A certain stimulus administered to each of 12 patients resulted in the following increases of BP 5, 2, 8, -1, 3, 0, -2, 1, 5, 0, 4, 6. Can it be concluded that the stimulus will in general be accompanied by an increase in BP.

→ ~~Let~~ us assume that the stimulus administered to all the 12 patients will

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Let us assume that stimulus will not ~~increase~~ accompanied by increase B.P. Taking pop^n to be normal with mean $\mu = 0$ & S.D σ

$$\bar{x} = \frac{5+2+8+(-1)+3+0+(-2)+1+5+0+4+6}{12}$$

$$\boxed{\bar{x} = 2.583}$$

$$s^2 = \frac{1}{n-1}\left[\sum (x_i - \bar{x})^2\right.$$

$$= \frac{1}{n-1}\left[\sum x_i^2 - 2x_i\bar{x} + (\bar{x})^2\right]$$

$$= \frac{1}{n-1}\left[\sum x_i^2 - 2\bar{x}\sum x_i + (\bar{x})^2 \sum 1\right]$$

$$= \frac{1}{n-1}\left[\sum x_i^2\right] - \frac{2\bar{x}\, n \sum x_i}{n(n-1)} + \frac{(\bar{x})^2\, n}{(n-1)}$$

$$= \frac{1}{n-1}\sum x_i^2 - \frac{2(\bar{x})^2 n}{n-1} + \frac{n}{n-1}(\bar{x})^2$$

$$= \frac{\sum x_i^2}{n-1} - \frac{n}{n-1}(\bar{x})^2$$

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$$= \frac{185}{11} - \frac{12}{11}(2.583)^2$$

$$= 16.81 - 7.27$$

$$= 9.54$$

$$s = \sqrt{9.54} = 3.088$$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{2.583 - 0}{3.088/\sqrt{12}} = 2.89$$

From table

$\nu = n-1 = 11$

$t_{0.05} = 2.20$

Here, $t_{cal} > t_{0.05}$

our assumption is rejected.

ie stimulus ~~does not~~ increase B.P

) The nine items of a sample have the following values 45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of these differ significantly from the assumed mean 47.5

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x $\quad$ ~~$x-\bar{x}$~~

Here $\bar{x} = \frac{45+47+50+52+48+47+49+53+51}{9}$

$= \frac{442}{9} = 49.11 \simeq 49$

x	$x-\bar{x}$	$(x_i-\bar{x})^2$
45	-4	16
47	-2	4
50	1	1
52	3	9
48	-1	1
47	-2	4
49	0	0
53	4	16
51	2	4
	1	55

$s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$

$= \frac{1}{8} 55 =$ ~~0.1125~~ $= 6.875$

$s =$ ~~$\sqrt{0.1125}$~~ 2.622

$=$ ~~0.3535~~

$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{(49 - 47.5) \cdot 3}{2.5222}$ $=$ ~~1.3022~~ $= 1.71$

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Here

$\nu = n-1 = 8$

from table

$t_{0.05} = 2.31$

$t_{cal} < t_{0.05}$

There is no significant difference betn

$\bar{x}$ & μ. Thus test provides no evidence

Chi square Test:

When a fair coin is tossed 80 times we expect from theoretical consideration that heads will appear 40 times & tails 40 times. But this never happens in practise ie the results obtained in an expt. do not agree exactly with the theoretical results. The magnitude of discrepency betn observation & theory is given by quantity χ^2. If $\chi^2 = 0$ the observed & theoretical frequencies completely agree. As the value of χ^2 increases the discrepency betn the observed

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* theoretical frequencies increases

Defn :-

If $O_1, O_2 \ldots O_n$ be set of observed (experimental) freq^n & $E_1, E_2 \ldots E_n$ be corresponding set of expected (theoretical) frequencies then χ^2 is

$$\chi^2 = \frac{(O_1 - E_1)^2}{E_1} + \frac{(O_2 - E_2)^2}{E_2} + \cdots + \frac{(O_n - E_n)^2}{E_n}$$

$$= \sum_{i=1} \frac{(O_i - E_i)^2}{E_i} \text{ with } n-1 \text{ d.f.}$$

$(\sum O_i = \sum E_i = n \text{ total freq}^n)$

* chi square distribution:

If $x_1, x_2 \ldots x_n$ be n independent normal variales with mean zero & S.D 1

Then χ^2 distribution has the

$x_1^2, x_2^2 - \cdots x_n^2$ random variates.

Eq^n of χ^2 curve is

$$y = y_0 \, e^{-\chi^2/2} \, (\chi^2)^{\frac{\nu - 1}{2}}$$

where $\nu = n-1$ is d.f.

since the eq^n does not have any parameter so it can be used for every problem of chi square.

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Properties of χ^2 distribution :-

i) If $\nu = 1$ then χ^2 curve reduces to $y = y_0 e^{-\chi^2/2}$ which is the exponential distribution

ii) If $\nu > 1$, this curve is tangential to x axis at origin & is positively skewed as mean at ν & mode at $\nu - 2$

iii) If $\nu > 30$, χ^2 curve approximates to normal curve. & we should refer to normal distribution tables for significant values of χ^2

iv) Since eq^n of χ^2 curve does not involve

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any parameter of popn, this distribution does not depend on the form of popn & is therefore useful in large no of problems.

v] Mean = ν & variance = 2ν

Goodness of fit

The value of χ^2 is used to test whether deviations of observed freqn from expected freqn are significant or not. It is also used to test how will a set of observation fit to given distribution, χ^2 so provides a test of goodness of fit & may be used to examine the validity of some hypothesis about observed freq^n distribution. As test of goodness of fit it can be used to study the correspondance bet^n theory & fact.

procedure to test significance & goodness of fit.

(I) Set up null hypothesis & calculate

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

(II) Find d.f & read the corresponding values of χ^2 at prescribed level of significance

(III) ~~If P < 0.05 the observed value of χ^2 is significant at 5% LOS~~

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If calculated value of χ^2 > tabulated value of χ^2 at α % level of significance then diff is significant betn observed & theoretical values.

If cal value of χ^2 < tab value of χ^2, the diff is not significant & it is good fit

① Suppose that a die is rolled 150 times & the no of times each face comes up is recorded & results are obtained as

Face	1	2	3	4	5	6
observed freq^n	29	19	19	27	26	30

Are these results consistent with the hypothesis that the die is fair at

1.1 LOS

→ let us assume that die is fair.

Theoretical $freq^n = NP_i$

E_i

$= 150 \frac{1}{6} = 25$

$i = 1, 2 \cdots 6$

Now,

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

Face	O_i	E_i	$O_i - E_i$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
1	29	25	4	16	0.64
2	19	25	-6	36	1.44
3	19	25	-6	36	1.44
4	27	25	2	4	0.16
5	26	25	1	1	0.04
6	30	25	5	25	1
	150	150			4.72

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ie cal. value of $\chi^2 = 4.72$

Here $\nu = n-1 = 6-1 = 5$

Tab. value of $\chi^2 = 15.09$.

cal value of $\chi^2 <$ tab. value of χ^2

ie our assumption true. we accept hypothesis.

ie die is fair.

(2) A sample analysis of exam results of 500 students was made. It was found that 220 had failed, 170 had secured third class, 90 were placed in 2nd class & 20 got first class. Are these fig. commensurate with general exam result which in the

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ratio of 4:3:2:1 for various categories resp.

→ Let us assume that given fig. are in the ratio 4:3:2:1

Theoretical freq^n are

$E_1 = \frac{4}{10} \times 500 = 200$

$E_2 : \frac{3}{10} \times 500 = 150$

$E_3 : \frac{2}{10} \times 500 = 100$

$E_4 : \frac{1}{10} \times 500 = 50$.

Now,

category	O_i	E_i	$O_i - E_i$	$\frac{(O_i - E_i)^2}{E_i}$
Failed	220	200	20	2
third class	170	150	20	2.66
2nd class	90	100	-10	1
1st class	20	50	-30	18
				23.66

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$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

ie cal. $\chi^2 = 23.66$

Now, $\nu = n-1 = 4-1 = 3$.

Take $\alpha = 5\%$ LOS.

From table

Tab value of $\chi^2 = 7.82$

$\therefore$ cal. $\chi^2 >$ Tab. χ^2

we reject hypothesis.

$\therefore$ Given fig. are not in ratio 4:3:2:1

(3) The table below gives the no. of customers visiting a certain post office on various days of week.

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Days	No. of customers.
Monday	130
Tue	120
wed	110
Thur	115
Fri	110
sat	135

Test whether the customers visiting the post office are uniformly distributed use 5% LOS.

→ Let us assume that customers visiting post ofc are uniformly distributed

Here

$$E_i = \frac{720}{6} = 120 \qquad i = 1, 2 \ldots 6$$

Days	O_i	E_i	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
mon	130	120		
Tue	120	120		
wed	110	120		
Thur	115	120		
fri	110	120		
sat	135	120		
		120		

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A. BSR 5

$\text{cal. } \chi^2 = \cancel{4.1275}$ 4.58

Here $\nu = n-1 = 6-1 = 5$ & $\alpha = 5\%$ LOS.

& $\text{Tab. } \chi^2 = \cancel{\text{11.070}}$ 11.07

$\text{cal. } \chi^2 < \text{Tab. } \chi^2$

we accept hypothesis.

ie customers visiting the post ofc are uniformly distributed.

Note :-

The chisquare distribution is a continuous prob. distribution with the values ranging from 0 to ∞ in +ve dir^n. The χ^2 never assume -ve values.

(ii) The shape of chisquare distribution depends on no. of degrees of freedom when ν is small, the shape of curve tends to be skewed to right & as ν gets larger, the shape becomes more symmetrical & can be approximated by normal distribution

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Chi square test for independence of Attributes :

Let the whole group of N individuals be divided into m classes $A_1, A_2 \dots A_m$ according to one character A & into n classes $B_1, B_2 \dots B_n$ according to another characteristic B.

B \ A	B_1	B_2	---	B_n	Total
A_1	O_{11}	O_{12}	--	O_{1n}	(A_1)
A_2	O_{21}	O_{22}	- ~	O_{2n}	(A_2)
⋮					
A_m	O_{m1}	O_{m2}	---	O_{mn}	(A_m)
Total	(B_1)	(B_2)	--	(B_n)	1

Here

$$(A_i) = \sum_{j=1}^{n} O_{ij} = \text{Total freq of } i^{th} \text{ row}$$

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$$(B_j) = \sum_{i=1}^{m} O_{ij} = \text{Total freq}^n \text{ of } j\text{th column.}$$

$$\sum A_i = \sum B_j = N$$

Here we want to test the statistic that attribute A & B are independent

H_0: Attribute A & B are independent

Expected freq^n

$$E_{ij} = \frac{(A_i)(B_j)}{N} \quad i = 1 \ldots m, \; j = 1 \ldots n$$

$$\& \; \chi^2 = \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

with df = $(m-1) \cdot (n-1)$ when m rows & n columns

obtain cal. value of χ^2.
Find Tab. value of χ^2 at α% LOS.
If cal. value of χ^2 > Tab. χ^2 we reject H_0 otherwise accept it

Ex:-

To determine ~~attribull~~ attitudes about prayers in public schools, a survey was conduded in 4 countries.

Attitude	Country A	B	C	D
Favour	65	66	40	34
Oppose	42	30	33	42
No opinion	93	54	27	24

Test for homogeneity of attitudes amoung 4 countries concerning prayers

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in public school.

→ Let us assume that There is homogeneity of attitudes among countries ie attitudes ~~of~~ are independent of countries

	B_1	B_2	B_3	B_4	Total
A_1	65	66	40	34	205
A_2	42	30	33	42	147
A_3	93	54	27	24	198
Total	200	150	100	100	550

$$E_{11} = \frac{(A_1)\cdot(B_1)}{N} = \frac{205 \times 200}{550} = 74.55$$

$$E_{12} = \frac{(A_1)\cdot(B_2)}{N} = \frac{205 \times 150}{550} = 55.91$$

and so on

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O_{ij}	E_{ij}	$O_{ij}-E_{ij}$	$\frac{(O_{ij}-E_{ij})^2}{E_{ij}}$
65	74.55	-9.55	
66	55.91	10.09	
40	37.27	2.73	
3~~0~~4	37.27	-3.27	
~~3~~42	53.45	-11.45	
30	40.09	-10.09	
33	26.73	6.27	
42	26.75	15.27	
93	72	21	
54	54	0	
27	36	-9	
24	36	-12	
550	550		31.0881

from χ^2 table for $(m-1)(n-1) = (3-1)(4-1) = 6$ d.f.

Tab. value of $\chi^2 =$ 12.592

cal. value of χ^2 > Tab. value of χ^2

We reject hypothesis.

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(2) A random sample of 90 adults is classified according to gender & no. of hrs they watch TV during week

Time spent watching TV during week	Gender M	F
Over 25 hrs	15	29
Under 25 hrs	27	19

Test whether Time spent in watching TV is independent of Gender at 5% LoS.

→ Let us assume that time spend in watching TV is independent of Gender

	B_1	B_2	T
A_1	15	29	44
A_2	27	19	48
T	42	48	90

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$$E_{11} = \frac{(A_1)(B_1)}{N}$$

$$E_{12} = \frac{(A_1)(B_2)}{N}$$

$$E_{21} = \frac{(A_2)(B_1)}{N}$$

$$E_{22} = \frac{(A_2)(B_2)}{N}$$

O_{ij}	E_{ij}	$\frac{(O_{ij} - E_{ij})^2}{E_{ij}}$

Σ = 5.4701

t from table

$df = (m-1)(n-1) = 1$

Tab. $\chi^2 = 3.841$

Here cal. value χ^2 > Tab value χ^2

we reject hypothesis

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ie watching TV depends on Gender.