

School of Computing

Creative Problem Solving

Polya's Four-Step Problem Solving Process

Part 1: Problem Solving is Cool & Polya 4S PS Process

Leong Hon Wai

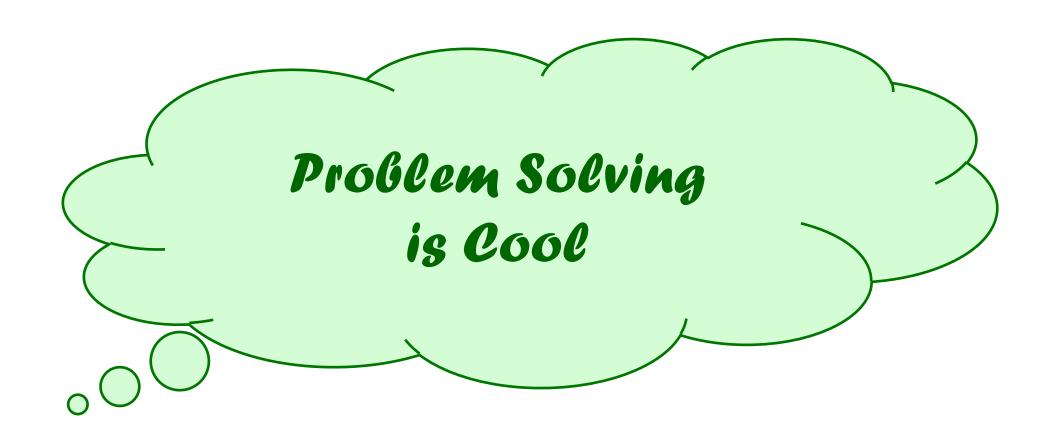
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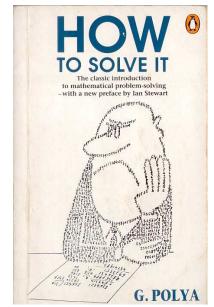


First, a Quote...

A great discovery solves a great problem, but there is a grain of discovery in the solution of any problem.

Your problem may be modest;
but if it challenges your curiosity and
bring into play your inventive faculties,
and if you solve it by your own means,
[then] you may experience the tension and
enjoy the triumph of discovery.

G. Polya, 1945



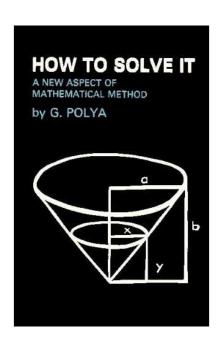


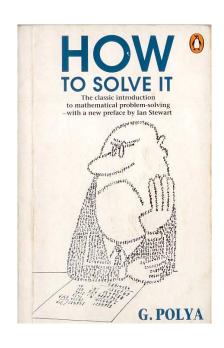
G. Polya (1887-1985)

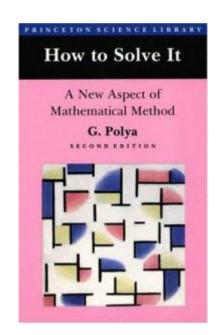
Experience the Aha! moment

and... you keep this triumph for a lifetime

"How to Solve It", by George Polya, 1945











and... it's still in demand today! > 70 years later

Still... the most widely cited reference for problem solving in all disciplines

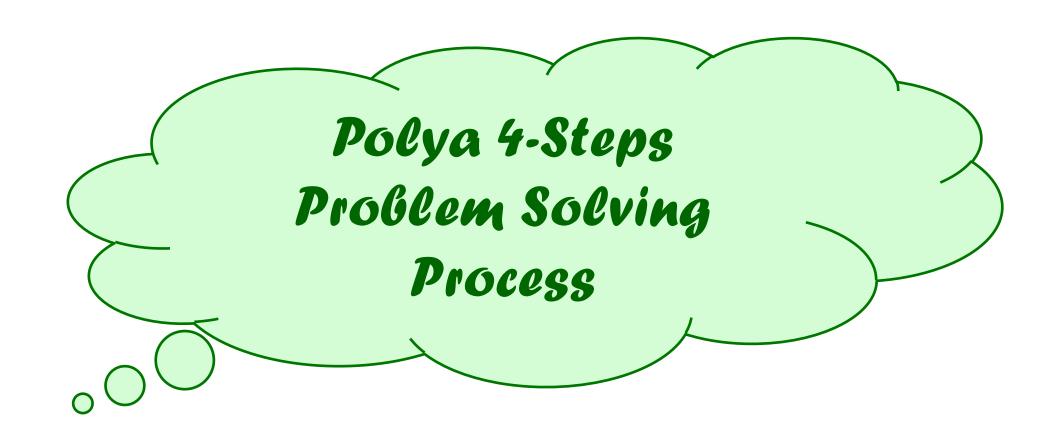
Why: Problem Solving

- There is discovery in the solution of any problem.
 - Problem challenges your curiosity, inventiveness, and creativity
 - Solving them by one's own efforts brings triumph of discovery
- Students in school/university have the opportunity to learn
 - Discover your talent for problem solving, ...Develop it further
 - Enjoy the problem solving process
- This opportunity is easily lost if student
 - is only after "the grade" or "the piece of paper"
 - remembers only before/during the exam, forgets after exams ASAP.

Develop a habit of ASKING...

- Do I understand the problem?
- Do I understand the solution?
- Why does the solution work?
- Why did he choose to do this or that?
- How can I invent such a solution? ...by myself?
- Can I do it differently?
- Can I do it more elegantly?

Don't just understand the solution, but also try to understand the ideas and the procedures of arriving at the solution.



Polya's Four-Step Problem Solving Process

I. UNDERSTANDING THE PROBLEM

You have to understand the problem

II. DEVISING A PLAN

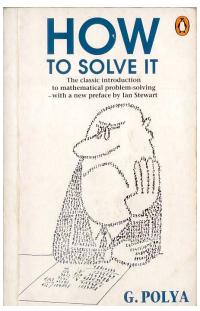
Find the connection between data and the unknown. You should obtain eventually a *plan* for the solution

III. CARRYING OUT THE PLAN

Carry out your plan

IV. LOOKING BACK

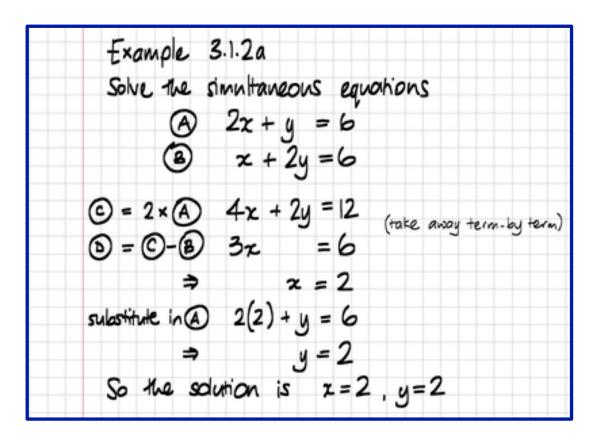
Re-examine the solution you obtained





"A Plan" vs "A Solution"

 Solution: everything is worked out.



 Plan: rough idea of solution. only main big steps.

Plan for Solving Simultaneous Equations: Given 2 equations, 2 variables

Step 1. First eliminate 1 variable (get 1 eqn and 1 var)

Step 2. Solve for this variable

Step 3: Solve for other variable

"A Plan" vs "A Solution"

 Plan: rough idea of solution. only main big steps.



Solution: everything is all worked out.



Why "Develop a Plan"...

- Plans are flexible (easy to change)
- Can try out many different plans in a short time
- Allows us to learn from mistakes quickly (fail early)
- Allows us to iterate to good plans QUICKLY

Don't waste lots of time with bad plans

I. UNDERSTANDING THE PROBLEM

What is the unknown?

...What are the data?

...What is the condition?

Polya suggests asking these questions to help you understand the problem

Create your own instances of the problem

Is it possible to satisfy the condition?

Is it sufficient? or redundant? or contradictory?

Draw a figure? Introduce suitable notations?

Separate the various parts of the condition?

Can you write them down?

II. DEVISING A PLAN (1 of 3)

Have you seen it before?

...or seen it in a slightly different form?

Have you solved a related problem?

Can you re-use it?

Re-use the results? Re-use the method?

Or add auxiliary element to make it possible?

Look at the unknown!

...problem with a similar unknown?

Step II is hard.
Needs more
questions to help...

II. DEVISING A PLAN (2 of 3)

More Questions

Can we solve some small instances? and look for a pattern

Can you re-state the problem?

Re-state it still differently? (view from different perspective) Go back to definitions

Try to solve a related problem first?

A more accessible related problem?

A more general problem? A special case?

An analogous problem?

II. DEVISING A PLAN (3 of 3)

Step II is the "hardest". So it has the most questions to help you get unstuck.

Can you Solve part of the problem first?

Keep only a part of the condition, drop the other parts; How far is the unknown then determined, how can it vary?

Can you derive something useful from the data? what other data appropriate to determine the unknown

Can you change the unknown or the data or both to get the new unknown & new data nearer to each other

Have you taken into account all essential notions involved? Did you use all the data? the whole condition?

III. CARRYING OUT THE PLAN

Carry out your plan of solution.

Check every step.

Can you see clearly that each step is correct?

Can you prove that it is correct?

The plan is rough, so here you carry out the plan carefully, step-by-step

IV. LOOKING BACK

Can you check the result?

Can you check the argument?

Can you derive the result differently?

Can you see it in a glance?

Can you use the result, or the method, for other problem?

Looking back helps you to gain insights into your solution/approach

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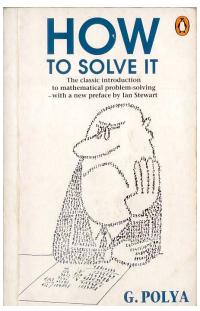
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Strategies for Step II: Devising a Plan

- Write down some instances
- Re-use a previous solution
- Guess and Check (trial and error)
- Solve some special cases first
- Look for a Pattern
- Solve a simpler problem first
- Enumerate/check all cases
- Use algebra!
- Look from different perspective

- Draw a picture
- Use a model
- Work backwards
- Use or reuse a formula
- Use symmetry, parity
- Direct reasoning
- Do a simulation
- MAGIC?
- Be ingenious!

Quick Summary:

- Learned the Polya's 4-Step PS Process
- The Key is "Asking Simpler Questions"
 - They point you in different directions
- Many people miss Step I. (...including me and you!)
- Step II is akin to "Divergent Thinking", then "Convergent Thinking"
- Step IV is important and most beneficial
 - Encourages self reflection
 - Enhances future problem solving...

End of

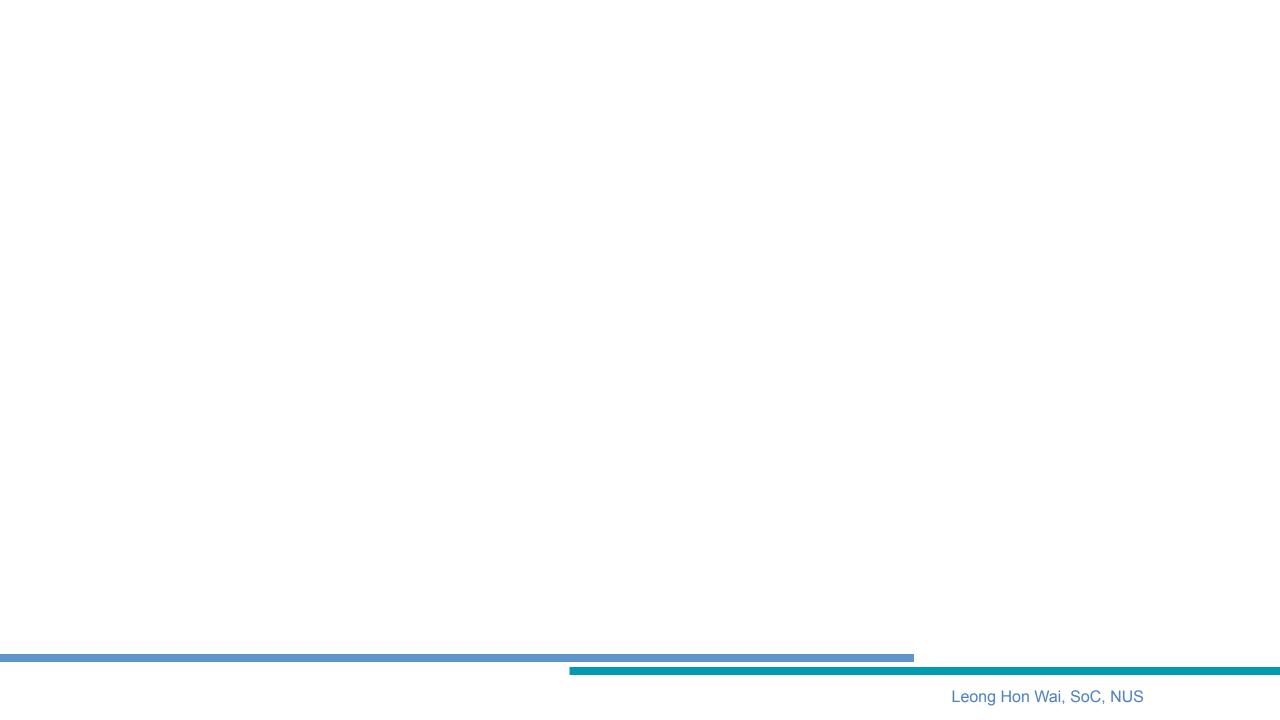
Polya 4-Steps Problem Solving Process

Contact me at





School of Computing



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Examples of Cool Solutions

- Can you think of Cool Solutions
 - Round Wheels
 - Cars, and soon to come (Self Driving Cars)
 - Currency
 - Airplanes, Air Travel
 - The Internet, WWW, Email
 - Phones, Mobile Phone,
 - Apps: MS Office, Games, Online-Games, Facebook, WhatsApp,