JOURNAL OF SPACECRAFT AND ROCKETS
Vol. 56, No. 4, July–August 2019

Multitarget Rendezvous for Active Debris Removal Using Multiple Spacecraft

Jun Bang* and Jaemyung Ahn†
Korea Advanced Institute of Science and Technology, Daejeon 34141, Republic of Korea

DOI: 10.2514/1.A34344

An optimal multitarget rendezvous problem for an active debris removal (ADR) mission using multiple chaser spacecraft is proposed. The problem is formulated mathematically as a variant of the vehicle routing problem with profits, which determines a set of rendezvous sequences and associated trajectories that maximize the sum of profits collected by conducting tasks under the constraints on the required ΔV and the duration of the mission. A two-phase framework is developed to solve the proposed multitarget rendezvous problem. The framework can reduce the complexity of the problem, by decomposing it into two different types of optimizations (trajectory optimization and combinatorial optimization), and obtain its solution efficiently. In the first phase of the framework, a series of trajectory optimization problems for all departure/arrival debris pairs is solved to generate the elementary solutions, a database of rendezvous trajectories. The second phase combines the elementary solutions prepared in the first phase to obtain the final solution of the problem. The column-generation technique is adopted to explore the routes (rendezvous sequences) that are relevant to the optimization problem. The validity of the proposed problem formulation and the optimization framework is demonstrated through ADR case studies.

Nomenclature

$\boldsymbol{A}$	=	set of arcs in a graph
C, c	=	cost associated with a route/arc in a graph
$\boldsymbol{G}$	=	graph representing a multitarget problem
K	=	number of chaser spacecraft
N	=	number of debris candidates
$\boldsymbol{P}_{M0}$	=	optimal multitarget rendezvous problem for a given set of debris
$\boldsymbol{P}_M$	=	optimal multitarget rendezvous problem using multiple spacecraft
p	=	profit assigned to a debris
$\boldsymbol{Q}$	=	set of debris candidates; $\{1, \ldots, N\}$
t_a	=	time to arrive at a node
t_d	=	time to depart from a node
$t_{\max}$	=	maximum mission duration
$\boldsymbol{V}$	=	set of nodes in a graph
$\boldsymbol{x}$	=	decision variable for $\boldsymbol{P}_{M0}$; $[\cdots, x_{(i,j)}, \cdots]$
$\boldsymbol{y}$	=	decision variable for $\boldsymbol{P}_M$; $[\cdots, y_k, \cdots]$
Δt_{ser}	=	service time required on a debris
λ	=	dual variable
Π, π	=	total profit associated with a set of routes or a route
$\boldsymbol{\tau}_a$	=	$[t_{a,0}, \ldots, t_{a,N}]$
$\boldsymbol{\tau}_d$	=	$[t_{d,0}, \ldots, t_{d,N}]$
Ω	=	index set of all possible routes
Ω_c	=	index set of routes in the current problem
Ω_f	=	index set of all feasible routes

Subscripts

i	=	index representing the departure debris
j	=	index representing the arrival debris
k	=	index representing a route (column)

Received 3 July 2018; revision received 15 January 2019; accepted for publication 31 January 2019; published online 28 February 2019.

*Postdoctoral Research Associate, Department of Aerospace Engineering, 291 Daehak-Ro.

†Associate Professor of Aerospace Engineering, 291 Daehak-Ro; jaemyung.ahn@kaist.ac.kr. (Corresponding Author).

I. Introduction

THE near-Earth space environment is becoming more and more polluted, as evidenced in part by the fact that about 75% of approximately 20,000 currently tracked on-orbit objects are space debris [1]. Recent studies on the evolution of the space debris population (e.g., the cascading effect referred to as the "Kessler syndrome") indicate that the density of debris, in low Earth orbit (LEO) in particular, has already reached a critical state in which the space environment is unstable. That is, the number of debris will keep increasing because of the mutual collisions between existing debris, even without any further launch activities [2–4].

Active debris removal (ADR) may be the only option to preserve the space environment for future generations. Liou and Johnson [5] and Lewis et al. [6] reported that at least 5–10 debris in LEO (larger than 10 cm) must be removed annually to prevent the further growth of the space debris population. In a multitarget rendezvous (MTRV) problem, one or multiple chaser spacecraft sequentially visit multiple debris moving in different orbits; this has received considerable attention for ADR mission planning because of its economic feasibility. The MTRV problem determines 1) the set of debris to remove, 2) their visiting sequence, and 3) associated rendezvous trajectories simultaneously.

The use of multiple chaser spacecraft can be an effective option to improve the performance of the mission. Previous studies on the distribution of space debris identified several regions with specific altitude/inclination ranges in which the density of debris is high and catastrophic collisions are likely to occur [7,8]. In addition, the debris belonging to an identified region are distributed so that their right ascension of ascending node (RAAN) values differ significantly and change over time because of the gravitational perturbation of the Earth (e.g., J_2 term). Bonnal et al. proposed a mission architecture composed of a large launch vehicle and four identical chaser spacecraft for removal of debris with similar inclination/altitude and diverse RAAN values [9]. The use of multiple spacecraft departing from orbits with different RAAN values can save the rendezvous cost required to offset large differences in node direction between debris pairs. For effective ADR mission planning, a preliminary study on the use of multiple spacecraft with respect to the number and size of spacecraft needs to be conducted.

Bang and Ahn formulated the MTRV as a variant of the traveling salesman problem (TSP) that obtains the minimum-cost rendezvous sequence and associated trajectories to visit all targets that are given as inputs [10]. This research extends the work by Bang and Ahn by proposing a vehicle routing problem with profits (VRPP) formulation of the MTRV that can select the debris based on the amount of profit obtainable by rendezvous and consider the use of multiple chaser spacecraft. A two-phase framework to solve the proposed problem by reducing the complexity inherent in it to find a good solution easily is developed. The first phase creates the database of "elementary solutions" that are the optimal solutions for single-target rendezvous problems associated with all departure/arrival debris pairs. The second phase combines the solutions prepared in the first phase to determine the optimal rendezvous sequences and associated costs for the MTRV.

The key contribution of this work is threefold. First, an optimal MTRV problem that maximizes the total profits obtainable by visiting target debris using multiple spacecraft is proposed and mathematically formulated. The proposed problem is distinguishable from the subjects discussed in previous studies (e.g., the Ninth Global Trajectory Optimization Competition, GTOC9) in that it does not necessarily remove all given set of debris and focuses on maximizing the risk reduction obtainable by the ADR mission [11]. Second, a solution procedure that can smartly handle the complexity of the proposed problem is developed. The proposed framework decomposes the original MTRV problem classified as the mixed integer nonlinear programming (MINLP) into nonlinear programming (NLP) problems (trajectory optimization) and an integer linear programming (ILP) problem (optimal target selection and sequencing) using two distinct phases. Finally, two realistic ADR case studies are conducted to demonstrate the effectiveness of the proposed problem. The effect of using multiple spacecraft is analyzed through the case study, whose results can provide a criterion on determining the number and sizes of spacecraft considering the characteristics of the mission.

The rest of this study is structured as follows. Section II presents a review of the past studies about the MTRV and/or ADR problems. Section III provides the description of the MTRV considered in this paper and its mathematical formulation, a variant of the VRPP. Section IV introduces the two-phase framework to solve the proposed problem. Two case studies that can demonstrate the validity of the problem formulation and solution procedure are presented in Sec. V (Iridium 33 debris cluster) and Sec. VI (the Kessler run). Finally, Sec. VII provides the conclusion of the study and discusses potential future work.

II. Literature Review

The optimal MTRV problem involves two different types of optimization problems: the optimal selection/sequencing of the rendezvous target out of a given candidate set (combinatorial optimization), and the trajectory optimization to determine rendezvous trajectories and associated cost (real-valued optimization). This problem is classified as a MINLP, which is one of most complicated and difficult-to-solve problem categories. Several previous studies on the MTRV problem tried to address the complexity of the problem using various approaches. Barbee et al. introduced a series method that can find a good, but not necessarily optimal, rendezvous sequence with relatively low computational load [12]. Braun et al. estimated the cost of the mission based on two rendezvous scenarios and explored the fixed-length rendezvous sequence using a brute-force approach [13]. Cerf introduced a three-step transfer strategy based on the RAAN drift and created the rendezvous cost model using the response surface method [14,15]. Simulated annealing was used to obtain the optimal rendezvous sequence for a multiple-target ADR mission. Yu et al. applied the hybrid optimal control framework composed of the inner loop and the outer loop to handle different variable types that appear in the optimal ADR scheduling problems [16,17]. The particle swarm optimization (PSO) and the exhaustive search methods were adopted for the two loops, respectively. Bérend and Olive [18] and Shen et al. [19] formulated the ADR problem as the time-dependent TSP and explored the optimal rendezvous sequence using the ant colony optimization and the branch-and-bound algorithm.

Most of these studies focused on minimizing the total cost to visit predetermined debris targets using a single chaser spacecraft; only a small fraction considered the profit-based debris selection or the use of multiple spacecraft for ADR. Izzo et al. introduced two types of TSP variants, the "classic" TSP and the (static and dynamic) "city selection" TSP, to maximize the sum of profit values assigned to the debris under a ΔV constraint [20]. Stuart et al. applied the multi-agent coordination procedure based on auction and bidding to consider multiple chaser spacecraft for an ADR mission, but they did not consider the J_2 effect and the time-varying rendezvous cost [21]. To the best of our knowledge, there is no previous study considering both the profit-based debris selection and the multiple-spacecraft issue.

III. Problem Definition

This section provides the description and the associated mathematical formulation of the maximum-profit multitarget rendezvous with multiple spacecraft, whose concept is shown in Fig. 1. Given a set of debris candidates, the problem seeks the sequences of visits to rendezvous with a subset of debris for their removal using multiple spacecraft to maximize the benefit (e.g., collection of risk reduction obtained by the mission).

The aforementioned optimization problem is complex and difficult to solve with a single-step formulation and solution procedure. The following subsections will define the problem using three steps: 1) the single-target optimal rendezvous problem, 2) the TSP version optimal MTRV problem, and 3) the VRPP version optimal MTRV problem. Two rendezvous strategies considered in this study and the procedure to determine the rendezvous trajectories between debris pairs are introduced. The formulation used to describe the minimum-cost

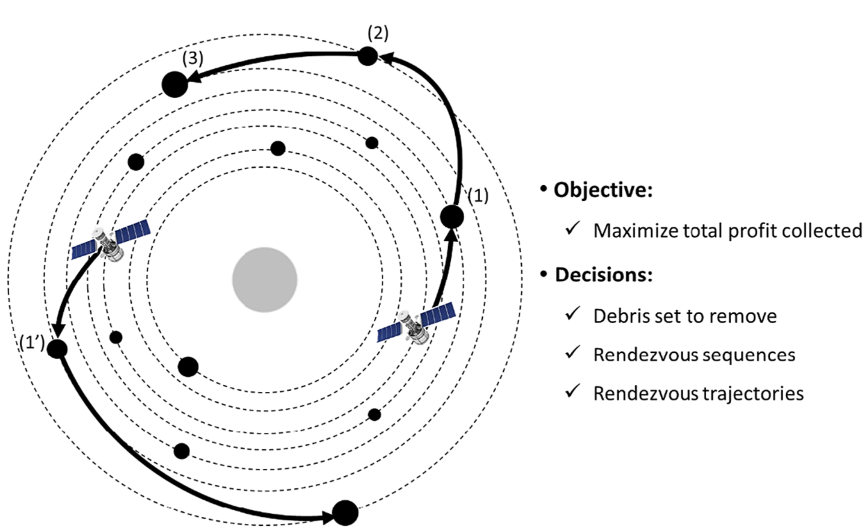


Fig. 1 Concept of maximum-profit multitarget rendezvous.

MTRV problem for a given debris set, which is TSP version ($\boldsymbol{P}_{M0}$) and the subproblem of the main problem, is presented. Finally, the maximum-profit MTRV using multiple spacecraft, the VRPP version ($\boldsymbol{P}_M$) and the main problem addressed in this study, is formulated.

A. Strategies for Optimal Single-Target Rendezvous

In-space rendezvous requires two different resource types: time and fuel (i.e., the velocity increment ΔV). ΔV is determined depending on the transfer geometry of the rendezvous and is the function of the departure and arrival times. In this study, it was presumed that the total mission duration (to complete all rendezvous transfers for multitarget ADR) is a key factor for selecting the rendezvous strategy. If the mission duration is sufficiently long, the strategy with a low fuel consumption with long transfer time will be a feasible option. Otherwise, the strategy that completes the rendezvous task within a short time, even with a relatively large ΔV value, should be adopted.

Two different mission types were assumed: the short mission (mission time no longer than one week) and the long mission (mission time a few weeks to years). A two-impulse Lambert rendezvous and a three-step transfer using the RAAN drift have been adopted as the strategies for the short and long missions, respectively.

Most debris of interest are orbiting in the LEO with altitudes ranging from 600 to 1000 km [7]. In this altitude range, the orbital perturbation caused by the flattening of the Earth is significant, and the rates of change in the orbital elements considering the first zonal coefficient (J_2) are expressed as follows [17]:

$$\dot{\Omega} = -\frac{3}{2} J_2 \sqrt{\mu} R_E^2 \frac{\cos i}{a^{7/2}(1-e^2)^2} \tag{1}$$

$$\dot{\omega} = \frac{3}{4} J_2 \sqrt{\mu} R_E^2 \frac{5\cos^2 i - 1}{a^{7/2}(1-e^2)^2} \tag{2}$$

$$\dot{M} = \frac{3}{4} J_2 \sqrt{\mu} R_E^2 \frac{3-\cos^2 i}{a^{7/2}(1-e^2)^{3/2}} \tag{3}$$

where μ and R_E are the gravitational parameter and the mean equatorial radius of Earth, respectively, and a, e, and i are the semimajor axis, eccentricity, and inclination angle of the debris orbit, respectively. These orbital precessions change the relative orbital orientation between debris orbits that accounts for a significant portion of the rendezvous cost. Izzo et al. [20] provide an analysis of the effectiveness of using the orbital precession for the rendezvous in missions with long duration. The analysis indicates that the cost of rendezvous (i.e., required ΔV) does not significantly change for the first week, but the cost variation increases significantly after a couple of weeks, which suggests that the effect of J_2 perturbation can be used to reduce the rendezvous cost of missions longer than two weeks.

A two-impulse Lambert rendezvous using the solution of the Lambert's boundary-value problem is selected as the rendezvous strategy for short-term ADR missions. At the departure time ($t = t_d$), the first impulse is applied on the chaser spacecraft to put it into the transfer orbit, which ensures that the chaser meets with the target at the arrival time ($t = t_a$). When the chaser meets with the target, the second impulse is imposed to make its velocity identical to that of the target. The total velocity increment required to complete the rendezvous is obtained by solving a multiple-revolution Lambert's problem [22]. The rendezvous trajectory between the chaser and the target is uniquely determined as a function of departure/arrival times of the rendezvous. A mathematical formulation of the optimal Lambert rendezvous problem for the ADR was presented in the authors' previous research [10].

For long-duration ADR missions, a three-step transfer using RAAN drift is adopted for rendezvous [15]. This strategy uses the drift orbit, which is an intermediate orbit that can accelerate or decelerate the natural RAAN drift because of the J_2 effect. It is composed of two independent orbital transfers and a waiting period between them. At the departure time ($t = t_d$), the chaser conducts a Hohmann transfer from the initial chaser orbit to the drift orbit, which ensures that the RAAN of the chaser aligns with that of the target orbit at the arrival time ($t = t_a$). When the alignment of RAAN is obtained, the chaser spacecraft transfers from the drift orbit to the target orbit using the second Hohmann transfer maneuver. This strategy provides the near-optimal ΔV (simplified as a function of departure/arrival times) by naturally minimizing the amount of plane change. Cerf [15] provides the details of the trajectory optimization problem based on this strategy.

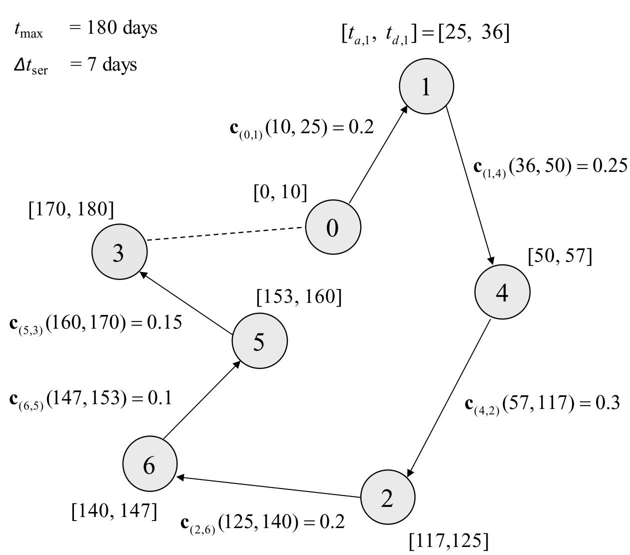
B. Minimum-Cost Multitarget Rendezvous

This subsection introduces a minimum-cost rendezvous sequence optimization problem to visit all given subset of debris using a single spacecraft ($\boldsymbol{P}_{M0}$), which is one of the key elements used to define the VRPP-type multitarget rendezvous using multiple chaser spacecraft ($\boldsymbol{P}_M$).

A graph representation of the MTRV problem is given that involves 1) nodes (target debris), 2) arcs (rendezvous trajectories), and 3) costs associated with the arcs (ΔV values for transfer). Let $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{A})$ be a directed graph where $\boldsymbol{V} (= \{0\} \cup \boldsymbol{Q})$ is the set of nodes (0: chaser at initial orbit, $\boldsymbol{Q} = \{1, \ldots, N\}$: set of target debris) and $\boldsymbol{A} = \bigcup_{(i,j)\in V\times V, i\neq j} a_{(i,j)}$, is the set of arcs connecting the nodes. Solving the MTRV problem for a given set of debris is equivalent to determining a route in the graph that starts at node 0 and visits all other nodes. Figure 2 illustrates an instance of the MTRV problem and its sample solution. The MTRV problem described here is different from the traditional TSP, primarily because the nodes are moving, and the cost associated with an arc varies with departure/arrival times of rendezvous, which necessitates the temporal constraint between successive rendezvous tasks.

The problem includes three sets of decision variables. The binary variable $x_{(i,j)}$ equals 1 if the spacecraft travels from target i to target j, and 0 otherwise. Variables $t_{a,i}$ and $t_{d,i}$ denote the arrival time and departure time to/from target i, respectively. The cost (i.e., ΔV) for the rendezvous trajectory from target i to target j with respect to departure time $t_{d,i}$ and arrival time $t_{a,j}$ is expressed as $c_{(i,j)}(t_{d,i}, t_{a,j})$.

The optimal MTRV problem to determine the minimum-cost rendezvous sequence and associated trajectories ($\boldsymbol{P}_{M0}$) is defined as follows.



$x_{(0,1)} = x_{(1,4)} = x_{(4,2)} = x_{(2,6)} = x_{(6,5)} = x_{(5,3)} = x_{(3,0)} = 1$, and 0 otherwise

$\boldsymbol{\tau}_{\mathbf{a}}^* = [0, 25, 117, 170, 50, 153, 140]$

$\boldsymbol{\tau}_{\mathbf{d}}^* = [10, 36, 125, 180, 57, 160, 147]$

$C^* = 1.2\ km/s$

Fig. 2 Sample solution of the MTRV problem for a given set of debris ($N = 6$).

$\boldsymbol{P}_{M0}$: optimal MTRV problem for a given set of debris (TSP formulation):

$$\min_{x,\tau_a,\tau_d} C = \sum_{i\in V}\sum_{j\in V} c_{(i,j)}(t_{d,i}, t_{a,j})x_{(i,j)} \tag{4}$$

subject to

$$\sum_{i\in V} x_{(i,j)} = 1, \quad \forall\ j \in \boldsymbol{V} \tag{5}$$

$$\sum_{j\in V} x_{(i,j)} = 1, \quad \forall\ i \in \boldsymbol{V} \tag{6}$$

$$t_{a,i} + \Delta t_{\text{ser}} \le t_{d,i}, \quad \forall\ i \in \boldsymbol{Q} \tag{7}$$

$$t_{d,i}x_{(i,j)} \le t_{a,j}, \quad \forall\ i \in \boldsymbol{V}, \quad \forall\ j \in \boldsymbol{Q} \tag{8}$$

$$0 \le t_{a,i} \le t_{\max} - \Delta t_{\text{ser}}, \quad \forall\ i \in \boldsymbol{V} \tag{9}$$

$$x_{(i,j)} \in \{0, 1\}, \quad \forall\ i, j \in \boldsymbol{V} \tag{10}$$

The objective function presented in Eq. (4) is to minimize the sum of rendezvous costs belonging to the path. Variables $\boldsymbol{x}(=[\cdots, x_{(i,j)}, \cdots]^T)$, $\boldsymbol{\tau}_a(=[t_{a,0}, \ldots t_{a,N}]^T)$, and $\boldsymbol{\tau}_d(=[t_{d,0}, \ldots t_{d,N}]^T)$ in Eq. (4) are decision vectors. Constraints expressed as Eqs. (5) and (6) ensure that the arrival at and departure from each target occur exactly once. Equations (7) and (8) represent the temporal constraints that the departure from each node should take place after the completion of service at the same node and no later than the arrival at the next node. Equation (9) is the constraint on the maximum mission duration, and Eq. (10) is the binary constraint on decision variable $x_{(i,j)}$.

C. Maximum-Profit Multitarget Rendezvous with Multiple Spacecraft

A VRPP-type multitarget rendezvous using multiple chaser spacecraft $\boldsymbol{P}_M$ is proposed in this subsection. In the graph $\boldsymbol{G}$ discussed in the previous subsection, a set of profit values p_i ($i = 1, \ldots, N$) is assigned to nodes (target debris). This indicates that visits to some targets are more important than visits to others (different rendezvous priority).

Solving the VRPP-type MTRV problem $\boldsymbol{P}_M$ is equivalent to the process to find the best combination of routes on the graph that maximizes total profit collected. Only the minimum-cost route is considered as a viable route for a given set of nodes; any other inefficient routes are excluded. The number of all possible routes equals the total number of subsets of $\boldsymbol{Q}$, which is 2^N. Let $\Omega = \{0, \ldots, 2^N - 1\}$ be an index set of all possible subsets of $\boldsymbol{Q}$ and $\boldsymbol{Q}_k$ be the subset corresponding to index k. The index set of all feasible routes (Ω_f) is defined as

$$\Omega_f = \{k \in \Omega | C^*(\boldsymbol{Q}_k) \le \Delta V_{\max}\} \tag{11}$$

where $C^*(\boldsymbol{Q}_k)$ is the minimum ΔV required to visit all nodes in $\boldsymbol{Q}_k$ (i.e., the objective function of $\boldsymbol{P}_{M0}$ associated with target set $\boldsymbol{Q}_k$), and $\Delta V_{\max}$ is the maximum velocity increment available. The VRPP-type optimal MTRV problem is formulated mathematically as follows.

$\boldsymbol{P}_M$: optimal MTRV problem using multiple spacecraft (VRPP formulation):

$$\max_y \Pi = \sum_{k\in\Omega_f} \pi_k y_k \tag{12}$$

subject to

$$\sum_{k\in\Omega_f} a_{i,k} y_k \le 1, \quad \forall\ i \in \boldsymbol{Q} \tag{13}$$

$$\sum_{k\in\Omega_f} y_k \le K \tag{14}$$

$$y_k \in \{0, 1\}, \quad \forall\ k \in \Omega_f \tag{15}$$

In this formulation, $\boldsymbol{y}(=[y_1, \cdots, y_{|\Omega_f|}]^T)$ is the vector collecting the decision variables. The binary decision variable y_k takes the value of 1 if the route with an index k (which is determined by the subset $\boldsymbol{Q}_k$) is included in the solution and 0 otherwise. Profit

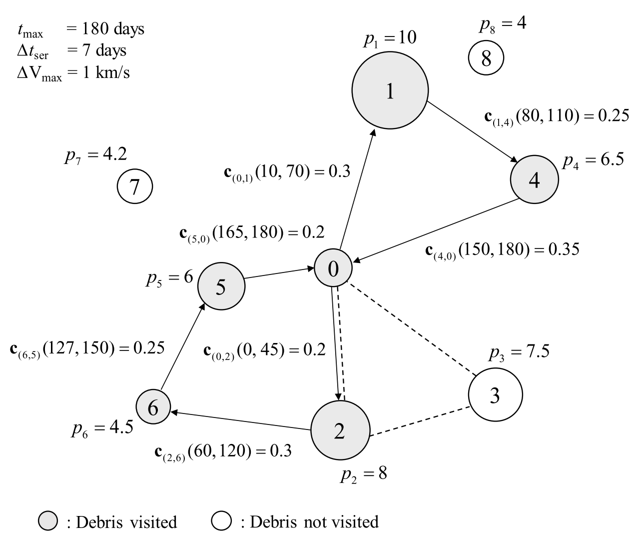
$$\pi_k = \sum_{i\in Q} a_{i,k} p_i$$

denotes the sum of profits for all nodes in $\boldsymbol{Q}_k$, and K is the number of spacecraft used for the mission. A constant $a_{i,k}$ equals 1 if node i is included in $\boldsymbol{Q}_k$ and 0 otherwise. The objective function of the problem expressed in Eq. (12) is to maximize the total profit associated with the selected routes. Equation (13) guarantees that each node is visited at most once, and Eq. (14) limits the number of available spacecraft. Finally, the binary constraint is expressed in Eq. (15). Figure 3 presents an instance of the VRPP-type optimal MTRV problem and its sample solution.

IV. Solution Procedure

Solving the VRPP-type MTRV problem $\boldsymbol{P}_M$ is challenging primarily because the size of route set increases exponentially with the number of debris candidates $|\Omega| = 2^N$. In addition, assessing the feasibility of each route requires solving independent $\boldsymbol{P}_{M0}$ that includes both binary variables $\boldsymbol{x}$ and continuous variables $\boldsymbol{\tau}_d$ and $\boldsymbol{\tau}_a$, which is classified as the MINLP. Consequently, an enormous number of complex optimization problems must be solved to obtain the final solution of $\boldsymbol{P}_M$.

In this study, two different techniques are adopted to solve $\boldsymbol{P}_M$ efficiently. The first technique is to reduce the complexity of an



$\Omega = \{0, \ldots, 2^8 - 1\}, \quad k \in \Omega$

$Q_{k_1} = \{0,1,4\}$	$\pi_{k_1} = 16.5$	$C^*(Q_{k_1}) = 0.9$	$k_1, k_2 \in \Omega_f$
$Q_{k_2} = \{0,2,5,6\}$	$\pi_{k_2} = 18.5$	$C^*(Q_{k_2}) = 0.95$	
$Q_{k_3} = \{0,2,3\}$	$\pi_{k_3} = 15.5$	$C^*(Q_{k_3}) > 1$	$k_3 \notin \Omega_f$

$\vdots$

$y_{k_1} = y_{k_2} = 1$, and 0 otherwise

$\Pi^* = 35$

Fig. 3 Sample solution of the MTRV problem using multiple spacecraft ($N = 8$ and $K = 2$).

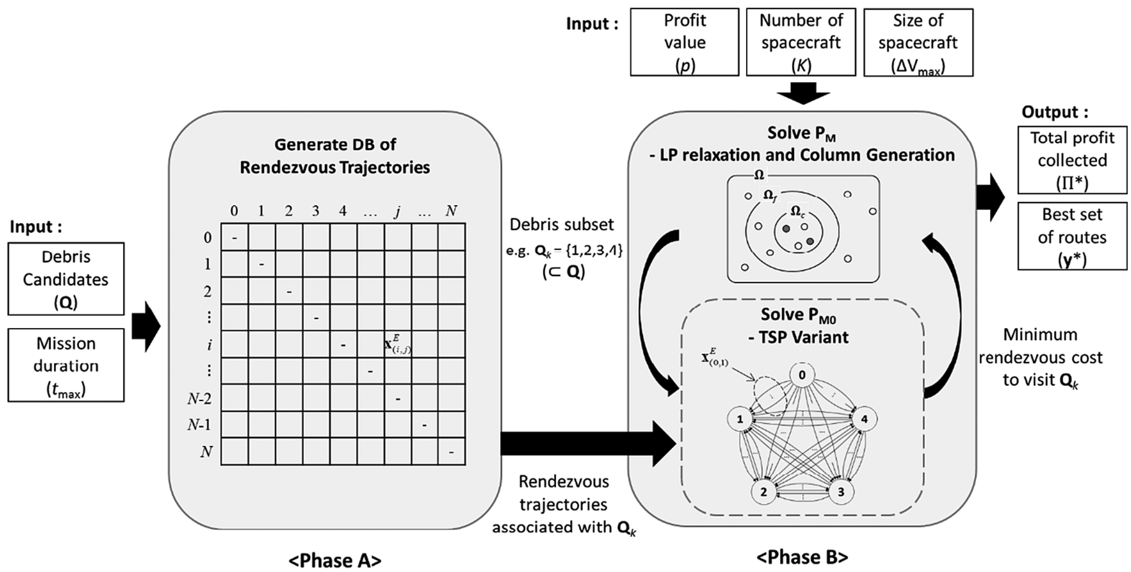


Fig. 4 Solution method to solve the MTRV problem using multiple spacecraft P_M.

individual P_{M0}. If the solution of P_{M0} can be obtained with less computational effort, the overall time to solve the main problem P_M can be reduced as well. In this research, a two-phase framework that can provide the near-optimal solution of P_{M0} using the database of rendezvous trajectories is applied. The second technique is to reduce the number of routes considered in the problem. Although the number of routes potentially considered for the problem is truly huge (2^N), only a few of them are included in the optimal solution. The column-generation technique, which is widely used for large-scale combinatorial optimization problems, is adopted to solve P_M without enumerating all columns. Figure 4 illustrates the outline of the proposed solution procedure used to solve the problem. The following subsections provide detailed explanations on the phases/steps of the solution procedure.

A. Solution Framework for Maximum-Profit Multitarget Rendezvous with Multiple Spacecraft

The two-phase framework and related algorithms proposed in [10] with proper modification is adopted and used to solve the optimal MTRV problem proposed here. The first phase solves a series of optimal single-target rendezvous problems for all departure/arrival debris pairs to generate the database of efficient rendezvous trajectories. The rendezvous cost is calculated as a function of departure/arrival times, as explained in Sec. III. The objective of the optimal single-target rendezvous problem is to minimize the sum of ΔV values associated with the rendezvous by determining the departure time t_d and the arrival time t_a. For each pair of debris, the solution space partitioning technique is used to split the whole exploration space, a two-dimensional space consisting of t_d and t_a, into multiple subspaces. A gradient-based algorithm is applied in each subspace to obtain the minimum-cost rendezvous trajectory within the boundary (the local minimum). This procedure identifies several locally optimal rendezvous trajectories having different departure/arrival times, which are stored in the database.

The second phase combines the rendezvous trajectories prepared in the first phase to obtain the solutions of subproblems P_{M0} associated with a debris subset and iteratively find the solution of the main problem P_M. There are multiple rendezvous trajectories for each departure/arrival debris, which are interpreted as "multiple arcs between two nodes" in the graph. In the framework, the subproblem P_{M0} is converted into a new variant of TSP that considers the multiple arcs and time window constraints, which is solved with an integer linear programming (ILP) solver. The mathematical formulation of the converted TSP variant and the validity of the solution framework are presented in [10].

The framework decomposes the original MINLP problem into a series of nonlinear programming (NLP) and an ILP. All computations to obtain optimal rendezvous trajectories, which is the NLP part of the framework, are conducted in advance. Once the database is prepared, there is no need to perform any trajectory optimization tasks again. Several heuristic algorithms to address the complexity of the original MTRV problem can be found in the literature. Although these algorithms successfully find the optimal solution of P_{M0}, all the design variables, including trajectory-related ones, must be reoptimized whenever evaluating the feasibility of a new route. However, the proposed approach adopts an ILP that is much easier to handle than the original problem.

It should be noted that there is a tradeoff between the optimality of the solution and the computational efficiency. Adding more trajectories to the database generated in the first phase can increase the possibility of finding the final solution closer to the true optimum but requires more computations to identify candidate trajectories (the first phase) and obtain the final solution (the second phase).

B. Column Generation

This subsection explains the procedure of the column-generation algorithm to obtain the solution of P_M without enumerating all routes (columns). The algorithm starts with formulating the linear programming (LP) relaxation of P_M, called the master problem (MP), by relaxing the binary constraints expressed as Eq. (15). Because the MP still includes a prohibitively large number of decision variables, a restricted master problem (RMP) including only a subset of feasible columns ($\Omega_c \subseteq \Omega_f$) is defined as follows:

Restricted master problem (RMP) for P_M:

$$\max_{y_c} \Pi = \sum_{k \in \Omega_c} \pi_k y_k \tag{16}$$

subject to

$$\sum_{k \in \Omega_c} a_{i,k} y_k \leq 1, \quad \forall\ i \in \boldsymbol{Q} \tag{17}$$

$$\sum_{k \in \Omega_c} y_k \leq K \tag{18}$$

$$y_k \geq 0, \quad \forall\ k \in \Omega_c \tag{19}$$

In this formulation, $\boldsymbol{y}_c (= [y_1, \cdots, y_{|\Omega_c|}]^T)$ is the vector collecting the decision variables associated with columns in Ω_c. Variable y_k is 1 if route k (i.e., column k) is used and 0 otherwise. Constraints

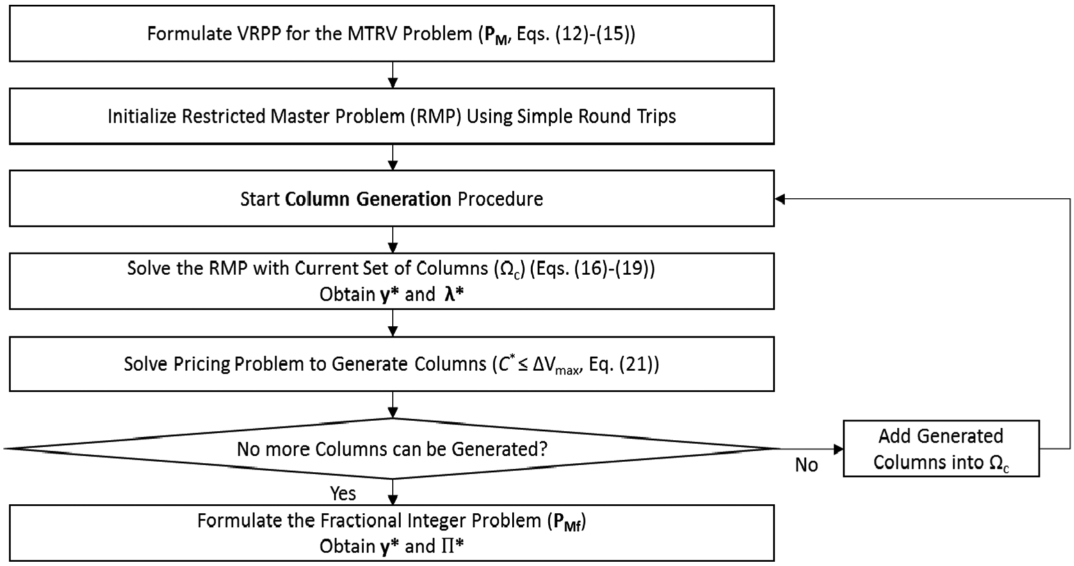


Fig. 5 Flowchart of column-generation procedure.

expressed in Eqs. (13–15) of the original MTRV problem $\boldsymbol{P}_M$ are imposed for RMP as Eqs. (17–19).

The next step is to add new columns that can improve the objective function of RMP into the current column subset (Ω_c). Let λ_i ($i = 1, \ldots, N$) and λ_0 denote the dual variables associated with constraints (17) and (18), respectively. According to the property of LP duality, the optimality of the RMP is not guaranteed if there exists any route in Ω_f that satisfies the following inequality:

$$\sum_{i\in Q} a_{i,k}\lambda_i + \lambda_0 < \pi_k \tag{20}$$

which can be rewritten as

$$\sum_{i\in Q} a_{i,k}\bar{p}_i + \lambda_0 < 0 \tag{21}$$

where $\bar{p}_i = (\lambda_i - p_i)$ is the "adjusted reward" of node i. The pricing problem aims at identifying columns that are not included in Ω_c and satisfying the inequality expressed in Eq. (21). The algorithm proposed by Butt and Ryan [23] is adopted to solve the pricing problem and generate new columns. In their algorithm, a subset of nodes is greedily constructed based on the order of adjusted reward representing the attractiveness of the node. Because the pricing problem is a minimization problem, nodes with lower $\bar{p}_i$ are more desirable to be included in a route. Therefore, all nodes are ranked in ascending order of their adjusted reward values, and only the nodes with negative adjusted reward are considered as follows:

$$\bar{p}_{(1)} \leq \bar{p}_{(2)} \leq \ldots \leq \bar{p}_{(n_p)} < 0 \tag{22}$$

where (i) denotes the index of a node with the ith lowest adjusted reward, and n_p is the number of nodes with negative adjusted reward. The routes consisting of these nodes are evaluated in the lexicographical order of i [24]. The feasibility of each route must be checked by solving the corresponding $\boldsymbol{P}_{M0}$. If the route is feasible ($\in \Omega_f$) and satisfies Eq. (21), then the column associated with the route is added into Ω_c. Otherwise, the column is discarded.

Figure 5 is the flowchart describing the column-generation procedure. Initially, the routes for simple round trips that visit only a single node (i.e., target debris) are included in the column set. For each iteration, the RMP associated with the current set of columns is solved to calculate dual variables. Then, the pricing problem defined with updated dual variables is solved to generate new columns. The procedure continues until no more additional columns can be identified. Finally, the fractional integer problem $\boldsymbol{P}_{Mf}$ is formulated by imposing binary variable constraints on the decision variables in Eq. (19). Although the solution of RMP with the final column set is the true optimal solution of MP, it does not guarantee that the solution of $\boldsymbol{P}_{Mf}$ is the optimal solution of $\boldsymbol{P}_M$. Instead, the optimality gap is calculated to measure the relative quality of the final integer solution as follows [24]:

$$G_{\text{opt}} = \frac{\Pi^*_{LP} - \Pi^*_{Mf}}{\Pi^*_{Mf}} \tag{23}$$

where Π^*_{LP} and Π^*_{Mf} denote the optimal objective function of RMP and $\boldsymbol{P}_{Mf}$ associated with the final column set, respectively.

V. Case Study 1: Iridium 33 Debris Cluster

We present two case studies that demonstrate the validity of the proposed problem formulation and solution framework: 1) multi-target rendezvous for IRIDIUM 33 debris cluster, and 2) the "Kessler run" case, which is the subject of the ninth Global Trajectory Optimization Competition (GTOC9) [11]. All the algorithms were implemented on a machine with an Intel Core i7 CPU processor and 8 GB RAM under the 64 bit Windows 7 operating system.

Each of the following subsections explains the problem parameters used for the first case study, the rendezvous trajectory database creation procedure, and the solutions of the optimal MTRV problem $\boldsymbol{P}_M$ with analyses of the effects of using multiple spacecraft and adopting different objective functions.

A. Problem Parameters

Randomly selected 100 debris in Iridium 33 debris cluster were used as the debris candidates for the case study. The orbital elements and the radar cross section (RCS) area for these debris were retrieved from the space-track catalog[‡] and are presented in Table 1. The debris are ranked in descending order of their RCS value; hence, the smaller debris index indicates the larger RCS value. The debris are distributed over a wide range of RAAN (between 69.2245 and 263.7066 deg), whereas their inclination angles are almost same (between 86.00 and 86.43 deg). The orbits are propagated using the simplified general perturbation 4 model to address the long-term J_2 effect on the orbital elements, especially on the RAAN [25].

Cases with two different mission durations, seven days (case A) and 360 days (case B), were considered to demonstrate the effects of mission duration and applied rendezvous strategy. The service times required on each debris for the two cases were set to 6 h and

[‡]Data available online at http://www.space-track.org [retrieved 1 December 2017].

Table 1 List of debris candidates and their RCS values

ID	NORAD ID	RCS, m^2	ID	NORAD ID	RCS, m^2	ID	NORAD ID	RCS, m^2	ID	NORAD ID	RCS, m^2
1	33886	0.7850	26	33854	0.0700	51	34517	0.0473	76	34097	0.0332
2	33777	0.5585	27	33965	0.0700	52	34521	0.0461	77	33853	0.0330
3	33776	0.5166	28	34107	0.0700	53	34105	0.0460	78	34643	0.0330
4	33773	0.4937	29	33876	0.0683	54	33881	0.0440	79	33954	0.0320
5	34071	0.3769	30	34160	0.0670	55	34146	0.0440	80	34535	0.0320
6	33850	0.3580	31	34350	0.0640	56	34529	0.0440	81	34987	0.0320
7	33775	0.1480	32	33887	0.0620	57	33860	0.0430	82	33961	0.0310
8	33862	0.1407	33	34143	0.0611	58	34079	0.0430	83	33962	0.0310
9	33772	0.1340	34	34098	0.0584	59	34351	0.0430	84	33950	0.0270
10	33873	0.1285	35	34095	0.0580	60	33878	0.0420	85	34099	0.0270
11	34088	0.1195	36	34106	0.0570	61	34378	0.0410	86	34147	0.0266
12	33867	0.1140	37	33879	0.0544	62	34490	0.0410	87	34101	0.0260
13	33875	0.1079	38	34159	0.0541	63	33960	0.0400	88	34532	0.0260
14	33874	0.1027	39	34082	0.0534	64	34363	0.0400	89	33953	0.0242
15	33967	0.1010	40	33888	0.0530	65	34368	0.0390	90	34525	0.0233
16	34077	0.0900	41	34520	0.0530	66	34518	0.0382	91	34376	0.0202
17	33966	0.0886	42	34508	0.0520	67	34524	0.0380	92	33855	0.0200
18	33955	0.0880	43	33870	0.0510	68	34091	0.0370	93	35628	0.0200
19	33959	0.0871	44	34104	0.0510	69	34511	0.0370	94	34774	0.0196
20	33866	0.0850	45	34148	0.0505	70	34522	0.0370	95	34366	0.0193
21	33858	0.0830	46	33849	0.0500	71	33865	0.0360	96	36028	0.0193
22	33884	0.0770	47	34102	0.0500	72	34076	0.0360	97	35297	0.0191
23	33869	0.0745	48	34367	0.0500	73	34361	0.0350	98	34486	0.0181
24	33859	0.0713	49	33864	0.0480	74	34081	0.0340	99	34487	0.0180
25	34103	0.0710	50	33956	0.0480	75	34526	0.0340	100	38020	0.0179

Table 2 Problem parameters for case study

Problem instance	Mission duration, $t_{\max}$, days	Profit p	Number of spacecraft (K)	Available velocity increment (ΔV max), km/s
Case A-1	7	A (RCS area, m^2)	{1, 2, 3, 4}	{0.5, 0.75, 1.0}
Case A-2	7	1 (number of debris)	{1, 2, 3, 4}	{0.5, 0.75, 1.0}
Case B-1	360	A	{1, 2, 3, 4}	{0.5, 0.75, 1.0}
Case B-2	360	1	{1, 2, 3, 4}	{0.5, 0.75, 1.0}

seven days, respectively. The objective of the proposed problem is to determine a set of rendezvous sequences and associated trajectories that maximize the total profit collected. In this research, two different profit values per debris, RCS area (cases A-1 and B-1, removal of large debris has higher priority) and a constant 1 (cases A-2 and B-2, try to remove as many debris as possible), were considered. For each problem instance, 12 mission alternatives that are combinations of different number of spacecraft ($\in \{1, 2, 3, 4\}$) and ΔV budget ($\in \{0.5, 0.75, 1.0\}$ km/s) were considered. The problem instances and associated parameters for the case study are summarized in Table 2.

B. Generating Database of Rendezvous Trajectories

The databases for rendezvous trajectories were prepared for each case based on the rendezvous strategies defined in Sec. III. For the missions shorter than a week (cases A-1 and A-2), the entire solution space was split into multiple subspaces whose sizes were identical to the orbital period of the departure debris. This size is sufficiently fine to consider all rendezvous opportunities for two-impulse Lambert rendezvous [10]. The subspace size for long-term ADR missions (cases B-1 and B-2) was set as 30 days to ensure that at least 12 opportunities for three-step transfer could be explored throughout the entire mission duration. The solution space partitioning algorithm was implemented in *C*, and the SNOPT solver was used to find locally optimal rendezvous trajectories in each subspace.

Postprocessing to remove expensive trajectories was conducted to reduce the size of the database based on two pruning conditions. The first pruning condition is the ΔV value of the rendezvous. Figures 6 and 7 show the minimum ΔV for every departure/arrival debris pair over the entire mission duration for short-term and long-term missions, respectively (the darker color represents the smaller ΔV). Only a fraction of each debris pair is transferable with a reasonable ΔV budget, and the trajectory whose rendezvous ΔV is higher than a predetermined value (set up as 1 km/s in this study) can be discarded. Second, a rendezvous trajectory dominated by another one in the same solution set is eliminated. Let $\boldsymbol{\tau}_1$ and $\boldsymbol{\tau}_2$ be the local optimal solutions of a single-target rendezvous problem associated with two debris. If $\boldsymbol{\tau}_1$ has earlier departure time t_d, later arrival time t_a, and larger associated ΔV compared with $\boldsymbol{\tau}_2$ simultaneously, $\boldsymbol{\tau}_1$ is dominated by $\boldsymbol{\tau}_2$ and can be discarded (not included in the final database).

Table 3 provides the statistics on the final database used for the case study. Long mission duration increases the number of reachable

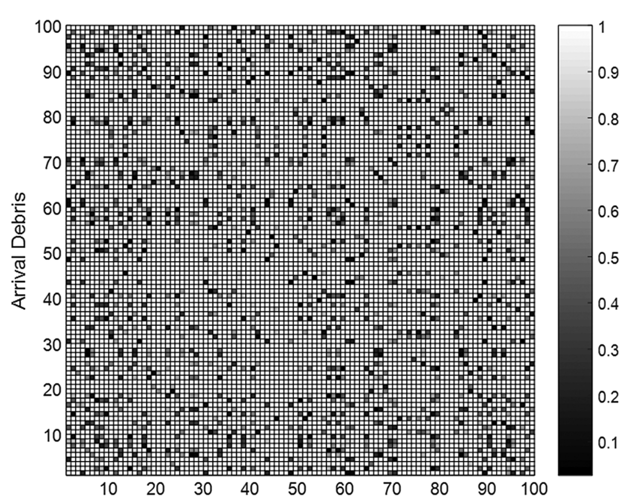


Fig. 6 Minimum rendezvous cost for each debris pair in case A (two-impulse Lambert rendezvous).

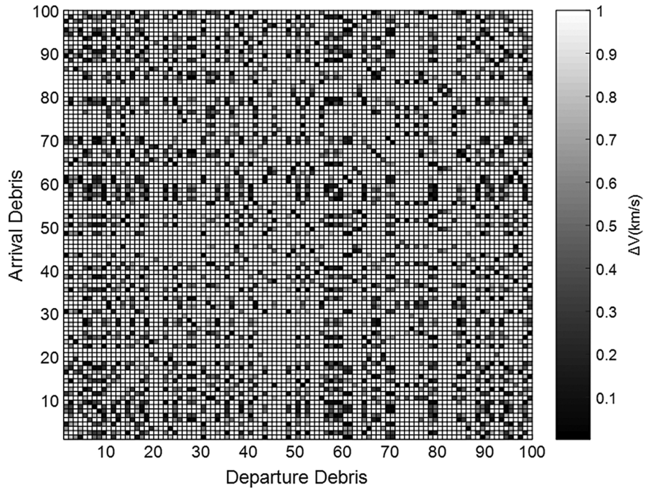


Fig. 7 Minimum rendezvous cost for each debris pair in case B (three-step transfer using RAAN drift).

debris from the current one. The average ΔV was significantly decreased when there was enough time to perform the RAAN drift. The three-step transfer requires a relatively small amount of computational load to identify a similar number of trajectories because the phasing cost is ignored in the calculation of ΔV.

C. Case Study Results

The solution method proposed in Sec. IV was applied for problem instances defined in previous subsections. To validate the effectiveness of the proposed method, two heuristic approaches were additionally considered. The first one is the "high-profit first" heuristic that sequentially selects the next rendezvous target based on the profit value assigned to each debris; the most profitable debris that has not been removed yet and can be reached with the remaining ΔV budget is added to the current sequence. The second approach is the "low-cost first" heuristic [12]. This approach iteratively evaluates ΔV for all extended rendezvous sequences created by adding one of nonvisited debris to the current sequence. The minimum-cost sequence is selected as the current one, and the process is repeated until no more extensions are feasible. The profit is not considered in the second heuristic. The column-generation algorithm was implemented in MATLAB with the GUROBI 7.5 solver.

Tables 4 and 5 comparatively exhibit the results of the case study that were obtained by using the proposed method and two heuristic approaches. For all test cases, the proposed method found better solutions than the heuristic approaches: the larger RCS area sum ($\sum A$) for cases A-1 and B-1, and the larger number of debris removed (N_d) for cases A-2 and B-2. There was a significant difference between the solutions of case A-1 and those of case A-2 (between the solutions of case B-1 and those of case B-2 as well), even though the same rendezvous trajectories were used to generate the final solution. This supports the importance of considering mission-specific profits in the problem formulation; simply maximizing the number of debris removed is not always the best way for ADR. The optimality gap G_{opt} was less than 4% for all test cases; hence, the solutions can be assumed to be near-optimal.

To validate the effectiveness of the ΔV estimation for long-duration missions, the ΔV values used in the individual arcs included in the final solution were compared with the results obtained by particle swarm optimization (PSO) with an assumption of four-impulse rendezvous presented in [19]. Figure 8 represents the differences between the estimations of ΔV based on the three-step transfer using RAAN drift and the optimal rendezvous cost obtained by PSO. Differences in ΔV values rarely exceed 20 m/s (only six out of 350 arcs exceed 20 m/s) and are small enough to demonstrate the effectiveness of the final solution.

Figures 9 and 10 show the effects of using multiple spacecraft for the mission. The objective function (the sum of the RCS area for cases A-1 and B-1, and the number of debris removed for cases A-2 and B-2) increases with the number of spacecraft in all test cases. Table 6 presents the details of one of the optimal solutions that use multiple spacecraft (case A-1/$K = 4$/$\Delta V_{\max} = 1.0$ km/s). The debris candidates are distributed over a wide range of RAANs; for example, the RAANs of the top-five largest debris range from 195.87 to 225.69 deg. In the optimal solution, four spacecraft were divided into four different RAAN areas: 215.64–219.10, 195.59–201.10, 206.86–211.55, and 225.69–231.74 deg. Each spacecraft visits a set of debris only within the RAAN range of less than 6 deg.

The effect of multiple spacecraft depends on the objective function of the problem. For cases A-1 and B-1, the increase in total RCS area decreases as the number of spacecraft increases because most of the profitable debris has already been removed. However, the number of debris removed in cases A-2 and B-2 increases almost linearly with the number of spacecraft. In addition, Figs. 9 and 10 can be applied as a criterion for evaluating various mission alternatives. For example,

Table 3 Information on the final database of rendezvous trajectories

Problem instance	Case A	Case B
Mission duration $t_{\max}$, days	7	360
Number of transferable debris pairs	1,806	3,137
Number of total rendezvous trajectories	21,384	23,926
Average ΔV, km/s	0.6004	0.3004
Computation time, s	134,161	18,978

Table 4 Objective functions for case A vs two heuristic approaches

		Case A-1: max $\sum A$				Case A-2: max N_d			
		Proposed method		Heuristic 1: high profit first		Proposed method		Heuristic 2: low cost first	
$\Delta V_{\max}$, km/s	Number of spacecraft (K)	$\sum A$, m^2	N_d	$\sum A$, m^2	N_d	$\sum A$, m^2	N_d	$\sum A$, m^2	N_d
0.5	1	1.4459 (17.9%)[a]	6	1.3207 (16.4%)	3	1.0373 (12.9%)	7	0.2253 (2.8%)	5
0.5	2	2.3532 (29.2%)	11	2.0272 (25.2%)	5	1.2677 (15.7%)	13	0.4776 (5.9%)	11
0.5	3	2.9954 (37.2%)	16	2.5828 (32.0%)	7	2.1103 (26.2%)	18	0.8514 (10.6%)	16
0.5	4	3.5624 (44.2%)	20	3.0737 (38.1%)	9	2.7375 (34.0%)	23	1.4065 (17.5%)	15
0.75	1	1.5320 (19.0%)	8	1.3648 (16.9%)	4	1.0793 (13.4%)	8	0.2434 (3.0%)	6
0.75	2	2.6448 (32.8%)	15	2.2120 (27.4%)	7	2.1018 (26.1%)	15	0.5377 (6.7%)	13
0.75	3	3.5907 (44.6%)	20	2.8186 (35.0%)	9	2.4065 (29.9%)	22	0.9545 (11.8%)	19
0.75	4	4.2689 (53.0%)	22	3.5975 (44.6%)	12	3.0332 (37.6%)	28	1.5096 (18.7%)	23
1.0	1	1.5730 (19.5%)	9	1.4747 (18.3%)	4	1.5730 (18.3%)	9	0.2834 (3.5%)	7
1.0	2	2.7935 (34.7%)	16	2.4792 (30.8%)	7	2.2796 (28.3%)	17	0.5970 (7.4%)	15
1.0	3	3.9204 (48.6%)	23	3.1358 (38.9%)	10	2.8962 (35.9%)	25	1.0138 (12.6%)	21
1.0	4	4.6252 (57.4%)	27	3.6967 (45.9%)	13	3.7707 (46.8%)	32	1.5959 (19.8%)	26

[a]The value in the parentheses represents the ratio of the RCS area sum of the removed debris to the total RCS area of all debris candidates.

Table 5 Objective functions for case B vs two heuristic approaches

		Case B-1: max $\sum A$				Case B-2: max N_d			
		Proposed method		Heuristic 1: high profit first		Proposed method		Heuristic 2: low cost first	
$\Delta V_{\max}$, km/s	Number of spacecraft (K)	$\sum A$, m^2	N_d	$\sum A$, m^2	N_d	$\sum A$, m^2	N_d	$\sum A$, m^2	N_d
0.5	1	1.8069 (22.4%)	8	1.4564 (18.1%)	4	0.4964 (6.2%)	9	0.8893 (11.0%)	7
0.5	2	3.1284 (38.8%)	15	2.3729 (29.4%)	6	2.0696 (25.7%)	18	2.3795 (29.5%)	14
0.5	3	4.1000 (50.9%)	24	2.9986 (37.2%)	9	2.7402 (34.0%)	27	2.9525 (36.6%)	20
0.5	4	4.8438 (60.1%)	29	3.5865 (44.5%)	12	2.9715 (36.9%)	35	3.2929 (40.9%)	25
0.75	1	2.1344 (26.5%)	6	1.3761 (17.1%)	3	0.5814 (7.2%)	11	0.8893 (11.0%)	7
0.75	2	3.6917 (45.8%)	14	2.5146 (31.2%)	7	2.9116 (36.1%)	21	2.3795 (29.5%)	14
0.75	3	4.5393 (56.3%)	22	3.4975 (43.4%)	11	3.2046 (39.8%)	30	3.0295 (37.6%)	21
0.75	4	5.1101 (63.4%)	32	3.9957 (37.6%)	15	4.1711 (51.8%)	39	3.3591 (41.7%)	28
1.0	1	2.4010 (29.8%)	8	1.4775 (18.3%)	3	0.9624 (11.9%)	12	0.8893 (11.0%)	7
1.0	2	3.9770 (49.3%)	17	2.6493 (32.9%)	7	1.7139 (21.3%)	22	2.3795 (29.5%)	14
1.0	3	4.7491 (58.9%)	27	3.5992 (44.7%)	11	2.6586 (33.0%)	32	3.0295 (37.6%)	21
1.0	4	5.3358 (66.2%)	35	4.1794 (51.9%)	21	4.3088 (53.5%)	40	3.3591 (41.7%)	28

if the mission aims at maximizing the total RCS area of debris removed within 360 days (case B-1), then the mission alternative composed of four spacecraft with 0.5 km/s of ΔV budgets can achieve a higher objective function than the alternative using three spacecraft with 1 km/s of ΔV (4.8438 versus 4.7491 m^2).

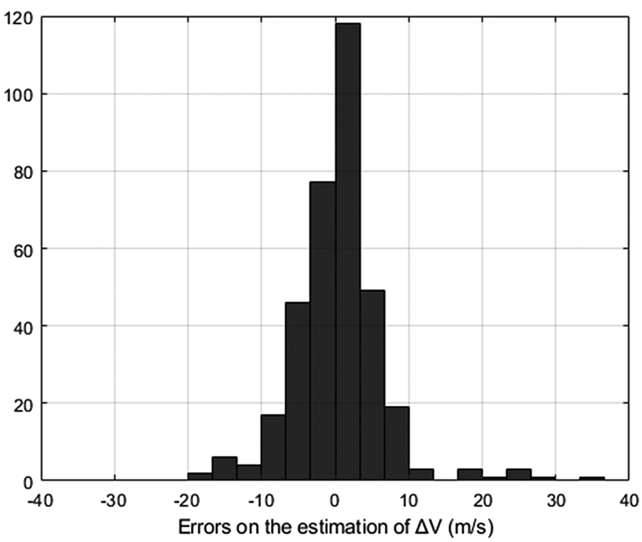


Fig. 8 Differences in ΔV for arcs included in the final solution (three-step transfer estimation vs PSO result).

VI. Case Study 2: The Kessler Run

We applied the proposed framework to obtain the rendezvous trajectory for removal of 123 debris used for the Kessler run problem (the subject of GTOC9), which received considerable attention in the community recently [11]. The goal of the competition was to design successive MTRV missions to remove all the given debris with the objective of minimizing total mission cost.

Note that the original problem in GTOC9 is not perfectly consistent with the problem formulation discussed in this paper, and we made the following modifications on it to adopt the proposed framework. First, the objective of the modified problem is to maximize the number of debris, which is realized by setting the profit values for all debris as 1 ($p_i = 1$). Note that the original problem minimizes the total cost, the sum of cost associated with missions ($\sum C_i$). The mission cost is composed of the base cost c_i and the part proportional to $(m_0 - m_{\text{dry}})^2$, where m_0 and m_{dry} are the initial mass and dry mass of a spacecraft used for the ith mission. Second, the constraints on the mass values ($m_{\text{dry}} = 2000$ kg, $m_p \leq 5000$ kg, $m_{de} = 30$ kg, where m_p is the propellant mass, and m_{de} is the mass of a deorbit package) in the original problem was translated into the constraint on the maximum available $\Delta V_{\max}$ for a spacecraft ($\Delta V_{\max} \leq 3454$ m/s). Finally, the time constraints between successive rendezvous/missions were removed because they are not considered in the proposed framework. All other parameters such as the orbital elements of debris, the time window of the entire mission, and the service time required to remove a debris were identical to those used in the original GTOC9 problem.

The best solution of the original problem obtained by the Jet Propulsion Laboratory removed all ($N = 123$) debris with 10 missions

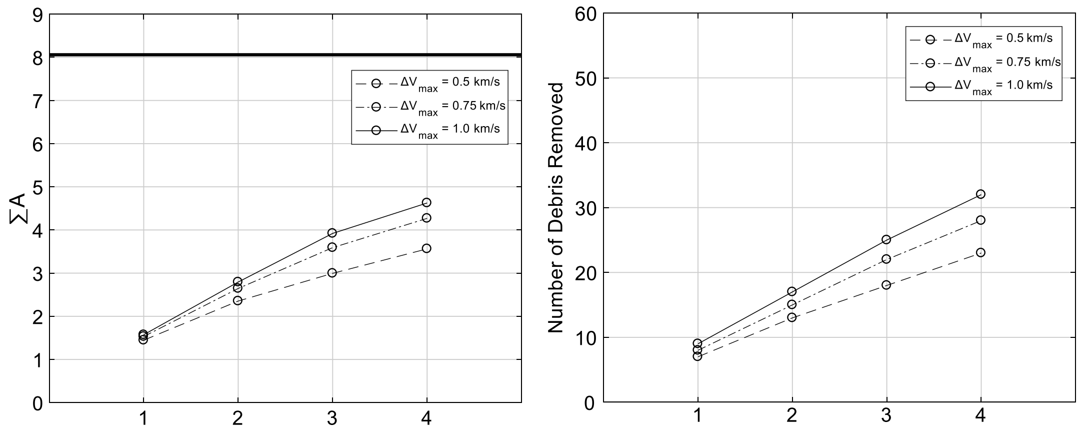


Fig. 9 Comparison of mission alternatives for case A-1 (left) and case A-2 (right).

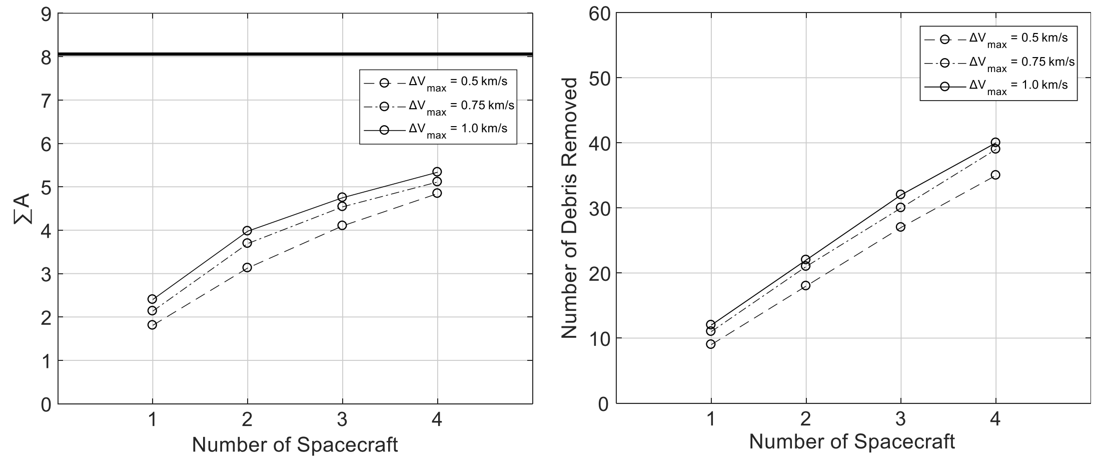


Fig. 10 Comparison of mission alternatives for case B-1 (left) and case B-2 (right).

Table 6 Details of an optimal solution for case A-1 ($K = 4$ and $\Delta V_{\mathbf{max}} = 1.0$ km/s)

Spacecraft 1					Spacecraft 2					Spacecraft 3					Spacecraft 4				
ID	t_a, days	t_d, days	ΔV, km/s	RAAN, deg	ID	t_a, days	t_d, days	ΔV, km/s	RAAN, deg	ID	t_a, days	t_d, days	ΔV, km/s	RAAN, deg	ID	t_a, days	t_d, days	ΔV, km/s	RAAN, deg
4	——	0.40	0.2912	215.64	6	——	0.46	0.0991	200.45	2	——	1.62	0.0598	206.86	19	——	2.69	0.1487	230.85
61	0.60	0.91	0.0581	217.70	18	0.63	1.18	0.0780	201.10	14	1.65	1.92	0.1018	207.22	86	2.87	3.84	0.2331	231.74
1	1.16	2.26	0.0603	217.33	58	1.21	3.01	0.2017	200.57	9	2.16	2.44	0.3008	207.59	23	4.03	4.56	0.5956	230.16
60	2.45	3.59	0.1528	217.09	22	3.21	3.66	0.2414	199.72	17	2.54	3.09	0.0703	209.83	3	4.80	——	——	225.69
48	3.78	4.06	0.0348	218.09	12	3.83	5.37	0.3083	197.89	50	3.33	5.30	0.0898	210.17	——	——	——	——	——
89	4.18	4.84	0.0554	218.01	28	5.59	5.99	0.0456	195.59	7	5.48	6.49	0.2824	210.54	——	——	——	——	——
43	4.94	6.13	0.1973	217.59	5	6.24	——	——	195.87	8	6.63	——	——	211.55	——	——	——	——	——
30	6.16	6.44	0.0366	219.07	——	——	——	——	——	——	——	——	——	——	——	——	——	——	——
97	6.48	——	——	219.10	——	——	——	——	——	——	——	——	——	——	——	——	——	——	——
Total			0.8865					0.9742					0.9049					0.9775	

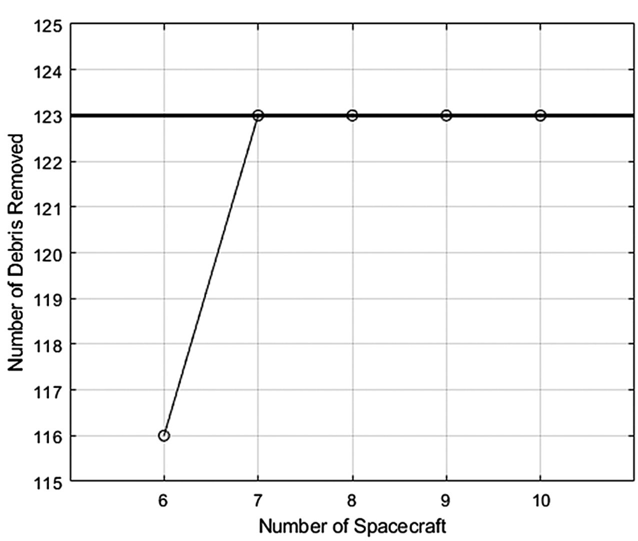


Fig. 11 Objective function ($J = \mathbf{number\ of\ removed\ debris}$) vs number of spacecraft (K) for case 2.

(or spacecraft) and removing all the debris for $K = 10$ would be the first criterion to demonstrate the validity of the proposed framework. Figure 11 shows the results of the modified problem obtained by the proposed method for different K values (i.e., number of spacecraft). We can notice that the objective function (the number of debris) is 123 for K values of 10, 9, 8, and 7. That is, seven spacecraft were sufficient to remove all debris within the entire mission period. The number of removed debris drops to 116 when we allow only six missions. The reduction in the required number of missions to remove all debris (from 10 to 7) is attributable to the relaxed time constraints on time between successive rendezvous/missions.

The optimal rendezvous sequences and corresponding ΔV values when the number of spacecraft is seven ($K = 7$) are provided in Table 7. Each spacecraft removed 15 to 19 debris within the given time window [23,467,26,419] expressed in modified Julian dates 2000 (MJD2000). The sum of ΔV values for all missions was 21.530 km/s.

VII. Conclusions

In this research, the optimal MTRV problem was formulated as the VRPP to address profits for each debris and the use of multiple

Table 7 Optimal rendezvous sequences to remove GTOC9 debris set ($K = 7$)

Spacecraft	Rendezvous sequence	Number of debris	ΔV, km/s	Departure (MJD2000)	Arrival (MJD2000)
1	9,16,22,15,95,50,27,86,98,1,85,54,40,104,121,14,52,56	18	3.066	23,472.00	26,357.91
2	84,64,110,36,5,87,111,79,42,58,103,101,78,106,60,80,94,63	18	3.353	23,474.79	26,398.34
3	28,109,65,91,25,4,116,47,20,17,35,23,59,43,99,120,82	17	3.134	23,513.57	26,399.61
4	76,11,41,61,72,39,66,57,77,33,90,69,113,88,31,96,81,73,46	19	3.437	23,527.61	26,303.56
5	13,24,8,108,48,119,0,105,51,114,112,53,3,38,29	15	2.575	23,556.75	26,399.70
6	12,19,107,45,26,7,21,93,55,30,49,44,100,97,6,92,117,10	18	2.608	23,613.32	26,364.57
7	71,70,37,67,75,122,2,32,89,74,18,83,34,118,115,102,68,62	18	3.357	23,628.29	26,393.25
Total		123	21.530		

spacecraft in an ADR mission. A two-phase framework that prepares and uses the database of rendezvous trajectories was applied to reduce the complexity of individual MTRV problems. A column-generation technique was adopted to find the best combination of rendezvous sequences with relatively low computational effort. Two case studies were conducted to demonstrate the effectiveness of the problem formulation and the proposed solution method.

Additional studies on improving the database of trajectories can be considered as future work. More trajectory options in the database can improve the quality of the solution but simultaneously increase the size of routing problems that must be solved. In this regard, an algorithm that smartly selects the rendezvous trajectory options according to the problem parameters can enhance the performance of the solution method. Considering the low-thrust trajectory would be helpful to improve the practicality of the framework, which is also a promising future work. To adopt the low-thrust option, a set of good (local optimal) low-thrust trajectories should be obtained and stored in phase A, whereas phase B of the framework can be applied without much modification. Consideration of heterogeneous spacecraft in an ADR mission, by relaxing the assumption that all spacecraft have identical ΔV budget, is another interesting extension of this study.

References

[1] Kelso, T. S., "Satellite Catalog (SATCAT)," *Celestrak*, http://www.celestrak.com/satcat/boxscore.php [retrieved 28 March 2018].

[2] Kessler, D. J., and Cour-Palais, B. G., "Collision Frequency of Artificial Satellites: The Creation of a Debris Belt," *Journal of Geophysical Research*, Vol. 83, No. A6, 1978, pp. 2637–2646. doi:10.1029/JA083iA06p02637

[3] Liou, J.-C., "Planetary Science: Risks in Space from Orbiting Debris," *Science*, Vol. 311, No. 5759, 2006, pp. 340–341. doi:10.1126/science.1121337

[4] Liou, J.-C., and Johnson, N. L., "Instability of the Present LEO Satellite Populations," *Advances in Space Research*, Vol. 41, No. 7, 2008, pp. 1046–1053. doi:10.1016/j.asr.2007.04.081

[5] Liou, J.-C., and Johnson, N. L., "A Sensitivity Study of the Effectiveness of Active Debris Removal in LEO," *Acta Astronautica*, Vol. 64, Nos. 2–3, 2009, pp. 236–243. doi:10.1016/j.actaastro.2008.07.009

[6] Lewis, H. G., White, A. E., Crowther, R., and Stokes, H., "Synergy of Debris Mitigation and Removal," *Acta Astronautica*, Vol. 81, No. 1, 2012, pp. 62–68. doi:10.1016/j.actaastro.2012.06.012

[7] Virgili, B. B., and Krag, H., "Strategies for Active Removal in LEO," *5th European Conference on Space Debris*, ESA Paper SP-672, July 2009.

[8] Castronuovo, M. M., "Active Space Debris Removal—A Preliminary Mission Analysis and Design," *Acta Astronautica*, Vol. 69, Nos. 9–10, 2011, pp. 848–859. doi:10.1016/j.actaastro.2011.04.017

[9] Bonnal, C., Ruault, J.-M., and Desjean, M.-C., "Active Debris Removal: Recent Progress and Current Trends," *Acta Astronautica*, Vol. 85, April 2013, pp. 51–60. doi:10.1016/j.actaastro.2012.11.009

[10] Bang, J., and Ahn, J., "Two-Phase Framework for Near-Optimal Multi-Target Lambert Rendezvous," *Advances in Space Research*, Vol. 61, No. 5, 2018, pp. 1273–1285. doi:10.1016/j.asr.2017.12.025

[11] Izzo, D., "Problem Description for the 9th Global Trajectory Optimisation Competition," *GTOC Portal*, May 2017, http://sophia.estec.esa.int/gtoc_portal. doi:10.5281/zenodo.570193

[12] Barbee, B. W., Alfano, S., Pinon, E., Gold, K., and Gaylor, D., "Design of Spacecraft Missions to Remove Multiple Orbital Debris Objects," *2011 Aerospace Conference*, IEEE Publ., Piscataway, NJ, 2011, pp. 1–14. doi:10.1109/AERO.2011.5747303

[13] Braun, V., Lüpken, A., Flegel, S., Gelhaus, J., Möckel, M., Kebschull, C., Wiedemann, C., and Vörsmann, P., "Active Debris Removal of Multiple Priority Targets," *Advances in Space Research*, Vol. 51, No. 9, 2013, pp. 1638–1648. doi:10.1016/j.asr.2012.12.003

[14] Cerf, M., "Multiple Space Debris Collecting Mission–Debris Selection and Trajectory Optimization," *Journal of Optimization Theory and Applications*, Vol. 156, No. 3, 2013, pp. 761–796. doi:10.1007/s10957-012-0130-6

[15] Cerf, M., "Multiple Space Debris Collecting Mission: Optimal Mission Planning," *Journal of Optimization Theory and Applications*, Vol. 167, No. 1, 2015, pp. 195–218. doi:10.1007/s10957-015-0705-0

[16] Yu, J., Chen, X., Chen, L., and Hao, D., "Optimal Scheduling of GEO Debris Removing Based on Hybrid Optimal Control Theory," *Acta Astronautica*, Vol. 93, Jan. 2014, pp. 400–409. doi:10.1016/j.actaastro.2013.07.015

[17] Yu, J., Chen, X., and Chen, L., "Optimal Planning of LEO Active Debris Removal Based on Hybrid Optimal Control Theory," *Advances in Space Research*, Vol. 55, No. 11, 2015, pp. 2628–2640. doi:10.1016/j.asr.2015.02.026

[18] Bérend, N., and Olive, X., "Bi-Objective Optimization of a Multiple-Target Active Debris Removal Mission," *Acta Astronautica*, Vol. 122, May 2016, pp. 324–335. doi:10.1016/j.actaastro.2016.02.005

[19] Shen, H.-X., Zhang, T.-J., Casalino, L., and Pastrone, D., "Optimization of Active Debris Removal Missions with Multiple Targets," *Journal of Spacecraft and Rockets*, Vol. 55, No. 1, 2018, pp. 181–189. doi:10.2514/1.A33883

[20] Izzo, D., Getzner, I., Hennes, D., and Simões, L. F., "Evolving Solutions to TSP Variants for Active Space Debris Removal," *Proceedings of the 2015 on Genetic and Evolutionary Computation Conference*, ACM Press, New York, 2015, pp. 1207–1214. doi:10.1145/2739480.2754727

[21] Stuart, J., Howell, K., and Wilson, R., "Application of Multi-Agent Coordination Methods to the Design of Space Debris Mitigation Tours," *Advances in Space Research*, Vol. 57, No. 8, 2016, pp. 1680–1697. doi:10.1016/j.asr.2015.05.002

[22] Gooding, R. H., "A Procedure for the Solution of Lambert's Orbital Boundary-Value Problem," *Celestial Mechanics and Dynamical Astronomy*, Vol. 48, No. 2, 1990, pp. 145–165. doi:10.1007/BF00049511

[23] Butt, S. E., and Ryan, D. M., "An Optimal Solution Procedure for the Multiple Tour Maximum Collection Problem Using Column Generation," *Computers & Operations Research*, Vol. 26, No. 4, 1999, pp. 427–441. doi:10.1016/S0305-0548(98)00071-9

[24] Ahn, J., De Weck, O., Geng, Y., and Klabjan, D., "Column Generation Based Heuristics for a Generalized Location Routing Problem with Profits Arising in Space Exploration," *European Journal of Operational Research*, Vol. 223, No. 1, 2012, pp. 47–59. doi:10.1016/j.ejor.2012.06.018

[25] Vallado, D., and Crawford, P., "SGP4 Orbit Determination," *AIAA/AAS Astrodynamics Specialist Conference and Exhibit*, AIAA Paper 2008-6770, Aug. 2008. doi:10.2514/6.2008-6770

M. Xin
Associate Editor