Solar Radiation on
a PV Generator

Oscar Perpiñán
Lamigueiro

Motivation

Solar Geometry

Angle of Incidence

Transposition
Procedure

Solar Radiation on a PV Generator
Fundamentals of PV Engineering

Oscar Perpiñán Lamigueiro

http://oscarperpinan.github.io

Inclination Angle

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▶ PV generators have an **inclination angle higher than zero** to maximize the performance.

▶ The generator **inclination angle** depends on the **latitude** of the location and on the **application**[*].

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[*]Rule of thumb: latitude minus 10º for a Grid Connected PV System; latitude plus 10º for a Standalone PV System.
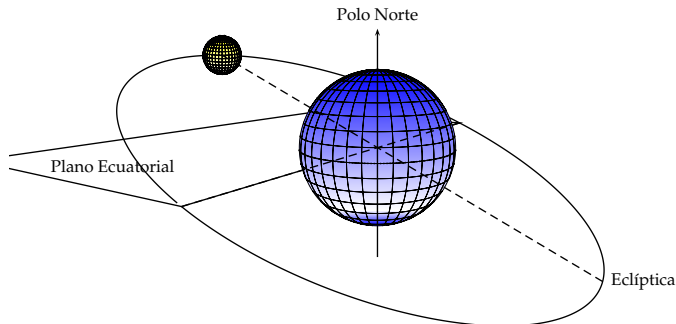
From Horizontal to Inclined

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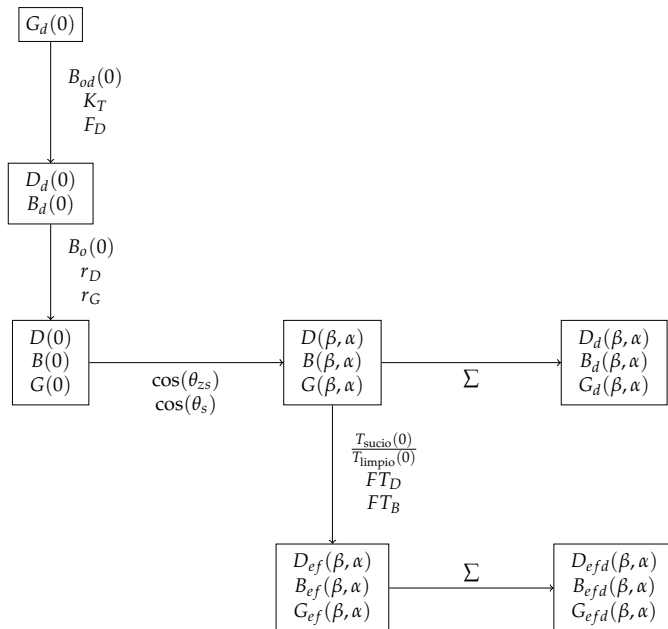
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Apparent Sun movement

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Declination Angle, δ

▶ Angle between the equatorial plane, and the line from the Sun to the center of the Earth.

$$\delta = 23.45° \cdot \sin(\frac{2\pi \cdot (d_n + 284)}{365})$$

Declination Angle and Seasons

▶ June solstice: δ_{max}
▶ December solstice: δ_{min}
▶ Equinoxes: $\delta = 0$

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Solar Hour Angle

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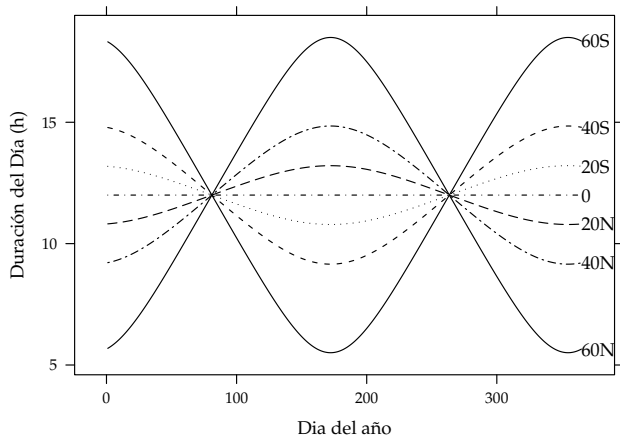
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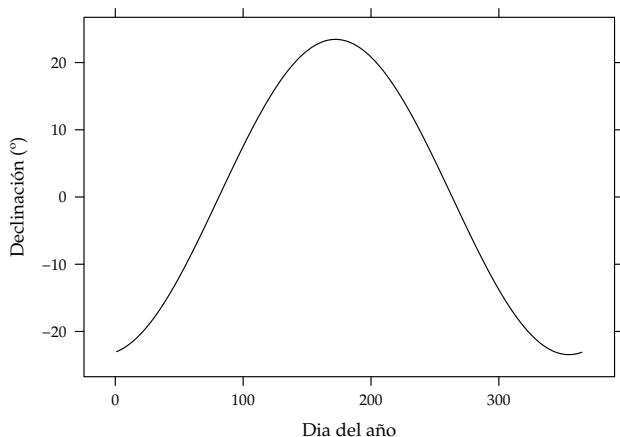
- ω: difference between the current instant and the noon or midday ($\omega = 0$).
- $\omega < 0$ before midday
- 1h = 15° (24h = 2π radians = 360°).
- $\boxed{\omega = -12, -11, -10, \ldots, -1, \mathbf{0}, 1, \ldots, 10, 11, 12}$ (hours)

Sunrise (ω_s)

$$\cos(\omega_s) = -\tan(\delta)\tan(\phi)$$

- $\omega_s < 0$
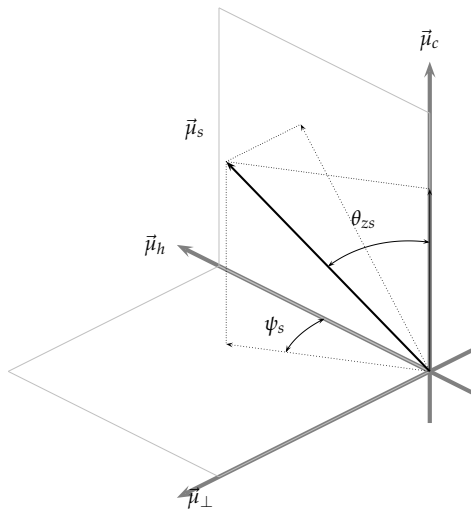- Day length, $|2 \cdot \omega_s|$, depends on the **latitude**, ϕ, and on the **day of year**.

Day length

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Zenith Angle

$$\cos(\theta_z) = \cos(\delta)\cos(\omega)\cos(\phi) + \sin(\delta)\sin(\phi)$$
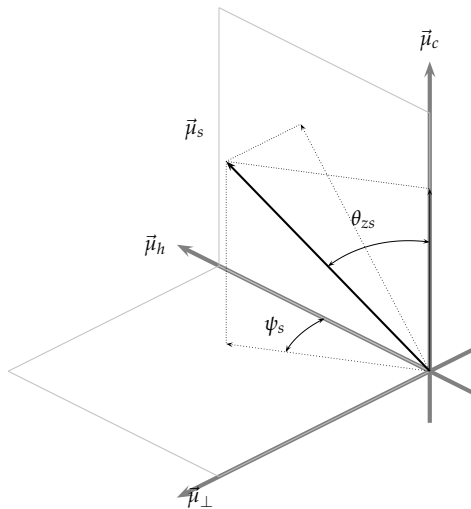
- θ_z, angle between the Sun and the zenith (vertical in a particular location).
- Depends on d_n, ω, and ϕ.

Azimuth Angle

$$\cos(\psi_s) = \text{sign}(\phi) \cdot \frac{\cos(\delta)\cos(\omega)\sin(\phi) - \cos(\phi)\sin(\delta)}{\sin(\theta_z)}$$

▶ ψ_s, angle between the projection of Sun onto the horizontal plane and the noon.

▶ Depends on d_n, ω, and ϕ.

Sun Trajectory (40°S)
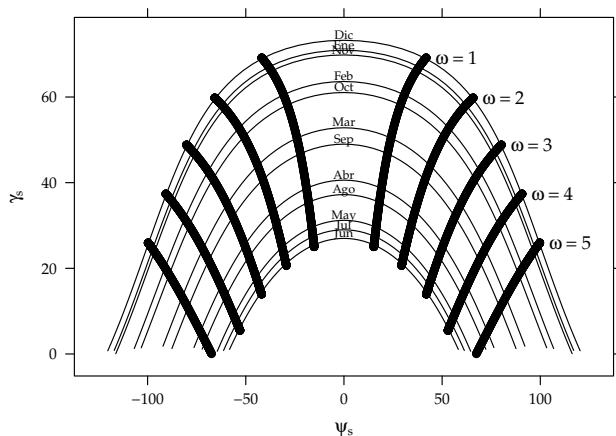
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Extraterrestrial Irradiation

- Solar radiation incident on a horizontal plane **at the top of the atmosphere** (TOA).
- Depends on the **latitude** and **day of the year**.

Extraterrestrial Irradiation

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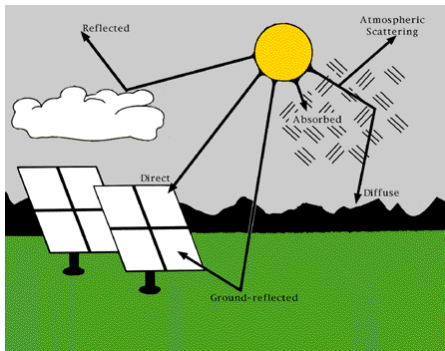
$$B_{0d}(0) = -\frac{24}{\pi} B_0 \epsilon_0 \cdot (\omega_s \sin \phi \sin \delta + \cos \delta \cos \phi \sin \omega_s)$$

- ω_s in radians (remember, $\omega_s < 0$)
- $B_0 \simeq 1367\,\mathrm{W\,m^{-2}}$ (Solar Constant)
- Eccentricity correction factor,
  $\epsilon_0 = 1 + 0,033 \cdot \cos(2\pi d_n/365)$

Definitions

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- θ_s, Angle of Incidence (AOI), angle between the solar rays and the line perpendicular to the generator surface
- α: Orientation of the generator (0° when oriented to the noon)
- β: Inclination of the generator.

Fixed System

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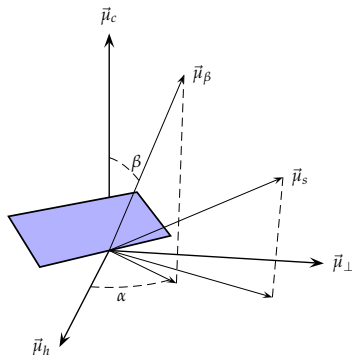Motivation

Solar Geometry

Angle of Incidence

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Fixed System

- If $\alpha = 0$

$$\cos(\theta_s) = \cos(\delta)\cos(\omega)\cos(\beta - |\phi|) - \text{sign}(\phi) \cdot \sin(\delta)\sin(\beta - |\phi|)$$

- Optimum Inclination $\beta_{opt} \simeq |\phi| - 10$.

Tracking System (1x axis, horizontal N-S)

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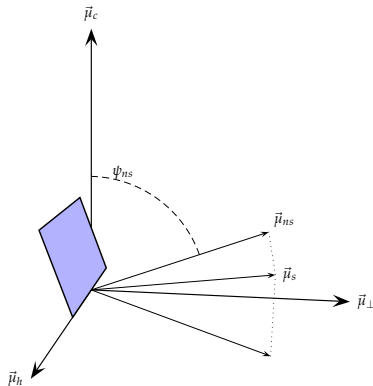Oscar Perpiñán
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Tracking System (1x axis, horizontal N-S)

$$\cos(\theta_s) = \cos(\delta)\sqrt{\sin^2(\omega) + (\cos(\omega)\cos(\phi) + \tan(\delta)\sin(\phi))^2}$$

Tracking System (2x axis)

Solar Radiation on
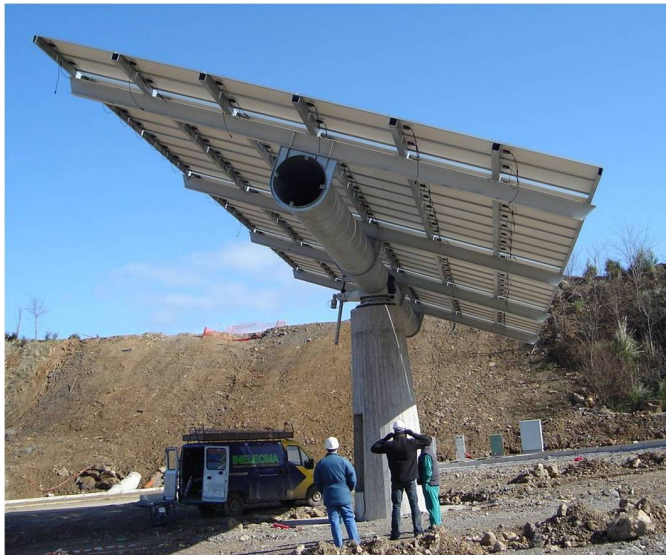a PV Generator

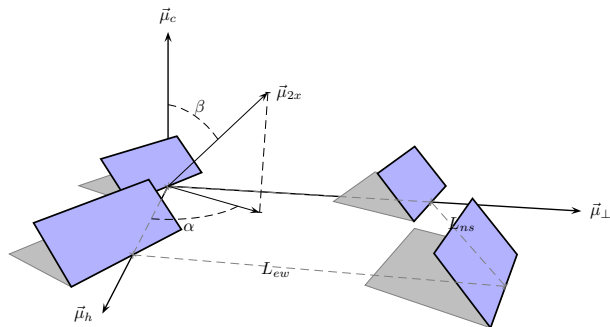Oscar Perpiñán
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Motivation

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Transposition
Procedure

Tracking System (2x axis)

$$\beta = \theta_z$$
$$\alpha = \psi_s$$
$$\cos(\theta_s) = 1$$

Diffuse and Beam Components

$G_d(0)$

$B_{od}(0)$
K_T
F_D

$D_d(0)$
$B_d(0)$

$B_o(0)$
r_D
r_G

$D(0)$
$B(0)$
$G(0)$

$\cos(\theta_{zs})$
$\cos(\theta_s)$

$D(\beta,\alpha)$
$B(\beta,\alpha)$
$G(\beta,\alpha)$

Σ

$D_d(\beta,\alpha)$
$B_d(\beta,\alpha)$
$G_d(\beta,\alpha)$

$\frac{T_{\text{sucio}}(0)}{T_{\text{limpio}}(0)}$
FT_D
FT_B

$D_{ef}(\beta,\alpha)$
$B_{ef}(\beta,\alpha)$
$G_{ef}(\beta,\alpha)$

Σ

$D_{efd}(\beta,\alpha)$
$B_{efd}(\beta,\alpha)$
$G_{efd}(\beta,\alpha)$
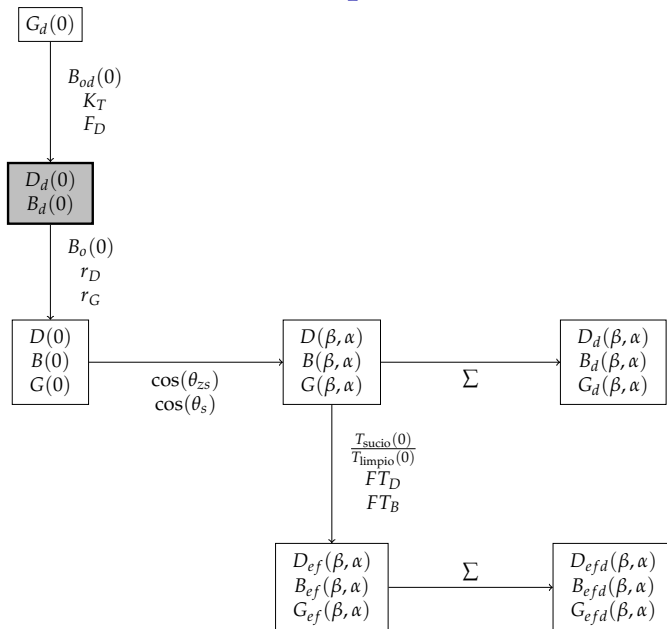
Extract Diffuse and Beam Components

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Clearness Index

► $K_{Td} = G_d(0) / B_{0d}(0)$

Diffuse Fraction

► $F_{Dd} = \frac{D_d(0)}{G_d(0)}$

Model

► Monthly values:

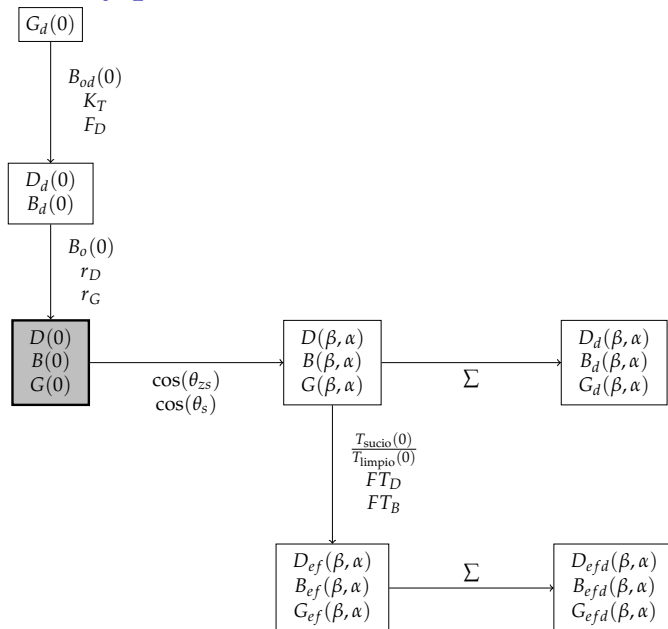$$F_{Dm} = 1 - 1.13 \cdot K_{Tm}$$

► Daily values:

$$F_{Dd} = \begin{cases} 0.99 & K_{Td} \leq 0.17 \\ 1.188 - 2.272 \cdot K_{Td} + 9.473 \cdot K_{Td}^2 - 21.856 \cdot K_{Td}^3 + 14.648 \cdot K_{Td}^4 & K_{Td} > 0.17 \end{cases}$$

Example

Let's $G_{d,m}(0) = 3150\,\text{Wh m}^{-2}$ in a month with $B_{0,dm}(0) = 4320\,\text{Wh m}^{-2}$. Thus:

- $K_{Tm} = \frac{3150}{4320} = 0.73$
- $F_{Dm} = 1 - 1.13 \cdot 0.73 = 0.175$
- $D_{d,m}(0) = 0.175 \cdot 3150 = 551{,}6\,\text{Wh m}^{-2}$
- $B_{d,m}(0) = 3150 - 551.6 = 2598{,}4\,\text{Wh m}^{-2}$

Intradaily profile

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Intradaily profile

Estimate irradiance from irradiation

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$$D(0) = r_D \cdot D_d(0)$$
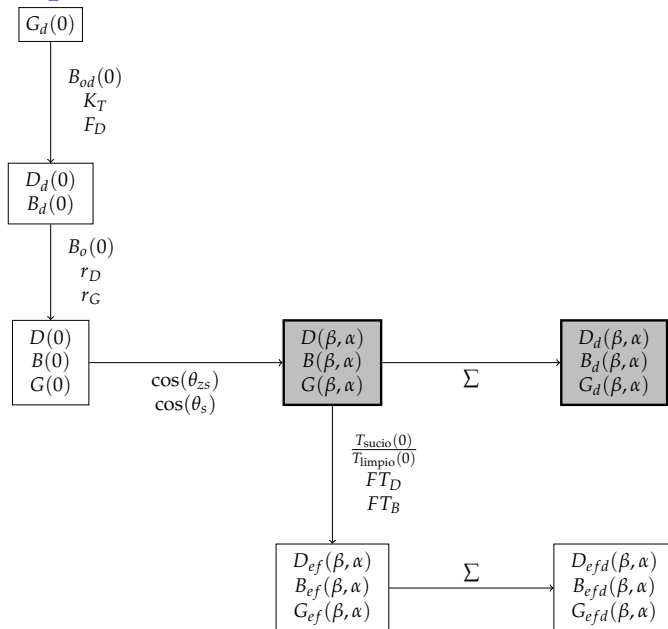$$G(0) = r_G \cdot G_d(0)$$
$$B(0) = G(0) - D(0)$$

$$r_D = \frac{\pi}{24} \cdot \frac{\cos(\omega) - \cos(\omega_s)}{\omega_s \cdot \cos(\omega_s) - \sin(\omega_s)}$$
$$r_G = r_D \cdot (a + b \cdot \cos(\omega))$$
$$a = 0.409 - 0.5016 \cdot \sin(\omega_s + \frac{\pi}{3})$$
$$b = 0.6609 + 0.4767 \cdot \sin(\omega_s + \frac{\pi}{3})$$

Transposition to the Plane of Generator

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Transposition to the Plane of Generator

▶ Beam radiation

$$B(\alpha, \beta) = B(0) \cdot \frac{\max(0, \cos(\theta_s))}{\cos(\theta_{zs})}$$

▶ Diffuse Radiation (isotropic model)

$$D(\alpha, \beta) = D(0) \cdot \frac{1 + \cos(\beta)}{2}$$

▶ Albedo

$$R(\beta, \alpha) = \rho \cdot G(0) \cdot \frac{1 - \cos(\beta)}{2}$$

$$\rho = 0.2$$

▶ Global

$$G(\alpha, \beta) = B(\alpha, \beta) + D(\alpha, \beta) + R(\alpha, \beta)$$

Back to daily values

Daily values are the sum of hourly values in a day

$$G_d(\alpha, \beta) = \sum_d G(\alpha, \beta)$$

$$D_d(\alpha, \beta) = \sum_d D(\alpha, \beta)$$

$$B_d(\alpha, \beta) = \sum_d B(\alpha, \beta)$$