Example 3 : In a distribution exactly normal 7 % of items are under 35 and 89 % are under 63. What are the mean and standard deviation ?

Sol. : Since 7 % items are below 35, 50 – 7 = 43 % items are between 35 and m, and since 89 % items are below 63, 89 – 50 = 39 % items are between m, and 63.

For area 0·43, $Z = 1·48$.

Since $35 < m$, $Z = -1·48$ and for area 0·39, $Z = 1·23$.

$$\therefore \quad \frac{35 - m}{\sigma} = -1·48 \quad \text{and} \quad \frac{63 - m}{\sigma} = 1·23$$

$$35 - m = -1·48\,\sigma$$

and $$63 - m = 1·23\,\sigma.$$

Subtracting $28 = 2·71\sigma$

$$\therefore \quad \sigma = \frac{28}{2·71} = 10·33$$

$$\therefore \quad m = 35 + 1·48\sigma = 35 + 1·48 \times 10·33$$
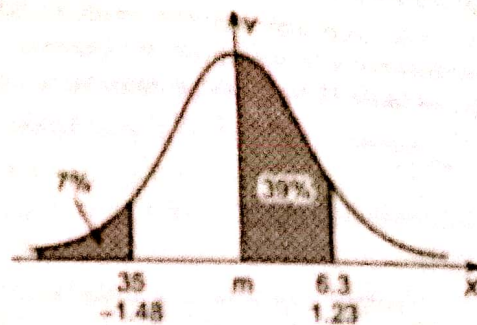
$$= 35 + 15·3 = 50·3.$$

Fig. 10.23

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EXERCISE - III

(A) 1. The weights of 4000 students are found to be normally distributed with mean 50 kilograms and standard deviation 5 kilograms. Find the probability that a student selected at random will have weight (i) less than 45 kilograms (ii) between 45 and 60 kilograms. (For a standard normal variate z, area under the curve between $z = 0$ and $z = 1$ is 0·3413 and that between $z = 0$ and $z = 2$ is 0·4772)

[**Ans. : (i)** 0·1587, **(ii)** 0·8185]

2. The sizes of 10,000 items are normally distributed with mean 20 cms and standard deviation 4 cms. Find the probability that an item selected at random will have size between (i) 18 cms and 23 cms. (ii) above 26 cms.

(Given : For a S.N.V. z area from $z = 0$ to $z = 0.5$ is 0·1915 , that from $z = 0$ to $z = 0.75$ is 0·2734 and that from $z = 0$ to $z = 1.5$ is 0·4332)   [**Ans. : (i)** 0·4649, **(ii)** 0·0668]

3. The customer accounts of a certain departmental store have an average balance of ₹ 120 and a standard deviation of ₹ 40. Assuming that the distribution of account balances is normal, find the proportion of accounts (i) over ₹ 150. (ii) between ₹ 100 and ₹ 150, (iii) between ₹ 60 and ₹ 90.

(Given : For S.N.V. z area from $z = 0$ to $z = 0.75$ is 0·2734, that from $z = 0$ to $z = 0.5$ is 0·1915 and that from $z = 0$ to $z = 1.5$ is 0·4332)   (S.U. 1982, 88)

[**Ans. : (i)** 0·2266, **(ii)** 0·4649, **(iii)** 0·1598]

(B) 1. If the mean and standard deviation of a normal distribution are 50 and 12. Find how many items out of 500 lie between 38 and 62 and also between 26 and 74.

(Given : For S.N.V. z area from $z = 0$ to $z = 1$ is 0·34135 and that from $z = 1$ to $z = 2$ is 0·1359.)

[**Ans. : (i)** 341, **(ii)** 477]

2. The income distribution of a group of 10,000 persons was found to be normal with mean ₹ 7500 and standard deviation ₹ 500. What is the number of persons of this group who have income (i) exceeding ₹ 6680, (ii) exceeding ₹ 8320 ?

(Given : Area under the standard normal curve between ordinates ± 1·64 is 0·8990.)

(S.U. 1987, 89, 99, 2006) [**Ans. : (i)** 9495, **(ii)** 505]

Engineering Mathematics - III
(Electrical, Electronics and E&TC)

(10-42)

Some Standard Distributions

Enginee
(Electrical,

3. Assuming that the diameter of 1000 plugs taken consecutively from a machine, form a normal distribution with mean 0·7515 and standard deviation 0·002, how many plugs are likely to be rejected if the approved diameter is 0·752 ± 0·004.

(Given : For a S.N.V. z area between $z = 0$ and $z = 1.75$ is 0·4599 and that between $z = 0$ and $z = 2.25$ is 0·4878.) **(S.U. 1985)** [**Ans. : 52·3**]

4. In a test of 2000 electric bulbs, it was found that the life time of the bulbs was normally distributed with the mean of 2040 hours and standard deviation of 60 hours. Estimate the number of bulbs likely to burn out in less than 1950 hours.

(Given : For S.N.V. z area between $z = 0$ and $z = 1.5$ is 0·4332) **(S.U. 1985)** [**Ans. : 134**]

5. The incomes of a group of 10,000 persons were distributed normally with mean ₹ 5200 and standard deviation of ₹ 600. Find the number of persons having income between ₹ 4000 and ₹ 5500.

(Given : For S.N.V. z, area between $z = 0$ and $z = 0.5$ is 0·1915 and that from $z = 0$ and $z = 2$ is 0·4772.) **(S.U. 1985)** [**Ans. : 6687**]

6. The size of hats is normally distributed with mean of 18·5 cms. and standard deviation of 2·5 cms. How many hats in a total of 2000 will have sizes between (i) 18 cms. and 20 and (ii) above 20 cms. ?

(Given : For S.N.V. z, area between $z = 0$ and $z = 0.6$ is 0·2257 and that between $z = 0$ and $z = 0.2$ is 0·0793.) **(S.U. 1985)** [**Ans. : (i)** 610, **(ii)** 549]

7. In an intelligence test administered to 1000 children the mean score was, 42 with s.d. 24. Find the number of children (i) scoring more than 60 and (ii) between 20 and 40 assuming the distribution to be normal.

(Given : For S.N.V. z area from $z = 0$ to $z = 0.75$ is 0·2734, that between $z = 0$ and $z = 0.9167$ is 0·3202 and that between $z = 0$ to $z = 0.08333$ is 0·0332.) **(S.U. 1988, 99)** [**Ans. : (i)** 227, **(ii)** 287]

8. In the above problem, find

(i) the number of students exceedign the score 50.

(ii) the number of students getting the score between 30 and 50.

(Area under S.N.V. z, from $z = 0$ to $z = 0.33$ is 0·1293 and from $z = 0$ to $z = 0.5$ is 0·195.)

(S.U. 2006) [**Ans. : (i)** 371, **(ii)** 324]

(C) 1. Diameters of ball bearings are normally distributed with mean 0·498 cms. and standard deviation 0·002 cms. If specifications require 0·500 ± 0·004 cm. what percentage of production will be rejected ?

(Given : For S.N.V. z area from $z = 0$ to $z = 1$ is 0·3413 and that from $z = 0$ to $z = 3$ is 0·4987.)

(S.U. 1986) [**Ans. : 16 %**]

2. The income distribution of a group of 10,000 persons was found to be normal with mean ₹ 750 and standard deviation ₹ 50. What percentage of this group had income (i) exceeding ₹ 668, (ii) exceeding ₹ 832 ?

(Given : Area under Standard Normal Curve between ordinates at ± 1·64 is 0·8990.)

(S.U. 1987, 89) [**Ans. : (i)** 94·95%, **(ii)** 5·05%]

3. The mean I.Q. of a large number of children of age 14 is 100 with s.d. 16. Assuming the distribution of I.Q. to be normal, find the percentage of the children having I.Q. between 70 and 120. (Area for S.N.V. z from $z = 0$ to $z = 1.875$ is 0·4696 and that from $z = 0$ to $z = -1.25$ is 0·3944.)

[**Ans. : 86·4%**]

4. The daily sales of a certain item are normally distributed with mean ₹ 8000 and variance ₹ 10,000. (i) What is the probability that on a certain day the sales will be less than ₹ 8210 ? (ii) What percentage of the days will the sales be between ₹ 8100 and ₹ 8210 ?

(Given : Area for S.N.V. z from $z = 0$ to $z = 2.1$ is 0·4821; that between $z = 0$ and $z = 1$ is 0·3413.)   [**Ans. : (i)** 0·9821, **(ii)** 14·08%]

5. The average selling price of houses in a city is ₹ 50,000 with standard deviation of ₹ 10,000. Assuming the distribution of selling price to be normal find (i) the percentage of houses that sell for more than ₹ 55,000, (ii) the percentage of houses selling between ₹ 45,000 and ₹ 60,000. (Area between $t = 0$ and $t = 1$ is 0·3413 and the area between $t = 0$ and $t = 0.5$ is 0·1915 where t is S.N.V.)   [**Ans. : (i)** 30·85%, **(ii)** 53·28%]

6. If the mean life time and S.D. of battery cells are 12 hrs, and 3 hrs what % of batteries will have life (i) between 10 and 14 hrs., (ii) more than 15 hrs., (iii) less than 6 hrs.

(Given : For S.N.V. z area from $z = 0$ to $z = 0.67$ is 0·2486, that from $z = 0$ to $z = 1$ is 0·3413 that from $z = 0$ to $z = 2$ is 0·4772.)   **(S.U. 1987)** [**Ans. : (i)** 49%, **(ii)** 16%, **(iii)** 2%]

7. The average number of units produced by a manufacturing concern per day is 355 with standard deviation of 50. It makes a profit of ₹ 1·50 per unit.

Determine the percentage of days when its total profit per day is (i) between ₹ 457·50 and ₹ 645·00, (ii) greater than ₹ 682·50. Assume the distribution to be normal.

(Area between $t = 0$ and $t = 1$ is 0·3413 and between $t = 0$ and $t = 1.5$ is 0·4332 and that between $t = 0$ and $t = 2$ is 0·4772.)

(Hint : Mean of the profit is 532·5 and S. D. of the profit is 75.)

[**Ans. : (i)** 77·45%, **(ii)** 2·27%]

(D)   1. The income distribution of workers in a certain factory was found to be normal with mean of ₹ 500 and standard deviation equal to ₹ 50. There were 228 persons above ₹ 600. How many persons were there in all ? (Area under the S.N. curve between height at 0 and 2 is 0·4772.)

(S.U. 1987) [**Ans. :** 10000]

2. In a factory a large number of workers have average income of ₹ 120. If 38·3% of them have income between ₹ 100 – 140 and 528 of them get more than ₹ 170, how many workers were interrogated ? (Area for S.N.V. between $z = 0$ and $z = 19.15$ is 0·5 and that between $z = 0$ and $z = 1.25$ is 0·3944.   (Hint : Find σ = 40.) [**Ans. :** 5000]

3. The arithmetic mean of purchases per day by a customer in a large store is ₹ 25 with standard deviation of ₹ 10. If on a particular day, 100 customers purchased for ₹ 37·80 or more, estimate the total number of customers who purchased from the store that day.

(Given that the normal area between $t = 0$ and $t = 1.28$ is 0·4000 where t is the S.N.V.)

[**Ans. :** 1000]

(E)   1. The heights of 1000 cakes baked with certain mix have a normal distribution with a mean of 5·75 cms, and a standard deviation of 0·75 cms. Find the number of cakes having heights between 5 cms, and 6·25 cms. Also find the maximum height of the flatest 200 cakes.

(For a standard variate t, the area between $t = -1$ and $t = 1$ is 0·6826, that between $t = 0$ and $t = 0.67$ is 0·2486 and that between $t = 0$ and $t = 0.84$ is 0·3).   [**Ans. :** 589·9, 5·12]

2. 1000 light bulbs with an average life of 120 days are installed in streets of Mumbai. Their length of life is normally distributed with variance 400 days. (i) How many will expire in less than 90 days? (ii) If it is decided to replace all the bulbs together what interval should be allowed between replacements if not more than 10 percent should expire before replacement ?

(Area between $z = 0$ and $z = 1.5$ is 0.4332 and 80% of the area lies between $z = \pm 1.28$).

[**Ans. : (i)** 67, **(ii)** 94.4]

3. Monthly salaries of 1000 workers have a normal distribution with mean of ₹ 575 and a standard deviation of ₹ 75. Find the number of workers having salaries between ₹ 500 and ₹ 625 p.m. Also find the minimum salary of the highest paid 200 workers.

(Given : For a standard normal variate t (i) area between $t = 0$ and $t = 1$ is 0.3413, (ii) area between $t = 0$ and $t = 0.67$ is 0.2486, (iii) area between $t = 0$ and $t = 0.84$ is 0.3.)

[**Ans. :** ₹ 5899 ; ₹ 638]

4. Incandescent electric bulbs of a certain make have a mean life of 800 hours with standard deviation of 150 hours. Out of 50.000 bulbs of that make used for street lighting how many of these would fuse in first 650 hours, if the distribution of the life of bulbs is assumed to be normal. In how many days will you find that only 7935 bulbs are burning ?

(Area of a normal curve lying between $z = 0$ to $z = 1$ is 0.3413.)   [**Ans. : (i)** 7935, **(ii)** 950]

(F) 1. In a distribution which is exactly normal, 7% of the items are under 35 and 89% items are under 63. Find the mean and standard deviation of the distribution.

(Given that for area 0.43 S.N.V. $z = 1.48$ and for area 0.39 S.N.V. $z = 1.23$.)   **(S.U. 1990)**

[**Ans. :** $m = 50.29$, $\sigma = 10.33$]

2. Find the mean and the standard deviation of a normal distribution of marks in an examination where 58% of the candidates obtained marks below 75, 4% got above 80 and the rest between 75 and 80.

(For a S.N.V. the area under the curve between $z = \pm 0.2$ is, 0.16 and between $z = \pm 1.8$ is 0.92.)
(S.U. 1993, 98) [**Ans. :** $m = 74.375$, $\sigma = 3.125$]

3. The qualifying marks for a certain examination are 35 and to secure distinction one has to score more than 74. If 25% of the students fail, whereas 6.681% obtained distinction, determine the mean and the standard deviation assuming that the distribution of marks is normal.

(Given : For S.N.V. z area from $z = 0$ to $z = 2/3$ is 0.25 and that between $z = 0$ to $z = 3/2$ is 0.43319.)

[**Ans. :** $m = 47$, $\sigma = 18$]

4. At a certain examination 10% students who appeared for the paper in Statistics got less than 30 marks and 97% of the students got less than 62 marks. Assuming the distribution to be normal, find the mean and the s.d. of the distribution.

(Given that 40% of the area of the normal curve is between the ordinates corresponding to $t = 0$ and $t = 1.3$ and 7% of the area is between $t = 1.3$ and $t = 1.9$ where, t is a S.N.V.)

[**Ans. :** $m = 43$, $\sigma = 10$]

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