Assignment 1

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Problem 1

Let $X = \{1,\ 2,\ a\}$*. Find the power set of* X*,* $P(X)$*.*

Problem 2

For each $n \in \mathbb{N}$*, let* $A_n = \{(n+1)k : k \in \mathbb{N}\}$

a)

What is $A_1 \cap A_2$*?*

b)

Determine the sets $\cup\{A_n : n \in \mathbb{N}\}$ *and* $\cap\{A_n : n \in \mathbb{N}\}$*.*

Problem 3

Let A *and* B *be two sets. Prove that* $A \subseteq B$ *iff* $A \cap B = A$

Problem 4

Let A*,* B*, and* C *be arbitrary sets. Prove that* $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$*.*

Problem 5

Let $\mathbb{N}$ *be the set of natural numbers, and* $|$ *be the relation of divisibility (i.e. we say* $y \in \mathbb{N}$ *divides* $x \in \mathbb{N}$*, denoted by* $y|x$*, if there exists an integer* n *such that* $x = ny$*). Prove that* $|$ *is an ordering relation on* $\mathbb{Z}$

Problem 6

Let $\leq$ *be an ordering relation on the set X. We define the inverse of* $\leq$*, denoted by* $\geq$ *, as follows:* $\forall x, y \in X, x \geq y$ *iff* $y \leq x$*. Prove that* $\geq$ *is an ordering relation on X.*

Problem 7

Let $(X, \leq)$ *be a totally-ordered space and* $Y \subseteq X$ *an nonempty subset of X. Let* α *be a lower bound of* Y *and* β *an upper bound of* Y*. Prove that* $\alpha \leq \beta$*.*