Random Experiment :

occurrences which can be repeated a number of times essentialy under the same conditions and whose result cannot be predicted before hand are known as random experiments.

e.g. rolling of a die, tossing a coin

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sample space :

The set of all possible outcomes of an experiment is called sample space

e.g. For example, if a coin is tossed, the possible outcomes are H(Head) and T(Tail)

$S = \{H, T\}$

Axioms

(i) With each event E is associated a real number bet^n 0 & 1 called the probability of that event and is denoted by P(E)

Thus $0 \le P(E) \le 1$

(ii) $P(S) = 1$ (The sum of the prob. of all events)

(iii) The probability of compound event is the sum of the probabilities of the simple events comprising the compound event

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$$P(E) = \frac{m}{n}$$

if event E consists of m sample points

$$= \frac{\text{Number of sample points in } E}{\text{Number of sample points in } S}$$

Ex. ① In a given race, the odds in favour of four horses A, B, C, D are 1:3, 1:4, 1:5, 1:6 respectively. Assuming that a dead heat is impossible; find the chance that one of them wins the race

Sol^n: Let P_1, P_2, P_3, P_4 be the probabilities of winning of the horses A, B, C, D respectively. Since a dead heat (in which all the four horses cover same distance in same time) is not possible, the events are mutually exclusive

odds in favour of A are 1:3

$$\therefore P_1 = \frac{1}{1+3} = \frac{1}{4}$$

$$P_2 = \frac{1}{5}, \quad P_3 = \frac{1}{6}, \quad P_4 = \frac{1}{7}$$

If P is the chance that one of them wins then

$$P = P_1 + P_2 + P_3 + P_4$$

$$= \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} = \frac{319}{420}$$

Q) A card is drawn from a well-shuffled pack of playing cards. What is the probability that it is either a spade or an ace?

Sol: Let

A = the event of drawing a spade

B = the event of drawing an ace

A and B are not mutually exclusive

AB = the event of drawing the ace of spades

$P(A) = \frac{13}{52}$, $P(B) = \frac{4}{52}$, $P(AB) = \frac{1}{52}$

$$P(A+B) = P(A) + P(B) - P(AB)$$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

Conditional Probability :-

The probability of the happening of an event E_1 when another event E_2 is known to have already happened is called conditional probability and is denoted by $P(E_1/E_2)$

Mutually Independent Events: An event E_1 is said to be independent of an event E_2 if

$$P(E_1/E_2) = P(E_1)$$

if the prob. of happening of E_1 is independent of E_2

* Multiplicative Law of probability
The probability of simeltaneous occurrence of two events is equal to the probability of one of the events multiplied by the conditional probability of the other i.e. for two events A & B

$$P(A \cap B) = P(A) + P(B/A)$$

where $P(B/A)$ represents the conditional probability of occurrence of B when the event A has already happened

Ex. ① A problem in mechanics is given to three students A, B, C whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively

what is the probability that the problem will be solved ?

Soln:- The probabilities of A, B, C solving the problem are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$

The probabilities of A, B, C not solving the problem are $1-\frac{1}{2}, 1-\frac{1}{3}, 1-\frac{1}{4}$ i.e.

$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$

∴ The probability that is not solved by any of them $= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$

Hence the probability that the problem is solved by at least one of them

$$= 1 - \frac{1}{4} = \frac{3}{4}$$

Ex: (2) The odds that a book will be favourably reviewed by three independent critics are 5 to 2, 4 to 3 and 3 to 4 respectively. What is the probability that, of the three reviews, a majority will be favourable?

Sol^n: Let the three critics be A, B, C. The prob. P_1, P_2, P_3 of the book being favourably reviewed by A, B, C are $\frac{5}{7}, \frac{4}{7}, \frac{3}{7}$ respectively

∴ The probabilities that the book is unfavourably reviewed by A, B, C are

$$1-\frac{5}{7} = \frac{2}{7}, \quad 1-\frac{4}{7} = \frac{3}{7}, \quad 1-\frac{3}{7} = \frac{4}{7}$$

A majority will be favourable if the reviews of at least two are favourable

i) If A, B, C all review favourably, the probability is

$$\frac{5}{7} \times \frac{4}{7} \times \frac{3}{7} = \frac{60}{343}$$

ii) If A, B review favourably and C review unfavourably, the probability is

$$\frac{5}{7} \times \frac{4}{7} \times \frac{4}{7} = \frac{80}{343} \quad (P_1 P_2 (1-P_3))$$

iii) If A, C review favourably & B reviews unfavourably, prob. is

$$\frac{5}{7} \times \frac{3}{7} \times \frac{3}{7} = \frac{45}{343} \quad [P_1 (1-P_2) P_3]$$

iv) If B, C review favourably and A reviews unfavourably the prob. is

$\frac{2}{7} \times \frac{4}{7} \times \frac{3}{7} = \frac{24}{343}$

Hence the probability that majority will be favourable is

$\frac{60}{343} + \frac{80}{343} + \frac{45}{343} + \frac{24}{343} = \frac{209}{343}$

Ex. 3 A can hit a target 4 times in 5 shots; B 3 times in 4 shots; C twice in 3 shots. They fire a volley. What is the probability that at least two shots hit?

Sol^n: Probability of A's hitting the target $= \frac{4}{5}$

prob. of B's hitting the target $= \frac{3}{4}$

prob. of C's hitting the target $= \frac{2}{3}$

For at least two hits we may have

i) A, B, C all hit the target.

prob. $= \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{24}{60}$

ii) A, B hit the target & C misses it,

prob. $= \frac{4}{5} \times \frac{3}{4} \times \left(1 - \frac{2}{3}\right) = \frac{4}{5} \times \frac{3}{4} \times \frac{1}{3} = \frac{12}{60}$

iii) A, C hit the target & B misses it

$$prob = \frac{4}{5}\times\left(1-\frac{3}{4}\right)\times\frac{2}{3} = \frac{4}{5}\times\frac{1}{4}\times\frac{2}{3} = \frac{8}{60}$$

(iv) B, C hit the target & A misses it, the probability for which is

$$\left(1-\frac{4}{5}\right)\times\frac{3}{4}\times\frac{2}{3} = \frac{1}{5}\times\frac{3}{4}\times\frac{2}{3} = \frac{6}{60}$$

since these are mutually exclusive events

$$req.\ Prob. = \frac{24}{60}+\frac{12}{60}+\frac{8}{60}+\frac{6}{60} = \frac{50}{60} = \frac{5}{6}$$

Ex(1) An urn contains 10 white and 3 black balls, while another urn contains 3 white and 5 black balls. Two balls are drawn from the first urn and put into the second urn and then a ball is drawn from the latter. What is the prob. that it is a white ball?

Soln. The two balls drawn from the first urn may be

i) both white ii) both black

iii) one white & one black

let these events be denoted by A, B, C respectively

$$P(A) = \frac{{}^{10}C_2}{{}^{13}C_2} = \frac{10\times 9}{13\times 12} = \frac{15}{26}$$

$$P(B) = \frac{{}^{3}C_2}{{}^{13}C_2} = \frac{3\times 2}{13\times 12} = \frac{1}{26}$$

$$P(C) = \frac{{}^{10}C_1 \times {}^{3}C_1}{{}^{13}C_2} = \frac{10}{26}$$

when two balls are transferred from first urn to second urn, the second urn will contain

i) 5 white & 5 black balls

ii) 3 white and 7 black balls

iii) 4 white and 6 black balls

Let W denote the event of drawing a white ball from the second urn in the three cases (i) (ii) (iii)

Now $P(W/A) = \frac{5}{10}$ $P(W/B) = \frac{3}{10}$

$P(W/C) = \frac{4}{10}$

Req. Prob. $= P(A)\cdot P(W/A) + P(B)\cdot P(W/B) + P(C)\cdot P(W/C)$

$$= \frac{15}{26}\cdot\frac{5}{10} + \frac{1}{26}\cdot\frac{3}{10} + \frac{10}{26}\cdot\frac{4}{10}$$

$$= \frac{75+3+40}{260} = \frac{118}{260} = \frac{59}{130}$$

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