# Ontology-based Data Access: Theory and Practice


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http://ontop.inf.unibz.it/ijcai-2018-tutorial

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no combined approach (no additional constants, no assumptions on data)

# Tree-Witness Rewriting: Recap

$$\begin{array}{lll} A_1(x) \to \exists y\, P(x,y) & P(x,y) \to R(x,y) & P(x,y) \to Q(y,x) \\ A_2(x) \to \exists y\, S_1(x,y) & S_1(x,y) \to S_2(x,y) & \\ A_3(x) \to \exists y\, Q(x,y) & \exists y\, Q(y,x) \to \exists y\, S_1(x,y) & \end{array}$$

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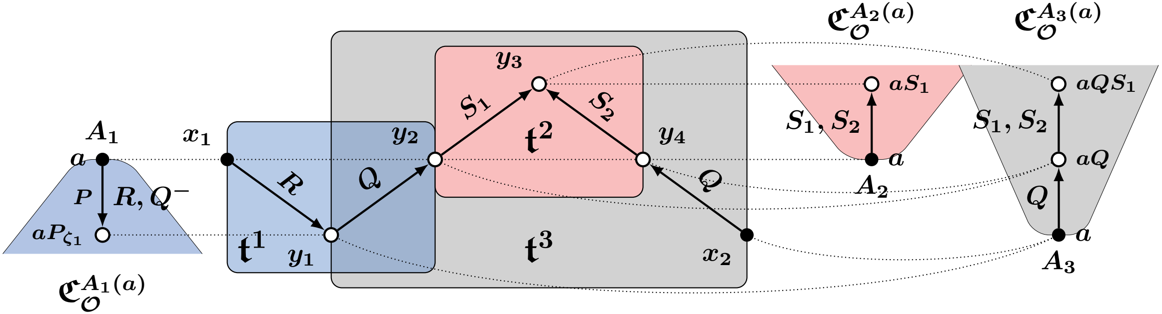

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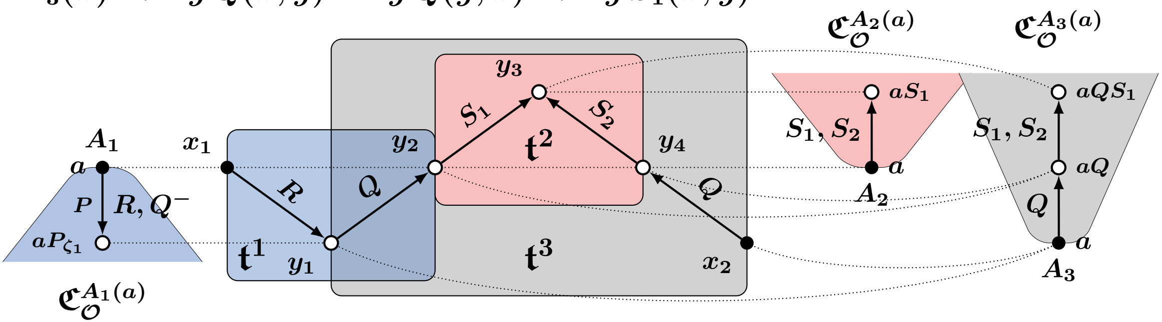


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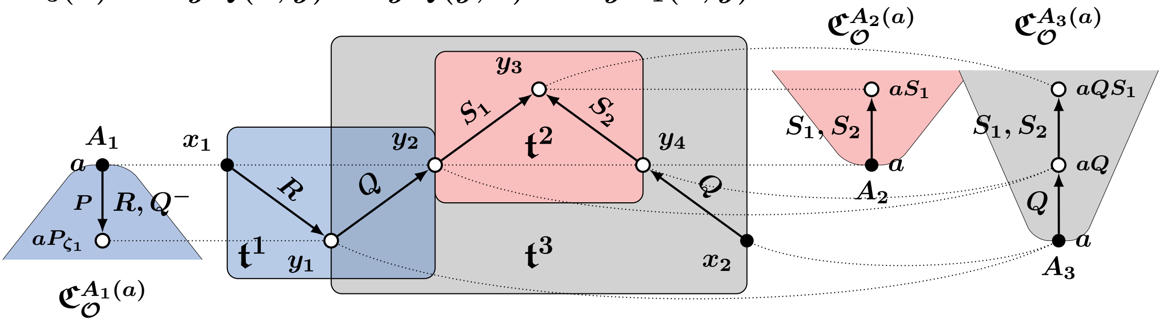


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$\emptyset$, $\{\mathfrak{t}_1\}$, $\{\mathfrak{t}_2\}$, $\{\mathfrak{t}_3\}$ and $\{\mathfrak{t}_1, \mathfrak{t}_2\}$
(exponentially many in general)

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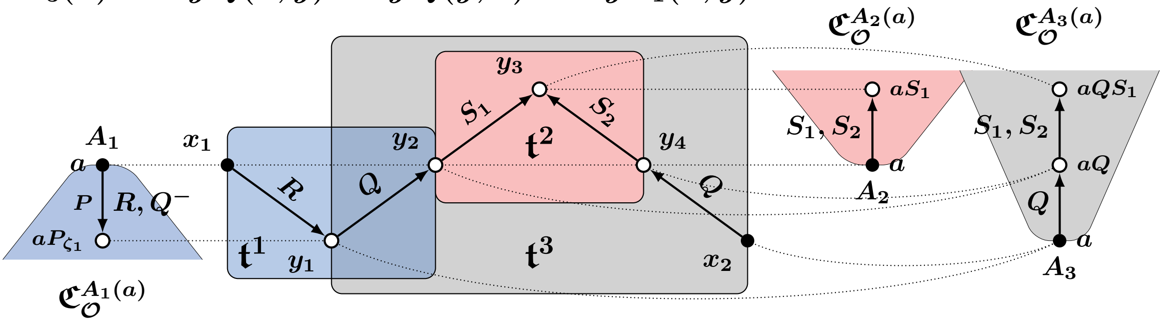


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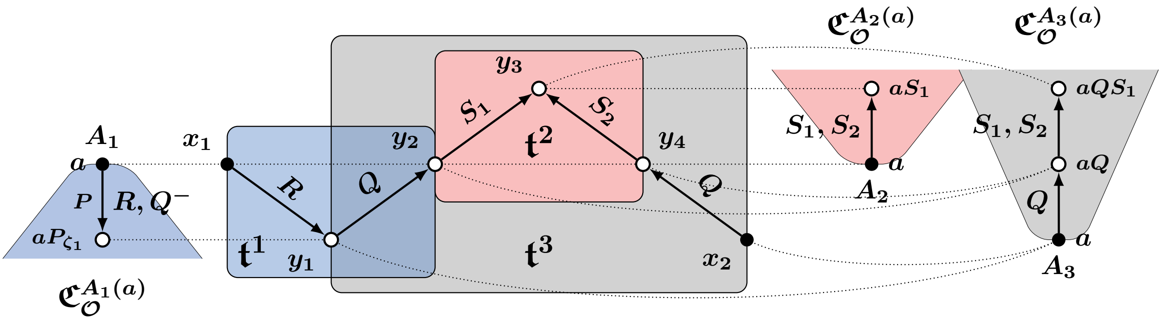
a **homomorphism** $q \to \mathfrak{C}_{\mathcal{O}}(\mathcal{D})$ = a map from variables of $q$ to **ind**$(\mathcal{A})$
\+
an **compatible subset** of $\Theta_Q$ such that
1. each tree witness is 'generated' by the data
2. each atom outside tree witnesses is 'present' in the data

# Tree Witness Rewritings as Hypergraphs



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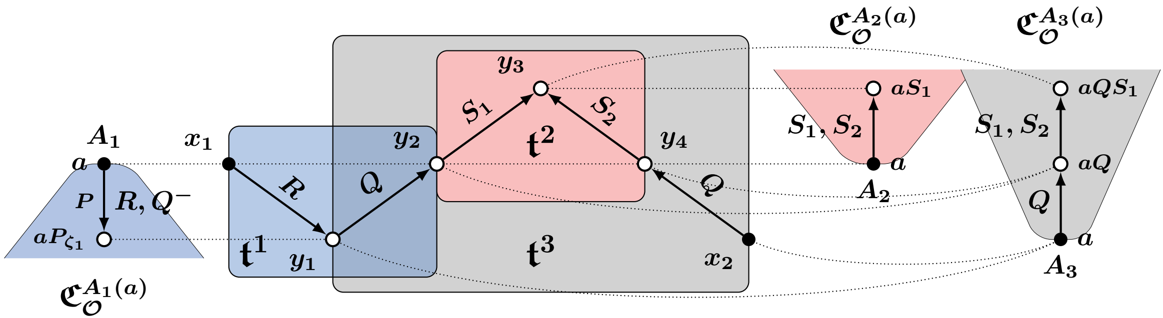
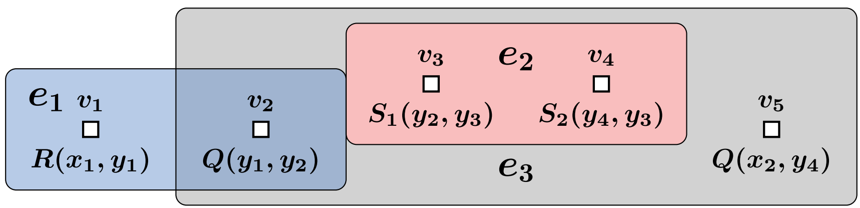



**OMQ hypergraph**

vertices = query atoms

hyperedges = sets of query atoms that can be mapped to trees



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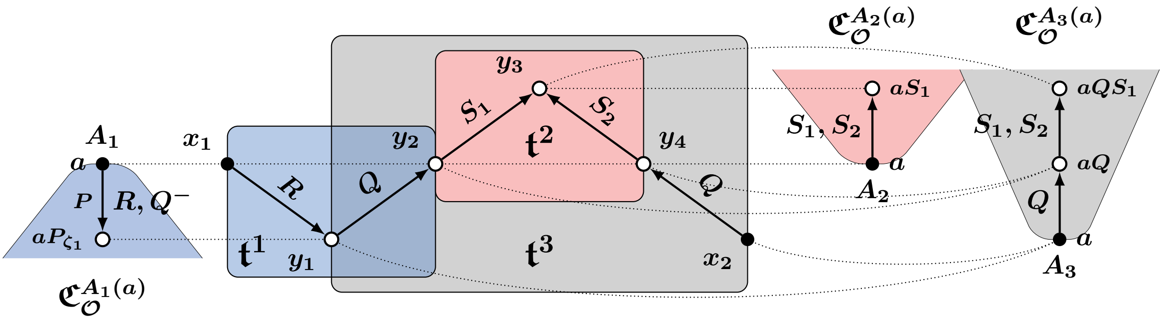


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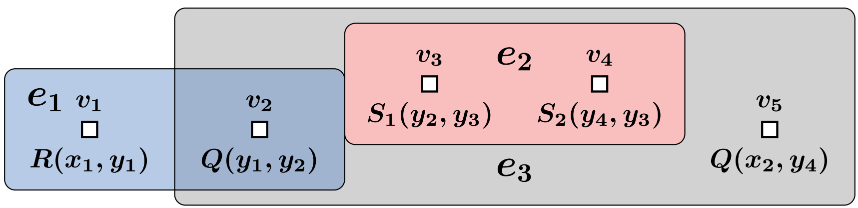


$$f_H = \bigvee_{E' \text{ independent}} \Big( \bigwedge_{v \in V \setminus V_{E'}} p_v \wedge \bigwedge_{e \in E'} p_e \Big)$$

**hypergraph function $f_H$**

# From OMQs to Hypergraph Programs (HGPs)

a **HGP** $P$ is a hypergraph $H$ whose vertices are labelled by
$0$, $1$, or a literal over $p_1, \dots, p_n$

$P$ returns $1$ on an assignment $\alpha\colon \{p_1, \dots, p_n\} \to \{0, 1\}$
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**class of OMQs**
all OMQs

size of rewritings

**(monotone) complexity class**
mNP/poly

**class of HGPs**
HGPs of degree 3

**class of circuits / NBPs**
nondeterministic Boolean circuits

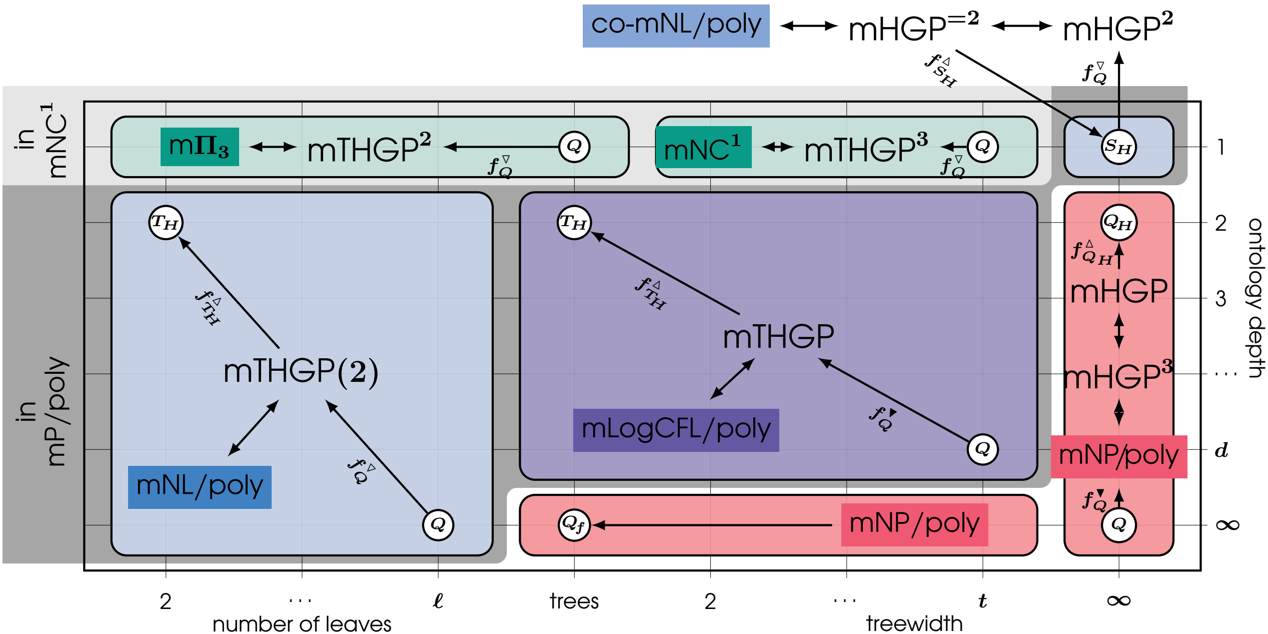
m$\Pi_3$ $\subsetneqq$ m$AC^0$ $\subsetneqq$ **m$NC^1$** $\subsetneqq$ mNL/poly $\subseteq$ mLogCFL/poly $\subsetneqq$ **mP/poly** $\subsetneqq$ **mNP/poly**

**monotone Boolean formulas (PE)**

**nondeterministic Boolean circuits (FO)**

**monotone circuits (NDL)**

# Roadmap for Succinctness Proofs

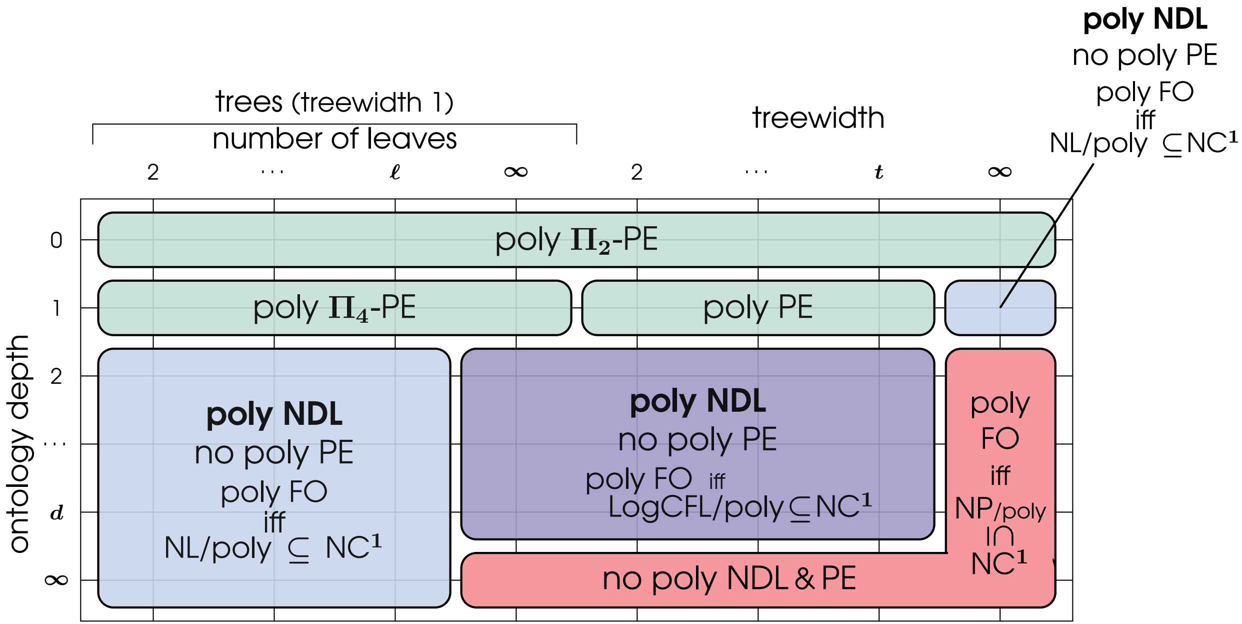



ontology depth

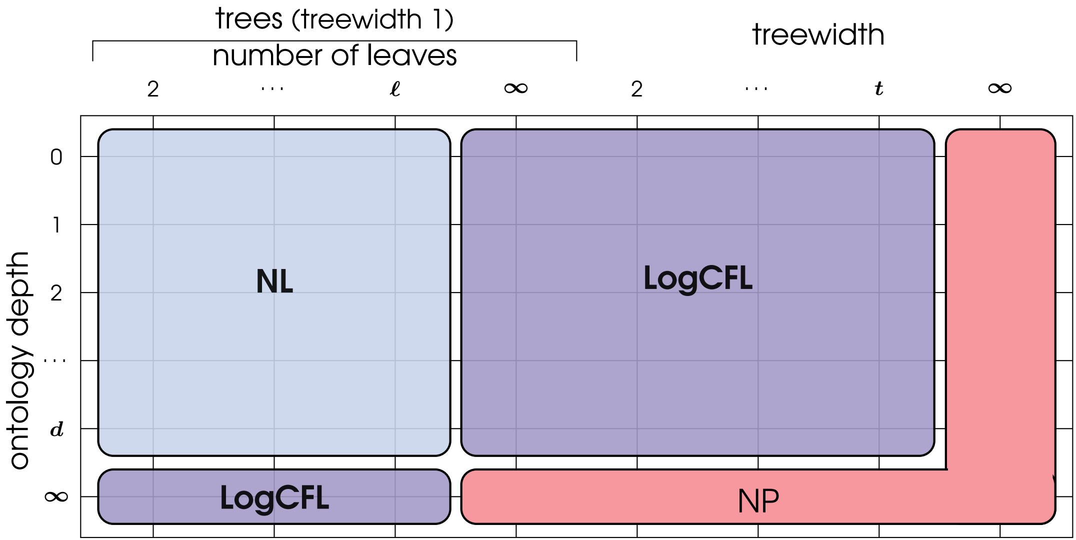
0 = no axioms with $\exists y$ on the right-hand side

$d$ $\approx$ trees $\mathfrak{C}^{\tau(a)}_{\mathcal{O}}$ of labelled nulls are of depth at most $d$

# Succinctness Landscape



# OMQ Answering Combined Complexity



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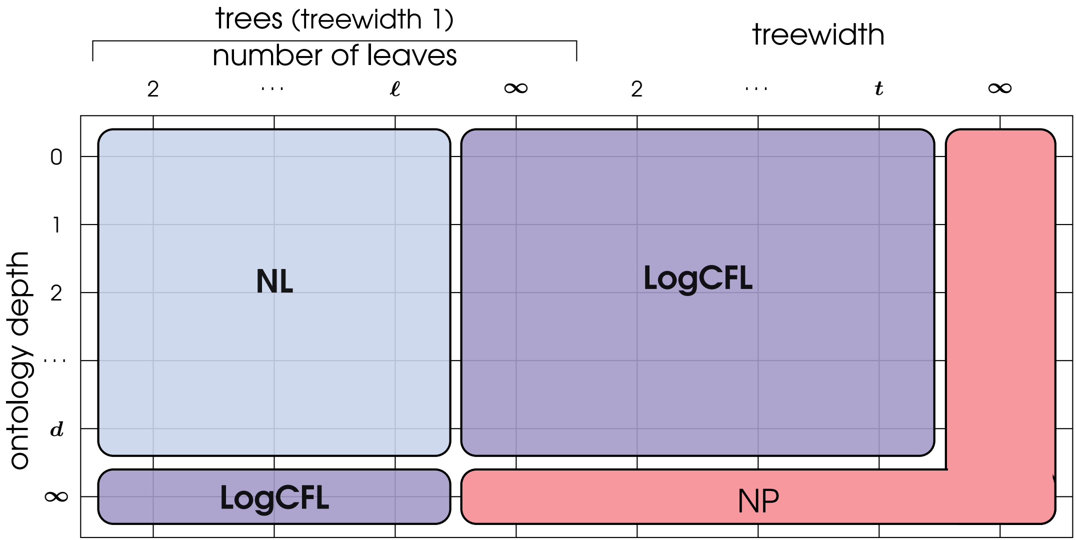


**NB:** polynomial-size NDL rewritings exist in all cases when OMQ answering is in LogCFL

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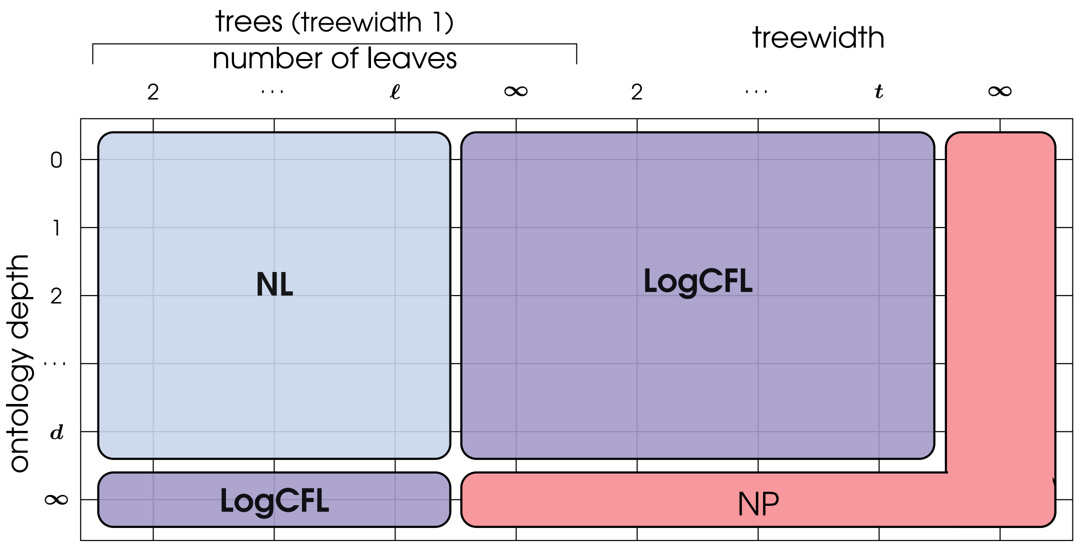


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**constructed polynomial-size NDL rewritings that can be evaluated in LogCFL/NL**

**LogCFL** means that they are **highly parallelisable**

# References


1. M. Bienvenu, S. Kikot, R. Kontchakov, V. V. Podolskii & M. Zakharyaschev. "Ontology-mediated queries: combined complexity and succinctness of rewritings via circuit complexity". In: JACM, 2018.

2. M. Bienvenu, S. Kikot, R. Kontchakov, V. V. Podolskii, V. Ryzhikov & M. Zakharyaschev. "The complexity of ontology-based data access with OWL 2 QL and bounded treewidth queries". In: Proc. of the 36th ACM Symposium on Principles of Database Systems, PODS 2017, 2017.

3. G. Gottlob, S. Kikot, R. Kontchakov, V. V. Podolskii, T. Schwentick & M. Zakharyaschev. "The Price of Query Rewriting in Ontology-Based Data Access". In: Artificial Intelligence, 213:42–59, 2014.

4. R. Rosati & A. Almatelli. "Improving query answering over DL-Lite ontologies". In: Proc. of the 12th Int. Conf. on Principles of Knowledge Representation & Reasoning, KR'10, pp. 290–300. AAAI, 2010.

5. M. Thomazo. "Compact rewritings for existential rules". In: Proc. of the 23rd Int. Joint Conf. on Artificial Intelligence IJCAI 2013, pp. 1125–1131. IJCAI/AAAI, 2013.