Similar as in the angle b/w 2 columns is zero i.e. their projection would fall on same axis.

K → rank K approx.
n → No. of document.
m → No. of terms.

$$AP_1(:,1) = c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \underset{\substack{\uparrow \\ \text{dominating column}}}{c_1(:,1)}$$

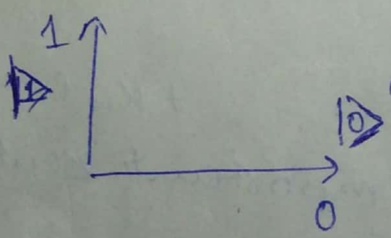
Quantum Algorithms

classical bits   0, 1.

Two bits.
00
01
10
11

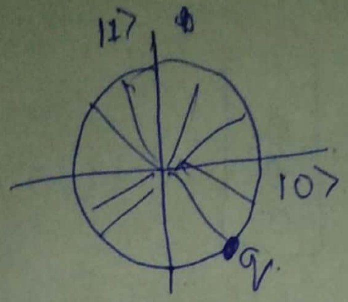
Quantum bits.

basic unit   qubit.



these basis vectors are Normalized.

$$|q\rangle = \alpha|0\rangle + \beta|1\rangle$$



We use the notations.

$|a\rangle$ = Column vector
↑ Ket

$\langle a|$ = row vector
↑ Bra

In a 1-qubit system $|0\rangle$ & $|1\rangle$ as basis states.

$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ → 2 dimensional space.

In a 2-qubit system, the basis states are.

$|00\rangle, |01\rangle, |10\rangle, |11\rangle$ → 4-dimensional space.

$\alpha |00\rangle + \beta |01\rangle + \gamma |10\rangle + \delta |11\rangle$

tensor product

Define:-

$|ab\rangle = |a\rangle \otimes |b\rangle$ (tensor product)

$$|00\rangle = |0\rangle \otimes |0\rangle$$

$$= \begin{bmatrix}1\\0\end{bmatrix} \otimes \begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\0\\0\\0\end{bmatrix}$$

$$= \begin{bmatrix}1\begin{bmatrix}1\\0\end{bmatrix}\\0\begin{bmatrix}1\\0\end{bmatrix}\end{bmatrix}$$

In a 1-quibit system.

$$|q\rangle = \alpha|0\rangle + \beta|1\rangle \qquad \alpha, \beta \in \mathbb{C}$$

Here $|q\rangle$ is normalized $\Rightarrow$.

conjugate transpose

Recall.

$$\langle x|y\rangle = \underbrace{(\langle x|)}_{\text{row vector}}\ \underbrace{(|y\rangle)}_{\text{column vector}}$$ is an inner product.

A vector is normalized if its inner product with itself is 1.

i.e. $\langle q|q\rangle = 1$.

Note. $\Rightarrow$ $\langle q| = \alpha\langle 0| + \beta\langle 1|$

Now; $\langle q|q\rangle = (\bar{\alpha}\langle 0| + \bar{\beta}\langle 1|)(\alpha|0\rangle + \cancel{\beta|1\rangle}\ \beta|1\rangle)$

$$= |\alpha|^2\langle 0|0\rangle + \bar{\alpha}\beta\langle 0|1\rangle +$$

$$\bar{\beta}\alpha\langle 1|0\rangle + |\beta|^2\langle 1|1\rangle.$$

Here, the mix terms $\langle 0|1\rangle$ & $\langle 1|0\rangle$ vanishes

bcoz,

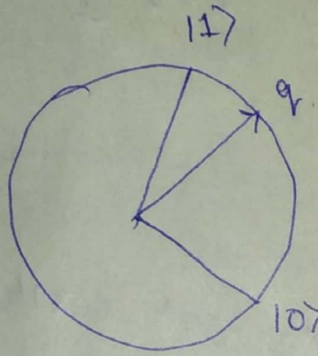
$$\langle 0|1\rangle = (\langle 0|)(|1\rangle) = [1\ 0]\begin{bmatrix}0\\1\end{bmatrix} = 0$$

similarly for $\langle 1|0\rangle$.

Here, $\langle q|q\rangle = 1 \Rightarrow |\alpha|^2 + |\beta|^2 = 1$.

(Here $\langle 0|0\rangle$ & $\langle 1|1\rangle = 1$)

given a state q.



70% towards $|1\rangle$.

$0.7 \leftarrow |\alpha|^2$.

30% towards $|0\rangle$.

$\underline{0.3} \uparrow |\beta|^2$

$$|q\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle.$$

Here $\alpha^2 = 1/2$ $\beta^2 = 1/2$.

State q is in between of $|0\rangle$ & $|1\rangle$

$|00\rangle = |0\rangle \otimes |0\rangle$

$= \begin{bmatrix}1\\0\end{bmatrix} \otimes \begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\0\\0\\0\end{bmatrix}$

$= \begin{pmatrix}1\begin{bmatrix}1\\0\end{bmatrix}\\0\begin{bmatrix}1\\0\end{bmatrix}\end{pmatrix} \rightarrow$

In a 1-quibit system.

$|q\rangle = \alpha|0\rangle + \beta|1\rangle \qquad \alpha, \beta \in \mathbb{C}$

Here $|q\rangle$ is normalized ∵

Conjugate transpose

Recall.

$\langle x|y\rangle = (\langle x|)(|y\rangle)$ is an inner product.

row vector, column vector.

A vector is normalized if its inner product with itself is 1.

i.e. $\langle q|q\rangle = 1$.

Note. $\Rightarrow \langle q| = \alpha\langle 0| + \beta\langle 1|$

Now; $\langle q|q\rangle = (\bar{\alpha}\langle 0| + \bar{\beta}\langle 1|)(\alpha|0\rangle + \beta|1\rangle)$

$= |\alpha|^2\langle 0|0\rangle + \bar{\alpha}\beta\langle 0|1\rangle +$
$\bar{\beta}\alpha\langle 1|0\rangle + |\beta|^2\langle 1|1\rangle.$

Here, the mix terms $\langle 0|1\rangle$ & $\langle 1|0\rangle$ ~~ca~~ vanishes
bcoz,

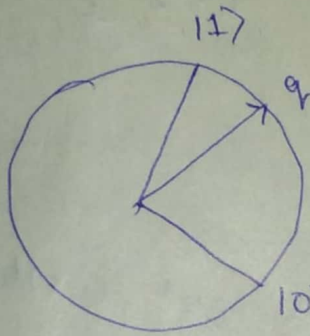
$$\langle 0|1\rangle = (\langle 0|)(|1\rangle) = [1\ 0]\begin{bmatrix}0\\1\end{bmatrix} = 0$$

Similarly for $\langle 1|0\rangle$.

Here, $\langle q|q\rangle = 1 \Rightarrow |\alpha|^2 + |\beta|^2 = 1.$

(Here $\langle 0|0\rangle$ & $\langle 1|1\rangle = 1$)

given a state q.



q ← 70% towards $|1\rangle$.

0.7 ← $|\alpha|^2$.

30% towards $|0\rangle$.

0.3

↑

$|\beta|^2$

$$|q\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle.$$

Here $\alpha^2 = 1/2 \quad \beta^2 = 1/2.$

State q is in between of $|0\rangle$ & $|1\rangle$

Bell States

2 qubit system. They are defined as,

$$|\beta_{00}\rangle \equiv \frac{1}{\sqrt{2}}\left[|00\rangle + |11\rangle\right].$$

$$|\beta_{01}\rangle \equiv \frac{1}{\sqrt{2}}\left[|01\rangle + |10\rangle\right].$$

$$|\beta_{10}\rangle \equiv \frac{1}{\sqrt{2}}\left[|00\rangle - |11\rangle\right].$$

$$|\beta_{11}\rangle \equiv \frac{1}{\sqrt{2}}\left[|01\rangle - |10\rangle\right].$$

---

1) Superposition state
2) entanglement state],

you can not write a tensor product state as a tensor product of 2 states → they are entangled with each other

---

These are called EPR pairs for Einstein, podolsky, Rosen.

These are called entangled states, i.e the states that can not be written as a tensor product of 2 1-qubit states.

$$|ab\rangle = |a\rangle \otimes |b\rangle$$

$$= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle + |0\rangle).$$

↳ this is not equal to any of above

4 equations.

Postulates of Quantum Mechanics. (4 Postulates)

Postulate-1)

Associated to any isolated physical system is a complex vector space with inner product ~~kno~~ known as the state space of the system. The system is completely described by its state vector.

Postulate-2)

Evolution of a quantum system

The evolution of a closed quantum system is described by a unitary transformation. That is, the state $|\psi\rangle$ of the system at the time t_1 is related to the state $|\psi'\rangle$ of the system at t_2 by a unitary operator U.

(a) $|\psi'\rangle = U|\psi\rangle$

for eg:- $|\psi\rangle = a|0\rangle + b|1\rangle$

$$U = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$U|\psi\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\left(a\begin{bmatrix} 1 \\ 0 \end{bmatrix} + b\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$$

$$= a\begin{bmatrix} 0 \\ 1 \end{bmatrix} + b\begin{bmatrix} 1 \\ 0 \end{bmatrix} = a|1\rangle + b|0\rangle$$

these unitary matrices work as quantum gates [similar to NOR, NAND etc in digital]

Postulate 3 (Measurement).

Quantum measurement are described by a collection $\{M_m\}$ of measurement operators. These are operators acting on the state space of the system being measured.

The index m refers to the measurement outcomes that may occur in the experiment.

If the state of the system is $|\psi\rangle$ immediately before the measurement, then the probability that result m occurs is given by

$$p(m) = \langle\psi| \underset{\substack{\uparrow \\ \text{transpose of } M}}{M_m^*} M_m |\psi\rangle$$

How?

After measurement $= M_m |\psi\rangle$

To calculate prob. :- Take inner product

$$\langle\psi| M_m^* M_m |\psi\rangle = |\alpha_m|^2.$$

M_m should eliminate other components & give result along with only one component

∴ we have $|\alpha_m|^2$ only, here.

~~$\Leftarrow \varphi$~~

$|\varphi_2\rangle = M|\psi\rangle$

but state vectors must be normalized.

$\therefore \quad |\varphi_2\rangle = \dfrac{M|\psi\rangle}{\langle \psi M^* | M \psi\rangle}$.

They satisfy the complete eq^n:-

$\cancel{\sum_m \langle\varphi | M_m^* M_m |\psi\rangle = I} \quad \cancel{\sum_m (M_m|\psi}$

$\cancel{1 = \sum_m p(m) = \sum_m \langle\psi | M_m^* M_m |\psi\rangle}$.

$\cancel{\sum_m (M_m|\psi\rangle)(M_m|\psi\rangle) = I}$

$\cancel{1 = \sum_m p(m) = \sum_m \langle\varphi | M_m^* M_m |\varphi\rangle}$.

$\sum_m (M_m |\psi\rangle \langle M_m | \psi) = I$.

$1 = \sum_m p(m) = \sum_m \langle\psi | M_m^* M_m |\psi\rangle$.

Eg:- $|q\rangle = \alpha|0\rangle + \beta|1\rangle$

Define:- $M_0 = |0\rangle\langle 0| = \begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}1 & 0\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}$

$M_1 = |1\rangle\langle 1| = \begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}0 & 1\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}$.

Measure the outcome 1:-

$$M_1|q\rangle = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\left(\alpha\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$$

$$= \beta|1\rangle$$

probability of getting out come 1)

$$\cancel{|\psi_2\rangle = \frac{M|\psi\rangle}{\langle\psi M^* M|\psi\rangle}}$$

$$p(1) = \langle q | M_1^* M_1 | q\rangle$$

$$= \left(\beta^* \langle 1|\right)\left(\beta|1\rangle\right)$$

$$= \beta^* \beta = |\beta|^2 .$$

Postulate - 4) (Multi - qubit system)

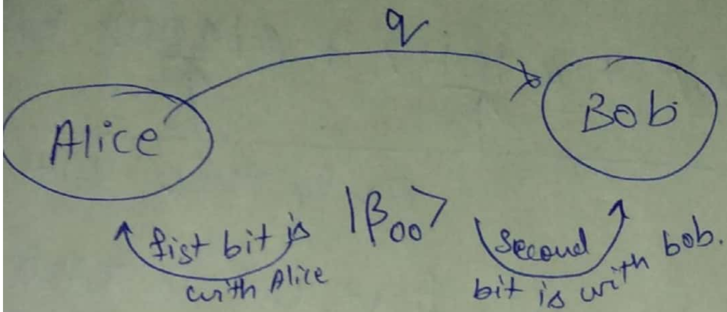
The state space of a composite physical system is the tensor product of the state spaces of the component - physical system.

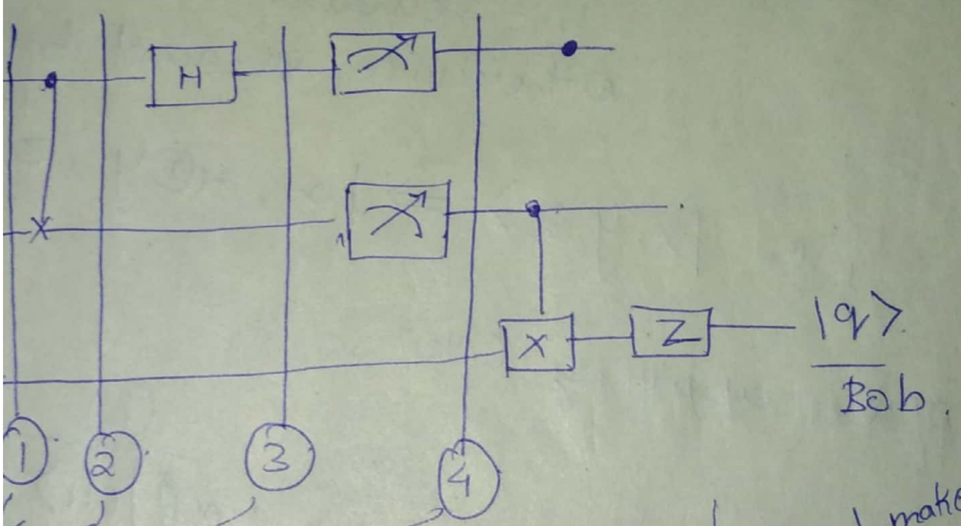
$$|01\rangle \otimes |1\rangle$$

$$|01\rangle \otimes |01\rangle$$

antum Teleportation

ansmit an unknown qubit instan-
ously over any ~~dt~~ distance.





teps.

step-0) make a entangled pair & give one bit to Alice other to bob.

But Alice & bob think they have both the bits.

Initial state.

$$|q\rangle \otimes |\beta_{00}\rangle$$

$$= \alpha|0\rangle + \beta|1\rangle$$

$$) \; |\beta_{00}\rangle = \left(\alpha|0\rangle + \beta|1\rangle\right) \otimes \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

$$\frac{1}{\sqrt{2}}\left(\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle\right)$$

Step-2)

Alice applies CNOT to her two qubit system

alice ~~says~~ knowns

$$\frac{1}{2}\left(\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle\right)$$

only these bits.

CNOT says:- if first bit is zero second bit remains as it is, otherwise second bit flips.

i.e. $|x, y, z\rangle \Rightarrow |x, x \oplus y, z\rangle$

After applying CNOT

$$|q\rangle \otimes |\beta_{00}\rangle = \frac{1}{\sqrt{2}}\Big(\overset{(a)}{\overline{\alpha|000\rangle}} + \overset{(b)}{\overline{\alpha|011\rangle}} + \underset{(c)}{\underline{\beta|110\rangle}} + \underset{(d)}{\underline{\beta|101\rangle}}\Big).$$

Hadamart gate

$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} = +$$

$$H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}} = -$$

(a) → after Ha

$|000\rangle$ —

(b) $|011\rangle$ —

(c) $|110\rangle$

(d) $|101\rangle$

Hence:-

$\frac{1}{2}(\alpha|$

a) → after Hadamard becomes.

$|000\rangle \rightarrow \frac{1}{\sqrt{2}}(|000\rangle + |100\rangle)$

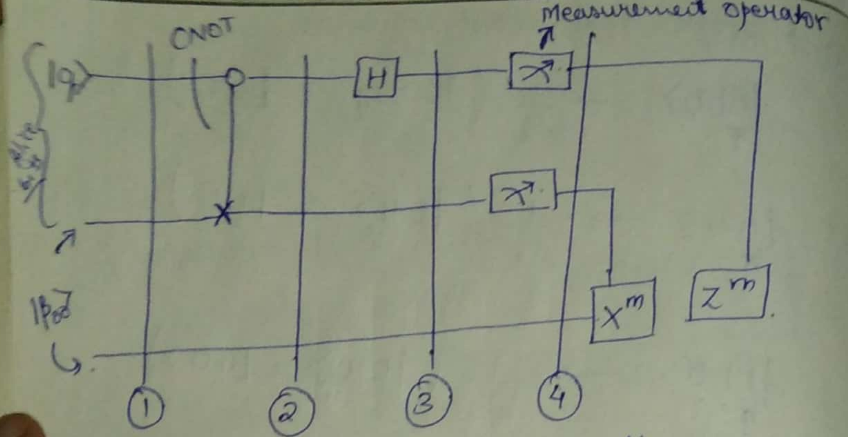
b) $|011\rangle \rightarrow \frac{1}{\sqrt{2}}(|011\rangle + |111\rangle)$

c) $|110\rangle \rightarrow \frac{1}{\sqrt{2}}(|010\rangle - |110\rangle)$

d) $|101\rangle \rightarrow \frac{1}{\sqrt{2}}(|001\rangle - |101\rangle)$.

Hence:-

$$\frac{1}{2}(\alpha|000\rangle + \alpha|100\rangle + \alpha|011\rangle + \alpha|111\rangle + \beta|010\rangle - \beta|110\rangle + \beta|001\rangle - \beta|101\rangle)$$



Circuit for Quantum teleportation.

1) $|q\rangle \otimes |\beta_{00}\rangle = \frac{1}{\sqrt{2}}\left(\alpha|\underline{0}00\rangle + \alpha|\underline{0}11\rangle + \beta|\underline{1}00\rangle + \beta|\underline{1}11\rangle\right)$

associated with alice.

$q = \alpha|0\rangle + \beta|1\rangle$

2) Apply CNOT.

$\frac{1}{\sqrt{2}}\left(\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle + \beta|101\rangle\right)$

3) Hadmard gate

$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$

$H\begin{bmatrix}1\\0\end{bmatrix} = \frac{1}{\sqrt{2}}\left(\begin{bmatrix}1\\0\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix}\right)$

$\underbrace{\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}}_{H}\begin{bmatrix}1\\0\end{bmatrix} = \frac{1}{\sqrt{2}}\left(\begin{bmatrix}1\\1\end{bmatrix}\right)$.

Check $\Rightarrow H^T H = I$

as H is unitary

i) Measurement operator → to measure $|00\rangle$ etc.

Outer product is used here.

$$M_{|00\rangle} = |00\rangle\langle 00|$$

$$= \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M_{|00\rangle} |00\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$M_{|00\rangle}$ measurement operator applied on $|00\rangle$ does not change anything but on any other it would give different measurement.

$$M_{|00\rangle} |01\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\rightarrow |0\rangle \otimes |1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

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given a boolean function how many steps do you need to determine if the function is constant or not ⇒ 2

Classically:- 2 steps are needed to find whether f is constant or not,

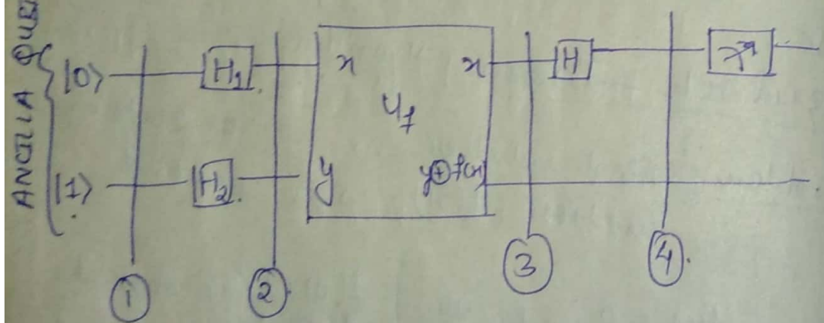
using parallel computing this can be done in 1 step. and 2 circuits.

given f is binary, where x, the input is also binary.

Assume, a unitary operator U_f that achieves

$$U_f |x, y\rangle = |x, y \oplus f(x)\rangle$$

$\oplus$ ← exclusive OR.



1) $|\varphi_1\rangle = |0\rangle \otimes |1\rangle = |01\rangle$

2) Apply H to both qubits.

$|\varphi_2\rangle = H_1 H_2 |\varphi_1\rangle = H_1|0\rangle \otimes H_2 \ldots$

$= |+\rangle \otimes |-\rangle = \frac{1}{2}\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}} \otimes \frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)$

↓ introduced To normalize.

$= \frac{1}{4}\left(|00\rangle - |01\rangle + |10\rangle - |11\rangle\right)$

3) $U_f |\varphi_2\rangle = \frac{1}{4}\Big(|0, 0 \oplus f(0)\rangle - |0, 1 \oplus f(0)\rangle + |1, 0 \oplus f(1)\rangle - |1, 1 \oplus f(1)\rangle\Big)$

⇓

$|\varphi_3\rangle$

4) Apply H:

~~$|\varphi_3\rangle$~~ =

$|\varphi_4\rangle = H|\varphi_3\rangle$

$= \frac{1}{4}\left(\cancel{\sqrt{2}}\right)\Big(|+, 0 \oplus f(0)\rangle - |+, 1 \oplus f(0)\rangle + |-, 0 \oplus f(1)\rangle - |-, 1 \oplus f(1)\rangle\Big)$.

$= \frac{1}{4\sqrt{2}}\Big[|0, 0 \oplus f(0)\rangle + |1, 0 \oplus f(0)\rangle - |0, 1 \oplus f(0)\rangle - |1, 1 \oplus f(0)\rangle + |0, 0 \oplus f(1)\rangle - |1, 0 \oplus f(1)\rangle - |0, 1 \oplus f(1)\rangle + |1, 1 \oplus f(1)\rangle\Big)$.

$$\frac{1}{4\sqrt{2}}\Big[\ \big(|0 \oplus f(0)\rangle + |0 \oplus f(1)\rangle - |1 \oplus f(1)\rangle$$
$$- |1 \oplus f(0)\rangle \ \cancel{- |1 \oplus f(1)\rangle} \big)$$
$$+ |1\rangle \big(|0 \oplus f(0)\rangle - |0 \oplus f(1)\rangle - |1 \oplus f(0)\rangle$$
$$+ |1 \oplus f(1)\rangle \big) \Big]$$

Two Cases

a) $a = f(0) = f(1)$.

b) $a = f(0) \neq f(1) = \bar{a}$

(a)

$$|\varphi_4\rangle_{f(0)=f(1)} = \frac{1}{2\sqrt{2}} \Big[|0\rangle \big(|a\rangle + |a\rangle - |\bar{a}\rangle - |\bar{a}\rangle \big)$$
$$+ |1\rangle \big(\cancel{|a\rangle} - \cancel{|a\rangle} - \cancel{|\bar{a}\rangle} + \cancel{|\bar{a}\rangle} \big) \Big]$$

$$= \frac{1}{\cancel{2}\sqrt{2}} \times \cancel{2} \Big[|0\rangle \big(|a\rangle - |\bar{a}\rangle \big) \Big]$$

$$= \frac{1}{2\sqrt{2}} \Big[|0\rangle \big(|a\rangle - |\bar{a}\rangle \big) \Big]$$

(b) $$|\varphi_4\rangle_{f(0) \neq f(1)} = \frac{|1\rangle \big(|a\rangle - |\bar{a}\rangle \big)}{2\sqrt{2}}$$

We note that

$f(0) \oplus f(1) = 0$ if $f(0) = f(1)$

$f(0) \oplus f(1) = 1$ if $f(0) \neq f(1)$

using this property

$$|a\rangle - |\bar{a}\rangle = \pm \big(|0\rangle - |1\rangle \big)$$

$$|\varphi_4\rangle = |f(0) \oplus f(1)\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

After Mid 1 uptil now.

+ LU & QR