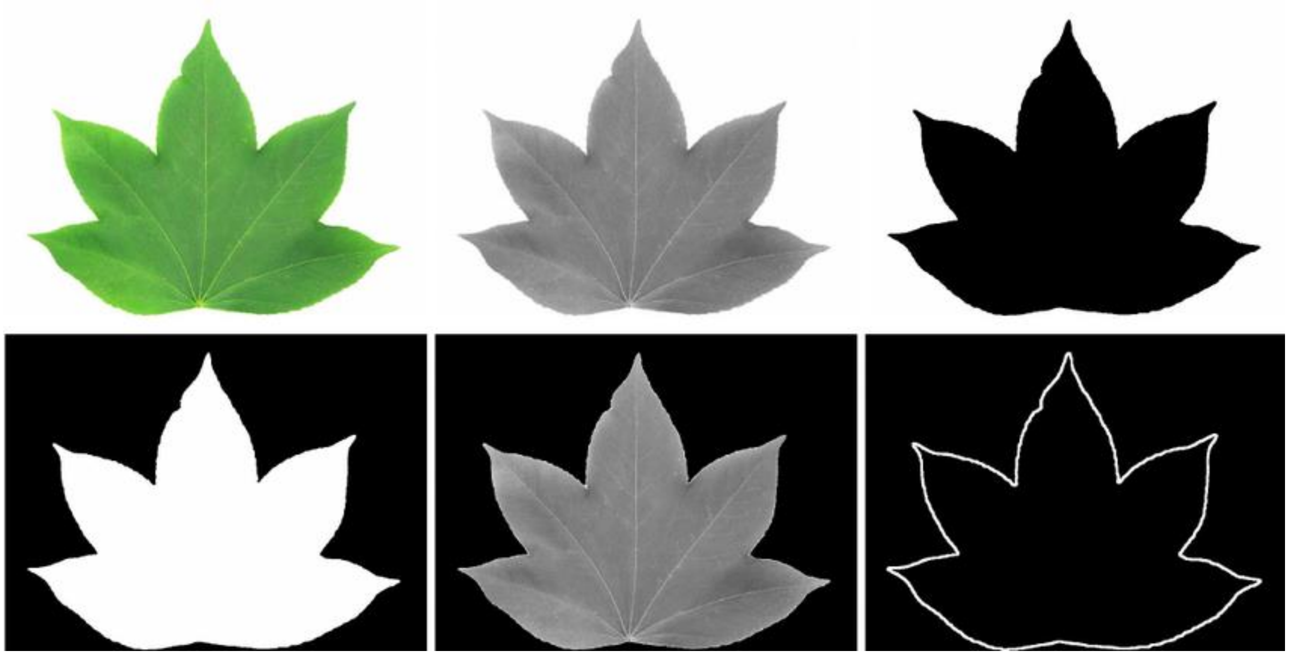
Binary Images



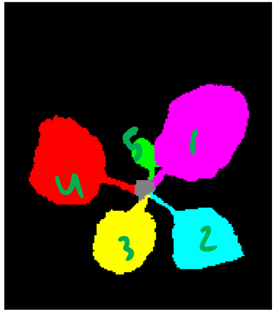
$I(x,y) < \underline{\theta_1} \rightarrow 0$

else $\rightarrow 255$

Plant Phenotyping



Plant Phenotyping



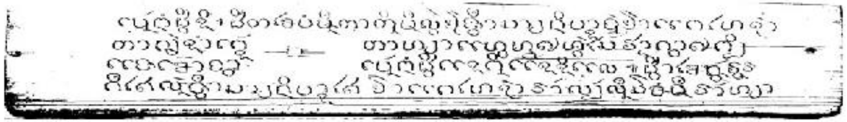
Recognizing Scene Text

https://cvit.iiit.ac.in/research/projects/cvit-projects/scene-text-understanding

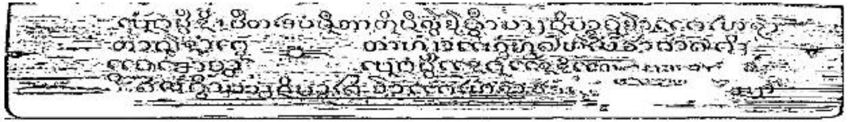
Document Image Analysis

a) RGB image

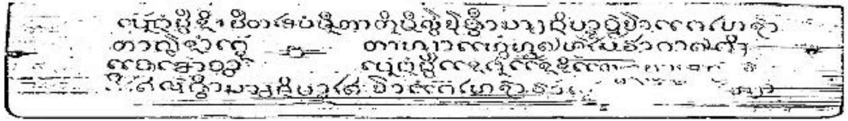
b) Noise reduction image



c) Binary image by Otsu's algorithm



d) Binary image by Niblack's algorithm



e) Binary image by Sauvola's algorithm

Figure 2. Samples of palm leaf images

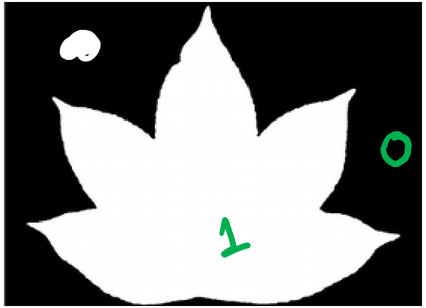
Background Subtraction

$- B$

Introduction to Morphological Operators

Image – Set of Pixels

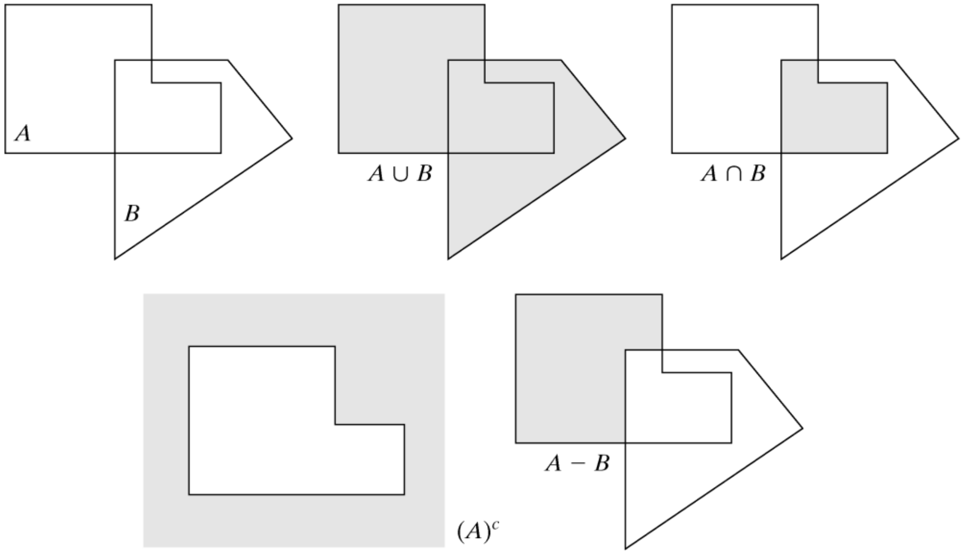
- Basic idea:
  - Object/Region = <u>set of pixels</u> (or coordinates of pixels)



$\{ (x,y) \ldots \}$

- 0 = background
- 1 = foreground

Object = set of pixels (or coordinates of pixels)



a b c
d e

FIGURE 9.1
(a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

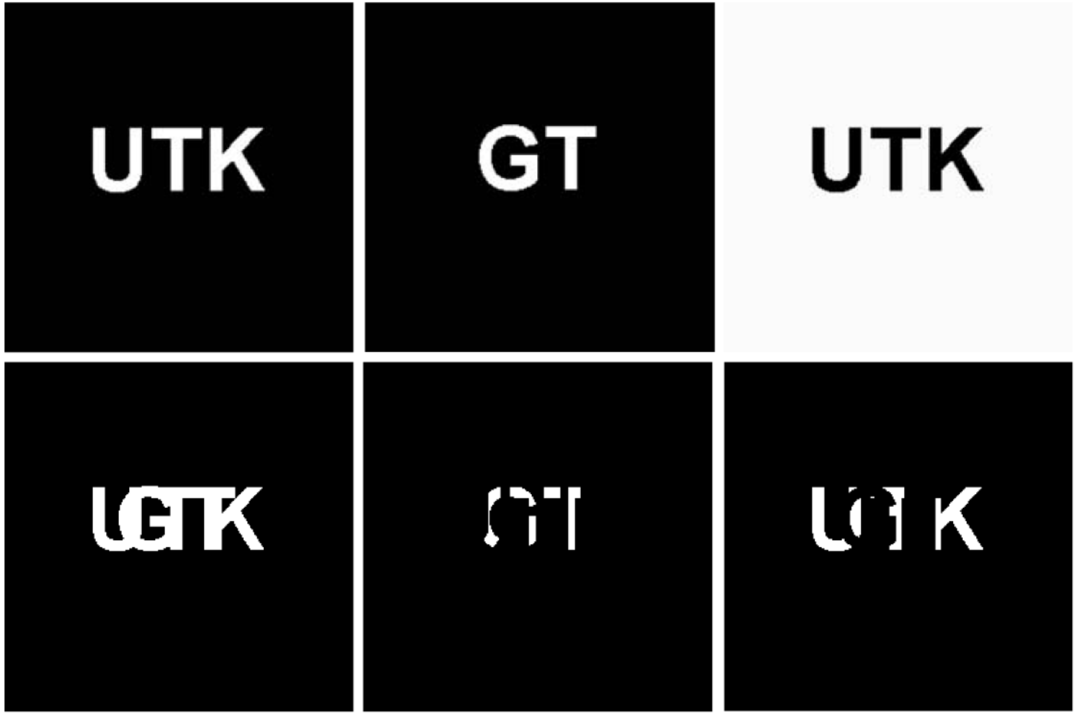
Basic operations on shapes

From: Digital Image Processing, Gonzalez, Woods And Eddins

Set Operations on Binary Images

255/0 → 1/0

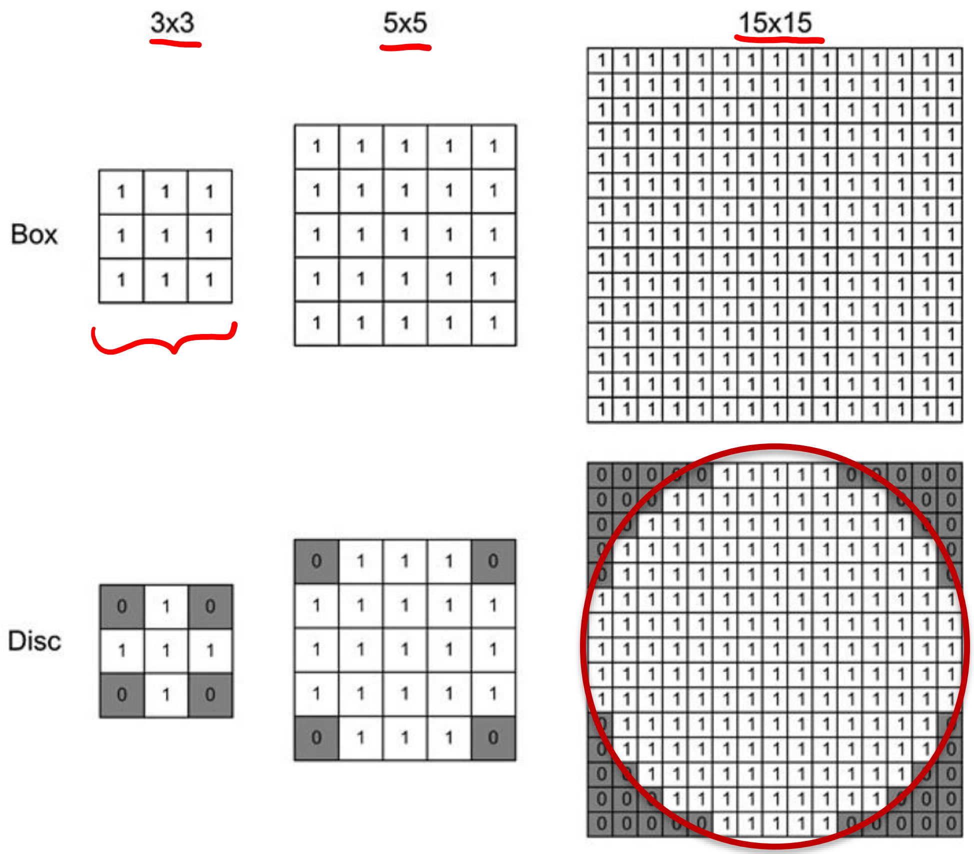
A B ~A



$A \cup B$ $A \cap B$ $A - B$

From: Digital Image Processing, Gonzalez,Woods And Eddins

Structuring Element



size shape

3x3 box

5 x5 disc

se = strel(3 3,
'disc');

Structuring Element (Kernel)

- Can have varying sizes
- Have an origin
- Usually, element values are 0,1 and none(!)
  - For thinning, other values are possible
- Empty spots in the Structuring Elements are *don't care*'s!

Box

1	1	1
1	1	1
1	1	1

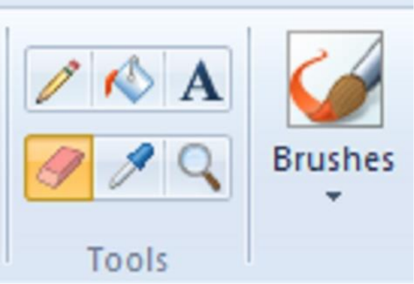
Disc

	1	
1	1	1
	1	

		1	1	1		
	1	1	1	1	1	
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
	1	1	1	1	1	
		1	1	1		

1	1	
1	0	
1		0

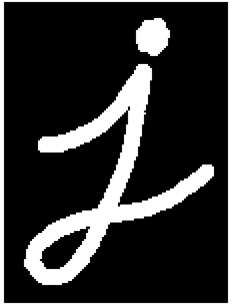
1	1	1
1	10	1
1	1	1



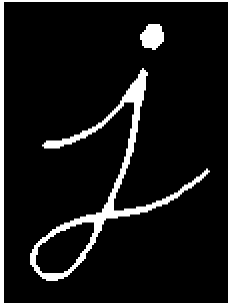
SE ← size shape origin

Erosion

Erosion



Thinning



Scribe List

2018102006
2018102007
2018102008
2018102009
2018102016
2018102017