MODULE – 2

2.1- Introduction:

- AC stands for alternating current, means voltage or current that changes polarity or direction over time.
- An alternating quantity changes continuously in magnitude and alternates in direction at regular intervals of time.
- AC voltage and current can be defined in two types namely single phase and three phase.
- The variation in the AC quantity plotted against time is known as a wave. And the pattern of the wave is known as the wave shape or wave form.
- The wave form is said to be sinusoidal if the variations in the magnitude and direction can be represented by the sine function.
- Generally the AC voltages generated at the large scale are sinusoidal voltages & corresponding currents are sinusoidal currents.
- Any alternating quantity can be represented by a simple sine wave as shown in fig below

2.2 – Advantages of Sine wave or AC

- Sine wave is one of the most natural wave form
- Sinusoidal voltage can be conveniently generated in the large AC generators
- Sinusoidal quantities can be mathematically expressed & analyzed in a simple manner
- The vector representation of a sine wave is useful in the solution of the complicated AC circuits.
- Sinusoidal voltages produce uniform torque, minimum vibration & least noise in the electric motors.

2.3 – Faraday's law of electromagnetic induction

This law can be stated as

- *"Whenever the magnetic flux linked with a circuit changes, an EMF is always induced in it*." OR it is also defined as "*whenever a conductor cuts magnetic flux, an EMF is induced in that conductor."*
- The magnitude of the induced EMF is equal to the rate of change of flux linkage and the direction of the induced EMF is such that the induced EMF opposes the cause of it production.

- Consider a coil with '*n*' turns & flux through it changes from initial value φ_1 to φ_2 in time t, thus the flux linked with coil of 'n' turns are $n\,\varphi_1$ & $n\,\varphi_2$ respectively.
  Initial flux linkage = $n\,\varphi_1$
  Final flux linkage = $n\,\varphi_2$
- The induced EMF is given by
- E = $-[\frac{n\varphi1-n\varphi2}{t}]$
- E = $-$ N $\frac{d\varphi}{dt}$
- The negative sign in the above equation indicates that the induced EMF sets up the current in such a direction that magnetic effect produced by it opposes the very cause producing it.
- The direction of induced EMF is given by Lenz's law.
- Lenz's law - The negative sign in the above equation indicates that the induced EMF sets up the current in such a direction that magnetic effect produced by it opposes the very cause producing it.

- Hence, the induced EMF always opposes the cause producing it.
- There are two possible methods by which EMF can be induced in the circuit.
- First one is moving a permanent magnet having magnetic field around near the coil.
- When magnet is moved relative to coil such a way that, the number of lines of force passing through the coil changes, hence magnetic flux changes with respect to time and EMF is induced.
- Second method is keeping magnet fix and rotating a coil or conductor in magnetic field.
- Here also, there is change in magnetic flux with respect to time and EMF is induced in the coil or conductor.

2.4 – Generation of Sinusoidal voltage

- Consider a single turn rectangular coil is kept in the magnetic field as shown in fig. The coil is so placed that it can be rotated in clockwise or anti-clockwise direction.
- The coil is made up of conducting material like copper or aluminum and two conductors a-b and c-d.
- Consider the coil is rotated in anticlockwise direction and while rotating, the conductors a-b and c-d cuts the line of flux in magnetic field.
- According to Faraday's law of electromagnetic induction, EMF is induced in the conductors and this EMF depends upon the position of the coil in the magnetic field which can be explained as follow.
- ө = 0°: The initial position of the coil as shown in fig.3.3 (a). The plane of the coil is perpendicular to the direction of magnetic field. The conductors move parallel to the magnetic field or magnetic lines of force. Hence the flux cutting is almost zero, so EMF induced is zero.

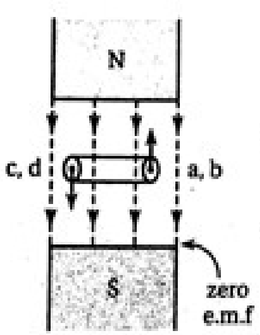


Fig 2.1 (a) - ө = 0°

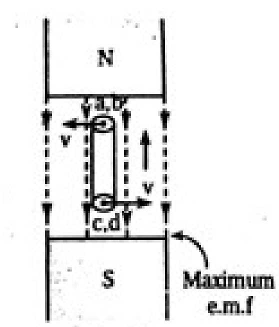


fig 2.2 (b) - ө = 90°

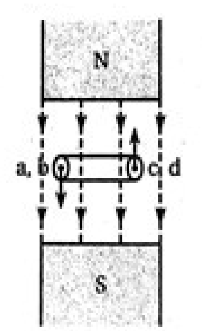


fig 2.3 (c) - ө = 180°

- ө = 90°: Now, the conductor is rotated by angle 90° in anticlockwise direction as shown in fig 2.3 (b).While rotating from ө = 0° to ө = 90° it will cut maximum flux. At ө = 90° maximum lines of flux are cut hence maximum EMF is induced.
- ө = 180°: When again coil rotates from ө = 90° to ө = 180° the flux cutting of the coil reduces. At ө = 180°as shown in fig 2.3(c), the conductors plane becomes perpendicular to magnetic lines of force. In this position conductors move parallel to magnetic lines of force hence no flux is cut and EMF induced is zero.
- The conductors move in same direction from ө = 0° to ө = 180°, so the induced EMF is having same polarity during this period called positive or first half cycle.

- As shown in fig. 2.3 (d), from ɵ = 180° to ɵ = 270° the direction of force on conductor a, b and c, d is changed. Conductor a, b is now going in downward direction and conductor c, d is going in upward direction. Due to this, the polarity of induced EMF changes.
- At ɵ = 270°,the conductors moves at right angle to magnetic lines of force hence maximum lines of force are cut by conductor and maximum EMF is induced in opposite direction.

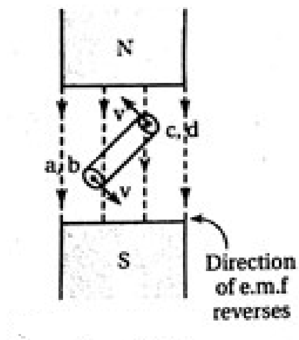


Fig. 2.3 (d) -180°< ɵ < 270°

- This set of variation repeats for every revolution as conductors rotate in a circular motion with certain speed.
- The instantaneous values of induced EMF in any conductor as it rotates from ɵ = 0° to ɵ = 360° is shown in fig. 2.4 and from figure it is clear that the waveform generated is pure sine wave.

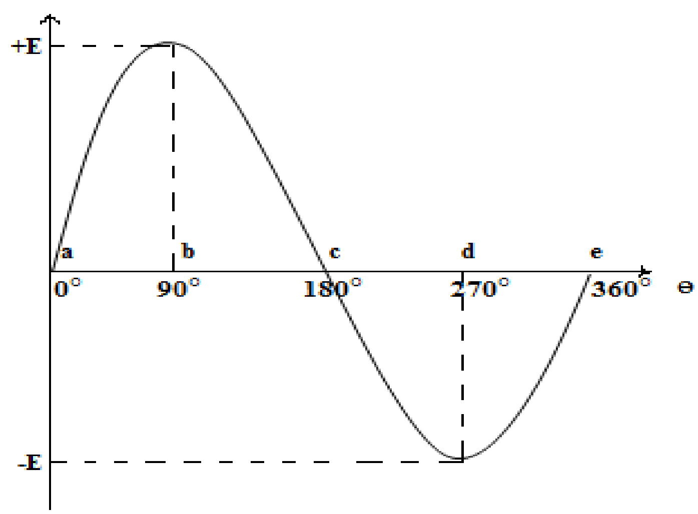


Fig. 2.4 – Graphical representation of induced EMF

2.5 - Some important terms related to AC circuit:

- **Instantaneous value –** The value of alternating quantity at a particular instant is known as instantaneous value.
- **Waveform –** The graph plotted by taking instantaneous values of alternating quantity against time is called as waveform. The graph of alternating quantity is sinusoidal.

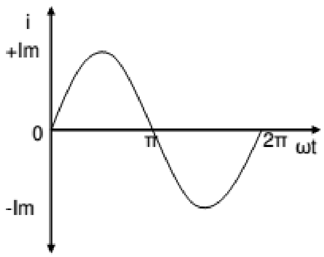
- **Cycle –** Each repetition of a complete set of changes undergone by the alternating quantity is called as cycle. Or complete set of positive and negative values of an alternating quantity is known as cycle. One cycle of alternating EMF is shown in fig. 2.4.
- **Time period (T):** The time taken by an alternating quantity to complete one cycle is called time period.
- **Frequency (f):** The number of cycles per second completed by an alternating quantity is called as frequency.

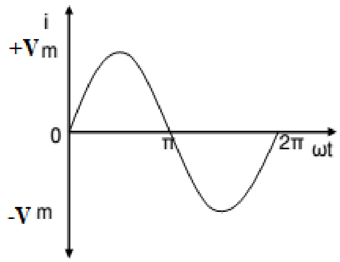
$$f = \frac{1}{T}$$

- **Amplitude:** The maximum value obtained by an alternating quantity during its positive or negative half cycle is called as amplitude or peak value.
- **Angular frequency:** It is defined as frequency expressed in electrical radians per second and it is denoted as 'ω'.

$$\omega = 2\pi f$$

- **Mathematical equation and waveform of alternating current and voltage:**





$i = I_m \sin \omega t$

$\mathbf{v} = \mathbf{V}_m \sin \omega t$

Fig. 2.5(a) – Alternating current fig. 2.5(b) – Alternating voltage

- **Phase difference**: When two alternating quantities of the same frequency have different zero points, they are said to have a phase difference. The angle between the zero points is the angle of phase difference. As shown in below fig, phase difference between two waveforms is given by angle 'φ'.

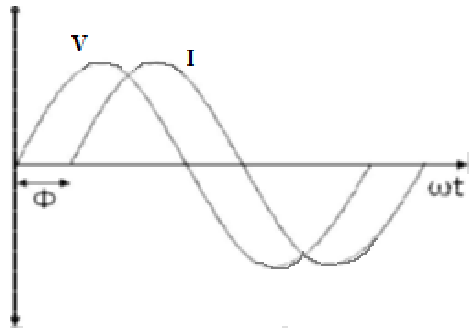


Fig. 2.6 – phase difference

- **In phase:** Two waveforms are said to be in phase, when the phase difference between them is zero. That is the zero points of both the waveforms are same. The waveform, phasor and equation representation of two sinusoidal quantities which are in phase is as shown. The figure shows that the voltage and current are in phase.

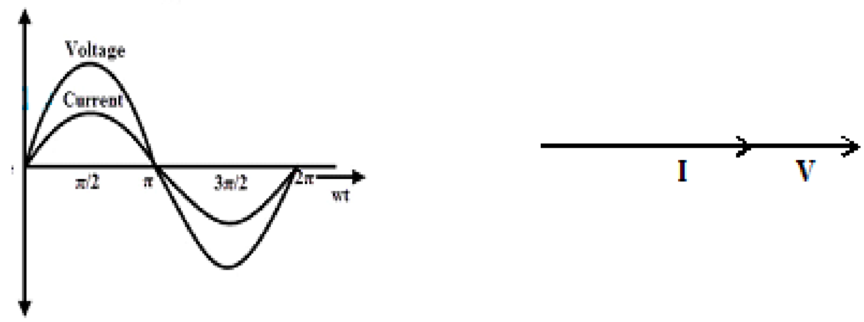


Fig. 2.7 – Voltage and current in phase

Mathematical expression when voltage and current are in phase can be written as,
$v = V_m \text{Sin}(\omega t)$
$i = I_m \sin(\omega t)$
In resistive circuits or load, current is in phase with voltage.

- **Lagging:** In the figure shown, the zero point of the current waveform is after the zero point of the voltage waveform. Hence the current is lagging behind the voltage. The waveform, phasor diagram is shown in below fig.

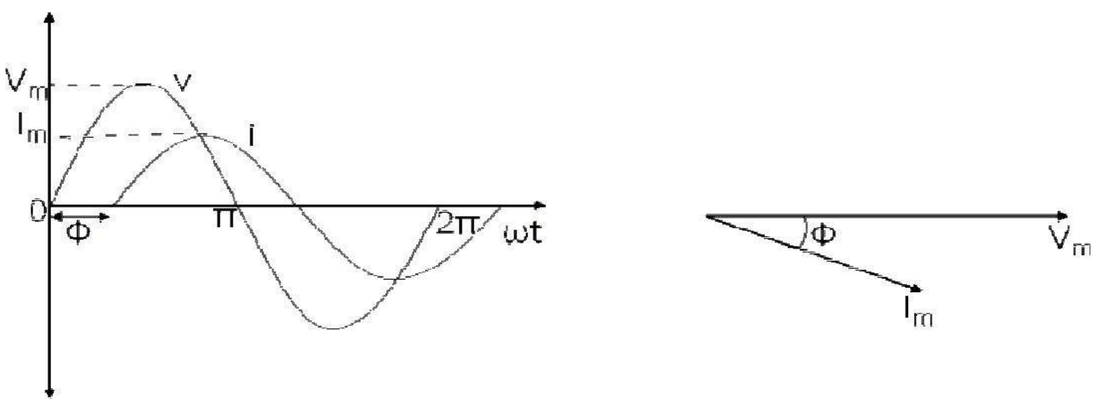


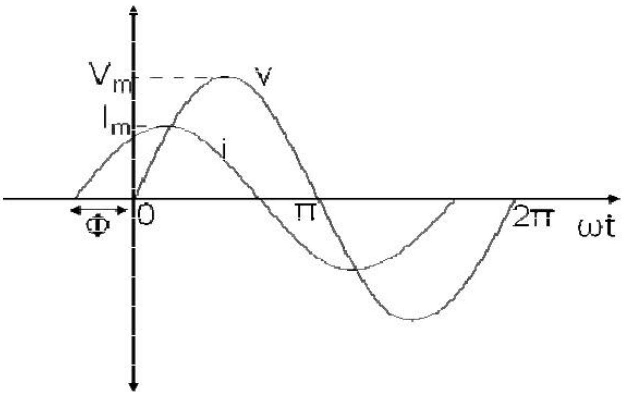
Fig. 2.8 – Current lagging supply voltage

Mathematical representation is as shown.
$V = V_m \text{Sin}(\omega t)$; $i = I_m \sin(\omega t - \varphi)$

From above equation we can say that current is lagging voltage by angle φ and hence there is phase difference of angle φ.

Generally, in inductive circuits or loads current is lagging the supply voltage.

- **Leading:** In the figure shown, the zero point of the current waveform is before the zero point of the voltage waveform. Hence the current is leading the voltage. The waveform, phasor diagram is as shown.



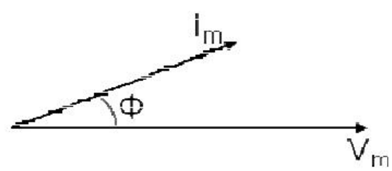


Fig. 2.9 – Current leading supply voltage

Mathematical representation is as shown.

$v = V_m \,\mathrm{Sin}(\omega t)$; $i = I_m \sin(\omega t + \varphi)$

From above equation we can say that current is leading voltage by angle φ and hence there is phase difference of angle φ.

In capacitive circuits or loads current is leading the supply voltage.

2.6 – Concept of Average and RMS value:

2.6.1 – Average Value of current or voltage: The arithmetic average of all the values of an alternating quantity over one cycle is called its average value of that quantity.

Consider, an alternating current waveform having maximum value (amplitude) Im as shown in fig

For Symmetrical waveforms, the average value calculated over one cycle becomes equal to zero because the positive area cancels the negative area. Hence for symmetrical waveforms, the average value is calculated for half cycle.

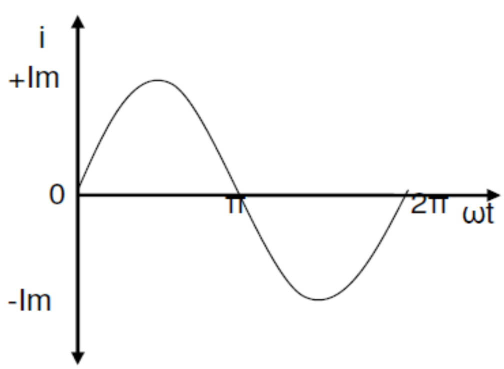


Fig. 2.10 – Alternating current waveform

Average value = $\frac{\text{Area under one half cycle}}{\text{Base}}$

So, from above fig. avg. value of current for half cycle for $\theta = \omega t$ can be written as,

$I_{avg} = \frac{\int_0^\pi i\, d\theta}{\pi}$

Putting, $i = I_m \sin\theta$ in above equation

$I_{avg} = \frac{1}{\pi} \int_0^\pi \text{Im} \sin \theta d\theta$

$= \frac{I_m}{\pi} \int_0^\pi \sin\theta\, d\theta$

$= \frac{I_m}{\pi} [-\cos\theta]_0^\pi$

$= \frac{-\text{Im}}{\pi} [-1 - 1]$

$= 2I_m / \pi$

$\mathbf{I_{avg} = 0.637\ I_m}$

Similarly, average voltage can be written as, $\mathbf{V_{avg} = 0.637\ V_m}$

2.6.2 – RMS Current or Voltage:

RMS value = $\frac{\sqrt{(\text{Area under squared wave one half cycle})}}{\text{Base}}$

Consider an alternating current waveform as shown in fig.3.10 above.

For this waveform, $i = I_m \sin(\omega t)$ RMS value of current can be find as,

Area under the squared wave for half cycle = $\int_0^\pi i^2\, d\theta$, and base = π

Mean value of current for a half cycle is,

$I = \frac{1}{\pi} \int_0^\pi i^2\, d\theta$

Put $i = I_m \sin\theta$

$I = \frac{1}{\pi} \int_0^\pi i_m^2 \sin^2 \theta d\theta$

$= \frac{I_m^2}{2\pi} [1 - cos2\theta]_0^\pi$

$= \frac{I_m^2}{2\pi} [\theta - \frac{sin2\theta}{2}]_0^\pi$

$= \frac{I_m^2}{2\pi} [\pi]$

$= \frac{I_m^2}{2}$

To find RMS value, we have to find square root of mean value.

$I_{rms} = \sqrt{\frac{I_m^2}{2}}$

$= \frac{I_m}{\sqrt{2}} = 0.707\ I_m$

Hence, $\mathbf{I_{rms} = 0.707\ I_m}$

Similarly, $\mathbf{V_{rms} = 0.707\ V_m}$

2.6.3 – Form Factor:

It is defined as the ratio of RMS value to the average value of an alternating quantity.

Form factor = $\frac{\text{RMS Value}}{\text{Average Value}}$

From above equations we can write,

RMS value = 0.707 × Maximum value
Average value = 0.637 × Maximum value
Hence, by taking the ratio form factor can be written as,

Form factor = $\frac{0.707 \times \text{Max.Value}}{0.637 \times \text{Max.Value}}$

Form factor = 1.11

2.6.4 – Peak Factor:

It is defined as the ratio of maximum value to the RMS value of an alternating quantity.

Peak factor = $\frac{\text{Max. Value}}{\text{RMS Value}}$

$= \frac{\text{Max.Value}}{0.707 \times \text{Max.Value}}$

$= \frac{1}{0.707}$

Peak factor = 1.414

2.7 – SINGLE PHASE AC CIRCUITS:

The resistor, inductor, capacitor are the basic elements of any electrical circuit. To analyze any electric circuit, it is necessary to understand the three cases,

a. AC through pure resistive circuit
b. AC through pure inductive circuit
c. AC through pure capacitive circuit

In each case, it is assumed that a purely sinusoidal alternating voltage given by equation $v = V_m \sin(\omega t)$ is applied to the circuit. The equation for current, power and phase shift is developed in each case.

2.7.1 – AC Circuit With Pure Resistance:

Consider a simple circuit containing a pure resistance 'R' ohms connected across an alternating voltage source $v = V_m \sin(\omega t)$ as shown in fig

According to Ohm's law, the current (i) in the circuit is given by,

$i = v/R$

$= \frac{V_m \sin\omega t}{R}$

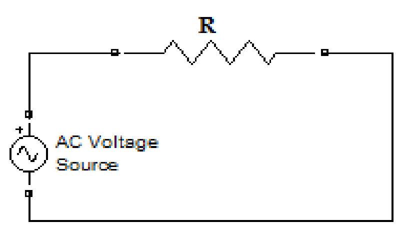


Fig. 2.11 – Purely resistive circuit

Comparing the above equation to $i = I_m \sin(\omega t + \varphi)$, we can write,

$I_m = V_m / R$ and $\varphi = 0$

Hence we can say that there no phase difference between voltage and current which means both voltage and current are in phase with each other.

Maximum value of current can be given by $I_m = V_m / R$

The waveform and phasor diagram is shown in below fig. 2.12

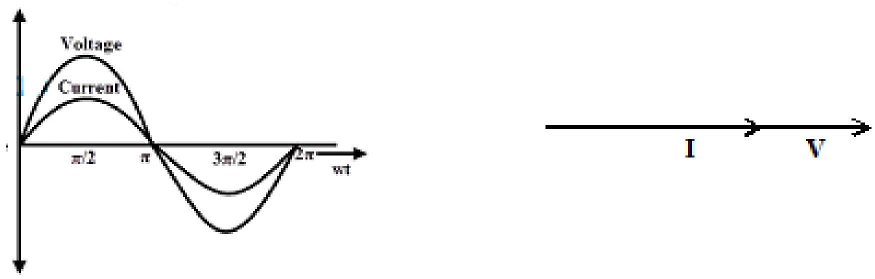


Fig. 2.12 – Phasor and waveform of purely resistive circuit

Power In Purely Resistive Circuit:

Power in electrical circuit is nothing but the multiplication of voltage and current.

Equation of voltage and current is

$v = V_m \sin \omega t$ and $i = I_m \sin \omega t$

Hence, power (P) is, $P = v \times i$

$P = V_m \sin \omega t \times I_m \sin \omega t = V_m I_m \sin^2(\omega t)$

By using trigonometric relation $\sin^2(\omega t)$ can be written as

$\sin^2(\omega t) = \frac{(1-cos2\omega t)}{2}$

Hence,

$P = V_m I_m \frac{(1-cos2\omega t)}{2}$

$P = [\frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t]$

From above equation of power we can say that the power is having two parts

a. $\frac{V_m I_m}{2}$ is s constant part which is the actual power consumed in the circuit.
b. $\frac{V_m I_m}{2}$ cos2ωt is variable part which is having the term 'cos(2ωt)' so, its average value over one cycle is zero.

Hence average power over one cycle is $\frac{V_m I_m}{2}$

Average Power = $\frac{V_m I_m}{2}$ = $V_{rms} \times I_{rms}$

2.7.2 – AC Circuit With Pure Inductance:

Consider a simple circuit containing a coil having inductance 'L' Henry, connected across an alternating voltage source $v = V_m \sin \omega t$ as shown in fig.2.13

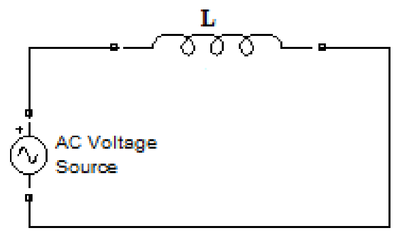


Fig.2.13 – Purely Inductive Circuit

When current is flowing through coil having inductance ‘L’, it sets up alternating magnetic field around the inductance. This alternating flux links with the coil and due to self-inductance, EMF is induced in the coil.

The voltage around the inductor which is same as supply voltage is given as,

$v = L\left(\frac{di}{dt}\right)$ 3.4

But, $v = V_m \sin \omega t$

$V_m \sin \omega t = L\left(\frac{di}{dt}\right)$

$di = \frac{Vm}{L} \sin \omega t \; dt$

Integrating both sides we get,

$i = \frac{-Vm}{L} \cos \omega t \cdot \frac{1}{\omega}$

Here, $-\cos \omega t$ can be written as $\sin(\omega t - \frac{\pi}{2})$

Hence,

$\mathbf{i = \frac{Vm}{\omega L} \sin(\omega t - \frac{\pi}{2})}$

The maximum value of current $\mathbf{I_m = \frac{Vm}{\omega L}}$

Putting in equation (3.9.5) we get,

$i = I_m \sin(\omega t - \frac{\pi}{2})$ and equation of voltage is

$v = V_m \sin \omega t$

Comparing these two equations we can say that, there is phase difference of $(-\frac{\pi}{2})$ rad or 90°

That is, ***current is lagging the supply voltage by*** $\boldsymbol{\frac{\pi}{2}}$ ***rad or 90°***

The phasor diagram and waveforms of this condition is shown below in fig 2.14.

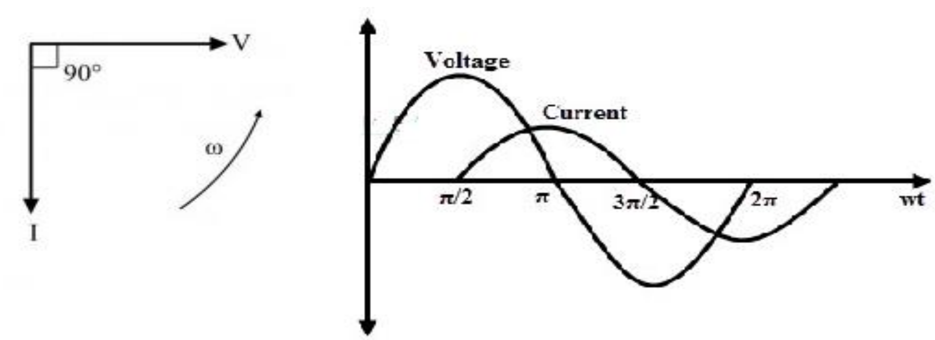


Fig.2.14 – Phasor and waveform of pure inductive circuit

Inductive reactance:

From equation 3.9.5, maximum value of current can be given as $I_m = \frac{Vm}{\omega L}$

Where the term 'ωL' is known as Inductive reactance (X_L) which is nothing but the opposition offered by inductance of a circuit to flow of alternating current.

It is given by.

$\mathbf{X_L = \omega L = 2\pi f L}$

Average Power In Purely inductive Circuit:

Equation of voltage and current in purely inductive circuit is written as

$v = V_m \sin \omega t$ and $i = I_m \sin(\omega t - \frac{\pi}{2})$

Hence, power (P) is, $P = v \times i$

$P = V_m \sin \omega t \times I_m \sin(\omega t - \frac{\pi}{2})$

$P = \frac{-V_m I_m}{2}(2 \sin \omega t . \cos \omega t)$

$P = \frac{-V_m I_m}{2}(\sin 2\omega t)$

Thus average power in inductive circuit is a sine wave with frequency doubles that of voltage and current wave.

Hence the mean value of power taken over one cycle is zero.

Avg power = 0

2.7.3 – AC with Pure Capacitor:

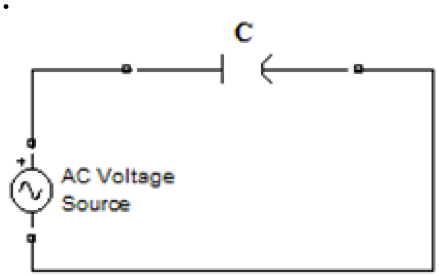


Fig.2.15 - Purely capacitive Circuit

Consider an AC circuit with a pure capacitance C as shown in the figure.

The alternating voltage 'v' is given by

$v = V_m \sin \omega t$

The current flowing in the circuit is 'i'. The voltage across the capacitor is given as 'V_C' which is the same as 'V'.

We can find the current through the capacitor as follows

$q = Cv$

$q = C V_m \sin \omega t$

$i = \frac{dq}{dt}$

Hence, differentiating equation 3.9.7 w.r.t 't'

$i = C V_m \cos \omega t \times \omega$

$= \omega CV_m \sin(\omega t + \frac{\pi}{2})$

$= \frac{V_m}{1/\omega C} \sin(\omega t + \frac{\pi}{2})$

In above equation

$I_m = \frac{V_m}{1/\omega C} = \frac{V_m}{X_C}$, and X_C **is called as capacitive reactance** given as

$\mathbf{X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}}$

Hence, equation 3.9.8 can be written as,

$i = I_m . \sin(\omega t + \frac{\pi}{2})$

and voltage equation is $v = V_m \sin \omega t$,

From these equations we can say that in purely capacitive circuit ***current is leading the supply voltage by*** $\boldsymbol{\frac{\pi}{2}}$ ***or 90°.*** This can be shown by phasor diagram and waveform

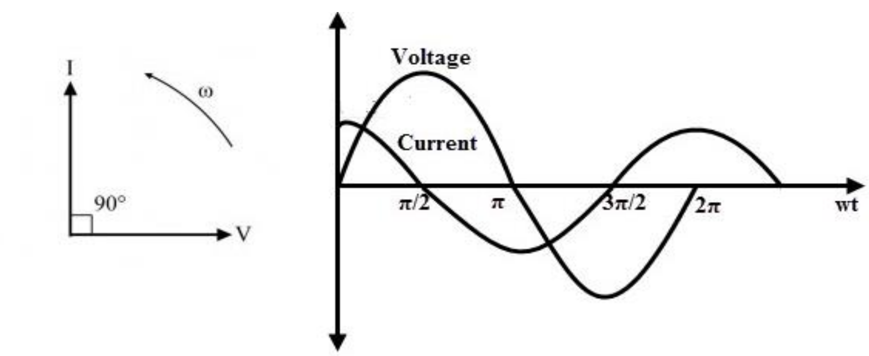


Fig.2.16 – Phasor diagram and waveform of pure capacitive circuit

Average power in pure capacitive circuit:

Equation of voltage and current in purely capacitive circuit is written as

$v = V_m \sin \omega t$ and $i = I_m \sin(\omega t + \frac{\pi}{2})$

Hence, power (P) is, $P = v \times i$

$P = V_m \sin \omega t \times I_m \sin(\omega t + \frac{\pi}{2})$

But, $\sin\left(\omega t + \frac{\pi}{2}\right) = \cos \omega t$ and $\sin \omega t . \cos \omega t = \frac{\sin(2\omega t)}{2}$

$P = \frac{V_m I_m}{2} (\sin 2\omega t)$

Thus average power in capacitive circuit is a sine wave with frequency doubles that of voltage and current wave.

Hence the mean value of power taken over one cycle is zero.

2.8 - Some Important Terms Related To AC Circuits:

2.8.1 – Power Factor:-

The power factor in an AC circuit is defined as the ***cosine of the angle*** between voltage and current. It is denoted as ‘cos(φ)’. The power factor can be lagging or leading depending upon the type of circuit and type of the load connected.

Power factor is also defined as the ratio of resistance to the impedance of the circuit
Or "The ratio of active power to the apparent power "

2.8.2 – Active power:-

It is the actual power or useful power that is consumed by the circuit. This power may be converted into heat or any useful form. It is denoted by ***'P'*** and its unit is 'Watt' (W) or larger unit is Kilo-watt (KW) or Mega Watt (MW)

Active power is given by,

$P = V \times I \times cos(\varphi)$ Or $P = I^2R$

2.8.3 – Reactive Power:-

It is the power developed in the components like inductor or capacitor in the circuit. This power is not useful power and it is not the actual output power but it circulates in the circuit. It is denoted by ***'Q'*** and its unit is Volt-Amperes- Reactive (VAR).

$Q = V \times I \times sin(\varphi)$ Or

$Q = I^2X_L$ for inductive circuit and $Q = I^2X_C$ for capacitive circuit.

2.8.4 – Apparent Power:-

It is defined as product of RMS voltage and RMS current. It is also defined as the total power in the circuit.

It is denoted by ***'S'*** and its unit is Volt- Ampere (VA) or Kilo-volt-ampere (KVA).

S= V×I

Or, $S = \sqrt{P^2 + Q^2}$

2.9 – Series R-L Circuit

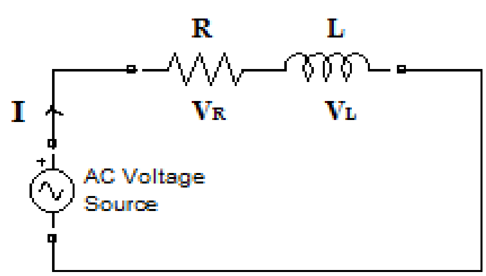


Fig. 2.17 – Series R – L circuit

Consider an AC circuit with a resistance R and an inductance L connected in series as shown in the figure. The alternating voltage v is given by,

$v = V_m \sin \omega t$

The current flowing in the circuit is 'i'.

The voltage across the resistor is V_R and that across the inductor is V_L. In above circuit,

V_R=I.R is in phase with I

V_L=I.X_L leads current by 90 degrees with the above information; the phasor diagram can be drawn as shown.

The current I is taken as the reference phasor. The voltage V_R is in phase with I and the voltage V_L leads the current by 90^0. The resultant voltage V can be drawn as shown in the figure 2.18 (a).

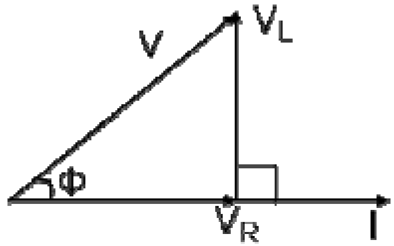


Fig. 2.18 (a) – phasor diagram

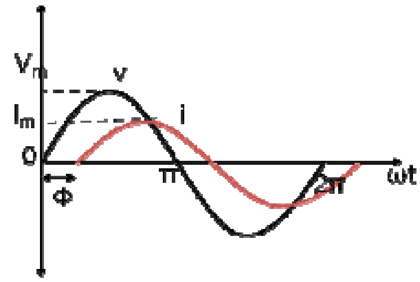


fig. 2.18 (b) – waveform

From the phasor diagram we observe that the voltage leads the current by an angle Φ or in other words the
current lags behind the voltage by an angle Φ. The waveform for an RL series circuit is also shown above.

Due to inductive nature of circuit, the current is lagging supply voltage by some angle 'φ'. The waveform is shown in fig. 2.18 (b)

The equations of voltage and current are given as,

$v = V_m \sin \omega t$

$i = I_m \sin (\omega t - \varphi)$

From the phasor diagram, the expressions for the resultant voltage V and the angle 'φ' can be derived as follows

$V = \sqrt{V_R^2 + V_L^2}$

Where, $V_R = IR$, $V_L = IX_L$

And $\mathbf{X_L = 2\pi f L}$

Also, $V = I.Z$

From phasor diagram,

$\tan(\varphi) = \frac{V_L}{V_R} = \frac{X_L}{R}$

Hence, $\boldsymbol{\varphi = \tan^{-1} \frac{X_L}{R}}$

Voltage triangle and Impedance triangle for R-L circuit:

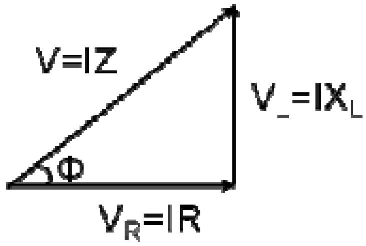


Fig. 2.19 (a) - Voltage triangle

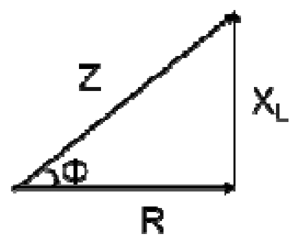


fig 2.19 (b) - Impedance triangle

From impedance triangle, power factor can be written as

$\boldsymbol{\cos(\varphi) = \frac{R}{Z}}$

and Impedance, $\mathbf{Z} = \sqrt{\boldsymbol{R^2 + X_L^2}}$ 3.17

Average power **$P = V \times I \times \cos(\varphi)$** Or $P = I^2R$
Hence the power in an RC series circuit is consumed only in the resistance. The capacitance does not consume any power.

Solved Example

Ex.3.1) A 200 V, 50 Hz, inductive circuit takes a current of 10A, lagging 30 degree. Find (i) Resistance (ii) Reactance (iii) Inductance of the coil.

Solution: Given data
V= 200 V
F = 50 Hz
I = 10 A
$\Phi = 30°$
By using equation 3.14 from above,
V = I.Z we can write,
$Z = \frac{V}{I}$; = $\frac{200}{10}$ = 20
$Z = 20\ \Omega$
By using equation 3.16,
$\cos(\varphi) = \frac{R}{Z}$
$\cos(30°) = \frac{R}{20}$
$R = 20 \times 0.866$
$R = 17.32\ \Omega$
From equation 3.17 we can write as,
$X_L = \sqrt{Z^2 - R^2} = \sqrt{20^2 - (17.32)^2}$
$X_L = 10\ \Omega$
$X_L = 2\pi f.L$
$L = X_L / 2\pi f$
$L = 0.031$ H
Ans – i) $R = 17.32\ \Omega$, ii) Reactance = $X_L = 10\ \Omega$, iii) Inductance $L = 0.031$ H

Ex.3.2) A circuit has resistance of 10 Ω and inductance of 50 mH. It is connected to 50 Hz , 230 V AC supply. Calculate: a) Inductive reactance b) Impedance c) Current in the circuit d) phase difference between voltage and current

Solution: Given data
$R = 10\ \Omega$
L = 50 mH = 50×10^{-3} H= 0.05 H (Converted mH into H)
V = 230 V
F= 50 Hz
$X_L = 2\pi f.L$
Putting the required values from given data

X_L = 2×3.142×50×0.005

X_L = 15.76 Ω

Z = $\sqrt{R^2 + X_L^2} = \sqrt{10^2 + (15.76)^2}$ = **18.66 Ω**

V = I.Z; I = $\frac{V}{Z} = \frac{230}{18.66}$ = **12.32 A**

Now, cos (φ) = $\frac{R}{Z}$; $\varphi = \cos^{-1}\frac{R}{Z}$

$\varphi = \cos^{-1}\frac{(10)}{(18.66)}$ = **57.76°**

Ans –a) Inductive reactance X_L = 15.76 Ω b) Impedance Z = 17.66 Ω c) Current I = 12.32 A d) Phase difference between voltage and current φ = 57.76°

2.10 – Series R-C Circuit

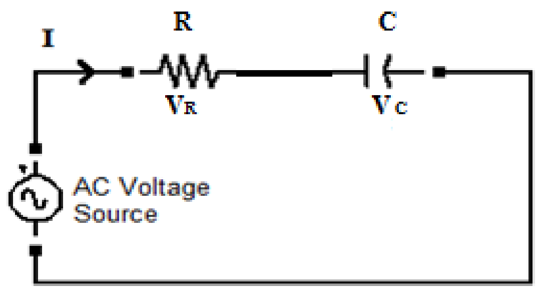


Fig. 2.20 – Series R-C circuit

Consider an AC circuit with a resistance R and an inductance L connected in series as shown in the figure. The alternating voltage v is given by,

v = $V_m \sin \omega t$

The current flowing in the circuit is 'i'.

The voltage across the resistor is V_R and that across the capacitor is V_C. In above circuit,

V_R=I.R is in phase with I

V_C=I.X_C lags current by 90 degrees as per the above information; the phasor diagram can be drawn as shown.

The current I is taken as the reference phasor. The voltage V_R is in phase with I and the voltage V_C lags the current by 90°. The resultant voltage V can be drawn as shown in the figure 2.21

Fig. 2.21

From the phasor diagram we observe that the voltage lags behind the current by an angle Φ or in other words the current leads the voltage by an angle Φ. The waveform for an RC series circuit is also shown above.

Due to capacitive nature of circuit, the current is leading supply voltage by some angle 'φ'.

The equations of voltage and current are given as,

$v = V_m \sin \omega t$

$i = I_m \sin(\omega t + \varphi)$

From the phasor diagram, the expressions for the resultant voltage V and the angle 'φ' can be derived as follows

$V = \sqrt{V_R^2 + V_C^2}$

Where, $V_R = IR$, $V_C = IX_C$, and $V = I.Z$

$X_C = \frac{1}{2\pi f c}$

$\tan(\varphi) = \frac{V_C}{V_R} = \frac{X_C}{R}$

Hence, $\boldsymbol{\varphi = \tan^{-1}\frac{X_C}{R}}$

Voltage triangle and Impedance triangle for R-C circuit:

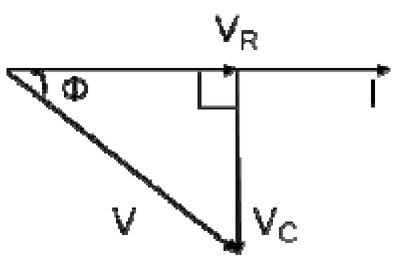


Fig. 2.22 (a)

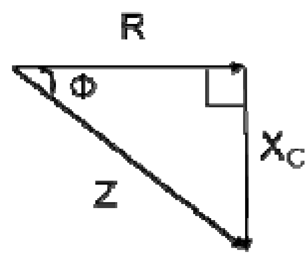


fig. 2.22 (b)

From impedance triangle fig.3.22 (a), power factor can be written as

$\mathbf{\cos(\varphi) = \frac{R}{Z}}$

and from fig 3.22 (b), Impedance, $\mathbf{Z} = \sqrt{\boldsymbol{R^2 + X_C^2}}$

Average power **P = V ×I× cos(φ)** Or **P = I^2R**

Hence the power in an RC series circuit is consumed only in the resistance. The capacitance does not consume any power.

<u>Solved Example</u>

Ex 3.3) A Capacitor of capacitance 79.5µF is connected in series with a non-inductive resistance of 30 Ω across a 100V, 50Hz supply. Find (i) Capacitive Reactance (ii) impedance (iii) current (iv) phase angle

Solution : Given data –

V = 100 V

R = 30 Ω

F= 50 Hz

C = 79.5 μF = 79.5×10^{-6} F (micro-farad converted into Farad)

Now, By using the formula for Capacitive Reactance (X_C)

$X_C = \frac{1}{2\pi fc} = \frac{1}{2\times3.142\times50\times79.5\times10^{-6}}$ = **40 Ω**

$Z = \sqrt{R^2 + X_C^2} = \sqrt{30^2 + 40^2}$ = **50 Ω**

Current is calculated by using the formula,

$I = \frac{V}{Z} = \frac{100}{50}$ = **2 A**

Now using equation 3.19 to calculate the phase angle,

$\varphi = \tan^{-1}\frac{X_C}{R} = \tan^{-1}\frac{40}{30}$

$\varphi = \tan^{-1}(1.33)$

φ = 53.3°

Ans: (i) Capacitive Reactance = X_C = **40 Ω (ii) impedance Z = 50 Ω (iii) current I = 2A**
(iv) phase angle φ = 53.3°

2.11 - R – L – C Series Circuit

Consider an AC circuit with a resistance R, an inductance L and a capacitance C connected in series as shown in the figure. The alternating voltage v is given by

$v = V_m \sin \omega t$

The current flowing in the circuit is 'i'. The voltage across the resistor is V_R, the voltage across the inductor is V_L and that across the capacitor is V_C

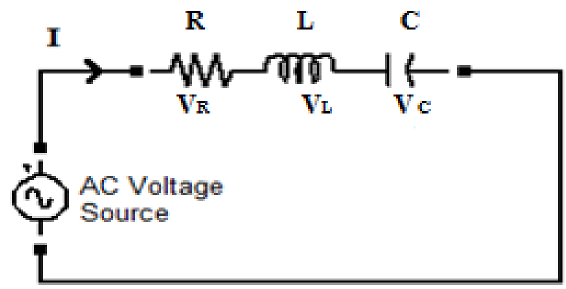


Fig. 2.23 – Series R-L-C circuit

As these components are in series, the current flowing through them is same but voltage across each component is different. According to the nature of resistance, inductance, and capacitance and as explained earlier, the voltage across each component can be given as,

$V_R=IR$ is in phase with I

$V_L=IX_L$ leads the current by 90 degrees

$V_C=IX_C$ lags behind the current by 90 degrees

The impedance of the circuit is given as,

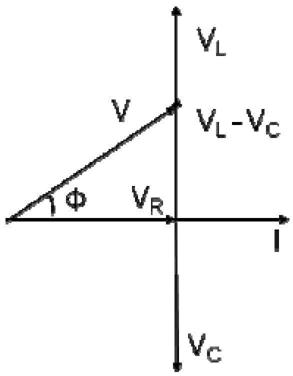
$Z = \sqrt{R^2 + X^2}$

Where $X = (X_L - X_C)$ or $(X_C - X_L)$

So, there can be three possible cases

i) **If $X_L > X_C$ –** When inductive reactance is greater than capacitive reactance then the voltage across inductor V_L is greater than V_C. Hence the resultant voltage across the L-C combination is

(V_L – V_C). The total voltage V is resultant of V_R and (V_L – V_C). Due to high inductive reactance the nature of circuit is inductive and the resultant voltage is leading the current I or current is lagging the supply voltage V. This is shown is phasor diagram in fig.2.24 (a)



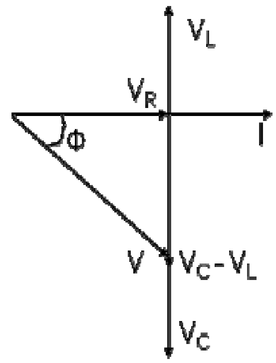


Fig. 2.24 (a) - $X_L > X_C$

fig. 2.24 (b) - $X_C > X_L$

ii) **If $X_C > X_L$:** When capacitive reactance is greater than inductive reactance then the voltage across capacitor V_C is greater than V_L. Hence the resultant voltage across the L-C combination is (V_C – V_L). The total voltage V is resultant of V_R and (V_C – V_L). Due to high capacitive reactance the nature of circuit is capacitive and the resultant voltage is lagging the current I or current is leading the supply voltage V. This is shown is phasor diagram in fig.2.24 (b)

From the phasor diagram, the expressions for the resultant voltage V and the angle φ can be derived as ,

$V = \sqrt{{V_R}^2 + (V_L - V_c)^2}$ Assuming $V_L > V_C$

$V = \sqrt{(IR)^2 + (IX_L - IX_c)^2}$

$V = I\,(\sqrt{(R)^2 + (X_L - X_c)^2})$

V = I.Z

Where, $\mathbf{Z} = \sqrt{(\boldsymbol{R})^2 + (\mathbf{X_L} - \mathbf{X_c})^2}$

Z is called as impedance of the circuit.

iii) **If $X_C = X_L$:** When inductive reactance is equal to capacitive reactance, voltage across inductor V_L is equal to V_C. Hence the resultant voltage across the L-C combination is (V_L – V_C) = 0 or $V_L = V_C$

Hence, from equation 2.22, the impedance Z = R (as $X_L - X_C = 0$).

This condition is called as **'resonance'**.

The power factor in resonance condition is $\cos(\varphi) = \frac{R}{Z} = \frac{Z}{Z} = 1$

Hence power factor is **unity** in resonance condition.

Sample Example:

Ex.3.4) A 230 V, 50 Hz ac supply is applied to a coil of 0.06 H inductance and 2.5 Ω resistance connected in series with a 6.8 µF capacitor. Calculate (i) Impedance (ii) Current (iii) Phase angle between current and voltage (iv) power factor (v) power consumed in the circuit.

Solution: Given data – V = 230 V, f = 50 Hz

L = 0.06H

R = 2.5 Ω

C = 6.8 µF = 6.8×10^{-6} F

Now, using formula for X_L and X_C

$X_L = 2\pi fL = 2\times3.142\times50\times0.06 = 18.84\ \Omega$

$X_C = \frac{1}{2\pi fC} = \frac{1}{2\times3.142\times50\times6.8\times10^{-6}} = 468\ \Omega$

(i) $Z = \sqrt{(\mathbf{R})^2 + (\mathbf{X_L} - \mathbf{X_c})^2} = \sqrt{(2.5)^2 + (18.84 - 468)^2} = \mathbf{449.2\ \Omega}$

(ii) $I = \frac{V}{Z} = \frac{230}{449.2} = \mathbf{0.512\ A}$

(iii) $\Phi = \cos^{-1}\frac{R}{Z} = \cos^{-1}\frac{2.5}{449.2} = \mathbf{89.70°}$

(iv) Power factor = cos (φ) = cos (89.70°) = **0.0055**

2.12 – Various Power's in AC circuit:

In an AC circuit, the various powers can be classified as

1. Real or Active power
2. Reactive power
3. Apparent power

- Real or active power in an AC circuit is the power that does useful work in the circuit.
- Reactive power flows in an AC circuit but does not do any useful work.
- Apparent power is the total power in an AC circuit.

A) Real Power : The power due to the active component of current is called as the active power or real power. It is denoted by P.

$P = V \times I \times \cos(\varphi) = I^2R$

Real power is the power that does useful power. It is the power that is consumed by the resistance. The unit of real power is Watt (W).

B) Reactive Power: The power due to the reactive component of current is called as the reactive power. It is denoted by Q.

$Q = V \times I \times \sin(\varphi) = I^2X_L$

Reactive power does not do any useful work. It is the circulating power in th L and C components. The unit for reactive power is Volt Amperes Reactive (VAR).

C) Apparent Power:

The apparent power is the total power in the circuit. It is denoted by S.

$S = V \times I = I^2Z$

S = $\sqrt{P^2 + Q^2}$
The unit for apparent power is Volt Amperes (VA).
Power Triangle:

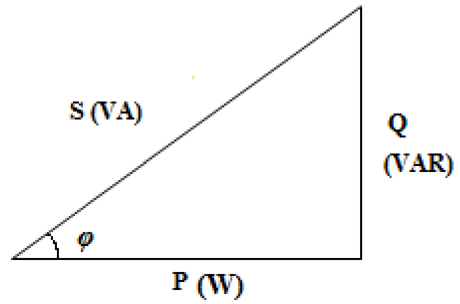


Fig.2.25 – Power triangle

2.13 - Power Factor:

- The power factor in an AC circuit is defined as the "***cosine of the angle between voltage and current"***. It is denoted as '**cos(φ)**'. The power factor can be lagging or leading depending upon the type of circuit and type of the load connected.
- Power factor is also defined as the ratio of resistance to the impedance of the circuit
- From power triangle, power factor can be defined as the ratio of active power to the apparent power. In above diagram of power triangle, taking cosine of angle 'φ'

$\cos(\varphi) = \frac{P}{S} = \frac{\text{Active power}}{\text{Apparant power}}$

2.14 - Disadvantages of low power factor:

- The power factor plays an important role in ac circuits depending upon the load. As we know that lower the power factor, higher is the load current and vice-versa.
- Lagging power factor has some disadvantages like large KVA rating because the KVA is inversely proportional to the power factor.
- Due to low power factor cost of generation, transmission, distribution increases.
- Another demerit is the large copper losses, at low power factor the conductor carries large current hence more copper losses occurs since copper loss is nothing but I^2R.
- Large voltage drops in transmission lines.