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Trajectory-oriented EKF-SLAM using the Fourier-Mellin Transform applied to Microwave Radar Images

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Abstract— **This paper is concerned with the Simultaneous Localization And Mapping (SLAM) application using data obtained from a microwave radar sensor. The radar scanner is based on the Frequency Modulated Continuous Wave (FMCW) technology. In order to overcome the complexity of radar image analysis, a trajectory-oriented EKF-SLAM technique using data from a $360°$ field of view radar sensor has been developed. This process makes no landmark assumptions and avoids the data association problem. The method of egomotion estimation makes use of the Fourier-Mellin Transform for registering radar images in a sequence, from which the rotation and translation of the sensor motion can be estimated. In the context of the scan-matching SLAM, the use of the Fourier-Mellin Transform is original and provides an accurate and efficient way of computing the rigid transformation between consecutive scans. Experimental results on real-world data are presented. Moreover a performance evaluation of the results is carried out. A comparative study between the output data of the proposed method and the data processed with smoothing approaches to SLAM is also achieved.**

I. INTRODUCTION

Environment mapping models have been studied intensively over the past two decades. In the literature, this problem is often referred to as Simultaneous Localization And Mapping (SLAM). For a broad and quick review of the different approaches developed to address this problem that represents a higher level of complexity in autonomous mobile robot applications, one can consult [2], [9], [10], [13] and [29]. Localization and mapping in large outdoor environments are applications related to the availability of efficient and robust perception sensors, particularly with regard to the problem of maximum range and the resistance to the environmental conditions. Even though lasers and cameras are well suited sensors for environment perception, their strong sensitivity to atmospheric conditions has, among other reasons, given rise to an interest for the development of a SLAM method starting from an active sensor like a radar or sonar sensor [25]. Microwave radar provides an alternative solution for environmental imaging.In this paper, a trajectory-oriented SLAM technique is presented using data from a $360°$ field of view radar sensor. This radar is based on Frequency Modulated Continuous Wave (FMCW) technology [19].

In Section II, a review of articles related to our research interests is carried out in order to position our work in relation to existing methods. Section III shortly presents the microwave radar scanner developed by a Cemagref research team (in the field of agricultural and environmental engineering research) [26]. Section IV gives the SLAM formulation used in this paper. There, the Fourier-Mellin Transform is applied to register images in a sequence and to estimate the rotation and translation of the radar system (see Section V). This process makes no landmark assumptions, and avoids the data association problem which is especially fragile to incorrect association of observations to landmarks. Section VI shows experimental results of this work which were implemented in Matlab and C/C++. Finally, in Section VII, a performance comparison of these results is carried out. They are compared with those obtained after a smoothing approach to SLAM as described in [8]. Section VIII concludes and introduces future work.

II. RELATED WORK

A. In the Field of Radar Mapping

In order to perform outdoor SLAM, laser sensors have been widely used [23] [14] [4] and a famous recent application with Velodyne HDL-64 3D LIDAR is presented in [16]. To provide localization and map building, the input range data is processed using geometric feature extraction and scan correlation techniques. Less research exists using sensors such as underwater sonar [25] and Frequency Modulated Continuous Wave (FMCW) radar. Interestingly, this last kind of sensor was already used by Clark in [6] at the end of the last century. In an environment containing a small number of well separated, highly reflective beacons, experiments were led with this sensor to provide a solution to the SLAM problem [9] using an extended Kalman filter framework and a landmark based approach. In [12], a model dedicated to the radar was developed to build a occupancy grid map. Finally, in [20], a method for building a map with sensors that return both range and received signal power information was presented. An outdoor occupancy grid map related to a 30 m vehicle's trajectory is analyzed. So far, there seems to have been no trajectory-oriented SLAM work based on radar information over important distances. However, vision-based, large-area SLAM has already been carried out successfully for underwater missions, using information filters over long distances [11] [18].

B. In the Field of Scan Matching SLAM

Since Lu and Milios presented their article [17] in search of a globally consistent solution to the 2D-SLAM problem with three degrees of freedom poses, many techniques have been proposed in the literature concerning robotics as well as computer vision.

A common method of pose estimation for mobile robots is scan matching. By solving the rigid transformation between consecutive scans from a range sensor, the robot's motion in the time period between the scans can be inferred. One of the

Fig. 1. The K2Pi FMCW radar.

most popular approaches for scan matching is the Iterative Closest Point (ICP) [3] algorithm. In ICP, the transformation between scans is found iteratively by assuming that every point in the first scan corresponds to its closest point in the second scan, and by calculating a closed form solution using these correspondences. However, sparse and noisy data, such as that from an imaging radar, can cause an ICP failure. A single noisy reading can significantly affect the computed transformation, causing the estimated robot pose to drift over time. Other recent trends in SLAM research are to apply probabilistic methods to 3D mapping. Cole et al. [7] used an extended Kalman filter on the mapping problem. Olson et al. presented in [22] a novel approach to solve the graph-based SLAM problem by applying stochastic gradient descent to minimize the error introduced by constraints.

Our algorithm is close to the method suggested by Cole et al. [7]. However, the Fourier-Mellin Transform for registering images in a sequence is used to estimate the rotation and translation of the radar sensor motion (see Section V). In the context of scan-matching SLAM, the use of the Fourier-Mellin Transform is original and provides an accurate and efficient way of computing the rigid transformation between consecutive scans (see Section V-C). It is a global method that takes into account the contributions of both range and power information of the radar image. In some sense, this “global” approach is also close to the histogram correlation approach used by Bosse and Zlot in [4].

III. A MICROWAVE RADAR SCANNER

The exploited radar uses the frequency modulation continuous wave (FMCW) technique [27][19]. The FMCW radar is called K2Pi (2π for panoramic - in K band) and is equipped with a rotating antenna in order to achieve a complete $360°$ per second monitoring around the vehicle, with an angular resolution of $3°$, in the 3-100 m range. A general view of the radar is presented in Fig. 1 and its main characteristics are listed in Table I. An example of radar images is presented in Fig. 2. Variations of shading indicate variations of amplitude in the power spectra.

IV. PROBLEM FORMULATION

A. SLAM Process

The used formulation of the SLAM problem is to estimate the vehicle trajectory defined by the estimated state $\mathbf{x}_k = \left[\mathbf{x}_{v_k}^T, \mathbf{x}_{v_{k-1}}^T, \ldots, \mathbf{x}_{v_1}^T\right]^T$. $\mathbf{x}_{v_i} = [x_i, y_i, \phi_i]^T$ is the state vector describing the location and orientation of the vehicle at time i. There is no explicit map; rather each pose estimate has an associated scan of raw sensed data that can be next aligned to form a global map.

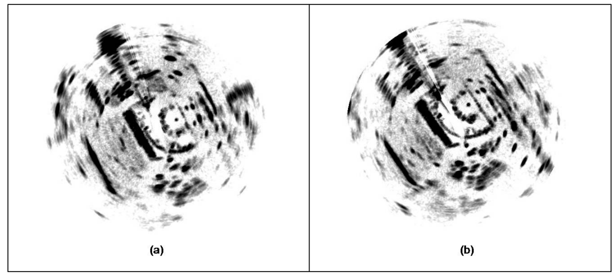


Fig. 2. Two consecutive radar images ((a) & (b)) obtained with the FMCW radar sensor (see Fig. 1).

TABLE I

CHARACTERISTICS OF THE K2PI FMCW RADAR.

Carrier frequency $F0$	24 GHz
Transmitter power Pt	20 dBm
Antenna gain G	20 dB
Bandwidth	250 MHz
Scanning rate	1 Hz
Angular resolution	$3°$
Angular precision	$0.1°$
Range Min/Max	3 m/100 m
Distance resolution	0.6 m
Distance precision (canonical target at 100 m)	0.05 m
Size (length-width-height)	27-24-30 cm
Weight	10 kg

B. Radar Scan Matching SLAM

Scan matching is the process of translating and rotating a radar scan such that a maximal overlap with another scan emerges. Assuming this alignment is approximately Gaussian, a new vehicle pose is added to the SLAM map by only adding the pose to the SLAM state vector.

So, observations are associated to each pose. They are compared and registered to offer potential constraints on the global map of vehicle poses. This is not only useful for odometry based state augmentation, but it is also an essential point for loop closing.

The estimator used here is the Extended Kalman Filter (EKF). This choice does not affect the reliability of our solution, even if we are aware that better alternatives could be used, especially when dealing with very large outdoor trajectories (e.g., [18]). But our goal was to focus our study on the behavior of the FMT in a SLAM and radar environment, using a well-known filter to make correct conclusions. Given a noisy control input $\mathbf{u}(k+1)$ at time $k+1$ measured from gyro and odometers, upon calculation of the new vehicle pose, $\mathbf{x}_{v_{n+1}}(k+1|k)$, and a corresponding covariance matrix, $\mathbf{P}_{v_{n+1}}(k+1|k)$, the global state vector, $\mathbf{x}$, and corresponding covariance matrix, $\mathbf{P}$, can be augmented as follows:

$$\mathbf{x}(k+1|k) = \begin{bmatrix} \mathbf{x}(k|k) \\ \mathbf{x}_{v_n} \oplus \mathbf{u}(k+1) \end{bmatrix} = \begin{bmatrix} \mathbf{x}_{v_0} \\ \mathbf{x}_{v_1} \\ \vdots \\ \mathbf{x}_{v_n} \\ \mathbf{x}_{v_{(n+1)}} \end{bmatrix} (k+1|k)$$

$$\mathbf{P}(k+1|k) = \begin{bmatrix} \mathbf{P}(k|k) & \mathbf{P}(k|k)\frac{\partial(\mathbf{x}_{v_n}\oplus\mathbf{u}(k+1))^T}{\partial\mathbf{x}_{v_n}} \\ \frac{\partial(\mathbf{x}_{v_n}\oplus\mathbf{u}(k+1))}{\partial\mathbf{x}_{v_n}}\mathbf{P}(k|k) & \mathbf{P}_{v_{n+1}}(k+1|k) \end{bmatrix}$$

The operator $\oplus$ is the displacement composition operator (e.g., [7]). $\mathbf{P}_{v_{n+1}}(k+1|k)$ is the covariance of the newly added vehicle state. Let us assume that two scans, $\mathbf{S}_i$, $\mathbf{S}_j$, have been registered. So, an observation $\mathbf{T}_{i,j}$ of the rigid transformation between poses and in the state vector exists. Therefore a predicted transformation between the two poses can be found from the observation model as follows:

$$\mathbf{T}_{i,j}(k+1|k) = \mathbf{h}\left(\mathbf{x}(k+1|k)\right)$$
$$= \ominus\left(\ominus\mathbf{x}_{v_j}(k+1|k) \oplus \mathbf{x}_{v_i}(k+1|k)\right)$$

where the operator $\ominus$ is the inverse transformation operator. Observations are assumed to be made according to a model of the form $\mathbf{T}_{i,j}(k+1) = \mathbf{h}\left(\mathbf{x}(k+1)\right) + \mathbf{v}(k+1) = \boldsymbol{\Psi}\left(\mathbf{S}_i, \mathbf{S}_j\right)$ in which $\boldsymbol{\Psi}$ and represents a registration algorithm, and $\mathbf{v}(k+1)$ is a vector of observation errors. The state update equations are then the classical EKF update equations. The search for a transformation $\mathbf{T}_{i,j}$ is achieved by maximizing a cross correlation function [1]. At present, the derived transformation covariance is set to a constant value. Of course, it would be preferable to use a better approach as suggested in [21] and [1].

V. FOURIER-MELLIN TRANSFORM FOR EGOMOTION ESTIMATION

A. Principle

The problem of registering two scans in order to determine the relative positions from which the scans were obtained, has to be solved. The choice of an algorithm is strongly influenced by the need for real-time operation. A FFT-based algorithm was chosen to perform scan matching.

Fourier-based schemes are able to estimate large rotations, scalings, and translations. Let us note that the scale factor is irrelevant in our case. Most of the DFT-based approaches use the shift property [24] [15] of the Fourier transform, which enables robust estimation of translations using normalized phase correlation [28].

To match two scans which are translated and rotated with respect to each other, the phase correlation method is used, stating that a shift in the coordinate frames of two functions is transformed in the Fourier domain as a linear phase difference. To deal with the rotation as a translational displacement, the images are previously transformed into an uniform polar Fourier representation.

It is known that if two images I_1 and I_2 differ only by a shift, $(\Delta x, \Delta y)$, (i.e., $I_2(x,y) = I_1(x-\Delta x, y-\Delta y)$), then their Fourier transforms are related by:

$$\hat{I}_1(w_x,w_y).e^{-i(w_x\Delta x + w_y \Delta y)} = \hat{I}_2(w_x,w_y).$$

Hence the normalized cross power spectrum is given by

$$\widehat{Corr}(w_x,w_y) = \frac{\hat{I}_2(w_x,w_y)}{\hat{I}_1(w_x,w_y)} = e^{-i(w_x\Delta x + w_y\Delta y)}. \quad (1)$$

Taking the inverse Fourier transform $Corr(x,y) = F^{-1}(\widehat{Corr}(w_x,w_y)) = \delta(x-\Delta x, y-\Delta y)$, which means that $Corr(x,y)$ is nonzero only at $(\Delta x, \Delta y) = \arg\max_{(x,y)}\{Corr(x,y)\}$. If the two images differ by rotational movement (θ_0) with translation $(\Delta x, \Delta y)$, then

$$I_2(x,y) = I_1(x\cos\theta_0 + y\sin\theta_0 - \Delta x, -x\sin\theta_0 + y\cos\theta_0 - \Delta y).$$

Converting from rectangular coordinates to polar coordinates makes it possible to represent rotation as shift: The Fourier Transform in polar coordinates is $\hat{I}_2(\rho,\theta) = e^{-i(w_x\Delta x + w_y\Delta y)}\hat{I}_1(\rho,\theta-\theta_0)$. Let M_1 and M_2 denote the magnitudes of $\hat{I}_1$ and $\hat{I}_2$ ($M_1 = \left|\hat{I}_1\right|$, $M_2 = \left|\hat{I}_2\right|$). So, M_1 and M_2 are related by $M_1(\rho,\theta) = M_2(\rho,\theta-\theta_0)$. The shift between the two images can now be resolved using Eq. 1.

B. Scan Registration

In order to perform a scan registration algorithm,the Fourier-Mellin Transform (FMT) has been chosen [5] [24]. The FMT is a global method that takes the contributions from all points in the images into account in order to provide a way to recover all rigid transformation parameters, i.e. rotation, translation. It is an efficient and accurate method to process a couple of images that are fairly similar (see Fig. 1). The steps of the scan registration algorithm are described in Alg. 1.

Algorithm 1 Steps of the Fourier-Mellin Transform algorithm applied to FMCW radar images

1) Get radar images I_k and I_{k-1}.
2) Apply thresholding filter to eliminate the speckle noise in both images.
3) Apply FFT to images $I_k \rightarrow \hat{I}_k$ and $I_{k-1} \rightarrow \hat{I}_{k-1}$.
4) Compute the magnitudes $M_k = \left|\hat{I}_k\right|$, $M_{k-1} = \left|\hat{I}_{k-1}\right|$
5) Transform the resulting values from rectangular to polar coordinates. $M() \rightarrow MP()$.
6) Apply the FFT to polar images, a bilinear interpolation is used. $MP() \rightarrow \widehat{MP}()$.
7) Compute $\widehat{Corr}(w_\rho,w_\theta)$ between $\widehat{MP}_k(w_\rho,w_\theta)$ and $\widehat{MP}_{k-1}(w_\rho,w_\theta)$ using Eq. 1.
8) Compute $Corr(\rho,\theta) = F^{-1}(\widehat{Corr}(w_\rho,w_\theta))$.
9) Find the location of the maximum of $Corr(\rho,\theta)$ and obtain the rotation value.
10) Construct Ir by applying reverse rotation to I_{k-1}.
11) Apply FFT to image Ir_{k-1}.
12) Compute the correlation $\widehat{Corr}(w_x,w_y)$ using Eq. 1.
13) Take inverse FFT $Corr(x,y)$ of $\widehat{Corr}(w_x,w_y)$.
14) Obtain the values $(\Delta x, \Delta y)$ of the shift.

C. Evaluation on Real Data

In order to prove the efficiency and the accuracy of the method, we present the errors obtained between the estimated rigid transformation and the one computed from the GPS and gyro recordings. On a sequence of 40 consecutive couples of real radar images, the errors are plotted in Fig. 3.

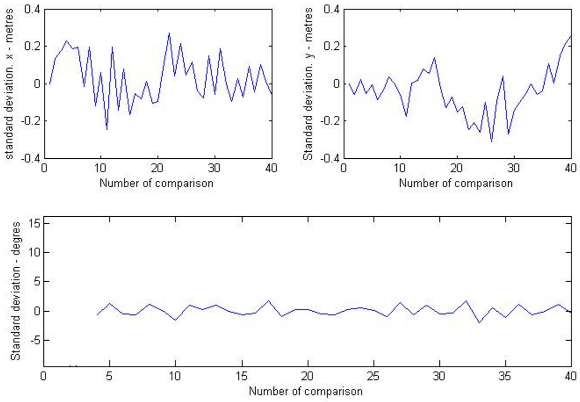


Fig. 3. Errors between the estimated rigid transformation using the Fourier-Mellin Transform and those which are estimated from the GPS and gyro recordings on a sequence of 40 consecutive couples of real radar images.

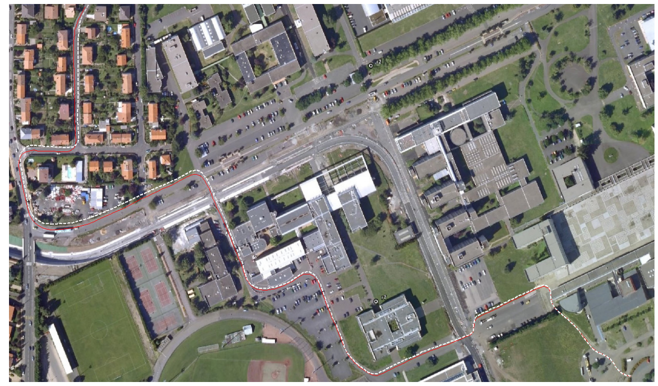


Fig. 4. Overlay of the estimated trajectory and the aerial image of Blaise Pascal University campus. The total traveled distance is around 1,000 m. The thin red line shows the trajectory of the vehicle measured with a centimetrically-precise GPS. The vehicle estimates are thick white dashes.

Fig. 5. Global map related to the trajectory depicted in Fig. 4.

Fig. 6. The total traveled distance is around 700 m. The thin red line shows the trajectory of the vehicle measured with a centimetrically-precise GPS. The vehicle estimates are in thick white dashes. The loop closure is performed.

VI. EXPERIMENTAL RESULTS

This section provides experimental results of the scan-matching SLAM application using the radar sensor previously described. The radar and the proprioceptive sensors (gyro, odometers) were mounted on a utility car moving at a speed ranging from 0 to 25 km/h. Here, two experimental runs are presented. They were performed in an outdoor field, Blaise Pascal University campus, with a complex environment (buildings, cars, trees, roads, road signs, etc.). The radar was on the top of the vehicle, 3 meters above the ground. The estimated trajectories obtained with the SLAM process are presented in Fig. 4 and 6. The successive positions of the radar are separated by an interval of one second. The photograph (see Fig. 4) is an aerial image of the experimental zone. The trajectory of the vehicle simultaneously measured with a centimetrically-precise RTK-GPS is overlayed. For these experiments, all data acquisitions have been realized in real time but SLAM processing has been realized off-line. One step of the process (scan registration, prediction and update) implemented in C/C++ is achieved in less one half-second with a QuadCore Intel Xeon (2.8 GHz) with 6 Go DDR2 FB-DIMM RAM (667 MHz) (no multi-threading used yet). A quantitative evaluation of the localization performances of the implemented process has been achieved.

The first experiment was made on a distance of 1,000 m without loop closure. Fig. 4 shows the trajectory of the vehicle. The global map is shown in Fig. 5. The second experiment was made on a distance of 700 m with loop closure (a circular trajectory around the campus sports-ground). Fig. 6 shows the estimated trajectory of the vehicle after loop closing. The global map is shown in Fig. 7.

VII. PERFORMANCE COMPARISON

Comparing two or more SLAM algorithms needs quantitative performance metrics like robustness, rate of convergence, quality of the results and specially of the maps. This last point is not addressed in this paper but it should be in future work using measures of quality based on entropy for instance. Here, we only focus on the rate of convergence and quality of results of three algorithms.

A. Pose Based Comparison

The pose based evaluations require "ground truth" data to compare to. In order to measure the quality of the output of SLAM algorithm, a ground truth trajectory is assumed to be available (here the GPS data). The SLAM algorithm gives out a set of final poses $\mathbf{x}_N$. Since each pose in 2D mapping has three components $x; y; \phi$ the average error in each of the components can be computed (see Fig. 8 and 9). Due to the lack of space, these errors are synthetized by the average distance. It is important that both the output of SLAM algorithm and the ground truth poses are in the same

Fig. 7. Global map related to the trajectory depicted in Fig. 6.

global frame. This could be done by rotating and translating the set of poses such that the first corresponding pose in each set is $(0; 0; 0)$. In order to show the behavior of the algorithms in terms of rate of convergence, the set of final poses of each algorithm is used. This leads to a graph that converges to an error of zero. It gives information about monotonic or jittering behavior of the algoritms.

B. Algorithms implemented for Comparison

In order to achieve the comparison, two trajectory-oriented SLAM algorithms were implemented. They are based on global optimization problem: the Levenberg-Marquardt algorithm and the *square root SAM* algorithm [8]. The goal is to recover the Maximum A Posteriori (MAP) estimate for the entire trajectory $X \triangleq \{\mathbf{x}_N\}$, given the measurements $Z \triangleq \{z_k\}$ and control inputs $U \triangleq \{u_i\}$. The MAP estimate is obtained by minimizing the non-linear least-squares problem:

$$\sum_{i=1}^{N} \|x_i - f_i\left(x_{i-1}, u_i\right)\|^2_{\Lambda_i} + \sum_{k=1}^{K} \|z_k - h_k\left(x_{ik}, x_{jk}\right)\|^2_{\Sigma_i}$$

Here $\|e\|^2_{\Sigma} \triangleq e^T \Sigma^{-1} e$ is defined as the squared Mahalanobis distance given a covariance matrix Σ. Gaussian measurement models are assumed. The process model $x_i = f_i(x_{i-1}, u_i) + w_i$ describes the odometry sensor process where w_i is normally distributed zero-mean measurement noise with covariance matrix Λ_i, and $z_k = h_k(x_{ik}, x_{jk}) + v_k$ is the Gaussian measurement equation, where v_k is normally distributed zero-mean measurement noise with covariance matrix Σ_i. The equations above model the robot's behavior in response to control input, and its sensors, respectively. For the mathematical details and the description of the algorithms, the reader can refer to [13] [8].

C. Results

For this comparative study, the three algorithms namely Radar Scan-SLAM using FMT (RS-SLAM-FMT which is our approach), Levenberg-Marquardt (LM) and *square root SAM* have been applied on a sequence of 289 poses related to the first experiment described in Section VI. The results are presented in Fig. 8 and 9. They attest the efficiency of the proposed method to perform outdoor SLAM using microwave radar images. In Fig. 8, the graph represents the estimated trajectories provided by the three algorithms. Near the end of trajectory (between poses 211 and 215), the recorded measurements were strongly noisy, disturbing the convergence to reference path at the end. RS-SLAM-FMT and *square root SAM* results are close. The LM algorithm suffers from the relative poor quality of the odometry data.

Fig. 9 (a) shows the behavior of the algorithms using error metrics. For each iteration of the algorithms, average total error between RTK-GPS and the estimates of RS-SLAM-FMT and *square root SAM* is computed (Cartesian distance between GPS and SLAM trajectories). Near the 200th iteration, the recorded measurements were strongly noisy, introducing a deviation between the two algorithms, previously close together. A jittering behavior appears in *square root SAM*. This phenomenon is reinforced in Fig. 9 (b) when observing the rate of convergence with respect to the sets of final poses of each algorithm. The difference between a global approach and an iterative algorithm appears on this trajectory without loop closure. Finally, in Fig. 9 (c), using final poses of the algorithms, the errors between GPS and estimated positions are plotted. Before the 200th pose, the results of RS-SLAM-FMT and *square root SAM* algorithms are close the "ground truth" trajectory. Our RS-SLAM-FMT approach provides similar results compared to those obtained with smoothing approaches.

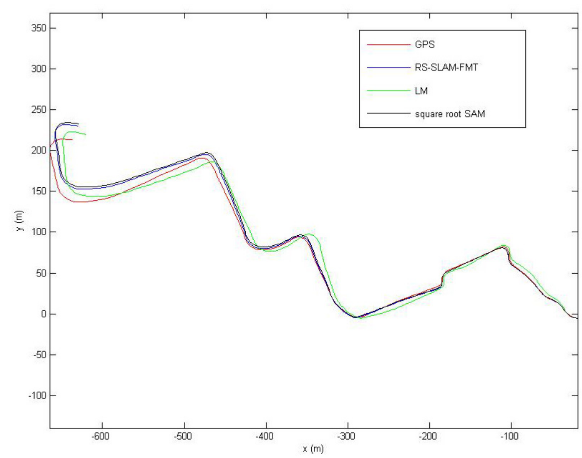
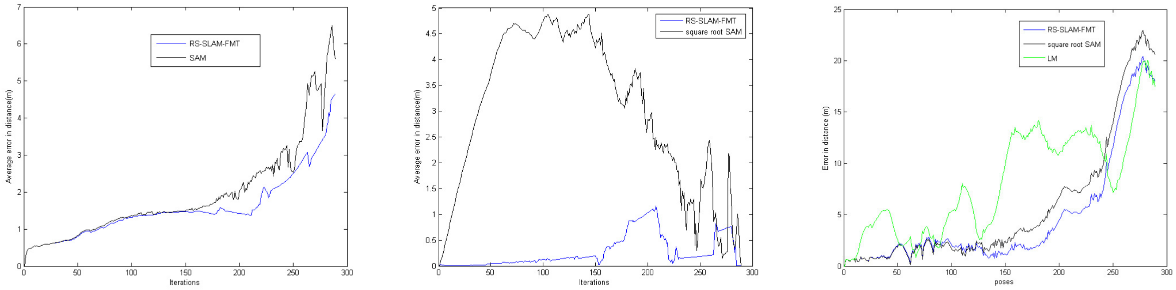


Fig. 8. Estimated trajectories by the three algorithms on data related to the first experiment described in Section VI.

VIII. CONCLUSION AND FUTURE WORK

This paper presented results of SLAM using a microwave radar sensor. Due to the complexity of radar target detection, identification, tracking and association, a trajectory-oriented SLAM process based on Fourier-Mellin Transform was developed; in this way, target assumptions about their position and nature were avoided. Experimental results on real-world data and a performance evaluation of the results were presented. A comparative study between the output data of the proposed method and the same data processed with two smoothing SLAM approaches attested the efficiency of our method.

Currently, this work considers only a static environment, assuming that there are no mobile elements around the radar. In order to develop a perception solution for high velocity robotics applications, future work will be devoted to:



(a) (b) (c)

Fig. 9. Comparative study. (a) Average total error per iteration. (b) Rate of convergence. (c) Error in location between RTK-GPS and the final estimates.

- the enhancement of the global map using methods such as the one described in [22];
- the evaluation of the map quality by inserting in an environment a small number of well separated, highly reflective beacons (LuneBerg lenses);
- the development of a new application of SLAM, integrating SLAM with Mobile Object Tracking (SLAM-MOT) once the sensor delivers the measurement of Doppler frequency to take the relative velocity of mobile targets into account [29].

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