Rigidly Aligning Non Overlapping Meshes using Minimal Surfaces

Hari Sridhar
Supervised by Etienne Vouga

Abstract— **In this paper, we study the problem of solving for an optimal alignment betwen two partial, non-overlapping range scans of a three-dimensional surface. Although existing alignment schemes, such as RigidICP or 4PCS, can solve for a pairwise rigid registration that yields an almost perfect alignment of two meshes, the need for corresponding points in an overlapping region limits their use in cases of minimal or zero overlap. Our approach overcomes this limitation by using an interpolating surface, instead of correspondences, to determining an optimal transformation matrix. We evaluate alignments by solving for an initial interpolating surface between the partial scans, via a greedy closest points zippering scheme, evolve this surface via mean curvature flow, and evaluate an energy function based on the area of said surface. Finding the optimal alignment reduces down to performing stochastic gradient descent on the transformation matrix, using the minimally interpolating surface as an energy metric.**

Keywords : computational geometry, rigid registration, mesh alignment, mean curvature flow, minimal surface

I. Introduction

The goal of rigid registration is to discover an optimal rigid-body transformation, that minimizes the distance between two input range scans. Rigid registration continues to remain a fundamental problem of computational geometry and computer vision, having been well-studied and documented across various papers and surveys. Use cases of registration include protein modeling [**4407841**], biomedical image analysis [**Zou:2015:TSR:2809654.2766976**], and field archeology [**Huang:2006:RFO:1179352.1141925**]. The Rigid Iterative Closest Points (RigidICP) algorithm, based solely on geometric information, remains the state-of-the-art for discovering rigid alignments between two three-dimensional models. RigidICP. The algorithm commences with two meshes and an initial guess for the transformation matrix, and solves for alignment as a minimization problem. This transformation matrix is iteratively refined by solving for pairs of corresponding points on the meshes, and minimizes an energy metric based on pairwise distances of these points [**Chen:1992:OMR:138628.138633**].

Despite its usefulness, the need to establish a set of corresponding points between the two input meshes, and the need for a good initial alignment, limits the applicability of RigidICP. Good corresponding points exist only when the two meshes share a large overlapping region. Additionally, although one could solve for an optimal initial alignment via a plethora of methods; tracking scanner positions, "spin-images", or computing principal axes, such steps beget additional preprocessing and computational resources [**Rusinkiewicz:2001:EVO**]. Consequently, RigidICP fails in situations where either the initial alignment proves dismal, or when the two scans possess very little to zero overlap [**Chen:1992:OMR:138628.138633**].

Our work circumnavigates these limitations by determining and refining alignments based on a minimal interpolating surface, instead of corresponding points. A minimal surface, by definition, is a surface that locally minimizes its area, equivalent to a surface of zero mean curvature. It can be likened to "gluing" the partial scans, using the least amount of glue. It is constructed from the scan boundaries, and is evolved over time using mean curvature flow. Using stochastic gradient descent, defined on an energy metric over this surface, the alignment is iteratively improved over time, such that we eventually reach an alignment that corresponds to a minimal surface that can no longer be minimized. This paper attempts to show that not only can an optimal alignment be discovered in cases of non-overlapping partial scans, but also that it performs better than the state-of-the-art .

II. Related Work

As established earlier, ICP and its variants, such as stable sampling, robust norm, and point-plane distance (refer to [**Rusinkiewicz:2001:EVO**] for more in-depth knowledge) will fail in cases of little to zero overlap, due to a lack of corresponding points. More ever, such methods may also yield penetrating rigid alignments. Nonetheless, we note that the state of the art, in the field of optimizing alignments, has been thoroughly documented with a multitude of approaches. Here, we summarize a set of different methodologies, and discuss their limitations to our problem domain.

A large class of alignment schemes, such as those based on geometric matching information [**Huang:2006:RFO:1179352.1141925**] [Huang], or congruent sets [**Aiger:2008:CSR:1399504.1360684**], require a set of corresponding points from their inputs. While [Huang et. Al]'s robust pairwise matching algorithm scales to multiple fragments and uses surface features to optimizing the fitting of fractured pieces, it is limited by the need to discover corresponding points from its inputs [**Huang:2006:RFO:1179352.1141925**]. Additionally, the 4-Points Congruent Sets algorithm, which extracts coplanar 4-points sets from the input surfaces to establish congruences among the surfaces, is limited by the need to for these 4-points sets to exist in regions of sufficient overlap. It too, proves unsuitable for cases of minimal or zero overlap [**Aiger:2008:CSR:1399504.1360684**].

Similar to the spirit of this paper, in [**7294872**], *Brandao et Al.* present a novel algorithm for joint alignment and stitching of two non-overlapping meshes, abbreviated JASNOM. This is achieved by optimizing an assignment matrix that corresponds to edges connecting the two mesh boundary, and then uses standard non-linear optimization algorithms to discover a rigid transformation that minimizes an associated cost function. Like our approach, they utilize an interpolating, stitched surface and non-linear optimization algorithms, to find an optimal alignment that minimizes a specific cost function. Nonetheless, it should be noted that applications of their approach are limited to only scans that possess complementary boundaries over the input mesh. More ever, their alignment remains highly sensitive to large gaps between boundaries. Another related paper, by Willis and Cooper, proposes a rigid alignment scheme for non-overlapping meshes, but restricted to cases of axially symmetric 3D datasets. Their approach too, mandates *a priori* knowledge of shared boundaries among broken pot pieces [**1333714**].