The Application of Carson's Equation to the Steady-State Analysis of Distribution Feeders

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***Abstract*—One of the primary purposes of performing the steady-state analysis of a distribution feeder is to determine the voltages at every node. Because these voltages are a function of the line voltage drops it is critical that the line impedances used are as exact as possible. In 1926 John Carson developed equations that would determine the self and mutual impedances of any number of overhead or underground conductors taking into account the effect of ground [1]. In recent years the application of Carson's equation has become the standard for the computation of line impedances. Because Carson's equation results in an infinite series, approximations have been made to ease in the computation of the impedances. The purpose of this paper is to investigate some of the more common approximations and determine what, if any, errors are made.**

***Index Terms*—line impedance calculations, distribution system analysis, overhead line impedances, underground impedances, Kron reduction, resistivity**

I. Introduction

When Carson's paper first appeared in 1926 it was not met with a lot of enthusiasm because of the complexity of the equations and the only tool for computations was the slide rule. With the advent of the digital computer the equations could now be incorporated into the computation of the impedances of overhead lines [2]. The original equations resulted in an infinite integral [3]. Because of this various approximations have been made in order to make the equations more useful.

In 1991 a paper giving the data for four distribution system test feeders was published [4]. The purpose of the test feeders was to give software developers a common set of data that could be used to verify the correctness of their programs. Since then the original four test feeders along with additional special purpose test feeders have been made available on the IEEE website [5].

As a result of developers and students using the test feeders to develop programs the question has come up as to what method should be used to compute the overhead and underground line impedances. Cason's equations are used by many but there have been several approximations used to simplify the calculations. For example, in Reference [7], the "modified Carson's equations" are developed. While these equations are simple and straight forward to apply there is some concern among developers if the full version of Carson's equations should be used. What are the errors introduced using the modified equations? This paper will address that question by applying the full and modified equations to a three-phase overhead line, a three-phase concentric neutral underground line and a four wire quadraplex cable. Since the concern is for the computation of line impedances for the steady-state analysis a frequency of 60 Hz will be used. The value of the earth resistivity will also be addressed.

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II. Carson's Equations

Carson's equations for the self-impedance with earth return and mutual impedances with common earth return are [1]:

$$z_{ii} = \left(r_i + 4\cdot\omega\cdot P\cdot G\right) + j2\cdot\omega\cdot G\left(x_i + \ln\frac{S_{ii}}{Radius_i} + 2\cdot Q\right)$$

$$z_{ij} = 4\cdot\omega\cdot P\cdot G + j2\cdot\omega\cdot G\left(\ln\frac{S_{ij}}{D_{ij}} + 2\cdot\omega\cdot Q\right)$$

where: $i = j = 1, 2 \cdots ncond$

$$ncond = \text{number of conductors} \tag{1}$$

$$G = 0.1609347\cdot 10^{-3}$$

$$\omega = 2\cdot\pi\cdot f$$

Equations 1 give the impedances in Ω/mile.

The spacing between conductors and their images used in Equations 1 are shown in Figure 1.

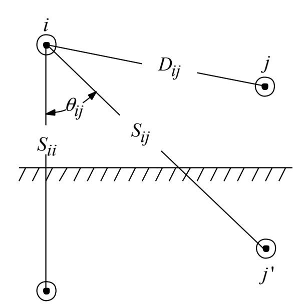

Figure 1 - Spacings

In Equations 1:

$$x_i = \ln\left[\frac{Radius_i}{GMR_i}\right]$$

where: $Radius_i$ = conductor radius in ft. (2)

GMR_i = conductor geometric mean radiusl in ft.

Substitute Equation 2 into Equations 1 and simplify:

$$z_{ii} = \left(r_i + 4\cdot\omega\cdot P\cdot G\right) + j2\cdot\omega\cdot G\left(\ln\frac{1}{GMR_i} + \ln\left(S_{ii}\right) + 2\cdot Q\right)$$

$$z_{ij} = 4\cdot\omega\cdot P\cdot G + j2\cdot\omega\cdot G\left(\ln\frac{1}{D_{ij}} + \ln\left(S_{ij}\right) + 2\cdot Q\right) \quad (3)$$

The difficulty in applying Equations 3 lies in the terms of P and Q. For purposes of this paper, P and Q are defined by Equations 4 and 5 [3],[6]. In the rest of this paper, these will be referred to as the "full" Carson's equations.

$$P_{ij} = \frac{\pi}{8} - \frac{1}{3\cdot\sqrt{2}}\cdot k_{ij}\cdot\cos\left(\theta_{ij}\right) + \frac{k_{ij}^{2}\cdot\cos\left(2\cdot\theta_{ij}\right)}{16}\cdot\left(0.6728 + \ln\frac{2}{k_{ij}}\right)$$
$$+\frac{k_{ij}^{2}}{16}\cdot\theta_{ij}\cdot\sin\left(\theta_{ij}\right) + \frac{k_{ij}^{3}}{\sqrt{2}\cdot 45}\cdot\cos\left(3\cdot\theta_{ij}\right) - \frac{\pi\cdot k_{ij}^{4}}{1536}\cdot\cos\left(4\cdot\theta_{ij}\right) \quad (4)$$

$$Q_{ij} = -0.0386 + \frac{1}{2}\cdot\ln\frac{2}{k_{ij}} + \frac{1}{3\cdot\sqrt{2}}\cdot k_{ij}\cdot\theta_{ij} - \frac{\pi\cdot k_{ij}^{2}}{64}\cdot\cos\left(2\cdot\theta_{ij}\right)$$
$$+\frac{k_{ij}^{3}}{45\cdot\sqrt{2}}\cdot\cos\left(3\cdot\theta_{ij}\right) - \frac{k_{ij}^{4}}{384}\cdot\theta_{ij}\cdot\sin\left(4\cdot\theta_{ij}\right) \quad (5)$$
$$-\frac{k_{ij}^{4}\cdot\cos\left(4\cdot\theta_{ij}\right)}{384}\cdot\left(\ln\frac{2}{k_{ij}} + 1.0895\right)$$

In Equations 4 and 5:

$$k_{ij} = 8.565\cdot 10^{-4}\cdot S_{ij}\cdot\sqrt{\frac{f}{\rho}}$$

Where: f = frequency = 60 Hz (6)

ρ = resistivity of earth in Ohm-meter

It is easy to see why there was reluctance in applying these equations using a slide rule in 1926.

In Reference [7], the "modified Carson's equations" are developed. This modification limits the number of terms used in P and Q in Equations 3. For this modification the terms used for P and Q are:

$$P_{ij} = \frac{\pi}{8}$$
$$Q_{ij} = -0.0386 + \frac{1}{2}\cdot\ln\left(\frac{2}{k_{ij}}\right) \quad (7)$$

The modified Carson's equations assuming a frequency of 60 Hz and a resistivity of 100 Ω-meters are:

$$z_{ii} = 0.09530 + r_i + j0.12134\cdot\left(\ln\frac{1}{GMR_i} + 7.93402\right)$$
$$z_{ij} = 0.09530 + j0.12134\cdot\left(\ln\frac{1}{D_{ij}} + 7.93402\right) \quad (8)$$

III. Kron Reduction

In applying both the "full" and the "modified" Carson's equations an *ncond x ncond* "primitive impedance matrix" is developed. The terms in this matrix include the self-impedance of each conductor and the mutual impedances between all conductors. For a four wire line the matrix will be 4 x 4. Included in each term, thanks to Carson, will be the effect of the self-impedance of ground and the mutual impedances between each conductor and ground. Since the neutral is grounded (the LG voltage is zero), the 4 x 4 primitive impedance matrix can be reduced to a 3 x 3 "phase impedance matrix". The 4 x 4 primitive impedance matrix is partitioned between the 3rd and 4th rows and columns to form:

$$\left[z_{primitive}\right] = \begin{bmatrix}\left[zij\right] & \left[zin\right]\\ \left[znj\right] & \left[znn\right]\end{bmatrix} \quad (9)$$

Applying the Kron reduction the 3 x 3 phase impedance matrix is determined by:

$$\left[z_{abc}\right] = \left[zij\right] - \left[zin\right]\cdot\left[znn\right]^{-1}\cdot\left[zjn\right] \quad (10)$$

IV. Four wire OH Line

A four-wire overhead line is shown in Figure 2. The phase impedance matrix will be developed for this line using the full and modified Carson's equations. The purpose is to determine how much of an error is made using the modified equations as opposed to the full equations.

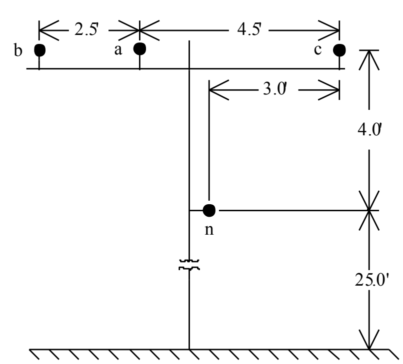

Figure 2 – Four Wire Overhead Line

Figure 2 represents configuration 400 in the IEEE Test Feeders [5]. The phase and neutral conductors are 1/0 ACSR 6/1 with the following:

Conductor resistance = 1.12 Ω/mile
GMR = 0.0045 ft.
Diameter = 0.398 inches

Note in Figure 2 that the phase sequence is b-a-c and that all impedance matrices that follow have units of Ω/mile. Applying the modified equations the primitive impedance matrix in is:

$$zp_{\text{mod}} = \begin{bmatrix} 1.2153+j1.6184 & 0.0953+j0.8515 & 0.0953+j0.7802 & 0.0953+j0.7865 \\ 0.0953+j0.8515 & 1.2153+j1.6184 & 0.0953+j0.7266 & 0.0953+j0.7524 \\ 0.0953+j0.7802 & 0.0953+j0.7266 & 1.2153+j1.6184 & 0.0953+j0.7674 \\ 0.0953+j0.7865 & 0.0953+j0.7543 & 0.0953+j0.7524 & 1.2153+j1.6184 \end{bmatrix} \tag{10}$$

Applying the full equations the primitive impedance matrix is:

$$zp_{full} = \begin{bmatrix} 1.2132+j1.6206 & 0.0932+j0.8537 & 0.0932+j0.7824 & 0.0933+j0.7886 \\ 0.0932+j0.8537 & 1.2132+j1.6206 & 0.0932+j0.7288 & 0.0933+j0.7545 \\ 0.0932+j0.7824 & 0.0932+j0.7288 & 1.2132+j1.6206 & 0.0953+j0.7674 \\ 0.0933+j0.7886 & 0.0933+j0.7545 & 0.0953+j0.7674 & 1.2132+j1.6206 \end{bmatrix} \tag{11}$$

In Equations 10 and 11 notice the small differences in the real and imaginary components of the matrix elements.

The Kron reduction is applied to the modified and full primitive impedance matrices to develop the phase impedance matrices. Those matrices are:

$$z_{\text{mod}} = \begin{bmatrix} 1.3369+j1.3331 & 0.2102+j0.5778 & 0.2132+j0.5014 \\ 0.2102+j0.5778 & 1.3239+j1.3557 & 0.2067+j0.4591 \\ 0.2132+j0.5014 & 0.2067+j0.4591 & 1.3295+j1.3459 \end{bmatrix} \tag{12}$$

$$z_{full} = \begin{bmatrix} 1.3366+j1.3346 & 0.2099+j0.5793 & 0.2128+j0.5029 \\ 0.2099+j0.5793 & 1.3235+j1.3573 & 0.2063+j0.4606 \\ 0.2128+j0.5029 & 0.2063+j0.4606 & 1.3292+j1.3474 \end{bmatrix} \tag{13}$$

In order to compare the two methods the mean error on both the real and imaginary parts are computed to be:

$$\begin{aligned} \text{Real}_{\text{error}} &= 0.1334\ \% \\ \text{Im}_{\text{error}} &= 0.2328\ \% \end{aligned} \tag{14}$$

This very small error is the result of the difference in specified resistance of "dirt" and the constant in the imaginary parts in the modified and full equations. Using the full equations a slightly different value of the dirt resistance and imaginary constant are computed for each term of the primitive impedance matrix as seen in Equations 10 and 11. With that in mind the mean value of the dirt resistance and the mean value of the constant are used in what will be referred to as the "new modified" equations. In the original modified equations the dirt resistance and constant are:

$$r_{dirt} = 0.09530 \qquad \text{constant} = 7.93402 \tag{15}$$

The mean values of the dirt resistance and constant for the full equations are:

$$r_{dirt} = 0.09327 \qquad \text{constant} = 7.95153 \tag{16}$$

Using these mean values the "new modified" equations are;

$$\begin{aligned} z_{ii} &= 0.09327 + r_i + j0.12134 \cdot \left(\ln \frac{1}{GMR_i} + 7.95153 \right) \\ z_{ij} &= 0.09327 + j0.12134 \cdot \left(\ln \frac{1}{D_{ij}} + 7.95153 \right) \end{aligned} \tag{17}$$

When these equations are used the errors compared to the full equations are:

$$\begin{aligned} \text{Real}_{\text{error}} &= 0.0342\ \% \\ \text{Im}_{\text{error}} &= 0.0183\ \% \end{aligned} \tag{18}$$

The errors have been reduced but it must be understood that the mean values of the new dirt resistance and constant are a function of the spacings between the conductors on the poles. Change the spacings and the new dirt resistance and constant will also change. Is it really worth the effort?

V. UNDERGROUND CONCENTRIC NEUTRAL CABLES

Configuration 425 in the IEEE Test Feeders is a three-phase underground concentric neutral line with an additional neutral conductor is shown in Figure 3.

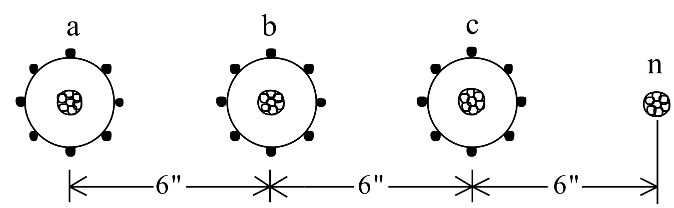

Figure 3 – UG Concentric Neutral Line

This line consists of 3 phase conductors, 3 equivalent concentric neutral conductors and the additional neutral conductor. In effect [7] there are 7 conductors which leads to a 7x7 primitive impedance matrix.

As was done with the three-phase OH line, the modified and full Carson's equations were used to compute the primitive impedance matrices. The matrices are partitioned between the 3rd and 4th rows and columns. The Kron reduction is used to develop the phase impedance matrices shown below.

$$z_{\text{mod}} = \begin{bmatrix} 1.1339+j0.6453 & 0.3189+j0.1770 & 0.2785+j0.0859 \\ 0.3189+j0.1770 & 1.1253+j0.5895 & 0.2991+j0.1327 \\ 0.2785+j0.0859 & 0.2991+j0.1327 & 1.0995+j0.5518 \end{bmatrix} \tag{19}$$

$$z_{full} = \begin{bmatrix} 1.1340 + j0.6454 & 0.3189 + j0.1770 & 0.2786 + j0.0850 \\ 0.3189 + j0.1770 & 1.1254 + j0.5895 & 0.2991 + j0.1327 \\ 0.2786 + j0.0850 & 0.2991 + j0.1327 & 1.0995 + j0.5518 \end{bmatrix} \quad (20)$$

As can be seen, there is very little difference between the two phase impedance matrices. Just to be consistent the percent errors are:

$$\begin{aligned} Real_{error} &= 0.0085\ \% \\ Im_{error} &= 0.0094\ \% \end{aligned} \quad (21)$$

This small of an error does not warrant computing new dirt resistance and constant values.

VI. QUADRAPLEX SECONDARY

Configuration 460 in the IEEE Test Feeders is a 4 wire quadraplex cable used in many three-phase secondary circuits. This cable is shown in Figure 4.

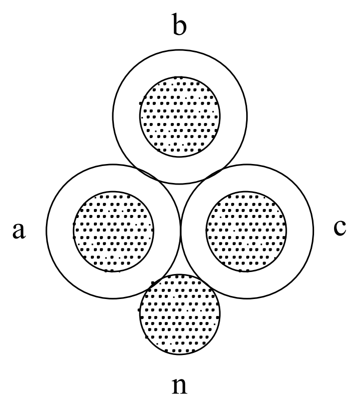

Figure 4 – Four wire quadraplex cable

The phase conductors are 2/0 AA phase and the neutral conductor is 2/0 ACSR. The thickness of the phase insulation is 60 mils.

For this case the primitive impedance matrices are 4x4 and the Kron reduction is used to compute the 3x3 phase impedance matrices for the modified and full Carson's equations.

$$z_{mod} = \begin{bmatrix} 1.2543 + j0.6031 & 0.4619 + j0.4936 & 0.4853 + j0.4490 \\ 0.4619 + j0.4936 & 1.2089 + j0.6901 & 0.4619 + j0.4936 \\ 0.4853 + j0.4490 & 0.4619 + j0.4936 & 1.2543 + j0.6031 \end{bmatrix} \quad (22)$$

$$z_{full} = \begin{bmatrix} 1.2547 + j0.6036 & 0.4623 + j.4941 & 0.4857 + j0.4495 \\ 0.4623 + j.4941 & 1.2093 + j0.6906 & 0.4623 + j0.4941 \\ 0.4857 + j0.4495 & 0.4623 + j0.4941 & 1.2547 + j0.6036 \end{bmatrix} \quad (23)$$

There is very little error between these two matrices. To be consistent the errors are:

$$\begin{aligned} Real_{error} &= 0.0688\ \% \\ Im_{error} &= 0.0980\ \% \end{aligned} \quad (24)$$

A correction for the dirt resistance and constant used in the modified Carson's equations could be made as was done for the OH line. However, with such a small error it doesn't seem to be necessary.

VII. VOLTAGE DROP ERRORS

It has been demonstrated that there is very little error in the phase impedance matrices for overhead, underground and secondary lines. The question at this point is what errors are made in the calculation of voltages and voltage drops using the full and modified phase impedance matrices? The overhead line will be used to demonstrate the possible errors.

The IEEE 34 Node Test Feeder nominal LL voltage is 24.9 kV. However, the source is set at 1.05 per-unit which means the source line-to-ground voltages are 15.095 kV. From Node 800 to Node 808 there is an overhead line of configuration 400. This configuration is the same as at the start of this paper. The total distance from Node 800 to Node 808 is 36,540 ft. There is a small distributed load from Node 802 to Node 806 that will be ignored. The complex power delivered to Node 808 is:

$$SL = \begin{bmatrix} 699 + j143 \\ 690 + j103 \\ 591 + j16 \end{bmatrix} \text{ kW+jkvar} \quad (25)$$

Using the modified Carson's equation phase impedance matrix, the voltages at Node 808 are computed to be:

$$\left[\text{VLN mod}_{808}\right] = \begin{bmatrix} 14606.60/\underline{-.62} \\ 14726.69/\underline{-121.0} \\ 14801.37/\underline{119.2} \end{bmatrix} \quad (26)$$

The source voltages are:

$$\left[VLN_{800}\right] = \begin{bmatrix} 15094.8/\underline{0} \\ 15094.8/\underline{-120} \\ 15094.8/\underline{120} \end{bmatrix} \quad (27)$$

For this operating condition the phase voltage drops from Node 800 to Node 808 are?

$$\left[vdrop_{mod}\right] = \frac{\left\| VLN_{800} \right| - \left| VLN_{808} \right\|}{\left| VLN_{800} \right|} \cdot 100 = \begin{bmatrix} 3.2344 \\ 2.3288 \\ 1.9441 \end{bmatrix} \ \% \quad (28)$$

For the same operating conditions the full Carson's equation phase impedance matrix is used and the voltages at Node 808 are computed to be:

$$\left[\text{VLN full}_{808}\right] = \begin{bmatrix} 14606.46/\underline{-.62} \\ 14726.78/\underline{-121.0} \\ 14801.42/\underline{119.2} \end{bmatrix} \quad (29)$$

The phase voltage drops are computed to be:

$$\left[vdrop_{full}\right] = \frac{\left||VLN_{800}| - |VLN_{808}|\right|}{|VLN_{800}|}\cdot 100 = \begin{bmatrix}3.2353\\2.3282\\1.9437\end{bmatrix}\ \% \qquad (30)$$

Computing the error is voltage drops:

$$vdrop_{error} = \frac{\left||vdrop_{full}| - |vdrop_{\text{mod}}|\right|}{|vdrop_{full}|}\cdot 100 = \begin{bmatrix}0.0278\\0.0237\\0.0166\end{bmatrix}\ \% \qquad (31)$$

Needless to say, the errors in using the modified vs. the full phase matrices are insignificant.

The errors in the computed load end voltages are:

$$V808_{error} = \frac{\left||\text{VLN full}_{808}| - |\text{VLN mod}_{808}|\right|}{|\text{VLN full}_{808}|}\cdot 100 = \begin{bmatrix}0.0009\\0.0006\\0.0002\end{bmatrix}\ \% \qquad (32)$$

Equations 31 and 32 demonstrate that there are virtually no errors between using the full or modified Carson's equations to compute the phase impedance matrix. This is important since there has been some concern between developers and students using the test feeders what phase impedance they should be using. Note that this paper has been addressing the phase impedance matrix as applied to the steady-state power flow studies.

VIII. THE EFFECT OF RESISTIVITY

For the computation of the phase impedance matrices the resistivity of "dirt" has traditionally been assumed to be 100 Ω-meter. The question is how sensitive are the impedances and node voltages changed using the assumed resistivity? Using the full Carson's equations with values of resistivity of 1000 Ω-meter and 10 Ω-meter (as compared to 100 Ω-meter), the effects on the phase impedances and load end voltages will be investigated.

For a resistivity of 1000 Ω-meter, the phase impedance matrix is:

$$z1000_{full} = \begin{bmatrix}1.3731+j1.3849 & 0.2459+j0.6310 & 0.2491+j0.5540\\0.2459+j0.6310 & 1.3591+j1.4105 & 0.2421+j0.5131\\0.2491+j0.5540 & 0.2421+j0.5131 & 1.3652+j1.3993\end{bmatrix} \qquad (33)$$

For ease of comparison, the phase impedance matrix for 100 Ω-meter is:

$$z100_{full} = \begin{bmatrix}1.3366+j1.3346 & 0.2099+j0.5793 & 0.2128+j0.5029\\0.2099+j0.5793 & 1.3235+j1.3573 & 0.2063+j0.4606\\0.2128+j0.5029 & 0.2063+j0.4606 & 1.3292+j1.3474\end{bmatrix} \qquad (34)$$

The phase impedance matrix of Equation 34 is used to compute the load end voltages for the same line and load conditions as in the previous section. The load end voltages with the new phase impedance matrix are:

$$\left[\text{VLN full}_{1000}\right] = \begin{bmatrix}14602.29\underline{/-.61}\\14725.79\underline{/-121.0}\\14806.68\underline{/119.2}\end{bmatrix} \qquad (35)$$

The load end voltages for 100 Ω-meters were found to be:

$$\left[\text{VLN full}_{100}\right] = \begin{bmatrix}14606.46\underline{/-.62}\\14726.78\underline{/-121.0}\\14801.42\underline{/119.2}\end{bmatrix} \qquad (36)$$

For this case the errors per phase between the 1000 and 100 Ω-meter lines are:

$$V_{error} = \frac{\left||\text{VLN full}_{1000}| - |\text{VLN full}_{100}|\right|}{|\text{VLN full}_{1000}|}\cdot 100 = \begin{bmatrix}0.0286\\0.0067\\0.0355\end{bmatrix}\ \% \qquad (37)$$

When the resistivity is changed to 10 Ω-meter the results are:

$$\left[\text{VLN full}_{10}\right] = \begin{bmatrix}14611.24\underline{/-.64}\\14727.50\underline{/-120.98}\\14795.80\underline{/119.22}\end{bmatrix} \qquad (38)$$

$$V_{error} = \frac{\left||\text{VLN full}_{10}| - |\text{VLN full}_{100}|\right|}{|\text{VLN full}_{10}|}\cdot 100 = \begin{bmatrix}0.0327\\0.0049\\0.0380\end{bmatrix}\ \% \qquad (39)$$

For both values of resistivity it is seen that the percentage errors are very small when compared to the 100 Ω-meter resistivity.

It is possible to change the constant in the imaginary part of the modified Carson's equations to take into account the change in resistivity. The constant term is given by [7]:

$$\begin{aligned}&\text{constant} = 7.6786 + \frac{1}{2}\cdot\ln\left(\frac{\rho}{60}\right)\\&\text{constant}_{1000} = 7.6786 + \frac{1}{2}\cdot\ln\left(\frac{1000}{60}\right) = 9.08531\\&\text{constant}_{10} = 7.6786 + \frac{1}{2}\cdot\ln\left(\frac{10}{60}\right) = 6.78272\end{aligned} \qquad (40)$$

Applying these new constants in the modified Carson's equations will yield the same very small percentage errors as with the full equations.

IX. CONCLUSIONS

The primary purposes of this paper were to investigate how the elements in the phase impedance matrix are affected by:

- The number of terms used in Carson's equations for:
    - Overhead lines

- Concentric neutral underground lines
- Secondary quadraplex cable

- The assumed value of resistivity

The phase impedance matrix was developed for the following IEEE Test Feeder configurations:

- Overhead - #400
- Concentric neutral underground - #425
- Secondary quadraplex - #460

For each configuration the phase impedance matrices were computed using the "modified" Carson's equations from reference [7] and the "full" Carson's equations from references [3] and [6] assuming a resistivity of 100 Ω-meters. A very small error (less than 0.3%) was found in comparing the individual elements in the two matrices. To test the error introduced on the node voltages using the modified and full equations a routine was developed to compute the node voltages at the end of a three-phase overhead line serving an unbalanced three-phase load. For this study it was found that the error in the node voltages was of the order 0.0005 %.

The effect of resistivity was studied for the overhead line for values of resistivity of 1000 and 10 Ω-meters. These values were used in computing the phase impedance matrices using the full equations. The new matrices were used to compute the load voltages for the unbalanced load. These voltages were then compared to the voltages computed using the modified equation that assumes 100 Ω-meters. The errors were found to be in the order of 0.03%. This very small error could be minimized by changing the constant in the imaginary term of the modified equations.

The IEEE Test Feeders are beginning to be used in many papers submitted for presentation at conferences and/or published in the transactions. There has been some concern that the values of the phase impedance matrices used by the authors might be the cause of paper results that do not match the published results. It is hoped that this paper has demonstrated that either the "modified" or "full" Carson's equations may be used in the computation of the phase impedance matrices.

It should be repeated that this paper has been limited to the application of the phase impedance matrices for the analysis of steady-state conditions at 60 Hz.

X. References

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