

Learning Machine Learning

Pt

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1 Glossary

1.1 Odds and Logit

In *Bi-Classification Problem*, the probability of *label* = 1 divided by the probability of *label* = 0 is called **Odds**.

$$y = P(\text{label} = 1)$$
$$\text{odds} = \frac{y}{1 - y}$$

Further, take the logarithm of both sides, we got **Log Odds/Logit**:

$$\text{logit} = \ln \frac{y}{1 - y}$$

2 Loss Functions and Objective Functions

2.1 Minimum Square Loss (MSE)

2.2 Cross-Entropy/Log Loss

For *Bi-classification Problem*. Given that x_i is one of the sample training data, y_i is the corresponding label, then

$$\hat{y}_i = \sigma(h(x_i|\theta)) \in \mathbb{R}$$
$$\mathcal{L}(y_i, \hat{y}_i) = \begin{cases} -\log(\hat{y}_i) & y_i = 1 \\ -\log(1 - \hat{y}_i) & y_i = 0 \end{cases}$$

Compress into one equation, then

$$\mathcal{L}(y_i, \hat{y}_i) = -[y_i * \log(\hat{y}_i) + (1 - y_i) * \log(1 - \hat{y}_i)]$$

More generally, for *Multi-Classification Problem*, given that

- x_i is one of the sample training data, which will be classified into one of k categories,
- $y_i \in \mathbb{R}^k$ is the **One-Hot Representation** of the corresponding label

the **Cross-Entropy Loss** is

$$\hat{y}_i = \text{softmax}(h(x_i|\theta)) \in \mathbb{R}^k$$
$$\mathcal{L}(y_i, \hat{y}_i) = - \sum_j^n y_i[j] * \log_2(\hat{y}_i[j])$$

2.3 Maximum (Log-)Likelihood

Given the probability of a series of accidents $A_i, i \in 1, 2, \dots, k$ is $P(A_i)$, then we want to **maximize** the probability that all of these accidents happen, thus

$$\max\{\prod_{i=0}^k P(A_i)\}$$

To simplify, we can take the logarithm of both sides, then

$$\max\{\ln \sum_{i=0}^k P(A_i)\}$$

which is namely **Maximum (Log) Likelihood**. While the **Loss Function** derived from above is naturally:

$$\mathcal{L}(y_i, \hat{y} \mid \theta) = -\ln \sum_{i=0}^k P(A_i)$$

Which we want to **Minimize**.

3 Probability

3.1 Basis

- Priori Probability: the probability which can be empirically inferred
- Posterior Probability: after A happening, sought the probability of the reason of A
- Bayesian Equation

3.2 Conditional Independent

$$p(A \mid C) * p(B \mid C) = p(AB \mid C)$$

4 Matrix

4.1 Differentiate/Derivation

$$\begin{aligned} Y &= A \cdot X \cdot B \\ \frac{\partial Y}{\partial X} &= A^T \cdot B^T \\ \frac{\partial Y}{\partial X^T} &= B \cdot A \end{aligned}$$

Another scenario,

$$\begin{aligned} Y &= X^T \cdot A \cdot X \\ \frac{\partial Y}{\partial X} &= (A + A^T) \cdot X \end{aligned}$$

5 Normalization

5.1 Scale

Given $p \in \mathbb{R}^k$, where p is the result of *Dot-Product* in *Dot-Product Attention Mechanism*, in order to counteract gradient vanishing in *softmax*, scale p by

$$p_{norm} = \frac{p}{\sqrt{k}}$$