(2)

Ex: (3) A random variable x has the follow
Probability function

x	:	1	2	3	4	5	6	7
$P(X=x)$	:	K	$2K$	$3K$	K^2	K^2+K	$2K^2$	$4K^2$

Find (i) K ii) $P(X<5)$
iii). $P(X>5)$

Sol^n Since $\Sigma P(x_i) = 1$

$$K + 2K + 3K + K^2 + K^2 + K + 2K^2 + 4K^2 =$$

$$8K^2 + 7K - 1 = 0$$

$$(8K-1)(K+1) = 0$$

ii) $P(X>5) = P(X=6) + P(X=7)$

$$= \frac{2}{64} + \frac{4}{64} = \frac{6}{64} = \frac{3}{32}$$

* Solve the following examples.

① If the probability density function $P(X=x)$ of a discrete random variate which assumes values x_1, x_2, x_3 such that $P(x_1) = 2P(x_2) = 3P(x_3)$ obtain the probability distribution of X

② The probability density function of a random variable X is

X :	0	1	2	3	4	5	6
$P(X=x)$:	K	$3K$	$5K$	$7K$	$9K$	$11K$	$13K$

Find $P(X<4)$, $P(3<X\le 6)$

③ A discrete random variable X has the following probability distribution

* Probability density function of a contin[uous] random variable X

A continuous function $y = f(x)$ such that

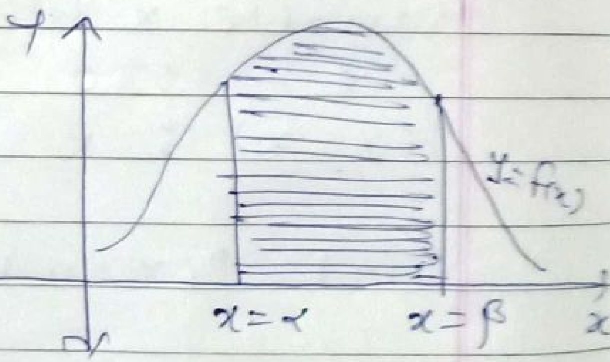
i) $f(x)$ is integrable

ii) $f(x) \geq 0 \quad \forall x$

iii) $\int_a^b f(x)\,dx = 1$ if $x \in [a, b]$ &

iv) $\int_\alpha^\beta f(x)\,dx = P(\alpha \leq x \leq \beta)$ where $a < \alpha < \beta < b$

is called probability density function of continuous random variable X



Thus for continuous random variable x,

$$P(\alpha \leq x \leq \beta) = \int_\alpha^\beta f(x)\,dx$$

represents the area under the curve $y=f(x)$ ~~and~~ between the ordinates $x=\alpha$ to $x=\beta$

* Properties of probability Density function

The probability density function $f(x)$ has the following properties.

① $f(x) > 0$, $-\infty < x < \infty$ (i.e. the curve $y = f(x)$ lies above the x-axis in the first & second quadrants only

ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$ (i.e. the total area under the curve & the x-axis is one)

iii) The probability that $\alpha \le X \le \beta$ is given by $P(\alpha \le X \le \beta) = \int_{\alpha}^{\beta} f(x)\,dx$

Ex ① Find k if the following function is a probability density function

$$f(x) = \begin{cases} k(1-x^2) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

also find i) $P(0.1 < X < 0.2)$ ii) $P(X > 0.5)$

Sol^n: Since $0 < x < 1$, $f(x) \ge 0$ for all x

Now $\int_0^1 f(x)\,dx = k\int_0^1 (1-x^2)\,dx$

$= k\left[x - \frac{x^3}{3}\right]_0^1 = k \cdot \frac{2}{3}$

but this must be equal to 1

$\frac{25}{3} = 1$

$k = \frac{3}{2}$

(i) $P(0.1 < x < 0.2) = \int_{0.1}^{0.2} \frac{3}{2}(1-x^2)\,dx$

$= \frac{3}{2}\left[x - \frac{x^3}{3}\right]_{0.1}^{0.2}$

$= \frac{3}{2}[0.0977]$

$= 0.146$

ii) $P(x > 0.5)$

$= \int_{0.5}^{1} \frac{3}{2}(1-x^2)\,dx$

$= \frac{3}{2}\left[x - \frac{x^3}{3}\right]_{0.5}^{1}$

$= \frac{3}{2}[0.667 - 0.458]$

sol^n: Since the total probability is one

$$\int_{-\infty}^{\infty} p(x)\,dx = \int_0^3 \left(\frac{x}{6} + k\right) dx = 1$$

$$= \left[\frac{x^2}{12} + kx\right]_0^3 = 1$$

$$= \frac{3}{4} + 3k = 1$$

$$= 3\left(\frac{1}{4} + k\right) = 1$$

$$\therefore \frac{1}{4} + k = \frac{1}{3}$$

$$k = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

$$P(x) = \begin{cases} \frac{x}{6} + \frac{1}{12} & \text{if } 0 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

$$\therefore P(1 \le x \le 2) = \int_1^2 \left(\frac{x}{6} + \frac{1}{12}\right) dx$$

$$= \left[\frac{x^2}{12} + \frac{x}{12}\right]_1^2$$

$$= \frac{1}{12}\left[(4+2) - (1+1)\right] = \frac{1}{12} \times 4 = \frac{1}{3}$$

Ex. (3) Let X be a contineous random variable with p.d.f $f(x) = kx(1-x)$, $0 \le x \le 1$. Find K and determine a number b such that $P(X \le b) = P(X \ge b)$

Solⁿ: Since $\int_{-\infty}^{\infty} f(x)dx = 1$ we have

$$K \int_0^1 (x - x^2)\, dx = 1$$

$$K \left[\frac{x^2}{2} - \frac{x^3}{3} \right] = 1$$

$$\therefore K \left[\frac{1}{2} - \frac{1}{3} \right] = 1$$

$$K = 6$$

Since, The total probability is 1 and $P(x \le b) = P(x \ge b)$, $P(x \le b) = 1/2$

$$\therefore \int_0^b f(x)\, dx = 1/2$$

$$6 \int_0^b (x - x^2)\, dx = \frac{1}{2}$$

$$\therefore \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^b = \frac{1}{12}$$

$$\frac{b^2}{2} - \frac{b^3}{3} = \frac{1}{12}$$

$$6b^2 - 4b^3 = 1$$

$$4b^3 - 6b^2 - 1 = 0$$
$$4b^3 - 2b^2 - 4b^2 - 2b + 2b - 1 = 0$$

$$(2b-1)(2b^2 - 2b + 1) = 0$$
$$\therefore \quad b = 1/2$$

Solve the following Examples

(1) A function is defined as

$$f(x) = \begin{cases} 0 & \text{for } x < 2 \\ \frac{2x+3}{18} & \text{for } 2 \le x \le 4 \\ 0 & \text{for } x > 4 \end{cases}$$

Show that $f(x)$ is a probability density function and find the probability that $2 < x < 3$

(2) A random variable x has the probability density function

$$f(x) = \begin{cases} 2e^{-2x} & \text{for } x \geq 0 \\ 0 & \text{for } x < 0 \end{cases}$$

Find (i) $P(1 \le x \le 3)$, (ii) $P(x \geq 0.5)$