Mapping Management and Expressive Ontologies in Ontology-Based Data Access

3. Theoretical Foundations of OBDA

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Outline

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Query answering via query rewriting

To compute the certain answers to a SPARQL query q over an OBDA instance $\mathcal{O} = \langle \mathcal{P}, \mathcal{D} \rangle$, with $\mathcal{P} = \langle \mathcal{T}, \mathcal{S}, \mathcal{M} \rangle$:

1. Compute the perfect rewriting of q w.r.t. $\mathcal{T}$.
2. Unfold the perfect rewriting wrt the mapping $\mathcal{M}$.
3. Evaluate the unfolded perfect rewriting over $\mathcal{D}$.

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To illustrate Step 1, we briefly describe *PerfectRef*, a simple query rewriting algorithm that requires to iterate over:

- rewriting steps that involve TBox inclusion assertions, and
- unification of query atoms.

The perfect rewriting of q is still a SPARQL query involving `UNION`.

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Note: disjointness assertions play a role in ontology satisfiability, but can be ignored during query rewriting (i.e., we have **separability**).

Query rewriting step: Basic idea

Intuition: an inclusion assertion corresponds to a logic programming rule.

Example

The inclusion assertion $\text{FullProf} \sqsubseteq \text{Prof}$
corresponds to the logic programming rule $\text{Prof}(z) \leftarrow \text{FullProf}(z)$.

Query rewriting step: Basic idea

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Basic rewriting step:

When an atom in the query unifies with the **head** of the rule, generate a new query by substituting the atom with the **body** of the rule.

We say that the inclusion assertion **applies to** the atom.

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Consider the query $\mathsf{q}(x) \leftarrow \mathsf{Prof}(x)$.

By applying the inclusion assertion to the atom $\mathsf{Prof}(x)$, we generate:

$$\mathsf{q}(x) \leftarrow \mathsf{FullProf}(x).$$

This query is added to the input query, and contributes to the perfect rewriting.

Query rewriting (cont'd)

Example

Consider the query $\mathsf{q}(x) \leftarrow \mathsf{teaches}(x, y), \mathsf{Course}(y)$

and the inclusion assertion $\exists \mathsf{teaches}^- \sqsubseteq \mathsf{Course}$
as a logic programming rule: $\mathsf{Course}(z_2) \leftarrow \mathsf{teaches}(z_1, z_2)$.

The inclusion applies to $\mathsf{Course}(y)$, and we add to the rewriting the query

$$\mathsf{q}(x) \leftarrow \mathsf{teaches}(x, y), \mathsf{teaches}(z_1, y).$$

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Consider now the query $\mathsf{q}(x) \leftarrow \mathsf{teaches}(x, y)$

and the inclusion assertion $\mathsf{FullProf} \sqsubseteq \exists\mathsf{teaches}$
as a logic programming rule: $\mathsf{teaches}(z, f(z)) \leftarrow \mathsf{FullProf}(z)$.

The inclusion applies to $\mathsf{teaches}(x, y)$, and we add to the rewriting the query

$$\mathsf{q}(x) \leftarrow \mathsf{FullProf}(x).$$

Query rewriting – Constants

Example

Conversely, for the query $\mathsf{q}(x) \leftarrow \mathsf{teaches}(x, \mathtt{matrix})$

and the same inclusion assertion as before $\mathsf{FullProf} \sqsubseteq \exists\mathsf{teaches}$
as a logic programming rule: $\mathsf{teaches}(z, f(z)) \leftarrow \mathsf{FullProf}(z)$

$\mathsf{teaches}(x, \mathtt{matrix})$ does not unify with $\mathsf{teaches}(z, f(z))$, since the **skolem term** $f(z)$ in the head of the rule **does not unify** with the constant $\mathtt{matrix}$.
Remember: We adopt the **unique name assumption**.

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Remember: We adopt the **unique name assumption**.

We say that the inclusion does **not** apply to the atom $\mathsf{teaches}(x, \mathtt{matrix})$.

Example

The same holds for the following query, where y is **distinguished**, since unifying $f(z)$ with y would correspond to returning a skolem term as answer to the query:

$$\mathsf{q}(x, y) \leftarrow \mathsf{teaches}(x, y).$$

Query rewriting – Join variables

An analogous behavior to the one with constants and with distinguished variables holds when the atom contains **join variables** that would have to be unified with skolem terms.

Example

Consider the query $\mathsf{q}(x) \leftarrow \mathsf{teaches}(x, y), \mathsf{Course}(y)$

and the inclusion assertion $\mathsf{FullProf} \sqsubseteq \exists \mathsf{teaches}$

as a logic programming rule: $\mathsf{teaches}(z, f(z)) \leftarrow \mathsf{FullProf}(z)$.

The inclusion assertion above does **not** apply to the atom $\mathsf{teaches}(x, y)$.

Query rewriting – Reduce step

Example

Consider now the query $\mathsf{q}(x) \;\leftarrow\; \mathsf{teaches}(x, y), \mathsf{teaches}(z, y)$

and the inclusion assertion $\mathsf{FullProf} \sqsubseteq \exists\mathsf{teaches}$
as a logic rule: $\mathsf{teaches}(z, f(z)) \leftarrow \mathsf{FullProf}(z)$.

This inclusion assertion does not apply to $\mathsf{teaches}(x, y)$ or $\mathsf{teaches}(z, y)$, since y is in join, and we would again introduce the skolem term in the rewritten query.

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Query rewriting – Reduce step

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as a logic rule: $\quad \mathsf{teaches}(z, f(z)) \leftarrow \mathsf{FullProf}(z).$

This inclusion assertion does not apply to $\mathsf{teaches}(x,y)$ or $\mathsf{teaches}(z,y)$, since y is in join, and we would again introduce the skolem term in the rewritten query.

Example

However, we can transform the above query by unifying the atoms $\mathsf{teaches}(x,y)$ and $\mathsf{teaches}(z,y)$. This rewriting step is called **reduce**, and produces the query

$$\mathsf{q}(x) \leftarrow \mathsf{teaches}(x,y).$$

Now, we can apply the inclusion above, and add to the rewriting the query

$$\mathsf{q}(x) \leftarrow \mathsf{FullProf}(x).$$

Query rewriting – Summary

To compute the perfect rewriting of a query q, start from q, iteratively get a CQ q' to be processed, and do one of the following:

- Apply to some atom of q' an inclusion assertion in $\mathcal{T}$ as follows:

$$
\begin{array}{rccc}
A_1 \sqsubseteq A_2 & \ldots, A_2(x), \ldots & \rightsquigarrow & \ldots, A_1(x), \ldots \\
\exists P \sqsubseteq A & \ldots, A(x), \ldots & \rightsquigarrow & \ldots, P(x, \_), \ldots \\
\exists P^- \sqsubseteq A & \ldots, A(x), \ldots & \rightsquigarrow & \ldots, P(\_, x), \ldots \\
A \sqsubseteq \exists P & \ldots, P(x, \_), \ldots & \rightsquigarrow & \ldots, A(x), \ldots \\
A \sqsubseteq \exists P^- & \ldots, P(\_, x), \ldots & \rightsquigarrow & \ldots, A(x), \ldots \\
\exists P_1 \sqsubseteq \exists P_2 & \ldots, P_2(x, \_), \ldots & \rightsquigarrow & \ldots, P_1(x, \_), \ldots \\
P_1 \sqsubseteq P_2 & \ldots, P_2(x, y), \ldots & \rightsquigarrow & \ldots, P_1(x, y), \ldots \\
P_1 \sqsubseteq P_2^- & \ldots, P_2(x, y), \ldots & \rightsquigarrow & \ldots, P_1(y, x), \ldots \\
& \ldots & &
\end{array}
$$

  ('_' denotes a variable that appears only once)

- Choose two atoms of q' that unify, and apply the unifier to q'.

Each time, the result of the above step is added to the queries to be processed.

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The UCQ resulting from this process is the **perfect rewriting** $r_{q,\mathcal{T}}$.

Query rewriting algorithm

Algorithm *PerfectRef*$(Q, \mathcal{T}_P)$
Input: union of conjunctive queries Q, set $\mathcal{T}_P$ of *DL-Lite* inclusion assertions
Output: union of conjunctive queries PR
$PR := Q$;
repeat
  $PR' := PR$;
  for each $q \in PR'$ **do**
    for each g in q **do**
      for each inclusion assertion I in $\mathcal{T}_P$ **do**
        if I is applicable to g **then** $PR := PR \cup \{\,$*ApplyPI*$(q, g, I)\,\}$;
    for each g_1, g_2 in q **do**
      if g_1 and g_2 unify **then** $PR := PR \cup \{\tau($*Reduce*$(q, g_1, g_2))\}$;
until $PR' = PR$;
return PR

Observations:

- Termination follows from having only finitely many different rewritings.
- Disjointness assertions and functionalities do not play any role in the rewriting of the query.

Query answering in *DL-Lite* – Example

TBox:

$$\begin{aligned} \mathsf{FullProf} &\sqsubseteq \mathsf{Prof} \\ \mathsf{Prof} &\sqsubseteq \exists\mathsf{teaches} \\ \exists\mathsf{teaches}^- &\sqsubseteq \mathsf{Course} \end{aligned}$$

Corresponding rules:

$$\begin{aligned} \mathsf{Prof}(x) &\leftarrow \mathsf{FullProf}(x) \\ \exists y(\mathsf{teaches}(x,y)) &\leftarrow \mathsf{Prof}(x) \\ \mathsf{Course}(x) &\leftarrow \mathsf{teaches}(y,x) \end{aligned}$$

Query: $\mathsf{q}(x) \leftarrow \mathsf{teaches}(x,y), \mathsf{Course}(y)$

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ABox: $\mathsf{teaches}(\mathtt{jim}, \mathtt{databases})$ $\mathsf{FullProf}(\mathtt{jim})$
$\mathsf{teaches}(\mathtt{julia}, \mathtt{security})$ $\mathsf{FullProf}(\mathtt{nicole})$

Evaluating the perfect rewriting over the ABox (seen as a DB) produces as answer $\{\mathtt{jim}, \mathtt{julia}, \mathtt{nicole}\}$.

Query answering in *DL-Lite* – An interesting example

TBox: $\mathsf{Person} \sqsubseteq \exists \mathsf{hasFather}$
$\exists \mathsf{hasFather}^- \sqsubseteq \mathsf{Person}$

ABox: $\mathsf{Person}(\mathtt{john})$

Query: $\mathsf{q}(x) \leftarrow \mathsf{Person}(x), \mathsf{hasFather}(x, y_1), \mathsf{hasFather}(y_1, y_2), \mathsf{hasFather}(y_2, y_3)$

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$\Downarrow$ **Apply** $\mathsf{Person} \sqsubseteq \exists\mathsf{hasFather}$ to the atom $\mathsf{hasFather}(y_2, \_)$

$\mathsf{q}(x) \leftarrow \mathsf{Person}(x), \mathsf{hasFather}(x, y_1), \mathsf{hasFather}(y_1, y_2), \mathsf{Person}(y_2)$

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$\mathsf{q}(x) \leftarrow \text{Person}(x), \text{hasFather}(x, y_1), \text{hasFather}(y_1, y_2), \text{hasFather}(\_, y_2)$

$\Downarrow$ **Unify** atoms $\text{hasFather}(y_1, y_2)$ and $\text{hasFather}(\_, y_2)$

$\mathsf{q}(x) \leftarrow \text{Person}(x), \text{hasFather}(x, y_1), \text{hasFather}(y_1, y_2)$

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Query answering in *DL-Lite* – An interesting example

TBox: $\mathsf{Person} \sqsubseteq \exists\mathsf{hasFather}$
$\exists\mathsf{hasFather}^- \sqsubseteq \mathsf{Person}$

ABox: $\mathsf{Person}(\mathtt{john})$

Query: $\mathsf{q}(x) \leftarrow \mathsf{Person}(x), \mathsf{hasFather}(x, y_1), \mathsf{hasFather}(y_1, y_2), \mathsf{hasFather}(y_2, y_3)$

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$\Downarrow$ **Apply** $\mathsf{Person} \sqsubseteq \exists\mathsf{hasFather}$ to the atom $\mathsf{hasFather}(y_2, \_)$

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$\mathsf{q}(x) \leftarrow \mathsf{Person}(x), \mathsf{hasFather}(x, y_1), \mathsf{hasFather}(y_1, y_2), \mathsf{hasFather}(\_, y_2)$

$\Downarrow$ **Unify** atoms $\mathsf{hasFather}(y_1, y_2)$ and $\mathsf{hasFather}(\_, y_2)$

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$\Downarrow$

$\ldots$

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Complexity of query answering in *DL-Lite*

Query answering for UCQs / SPARQL queries is:

- Efficiently tractable in the size of the TBox, i.e., PTime.
- Very efficiently tractable in the size of the ABox, i.e., AC^0.
- Exponential in the size of the **query**, more precisely NP-complete.

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Can we go beyond *DL-Lite*?

Essentially no! By adding essentially any additional DL constructor we lose first-order rewritability and hence these nice computational properties.

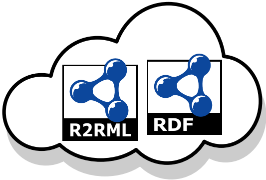
Outline

1. Query rewriting wrt an OWL 2 QL ontology
2. Saturation and optimization of the mapping
3. Query reformulation

Querying the OBDA system

OBDA system $\mathcal{K} = \langle \mathcal{T}, \mathcal{M}, \mathcal{D} \rangle$

- *DL-Lite*$_\mathcal{R}$ TBox $\mathcal{T}$
- RDF graph $\mathcal{G}$ obtained from the mapping $\mathcal{M}$ and the data sources $\mathcal{D}$
- $\mathcal{G}$ can be viewed as the ABox



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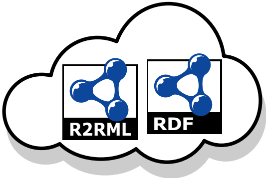

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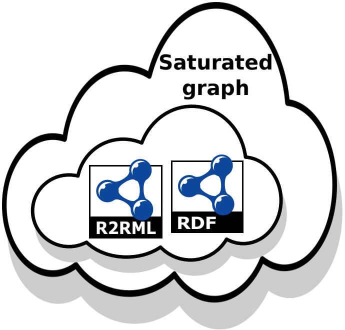

Saturated RDF graph $\mathcal{G}_{\text{sat}}$

- Saturation of $\mathcal{G}$ w.r.t. $\mathcal{T}$
- H-complete ABox

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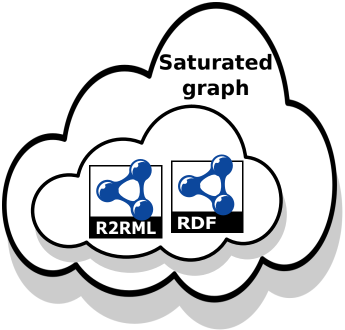
Query answering

- SPARQL query $\boldsymbol{q}$ over $\mathcal{K}$
- If there is no existential restriction $B \sqsubseteq \exists R.C$ in $\mathcal{T}$, $\boldsymbol{q}$ can be directly evaluated over $\mathcal{G}_{\mathsf{sat}}$



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How to handle the RDF graph $\mathcal{G}_{\text{sat}}$ in practice?

By materializing it

- Materialization of $\mathcal{G}$ (ETL) + saturation
- – Large volume
- – Maintenance
- Typical profile: OWL 2 RL

By keeping it virtual

- Query rewriting
- + No materialization required
- Saturated mapping $\mathcal{M}_{\text{sat}}$
- Typical profile: OWL 2 QL

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H-complete ABox

[Rodriguez-Muro, Kontchakov, and Zakharyaschev 2013; Kontchakov and Zakharyaschev 2014]

ABox saturation

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- Let $\mathcal{K}$ be a *DL-Lite*$_\mathcal{R}$ knowledge base, and let $\mathcal{K}'$ be the result of saturating $\mathcal{K}$. For every ABox assertion α, we have:

$$\mathcal{K} \models \alpha \quad \text{iff} \quad \alpha \in \mathcal{K}'$$

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Saturated mapping $\mathcal{M}_{\mathsf{sat}}$ (also called *T-mapping*)

- Composition of the mapping $\mathcal{M}$ and the *DL-Lite*$_\mathcal{R}$ TBox $\mathcal{T}$
- $\mathcal{M}_{\mathsf{sat}} + \mathcal{D} \rightarrow \mathcal{G}_{\mathsf{sat}}$ (H-complete ABox)

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- $\mathcal{M}_{\text{sat}} + \mathcal{D} \rightarrow \mathcal{G}_{\text{sat}}$ (H-complete ABox)
- Independent of the SPARQL query $\boldsymbol{q}$ (can be pre-computed)
- Can be optimized (query containment)

TBox, user-defined mapping assertions and foreign key

Student $\sqcup$ PostDoc $\sqcup$ AssociateProfessor $\sqcup$ $\exists$teaches $\sqsubseteq$ Person

$$\mathsf{Student}(\mathrm{URI}_1(p)) \leftarrow \texttt{uni1-student}(p, f, l) \tag{1}$$

$$\mathsf{PostDoc}(\mathrm{URI}_2(a)) \leftarrow \texttt{uni1-academic}(a, f, l, s), s = 9 \tag{2}$$

$$\mathsf{AssociateProfessor}(\mathrm{URI}_2(a)) \leftarrow \texttt{uni1-academic}(a, f, l, s), s = 2 \tag{3}$$

$$\mathsf{FacultyMember}(\mathrm{URI}_2(a)) \leftarrow \texttt{uni1-academic}(a, f, l, s) \tag{4}$$

$$\mathsf{teaches}(\mathrm{URI}_2(a), \mathrm{URI}_3(c)) \leftarrow \texttt{uni1-teaching}(c, a) \tag{5}$$

FK: $\exists y_1.\texttt{uni1-teaching}(y_1, x) \rightarrow \exists y_2 y_3 y_4.\texttt{uni1-academic}(x, y_2, y_3, y_4)$

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Non-optimized saturated mapping assertions for Person

$$\mathsf{Person}(\mathrm{URI}_1(p)) \leftarrow \texttt{uni1-student}(p, f, l) \quad (6)$$

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Mapping assertions for Person after optimization (query containment)

$$\mathsf{Person}(\mathrm{URI}_1(p)) \leftarrow \texttt{uni1-student}(p, f, l) \tag{11}$$
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Outline

1. Query rewriting wrt an OWL 2 QL ontology
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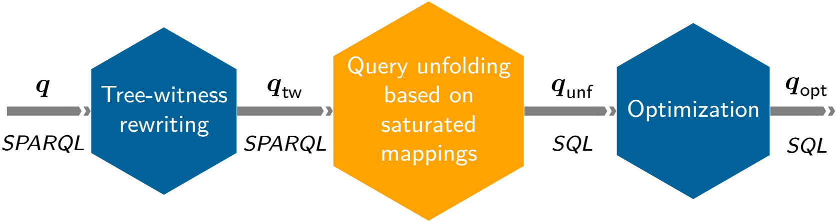
   - Tree-witness rewriting
   - SQL query optimization

Query reformulation

Implemented by Ontop



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Example: Existential reasoning (I)

Example

Suppose that every graduate student is supervised by some professor, i.e.

$$\texttt{GraduateStudent} \sqsubseteq \exists\, \texttt{isSupervisedBy.Professor}$$

and john is a graduate student:

$$\texttt{GraduateStudent(john)}$$

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Yes. Even though we don't know who is john's supervisor (under existential reasoning).

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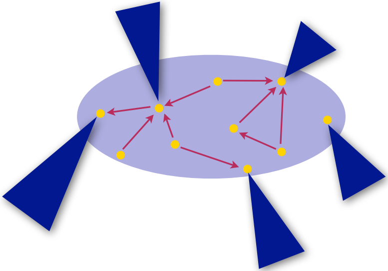
Existential reasoning and tree-witness rewriting

Fact: every consistent DL-Lite KB $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ has a **canonical model** $\mathcal{I}_{\mathcal{K}}$, which **gives the right answers to all CQs**, i.e., $\mathsf{cert}(q, \mathcal{K}) = \mathsf{ans}(q, \mathcal{I}_{\mathcal{K}})$



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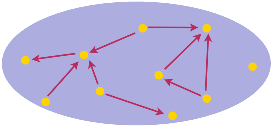
Core

individuals from $\mathcal{A}$



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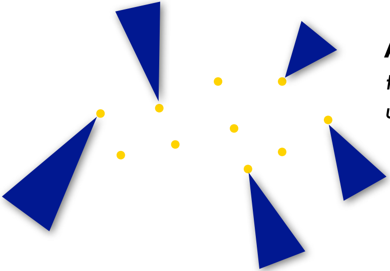

Anonymous part

trees rooted at individuals, using unnamed objects

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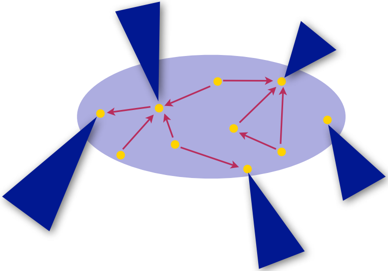

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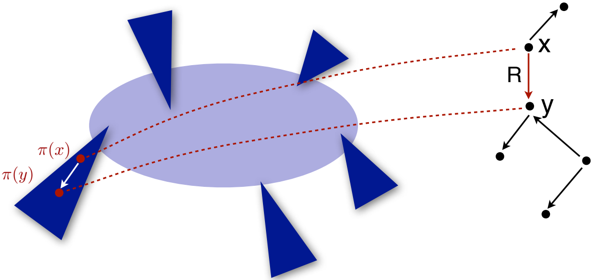

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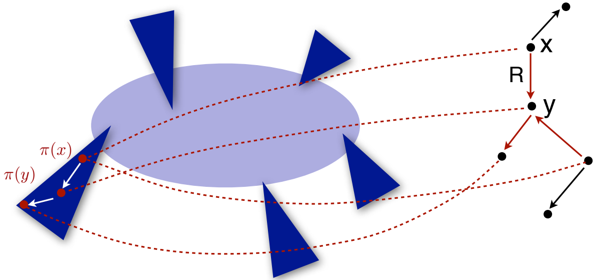

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Tree-witness rewriting option in *Ontop*

Note that if users want to enable existential reasoning, the option of tree-witness rewriting algorithm needs to be switched explicitly.

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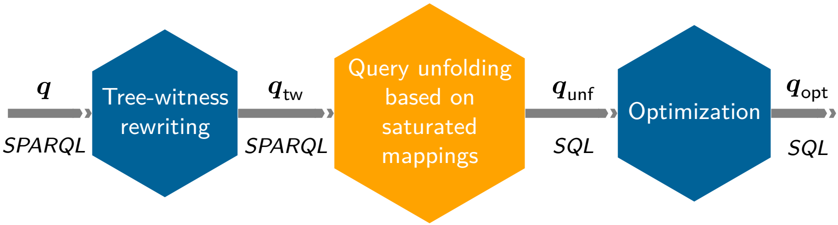

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SQL query optimization

Objective : produce SQL queries that are ...

- similar to manually written ones
- adapted to existing query planners

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Semantic optimization

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- Redundant union elimination
- Using functional constraints

Integrity constraints

- Primary and foreign keys, unique constraints
- Sometimes implicit
- Vital for query reformulation!

Reformulation example - 1. Unfolding

Saturated mappings

$$\begin{aligned}
\mathsf{firstName}(\mathrm{URI}_1(p), f) &\leftarrow \texttt{uni1-student}(p, f, l)\\
\mathsf{firstName}(\mathrm{URI}_2(a), f) &\leftarrow \texttt{uni1-academic}(a, f, l, s)\\
\mathsf{lastName}(\mathrm{URI}_1(p), l) &\leftarrow \texttt{uni1-student}(p, f, l)\\
\mathsf{lastName}(\mathrm{URI}_2(a), l) &\leftarrow \texttt{uni1-academic}(a, f, l, s)\\
\mathsf{Teacher}(\mathrm{URI}_2(a)) &\leftarrow \texttt{uni1-academic}(a, f, l, s), s \in [1, 8]\\
\mathsf{Teacher}(\mathrm{URI}_2(a)) &\leftarrow \texttt{uni1-teaching}(c, a)
\end{aligned}$$

Query

$$\boldsymbol{q}(x, y, z) \leftarrow \mathsf{Teacher}(x), \mathsf{firstName}(x, y), \mathsf{lastName}(x, z)$$

$$\boldsymbol{q}_{\mathsf{tw}} = \boldsymbol{q}$$

Reformulation example - 1. Unfolding

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\mathsf{firstName}(\mathrm{URI}_1(p), f) &\leftarrow \texttt{uni1-student}(p, f, l)\\
\mathsf{firstName}(\mathrm{URI}_2(a), f) &\leftarrow \texttt{uni1-academic}(a, f, l, s)\\
\mathsf{lastName}(\mathrm{URI}_1(p), l) &\leftarrow \texttt{uni1-student}(p, f, l)\\
\mathsf{lastName}(\mathrm{URI}_2(a), l) &\leftarrow \texttt{uni1-academic}(a, f, l, s)\\
\mathsf{Teacher}(\mathrm{URI}_2(a)) &\leftarrow \texttt{uni1-academic}(a, f, l, s), s \in [1, 8]\\
\mathsf{Teacher}(\mathrm{URI}_2(a)) &\leftarrow \texttt{uni1-teaching}(c, a)
\end{aligned}$$

Query

$$\boldsymbol{q}(x, y, z) \leftarrow \mathsf{Teacher}(x), \mathsf{firstName}(x, y), \mathsf{lastName}(x, z)$$

$$\boldsymbol{q}_{\mathsf{tw}} = \boldsymbol{q}$$

Query unfolding

$$\begin{aligned}
\boldsymbol{q}_{\mathsf{unf}}(x, y, z) &\leftarrow \boldsymbol{q}_{\mathsf{unf1}}(x), \boldsymbol{q}_{\mathsf{unf2}}(x, y), \boldsymbol{q}_{\mathsf{unf3}}(x, z)\\
\boldsymbol{q}_{\mathsf{unf1}}(\mathrm{URI}_2(a)) &\leftarrow \texttt{uni1-academic}(a, f, l, s), s \in [1, 8]\\
\boldsymbol{q}_{\mathsf{unf1}}(\mathrm{URI}_2(a)) &\leftarrow \texttt{uni1-teaching}(c, a)\\
\boldsymbol{q}_{\mathsf{unf2}}(\mathrm{URI}_1(p), f) &\leftarrow \texttt{uni1-student}(p, f, l)\\
\boldsymbol{q}_{\mathsf{unf2}}(\mathrm{URI}_2(a), f) &\leftarrow \texttt{uni1-academic}(a, f, l, s)\\
\boldsymbol{q}_{\mathsf{unf3}}(\mathrm{URI}_1(p), l) &\leftarrow \texttt{uni1-student}(p, f, l)\\
\boldsymbol{q}_{\mathsf{unf3}}(\mathrm{URI}_2(a), l) &\leftarrow \texttt{uni1-academic}(a, f, l, s)
\end{aligned}$$

Reformulation example - 2. Structural optimization

Query unfolding

$$
\begin{aligned}
\boldsymbol{q}_{\mathsf{unf}}(x,y,z) &\leftarrow \boldsymbol{q}_{\mathsf{unf1}}(x), \boldsymbol{q}_{\mathsf{unf2}}(x,y), \boldsymbol{q}_{\mathsf{unf3}}(x,z) \\
\boldsymbol{q}_{\mathsf{unf1}}(\mathrm{URI}_2(a)) &\leftarrow \texttt{uni1-academic}(a,f,l,s), s \in [1,8] \\
\boldsymbol{q}_{\mathsf{unf1}}(\mathrm{URI}_2(a)) &\leftarrow \texttt{uni1-teaching}(c,a) \\
\boldsymbol{q}_{\mathsf{unf2}}(\mathrm{URI}_1(p), f) &\leftarrow \texttt{uni1-student}(p,f,l) \\
\boldsymbol{q}_{\mathsf{unf2}}(\mathrm{URI}_2(a), f) &\leftarrow \texttt{uni1-academic}(a,f,l,s) \\
\boldsymbol{q}_{\mathsf{unf3}}(\mathrm{URI}_1(p), l) &\leftarrow \texttt{uni1-student}(p,f,l) \\
\boldsymbol{q}_{\mathsf{unf3}}(\mathrm{URI}_2(a), l) &\leftarrow \texttt{uni1-academic}(a,f,l,s)
\end{aligned}
$$

Reformulation example - 2. Structural optimization

Query unfolding

$$
\begin{aligned}
\boldsymbol{q}_{\mathsf{unf}}(x,y,z) &\leftarrow \boldsymbol{q}_{\mathsf{unf1}}(x), \boldsymbol{q}_{\mathsf{unf2}}(x,y), \boldsymbol{q}_{\mathsf{unf3}}(x,z)\\
\boldsymbol{q}_{\mathsf{unf1}}(\mathrm{URI}_2(a)) &\leftarrow \texttt{uni1-academic}(a,f,l,s), s \in [1,8]\\
\boldsymbol{q}_{\mathsf{unf1}}(\mathrm{URI}_2(a)) &\leftarrow \texttt{uni1-teaching}(c,a)\\
\boldsymbol{q}_{\mathsf{unf2}}(\mathrm{URI}_1(p),f) &\leftarrow \texttt{uni1-student}(p,f,l)\\
\boldsymbol{q}_{\mathsf{unf2}}(\mathrm{URI}_2(a),f) &\leftarrow \texttt{uni1-academic}(a,f,l,s)\\
\boldsymbol{q}_{\mathsf{unf3}}(\mathrm{URI}_1(p),l) &\leftarrow \texttt{uni1-student}(p,f,l)\\
\boldsymbol{q}_{\mathsf{unf3}}(\mathrm{URI}_2(a),l) &\leftarrow \texttt{uni1-academic}(a,f,l,s)
\end{aligned}
$$

Normalization: explicit joining conditions

$$
\begin{aligned}
\boldsymbol{q}_{\mathsf{exp}}(x,y,z) &\leftarrow \boldsymbol{q}_{\mathsf{unf1}}(x), \boldsymbol{q}_{\mathsf{unf2}}(x_1,y), \boldsymbol{q}_{\mathsf{unf3}}(x_2,z), x = x_1, x = x_2\\
\boldsymbol{q}_{\mathsf{unf1}}(\mathrm{URI}_2(a)) &\leftarrow \texttt{uni1-academic}(a,f,l,s), s \in [1,8]\\
\boldsymbol{q}_{\mathsf{unf1}}(\mathrm{URI}_2(a)) &\leftarrow \texttt{uni1-teaching}(c,a)\\
\boldsymbol{q}_{\mathsf{unf2}}(\mathrm{URI}_1(p),f) &\leftarrow \texttt{uni1-student}(p,f,l)\\
\boldsymbol{q}_{\mathsf{unf2}}(\mathrm{URI}_2(a),f) &\leftarrow \texttt{uni1-academic}(a,f,l,s)\\
\boldsymbol{q}_{\mathsf{unf3}}(\mathrm{URI}_1(p),l) &\leftarrow \texttt{uni1-student}(p,f,l)\\
\boldsymbol{q}_{\mathsf{unf3}}(\mathrm{URI}_2(a),l) &\leftarrow \texttt{uni1-academic}(a,f,l,s)
\end{aligned}
$$

Reformulation example - 2. Structural optimization

Normalization: explicit joining conditions

$$
\begin{aligned}
\boldsymbol{q}_{\mathsf{exp}}(x, y, z) &\leftarrow \boldsymbol{q}_{\mathsf{unf1}}(x), \boldsymbol{q}_{\mathsf{unf2}}(x_1, y),\\
&\quad \boldsymbol{q}_{\mathsf{unf3}}(x_2, z),\\
&\quad x = x_1, x = x_2\\
\boldsymbol{q}_{\mathsf{unf1}}(\mathrm{URI}_2(a)) &\leftarrow \texttt{uni1-academic}(a, f, l, s),\\
&\quad s \in [1, 8]\\
\boldsymbol{q}_{\mathsf{unf1}}(\mathrm{URI}_2(a)) &\leftarrow \texttt{uni1-teaching}(c, a)\\
\boldsymbol{q}_{\mathsf{unf2}}(\mathrm{URI}_1(p), f) &\leftarrow \texttt{uni1-student}(p, f, l)\\
\boldsymbol{q}_{\mathsf{unf2}}(\mathrm{URI}_2(a), f) &\leftarrow \texttt{uni1-academic}(a, f, l, s)\\
\boldsymbol{q}_{\mathsf{unf3}}(\mathrm{URI}_1(p), l) &\leftarrow \texttt{uni1-student}(p, f, l)\\
\boldsymbol{q}_{\mathsf{unf3}}(\mathrm{URI}_2(a), l) &\leftarrow \texttt{uni1-academic}(a, f, l, s)
\end{aligned}
$$

Flattening (URI template lifting) - part 1/2

$$
\begin{aligned}
\boldsymbol{q}_{\mathsf{lift}}(\mathrm{URI}_2(a), y, z) &\leftarrow \texttt{uni1-academic}(a, f_1, l_1, s_1),\\
&\quad \texttt{uni1-student}(p, f_2, l_2),\\
&\quad \texttt{uni1-student}(p_1, f_3, l_3),\\
&\quad \mathrm{URI}_2(a) = \mathrm{URI}_1(p),\\
&\quad \mathrm{URI}_2(a) = \mathrm{URI}_1(p_1),\\
&\quad s_1 \in [1, 8]\\
\boldsymbol{q}_{\mathsf{lift}}(\mathrm{URI}_2(a), y, z) &\leftarrow \texttt{uni1-academic}(a, f_1, l_1, s_1),\\
&\quad \texttt{uni1-student}(p, f_2, l_2),\\
&\quad \texttt{uni1-academic}(a_2, f_3, z, s_3),\\
&\quad \mathrm{URI}_2(a) = \mathrm{URI}_1(p),\\
&\quad \mathrm{URI}_2(a) = \mathrm{URI}_2(a_2), \ldots
\end{aligned}
$$

(One sub-query ignored)

$$
\begin{aligned}
\boldsymbol{q}_{\mathsf{lift}}(\mathrm{URI}_2(a), y, z) &\leftarrow \texttt{uni1-academic}(a, f_1, l_1, s_1),\\
&\quad \texttt{uni1-academic}(a_1, y, l_2, s_2),\\
&\quad \texttt{uni1-academic}(a_2, f_3, z, s_3),\\
&\quad \mathrm{URI}_2(a) = \mathrm{URI}_2(a_1),\\
&\quad \mathrm{URI}_2(a) = \mathrm{URI}_2(a_2),\\
&\quad s_1 \in [1, 8]
\end{aligned}
$$

Reformulation example - 2. Structural optimization

Normalization: explicit joining conditions

$$\begin{aligned}
\boldsymbol{q}_{\mathsf{exp}}(x,y,z) &\leftarrow \boldsymbol{q}_{\mathsf{unf1}}(x), \boldsymbol{q}_{\mathsf{unf2}}(x_1,y),\\
&\quad \boldsymbol{q}_{\mathsf{unf3}}(x_2,z),\\
&\quad x = x_1, x = x_2\\
\boldsymbol{q}_{\mathsf{unf1}}(\mathrm{URI}_2(a)) &\leftarrow \texttt{uni1-academic}(a,f,l,s),\\
&\quad s \in [1,8]\\
\boldsymbol{q}_{\mathsf{unf1}}(\mathrm{URI}_2(a)) &\leftarrow \texttt{uni1-teaching}(c,a)\\
\boldsymbol{q}_{\mathsf{unf2}}(\mathrm{URI}_1(p),f) &\leftarrow \texttt{uni1-student}(p,f,l)\\
\boldsymbol{q}_{\mathsf{unf2}}(\mathrm{URI}_2(a),f) &\leftarrow \texttt{uni1-academic}(a,f,l,s)\\
\boldsymbol{q}_{\mathsf{unf3}}(\mathrm{URI}_1(p),l) &\leftarrow \texttt{uni1-student}(p,f,l)\\
\boldsymbol{q}_{\mathsf{unf3}}(\mathrm{URI}_2(a),l) &\leftarrow \texttt{uni1-academic}(a,f,l,s)
\end{aligned}$$

Flattening (URI template lifting) - part 2/2

$$\begin{aligned}
\boldsymbol{q}_{\mathsf{lift}}(\mathrm{URI}_2(a),y,z) &\leftarrow \texttt{uni1-teaching}(c,a),\\
&\quad \texttt{uni1-student}(p,f_2,l_2),\\
&\quad \texttt{uni1-student}(p_1,f_3,l_3),\\
&\quad \mathrm{URI}_2(a) = \mathrm{URI}_1(p),\\
&\quad \mathrm{URI}_2(a) = \mathrm{URI}_1(p_1)
\end{aligned}$$

(One sub-query ignored)

$$\begin{aligned}
\boldsymbol{q}_{\mathsf{lift}}(\mathrm{URI}_2(a),y,z) &\leftarrow \texttt{uni1-teaching}(c,a),\\
&\quad \texttt{uni1-academic}(a_1,y,l_2,s_2),\\
&\quad \texttt{uni1-student}(p,f_3,l_3),\\
&\quad \mathrm{URI}_2(a) = \mathrm{URI}_2(a_1),\\
&\quad \mathrm{URI}_2(a) = \mathrm{URI}_1(p)\\
\boldsymbol{q}_{\mathsf{lift}}(\mathrm{URI}_2(a),y,z) &\leftarrow \texttt{uni1-teaching}(c,a),\\
&\quad \texttt{uni1-academic}(a_1,y,l_2,s_2),\\
&\quad \texttt{uni1-academic}(a_2,f_3,z,s_3),\\
&\quad \mathrm{URI}_2(a) = \mathrm{URI}_2(a_1),\\
&\quad \mathrm{URI}_2(a) = \mathrm{URI}_2(a_2)
\end{aligned}$$

Reformulation example - 2. Structural optimization

Simplification and implicit equality normalization

$$
\begin{aligned}
\boldsymbol{q}_{\mathsf{struct}}(\mathrm{URI}_2(a), y, z) \leftarrow & \texttt{uni1-academic}(a, f_1, l_1, s_1), \\
& \texttt{uni1-academic}(a, y, l_2, s_2), \\
& \texttt{uni1-academic}(a, f_3, z, s_3), s_1 \in [1, 8] \\
\boldsymbol{q}_{\mathsf{struct}}(\mathrm{URI}_2(a), y, z) \leftarrow & \texttt{uni1-teaching}(c, a), \\
& \texttt{uni1-academic}(a, y, l_2, s_2), \\
& \texttt{uni1-academic}(a, f_3, z, s_3)
\end{aligned}
$$

Reformulation example - 2. Structural optimization

Simplification and implicit equality normalization

$$\boldsymbol{q}_{\mathsf{struct}}(\mathrm{URI}_2(a), y, z) \leftarrow \texttt{uni1-academic}(a, f_1, l_1, s_1), \texttt{uni1-academic}(a, y, l_2, s_2), \texttt{uni1-academic}(a, f_3, z, s_3), s_1 \in [1, 8]$$

$$\boldsymbol{q}_{\mathsf{struct}}(\mathrm{URI}_2(a), y, z) \leftarrow \texttt{uni1-teaching}(c, a), \texttt{uni1-academic}(a, y, l_2, s_2), \texttt{uni1-academic}(a, f_3, z, s_3)$$

Remarks on the flattening step:

- Possible exponential blowup!
- Usually avoided thanks to incompatible URI templates.

Reformulation example - 3. Semantic optimization

Simplification and implicit equality normalization

$$\begin{aligned} \boldsymbol{q}_{\mathsf{struct}}(\mathrm{URI}_2(a), y, z) \leftarrow & \mathtt{uni1\text{-}academic}(a, f_1, l_1, s_1), \\ & \mathtt{uni1\text{-}academic}(a, y, l_2, s_2), \\ & \mathtt{uni1\text{-}academic}(a, f_3, z, s_3), s_1 \in [1, 8] \\ \boldsymbol{q}_{\mathsf{struct}}(\mathrm{URI}_2(a), y, z) \leftarrow & \mathtt{uni1\text{-}teaching}(c, a), \\ & \mathtt{uni1\text{-}academic}(a, y, l_2, s_2), \\ & \mathtt{uni1\text{-}academic}(a, f_3, z, s_3) \end{aligned}$$

Self-join elimination (semantic optimization)

PK: $\mathtt{uni1\text{-}academic}(a, b, c, d) \wedge \mathtt{uni1\text{-}academic}(a, b', c', d')$
$\rightarrow (b = b') \wedge (c = c') \wedge (d = d')$

$$\boldsymbol{q}_{\mathsf{opt}}(\mathrm{URI}_2(a), y, z) \leftarrow \mathtt{uni1\text{-}academic}(a, y, z, s_1), s_1 \in [1, 8]$$

$$\boldsymbol{q}_{\mathsf{opt}}(\mathrm{URI}_2(a), y, z) \leftarrow \mathtt{uni1\text{-}teaching}(c, a), \mathtt{uni1\text{-}academic}(a, y, z, s_2)$$

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