Tutorial - 1 (DAA)

Ans-1 **Asymptotic Notations:** Asymptotic Notations are the mathematical notations used to describe the running time of an algorithm.

Different types of Asymptotic Notations:

1. **Big-o Notation (O):** It represents upper Bound of algorithm.
   $f(n) = O(g(n))$ if $f(n) \leq c * g(n)$
2. **Omega Notation (Ω):** It represents lower bound of Algorithm.
   $f(n) = \Omega(g(n))$ if $f(n) \geq c * g(n)$
3. **Theta Notation (Θ):** It represents upper and lower bound of algorithm.
   $f(n) = \Theta(g(n))$ if $c_1 g(n) \leq f(n) \leq c_2 g(n)$

Ans-2

```
for(i=1 to n)
{  i=i*2
}
```

```
i = 1
i = 2
i = 4
i = 8
i = 16
i = n
```

It is forming GP

$a_n = a r^{n-1}$

$n = a r^{k-1}$

$n = 1 \times (2)^{k-1}$

$\log n = \log 2^{k-1}$

$\log n = (k-1) \log 2$

$\boxed{k = \log n + 1}$

$O(\log n)$

$(a_n = n,\ r = 2,\ a = 1)$

Am-3 $T(n) = 3T(n-1)$ if $n>0$, otherwise 1

$T(1) = 3T(0)$ $[T(0)=1]$

$T(1) = 3\times1$

$T(2) = 3T(1) = 3\times3\times1$

$T(3) = 3\times T(2) = 3\times3\times3$

⋮

$T(n) = 3\times3\times3\cdots$

$= 3^n$ $= O(3^n)$

Am-4 $T(n) = 2T(n-1)-1$ if $n>0$, otherwise 1

$T(0) = 1$

$T(1) = 2T(0)-1$

$T(1) = 2-1 = 1$

$T(2) = 2T(1)-1$

$T(2) = 2-1 = 1$

$T(3) = 2T(2)-1$

$= 2-1 = 1$

⋮

$T(n) = 1$ $O(1)$

Am-5

```
int i=1, s=1
while (s<=n)
{
    i++;
    s = s+i;
    printf("#");
}
```

i=1	s=1
i=2	s=i+2
i=3	s=1+2+3
i=4	s=1+2+3+4
⋮	⋮

Loop ends when $s > n$

$1+2+3+4 \cdots k > n$

$\frac{k(k+1)}{2} > n$

$k^2 > n$

$k > \sqrt{n}$

$O = O(\sqrt{n})$

Ans-6

```
Void function (int n)
{ int i, count = 0;
  for(int i=1; i*i<=n; i++)
      count++;
}
```

i=1
i=2
i=3
i=4
⋮
i=k

Loop ends when $i * i > n$

$k \times k > n$

$k^2 > n$

$k > \sqrt{n}$

$O(n) = \sqrt{n}$

Ans 7

```
Void function(int n)
{  int i, j, k, count=0;
   for (i=n/2; i<=n; i++)
   {  for (j=1; j<=n; j=j*2)
      {  for (k=1; k<=n; k=k*2)
            count++;
      }
   }
```

- **1st Loop** : $i = \frac{n}{2}$ to n , i++

  $= O\left(\frac{n}{2}\right) = O(n)$

- **2nd Nested Loop** : $j = 1$ to n , $j = j * 2$

  $j = 1$
  $j = 2$
  $j = 4$
  $j = n$

  $= O(\log n)$

- **3rd Nested Loop**: $k = 1$ to n , $k = k * 2$

  $k = 1$
  $k = 2$
  $k = 4$

  $= O(\log n)$

Total Complexity = $O(n \times \log n \times \log n) = O(n \log^2 n)$

Ans 8

```
function(int n)
{  if (n==1) return;          ---- 1
   for (int i=1 to n)
   {  for (int j=1 to n)      ---- n^2
      {  printf("*");
      }
   }
   function(n-3);             ---- T(n-3)
}
```

$$\boxed{T(n) = T(n-3) + n^2}$$

$T(1) = 1$

$\rightarrow T(1) = 1$

$\rightarrow T(4) = T(4-3) + 4^2$

$= T(1) + 4^2 = 1^2 + 4^2$

$\rightarrow T(7) = T(7-3) + 7^2$

$= 1^2 + 4^2 + 7^2$

$\rightarrow T(10) = T(10-3) + 10^2$

$= 1^2 + 4^2 + 7^2 + 10^2$

So, $T(n) = 1^2 + 4^2 + 7^2 + 10^2 \ldots n^2 = \frac{n(n+1)(2n+1)}{6} = O(n^3)$

Also for terms like $T(2)$, $T(3)$, $T(5)$

So, $T(n) = O(n^3)$

Ans-9

```
Void function (int n)
{   for(int i=1 to n) ——— n
    {  for (j=1 ; j<=n ; j=j+1) — n
       {  Printf("*");
       }
    }
}
```

i=1 —— j=1 to n

i=2 —— j=1 to n

i=3 —— j=1 to n

i=4 —— j=1 to n

So, for i upto n it will take n^2

So, $T(n) = O(n^2)$

Ans to $f_1(n) = n^K$, $f_2(n) = c^n$

$K \geq 1$, $c > 1$

Asymptotic relationship between f_1 and f_2 is Big O i.e $f_1(n) = O(f_2(n)) = O(c^n)$

or $n^K \leq \eta * c^n$ [η is some constant]