MONASH University

MONASH BUSINESS SCHOOL

ETC3550 Applied forecasting for business and economics

Ch5. The forecasters' toolbox

OTexts.org/fpp3/

Outline

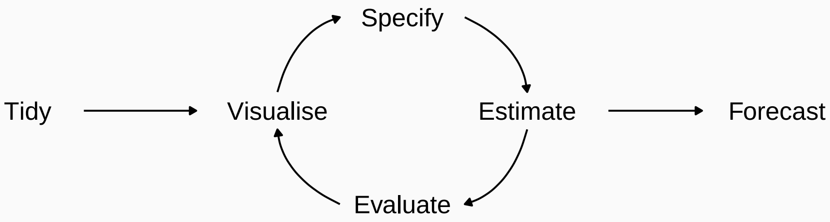
Outline

A tidy forecasting workflow

The process of producing forecasts can be split up into a few fundamental steps.

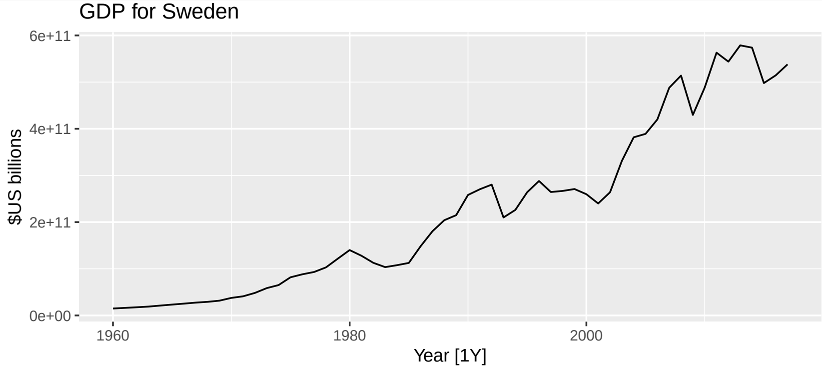
1. Preparing data
2. Data visualisation
3. Specifying a model
4. Model estimation
5. Accuracy & performance evaluation
6. Producing forecasts

A tidy forecasting workflow



Data preparation and visualisation

```r
global_economy %>%
  filter(Country=="Sweden") %>%
  autoplot(GDP) +
    ggtitle("GDP for Sweden") + ylab("$US billions")
```



Model estimation

The `model()` function trains models to data.

```
fit <- global_economy %>%
  model(trend_model = TSLM(GDP ~ trend()))
fit
```

```
## # A mable: 263 x 2
## # Key:     Country [263]
##    Country              trend_model
##    <fct>                <model>
##  1 Afghanistan          <TSLM>
##  2 Albania              <TSLM>
##  3 Algeria              <TSLM>
##  4 American Samoa       <TSLM>
##  5 Andorra              <TSLM>
```

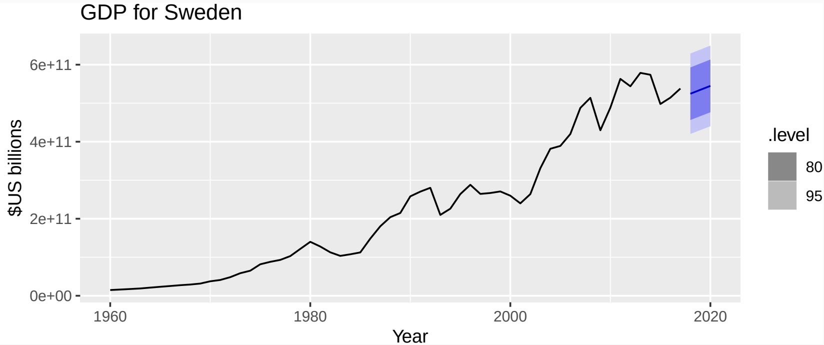
Producing forecasts

```r
fit %>% forecast(h = "3 years")
```

```
## # A fable: 789 x 5 [1Y]
## # Key:     Country, .model [263]
##    Country     .model      Year       GDP .distribution
##    <fct>       <chr>      <dbl>     <dbl> <dist>
##  1 Afghanistan trend_mod~  2018    1.62e10 N(1.6e+10, 1.3e~
##  2 Afghanistan trend_mod~  2019    1.65e10 N(1.7e+10, 1.3e~
##  3 Afghanistan trend_mod~  2020    1.68e10 N(1.7e+10, 1.3e~
##  4 Albania     trend_mod~  2018    1.37e10 N(1.4e+10, 3.9e~
##  5 Albania     trend_mod~  2019    1.42e10 N(1.4e+10, 3.9e~
##  6 Albania     trend_mod~  2020    1.46e10 N(1.5e+10, 3.9e~
##  7 Algeria     trend_mod~  2018    1.58e11 N(1.6e+11, 9.4e~
##  8 Algeria     trend_mod~  2019    1.61e11 N(1.6e+11, 9.4e~
##  9 Algeria     trend_mod~  2020    1.64e11 N(1.6e+11, 9.4e~
```

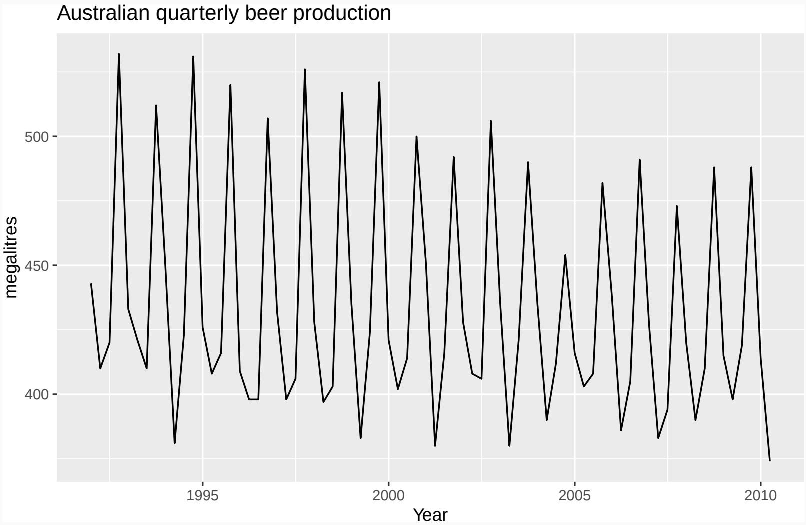
Visualising forecasts

```
fit %>% forecast(h = "3 years") %>%
  filter(Country=="Sweden") %>%
  autoplot(global_economy) +
    ggtitle("GDP for Sweden") + ylab("$US billions")
```



Outline

Some simple forecasting methods

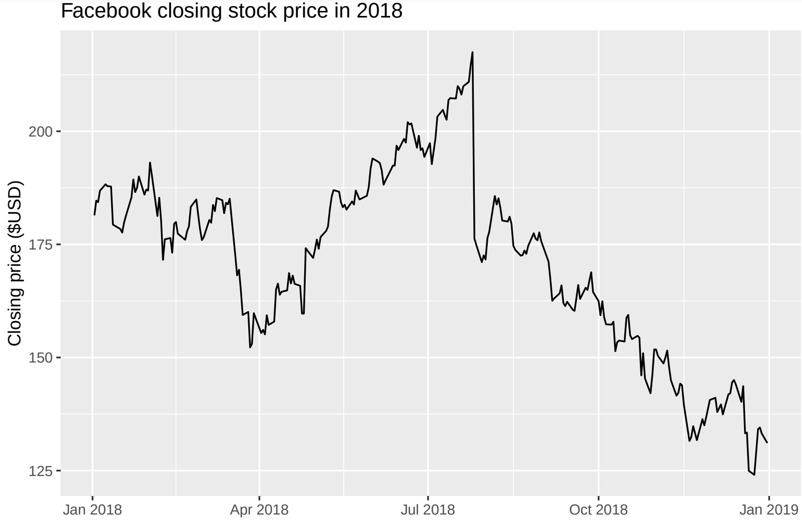


How would you forecast these series?

Some simple forecasting methods

How would you forecast these series?

Some simple forecasting methods

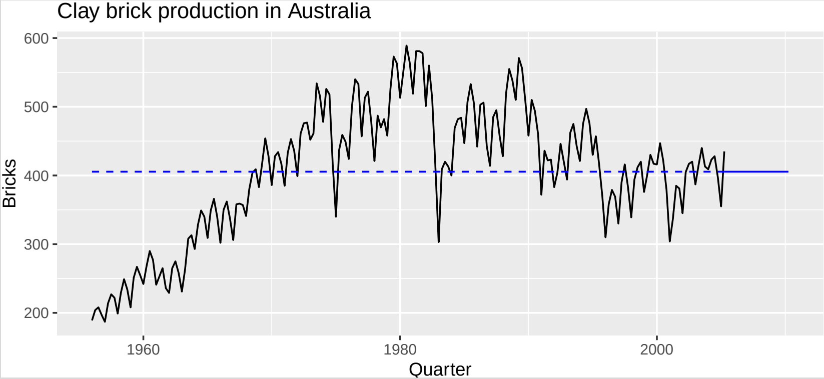


How would you forecast these series?

Some simple forecasting methods

`MEAN(y)`: Average method

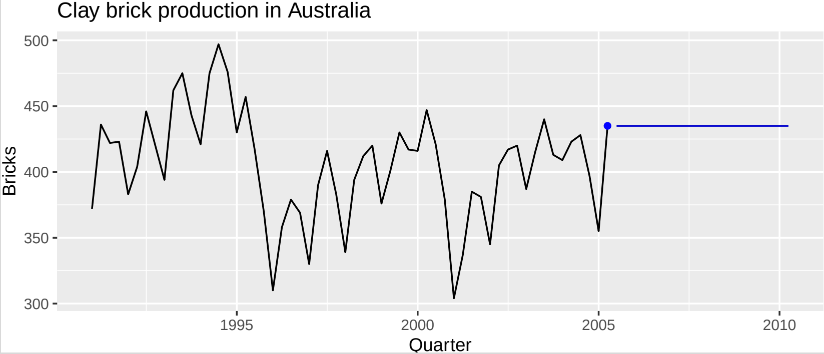
- Forecast of all future values is equal to mean of historical data $\{y_1, \ldots, y_T\}$.
- Forecasts: $\hat{y}_{T+h|T} = \bar{y} = (y_1 + \cdots + y_T)/T$



Some simple forecasting methods

`NAIVE(y)`: Naïve method

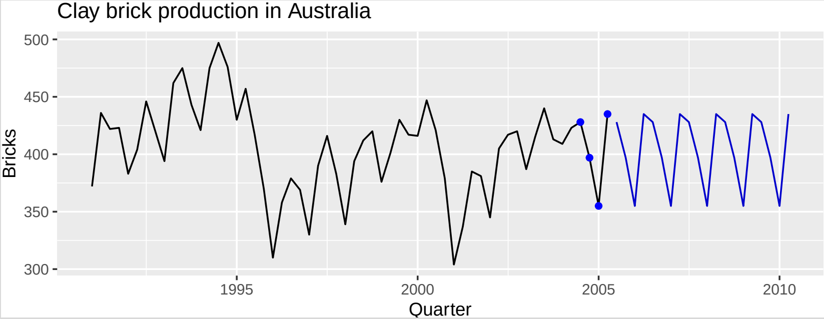
- Forecasts equal to last observed value.
- Forecasts: $\hat{y}_{T+h|T} = y_T$.
- Consequence of efficient market hypothesis.



Some simple forecasting methods

`SNAIVE(y ~ lag(m))`: Seasonal naïve method

- Forecasts equal to last value from same season.
- Forecasts: $\hat{y}_{T+h|T} = y_{T+h-m(k+1)}$, where m = seasonal period and k is the integer part of $(h-1)/m$.



Some simple forecasting methods

`RW(y ~ drift())`: Drift method

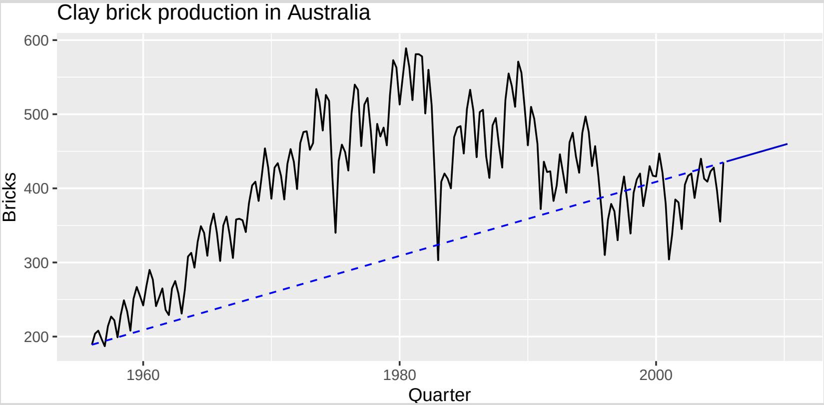
- Forecasts equal to last value plus average change.
- Forecasts:

$$\hat{y}_{T+h|T} = y_T + \frac{h}{T-1}\sum_{t=2}^{T}(y_t - y_{t-1})$$
$$= y_T + \frac{h}{T-1}(y_T - y_1).$$

- Equivalent to extrapolating a line drawn between first and last observations.

Some simple forecasting methods

Drift method



Model fitting

The `model()` function trains models to data.

```
brick_fit <-  aus_production %>%
  filter(!is.na(Bricks)) %>%
  model(
    Seasonal_naive = SNAIVE(Bricks),
    Naive = NAIVE(Bricks),
    Drift = RW(Bricks ~ drift()),
    Mean = MEAN(Bricks)
  )
```

```
## # A mable: 1 x 4
##   Seasonal_naive Naive   Drift         Mean
##   <model>        <model> <model>       <model>
## 1 <SNAIVE>       <NAIVE> <RW w/ drift> <MEAN>
```

A `mable` is a model table, each cell corresponds to a fitted model.

Producing forecasts

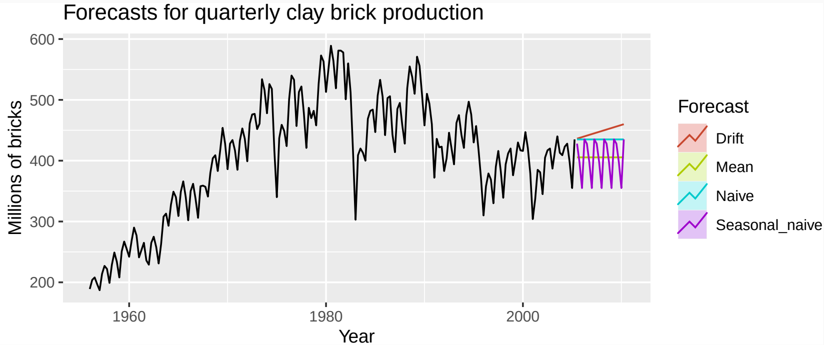
```r
brick_fc <- brick_fit %>%
  forecast(h = "5 years")
```

```
## # A fable: 80 x 4 [1Q]
## # Key:     .model [4]
##   .model         Quarter Bricks .distribution
##   <chr>            <qtr>  <dbl> <dist>
## 1 Seasonal_naive 2005 Q3    428 N(428, 2336)
## 2 Seasonal_naive 2005 Q4    397 N(397, 2336)
## 3 Seasonal_naive 2006 Q1    355 N(355, 2336)
## 4 Seasonal_naive 2006 Q2    435 N(435, 2336)
## # ... with 76 more rows
```

A `fable` is a forecast table with point forecasts and distributions.

Visualising forecasts

```r
brick_fc %>%
  autoplot(aus_production, level = NULL) +
  ggtitle("Forecasts for quarterly clay brick production") +
  xlab("Year") + ylab("Millions of bricks") +
  guides(colour = guide_legend(title = "Forecast"))
```



Facebook closing stock price

```r
# Extract training data
fb_stock <- gafa_stock %>%
  group_by(Symbol) %>%
  mutate(trading_day = row_number()) %>%
  update_tsibble(index=trading_day, regular=TRUE) %>%
  ungroup() %>%
  filter(Symbol == "FB")
# Specify, estimate and forecast
fb_stock %>%
  model(
    Mean = MEAN(Close),
    Naive = NAIVE(Close),
    Drift = RW(Close ~ drift())
  ) %>%
  forecast(h=42) %>%
  autoplot(fb_stock, level = NULL) +
  ggtitle("Facebook closing stock price") +
  xlab("Day") + ylab("") +
  guides(colour=guide_legend(title="Forecast"))
```

Facebook closing stock price

Your turn

- Produce forecasts using an appropriate benchmark method for household wealth (`hh_budget`). Plot the results using `autoplot()`.
- Produce forecasts using an appropriate benchmark method for Australian takeaway food turnover (`aus_retail`). Plot the results using `autoplot()`.

Outline

Fitted values

- $\hat{y}_{t|t-1}$ is the forecast of y_t based on observations $y_1, \dots, y_{t-1}$.
- We call these "fitted values".
- Sometimes drop the subscript: $\hat{y}_t \equiv \hat{y}_{t|t-1}$.
- Often not true forecasts since parameters are estimated on all data.

For example:

- $\hat{y}_t = \bar{y}$ for average method.
- $\hat{y}_t = y_{t-1} + (y_T - y_1)/(T-1)$ for drift method.

Forecasting residuals

Residuals in forecasting: difference between observed value and its fitted value: $e_t = y_t - \hat{y}_{t|t-1}$.

Forecasting residuals

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Assumptions

1. $\{e_t\}$ uncorrelated. If they aren't, then information left in residuals that should be used in computing forecasts.
2. $\{e_t\}$ have mean zero. If they don't, then forecasts are biased.

Forecasting residuals

Residuals in forecasting: difference between observed value and its fitted value: $e_t = y_t - \hat{y}_{t|t-1}$.

Assumptions

1. $\{e_t\}$ uncorrelated. If they aren't, then information left in residuals that should be used in computing forecasts.
2. $\{e_t\}$ have mean zero. If they don't, then forecasts are biased.

Useful properties (for distributions & prediction intervals)

3. $\{e_t\}$ have constant variance.
4. $\{e_t\}$ are normally distributed.

Facebook closing stock price

```
fb_stock <- gafa_stock %>%
  filter(Symbol == "FB") %>%
  mutate(trading_day = row_number()) %>%
  update_tsibble(index = trading_day, regular = TRUE)
fb_stock %>% autoplot(Close)
```

Facebook closing stock price

```r
fit <- fb_stock %>% model(NAIVE(Close))
augment(fit)
```

```
## # A tsibble: 1,258 x 6 [1]
## # Key:       Symbol, .model [1]
##    Symbol .model       trading_day Close .fitted .resid
##    <chr>  <chr>              <int> <dbl>   <dbl>  <dbl>
##  1 FB     NAIVE(Close)           1  54.7    NA   NA
##  2 FB     NAIVE(Close)           2  54.6    54.7 -0.150
##  3 FB     NAIVE(Close)           3  57.2    54.6  2.64
##  4 FB     NAIVE(Close)           4  57.9    57.2  0.720
##  5 FB     NAIVE(Close)           5  58.2    57.9  0.310
##  6 FB     NAIVE(Close)           6  57.2    58.2 -1.01
##  7 FB     NAIVE(Close)           7  57.9    57.2  0.720
##  8 FB     NAIVE(Close)           8  55.9    57.9 -2.03
##  9 FB     NAIVE(Close)           9  57.7    55.9  1.83
## 10 FB     NAIVE(Close)          10  57.6    57.7 -0.140
## # ... with 1,248 more rows
```

Facebook closing stock price

```r
fit <- fb_stock %>% model(NAIVE(Close))
augment(fit)
```

$\hat{y}_{t|t-1}$ e_t

```
## # A tsibble: 1,258 x 6 [1]
## # Key:       Symbol, .model [1]
##    Symbol .model       trading_day Close .fitted .resid
##    <chr>  <chr>              <int> <dbl>   <dbl>  <dbl>
##  1 FB     NAIVE(Close)           1  54.7    NA    NA
##  2 FB     NAIVE(Close)           2  54.6    54.7 -0.150
##  3 FB     NAIVE(Close)           3  57.2    54.6  2.64
##  4 FB     NAIVE(Close)           4  57.9    57.2  0.720
##  5 FB     NAIVE(Close)           5  58.2    57.9  0.310
##  6 FB     NAIVE(Close)           6  57.2    58.2 -1.01
                                     7  57.9    57.2  0.720
                                     8  55.9    57.9 -2.03
                                     9  57.7    55.9  1.83
                                    10  57.6    57.7 -0.140
```

Naïve forecasts:

$$\hat{y}_{t|t-1} = y_{t-1}$$

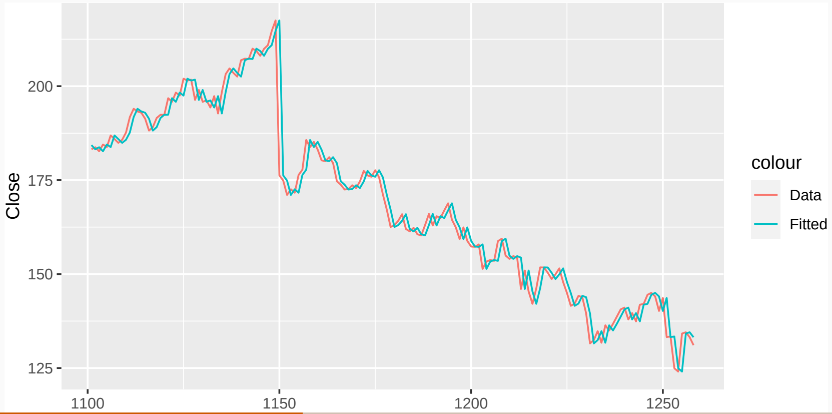
$$e_t = y_t - \hat{y}_{t|t-1} = y_t - y_{t-1}$$

Facebook closing stock price

```r
augment(fit) %>%
  ggplot(aes(x = trading_day)) +
  geom_line(aes(y = Close, colour = "Data")) +
  geom_line(aes(y = .fitted, colour = "Fitted"))
```

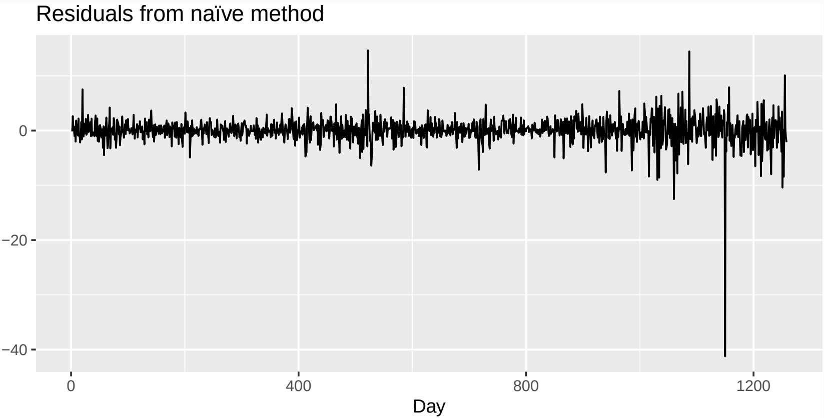
Facebook closing stock price

```r
augment(fit) %>%
  filter(trading_day > 1100) %>%
  ggplot(aes(x = trading_day)) +
  geom_line(aes(y = Close, colour = "Data")) +
  geom_line(aes(y = .fitted, colour = "Fitted"))
```



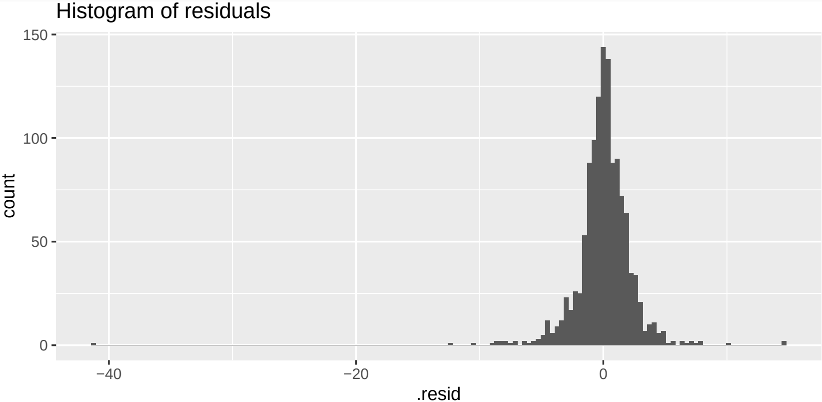
Facebook closing stock price

```r
augment(fit) %>%
  autoplot(.resid) + xlab("Day") + ylab("") +
  ggtitle("Residuals from naïve method")
```



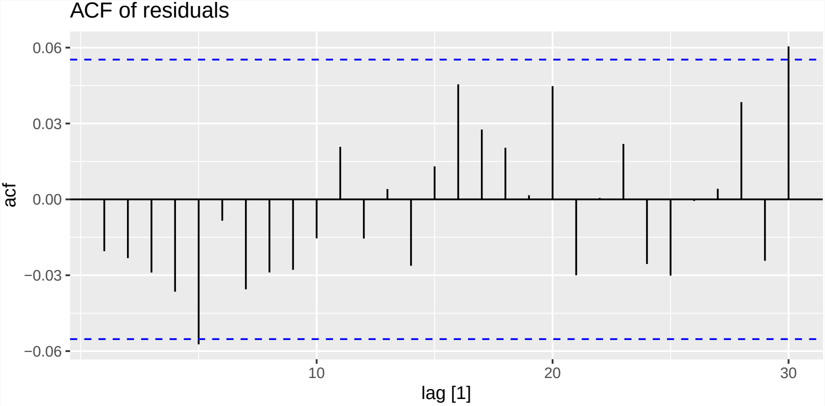
Facebook closing stock price

```r
augment(fit) %>%
  ggplot(aes(x = .resid)) +
  geom_histogram(bins = 150) +
  ggtitle("Histogram of residuals")
```



Facebook closing stock price

```r
augment(fit) %>%
  ACF(.resid) %>%
  autoplot() + ggtitle("ACF of residuals")
```



ACF of residuals

- We assume that the residuals are white noise (uncorrelated, mean zero, constant variance). If they aren't, then there is information left in the residuals that should be used in computing forecasts.
- So a standard residual diagnostic is to check the ACF of the residuals of a forecasting method.
- We *expect* these to look like white noise.

Portmanteau tests

Consider a *whole set* of r_k values, and develop a test to see whether the set is significantly different from a zero set.

Portmanteau tests

Consider a *whole set* of r_k values, and develop a test to see whether the set is significantly different from a zero set.

Box-Pierce test

$$Q = T\sum_{k=1}^{h} r_k^2$$

where h is max lag being considered and T is number of observations.

- If each r_k close to zero, Q will be **small**.
- If some r_k values large (positive or negative), Q will be **large**.

Portmanteau tests

Consider a *whole set* of r_k values, and develop a test to see whether the set is significantly different from a zero set.

Ljung-Box test

$$Q^* = T(T+2)\sum_{k=1}^{h}(T-k)^{-1}r_k^2$$

where h is max lag being considered and T is number of observations.

- My preferences: $h = 10$ for non-seasonal data, $h = 2m$ for seasonal data.
- Better performance, especially in small samples.

Portmanteau tests

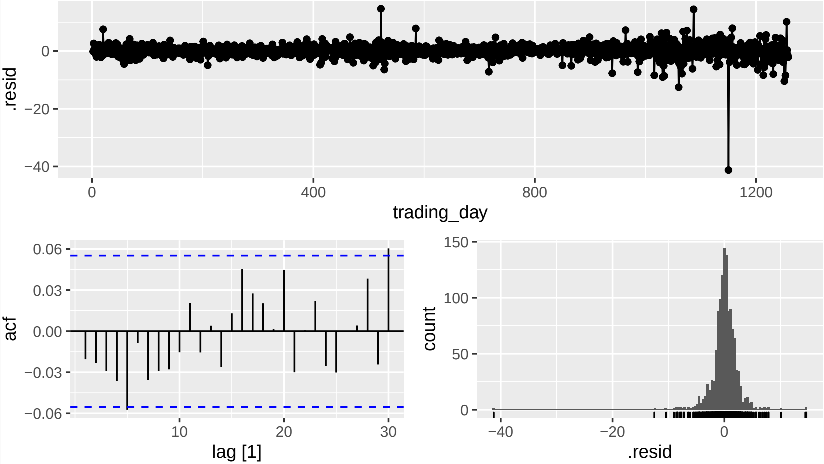
- If data are WN, Q^* has χ^2 distribution with $(h - K)$ degrees of freedom where K = no. parameters in model.
- When applied to raw data, set $K = 0$.

```r
augment(fit) %>%
  features(.resid, ljung_box, lag=10, dof=0)
```

```
## # A tibble: 1 x 4
##   Symbol .model       lb_stat lb_pvalue
##   <chr>  <chr>          <dbl>     <dbl>
## 1 FB     NAIVE(Close)    12.1     0.276
```

gg_tsresiduals function

```r
gg_tsresiduals(fit)
```



Your turn

Compute seasonal naïve forecasts for quarterly Australian beer production from 1992.

```
recent <- aus_production %>% filter(year(Quarter) >= 1992)
fit <- recent %>% model(SNAIVE(Beer))
fit %>% forecast() %>% autoplot(recent)
```

Test if the residuals are white noise.

```
augment(fit) %>% features(.resid, ljung_box, lag=10, dof=0)
gg_tsresiduals(fit)
```

What do you conclude?

Outline

Forecast distributions

- A forecast $\hat{y}_{T+h|T}$ is (usually) the mean of the conditional distribution $y_{T+h} \mid y_1, \dots, y_T$.
- Most time series models produce normally distributed forecasts.
- The forecast distribution describes the probability of observing any future value.

Forecast distributions

Assuming residuals are normal, uncorrelated, sd = $\hat{\sigma}$:

Mean:	$\hat{y}_{T+h\|T} \sim N(\bar{y}, (1 + 1/T)\hat{\sigma}^2)$
Naïve:	$\hat{y}_{T+h\|T} \sim N(y_T, h\hat{\sigma}^2)$
Seasonal naïve:	$\hat{y}_{T+h\|T} \sim N(y_{T+h-m(k+1)}, (k + 1)\hat{\sigma}^2)$
Drift:	$\hat{y}_{T+h\|T} \sim N(y_T + \frac{h}{T-1}(y_T - y_1), h\frac{T+h}{T}\hat{\sigma}^2)$

where k is the integer part of $(h - 1)/m$.

Note that when $h = 1$ and T is large, these all give the same approximate forecast variance: $\hat{\sigma}^2$.

Prediction intervals

- A prediction interval gives a region within which we expect y_{T+h} to lie with a specified probability.
- Assuming forecast errors are normally distributed, then a 95% PI is

$$\hat{y}_{T+h|T} \pm 1.96\hat{\sigma}_h$$

  where $\hat{\sigma}_h$ is the st dev of the h-step distribution.
- When $h = 1$, $\hat{\sigma}_h$ can be estimated from the residuals.

Prediction intervals

```r
brick_fc %>% hilo(level = 95)
```

```
## # A tsibble: 80 x 4 [1Q]
## # Key:       .model [4]
##    .model         Quarter Bricks                     95%
##    <chr>            <qtr>  <dbl>                  <hilo>
##  1 Seasonal_naive 2005 Q3    428 [333.2737, 522.7263]95
##  2 Seasonal_naive 2005 Q4    397 [302.2737, 491.7263]95
##  3 Seasonal_naive 2006 Q1    355 [260.2737, 449.7263]95
##  4 Seasonal_naive 2006 Q2    435 [340.2737, 529.7263]95
##  5 Seasonal_naive 2006 Q3    428 [294.0368, 561.9632]95
##  6 Seasonal_naive 2006 Q4    397 [263.0368, 530.9632]95
##  7 Seasonal_naive 2007 Q1    355 [221.0368, 488.9632]95
##  8 Seasonal_naive 2007 Q2    435 [301.0368, 568.9632]95
##  9 Seasonal_naive 2007 Q3    428 [263.9292, 592.0708]95
## 10 Seasonal_naive 2007 Q4    397 [232.9292, 561.0708]95
```

Prediction intervals

- Point forecasts are often useless without a measure of uncertainty (such as prediction intervals).
- Prediction intervals require a stochastic model (with random errors, etc).
- Multi-step forecasts for time series require a more sophisticated approach (with PI getting wider as the forecast horizon increases).

Prediction intervals

- Computed automatically from the forecast distribution.
- Use `level` argument to control coverage.
- Check residual assumptions before believing them (we will see this next class).
- Usually too narrow due to unaccounted uncertainty.

Outline

Modelling with transformations

Transformations used in the left of the formula will be automatically back-transformed. To model log-transformed food retailing turnover, you could use:

```
food <- aus_retail %>%
  filter(Industry == "Food retailing") %>%
  summarise(Turnover = sum(Turnover))
```

```
fit <- food %>%
  model(SNAIVE(log(Turnover)))
```

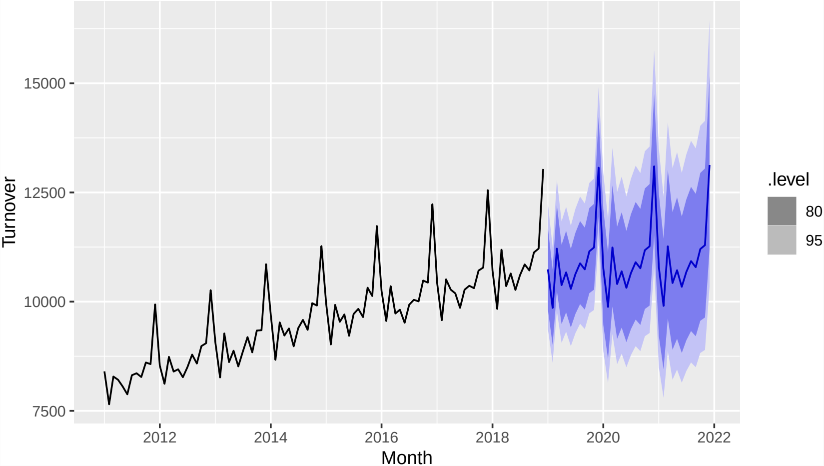
Forecasting with transformations

```r
fc <- fit %>%
  forecast(h = "3 years")
```

```
## # A fable: 36 x 4 [1M]
## # Key:     .model [1]
##   .model                      Month Turnover .distribution
##   <chr>                       <mth>    <dbl> <dist>
## 1 SNAIVE(log(Turnover)) 2019 Jan   10738. t(N(9.3, 0.0047))
## 2 SNAIVE(log(Turnover)) 2019 Feb    9856. t(N(9.2, 0.0047))
## 3 SNAIVE(log(Turnover)) 2019 Mar   11214. t(N(9.3, 0.0047))
## 4 SNAIVE(log(Turnover)) 2019 Apr   10378. t(N(9.2, 0.0047))
## 5 SNAIVE(log(Turnover)) 2019 May   10670. t(N(9.3, 0.0047))
## 6 SNAIVE(log(Turnover)) 2019 Jun   10292. t(N(9.2, 0.0047))
## # ... with 30 more rows
```

Forecasting with transformations

```
fc %>% autoplot(filter(food, year(Month) > 2010))
```



Bias adjustment

- Back-transformed point forecasts are medians.
- Back-transformed PI have the correct coverage.

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- Back-transformed point forecasts are medians.
- Back-transformed PI have the correct coverage.

Back-transformed means

Let X be have mean μ and variance σ^2.

Let $f(x)$ be back-transformation function, and $Y = f(X)$.

Taylor series expansion about μ:

$$f(X) = f(\mu) + (X - \mu)f'(\mu) + \frac{1}{2}(X - \mu)^2 f''(\mu).$$

Bias adjustment

- Back-transformed point forecasts are medians.
- Back-transformed PI have the correct coverage.

Back-transformed means

Let X be have mean μ and variance σ^2.

Let $f(x)$ be back-transformation function, and $Y = f(X)$.

Taylor series expansion about μ:

$$f(X) = f(\mu) + (X - \mu)f'(\mu) + \frac{1}{2}(X - \mu)^2 f''(\mu).$$

$$\mathrm{E}[Y] = \mathrm{E}[f(X)] = f(\mu) + \tfrac{1}{2}\sigma^2 f''(\mu)$$

Bias adjustment

Box-Cox back-transformation:

$$y_t = \begin{cases} \exp(w_t) & \lambda = 0; \\ (\lambda W_t + 1)^{1/\lambda} & \lambda \neq 0. \end{cases}$$

$$f(x) = \begin{cases} e^x & \lambda = 0; \\ (\lambda x + 1)^{1/\lambda} & \lambda \neq 0. \end{cases}$$

$$f''(x) = \begin{cases} e^x & \lambda = 0; \\ (1 - \lambda)(\lambda x + 1)^{1/\lambda - 2} & \lambda \neq 0. \end{cases}$$

Bias adjustment

Box-Cox back-transformation:

$$y_t = \begin{cases} \exp(w_t) & \lambda = 0; \\ (\lambda W_t + 1)^{1/\lambda} & \lambda \neq 0. \end{cases}$$

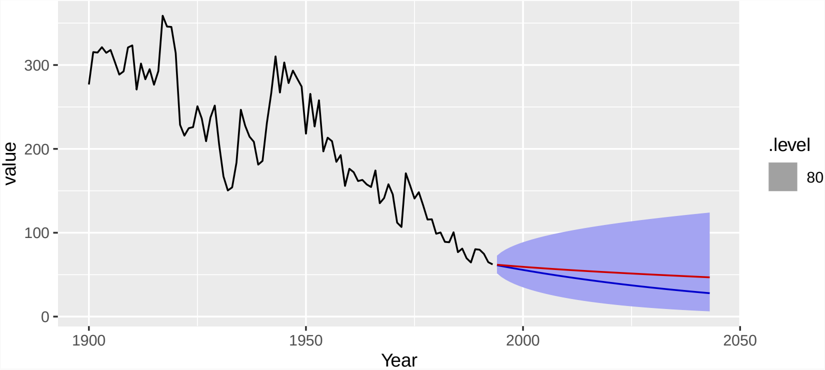
$$f(x) = \begin{cases} e^x & \lambda = 0; \\ (\lambda x + 1)^{1/\lambda} & \lambda \neq 0. \end{cases}$$

$$f''(x) = \begin{cases} e^x & \lambda = 0; \\ (1-\lambda)(\lambda x + 1)^{1/\lambda - 2} & \lambda \neq 0. \end{cases}$$

$$\mathrm{E}[Y] = \begin{cases} e^{\mu}\left[1 + \frac{\sigma^2}{2}\right] & \lambda = 0; \\ (\lambda\mu + 1)^{1/\lambda}\left[1 + \frac{\sigma^2(1-\lambda)}{2(\lambda\mu+1)^2}\right] & \lambda \neq 0. \end{cases}$$

Bias adjustment

```
eggs <- as_tsibble(fma::eggs)
fit <- eggs %>% model(RW(log(value) ~ drift()))
fc <- fit %>% forecast(h=50)
fc_biased <- fit %>% forecast(h=50, bias_adjust = FALSE)
eggs %>% autoplot(value) + xlab("Year") +
  autolayer(fc_biased, level = 80) +
  autolayer(fc, colour = "red", level = NULL)
```



Outline

Forecasting and decomposition

- Forecast seasonal component by repeating the last year
- Forecast seasonally adjusted data using non-seasonal time series method.
- Combine forecasts of seasonal component with forecasts of seasonally adjusted data to get forecasts of original data.
- Sometimes a decomposition is useful just for understanding the data before building a separate forecasting model.

US Retail Employment

```r
us_retail_employment <- us_employment %>%
  filter(year(Month) >= 1990, Title == "Retail Trade") %>%
  select(-Series_ID)
us_retail_employment
```

```
## # A tsibble: 357 x 3 [1M]
##        Month Title        Employed
##        <mth> <chr>           <dbl>
##  1 1990 Jan Retail Trade   13256.
##  2 1990 Feb Retail Trade   12966.
##  3 1990 Mar Retail Trade   12938.
##  4 1990 Apr Retail Trade   13012.
##  5 1990 May Retail Trade   13108.
##  6 1990 Jun Retail Trade   13183.
##  7 1990 Jul Retail Trade   13170.
##  8 1990 Aug Retail Trade   13160.
##  9 1990 Sep Retail Trade   13113.
## 10 1990 Oct Retail Trade   13185.
```

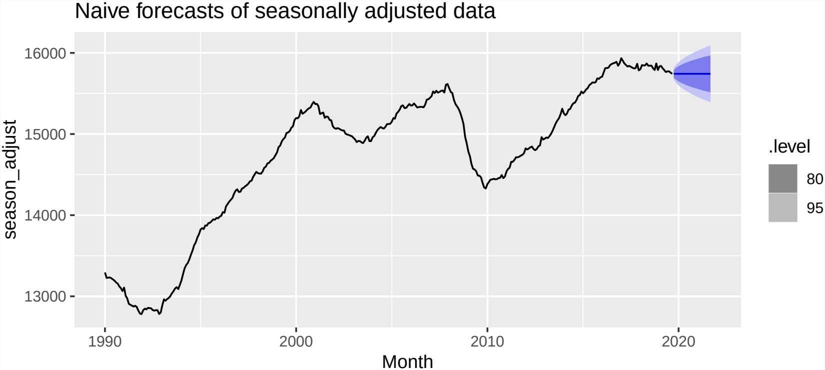
US Retail Employment

```r
dcmp <- us_retail_employment %>%
  model(STL(Employed)) %>%
  components() %>% select(-.model)
dcmp
```

```
## # A tsibble: 357 x 6 [1M]
##          Month Employed  trend season_year remainder
##          <mth>    <dbl>  <dbl>       <dbl>     <dbl>
##  1    1990 Jan   13256. 13291.       -38.1    3.08
##  2    1990 Feb   12966. 13272.      -261.   -44.2
##  3    1990 Mar   12938. 13252.      -291.   -23.0
##  4    1990 Apr   13012. 13233.      -221.     0.0892
##  5    1990 May   13108. 13213.      -115.     9.98
##  6    1990 Jun   13183. 13193.       -25.6   15.7
##  7    1990 Jul   13170. 13173.       -24.4   22.0
##  8    1990 Aug   13160. 13152.       -11.8   19.5
##  9    1990 Sep   13113. 13131.       -43.4   25.7
## 10    1990 Oct   13185. 13110.        62.5   12.3
```

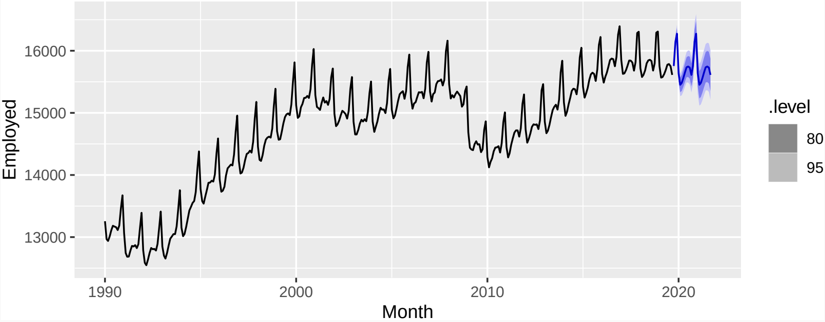
US Retail Employment

```r
dcmp %>%
  model(NAIVE(season_adjust)) %>%
  forecast() %>%
  autoplot(dcmp) +
  ggtitle("Naive forecasts of seasonally adjusted data")
```



US Retail Employment

```r
us_retail_employment %>%
  model(stlf = decomposition_model(
    STL(Employed ~ trend(window = 7), robust = TRUE),
    NAIVE(season_adjust)
  )) %>%
  forecast() %>%
  autoplot(us_retail_employment)
```



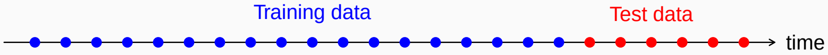
Decomposition models

`decomposition_model()` creates a decomposition model

- You must provide a method for forecasting the `season_adjust` series.
- A seasonal naive method is used by default for the `seasonal` components.
- The variances from both the seasonally adjusted and seasonal forecasts are combined.

Outline

Training and test sets



- A model which fits the training data well will not necessarily forecast well.
- A perfect fit can always be obtained by using a model with enough parameters.
- Over-fitting a model to data is just as bad as failing to identify a systematic pattern in the data.
- The test set must not be used for *any* aspect of model development or calculation of forecasts.
- Forecast accuracy is based only on the test set.

Forecast errors

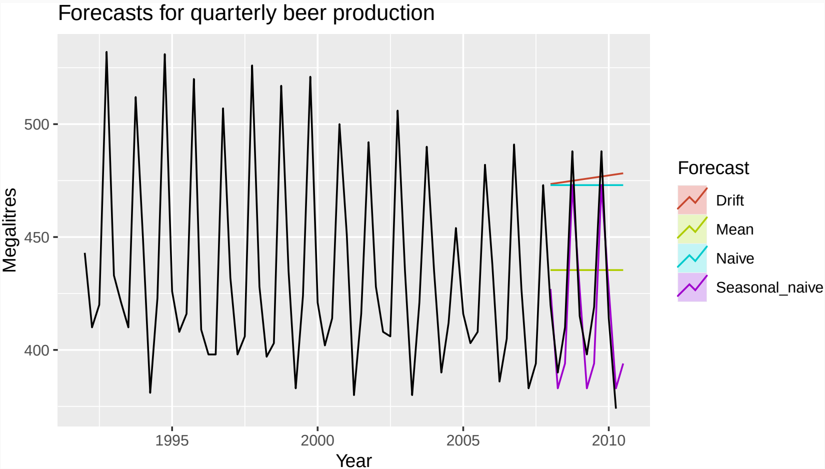
Forecast “error”: the difference between an observed value and its forecast.

$$e_{T+h} = y_{T+h} - \hat{y}_{T+h|T},$$

where the training data is given by $\{y_1, \dots, y_T\}$

- Unlike residuals, forecast errors on the test set involve multi-step forecasts.
- These are *true* forecast errors as the test data is not used in computing $\hat{y}_{T+h|T}$.

Measures of forecast accuracy



Measures of forecast accuracy

$$
\begin{aligned}
y_{T+h} &= (T+h)\text{th observation}, h = 1, \dots, H \\
\hat{y}_{T+h|T} &= \text{its forecast based on data up to time } T. \\
e_{T+h} &= y_{T+h} - \hat{y}_{T+h|T}
\end{aligned}
$$

$$
\begin{aligned}
\text{MAE} &= \text{mean}(|e_{T+h}|) \\
\text{MSE} &= \text{mean}(e^2_{T+h}) \qquad \text{RMSE} = \sqrt{\text{mean}(e^2_{T+h})} \\
\text{MAPE} &= 100\text{mean}(|e_{T+h}|/|y_{T+h}|)
\end{aligned}
$$

Measures of forecast accuracy

$$\begin{aligned} y_{T+h} &= (T+h)\text{th observation, } h = 1, \dots, H \\ \hat{y}_{T+h|T} &= \text{its forecast based on data up to time } T. \\ e_{T+h} &= y_{T+h} - \hat{y}_{T+h|T} \end{aligned}$$

$$\begin{aligned} \text{MAE} &= \text{mean}(|e_{T+h}|) \\ \text{MSE} &= \text{mean}(e^2_{T+h}) \qquad \qquad \text{RMSE} = \sqrt{\text{mean}(e^2_{T+h})} \\ \text{MAPE} &= 100\text{mean}(|e_{T+h}|/|y_{T+h}|) \end{aligned}$$

- MAE, MSE, RMSE are all scale dependent.
- MAPE is scale independent but is only sensible if $y_t \gg 0$ for all t, and y has a natural zero.

Measures of forecast accuracy

Mean Absolute Scaled Error

$$\text{MASE} = \text{mean}(|e_{T+h}|/Q)$$

where Q is a stable measure of the scale of the time series $\{y_t\}$.

Proposed by Hyndman and Koehler (IJF, 2006).

For non-seasonal time series,

$$Q = (T-1)^{-1}\sum_{t=2}^{T}|y_t - y_{t-1}|$$

works well. Then MASE is equivalent to MAE relative to a naïve method.

Measures of forecast accuracy

Mean Absolute Scaled Error

$$\text{MASE} = \text{mean}(|e_{T+h}|/Q)$$

where Q is a stable measure of the scale of the time series $\{y_t\}$.

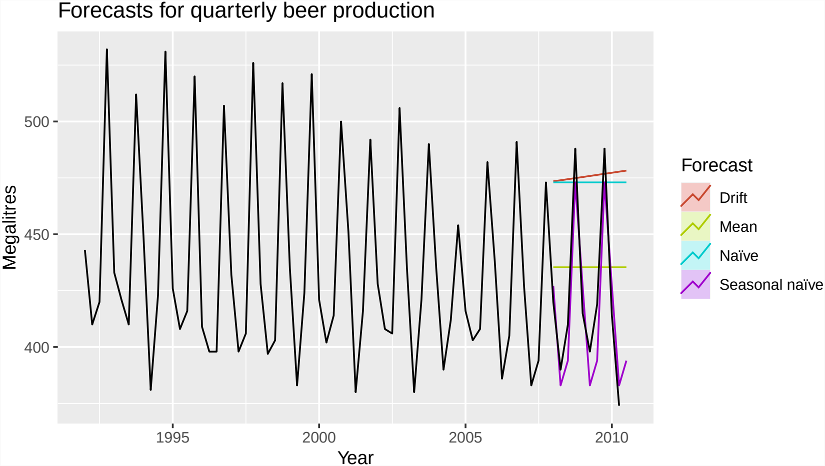
Proposed by Hyndman and Koehler (IJF, 2006).

For seasonal time series,

$$Q = (T-m)^{-1} \sum_{t=m+1}^{T} |y_t - y_{t-m}|$$

works well. Then MASE is equivalent to MAE relative to a seasonal naïve method.

Measures of forecast accuracy



Measures of forecast accuracy

```
recent_production <- aus_production %>%
  filter(year(Quarter) >= 1992)
train <- recent_production %>%
  filter(year(Quarter) <= 2007)
beer_fit <- train %>%
  model(
    Mean = MEAN(Beer),
    Naive = NAIVE(Beer),
    Seasonal_naive = SNAIVE(Beer),
    Drift = RW(Beer ~ drift())
  )
beer_fc <- beer_fit %>%
  forecast(h = 10)
```

Measures of forecast accuracy

```r
accuracy(beer_fit)
```

```
## # A tibble: 4 x 6
##   .model         .type     RMSE   MAE  MAPE  MASE
##   <chr>          <chr>    <dbl> <dbl> <dbl> <dbl>
## 1 Drift          Training  65.3  54.8 12.2   3.83
## 2 Mean           Training  43.6  35.2  7.89  2.46
## 3 Naïve          Training  65.3  54.7 12.2   3.83
## 4 Seasonal naïve Training  16.8  14.3  3.31  1
```

```r
accuracy(beer_fc, recent_production)
```

```
## # A tibble: 4 x 6
##   .model         .type  RMSE   MAE  MAPE  MASE
##   <chr>          <chr> <dbl> <dbl> <dbl> <dbl>
## 1 Drift          Test   64.9  58.9 14.6  4.12
## 2 Mean           Test   38.4  34.8  8.28 2.44
## 3 Naïve          Test   62.7  57.4 14.2  4.01
## 4 Seasonal naïve Test   14.3  13.4  3.17 0.937
```

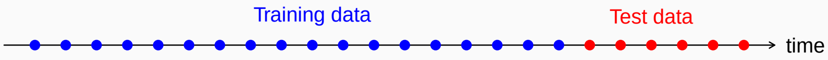
Poll: true or false?

1. Good forecast methods should have normally distributed residuals.
2. A model with small residuals will give good forecasts.
3. The best measure of forecast accuracy is MAPE.
4. If your model doesn’t forecast well, you should make it more complicated.
5. Always choose the model with the best forecast accuracy as measured on the test set.

Outline

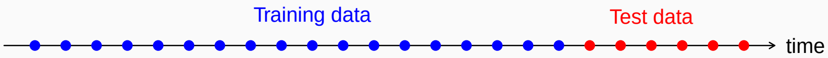
Time series cross-validation

Traditional evaluation

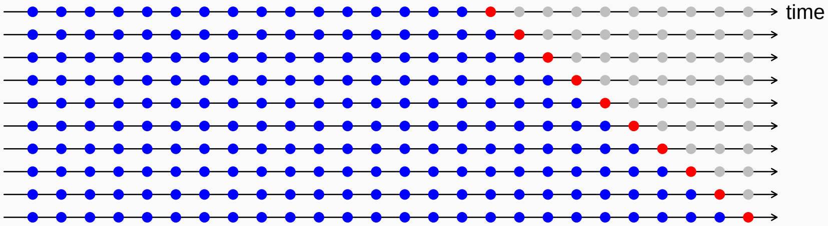


Time series cross-validation

Traditional evaluation

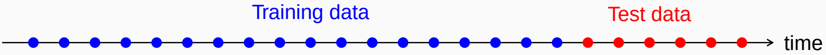


Time series cross-validation

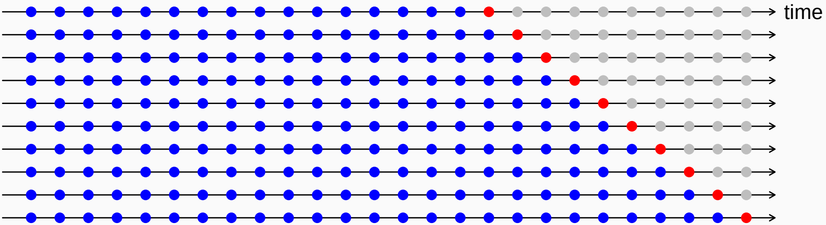


Time series cross-validation

Traditional evaluation



Time series cross-validation



- Forecast accuracy averaged over test sets.
- Also known as "evaluation on a rolling forecasting origin"

Creating the rolling training sets

There are three main rolling types which can be used.

- Stretch: extends a growing length window with new data.
- Slide: shifts a fixed length window through the data.
- Tile: moves a fixed length window without overlap.

Three functions to roll a tsibble: `stretch_tsibble()`, `slide_tsibble()`, and `tile_tsibble()`.

For time series cross-validation, stretching windows are most commonly used.

Creating the rolling training sets

Time series cross-validation

Stretch with a minimum length of 3, growing by 1 each step.

```
fb_stretch <- fb_stock %>%
  stretch_tsibble(.init = 3, .step = 1) %>%
  filter(.id != max(.id))
```

```
## # A tsibble: 790,650 x 4 [1]
## # Key:       .id [1,255]
##   Date       Close trading_day   .id
##   <date>     <dbl>       <int> <int>
## 1 2014-01-02  54.7           1     1
## 2 2014-01-03  54.6           2     1
## 3 2014-01-06  57.2           3     1
## 4 2014-01-02  54.7           1     2
## 5 2014-01-03  54.6           2     2
## 6 2014-01-06  57.2           3     2
## 7 2014-01-07  57.9           4     2
```

Time series cross-validation

Estimate RW w/ drift models for each window.

```r
fit_cv <- fb_stretch %>%
  model(RW(Close ~ drift()))
```

```
## # A mable: 1,255 x 3
## # Key:     .id, Symbol [1,255]
##      .id Symbol RW(Close ~ drift())
##    <int> <chr>  <model>
## 1      1 FB     <RW w/ drift>
## 2      2 FB     <RW w/ drift>
## 3      3 FB     <RW w/ drift>
## 4      4 FB     <RW w/ drift>
## # ... with 1,251 more rows
```

Time series cross-validation

Produce one step ahead forecasts from all models.

```r
fc_cv <- fit_cv %>%
  forecast(h=1)
```

```
## # A fable: 1,255 x 5 [1]
## # Key:     .id, Symbol [1,255]
##     .id Symbol trading_day Close .distribution
##   <int> <chr>        <dbl> <dbl> <dist>
## 1     1 FB               4  58.4 N(58, 3.9)
## 2     2 FB               5  59.0 N(59, 2)
## 3     3 FB               6  59.1 N(59, 1.5)
## 4     4 FB               7  57.7 N(58, 1.8)
## # ... with 1,251 more rows
```

Time series cross-validation

```r
# Cross-validated
fc_cv %>% accuracy(fb_stock)
# Training set
fb_stock %>% model(RW(Close ~ drift())) %>% accuracy()
```

	RMSE	MAE	MAPE
Cross-validation	2.418	1.469	1.266
Training	2.414	1.465	1.261

A good way to choose the best forecasting model is to find the model with the smallest RMSE computed using time series cross-validation.