MONASH University

MONASH BUSINESS SCHOOL

ETC3550 Applied forecasting for business and economics

Ch3. The forecasters' toolbox

OTexts.org/fpp3/

Outline

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A tidy forecasting workflow

The process of producing forecasts can be split up into a few fundamental steps.

1. Data preparation
2. Visualise
3. Specify a model
4. Estimating the model
5. Evaluate model
6. Producing forecasts

A tidy forecasting workflow

Outline

Some simple forecasting methods

How would you forecast these time

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Average method

- Forecast of all future values is equal to mean of historical data $\{y_1, \dots, y_T\}$.
- Forecasts: $\hat{y}_{T+h|T} = \bar{y} = (y_1 + \cdots + y_T)/T$

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Naïve method

- Forecasts equal to last observed value.
- Forecasts: $\hat{y}_{T+h|T} = y_T$.
- Consequence of efficient market hypothesis.

Some simple forecasting methods

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Naïve method

- Forecasts equal to last observed value.
- Forecasts: $\hat{y}_{T+h|T} = y_T$.
- Consequence of efficient market hypothesis.

Seasonal naïve method

- Forecasts equal to last value from same season.
- Forecasts: $\hat{y}_{T+h|T} = y_{T+h-m(k+1)}$, where m = seasonal period and k is the integer part of $(h-1)/m$.

Some simple forecasting methods

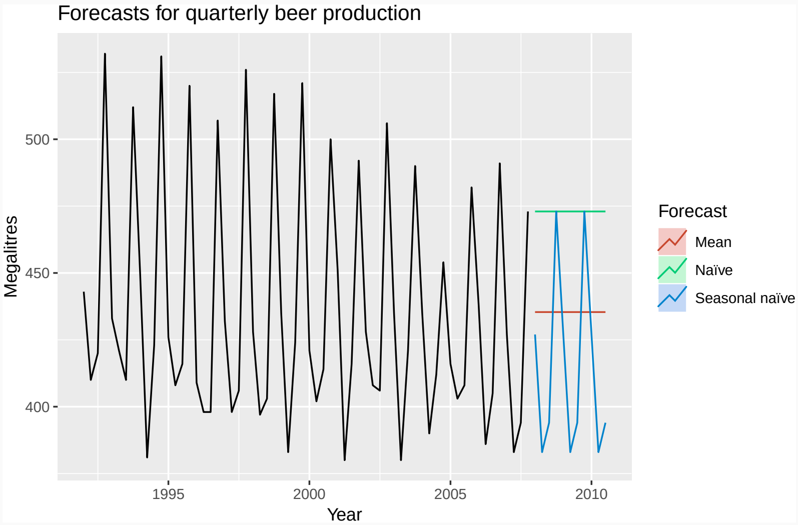
Drift method

- Forecasts equal to last value plus average change.
- Forecasts:

$$\hat{y}_{T+h|T} = y_T + \frac{h}{T-1}\sum_{t=2}^{T}(y_t - y_{t-1})$$
$$= y_T + \frac{h}{T-1}(y_T - y_1).$$

- Equivalent to extrapolating a line drawn between first and last observations.

Some simple forecasting methods



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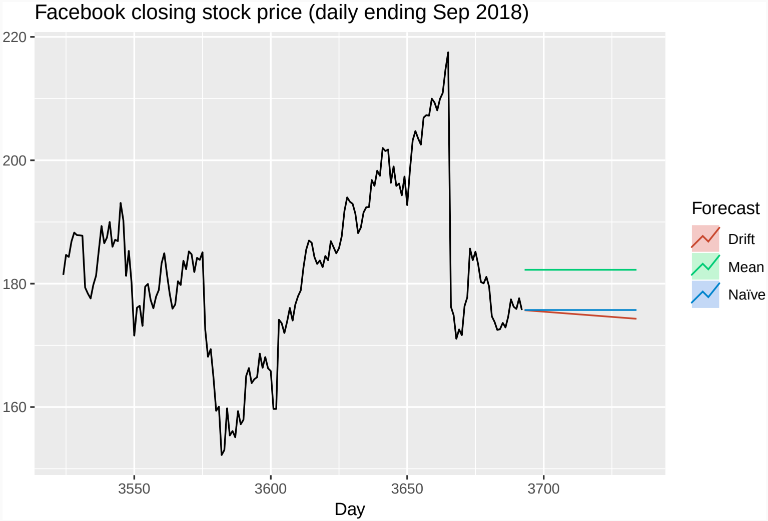

Some simple forecasting methods

Code for previous graph

```
fb_stock <- gafa_stock %>%
  mutate(trading_day = row_number()) %>%
  update_tsibble(index=trading_day, regular=TRUE) %>%
  filter(Symbol == "FB",
         between(Date, ymd("2018-01-01"), ymd("2018-09-01")))

fb_stock %>%
  model(
    Mean = MEAN(Close),
    Naïve = NAIVE(Close),
    Drift = RW(Close ~ drift())
  ) %>%
  forecast(h=42) %>%
  autoplot(fb_stock, level = NULL) +
    ggtitle("Facebook closing stock price (daily ending Sep 2018)") +
    xlab("Day") + ylab("") +
    guides(colour=guide_legend(title="Forecast"))
```

Some simple forecasting methods

- Mean: `MEAN(y)`
- Naïve: `NAIVE(y)`
- Seasonal naïve: `SNAIVE(y)`
- Drift: `RW(y ~ drift())`

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Your turn

- Use these four functions to produce forecasts for Facebook closing price (`gafa_stock`) and Australian takeaway food turnover (`aus_retail`).
- Plot the results using `autoplot()`.

Outline

Specifying a model

```
model_fn(t(LHS) ~ specials, extras)
```

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Mathematical transformations for stabilizing variation

Square root	$w_t = \sqrt{y_t}$	$\downarrow$
Cube root	$w_t = \sqrt[3]{y_t}$	Increasing
Logarithm	$w_t = \log(y_t)$	strength

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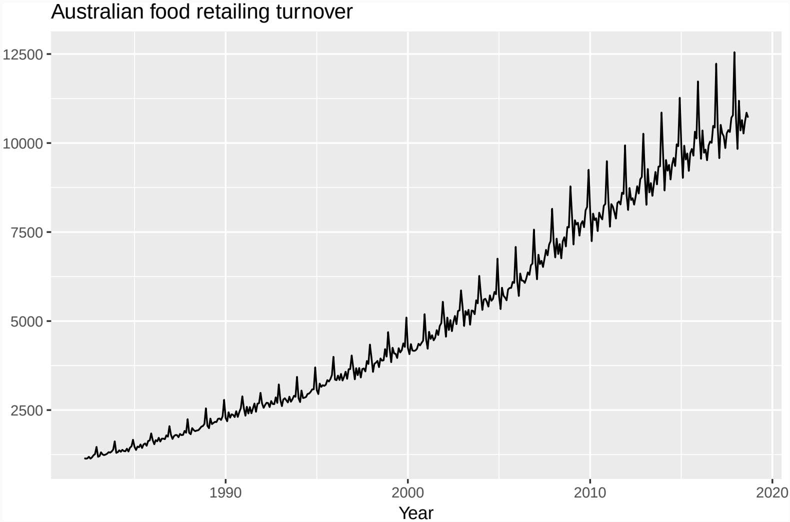
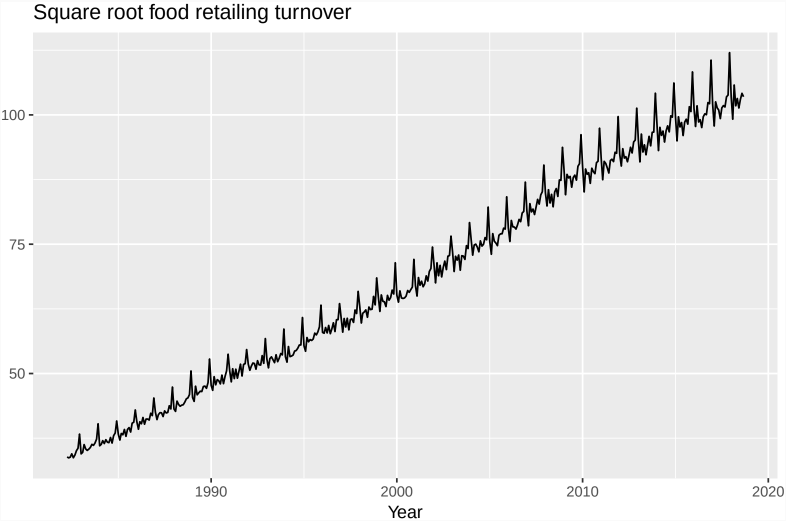
Logarithms, in particular, are useful because they are more interpretable: changes in a log value are **relative (percent) changes on the original scale**.

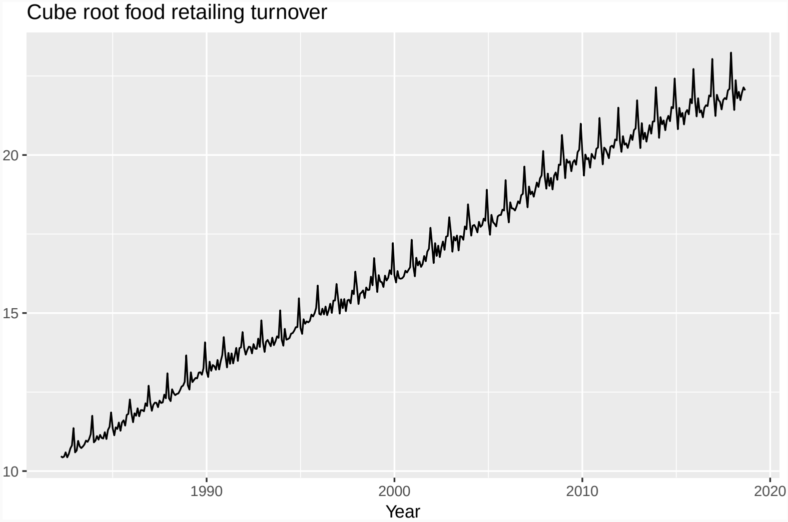
Variance stabilization



Variance stabilization



Variance stabilization



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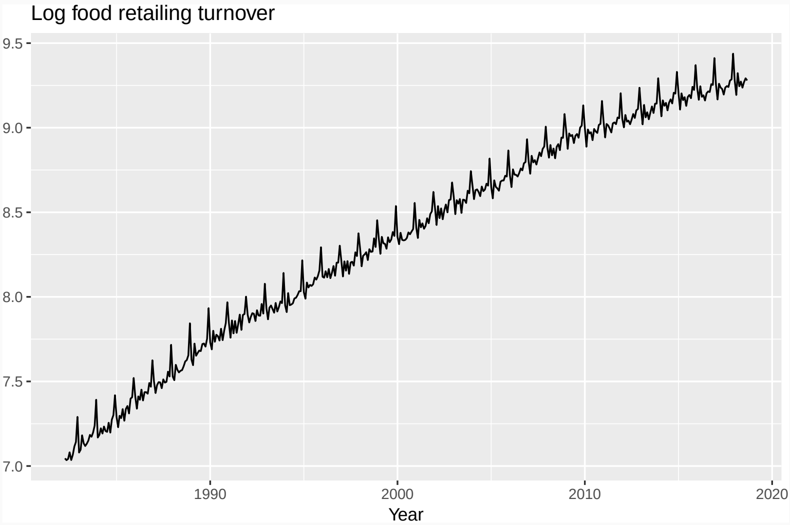

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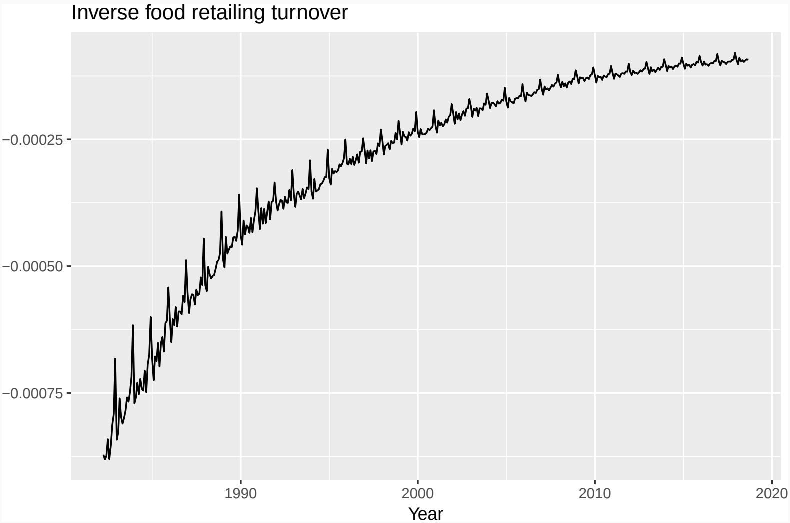

Box-Cox transformations

Each of these transformations is close to a member of the family of **Box-Cox transformations**:

$$w_t = \begin{cases} \log(y_t), & \lambda = 0; \\ (y_t^\lambda - 1)/\lambda, & \lambda \neq 0. \end{cases}$$

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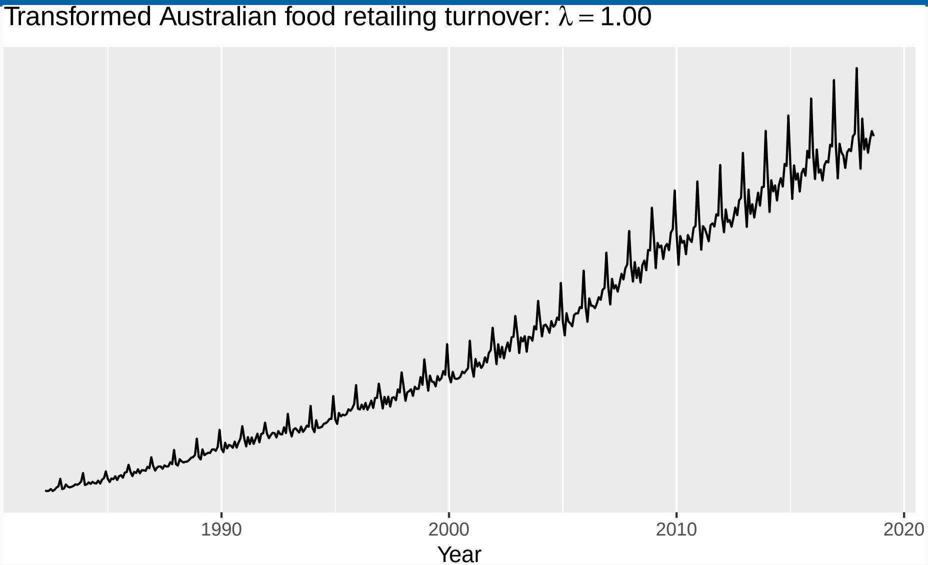
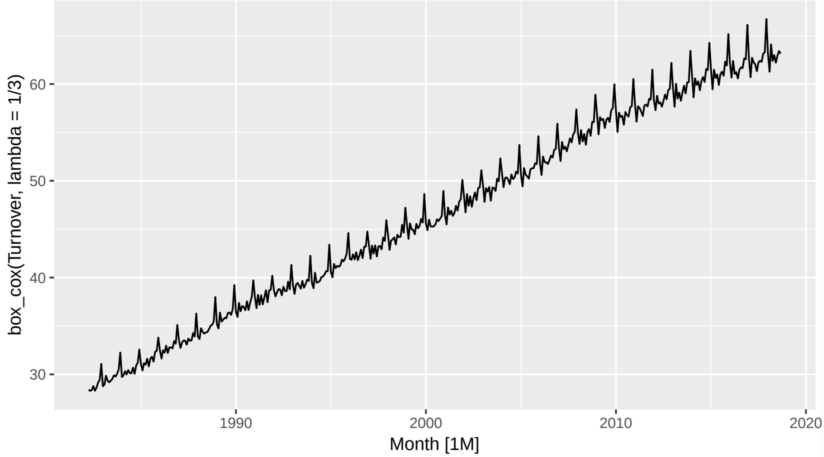
- $\lambda = 1$: (No substantive transformation)
- $\lambda = \frac{1}{2}$: (Square root plus linear transformation)
- $\lambda = 0$: (Natural logarithm)
- $\lambda = -1$: (Inverse plus 1)

Box-Cox transformations



Box-Cox transformations

```r
food %>% autoplot(box_cox(Turnover,lambda=1/3))
```



Box-Cox transformations

- y_t^λ for λ close to zero behaves like logs.
- If some $y_t = 0$, then must have $\lambda > 0$
- if some $y_t < 0$, no power transformation is possible unless all y_t adjusted by **adding a constant to all values**.
- Simple values of λ are easier to explain.
- Results are relatively insensitive to λ.
- Often no transformation ($\lambda = 1$) needed.
- Transformation can have very large effect on PI.
- Choosing $\lambda = 0$ is a simple way to force forecasts to be positive

Box-Cox transformations

```
food %>% features(Turnover, features = guerrer
```

```
## # A tibble: 1 x 1
##   lambda_guerrero
##             <dbl>
## 1         0.00762
```

Box-Cox transformations

```
food %>% features(Turnover, features = guerrer
```

```
## # A tibble: 1 x 1
##   lambda_guerrero
##             <dbl>
## 1         0.00762
```

- This attempts to balance the seasonal fluctuations and random variation across the series.
- Always check the results.
- A low value of λ can give extremely large

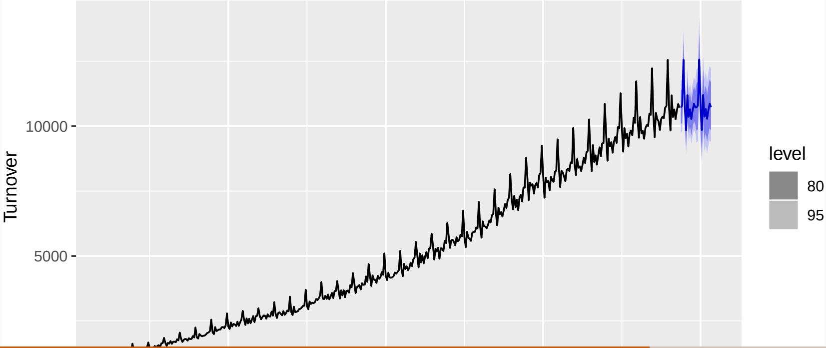
Back-transformation

We must reverse the transformation (or *back-transform*) to obtain forecasts on the original scale. The reverse Transformations are given by

$$y_t = \begin{cases} \exp(w_t), & \lambda = 0; \\ (\lambda W_t + 1)^{1/\lambda}, & \lambda \neq 0. \end{cases}$$

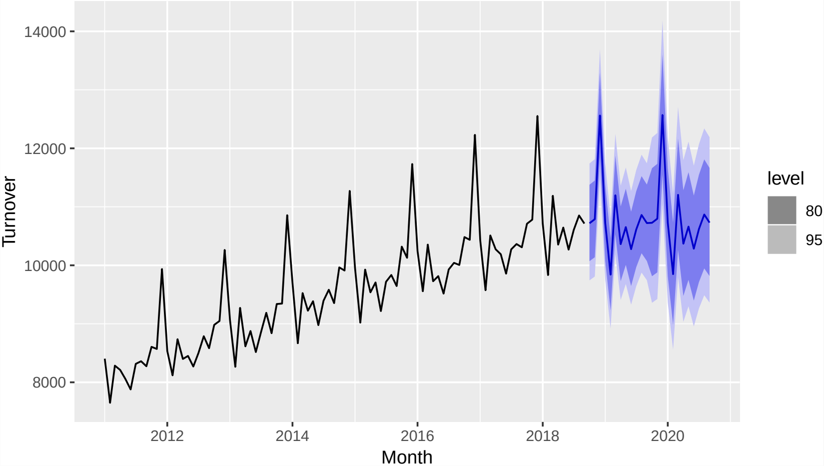
Back-transformation

```
fc <- food %>%
  model(SNAIVE(box_cox(Turnover, lambda=1/3))) %>%
  forecast()
fc %>% autoplot(food)
```



Back-transformation

```
fc %>% autoplot(filter(food, year(Month) > 201
```



Your turn

Find a Box-Cox transformation that works for the Australian gas production (`aus_production`).

Bias adjustment

- Back-transformed point forecasts are medians.
- Back-transformed PI have the correct coverage.

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Back-transformed means

Let X be have mean μ and variance σ^2.

Let $f(x)$ be back-transformation function, and $Y = f(X)$.

Taylor series expansion about μ:

$$f(X) = f(\mu) + (X - \mu)f'(\mu) + \frac{1}{2}(X - \mu)^2 f''(\mu).$$

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$$\text{E}[Y] = \text{E}[f(X)] = f(\mu) + \tfrac{1}{2}\sigma^2 f''(\mu)$$

Bias adjustment

Box-Cox back-transformation:

$$y_t = \begin{cases} \exp(w_t) & \lambda = 0; \\ (\lambda W_t + 1)^{1/\lambda} & \lambda \neq 0. \end{cases}$$

$$f(x) = \begin{cases} e^x & \lambda = 0; \\ (\lambda x + 1)^{1/\lambda} & \lambda \neq 0. \end{cases}$$

$$f''(x) = \begin{cases} e^x & \lambda = 0; \\ (1 - \lambda)(\lambda x + 1)^{1/\lambda - 2} & \lambda \neq 0. \end{cases}$$

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$$\text{E}[Y] = \begin{cases} e^{\mu}\left[1 + \frac{\sigma^2}{2}\right] & \lambda = 0; \\ (\lambda\mu + 1)^{1/\lambda}\left[1 + \frac{\sigma^2(1-\lambda)}{2(\lambda\mu+1)^2}\right] & \lambda \neq 0. \end{cases}$$

Bias adjustment

```
eggs <- as_tsibble(fma::eggs)
fit <- eggs %>% model(RW(log(value) ~ drift()))
fc <- fit %>% forecast(h=50)
fc_biased <- fit %>% forecast(h=50, bias_adjust = FALSE)
eggs %>% autoplot(value) +
  autolayer(fc_biased, series="Simple back transformation", level
  autolayer(fc, series="Bias adjusted", level = NULL) +
  guides(colour=guide_legend(title="Forecast"))
```

Outline

Prediction intervals

- A forecast $\hat{y}_{T+h|T}$ is (usually) the mean of the conditional distribution $y_{T+h} \mid y_1, \dots, y_T$.
- A prediction interval gives a region within which we expect y_{T+h} to lie with a specified probability.
- Assuming forecast errors are normally distributed, then a 95% PI is

$$\hat{y}_{T+h|T} \pm 1.96\hat{\sigma}_h$$

  where $\hat{\sigma}_h$ is the st dev of the h-step distribution.
- When $h = 1$, $\hat{\sigma}_h$ can be estimated from the residuals.

Prediction intervals

Naive forecast with prediction interval:

```
fit <- fb_stock %>% model(NAIVE(Close))
res_sd <- sd(augment(fit)$.resid, na.rm = TRUE)
last(fb_stock$Close) + 1.96 * res_sd * c(-1,1)
```

```
## [1] 166.6929 184.7670
```

```
forecast(fit, h = 1) %>%
  mutate(interval = hilo(.distribution, 95))
```

```
## # A fable: 1 x 6 [?]
## # Key:     Symbol, .model [1]
##   Symbol .model trading_day Close
##   <fct>  <chr>        <int> <dbl>
```

Prediction intervals

- Point forecasts are often useless without prediction intervals.
- Prediction intervals require a stochastic model (with random errors, etc).
- Multi-step forecasts for time series require a more sophisticated approach (with PI getting wider as the forecast horizon increases).

Prediction intervals

Assume residuals are normal, uncorrelated, sd = $\hat{\sigma}$:

Mean forecasts:	$\hat{\sigma}_h = \hat{\sigma}\sqrt{1 + 1/T}$
Naïve forecasts:	$\hat{\sigma}_h = \hat{\sigma}\sqrt{h}$
Seasonal naïve forecasts	$\hat{\sigma}_h = \hat{\sigma}\sqrt{k + 1}$
Drift forecasts:	$\hat{\sigma}_h = \hat{\sigma}\sqrt{h(1 + h/T)}.$

where k is the integer part of $(h - 1)/m$.

Note that when $h = 1$ and T is large, these all give the same approximate value $\hat{\sigma}$.

Prediction intervals

- Computed automatically from the forecast distribution.
- Use `level` argument to control coverage.
- Check residual assumptions before believing them.
- Usually too narrow due to unaccounted uncertainty.