$= 0.09523 + 0.05 [\ldots + 0.8186]$

$= 0.09523 + 0.05 \times 1.72337$

$= 0.09523 + 0.08616$

$= 0.181398$

$y_1^{(3)} = y_1^{(4)}$

$\therefore y(0.2) = 0.1813$

Runge-Kutta Method :-

Runge-kutta method is a general method Euler's & Modified Euler's method are particular method of Runge-kutta method. Runge-kutta method is called as fourth order Runge kutta method.

Procedure :-

Let the given differential equation be $\frac{dy}{dx} = f(x, y)$. with initial conditions $x = x_0, y = y_0$

To find the value of $y = y_0 + k$ at $x = x_0 + h$, we

Calculate

$$K_1 = h \cdot f(x_0, y_0)$$

$$K_2 = h \cdot f\left[x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right]$$

$$K_3 = h \cdot f\left[x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right]$$

$$K_4 = h \cdot f(x_0 + h,\ y_0 + K_3)$$

Then we calculate

$$K = \frac{1}{6}\left[K_1 + 2K_2 + 2K_3 + K_4\right]$$

$\therefore$ The required value of y

$$\boxed{y = y_0 + K}$$

Ex. (i)

Solve $\frac{dy}{dx} = xy$ with initial condition $y(1) = 2$ and find y at $x = 1.2$ and at $x = 1.4$

Sol^n: We have $\frac{dy}{dx} = xy$

$\therefore f(x, y) = xy$ & $x_0 = 1$, $y_0 = 2$, $h = 0.2$

(a) $K_1 = h \cdot f(x_0, y_0)$
$= 0.2(1 \times 2) = 0.4$

$$K_2 = h \cdot f\left[x_0 + \frac{h}{2},\ y_0 + \frac{K_1}{2}\right]$$

$$= 0.2\left[(1 + 0.1)(2 + 0.2)\right] = 0.484$$

$$K_3 = h \cdot f\left[x_0 + \frac{h}{2},\ y_0 + \frac{K_2}{2}\right]$$

$$= 0.2\left[(1 + 0.1)(2 + 0.242)\right]$$

$$= 0.49324$$

$$K_4 = h \cdot f[x_0 + h, y_0 + k_3]$$

$$= 0.2[(1+0.2)(2+0.49324)]$$

$$= 0.598378$$

$$\therefore K = \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$$

$$= \frac{1}{6}[0.4 + 2(0.484) + 2(0.49324) + 0.59837]$$

$$= 0.492143$$

Hence, the approximate value of y is

$$= y_0 + K = 2 + 0.492143$$

$$= 2.492143$$

(b) Again to find y at $x = 1.4$ we have
$x_0 = 1.2$, $y_0 = 2.492143$, $h = 0.2$

$$K_1 = h \cdot f(x_0, y_0) = 0.2(1.2 \times 2.492143)$$

$$= 0.598114$$

$$K_2 = h \cdot f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$$

$$= 0.2\left[(1.2 + 0.1)\left(2.492143 + \frac{0.598114}{2}\right)\right]$$

$$= 0.725712$$

$$K_3 = 0.7423$$

$$K_4 = h \cdot f(x_0 + h, y_0 + K_3)$$

$$= 0.2\left[(1.2 + 0.2)(2.492143 + 0.7423)\right]$$

$$= 0.905644$$

$$\therefore K = \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$$

$$= \frac{1}{6}\left[0.598114 + 2(0.725712) + 2(0.7423) + 0.905644\right]$$

$$= 0.739964$$

Hence, the approximate value of y is

$$y = y_0 + K = 2.492143 + 0.739964$$
$$= 3.23107$$

Ex② Apply Runge-Kutta method to find an approximate value of y at $x = 0.2$ if $\frac{dy}{dx} = x + y^2$ given that $y = 1$ when $x = 0$ in steps of $h = 0.1$

Sol^n: We have $\frac{dy}{dx} = x + y^2$

$\therefore f(x,y) = x + y^2$ & $x_0 = 0$, $y_0 = 1$, $h = 0.1$

(a) $k_1 = h \cdot f(x_0, y_0) = 0.1\left[0^2 + 1^2\right] = 0.1$

$$k_2 = h \cdot f\left[x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right] = \left[0.1\left[(0 + 0.05) + (1 + 0.05)^2\right]\right]$$

$$= 0.11525$$

$$k_3 = h \cdot f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$= 0.1\left[(0.05) + (1 + 0.057625)^2\right]$$

$$= 0.11686$$

$$k_4 = h \cdot f(x_0 + h, y_0 + k_3)$$

$$= 0.1\left[(0 + 0.1) + (1 + 0.11686)^2\right]$$

$$= 0.13474$$

$$K = \frac{1}{6}\left[K_1 + 2K_2 + 2K_3 + K_4\right]$$

$$= \frac{1}{6}\left[0.1 + 2(0.11525) + 2(0.11686) + 0.13474\right]$$

$$= 0.1165$$

∴ The approximate value of y

$$y = y_0 + K = 1 + 0.1165 = 1.1165$$

(b) Again to find y at $x = 0.2$ we have

$x_0 = 0.1$, $y_0 = 1.1165$ $h = 0.1$ $f(x,y) = x + y^2$

$$K_1 = h \cdot f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$= 0.1\left[(0.1 + 0.05) + (1.1165 + 0.06733)^2\right]$$

$$= 0.15514$$

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$k_3 = 0.15758$

$k_4 = 0.18233$

$\therefore K = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= \frac{1}{6}[0.13466 + 2(0.15514) + 2(0.15758) + 0.18233]$

$= 0.1571$

$\therefore$ The approximate value of y

$y = y_0 + K = 1.1165 + 0.1571 = 1.2736$

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