Simulating your income (10 points)

Assume that all of you will graduate from MACSS program at the University of Chicago in June 2019. Your annual income from the time you graduate to the end of your life is generated by the following process:

$$\ln(inc_{2019}) = \ln(inc_0) + \ln(\epsilon_{2019})$$

$$\ln(inc_t) = (1-\rho)[\ln(inc_0) + g(t-2019)] + \rho\ln(inc_{t-1}) + \ln(\epsilon_t) \text{ for } 2020 \leq t \leq 2058$$

where the variable inc_t is your annual income in year $t \geq 2019$, inc_0 is the average starting income ($t = 2019$) for a MACSS student, $\rho \in [0,1)$ reflects some positive dependence of today's income on last period's income, g is a long-run annual growth rate for your annual salary, and ϵ_t is an error term that is distributed log-normally $LN(0,\sigma^2)$ where σ^2 is the variance of the log of the error term. That is, $\ln(\epsilon_t)$ is distributed normal $N(0,\sigma^2)$.

1. (3 points) Let the standard deviation of your income process be $\sigma = 0.1$, let the persistence be $\rho = 0.2$, let the long-run growth rate of income be $g = 0.03$, and let the average initial income be $inc_0 = \$80,000$. Assume you will work for 40 years after you graduate (2019 to 2058). Simulate 10,000 different realizations of your lifetime income. Do this by first drawing 10,000 sets of 40 normally distributed errors with mean 0 and standard deviation $\sigma = 0.1$. Then plug those into the income process defined above to simulate your lifetime income. Plot one of the lifetime income paths. Make sure your axes are correctly labeled and your plot has a title.
2. (2 points) Plot a histogram with 50 bins of year $t = 2019$ initial income for each of the 10,000 simulations. What percent of your class will earn more than $100,000 in the first year out of the program? What percent of the class will earn less than $70,000? Is the distribution normally distributed (i.e. symmetric and bell-curved)?
3. (3 points) Suppose you graduate from the MACSS program with $95,000 of zero-interest debt. You will use 10% of your annual salary after you graduate to pay off this loan. Plot the histogram of how many years it takes to pay off the loan in each of your 10,000 simulations. This histogram will only have as many bins as you have unique years in which people pay off their debt. In what percent of the simulations are you able to pay off the loan in 10 years (on or before $t = 2028$)?
4. (2 points) Now suppose that the UChicago MACSS program becomes very well known in the next year, and the skills you are learning are demanded more by employers. This increases the average starting salary to $inc_0 = \$85,000$, but the standard deviation in incomes increases also to $\sigma = 0.15$. Plot the new histogram of how many years it takes to pay off your loan of $95,000 in your new 10,000 simulations with the new standard deviation and the new average initial salary. In what percent of the simulations are you able to pay off the loan in 10 years (on or before $t = 2028$)?

Submitting your assignment

See here for instructions on submitting course assignments.

Assignment format

Submit your assignment as either a Jupyter Notebook (`.ipynb`) or R Markdown document (`.Rmd` knitted with `output: md_document` in the front matter).

Submission deadline

Submit your pull request before class on Monday, November 6 (11:30 am).