22/07/19

1. Mathematical logic & Preposition

* Preposition: It is statement which is either true/false, having truth value

* Logical connectives: 1) Negation (~) - statement is form by introducing not at proper place. e.g. I'm going for a walk.
⇒ I'm not going for a walk
⇒ It is not the case that I'm going for a walk.

2) AND (Conjuction): If P & Q are statements then compound statement "P and Q" is called as 'conjuction of P & Q' denoted as 'P ∧ Q'.

P: The sun is shiny.
Q: Birds are singing.

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

3) OR (Disjunction): If P & Q are two statement then compound statement "P or Q" is called as 'disjunction of P and Q'. denoted as P ∧ Q.

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

4) Conditional: If P & Q are two statements then compound statement 'If P then Q (p → q) is called conditional statement or 'implecation'.

P: It rains.
Q: I carry an umbrella.

⇒ If it rains, then I will carry an umbrella.

P	Q	P → q
T	T	T
T	F	F
F	T	T
F	F	T

5) Biconditional: If P & Q are two statements then compound statement P if and only if Q (P ↔ Q) is called as biconditional statement.

eg an integer is even if and only if it is divisible by 2.

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

Statement Formula: If statement is having connectives that statement becomes a formula.

Problems:

Q.1 P: Rajni is tall
Q: Rajani is beautiful.

Write given statement in symbolic form
1) Rajani is tall & beautiful.
2) It is false that Rajani is short or tall and beautiful.
3) Rajani is tall but not beautiful.

→ ① $P \wedge Q$ ② $\sim(\sim P \vee Q)$ ③ $P \wedge \sim Q$

Q.2 P: I will study discrete structures.
Q: I will go to a movie
R: I'm not in a good mood.

Write given statement in symbolic form
① If I'm not in good mood then I will go to movie.
② I will go to a movie only if I will not study discrete structures then
③ If I will not study then I'm not in a good mood.

→ ① $\sim r \to q$ ② $q \leftrightarrow \sim p$ ③ $\sim p \to \sim r$.

Q Truth tables: for ① $(\sim p \vee q) \to q$
② $((\sim p \wedge q) \vee (q \wedge r)) \to r$

①

p	q	$\sim p$	$\sim p \vee q$	$(\sim p \vee q) \to q$
T	T	F	T	F
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

②

p	q	~~~p~~ r	$\sim p$	$(\sim p \wedge q)$	$(q \wedge r)$	$(\sim p \wedge q) \vee (q \wedge r)$	$(\sim p \wedge q) \vee (q \wedge r) \to r$
F	F	F	T	F	F	F	T
F	F	T	T	F	F	F	T
F	T	F	T	T	F	T	F
F	T	T	T	T	T	T	T
T	F	F	F	F	F	F	T
T	F	T	F	F	F	F	T
T	T	F	F	F	F	F	T
T	T	T	F	F	T	T	T

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Q. $p \to q$ is false Determine truth value for $(\sim(p \wedge q)) \to q$. = $\boxed{F}$.
for $P = T$ & $q = F$ $p \to q \Rightarrow$ False.

Q If p & q are false prepositions find truth value for $(p \vee q) \wedge (\sim p \vee \sim q)$
$\Rightarrow \boxed{F}$

Q If $p \to q$ is true Can we determine truth value of $\sim p \vee (p \to q)$
$\Rightarrow$ for $P = T, Q = T$ Ans. $\boxed{T}$
$P = F, Q = T$ $\boxed{T}$
$P = F, Q = F$ $\boxed{T}$

Q A certain country is inhabited only by people who either tales the truth or always tells lies and who will respond to question only with yes/No. A touriust comes to a fork in the road where one branch leads to capital & other doesn't. There is no sign indicating which branch to take. But there is an inhabitant Mr. Z standing at fork. What single question should tourist ask him to determine which branch to take?

→ p: Mr. Z tells truth
q: left hand branch leads capital.
A: $p \leftrightarrow q$ = ?

Case I: Z always speaks true.

p	q	A	Ans.
T	T	T	Yes
T	F	F	No

Case II: Z always tells false

p	q	A	Ans.
F	T	F	No
F	F	T	Yes.

- Tautology: A statement formula is called as tautology if it always assumes truth value T irrespective of value assigned to its values.
- contradiction: A statement formula is called as controdiction if it assumes false value F irrespective of value assigned to its values.
- statement formula: which is neither tautology nor contradiction called as contingency.

Q Construct truth table for ① $(p \wedge q) \wedge \sim(p \vee q)$ ② $(p \wedge q) \to p$.

① p q $p \wedge q$

Q) A: WFF (Well formed formula) & $P_1, P_2, \ldots, P_n$
Find how many set of truth values will be assigned to variable $P_1, P_2, \ldots$

P			
T	T	T	F
F	T	F	F

* If A and B two statement formula truth value of A is truth value of B then we say A is equivalent B $(A \Leftrightarrow B)$ $P \Leftrightarrow \sim\sim P$

1) $P \vee P \Leftrightarrow P$ $\quad P \wedge P \Leftrightarrow P$ Idempotent law
2) $(P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$ Associative law $(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R)$
3) $(P \wedge Q) \Leftrightarrow (Q \wedge P)$ commutative law $(P \vee Q) \Leftrightarrow (Q \vee P)$
4) $P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$ distributive law $P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$
5) $P \vee F \Leftrightarrow P$
6) $P \vee (P \wedge Q) = P$, $P \wedge (P \vee Q) = P$ } Absorption law
7) $\sim(P \vee Q) \Leftrightarrow \sim P \wedge \sim Q$, $\sim(P \wedge Q) \Leftrightarrow \sim P \vee \sim Q$ } DeMorgan's law

Q. $(\sim P \wedge (\sim Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$

distributive

$\Rightarrow$ L.H.S $= (\sim P \wedge (\sim Q \wedge R)) \vee (Q \vee P) \wedge R$
$= [(\sim P \wedge \sim Q) \wedge R] \vee [(Q \vee P) \wedge R]$ ← Associative
$= [\sim(P \vee Q) \wedge R] \vee [(Q \vee P) \wedge R]$ ← DeMorgan's
$= [\sim(P \vee Q) \vee (P \vee Q)] \wedge R$
$= R$
$=$ R.H.S

Q. $((P \vee Q) \wedge \sim(\sim P \wedge (\sim Q \vee \sim R))) \vee (\sim P \wedge \sim Q) \vee (\sim P \wedge \sim R)$
$\therefore = (P \vee Q) \wedge \sim[(\sim P \wedge \sim Q) \vee (\sim P \wedge \sim R)] \vee (\sim P \wedge \sim Q) \vee (\sim P \wedge \sim R)$
$= ((P \vee Q) \wedge \sim[\sim(P \vee Q) \vee \sim(P \vee R)]) \vee (\sim P \wedge \sim Q) \vee (\sim P \wedge \sim R)$
$= ((P \vee Q) \wedge (P \vee Q) \wedge (P \vee R)) \vee \sim(P \vee Q) \vee \sim(P \vee R)$
$= [(P \vee Q) \wedge (P \vee R)] \vee [\sim(P \vee Q)] \vee [\sim(P \vee R)]$
$= [P \vee (Q \wedge R)] \vee [\sim(P \vee Q)] \vee [\sim(P \vee R)]$
$= [P \vee (Q \wedge R)] \vee (\sim P \wedge (\sim Q \vee \sim R))$
$= [P \vee (Q \wedge R)] \vee (\sim P \wedge \sim(Q \wedge R))$
$= [P \vee (Q \wedge R)] \vee \sim(P \vee (Q \wedge R))$
$= P \vee (Q \wedge R)$

* Two Formulas:
Duality law: A and A^* are said to be duals of each other if either one can be obtained by replacing '$\vee$' by '$\wedge$' and '$\wedge$' by '$\vee$'. In similar way truth value 'T' can be replaced by 'F' and 'F' by 'T'.

e.g. $(P \wedge Q) \vee T = A \Rightarrow A^* = (P \vee Q) \wedge F$
$\sim(P \vee Q) = \sim P \wedge \sim Q$
$\sim(P \wedge Q) = \sim P \vee \sim Q$

Negation of any formula is equals to its dual when every variable is replaced by its dual
1) $(P \vee Q) = \sim(\sim P \wedge \sim Q)$
2) $(P \wedge Q) = \sim(\sim P \vee \sim Q)$

* Any two statement formulas are equivalent then their duals are equivalent
i.e $A \Leftrightarrow B$ then $A^* \Leftrightarrow B^*$

* If A is equivalent to B i.e $A \Leftrightarrow B$ is tautology
e.g $P \to Q \Leftrightarrow \sim P \vee Q$

p	q	$p \to q$	$\sim p$	$\sim p \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

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$P \to (Q \to R) \Leftrightarrow P \to (\sim Q \vee R) \Leftrightarrow (P \wedge Q) \to R$
$P \to (\sim Q \vee R) \Leftrightarrow \sim P \vee (\sim Q \vee R) \Leftrightarrow \sim(P \wedge Q) \vee R$
(A.L) $\sim P \vee (\sim Q \vee R) \Leftrightarrow \sim P \vee (\sim Q \vee R) \Leftrightarrow \sim(P \wedge Q) \vee R$
$(\sim P \vee \sim Q) \vee R \Leftrightarrow (\sim P \vee \sim Q) \vee R \Leftrightarrow (\sim P \vee \sim Q) \vee R$
(D.L) $\sim(P \wedge Q) \vee R \Leftrightarrow \sim(P \wedge Q) \vee R \Leftrightarrow \sim(P \wedge Q) \vee R$

Q. Which of the following connectives are symmetric
1) $\vee$ $\quad$ 2) $\wedge$ $\quad$ 3) $\to$ $\quad$ 4) $\leftrightarrow$

Converse, Inverse and contrapositive:
$P \to Q$ (formula) : Converse :- $Q \to P$
inverse : $\sim P \to \sim Q$
contrapositive: $\sim Q \to \sim P$

- A statement A is said to be tautologically imply a statement B if and only if $A \to B$ is tautology. denoted by $A \Rightarrow B$.

Q. $\underbrace{\underbrace{(P \vee Q) \wedge (P \to R)}_{1} \wedge (Q \to R)}_{A} \Rightarrow \underbrace{R}_{B}$ $A \to B$ is tautology.

P	q	r	(PVq)	(P→R)	(Q→R)	1	2	A→B.
F	F	F	F	T	T	F	F	T
F	F	T	F	T	T	F	F	T
F	T	F	T	T	F	T	F	T
F	T	T	T	T	T	T	T	T
T	F	F	T	F	T	F	F	T
T	F	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	T	T	T	T	T	T	T	T

Q. $P \wedge Q \Rightarrow Q$
Q. $\sim P \Rightarrow P \to Q$ } Verify.

• Functionally complete set of connectives:

$P \leftrightarrow Q \Leftrightarrow (P \to Q) \wedge (Q \to P)$

P	Q	P↔Q	P→Q	Q→P	(1∧2)
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

Note:
$P \to Q$
$\Leftrightarrow \sim P \vee Q$

Let P & Q be any two formulas then $P \bar{\vee} Q$ whenever P or Q but noth.
(P exclusive or Q)

P	Q	$P \bar{\vee} Q \Leftrightarrow Q \bar{\vee} P$ (symmetric)
T	T	F
T	F	T
F	T	T
F	F	F

Associative
distributive

$$\boxed{\begin{array}{ll} P \uparrow Q \Leftrightarrow \sim(P \wedge Q) & \text{NAND} \\ P \downarrow Q \Leftrightarrow \sim(P \vee Q) & \text{NOR.} \end{array}}$$ } :- symmetric

Q. Express in terms of NOR 1) $P \uparrow Q$. (NAND)

$\Rightarrow (P \uparrow Q)$
$\Rightarrow \sim(P \wedge Q)$ .. (DM)
$\Rightarrow \sim P \vee \sim Q$
$\Rightarrow \sim(\sim P \downarrow \sim Q)$

Q. If A is statement formula A(P,Q,R) represented as $A = P \uparrow (Q \wedge \sim(R \downarrow P))$ find out its dual A^* Also find formulas which are equivalent to A and A^* which contains connectives AND, OR, ~ only.

$\longrightarrow$ $A^* = P \downarrow (Q \vee \sim(R \uparrow P))$
$= \sim(P \vee (Q \vee \sim(R \uparrow P)))$
$= \sim(P \vee (Q \vee (P \wedge Q)))$

Q Prove that $P \to (Q \to R) \Leftrightarrow (P \wedge Q) \to R$

L.H.S $= \sim P \vee (Q \to R)$
$= \sim P \vee (\sim Q \vee R)$
$= (\sim P \vee \sim Q) \vee R$
$= \sim(P \wedge Q) \vee R$
$= (P \wedge Q) \to R$
$=$ R.H.S

$\boxed{(P \vee Q) \vee \sim P \Leftrightarrow \text{Tautology}}$

$= (Q \vee P) \vee \sim P$
$= Q \vee (P \vee \sim P)$
$= Q \vee T$
$= T$

$\boxed{\sim(P \vee Q) \vee (\sim P \wedge Q) \vee P \Leftrightarrow \text{Tautology}}$

$= (\sim P \wedge \sim Q) \vee (\sim P \wedge Q) \vee P$
$= \sim P \wedge (\sim Q \vee Q) \vee P$
$= \sim P \wedge T \vee P$
$= \sim P \vee P \wedge T$
$= T$

* Normal forms : 1) Elementary Product: Product of variables and their negation in formula is called as an elementary product. e.g $\sim P \wedge Q$, $\sim P \wedge \sim Q$
2) Elementary Sum: Sum of variables and their negation in formula is called elementary sum. e.g. $P \vee \sim Q$.
3) DNF (Disjunctive Normal Formula) Formula equivalent to given formula which consist of sum of elementary product is called as 'DNF'.

Q. Find DNF of $P \wedge (P \to Q)$
$= P \wedge (\sim P \vee Q)$
$= (P \wedge \sim P) \vee (Q \wedge P)$

CNF (conjuctive Normal form): A formula which is equivalent to given formula which consists which consist of product of elementary sum, is called as 'CNF'.

Q. Find CNF of $P \to \cap (P \to Q)$
$= P \wedge (\sim P \vee Q)$

single variable can be elementary product/sum.

Q. Find DNF of 1) $P \to Q$ 2) $P \vee Q$ 3) $\sim (P \wedge Q)$
① $P \to Q$ ② $P \vee Q$ ③ $\sim P \vee \sim Q$
$= \sim P \vee Q$

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Valid Argument: is finite sequence of statements $P_1, P_2, \ldots P_n$ which is called as premises. Written statement 'c' called as conclusion such that $P_1 \wedge P_2 \wedge P_3 \ldots \wedge P_n \to c$ is tautology.

Rules of inference:
1) Modus Ponens: $(P_1 \to q) \wedge P_2 \to q$ (Tautology)
2) Modus Tollens: $(P \to q) \wedge \sim q \to \sim p$ (—"—)
3) Disjunctive Syllogism: $(p \vee q) \wedge \sim p \to q$ (—"—)
4) Hypothetical syllogism: $(p \to q) \wedge (q \to r) \to (p \to r)$

Q. Determine following Argument is valid/not.
If John goes to class he is on time. but John is late. therefore he will miss the class.

Ans. → P_1: John goes to class
q: John is on time

$p \to q$
$\sim q$
$\sim p$ (conclusion)

According to Modus Tollens rule this argument is valid.

Q. I'm happy if my prg runs. A necessary condition for prg to run is it should be error free. I'm not happy. therefore prg is not error free.

→ p: I'm happy
q: my prg runs
r: It should be error free

$q \to p$
$q \to r$
$\sim p$
$\sim r$

$(q \to p) \wedge (q \to r) \wedge \sim p \to \sim r$

2. Set Theory

Empty set: If it contains no elements.
N is notation for set of natural Numbers.
Z —"— integers $Z: \{\ldots -1, 0, 1, \ldots\}$
Z^+ —"— +ve integers $Z^+: \{1, 2, \ldots\}$
Q —"— Rational numbers
A is given set then $\bar{A}$ is compliment of that set. $\bar{A} = \{x \mid x \notin A\}$

- Difference of set: The difference A-B defined as $A-B = \{x \mid x \in A \ \& \ x \notin B\}$
$B-A = \{x \mid x \in B \ \& \ x \notin A\}$

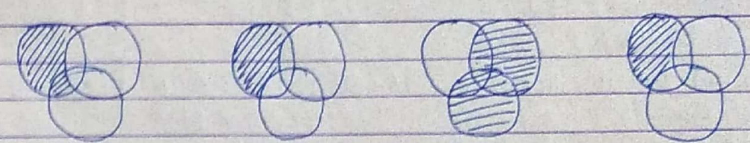
Q. U is universal set consists of n natural numbers upto 15. A is set which greater than 4 & less than 12. B is set of natural numbers which greater >8 & <15, $C \to >5$ & <10 find $\bar{A}-\bar{B}$, $\bar{C}-\bar{A}$

→ $U = \{n \mid n \in N, n \le 15\}$
$A = \{n \mid n \in N, 4 < n < 12\}$
$B = \{n \mid n \in N, 8 < n < 15\}$
$C = \{n \mid n \in N, 5 < n < 10\}$

$\bar{A} = \{1, 2, 3, 4, 12, 13, 14, 15\}$
$\bar{B} = \{1, 2, 3, 4, 5, 6, 7, 8, 15\}$
$\bar{C} = \{1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 15\}$

$\therefore \bar{A} - \bar{B} = \{12, 13, 14\}$
$\therefore \bar{C} - \bar{A} = \{5, 10, 11\}$

Q. show that $(A-B)-C = A-(B \cup C)$ using venn diagram.



A-B (A-B)-C B∪C A-(B∪C)

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- Ternery: -1, 0, 1
- discrete is equal to distinct differentiation
- Base 10 - digital, Base 2 - discrete.
- Complete not fraction
- Redesigning soln
- Apply theory to solve real life problems.
- Understand, Apply, evaluate
- Tristate - floating
- $a \to b = \sim a \vee b$ * Graph theory problems.

• If A and B are finite sets which are disjoint

$|A|$ = no. of elements in A

~~$|B|$ = no. of elements in B~~

$\boxed{\begin{array}{l} |A \cup B| = |A| + |B| \\ |A - B| = |A| - |A \cap B| \end{array}}$

• Principle of inclusion and exclusion: If A and B are finite sets then $|A \cup B| = |A| + |B| - |A \cap B|$ called as principle of inclusion-exclusion.

Q. In a survey 2000 people they were asked whether they nee read India today/ Business times It was found that 1200 people India today, 900 → Business times, 400 → both Find how many read atleast one magazine and how many read none?

→ $|A| = 1200$ 2) $|A \cup B| = |A| + |B| - |A \cap B|$

$|B| = 900$ $= 2100 - 400$

$|A \cap B| = 400$ $= 1700$

1) $U - |A \cup B| = 300$

Q. Among integers 1 to 300 find how many are not divisible by 3, nor by 5 Find also how many are divisible by 3 but not by 7

→ $|A|$ = no. divisible by 3

$|B|$ = no. divisible by 5

$|\bar{A} \cap \bar{B}| = U - |A \cup B|$

$= 300 - ((100) + 60 - 20)$

$= 300 - 140$

$= 160$

A = no divisible by 3 $|A - B| = |A| - |A \cap B|$

B = —"— by 7 $= 100 - 14$

$= 86$

Q If A be any set power set of A is denoted by P(A) & it is set of all subsets of A.

2. Relations and Functions

• Let $\{A_1, A_2, \dots A_n\}$ be a finite collection of sets then subset R of $A_1 \times A_2 \times \dots \times A_n$ is called as an 'n-ary Relation'

• If n = 1, 2, 3, ... called as unary, binary, ternary relⁿ resp.

Q. • Let z be set of integers, x is an even integer can be characterized as relation $R = \{x \in z \mid x \text{ is even}\}$ which is unary

• Binary relation: x is divisible by y

$R = \{(x, y) \mid x \text{ is divisible by } y\}$

• Ternary relation: $(x+y)$ divisible by z

$R = \{(x, y, z) \mid (x+y) \text{ is divisible by } z\}$

• A and B are two non-empty set and binary relations ~~are~~ R from A to B is actually subset of $A \times B$. $R \subseteq A \times B$

$A = \{a, b\}$ $B = \{x, y\}$

$A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$

then domain of R denoted by $D(R)$ = elements in A that are related to some elements in B

• Range of R is denoted by $R_n(R)$ is set of elements in B that are related to some elements in B

$R_n(R) = \{b \in B \mid \text{for some } a \in A, (a, b) \in R\}$

Q. Let $A = \{2, 3, 4, 5\}$ and R be an relation defined as (A relates to B) aR_b iff $a < b$. Find $D(R)$ and $R_n(R)$.

→ $R = \{(2,3), (3,4), (4,5), (2,4), (2,5), (3,5)\}$

$D(R) = \{2, 3, 4\}$ $R_n R = \{3, 4, 5\}$

• Complement of Relation:

$\bar{R} = \{(a, b) \mid (a, b) \in \bar{R}\}$ $a\bar{R}b$

Q. $A = \{1, 2, 3, 4\}$ $B = \{a, b, c\}$ $\underset{\text{subset of } A \times B}{R} = \{(1,a), (1,b), (2,c), (3,a), (4,b)\}$

$S = \{(1,b), (1,c), (2,a), (3,b), (4,b)\}$ Find $\bar{R}$ and $\bar{S}$

→ $A \times B = \{(1,a), (1,b), (1,c), (2,b), (2,c), (2,a), (3,a), (3,b), (3,c)\}$

$\bar{R} = \{(1,c), (2,b), (2,a), (3,b), (3,c), (4,a), (4,c)\}$

$\bar{S} = \{(1,b), (2,b), (2,c), (3,a), (3,c), (4,a), (4,c)\}$

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• **Converse of Relation:** Let R be relation from A to B then co... of R denoted as R^c is relation from B...

$R^c = \{(b,a) \mid (a,b) \in R\}$

$(R^c)^c = R$

e.g. $A = \{2,3,4,6\}$ and R and S be the relations on A such that

$R = \{(a,b) \mid a = b+1 \text{ or } b = 2a\}$

$S = \{(a,b) \mid a \text{ divides } b\}$ Find $(R \cap S)^c$

$\therefore R = \{(3,2), (4,3), (4,2), (3,6)\}$

$S = \{(2,4), (3,6), (2,6)\} (2,2), (3,3), (4,4), (6,6)\}$

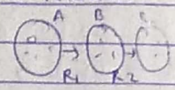
$R \cap S = \{(2,4), (3,6)\}$

$(R \cap S)^c = \{(4,2), (6,3)\}$

• **Composition of Binary Relations:** sequence of relation

Q. Let $A = \{a,b,c,d\}$, $R_1 = \{(a,a), (a,b), (b,d)\}$

$R_2 = \{(a,d), (b,c), (c,b)\}$



Find composite relation from R_1 to R_2, from R_2 to R_1, from R_2^2.

→ R_1 = A to B

$R_1 \cdot R_2 = \{(a,d), (a,c)\}$

$R_2 \cdot R_1 = \{(c,d)\}$

$R_2^2 = \{(b,b), (c,c)\}(c,d)\}$

• **Matrix representation of Relation:**

Q. Let A be set $A = \{a,b,c,d\}$, $B = \{1,2,3\}$, R be relation

$R = \{(a,1), (a,2), (b,1), (c,2), (d,1)\}$

Find relation Matrix M_R.

→ M = 4 rows
= 3 columns

$$M = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline a & 1 & 1 & 0 \\ b & 1 & 0 & 0 \\ c & 0 & 1 & 0 \\ d & 1 & 0 & 0 \end{array}$$

Q. Let $A = \{1,2,3,4,8\}$ and equal to B and aRb, $a+b \leq 9$

Find relation Matrix M_R. (2,1) (3,1) (3,2) (8

$R = \{(1,2), (1,3), (1,4), (1,8), (2,3), (2,4), (3,4)\} (1,1) (2,2) (3,3)$

(2,1) (4,4)

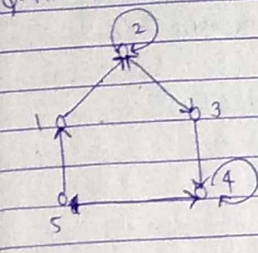
~~(3,1)~~ (4,1) ~~(8,1)~~ (

(4,2) (4,3)

$$M_R = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 8 \\ \hline 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 & 1 & 0 \\ 3 & 1 & 1 & 1 & 1 & 0 \\ 4 & 1 & 1 & 1 & 1 & 0 \\ 8 & 1 & 0 & 0 & 0 & 0 \end{array}$$

• **Graphical representation of Relation:** A is finite set and R is relation on A it is possible to represent R with picture by means of a graph

Q. Find relation determined by digraph. Find its matrix.



$R = \{(1,2), (2,2) (2,3) (3,2) (3,4) (5,4) (5,1) (4,4)\}$

$$M_R = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 1 & 0 & 0 & 0 \\ 2 & 0 & 1 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 & 1 & 0 \\ 4 & 0 & 0 & 0 & 1 & 0 \\ 5 & 1 & 0 & 0 & 1 & 0 \end{array}$$

30/08/19

Special properties of Binary relation:

1) **Reflexive relation:** R is reflexive for every element a where $a \in A$ $(a,a) \in R$. R is not reflexive if for some elements $a \in A$ $(a,a) \notin A$. e.g. $A = \{x,y,z\}$ $R = \{(x,x), (y,y), (z,z)\}$ → every element should form rel^n.

e.g. $A = \{1,2\}$ $R = \{(1,1), (2,2), (1,2)\}$

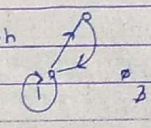
2) **Irreflexive Relation:** R is said to be irreflexive if for every element $a \in A$, $(a,a) \notin R$.

e.g. $A = \{1,2\}$ $R = \{(1,2), (2,1)\}$

$A = \{1,2\}$ $R = \{(1,2), (2,1), (1,1)\}$ → Not reflexive nor irreflexive

↓

digraph



3) **Symmetrix Rel^n:** R is said to be symmetric if whenever a relates to b (aRb) then b relates to a (bRa), it is then follow that R is not symmetric when aRb then $b \not R a$ ⇒ Not symmetric.

$A = \{a,b\}$, $R = \{(a,b)\}$, $(a,a), (b,b)\}$ $(b,a)\}$

$A = \{1,2\}$, $R = \{(1,1), (a,2)\}$ → ~~symmetric~~

4) **Antisymmetric:** R is said to be antisymmetric if whenever a relates b (aRb) and b relates to a (bRa) then a=b it follows that R is not antisymmetric if we have elements $a,b \in A$ such that $a \neq b$.

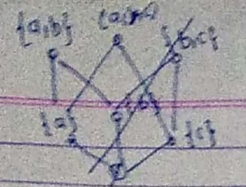
Let A is equal to ℝ, R is rel^n ≤

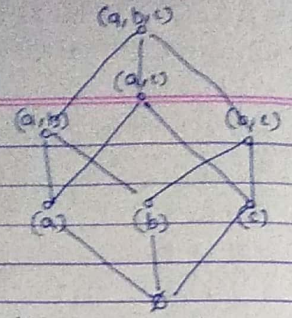
$R = \{(2,3), (2,2), (3,5)\}$

5) **Asymmetric:** R is said to be asymmetric if whenever aRb then $b \not R a$.

A = ℝ then relation <, >

$R = \{(1,2), (3,5)\}$





6/3/18

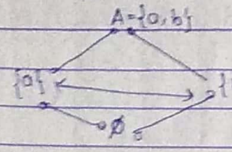
- Chain

$A = \{1,2,3,4,5,\leq\}$

Antichain $A = \{a,b\}$

POSET = $\{P(A), \subseteq\}$

$P(A) = \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$



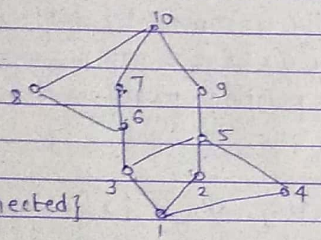
Q. $A = \{1,2,3,4,5,6,7,8,9,10\}$

GLB (2,3) = 1

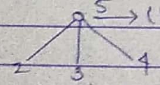
GLB (2,7) = 1

GLB (5,8) = 3

LUB (3,5) = 5 {Already connected}



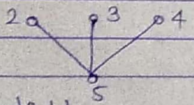
* **Join Semi Lattice :**



5 → (Upper Bound)

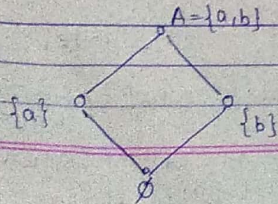
(Minimal elements)

* **Meet Semilattice :**



(Maximal elements

* **Lattice**



(Every pair has upper & Lower bound)

Q A is set of real numbers then {POSET A, ≤} is lattice Find out GLB and LUB for every ordered pair.

→ $A = \{1,2,3,4,\ldots n\}$

$(3,4) \Rightarrow$ LUB = 4 = max(a,b)

GLB = 3 = min(a,b)

Q. $A = \{a,b,c\}$ POSET = $\{P(A), \subseteq\}$

Lattice

* Compliment of element: For every $a \in L$ there exist element $b \in L$ such that "$a \cup b = I$", "$a \cap b = 0$"

$\underset{\text{LUB}}{\cup}$ $\underset{\text{GLB}}{\cap}$

then a is compliment of b.

$\emptyset$ and {A} are also compliments of each other.

* Types of lattice

1) Distributive : At most 1 compliment

2) Complimented : At least 1 compliment

3) Boolean Algebra : Distributive and complimented

Q. $A = \{1,2,3,9,18\}$ Find no. of edges in hasse dig. " a | b " divides Upper bound and lower bound for (2,3). and type of lattice.

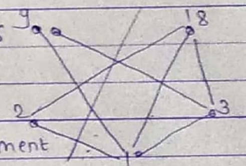
→

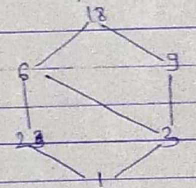
LUB ⇒ (2,3) = ~~18~~ 6

GLB ⇒ (2,3) = 1

edges = 7

1 and 18 compliment of each other





- sublattice: $(L, \underset{\text{Join}}{\cup}, \underset{\text{meet}}{\cap})$ $(M, \cup, \cap)$ subset

$(a,b) = L \& M$

$a \cup b$ = same in L as well as M

$a \cap b$ = same in L as well as M.

$L = \{x,y,z,a\}$

$M = \{x,y\}$

$M_2 = \{x,z\}$

$M_3 = \{x,a\}$

Q. $L = \{x,a,b,c,d,e,y\}$ find which of the following subsets are sublattice of L?

x

* compatibility: 1) Reflexive & symmetric

e.g. "being friend of" (x, y)

$X = \{x_1, x_2, x_3, \ldots x_n\}$

$X_1 = \{x_1, x_2, x_3\}$ ⟶ maximum compatibility block

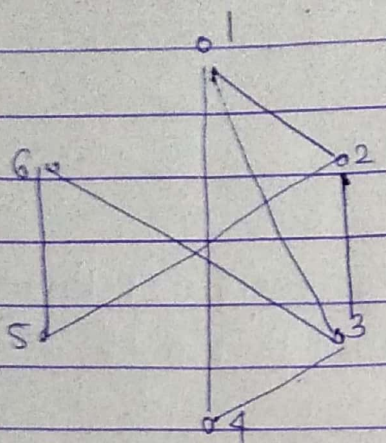
$X_2 = \{x_4, x_5\}$

Q. Compatibility relation matrix is given. Draw a graph and find max compatible block.

⟶ $1 \approx 4$,

$1 \approx 3$,

$1 \approx 2$,



digraph

2	1				
3	1	1			
4	1	1	1		
5	0	1	0	0	
6	0	0	1	0	1
	1	2	3	4	5