Lab - 6

Rushi Rajpara (201801410)* and Ayan Khokhar (201801057)†
Dhirubhai Ambani Institute of Information & Communication Technology,
Gandhinagar, Gujarat 382007, India
CS-302, Modeling and Simulation

In this lab we numerically and analytically analyze spread of malaria and effect of various types of intervention on it.

Model

Introduction

We can estimate spread of malaria, by studying interaction between human and mosquitos, through this model. Figure(1) represents Flow diagram of this model,

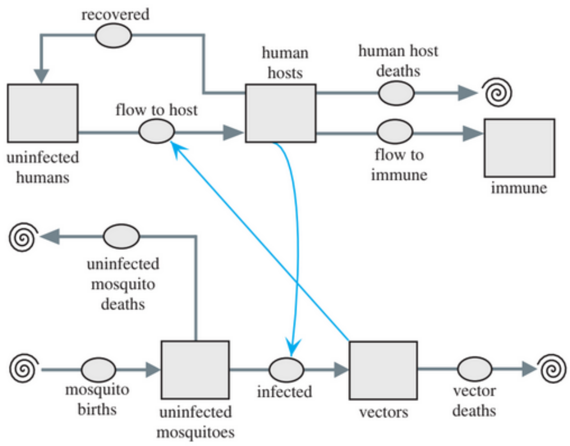

FIG. 1: Flow Diagram

Assumptions

1. Humans don't die due to any other reason than that of malaria.
2. No natural births in human population.
3. If an infected mosquito bites an uninfected human, then human will surely get infected.
4. If an uninfected mosquito bites and infected human, then mosquito will surely get infected.

Mathematical Model

There are total 7 compartments in our model, they are as follows:

S_H : Susceptible Humans.

I_H : Infected Humans.

I_{mH} : Immune Humans.

D_H : Dead Humans.

S_M : Susceptible Mosquitoes.

I_M : Infected Mosquitoes.

D_M : Dead Mosquitoes.

Also, assume that N_H and N_M represents human and mosquito population, respectively, at any time.

Following are the differential equations for our model,

$$\frac{dS_H}{dt} = kI_H - \frac{PI_M S_H}{N_M}$$

$$\frac{dI_H}{dt} = \frac{PI_M S_H}{N_M} - (i_R + k + d_h)I_H$$

$$\frac{dI_{mH}}{dt} = i_R I_H$$

$$\frac{dD_H}{dt} = d_h I_H$$

$$\frac{dN_H}{dt} = -d_h I_H$$

$$\frac{dS_M}{dt} = bN_M - d_m S_M - \frac{PS_M I_H}{N_H}$$

$$\frac{dI_M}{dt} = \frac{PS_M I_H}{N_H} - d_m I_M$$

*Electronic address: 201801410@daiict.ac.in
†Electronic address: 201801057@daiict.ac.in

$$\frac{dD_M}{dt} = d_m(I_M + S_M)$$

$$\frac{dN_M}{dt} = bN_M - d_m(I_M + S_M)$$

Here,

k : Rate of becoming susceptibles for human.

P : Average number of bites from mosquitoes an human have per unit time.

d_h : Human death rate due to malaria.

i_R : Rate of becoming immune to malaria for human.

b : Birth rate of mosquitoes.

d_m : death rate of mosquitoes.

Reproduction Number

We can use Next Generation method to obtain reproduction number for our model. We will follow standard approach to get generation matrix. We have 2 types of infected classes in our model i.e., infected humans and infected mosquitos. So, we can define $\frac{dI_H}{dt}$ and $\frac{dI_M}{dt}$ as follows,

$$\frac{dI_H}{dt} = \mathcal{F}_1(I_M, I_H) - \upsilon_1(I_M, I_H)$$

$$\frac{dI_M}{dt} = \mathcal{F}_2(I_M, I_H) - \upsilon_2(I_M, I_H)$$

Here,

$$\mathcal{F}_1(I_M, I_H) = \frac{PS_H I_M}{N_M}$$

$$\mathcal{F}_2(I_M, I_H) = \frac{PS_M I_H}{N_H}$$

$$\upsilon_1(I_M, I_H) = (i_R + k + d_h)I_H$$

$$\upsilon_2(I_M, I_H) = d_m I_M$$

From above equations we can obtain F and V matrix by considering equilibrium as $I_M = I_H = 0$,

$$F = \begin{bmatrix} \frac{PS_H(0)}{N_M(0)} & 0 \\ 0 & \frac{PS_M(0)}{N_H(0)} \end{bmatrix}$$

$$V = \begin{bmatrix} i_R + k + d_h & 0 \\ 0 & d_m \end{bmatrix}$$

Spectral radius of $FV^{-1}$is reproduction number (R), that is,

$$R = \sqrt{\frac{P^2 * S_M(0) * S_H(0)}{(i_R + k + d_h) * d_m * N_H(0) * N_M(0)}}$$

$$R = \sqrt{\frac{P^2}{d_m * (i_R + k + d_h)}}$$

Note, that there is an easier method to obtain generation matrix G (i.e., FV^{-1}) by putting expected number of secondary infections of type i caused by a single infected individual of type j at position (i, j) in matrix, assuming that the population of type i is entirely susceptible. In our case, one infected human can infect $\frac{PS_M(0)}{(i_R+k+d_h)*N_H(0)}$ mosquitoes and one infected mosquitoes can infect $\frac{PS_H(0)}{d_m N_M(0)}$ humans. So, we can directly define G as,

$$G = \begin{bmatrix} 0 & \frac{PS_H(0)}{d_m N_M(0)} \\ \frac{PS_M(0)}{(i_R+k+d_h)N_H(0)} & 0 \end{bmatrix}$$

Observations

Human population, number of infected humans, number of susceptible humans, number of immune humans and number of deaths vary as shown in figure (2).

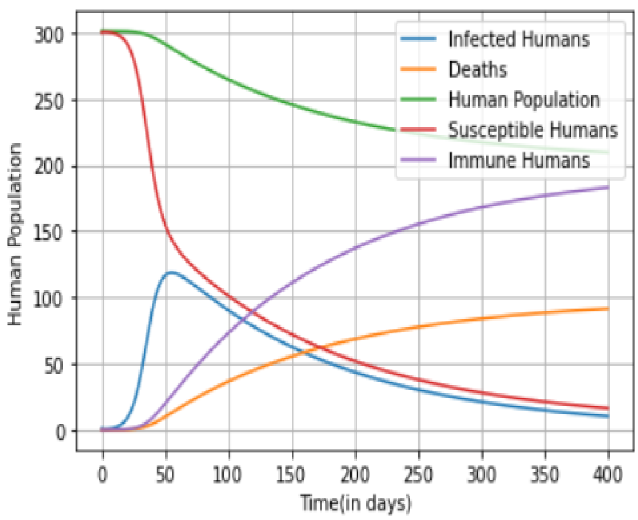

FIG. 2: Population in Various Compartment v/s Time. Here, $P = 0.3\ d^{-1}$, $k = 0.3\ d^{-1}$, $b = 0.01\ d^{-1}$, d_h= $0.005\ d^{-1}$, d_m= $0.01\ d^{-1}$and i_R= $0.01\ d^{-1}$.

Now, we will try to figure out effect of various parameters on maximum infected humans (among all time) and total deaths.

Effect of Change in P

We observed that if we increase P then maxima of curve of infected humans will increase and also there will be more number of deaths of humans.

Effect of change in d_m

We observed very slight dependence of d_m on maxima of curve of infected humans and total number of deaths. increasing slightly reduced both of those things.

Effect of other parameters

If we increase d_h and keep every other parameter to be the same, then maxima of infected curve will decrease but there will be more deaths overall. If we increase i_R and keep every other parameters to be the same then both maxima of infected curve and total deaths will decrease and same is for k.

Health Interventions

To prevent Malaria spread among the population we will discuss here two health interventions,

1. Fumigation
2. Vaccination

Fumigation

Fumigation will have effect only on mosquitoes and thus the number of mosquitoes from susceptible and infected compartment decreases and the resultant gets added to the dead mosquitoes. Considering the fumigation rate to be α the modified equations to account the fumigation are as follows:

$$\frac{dS_M}{dt} = bN_M - d_m S_M - \frac{PS_M I_H}{N_H} - \alpha S_M$$

$$\frac{dI_M}{dt} = \frac{PS_M I_H}{N_H} - d_m I_M - \alpha I_M$$

$$\frac{dN_M}{dt} = bN_M - d_m(S_M + I_M) - \alpha(S_M + I_M)$$

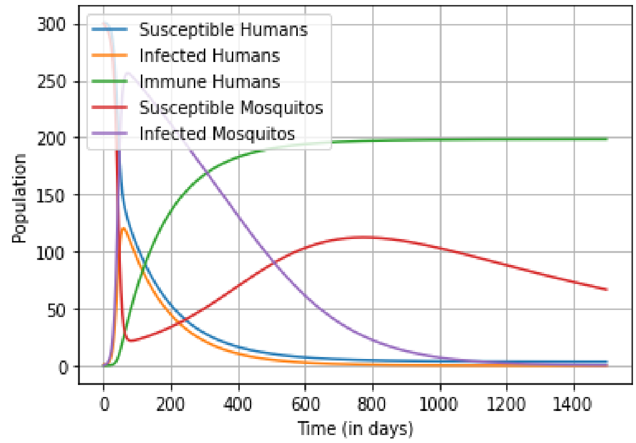

FIG. 3: Population vs Time (in days) where fumigation rate ($\alpha = 0.001$)

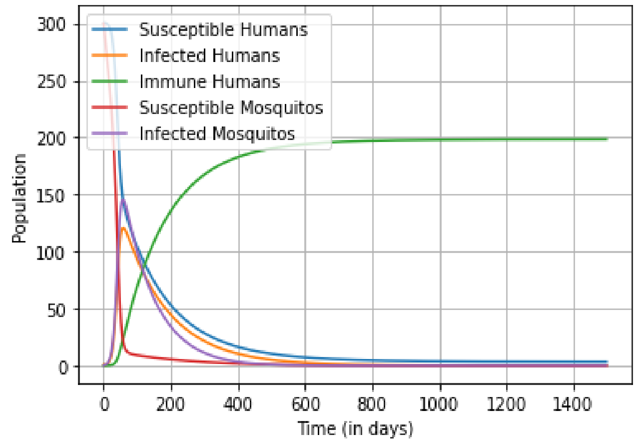

FIG. 4: Population vs Time (in days) where fumigation rate ($\alpha = 0.01$)

From the figure (3) and (4) we can see that, as the fumigation rate increases, the deaths of susceptible and infected mosquitoes increases. Thus fumigation increases the death rate of the vectors (mosquitoes).

The time taken to reach the maxima of infected human increases due to fumigation thus we get sufficient time to take the precautions to lower the value of infected humans. But the maxima of number of infected humans does not change significantly.

In the case of figure (3) assuming that the deaths caused due to fumigation and the natural deaths of the mosquitoes are lesser than the birth rate of the mosquitoes then the number of susceptible mosquitoes decreases to a minima and then tries to increase. As the new born mosquitoes will be susceptible.

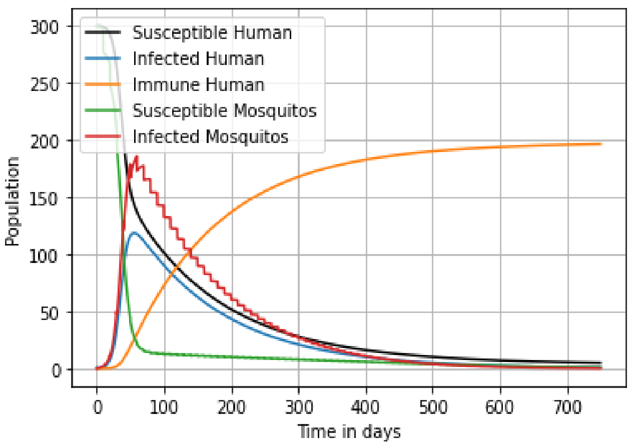

FIG. 5: Population vs Time (in days) where fumigation rate ($\alpha = 0.07$) and the fumigation is done in time intervals of 10 days

Thus, the effect of the fumigation depends on the fumigation rate(α), mosquitoes birth (b) and death rate(d_m).

A more realistic situation is when the fumigation takes place in some intervals of time. Figure (5) shows one such example where the fumigation. Thus in the time interval between the successive fumigation the disease tries to spread and thus the epidemic lasts longer.

Thus, it is better to do effective fumigation in a shorter intervals of time, it will have a greater effect to stop the epidemic.

Vaccination

Vaccination will cause some of the susceptible humans to become immune thus the number of people from susceptible compartment reduces and the resultant flows into the immune compartment. Considering the vaccination rate as β, the modified equations to account the vaccination are as follows:

$$\frac{dS_H}{dt} = kI_H - \frac{PS_H I_M}{N_M} - \beta S_H$$

$$\frac{dI_H}{dt} = \frac{PS_H I_M}{N_M} - (i_R + k + d_h)I_H$$

$$\frac{dI_{mH}}{dt} = i_R I_H + \beta S_H$$

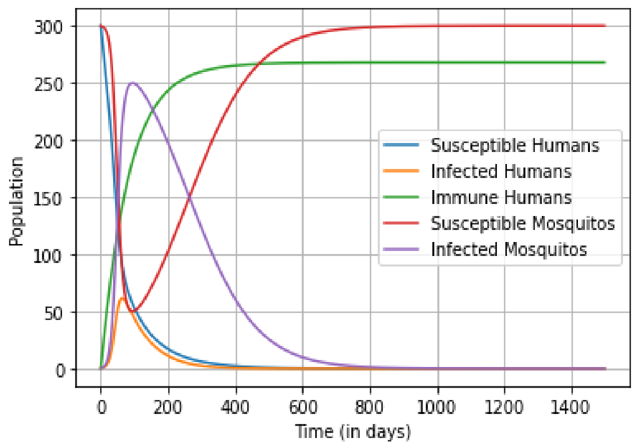

FIG. 6: Population vs Time (in days) where vaccination rate ($\beta = 0.01$)

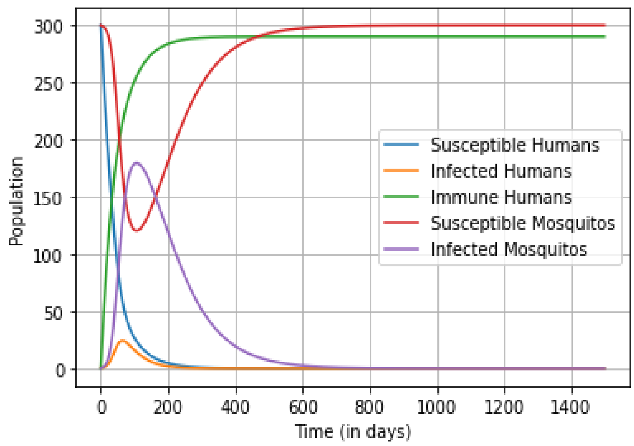

FIG. 7: Population vs Time (in days) where vaccination rate ($\beta = 0.02$)

From figures (6) and (7) we can see that, as the vaccination rate increases the humans move from susceptible compartment to the immune compartment faster. Thus vaccination decreases the number of infected humans and due to which the number of infected mosquitoes also decreases. Thus due to less number of susceptible human available the epidemic dies out quickly causing less number of deaths.

Figure (8) shows a situation where vaccination is carried out in regular intervals. In a such a situation it will take longer time for the susceptible humans to get immunized but it will cause lesser deaths compared to a situation where no health intervention is done.

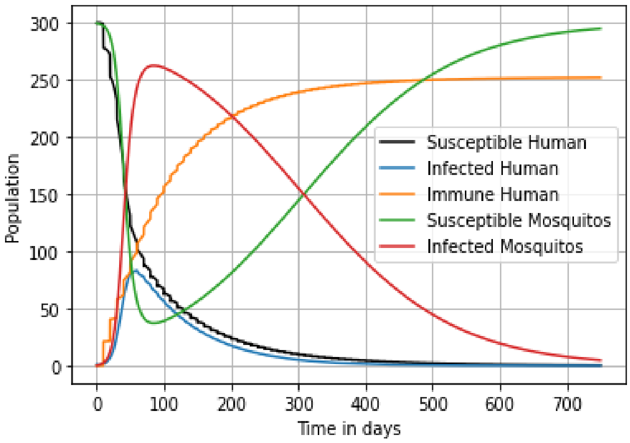

FIG. 8: Population vs Time (in days) where vaccination rate ($\beta = 0.07$) and vaccination is done in time intervals of 10 days

Conclusion

We saw effects of different public health interventions such as fumigation and vaccination on the spread of malaria. Fumigation can be effective if done before the infected population increases beyond a certain number after that it might not prove effective. Vaccination has a significant effect if done as soon as possible. Accumulation of water in the surrounding areas must be prevented as that reduces the birth rate of mosquitoes. Thus can help reduce the further increase in the number of infected humans. Such interventions if used together can reduce the disease spread.