Lab - 4

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CS-302, Modeling and Simulation

In this lab, we will analyse models of diffusion of innovation. We will analyze how different conditions changes the rate of adoption of a innovation.

Introduction

Diffusion of Innovations is a theory that explains how, why and at what rate the new innovations, ideas and technology spread. Technology adoption processes typically follow a normal distribution or a bell curve over a long time. The model describes how people adopt new product or technology and how the product's captures market share and increases it's sales.

Assumptions

The population remains constant throughout time period. Maximum number of the buyers remains constant. All the people will buy the product eventually. Influence by external and internal factors is assumed to be same for the entire population.

Model

Typically such problems are modelled using a differential equation as follows:

$$\frac{dN}{dt} = \alpha(t)(N_a - N(t)) \tag{1}$$

Where $\alpha(t)$ is the diffusion coefficient and N_a is the maximum number of potential users and N(t) represents the number of users that have adopted the product till time t.

$\alpha(t)$ can be modelled as a function of two constants p and q, where p is the coefficient of innovation that describes the effects due to external factors such as advertisements and marketing whereas q is the coefficient of imitation that represents the effect of internal factor such as rumours and word of mouth.

External Influence Model

In this model $\alpha(t) = p$. It captures the influence of the innovators and the people who adopt the product on their own without the external influence.

The analytical solution for an External Influence Model will be:

$$N = N_a - (N_a - N_0) \times exp(-tp)$$

Where N_0 is the initial number of users at t = 0.

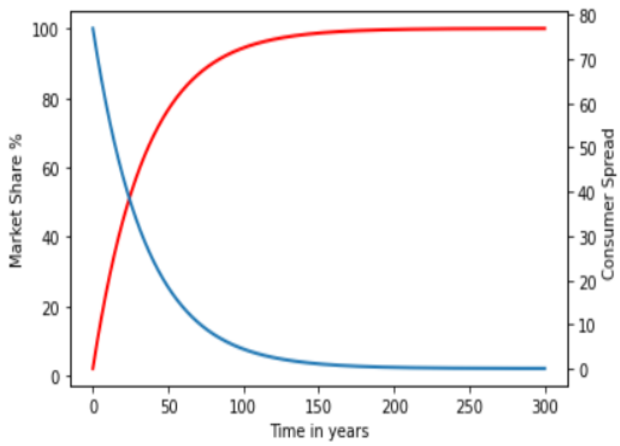

Figure 1: External Influence Model

The red line in the above graph represents the product's market share whereas the blue line represents product's sales. We can see from above equations and graphs that, as the time increases, the number of new adopters($\frac{dN}{dt}$) decreases.

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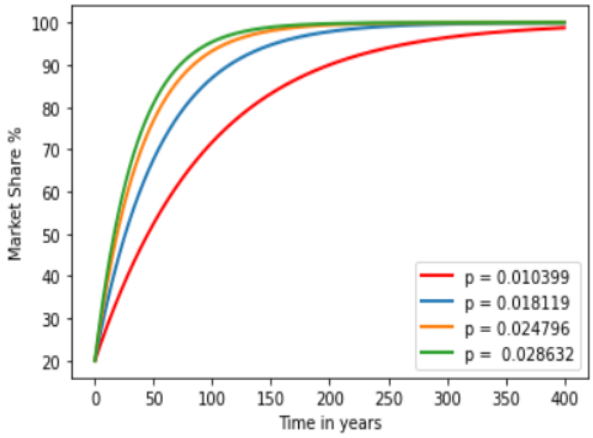

Figure 2: External Influence Model with varying parameter p

As the value of p increases, the effect of the external influence becomes strong and more number of people adopt the product in the early stage. Thus the market share saturates at a faster rate.

Internal Influence Model

In this model $\alpha(t) = q \times \frac{N(t)}{N_a}$. This model now captures the adoption effect of other users. The analytical solution for an Internal Influence Model will be:

$$N = \frac{N_0 N_a}{N_0 + (N_a - N_0) \times e^{-qt}}$$

Where N_0 is the initial number of users at t = 0.

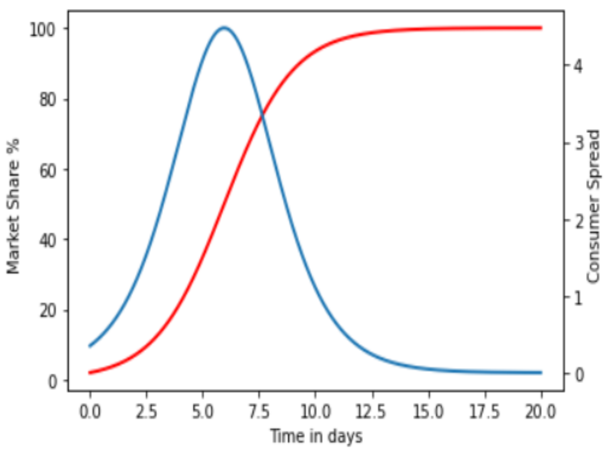

Figure 3: Internal Influence Model

We can see from the above graph that the blue line represents rate of market share increase ($\frac{dN}{dt}$) in the initial stage increases, attains a maxima and then decreases.

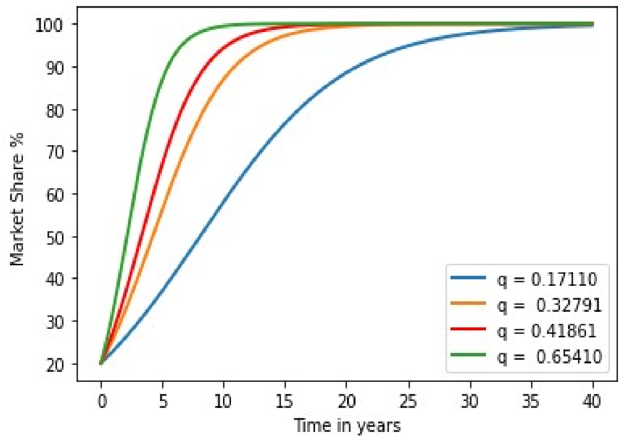

Figure 4: Internal Influence Model with varying parameter q

As the value of q increases, more people adopt the product or technology in the initial stage. Hence the market share saturates quickly.

As a limitation of this model, initially there must be some product users which can influence other users.

Bass Model

In this model $\alpha(t) = p + q \times \frac{N(t)}{N_a}$. This model captures the adoption of all users.

The analytical solution for the Bass Model will be:

$$N(t) = \frac{1 - e^{-(p+q)t}}{1 + \frac{q}{p} \times e^{-(p+q)t}} \times N_a$$

The peak rate of adoption occurs at the time t^*, which is defined as follows,

$$t^* = \frac{1}{p+q} \times ln(\frac{q}{p})$$

. Thus if q > p, the peak time is positive. If q < p then the peak time will be negative and thus the sales will be declining since the launch time (t = 0).

Bass Model removes the limitation on the initial number of users as in the Internal Influence Model.

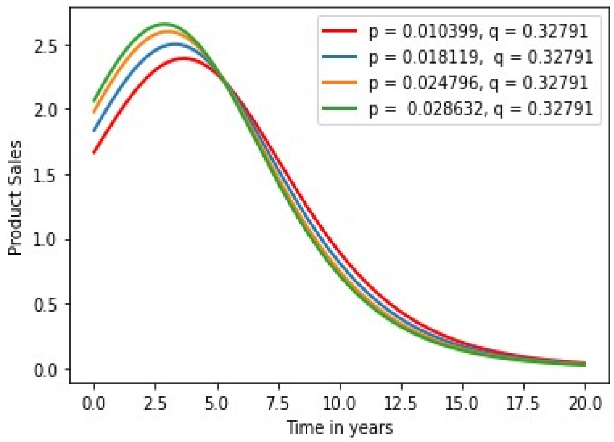

Figure 5: Mixed Influence(Bass) Model of Product Sales vs Time with fixed q and varying p

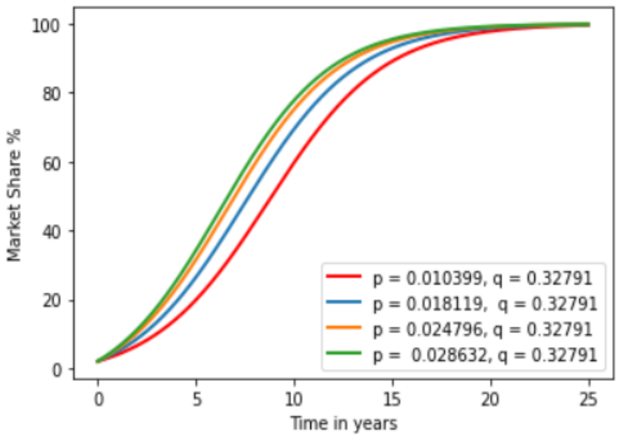

Figure 6: Mixed Influence(Bass) Model of Market Share vs Time with fixed q and varying p

We can see from figure(5) and figure(6) that as the p value (external factor) increases then the peak sales of the product occurs earlier. The p value can be increased using advertisements and marketing

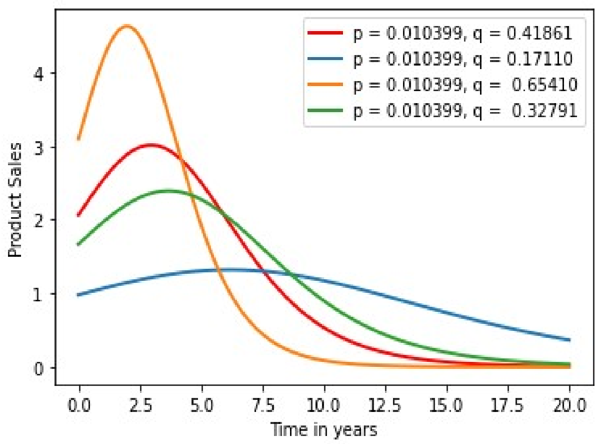

Figure 7: Mixed Influence(Bass) Model of Product Sales vs Time with fixed p and varying q

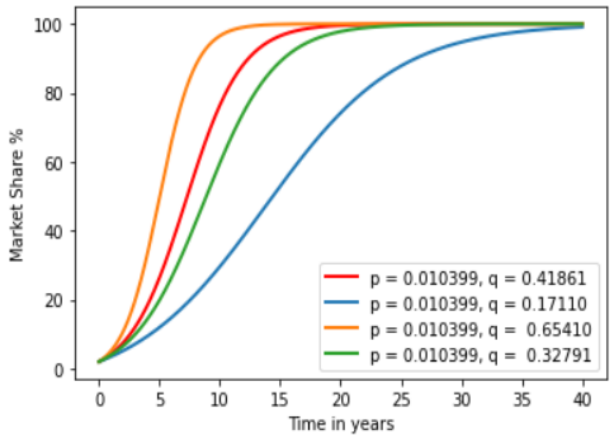

Figure 8: Mixed Influence(Bass) Model of Market Share vs Time with fixed p and varying q

We can see from figure(7) and (8) that as the q value (internal factor) increases the time taken to achieve peak sales of the product occurs earlier and the graph also decreases at a faster rate . The q value depends on the people's perception and the word of mouth.

Modified Bass Model:

Some of the limitations of the bass model are as follows:

- In bass model, it is assumed that adoption due to interaction remains constant over the entire time period which is generally not true.

- Also, maximum rate of adoption of a technology occurs when technology is already been adopted by N_m people, where

$$N_m = N_a(\frac{q-p}{2q})$$

Here, N_m can never be grater than $\frac{Na}{2}$, which is again not always true.

To overcome some of the limitations of bass model, we can modify equation of bass model as follows,

$$\frac{dN}{dt} = \left(p + q\left(\frac{N}{N_a}\right)^{1+\beta}\right)(N_a - N) \tag{2}$$

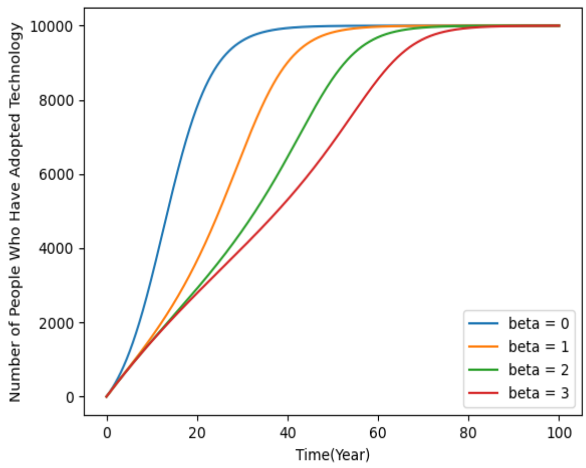

Figure 9: Number of people who have adopted a technology v/s Time. Here we should observe how plot changes with change in β.

From figure(9), we can clearly see that at any time, number of people who have adopted a technology is greater when β is smaller.

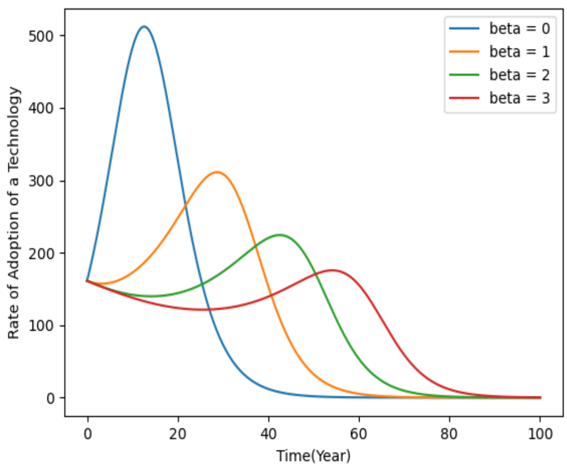

Figure 10: Rate of adoption of a technology v/s Time.

From figure(10), we can observe that as β increases, maximum value of rate of adoption of the technology decreases until it reaches the value of pN_a(rate at t = 0) after which it will remain the same. We can also observe that position of spike in rate of adoption is changing with change in β. This spike occurs roughly when N_k people have already adopted the technology, where

$$N_k = N_a \left(\frac{1 + \beta - p}{2 + \beta} \right) \tag{3}$$

Equation (3) is approximation which is close to actual answer for $p << 1$.

Thus, we can see that in this modified bass model, the spike in curve of rate of adoption can occur after 50% of population has already adopted the technology. Also, we can say that $1 + \beta$ is representative of average number of spreaders that are required to come in contact with one ignorant so that ignorant adopts the technology. As higher number of spreaders are required to come in contact with a ignorant, spike in sales occurs at a later point when N is sufficiently large and sufficient amount of ignorant are present. After, this point due to decrease in number of ignorant, sales will go down.

Conclusion:

We looked at how values of different parameters can affect the rate of adoption of a innovation. Firms can somewhat control value of p through advertisements and endorsements. The value of q depends upon the people's perspective. The firms can advertise and thus can create or change people's perception about their product. By observing the outputs of model for different values of p and q, they can select a strategy to control rate of adoption of an innovation which is best suited to them.