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Optimal scheduling of GEO debris removing based on hybrid optimal control theory

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ABSTRACT

The scheduling of debris removing in a coplanar GEO orbit is studied in this paper. Specifically, the servicing spacecraft is considered to be initially on the circular orbit of the debris to be removed, and it should rendezvous with each debris, remove them to the graveyard orbit and finally return to its initial location. The minimum-cost, two-impulse phasing maneuver is used for each rendezvous. The objective is to find the best sequence with the minimum total Δv to service all debris in the constellation. Considering this mission as a hybrid optimal control problem, a mathematical model is proposed. Based on the analysis and numerous experiments, two heuristic laws of the problem are found out: (1) when targets are sparsely distributed (i.e. each phase angle between the targets is larger than certain threshold), the optimal sequence is the counter-orbit-wise one starting from either the closest leading or the closest lagging target; (2) in general cases, the optimal sequence is the combination of the counter-orbit-wise segments in which each phase angle is larger than a threshold. Then a Rapid Method (RM) is presented based on these laws. It can be concluded from the experiments that the RM is effective and efficient in dealing with this problem especially for the cases with numerous targets.

1. Introduction

The geostationary ring is a valuable resource and the monotonically growing debris population in the Geosynchronous Equatorial Orbit (GEO) increases the probability of accidental collision with known objects during the system's orbital lifetime [1,2]. Space debris mitigation guidelines and directives argue in favor of moving satellites at end of life to a graveyard zone above the densely populated geosynchronous ring [2]. There are two solutions to re-orbit the debris [1]. One makes use of the propellant remaining at the end-of-life, and the other resorts to the space capture system or the electrostatic tractor force [3]. Generally the remaining propellant of the spacecraft may not be known accurately and may not be sufficient for the required task. Therefore, the latter approach is preferable, and it would extend the satellite's useful life to the fullest possible.

NASA, ESA, and other relevant authorities have begun to respond to the orbital debris problem by placing requirements [4] and issuing new projects for debris mitigation upon new space systems. Examples of these projects are NASA's “Momentum eXchange/Electrodynamic Reboost (MXER) tether” [5] which can provide a cheap way to de-orbit a satellite at the end of its useful life and prevent it from becoming a piece of space debris, and ESA's “ROGER—the RObotic GEostationary orbit Restorer” [6] which is a new concept for an in-orbit roving debris removal system. ROGER can be tasked to approach and capture a redundant or non-operational satellite in the geostationary orbit and tow it into a parking or graveyard orbit.

In the process of debris removing by tether system, the Servicing Spacecraft (SSc) repeatedly rendezvous with the debris, capture it and then fly to the graveyard orbit to

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release it until all debris are removed. Essentially, it is a multi-spacecraft rendezvous mission. Despite the success in servicing a single spacecraft and the numerous papers studying optimal rendezvous between two spacecraft (e.g. [7–9]), so far there has been only a little reported work devoted in the area of developing optimal sequence and trajectories for servicing multiple targets in a constellation, especially for the area of debris removing. In this paper we consider the problem of servicing multiple debris in a GEO orbit with one SSc. The goal is to find the best sequence of targets with the minimum cost (measured in terms of Δv).

Similar work has been done based on the application of on-orbit refueling. Shen [10] studies the scheduling of servicing multiple satellites in a circular orbit. The minimum-cost, two-impulse lambert maneuver is used for each rendezvous between the SSc and the satellite to be refueled. Ouyang et al. [11] transformed the refueling problem into a Travel Salesman Problem (TSP), defined the fuel cost of each rendezvous was only determined by the orbital plane difference between the two spacecraft, and then combat it using genetic algorithm. Considering the J_2 perturbation and the time window constraints based on lighting condition, Zhang et al. [12] focused on the LEO long-duration multi-spacecraft rendezvous mission, and built a Mixed Integer Nonlinear Programming (MINLP) model for the problem. Bo et al. [13] presented a new refueling patterns based on formation flying, and proposed two strategies to address the problem. Despite the success in on-orbit refueling, these studies could not be applied to the debris removing scenario directly, but may be used as references.

Recently, Hybrid Optimal Control (HOC) theory has been applied to the solution of space mission planning. HOC problems are problems that include both continuous-valued variables and categorical variables in the problem formulation [14]. In [15], Ross et al. introduce HOC method to tackle the increasing sophistication of space missions, and proposed a formalism that can free mission planners to focus on high-level decision making by automating and optimizing the details of the inner loops. In [14], Conway et al. presented a mathematical framework for describing the asteroids-visit problem based on the HOC theory, and proposed solution methods based on evolutionary principles. In [16,17], Chilan et al. applied the HOC method to solve the multiphase mission planning problem. In [18], a problem of autonomous interplanetary mission design with several gravity assists was solved with a HOC approach by Englander. In [19], Wall et al. developed a HOC problem solution algorithm, in which GA's are employed for both the inner-loop and outer-loop problem solvers, and it can efficiently solve a number of mission planning problems in astrodynamics.

In this paper we consider the scheduling of the space debris removing mission as a HOC problem, and use hybrid automaton to model and address it. This paper is organized as follows. Section 2 analyzes the process of the debris removing in detail and describes the mission scenario researched in this paper. Section 3 presents the model of the HOC problem. Section 4 gives the solution method for the problem. Subsequently, a heuristic study leads to a rapid method. And in Section 5, numerous examples are carried out to demonstrate the effectiveness of the model and solution method.

2. Mission scenario

Taking ROGER as an example [6], in the process of debris removing (see Fig. 1), the mission scenario begins with the launch of the ROGER servicing satellite into a geostationary orbit. And then phase to an orbit position, where the rendezvous maneuver to the first target satellite can start. After a series of Rendezvous Proximity Operations (RPO), the ROGER will be pointed to the center of the target and the capture mechanism will be released. After the capturing and stabilization maneuvers, ROGER will inject the debris into the graveyard orbit, where a separation will be performed. Finally ROGER starts the next phase of RPO, and repeats the removing actions until all of the debris are removed.

Ignoring the first phase of launching, the debris removing process contains: RPO, capture operations and release operations. Capture and release operations are serial activities that out of our consideration. Therefore, the emphasis of the mission planning is put on finding the best mission sequence and the optimal RPO trajectories. RPO is an appealing and challenging area of study and much work has been done to address it. In order to simplify the problem, we take two-impulse phasing maneuver to perform

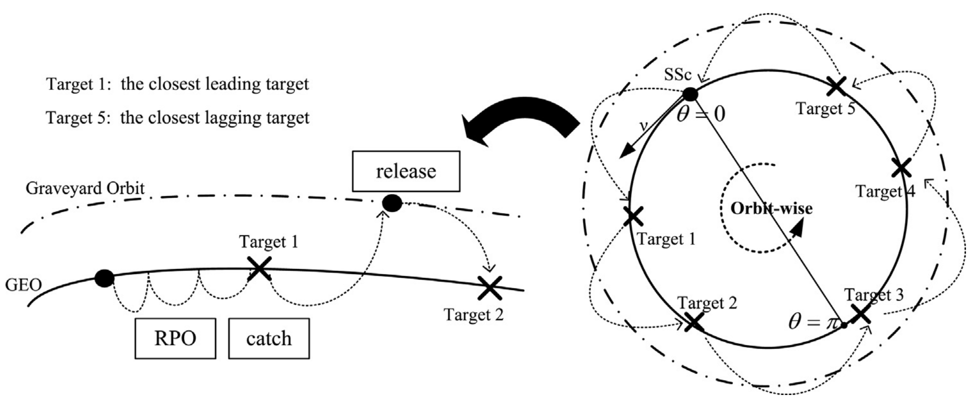

Fig. 1. Mission scenario.

RPO, and sufficient time is given for the mission. In addition, we assume that once the SSc passed the graveyard orbit, it can release the debris immediately. Meanwhile, it is defined that the SSc originally stays on a maintenance station in GEO belt. When the whole mission is completed, the SSc should finally return to its original location as it is required to have certain routine maintenance (e.g. refueling, fixing).

Summing up, the problem studied in this paper can be stated as, there are N debris and one SSc running on the same GEO orbit with different orbital slots. The task of the SSc is required to rendezvous with each of the debris. After the SSc finishes removing one debris, it visits the next one until all the targets are removed. Finally the SSc should return to its original location. As a practical concern, the total time to complete the mission is sufficiently offered. The goal is to find the mission sequence to visit all debris in the constellation such that the total rendezvous cost (Δv) is minimized.

3. Mathematical description of the HOC problem

The first challenge is to create a mathematical formalism for description of the problem. The formalism presented by Ross is flexible and suited to aerospace trajectory/mission planning, and it introduces a method for categorizing unallowed event transitions [14,15]. A succinct description of the mathematical formalism for the HOC problem follows.

3.1. Categorical state space

A maneuver automaton can be described in a directed graph or "digraph" [15]. Fig. 2 shows a digraph for an example problem in which there are three possible events that can be combined in some order to qualitatively describe the mission plan. The categorical state space, that is, the totality of events, is graphically depicted, as are the allowed transitions.

Suppose that the categorical state space for the problem is $Q = \{q_1, q_2, q_3\}$, with cardinality N_Q ($N_Q = 3$ for the example in Fig. 2). Each vertex q_i is the state corresponding to the SSc flying at the slot of the ith debris with zero thrust. Each edge represents a switch of the SSc's dynamics from one vector field to another. $\boldsymbol{u}_{ij}$ denotes the system input that triggers the switch from state q_i to q_i.

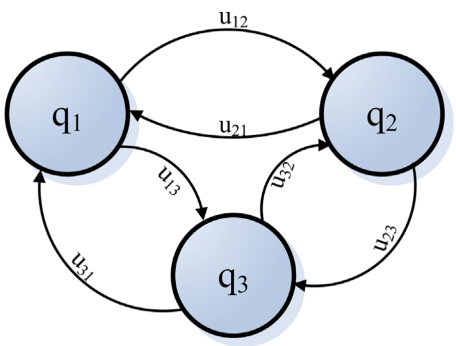

Fig. 2. Digraph for a simple mission.

Depending on the nature of events the edges may require changes in the system dynamics from one vector field to another.

A mission plan is described qualitatively by an event sequence, e.g. $q = (q_2, q_1, q_3)$, with events chosen from the categorical state space. This mission plan sequence describes that starting from the original location, the SSc remove the debris numbered 2, 1, and 3 one by one, and finally return to its original slot. The real trajectory of the SSc is $\boldsymbol{q} = (q_0, q_2, q_1, q_3, q_0\}$, where q_0 denotes the initial fly state of the SSc. In this paper, we tacitly approve that q_0 is the original and final state of the SSc, so q_0 won't appear in the mission plan.

3.2. Continuous-time dynamics

Associated with each state $q \in Q$ is a continuous-time controlled dynamical system: $\dot{\boldsymbol{x}} = f(\boldsymbol{x}, \boldsymbol{u}, t, q)$. As mentioned previously, the SSc and all of the debris are initially running on the same circular GEO orbit, and each discrete state represents the SSc flying at a defined orbital slot with zero thrust. So, at each discrete state, we have the dynamics of SSc with $\boldsymbol{x} = [\boldsymbol{r}, \boldsymbol{v}, \theta]$. $\boldsymbol{r}$, $\boldsymbol{v}$, and θ respectively denotes the radius vector, velocity vector and true anomaly of the SSc.

$$\begin{aligned} \frac{d\mathbf{r}}{dt} &= \mathbf{v} \\ \frac{d\theta}{dt} &= \omega_\theta \\ \frac{d\mathbf{v}}{dt} &= -\frac{\mu}{r^3}\mathbf{r} \end{aligned} \tag{1}$$

3.3. Discrete events

Arguably, the most important notion in hybrid control is the formalization and generalization of a switch [15]. Let $(\boldsymbol{x}, \boldsymbol{u})$ and $(\boldsymbol{x}', \boldsymbol{u}')$ denote the continuous-valued state and control variables associated with any two vertices $q, q' \in Q$. The generalized switching set, which may be empty, is called the event set $\boldsymbol{E}(q, q')$ and is defined (when nonempty) by means of an inequality constraint on a function $\boldsymbol{e}(\cdot, q, q')$.

$$\mathbf{E}(q, q') = \{(\boldsymbol{x}, \boldsymbol{u}, \tau, \boldsymbol{x}', \boldsymbol{u}', \tau') : \mathbf{e}^L \le \boldsymbol{e}(\boldsymbol{x}, \boldsymbol{u}, \tau, \boldsymbol{x}', \boldsymbol{u}', \tau, q, q') \le \mathbf{e}^U\} \tag{2}$$

$\boldsymbol{e}^L$ and $\boldsymbol{e}^U$ are the lower and upper bounds on the values of the function $\boldsymbol{e}(\cdot, q, q')$, respectively. The function $\boldsymbol{e}(\cdot, q, q')$ is called the event function associated with the discrete states q and q'. In investigating a transition of a hybrid system from one location $q \in Q$ to another location $q' \in Q$, at some (possibly unknown) time t_e, we define a switching set $\boldsymbol{S}(q, q') \subset \boldsymbol{E}(q, q')$ as

$$\boldsymbol{S}(q, q') = \{(\boldsymbol{x}, \boldsymbol{u}, \tau, \boldsymbol{x}', \boldsymbol{u}', \tau') \in \boldsymbol{E}(q, q') : \tau = \tau' \equiv t_e \in \boldsymbol{R}\} \tag{3}$$

If $\boldsymbol{S}(q, q') \ne \varnothing$ then the transition from q to q' is allowed, i.e. there is a directed edge from q to q' in the digraph. The system adjacency matrix $\boldsymbol{A} = [A_{ij}]$ can be used to encode the digraph [15].

$$A_{ij}=\begin{cases}1 & \text{if } S(q_i,q_j)\neq\varnothing\\ 0 & \text{if } S(q_i,q_j)=\varnothing\end{cases} \tag{4}$$

Essentially, we use $\boldsymbol{A}$ as a computational means to encode the digraph of the hybrid automaton.

3.4. Cost functions

In this paper, we only take the maneuver cost into consideration, which can be described as the absolute value of the velocity increment Δv. When the SSc is running on the discrete state, there is no thrust, so the velocity increment is zero. Associated with any pair $(q,q')\in Q\times Q$, there is an event cost that brought about by the transition. In other words in the whole system only the switching event could bring about maneuver cost. So the cost of the system is determined by the switch set $\boldsymbol{E}(q,q')$ and the mission sequence $\boldsymbol{q}$.

Let $C(\cdot,q,q')$ denotes the cost from the state q to q':

$$C(\cdot,q,q'):\mathbf{E}(q,q')\to\boldsymbol{R}\cup\{\infty\} \tag{5}$$

and the whole cost of the system can be given as:

$$J(\boldsymbol{x}(\cdot),\boldsymbol{u}(\cdot),\boldsymbol{q},N_Q)=\sum C(\cdot,q^i,q^{i+1}) \tag{6}$$

(Note that, the subscript of the state q represents its location in the state space Q, and the superscript denotes its location in the mission sequence $\boldsymbol{q}$.)

3.5. Summary of the HOC problem

In summary, the space debris removing mission researched in this paper could simply be described as a HOC problem, and can be formulated as:

$$\begin{array}{ll}\text{find} & \boldsymbol{q},\boldsymbol{u}\\ \text{minimize} & J(\mathbf{x}(\cdot),\boldsymbol{u}(\cdot),\boldsymbol{q},N_Q)\\ \text{subject to} & \dot{\boldsymbol{x}}=f(\boldsymbol{x},\boldsymbol{u},t,q)\\ & \boldsymbol{q}\in U_D\\ & \boldsymbol{q}\in U_A\\ & q\in\boldsymbol{q},q\in Q\end{array} \tag{7}$$

where U_D denotes the plan sets that meet the constraint: the SSc should rendezvous with each space debris once and only once, and U_A denotes the plan sets that meet the constraint of the digraph. $\boldsymbol{u}$ is the input vector of the system.

4. Solution of the HOC problem

4.1. General method

Obviously, the HOC problem of this paper is combined by an outer-loop and an inner-loop optimal problems. The outer one determines the optimal mission sequence $\boldsymbol{q}$, while the inner one finds the optimal control input $\boldsymbol{u}$ that could trigger the rendezvous trajectory with minimum cost. These two loops interact with each other. To determine the optimal mission plan sequence, one must be able to determine the cost of each rendezvous first. That is to say before we determine the best sequence, the inner-loop should be solved first. In order to solve the HOC problem efficiently, we model the outer-loop and the inner-loop problems respectively, and then present the approach of solution.

In the inner-loop, for each rendezvous segment, given two states q^i and q^{i+1}, its purpose is to find the optimal transfer orbit from q^i to q^{i+1}. It is a typical optimal rendezvous problem. There are several methods to carry out the path, such as Hohmann transfer, Lambert transfer, Phasing maneuvers and so on. In this paper, we apply phasing maneuvers to transferring. In a two-impulse phasing maneuver, the SSc transfers from one point to another in a circular orbit, such that the velocity changes occur at the same point in the transfer orbit. So we can suppose that the switching control input from q^i to q^{i+1} is $\boldsymbol{u}_{i,i+1}$, and $\boldsymbol{u}_{i,i+1}=(t_i,t_{i+1},\Delta v_i,\Delta v_{i+1})$, where Δv_i represents the velocity change at time t_i. To solve $\boldsymbol{u}_{i,i+1}$, we can utilize the formulations below:

$$\Delta v=\sqrt{\mu}\left|\sqrt{\frac{2}{r}-\frac{1}{a}}-\sqrt{\frac{1}{a}}\right| \tag{8}$$

$$a=r\left(\frac{2\pi n_t+\Delta\theta}{2\pi n_s}\right)^{2/3} \tag{9}$$

where r is the orbit radius of the debris, a is the semi-major axis of the transfer orbit, n_t and n_s are the number of revolutions of the debris and the SSc during phasing, $\Delta\theta$ is the phase angle which is measured from the target to the SSc with the direction of the target's motion defined as the positive direction, and $\Delta\theta\in[-\pi,\pi]$.

Specially, the altitude of the transfer orbit is constrained except for the first rendezvous. The U.S. Government Debris Mitigation Standard Practice recommends that all vehicles should be moved to a graveyard orbit with a perigee that is at least 300 km above the exact GEosynchronous circular Orbit altitude (GEO) [20]. Assume that the graveyard orbit is a circular orbit. Then the transfer orbit of phasing maneuvers should have an apogee that is at least higher than the radius of the graveyard orbit. This constraint could be formulated as $r+R\le 2a\le r+R_{\max}$, where R is the radius of the graveyard orbit and $R_{\max}$ is the maximum constraint of a. Obviously, when the number of revolutions n_t and n_s are constrained, $R_{\max}$ is defined.

Let $X(t)=(\boldsymbol{r},\boldsymbol{v},\theta,\dot{\boldsymbol{r}},\dot{\boldsymbol{v}},\dot{\theta},t)$ describes the dynamics of the SSc at time t, $X(t_i^-)$ represents the dynamics before the velocity change Δv_i, and $X(t_i{}^+)$ denotes that of SSc after Δv_i. Let the input of the system is $\boldsymbol{u}=(\boldsymbol{u}_{0,1},\boldsymbol{u}_{1,2},\ldots,\boldsymbol{u}_{N,0})$. In summary, the inner-loop optimization can be modeled as

$$\begin{array}{ll}\text{find} & \boldsymbol{u}\\ \text{min} & J=\sum|\Delta v_i|\\ \text{subject to} & X(t_i^-)=X_{q^i}\\ & X(t_{i+1}^+)=X_{q^{i+1}}\\ & X(t_0)=X_{q_0}\\ & X(t_f)=X_{q_0}\\ & \boldsymbol{u}^L\le\boldsymbol{u}\le\boldsymbol{u}^U\\ & \Delta t\le t_{\min}\end{array} \tag{10}$$

where $\boldsymbol{u}^L$ and $\boldsymbol{u}^U$ represents the lower and upper bound of the input, and $t_{\min}$ denotes the minimum time constraint of the SSc staying at one state q for debris capturing (every time the SSc arrives at the debris, it could not fly away immediately, since abundant time should be shared for the SSc to capture the debris on orbit.).

Assuming that there exists an inner-loop solver capable of determining the optimal trajectory for each feasible sequence $\boldsymbol{q}$, the outer-loop could be formulated as:

$$\begin{array}{ll} \text{find} & \boldsymbol{q} \\ \text{subject to} & \boldsymbol{q} \in U_D \\ & \boldsymbol{q} \in U_A \\ & q \in \boldsymbol{q},\ q \in Q \end{array} \tag{11}$$

For the outer-loop, when the amount of the targets is small, we can enumerate all feasible sequences (exhaustive search) and the optimal choice would then be the one with the smallest cost. When the amount is large, intelligent algorithms are preferable, such as genetic algorithm, simulated annealing and so on.

4.2. CO law and CS law for the HOC problem

In [10], Shen have investigated the minimum-cost scheduling problem for one SSc to service satellites in a circular orbit, and the minimum-cost, two-impulse lambert maneuver is used for each rendezvous. After numerous experiments, Shen concluded that the best rendezvous sequence is one of the orbit-wise and counter-orbit-wise sequential orders for the case where the SSc returns to its original orbital slot (the direction of orbit-wise can be seen in Fig. 1). Phasing maneuver is the exceptional case of lambert transfer, so this conclusion is also applicable to this paper.

Different from the problem investigated by Shen, there is a special altitude constraint in this paper for the transfer orbits. Specifically to say, once the SSc caught the debris, it should transfer to a higher orbit to release it. The altitude of the transfer orbit is constrained by the height of the graveyard orbit. We mainly take phasing maneuver to perform transferring. In phasing maneuver, depending on whether the transfer orbit has a lesser semi-major axis or not, it is termed sub-synchronous or super-synchronous. When the mission time is given, the former one is cost-saving for the SSc to chase while the latter one is cost-saving for waiting. In our study, due to the altitude constraint, the transfer orbit should always be the super-synchronous one except for the first rendezvous. That is to say the SSc always waits for targets except for the first one. So it can be guessed that the optimal mission sequence studied here would be in the counter-orbit-wise order except for the first target.

In this section, the optimal sequence problem is investigated numerically. For this reason, only constellations with a small number of targets to be serviced ($n \le 6$) are considered. Even for a constellation with six targets to be removed, there are $6! = 720$ different sequences. The minimum cost is calculated for each sequence using the method described above. Before the experiments, we introduce the notion of the Chasing Angle (CA). CA is the angle which is measured from the SSc to the target with the direction of the SSc's motion defined as the positive direction, and CA $\in [0, 2\pi]$. The Total Chasing Angle (TCA) is defined for a given sequence of mission as the sum of the CAs of all the rendezvous segments. In the following, we present 4 of the numerous case studies that have been conducted. In all of the examples, SSc is always at the location with $\theta = 0$ rad, and starting from the location of SSc, all of the targets are numbered one by one along the orbit-wise direction (e.g. Fig. 1). The relevant constraints for these experiments are given in Table 1.

In this paper, the cost is given as $\Delta\overline{v}$, normalized Δv ($\Delta\overline{v} = \Delta v\sqrt{r/\mu}$). In all experiments, the value of $\Delta\overline{v}$ is accurate to four decimal places. The difference of 0.0001 in $\Delta\overline{v}$ would cause the difference of 0.34 m/s in Δv. Actually, 0.34 m/s is small and negligible. Therefore $\Delta\overline{v}$ could clearly describe the differences among the various sequences.

Case 1. In this case, we have one SSc and six targets that must be removed. These six targets are evenly distributed along the circular orbit, i.e., the separation angle between any two neighboring targets is 60°. The SSc is initially at the slot of target 6.

Case 2. In this case, we have one SSc and six targets. The SSc is initially at the slot of target 6. The initial slot of each target is given below.

$$\theta_1 = \pi/6,\ \theta_2 = \pi/2,\ \theta_3 = \pi,\ \theta_4 = 4\pi/3,\ \theta_5 = 11\pi/6, \text{ and } \theta_6 = 2\pi$$

Assume that the first target to be visited must be target 6, that is to say the first rendezvous is defined previously. And the results of $5! = 120$ different sequences would be showed below.

Figs. 3 and 4 for Cases 1 and 2 respectively, show the plots of the TCA vs. Cost on condition that the first rendezvous is out of consideration and the targets are

Table 1
Constraints of the examples.

parameters	n_t	n_s	t_{min}	r	R
value	≤ 6	≤ 6	1 day	35786 km	36086 km

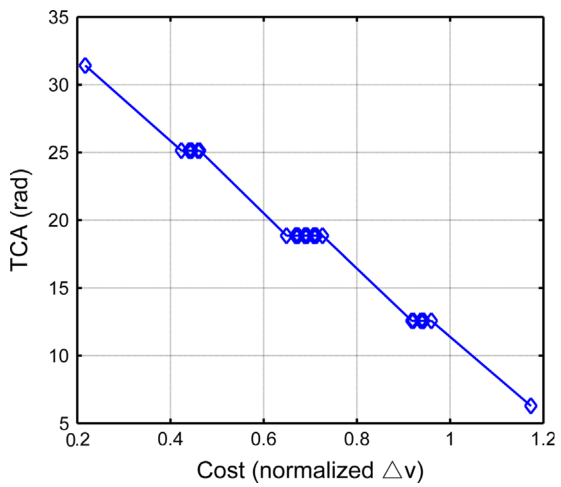

Fig. 3. TCA vs. cost for Case 1.

evenly and unevenly distributed. Each rhombus on the plot corresponds to a mission sequence. It is seen that the TCA decreases monotonically as the total cost increases and the minimum cost sequence is the one with maximal TCA in both distributions. The TCA reflects the characteristic of the arrangements. The sequence with maximal TCA is the one of totally counter-orbit-wise sequential order, i.e. (6,5,4,3,2,1). And the sequence with great TCA means that a large proportion of this sequence is sequentially counter-orbit-wise. It can be concluded from both Figs. 3 and 4 that when the first rendezvous is given, the remaining targets arranged in total counter-orbit-wise order may be the best.

A sequence index is used to identify a particular sequence. Table 2 gives the best and the worst 6 sequences out of the 120 sequences for Cases 1 and 2, respectively.

As shown in Table 2, sequence number 1 is the minimum cost sequence which is corresponding to (6,5,4,3,2,1) both in Cases 1 and 2. It is also illustrated that the best sequences are always in totally counter-orbit-wise or partially counter-orbit-wise style, while the worst ones are always in totally orbit-wise or partially orbit-wise style (the first rendezvous is out of consideration). And the sequences (ignoring target 6), starting from any target, strictly arranged in counter-orbit-wise order, are not necessarily to be the optimal arrangements (e.g. (6,4,3,2,1,5)). That is partly due to that the original and the final location of the SSc must be taken into consideration. It can be concluded from Cases 1 and 2 that, when the first rendezvous is given, the remaining alignment which starts from the closest lagging target, and is arranged in total counter-orbit-wise order, would be the best sequence.

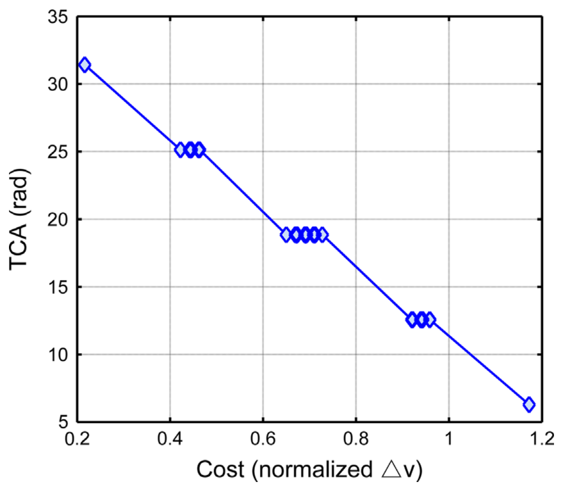

Fig. 4. TCA vs. cost for Case 2.

Table 2
Best and worst six sequences of Cases 1 and 2.

	Index	Best sequences		Index	Worst sequences	
		$\Delta\bar{v}$	Arrangement		$\Delta\bar{v}$	Arrangement
Case 1	1	0.2172	(6,5,4,3,2,1)	109	1.1734	(6,1,2,3,4,5)
	12	0.4234	(6,5,3,1,4,2)	107	0.9597	(6,1,3,5,2,4)
	39	0.4234	(6,4,2,5,3,1)	78	0.9597	(6,2,4,1,3,5)
	41	0.4234	(6,4,2,1,5,3)	59	0.9597	(6,3,5,1,2,4)
	17	0.4397	(6,5,2,1,4,3)	111	0.9434	(6,1,2,4,3,5)
	36	0.4427	(6,5,2,1,4,3)	110	0.9434	(6,1,2,3,5,4)
Case 2	1	0.2164	(6,5,4,3,2,1)	109	1.1726	(6,1,2,3,4,5)
	12	0.4223	(6,5,3,1,4,2)	107	0.9589	(6,1,3,5,2,4)
	41	0.4223	(6,4,2,1,5,3)	78	0.9585	(6,2,4,1,3,5)
	39	0.4226	(6,4,2,5,3,1)	59	0.9584	(6,3,5,1,2,4)
	4	0.4417	(6,5,4,2,1,3)	79	0.9437	(6,2,3,4,5,1)
	68	0.4419	(6,3,1,5,4,2)	21	0.9433	(6,3,5,1,2,4)

Case 3. In this case, we have one SSc and six targets. These seven objects are randomly distributed along the GEO circular orbit. The SSc is initially at the slot of $\theta=0$. And the initial locations of the targets for examples are given in Table 3.

In the examples of Case 3, different distributions of the targets are deliberately given, involving uniformly distributed (see 3-1), randomly distributed between $(0,\pi]$ (see 3-2), randomly distributed between $[\pi,2\pi)$ (see 3-3), and randomly distributed in the whole circle (see 3-4).

The best 2 sequences out of 720 arrangements for each example are given in Table 4. Fig. 5 shows the plot of the TCA vs. cost on condition that the first rendezvous is taken into consideration and the targets are evenly and unevenly distributed. As illustrated in Fig. 5, the TCA decreases nonlinearly as the total cost increases, and the sequence with maximal TCA is no longer the best (in 3-2). As indicated in Table 4, the best sequence for each example is either (6,5,4,3,2,1) or (1,6,5,4,3,2), so do the second best sequence. It is necessary to compute the costs for both sequences and the one with the smaller cost is the best. As we know, when the first rendezvous is defined, the remaining arrangement can be obtained along the counter-orbit-wise direction. Based on the minimum-cost principle, the first rendezvous target would theoretically be the one either closely leads or lags the SSc. So, in general, the best mission sequence would be the one in

Table 3
Initial locations of the targets for Case 3.

Target number (θ rad)	1	2	3	4	5	6
Examples						
3-1	$2\pi/7$	$4\pi/7$	$6\pi/7$	$8\pi/7$	$10\pi/7$	$12\pi/7$
3-2	$\pi/12$	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	π
3-3	π	$13\pi/12$	$7\pi/6$	$4\pi/3$	$3\pi/2$	$5\pi/3$
3-4	$\pi/3$	$\pi/2$	$2\pi/3$	π	$3\pi/2$	$23\pi/12$

Table 4
Best two sequences for examples in Case 3.

Examples	The optimal sequence	The suboptimal sequence
3-1	(6,5,4,3,2,1)	(1,6,5,4,3,2)
3-2	(1,6,5,4,3,2)	(6,5,4,3,2,1)
3-3	(6,5,4,3,2,1)	(1,6,5,4,3,2)
3-4	(6,5,4,3,2,1)	(1,6,5,4,3,2)

counter-orbit-wise order starting from either the closest leading or the closest lagging target.

Specially, when the apogee altitude constraint of the transfer orbit is definitely given, the minimum transfer time is determined. When the phase angle between the SSc and the target is smaller than some critical value that is associated with the altitude constraint, even though the SSc transferred in minimal time, the neighboring target that is closely lagging behind would be missed out, and the SSc should take more time and energy to chase it afterwards. As a result, the best target that would be visited next is not necessarily the closest one lagging behind the SSc but may be the one lags with a minimal phase angle that is larger than the threshold. Next, experiments will be carried out to study the cases with small phase angles. To distinguish the following case from the previous ones, we define that if each phase angle between the targets is larger than certain threshold, these targets are sparsely distributed. That is to say, Cases 1–3 represent the sparsely distributed cases. In the following, we will study the general cases, i.e. the phase angle between the targets may be smaller than the threshold.

Case 4. In this case, we have one SSc and six targets. These seven objects are intensively distributed among some small intervals with some of the phase angles between the neighboring targets are smaller than the threshold. The SSc is initially at the slot of $\theta = 0$. And the initial locations of the targets for examples are given in Table 5. (The threshold of the phase angle can be calculated by Eq. (9), and based on the constraints in Table 1, the threshold of these experiments is calculated to be 0.0395 rad). The best six sequences for these three examples are given in Table 6.

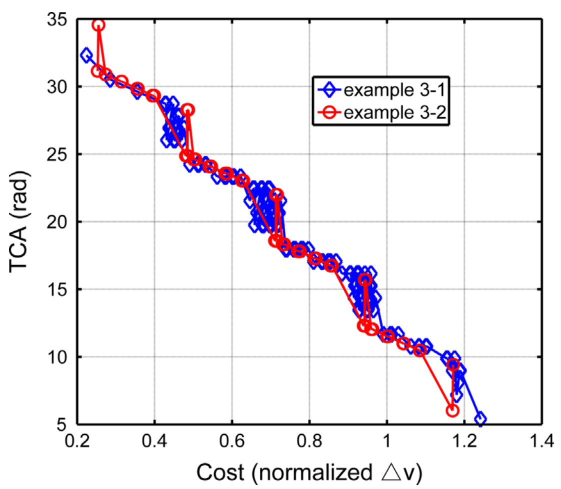

Fig. 5. TCA vs. cost for examples 3-1 and 3-2.

As indicated in Table 6, when some of the phase angles between the targets are smaller than the threshold, the optimal sequence is not necessary to be the counter-orbit-wise one. Instead, the optimal sequences must be the combinations of the counter-orbit-wise segments, and in each segment the phase angles between each target are larger than the threshold.

Summarizing the heuristic study above, two conclusions can be drawn. In the HOC problem studied in this paper, (1) when the targets are sparsely distributed on the orbit (each phase angle between the targets is larger than the threshold), the best mission sequence would be the one of counter-orbit-wise sequential order starting from either the closest leading target or the closest lagging target (CO law); (2) in general cases, the best mission sequence would be the combination of the counter-orbit-wise segments of targets, and in each segment, the phase angles between each target are larger than the threshold (CS law). These two laws are verified by numerous experiments above.

4.3. Rapid method for the HOC problem

Based on the CO law and the CS law, we present a Rapid Method (RM) to solve the HOC problem (see Fig. 6).

Note that, $q_1, q_2, \ldots, q_N$ are sequentially numbered in orbit-wise direction (see Fig. 1), and the elements in S_i are always arranged in counter-orbit-wise order.

5. Experiments

The SSc will visit the targets one by one. All of the objects initially run on a coplanar GEO orbit with the radius of 35,786 km. In this section, methods are employed to carry out the results. These methods are Exhaustive Search (ES), Simulated Annealing (SA), and Rapid Method (RM). And the CS law is used as a heuristic factor in SA.

The relevant constraints are given in Table 1. And the initial locations of the targets are given in Tables 3 and 5. The threshold of the phase angle in these experiments is calculated to be 0.0395 by Eq. (9). In Table 7, seven examples are carried out by ES, SA and RM, respectively. $\Delta\bar{v}$ is the normalized value of Δv ($\Delta\bar{v} = \Delta v\sqrt{r/\mu}$).

Obviously, ES can obtain the optimal result since it searches all of the possible arrangements. Comparing the results, we can find that (1) the SA programmed in this

Table 5
Initial locations of the targets for Case 4.

Target number (θ rad)	1	2	3	4	5	6
Examples						
4-1	1.6	1.62	1.64	4.7832	4.7932	4.8032
4-2	3.4832	3.6832	3.7032	3.7332	3.7432	3.7732
4-3	0.8	0.81	0.82	0.9	0.91	0.93

Table 6
Best six sequences of examples in Case 4.

Index	4-1		4-2		4-3	
	$\Delta\overline{v}$	Sequence	$\Delta\overline{v}$	Sequence	$\Delta\overline{v}$	Sequence
1	0.6950	$(6,2\mid 4,3\mid 5,1)$	0.4995	$(5,3\mid 6,4,2,1)$	0.5470	$(4,3\mid 5,2\mid 6,1)$
2	0.6950	$(6,2\mid 5,3\mid 4,1)$	0.4995	$(6,4,2\mid 5,3,1)$	0.5470	$(4,2\mid 5,3\mid 6,1)$
3	0.6950	$(6,3\mid 4,2\mid 5,1)$	0.5001	$(6,4,2,1\mid 5,3)$	0.5470	$(4,3\mid 6,1\mid 5,2)$
4	0.6950	$(6,3\mid 5,2\mid 4,1)$	0.5004	$(5,3,1\mid 6,4,2)$	0.5471	$(4,2\mid 6,1\mid 5,3)$
5	0.6950	$(6,1\mid 5,3\mid 4,2)$	0.7179	$(6,2\mid 5,3\mid 4,1)$	0.5473	$(4,3\mid 6,2\mid 5,1)$
6	0.6950	$(6,1\mid 4,3\mid 5,2)$	0.7180	$(5,3\mid 6,2\mid 4,1)$	0.5474	$(4,3\mid 5,1\mid 6,2)$

Algorithm: Rapid Method (RM)
Input: $Q=\{q_1,q_2,\ldots,q_N\}$
Output: the best plan $\mathbf{q}$

If the targets are sparsely distributed
  Compare $\mathbf{q}_1=(q_1,q_N,q_{N-1},\ldots,q_2)$ with $\mathbf{q}_2=(q_N,q_{N-1},\ldots,q_2,q_1)$, and $\mathbf{q}$ is the one with smaller cost
else
  $i=0$;
  while $Q\neq\varnothing$
    $i{+}{+}$; $S_i=\varnothing$;
    for each q_i **in** Q
      if each phase angle in $S_i=S_i+\{q_i\}$ is larger than the threshold
        $S_i=S_i+\{q_i\}$; $Q=Q-\{q_i\}$
      end if
    end for
  end while
  enumerate the possible combinations of $\{S_1,S_2,\ldots,S_n\}$, and find the best.
end if

Fig. 6. Algorithm of rapid method.

paper is effective to find out the optimal answer when the amount of the targets is small; (2) if targets are sparsely distributed, RM could find out the best result; if not, it still could provide a suboptimal result (see 3-1 and 3-3), and this is mainly due to that RM has only searched part of the feasible combinations. In RM, based on the classification principle, RM always searches the closest one that meet the phase angle constraint to extend the segment. However, maybe the combination of segments of larger phase angles is preferable. For instance, in 4-3, target 3 is firstly added to the segment starting with 6, but actually, the combination comprised of segment (6,1) is better.

Next, we will carry out the examples from Case 5 to test the effectiveness and the availability of the SA and RM in dealing with a larger amount of targets. The algorithm SA is the one presented in Table 7.

Case 5. We have 1 SSc and multiple targets. These objects are randomly distributed along the same GEO orbit (i.e. each phase angle is randomly given).

Fig. 7 compares the results computed by SA and RM, while Fig. 8 compares the computing time of two methods. Fig. 7 depicts that: (1) the results of SA are close to those of RM; (2) in most of the cases, the results of RM are better than those of SA. Fig. 8 illustrates that the time consumed by RM is awfully shorter than that of SA. To sum up, RM could get a preferable result in short time. We can draw a conclusion from the experiments that the Rapid Method, which is easy to implement, is effective and efficient in dealing with the HOC problem studied in this paper, especially for the cases with numerous targets.

6. Conclusions

The increasingly growing debris in GEO orbit has attracted the notice of the relevant authorities, and new projects have been issued to tackle these debris. The scheduling of debris removing in a coplanar GEO orbit is studied in this paper. Specifically, the SSc is considered to be initially on the circular orbit of the debris to be removed, and it should repeatedly rendezvous with the debris, capture it and then fly to the graveyard orbit to release it until all debris are removed. The minimum-cost, two-impulse phasing maneuver is used for each rendezvous. The objective is to find the best sequence with the minimum total Δv. Considering this mission as a hybrid optimal control problem, a mathematical model is proposed. Two heuristic laws of the problem are pointed out by analysis and experiments, (1) when targets are sparsely distributed, the optimal sequence is the counter-orbit-wise one starting from either the first target or the last target; (2) in general cases, the optimal result is the combination of the counter-orbit-wise segments in which each phase angle is larger than a threshold. A rapid method is then presented based on these laws. And numerous experiments show that the RM is effective and efficient in dealing with this problem especially for the cases with numerous targets. The CO law and the CS law are beneficial to the future work, e.g. in the cases of emergency, we can quickly reschedule the mission sequences based on these laws.

Note that, RM developed here is based on the heuristic laws (CO law and CS law). It is applicable only on condition that the disposal orbit is higher than the debris' orbit. Conventionally, GEO debris is disposed by way of tugging higher [2], while LEO debris by way of sending back to the atmosphere [4]. The former one is to heighten the debris' orbit, while the latter one is to lower. Therefore, RM is only applicable to GEO cases. LEO debris removing is another interesting topic. It would be studied in our future work.

No perturbations are considered in this paper. According to Shi and Friesen's work [21,22], in short time (less

Table 7
Results of the examples.

Examples	$\Delta\bar{v}$			Optimal sequence		
	ES	SA	RM	ES	SA	RM
3-1	0.2240	0.2240	0.2240	(6,5,4,3,2,1)	(6,5,4,3,2,1)	(6,5,4,3,2,1)
3-2	0.2532	0.2532	0.2532	(1,6,5,4,3,2)	(1,6,5,4,3,2)	(1,6,5,4,3,2)
3-3	0.4363	0.4363	0.4363	(6,5,4,3,2,1)	(6,5,4,3,2,1)	(6,5,4,3,2,1)
3-4	0.2188	0.2188	0.2188	(6,5,4,3,2,1)	(6,5,4,3,2,1)	(6,5,4,3,2,1)
4-1	**0.6950**	**0.6950**	**0.7033**	**(6,2ǀ4,3ǀ5,1)**	**(6,2ǀ4,3ǀ5,1)**	**(6,3,1ǀ5,2ǀ4)**
4-2	0.4995	0.4995	0.4995	(5,3ǀ6,4,2ǀ1)	(5,3ǀ6,4,2ǀ1)	(5,3ǀ6,4,2ǀ1)
4-3	**0.5470**	**0.5470**	**0.5516**	**(4,3ǀ5,2ǀ6,1)**	**(4,3ǀ5,2ǀ6,1)**	**(4,1ǀ6,3ǀ5,2)**

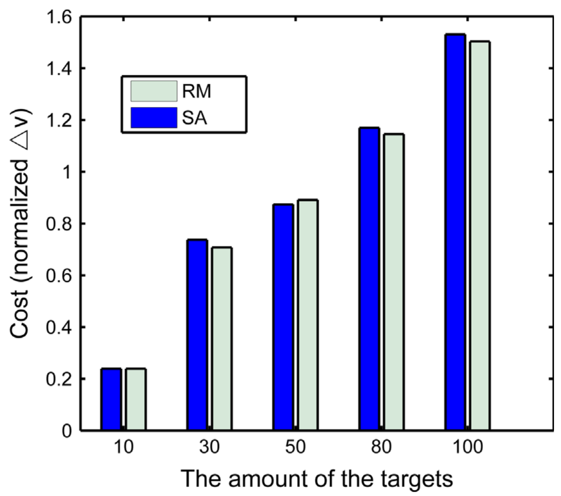

Fig. 7. Comparison of results.

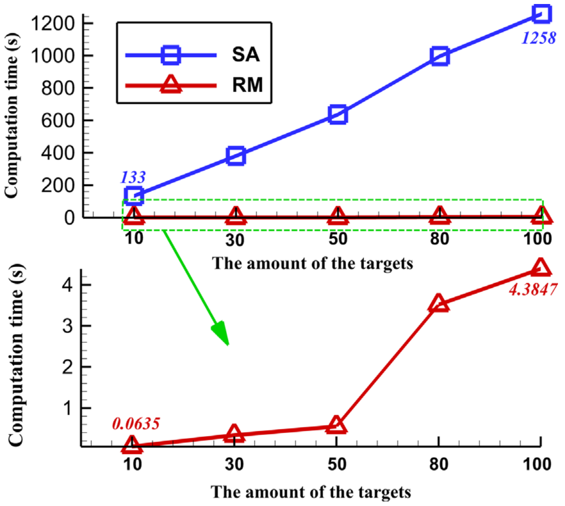

Fig. 8. Comparison of computation time.

than 1 year), the perturbation effect is small and insignificant on near GEO objects. Therefore, in our study, if the perturbations were considered, the conclusion should have nothing different. However, if the amount of the targets is remarkably large and the time for the mission is long, it is necessary to update the locations of the debris in time and then slightly modify the mission sequence.

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