ECE339 ECE Lab **Spring 2018**

Experiment 1: Random Variables

Lecturer: Prof. Ramamurthy, Dr. Lalitha Vadlamani *TA: Navyasri Reddy*

1.1 Objectives

1. Plot the probability density function of a random variable.
2. Compute the mean and variance of uniformly and normally distributed random variables.
3. Verify the Central Limit Theorem.
4. Generate an exponentially distributed random variable from uniformly distributed random variable.
5. Generate a Rayleigh distributed random variable from a Gaussian distributed random variable.

1.2 MATLAB Commands

rand, randn, hist

1.3 Steps to be followed

- **For Part 1:**
  (a) Generate random numbers that are (i) uniformly distributed (ii) normally distributed
  (b) Plot the probability density function of the above two random variables using the histogram command in MATLAB.
- **For Part 2:**
  (a) Compute the mean and variance of the above generated (two) random variables. Do not use inbuilt MATLAB commands of mean and variance).
- **For Part 3:**
  (a) Generate 3 independent and identically distributed normal random variables.
  (b) Verify that the sum of these random variables is also normally distributed.
  (c) Compute the mean and the variance of the random variable generated in Step b.
  (d) Generate 12 independent and identically distributed uniform random variables.
  (e) Verify that the sum of these random variables in normally distributed.
  (f) Compute the mean and the variance of the random variable generated in Step e.
- **For Parts 4 and 5:**
  Apply an appropriate transformation to generate random variable with a desired distribution starting from random variable with the given distribution.

1.4 Theory for Part 4

Exponential cumulative distribution function (cdf) with parameter λ is given by

$$F(x) = P(X \leq x) = 1 - e^{-\lambda x}, x \geq 0.$$

Consider a uniform random variable $U \sim unif(0,1)$. Consider the following function of the random variable U,

$$X = F^{(-1)}(U).$$

Using the following arguments, we can show that X has exponential cdf

$$\begin{aligned} F_X(x) &= P(X \leq x) \\ &= P(F^{(-1)}(U) \leq x) \\ &= P(U \leq F(x)) \quad \text{(Since } F \text{ is monotone increasing function)} \\ &= F(x). \end{aligned}$$

Also, note that $F^{(-1)}(U) = -\frac{1}{\lambda}\log_e(1-U)$. So, the procedure to generate an exponential r.v. with parameter λ from a uniform random variable is as follows:

(i) Generate $U \sim unif(0,1)$.

(ii) Set $X = -\frac{1}{\lambda}\log_e(1-U)$.

(iii) Verify that X has exponential cdf and pdf.

1.5 Theory for Part 5

Consider two independent random variables U and V, which are both normally distributed, mean zero, variance σ^2. Then, the joint pdf of U and V is given by

$$\begin{aligned} f_{U,V}(u,v) &= f_U(u)f_V(v) \\ &= \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{u^2}{2\sigma^2}}\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{v^2}{2\sigma^2}} = \frac{1}{2\pi\sigma^2}e^{-\frac{u^2+v^2}{2\sigma^2}} \end{aligned}$$

Consider a random variable X, given by $X = \sqrt{U^2+V^2}$. Then, cdf of X is given by

$$\begin{aligned} F_X(x) = P(X \leq x) &= P(\sqrt{U^2+V^2} \leq x) \\ &= \iint_{D_x} f_U(u)f_V(v)dudv \qquad \text{where } \; D_x = \{(u,v) : \sqrt{u^2+v^2} \leq x\} \\ &= \iint_{D_x} \frac{1}{2\pi\sigma^2}e^{-\frac{u^2+v^2}{2\sigma^2}}dudv. \end{aligned}$$

Expressing the above double integral in polar coordinates, we have

$$F_X(x) = \frac{1}{2\pi\sigma^2}\int_0^{2\pi}\int_0^r re^{-\frac{r^2}{2\sigma^2}}drd\theta = \frac{1}{\sigma^2}\int_0^r re^{-\frac{r^2}{2\sigma^2}}dr.$$

Pdf of X is the derivative of cdf and is given by

$$f_X(x) = \frac{dF_X(x)}{dx} = \frac{x}{\sigma^2}e^{-\frac{x^2}{2\sigma^2}},$$

which is the Rayleigh distribution.
Based on this theory, write a procedure equivalent to that in part 4 for generating a Rayleigh random variable.