

ETC3550

Applied forecasting for business and economics

Ch5. The forecasters' toolbox

OTexts.org/fpp3/

Outline

- 1 A tidy forecasting workflow
- 2 Some simple forecasting methods
- 3 Residual diagnostics
- 4 Distributional forecasts and prediction intervals
- 5 Forecasting with transformations
- 6 Forecasting and decomposition
- 7 Evaluating forecast accuracy
- 8 Time series cross-validation

Outline

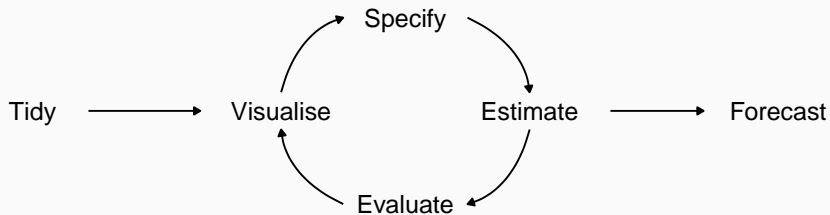
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A tidy forecasting workflow

The process of producing forecasts can be split up into a few fundamental steps.

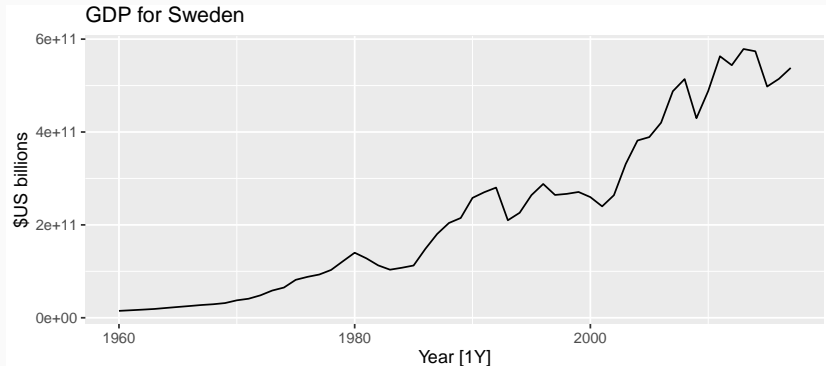
- 1 Preparing data
- 2 Data visualisation
- 3 Specifying a model
- 4 Model estimation
- 5 Accuracy & performance evaluation
- 6 Producing forecasts

A tidy forecasting workflow



Data preparation and visualisation

```
global_economy %>%  
  filter(Country=="Sweden") %>%  
  autoplot(GDP) +  
  ggtitle("GDP for Sweden") + ylab("$US billions")
```



Model estimation

The `model()` function trains models to data.

```
fit <- global_economy %>%  
  model(trend_model = TSLM(GDP ~ trend()))  
fit
```

```
## # A mable: 263 x 2  
## # Key:      Country [263]  
##   Country      trend_model  
##   <fct>        <model>  
## 1 Afghanistan <TSLM>  
## 2 Albania     <TSLM>  
## 3 Algeria     <TSLM>  
## 4 American Samoa <TSLM>  
## 5 Andorra     <TSLM>
```

Producing forecasts

```
fit %>% forecast(h = "3 years")
```

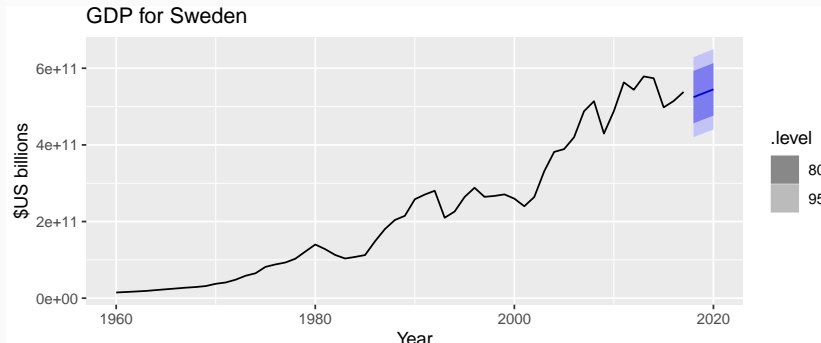
```
## # A tibble: 789 x 5 [1Y]
```

```
## # Key:   Country, .model [263]
```

##	Country	.model	Year	GDP	.distribution
##	<fct>	<chr>	<dbl>	<dbl>	<dist>
##	1 Afghanistan	trend_mod~	2018	1.62e10	N(1.6e+10, 1.3e~
##	2 Afghanistan	trend_mod~	2019	1.65e10	N(1.7e+10, 1.3e~
##	3 Afghanistan	trend_mod~	2020	1.68e10	N(1.7e+10, 1.3e~
##	4 Albania	trend_mod~	2018	1.37e10	N(1.4e+10, 3.9e~
##	5 Albania	trend_mod~	2019	1.42e10	N(1.4e+10, 3.9e~
##	6 Albania	trend_mod~	2020	1.46e10	N(1.5e+10, 3.9e~
##	7 Algeria	trend_mod~	2018	1.58e11	N(1.6e+11, 9.4e~
##	8 Algeria	trend_mod~	2019	1.61e11	N(1.6e+11, 9.4e~
##	9 Algeria	trend_mod~	2020	1.64e11	N(1.6e+11, 9.4e~

Visualising forecasts

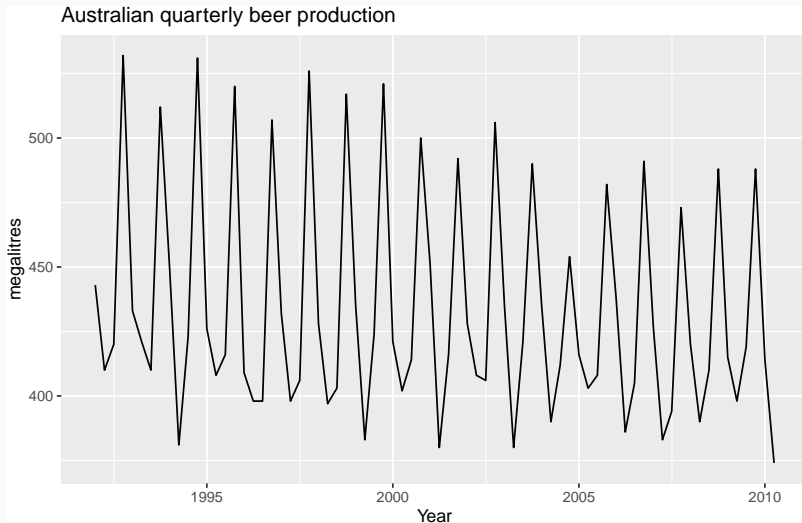
```
fit %>% forecast(h = "3 years") %>%  
  filter(Country=="Sweden") %>%  
  autoplot(global_economy) +  
    ggtitle("GDP for Sweden") + ylab("$US billions")
```



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Some simple forecasting methods



How would you forecast these series?

Some simple forecasting methods



How would you forecast these series?

Some simple forecasting methods

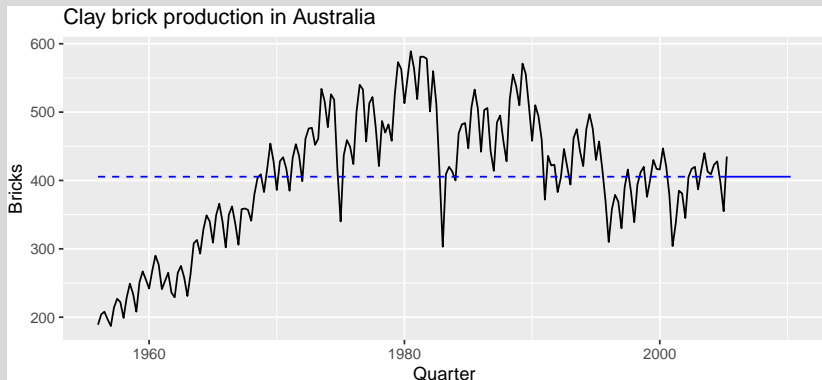


How would you forecast these series?

Some simple forecasting methods

MEAN(y): Average method

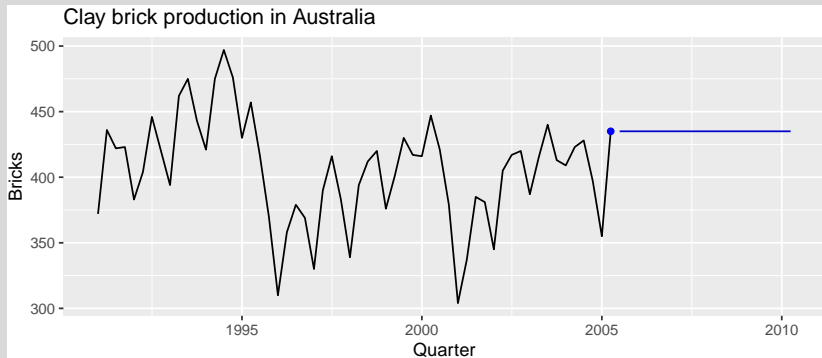
- Forecast of all future values is equal to mean of historical data $\{y_1, \dots, y_T\}$.
- Forecasts: $\hat{y}_{T+h|T} = \bar{y} = (y_1 + \dots + y_T)/T$



Some simple forecasting methods

NAIVE(y): Naïve method

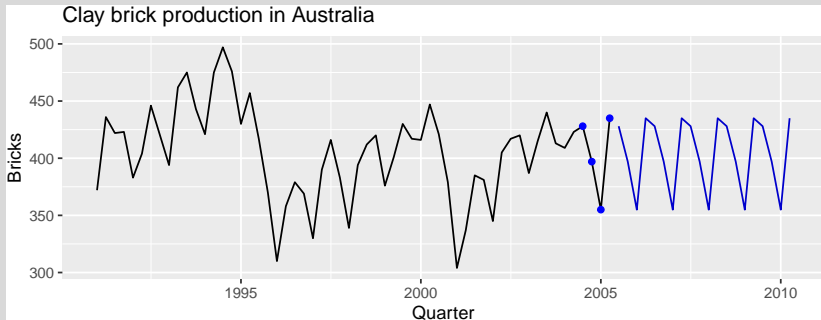
- Forecasts equal to last observed value.
- Forecasts: $\hat{y}_{T+h|T} = y_T$.
- Consequence of efficient market hypothesis.



Some simple forecasting methods

SNAIVE ($y \sim \text{lag}(m)$): Seasonal naïve method

- Forecasts equal to last value from same season.
- Forecasts: $\hat{y}_{T+h|T} = y_{T+h-m(k+1)}$, where m = seasonal period and k is the integer part of $(h - 1)/m$.



Some simple forecasting methods

`RW(y ~ drift())`: Drift method

- Forecasts equal to last value plus average change.
- Forecasts:

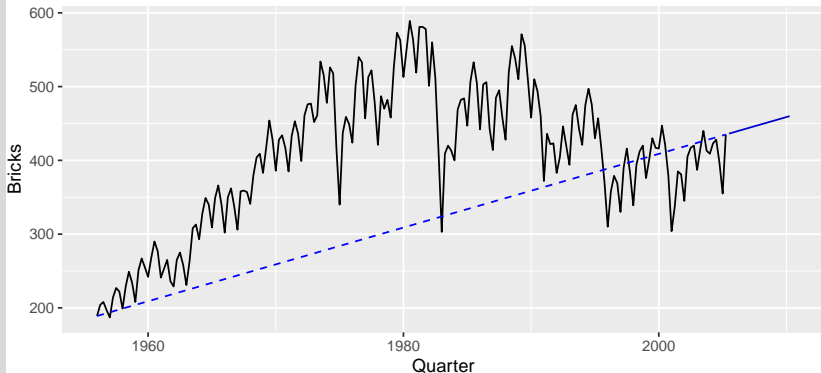
$$\begin{aligned}\hat{y}_{T+h|T} &= y_T + \frac{h}{T-1} \sum_{t=2}^T (y_t - y_{t-1}) \\ &= y_T + \frac{h}{T-1} (y_T - y_1).\end{aligned}$$

- Equivalent to extrapolating a line drawn between first and last observations.

Some simple forecasting methods

Drift method

Clay brick production in Australia



Model fitting

The `model()` function trains models to data.

```
brick_fit <- aus_production %>%  
  filter(!is.na(Bricks)) %>%  
  model(  
    `Seasonal_naïve` = SNAIVE(Bricks),  
    `Naïve` = NAIVE(Bricks),  
    Drift = RW(Bricks ~ drift()),  
    Mean = MEAN(Bricks)  
  )
```

```
## # A mable: 1 x 4  
##   Seasonal_naïve Naïve   Drift         Mean  
##   <model>         <model> <model>         <model>  
## 1 <SNAIVE>        <NAIVE> <RW w/ drift> <MEAN>
```

A mable is a model table, each cell corresponds to a fitted model.

Producing forecasts

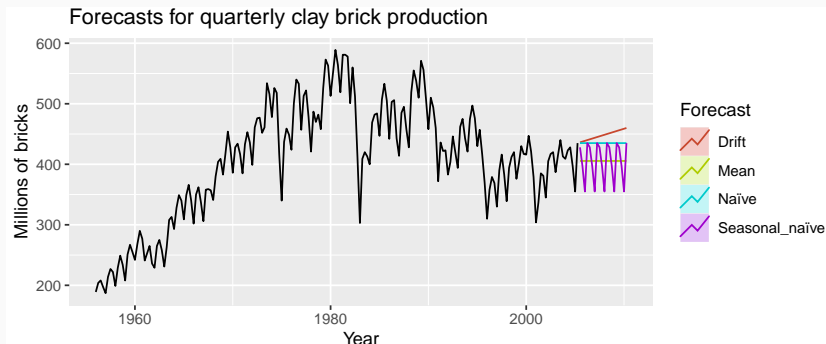
```
brick_fc <- brick_fit %>%  
  forecast(h = "5 years")
```

```
## # A fable: 80 x 4 [1Q]  
## # Key:      .model [4]  
##   .model      Quarter Bricks .distribution  
##   <chr>         <qtr>  <dbl> <dist>  
## 1 Seasonal_naïve 2005 Q3    428 N(428, 2336)  
## 2 Seasonal_naïve 2005 Q4    397 N(397, 2336)  
## 3 Seasonal_naïve 2006 Q1    355 N(355, 2336)  
## 4 Seasonal_naïve 2006 Q2    435 N(435, 2336)  
## # ... with 76 more rows
```

A fable is a forecast table with point forecasts and distributions.

Visualising forecasts

```
brick_fc %>%  
  autoplot(aus_production, level = NULL) +  
  ggtitle("Forecasts for quarterly clay brick production") +  
  xlab("Year") + ylab("Millions of bricks") +  
  guides(colour = guide_legend(title = "Forecast"))
```



Facebook closing stock price

```
# Extract training data
fb_stock <- gafa_stock %>%
  group_by(Symbol) %>%
  mutate(trading_day = row_number()) %>%
  update_tsibble(index=trading_day, regular=TRUE) %>%
  ungroup() %>%
  filter(Symbol == "FB")

# Specify, estimate and forecast
fb_stock %>%
  model(
    Mean = MEAN(Close),
    `Naïve` = NAIVE(Close),
    Drift = RW(Close ~ drift())
  ) %>%
  forecast(h=42) %>%
  autoplot(fb_stock, level = NULL) +
  ggtitle("Facebook closing stock price") +
  xlab("Day") + ylab("") +
  guides(colour=guide_legend(title="Forecast"))
```

Facebook closing stock price



Your turn

- Produce forecasts using an appropriate benchmark method for household wealth (`hh_budget`). Plot the results using `autoplot()`.
- Produce forecasts using an appropriate benchmark method for Australian takeaway food turnover (`aus_retail`). Plot the results using `autoplot()`.

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Fitted values

- $\hat{y}_{t|t-1}$ is the forecast of y_t based on observations y_1, \dots, y_t .
- We call these “fitted values”.
- Sometimes drop the subscript: $\hat{y}_t \equiv \hat{y}_{t|t-1}$.
- Often not true forecasts since parameters are estimated on all data.

For example:

- $\hat{y}_t = \bar{y}$ for average method.
- $\hat{y}_t = y_{t-1} + (y_T - y_1)/(T - 1)$ for drift method.

Forecasting residuals

Residuals in forecasting: difference between observed value and its fitted value: $e_t = y_t - \hat{y}_{t|t-1}$.

Forecasting residuals

Residuals in forecasting: difference between observed value and its fitted value: $e_t = y_t - \hat{y}_{t|t-1}$.

Assumptions

- 1 $\{e_t\}$ uncorrelated. If they aren't, then information left in residuals that should be used in computing forecasts.
- 2 $\{e_t\}$ have mean zero. If they don't, then forecasts are biased.

Forecasting residuals

Residuals in forecasting: difference between observed value and its fitted value: $e_t = y_t - \hat{y}_{t|t-1}$.

Assumptions

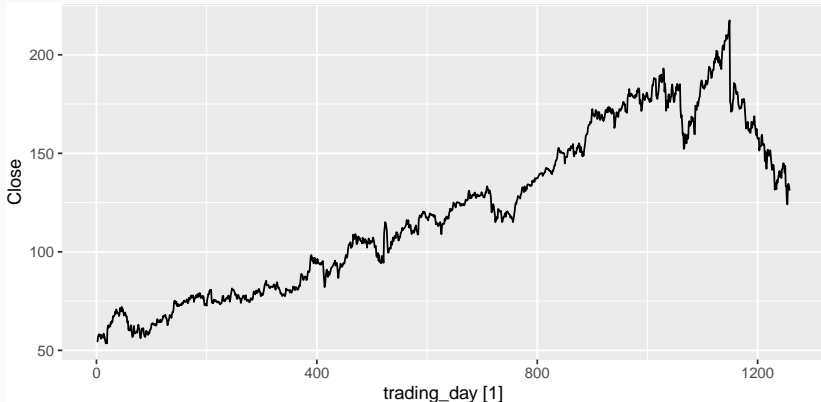
- 1 $\{e_t\}$ uncorrelated. If they aren't, then information left in residuals that should be used in computing forecasts.
- 2 $\{e_t\}$ have mean zero. If they don't, then forecasts are biased.

Useful properties (for distributions & prediction intervals)

- 3 $\{e_t\}$ have constant variance.
- 4 $\{e_t\}$ are normally distributed.

Facebook closing stock price

```
fb_stock <- gafa_stock %>%  
  filter(Symbol == "FB") %>%  
  mutate(trading_day = row_number()) %>%  
  update_tsibble(index = trading_day, regular = TRUE)  
fb_stock %>% autoplot(Close)
```



Facebook closing stock price

```
fit <- fb_stock %>% model(NAIVE(Close))  
augment(fit)
```

```
## # A tibble: 1,258 x 6 [1]  
## # Key:      Symbol, .model [1]  
##   Symbol .model      trading_day Close .fitted .resid  
##   <chr>  <chr>          <int> <dbl>   <dbl>  <dbl>  
## 1 FB     NAIVE(Close)         1  54.7    NA     NA  
## 2 FB     NAIVE(Close)         2  54.6    54.7 -0.150  
## 3 FB     NAIVE(Close)         3  57.2    54.6  2.64  
## 4 FB     NAIVE(Close)         4  57.9    57.2  0.720  
## 5 FB     NAIVE(Close)         5  58.2    57.9  0.310  
## 6 FB     NAIVE(Close)         6  57.2    58.2 -1.01  
## 7 FB     NAIVE(Close)         7  57.9    57.2  0.720  
## 8 FB     NAIVE(Close)         8  55.9    57.9 -2.03  
## 9 FB     NAIVE(Close)         9  57.7    55.9  1.83  
## 10 FB    NAIVE(Close)        10  57.6    57.7 -0.140  
## # ... with 1,248 more rows
```

Facebook closing stock price

```
fit <- fb_stock %>% model(NAIVE(Close))  
augment(fit)
```

```
## # A tsibble: 1,258 x 6 [1]  
## # Key:      Symbol, .model [1]  
##   Symbol .model      trading_day Close .fitted .resid  
##   <chr>   <chr>          <int> <dbl>   <dbl>   <dbl>  
## 1 FB     NAIVE(Close)         1  54.7    NA      NA  
## 2 FB     NAIVE(Close)         2  54.6    54.7  -0.150  
## 3 FB     NAIVE(Close)         3  57.2    54.6   2.64  
## 4 FB     NAIVE(Close)         4  57.9    57.2   0.720  
## 5 FB     NAIVE(Close)         5  58.2    57.9   0.310  
## 6 FB     NAIVE(Close)         6  57.2    58.2  -1.01  
## 7 57.9    57.2   0.720  
## 8 55.9    57.9  -2.03  
## 9 57.7    55.9   1.83  
## 10 57.6    57.7  -0.140
```

Naïve forecasts:

$$\hat{y}_{t|t-1} = y_{t-1}$$

$$e_t = y_t - \hat{y}_{t|t-1} = y_t - y_{t-1}$$

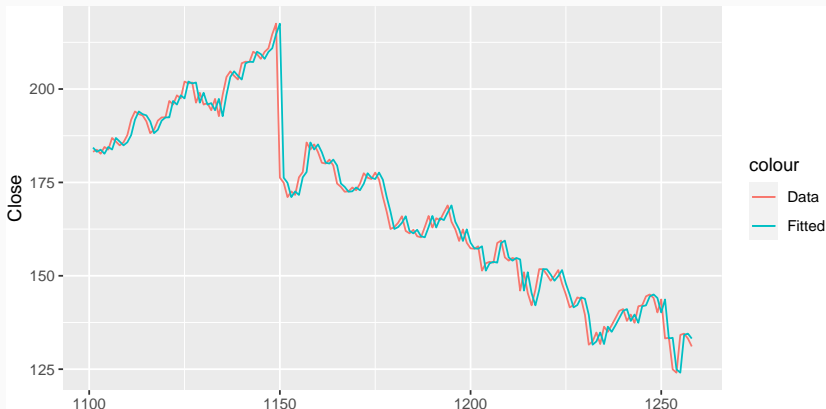
Facebook closing stock price

```
augment(fit) %>%  
  ggplot(aes(x = trading_day)) +  
  geom_line(aes(y = Close, colour = "Data")) +  
  geom_line(aes(y = .fitted, colour = "Fitted"))
```



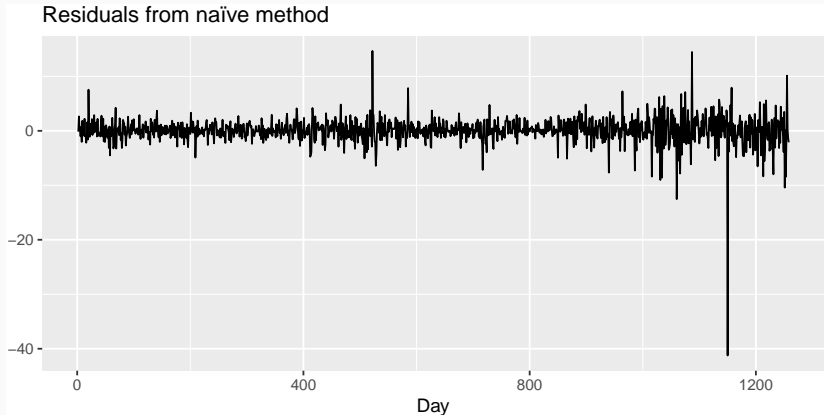
Facebook closing stock price

```
augment(fit) %>%  
  filter(trading_day > 1100) %>%  
  ggplot(aes(x = trading_day)) +  
  geom_line(aes(y = Close, colour = "Data")) +  
  geom_line(aes(y = .fitted, colour = "Fitted"))
```



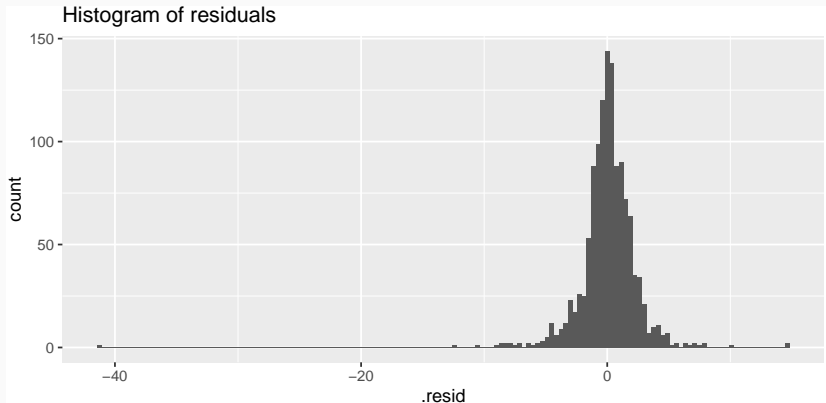
Facebook closing stock price

```
augment(fit) %>%  
  autoplot(.resid) + xlab("Day") + ylab("") +  
  ggtitle("Residuals from naïve method")
```



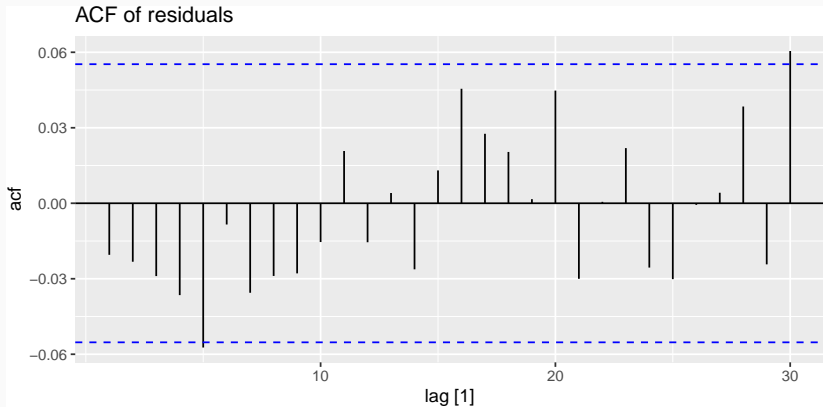
Facebook closing stock price

```
augment(fit) %>%  
  ggplot(aes(x = .resid)) +  
  geom_histogram(bins = 150) +  
  ggtitle("Histogram of residuals")
```



Facebook closing stock price

```
augment(fit) %>%  
  ACF(.resid) %>%  
  autoplot() + ggtitle("ACF of residuals")
```



ACF of residuals

- We assume that the residuals are white noise (uncorrelated, mean zero, constant variance). If they aren't, then there is information left in the residuals that should be used in computing forecasts.
- So a standard residual diagnostic is to check the ACF of the residuals of a forecasting method.
- We *expect* these to look like white noise.

Portmanteau tests

Consider a *whole set* of r_k values, and develop a test to see whether the set is significantly different from a zero set.

Portmanteau tests

Consider a *whole* set of r_k values, and develop a test to see whether the set is significantly different from a zero set.

Box-Pierce test

$$Q = T \sum_{k=1}^h r_k^2$$

where h is max lag being considered and T is number of observations.

- If each r_k close to zero, Q will be **small**.
- If some r_k values large (positive or negative), Q will be **large**.

Portmanteau tests

Consider a *whole* set of r_k values, and develop a test to see whether the set is significantly different from a zero set.

Ljung-Box test

$$Q^* = T(T+2) \sum_{k=1}^h (T-k)^{-1} r_k^2$$

where h is max lag being considered and T is number of observations.

- My preferences: $h = 10$ for non-seasonal data, $h = 2m$ for seasonal data.
- Better performance, especially in small samples.

Portmanteau tests

- If data are WN, Q^* has χ^2 distribution with $(h - K)$ degrees of freedom where K = no. parameters in model.
- When applied to raw data, set $K = 0$.

```
# lag=h and fitdf=K
```

```
Box.test(augment(fit)$resid,  
lag = 10, fitdf = 0, type = "Lj")
```

```
##
```

```
## Box-Ljung test
```

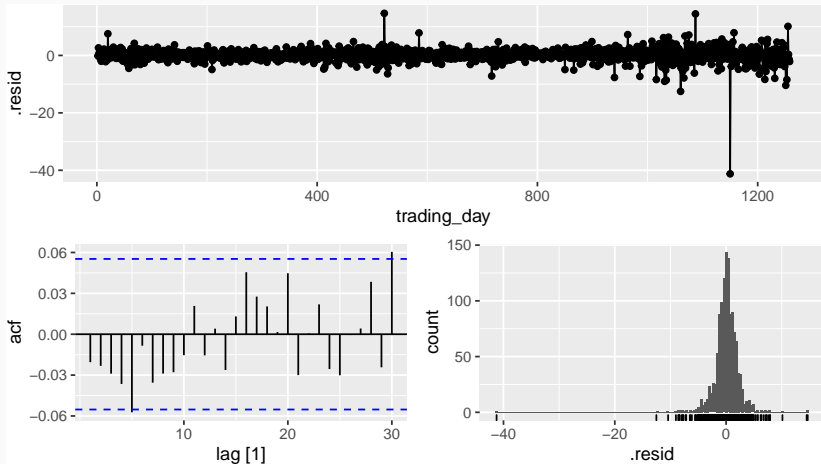
```
##
```

```
## data: augment(fit)$resid
```

```
## X-squared = 12.136, df = 10, p-value = 0.276
```

gg_tsresiduals function

```
gg_tsresiduals(fit)
```



Your turn

Compute seasonal naïve forecasts for quarterly Australian beer production from 1992.

```
recent <- aus_production %>% filter(year(Quarter) >= 1992)
fit <- recent %>% model(SNAIVE(Beer))
fit %>% forecast() %>% autoplot(recent)
```

Test if the residuals are white noise.

```
Box.test(augment(fit)$resid, lag=10, fitdf=0, type="Lj")
gg_tsresiduals(fit)
```

What do you conclude?

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Forecast distributions

- A forecast $\hat{y}_{T+h|T}$ is (usually) the mean of the conditional distribution $y_{T+h} \mid y_1, \dots, y_T$.
- Most time series models produce normally distributed forecasts.
- The forecast distribution describes the probability of observing any future value.

Forecast distributions

Assuming residuals are normal, uncorrelated, $\text{sd} = \hat{\sigma}$:

Mean: $\hat{y}_{T+h|T} \sim N(\bar{y}, (1 + 1/T)\hat{\sigma}^2)$

Naïve: $\hat{y}_{T+h|T} \sim N(y_T, h\hat{\sigma}^2)$

Seasonal naïve: $\hat{y}_{T+h|T} \sim N(y_{T+h-m(k+1)}, (k+1)\hat{\sigma}^2)$

Drift: $\hat{y}_{T+h|T} \sim N(y_T + \frac{h}{T-1}(y_T - y_1), h\frac{T+h}{T}\hat{\sigma}^2)$

where k is the integer part of $(h - 1)/m$.

Note that when $h = 1$ and T is large, these all give the same approximate forecast variance: $\hat{\sigma}^2$.

Prediction intervals

- A prediction interval gives a region within which we expect y_{T+h} to lie with a specified probability.
- Assuming forecast errors are normally distributed, then a 95% PI is

$$\hat{y}_{T+h|T} \pm 1.96\hat{\sigma}_h$$

where $\hat{\sigma}_h$ is the st dev of the h -step distribution.

- When $h = 1$, $\hat{\sigma}_h$ can be estimated from the residuals.

Prediction intervals

```
brick_fc %>% hilo(level = 95)
```

```
## # A tsibble: 80 x 4 [1Q]
```

```
## # Key:           .model [4]
```

##	.model	Quarter	Bricks	`95%`
##	<chr>	<qtr>	<dbl>	<hilo>
##	1 Seasonal_naïve	2005 Q3	428	[333.2737, 522.7263]95
##	2 Seasonal_naïve	2005 Q4	397	[302.2737, 491.7263]95
##	3 Seasonal_naïve	2006 Q1	355	[260.2737, 449.7263]95
##	4 Seasonal_naïve	2006 Q2	435	[340.2737, 529.7263]95
##	5 Seasonal_naïve	2006 Q3	428	[294.0368, 561.9632]95
##	6 Seasonal_naïve	2006 Q4	397	[263.0368, 530.9632]95
##	7 Seasonal_naïve	2007 Q1	355	[221.0368, 488.9632]95
##	8 Seasonal_naïve	2007 Q2	435	[301.0368, 568.9632]95
##	9 Seasonal_naïve	2007 Q3	428	[263.9292, 592.0708]95
##	10 Seasonal_naïve	2007 Q4	397	[232.9292, 561.0708]95

Prediction intervals

- Point forecasts are often useless without a measure of uncertainty (such as prediction intervals).
- Prediction intervals require a stochastic model (with random errors, etc).
- Multi-step forecasts for time series require a more sophisticated approach (with PI getting wider as the forecast horizon increases).

Prediction intervals

- Computed automatically from the forecast distribution.
- Use `level` argument to control coverage.
- Check residual assumptions before believing them (we will see this next class).
- Usually too narrow due to unaccounted uncertainty.

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Modelling with transformations

Transformations used in the left of the formula will be automatically back-transformed. To model log-transformed food retailing turnover, you could use:

```
food <- aus_retail %>%  
  filter(Industry == "Food retailing") %>%  
  summarise(Turnover = sum(Turnover))
```

```
fit <- food %>%  
  model(SNAIVE(log(Turnover)))
```

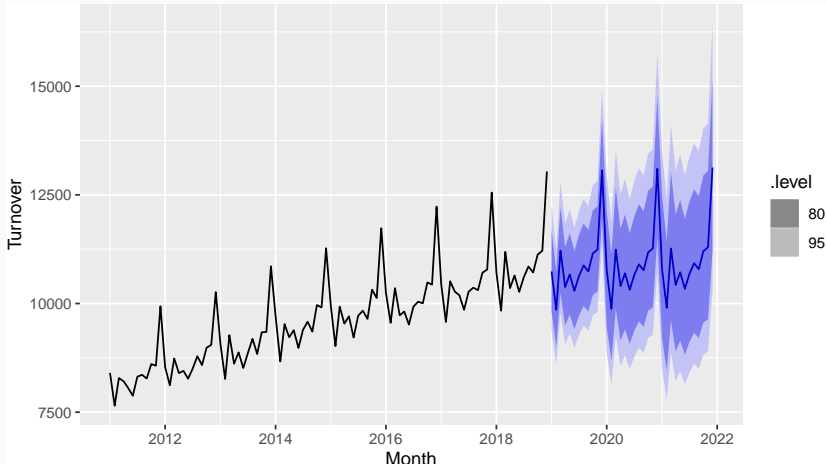
Forecasting with transformations

```
fc <- fit %>%  
  forecast(h = "3 years")
```

```
## # A tibble: 36 x 4 [1M]  
## # Key:   .model [1]  
##   .model                Month Turnover .distribution  
##   <chr>                 <mth>      <dbl> <dist>  
## 1 SNAIVE(log(Turnover)) 2019 Jan    10738. t(N(9.3, 0.0047))  
## 2 SNAIVE(log(Turnover)) 2019 Feb     9856. t(N(9.2, 0.0047))  
## 3 SNAIVE(log(Turnover)) 2019 Mar    11214. t(N(9.3, 0.0047))  
## 4 SNAIVE(log(Turnover)) 2019 Apr    10378. t(N(9.2, 0.0047))  
## 5 SNAIVE(log(Turnover)) 2019 May    10670. t(N(9.3, 0.0047))  
## 6 SNAIVE(log(Turnover)) 2019 Jun    10292. t(N(9.2, 0.0047))  
## # ... with 30 more rows
```

Forecasting with transformations

```
fc %>% autoplot(filter(food, year(Month) > 2010))
```



Bias adjustment

- Back-transformed point forecasts are medians.
- Back-transformed PI have the correct coverage.

Bias adjustment

- Back-transformed point forecasts are medians.
- Back-transformed PI have the correct coverage.

Back-transformed means

Let X be have mean μ and variance σ^2 .

Let $f(x)$ be back-transformation function, and $Y = f(X)$.

Taylor series expansion about μ :

$$f(X) = f(\mu) + (X - \mu)f'(\mu) + \frac{1}{2}(X - \mu)^2f''(\mu).$$

Bias adjustment

- Back-transformed point forecasts are medians.
- Back-transformed PI have the correct coverage.

Back-transformed means

Let X be have mean μ and variance σ^2 .

Let $f(x)$ be back-transformation function, and $Y = f(X)$.

Taylor series expansion about μ :

$$f(X) = f(\mu) + (X - \mu)f'(\mu) + \frac{1}{2}(X - \mu)^2f''(\mu).$$

$$E[Y] = E[f(X)] = f(\mu) + \frac{1}{2}\sigma^2f''(\mu)$$

Bias adjustment

Box-Cox back-transformation:

$$y_t = \begin{cases} \exp(w_t) & \lambda = 0; \\ (\lambda W_t + 1)^{1/\lambda} & \lambda \neq 0. \end{cases}$$

$$f(x) = \begin{cases} e^x & \lambda = 0; \\ (\lambda x + 1)^{1/\lambda} & \lambda \neq 0. \end{cases}$$

$$f''(x) = \begin{cases} e^x & \lambda = 0; \\ (1 - \lambda)(\lambda x + 1)^{1/\lambda - 2} & \lambda \neq 0. \end{cases}$$

Bias adjustment

Box-Cox back-transformation:

$$y_t = \begin{cases} \exp(w_t) & \lambda = 0; \\ (\lambda W_t + 1)^{1/\lambda} & \lambda \neq 0. \end{cases}$$

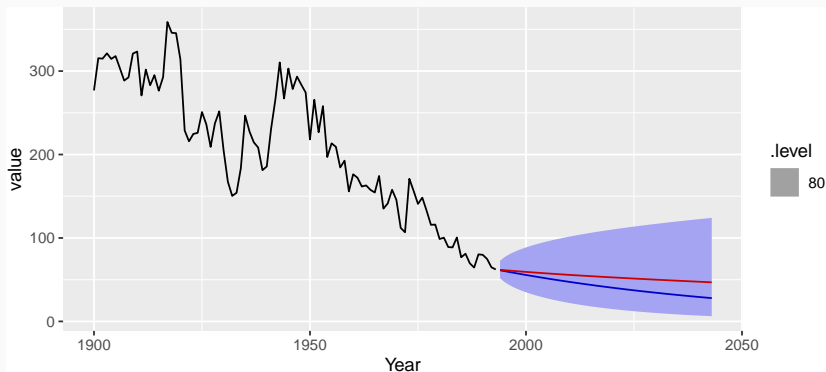
$$f(x) = \begin{cases} e^x & \lambda = 0; \\ (\lambda x + 1)^{1/\lambda} & \lambda \neq 0. \end{cases}$$

$$f''(x) = \begin{cases} e^x & \lambda = 0; \\ (1 - \lambda)(\lambda x + 1)^{1/\lambda - 2} & \lambda \neq 0. \end{cases}$$

$$E[Y] = \begin{cases} e^{\mu} \left[1 + \frac{\sigma^2}{2} \right] & \lambda = 0; \\ (\lambda \mu + 1)^{1/\lambda} \left[1 + \frac{\sigma^2(1-\lambda)}{2(\lambda \mu + 1)^2} \right] & \lambda \neq 0. \end{cases}$$

Bias adjustment

```
eggs <- as_tsibble(fma::eggs)
fit <- eggs %>% model(RW(log(value) ~ drift()))
fc <- fit %>% forecast(h=50)
fc_biased <- fit %>% forecast(h=50, bias_adjust = FALSE)
eggs %>% autoplot(value) + xlab("Year") +
  autolayer(fc_biased, level = 80) +
  autolayer(fc, colour = "red", level = NULL)
```



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Forecasting and decomposition

- Forecast seasonal component by repeating the last year
- Forecast seasonally adjusted data using non-seasonal time series method.
- Combine forecasts of seasonal component with forecasts of seasonally adjusted data to get forecasts of original data.
- Sometimes a decomposition is useful just for understanding the data before building a separate forecasting model.

US Retail Employment

```
us_retail_employment <- us_employment %>%  
  filter(year(Month) >= 1990, Title == "Retail Trade") %>%  
  select(-Series_ID)  
us_retail_employment
```

```
## # A tibble: 357 x 3 [1M]  
##       Month Title      Employed  
##       <mth> <chr>      <dbl>  
## 1 1990 Jan Retail Trade 13256.  
## 2 1990 Feb Retail Trade 12966.  
## 3 1990 Mar Retail Trade 12938.  
## 4 1990 Apr Retail Trade 13012.  
## 5 1990 May Retail Trade 13108.  
## 6 1990 Jun Retail Trade 13183.  
## 7 1990 Jul Retail Trade 13170.  
## 8 1990 Aug Retail Trade 13160.  
## 9 1990 Sep Retail Trade 13113.  
## 10 1990 Oct Retail Trade 13185.
```


US Retail Employment

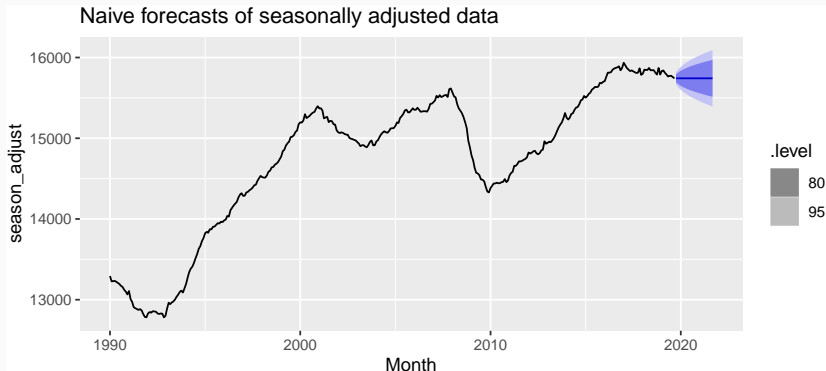
```
dcmp <- us_retail_employment %>%  
  model(STL(Employed)) %>%  
  components() %>% select(-.model)  
dcmp
```

```
## # A tsibble: 357 x 6 [1M]
```

##		Month	Employed	trend	season_year	remainder
##		<mth>	<dbl>	<dbl>	<dbl>	<dbl>
##	1	1990 Jan	13256.	13291.	-38.1	3.08
##	2	1990 Feb	12966.	13272.	-261.	-44.2
##	3	1990 Mar	12938.	13252.	-291.	-23.0
##	4	1990 Apr	13012.	13233.	-221.	0.0892
##	5	1990 May	13108.	13213.	-115.	9.98
##	6	1990 Jun	13183.	13193.	-25.6	15.7
##	7	1990 Jul	13170.	13173.	-24.4	22.0
##	8	1990 Aug	13160.	13152.	-11.8	19.5
##	9	1990 Sep	13113.	13131.	-43.4	25.7
##	10	1990 Oct	13185.	13110.	-62.5	12.2

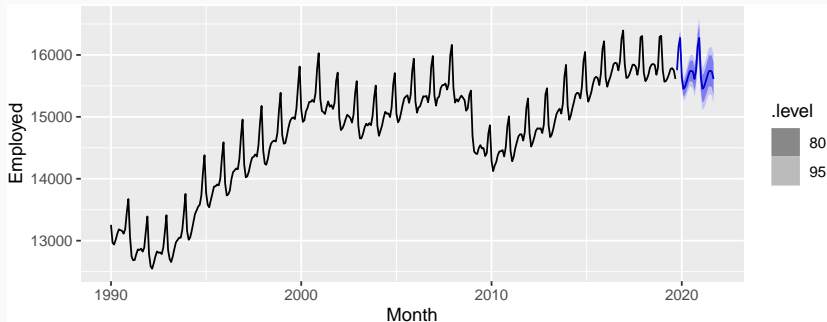
US Retail Employment

```
dcmp %>%  
  model(NAIVE(season_adjust)) %>%  
  forecast() %>%  
  autoplot(dcmp) +  
  ggtitle("Naive forecasts of seasonally adjusted data")
```



US Retail Employment

```
us_retail_employment %>%  
  model(stlf = decomposition_model(  
    STL(Employed ~ trend(window = 7), robust = TRUE),  
    NAIVE(season_adjust)  
  )) %>%  
  forecast() %>%  
  autoplot(us_retail_employment)
```



Decomposition models

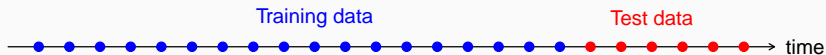
`decomposition_model()` creates a decomposition model

- You must provide a method for forecasting the `season_adjust` series.
- A seasonal naive method is used by default for the `seasonal` components.
- The variances from both the seasonally adjusted and seasonal forecasts are combined.

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Training and test sets



- A model which fits the training data well will not necessarily forecast well.
- A perfect fit can always be obtained by using a model with enough parameters.
- Over-fitting a model to data is just as bad as failing to identify a systematic pattern in the data.
- The test set must not be used for *any* aspect of model development or calculation of forecasts.
- Forecast accuracy is based only on the test set.

Forecast errors

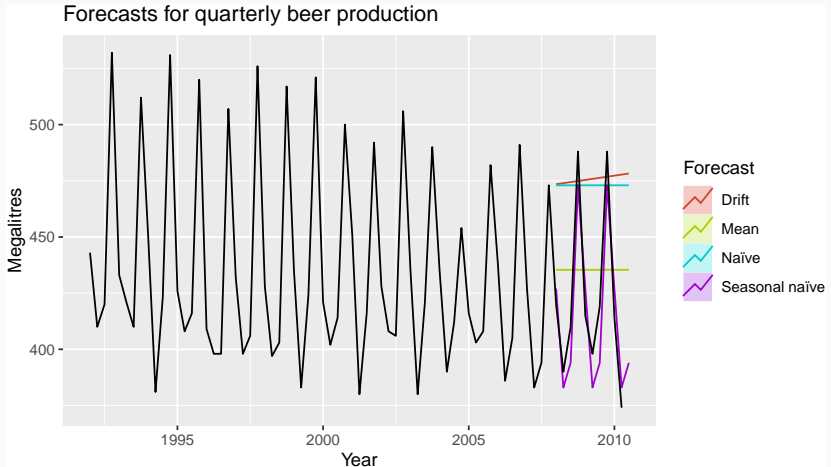
Forecast “error”: the difference between an observed value and its forecast.

$$e_{T+h} = y_{T+h} - \hat{y}_{T+h|T},$$

where the training data is given by $\{y_1, \dots, y_T\}$

- Unlike residuals, forecast errors on the test set involve multi-step forecasts.
- These are *true* forecast errors as the test data is not used in computing $\hat{y}_{T+h|T}$.

Measures of forecast accuracy



Measures of forecast accuracy

y_{T+h} = $(T + h)$ th observation, $h = 1, \dots, H$

$\hat{y}_{T+h|T}$ = its forecast based on data up to time T .

$$e_{T+h} = y_{T+h} - \hat{y}_{T+h|T}$$

$$\text{MAE} = \text{mean}(|e_{T+h}|)$$

$$\text{MSE} = \text{mean}(e_{T+h}^2)$$

$$\text{RMSE} = \sqrt{\text{mean}(e_{T+h}^2)}$$

$$\text{MAPE} = 100\text{mean}(|e_{T+h}|/|y_{T+h}|)$$

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$$\text{MAPE} = 100\text{mean}(|e_{T+h}|/|y_{T+h}|)$$

- MAE, MSE, RMSE are all scale dependent.
- MAPE is scale independent but is only sensible if $y_t \gg 0$ for all t , and y has a natural zero.

Measures of forecast accuracy

Mean Absolute Scaled Error

$$\text{MASE} = \text{mean}(|e_{T+h}|/Q)$$

where Q is a stable measure of the scale of the time series $\{y_t\}$.

Proposed by Hyndman and Koehler (IJF, 2006).

For non-seasonal time series,

$$Q = (T - 1)^{-1} \sum_{t=2}^T |y_t - y_{t-1}|$$

works well. Then MASE is equivalent to MAE relative to a naïve method.

Measures of forecast accuracy

Mean Absolute Scaled Error

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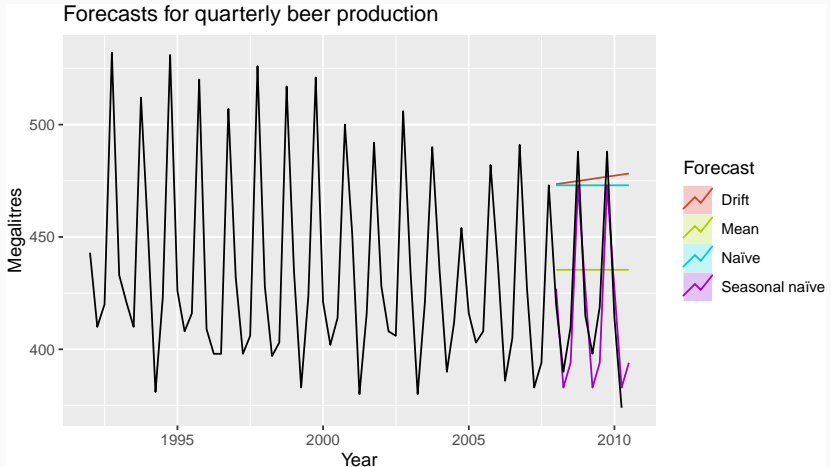
Proposed by Hyndman and Koehler (IJF, 2006).

For seasonal time series,

$$Q = (T - m)^{-1} \sum_{t=m+1}^T |y_t - y_{t-m}|$$

works well. Then MASE is equivalent to MAE relative to a seasonal naïve method.

Measures of forecast accuracy



Measures of forecast accuracy

```
recent_production <- aus_production %>%  
  filter(year(Quarter) >= 1992)  
train <- recent_production %>%  
  filter(year(Quarter) <= 2007)  
beer_fit <- train %>%  
  model(  
    Mean = MEAN(Beer),  
    `Naïve` = NAIVE(Beer),  
    `Seasonal naïve` = SNAIVE(Beer),  
    Drift = RW(Beer ~ drift())  
  )  
beer_fc <- beer_fit %>%  
  forecast(h = 10)
```

Measures of forecast accuracy

```
accuracy(beer_fit)
```

```
## # A tibble: 4 x 6
##   .model      .type    RMSE    MAE    MAPE    MASE
##   <chr>      <chr>    <dbl> <dbl> <dbl> <dbl>
## 1 Drift      Training  65.3   54.8  12.2   3.83
## 2 Mean      Training  43.6   35.2   7.89   2.46
## 3 Naïve     Training  65.3   54.7  12.2   3.83
## 4 Seasonal naïve Training  16.8   14.3   3.31   1
```

```
accuracy(beer_fc, recent_production)
```

```
## # A tibble: 4 x 6
##   .model      .type    RMSE    MAE    MAPE    MASE
##   <chr>      <chr>    <dbl> <dbl> <dbl> <dbl>
## 1 Drift      Test     64.9   58.9  14.6   4.12
## 2 Mean      Test     38.4   34.8   8.28   2.44
## 3 Naïve     Test     62.7   57.4  14.2   4.01
## 4 Seasonal naïve Test     14.3   13.4   3.17  0.937
```

Poll: true or false?

- 1 Good forecast methods should have normally distributed residuals.
- 2 A model with small residuals will give good forecasts.
- 3 The best measure of forecast accuracy is MAPE.
- 4 If your model doesn't forecast well, you should make it more complicated.
- 5 Always choose the model with the best forecast accuracy as measured on the test set.

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Time series cross-validation

Traditional evaluation

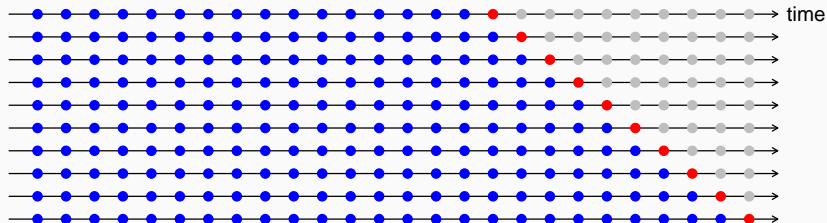


Time series cross-validation

Traditional evaluation



Time series cross-validation

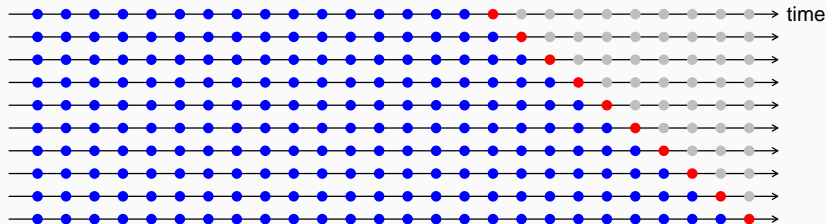


Time series cross-validation

Traditional evaluation



Time series cross-validation



- Forecast accuracy averaged over test sets.
- Also known as “evaluation on a rolling forecasting origin”

Creating the rolling training sets

There are three main rolling types which can be used.

- Stretch: extends a growing length window with new data.
- Slide: shifts a fixed length window through the data.
- Tile: moves a fixed length window without overlap.

Three functions to roll a tsibble: `stretch_tsibble()`, `slide_tsibble()`, and `tile_tsibble()`.

For time series cross-validation, stretching windows are most commonly used.

Creating the rolling training sets

Time series cross-validation

Stretch with a minimum length of 3, growing by 1 each step.

```
fb_stretch <- fb_stock %>%  
  stretch_tsibble(.init = 3, .step = 1) %>%  
  filter(.id != max(.id))
```

```
## # A tsibble: 790,650 x 4 [1]  
## # Key:           .id [1,255]  
##   Date      Close trading_day  .id  
##   <date>    <dbl>         <int> <int>  
## 1 2014-01-02  54.7             1     1  
## 2 2014-01-03  54.6             2     1  
## 3 2014-01-06  57.2             3     1  
## 4 2014-01-02  54.7             1     2  
## 5 2014-01-03  54.6             2     2  
## 6 2014-01-06  57.2             3     2  
## 7 2014-01-07  57.9             4     2
```

Time series cross-validation

Estimate RW w/ drift models for each window.

```
fit_cv <- fb_stretch %>%  
  model(RW(Close ~ drift()))
```

```
## # A mable: 1,255 x 3  
## # Key:      .id, Symbol [1,255]  
##      .id Symbol `RW(Close ~ drift())`  
##    <int> <chr>  <model>  
## 1      1 FB      <RW w/ drift>  
## 2      2 FB      <RW w/ drift>  
## 3      3 FB      <RW w/ drift>  
## 4      4 FB      <RW w/ drift>  
## # ... with 1,251 more rows
```


Time series cross-validation

Produce one step ahead forecasts from all models.

```
fc_cv <- fit_cv %>%  
  forecast(h=1)
```

```
## # A tibble: 1,255 x 5  
## # Key:   .id, Symbol [1,255]  
##       .id Symbol trading_day Close .distribution  
##   <int> <chr>          <dbl> <dbl> <dist>  
## 1     1  1 FB              4  58.4 N(58, 3.9)  
## 2     2  2 FB              5  59.0 N(59, 2)  
## 3     3  3 FB              6  59.1 N(59, 1.5)  
## 4     4  4 FB              7  57.7 N(58, 1.8)  
## # ... with 1,251 more rows
```

Time series cross-validation

```
# Cross-validated  
fc_cv %>% accuracy(fb_stock)  
# Training set  
fb_stock %>% model(RW(Close ~ drift())) %>% accuracy()
```

	RMSE	MAE	MAPE
Cross-validation	2.418	1.469	1.266
Training	2.414	1.465	1.261

A good way to choose the best forecasting model is to find the model with the smallest RMSE computed using time series cross-validation.