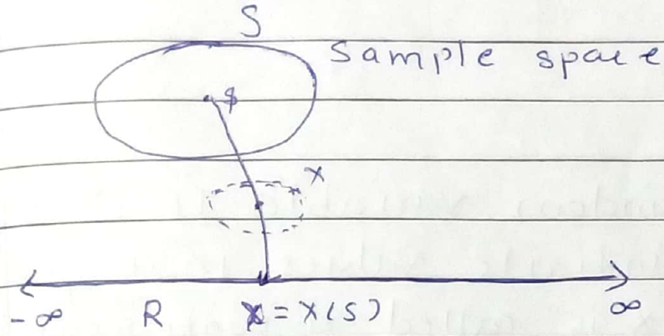
① Random variable :-

Let E be an experiment and s be the sample space associated with it. A function x assigning to every element s of S one & only one real number x = x(s) of R is called a random variable



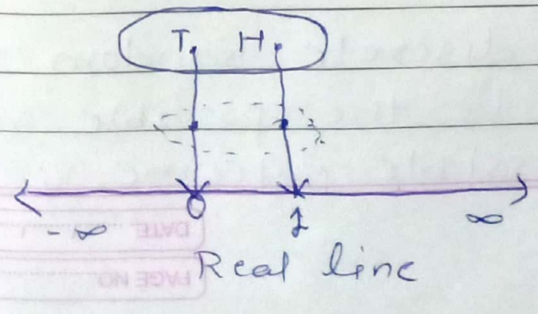
Since x is a function whose domain is the set of outcomes of an experiment and whose range is a part or the whole of 3 line (-∞ < x < ∞)

The random variable x can be discrete or contineous depending upon the nature of it's domain

e.g. ① Tossing of coin geting Head or tail

~~Tossing of~~

then S = {H, T} If x is the random variable denoting the number of heads then we have X(H) = 1 & X(T) = 0



① Definition

Let x be a random variable. If x takes finite or countably infinite values $x_0, x_1, x_2 \ldots$ then x is called a discreate random variable

e.g. Tossing of coin

$S = \{H, T\}$

② Let x be a random variable If x takes uncountably infinite values in a given interval then x is called a contineous random variable

e.g. Height of person or weight of person
Here x takes contineously all values between a specified interval

* probability Distribution of a Discrete Random variable

→ We know that with every possible outcome of an experiment there will be associated it's probability

If x_i is the value of x and $P(x_i)$ is the probability of x_i then set of pairs $(x_i, P(x_i))$ is called the probability distribution

Definition :- Let x be discrete random variable. Let $x_1, x_2, \ldots x_n$ be the possible values of x. With each possible outcome x_i we associate a number $P(x_i) = P(X = x_i)$ called the probability of x_i. The numbers $P(x_i)$, $i = 1, 2, \dots n \dots$ must satisfy the following conditions

1) $P(x_i) \geq 0 \;\forall\; i$

2) $\sum_{i=1}^{n} P(x_i) = 1$

The function p is called the probability function or probability mass function or probability density function

X	x_1	x_2	x_3	$\dots$	x_n
$P(x_i)$	P_1	P_2	P_3	$\dots$	P_n

e.g. ① A random variable X takes values 0, 1, 2 and 3 then $P(X = x) = \frac{x-1}{2}$ can be it's probability distribution

Soln. $P(x_i) \geq 0 \;\forall\; i$

putting $x = 0, 1, 2, 3$ we get

$P(0) = -\frac{1}{2}$, $P(1) = 0$, $P(2) = \frac{1}{2}$, $P(3) = 1$

since the probability cannot be negative $P(X = x) = \frac{x-1}{2}$ cannot be a probability distribution.

e.g. (2) A random variable takes values 0, 1, 2 & is it's probability distribution $P(x) = \frac{x+1}{3}$

→ putting $x = 0, 1, 2$ in $P(x)$ we get

$P(0) = \frac{1}{3}$, $P(1) = \frac{2}{3}$, $P(2) = 1$

Although all probabilities are positive the sum of all the probabilities is 2. Hence $P(X=x) = \frac{x+1}{3}$ also cannot be a probability distribution

EX. (1) From the past experience it was found that the daily demand at an autogarage was as under

Daily Demand :	5	6	7
probability :	0.25	0.65	0.10

check ~~whether~~ if this is a probability distribution. Find also the probability that over a period of two days the number of demands would be 11 or 12

Sol^n. Since the sum of all probabilities $= 0.25 + 0.65 + 0.10 = 1$, it is a probability distribution

P(11 requests over two days)
= P(5 requests on first day and 6 on the second)
+ P(6 requests on first day and 5 on the second)

$= (0.25 \times 0.65) + (0.65 \times 0.25)$
$= 0.1625 + 0.1625$
$= 0.325$

P(12 requests over two days)
= P(5 requests on the first day and 7 on the second)
+ P(6 requests on the first day and 6 on the second)
+ P(7 requests on the first day & 5 on the second)

$= (0.25 \times 0.10) + (0.65 \times 0.65) + (0.10 \times 0.25)$
$= 0.025 + 0.4224 + 0.025 = 0.7975$

Ex. The probability mass function of a random variable X is zero except at the points $X = 0, 1, 2$. At these points it has the values

$P(0) = 3c^3$, $P(1) = 4c - 10c^2$, $P(2) = 5c - 1$
i) Determine c, ii) Find $P(X<1)$, $P(1 < X \leq 2)$, $P(0 < X \leq 2)$

Soln. Since $\Sigma p_i = 1$ we have

$P(0) + P(1) + P(2) = 1$
$3c^3 - 10c^2 + 4c + 5c - 1 = 1$
$\therefore 3c^3 - 10c^2 + 9c - 2 = 0$
$(3c-1)(c-2)(c-1) = 0$

$\therefore c = 1/3$
the other values are not admissible

$\therefore$ The probability distribution is

x	0	1	2
$P(X=x)$	$1/9$	$2/9$	$2/3$

$\therefore$ $P(X<1) = P(X=0) = 1/9$

$P(1 < X \leq 2) = P(X=2) = 2/3$

$P(0 < X \leq 2) = P(X=1) + P(X=2)$

$= \frac{2}{9} + \frac{2}{3} = \frac{8}{9}$