

5

AIR-STANDARD CYCLES

5.1 Introduction

A heat engine cycle is a series of thermodynamic processes through which a working fluid (working substance) passes in a certain sequence. At the completion of the cycle, the working fluid returns to its original condition, i.e., the working fluid at the end of the cycle has the same pressure, volume, temperature and internal energy that it had at the beginning of the cycle. Somewhere during every cycle, heat is received by the working fluid. It is, then, the object of the cycle to convert as much of this heat energy as possible into useful work. The heat energy which is not converted, is rejected by the working fluid during some process of the cycle.

5.2 Heat Engine

Any machine designed to carry out a thermodynamic cycle, and thus converts heat energy supplied to it into mechanical energy, is called a *heat engine*. Hence, the cycle it operates on is known as a *heat engine cycle*. Heat engine is generally made up of a piston and cylinder, together with the following main elements :

- (i) a *hot body*, serving as a source of heat which is received during the cycle,
- (ii) a *cold body*, whose function is to receive the heat rejected during the cycle, and
- (iii) a *working fluid (working substance)*, which receives heat directly from the hot body, rejects to the cold body, does external work on the piston during expansion, and have work done upon it by the piston during compression. The working substance may be steam, air, or mixture of fuel and air.

5.2.1 Types of heat engines : Heat engines may be of the following types :

- (i) *Steam engine, and steam turbine*, in which the working fluid (working substance) is steam,
- (ii) *Hot air engine*, in which the working fluid is air, and
- (iii) *Internal combustion engine, and gas turbine*, in which the working fluid is a mixture of gases and air, or products of combustion of fuel oil and air.

The cycles which will be presented in this chapter are ideal cycles which will apply to the last two types of heat engines, i.e., hot air engines and internal combustion engines including gas turbines. The ideal cycles which apply to the first type (i.e., steam engine) are described in chapter 9 of Volume I.

5.2.2 Available work of cycle : As stated above, it is evident that the function of any heat engine cycle is to receive heat from some external source -- the hot body, and transform as much of this heat as possible into mechanical energy. The amount of heat which is transformed into mechanical energy is known as *available energy* of the cycle. It is equal to the difference between the heat received during the cycle from the hot body and the heat rejected during the cycle to the cold body, in the absence of any other losses. This statement is of course a direct consequence of the law of conservation of energy.

Let Q = available energy for doing work per cycle in heat units,

Q_1 = heat received during each cycle from the hot body in heat units, and

Q_2 = heat rejected during each cycle to the cold body in heat units.

Then, $Q = Q_1 - Q_2$ in heat units. ... (5.1)

Every cycle contains thermodynamic processes involving both expansion and compression processes. During the former (expansion), work is done upon the piston by the gas while during the latter (compression), work is done on the gas by the piston. The difference between the work done by the gas and the work done on the gas during the complete cycle is called the *net available work* of the cycle. It is necessarily equal to the available energy for doing work per cycle.

If W = net work done during the cycle in heat units,

then, $W = Q = Q_1 - Q_2$ in heat units. ... (5.2)

5.2.3 Efficiency of a cycle : The thermal efficiency of a heat engine cycle is defined as the ratio of the available heat energy of the cycle for doing work to the heat received during the cycle from the hot body. It is usually denoted by the letter η (eta),

$$\text{i.e., Efficiency, } \eta = \frac{\text{Heat equivalent of the net work done per cycle}}{\text{Heat received during the cycle from the hot body}} \quad \dots (5.3)$$

$$\text{Thus, using eqns. (5.1) and (5.2), } \eta = \frac{Q}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = \frac{W}{Q_1}$$

The definition of efficiency given above is applicable to any type of heat engine cycle.

Hence, the expression for the efficiency given by the eqn. (5.3) is known as *theoretical or ideal thermal efficiency* of the cycle, as it does not take into account any practical losses which do occur in the actual running of the engine.

5.2.4 Air-standard efficiency of a cycle : In order to compare the thermal efficiency of actual internal combustion engine cycles, the engineer needs some standard to serve as a yard-stick. The yard-stick used is the theoretical thermal efficiency of the engine working on ideal cycle, using air as the working fluid. The theoretical thermal efficiency of the ideal cycle is known as the *air-standard efficiency*, since it is worked out on the basis of the working fluid being air throughout the cycle, i.e., the effect of calorific value of fuel used is eliminated, and the heat is supplied by bringing a hot body in contact with the end of the cylinder. Thermal efficiency of the ideal cycle can be worked out before the engine is constructed and hence indicates the maximum approachable efficiency of the completed engine. Should the actual indicated thermal efficiency of the completed engine not closely approach this efficiency (air-standard efficiency), alterations and improvements may be made to bring about the desired result. It may be noted that actual engine can never give thermal efficiency as high as the air-standard efficiency when operated on the same cycle as air engine. Actual indicated thermal efficiency of a well designed and well constructed internal combustion engine, when properly operated, should be atleast two-third of air-standard efficiency.

5.3 Thermodynamic Reversibility

In chapter 2 of volume 1, we have defined eight thermodynamic process; any one of these processes which can be operated in a reverse direction is known as *reversible* process. The factors which make a process irreversible are : (i) temperature difference required for heat to flow, and (ii) fluid friction. Thus, for an operation to be thermodynamically reversible, following conditions should be satisfied :

- (i) The temperature of the hot body supplying the heat must at any instant be the same as that of working fluid which receives the heat. If the source of heat is at a

higher temperature than the working fluid, heat will be transferred to the latter (working fluid), but when the process is reversed, heat must flow from the working fluid back into the hot body, which is at a higher temperature. This is contrary to the second law of thermodynamics. It follows that the operation could not be reversed. Thus, for an operation to be thermodynamically reversible, there cannot be temperature difference between the hot body and the working fluid during the transfer of heat. Flow of heat to or from the working fluid without finite temperature drop implies perfect exchange or infinitely slow process. This is known as *external reversibility*.

(ii) Friction between the fluid and the walls of the container and viscous friction of the fluid can never be eliminated in a thermodynamic process. The energy lost in overcoming the frictional forces is regenerated into heat. When any thermodynamic process is reversed, say from expansion to compression, friction effect cannot be reversed, i.e., friction heat cannot be absorbed back into the fluid. Thus, for an operation to be thermodynamically reversible, fluid friction must be absent. This is known as *internal reversibility*.

It will be seen from the above that the concept of thermodynamic reversibility is purely hypothetical, because the transfer of heat becomes less as the condition of reversibility is approached, and fluid friction can never be completely eliminated. Thus any thermodynamic process can be reversed, if external and internal reversibility is assumed. It may be noted that the basic requirement for throttling process is friction. Thus, throttling process is not reversible. A frictionless adiabatic operation is reversible. All other thermodynamic processes are reversible if external and internal reversibility is assumed.

5.3.1 Reversible cycle : For thermodynamic cycle to be reversible, it must consist of reversible processes only. When a cycle is reversed, all the processes are performed in the reversed direction. A heat engine cycle takes heat from the hot body and rejects portion of it to a cold body, and converts remaining quantity of heat into mechanical work. When this cycle is reversed, heat will be absorbed from the cold body and rejected to a hot body. This will necessitate external work to be supplied. This reversed cycle is known as *heat pump* or *refrigerating machine*.

A reversible cycle should not be confused with a mechanically reversible engine. Steam engine can be made to revolve in a reversed direction by mechanically altering the valve settings but this does not reverse the cycle on which the engine works. A reversed engine merely rotates in the opposite direction, but a reversed cycle converts a power producing engine into a heat pump or refrigerator.

It may be noted that an engine working on reversible cycle is the most efficient engine.

5.4 Ideal Heat Engine Cycle

There are number of ideal heat engine cycles made up of some of the following processes in which

- (a) heat is taken in or rejected at constant temperature (isothermal compression or expansion),
- (b) heat is taken in or rejected at constant pressure,
- (c) heat is taken in or rejected at constant volume, and
- (d) compression and expansion are frictionless adiabatic (isentropic).

Only five principal ideal heat engine cycles will be described in this chapter which may be summarised as follows :

- (i) *The constant temperature cycle* : Here, heat is taken in and rejected at constant temperature, and compression and expansion being frictionless adiabatic or isentropic.

This cycle is known as *Carnot cycle*.

- (ii) *The constant volume cycle* : In this, heat is taken in and rejected at constant volume, and compression and expansion being frictionless adiabatic. This cycle is known as *Otto cycle*.
- (iii) *The modified constant pressure cycle* : In this, heat is taken in at constant pressure and rejected at constant volume, and compression and expansion being frictionless adiabatic. This cycle is known as *Diesel cycle*.
- (iv) *The dual-combustion or mixed cycle* : Here, heat is partly taken in at constant volume and then at constant pressure and heat is rejected at constant volume, and compression and expansion being frictionless adiabatic. This cycle is sometimes known as *semi-Diesel cycle*.
- (v) *The (true) constant pressure cycle* : In this, heat is taken in and rejected at constant pressure, and compression and expansion being frictionless adiabatic. This cycle is known as *Joule cycle*.

5.5 Carnot Cycle

This cycle was brought out in 1824 by a French engineer named Sadi Carnot. Although its limitations are such that no heat engine has ever been constructed to use it. This cycle theoretically permits the conversion of the maximum quantity of heat energy into mechanical energy, as being a reversible cycle. In other words, it gives the maximum efficiency that is possible to obtain in a heat engine. Hence, its usefulness lies in the comparison which it affords with other heat engines, giving as it does under the conditions, the maximum efficiency that they would like to approach.

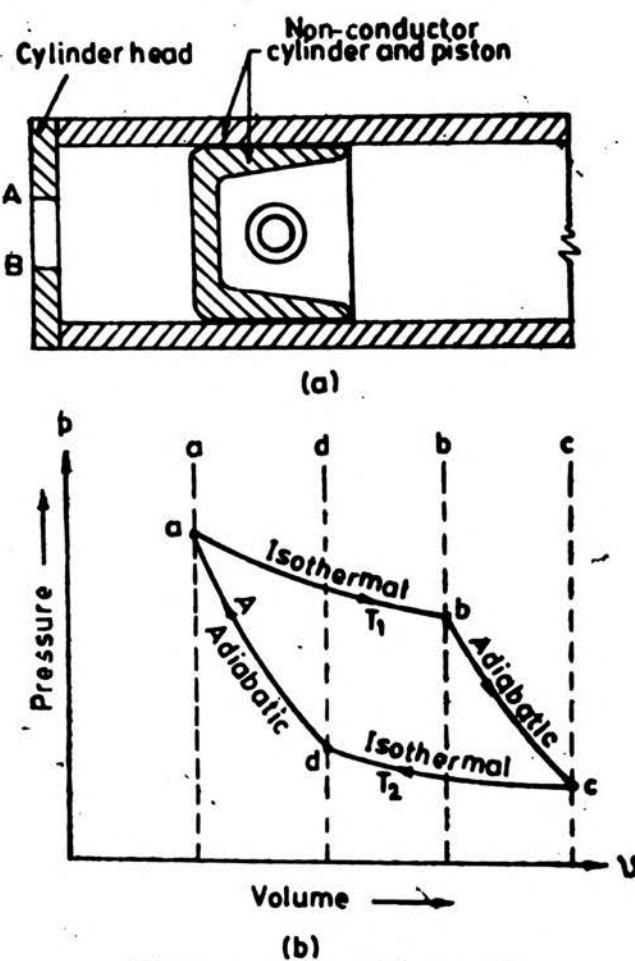


Fig. 5-1. p-v diagram of Carnot cycle.

An engine operating on this ideal cycle, would require a cylinder and a piston of perfectly non-conducting material, a cylinder head that will conduct heat perfectly, and three other elements that can be brought into contact with the conducting cylinder head at AB as shown in fig. 5-1, as occasion demands. These three elements are : (a) the *hot body*, always at temperature T_1 , the source of heat energy supplied to the working fluid (air), (b) the *non-conducting cover* to fit the cylinder at AB, and (c) the *cold body*, maintained at a temperature T_2 , the minimum temperature of the cycle. The element (cold body) receives the heat that is rejected from the working fluid (air).

Consider one kg of air at temperature T_1 as the working fluid in the engine cylinder. Let point a (fig. 5-1b) represent the state of the working fluid as regards pressure p_a and volume v_a at absolute temperature T_1 .

Isothermal expansion : At point a, the body at temperature T_1 is brought in contact with the cylinder head at AB and heat is supplied at temperature T_1 to the working fluid (air). This causes the air to expand

isothermally along the curve $a-b$ from volume v_a to v_b until point b is reached. This point is the end of isothermal expansion. The temperature through this process ab has been maintained constant at T_1 . As the air expands, it forces the piston outward thus doing work on the piston.

Adiabatic expansion : At point b , the hot body is removed and replaced by the non-conductor cover. Since all the elements of the engine which are now in contact with the working fluid (air) are non-conductors, no heat can be added or abstracted from the air. The air now expands adiabatically along curve $b-c$; doing work on the piston at the expense of its internal energy. Consequently the temperature falls from T_1 to T_2 and the volume increases from v_b to v_c . At point c , the piston is at the end of the outward stroke.

Isothermal compression : At point c , the non-conducting cover is removed and the cold body at temperature T_2 is brought in contact with the conducting cylinder head at AB. The piston now moves inward compressing the air isothermally along the curve $c-d$ from volume v_c to v_d , until point d is reached. During this compression, the heat which is rejected by the air goes into the cold body. This makes the isothermal compression at constant temperature T_2 possible.

Adiabatic compression : At point d the cold body is removed and the non-conducting cover again takes the position at AB. The air is now adiabatically compressed along the curve $d-a$, until it reaches the starting point a of the cycle, where it resumes its initial conditions of temperature, pressure and volume, and the piston is returned to the end of the stroke.

Since no transfer of heat occurs during both adiabatic operations, then by the law of conservation of energy, the difference between the heat received and heat rejected must be equal to the net work done. Now for any non-flow thermodynamic process,

$$\text{Heat added} = \text{work done} + \text{change in internal energy.}$$

Since during isothermal expansion process $a-b$ the temperature does not change neither will the internal energy change.

Heat added (supplied) during operation $a-b$ = work done

$$= p_a v_a \log_e \left(\frac{v_b}{v_a} \right) = RT_1 \log_e (r_1) \text{ per kg of air}$$

where r_1 = isothermal expansion ratio $\frac{v_b}{v_a}$.

Heat rejected during operation $c-d$

$$= p_c v_c \log_e \left(\frac{v_d}{v_c} \right) = RT_2 \log_e (r_2) \text{ per kg of air}$$

where r_2 = isothermal compression ratio $\frac{v_c}{v_d}$.

As stated above, the net work done per kg of air is the difference between the heat supplied and heat rejected.

$$\begin{aligned} \therefore \text{Net work done} &= RT_1 \log_e (r_1) - RT_2 \log_e (r_2) \\ &= R [T_1 \log_e (r_1) - T_2 \log_e (r_2)] \text{ per kg of air} \end{aligned} \quad \dots (5.4)$$

Using temperature and volume relationship for adiabatic process and considering adiabatic expansion $b-c$ (fig. 5-1),

$$\frac{\text{Higher temperature}}{\text{Lower temperature}} = (r)^{\gamma - 1} \quad \text{i.e. } \frac{T_b}{T_c} = \left(\frac{v_c}{v_b} \right)^{\gamma - 1}$$

Since, $T_a = T_b = T_1$ and $T_c = T_d = T_2$

$$\therefore \frac{T_1}{T_2} = \left(\frac{V_c}{V_b} \right)^{\gamma - 1} \quad \dots(i)$$

Similarly, for adiabatic compression $d \rightarrow a$, $\frac{T_a}{T_d} = \left(\frac{V_d}{V_a} \right)^{\gamma - 1}$

Since, $T_a = T_1$ and $T_d = T_2$,

$$\therefore \frac{T_1}{T_2} = \left(\frac{V_d}{V_a} \right)^{\gamma - 1} \quad \dots(ii)$$

From (i) and (ii), $\frac{V_c}{V_b} = \frac{V_d}{V_a}$ or $\frac{V_c}{V_d} = \frac{V_b}{V_a}$ i.e., $r_2 = r_1$

$$\begin{aligned} \text{From eqn. (5.4), Net work done} &= R [T_1 \log_e(r_1) - T_2 \log_e(r_2)] \text{ per kg of air} \\ &= R(T_1 - T_2) \log_e(r_1) \text{ per kg of air.} \end{aligned} \quad \dots(5.5)$$

But, Efficiency = $\frac{\text{Net work done per kg of air}}{\text{Heat supplied per kg of air}}$

$$= \frac{R(T_1 - T_2) \log_e(r_1)}{R T_1 \log_e(r_1)} = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1} \quad \dots(5.6)$$

Problem – 1 : While undergoing a Carnot cycle, the working fluid receives heat at a temperature of 317°C and rejects heat at a temperature of 22°C . Find the theoretical efficiency of the cycle. If the engine working on this cycle absorbs 2,100 kJ/min. from the hot body, calculate the net work done in kJ per sec. and the theoretical power of the engine.

Referring to fig. 5–1, $T_1 = 317 + 273 = 590$ K; $T_2 = 22 + 273 = 295$ K

Using eqn. (5.6),

$$\text{Theoretical efficiency of the Carnot cycle, } \eta = 1 - \frac{T_2}{T_1} = 1 - \frac{295}{590} = 0.5 \text{ i.e. 50\%}$$

Efficiency = $\frac{\text{Work done per minute}}{\text{Heat supplied per minute}}$

$$\text{i.e., } 0.5 = \frac{\text{Work done per minute}}{2,100}$$

$$\therefore \text{Work one per minute} = 0.5 \times 2,100 = 1,050 \text{ kJ/min. or } 17.5 \text{ kJ/sec}$$

Since, one kW = 1 kJ/sec,

Theoretical power of the engine = 17.5 kW.

5.6 Otto Cycle (Constant Volume Cycle)

This ideal heat engine cycle was proposed in 1862 by Beau de Rochas. In 1876 Dr. Otto designed an engine to operate on this cycle. The Otto engine immediately became so successful from a commercial stand point, that its name was affixed to the cycle used by it. This cycle is in use in all gas and petrol engines together with many types of oil engines.

The ideal $p-v$ and $T-\vartheta$ diagrams of this cycle are shown in fig 5-2. In working out the air-standard efficiency of the cycle, the following assumptions are made :

- (i) The working fluid (working substance) in the engine cylinder is air, and it behaves as a perfect gas, i.e., it obeys the gas laws and has constant specific heats.
- (ii) The air is compressed adiabatically (without friction) according to law

$pV^\gamma = C$ (where $\gamma = \frac{K_p}{K_v}$) in the engine cylinder during the compression stroke.

(iii) The heat is supplied to the air at constant volume by bringing a hot body in contact with the end of the engine cylinder.

(iv) The air expands in the engine cylinder adiabatically (without friction) during the expansion stroke.

(v) The heat is rejected from the air at constant volume by bringing a cold body in contact with the end of the engine cylinder.

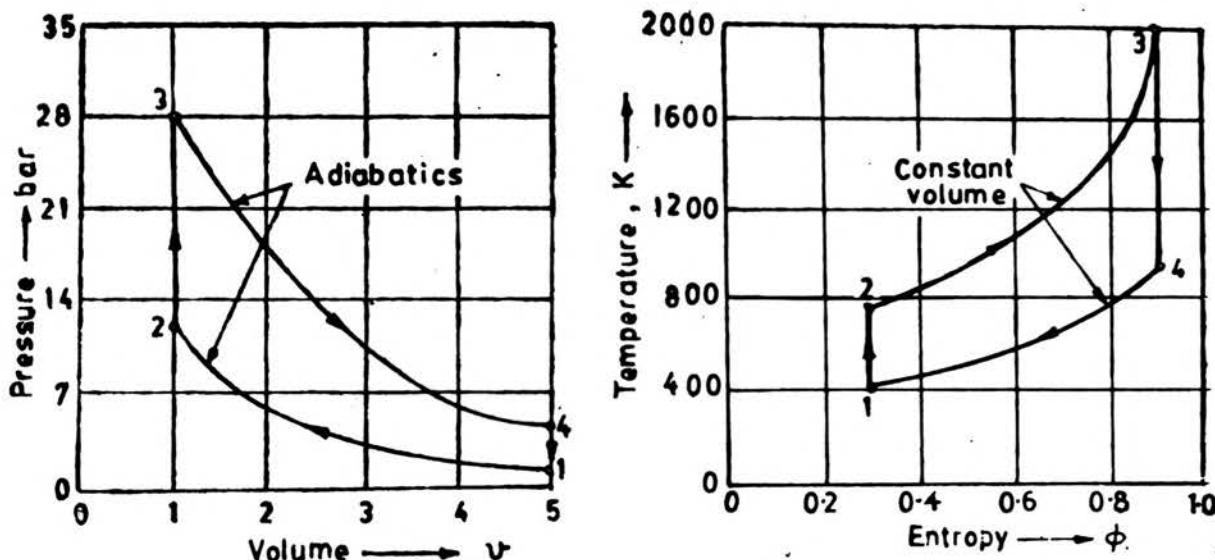


Fig. 5.2. p - v and T - ϕ diagrams of Otto cycle.

Consider one kilogram of air in the engine cylinder at point (1). This air is compressed adiabatically to point (2), at which condition the hot body is placed in contact with the end of the cylinder. Heat is now supplied at constant volume, and temperature and pressure rise; this operation is represented by (2-3). The hot body is then removed and the air expands adiabatically to point (4). During this process, work is done on the piston. At point (4), the cold body is placed at the end of the cylinder. Heat is now rejected at constant volume, resulting in drop of temperature and pressure. This operation is represented by (4-1). The cold body is then removed after the air is brought to its original state (condition). The cycle is thus completed.

The cycle consists of two constant volume processes and two frictionless adiabatic processes. The heat is supplied during constant volume process (2-3) and rejected during constant volume process (4-1). There is no exchange of heat during the two frictionless adiabatics (1-2) and (3-4).

Heat supplied during constant volume operation (2-3) = $K_v (T_3 - T_2)$ heat units/kg of air.

Heat rejected during constant volume operation (4-1) = $K_v (T_4 - T_1)$ heat units/kg of air.

$$\text{Net work done} = \text{Heat supplied} - \text{Heat rejected}$$

$$= K_v (T_3 - T_2) - K_v (T_4 - T_1) \text{ heat units per kg of air.}$$

$$\text{Efficiency, } \eta = \frac{\text{Net work done per kg of air}}{\text{Heat supplied per kg of air}}$$

$$= \frac{K_v (T_3 - T_2) - K_v (T_4 - T_1)}{K_v (T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

... (5.7)

Considering adiabatic compression (1-2),

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = (r)^{\gamma-1} \quad (\text{where } r = \text{ratio of compression})$$

or $\frac{\text{Higher temperature } (T_2)}{\text{Lower temperature } (T_1)} = (r)^{\gamma-1}$

$$\therefore T_2 = T_1 (r)^{\gamma-1} \quad \dots \text{(i)}$$

Again considering adiabatic expansion (3-4),

$$\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{\gamma-1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = (r)^{\gamma-1} \quad (\text{as } v_4 = v_1 \text{ and } v_3 = v_2)$$

or $\frac{\text{Higher temperature } (T_3)}{\text{Lower temperature } (T_4)} = (r)^{\gamma-1}$

$$\therefore T_3 = T_4 (r)^{\gamma-1} \quad \dots \text{(ii)}$$

Substituting values of T_2 and T_3 from (i) and (ii) in eqn. (5.7), we get,

$$\begin{aligned} \text{Air-standard efficiency} &= 1 - \frac{T_4 - T_1}{T_4 (r)^{\gamma-1} - T_1 (r)^{\gamma-1}} \\ (\text{A. S. E.}) &= 1 - \frac{T_4 - T_1}{(r)^{\gamma-1} (T_4 - T_1)} = 1 - \frac{1}{(r)^{\gamma-1}} \end{aligned} \quad \dots \text{(5.8)}$$

From eqn. (5.8), it is seen that the air-standard efficiency of I.C. engines working on Otto cycle is a function of the compression ratio (r) only. The following table gives the value of air-standard efficiency for various ratios of compression :

Ratio of compression, r	2.0	3.0	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0
Percentage air-standard efficiency ($\gamma = 1.4$)	24.51	35.42	42.56	45.21	47.47	49.44	51.16	52.70	54.00	55.34	56.46

The air-standard efficiency expression, $\eta = 1 - \frac{1}{(r)^{\gamma-1}}$ can also be expressed in terms of temperatures T_1 and T_2 .

$$\text{From (i), } \frac{T_2}{T_1} = (r)^{\gamma-1}$$

$$\therefore \text{Air-standard efficiency} = 1 - \frac{1}{\frac{T_2}{T_1}} = 1 - \frac{T_1}{T_2} = \frac{T_2 - T_1}{T_2} \quad \dots \text{(5.9)}$$

From eqn. (5.9), it should be observed that T_2 is not the highest temperature of the cycle, and therefore the efficiency is less than Carnot, which, for the temperature range obtaining, would be $1 - \frac{T_1}{T_3}$ or $\frac{T_3 - T_1}{T_3}$ where T_3 is the highest temperature of the Otto cycle.

The eqn. (5.8) shows, that higher thermal efficiency can be obtained with higher compression ratio, and smaller the difference between T_3 and T_2 , the more closely is the Carnot efficiency approached, but at the expense of reduction in net work done per kg of air.

Problem – 2 : In an ideal Otto cycle engine, the temperature and pressure at the beginning of compression are 43°C and 100 kPa respectively and the temperature at the

end of adiabatic compression is 323°C . If the temperature at the end of constant volume heat addition is $1,500^{\circ}\text{C}$, calculate : (a) the compression ratio, (b) the air-standard efficiency, and (c) the temperature and pressure at the end of adiabatic expansion. Assume γ as 1.4 for air.

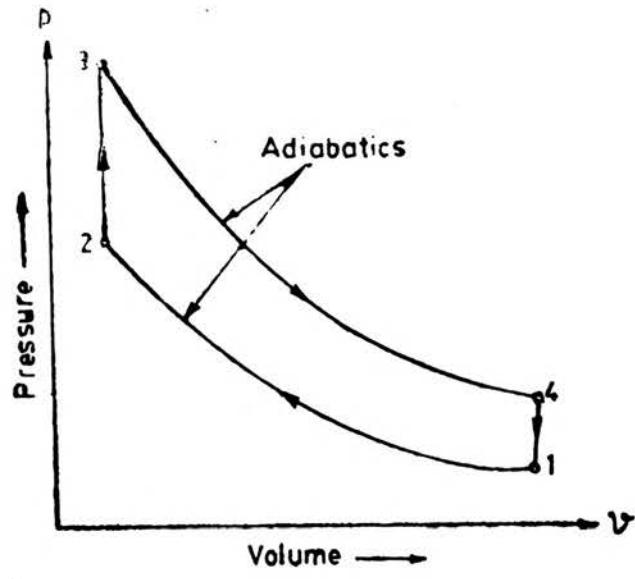
(a) Referring to fig. 5-3, $p_1 = 100 \text{ kPa}$,

$$T_1 = 43 + 273 = 316 \text{ K};$$

$$T_2 = 323 + 273 = 596 \text{ K};$$

$$T_3 = 1,500 + 273 = 1,773 \text{ K}.$$

Referring to fig. 5-3 and considering adiabatic compression (1-2),



$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (r)^{\gamma-1}$$

$$\text{i.e. } (r)^{\gamma-1} = \frac{T_2}{T_1}$$

∴ Compression ratio,

$$r = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}} = \left(\frac{596}{316}\right)^{\frac{1}{1.4-1}} \\ = (1.886)^{2.5} = 4.87$$

(b) Using eqn. (5.8), Air-standard efficiency (A.S.E.)

$$= 1 - \frac{1}{(r)^{\gamma-1}} = 1 - \frac{1}{(4.87)^{0.4}}$$

$$= 1 - \frac{1}{1.884} = 0.4695 \text{ or } 46.95\%$$

Alternatively, using eqn. (5.9),

Fig. 5-3. p-v diagram of Otto cycle.

Air-standard efficiency $1 - \frac{T_1}{T_2} = 1 - \frac{316}{596} = 0.4695 \text{ or } 46.95\% \text{ (same as before)}$

$$(c) \text{ Now, } \frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} = (r)^{\gamma} = (4.87)^{1.4} = 9.173$$

$$\therefore p_2 = p_1 \times 9.173 = 100 \times 9.173 = 917.3 \text{ kPa}$$

$$\text{From constant volume heat addition (2-3), } \frac{p_2 V_2}{T_2} = \frac{p_3 V_3}{T_3}$$

$$\text{Hence, as } V_2 = V_3, \frac{p_3}{p_2} = \frac{T_3}{T_2}$$

$$\therefore p_3 = p_2 \times \frac{T_3}{T_2} = 917.3 \times \frac{(1,500 + 273)}{(323 + 273)} = 2,729 \text{ kPa}$$

Considering adiabatic expansion (3-4) of fig. 5-3, $p_3 V_3^{\gamma} = p_4 V_4^{\gamma}$

$$\therefore p_4 = \frac{p_3}{\left(\frac{V_4}{V_3}\right)^{\gamma}} = \frac{p_3}{(r)^{\gamma}} = \frac{2,729}{(4.87)^{1.4}} = \frac{2,729}{9.173} = 297.5 \text{ kPa}$$

Considering adiabatic expansion (3-4) of fig. 5-3, $\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1}$

$$\therefore T_4 = \frac{T_3}{\left(\frac{V_4}{V_3}\right)^{\gamma-1}} = \frac{T_3}{(r)^{\gamma-1}} = \frac{1,773}{(4.87)^{0.4}} = \frac{1,773}{1.884} = 941\text{K or } t_4 = 668^\circ\text{C}$$

Problem – 3 : In an engine working on the ideal Otto cycle, the pressure and temperature at the beginning of compression are 100 kPa and 40°C respectively. If the air-standard efficiency of the engine is 50%, determine; (i) the compression ratio, and (ii) the pressure and temperature at the end of adiabatic compression. Assume γ for air as 1.4.

Referring to fig. 5.3, $p_1 = 100 \text{ kPa}$, $T_1 = 40 + 273 = 313 \text{ K}$.

(i) Using eqn. (5.8),

$$\text{Air-standard efficiency} = 1 - \frac{1}{(r)^{\gamma-1}} \quad (\text{where } r = \text{compression ratio})$$

$$\text{i.e. } 0.5 = 1 - \frac{1}{(r)^{0.4}}$$

$$\therefore (r)^{0.4} = 2$$

$$\text{Taking logs of both the sides, } \log r = \frac{\log 2}{0.4} = \frac{0.301}{0.4} = 0.7525$$

$$\therefore r = 5.656 \text{ (compression ratio)}$$

$$(ii) \text{ Referring to fig. 5-3 and considering adiabatic compression (1-2), } \frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^\gamma$$

$$\therefore p_2 = p_1 \times \left(\frac{V_1}{V_2}\right)^\gamma = p_1 \times (r)^\gamma$$

$$= 100 \times (5.656)^{1.4} = 1,131 \text{ kPa}$$

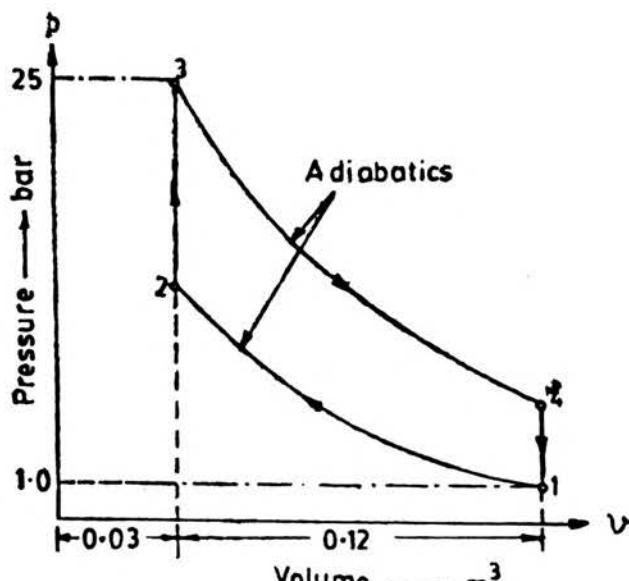


Fig. 5-4. *p-v* diagram of Otto cycle.

Again referring to fig. 5-3 and considering adiabatic compression (1-2), $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$

$$\therefore T_2 = T_1 \times \left(\frac{V_1}{V_2}\right)^{\gamma-1} = T_1 \times (r)^{\gamma-1}$$

$$= 313 \times (5.656)^{0.4} = 626 \text{ K or } t_2 = 353^\circ\text{C}$$

Problem – 4 : An engine working on the ideal Otto cycle, has a clearance volume of 0.03m^3 and swept volume of 0.12 m^3 . The pressure and temperature at the beginning of compression are 100 kPa and 100°C respectively. If the pressure at the end of constant volume heat addition is 2,500 kPa, calculate : (a) the air-standard efficiency of the cycle, and (b) the temperatures at the salient (key) points of the cycle. Assume $\gamma = 1.4$ for air.

- (a) Referring to fig. 5-4, $p_1 = 1$ bar; $p_3 = 25$ bar;
 $T_1 = 100 + 273 = 373$ K; $v_2 = 0.03 \text{ m}^3$; $v_1 - v_2 = 0.12 \text{ m}^3$.
 Compression ratio,

$$r = \frac{v_1}{v_2} = \frac{\text{clearance volume} + \text{swept volume}}{\text{clearance volume}}$$

$$= \frac{0.03 + 0.12}{0.03} = 5$$

Using eqn. (5.8), Air-standard efficiency

$$\eta = 1 - \frac{1}{(r)^{\gamma-1}} = 1 - \frac{1}{(5)^{0.4}}$$

$$= 0.475 \text{ or } 47.5\%$$

- (b) Referring to fig. 5-4 and considering adiabatic compression (1-2),

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^\gamma$$

$$\therefore p_2 = p_1 \times \left(\frac{v_1}{v_2}\right)^\gamma = p_1 \times (r)^\gamma = 10 \times (5)^{1.4} = 9.52 \text{ bar}$$

$$\text{Again from adiabatic compression (1-2), } \frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = (r)^{\gamma-1}$$

$$\therefore T_2 = T_1 \times (r)^{\gamma-1} = 373 \times (5)^{0.4} = 710 \text{ K or } t_2 = 437^\circ\text{C}$$

$$\text{From constant volume heat addition (2-3), } \frac{p_2 v_2}{T_2} = \frac{p_3 v_3}{T_3}$$

$$\text{Hence, as } v_2 = v_3, \frac{T_3}{T_2} = \frac{p_3}{p_2}$$

$$\therefore T_3 = T_2 \times \frac{p_3}{p_2} = 710 \times \frac{25}{9.52} = 1,865 \text{ K or } t_3 = 1,592^\circ\text{C}$$

Considering adiabatic expansion (3-4),

$$\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{\gamma-1}$$

$$\therefore T_4 = \frac{T_3}{\left(\frac{v_4}{v_3}\right)^{\gamma-1}} = \frac{T_3}{(r)^{\gamma-1}}$$

$$= \frac{1,865}{(5)^{0.4}} = 980 \text{ K or } t_4 = 707^\circ\text{C}$$

Problem – 5 : An air engine works on the ideal cycle in which heat is received and rejected at constant volume. The pressure and temperature at the beginning of compression are 100 kPa and 40°C respectively. The pressure at the end of adiabatic compression is 15 times that at start. If the temperature reached at the end of constant volume heat addition is 1,947°C, find : (a)

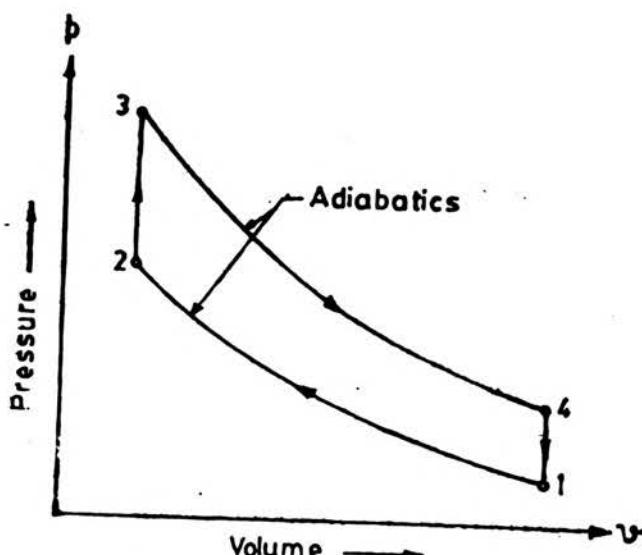


Fig. 5.5. p - v diagram of Otto cycle.

the heat supplied per kg of air, (b) the air-standard efficiency, (c) the work done per kg of air, and (d) the pressure and temperature at the end of adiabatic expansion. Take $K_v = 0.7165 \text{ kJ/kg K}$ and $\gamma = 1.4$ for air.

Referring to fig. 5.5, $p_1 = 100 \text{ kPa}$;
 $p_2 = 15p_1 = 15 \times 100 = 1,500 \text{ kPa}$;
 $T_1 = 40 + 273 = 313 \text{ K}$;
 $T_3 = 1,947 + 273 = 2,220 \text{ K}$; and $\gamma = 1.4$

(a) Considering adiabatic compression (1-2),

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{i.e. } T_2 = T_1 \times \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore T_2 = 313 \times \left(\frac{1,500}{100}\right)^{\frac{0.4}{1.4}} = 313 \times 2.169 = 678.9 \text{ K}$$

$$\begin{aligned} \text{Heat supplied per kg of air} &= K_v (T_3 - T_2) \\ &= 0.7165 (2,200 - 678.9) = 1,104.2 \text{ kJ/kg of air} \end{aligned}$$

$$(b) \text{ Using eqn. (5.9), Air-standard efficiency} = 1 - \frac{T_1}{T_2} = 1 - \frac{313}{678.9} = 0.535 \text{ or } 53.5\%$$

$$(c) \text{ Now, Air-standard efficiency (A.S.E.)} = \frac{\text{Work done per kg of air}}{\text{Heat supplied per kg of air}}$$

$$\begin{aligned} \therefore \text{Work done per kg of air} &= \text{A.S.E.} \times \text{Heat supplied per kg of air} \\ &= 0.535 \times 1,104.2 = 590.75 \text{ kJ/kg of air.} \end{aligned}$$

$$(d) \text{ From constant volume heat addition (2-3), } \frac{p_2 v_2}{T_2} = \frac{p_3 v_3}{T_3}$$

$$\text{Hence as } v_2 = v_3, \frac{p_3}{p_2} = \frac{T_3}{T_2} \quad \text{i.e., } p_3 = p_2 \times \frac{T_3}{T_2}$$

$$\therefore p_3 = 1,500 \times \frac{2,220}{678.9} = 4,948 \text{ kPa}$$

From adiabatic compression (1-2) and adiabatic expansion (3 - 4),

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^\gamma \text{ and } \frac{p_3}{p_4} = \left(\frac{v_4}{v_3}\right)^\gamma \quad \therefore \frac{p_2}{p_1} = \frac{p_3}{p_4} \quad (\text{as } v_1 = v_4 \text{ and } v_2 = v_3)$$

$$\therefore p_4 = \frac{p_1}{p_2} \times p_3 = \frac{1}{15} \times 4,948 = 329.6 \text{ kPa.}$$

$$\text{From constant volume heat rejection (4-1), } \frac{p_4 v_4}{T_4} = \frac{p_1 v_1}{T_1}$$

$$\text{Hence, as } v_4 = v_1, \frac{p_4}{T_4} = \frac{p_1}{T_1}$$

$$\therefore T_4 = \frac{p_4}{p_1} \times T_1 = \frac{329.6}{100} \times 313 = 1,031.6 \text{ K or } t_4 = 758.6^\circ\text{C}$$

5.7 Diesel Cycle (Constant Pressure Cycle)

Internal combustion engines of today, operate on heat engine cycles which approximate either the ideal Otto cycle or the ideal Diesel cycle. In 1897, Dr. Rudolph Diesel constructed

the first successful Diesel engine. This engine was designed to operate on a new heat engine cycle devised by him and hence is known as *Diesel cycle*. Diesel engines have been used to a considerable extent in stationary marine and locomotive practice.

The ideal p - v and T - ϕ diagrams of the cycle are shown in fig. 5-6.

In working out the air-standard efficiency of this cycle, the following assumptions are made :

- (i) The working fluid in the engine cylinder is air and it behaves as a perfect gas, i.e., it obeys the gas laws and has constant specific heats.
- (ii) The air is compressed adiabatically (without friction) in the engine cylinder during the compression stroke.
- (iii) Heat is supplied to the air at constant pressure by bringing a hot body in contact with the end of the cylinder.
- (iv) The air expands in the engine cylinder adiabatically (without friction) during expansion stroke.
- (v) Heat is abstracted from the working substance at constant volume by bringing a cold body in contact with the end of the cylinder.

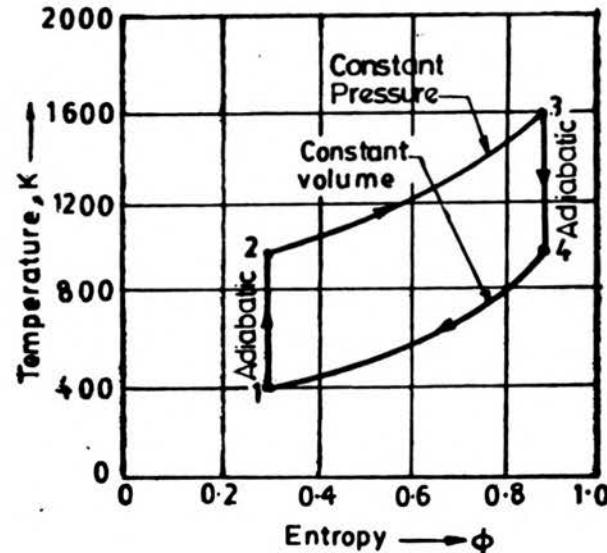
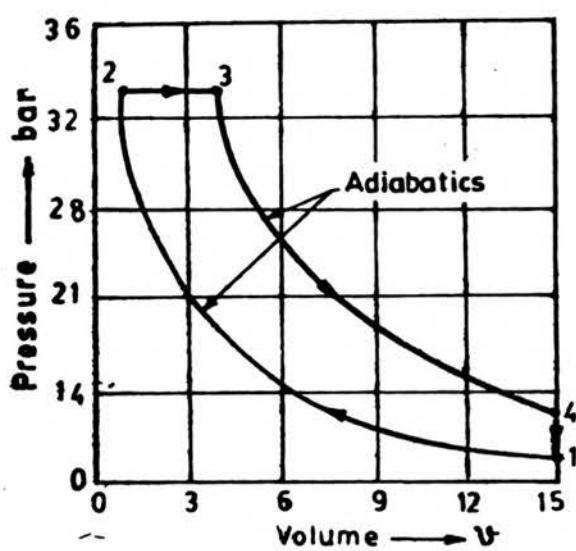


Fig. 5.6. p - v and T - ϕ diagrams of Diesel cycle.

Imagine the cylinder to contain 1 kg of air at point (1). This air is compressed adiabatically to point (2) by the piston during its inward stroke. The air now occupies the clearance volume. The heat is then supplied at constant pressure by bringing a hot body in contact with the end of the cylinder. At point (3) the hot body is removed and the supply of heat is stopped. This point is known as the point of *cut-off*. The air now expands adiabatically to point (4). During this process, work is done on the piston. The air now occupies the whole cylinder volume. The cold body is then placed at the end of the cylinder and heat is abstracted from the working substance at constant volume until the pressure falls to point (1). This operation is represented by (4-1). The cold body is removed after the air is brought to its original condition (1). The cycle is thus completed.

The cycle consists of two adiabatic processes, one constant pressure process and one constant volume process. Heat is supplied during constant pressure process (2-3)

and heat is rejected during constant volume process (4-1). There is no exchange of heat during adiabatic processes (1-2) and (3-4)

Heat supplied during constant pressure process (2-3) = $K_p (T_3 - T_2)$ heat units per kg of air.

Heat rejected during constant volume process (4-1) = $K_v (T_4 - T_1)$ heat units per kg of air.

Net work done by the air = Heat supplied - Heat rejected

= $K_p (T_3 - T_2) - K_v (T_4 - T_1)$ heat units per kg of air.

$$\begin{aligned} \text{A.S. efficiency, } \eta &= \frac{\text{Net work done per kg of air}}{\text{Heat supplied per kg of air}} \\ &= \frac{k_p (T_3 - T_2) - k_v (T_4 - T_1)}{k_p (T_3 - T_2)} \\ &= 1 - \frac{k_v}{k_p} \times \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{1}{\gamma} \times \frac{T_4 - T_1}{T_3 - T_2} \end{aligned} \quad \dots(5.10)$$

Let compression ratio, $\frac{V_1}{V_2} = r$ and cut-off ratio, $\frac{V_3}{V_2} = \rho$

$$\text{Then, expansion ratio, } \frac{V_4}{V_3} = \frac{V_4}{V_2} \times \frac{V_2}{V_3} = \frac{V_1}{V_2} \times \frac{V_2}{V_3} = \frac{r}{\rho} \quad \dots(5.11)$$

$$\text{From constant pressure heat addition (2-3), } \frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3}$$

$$\text{Hence, as } P_2 = P_3, \frac{T_3}{T_2} = \frac{V_3}{V_2} = \rho \quad \therefore T_2 = \frac{T_3}{\rho} \quad \dots(i)$$

$$\text{From adiabatic compression (1-2), } \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (r)^{\gamma-1}$$

$$\therefore T_1 = \frac{T_2}{(r)^{\gamma-1}} \quad \dots(ii)$$

$$\text{Substituting value of } T_2 \text{ from (i) in (ii), we get, } T_1 = \frac{T_3}{\rho(r)^{\gamma-1}} \quad \dots(iii)$$

$$\text{From adiabatic expansion (3-4), } \frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = \left(\frac{r}{\rho}\right)^{\gamma-1}$$

$$\therefore T_4 = \frac{T_3 (\rho)^{\gamma-1}}{(r)^{\gamma-1}} \text{ or } T_4 = \frac{T_3 (\rho)^\gamma}{\rho (r)^{\gamma-1}} \quad \dots(iv)$$

Substituting values of T_2 , T_1 and T_4 from (i), (iii) and (iv) in eqn. (5.10), we get,

$$\begin{aligned} \text{Air-standard efficiency} &= 1 - \frac{1}{\gamma} \times \frac{\frac{T_3 (\rho)^\gamma}{\rho (r)^{\gamma-1}} - \frac{T_3}{\rho}}{T_3 - \frac{T_3}{\rho}} \\ &= 1 - \frac{1}{\gamma} \times \frac{1}{(r)^{\gamma-1}} \left\{ \frac{\frac{T_3 (\rho)^\gamma}{\rho} - \frac{T_3}{\rho}}{T_3 - \frac{T_3}{\rho}} \right\} \end{aligned}$$

$$\begin{aligned}
 \text{A.S.E.} &= 1 - \frac{1}{\gamma} \times \frac{1}{(r)^{\gamma-1}} \left\{ \frac{\frac{(\rho)^{\gamma}}{\rho} - \frac{1}{\rho}}{1 - \frac{1}{\rho}} \right\} \\
 &= 1 - \frac{1}{\gamma} \times \frac{1}{(r)^{\gamma-1}} \left\{ \frac{(\rho)^{\gamma} - 1}{\rho - 1} \right\} \\
 &= 1 - \frac{1}{(r)^{\gamma-1}} \times \left\{ \frac{(\rho)^{\gamma} - 1}{\gamma(\rho - 1)} \right\} \quad \dots(5.12)
 \end{aligned}$$

This expression is the air-standard efficiency of the Diesel cycle. It will be noted from the expression (5.12), that the air-standard efficiency of the Diesel cycle depends upon the value of r and ρ . The efficiency increases as r is increased and decreases as ρ is increased.

The factor $\left[\frac{(\rho)^{\gamma} - 1}{\gamma(\rho - 1)} \right]$ depends upon the value of cut-off ratio and is greater than unity; hence the air-standard efficiency of the Diesel cycle for a given compression ratio is less than $1 - \left(\frac{1}{r} \right)^{\gamma-1}$, which is the efficiency of Otto cycle.

Problem – 6 : An engine working on Diesel cycle has a compression ratio of 15 and cut-off takes place at 5% of the stroke. Find the air-standard efficiency. Assume value of $\gamma = 1.4$ for air.

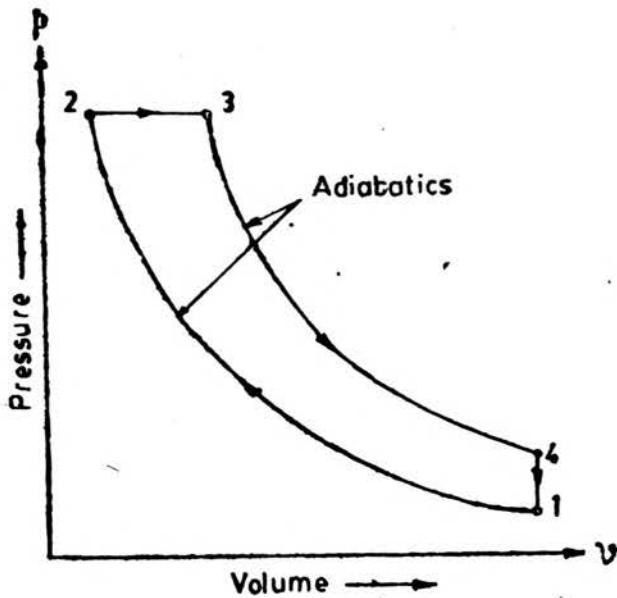


Fig. 5-7. p - v diagram of Diesel cycle.

Referring to fig. 5-7,

Compression ratio, r

$$= \frac{v_1}{v_2} = 15 \therefore v_1 = 15v_2$$

Now, stroke volume

$$= v_1 - v_2 = 15v_2 - v_2 = 14v_2$$

$v_3 = 5\%$ of stroke volume + clearance volume.

$$\therefore v_3 = \left(\frac{5}{100} \times 14v_2 \right) + v_2 = 1.7v_2$$

$$\text{Cut-off ratio, } \rho = \frac{v_3}{v_2} = \frac{1.7v_2}{v_2} = 1.7$$

Using eqn. (5.12), Air-standard efficiency

$$\begin{aligned}
 &= 1 - \frac{1}{(r)^{\gamma-1}} \left\{ \frac{(\rho)^{\gamma} - 1}{\gamma(\rho - 1)} \right\} \\
 &= 1 - \frac{1}{(15)^{0.4}} \times \left[\frac{(1.7)^{1.4} - 1}{1.4(1.7 - 1)} \right] \\
 &= 0.625 \text{ or } 62.5\%
 \end{aligned}$$

Problem – 7 : An air engine works on the following cycle : Air is taken in at atmospheric pressure of 110 kPa and temperature of 16°C, and is compressed adiabatically, the pressure at the end of the stroke being 3,500 kPa. Heat is taken in at constant pressure, the expansion afterwards takes place adiabatically, the ratio of expansion being 5. The air is exhausted at the end of the

stroke, the heat is assumed to be rejected at constant volume. Find the ideal thermal efficiency. Take the specific heats of air as $k_p = 1.0035 \text{ kJ/kg K}$ and $k_v = 0.7165 \text{ kJ/kg K}$.

Here, $\gamma = \frac{k_p}{k_v} = \frac{1.0035}{0.7165} = 1.4$ and $\frac{\gamma - 1}{\gamma} = \frac{0.4}{1.4} = 0.286$, $\frac{1}{\gamma} = \frac{1}{1.4} = 0.715$;

$p_1 = 110 \text{ kPa}$; $p_2 = p_3 = 3,500 \text{ kPa}$; $T_1 = 273 + 16 = 289 \text{ K}$.

Referring to fig. 5-8 and considering adiabatic compression (1-2),

$$\begin{aligned}\frac{p_2}{p_1} &= \left(\frac{V_1}{V_2}\right)^\gamma = (r)^\gamma \\ \therefore r &= \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} = \left(\frac{3,500}{110}\right)^{0.715} \\ &= 11.8 \text{ (compression ratio)}\end{aligned}$$

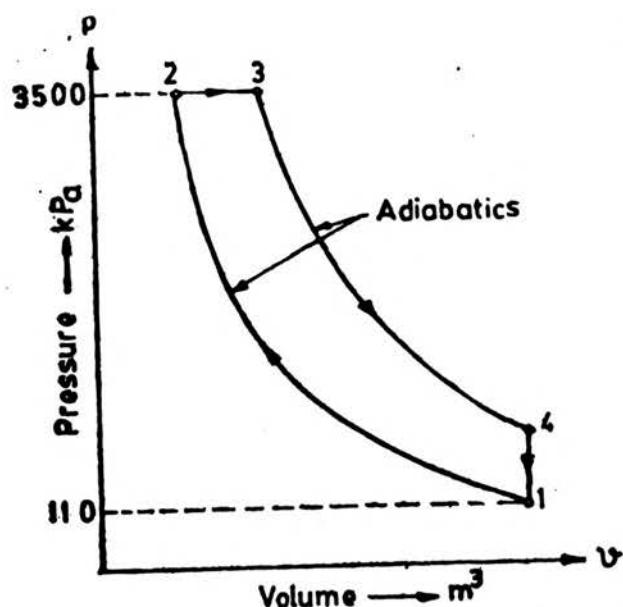


Fig. 5-8. p - v diagram of Diesel cycle.

$$\text{Hence, as } p_2 = p_3, \frac{T_3}{T_2} = \frac{V_3}{V_2}$$

$$\therefore T_3 = \frac{V_3}{V_2} \times T_2 = 2.36 \times 778 = 1,838 \text{ K.}$$

$$\text{Considering adiabatic expansion (3-4), } \frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma - 1}$$

$$\therefore T_4 = T_3 \times \left(\frac{V_3}{V_4}\right)^{\gamma - 1} = 1,838 \times \left(\frac{1}{5}\right)^{0.4} = 1,838 \times 0.524 = 960 \text{ K.}$$

Now, heat supplied per kg of air

$$= k_p (T_3 - T_2) = 1.0035 (1,838 - 778) = 1,063.71 \text{ kJ per kg of air, and heat rejected per kg of air} = k_v (T_4 - T_1) = 0.7165 (960 - 289) = 480.7 \text{ kJ per kg of air.}$$

Hence, heat converted to work or work done per kg of air

$$= k_p (T_3 - T_2) - k_v (T_4 - T_1) = 1,063.71 - 480.7 = 583.01 \text{ kJ/kg of air.}$$

Also the ratio of expansion, $\frac{V_4}{V_3} = 5$ (given).

Using eqn. (5.11), cut-off ratio $\left(\frac{V_3}{V_2}\right) =$

$$\begin{aligned}\rho &= \frac{\text{Ratio of compression}}{\text{Ratio of expansion}} \\ &= \frac{11.8}{5} = 2.36\end{aligned}$$

Considering adiabatic compression (1-2),

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}} = \left(\frac{3,500}{110}\right)^{0.286} = 2.69$$

$$\therefore T_2 = T_1 \times 2.69 = 289 \times 2.69 = 778 \text{ K.}$$

From constant pressure heat addition

$$(2-3), \frac{p_2 V_2}{T_2} = \frac{p_3 V_3}{T_3}$$

Ideal thermal efficiency of Air standard efficiency

$$= \frac{\text{work done per kg. of air}}{\text{heat supplied per kg. of air}} = \frac{583.01}{1,063.7} = 0.5483 \text{ or } 54.83\%$$

Alternatively, using eqn. (5.12), the ideal thermal efficiency or air-standard efficiency.

$$\text{A.S.E.} = 1 - \frac{1}{(r)^{\gamma-1}} \times \left[\frac{(p)^{\gamma} - 1}{\gamma(p - 1)} \right]$$

Here, ratio of compression, $r = 11.8$, cut-off ratio, $p = 2.36$, $\gamma = 1.4$.

On substitution of values in eqn. (5.12), we get,

$$\begin{aligned} \text{Air standard efficiency} &= 1 - \frac{1}{(11.8)^{1.4-1}} \times \left[\frac{(2.36)^{1.4} - 1}{1.4(2.36 - 1)} \right] \\ &= 1 - 0.371 \left[\frac{3.32 - 1}{1.904} \right] \\ &= 1 - 0.4517 = 0.5483 \text{ i.e. } 54.83\% \text{ (same as before) } \end{aligned}$$

Problem – 8 : The following data relate to a theoretical Diesel cycle, using air as the working fluid :

Pressure at the end of suction stroke ... 100 kPa

Temperature at the end of suction stroke ... 30°C

Temperature at the end of constant pressure heat addition ... 1,500°C

Compression ratio ... 16

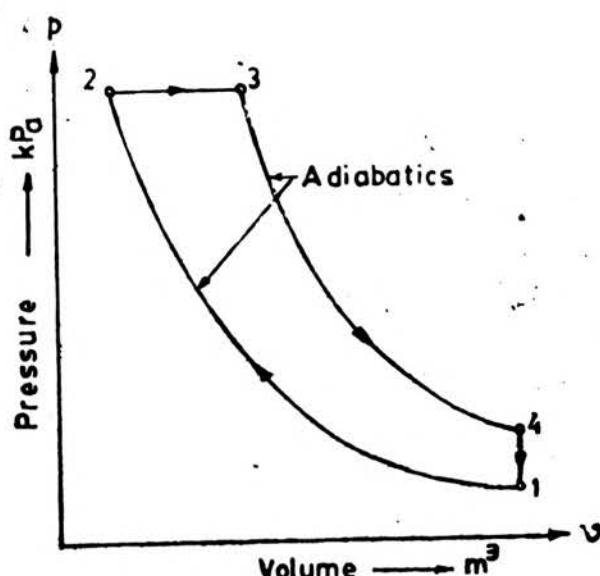
Specific heat of air at constant pressure ... 1.005 kJ/Kg K

Specific heat of air at constant volume ... 0.7115 kJ/kg K

Find : (a) the percentage of stroke at which cut-off takes place, (b) the temperature at the end of expansion stroke, and (c) the ideal thermal efficiency.

Here, $p_1 = 100 \text{ kPa}$; $t_1 = 30^\circ \text{ C}$; $t_3 = 1,500^\circ \text{ C}$; $\frac{V_1}{V_2} = r = 16$; $\gamma = \frac{k_p}{k_v} = \frac{1.005}{0.7115} = 1.41$

(a) Referring to fig. 5-9 and considering adiabatic compression 1-2,



$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\gamma-1} = (r)^{\gamma-1} \text{ or}$$

$$T_2 = T_1 \times (r)^{\gamma-1}$$

$$\therefore T_2 = (30 + 273) \times (16)^{0.41} = 303 \times 3.117 = 945 \text{ K.}$$

From constant pressure heat addition (2-3),

$$\frac{p_2 V_2}{T_2} = \frac{p_3 V_3}{T_3}$$

$$\text{Hence, as } p_2 = p_3, \frac{V_3}{V_2} = \frac{T_3}{T_2}$$

$$\therefore \frac{V_3}{V_2} = \frac{(1,500 + 273)}{945} = 1.876 \text{ or } V_3 = 1.876 V_2$$

Fig. 5-9. p - v diagram of Diesel cycle

Now percentage of the stroke at which cut-off takes place

$$= \frac{\text{Volume at cut-off} - \text{Clearance volume}}{\text{Stroke volume}} \times 100$$

$$= \frac{V_3 - V_2}{V_1 - V_2} \times 100 = \frac{1.876V_2 - V_2}{16V_2 - V_2} \times 100 = 5.84\%$$

$$(b) \frac{V_4}{V_3} = \frac{V_4}{V_2} \times \frac{V_2}{V_3} = \frac{V_1}{V_2} \times \frac{V_2}{V_3}$$

(as $V_4 = V_1$)

$$\therefore \text{Expansion ratio, } \frac{V_4}{V_3} = 16 \times \frac{1}{1.876} = 8.529$$

From adiabatic expansion (3 - 4),

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} \quad \therefore T_4 = \frac{T_3}{\left(\frac{V_4}{V_3}\right)^{\gamma-1}}$$

$$\therefore T_4 = \frac{1,773}{(8.529)^{0.41}} = \frac{1,773}{2.408} = 736 \text{ K or } T_4 = 463^\circ\text{C}$$

(c) Heat supplied = $k_p (T_3 - T_2) = 1.005 (1,773 - 945) = 832.14 \text{ kJ/kg of air}$.

Heat rejected = $k_v (T_4 - T_1) = 0.7115 (736 - 303) = 308.07 \text{ kJ/kg of air}$.

\therefore Heat converted to work or work done = $832.14 - 308.07 = 524.07 \text{ kJ/kg of air}$.

Now, ideal thermal eff.] = $\frac{\text{Work done per kg of air}}{\text{Heat supplied per kg of air}} = \frac{524.07}{832.14} = 0.6286 \text{ or } 62.86\%$
or A.S. efficiency

5.8 Dual-combustion Cycle

This cycle is known as a dual-combustion or mixed cycle because the heat is taken in partly at constant volume and partly at constant pressure. This cycle is used in modern high speed oil engines and is a combination of Otto and Diesel cycles. Engines working on this cycle are sometimes called *Semi-Diesel* engines.

The ideal p - v and T - ϕ diagrams for this cycle are shown in fig. 5-10. In working out the air-standard efficiency of this cycle the following assumptions are made :

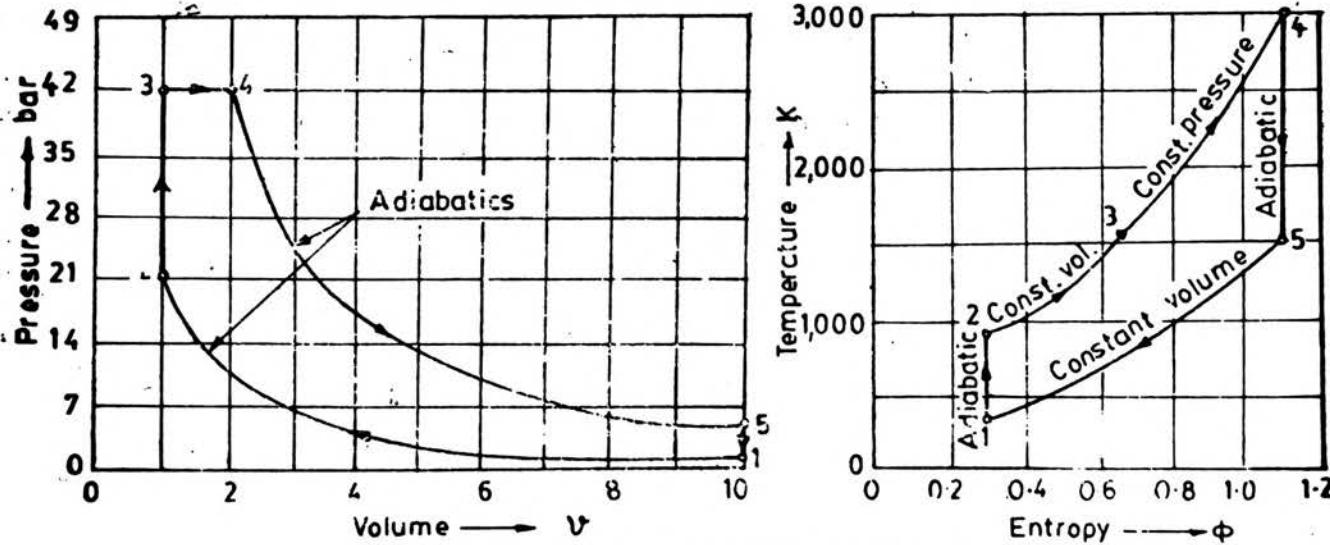


Fig. 5-10. p - v and T - ϕ diagrams of dual-combustion cycle.

- (i) The working fluid in the engine cylinder is air and behaves as a perfect gas, i.e., it obeys gas laws and has constant specific heats.
- (ii) The working fluid is compressed in the engine cylinder adiabatically (without friction) during compression stroke.
- (iii) Heat is partly supplied at constant volume to the working fluid by bringing a hot body in contact with the end of the cylinder. The source of heat is still maintained while the piston moves outward during the working stroke and the remaining heat is supplied to the working fluid at constant pressure. The hot body is then removed after the first portion of the working stroke is completed.
- (iv) The working fluid expands in the engine cylinder adiabatically (without friction) during the expansion stroke.
- (v) Heat is abstracted from the working fluid (substance) at constant volume by bringing a cold body in contact with the end of the cylinder.

Imagine the cylinder to contain one kg of air at point (1). This air is compressed adiabatically to point (2) by the piston during its inward stroke. The air now occupies the clearance volume. The heat is then supplied at constant volume by bringing a hot body in contact with the end of the cylinder. This operation is represented by line (2-3). The hot body is still maintained at the end of the cylinder during first portion of the working stroke and heat is supplied at constant pressure. This operation is represented by line (3-4). The hot body is then removed and the supply of heat is stopped at point 4. The air now expands adiabatically to point (5). During this process work is done on the piston. The air now occupies the whole cylinder volume. The cold body is then placed at the end of the cylinder and heat is abstracted from the working fluid at constant volume until the pressure falls to point (1). This operation is represented by line (5-1). The cold body is then removed and air is brought to its original condition (1). The cycle is thus completed.

This cycle consists of two adiabatic processes, two constant volume processes and one constant pressure process. Heat is supplied during constant volume process (2-3) and constant pressure process (3-4). Heat is rejected during constant volume process (5-1). There is no exchange of heat during the adiabatics (1-2) and (4-5). Referring to fig. 5-10,

$$\text{Heat supplied} = k_v (T_3 - T_2) + k_p (T_4 - T_3) \text{ heat units/kg of air and}$$

$$\text{Heat rejected} = k_v (T_5 - T_1) \text{ heat units/kg of air.}$$

$$\therefore \text{Net work done} = k_v (T_3 - T_2) + k_p (T_4 - T_3) - k_v (T_5 - T_1) \text{ heat units/kg of air.}$$

$$\text{A.S.E.} = \frac{\text{Work done/kg of air}}{\text{Heat supplied/kg of air}}$$

$$\begin{aligned}
 &= \frac{k_v (T_3 - T_2) + k_p (T_4 - T_3) - k_v (T_5 - T_1)}{k_v (T_3 - T_2) + k_p (T_4 - T_3)} \\
 &= 1 - \frac{k_v (T_5 - T_1)}{k_v (T_3 - T_2) + k_p (T_4 - T_3)} \\
 &= 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma (T_4 - T_3)}
 \end{aligned} \tag{5.13}$$

$$\text{Let compression ratio, } \frac{V_1}{V_2} = r \text{ and cut-off ratio, } \frac{V_4}{V_3} = \rho,$$

$$\text{Then, expansion ratio, } \frac{V_5}{V_4} = \frac{V_1}{V_2} \times \frac{V_3}{V_4} = \frac{r}{\rho} \quad (\text{as } V_5 = V_1 \text{ and } V_2 = V_3)$$

Let pressure ratio or explosion ratio, $\frac{p_3}{p_2} = \beta$

From constant volume heat addition (2-3), $\frac{p_2 v_2}{T_2} = \frac{p_3 v_3}{T_3}$

$$\text{Hence, as } v_2 = v_3, \frac{T_3}{T_2} = \frac{p_3}{p_2} = \beta \quad \therefore T_2 = \frac{T_3}{\beta} \quad \dots(i)$$

$$\text{From constant pressure heat addition (3-4), } \frac{T_4}{T_3} = \frac{v_4}{v_3} = \rho \text{ or } T_4 = \rho T_3 \quad \dots(ii)$$

$$\text{From adiabatic compression (1-2), } \frac{T_2}{T_1} = (r)^{\gamma-1} \quad \text{or} \quad T_1 = \frac{T_2}{(r)^{\gamma-1}} \quad \dots(iii)$$

$$\text{Substituting value of } T_2 \text{ from (i) in (iii), we get, } T_1 = \frac{T_3}{\beta (r)^{\gamma-1}} \quad \dots(iv)$$

$$\text{From adiabatic expansion (4-5), } \frac{T_4}{T_5} = \left(\frac{v_5}{v_4}\right)^{\gamma-1} = \left(\frac{r}{\rho}\right)^{\gamma-1}$$

$$\therefore T_5 = \frac{T_4}{\left(\frac{r}{\rho}\right)^{\gamma-1}} = \frac{T_4 (\rho)^{\gamma-1}}{(r)^{\gamma-1}} \quad \dots(v)$$

Substituting value of T_4 from (ii) in (v), we get,

$$T_5 = \frac{\rho T_3 (\rho)^{\gamma-1}}{(r)^{\gamma-1}} = \frac{\rho^\gamma T_3}{(r)^{\gamma-1}} \quad \dots(vi)$$

Substituting values of T_2 , T_4 , T_1 and T_5 from (i), (ii), (iv) and (vi) in eqn. (5.13), we get,

$$\begin{aligned} \text{Air-standard efficiency} &= 1 - \frac{\frac{\rho^\gamma T_3}{(r)^{\gamma-1}} - \frac{T_3}{\beta (r)^{\gamma-1}}}{\left(T_3 - \frac{T_3}{\beta}\right) + \gamma(\rho T_3 - T_3)} \\ &= 1 - \frac{\frac{\rho^\gamma}{(r)^{\gamma-1}} - \frac{1}{\beta (r)^{\gamma-1}}}{\left(1 - \frac{1}{\beta}\right) + \gamma(\rho - 1)} \\ &= 1 - \frac{1}{(r)^{\gamma-1}} \left\{ \frac{\left(\frac{\rho^\gamma}{(r)^{\gamma-1}} - \frac{1}{\beta (r)^{\gamma-1}}\right)}{\left(1 - \frac{1}{\beta}\right) + \gamma(\rho - 1)} \right\} \\ &= 1 - \frac{1}{(r)^{\gamma-1}} \left[\frac{\beta(\rho)^\gamma - 1}{(\beta - 1) + \beta\gamma(\rho - 1)} \right] \quad \dots(5.14) \end{aligned}$$

This is the air-standard efficiency of the dual-combustion cycle.

Now, in eqn. (5.14), if pressure ratio, $\beta = 1$, i.e. $p_3 = p_2$,

$$\text{A.S.E.} = 1 - \frac{1}{(r)^{\gamma-1}} \left\{ \frac{(\rho)^\gamma - 1}{\gamma(\rho - 1)} \right\}$$

which is the expression of the A.S.E. of the Diesel cycle.

Again, in eqn. (5.14), if cut-off ratio, $\rho = 1$, i.e. $v_3 = v_4$,

$$\begin{aligned} A.S.E. &= 1 - \frac{1}{(r)^{\gamma-1}} \left[\frac{(\beta-1)}{(\beta-1) + \beta\gamma(0)} \right] \\ &= 1 - \frac{1}{(r)^{\gamma-1}} \left[\frac{\beta-1}{\beta-1} \right] = 1 - \frac{1}{(r)^{\gamma-1}} \end{aligned}$$

which is the expression of the Air-standard efficiency (A.S.E.) of the Otto cycle.

Problem – 9 : An oil engine working on the dual-combustion cycle has a cylinder diameter of 25 cm and stroke of 36 cm. The clearance volume is 1,600 cm³ and cut-off takes place at 5 per cent of the stroke. The explosion pressure ratio is 1.4. Find the air-standard efficiency of the engine. Assume $\gamma = 1.4$ for air.

$$\text{Stroke volume, } v_s = \frac{\pi}{4} d^2 \times l = \frac{\pi}{4} (25)^2 \times 36 = 17,600 \text{ cm}^3,$$

$$\text{Clearance volume, } v_c = 1,600 \text{ cm}^3 \text{ (given).}$$

$$\text{Compression ratio, } r = \frac{v_c + v_s}{v_c} = \frac{1,600 + 17,600}{1,600} = 12$$

$$\text{Cut-off ratio, } \rho = \frac{v_c + (0.05 v_s)}{v_s} = \frac{1,600 + (0.05 \times 17,600)}{1,600} = 1.55$$

$$\text{Explosion pressure ratio, } \beta = 1.4 \text{ (given).}$$

$$\begin{aligned} \text{Using eqn. (5.14), Air-standard efficiency} &= 1 - \frac{1}{(r)^{\gamma-1}} \left[\frac{\beta(\rho)^{\gamma-1}}{(\beta-1) + \beta\gamma(\rho-1)} \right] \\ &= 1 - \frac{1}{(12)^{0.4}} \left[\frac{1.4(1.55)^{1.4}-1}{(1.4-1) + 1.4 \times 1.4 (1.55-1)} \right] \\ &= 1 - \frac{1}{2.7} \left[\frac{2.68-1}{0.4+1.08} \right] = 0.605 \text{ or } 60.5\%. \end{aligned}$$

Problem – 10 : In an engine working on the ideal dual-combustion cycle, the temperature and pressure at the beginning of compression are 100°C and 100 kPa respectively. The compression ratio is 10:1. If the maximum pressure is limited to 7,000 kPa and 1,675 kJ of heat is supplied per kg of air, determine the temperatures at salient (key) points of the cycle and the air standard efficiency of the engine. Assume $k_p = 1.01 \text{ kJ/kg K}$ and $K_v = 0.716 \text{ kJ/kg K}$ for air.

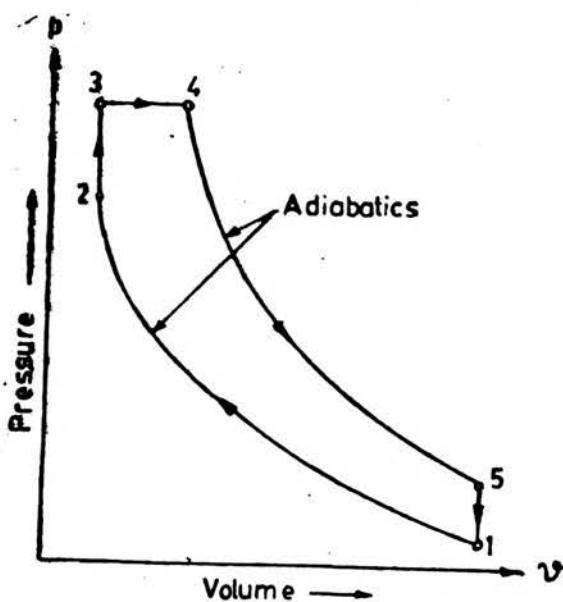


Fig. 5.11. p-v diagram of dual-combustion cycle.

$$\begin{aligned} \text{Here, } p_1 &= 100 \text{ kPa}; \\ T_1 &= 100 + 273 = 373 \text{ K}; \\ p_3 &= p_4 = 7,000 \text{ kPa}; \\ r &= \frac{v_1}{v_2} = 10; \gamma = \frac{k_p}{k_v} = \frac{1.01}{0.716} = 1.41 \end{aligned}$$

Referring to fig 5-11, and considering adiabatic compression (1-2),

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\gamma-1} = (r)^{\gamma-1}$$

$$\therefore T_2 = T_1 \times (r)^{\gamma-1}$$

$$= 373 \times (10)^{0.41} = 958.8 \text{ K} \quad \text{or} \quad t_2 = 685.8^\circ\text{C}.$$

Again, $\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma$ or $P_2 = P_1 \left(\frac{V_1}{V_2}\right)^\gamma$

$$\therefore P_2 = P_1(r)^\gamma = 100 \times (10)^{1.41} = 2,570 \text{ kPa}$$

From constant volume heat addition (2-3), $\frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3}$

Hence, as $V_2 = V_3$, $\frac{T_3}{T_2} = \frac{P_3}{P_2}$ or $T_3 = T_2 \times \frac{P_3}{P_2}$

$$\therefore T_3 = 958.8 \times \frac{7,000}{2,570} = 2,612 \text{ K} \quad \text{or} \quad t_3 = 2,339^\circ\text{C}$$

Heat added at *constant volume* (2-3) per kg. of air

$$= K_v (T_3 - T_2) = 0.716 (2,612 - 958.8) = 1,183.7 \text{ kJ/kg of air.}$$

\therefore Heat added at *constant pressure* (3-4) per kg of air

$$= 1,675 - 1,183.7 = 491.3 \text{ kJ per kg of air} = k_p (T_4 - T_3)$$

$$\text{i.e., } T_4 = T_3 + \frac{491.3}{k_p} = 2,612 + \frac{491.3}{1.01} = 3,098 \text{ K} \quad \text{or} \quad t_4 = 2,825^\circ\text{C.}$$

From constant pressure heat addition (3-4), $\frac{P_3 V_3}{T_3} = \frac{P_4 V_4}{T_4}$

Hence, as $P_3 = P_4$, $\frac{V_4}{V_3} = \frac{T_4}{T_3} = \frac{3,098}{2,612} = 1.186 = \rho$ (cut-off ratio).

From adiabatic expansion (4-5), $\frac{T_4}{T_5} = \left(\frac{V_5}{V_4}\right)^{\gamma-1} = \left(\frac{r}{\rho}\right)^{\gamma-1}$

$$\therefore T_5 = \frac{T_4}{\left(\frac{r}{\rho}\right)^{\gamma-1}} = \frac{3,098}{\left(\frac{10}{1.186}\right)^{0.41}} = 1,293 \text{ K} \quad \text{or} \quad t_5 = 1,020^\circ\text{C}$$

Heat rejected at *constant volume* (5-1) per kg of air

$$= K_v (T_5 - T_1) = 0.716 (1,293 - 373) = 658.72 \text{ kJ/kg of air}$$

\therefore Work done per kg of air = Heat supplied per kg of air - Heat rejected per kg of air

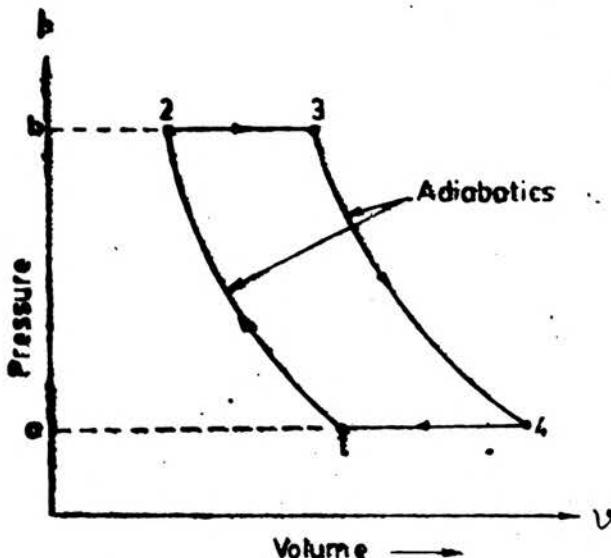
$$= 1,675 - 658.72 = 1,016.28 \text{ kJ/kg of air.}$$

$$\therefore \text{Air-standard efficiency} = \frac{\text{Work done per kg of air}}{\text{Heat supplied per kg of air}} = \frac{1,016.28}{1,675} = 0.6067 \text{ or } 60.67\%$$

5.9 Joule Cycle

In 1851, Dr. Joule proposed to use a cycle in which heat was received and rejected at constant pressure and called this cycle as *constant pressure cycle*. This cycle is used in gas turbine plant of the constant pressure type. In the year 1873, Mr. Brayton used Joule air cycle in open cycle constant pressure gas turbine plants and hence Joule cycle is also known as *Brayton cycle*. This Joule (or Brayton) cycle, used in gas turbine plants, is described in Volume III.

The cycle, consisting of two constant pressure processes (2-3) and (4-1), and two adiabatic processes (3-4) and (1-2), was suggested by Joule for use in hot air engine. The engine consists of an expansion cylinder, a compression cylinder, a heating chamber maintained at temperature T_3 by means of a heater, and cooling chamber

Fig. 5-12. p - v diagram of Joule cycle.

this operation, work is done on the piston, provides excess work and also drives the compressor. This operation is represented by (3-4). During the return stroke of the piston of the expansion cylinder, the low temperature air at T_4 is delivered through a valve to a cooling chamber where it is further cooled at constant pressure from T_4 to T_1 . The cycle is thus completed.

Work done by the air on the piston of the expansion cylinder is given by the area $b\text{-}3\text{-}4\text{-}a$; the work done on the air in the compression cylinder is given by the area $b\text{-}2\text{-}1\text{-}a$. Hence, the net work done per cycle is represented by the area $1\text{-}2\text{-}3\text{-}4$. Heat supplied per kg of air = $k_p (T_3 - T_2)$ heat units. Heat rejected per kg of air

$$= K_p (T_4 - T_1) \text{ heat units.}$$

Since there is no exchange of heat during the frictionless adiabatic processes,

Net work done = Heat supplied - Heat rejected

$$= k_p (T_3 - T_2) - k_p (T_4 - T_1) \text{ heat units per kg of air.}$$

$$\text{Air-standard efficiency} = \frac{\text{Work done per kg of air}}{\text{Heat supplied per kg of air}}$$

$$= \frac{k_p (T_3 - T_2) - k_p (T_4 - T_1)}{k_p (T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2} \quad \dots 5.15$$

Now, since the adiabatic expansion and adiabatic compression both take place between the same terminal pressure (same pressure ratio), the ratio of compression and expansion are equal. Calling this common ratio r , we have,

$$\frac{T_2}{T_1} = (r)^{\gamma-1} \quad \text{or} \quad T_2 = T_1 (r)^{\gamma-1}$$

$$\text{Similarly, } \frac{T_3}{T_4} = (r)^{\gamma-1} \quad \text{or} \quad T_3 = T_4 (r)^{\gamma-1}$$

Substituting the values of T_2 and T_3 in the eqn. (5.15) we get,

$$\begin{aligned} \text{Air-standard efficiency} &= 1 - \frac{T_4 - T_1}{T_4(r)^{\gamma-1} - T_1(r)^{\gamma-1}} \\ &= 1 - \frac{1}{(r)^{\gamma-1}} \end{aligned} \quad \dots (5.16)$$

maintained at temperature T_1 by means of cooling water.

The cycle of operation is as follows :

Cold air from cooling chamber at temperature T_1 is drawn in the compression cylinder. This operation is represented by (a-1) on the p - v diagram as shown in fig. 5-12. The air is now compressed adiabatically to temperature T_2 and delivered through a valve to a heater where it is heated from temperature T_2 to T_3 at constant pressure, increasing its volume from v_2 to v_3 . This hot air passes into the working or expansion cylinder where it is allowed to expand adiabatically until its pressure and temperature fall to p_4 and T_4 respectively. During

The efficiency expression, $1 - \frac{1}{(r)^{\gamma-1}}$ can also be expressed in terms of temperatures T_3 and T_4

$$\text{We know that } \frac{T_3}{T_4} = (r)^{\gamma-1} \text{ or } \frac{T_4}{T_3} = \frac{1}{(r)^{\gamma-1}}$$

Substituting the value of $\frac{1}{(r)^{\gamma-1}}$ in the eqn. (5.15), we get,

$$\text{Air-standard efficiency} = 1 - \frac{T_4}{T_3} = \frac{T_3 - T_4}{T_3} \quad \dots(5.17a)$$

Now, though T_3 is the maximum temperature, T_4 is greater than the minimum temperature T_1 , so that efficiency of this cycle is less than Carnot cycle efficiency when operating between same limits of maximum and minimum temperatures.

The expression of air-standard efficiency, as given in eqn. 5.16, is in terms of volume ratio $(r = \frac{V_1}{V_2})$. It can also be given in terms of pressure ratio $(r_p = \frac{P_2}{P_1})$ as under :

$$\text{Air-standard efficiency} = 1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}} \quad \dots(5.17b)$$

$$\text{as } \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \text{ or } \frac{T_2}{T_1} = (r)^{\gamma-1} = (r_p)^{\frac{\gamma-1}{\gamma}}$$

Although no engine was constructed to work on this cycle, the reversed cycle, i.e., Joule air engine reversed in direction was extensively used in refrigeration for a number of years. The reversed Joule cycle, known as *Bell-Coleman cycle*, is described in volume III.

Problem – 11 : A gas turbine working on Joule cycle takes in air at an atmospheric pressure of 110 kPa and 20°C. The air is compressed adiabatically to a pressure of 300 kPa in the compressor. Heat is then added at constant pressure in combustion chamber and then expanded adiabatically to atmospheric pressure in the turbine. If the maximum temperature is limited to 550°C, find the air-standard efficiency of the cycle. Assume $k_p = 0.9965 \text{ kJ/kg}$ and $\gamma = 1.4$ for air.

Referring to fig. 5-12, $p_1 = 110 \text{ kPa}$; $T_1 = 20 + 273 = 293 \text{ K}$;
 $p_2 = p_3 = 300 \text{ kPa}$; $T_3 = 550 + 273 = 823 \text{ K}$.

$$\frac{p_3}{p_4} = (r)^\gamma \text{ and } \frac{p_2}{p_1} = (r)^\gamma$$

$$\therefore \frac{p_3}{p_4} = \frac{p_2}{p_1} = \frac{300}{110} \quad (\text{ratio of expansion} = \text{ratio of compression})$$

$$\text{From adiabatic expansion (3 – 4), } \frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{300}{100}\right)^{\frac{0.4}{1.4}} = 1.332$$

$$\therefore T_4 = \frac{T_3}{1.332} = \frac{823}{1.332} = 617.7 \text{ K}$$

$$\text{Using eqn. (5.17a), Air-standard efficiency} = \frac{T_3 - T_4}{T_3} = \frac{823 - 617.7}{823} = 0.25 \text{ or } 25\%$$

5.10 Mean Effective Pressure (M.E.P.)

The mean effective pressure (M.E.P.) of a cycle or heat engine is the average net pressure in newtons per unit area that operates on the piston throughout its stroke. It is then the average height of the p - v diagram of the cycle or indicator diagram of any actual engine.

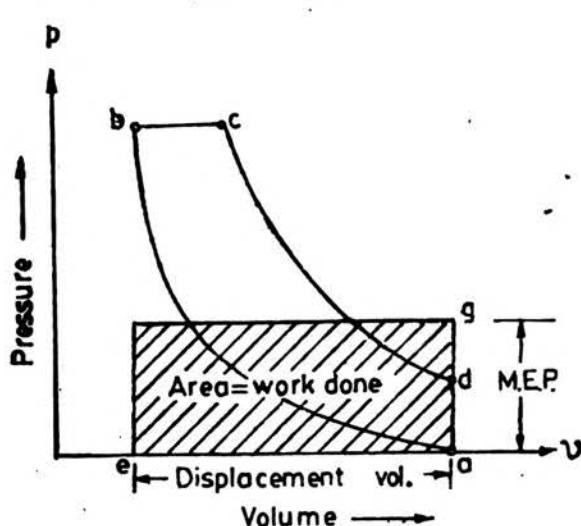


Fig. 5-13. Explanation of mean effective pressure.

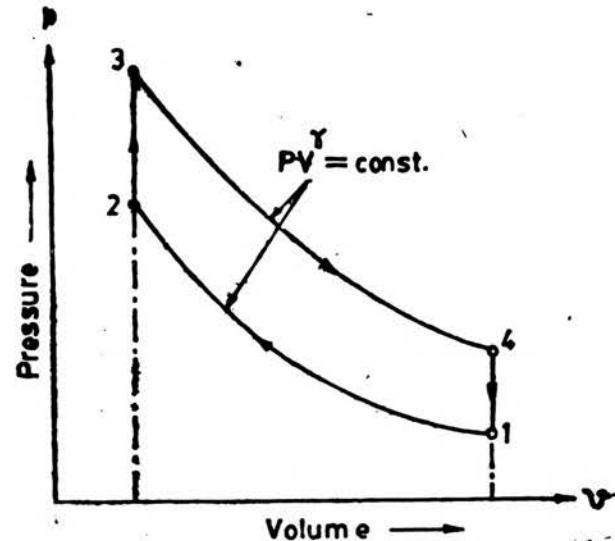


Fig. 5-14. p - v diagram of Otto cycle.

In fig. 5-13, $a-b-c-d$ shows the p - v diagram and $a-e-f-g$ the equivalent rectangular diagram. The area of the rectangle $a-e-f-g$ is equal to area of indicator diagram $a-b-c-d$. Since the area of the p - v diagram is equal to the net work of the cycle in kJ, it is evident that,

$$\begin{aligned} \text{M.E.P.} &= \frac{\text{Work done per cycle in kJ}}{\text{Displacement volume in m}^3} \text{ kPa or KN/m}^2 \\ &= \frac{\text{Work done per cycle in kJ}}{v_a - v_e \text{ in m}^3} \text{ kPa or KN/m}^2 \end{aligned} \quad \dots(5.18)$$

where, v_a = total cylinder volume in m^3 ,

v_e = clearance volume in m^3 , and

$v_a - v_e$ = piston displacement volume in m^3 .

The mean effective pressure of the ideal cycles used in modern internal combustion engines is obtained as follows :

5.10.1 Otto cycle : The p - v diagram of the ideal Otto cycle is shown in fig. 5-14.

Work done per cycle = Area 1-2-3-4

$$\begin{aligned} &= \text{area under adiabatic expansion (3-4)} \text{ minus area under adiabatic compression (1-2)} \\ &= \left[\frac{p_3 v_3 - p_4 v_4}{\gamma - 1} - \frac{p_2 v_2 - p_1 v_1}{\gamma - 1} \right] \text{ kJ} \end{aligned}$$

where, pressure are in kPa and volumes in m^3 .

$$\text{Ideal M.E.P.} = \frac{\text{Work done per cycle in kJ}}{v_1 - v_2 \text{ in m}^3} \text{ kPa or KN/m}^2 \quad \dots(5.19)$$

where, $v_1 - v_2$ = piston displacement volume in m^3 .

Problem – 12 : Show that the ideal M.E.P. of the Otto cycle is given by

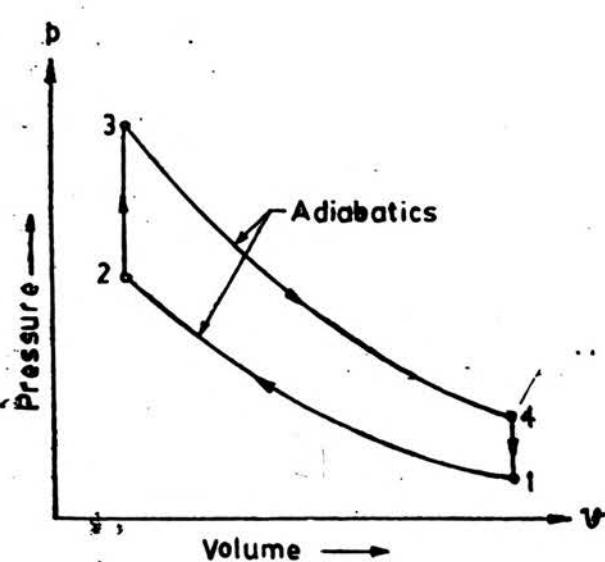


Fig. 5-15. p-v diagram of Otto cycle.

$$\frac{p_1 r (\beta - 1) (r^{\gamma} - 1 - 1)}{(r - 1) (\gamma - 1)}$$

where, p_1 = pressure at the beginning of compression,

r = compression ratio, and

β = ratio of maximum pressure to compression pressure.

Referring to fig. 5-15,

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} = (r)^{\gamma} \quad \therefore p_2 = p_1 (r)^{\gamma}$$

$$\text{Now, } \frac{p_3}{p_2} = \beta \quad \therefore p_3 = p_2 \beta = p_1 (r)^{\gamma} \beta$$

$$\frac{p_3}{p_4} = \left(\frac{v_4}{v_3}\right)^{\gamma} = (r)^{\gamma}$$

$$\therefore p_4 = \frac{p_3}{(r)^{\gamma}} = \frac{p_1 r^{\gamma} \beta}{(r)^{\gamma}} = p_1 \beta$$

$$\text{Now, } \frac{v_1}{v_2} = r \quad \therefore v_1 = r v_2 = v_4$$

$$v_3 = v_2$$

$$\text{Work done per cycle} = \text{area } 1-2-3-4$$

$$= \text{area under (3-4)} \text{ minus area under (2-1)}$$

$$= \frac{p_3 v_3 - p_4 v_4}{(\gamma - 1)} - \frac{p_2 v_2 - p_1 v_1}{(\gamma - 1)}$$

$$\text{M.E.P.} = \frac{\text{area of the diagram}}{\text{length of diagram}} = \frac{\text{Work done per cycle}}{(v_1 - v_2)}$$

$$\begin{aligned} &= \frac{1}{(v_1 - v_2)} \left[\frac{(p_3 v_3 - p_4 v_4) - (p_2 v_2 - p_1 v_1)}{(\gamma - 1)} \right] \\ &= \frac{1}{v_2 (r - 1)} \left[\frac{(p_1 (r)^{\gamma} \beta v_2 - p_1 \beta v_2 r) - (p_1 (r)^{\gamma} v_2 - p_1 r v_2)}{(\gamma - 1)} \right] \\ &= \frac{1}{(r - 1)} \left[\frac{(p_1 (r)^{\gamma} \beta - p_1 \beta r) - (p_1 (r)^{\gamma} - p_1 r)}{p_1 r \left[(r)^{\gamma - 1} - 1 \right] - \left((r)^{\gamma - 1} - 1 \right)} \right] \\ &= \frac{p_1 r (\beta - 1) (r^{\gamma - 1} - 1)}{(r - 1) (\gamma - 1)} \end{aligned}$$

Problem – 13 : In an ideal Otto cycle the charge taken in is assumed to be air at a temperature of 20°C and a pressure of 110 kPa. If the clearance volume is 25 per

cent of the swept volume and the temperature at the end of the constant volume heat addition is $1,440^{\circ}\text{C}$, find the ideal mean effective pressure in kPa. Take $\gamma = 1.4$ for air.

Referring to fig. 5-16.

$$p_1 = 110 \text{ kPa}; t_1 = 20^{\circ}\text{C}; t_3 = 1,440^{\circ}\text{C}.$$

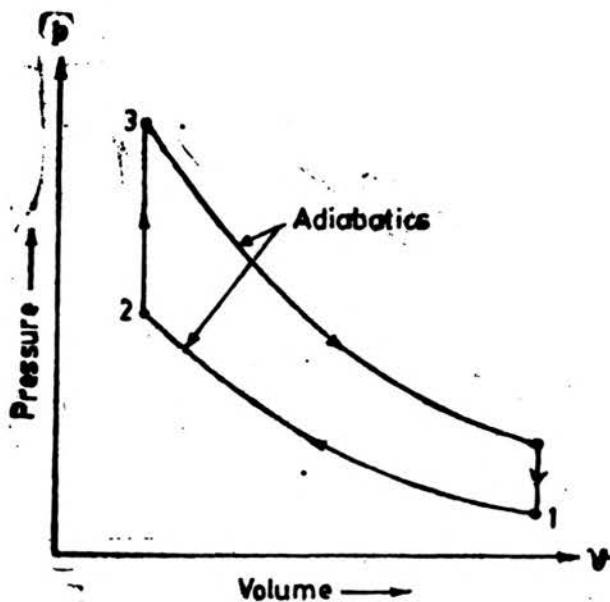


Fig. 5.16. p - v diagram of Otto cycle.

Swept volume = $v_1 = v_2$ and clearance volume = v_2 .

$$\text{Now, } v_2 = 0.25(v_1 - v_2) \text{ i.e., } 1.25v_2 = 0.25v_1$$

$$\therefore \frac{v_1}{v_2} = 5 \text{ (Compression ratio)}$$

Considering adiabatic compression (1.2),

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} = (r)^{\gamma} = (5)^{1.4}$$

$$\therefore p_2 = p_1 \times (5)^{1.4} \\ = 110 \times 9.518 = 1,047 \text{ kPa}$$

Considering adiabatic compression (1-2),

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = (r)^{\gamma-1} = (5)^{1.4-1}$$

$$\therefore T_2 = T_1 \times (5)^{0.4} = 293 \times 1.905 = 558 \text{ K}$$

$$\text{From constant volume process (2-3), } \frac{p_3 V_3}{T_3} = \frac{p_2 V_2}{T_2}$$

$$\text{Hence, as } v_3 = v_2, \frac{p_3}{p_2} = \frac{T_3}{T_2}$$

$$\therefore p_3 = p_2 \times \frac{T_3}{T_2} = 1,047 \times \frac{(1,440 + 273)}{558} = 3,214.17 \text{ kPa}$$

$$\text{Considering adiabatic expansions (3-4), } \frac{p_3}{p_4} = \left(\frac{v_4}{v_3}\right)^{\gamma} = (r)^{\gamma}$$

$$\therefore p_4 = \frac{p_3}{(r)^{\gamma}} = \frac{3,214.17}{(5)^{1.4}} = \frac{3,214.17}{9.518} = 337.69 \text{ kPa}$$

$$\begin{aligned} \text{Work done per cycle} &= \left[\frac{p_3 V_3 - p_4 V_4}{\gamma - 1} - \frac{p_2 V_2 - p_1 V_1}{\gamma - 1} \right] \text{ kJ} \\ &= \frac{3,214.17 v_2 - 337.69 \times 5v_2}{1.4 - 1} - \frac{1,047v_2 - 110 \times 5v_2}{1.4 - 1} \\ &= v_2 \times 2,580.42 \text{ kJ} \end{aligned}$$

$$\text{Ideal M.E.P.} = \frac{\text{Work done per cycle in kJ}}{(v_1 - v_2) \text{ in m}^3} \text{ kPa}$$

$$= \frac{v_2 \times 2,580.42}{v_2 \times 4} = 645.1 \text{ kPa}$$

Problem - 14 : An engine working on the ideal Otto cycle has a clearance volume

of 0.03 m^3 and swept volume of 0.12 m^3 . If the heat supplied at constant volume is 145 kJ per cycle, calculate the ideal mean effective pressure in kPa. Take $\gamma = 1.4$ for air.

$$\text{Compression ratio, } r = \frac{\text{Clearance volume} + \text{Swept volume}}{\text{Clearance volume}} = \frac{0.03 + 0.12}{0.03} = 5$$

$$\text{Air-standard efficiency} = 1 - \frac{1}{(r)^{\gamma-1}} = 1 - \frac{1}{(5)^{0.4}} = 0.475 \text{ or } 47.5\%$$

$$\text{Again, air-standard efficiency} = \frac{\text{Work done per cycle in kJ}}{\text{Heat supplied per cycle in kJ}}$$

$$\text{i.e., } 0.475 = \frac{\text{Work done per cycle in kJ}}{145}$$

$$\therefore \text{Work done per cycle} = 145 \times 0.475 = 68.88 \text{ kJ}$$

$$\text{Ideal M.E.P.} = \frac{\text{Work done per cycle in kJ}}{\text{Swept volume in m}^3} = \frac{68.88}{0.12} = 574 \text{ kPa}$$

5.10.2 Diesel cycle : The $p-v$ diagram of the ideal Diesel cycle is shown in fig. 5-17.

$$\text{Work done per cycle} = \text{Area 1-2-3-4}$$

$$= \text{area under (2-3)} + \text{area under (3-4)} \text{ minus area under (2-1)}$$

$$= \left[p_2 (v_3 - v_2) + \left(\frac{p_3 v_3 - p_4 v_4}{\gamma - 1} \right) - \left[\left(\frac{p_2 v_2 - p_1 v_1}{\gamma - 1} \right) \right] \right] \text{ kJ}$$

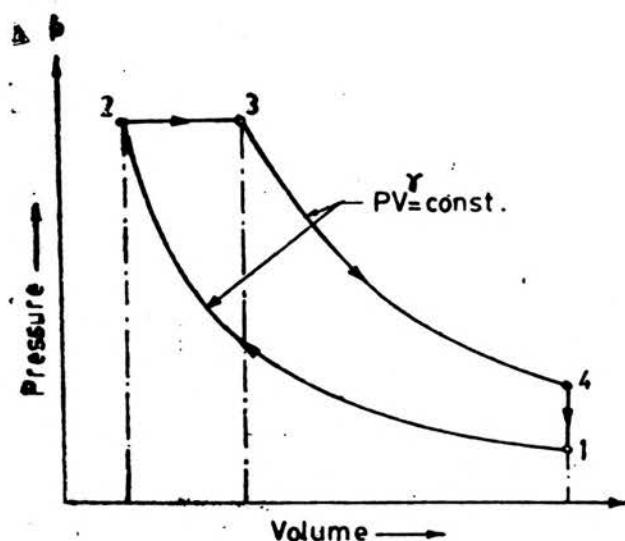


Fig. 5-17. $p-v$ diagram of ideal Diesel cycle for air.

Referring to fig. 5-17, $p_1 = 108 \text{ kPa}$, $T_1 = 30 + 273 = 303 \text{ K}$.

$$(i) \quad \gamma = \frac{k_p}{k_v} \text{ i.e. } 1.4 = \frac{K_p}{0.718} \quad \therefore \quad K_p = 1.4 \times 0.718 = 1.005 \text{ kJ/kg K}$$

$$\text{Let clearance volume} = v_2; \frac{v_1}{v_2} = r = 14 \quad \therefore \quad v_1 = 14 v_2 = v_4$$

$$\text{Again, } v_3 = v_2 + \frac{5}{100} (v_1 - v_2) = v_2 + \frac{5}{100} (13 v_2) = 1.65 v_2; \frac{v_3}{v_2} = p = 1.65$$

where, pressures are in kPa and volumes in m^3 .

Ideal M.E.P.

$$= \frac{\text{Work done per cycle in kJ}}{(v_1 - v_2) \text{ in m}^3} \text{ kPa} \quad \dots (5.20)$$

where, $v_1 - v_2$ = piston displacement volume in m^3 .

Problem – 15 : A Diesel engine, working on an ideal cycle, has a compression ratio of 14 and takes in a charge of air at a pressure of 108 kPa and temperature of 30°C . If the cut-off takes place at 5 per cent of the stroke, find : (i) the ideal thermal efficiency, and (ii) the ideal mean effective pressure of the cycle in kPa.

Take $\gamma = 1.4$ and $k_v = 0.718 \text{ kJ/kg K}$.

From adiabatic compression (1-2), $\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma$

$$\therefore P_2 = P_1 \times \left(\frac{V_1}{V_2}\right)^\gamma = P_1 \times (r)^\gamma = 108 \times (14)^{1.4} = 108 \times 40.23 = 4,345 \text{ kPa}$$

From adiabatic compression (1-2), $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (r)^{\gamma-1}$

$$\therefore T_2 = T_1 (r)^{\gamma-1} = 303 \times (14)^{0.4} = 871 \text{ K}$$

From constant pressure process (2-3), $\frac{P_3 V_3}{T_3} = \frac{P_2 V_2}{T_2}$

$$\text{Hence, as } P_3 = P_2, \frac{T_3}{T_2} = \frac{V_3}{V_2} = 1.65$$

$$\therefore T_3 = T_2 \times 1.65 = 871 \times 1.65 = 1,437 \text{ K}$$

From adiabatic expansion (3-4), $\frac{P_3}{P_4} = \left(\frac{V_4}{V_3}\right)^\gamma = \left(\frac{14 V_2}{1.65 V_2}\right)^{1.4} = (8.485)^{1.4} = 19.95$

$$\therefore P_4 = \frac{P_3}{19.95} = \frac{4,345}{19.95} = 217.8 \text{ kPa}$$

Considering adiabatic expansion (3-4), $\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = \left(\frac{r}{\rho}\right)^{\gamma-1}$

$$\therefore T_4 = \frac{T_3}{\left(\frac{r}{\rho}\right)^{\gamma-1}} = \frac{1,437}{\left(\frac{14}{1.65}\right)^{0.4}} = \frac{1,437}{2.355} = 610 \text{ K}$$

Heat supplied per kg of air = $k_p (T_3 - T_2) = 1.005 (1,437 - 871) = 868.8 \text{ kJ/kg of air}$

Heat rejected per kg of air = $k_v (T_4 - T_1) = 0.718 (610 - 303) = 220.4 \text{ kJ/kg of air}$

$$\begin{aligned} \text{Ideal thermal efficiency} &= \frac{\text{Heat converted into work in kJ per kg of air}}{\text{Heat supplied in kJ per kg of air}} \\ &= \frac{868.8 - 220.4}{868.8} = 0.6125 \text{ or } 61.25\% \end{aligned}$$

(ii) Referring to fig. 5-17, work done per cycle = Area 1-2-3-4

= area under (2-3) plus area under (3-4) minus area under (2-1)

$$\begin{aligned} \text{Work done/cycle} &= \left[P_2 (V_3 - V_2) + \frac{P_3 V_3 - P_4 V_4}{\gamma - 1} - \frac{P_2 V_2 - P_1 V_1}{\gamma - 1} \right] \text{ kJ} \\ &= V_2 \left[4,345 \times 0.65 + \frac{4,345 \times 1.65 - 217.8 \times 14}{1.4 - 1} - \frac{4,345 \times 1 - 108 \times 14}{1.4 - 1} \right] \\ &= V_2 \times 6,041.75 \text{ kJ} \end{aligned}$$

$$\begin{aligned} \text{Using eqn. (5.20), Ideal M.E.P.} &= \frac{\text{Work done per cycle in kJ}}{\text{stroke volume } (V_1 - V_2) \text{ in m}^3} \text{ kPa} \\ &= \frac{V_2 \times 6,041.75}{V_2 \times 13} = 464.75 \text{ kPa} \end{aligned}$$

Problem – 16: In an ideal Diesel cycle, the pressure and temperature at the commencement of the compression stroke are 95 kPa and 15°C. The pressure at the end of the adiabatic

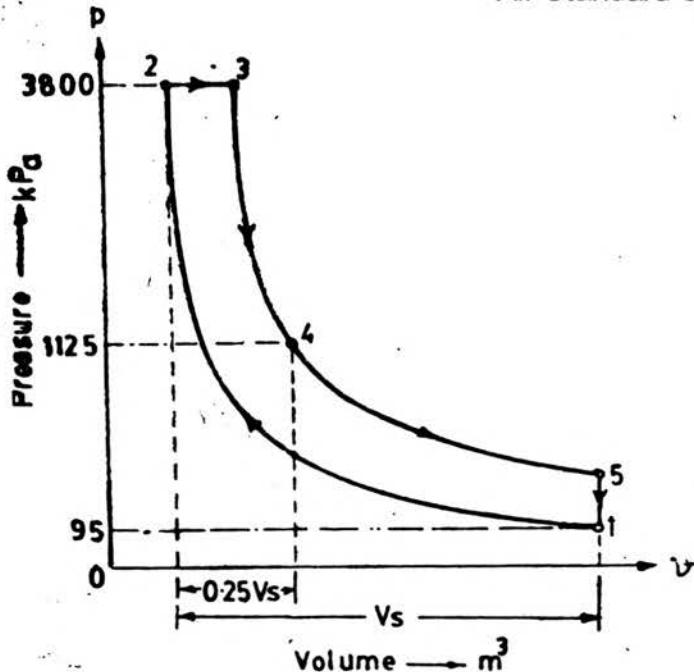


Fig. 5-18. p-v diagram of ideal Diesel cycle

$$\therefore k_p = 1.4 \times 0.718 = 1.005 \text{ kJ/kg K}$$

Referring to fig. 5-18 and considering adiabatic compression (1-2),

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} = (r)^{\gamma}$$

$$\therefore r = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} = \left(\frac{3,800}{95}\right)^{\frac{1}{1.4}} = 14$$

Let the clearance volume be unity then $v_1 = 14$, $v_2 = 1$, $v_5 = 14$, and stroke volume $= v_1 - v_2 = 13$

25% of the stroke volume $= 0.25 \times 13 = 3.25$. Hence, $v_4 = 3.25 + 1 = 4.25$

Considering points 3 and 4 adiabatic expansion (3-4), $p_3 v_3^{\gamma} = p_4 v_4^{\gamma}$

$$\therefore \frac{p_3}{p_4} = \left(\frac{v_4}{v_3}\right)^{\gamma} \quad \text{or} \quad \frac{v_4}{v_3} = \left(\frac{p_3}{p_4}\right)^{\frac{1}{\gamma}}$$

$$\therefore v_3 = \frac{v_4}{\left(\frac{p_3}{p_4}\right)^{\frac{1}{\gamma}}} = \frac{4.25}{\left(\frac{3,800}{95}\right)^{\frac{1}{1.4}}} = \frac{4.25}{2.382} = 1.78$$

Considering adiabatic compression (1-2), $\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = (r)^{\gamma-1}$

$$\therefore T_2 = T_1 \times (r)^{\gamma-1} = 288 \times (14)^{1.4-1} = 827 \text{ K}$$

From constant pressure heat addition (2-3), $\frac{p_2 v_2}{T_2} = \frac{p_3 v_3}{T_3}$

$$\text{Hence, as } p_2 = p_3, \frac{T_3}{T_2} = \frac{v_3}{v_2}$$

end of the adiabatic compression is 3,800 kPa, and it is 1,125 kPa when the piston displaces 25% of the stroke volume. Find :

- (a) the maximum temperature reached in the cycle,
- (b) the ideal thermal efficiency,
- (c) the ideal m.e.p., and
- (d) approximate percentage of expansion stroke at which cut-off takes place.

Take $\gamma = 1.4$ and $k_v = 0.718 \text{ kJ/kg K}$ for air.

Referring to fig. 5-18,

$$p_1 = 95 \text{ kPa}; t_1 = 15^\circ\text{C};$$

$$p_2 = p_3 = 3,800 \text{ kPa};$$

$$p_4 = 1,125 \text{ kPa}.$$

$$(a) \gamma = \frac{k_p}{k_v} \text{ i.e., } 1.4 = \frac{k_p}{0.718}$$

$$\therefore T_3 = T_2 \times \frac{V_3}{V_2} = 827 \times \frac{1.78}{1} = 1,472\text{K or } T_3 = 1,199^\circ\text{C}$$

(b) Considering adiabatic expansions (3-5), $\frac{T_3}{T_5} = \left(\frac{V_5}{V_3}\right)^{\gamma-1}$

$$\therefore T_5 = \frac{T_3}{\left(\frac{V_5}{V_3}\right)^{\gamma-1}} = \frac{1,472}{\left(\frac{14}{1.78}\right)^{0.4}} = 645\text{ K}$$

Heat supplied per kg of air = $k_p (T_3 - T_2) = 1.005 (1,472 - 827) = 648.2\text{ kJ/kg of air}$

Heat rejected per kg of air = $k_v (T_5 - T_1) = 0.718 (645 - 288) = 256.2\text{ kJ/kg of air}$

Ideal thermal efficiency = $\frac{\text{Heat converted work in kJ per kg of air}}{\text{Heat supplied in kJ per kg of air}}$

$$= \frac{648.2 - 256.2}{648.2} = 0.6047 \text{ or } 60.47\%$$

(c) From adiabatic expansion (3-5), $\frac{P_3}{P_5} = \left(\frac{V_5}{V_3}\right)^\gamma$

$$\therefore P_5 = \frac{P_3}{\left(\frac{V_5}{V_3}\right)^\gamma} = \frac{3,800}{\left(\frac{14}{1.78}\right)^{1.4}} = \frac{3,800}{17.95} = 212\text{ kPa}$$

Referring to fig. 5-18, ideal work done per cycle = Area 1-2-3-5

= area under (2-3) plus area under (3-5) minus area under (2-1)

$$= \left[P_2 (V_3 - V_2) + \frac{P_3 V_3 - P_5 V_5}{\gamma - 1} - \frac{P_2 V_2 - P_1 V_1}{\gamma - 1} \right] \text{ kJ}$$

$$= V_2 \left[3,800 (1.78 - 1) + \frac{3,800 \times 1.78 - 212 \times 14}{0.4} - \frac{3,800 \times 1 - 95 \times 14}{0.4} \right]$$

$$= V_2 \times 6,029 \text{ kJ}$$

Using eqn. (5.20), Ideal M.E.P. = $\frac{\text{Work done per cycle in kJ}}{\text{Stroke volume } (V_1 - V_2) \text{ in m}^3} \text{ kPa}$

$$= \frac{V_2 \times 6,029}{V_2 \times 13} = 463.77 \text{ kPa}$$

(d) Percentage of expansion stroke at which cut-off takes place

$$= \frac{\text{volume at cut-off clearance volume}}{\text{stroke volume}} \times 100$$

$$= \frac{V_3 - V_2}{V_1 - V_2} \times 100 = \frac{1.78 - 1}{13} \times 100 = 6\%$$

Problem - 17 : Show that ideal M.E.P. of the Diesel cycle is given by ;

$$\frac{P_1 (r)^\gamma \left[\gamma (\rho - 1) - (r)^{1-\gamma} (\rho^\gamma - 1) \right]}{(\gamma - 1) (r - 1)}$$

where, P_1 = Pressure at the beginning of the compression,

r_1 = Compression ratio, and ρ = Cut-off ratio.

Referring to fig. 5-19,

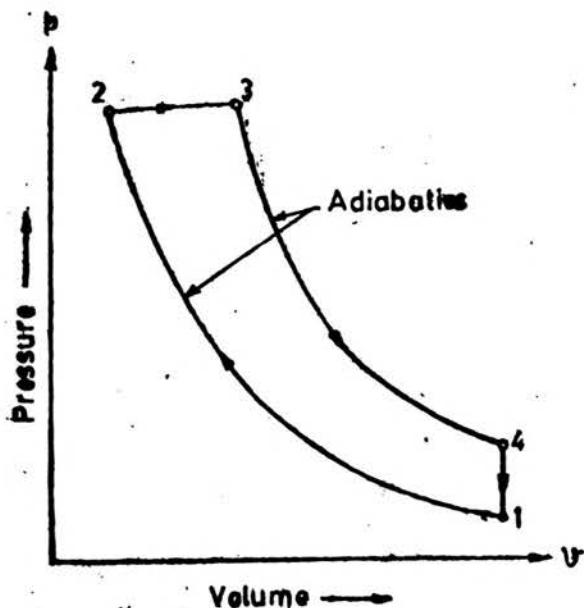


Fig. 5-19. p-v diagram of ideal Diesel cycle

$$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} = (r)^{\gamma}$$

$$\therefore p_2 = p_1 (r)^{\gamma} = p_3$$

$$\frac{p_3}{p_4} = \left(\frac{V_4}{V_3}\right)^{\gamma} = \left(\frac{r}{\rho}\right)^{\gamma}$$

$$\therefore p_4 = \frac{p_3 (\rho)^{\gamma}}{(r)^{\gamma}}$$

$$\frac{V_1}{V_2} = r \quad \therefore V_1 = r V_2 = V_4$$

$$\frac{V_3}{V_2} = \rho \quad \therefore V_3 = \rho V_2$$

Work done per cycle = Area 1-2-3-4
= area under (2-3) plus area
under (3-4) minus area under (2-1)

$$= p_3 (V_3 - V_2) + \frac{p_3 V_3 - p_4 V_4}{\gamma - 1} - \frac{p_2 V_2 - p_1 V_1}{\gamma - 1}$$

$$\begin{aligned} \text{Ideal M.E.P.} &= \frac{\text{Area of diagram}}{\text{Length of diagram}} = \frac{\text{Work done per cycle}}{V_1 - V_2} \\ &= \frac{1}{V_1 - V_2} \left[p_3 (V_3 - V_2) + \frac{p_3 V_3 - p_4 V_4}{\gamma - 1} - \frac{p_2 V_2 - p_1 V_1}{\gamma - 1} \right] \\ &= \frac{1}{V_2 (r - 1)} \left[p_3 (\rho V_2 - V_2) + \frac{p_3 \rho V_2 - p_4 V_2 r}{\gamma - 1} - \frac{p_2 V_2 - p_1 r V_2}{\gamma - 1} \right] \\ &= \frac{1}{r - 1} \left[p_3 (\rho - 1) + \frac{p_3 \rho - p_4 r}{\gamma - 1} - \frac{p_2 - p_1 r}{\gamma - 1} \right] \\ &= \frac{1}{(r - 1)} \left[\frac{p_3 (\rho - 1)(\gamma - 1) + (p_3 \rho - p_4 r) - (p_2 - p_1 r)}{\gamma - 1} \right] \\ &= \frac{1}{(r - 1)} \left[\frac{p_3 (\rho - 1)(\gamma - 1) + (p_3 \rho - p_3 \rho^{\gamma} r^{1-\gamma}) - (p_2 - p_2 r^{1-\gamma})}{\gamma - 1} \right] \\ &= \frac{1}{(r - 1)} \left[\frac{p_3 (\rho \gamma - \rho - \gamma + 1 + \rho - \rho^{\gamma} r^{1-\gamma} - 1 + r^{1-\gamma})}{(\gamma - 1)} \right] \\ &= \frac{1}{(r - 1)} \left[\frac{p_3 (\rho \gamma - \gamma - \rho^{\gamma} r^{1-\gamma} + r^{1-\gamma})}{(\gamma - 1)} \right] \\ &= \frac{p_3 [\gamma(\rho - 1) - r^{1-\gamma}(\rho^{\gamma} - 1)]}{(\gamma - 1)(r - 1)} \\ &= \frac{p_1 (r)^{\gamma} [\gamma(\rho - 1) - (r)^{1-\gamma}(\rho^{\gamma} - 1)]}{(\gamma - 1)(r - 1)} \end{aligned}$$

Problem - 18 : The mean effective pressure of ideal Diesel cycle engine is 6.1 bar.

If the pressure at the beginning of compression is 1 bar and the compression ratio is 13, determine the cut-off ratio. Assume $\gamma = 1.4$ for air.

Using the expression of ideal M.E.P. of diesel cycle derived earlier (vide problem-17) Ideal M.E.P.

$$= \frac{p_1 (r)^\gamma [\gamma(p - 1) - (r)^{1-\gamma} (p^\gamma - 1)]}{(\gamma - 1)(r - 1)}$$

where, p_1 = Pressure at the beginning of compression,
 r = Compression ratio, and ρ = Cut-off ratio.

Referring to fig. 5-19, $p_1 = 1$ bar; $\frac{V_1}{V_2} = r = 13$; ideal M.E.P. = 6.1 bar; $\gamma = 1.4$

On substitution of the values, we get

$$\begin{aligned} 6.1 &= \frac{1 \times (13)^{1.4} [1.4(p - 1) - (13)^{-0.4} (p^{1.4} - 1)]}{(1.4 - 1)(13 - 1)} \\ &= \frac{36.27 [1.4(p - 1) - 0.3584 (p^{1.4} - 1)]}{4.8} \end{aligned}$$

$$\text{or } \frac{6.1 \times 4.8}{36.27} = 1.4p - 1.4 - 0.3584 p^{1.4} + 0.3548$$

$$\text{or } 1.8493 = 1.4p - 0.3584 p^{1.4}$$

$$\text{or } 1.4p - 0.3584 p^{1.4} - 1.8493 = 0$$

By trial and error method putting $p = 2$, we get

$$(1.4 \times 2) - 0.3584 (2)^{1.4} - 1.8493 = 0 \text{ or } 2.8 - 0.9458 - 1.8493 \approx 0$$

Thus cut-off ratio, $\rho \approx 2$

5.10.3 Dual-Combustion cycle : The p - v diagram of ideal cycle is shown in fig. 5-20. Work done per cycle = Area 1-2-3-4-5 = area under (3-4) + area under (4-5) minus area under (2-1).

$$= \left[p_3 (v_4 - v_3) + \frac{p_4 v_4 - p_5 v_5}{\gamma - 1} - \frac{p_2 v_2 - p_1 v_1}{\gamma - 1} \right] \text{ kJ}$$

where pressures are in kPa and volumes in m^3 .

$$\therefore \text{Ideal M.E.P.} = \frac{\text{Work done per cycle in kJ}}{(v_1 - v_2) \text{ is } \text{m}^3} \text{ kPa} \quad \dots(5.21)$$

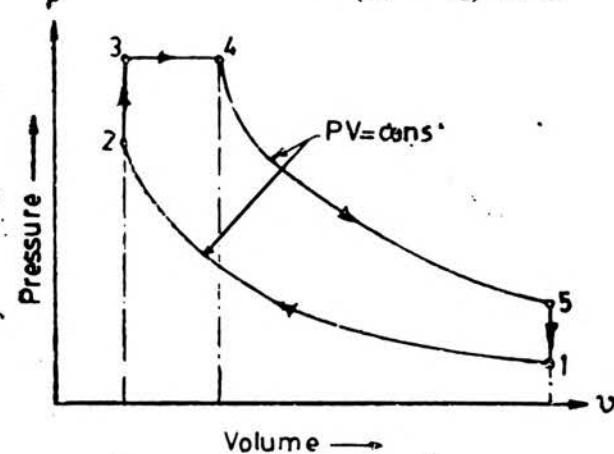


Fig. 5-20. p-v diagram of ideal dual-combustion cycle

where $(v_1 - v_2)$ = piston displacement volume in m^3 .

Problem - 19 : A compression-ignition engine, working on the dual-combustion cycle, has a compression ratio of 10 and two-thirds of the total heat supplied is taken in at constant volume and the remainder at constant pressure. The maximum pressure in the cycle is 4200 kPa and the pressure and temperature at the beginning of compression are 105 kPa and 303 K respectively. Assuming the working fluid to be air

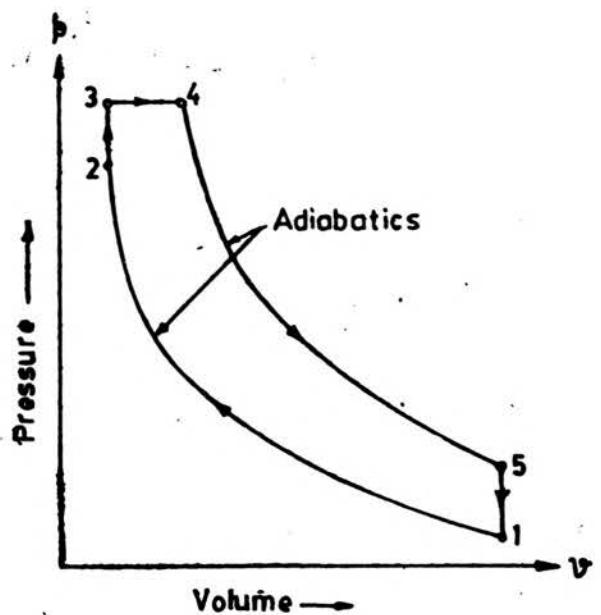


Fig. 5-21. p-v diagram of ideal dual-combustion cycle

throughout the cycle, find the ideal mean effective pressure of the cycle in kPa. Assume $k_p = 1.0035 \text{ kJ/kg K}$, $K_v = 0.7165 \text{ kJ/kg K}$ and $\gamma = 1.4$ for air.

Referring to fig. 5-21,

$$p_3 = p_4 = 4,200 \text{ kPa}; p_1 = 105 \text{ kPa};$$

$$T_1 = 303 \text{ K}; \frac{v_1}{v_2} = r = 10$$

Then, $v_1 = 10 v_2 = r$ and
swept volume = $v_1 - v_2 = 9 v_2$.

From adiabatic compression (1-2),

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2} \right)^{\gamma-1} = (r)^{\gamma-1}$$

$$\therefore T_2 = T_1 \times (r)^{\gamma-1} = 303 \times (10)^{0.4} = 761.1 \text{ K}$$

From adiabatic compression (1-2),

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2} \right)^\gamma = (r)^\gamma$$

$$\therefore p_2 = p_1 \times (r)^\gamma = 105 \times (10)^{1.4} = 2,637.6 \text{ kPa}$$

$$\text{From constant volume heat addition (2-3), } \frac{p_2 v_2}{T_2} = \frac{p_3 v_3}{T_3}$$

$$\text{Hence, as } v_2 = v_3, \frac{p_3}{p_2} = \frac{T_3}{T_2} \text{ i.e., } \frac{4,200}{2,637.6} = \frac{T_3}{761.1} \quad \therefore T_3 = 1,211.9 \text{ K}$$

Heat supplied at constant volume per kg of air

$$= k_v (T_3 - T_2) = 0.7165 (1211.9 - 761.1) = 323 \text{ kJ/kg of air}$$

Heat supplied at constant pressure per kg of air

$$= k_p (T_4 - T_3) = \frac{1}{2} \times 323 = 161.5 \text{ kJ/kg of air.}$$

$$\therefore 161.5 = k_p (T_4 - T_3) = 1.0035 (T_4 - 1,211.9)$$

$$\therefore T_4 = 1,372.8 \text{ K}$$

$$\text{From constant pressure heat addition (3-4), } \frac{p_3 v_3}{T_3} = \frac{p_4 v_4}{T_4}$$

$$\text{Hence, as } p_3 = p_4, \frac{v_4}{v_3} = \frac{T_4}{T_3} = \frac{1,372.8}{1,211.9} = 1.132 = \rho \text{ (cut-off ratio)}$$

$$\therefore \rho = 1.132 v_3 = 1.132 v_2$$

$$\text{From adiabatic expansion (4-5), } \frac{p_4}{p_5} = \left(\frac{v_5}{v_4} \right)^\gamma$$

$$\therefore p_5 = \frac{p_4}{\left(\frac{v_5}{v_4} \right)^\gamma} = \frac{p_4}{\left(\frac{r}{\rho} \right)^\gamma} = \frac{4,200}{\left(\frac{10}{1.132} \right)^{1.4}} = 199.14 \text{ kPa}$$

Work done per cycle = area 1 - 2 - 3 - 4 - 5

= area under (3-4) plus area under (4-5) minus area under (2-1).

$$\begin{aligned}
 &= \left[p_3(v_4 - v_3) + \frac{p_4v_4 - p_5v_5}{\gamma - 1} - \frac{p_2v_2 - p_1v_1}{\gamma - 1} \right] \text{ kJ} \\
 &= v_2 \left[4,200 \times 0.132 + \frac{4,200 \times 1.132 - 199.14 \times 10}{0.4} - \frac{2,637.6 \times 1 - 105 \times 10}{0.4} \right] \\
 &= 3,507.6 v_2 \text{ kJ}
 \end{aligned}$$

$$\begin{aligned}
 \text{Ideal M.E.P.} &= \frac{\text{Work done per cycle in kJ}}{\text{Swept volume in m}^3(v_1 - v_2)} \text{ kPa} \\
 &= \frac{3,507.6v_2}{9v_2} = 389.73 \text{ kPa.}
 \end{aligned}$$

5.11 Actual Cycle Thermal Efficiency

Most of the internal combustion engines of today are designed to follow as closely as possible the Ideal Otto and Diesel cycles. That they can not follow exactly these ideal cycles and hence operate with thermal efficiencies lower than those of ideal cycle is evident when one thinks of some of the practical limitations that are involved. For instance, both ideal cycles use an adiabatic compression and an adiabatic expansion. The adiabatic process requires that no heat be added or rejected throughout its duration and hence requires that the working substance be surrounded by a material that is a perfect non-conductor of heat. The cast iron cylinders of actual engines are conductors of heat and hence the process carried on within them can only approach or approximate the adiabatic. For this reason combined with others, an expansion or compression carried out in the actual engine follows a polytropic process whose value of index n in the equation $pV^n = \text{constant}$ is about 1.35 instead of 1.4 as in a ideal adiabatic (isentropic) process.

The combustion in the actual Otto cycle engines cannot be instantaneous as it is in the ideal Otto cycle. This results in a sloping combustion line on p - v diagram (indicator diagram), that tends to decrease the area of the diagram and the effect of this is to produce a lower actual thermal efficiency. Similarly, in the actual Diesel engine cycle, there is a similar tendency in that the combustion line instead of being maintained horizontal as in the ideal Diesel cycle, tends to slope downward.

The efficiency of the ideal cycle is known as the air-standard efficiency, since it is worked out, on the basis of the working substance being air throughout the cycle. Therefore, the specific heats for air are used. It is evident that this is in variance with the actual cycle. In the actual engine, the working substance is not air but may contain a proportion of gases whose specific heats at constant pressure and constant volume do not bear the same relation as in the case of air.

In the ideal cycle, the specific heat of working substance (air) is considered constant throughout the whole range of temperature. The specific heat of any gas varies with temperature. Therefore, in actual engine the temperature and pressure to which the working substance will be raised, will be lower than would be the case with constant specific heats. Therefore, the area of indicator diagram will be less and the effect of this is to produce lower actual thermal efficiency.

The total effect of all these differences is to produce thermal efficiency of the actual cycle much less than the ideal or air-standard efficiency.

Tutorial - 5

1. Delete the phrase which is not applicable in the following statements :

- (i) The air-standard efficiency of a Diesel cycle having fixed compression ratio will decrease / increase with increase in cut-off ratio.
- (ii) The air-standard efficiency of an I.C. engine decreases / increases with increase in compression ratio.
- (iii) In a Diesel cycle, the ratio of volume of cylinder at the point of cut-off and clearance volume is known as cut-off ratio/compression ratio.
- (iv) In a Diesel cycle, heat is rejected at constant volume/constant pressure.
- (v) Indicated thermal efficiency of well designed and well constructed I.C. engine, when properly operated, will be about one-third/two-third of air-standard efficiency.
- (vi) The most efficient engine is that which works on a reversible/an irreversible cycle.

[(i) increase, (ii) decreases, (iii) compression ratio,
(iv) constant pressure, (v) one-third, (vi) an irreversible]

2. Fill in the blanks in the following statements :

- (i) For thermodynamic cycle to be reversible it must consists of _____ processes only.
- (ii) In Otto cycle, heat is added at constant _____.
- (iii) In Diesel cycle, heat is added at constant _____.
- (iv) The theoretical thermal efficiency of the ideal cycle using air as the working fluid is known as _____.
- (v) Joule cycle is used in gas turbine plant of _____ type.
- (vi) The mean effective pressure of a cycle or heat engine is the _____ pressure in newtons per unit area that operates on the piston throughout its stroke.
- (vii) Cut-off ratio of a Diesel cycle is _____ than unity.
- (viii) Dual-combustion cycle is used in _____ engines.

[(i) reversible, (ii) volume, (iii) pressure, (iv) air-standard efficiency, (v) the constant pressure, (vi) average net, (vii) greater, (viii) modern high speed oil]

3. Indicate the correct answer by choosing correct phrase out of the following statements:

- (i) Air-standard efficiency of Otto cycle is expressed as

$$(a) 1 - \left(\frac{1}{r}\right)^{\frac{\gamma}{\gamma-1}}$$

$$(b) 1 - \left(\frac{1}{r}\right)^{\frac{\gamma+1}{\gamma-1}}$$

$$(c) 1 - \left(\frac{1}{r}\right)^{\frac{\gamma-1}{\gamma}}$$

$$(d) 1 - \left(\frac{1}{r}\right)^{\frac{1}{\gamma}}$$

where, r is the compression ratio and γ is the ratio of the specific heats of air.

- (ii) For the same compression ratio,
 - (a) both Otto cycle and Diesel cycle are equally efficient.
 - (b) Efficiency of Otto cycle is more than that of the Diesel cycle.
 - (c) Efficiency of Diesel cycle is more than that of the Otto cycle.
- (iii) In an engine working on ideal Otto cycle, combustion takes place
 - (a) at constant pressure.
 - (b) at constant volume.
 - (c) partly at constant volume and partly at constant pressure.
 - (d) at constant temperature.
- (iv) Dual-combustion cycle is also known as
 - (a) Otto cycle. (b) Diesel cycle. (c) Semi-Diesel cycle. (d) Joule cycle.
- (v) Thermal efficiency of the Carnot cycle is
 - (a) $\frac{T_1 - T_2}{T_2}$
 - (b) $\frac{T_1 - T_2}{T_1}$
 - (c) $\frac{T_2}{T_1 - T_2}$
 - (d) $\frac{T_1}{T_1 - T_2}$
- (vi) In an air-standard cycle,

- (a) all processes are reversible.
- (b) all processes are irreversible.
- (c) two processes are reversible and other processes are irreversible.
- (d) reversibility and irreversibility is not important.
- (vii) Efficiency of a Diesel cycle
 - (a) increases as compression ratio is increased and decreases as cut-off ratio is increased.
 - (b) increases with increase in both compression ratio and cut-off ratio.
 - (c) decreases with increase in both compression ratio and cut-off ratio.
 - (d) decreases as compression ratio is increased and increases as cut-off ratio is increased.
- (viii) Joule cycle consists of
 - (a) two isentropic and two isothermal operations.
 - (b) two isentropic and two constant pressure operations.
 - (c) two isentropic and two constant volume operations.
 - (d) two isothermal and two constant pressure operations.
- (ix) Compression ratio of an I.C. engine is the ratio of
 - (a) the volume of air in the cylinder before compression stroke and volume of air in the cylinder after compression stroke.
 - (b) volume displaced by piston per stroke and clearance volume.
 - (c) pressure after compression and pressure before compression.
 - (d) temperature after compression and temperature before compression.
- (x) Joule cycle is used in :
 - (a) petrol. engine.
 - (b) diesel engine.
 - (c) constant pressure type gas turbine plant.
 - (d) gas engine.

[(i) c, (ii) b, (iii) b, (iv) c, (v) b, (vi) a, (vii) a, (viii) b, (ix) a, (x) c]

4. The temperature limits for a Carnot cycle using air as working fluid are 420°C and 10°C . Calculate the efficiency of the cycle and the ratio of adiabatic expansion. Take $\gamma = 1.4$ for air

[59.16%; 9.38 : 1]

5. Define the term 'Air-Standard Efficiency' as applied to an internal combustion engine.

A petrol engine working on Otto cycle has a cylinder diameter of 10 cm and stroke of 15 cm. The clearance volume is 250 cm^3 . Find the ideal thermal efficiency (air-standard efficiency) of the engine. Take $\gamma = 1.4$ for air.

[50.17%]

6. (a) Obtain an expression for the air-standard efficiency of an internal combustion engine working on the Otto cycle in terms of the ratio of compression r and the ratio of the specific heats of air γ .

In an engine working on the ideal Otto cycle, the temperatures at the beginning and the end of adiabatic compression are 100°C and 473°C respectively. Find the compression ratio and the air-standard efficiency of the engine. Take $\gamma = 1.4$ for air.

[$r = 5.656$; 50%]

- (b) Establish an expression for the air-standard efficiency of an engine working on the Otto cycle. If an engine working on this cycle and using air as the working fluid has its compression ratio raised from 5 to 6, find the percentage increase in ideal thermal efficiency. Take $\gamma = 1.4$ for air.

[7.75%]

7. Show that the efficiency of an air engine working on the constant volume cycle is given by $1 - \left(\frac{1}{r}\right)^{\gamma-1}$ where, r is the compression ratio and γ is the ratio of the specific heats of air.

8. Describe the constant volume cycle for an air engine.

Calculate the air-standard efficiency (theoretical thermal efficiency) of this cycle when the pressure at the end of compression is 15 times that at the start.

If in the above case the initial temperature of air is 40°C and maximum temperature is $1,677^{\circ}\text{C}$, find : (i) the heat supplied per kg of air, and (ii) the work done per kg of air. Take $\gamma = 1.4$ and $k_v = 0.7165 \text{ kJ/kg}$

K for air.

[53.88%; (i) 910.6 kJ/kg; (ii) 490.6 kJ/kg]

9. In an ideal Otto cycle engine the compression and expansion follow the adiabatic law with the value of γ as 1.4. The pressure, temperature, and volume at the beginning of the compression are 100 kPa, 40°C and 0.03 m³ respectively. The pressure at the end of compression is 750 kPa and that at the end of constant volume heat addition is 1,900 kPa. Calculate the temperatures at the end of (i) adiabatic compression, (ii) constant volume heat addition, and (iii) adiabatic expansion. Also find the compression ratio and the air-standard efficiency of the engine. Take $k_v = 0.7165$ kJ/kg K for air.

Sketch the pressure - volume and temperature - entropy diagrams for the cycle.

[(i) 284°C; (ii) 1,138°C; (iii) 519.7°C; 4.224; 43.82%]

10. (a) Sketch the ideal indicator diagrams for the Otto, Diesel and dual-combustion cycles.

A Diesel engine has a cylinder diameter of 17 cm and a stroke of 25 cm. The clearance volume is 450 cm³ and cut-off takes place at 6% of the stroke. Find the air-standard efficiency of the engine. Take $\gamma = 1.4$ for air.

[60.1%]

- (b) Obtain the formula for the ideal efficiency of the Diesel cycle in terms of the volume ratios, assuming constant specific heats.

Find the percentage loss in the ideal thermal efficiency of a Diesel cycle engine with compression ratio of 15, by delaying the cut-off from 5 per cent to 10 per cent of the stroke. Take $\gamma = 1.4$ for air.

[6.6%]

11. Derive an expression for the thermal efficiency of an internal combustion engine working on the ideal Diesel cycle.

12. A Diesel engine working on the ideal cycle draws in air at a pressure of 110 kPa and temperature of 288 K. The air is compressed adiabatically to 3.5 MPa (3,500 kPa). Heat is taken in at constant pressure and expansion takes place adiabatically, the ratio of expansion being 5. The air is exhausted at the end of the stroke at constant volume. Calculate : (i) the temperatures at the salient (key) points of the cycle, (ii) the heat received per kg of working fluid, (iii) the heat rejected per kg of working fluid, (iv) the work done per kg of working fluid, and (v) the ideal thermal efficiency. Take $k_p = 1.0035$ kJ/kg K, $k_v = 0.7165$ kJ/kg K and $\gamma = 1.4$ for air.

[(i) 500.9°C, 1,559.6°C; 689.5°C; (ii) 1,062.41 kJ/kg; (iii) 483.28 kJ/kg; (iv) 579.13 kJ/kg; (v) 54.51%]

13. Describe the ideal air cycle for the Diesel engine receiving heat at constant pressure and rejecting heat at constant volume. Show that the efficiency of this cycle is less than that of the constant volume cycle for the same compression ratio.

14. (a) In an ideal Diesel cycle the temperatures at the beginning and end of compression are 57°C and 603°C respectively, whilst those at the beginning and end of expansion are 1,950°C and 870°C respectively. Determine per kg of working fluid for which $R = 0.287$ kJ/kg K and $\gamma = 1.4$, (a) the heat received in kJ, (b) the heat rejected in kJ, (c) the work done in kJ, and (d) the ideal thermal efficiency.

If the compression ratio is 14:1 and the pressure at the beginning of compression is 100 kPa, determine the maximum pressure in the cycle.

[(a) 1,343 kJ/kg; (b) 583 kJ/kg; (c) 760 kJ/kg; (d) 56.59%; 4,023 kPa]

- (b) Sketch the pressure-volume and temperature-entropy diagrams for the ideal Diesel cycle and describe the sequence of operations.

In an ideal Diesel cycle, the temperatures at the beginning and end of compression are 32°C and 615°C respectively. If the temperature at the end of constant pressure heat addition is, 1,780°C, determine : (a) the value of the compression ratio, (b) the percentage of the working stroke at which cut-off takes place, and (c) the ideal thermal efficiency. Assume $\gamma = 1.4$ and $k_p = 0.997$ kJ/kg K for air.

[(a) $r = 14.4$; (b) 9.78%; (c) 58.19%]

15. Show that the efficiency of an air engine working on the Diesel cycle may be expressed as :

$$1 - \frac{1}{(r)^{\gamma-1}} \cdot \frac{1}{\gamma} \cdot \frac{(\rho)^{\gamma}-1}{\rho-1}$$

where, r is the compression ratio, ρ is the cut-off ratio, and γ is the ratio of the specific heats of air.

16. Derive an expression for the air-standard efficiency of an oil engine working on the Diesel cycle, stating clearly the assumptions made.

State the reasons why the actual thermal efficiency of an internal combustion engine is lower than its air-standard efficiency.

17. Show that in an engine working on the dual-combustion cycle and using air as the working fluid, the

air-standard efficiency is given by the expression :

$$1 - \frac{1}{(r)^{\gamma-1}} \left[\frac{\beta(\rho)^{\gamma} - 1}{(\beta - 1) + \beta\gamma(\rho - 1)} \right]$$

where, r = compression ratio, β = explosion ratio, ρ = cut-off ratio, and γ = ratio of specific heats of air.

18. An oil engine working on the dual-combustion cycle has a cylinder diameter of 20 cm and a stroke of 40 cm. The compression ratio is 13.5 and the explosion or pressure ratio obtained from indicator diagram is 1.42. From the indicator diagram it was found that cut-off occurred at 5.1% of the stroke. Find the air-standard efficiency of the engine. Assume $\gamma = 1.4$ for air.

[61.58%]

19. A high speed Diesel engine working on ideal dual-combustion cycle, takes in air at a pressure of 100 kPa and the temperature of 50°C and compresses it adiabatically to $1/14$ th of its original volume. At the end of the compression the heat is added in such a manner that during the first stage, the pressure increases at constant volume to twice the pressure of the adiabatic compression, and during the second stage following the constant volume heat addition, the volume is increased twice the clearance volume at constant pressure. The air is then allowed to expand adiabatically to the end of the stroke where it is exhausted, heat being rejected at constant volume. Calculate (i) the temperatures at the salient (key) points of the cycle, and (ii) the ideal thermal efficiency. Take $k_p = 0.9923$ kJ/kg K and $k_v = 0.7076$ kJ/kg K for air.

Sketch the pressure-volume and temperature-entropy diagrams for the cycle.

[(i) 655.3°C, 1,583.6°C, 3,440.2°C, 1,432°C, (ii) 60.87%]

20. In a compression-ignition engine working on ideal dual-combustion cycle, the pressure and temperature at the beginning of compression are 1 bar and 127°C respectively. The pressure at the end of compression is 30 bar and the maximum pressure of the cycle is 50 bar. During combustion, half of the heat is added at constant volume and half at constant pressure. Both the compression and expansion curves are adiabatic and heat is rejected at constant volume. Calculate the temperatures at the salient (key) points of the cycle and the ideal thermal efficiency. Take $k_p = 0.9965$ kJ/kg K and $k_v = 0.7118$ kJ/kg K for air throughout the cycle.

[783.8°C, 1,488°C, 1,991°C, 674.3°C; 61.14%]

21. A high speed Diesel engine working on the ideal dual-combustion cycle has compression ratio of 11. The pressure and temperature before compression are 100 kPa and 90°C respectively. If the maximum pressure in the cycle is 5,000 kPa and the constant pressure heat addition continues for $1/20$ th of the stroke, find the work done per kg of air and the ideal thermal efficiency. Take $k_p = 0.9965$ kJ/kg K and $k_v = 0.7118$ kJ/kg K for air.

Sketch the pressure-volume and temperature-entropy diagrams for the cycle. [787 kJ/kg; 59.52%]

22. Calculate the air-standard efficiency of a gas turbine plant working on Joule cycle between 103 kPa and 412 kPa. If minimum and maximum temperatures in the cycle are 27°C and 527°C respectively, find temperatures after isentropic compression and after isentropic expansion. Take $\gamma = 1.4$ for air.

[32.7%, 172.5°C, 265.72°C]

23. Show that the ideal M.E.P. of the Otto cycle is given by the expression :

$$\frac{p_1 r (\beta - 1)(r^{\gamma-1} - 1)}{(r - 1)(\gamma - 1)}$$

where, p_1 = pressure at the beginning of compression, r = compression ratio, and β = ratio of maximum pressure to compression pressure.

24. A petrol engine with supply pressure and temperature of 100 kPa and 40°C respectively and working on ideal Otto cycle has a compression ratio of 5.8. Heat supplied at constant volume per kilogram of charge is 586 kJ. Find the pressures and temperatures at the salient (key) points of the ideal cycle, if the compression and expansion law is $pV^{1.4} = \text{constant}$. Calculate also the theoretical mean effective pressure. Take $k_v = 0.712$ kJ/kg K for air.

[1,171 kPa, 2,696 kPa; 230.2 kPa; 359.26°C, 1,182.62°C, 447.6°C; M.E.P. = 401 kPa]

25. Show that the ideal M.E.P. of the Diesel cycle may be expressed as

$$\frac{p_1 (r)^{\gamma} [\gamma(\rho - 1) - (r)^{1-\gamma}(\rho^{\gamma} - 1)]}{(\gamma - 1)(r - 1)}$$

where, p_1 = pressure at the beginning of compression, r = compression ratio, and ρ = cut-off ratio,

26. An air engine works on ideal cycle in which heat is received at constant pressure and rejected at constant volume. The pressure and temperature at the beginning of the compression stroke are 100 kPa and 15°C

respectively. The compression ratio is 15.3 and expansion ratio is 7. If the law of adiabatic compression and expansion is $pv^{1.4} = \text{constant}$, calculate : (i) the ideal thermal efficiency, and (ii) the ideal mean effective pressure of the cycle. Take $k_p = 0.994 \text{ kJ/kg K}$ and $k_v = 0.709 \text{ kJ/kg K}$ for air.

$$[(i) 59.87\% ; (ii) \text{M.E.P.} = 791.3 \text{ kPa}]$$

27. An air engine working on ideal cycle in which heat is received at constant pressure and rejected at constant volume. The pressure and temperature at the beginning of compression stroke are 100 kPa and 40°C respectively. The compression ratio is 13 and cut-off ratio is 2. If the compression and expansion curves are adiabatic, calculate the ideal mean effective pressure of the cycle and its ideal thermal efficiency.

Take $k_p = 1.0035 \text{ kJ/kg K}$, $k_v = 0.7165 \text{ kJ/kg K}$ and $\gamma = 1.4$ for air.

$$[\text{M.E.P.} = 613.68 \text{ kPa} ; 58.08\%]$$

28. A Diesel engine works on the ideal cycle with a compression ratio of 14 and with cut-off taking place at 10% of the stroke. The pressure at the beginning of compression is 100 kPa. Calculate the ideal thermal efficiency and ideal mean effective pressure of the cycle. Take $\gamma = 1.4$ for air.

$$[57.86\% ; \text{M.E.P.} = 813.58 \text{ kPa}]$$

29. The compression ratio of an engine working on the dual-combustion cycle is 9 to 1 and the maximum pressure is 3,900 kPa. The temperature at the beginning of compression is 95°C and at the end of expansion is 545°C. Considering the ideal cycle with air as the working fluid and assuming that the pressure at the beginning of compression is 100 kPa, find (a) the ideal thermal efficiency, and (b) the ideal mean effective pressure of the cycle. Take $k_p = 0.9965 \text{ kJ/kg K}$ and $k_v = 0.7118 \text{ kJ/kg K}$ for air.

$$[(a) 57.98\% ; (b) \text{M.E.P.} = 475.6 \text{ kPa}]$$