# Solar Radiation on a PV Generator

## Fundamentals of PV Engineering

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# Inclination Angle

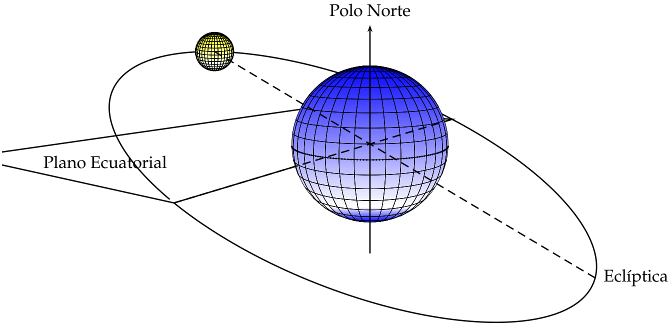
- PV generators have an **inclination angle higher than zero** to maximize the performance.
- The generator inclination angle depends on the latitude of the location and on the application*.

*Rule of thumb: latitude minus 10º for a Grid Connected PV System; latitude plus 10º for a Standalone PV System.

# From Horizontal to Inclined

$$
\begin{tikzcd}
\boxed{G_d(0)} \arrow[d] & & \\
\boxed{\begin{array}{c} D_d(0) \\ B_d(0) \end{array}} \arrow[d] & & \\
\boxed{\begin{array}{c} D(0) \\ B(0) \\ G(0) \end{array}} \arrow[r] & \boxed{\begin{array}{c} D(\beta,\alpha) \\ B(\beta,\alpha) \\ G(\beta,\alpha) \end{array}} \arrow[r] \arrow[d] & \boxed{\begin{array}{c} D_d(\beta,\alpha) \\ B_d(\beta,\alpha) \\ G_d(\beta,\alpha) \end{array}} \\
& \boxed{\begin{array}{c} D_{ef}(\beta,\alpha) \\ B_{ef}(\beta,\alpha) \\ G_{ef}(\beta,\alpha) \end{array}} \arrow[r] & \boxed{\begin{array}{c} D_{efd}(\beta,\alpha) \\ B_{efd}(\beta,\alpha) \\ G_{efd}(\beta,\alpha) \end{array}}
\end{tikzcd}
$$

# Apparent Sun movement



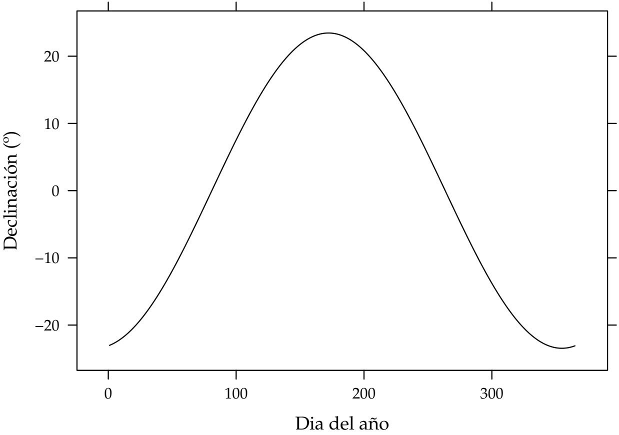

## Declination Angle, $\delta$

- Angle between the equatorial plane, and the line from the Sun to the center of the Earth.

$$\delta = 23.45^\circ \cdot \sin(\frac{2\pi \cdot (d_n + 284)}{365})$$

# Declination Angle and Seasons

- June solstice: $\delta_{max}$
- December solstice: $\delta_{min}$
- Equinoxes: $\delta = 0$



# Solar Hour Angle

- $w$, difference between the current instant and the noon or midday ($w = 0$).
  - (Hours) $-12, -11, -10, \ldots, -1, \mathbf{0}, 1, \ldots, 10, 11, 12$
  - 1h = 15º ($24h = 2\pi$ radians $= 360$).
- Sunrise ($w_s < 0$):

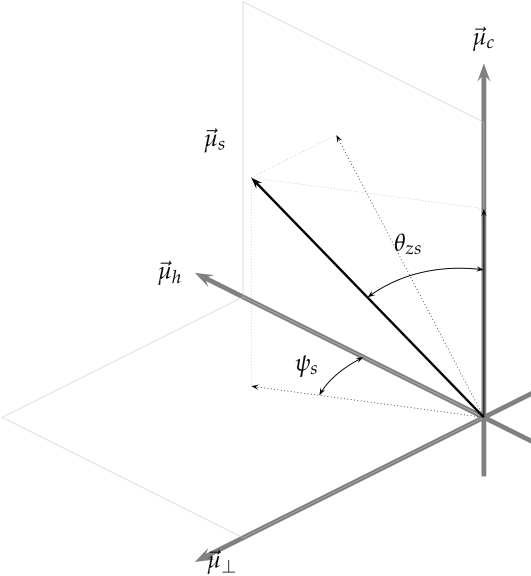
$$\cos(\omega_s) = -\tan(\delta)\tan(\phi)$$

- Day length, $|2 \cdot \omega_s|$, depends on the **latitude**, $\phi$, and on the **day of year**.

# Zenith Angle

$$\cos(\theta_z) = \cos(\delta)\cos(\omega)\cos(\phi) + \sin(\delta)\sin(\phi)$$

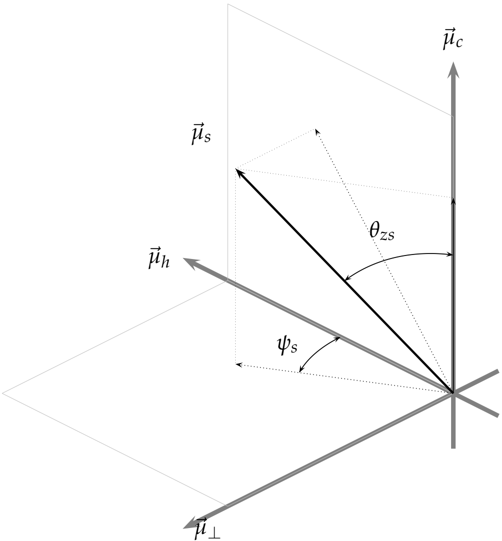
- $\theta_z$, angle between the Sun and the zenith (vertical in a particular location).
- Depends on $d_n$, $\omega$, and $\phi$.



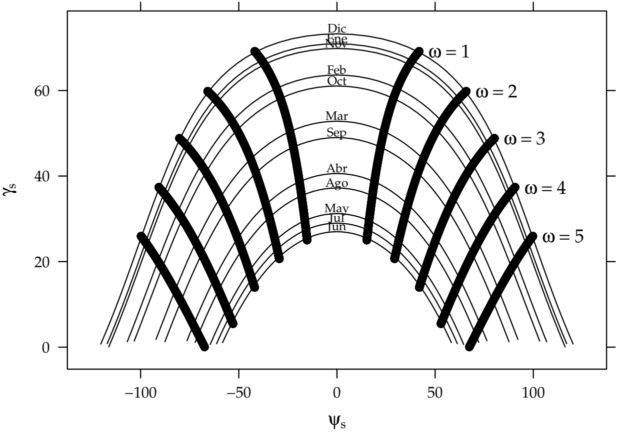
# Azimuth Angle

$$\cos(\psi_s) = \text{sign}(\phi) \cdot \frac{\cos(\delta)\cos(\omega)\sin(\phi) - \cos(\phi)\sin(\delta)}{\sin(\theta_z)}$$

- $\psi_s$, angle between the projection of Sun onto the horizontal plane and the noon.
- Depends on $d_n$, $\omega$, and $\phi$.

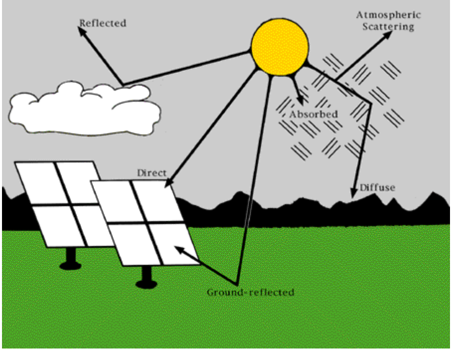


# Sun Trajectory (40°S)



# Extraterrestrial Irradiation

- Solar radiation incident on a horizontal plane at top of the atmosphere.
- Depends on the latitude and day of the year.



# Extraterrestrial Irradiation

- $B_{0d}(0) = -\frac{24}{\pi} B_0 \epsilon_0 \cdot (\omega_s \sin\phi \sin\delta + \cos\delta \cos\phi \sin\omega_s)$
  - $\omega_s$ in radians
  - $B_0 \simeq 1367\,\mathrm{W\,m^{-2}}$ (Solar Constant)
  - Eccentricity correction factor, $\epsilon_0 = 1 + 0,033 \cdot \cos(2\pi d_n / 365)$

# Definitions

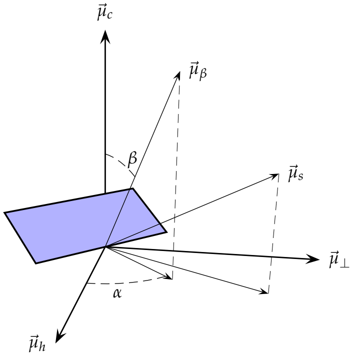
- $\theta_s$, Angle of Incidence (AOI), angle between the solar rays and the line perpendicular to the generator surface
- $\alpha$: Orientation of the generator ($0^{\circ}$ when oriented to the noon)
- $\beta$: Inclination of the generator.

# Fixed System

# Fixed System

- If $\alpha = 0$

$$\cos(\theta_s) = \cos(\delta)\cos(\omega)\cos(\beta - |\phi|) - \text{sign}(\phi) \cdot \sin(\delta)\sin(\beta - |\phi|)$$




- Optimum Inclination $\beta_{opt} \simeq |\phi| - 10$.

# Tracking System (1x axis, horizontal N-S)

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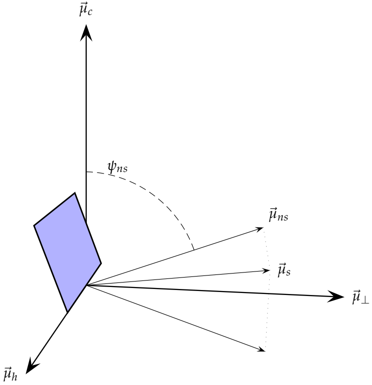
$$\cos(\theta_s) = \cos(\delta)\sqrt{\sin^2(\omega) + \left(\cos(\omega)\cos(\phi) + \tan(\delta)\sin(\phi)\right)^2}$$



# Tracking System (2x axis)

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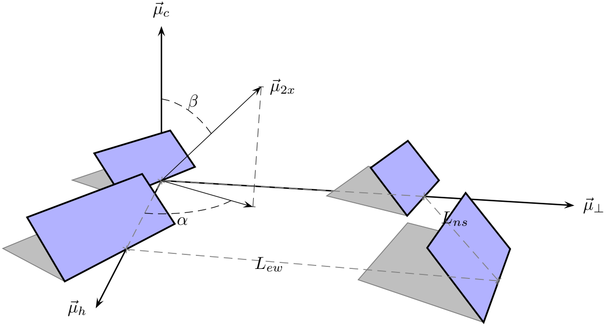


$$\beta = \theta_z$$
$$\alpha = \psi_s$$
$$\cos(\theta_s) = 1$$

# Transposition Procedure

$$
\begin{tikzcd}
\boxed{G_d(0)} \arrow[d] & & \\
\boxed{\begin{array}{c} D_d(0) \\ B_d(0) \end{array}} \arrow[d] & & \\
\boxed{\begin{array}{c} D(0) \\ B(0) \\ G(0) \end{array}} \arrow[r] & \boxed{\begin{array}{c} D(\beta,\alpha) \\ B(\beta,\alpha) \\ G(\beta,\alpha) \end{array}} \arrow[r] \arrow[d] & \boxed{\begin{array}{c} D_d(\beta,\alpha) \\ B_d(\beta,\alpha) \\ G_d(\beta,\alpha) \end{array}} \\
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\end{tikzcd}
$$

# Extract Diffuse and Beam Components

## Clearness Index

- $K_{Td} = G_d(0)/B_{0d}(0)$

## Diffuse Fraction

- $F_{Dd} = \frac{D_d(0)}{G_d(0)}$

## Model

- Monthly values:

$$F_{Dm} = 1 - 1.13 \cdot K_{Tm}$$

- Daily values:

$$F_{Dd} = \begin{cases} 0.99 & K_{Td} \leq 0.17 \\ 1.188 - 2.272 \cdot K_{Td} + 9.473 \cdot K_{Td}^2 - 21.856 \cdot K_{Td}^3 + 14.648 \cdot K_{Td}^4 & K_{Td} > 0.17 \end{cases}$$

# Example

Let's $G_{d,m}(0) = 3150\,\mathrm{Wh\,m^{-2}}$ in a month with $B_{0,dm}(0) = 4320\,\mathrm{Wh\,m^{-2}}$. Thus:

- $K_{Tm} = \frac{3150}{4320} = 0.73$
- $F_{Dm} = 1 - 1.13 \cdot 0.73 = 0.175$
- $D_{d,m}(0) = 0.175 \cdot 3150 = 551{,}6\,\mathrm{Wh\,m^{-2}}$
- $B_{d,m}(0) = 3150 - 551.6 = 2598{,}4\,\mathrm{Wh\,m^{-2}}$

# Intradaily profile

## Estimate irradiance from irradiation

$$D(0) = r_D \cdot D_d(0)$$
$$G(0) = r_G \cdot G_d(0)$$
$$B(0) = G(0) - D(0)$$

$$r_D = \frac{\pi}{T} \cdot \frac{\cos(\omega) - \cos(\omega_s)}{\omega_s \cdot \cos(\omega_s) - \sin(\omega_s)}$$
$$r_G = r_D \cdot (a + b \cdot \cos(\omega))$$
$$a = 0.409 - 0.5016 \cdot \sin(\omega_s + \frac{\pi}{3})$$
$$b = 0.6609 + 0.4767 \cdot \sin(\omega_s + \frac{\pi}{3})$$

# Transposition to the Plane of Generator

- Beam radiation

$$B(\alpha,\beta) = B(0)\cdot\frac{\max(0,\cos(\theta_s))}{\cos(\theta_{zs})}$$

- Diffuse Radiation (isotropic model)

$$D(\alpha,\beta) = D(0)\cdot\frac{1+\cos(\beta)}{2}$$

- Albedo

$$R(\beta,\alpha) = \rho\cdot G(0)\cdot\frac{1-\cos(\beta)}{2}$$

$$\rho = 0.2$$

- Global

$$G(\alpha,\beta) = B(\alpha,\beta) + D(\alpha,\beta) + R(\alpha,\beta)$$

# Back to daily values

## Daily values are the sum of hourly values in a day

$$G_d(\alpha, \beta) = \sum_d G(\alpha, \beta)$$
$$D_d(\alpha, \beta) = \sum_d D(\alpha, \beta)$$
$$B_d(\alpha, \beta) = \sum_d B(\alpha, \beta)$$