



Feature Detection

Automatic 점 매칭 방법



Image Matching



by [Diva Sian](#)



by [swashford](#)



Image Matching



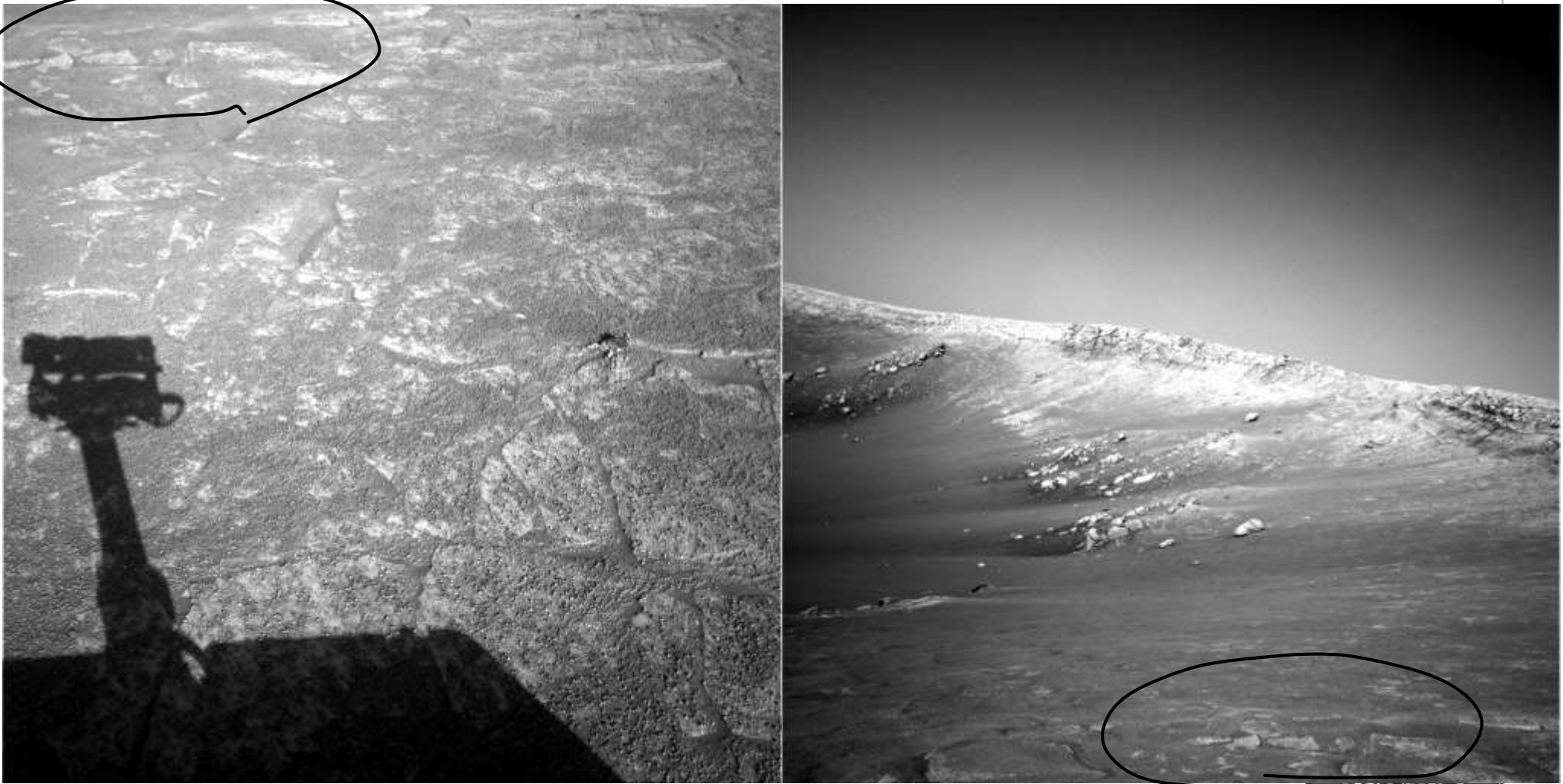
by [Diva Sian](#)



by [scgbt](#)



Image Matching

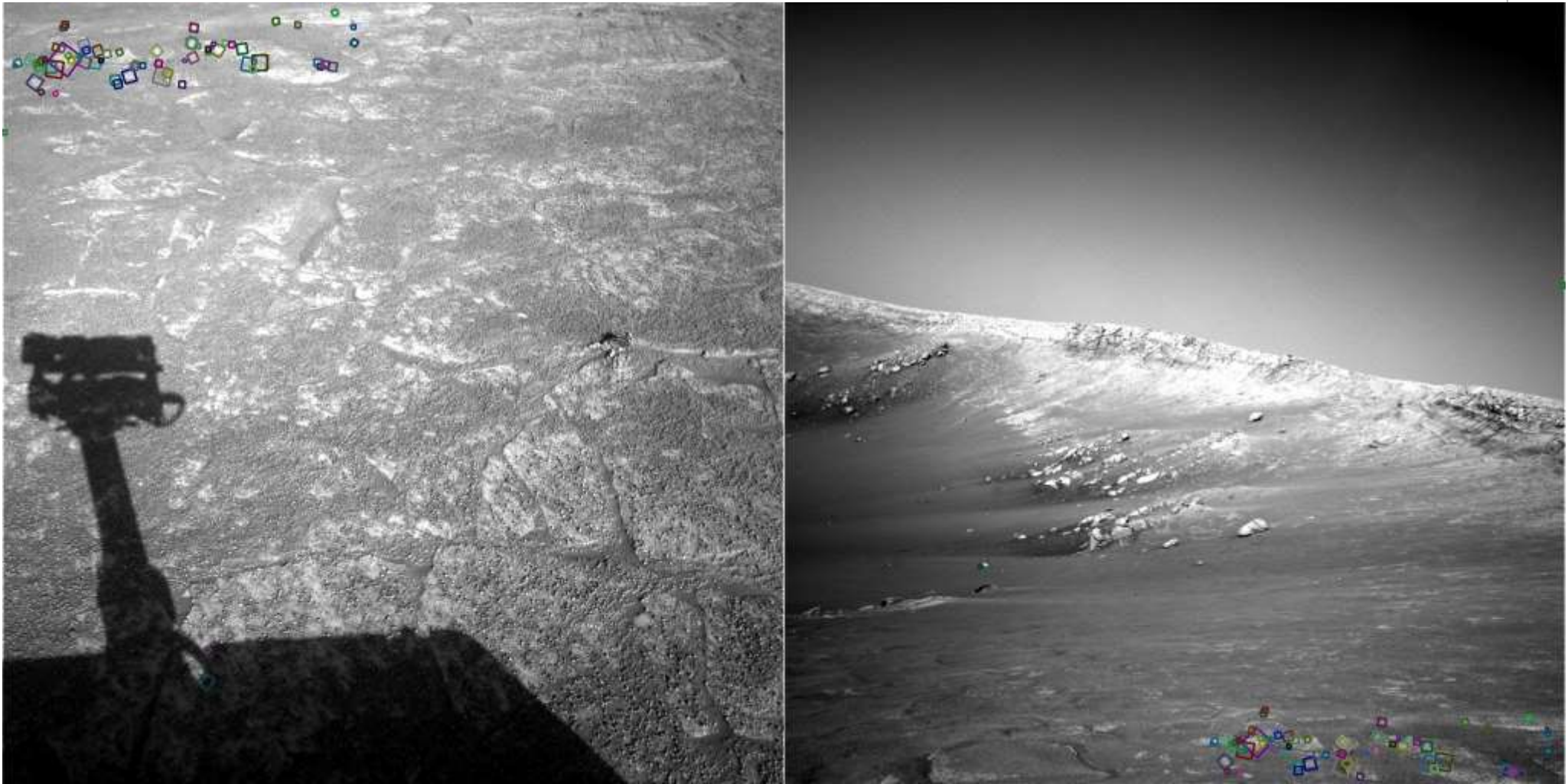


NASA Mars Rover images

overlapping 영역이 있다면 어느 점이 같은 점인지 찾아내기



Image Matching



NASA Mars Rover images with SIFT feature matches

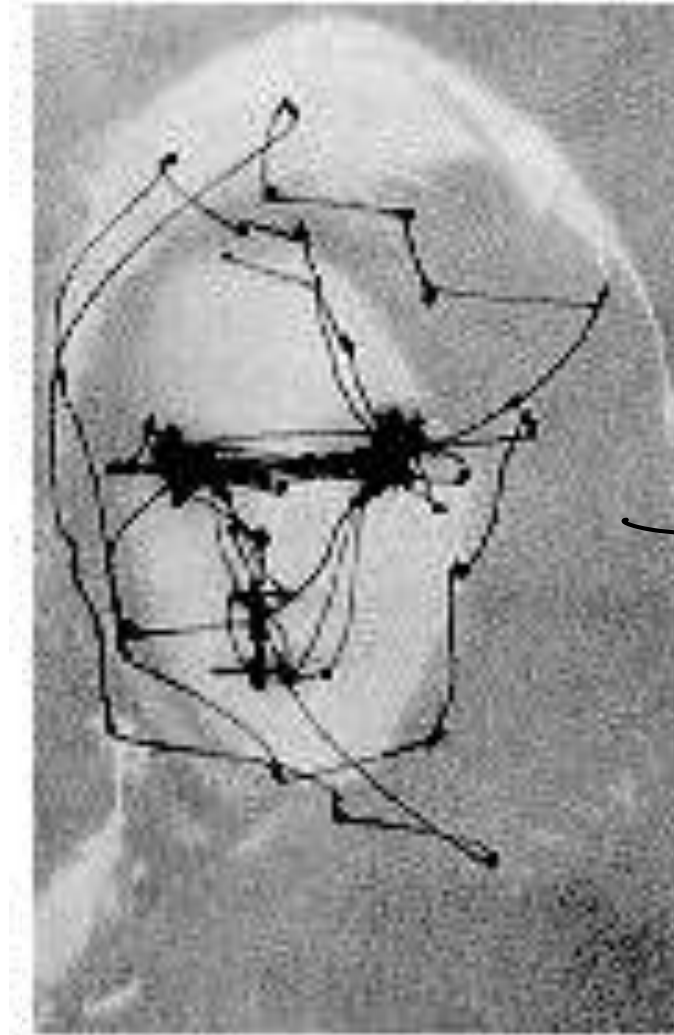
Figure by Noah Snaveley



How to Match?

- Human eye movements

What catches your interest?



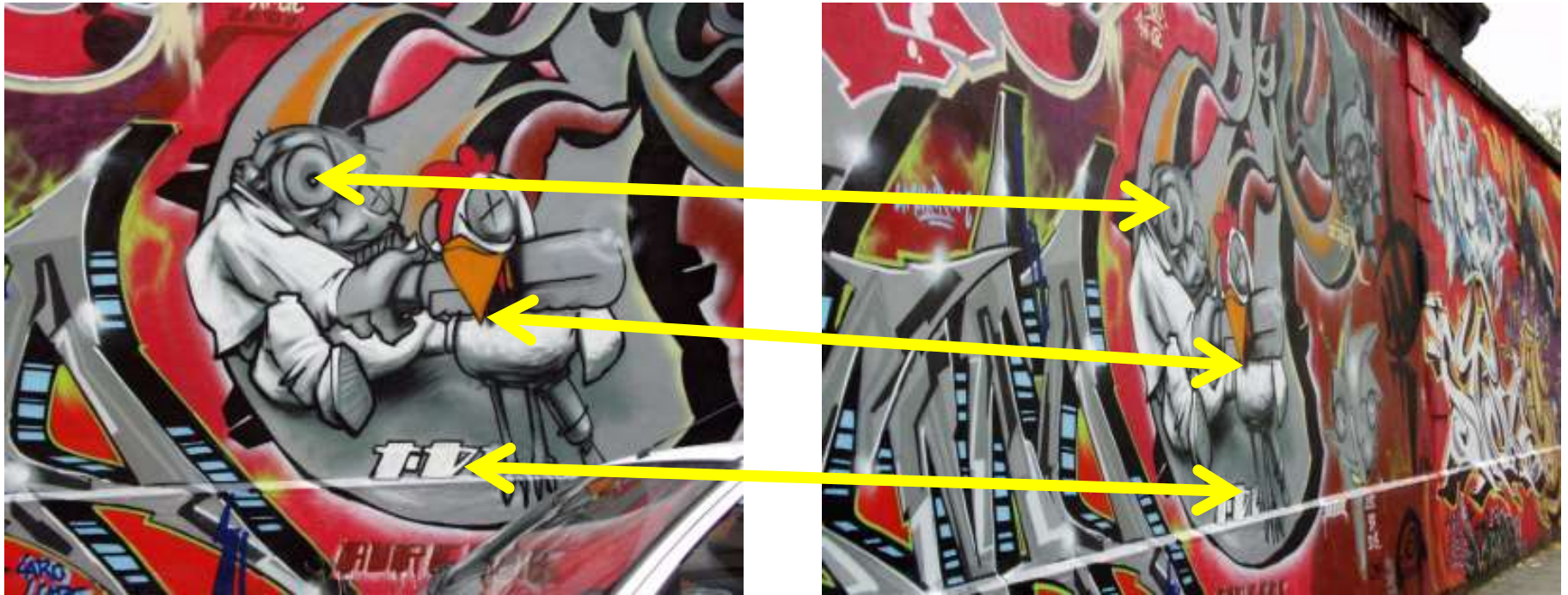
이런 점들을
feature라고 할
특징점 point

Yarbus eye tracking

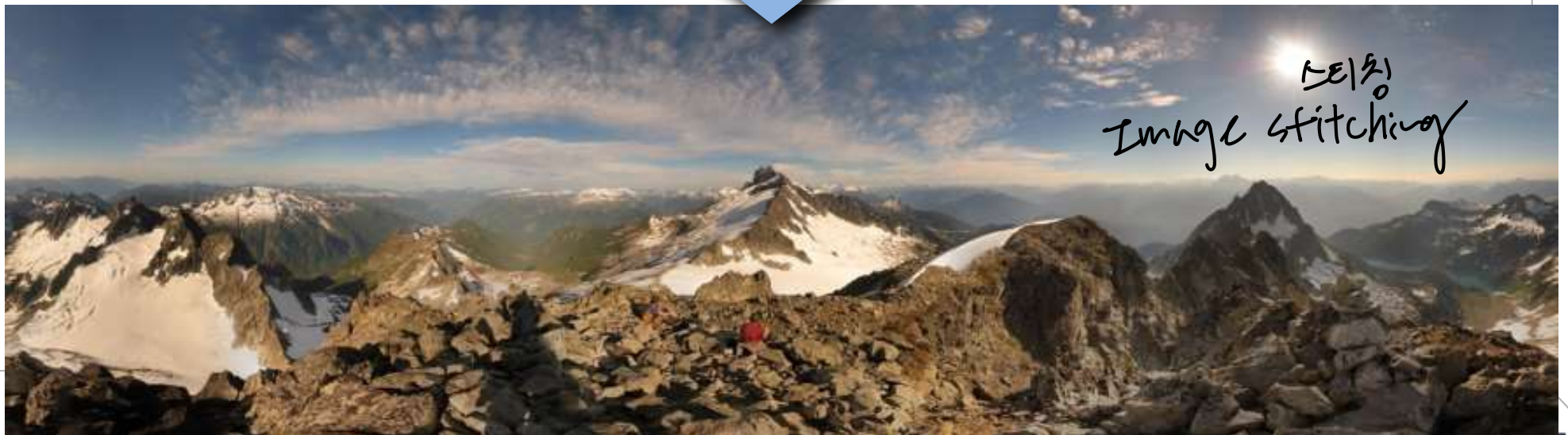


How to Match?

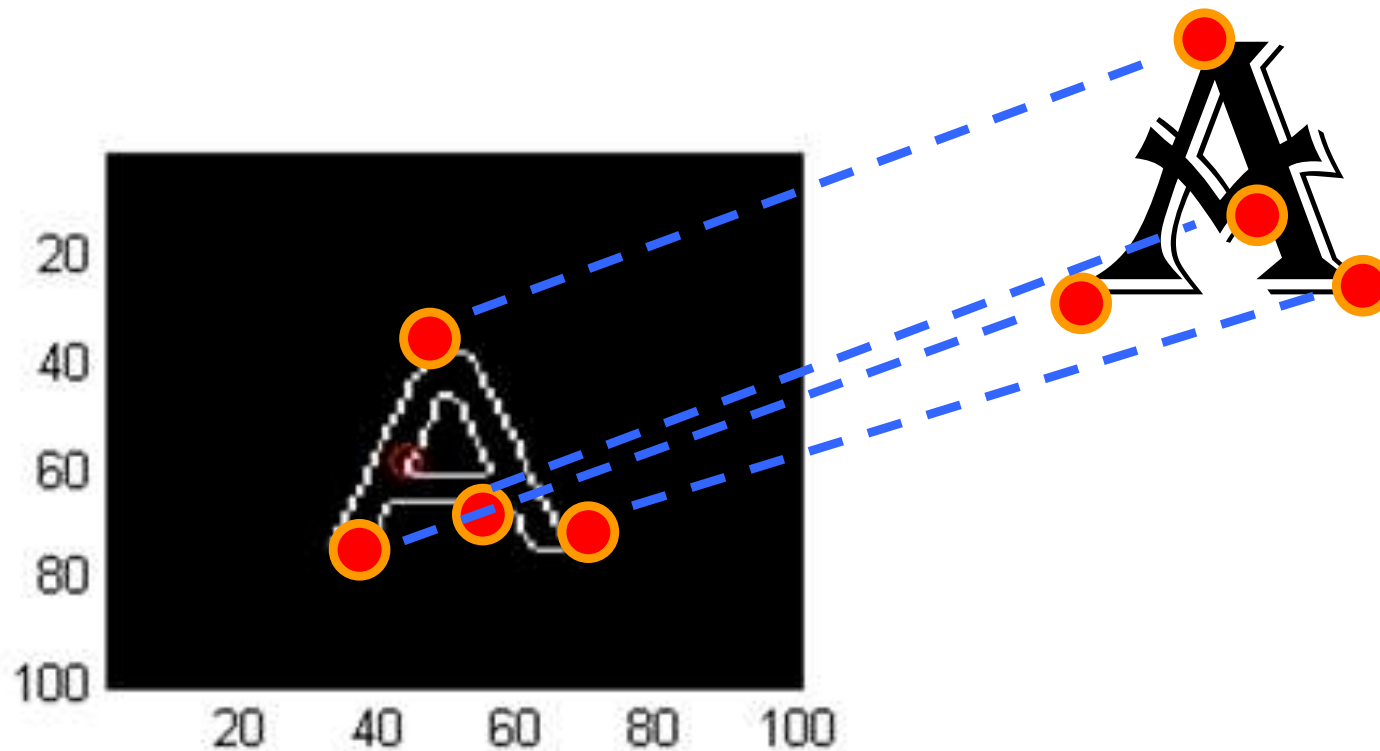
- Corresponding points



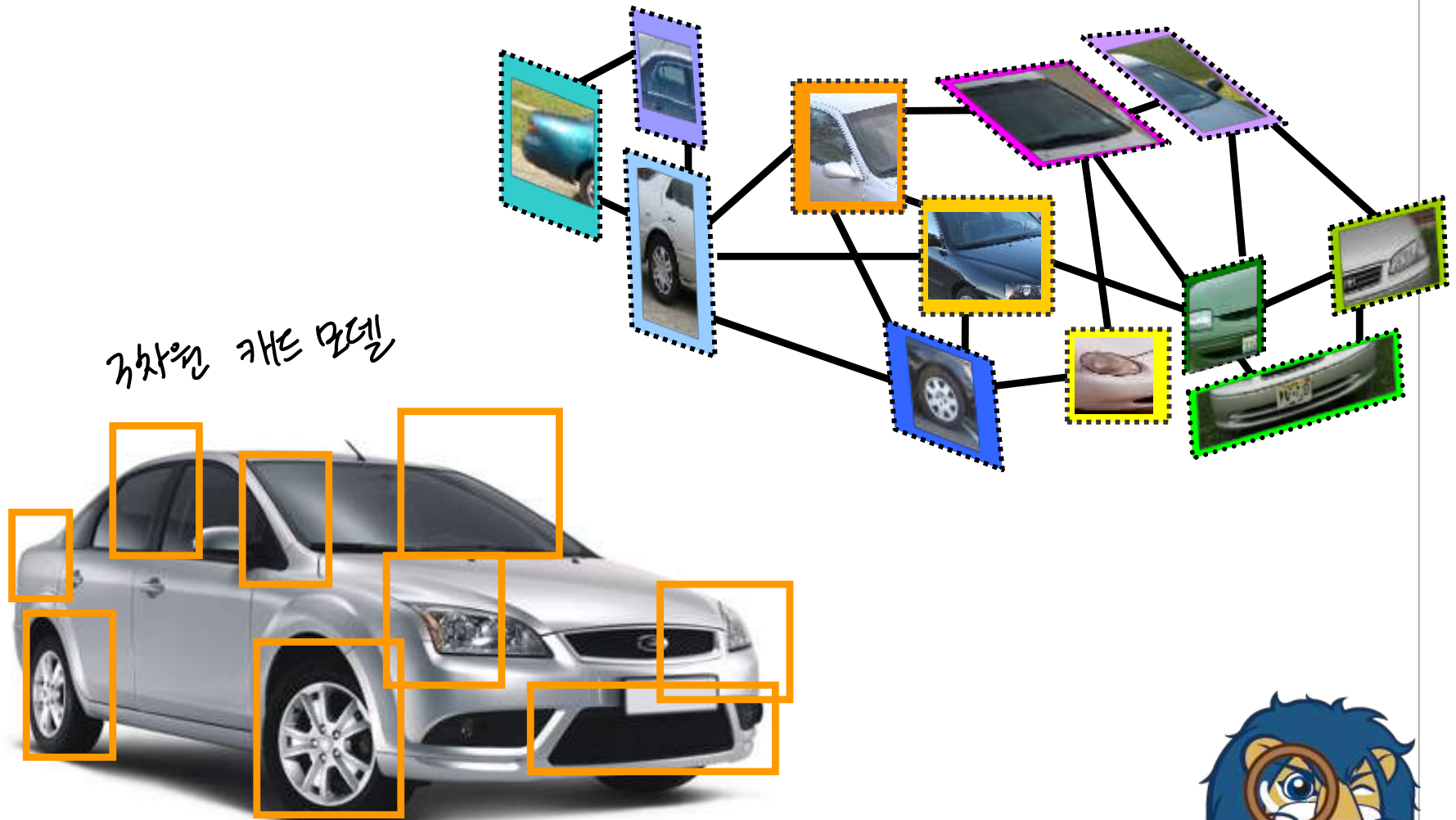
Example: automatic panoramas



Example: fitting a 2D shape template



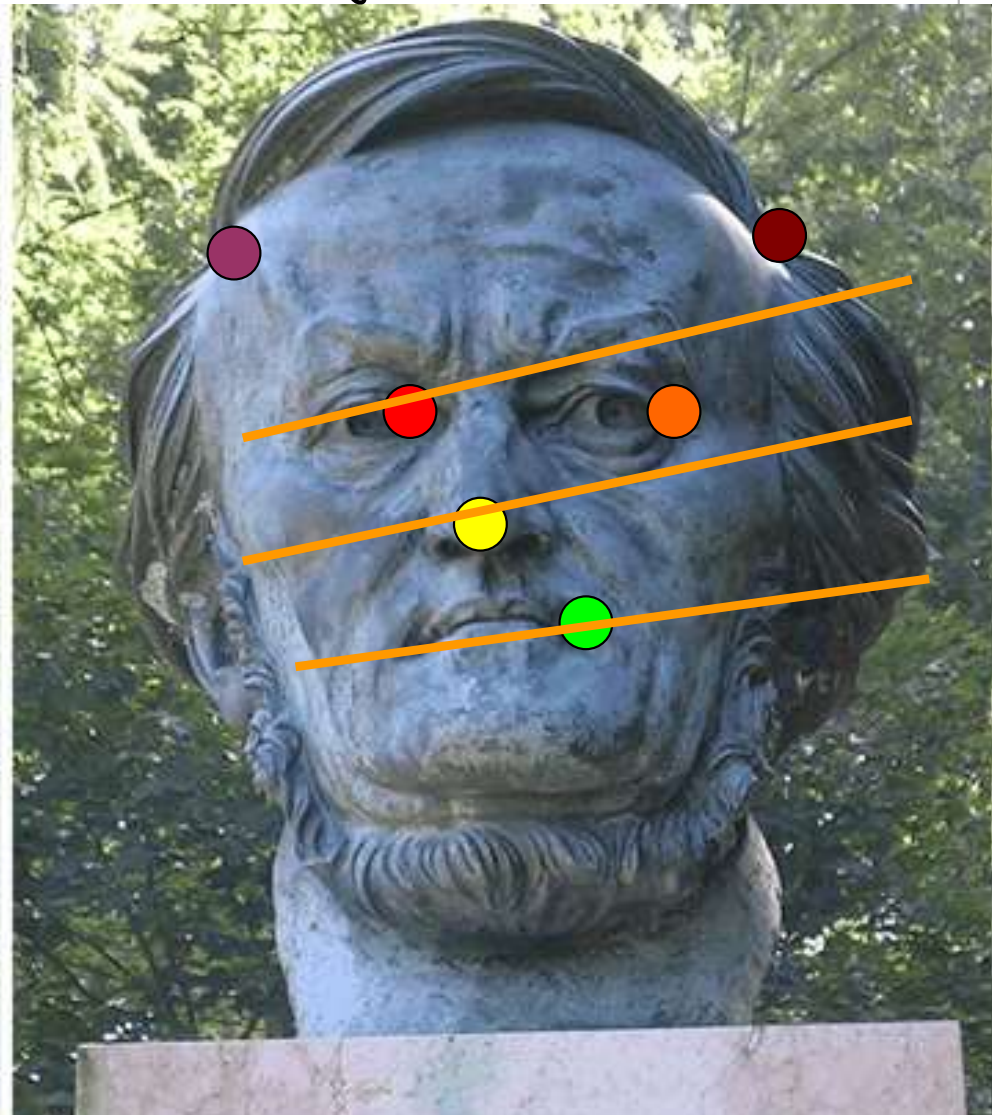
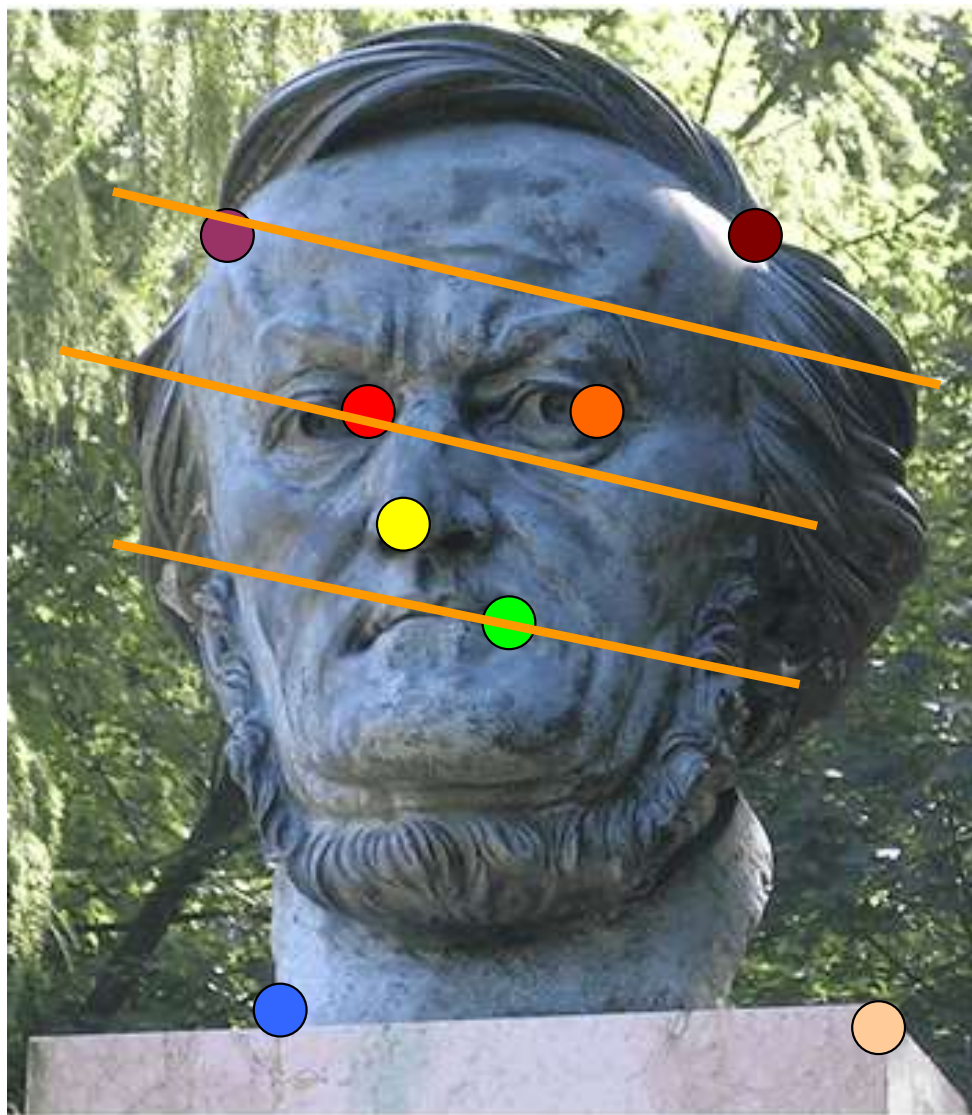
Example: fitting a 3D object model



Example: estimating homography/fundamental matrix that corresponds two views

매치가 된 피쳐 포인트들이 주어진

이미지의 3차원 Geometry 찾아낼수있음



Example: visual tracking



Tracking over consecutive video frames

좋은 feature 찾기 (색, shape...) ← 사전지식에 기반
→ 중요한 피쳐를 NN이 찾을 수 있도록 automatic하게



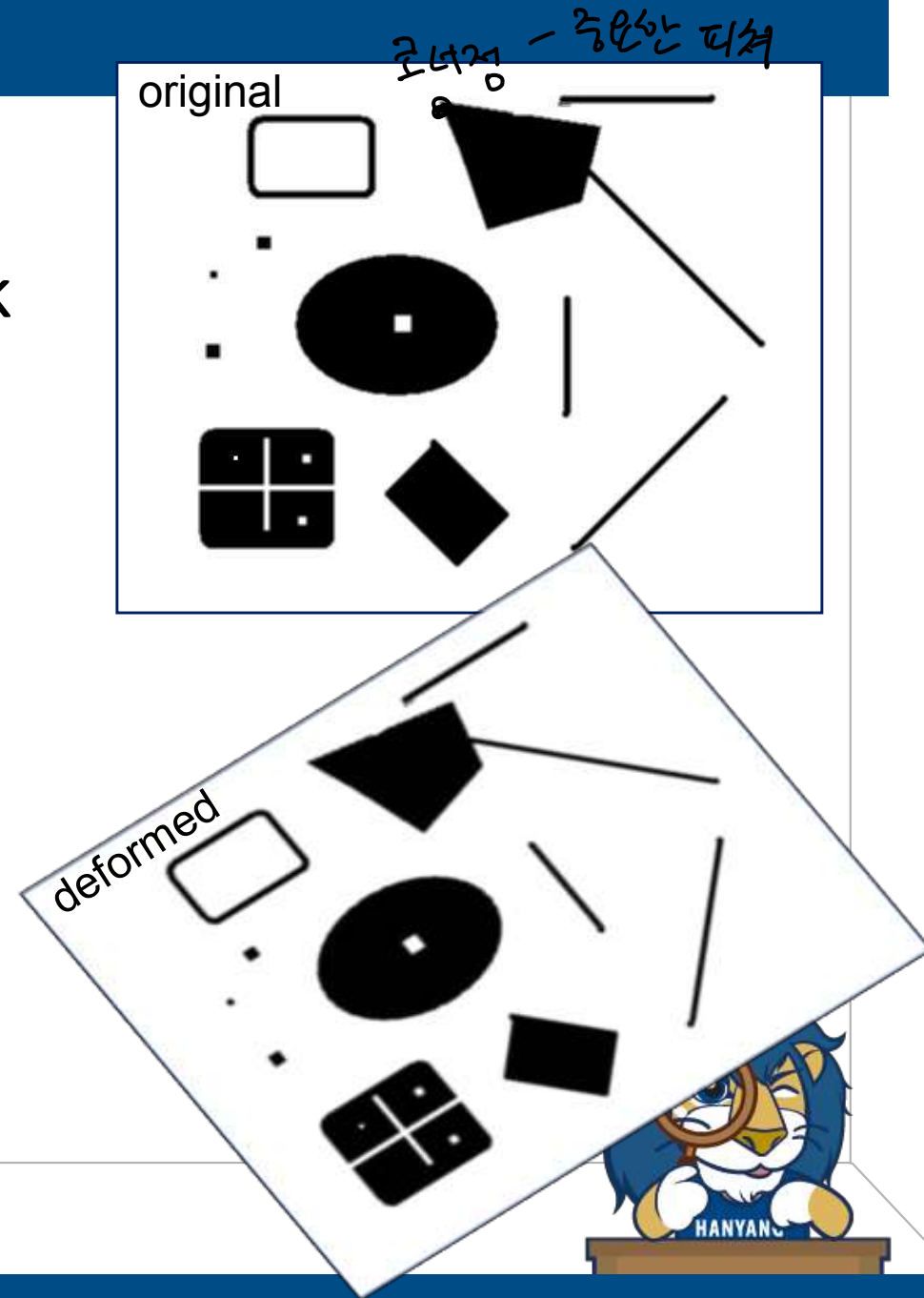
Other Applications

- Key (feature) points are used for:
 - Image alignment
 - 3D reconstruction
 - Motion tracking
 - Robot navigation
 - Indexing and database retrieval
 - Object recognition
- Almost *everywhere* in computer vision



Choosing Interest Points

- Which points would you choose?
 - Suppose you have to click on some point, go away and come back after I deform the image, and click on the same points again.



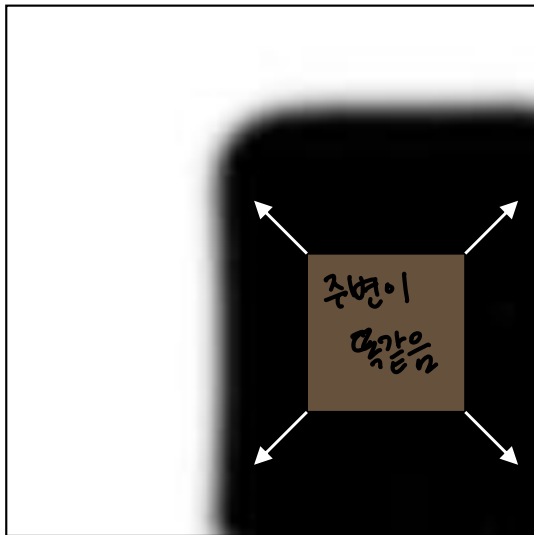
Intuition

- Good features to match

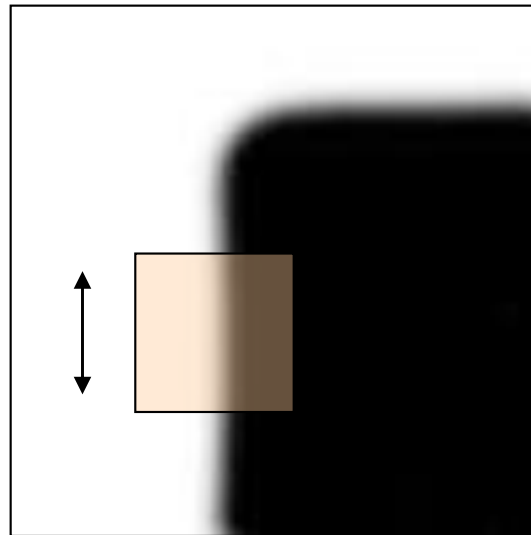


Corners

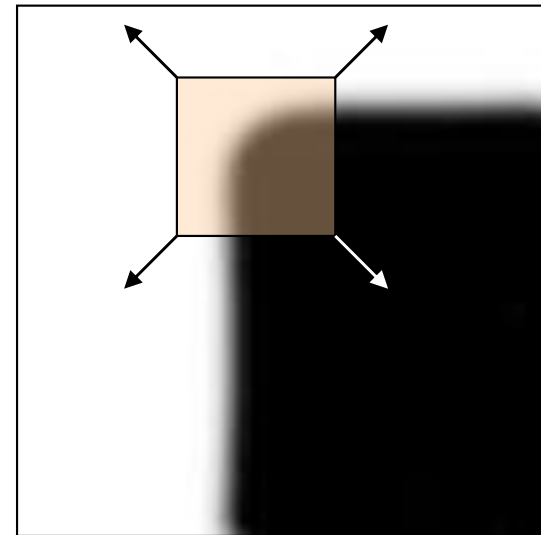
- How does the window change when you shift it?



“flat” region:
no change in
all directions



“edge”:
no change along
the edge direction



“corner”:
significant
change in all
directions



Corners

- Why Corners?

$A \neq B \neq C$
 $\theta = D$
 \rightarrow edge 특징

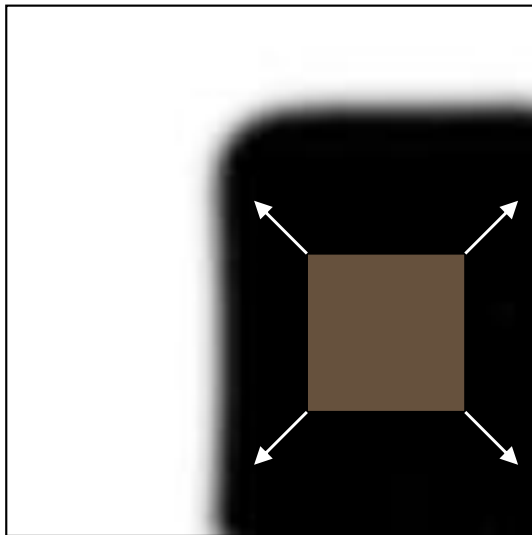


$A \neq B \neq C$
 \rightarrow corner는 어떤 점을 잡더라도 다 다름

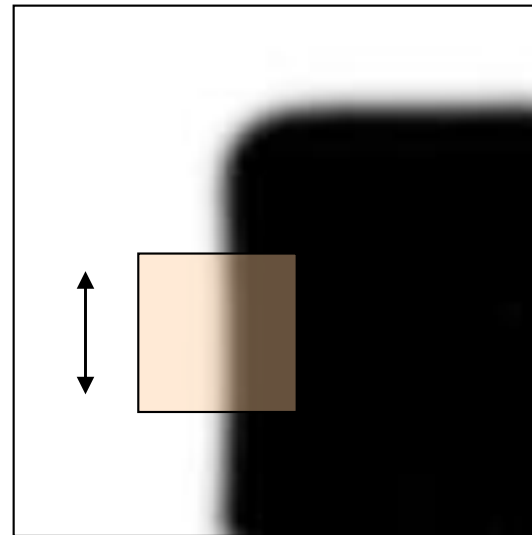
A, B, C patch game
 \rightarrow flat region 특징

이런 바탕은
 corner detection

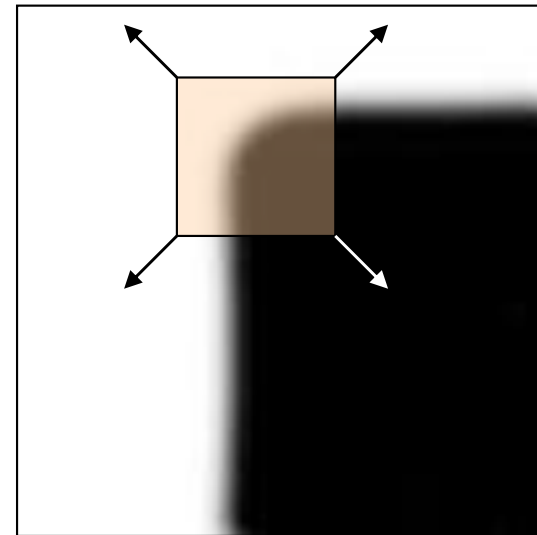
– Shifting the window in any direction causes a big change



“flat” region:
 no change in
 all directions



“edge”:
 no change along
 the edge direction

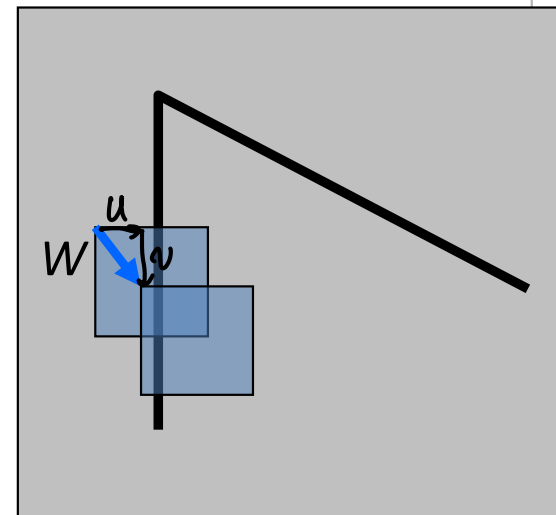


“corner”:
 significant
 change in all
 directions



Harris corner detection: the math

- Consider shifting the window W by (u,v)
 - how do the pixel values in W change?
 - compare each pixel before and after by summing up the squared differences (SSD)
 - this defines an SSD “error” $E(u,v)$:



즉 pixel의 difference 제곱

$$E(u, v) = \sum_{(x, y) \in W} (I(x + u, y + v) - I(x, y))^2$$

Error

변동성간의 차이



Harris corner detection: the math

- *Small motion assumption* (small u, v)

이미지에서 특징위치가
코너인지 아닌지 검사

Taylor Series expansion of I :

$$I(x+u, y+v) = I(x, y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \text{higher order terms}$$

If the motion (u, v) is small, then first order approximation is good enough;

$$I(x+u, y+v) \approx I(x, y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v$$
$$\approx I(x, y) + [I_x \ I_y] \begin{bmatrix} u \\ v \end{bmatrix}$$

검사 위치가 조금

$x+u$ 가 굉장히 가까우면 검사가 잘됨

linear, 2차항까지 무시 가능

$$\text{shorthand: } I_x = \frac{\partial I}{\partial x}$$



Harris corner detection: the math

근사가 유효할 때 : $x+u \approx u$: u 가 굉장히 작을 때

- Under small motion assumption

$$E(u, v) = \sum_{(x,y) \in W} (I(x+u, y+v) - I(x, y))^2$$

$$\approx \sum_{(x,y) \in W} (I(x, y) + I_x(x, y)u + I_y(x, y)v - I(x, y))^2$$

$$\approx \sum_{(x,y) \in W} (I_x(x, y)u + I_y(x, y)v)^2$$

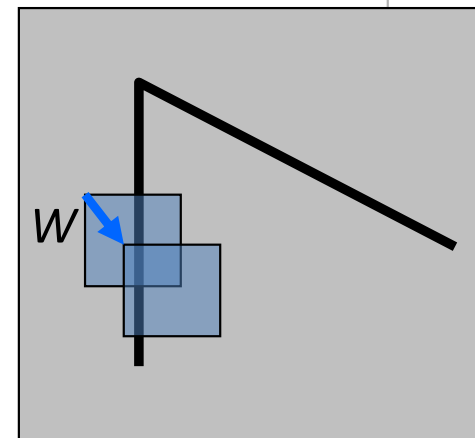


Harris corner detection: the math

- $E(u,v)$ is locally approximated as a *quadratic form*

$$\begin{aligned} E(u,v) &\approx \sum_{(x,y) \in W} (I_x(x,y)u + I_y(x,y)v)^2 \\ &\approx \sum_{(x,y) \in W} (I_x^2 u^2 + 2I_x I_y uv + I_y^2 v^2) \\ &\approx Au^2 + 2Buv + Cv^2 \end{aligned}$$

$$A = \sum_{(x,y) \in W} I_x^2 \quad B = \sum_{(x,y) \in W} I_x I_y \quad C = \sum_{(x,y) \in W} I_y^2$$



Harris corner detection: the math

u, v가 영점 값을 때 유용함



가까운 픽처들에서 가능

밝기값 1:1 비교



없어야 하는데 \rightarrow 픽처의 밝기 정보 알 필요 없음

$$E(u, v) \approx Au^2 + 2Buv + Cv^2$$

$$\approx \begin{bmatrix} u & v \end{bmatrix} \underbrace{\begin{bmatrix} A & B \\ B & C \end{bmatrix}}_H \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{(x,y) \in W} I_x^2$$

지속방향의 밝기값²
인접한 두 픽셀의 밝기값 차이²

H

$$B = \sum_{(x,y) \in W} I_x I_y$$

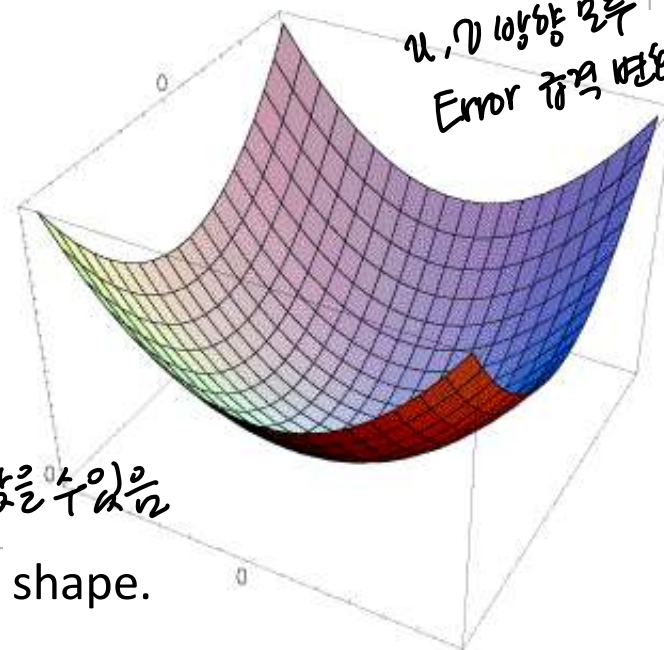
$$C = \sum_{(x,y) \in W} I_y^2$$

decompose

(두개의 eigen vector)
" value) 찾을 수 있음

Let's try to understand its shape.

corner 점.
u, v 방향 모두
Error 급격 변화



Harris corner detection: the math

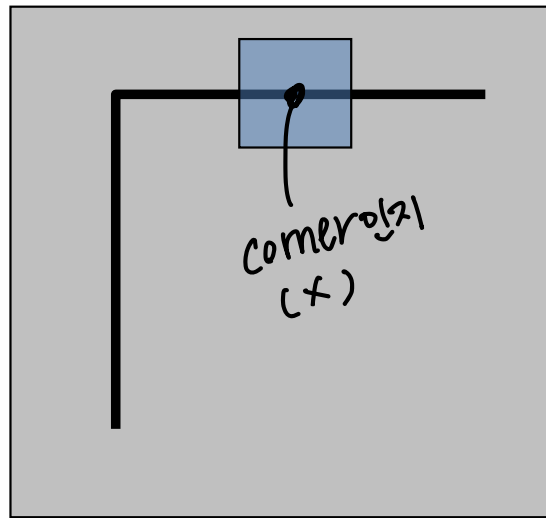
- At horizontal edges

$$E(u, v) \approx \begin{bmatrix} u & v \end{bmatrix} \underbrace{\begin{bmatrix} A & B \\ B & C \end{bmatrix}}_H \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{(x,y) \in W} I_x^2$$

$$B = \sum_{(x,y) \in W} I_x I_y$$

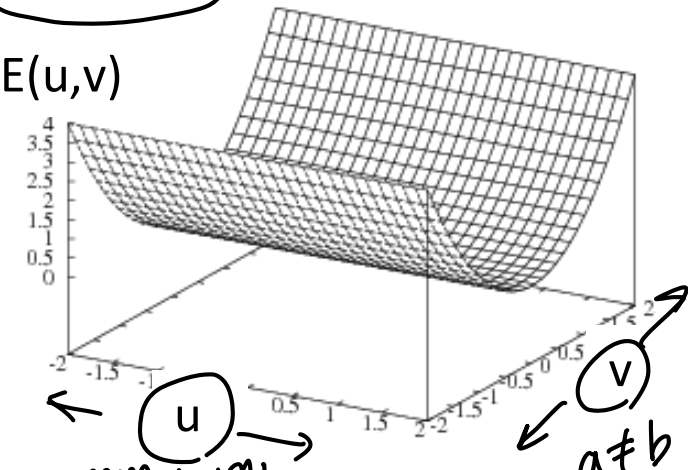
$$C = \sum_{(x,y) \in W} I_y^2$$



Horizontal edge: $I_x = 0$

$$H = \begin{bmatrix} 0 & 0 \\ 0 & C \end{bmatrix}$$

$E(u,v)$



Harris corner detection: the math

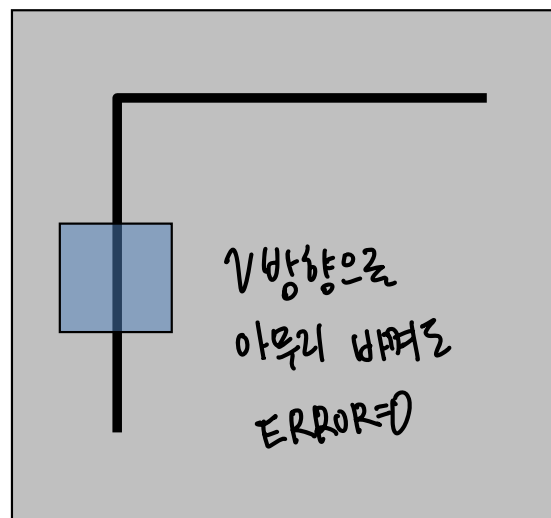
- At vertical edges

$$E(u, v) \approx \begin{bmatrix} u & v \end{bmatrix} \underbrace{\begin{bmatrix} A & B \\ B & C \end{bmatrix}}_H \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{(x,y) \in W} I_x^2$$

$$B = \sum_{(x,y) \in W} I_x I_y$$

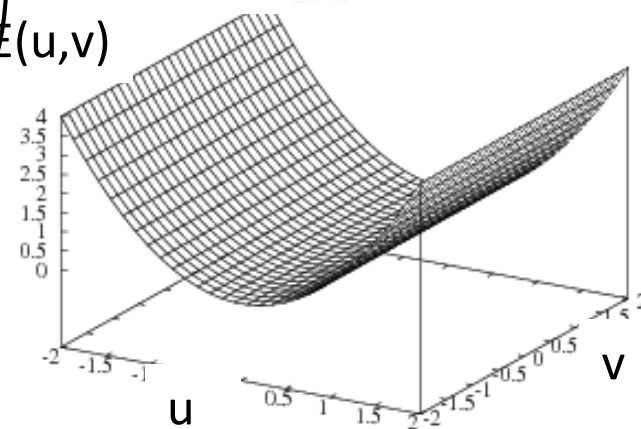
$$C = \sum_{(x,y) \in W} I_y^2$$



Vertical edge: $I_y = 0$

$$H = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}$$

$E(u, v)$



Harris corner detection

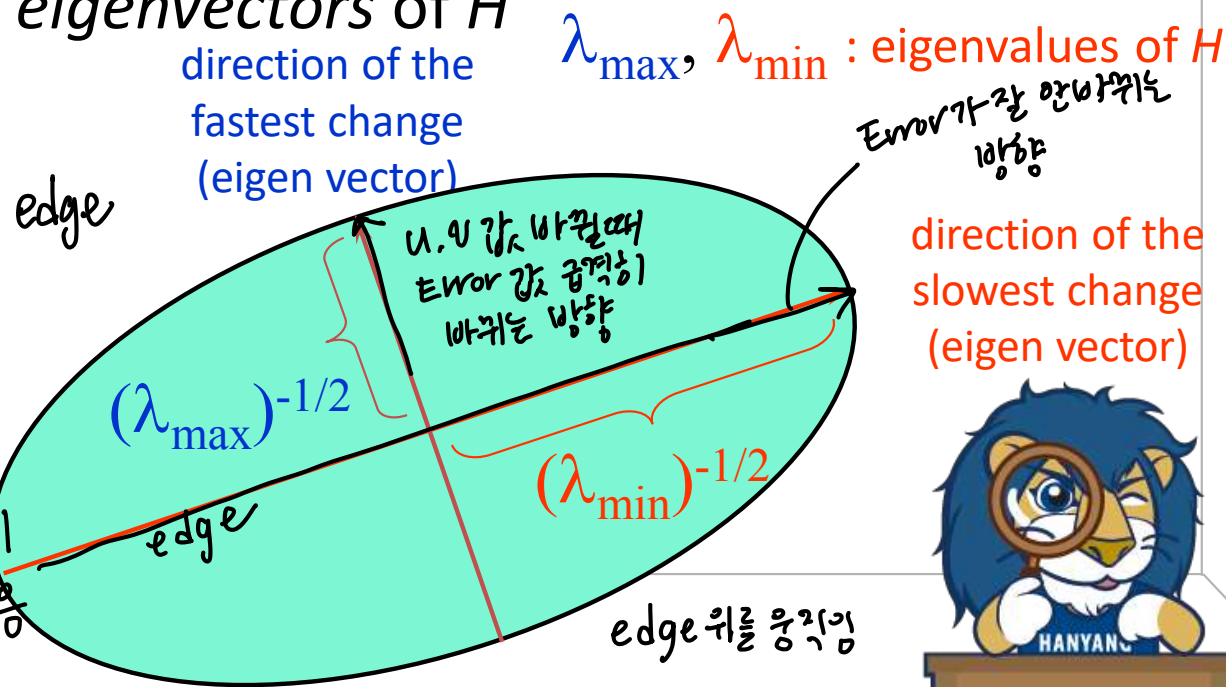
- General case

- Shape of H tells us something about the *distribution of gradients* around a pixel
- We can visualize H as an ellipse where the axis lengths determined by the *eigenvalues* of H , and orientations determined by the *eigenvectors* of H

Ellipse equation:

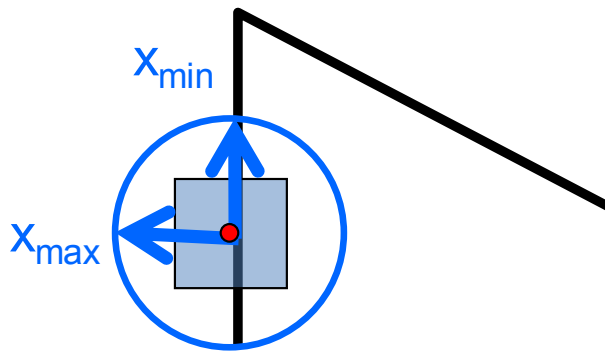
$$\begin{bmatrix} u & v \end{bmatrix} H \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$

H $\xrightarrow{\text{decompose}}$ eigenvalue \rightarrow 코너인지
vector 알지 모르



Harris corner detection

$$E(u, v) \approx \begin{bmatrix} u & v \end{bmatrix} \underbrace{\begin{bmatrix} A & B \\ B & C \end{bmatrix}}_H \begin{bmatrix} u \\ v \end{bmatrix}$$



$$H_{2 \times 2}$$

$$H x_{\max} = \lambda_{\max} x_{\max}$$

$$H x_{\min} = \lambda_{\min} x_{\min}$$

- Eigenvalues and eigenvectors of H

- Define shift directions with the smallest and largest change in error

- x_{\max} = direction of largest change in E eigen vector x :
Error가 엄청 빨리/천천히 바뀌는 방향

- λ_{\max} = amount of change in direction x_{\max} eigenvalue λ :
타원의 장축/단축 길이 결정

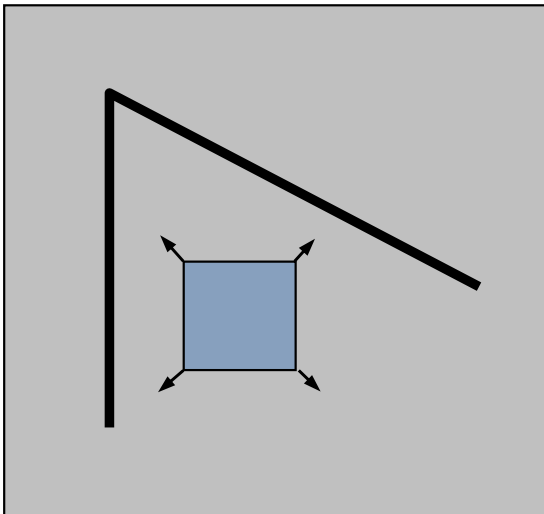
- x_{\min} = direction of smallest change in E

- λ_{\min} = amount of change in direction x_{\min}



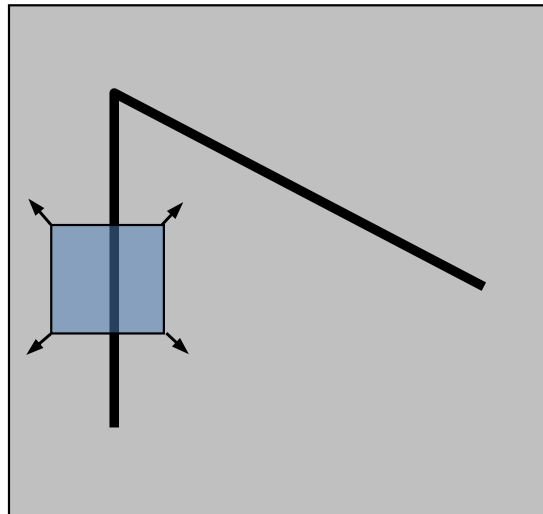
Harris corner detection

Small λ_{\max} & small λ_{\min}



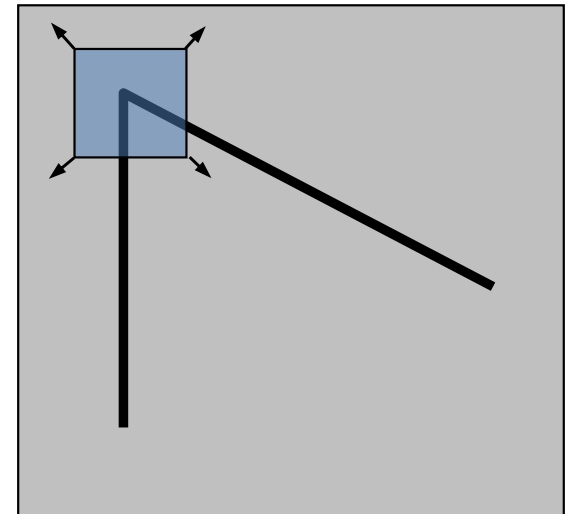
“flat” region:
no change in all
directions

Large λ_{\max} & small λ_{\min}



“edge”:
no change along the
edge direction

Large λ_{\max} & large λ_{\min}



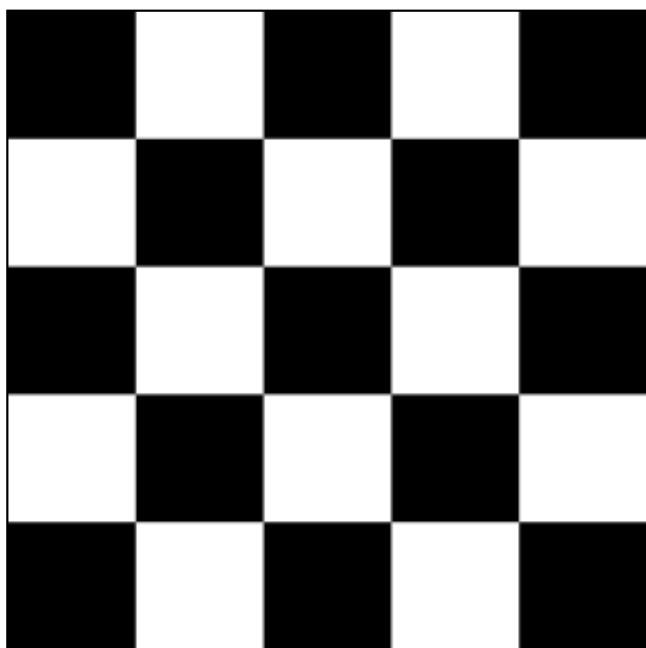
“corner”:
significant change in
all directions



Harris corner detection

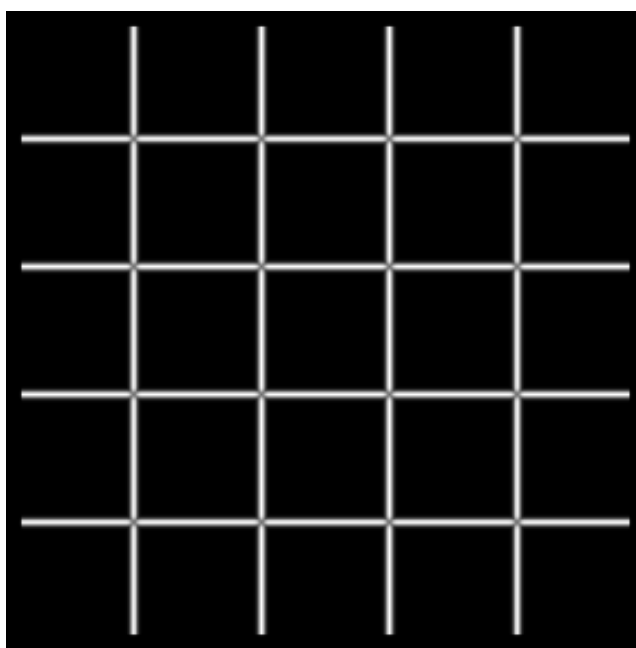
- Real Example

– At corner points, both λ_{\max} and λ_{\min} are large



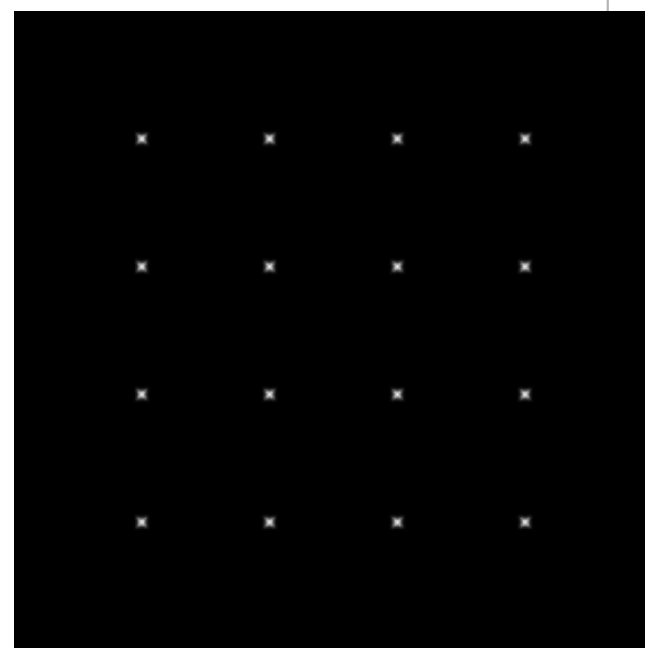
I

eigenvalue sorting



λ_{\max}

→ edge, corner 등에서
값이 큼



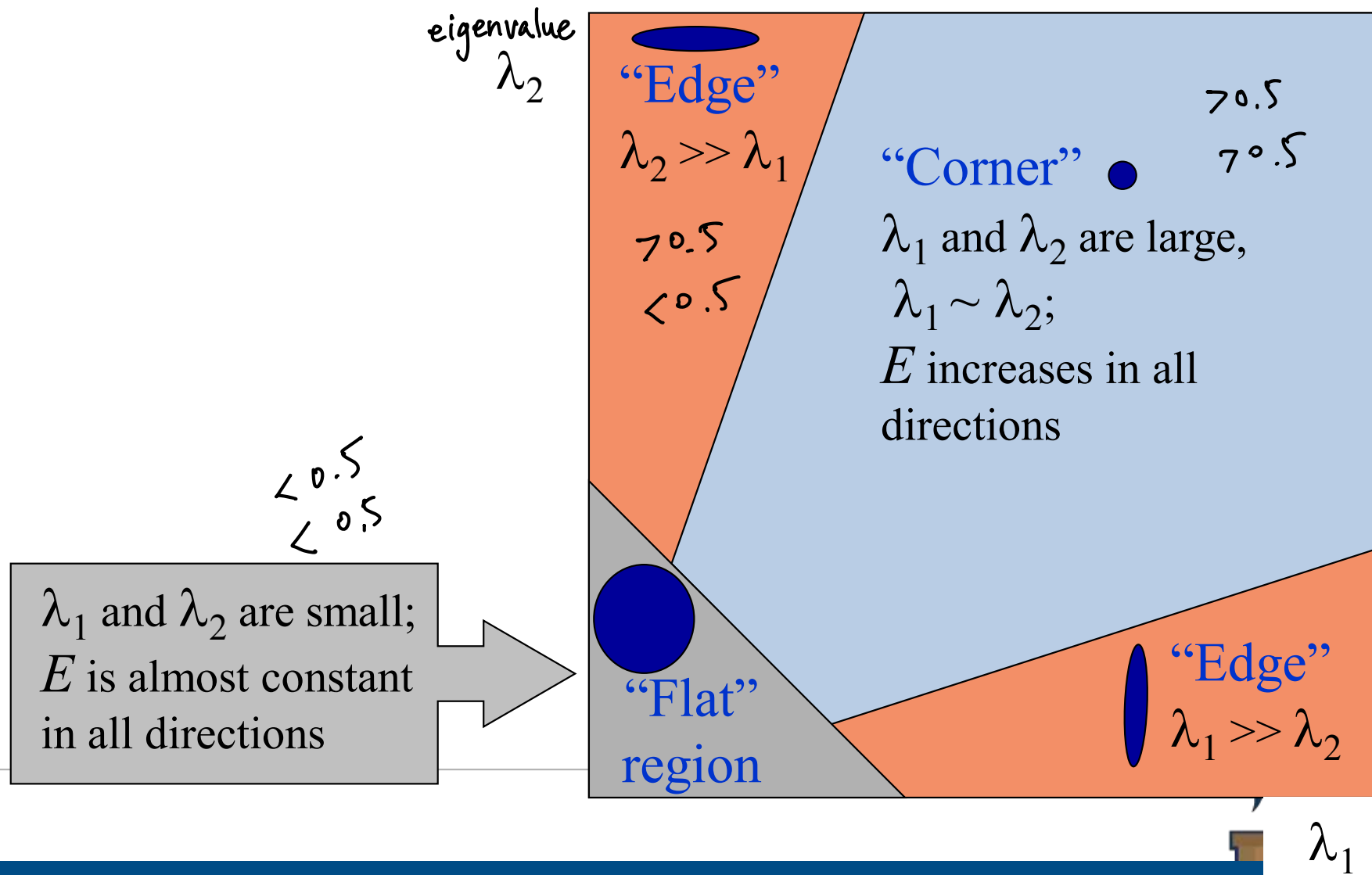
λ_{\min}

corner에서만
살아남음
⇒ λ_{\min} 값이 크면 코너



Harris corner detection

- Two eigen values and corresponding region in the given image



Harris corner detection

- Harris operator (metric)
 - Called the “Harris Corner Detector” or “Harris Operator”
 - Shi-Tomasi corner detector: λ_{\min}
 - Harris and Stephens: $\det(H) - \alpha \cdot \text{trace}(H)$
 - Harmonic mean: $\det(H) / \text{trace}(H)$
 - Among lots metrics, this is one of the most popular
 - Harris response

$$f = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2} = \frac{\text{determinant}(H)}{\text{trace}(H)}$$

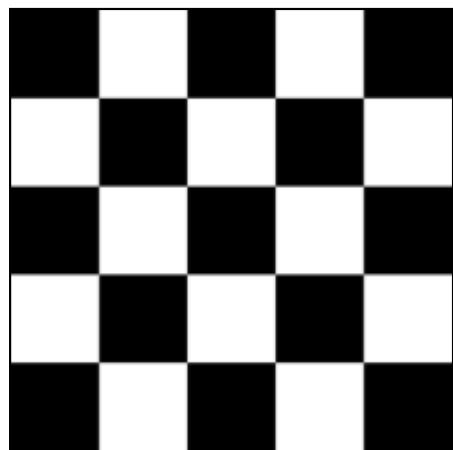
(λ 가 둘다 크면 값이 큼)



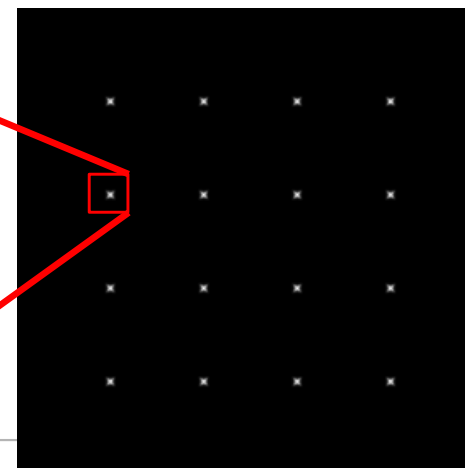
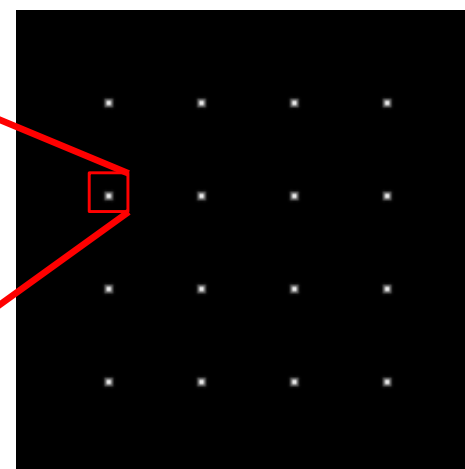
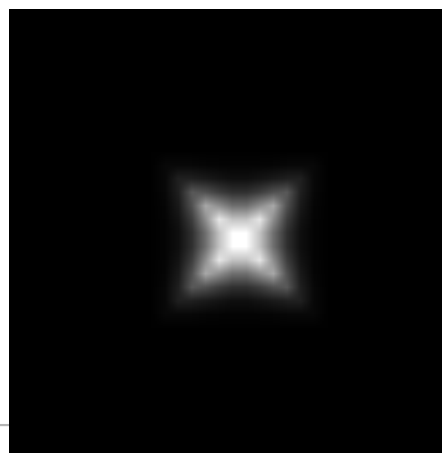
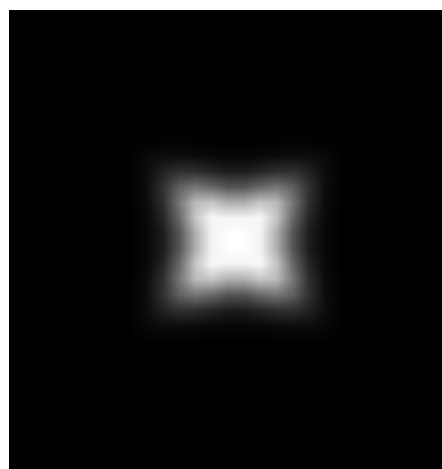
Harris corner detection

- Harris operator value $f \approx \lambda_{\min}$

코너검출에 사용 f 가 특정값 이상이면 코너



I
각 픽셀 위치에서
eigenvalue 찾고



Harris
operator

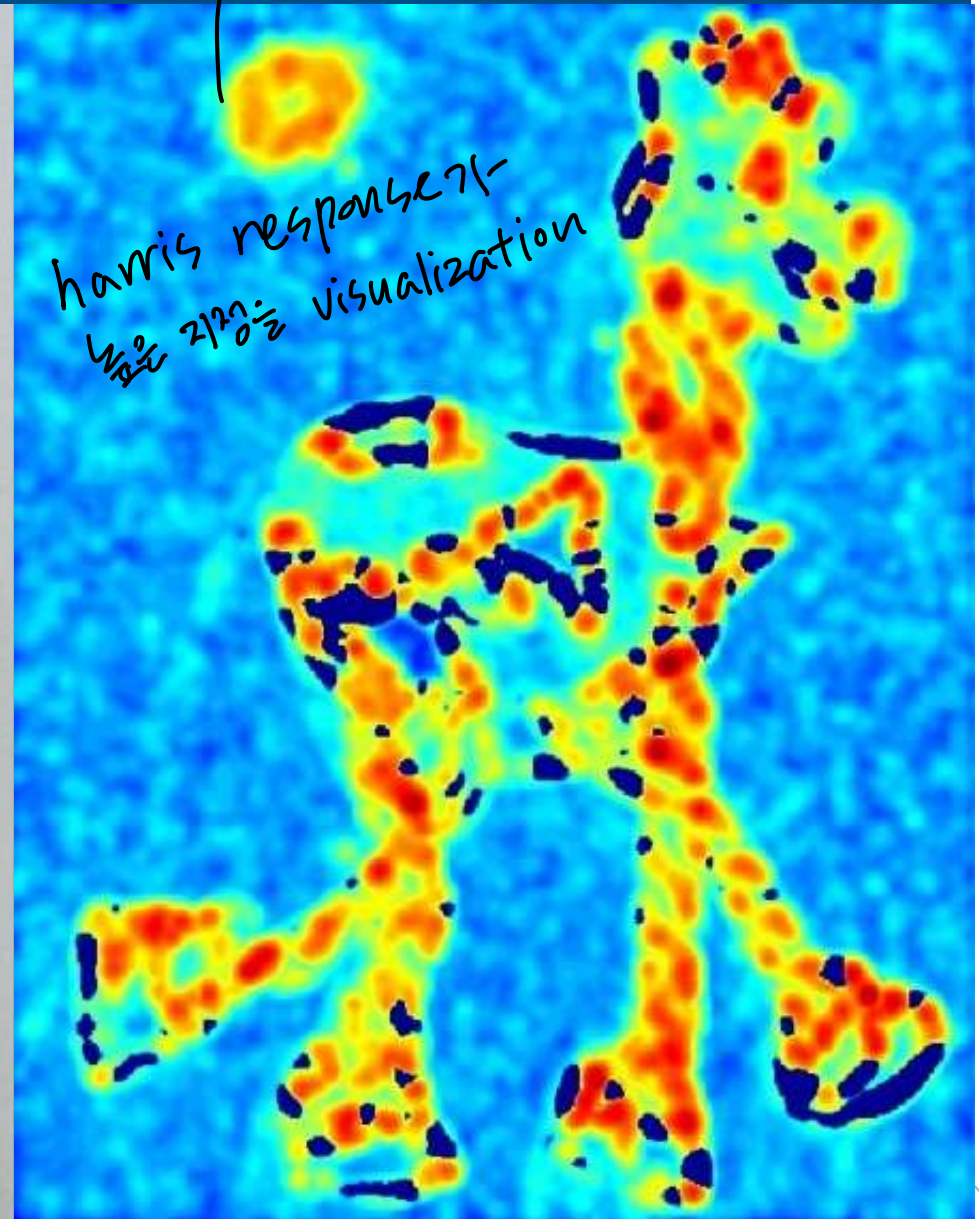


HANYANG

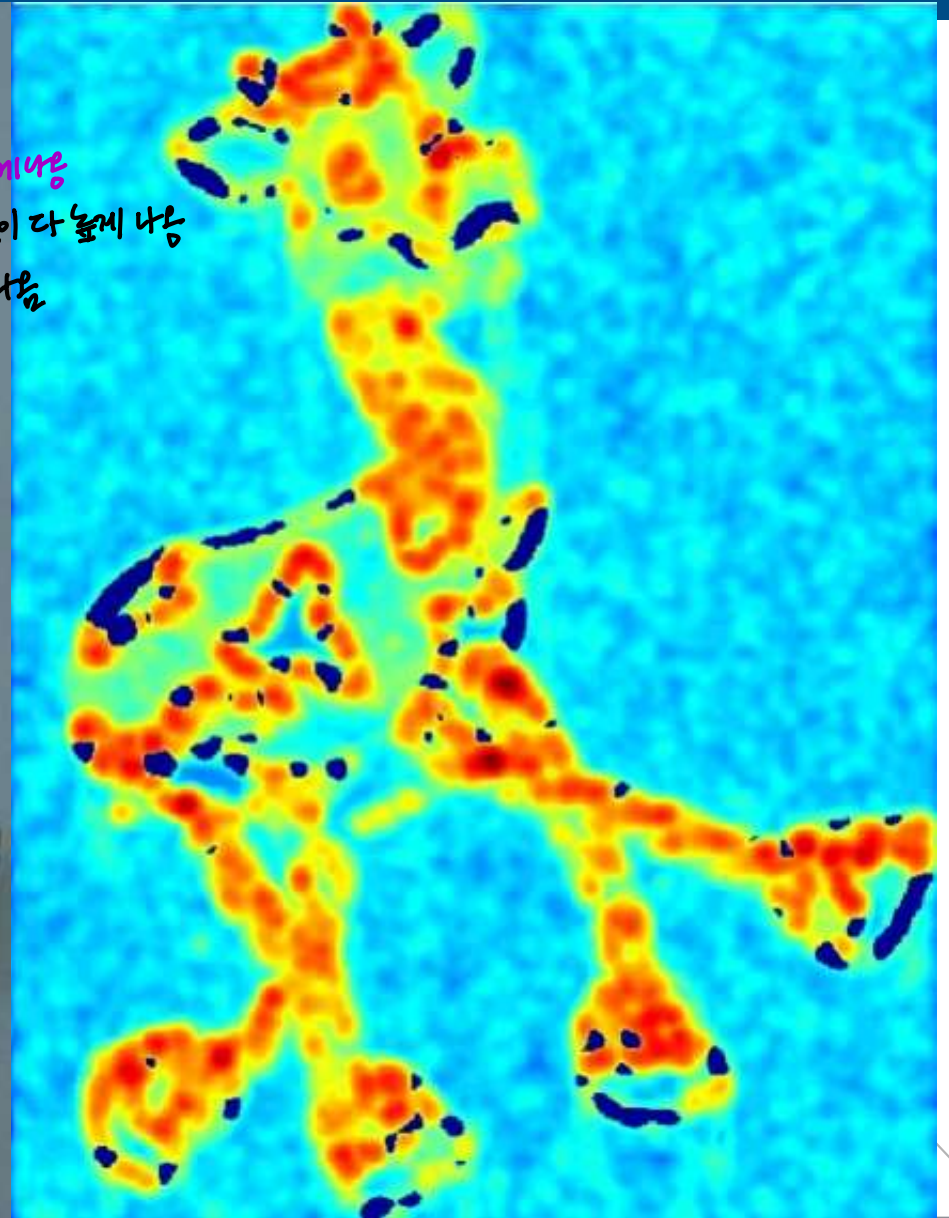
Harris detector example



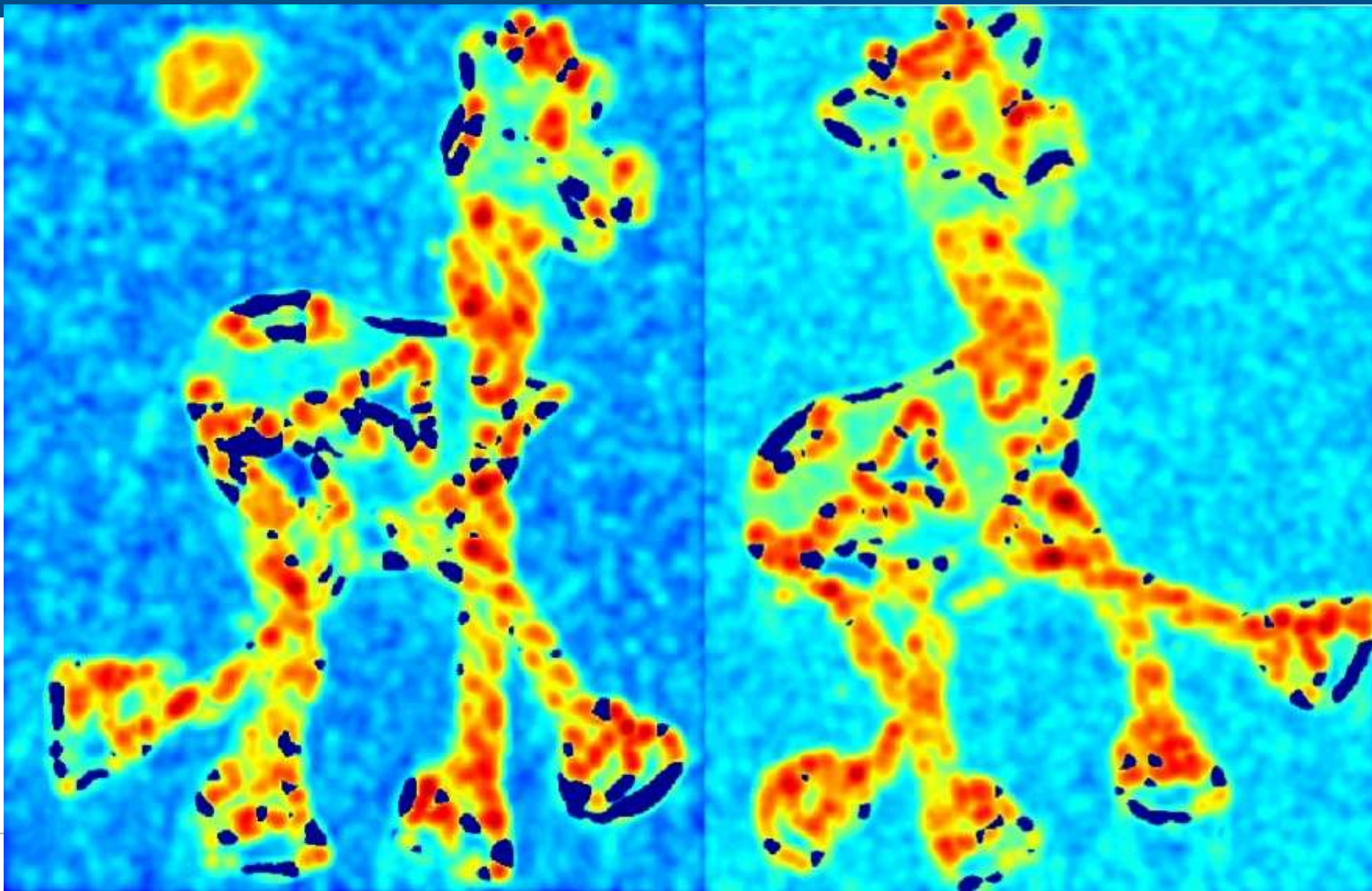
Harris detector example



Harris detector example



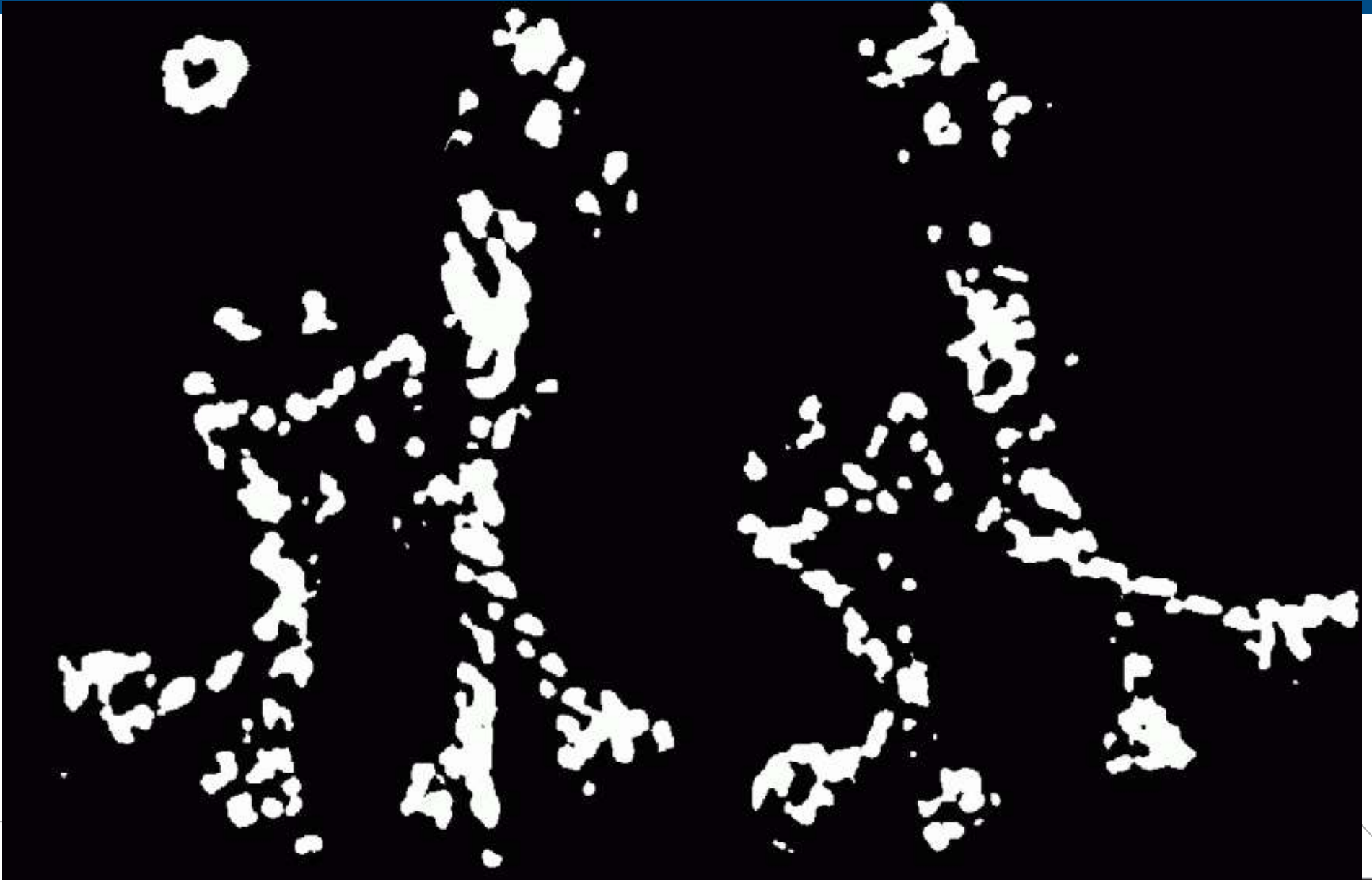
Harris detector example



Harris detector example

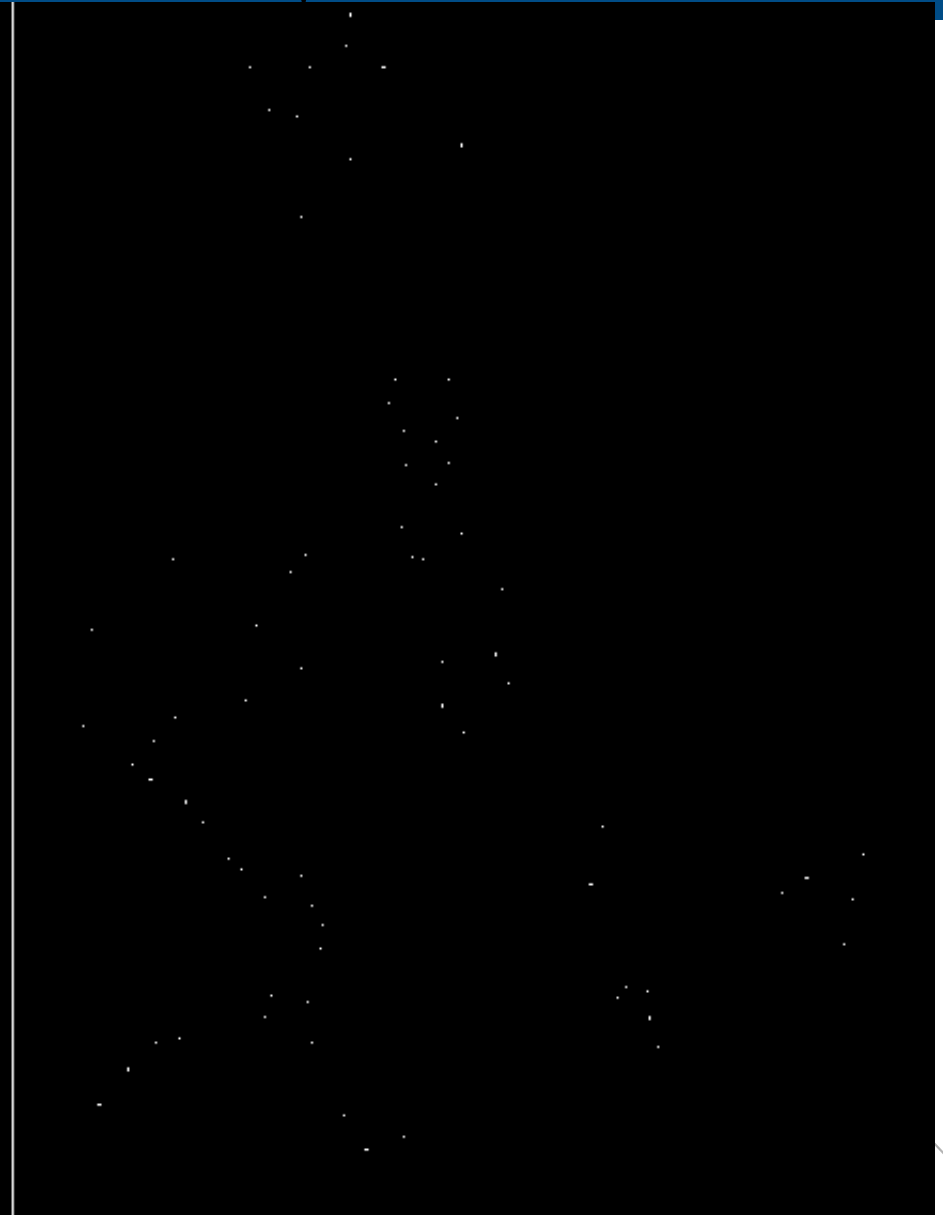
특정 threshold로 harris response...

corner 검출을 위한 영역으로 cornerness 가 높게 나옴



Local maxima of f

↓ NMS
Non Maximal Suppression
젤 코너값이 높은 것 빼고 다 버림
harrison response 값이 큰 애들만 남김



How to improve Harris detector?

4kip

- Weighting the derivatives

- In practice, using a simple and equally weighted window W doesn't work too well

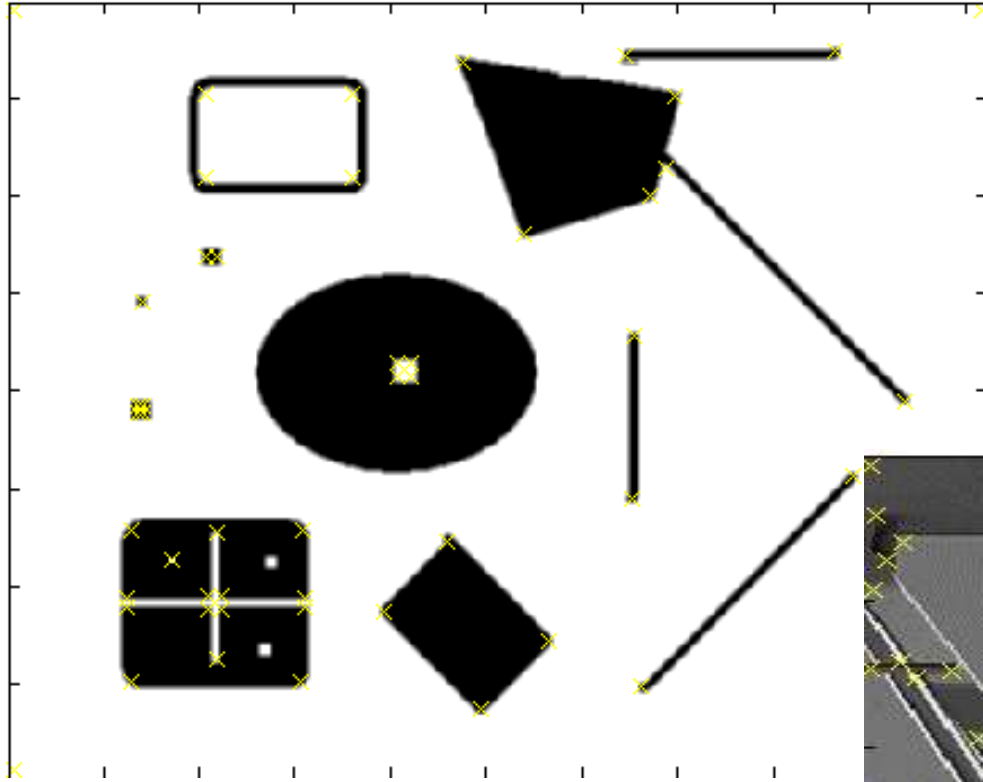
$$H = \sum_{(x,y) \in W} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

- Instead, we'll *weight* each derivative value based on its distance from the center pixel

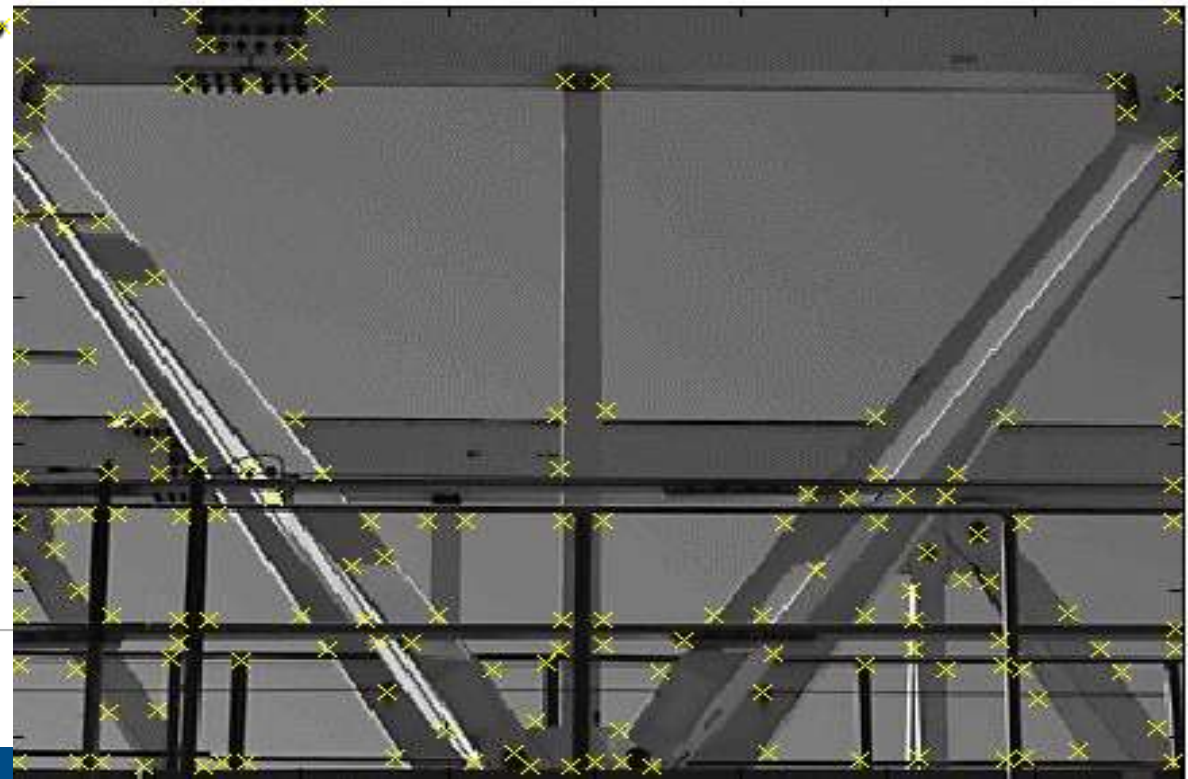
$$H = \sum_{(x,y) \in W} w_{x,y} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



Harris detector: results



Effect: A very precise corner detector.

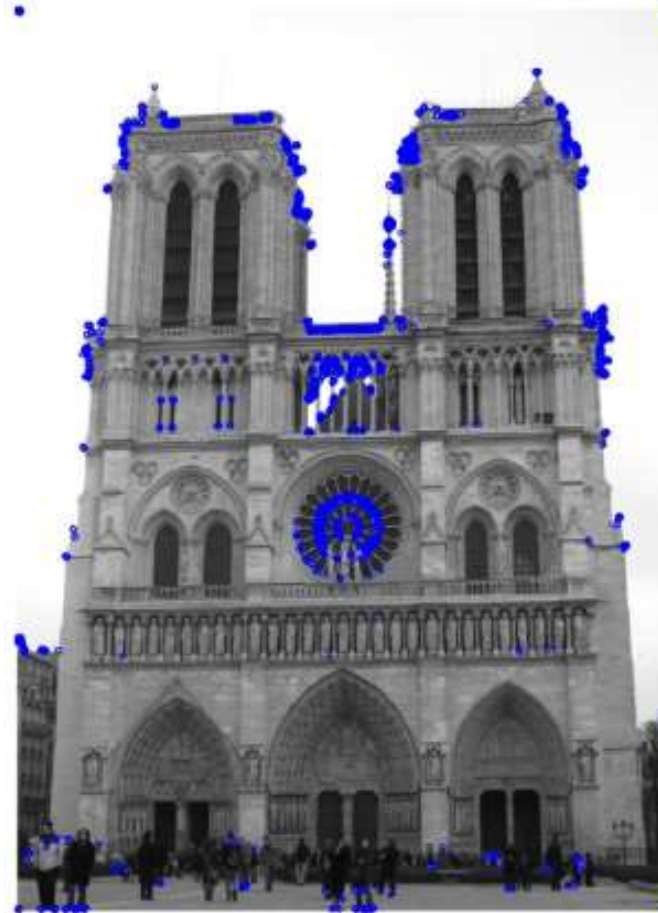


Harris detector: results



Adaptive Non-maximal Suppression (CVPR'05)

- To be a feature, it must be
 - A local maxima
 - Response (f) value is significantly greater than neighbors



Harris corner detection result



Adaptive Non-maximal Suppression (CVPR'05)

- Local Non-Maximum suppression
 - Slide a 3x3 window through the image and suppressing all the key points except for the key point with maximum confidence



Local Non-Maximum suppression



Adaptive Non-maximal Suppression (CVPR'05)

- Adaptive Non-Maximal Suppression
 - Apply Local non-maximum suppression
 - Keep track of the minimum distance to a larger magnitude interest point per point
 - Sort the list of interest points by descending radius and take the top N

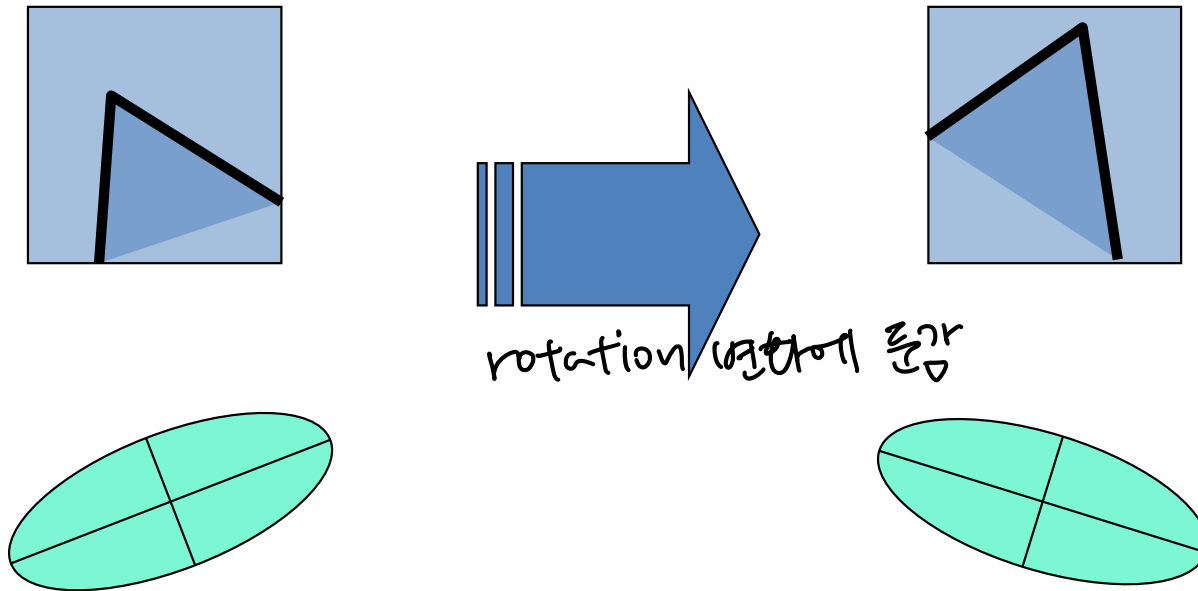


Adaptive Non-Maximal Suppression



Harris Detector: Invariance Properties

- Translation and rotation invariance



Harris Corner Detector 는 rotation, translation
에 관계없이 코너 검출
잘됨

rotation 변환에 민감

Ellipse rotates but its shape (i.e. eigenvalues)
remains the same

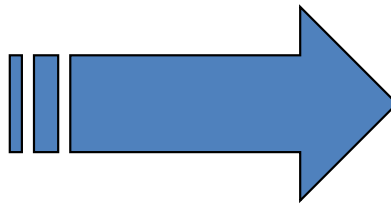
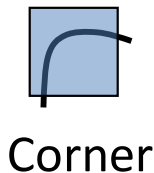
Corner response is invariant to image rotation



Harris Detector: Invariance Properties

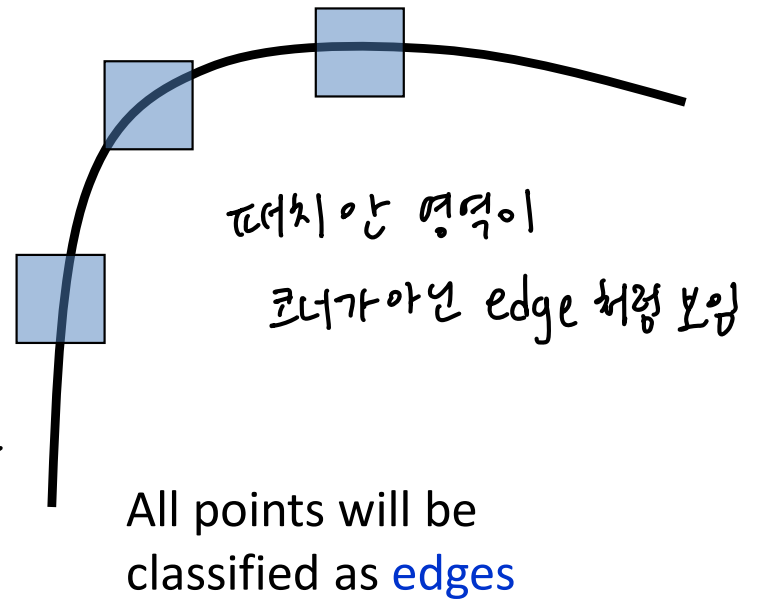
- Scaling

① Corner 또는 ② 패치 크기 변경



코너 커지면 패치 사이즈도 커져야 함

$$H = \begin{bmatrix} A & B \\ B & C \end{bmatrix} \text{ 계산}$$



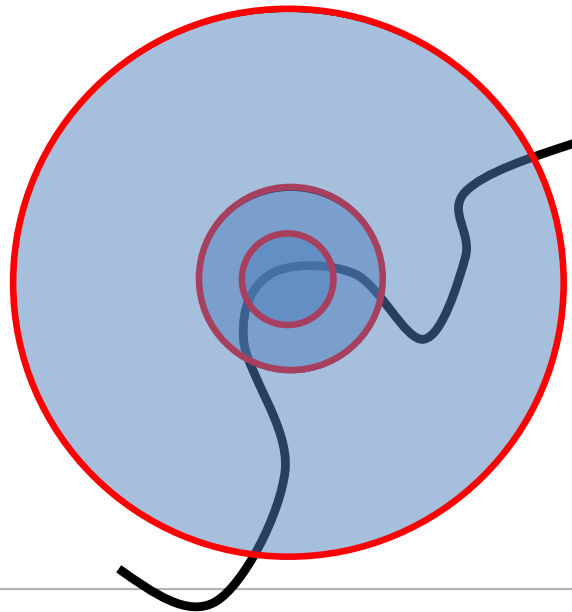
scale 변함에 민감

Not invariant to scaling



Harris Detector: scale invariance

- Suppose you're looking for corners
 - Key idea: find scale that gives local maximum of f in both position and scale
 - One definition of f : the Harris operator



큰 코너는 큰 패치로 찾아야 함



Harris Detector: scale invariance

- Real example

original size



확대됨

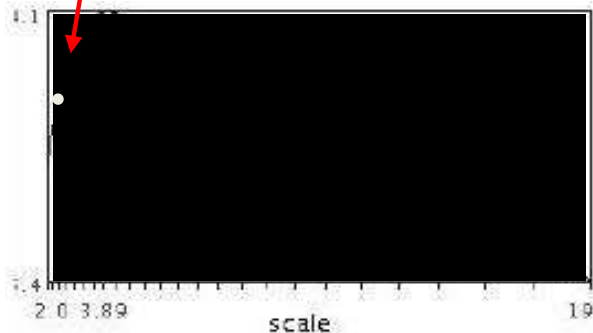
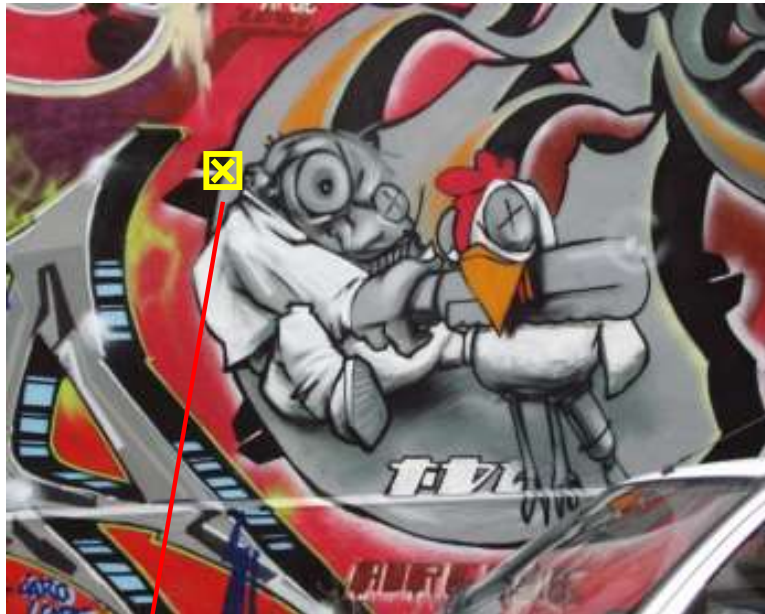


$$f(I_{i_1 \dots i_m}(x, \sigma)) = f(I_{i_1 \dots i_m}(x', \sigma'))$$

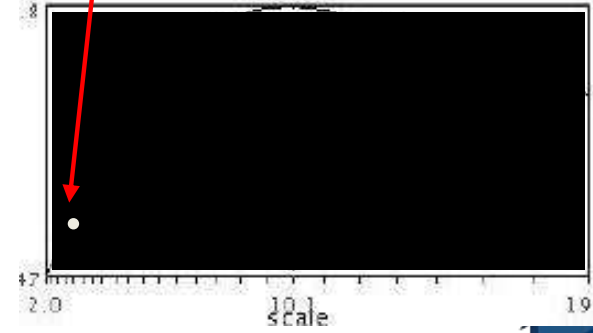


Harris Detector: scale invariance

- Automatic scale detection: compare corneriness



$$f(I_{i_1 \dots i_m}(x, \sigma))$$



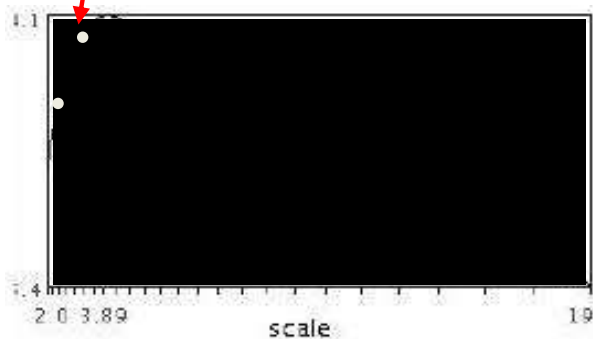
$$f(I_{i_1 \dots i_m}(x', \sigma))$$

동일한 패치 사이즈를
사용해서 harris
response를 보면

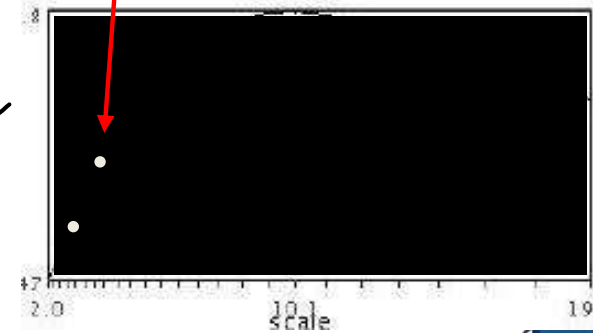


Harris Detector: scale invariance

- Automatic scale detection: compare corneriness



$$f(I_{i_1 \dots i_m}(x, \sigma))$$



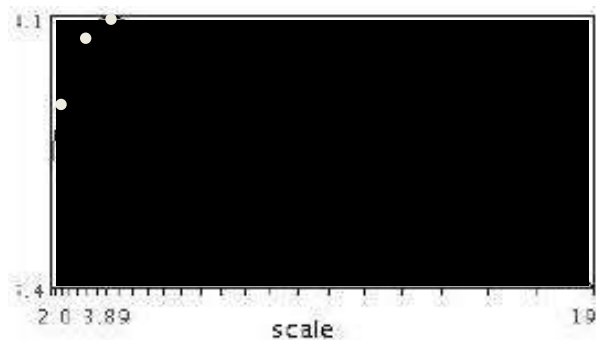
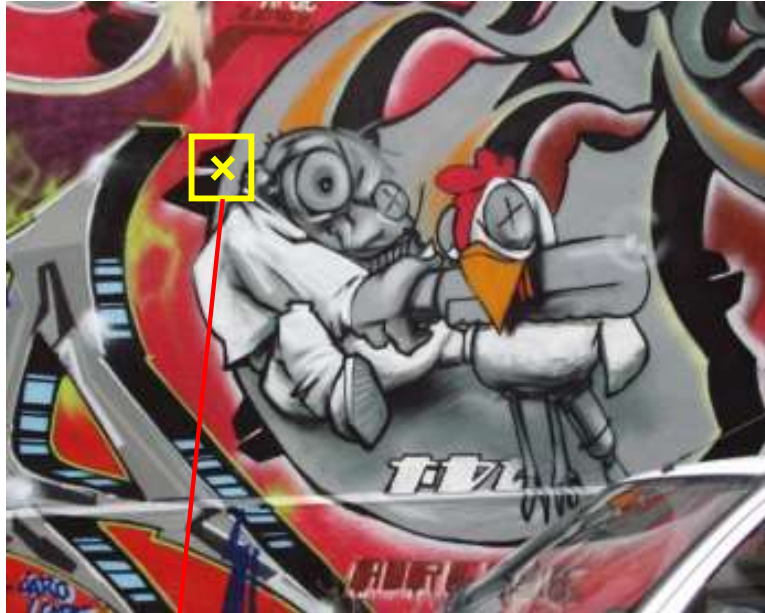
$$f(I_{i_1 \dots i_m}(x', \sigma))$$

큰 패치를 쓰면
harris response
더 좋아짐

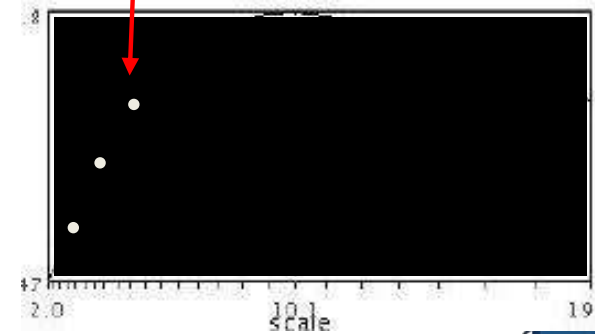


Harris Detector: scale invariance

- Automatic scale detection: compare corneriness



$$f(I_{i_1 \dots i_m}(x, \sigma))$$

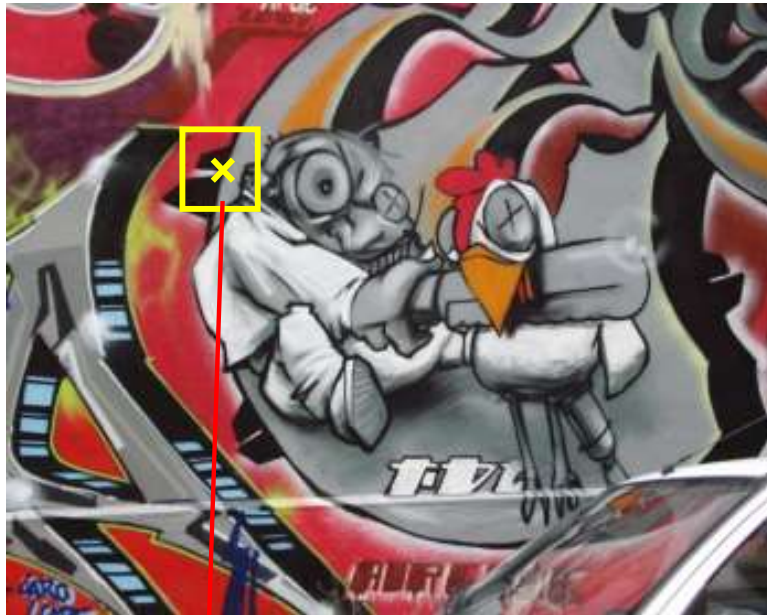


$$f(I_{i_1 \dots i_m}(x', \sigma))$$

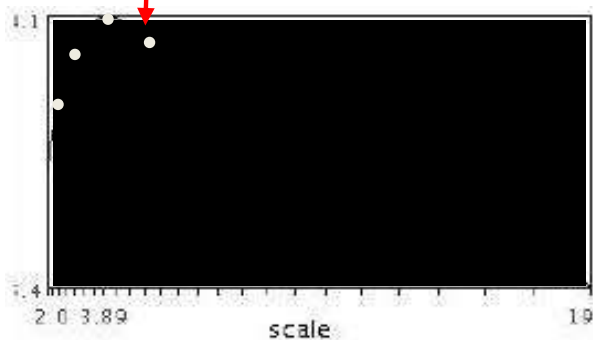


Harris Detector: scale invariance

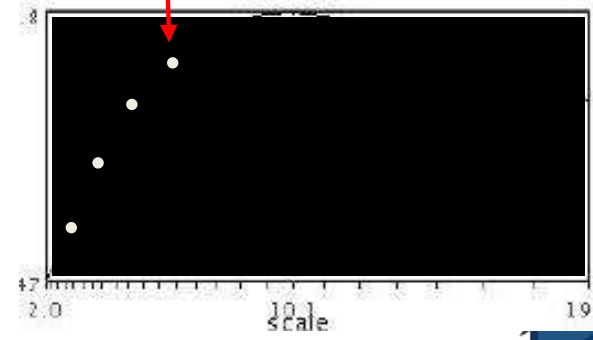
- Automatic scale detection: compare corneriness



더이상
코너처럼
안보임



$$f(I_{i_1...i_m}(x, \sigma))$$

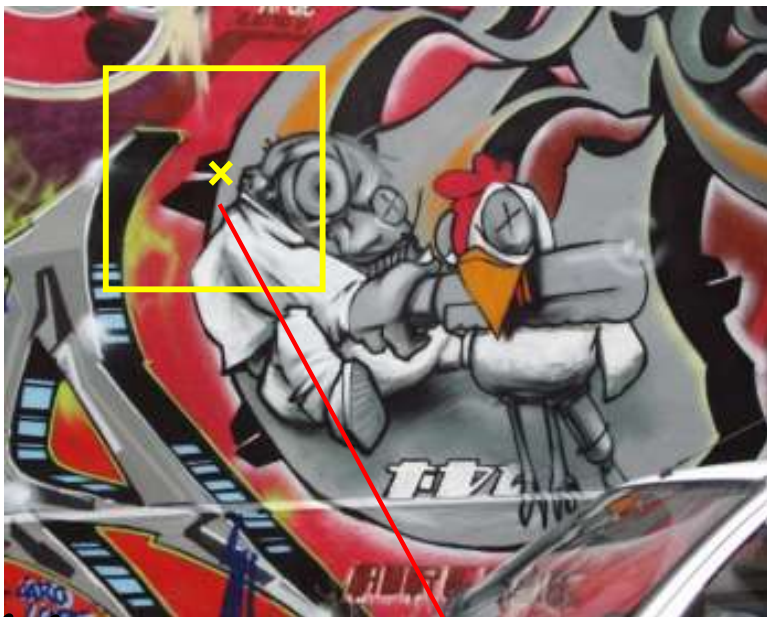


$$f(I_{i_1...i_m}(x', \sigma))$$

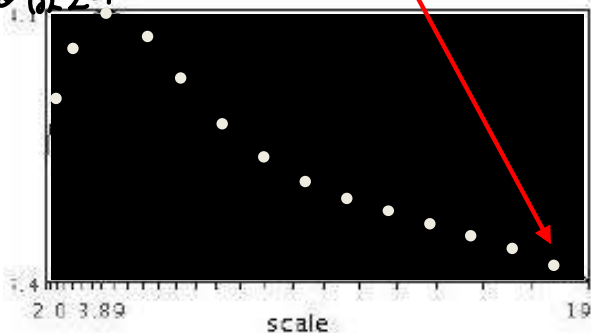


Harris Detector: scale invariance

- Automatic scale detection: compare cornerness

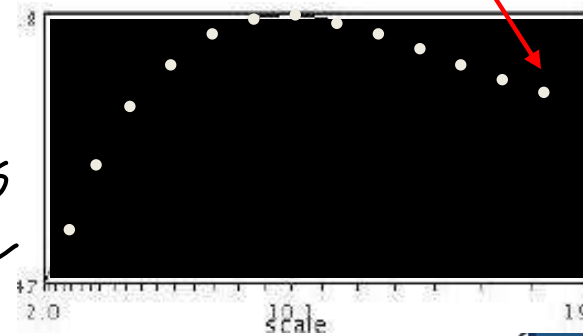


가장 높은 값을 기준으로
내가 정한 threshold 값보다
크면 코너라고 짐작



$$f(I_{i_1 \dots i_m}(x, \sigma))$$

cornerness가
가장 높게 나오는
패치 사이즈가 있을
큰 코너는 큰 패치로
인식해야 함

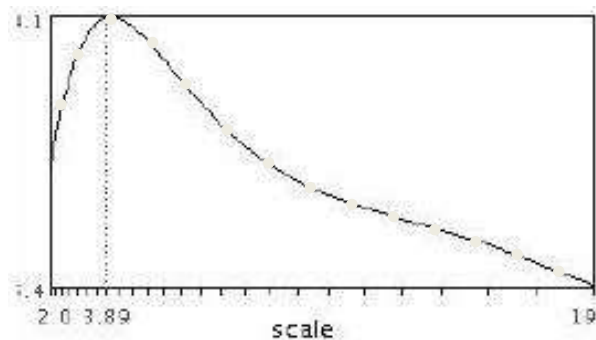
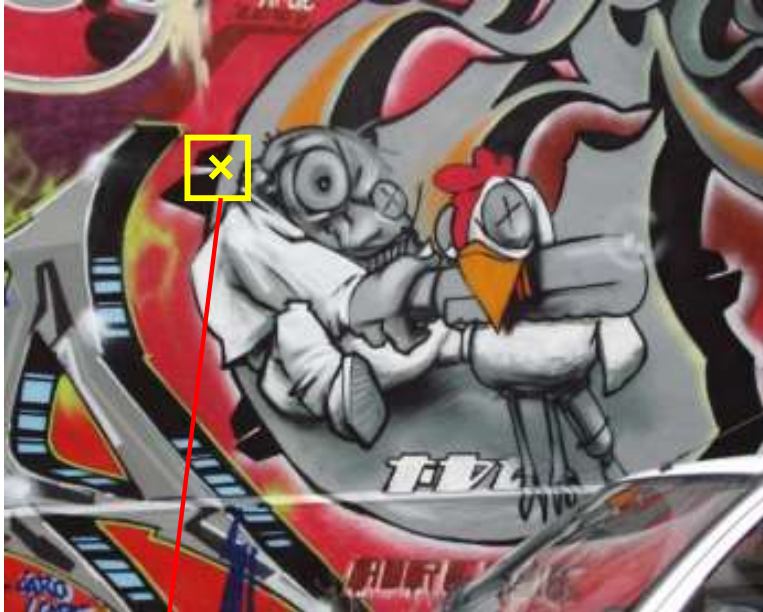


$$f(I_{i_1 \dots i_m}(x', \sigma))$$

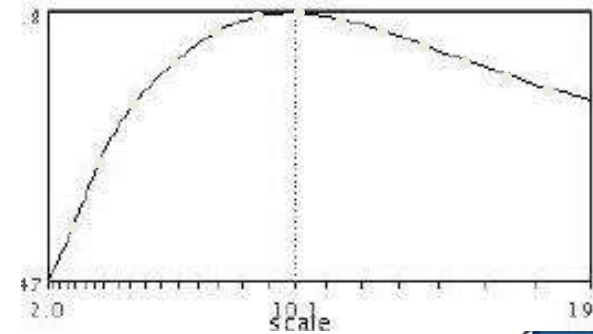


Harris Detector: scale invariance

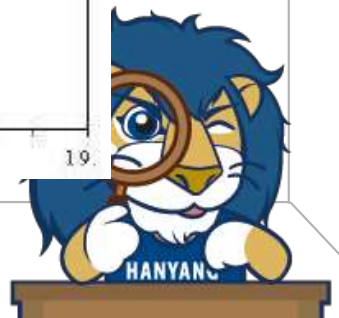
- Automatic scale detection: compare corneriness



$$f(I_{i_1...i_m}(x, \sigma))$$



$$f(I_{i_1...i_m}(x', \sigma'))$$



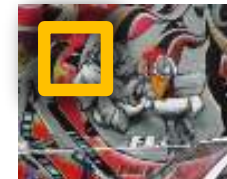
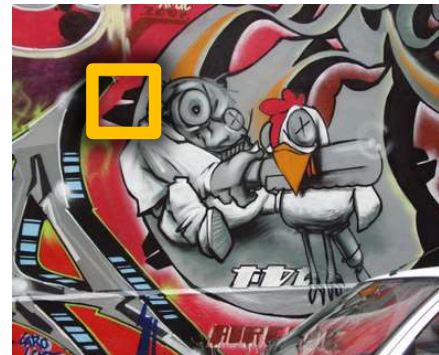
Harris Detector: scale invariance

- Practical implementation 그림 사이즈를 줄임 \rightarrow 계산 속도 향상
 - Instead of computing f with larger windows, implement using a fixed window size with an image pyramid

5x5 harris detector



Original image size



Other Detectors

- Scale-invariant feature transform (SIFT)
 - Lowe. ICCV'99
 - Find local maxima in position-scale space of Difference-of-Gaussian
- Maximally Stable Extremal Regions (MSER)
 - Matas et al. BMCV'02
 - Based on Watershed segmentation algorithm
 - Select regions that stay stable over a large parameter range



SIFT result



MSER result



Summary

- Why interest (feature) points?
 - Used in many applications for matching
- Harris corner detector
 - Traditional approach
 - Give motivation in many follow-up studies
- Invariant properties of Harris corner feature
 - Rotation invariant
 - Partially invariant to affine changes of brightness
 - Originally not invariant to scaling
 - But can be overcome by comparing corneriness in different scale



Matlab

- Download (licensed ver.)
 - <https://iic.hanyang.ac.kr/office365>
- Quick guide (official)
 - FYI
 - <https://kr.mathworks.com/videos/introduction-to-matlab-81592.html>
- HW1 (09/20~09/25)
 - Check out our course website
 - Linear image filtering



Thank you!

