M365C: Real Analysis I
Homework # 07
Handout: 03/01/2018, Thursday
Due: 03/08/2018, Thursday

Submission. Please make your homework neat and stapled. Note that *no late homework will be accepted without compelling reasons.*

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1 To be Graded

Problem 1. Let $x, y \in \mathbb{R}$ be such that $x \neq y$. Prove that $\exists \varepsilon > 0$ such that $B_\varepsilon(x) \cap B_\varepsilon(y) = \emptyset$.

Problem 2. Let $X = [0, \infty)$ and take $\mathcal{O}_k = (-10, k)$, $k \geq 1$. Prove that:
(i) $O = \{\mathcal{O}_k\}_{k=1}^{\infty}$ is a open cover of X;
(ii) O has no finite subcover, i.e. X is not compact.

Problem 3. Let (X, ρ) be a metric space with ρ the discrete metric. Prove that (X, ρ) is compact if and only if X is a finite set.

Problem 4. Let (X, σ) be a metric space, and $f(x) : Dom(f) = X \mapsto \mathbb{R}$ be a continuous function (under the usual Euclidean metric $\rho(x, y) = |x - y|$ on $\mathbb{R}$). Prove that $|f(x)|$ is a continuous function on $Dom(f)$.

Problem 5. Let (X, σ) be a metric space, $f : X \to \mathbb{R}$ and $g : X \to \mathbb{R}$ be both Lipschitz on X (under the usual Euclidean metric $\rho(x, y) = |x - y|$ on $\mathbb{R}$). Prove that $f + g$ is also Lipschitz on X.

Problem 6. Let (X, σ) be a metric space, $f : X \to \mathbb{R}$ and $g : X \to \mathbb{R}$ be both uniformly continous on X (under the usual Euclidean metric $\rho(x, y) = |x - y|$ on $\mathbb{R}$). Prove that $f + g$ is also uniformly continous on X.

2 Reading Assignments

- Review Lecture Notes # 10 and # 11;
- Review Sections 2.3, 4.1, 4.2 and 4.3 of the textbook;