Probability

(a) Trial and event:
Let an experiment be repeated under essentially the same conditions and let it result in any one of the several possible outcomes. Then, the experiment is called a trial & the possible outcomes are known as events or cases.

e.g. (i) Tossing of a coin is a trial and the turning up of head or tail is an event.
ii) Throwing a die is a trial & getting 1 or 2 or 3 or 4 or 5 or 6 is an event.

(b) Exhaustive events: The total number of all possible outcomes in any trial is known as exhaustive events or cases.

e.g. (1) In tossing of a coin there are two exhaustive cases head and tail.

(2) In throwing of two dice, the exhaustive cases are $6 \times 6 = 6^2$

(c) Favourable events or cases: The cases which entail the happening of an event are said to be favourable to the event. It is the total number of possible outcomes in which the specified event happens.

e.g. (i) In throwing a die, the number of cases favourable to the appearance of a multiple of 3 are two viz. 3 & 6 while the number of cases favourable to the appearance of an even number are three viz. 2, 4 and 6

(ii) In a throw of two dice, the number of cases favourable to getting a sum 6 is 5
viz., (1,5); (5,1); (2,4); (4,2); (3,3)

(d) Mutually exclusive events: Events are said to be mutually exclusive or incompatible if the happening of any one of them precludes the happening of all other

e.g. In tossing a coin, the events head and tail are mutually exclusive since if the outcome is head, the possibility of getting tail in the same trial is ruled out.

(e) Equally likely events: Events are said to b equally likely if there is no reason t expect any one in preference to any other

e.g. In throwing a die all the six face are equally likely to come

(i) Independent & dependent events:
Two or more events are said to be independent if the happening or non-happening of any one does not depend or is not affected by the happening or non-happening of any other.

e.g. If a card is drawn from a pack of well shuffled cards and replaced before drawing the second card, the result of the second draw is independent of the first draw. However, if the first card drawn is not replaced, then the second draw is dependent on the first draw.

Mathematical definition of probability:

If a trial results in 'n' exhaustive, mutually exclusive and equally likely cases and 'm' of them are favourable to the happening of an event E, then the probability of happening of E is given by

$$p = P(E) = \frac{\text{Favourable no. of cases}}{\text{Exhaustive no. of cases}} = \frac{m}{n}$$

The probability that event E will not happen is given by

$$q = P(\bar{E}) = \frac{\text{unfavourable no. of cases}}{\text{Exhaustive no. of cases}}$$

$$= \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - p$$

$\therefore \quad p + q = 1 \qquad P(E) + P(\bar{E}) = 1$

Ex. ① A bag contains 7 white, 6 red and 5 black balls. Two balls are drawn at random. Find the probability that they will both be white

sol^n: Tot. no. of balls $= 7 + 6 + 5 = 18$

out of 18 balls, 2 can be dran in $^{18}C_2$ ways.

$$\therefore \quad ^{18}C_2 = \frac{18 \times 17}{2 \times 1} = 153 = \text{Exhaustive no. of cases}$$

out of 7 white balls 2 can be drawn.

$$\text{in } \; ^{7}C_2 = \frac{7 \times 6}{2 \times 1} = 21 \text{ ways}$$

$\therefore$ ~~Exh~~ Favourable No. of cases $= 21$

$$\therefore \quad \text{Probability} = \frac{21}{153} = \frac{7}{51}$$

Ex. ② Four cards are drawn from a pack of cards. Find the probability that i) all are diamonds ii) there is one card of each suit, and iii) there are two spades and two hearts

$\longrightarrow$)

4 cards can be drawn from 52 cards $^{52}C_4$ ways.

$\therefore$ Exhaustive number of cases $= {}^{52}C_4 = 27\ldots$

(i) There are 13 diamonds

$\therefore$ $^{13}C_4$ ways

$\therefore$ Favourable cases $= {}^{13}C_4 = 715$

$\therefore$ Required probability $= \frac{715}{270725} = \frac{11}{4165}$

ii) There are 4 suits each containing 13 cards

Favourable no. of cases $= {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1$
$= 13 \times 13 \times 13 \times 13$

$\therefore$ Required prob. $= \frac{13 \times 13 \times 13 \times 13}{270725} = \frac{2197}{20825}$

(iii) 2 spades out of 13 : $^{13}C_2$ ways
2 hearts out of 13 : $^{13}C_2$ ways.

$\therefore$ Favo. no. of cases $= {}^{13}C_2 \times {}^{13}C_2 = 78 \times 78$

Required prob $= \frac{78 \times 78}{270725} = \frac{468}{20825}$