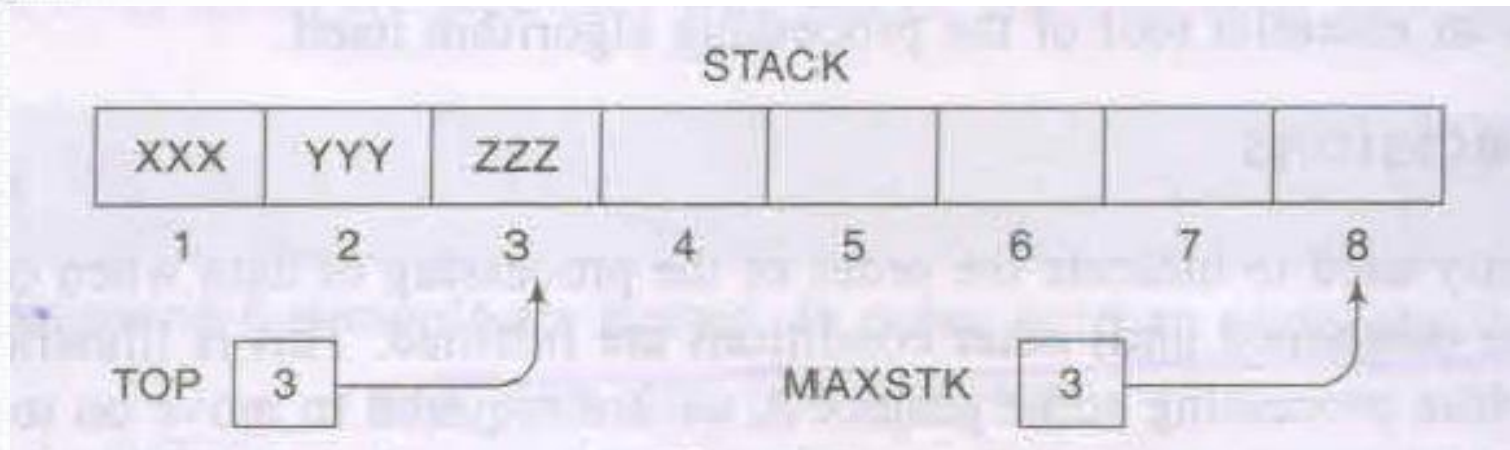




# **Stacks and queues**



# Array Representation of Stack



- Stack Overflow
- Stack Underflow



# PUSH and POP Operations

**PUSH(STACK, TOP, MAXSTK, ITEM)**

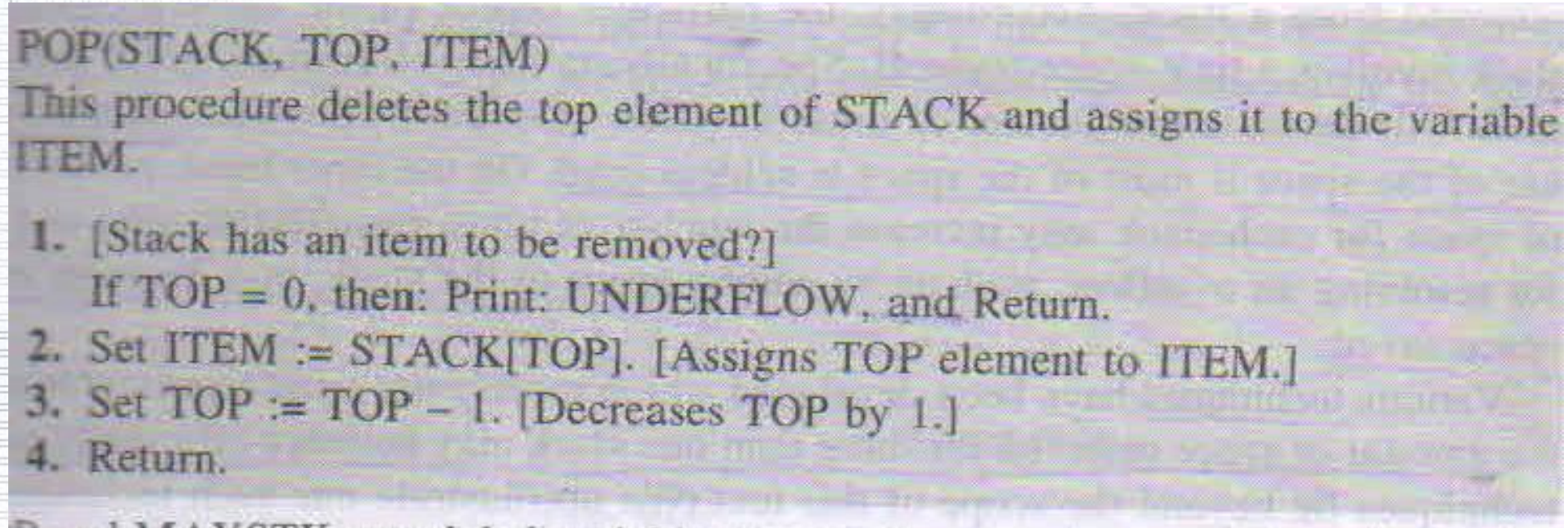
This procedure pushes an ITEM onto a stack.

1. [Stack already filled?]  
If  $TOP = MAXSTK$ , then: Print: OVERFLOW, and Return.
2. Set  $TOP := TOP + 1$ . [Increases TOP by 1.]
3. Set  $STACK[TOP] := ITEM$ . [Inserts ITEM in new TOP position.]
4. Return.





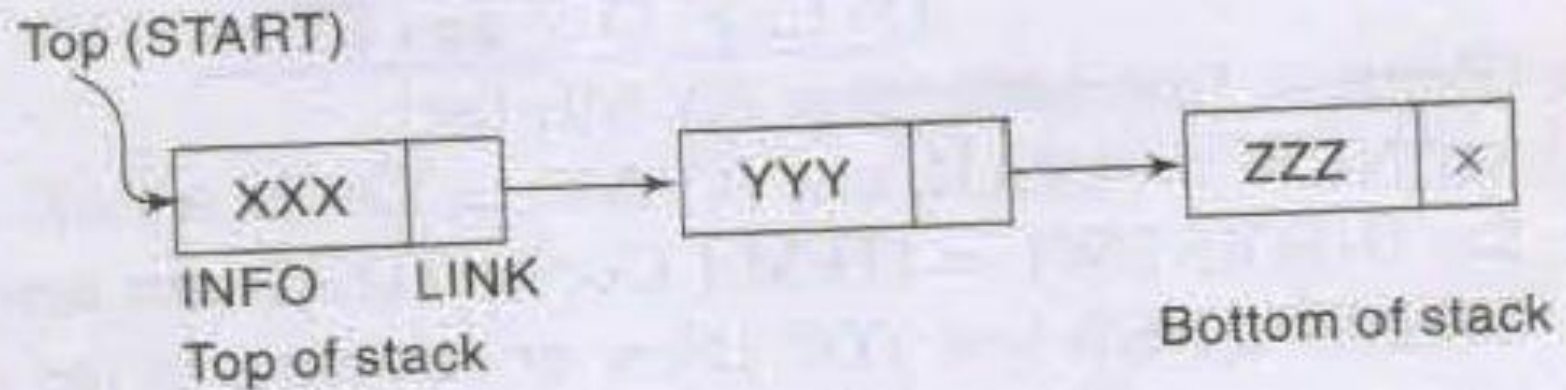
# PUSH and POP Operations



Note: The value of TOP is changed before the insertion in PUSH  
but the value of TOP is changed after deletion in POP



# Linked representation of STACK





# Linked representation of STACK

Push 'WWW' into STACK  
STACK before Push operation:



STACK after Push operation

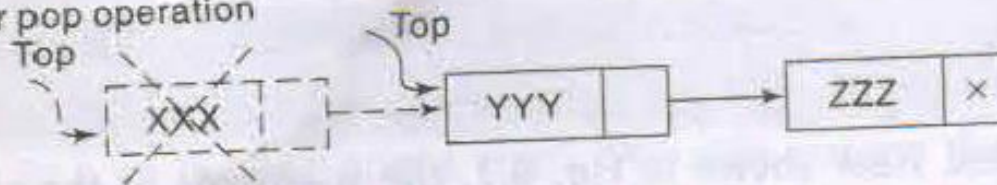


Fig. 6.8

Pop from STACK  
STACK before pop operation:



STACK after pop operation







# Linked representation of STACK

PUSH\_LINKSTACK(INFO, LINK, TOP, AVAIL, ITEM)

This procedure pushes an ITEM into a linked stack

1. [Available space?] If  $AVAIL = NULL$ , then Write OVERFLOW and Exit
2. [Remove first node from AVAIL list]  
Set  $NEW := AVAIL$  and  $AVAIL := LINK[AVAIL]$ .
3. Set  $INFO[NEW] := ITEM$  [Copies ITEM into new node]
4. Set  $LINK[NEW] := TOP$  [New node points to the original top node in the stack]
5. Set  $TOP = NEW$  [Reset TOP to point to the new node at the top of the stack]
6. Exit.





# Linked representation of STACK

POP\_LINKSTACK(INFO, LINK, TOP, AVAIL, ITEM)

This procedure deletes the top element of a linked stack and assigns it to the variable ITEM

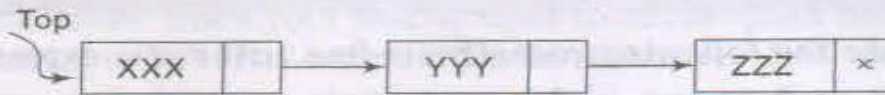
1. [Stack has an item to be removed?]  
IF TOP = NULL then Write: UNDERFLOW and Exit.
2. Set ITEM := INFO[TOP] [Copies the top element of stack into ITEM ]
3. Set TEMP := TOP and TOP = LINK[TOP]  
[Remember the old value of the TOP pointer in TEMP  
and reset TOP to point to the next element in the stack ]
4. [Return deleted node to the AVAIL list]  
Set LINK[TEMP] = AVAIL and AVAIL = TEMP.
5. Exit.





# Linked representation of STACK

(i) Push BBB (ii) Pop (iii) Pop (iv) Push MMM  
Original linked stack:



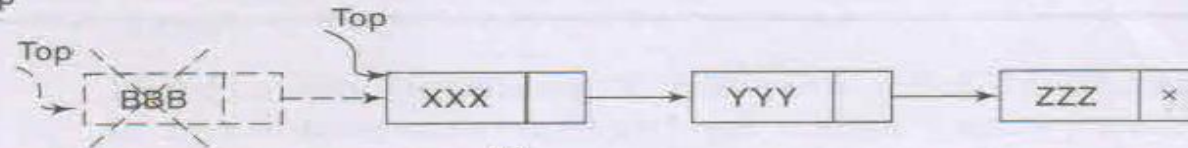
(a)

(i) Push BBB



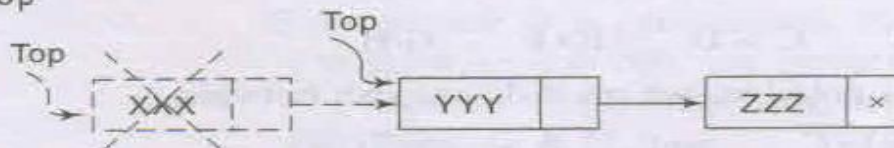
(b)

(ii) Pop



(c)

(iii) Pop



(d)

(iv) Push MMM



(e)



# QUICKSORT, an application of STACK

Suppose A is the following list of 12 numbers:

(44), 33, 11, 55, 77, 90, 40, 60, 99, 22, 88, (66)

(22), 33, 11, 55, 77, 90, 40, 60, 99, (44), 88, 66

22, 33, 11, (44), 77, 90, 40, 60, 99, (55), 88, 66

22, 33, 11, (40), 77, 90, (44), 60, 99, 55, 88, 66

22, 33, 11, (40), 44, 90, (77), 60, 99, 55, 88, 66

22, 33, 11, 40, (44), 90, 77, 60, 99, 55, 88, 66

First sublist

Second sublist





# Polish Notation: Evaluation of Postfix expression

: This algorithm finds the VALUE of an arithmetic expression P written in postfix notation.

1. Add a right parenthesis “)” at the end of P. [This acts as a sentinel.]
2. Scan P from left to right and repeat Steps 3 and 4 for each element of P until the sentinel “)” is encountered.
3. If an operand is encountered, put it on STACK.
4. If an operator  $\otimes$  is encountered, then:
  - (a) Remove the two top elements of STACK, where A is the top element and B is the next-to-top element.
  - (b) Evaluate  $B \otimes A$ .
  - (c) Place the result of (b) back on STACK.[End of If structure.]
- [End of Step 2 loop.]
5. Set VALUE equal to the top element on STACK.
6. Exit.



# Polish Notation: Evaluation of Postfix expression

<i>Symbol Scanned</i>		<i>STACK</i>
(1)	5	5
(2)	6	5, 6
(3)	2	5, 6, 2
(4)	+	5, 8
(5)	*	40
(6)	12	40, 12
(7)	4	40, 12, 4
(8)	/	40, 3
(9)	-	37
(10)	)	

$5 * ( 6 + 2 ) - 12 / 4$

P: 5, 6, 2, +, \*, 12, 4, /, -





# Polish Notation: Transforming Infix into Postfix

## POLISH(Q, P)

Suppose Q is an arithmetic expression written in infix notation. This algorithm finds the equivalent postfix expression P.

1. Push "(" onto STACK, and add ")" to the end of Q.
2. Scan Q from left to right and repeat Steps 3 to 6 for each element of Q until the STACK is empty:
3. If an operand is encountered, add it to P.
4. If a left parenthesis is encountered, push it onto STACK.
5. If an operator  $\otimes$  is encountered, then:



# Polish Notation: Transforming Infix into Postfix

(a) Repeatedly pop from STACK and add to P each operator (on the top of STACK) which has the same precedence as or higher precedence than  $\otimes$ .

(b) Add  $\otimes$  to STACK.

[End of If structure.]

6. If a right parenthesis is encountered, then:

(a) Repeatedly pop from STACK and add to P each operator (on the top of STACK) until a left parenthesis is encountered.

(b) Remove the left parenthesis. [Do not add the left parenthesis to P.]

[End of If structure.]

[End of Step 2 loop.]

7. Exit.



# Polish Notation: Transforming Infix into Postfix

Symbol Scanned		STACK	Expression P
(1)	A	(	A
(2)	+	( +	A
(3)	(	( + (	A
(4)	B	( + (	A B
(5)	*	( + ( *	A B
(6)	C	( + ( *	A B C
(7)	-	( + ( -	A B C *
(8)	(	( + ( - (	A B C *
(9)	D	( + ( - (	A B C * D
(10)	/	( + ( - ( /	A B C * D
(11)	E	( + ( - ( /	A B C * D E
(12)	↑	( + ( - ( / ↑	A B C * D E E
(13)	F	( + ( - ( / ↑	A B C * D E E F
(14)	)	( + ( -	A B C * D E E F ↑ /
(15)	*	( + ( - *	A B C * D E E F ↑ /
(16)	G	( + ( - *	A B C * D E E F ↑ / G
(17)	)	( +	A B C * D E E F ↑ / G * -
(18)	*	( + *	A B C * D E E F ↑ / G * -
(19)	H	( + *	A B C * D E E F ↑ / G * - H
(20)	)		A B C * D E E F ↑ / G * - H * +

$A + (B * C - (D / E \uparrow F) * G) * H$

A B C \* D E F ↑ / G \* - H \* +



# Queues

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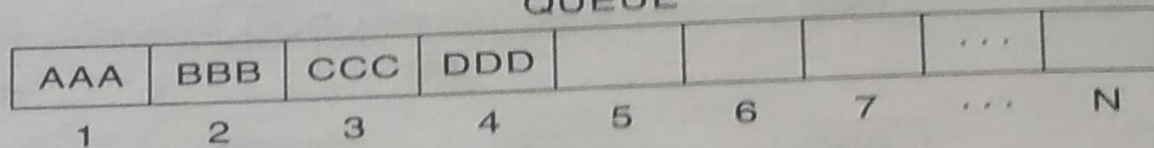
- Linear DS
- FIFO
- Can be implemented using Arrays or LL
- Front Pointer
- Rear Pointer





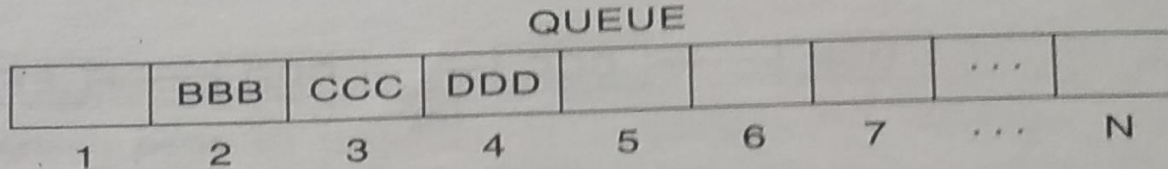
# Queues : Array Representation

FRONT: 1  
REAR: 4



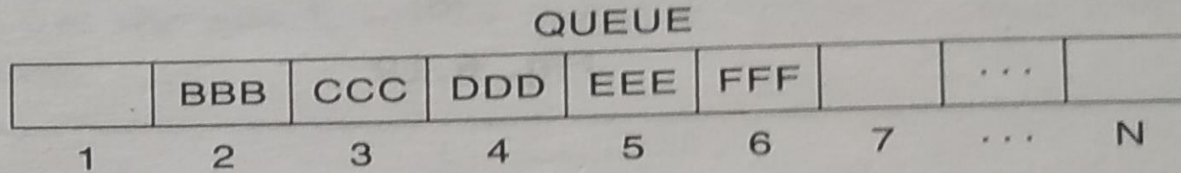
(a)

FRONT: 2  
REAR: 4



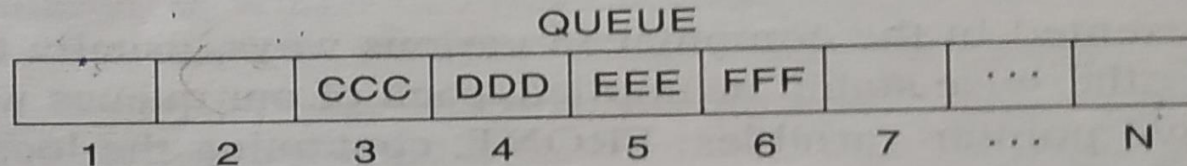
(b)

FRONT: 2  
REAR: 6



(c)

FRONT: 3  
REAR: 6





# Queues : Circular Array Representation

(a) Initially empty:	FRONT: 0 REAR: 0	<table><tr><th colspan="5">QUEUE</th></tr><tr><td></td><td></td><td></td><td></td><td></td></tr><tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>A</td><td>B</td><td>C</td><td></td><td></td></tr></table>	QUEUE										1	2	3	4	5	A	B	C		
QUEUE																						
1	2	3	4	5																		
A	B	C																				
(b) A, B and then C inserted:	FRONT: 1 REAR: 3	<table><tr><td></td><td>B</td><td>C</td><td></td><td></td></tr></table>		B	C																	
	B	C																				
(c) A deleted:	FRONT: 2 REAR: 3	<table><tr><td></td><td>B</td><td>C</td><td></td><td></td></tr></table>		B	C																	
	B	C																				
(d) D and then E inserted:	FRONT: 2 REAR: 5	<table><tr><td></td><td>B</td><td>C</td><td>D</td><td>E</td></tr></table>		B	C	D	E															
	B	C	D	E																		
(e) B and C deleted:	FRONT: 4 REAR: 5	<table><tr><td></td><td></td><td></td><td>D</td><td>E</td></tr></table>				D	E															
			D	E																		
(f) F Inserted:	FRONT: 4 REAR: 1	<table><tr><td>F</td><td></td><td></td><td>D</td><td>E</td></tr></table>	F			D	E															
F			D	E																		
(g) D deleted:	FRONT: 5 REAR: 1	<table><tr><td>F</td><td></td><td></td><td></td><td>E</td></tr></table>	F				E															
F				E																		
(h) G and then H inserted:	FRONT: 5 REAR: 3	<table><tr><td>F</td><td>G</td><td>H</td><td></td><td>E</td></tr></table>	F	G	H		E															
F	G	H		E																		
(i) E deleted:	FRONT: 1 REAR: 3	<table><tr><td>F</td><td>G</td><td>H</td><td></td><td></td></tr></table>	F	G	H																	
F	G	H																				
(j) F deleted:	FRONT: 2 REAR: 3	<table><tr><td></td><td>G</td><td>H</td><td></td><td></td></tr></table>		G	H																	
	G	H																				
(k) K inserted:	FRONT: 2 REAR: 4	<table><tr><td></td><td>G</td><td>H</td><td>K</td><td></td></tr></table>		G	H	K																
	G	H	K																			
(l) G and H deleted:	FRONT: 4 REAR: 4	<table><tr><td></td><td></td><td></td><td>K</td><td></td></tr></table>				K																
			K																			
(m) K deleted, QUEUE empty:	FRONT: 0 REAR: 0	<table><tr><td></td><td></td><td></td><td></td><td></td></tr></table>																				





# Queues : Insert

QINSERT(Queue, N, FRONT, REAR, ITEM)

This procedure inserts an element ITEM into a queue.

1. [Queue already filled?]

If  $FRONT = 1$  and  $REAR = N$ , or if  $FRONT = REAR + 1$ , then:

Write: OVERFLOW, and Return.

2. [Find new value of REAR.]

If  $FRONT := NULL$ , then: [Queue initially empty.]

Set  $FRONT := 1$  and  $REAR := 1$ .

Else if  $REAR = N$ , then:

Set  $REAR := 1$ .

Else:

Set  $REAR := REAR + 1$ .

[End of If structure.]

3. Set  $QUEUE[REAR] := ITEM$ . [This inserts new element.]

4. Return.



# Queues : Delete

QDELETE(Queue, N, FRONT, REAR, ITEM)

This procedure deletes an element from a queue and assigns it to the variable ITEM.

1. [Queue already empty?]

    If FRONT := NULL, then: Write: UNDERFLOW, and Return.

2. Set ITEM := QUEUE[FRONT].

3. [Find new value of FRONT.]

    If FRONT = REAR, then: [Queue has only one element to start.]

        Set FRONT := NULL and REAR := NULL.

    Else if FRONT = N, then:

        Set FRONT := 1.

    Else:

        Set FRONT := FRONT + 1.

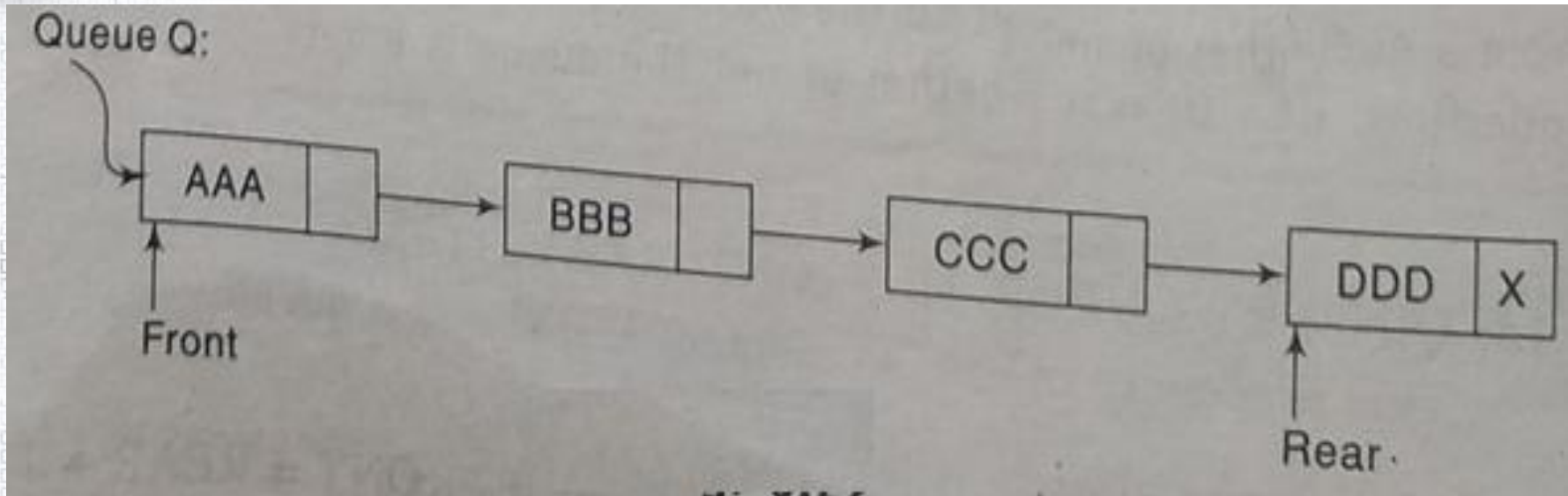
    [End of If structure.]

4. Return.





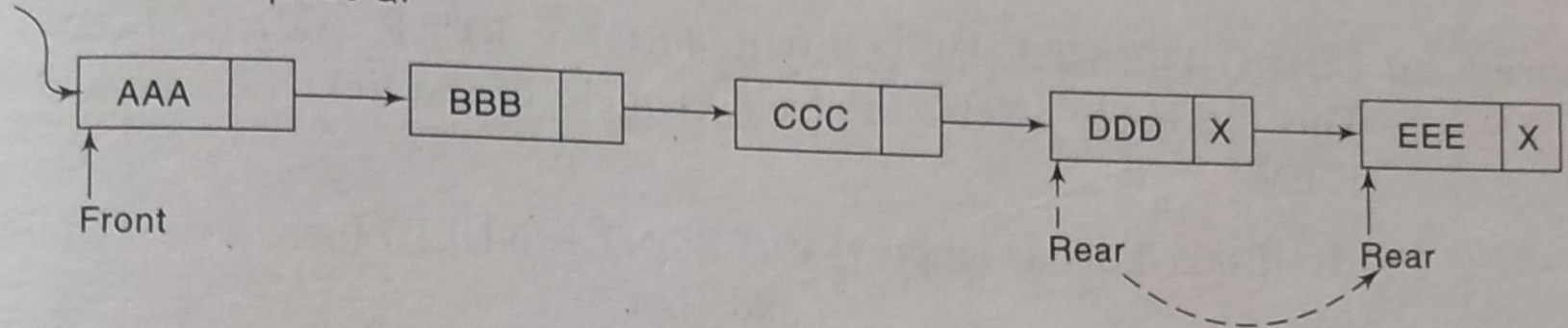
# Queues : Linked Representation



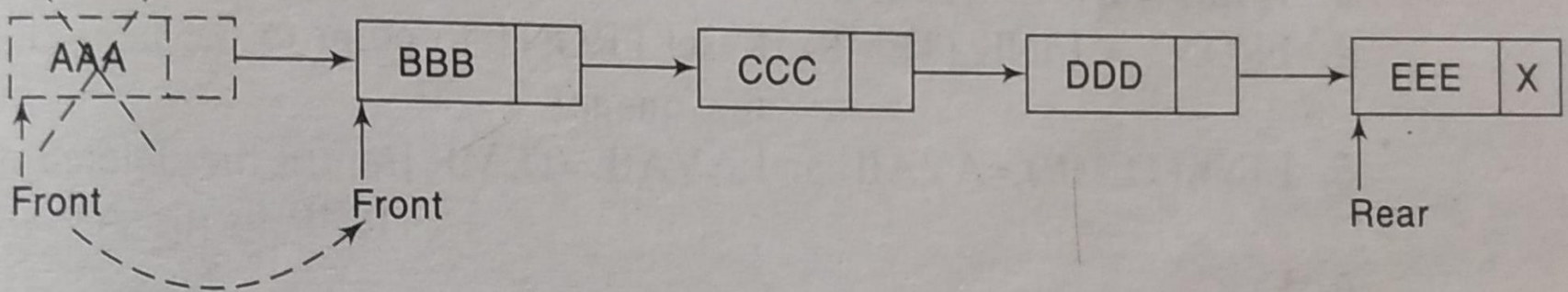


# Queues : Linked Representation- Operations

Insert 'EEE' into queue Q:



Delete from queue Q







# Queues : Linked Representation- Insert

LINKQ\_INSERT(INFO, LINK, FRONT, REAR, AVAIL, ITEM)

This procedure inserts an ITEM into a linked queue

1. [Available space?] If  $AVAIL = NULL$ , then Write OVERFLOW and Exit
2. [Remove first node from AVAIL list]  
Set  $NEW := AVAIL$  and  $AVAIL := LINK[AVAIL]$
3. Set  $INFO[NEW] := ITEM$  and  $LINK[NEW] = NULL$   
[Copies ITEM into new node]
4. If  $(FRONT = NULL)$  then  $FRONT = REAR = NEW$   
[If Q is empty then ITEM is the first element in the queue Q]  
else set  $LINK[REAR] := NEW$  and  $REAR = NEW$   
[REAR points to the new node appended to the end of the list]
5. Exit.



# Queues : Linked Representation- Delete

LINKQ\_DELETE (INFO, LINK, FRONT, REAR, AVAIL, ITEM)

This procedure deletes the front element of the linked queue and stores it in ITEM

1. [Linked queue empty?] if (FRONT = NULL) then Write: UNDERFLOW and Exit
2. Set TEMP = FRONT [If linked queue is nonempty, remember FRONT in a temporary variable TEMP]
3. ITEM = INFO (TEMP)
4. FRONT = LINK (TEMP) [Reset FRONT to point to the next element in the queue]
5. LINK(TEMP) = AVAIL and AVAIL = TEMP [return the deleted node TEMP to the AVAIL list]
6. Exit.





# Queues : Deques

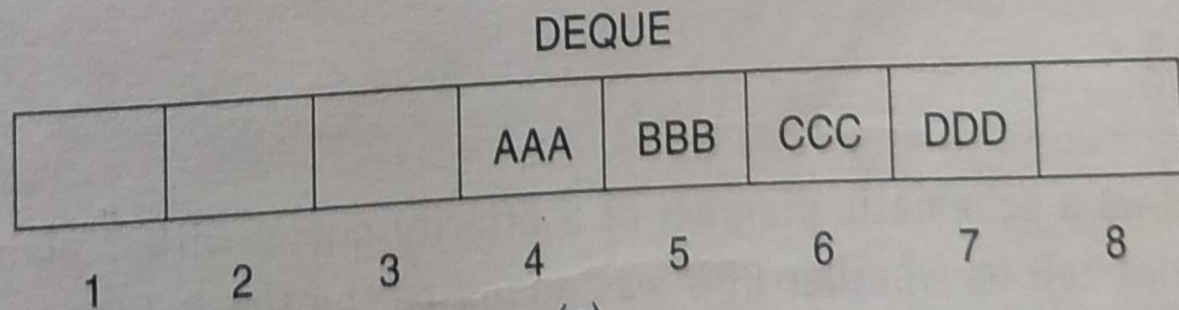
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- Double ended queue
- Elements can be added or removed at either end but not in the middle
- Representation using circular array DEQUE with pointer LEFT and RIGHT
- Two variants of deque, Input-restricted deque and Output-restricted deque



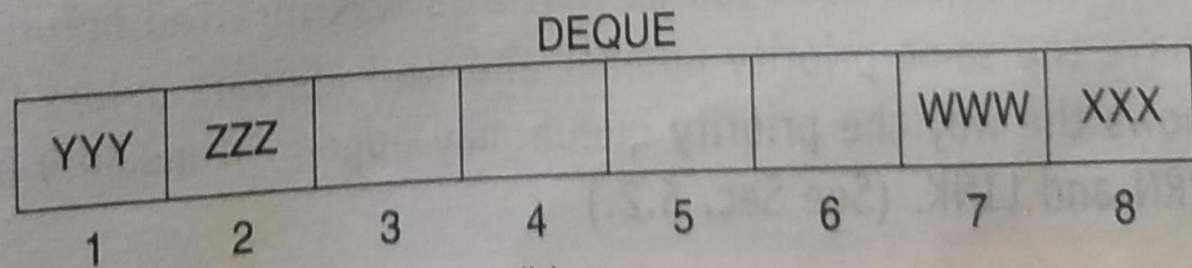
# Queues : Deques

LEFT: 4  
RIGHT: 7



(a)

LEFT: 7  
RIGHT: 2



(b)





# Queues : Priority Queue

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