* BAYE'S THEOREM

If $E_1, E_2 \cdots E_n$ are mutually exclusive and exhaustive events with $P(E_i) \neq 0$ $(i = 1, 2, \cdots n)$ of a random experiment then for any arbitrary event $A$ of the sample space of the above experiment with $P(A) > 0$, we have

$$P(E_i/A) = \frac{P(E_i)\,P(A|E_i)}{\sum_{i=1}^{n} P(E_i)\,P(A/E_i)}$$

proof:- Let $S$ be the sample space of the random experiment

The events $E_1, E_2 \cdots E_n$ being exhaustive

$$S = E_1 \cup E_2 \cup \cdots \cup E_n$$

$$A = A \cap S \qquad (\because A \subset S)$$

$$= A \cap (E_1 \cup E_2 \cdots \cup E_n)$$

$$= (A \cap E_1) \cup (A \cap E_2) \cup \cdots \cup (A \cap E_n)$$

$$P(A) = P(A \cap E_1) + P(A \cap E_2) + \cdots + P(A \cap E_n)$$

$$= P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) + \cdots P(E_n) \cdot P(A|E_n)$$

$$= \sum_{i=1}^{n} P(E_i)\,P(A/E_i)$$

Now $P(A \cap E_i) = P(A) \cdot P(E_i/A)$

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$$\Rightarrow P(E_i/A) = \frac{P(A \cap E_i)}{P(A)}$$

$$= \frac{P(E_i)\,P(A/E_i)}{\sum_{i=1}^{n} P(E_i)\,P(A/E_i)}$$

Ex. ① A bag X contains 2 white and 3 red balls and a bag Y contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag Y.

Sol$^n$. Let $E_1$: the ball is drawn from bag X

$E_2$: The ball is drawn from bag Y

$A$: the ball is red

we have to find $P(E_2/A)$

By Baye's theorem.

$$P(E_2/A) = \frac{P(E_2)\,P(A/E_2)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)} \quad \text{--- (1)}$$

Since the two bags are equally likely to be selected.

$$P(E_1) = P(E_2) = \frac{1}{2}$$

Also $P(A/E_1) = P(\text{a red ball is drawn from bag X}) = \frac{3}{5}$

$P(A/E_2) = P(\text{a red ball is drawn from bag Y}) = \frac{5}{9}$

$\therefore$ From (1) we have.

$$P(E_2/A) = \frac{\frac{1}{2}\times\frac{5}{9}}{\frac{1}{2}\times\frac{3}{5}+\frac{1}{2}\times\frac{5}{9}} = \frac{25}{52}$$

Ex. (2) In a bolt factory machine A, B, C manufactures respectively 25 %, 35 % and 40 % of the total, of their outputs 5, 4 and 2 percent are defective balls. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine B.

→ Let $E_1, E_2, E_3$ denote the events that a bolt selected at random is manufactured by the machines A, B and C respectively and let H denote the event of it's being defective. Then

$P(E_1) = 0.25$, $P(E_2) = 0.35$, $P(E_3) = 0.40$

The prob. of drawing defective bolt manufactured by machine A is

$P(H/E_1) = 0.05$

$\text{||}^{ly}$ $P(H/E_2) = 0.04$

$P(H|E_3) = 0.02$

By Baye's theorem we have

$$P(E_2/H) = \frac{P(E_2)\,P(H/E_2)}{P(E_1)\cdot P(H/E_1) + P(E_2)\cdot P(H/E_2) + P(E_3)\cdot P(H/E_3)}$$

$$= \frac{0.35 \times 0.04}{0.25\times 0.05 + 0.35 \times 0.04 + 0.40 \times 0.02}$$

$$= \frac{0.0140}{0.0345} = 0.41$$

Ex ③ The contents of urns I, II, III are as follows

1 white, 2 black and 3 red balls

2 white 1 black and 1 red balls &

4 white 5 black and 3 red balls

One urn is chosen at random and two balls drawn. They happen to be white and red. What is the probability that they come from urns I, II or III?

Soln: $E_1$: urn I is chosen

$E_2$: Urn II is chosen

$E_3$: urn III is "

A : the two balls are white & red

We have to find

$P(E_1/A)$, $P(E_2/A)$ & $P(E_3/A)$

$\therefore P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

$P(A/E_1) = P($a white and a red ball are drawn from urn I$)$

$$= \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2} = \frac{1}{5}$$

$$P(A/E_2) = \frac{{}^2C_1 \times {}^1C_1}{{}^4C_2} = \frac{1}{3}$$

$$P(A/E_3) = \frac{{}^4C_1 \times {}^3C_1}{{}^{12}C_2} = \frac{2}{11}$$

By Baye's theorem We have

$$P(E_1/A) = \frac{P(E_1)\,P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3)\,P(A/E_3)}$$

$$= \frac{\frac{1}{3} \times \frac{1}{5}}{\frac{1}{3} \times \frac{1}{5} + \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{11}}$$

$$= \frac{33}{118}$$

IIly $P(E_2/A) = \frac{55}{118}$, $P(E_3/A) = \frac{15}{59}$

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