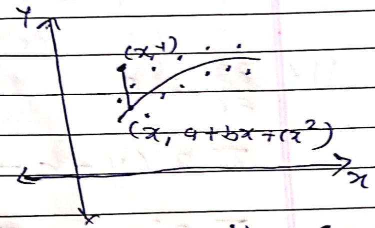
* <u>Curve fitting</u> :-

Fitting of curve to a given set of values means finding functional relationship between x & y

* Fitting of parabola by the principle of least square.



→ Let $y = a + bx + cx^2$ be the equation of parabola

∴ By principle of least square, the sum of distances of all points from parabola must be minimum

$$\therefore S = \sum (y - a - bx - cx^2)^2 \text{ must be minimum}$$

$$\therefore \frac{\partial S}{\partial a} = 0, \quad \frac{\partial S}{\partial b} = 0, \quad \frac{\partial S}{\partial c} = 0 \quad \text{(cond}^n \text{ to be minimum)}$$

$$\therefore \frac{\partial S}{\partial a} = 0$$

$$\sum 2(y - a - bx - cx^2)(-1) = 0$$

$$\Rightarrow \sum (y - a - bx - cx^2) = 0$$

$$\Rightarrow \sum y - a\sum 1 - b\sum x - c\sum x^2 = 0$$

$$\Rightarrow \sum y - an - b\sum x - c\sum x^2 \quad \text{— (2)}$$

$$\sum y = an + b\sum x + c\sum x^2 \quad \text{— (1)}$$

$$\frac{\partial S}{\partial b} = 0$$

$$\Rightarrow \Sigma 2(y-a-bx-cx^2)(-x) = 0$$

$$\Rightarrow \Sigma xy - a\Sigma x - b\Sigma x^2 - c\Sigma x^3 = 0$$

$$\Rightarrow \Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3 \quad \text{—(2)}$$

||ly $\frac{\partial S}{\partial c} = 0$

$$\Sigma 2(y-a-bx-cx^2)(-x^2) = 0$$

$$\Sigma x^2y - a\Sigma x^2 - b\Sigma x^3 - c\Sigma x^4 = 0$$

$$\therefore \Sigma x^2y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4 \quad \text{—(3)}$$

To find a, b, c solve eqn (1)(2)(3) & put in the parabola

Ex. (1) Fit a straight line to the following data

x: 0, 1, 2, 3, 4

y: 1, 1.8, 3.3, 4.5, 6.3

→ Let $y = a + bx$ be straight line

Normal equations are

$$\Sigma y = na + b\Sigma x = na + b\Sigma x \qquad n = \text{No of observation.}$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2$$

x	y	xy	x^2
0	1	0	0
1	1.8	1.8	1
2	3.3	6.6	4
3	4.5	13.5	9
4	6.3	25.2	16

$\Sigma x = 10$, $\Sigma y = 16.9$, $\Sigma xy = 47.1$

$\Sigma x^2 = 30$

$\therefore$ $16.9 = 6a + 10b$

$47.1 = 10a + 30b$

Solving $a = 0.72$, $b = 1.33$

EX. Fit the curve of the form $y = ab^x$ for the following data

x : 1, 2, 3, 4, 5, 6, 7, 8

y : 1, 1.2, 1.8, 2.5, 3.6, 4.7, 6.6, 9.1

$\rightarrow$ $y = ab^x$

Taking log on both sides.

$\log y = \log a + x \log b$

$V = A + Bx$

∴ Normal eqn.

$$\Sigma V = An + B\Sigma x$$

$$\Sigma xV = A\Sigma x + B\Sigma x^2$$

x	y	$V = \log y$	x^2	$x \cdot V$
1	1	0	1	0
2	1.2	0.0791	4	0.1582
3	1.8	0.2552	9	0.7656
4	2.5	0.3979	16	1.5916
5	3.6	0.5563	25	2.78
6	4.7	0.6720	36	4.03
7	6.6	0.8195	49	5.73
8	9.1	0.9590	64	7.67

$x = 36 \quad \Sigma V = 3.74$

$x^2 = 204 \quad \Sigma xV = 22.72$

$\therefore \quad 3.74 = 8A + 36B$

$22.72 = 36A + 204B$

$\therefore A = -0.1625 \quad B = 0.14$

If $A = \log a \quad B = \log b$.

$\log a = -0.1625 \Rightarrow a = 0.85$

$\log b = 0.14 \Rightarrow b = 1.15$

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$y = 0.85 (1.15)^x$

Ex(5) Fit a second degree parabola to the following data and estimate y if $x=6$

x: 1 2 3 4 5

y: 25, 28, 33, 39, 46

Sol^n: Let $y = a + bx + cx^2$ (second deg. parabola)

Now Let $u = x - \bar{x}$

$\therefore u = x - 3$

x	y	$u=x-3$	u^2	u^3	u^4	uy	u^2y
1	25	-2	4	-8	16	-50	100
2	28	-1	1	-1	1	-28	28
3	33	0	0	0	0	0	0
4	39	1	1	1	1	39	39
5	46	2	4	8	16	92	184

$\Sigma y = 171$ $\Sigma u = 0$ $\Sigma u^2 = 10$

$\Sigma u^3 = 0$, $\Sigma u^4 = 34$, $\Sigma uy = 53$, $\Sigma u^2y = 351$

$y = a + bu + cu^2$ where $u = x-3$

$\Sigma y = na + b\Sigma u + c\Sigma u^2$

$\Sigma uy = a\Sigma u + b\Sigma u^2 + c\Sigma u^3$

$\Sigma u^2y = a\Sigma u^2 + b\Sigma u^3 + c\Sigma u^4$

$\therefore 171 = 5a + 10c$

$53 = 10b$

$351 = 10a + 34c$

$a = 32.91, \quad b = 5.3, \quad c = 0.6428$

$y = 32.91 + 5.3(x-3) + 0.6428\,(x-3)^2$

$y = 0.6428x^2 + 1.44x + 22.79$

At $x = 6 \Rightarrow y = 54.5952$

Ex. Fit a second degree parabola for the data

x: 0, 1, 2, 3, 4
y: 1, 4, 10, 17, 30

Sol^n. Let $y = a + bx + cx^2$ be second degree parabola

$\rightarrow \quad \bar{x} = \frac{\Sigma x}{5} = \frac{10}{5} = 2$

Let $u = x - \bar{x}$

$u = x - 2$

Normal equations are.

$\Sigma y = an + b\Sigma u + c\Sigma u^2$

$\Sigma uy = a\Sigma u + b\Sigma u^2 + c\Sigma u^3$

$\Sigma u^2 y = a\Sigma u^2 + b\Sigma u^3 + c\Sigma u^4$

$y = a + bu + cu^2$

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i.e. $y = a + b(x-2) + c(x-2)^2$

x	y	$u = x-3$	u^2	u^3	u^4	uy	u^2y
0	1	-2	4	-8	16	-2	4
1	4	-1	1	-1	1	-4	4
2	10	0	0	0	0	0	0
3	17	1	1	1	1	17	17
4	30	2	4	8	16	60	120

$\Sigma y = 62$, $\Sigma u = 0$, $\Sigma u^2 = 10$ $\Sigma u^3 = 0$
$\Sigma u^4 = 34$, $\Sigma uy = 71$, $\Sigma u^2y = 145$

From normal equations

$62 = 5a + 10c$
$71 = 10b + 0$
$145 = 10a + 34c$

$\therefore$ solving $a = 9.4$, $b = 7.1$ $c = 1.5$

$\therefore$ $y = 9.4 + 7.1(x-2) + 1.5(x-2)^2$

Problems for H.W.

① Find a, b, c if the curve $y = a + bx + cx^2$ is best fit for the data.

x : 3, 2, 1, 0, -1, -2, -3
y : 10, 8, 3, 1, 2, 6, 8

Note: If the data is of equal interval in large numbers then we change the scale as

$$u = \frac{x - x_0}{h}$$

Ex. fit a least square geometric curve $y = ax^b$ to the following data

x : 1, 2, 3, 4, 5
y : 0.5, 2, 4.5, 8, 12.5

Sol^n:

$$y = ax^b$$

$$\log y = \log a + b \log x$$

$$Y = A + BX$$

$A = \log a$

$$\Sigma Y = nA + B\Sigma X$$
$$\Sigma X \cdot Y = A\Sigma X + B\Sigma X^2$$

x	y	$Y = \log y$	$X = \log x$	$X^2 = (\log x)^2$	$X \cdot Y$
1	0.5	-0.6931	0	0	
2	2	0.6931	0.6931	0.4803	0
3	4.5	1.5040	1.0986	1.2069	0.4803
4	8	2.0794	1.3862	1.9215	1.65229
5	12.5	2.5257	1.6094	2.5901	2.8824
		6.1091	4.7873	6.1988	4.0648
					9.0797

$$6.1091 = 5A + B\,4.7873 \quad \times 4.7873$$
$$9.0797 = A\,4.7873 + B\,6.1988 \quad \times 5$$

$$29.24 = 23.9365A + 22.9182B$$
$$45.3985 = 23.9365A + 30.994B$$

$$-16.1585 = -8.0752B$$

$$B = \frac{16.1585}{8.0752} = 2.001 \simeq 2$$

$$6.1091 = 5A + B\,4.7873$$

$$6.1091 = 5A + 2 \times 4.7873$$

$$6.1091 = 5A + 9.5746$$

$$6.5746 - 9.5746 = 5A$$

$$-3 = 5A$$

$$A = -\frac{3}{5} = -0.6$$

$$\therefore A = -0.6 = \log a$$

$$\therefore a = 0.5488 \qquad b = 2$$

$$\therefore y = (0.5488)x^2$$