M365C: Real Analysis I
Homework # 05
Handout: 02/15/2018, Thursday
Due: 02/22/2018, Thursday

Submission. Please make your homework neat and stapled. Note that *no late homework will be accepted without compelling reasons.*

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1 To be Graded

Problem 1. Prove directly, using the definition of convergence, that each of the following sequences converges in the metric space (X, ρ) with $X = \mathbb{R}$ and $\rho(x, y) = |x - y|$:
(a) The sequence $\{x_n\}$ with $x_n = 1 + \dfrac{10}{\sqrt{n}}$;
(b) The sequence $\{x_n\}$ with $x_n = 3 + 2^{-n}$;
(c) The sequence $\{x_n\}$ with $x_n = \dfrac{2n+3}{n+1}$.

Problem 2. Let (X, ρ) be a discrete metric space, and $\{x_n\}$ a sequence in X. Prove that $x_n \to x$ if and only if there exists a $N \in \mathbb{N}$ such that $x_n = x \ \forall n \geq N$.

Problem 3. Let ρ and σ be two uniformly equivalent metrics defined on X and $\{x_n\}$ be a sequence in X. Show that $x_n \to x$ in metric ρ iff $x_n \to x$ in metric σ.

2 Reading Assignments

- Review Lecture Notes # 7 and # 8;
- Review Sections 2.2, 3.1 and 3.2 of the textbook;