Poisson Distribution :-

Poisson distribution was discovered by the French Mathematician Poisson in 1837. Poisson distribution is the limitting case of the binomial distribution under the following conditions.

(i) n, the number of trials is infinitely large i.e. $n \to \infty$

(ii) p the probability of success in each trial is constant and infinitely small i.e. $p \to 0$

(iii) np, the average success is finite say $m = n \cdot p$

(b) Definition :-

A random variable x is said to follow Poisson distribution if the probability of x is given by

$$P(X = x) = \frac{e^{-m} m^{x}}{x!}, \quad x = 0, 1, 2 \ldots$$

& $(m > 0)$ is called the parameter of the distribution

When do we get Poisson distribution

- We get a poisson distribution if the following conditions are satisfied

① The number of trials n is infinitely large i.e. $n \to \infty$

(2) A trial results in only two ways success or failure

(3) If p and q are probabilities of success & failure, then $p+q=1$

(4) The probability p of success is very small i.e. $p \to 0$

(5) $n \to \infty$ & $p \to 0$ such that $np = m > 0$ a consta.

Uses :-

Poisson distribution is used in problems involving :

(1) the number of deaths due to a disease such as heart attak, cancer etc.

(2) the number of accidents during a week or a month etc.

(3) the number of phone calls received at a particular telephone exchange during period of time.

(4) the number of cars passing a particular point on a road during a period of time

(5) the number of printing mistakes on a page of a book etc.

* Constants of Poission Distribution

(1) The mean $= m = n \cdot p$

(2) The variance $= m = n \cdot p$

mean & variance are same for the poisson distribution.

Ex① Find out the fallacy if any in the following Statement

"If X is a poisson variate such that $P(X=2) = 9P(X=4) + 90P(X=6)$ then mean of $X = 1$"

Sol^n: Let m be the mean of X

$$\therefore P(X=x) = \frac{e^{-m} m^x}{x!}$$

By data $$e^{-m} \frac{m^2}{2!} = 9 \cdot e^{-m} \cdot \frac{m^4}{4!} + 90 e^{-m} \frac{m^6}{6!}$$

$$\therefore \frac{1}{2} = \frac{3m^2}{8} + \frac{m^4}{8}$$

$$m^4 + 3m^2 - 4 = 0$$

$$(m^2+4)(m^2-1) = 0$$

$$m^2 = -4 \quad \text{or} \quad m^2 = 1$$

$\therefore$ The mean is 1 since $m > 0$

$\therefore$ the statement is correct

Ex② A car hire firm has two cars which it hires out day by day. The number of demand for a car on each day is distributed as poisson variate with mean 1.5. Calculate the proportion of days on which (i) neither car is used (ii) some demand is refused

Solⁿ: We have

$$P(X=x) = \frac{e^{-m} m^x}{x!} = \frac{e^{-1.5}(1.5)^x}{x!}, \quad x = 0, 1, 2$$

(i) probability that there is no demand is

$$P(X=0) = \frac{e^{-1.5}(1.5)^0}{0!} = 0.2231$$

(ii) Probability that some demand is refused

$$\therefore P(X>2) = 1 - [P(X=0) + P(X=1) + P(X=2)]$$

$$= 1 - \left[\frac{e^{-1.5}(1.5)^0}{0!} + \frac{e^{-1.5}(1.5)^1}{1!} + \frac{e^{-1.5}(1.5)^2}{2!}\right]$$

$$= 1 - [0.2231 + 0.3347 + 0.2510]$$

$$= 0.1912$$

∴ Proportion of days on which
i) neither car is used is 0.2231
ii) some demand is refused is 0.1912

Ex ③ If a random variable x follows poisson distribution such the $P(X=1) = 2P(X=2)$, find the mean and the variance of the distribution. Also find $P(X=3)$

Solⁿ: Let the parameter of the poisson distribution be m

$$\therefore P(X=x) = \frac{e^{-m} m^x}{x!}$$

We are given that

$$P(X=1) = 2P(X=2)$$

$$\frac{e^{-m} m^1}{1!} = 2 \frac{e^{-m} m^2}{2!}$$

$$\therefore 1 = \frac{2m}{2!}$$

$$m = 1$$

$\therefore$ mean = variance = 1

Now $P(X=3) = \frac{e^{-m} m^x}{x!} = \frac{e^{-1} 1^3}{3!} = 0.0613$

Ex ⑦ A hospital switch board receives an average of 4 emergency calls in a 10 minutes interval. What is the probability that (i) there are atleast two emergency calls (ii) there are exactly 3 emergency calls in an interval of 10 minutes

$\rightarrow$ We have $P(X) = \frac{e^{-m} m^x}{x!}$ Here, $m = 4$

(i) P(At least two emergency calls)

$$= 1 - [P(X=0) + P(X=1)]$$

$$= 1 - \left[\frac{e^{-4} 4^0}{0!} + \frac{e^{-4} 4^1}{1!}\right]$$

$$= 1 - e^{-4}[1+4]$$

$$= 1 - e^{-4} \cdot 5 = 1 - 0.0915$$

$$= 0.9085$$

$$P(X=3) = \frac{e^{-m} m^x}{x!} = \frac{e^{-4} 4^3}{3!} = 0.195$$

Ex ⑤ An insurance company found that only 0.01 % of the population is involved in a certain type of accident each year. If it's 1000 policy holders were randomly selected from the population what is the probability that no more than two of it's clients are involved in such accident next year?

We have $p = \frac{0.01}{100} = 0.0001$, $n = 1000$

$m = n \cdot p = 1000 \times 0.0001 = 0.1$

$P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$

$$= e^{-0.1}\left[\frac{(0.1)^0}{0!} + \frac{(0.1)^1}{1!} + \frac{(0.1)^2}{2!}\right]$$

$= 0.9998$

Ex. ⑥ The number of accidents in a year attributed to taxi drivers in a city follows poisson distribution with mean 3. out of 1000 taxi drivers, find approximately the number of drivers with

i) no accident in a year

ii) more than 3 accident in a year

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Sol For a poisson distribution

$m = 3$

$P(X=x) \; e^{-m} \frac{m^x}{x!}$

i) P(no accident in a year) $= P(X=0)$

$= \frac{e^{-3} 3^0}{0!}$

$= 0.0498$

$\therefore$ Expected no. of drivers with no accidents

$= N \cdot P(X=0)$

$= 1000 \times 0.0498 = 49.8 \cong 50$ nearly

P(More than three accidents) $= 1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3)]$

$= 1 - [0.0498 + 0.1494 + 0.2241]$

$= 1 - 0.4233$

$= 0.5767$

$\therefore$ Expected No. of drivers more than three accidents

$= N \times p = 1000 \times 0.5767 = 576.7$

$= 577$ nearly

Ex. 7 Fit a poisson distribution to the following data

x :	0	1	2	3	4	Tot
f :	109	65	22	3	1	200

Sol^n: Fitting of poisson distribution means finding expected frequencies using poisson's distribution

Now, $m = \text{mean} = \frac{\Sigma f x}{\Sigma f}$

$$\therefore m = \frac{109 \times 0 + 65 \times 1 + 22 \times 2 + 3 \times 3 + 1 \times 4}{200}$$

$$= \frac{122}{200} = 0.61$$

Now expected frequency $= N \cdot P(X)$

$$\therefore N \cdot \frac{e^{-m} m^x}{x!} = 200 \cdot e^{-0.61} \frac{(0.61)^x}{x!}$$

putting $x = 0, 1, 2, 3, 4$

we get the expected frequencies as.

109, 66 20, 4, 1

Ex. 8 In a screw making machine there are on an average two defective screws out of 100. The screws are packed in boxes of 100. Find the probability that a box contains (i) no defective (ii) 5 defective screws

Sol^n $m=2$

$$P(X=x) = \frac{e^{-m} m^x}{x!}$$

(i) $P(X=0) = \frac{e^{-2} 2^0}{0!} = e^{-2} = 0.1353$

(ii) $P(X=5) = \frac{e^{-2} \cdot 2^5}{5!} = 0.2667 \times e^{-2}$
$= 0.036$

Ex. 9 Letters were received in an office on each of 100 days. Fit a poisson distribution and find the expected frequencies for $x=0,1$

No. of letters:	0	1	2	3	4	5	6	7	8	9	10
Freq^n:	1	4	15	22	21	20	8	6	2	0	1

Sol^n

$\Sigma f_i = 1+4+15+ \cdots +1 = 100$

$$m = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{1\times 0 + 4\times 1 + \cdots 1\times 10}{100} = \frac{400}{100} = 4$$

$$P(X=x) = \frac{e^{-m} m^x}{x!} = \frac{e^{-4} 4^x}{x!}$$

$P(0) = e^{-4} = 0.0183$

$F(0) = N \cdot P(0) = 100 \times 0.0183 = 1.83 \simeq 2$

$$P(1) = \frac{e^{-4} 4^1}{1!} = 0.0732$$

$\therefore E(4) = N \times P(1)$

$= 100 \times 0.0732 = 7.32 \simeq 7$

Ex. (10) A book contains 100 misprints distributed randomly throughout 100 pages. What is the probability that a page observed at random contains at least two misprints

Sol^n :- Here $m = 1$

$\therefore$ P(At least two misprints)

$= 1 - [P(X=0) + P(X=1)]$

$$= 1 - \left[\frac{e^{-1} 1^0}{0!} + \frac{e^{-1} 1^1}{1!} \right]$$

$= 1 - e^{-1}[1+1]$

$= 1 - e^{-1} \times 2$

$= 1 - 0.7357$

$= 0.2643$

Ex. (11) If the probability that an individual suffers a bad reaction from a certain injection is 0.001 determine the probability that out of 2000 individuals (i) exactly 3 (ii) more than two will suffer a bad reaction

Sol^n :- We are given $n = 2000$, $p = 0.001$

but $m = n \cdot p = 2000 \times 0.001 = 2$

By poisson distribution

$$P(x) = \frac{e^{-m} m^{x}}{x!}$$

$$P(x) = \frac{e^{-2} 2^{x}}{x!}$$

(ii) P(More than two suffers a bad reaction)

$$= 1 - [P(X=0) + P(X=1) + P(X=2)]$$

$$= 1 - \left[\frac{e^{-2} 2^{0}}{0!} + \frac{e^{-2} 2^{1}}{1!} + \frac{e^{-2} 2^{2}}{2!}\right]$$

$$= 1 - [0.1353 + 0.2707 + 0.2707]$$

$$= 0.3233$$

(i) $P(X=3) = \frac{e^{-2} 2^{3}}{3!} = 0.1804$

Ex. 12 A firm produces articles of which 0.1 percent are defective. It packs them in cases each containing 500 articles If a whole-saler purcheses 100 such a cases, how many cases can be expected to be free from defectives, how many can be expected to have one defective

→ We have $p = \frac{0.1}{100} = 0.001$, $n = 500$

Mean $= m = n \cdot p = 500 \times 0.001 = 0.5$

By poisson distribution

$$P(x) = \frac{e^{-m} m^x}{x!} - \frac{e^{-0.5} (0.5)^x}{x!}$$

$\therefore$ $p(\text{no detective}) = P(x=0) = \frac{e^{-0.5} (0.5)^0}{0!}$

$= 0.6065$

$P(\text{one detective}) = P(x=1) = \frac{e^{-0.5} (0.5)^1}{1!}$

$= 0.3033$

$\therefore$ Expected No. of no detective $= N \cdot P$

$= 100 \times 0.6065$

$= 61$

Expected No. of one detective $= N \cdot P$

$= 100 \times 0.3033$

$= 30$