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School of Computing

Creative Problem Solving: Inverting a Triangle of Coins

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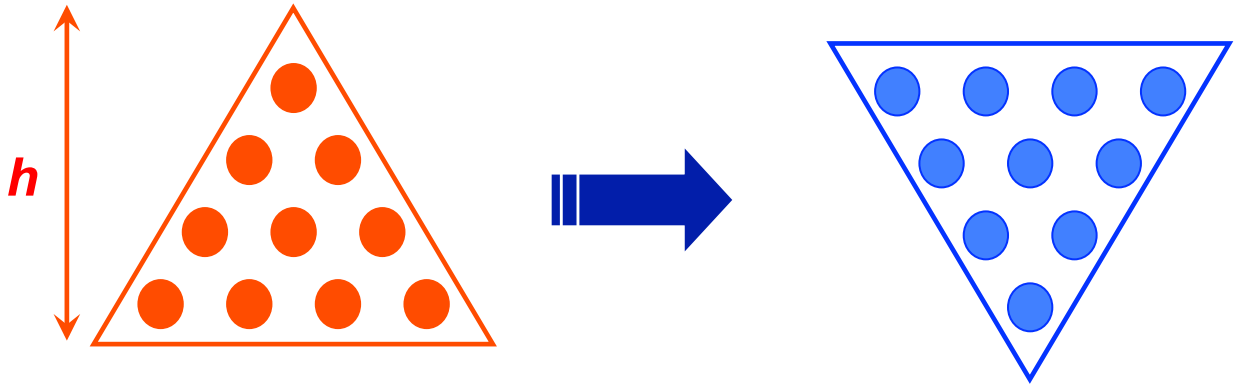
Experience the joy of problem solving

Inverting a Triangle-of-Coins

Problem: Inverting a Triangle of Coins

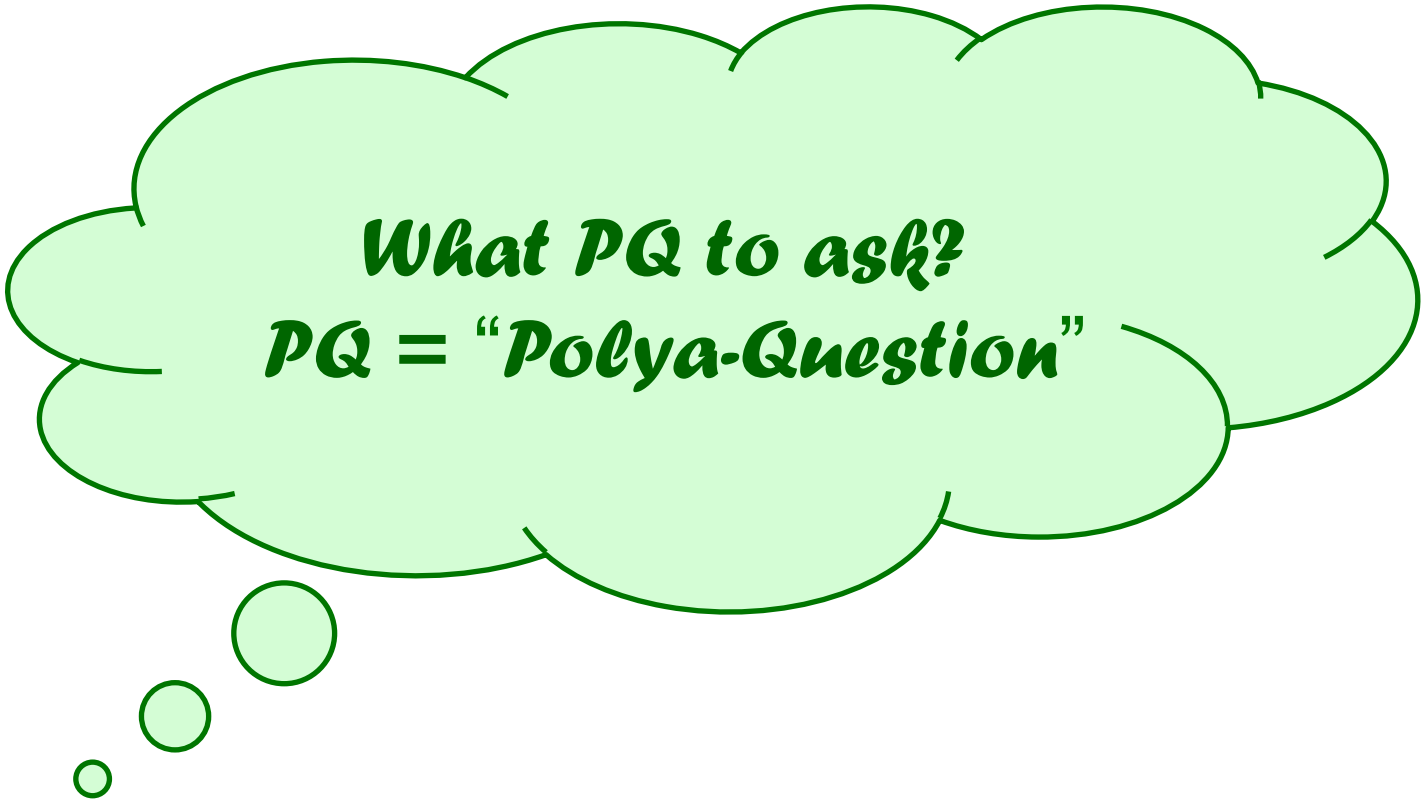


- "Invert" a triangle of coins by moving as few coins as possible
- Let *h* := "height of triangle"
  m := "# of coins to move"

Question: What is the value of *m*, when *h* = 191 ?

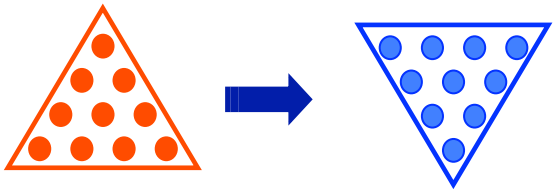
Today is
19 Jan

Ask PQ…



Inverting a Triangle of Coins (1)

- **PQ:** Try some small instances…

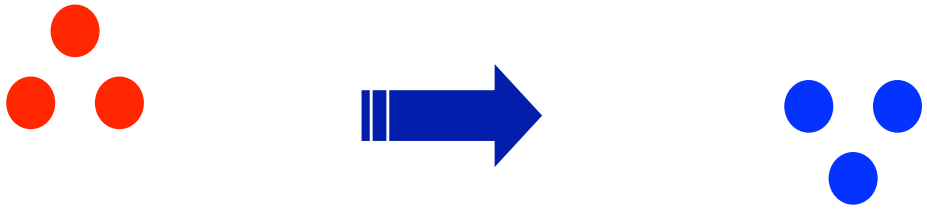


  - $h = 1$

- **TRIVIAL:**
  - $h = 1$ $m = 0$ (Trivial, but still important.)

Inverting a Triangle of Coins (2)

- **Now,** try something bigger…
  - *h* = 2



- **TRIVIAL:**
  - *h* = 1 *m* = 0 (Trivial, but still important.)
  - *h* = 2 *m* = 1 (Simple, isn't it?)

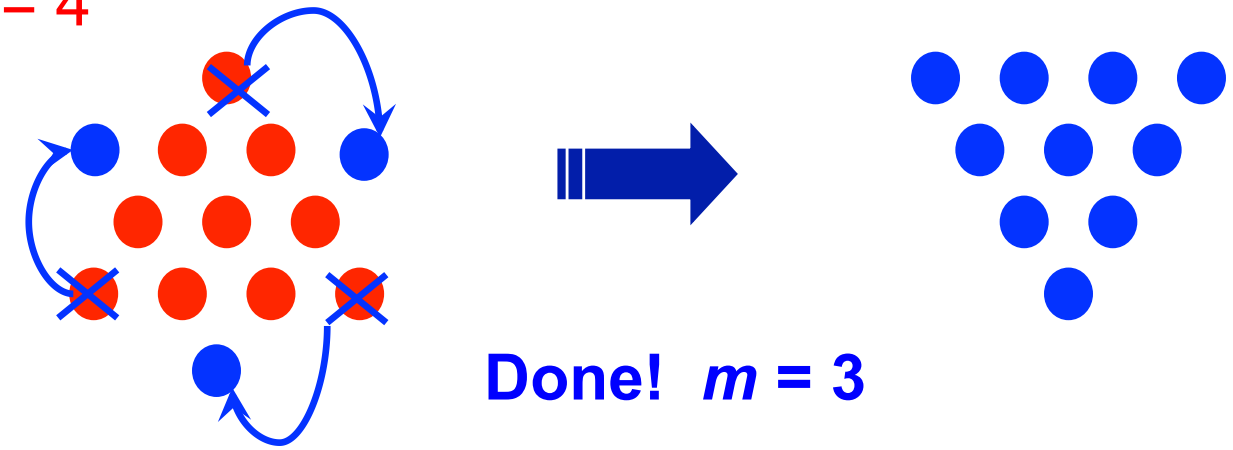
Inverting a Triangle of Coins (3)

- **Now,** try something bigger…
  - $h = 3$

- **RESULTS:**
  - $h = 1$ $m = 0$ (Trivial, but still important.)
  - $h = 2$ $m = 1$ (Simple, isn't it?)
  - $h = 3$ $m = 2$ (Getting interesting…)

Inverting a Triangle of Coins (4)

- Now, try the original problem…
  - $h = 4$

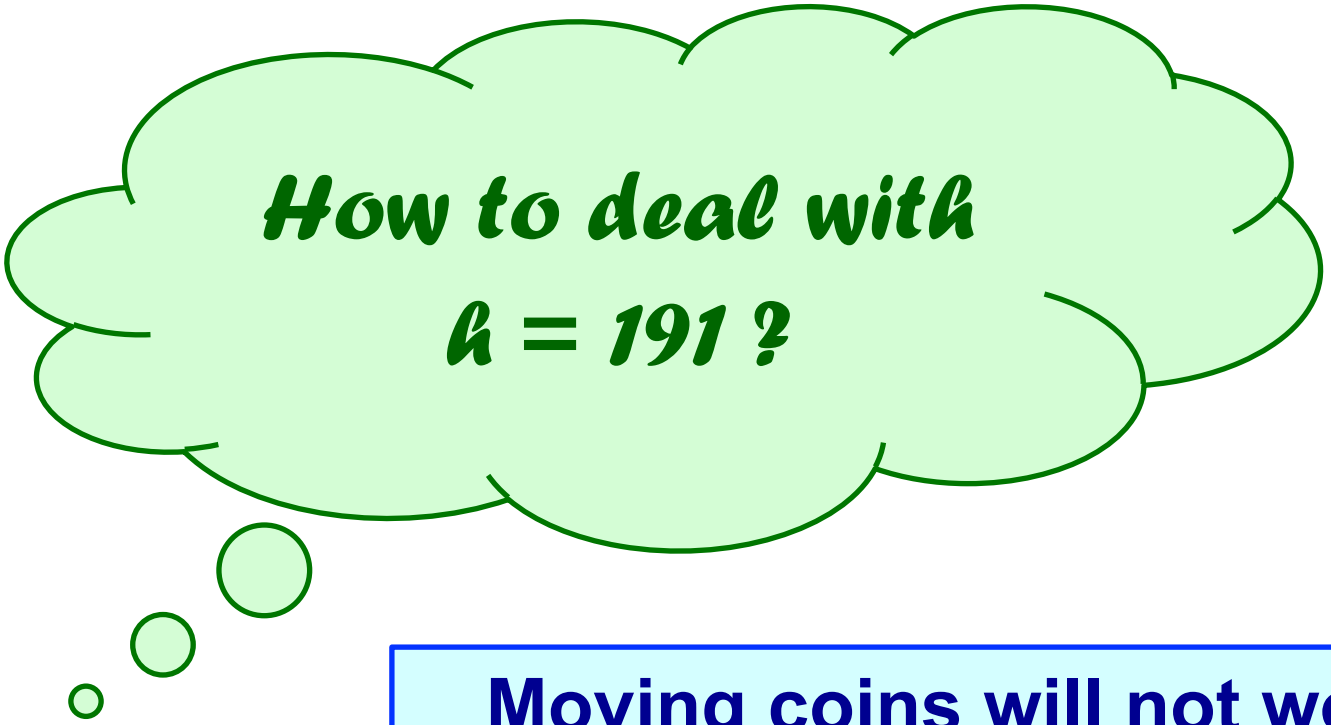


Done! $m = 3$

- **RESULTS:**
  - $h = 1$ $m = 0$
  - $h = 2$ $m = 1$
  - $h = 3$ $m = 2$
  - $h = 4$ $m = 3$

Is there a pattern?

What about $h = 5$? $h = 191$

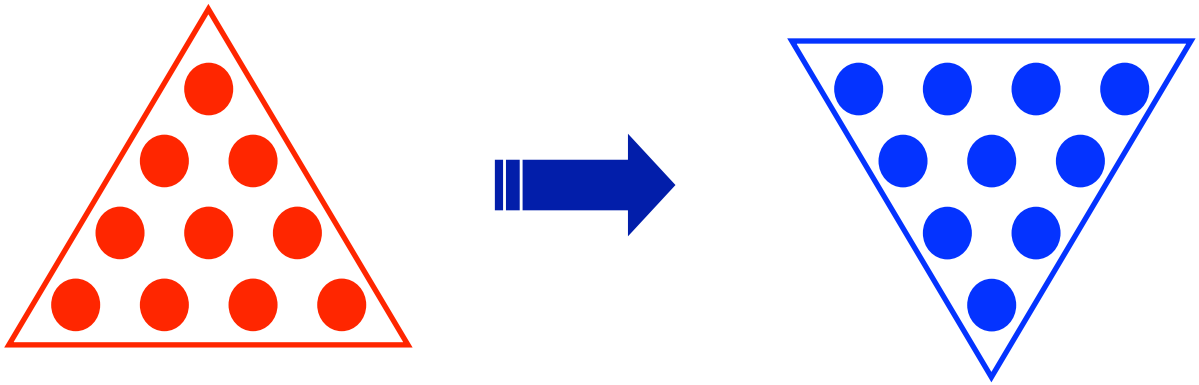


Moving coins will not work

PQ: Can we view problem from a different perspective?

We Need a Different Perspective!!

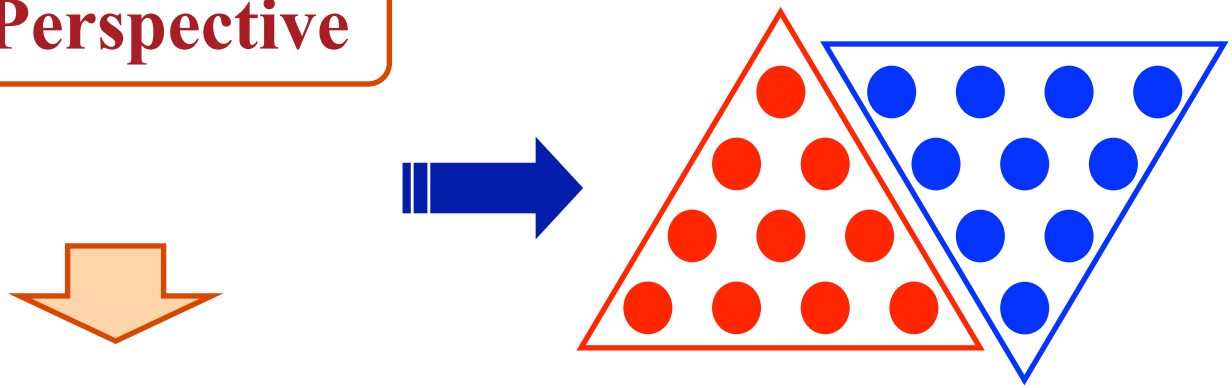
- Let's see….



- How about… seeing them *together?*
- Let's move *THEM*…

We Need a Different Perspective (2)

New Perspective

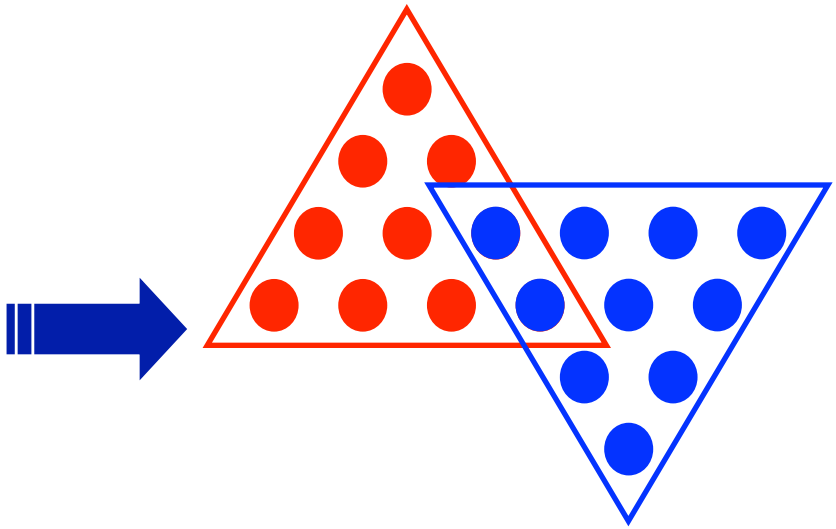


New Question:

How to turn the ”orange coins” into the “blue coins”?

Need to move 10 coins!

We Need a Different Perspective (3)

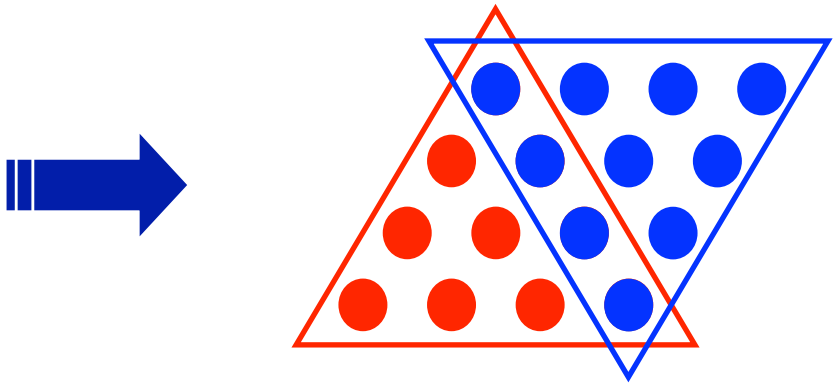


New Operation:

Move the orange triangle around the blue triangle.

Need to move 8 coins!

We Need a Different Perspective (4)

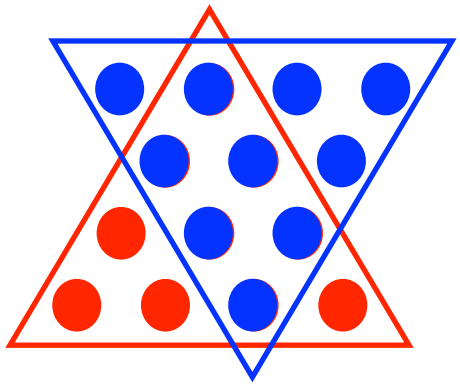


New Operation:

Move the orange triangle around the blue triangle.

Need to move 6 coins!

We Need a Different Perspective (5)

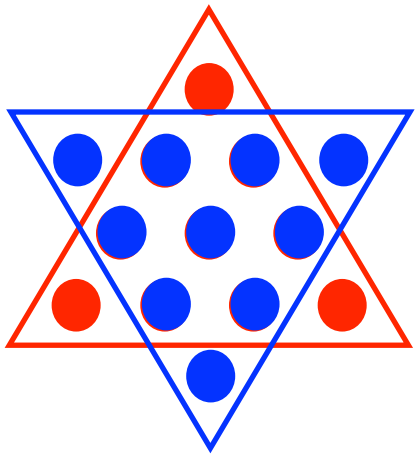


New Insight

Minimize "moves" = Maximize overlap

Need to move 4 coins!

We Need a Different Perspective (6)

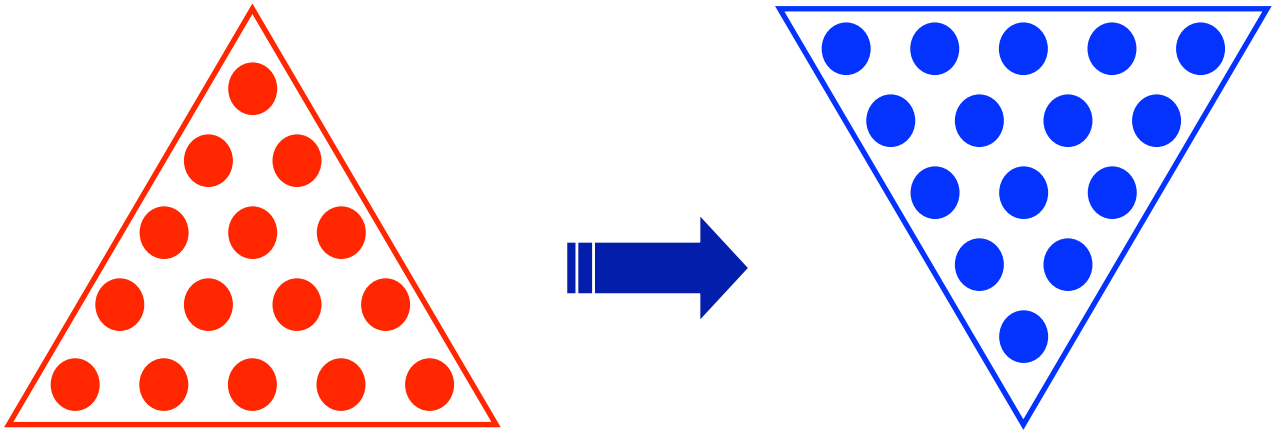


Is this the maximum overlap?

Need to move 3 coins!

Try new insight on larger problem

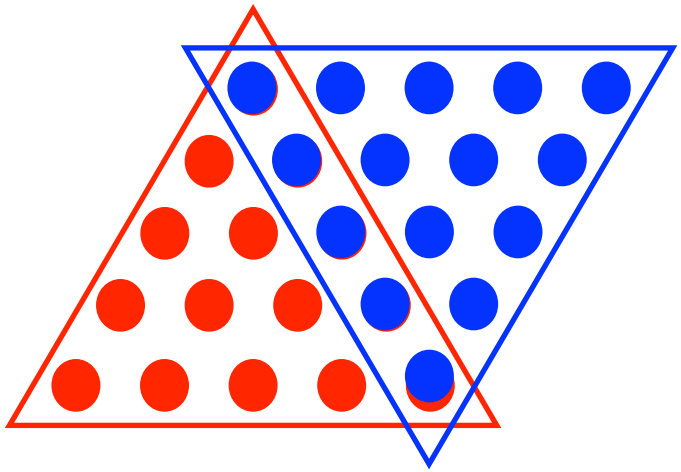
Now try $h = 5$



Move triangle & maximum overlap

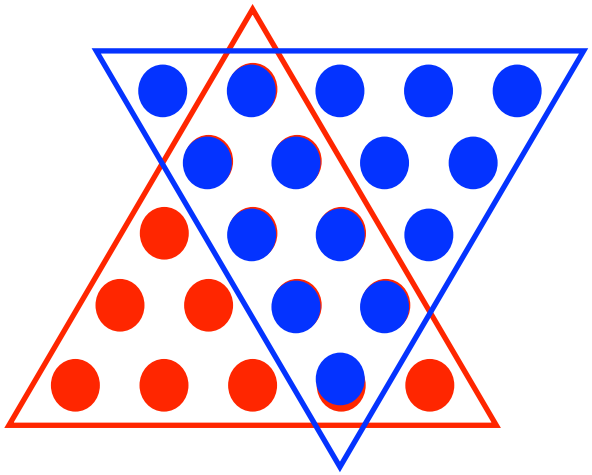
Maximizing Overlap for h=5 (1)

- There are 3 sides in each triangle.
  - At each side, we have a "non-overlapping" triangle!
  - Denote their heights by (*a*, *b*, *c*)



(0,0,4)

10 coins!

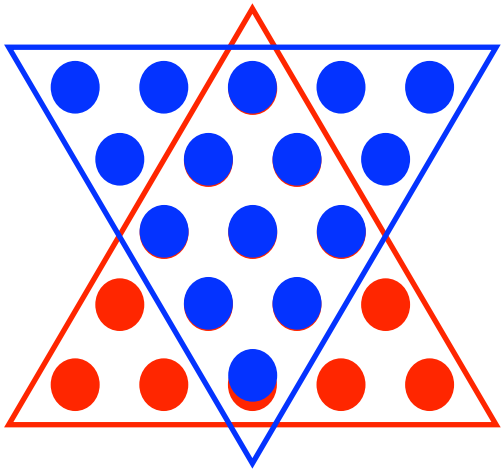


(0,1,3)

7 coins!

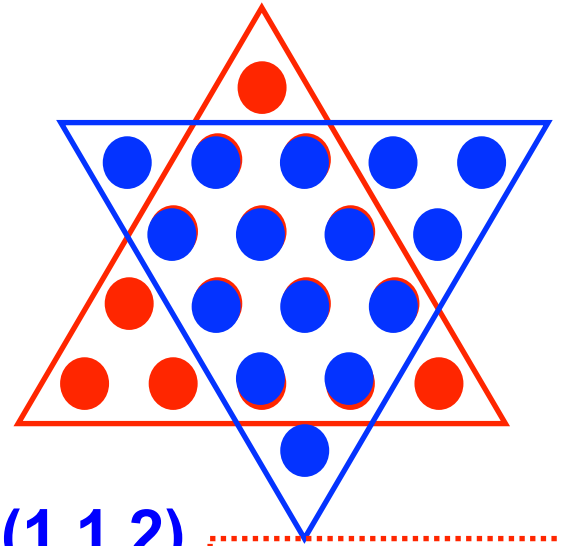
Maximizing Overlap for h=5 (2)

- More configurations…



(0,2,2)

6 coins!



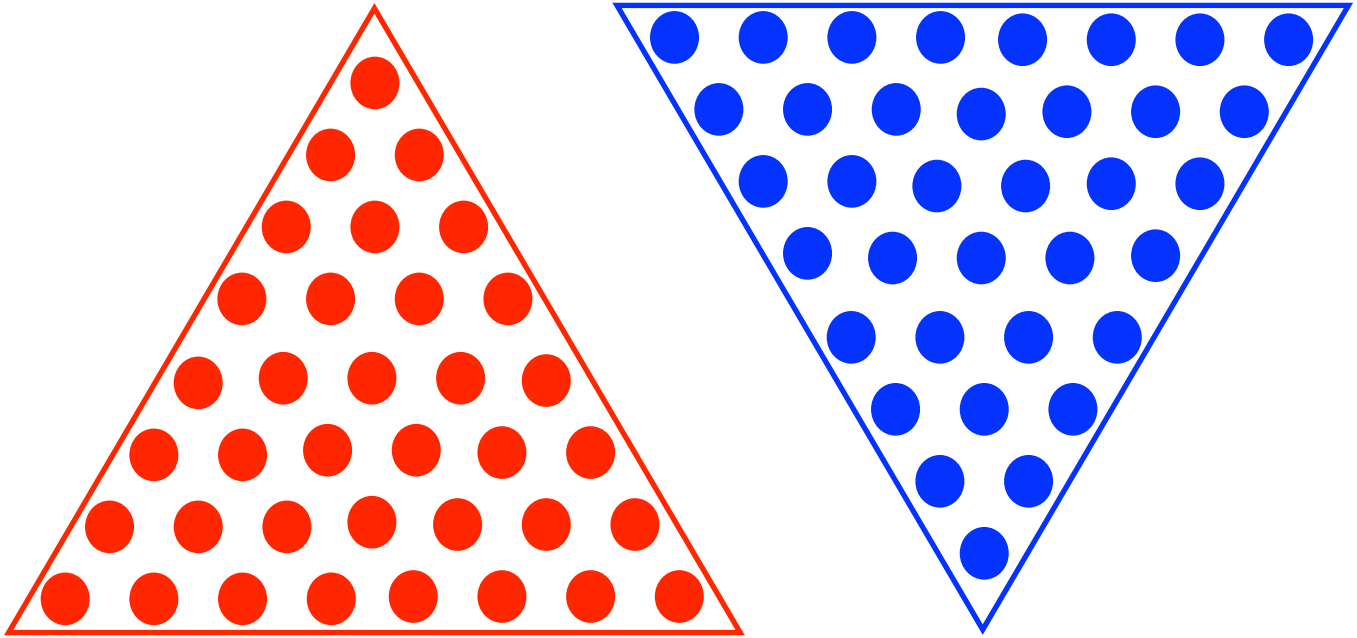
(1,1,2)

5 coins!

- Note that (a + b + c) = 4 (an invariant)
- Which configuration gives minimum *m*?

Activity Period #4:

"Invert Triangle of Coins" for *h*=8 (DIY)
(5 minutes)

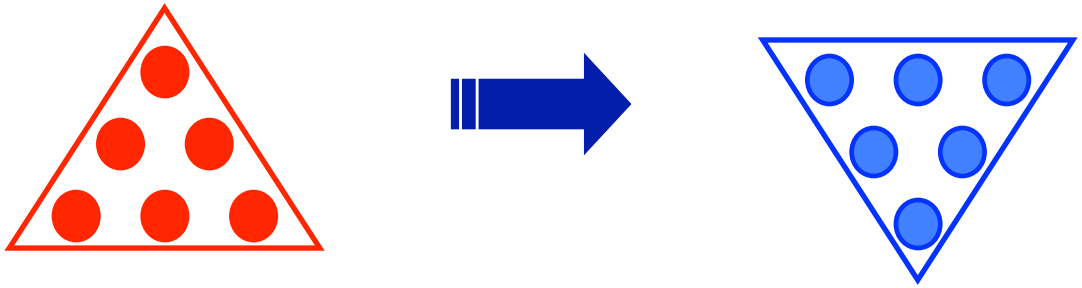


Review of Activity #4

- **Did you try with real coins?**
  - Yes: _____ No: _____
- **What is the value of (a + b + c)?**
- **What configuration (a, b, c) is *minimum*?**
  - (a, b, c) = (__ , __ ,__)
  - How many coins moved? _____ coins
- **Was it easy?**
- ***Was it fun?*** ***h* = 191**

Maximizing Overlap is Easy…

- Try for previous cases h = 3



- "Non-overlapping triangles with hts (a,b,c)
  - *For h=2, we have* (0,0,1) *m=1*
  - *For h=3, we have* (0,1,1) *m=2*
  - *For h=4, we have* (1,1,1) *m=3*
  - *For h=5, we have* (1,1,2) *m=5*
  - *For h=6, we have* (1,2,2) *m=7*
  - *For h=7, we have* (2,2,2) *m=9*
  - *For h=8, we have* (2,2,3) *m=12*

Looking Back…

- Can work out for h = 9, 10, 11, …, 191

- The ANSWER is NOT so important,
  the *METHOD IS more important !*

- Where is the *Key Step*?

The Aha! moment

- Why was it *not apparent* to us *at the beginning* ?

- How *did* we get to this step?

Reflections…

- **Creative Problem Solving**
  - **Looking from a Different Perspective**
  - **Problem Transformation**
  - **Importance of Asking Questions!**

  - **The actual answer is not so important.**
  - **The process is key and**
  - **It is FUN!**

Thank you!

If you want to contact me,
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