pgRouting Manual

Release v2.5.1

pgRouting Contributors

Nov 03, 2017

Contents

pgRouting extends the PostGIS[1]/PostgreSQL[2] geospatial database to provide geospatial routing and other network analysis functionality.

This is the manual for pgRouting v2.5.1.

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[1] http://postgis.net
[2] http://postgresql.org
[3] http://creativecommons.org/licenses/by-sa/3.0/

CHAPTER 1

General

1.1 Introduction

pgRouting is an extension of PostGIS[4] and PostgreSQL[5] geospatial database and adds routing and other network analysis functionality. A predecessor of pgRouting – pgDijkstra, written by Sylvain Pasche from Camptocamp[6], was later extended by Orkney[7] and renamed to pgRouting. The project is now supported and maintained by Georepublic[8], iMaptools[9] and a broad user community.

pgRouting is part of OSGeo Community Projects[10] from the OSGeo Foundation[11] and included on OSGeo Live[12].

1.1.1 Licensing

The following licenses can be found in pgRouting:

License	
GNU General Public License, version 2	Most features of pgRouting are available under GNU General Public License, version 2[13].
Boost Software License - Version 1.0	Some Boost extensions are available under Boost Software License - Version 1.0[14].
MIT-X License	Some code contributed by iMaptools.com is available under MIT-X license.
Creative Commons Attribution-Share Alike 3.0 License	The pgRouting Manual is licensed under a Creative Commons Attribution-Share Alike 3.0 License[15].

In general license information should be included in the header of each source file.

4 http://postgis.net
5 http://postgresql.org
6 http://camptocamp.com
7 http://www.orkney.co.jp
8 http://georepublic.info
9 http://imaptools.com/
10 http://wiki.osgeo.org/wiki/OSGeo_Community_Projects
11 http://osgeo.org
12 http://live.osgeo.org/
13 http://www.gnu.org/licenses/gpl-2.0.html
14 http://www.boost.org/LICENSE_1_0.txt
15 http://creativecommons.org/licenses/by-sa/3.0/

1.1.2 Contributors

This Release Contributors

Individuals (in alphabetical order)

Maoguang Wang, Vidhan Jain, Virginia Vergara

And all the people that give us a little of their time making comments, finding issues, making pull requests etc.

Corporate Sponsors (in alphabetical order)

These are corporate entities that have contributed developer time, hosting, or direct monetary funding to the pgRouting project:

- Georepublic[16]
- Google Summer of Code[17]
- iMaptools[18]
- Paragon Corporation[19]

Contributors Past & Present:

Individuals (in alphabetical order)

Akio Takubo, Andrea Nardelli, Anton Patrushev, Ashraf Hossain, Christian Gonzalez, Daniel Kastl, Dave Potts, David Techer, Denis Rykov, Ema Miyawaki, Florian Thurkow, Frederic Junod, Gerald Fenoy, Jay Mahadeokar, Jinfu Leng, Kai Behncke, Kishore Kumar, Ko Nagase, Manikata Kondeti, Mario Basa, Martin Wiesenhaan, Maxim Dubinin, Maoguang Wang, Mohamed Zia, Mukul Priya, Razequl Islam, Regina Obe, Rohith Reddy, Sarthak Agarwal, Stephen Woodbridge, Sylvain Housseman, Sylvain Pasche, Vidhan Jain, Virginia Vergara

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- Camptocamp
- CSIS (University of Tokyo)
- Georepublic
- Google Summer of Code
- iMaptools
- Orkney
- Paragon Corporation

[16] https://georepublic.info/en/
[17] https://developers.google.com/open-source/gsoc/
[18] http://imaptools.com
[19] http://www.paragoncorporation.com/

1.1.3 More Information

- The latest software, documentation and news items are available at the pgRouting web site http://pgrouting.org.
- PostgreSQL database server at the PostgreSQL main site http://www.postgresql.org.
- PostGIS extension at the PostGIS project web site http://postgis.net.
- Boost C++ source libraries at http://www.boost.org.
- Computational Geometry Algorithms Library (CGAL) at http://www.cgal.org.
- The Migration guide can be found at https://github.com/pgRouting/pgrouting/wiki/Migration-Guide.

1.2 Installation

Table of Contents

Instructions for downloading and installing binaries for different Operative systems instructions and additional notes and corrections not included in this documentation can be found in Installation wiki[20]

To use pgRouting postGIS needs to be installed, please read the information about installation in this Install Guide[21]

1.2.1 Short Version

Extracting the tar ball

```
tar xvfz pgrouting-2.4.0.tar.gz
cd pgrouting-2.4.0
```

To compile assuming you have all the dependencies in your search path:

```
mkdir build
cd build
cmake  ..
make
sudo make install
```

Once pgRouting is installed, it needs to be enabled in each individual database you want to use it in.

```
createdb routing
psql routing -c 'CREATE EXTENSION postGIS'
psql routing -c 'CREATE EXTENSION pgRouting'
```

[20] https://github.com/pgRouting/pgrouting/wiki/Notes-on-Download%2C-Installation-and-building-pgRouting
[21] http://www.postgis.us/presentations/postgis_install_guide_22.html

1.2.2 Get the sources

The pgRouting latest release can be found in https://github.com/pgRouting/pgrouting/releases/latest

wget

To download this release:

```
wget -O pgrouting-v2.4.0.tar.gz https://github.com/pgRouting/pgrouting/archive/v2.
↪4.0.tar.gz
```

Goto *Short Version* to the extract and compile instructions.

git

To download the repository

```
git clone git://github.com/pgRouting/pgrouting.git
cd pgrouting
git checkout |release|
```

Goto *Short Version* to the compile instructions (there is no tar ball).

1.2.3 Enabling and upgrading in the database

Enabling the database

pgRouting is an extension and depends on postGIS. Enabling postGIS before enabling pgRouting in the database

```
CREATE EXTENSION postgis;
CREATE EXTENSION pgrouting;
```

Upgrading the database

To upgrade pgRouting in the database to version 2.4.0 use the following command:

```
ALTER EXTENSION pgrouting UPDATE TO "2.4.0";
```

More information can be found in https://www.postgresql.org/docs/current/static/sql-createextension.html

1.2.4 Dependencies

Compilation Dependencies

To be able to compile pgRouting, make sure that the following dependencies are met:

- C and C++0x compilers * g++ version >= 4.8
- Postgresql version >= 9.2
- PostGIS version >= 2.0
- The Boost Graph Library (BGL). Version >= 1.46
- CMake >= 3.2
- CGAL >= 4.2

optional dependencies

For user's documentation

- Sphinx >= 1.1
- Latex

For developer's documentation

- Doxygen >= 1.7

For testing

- pgtap
- pg_prove

Example: Installing dependencies on linux

Installing the compilation dependencies

```
sudo apt-get install
    cmake \
    g++ \
    postgresql-9.3 \
    postgresql-server-dev-9.3 \
    libboost-graph-dev \
    libcgal-dev
```

Installing the optional dependencies

```
sudo apt-get install -y python-sphinx \
    texlive \
    doxygen \
    libtap-parser-sourcehandler-pgtap-perl \
    postgresql-9.3-pgtap
```

1.2.5 Configuring

pgRouting uses the *cmake* system to do the configuration.

The build directory is different from the source directory

Create the build directory

```
$ mkdir build
```

Configurable variables

To see the variables that can be configured

```
$ cd build
$ cmake -L ..
```

Configuring The Documentation

Most of the effort of the documentation has being on the HTML files. Some variables for the documentation:

Variable	Default	Comment
WITH_DOC	BOOL=OFF	Turn on/off building the documentation
BUILD_HTML	BOOL=ON	If ON, turn on/off building HTML for user's documentation
BUILD_DOXY	BOOL=ON	If ON, turn on/off building HTML for developer's documentation
BUILD_LATEX	BOOL=OFF	If ON, turn on/off building PDF
BUILD_MAN	BOOL=OFF	If ON, turn on/off building MAN pages
DOC_USE_BOOTSTRAP	BOOL=OFF	If ON, use sphinx-bootstrap for HTML pages of the users documentation

Configuring with documentation

```
$ cmake -DWITH_DOC=ON ..
```

Note: Most of the effort of the documentation has being on the html files.

1.2.6 Building

Using `make` to build the code and the documentation

The following instructions start from *path/to/pgrouting/build*

```
$ make          # build the code but not the documentation
$ make doc      # build only the documentation
$ make all doc  # build both the code and the documentation
```

We have tested on several platforms, For installing or reinstalling all the steps are needed.

Warning: The sql signatures are configured and build in the `cmake` command.

MinGW on Windows

```
$ mkdir build
$ cd build
$ cmake -G"MSYS Makefiles" ..
$ make
$ make install
```

Linux

The following instructions start from *path/to/pgrouting*

```
mkdir build
cd build
cmake  ..
make
sudo make install
```

When the configuration changes:

```
rm -rf build
```

and start the build process as mentioned above.

1.2.7 Testing

Currently there is no `make test` and testing is done as follows

The following instructions start from *path/to/pgrouting/*

```
tools/testers/algorithm-tester.pl
createdb  -U <user> ___pgr___test___
sh ./tools/testers/pg_prove_tests.sh <user>
dropdb  -U <user> ___pgr___test___
```

1.2.8 See Also

Indices and tables

- genindex
- search

1.3 Support

pgRouting community support is available through the pgRouting website[22], documentation[23], tutorials, mailing lists and others. If you're looking for *commercial support*, find below a list of companies providing pgRouting development and consulting services.

1.3.1 Reporting Problems

Bugs are reported and managed in an issue tracker[24]. Please follow these steps:

1. Search the tickets to see if your problem has already been reported. If so, add any extra context you might have found, or at least indicate that you too are having the problem. This will help us prioritize common issues.
2. If your problem is unreported, create a new issue[25] for it.
3. In your report include explicit instructions to replicate your issue. The best tickets include the exact SQL necessary to replicate a problem.
4. If you can test older versions of PostGIS for your problem, please do. On your ticket, note the earliest version the problem appears.
5. For the versions where you can replicate the problem, note the operating system and version of pgRouting, PostGIS and PostgreSQL.
6. It is recommended to use the following wrapper on the problem to pin point the step that is causing the problem.

[22] http://pgrouting.org/support.html
[23] http://docs.pgrouting.org
[24] https://github.com/pgrouting/pgrouting/issues
[25] https://github.com/pgRouting/pgrouting/issues/new

```
SET client_min_messages TO debug;
  <your code>
SET client_min_messages TO notice;
```

1.3.2 Mailing List and GIS StackExchange

There are two mailing lists for pgRouting hosted on OSGeo mailing list server:

- User mailing list: http://lists.osgeo.org/mailman/listinfo/pgrouting-users
- Developer mailing list: http://lists.osgeo.org/mailman/listinfo/pgrouting-dev

For general questions and topics about how to use pgRouting, please write to the user mailing list.

You can also ask at GIS StackExchange[26] and tag the question with `pgrouting`. Find all questions tagged with `pgrouting` under http://gis.stackexchange.com/questions/tagged/pgrouting or subscribe to the pgRouting questions feed[27].

1.3.3 Commercial Support

For users who require professional support, development and consulting services, consider contacting any of the following organizations, which have significantly contributed to the development of pgRouting:

Company	Offices in	Website
Georepublic	Germany, Japan	https://georepublic.info
iMaptools	United States	http://imaptools.com
Paragon Corporation	United States	http://www.paragoncorporation.com
Camptocamp	Switzerland, France	http://www.camptocamp.com

- *Sample Data* that is used in the examples of this manual.

1.4 Sample Data

The documentation provides very simple example queries based on a small sample network. To be able to execute the sample queries, run the following SQL commands to create a table with a small network data set.

Create table

```
CREATE TABLE edge_table (
    id BIGSERIAL,
    dir character varying,
    source BIGINT,
    target BIGINT,
    cost FLOAT,
    reverse_cost FLOAT,
    capacity BIGINT,
    reverse_capacity BIGINT,
    category_id INTEGER,
    reverse_category_id INTEGER,
    x1 FLOAT,
    y1 FLOAT,
    x2 FLOAT,
    y2 FLOAT,
```

[26] http://gis.stackexchange.com/
[27] http://gis.stackexchange.com/feeds/tag?tagnames=pgrouting&sort=newest

```
    the_geom geometry
);
```

Insert data

```
INSERT INTO edge_table (
    category_id, reverse_category_id,
    cost, reverse_cost,
    capacity, reverse_capacity,
    x1, y1,
    x2, y2) VALUES
(3, 1,     1,  1,  80, 130,   2,   0,     2, 1),
(3, 2,    -1,  1,  -1, 100,   2,   1,     3, 1),
(2, 1,    -1,  1,  -1, 130,   3,   1,     4, 1),
(2, 4,     1,  1, 100,  50,   2,   1,     2, 2),
(1, 4,     1, -1, 130,  -1,   3,   1,     3, 2),
(4, 2,     1,  1,  50, 100,   0,   2,     1, 2),
(4, 1,     1,  1,  50, 130,   1,   2,     2, 2),
(2, 1,     1,  1, 100, 130,   2,   2,     3, 2),
(1, 3,     1,  1, 130,  80,   3,   2,     4, 2),
(1, 4,     1,  1, 130,  50,   2,   2,     2, 3),
(1, 2,     1, -1, 130,  -1,   3,   2,     3, 3),
(2, 3,     1, -1, 100,  -1,   2,   3,     3, 3),
(2, 4,     1, -1, 100,  -1,   3,   3,     4, 3),
(3, 1,     1,  1,  80, 130,   2,   3,     2, 4),
(3, 4,     1,  1,  80,  50,   4,   2,     4, 3),
(3, 3,     1,  1,  80,  80,   4,   1,     4, 2),
(1, 2,     1,  1, 130, 100,   0.5, 3.5,   1.999999999999,3.5),
(4, 1,     1,  1,  50, 130,   3.5, 2.3,   3.5,4);

UPDATE edge_table SET the_geom = st_makeline(st_point(x1,y1),st_point(x2,y2)),
dir = CASE WHEN (cost>0 AND reverse_cost>0) THEN 'B'   -- both ways
           WHEN (cost>0 AND reverse_cost<0) THEN 'FT'  -- direction of the␣
↪LINESSTRING
           WHEN (cost<0 AND reverse_cost>0) THEN 'TF'  -- reverse direction of the␣
↪LINESTRING
           ELSE '' END;                                -- unknown
```

Topology

- Before you test a routing function use this query to create a topology (fills the `source` and `target` columns).

```
SELECT pgr_createTopology('edge_table',0.001);
```

Points of interest

- When points outside of the graph.
- Used with the *withPoints - Family of functions* functions.

```
CREATE TABLE pointsOfInterest(
    pid BIGSERIAL,
    x FLOAT,
    y FLOAT,
```

```
    edge_id BIGINT,
    side CHAR,
    fraction FLOAT,
    the_geom geometry,
    newPoint geometry
);

INSERT INTO pointsOfInterest (x, y, edge_id, side, fraction) VALUES
(1.8, 0.4,   1, 'l', 0.4),
(4.2, 2.4,  15, 'r', 0.4),
(2.6, 3.2,  12, 'l', 0.6),
(0.3, 1.8,   6, 'r', 0.3),
(2.9, 1.8,   5, 'l', 0.8),
(2.2, 1.7,   4, 'b', 0.7);
UPDATE pointsOfInterest SET the_geom = st_makePoint(x,y);

UPDATE pointsOfInterest
    SET newPoint = ST_LineInterpolatePoint(e.the_geom, fraction)
    FROM edge_table AS e WHERE edge_id = id;
```

Restrictions

- Used with the *pgr_trsp - Turn Restriction Shortest Path (TRSP)* functions.

```
CREATE TABLE restrictions (
    rid BIGINT NOT NULL,
    to_cost FLOAT,
    target_id BIGINT,
    from_edge BIGINT,
    via_path TEXT
);

INSERT INTO restrictions (rid, to_cost, target_id, from_edge, via_path) VALUES
(1, 100,  7,  4, NULL),
(1, 100, 11,  8, NULL),
(1, 100, 10,  7, NULL),
(2,   4,  8,  3, 5),
(3, 100,  9, 16, NULL);
```

Categories

- Used with the *Flow - Family of functions* functions.

```
/*
CREATE TABLE categories (
    category_id INTEGER,
    category text,
    capacity BIGINT
);

INSERT INTO categories VALUES
(1, 'Category 1', 130),
(2, 'Category 2', 100),
(3, 'Category 3',  80),
(4, 'Category 4',  50);
*/
```

Vertex table

- Used in some deprecated signatures or deprecated functions.

```
-- TODO check if this table is still used
CREATE TABLE vertex_table (
    id SERIAL,
    x FLOAT,
    y FLOAT
);
INSERT INTO vertex_table VALUES
(1,2,0), (2,2,1), (3,3,1), (4,4,1), (5,0,2), (6,1,2), (7,2,2),
(8,3,2), (9,4,2), (10,2,3), (11,3,3), (12,4,3), (13,2,4);
```

1.4.1 Images

- Red arrows correspond when `cost` > 0 in the edge table.
- Blue arrows correspond when `reverse_cost` > 0 in the edge table.
- Points are outside the graph.
- Click on the graph to enlarge.

Network for queries marked as `directed` and `cost` and `reverse_cost` columns are used

When working with city networks, this is recommended for point of view of vehicles.

Network for queries marked as `undirected` and `cost` and `reverse_cost` columns are used

When working with city networks, this is recommended for point of view of pedestrians.

Network for queries marked as `directed` and only `cost` column is used

Network for queries marked as `undirected` and only `cost` column is used

Pick & Deliver Data

```
DROP TABLE IF EXISTS customer CASCADE;
CREATE table customer (
    id BIGINT not null primary key,
    x DOUBLE PRECISION,
    y DOUBLE PRECISION,
    demand INTEGER,
    opentime INTEGER,
    closetime INTEGER,
    servicetime INTEGER,
    pindex BIGINT,
    dindex BIGINT
);


INSERT INTO customer(
  id,     x,     y, demand, opentime, closetime, servicetime, pindex, dindex) VALUES
(  0,    40,    50,      0,       0,   1236,      0,      0,      0),
```

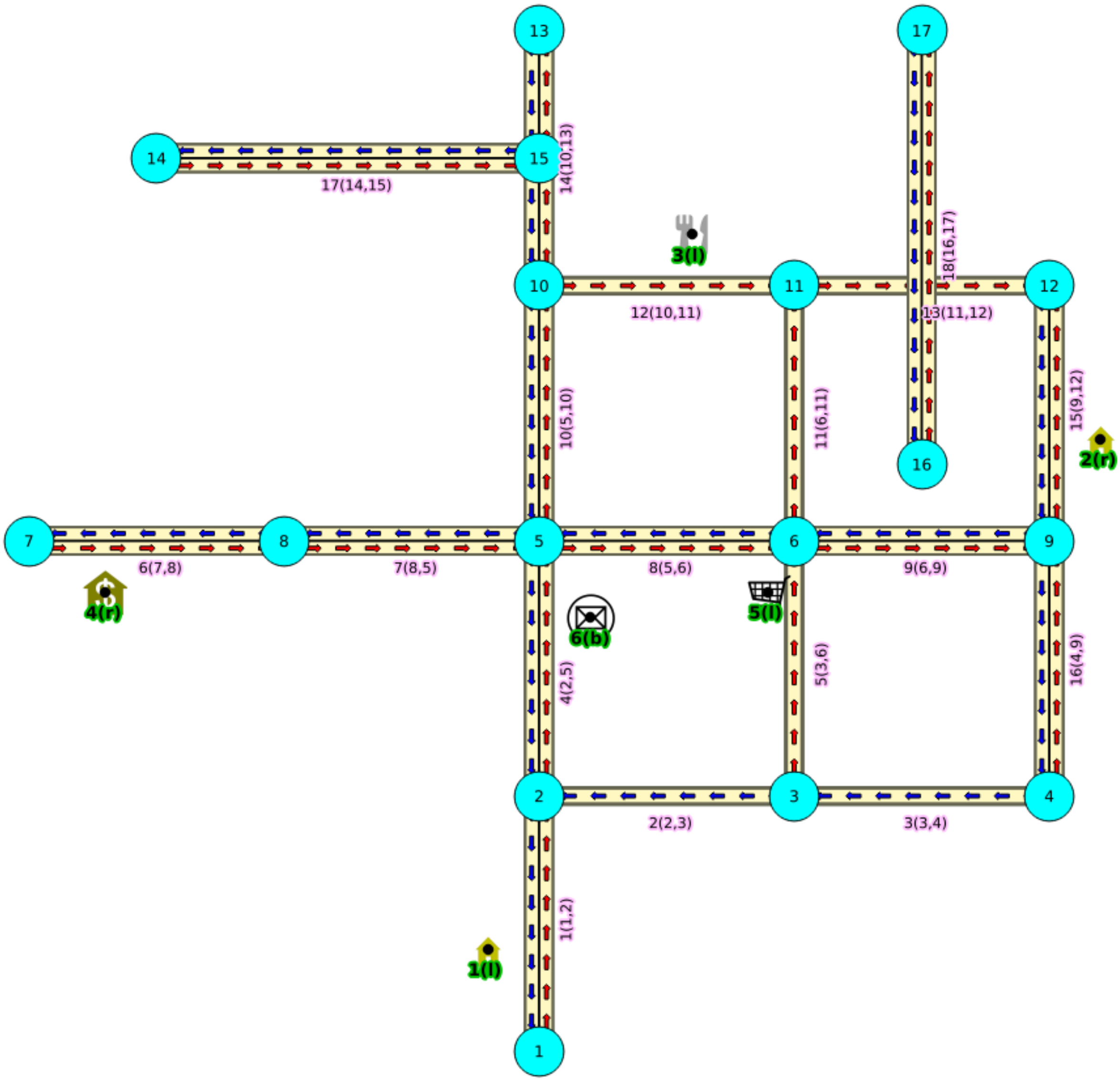

Fig. 1.1: **Graph 1: Directed, with cost and reverse cost**

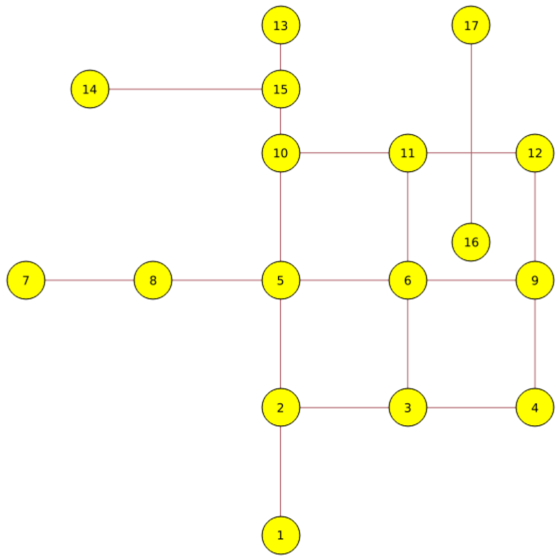

Fig. 1.2: **Graph 2: Undirected, with cost and reverse cost**

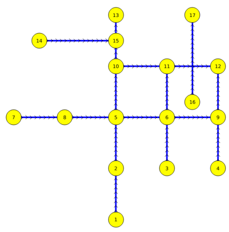

Fig. 1.3: **Graph 3: Directed, with cost**

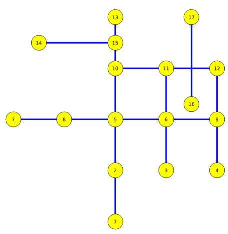

Fig. 1.4: **Graph 4: Undirected, with cost**

```
(  1,    45,    68,    -10,    912,    967,    90,    11,     0),
(  2,    45,    70,    -20,    825,    870,    90,     6,     0),
(  3,    42,    66,     10,     65,    146,    90,     0,    75),
(  4,    42,    68,    -10,    727,    782,    90,     9,     0),
(  5,    42,    65,     10,     15,     67,    90,     0,     7),
(  6,    40,    69,     20,    621,    702,    90,     0,     2),
(  7,    40,    66,    -10,    170,    225,    90,     5,     0),
(  8,    38,    68,     20,    255,    324,    90,     0,    10),
(  9,    38,    70,     10,    534,    605,    90,     0,     4),
( 10,    35,    66,    -20,    357,    410,    90,     8,     0),
( 11,    35,    69,     10,    448,    505,    90,     0,     1),
( 12,    25,    85,    -20,    652,    721,    90,    18,     0),
( 13,    22,    75,     30,     30,     92,    90,     0,    17),
( 14,    22,    85,    -40,    567,    620,    90,    16,     0),
( 15,    20,    80,    -10,    384,    429,    90,    19,     0),
( 16,    20,    85,     40,    475,    528,    90,     0,    14),
( 17,    18,    75,    -30,     99,    148,    90,    13,     0),
( 18,    15,    75,     20,    179,    254,    90,     0,    12),
( 19,    15,    80,     10,    278,    345,    90,     0,    15),
( 20,    30,    50,     10,     10,     73,    90,     0,    24),
( 21,    30,    52,    -10,    914,    965,    90,    30,     0),
( 22,    28,    52,    -20,    812,    883,    90,    28,     0),
( 23,    28,    55,     10,    732,    777,     0,     0,   103),
( 24,    25,    50,    -10,     65,    144,    90,    20,     0),
( 25,    25,    52,     40,    169,    224,    90,     0,    27),
( 26,    25,    55,    -10,    622,    701,    90,    29,     0),
( 27,    23,    52,    -40,    261,    316,    90,    25,     0),
( 28,    23,    55,     20,    546,    593,    90,     0,    22),
( 29,    20,    50,     10,    358,    405,    90,     0,    26),
( 30,    20,    55,     10,    449,    504,    90,     0,    21),
( 31,    10,    35,    -30,    200,    237,    90,    32,     0),
( 32,    10,    40,     30,     31,    100,    90,     0,    31),
( 33,     8,    40,     40,     87,    158,    90,     0,    37),
( 34,     8,    45,    -30,    751,    816,    90,    38,     0),
( 35,     5,    35,     10,    283,    344,    90,     0,    39),
( 36,     5,    45,     10,    665,    716,     0,     0,   105),
( 37,     2,    40,    -40,    383,    434,    90,    33,     0),
( 38,     0,    40,     30,    479,    522,    90,     0,    34),
( 39,     0,    45,    -10,    567,    624,    90,    35,     0),
( 40,    35,    30,    -20,    264,    321,    90,    42,     0),
( 41,    35,    32,    -10,    166,    235,    90,    43,     0),
( 42,    33,    32,     20,     68,    149,    90,     0,    40),
( 43,    33,    35,     10,     16,     80,    90,     0,    41),
( 44,    32,    30,     10,    359,    412,    90,     0,    46),
( 45,    30,    30,     10,    541,    600,    90,     0,    48),
( 46,    30,    32,    -10,    448,    509,    90,    44,     0),
( 47,    30,    35,    -10,   1054,   1127,    90,    49,     0),
( 48,    28,    30,    -10,    632,    693,    90,    45,     0),
( 49,    28,    35,     10,   1001,   1066,    90,     0,    47),
( 50,    26,    32,     10,    815,    880,    90,     0,    52),
( 51,    25,    30,     10,    725,    786,     0,     0,   101),
( 52,    25,    35,    -10,    912,    969,    90,    50,     0),
( 53,    44,     5,     20,    286,    347,    90,     0,    58),
( 54,    42,    10,     40,    186,    257,    90,     0,    60),
( 55,    42,    15,    -40,     95,    158,    90,    57,     0),
( 56,    40,     5,     30,    385,    436,    90,     0,    59),
( 57,    40,    15,     40,     35,     87,    90,     0,    55),
( 58,    38,     5,    -20,    471,    534,    90,    53,     0),
( 59,    38,    15,    -30,    651,    740,    90,    56,     0),
( 60,    35,     5,    -40,    562,    629,    90,    54,     0),
( 61,    50,    30,    -10,    531,    610,    90,    67,     0),
( 62,    50,    35,     20,    262,    317,    90,     0,    68),
( 63,    50,    40,     50,    171,    218,    90,     0,    74),
```

```
( 64,     48,    30,     10,    632,    693,     0,      0,   102),
( 65,     48,    40,     10,     76,    129,    90,      0,    72),
( 66,     47,    35,     10,    826,    875,    90,      0,    69),
( 67,     47,    40,     10,     12,     77,    90,      0,    61),
( 68,     45,    30,    -20,    734,    777,    90,     62,     0),
( 69,     45,    35,    -10,    916,    969,    90,     66,     0),
( 70,     95,    30,    -30,    387,    456,    90,     81,     0),
( 71,     95,    35,     20,    293,    360,    90,      0,    77),
( 72,     53,    30,    -10,    450,    505,    90,     65,     0),
( 73,     92,    30,    -10,    478,    551,    90,     76,     0),
( 74,     53,    35,    -50,    353,    412,    90,     63,     0),
( 75,     45,    65,    -10,    997,   1068,    90,      3,     0),
( 76,     90,    35,     10,    203,    260,    90,      0,    73),
( 77,     88,    30,    -20,    574,    643,    90,     71,     0),
( 78,     88,    35,     20,    109,    170,     0,      0,   104),
( 79,     87,    30,     10,    668,    731,    90,      0,    80),
( 80,     85,    25,    -10,    769,    820,    90,     79,     0),
( 81,     85,    35,     30,     47,    124,    90,      0,    70),
( 82,     75,    55,     20,    369,    420,    90,      0,    85),
( 83,     72,    55,    -20,    265,    338,    90,     87,     0),
( 84,     70,    58,     20,    458,    523,    90,      0,    89),
( 85,     68,    60,    -20,    555,    612,    90,     82,     0),
( 86,     66,    55,     10,    173,    238,    90,      0,    91),
( 87,     65,    55,     20,     85,    144,    90,      0,    83),
( 88,     65,    60,    -10,    645,    708,    90,     90,     0),
( 89,     63,    58,    -20,    737,    802,    90,     84,     0),
( 90,     60,    55,     10,     20,     84,    90,      0,    88),
( 91,     60,    60,    -10,    836,    889,    90,     86,     0),
( 92,     67,    85,     20,    368,    441,    90,      0,    93),
( 93,     65,    85,    -20,    475,    518,    90,     92,     0),
( 94,     65,    82,    -10,    285,    336,    90,     96,     0),
( 95,     62,    80,    -20,    196,    239,    90,     98,     0),
( 96,     60,    80,     10,     95,    156,    90,      0,    94),
( 97,     60,    85,     30,    561,    622,     0,      0,   106),
( 98,     58,    75,     20,     30,     84,    90,      0,    95),
( 99,     55,    80,    -20,    743,    820,    90,    100,     0),
( 100,    55,    85,     20,    647,    726,    90,      0,    99),
( 101,    25,    30,    -10,    725,    786,    90,     51,     0),
( 102,    48,    30,    -10,    632,    693,    90,     64,     0),
( 103,    28,    55,    -10,    732,    777,    90,     23,     0),
( 104,    88,    35,    -20,    109,    170,    90,     78,     0),
( 105,     5,    45,    -10,    665,    716,    90,     36,     0),
( 106,    60,    85,    -30,    561,    622,    90,     97,     0);
```

CHAPTER 2

Pgrouting Concepts

2.1 pgRouting Concepts

Contents

2.1.1 Getting Started

This is a simple guide to walk you through the steps of getting started with pgRouting. In this guide we will cover:

Create a routing Database

The first thing we need to do is create a database and load pgrouting in the database. Typically you will create a database for each project. Once you have a database to work in, your can load your data and build your application in that database. This makes it easy to move your project later if you want to to say a production server.

For Postgresql 9.2 and later versions

```
createdb mydatabase
psql mydatabase -c "create extension postgis"
psql mydatabase -c "create extension pgrouting"
```

Load Data

How you load your data will depend in what form it comes it. There are various OpenSource tools that can help you, like:

osm2pgrouting

- this is a tool for loading OSM data into postgresql with pgRouting requirements

shp2pgsql

- this is the postgresql shapefile loader

ogr2ogr

- this is a vector data conversion utility

osm2pgsql

- this is a tool for loading OSM data into postgresql

So these tools and probably others will allow you to read vector data so that you may then load that data into your database as a table of some kind. At this point you need to know a little about your data structure and content. One easy way to browse your new data table is with pgAdmin3 or phpPgAdmin.

Build a Routing Topology

Next we need to build a topology for our street data. What this means is that for any given edge in your street data the ends of that edge will be connected to a unique node and to other edges that are also connected to that same unique node. Once all the edges are connected to nodes we have a graph that can be used for routing with pgrouting. We provide a tool that will help with this:

Note: this step is not needed if data is loaded with *osm2pgrouting*

```
select pgr_createTopology('myroads', 0.000001);
```

- *pgr_createTopology*

Check the Routing Topology

There are lots of possible sources for errors in a graph. The data that you started with may not have been designed with routing in mind. A graph has some very specific requirements. One is that it is *NODED*, this means that except for some very specific use cases, each road segment starts and ends at a node and that in general is does not cross another road segment that it should be connected to.

There can be other errors like the direction of a one-way street being entered in the wrong direction. We do not have tools to search for all possible errors but we have some basic tools that might help.

```
select pgr_analyzegraph('myroads', 0.000001);
select pgr_analyzeoneway('myroads',  s_in_rules, s_out_rules,
                                     t_in_rules, t_out_rules
                                     direction)
select pgr_nodeNetwork('myroads', 0.001);
```

- *pgr_analyzeGraph*
- *pgr_analyzeOneway*
- *pgr_nodeNetwork*

Compute a Path

Once you have all the preparation work done above, computing a route is fairly easy. We have a lot of different algorithms that can work with your prepared road network. The general form of a route query is:

```
select pgr_dijkstra(`SELECT * FROM myroads', 1, 2)
```

As you can see this is fairly straight forward and you can look and the specific algorithms for the details of the signatures and how to use them. These results have information like edge id and/or the node id along with the cost or geometry for the step in the path from *start* to *end*. Using the ids you can join these result back to your edge table to get more information about each step in the path.

- *pgr_dijkstra*

2.1.2 Inner Queries

There are several kinds of valid inner queries and also the columns returned are depending of the function. Which kind of inner query will depend on the function(s) requirements. To simplify variety of types, `ANY-INTEGER` and `ANY-NUMERICAL` is used.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the edges_sql query (id is not necessary)

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Parameter	Type	Default	Description
edges_sql	`TEXT`		SQL query as described above.
via_vertices	`ARRAY[ANY-INTEGER]`		Array of ordered vertices identifiers that are going to be visited.
directed	`BOOLEAN`	`true`	• When `true` Graph is considered *Directed* • When `false` the graph is considered as Undirected.
strict	`BOOLEAN`	`false`	• When `false` ignores missing paths returning all paths found • When `true` if a path is missing stops and returns *EMPTY SET*
U_turn_on_edge	`BOOLEAN`	`true`	• When `true` departing from a visited vertex will not try to avoid using the edge used to reach it. In other words, U turn using the edge with same *id* is allowed. • When `false` when a departing from a visited vertex tries to avoid using the edge used to reach it. In other words, U turn using the edge with same *id* is used when no other path is found.

Description of the edges_sql query for astar like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.
x1	`ANY-NUMERICAL`		X coordinate of *source* vertex.
y1	`ANY-NUMERICAL`		Y coordinate of *source* vertex.
x2	`ANY-NUMERICAL`		X coordinate of *target* vertex.
y2	`ANY-NUMERICAL`		Y coordinate of *target* vertex.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the edges_sql query for Max-flow like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
capacity	`ANY-INTEGER`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_capacity	`ANY-INTEGER`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

Description of the Points SQL query

points_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Description
pid	`ANY-INTEGER`	(optional) Identifier of the point. • If column present, it can not be NULL. • If column not present, a sequential identifier will be given automatically.
edge_id	`ANY-INTEGER`	Identifier of the "closest" edge to the point.
fraction	`ANY-NUMERICAL`	Value in <0,1> that indicates the relative postition from the first end point of the edge.
side	`CHAR`	(optional) Value in ['b', 'r', 'l', NULL] indicating if the point is: • In the right, left of the edge or • If it doesn't matter with 'b' or NULL. • If column not present 'b' is considered.

Where:

ANY-INTEGER smallint, int, bigint

ANY-NUMERICAL smallint, int, bigint, real, float

2.1.3 Return columns & values

There are several kinds of columns returned are depending of the function.

Description of the return values for a path

Returns set of `(seq, path_seq [, start_vid] [, end_vid], node, edge, cost, agg_cost)`

Column	Type	Description
seq	`INT`	Sequential value starting from **1**.
path_id	`INT`	Path identifier. Has value **1** for the first of a path. Used when there are multiple paths for the same `start_vid` to `end_vid` combination.
path_seq	`INT`	Relative position in the path. Has value **1** for the beginning of a path.
start_vid	`BIGINT`	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	`BIGINT`	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
node	`BIGINT`	Identifier of the node in the path from `start_vid` to `end_vid`.
edge	`BIGINT`	Identifier of the edge used to go from `node` to the next node in the path sequence. `-1` for the last node of the path.
cost	`FLOAT`	Cost to traverse from `node` using `edge` to the next node in the path sequence.
agg_cost	`FLOAT`	Aggregate cost from `start_v` to `node`.

Description of the return values for a Cost function

Returns set of `(start_vid, end_vid, agg_cost)`

Column	Type	Description
start_vid	`BIGINT`	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	`BIGINT`	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
agg_cost	`FLOAT`	Aggregate cost from `start_vid` to `end_vid`.

Description of the Return Values

Column	Type	Description
seq	`INT`	Sequential value starting from **1**.
edge_id	`BIGINT`	Identifier of the edge in the original query(edges_sql).
source	`BIGINT`	Identifier of the first end point vertex of the edge.
target	`BIGINT`	Identifier of the second end point vertex of the edge.
flow	`BIGINT`	Flow through the edge in the direction (source, target).
residual_capacity	`BIGINT`	Residual capacity of the edge in the direction (source, target).

2.1.4 Advanced Topics

Routing Topology

Overview

Typically when GIS files are loaded into the data database for use with pgRouting they do not have topology information associated with them. To create a useful topology the data needs to be "noded". This means that where two or more roads form an intersection there it needs to be a node at the intersection and all the road segments need to be broken at the intersection, assuming that you can navigate from any of these segments to any other segment via that intersection.

You can use the *graph analysis functions* to help you see where you might have topology problems in your data. If you need to node your data, we also have a function *pgr_nodeNetwork()* that might work for you. This function splits ALL crossing segments and nodes them. There are some cases where this might NOT be the right thing to do.

For example, when you have an overpass and underpass intersection, you do not want these noded, but pgr_nodeNetwork does not know that is the case and will node them which is not good because then the router will be able to turn off the overpass onto the underpass like it was a flat 2D intersection. To deal with this problem some data sets use z-levels at these types of intersections and other data might not node these intersection which would be ok.

For those cases where topology needs to be added the following functions may be useful. One way to prep the data for pgRouting is to add the following columns to your table and then populate them as appropriate. This example makes a lot of assumption like that you original data tables already has certain columns in it like `one_way`, `fcc`, and possibly others and that they contain specific data values. This is only to give you an idea of what you can do with your data.

```
ALTER TABLE edge_table
    ADD COLUMN source integer,
    ADD COLUMN target integer,
    ADD COLUMN cost_len double precision,
    ADD COLUMN cost_time double precision,
    ADD COLUMN rcost_len double precision,
    ADD COLUMN rcost_time double precision,
    ADD COLUMN x1 double precision,
    ADD COLUMN y1 double precision,
    ADD COLUMN x2 double precision,
    ADD COLUMN y2 double precision,
    ADD COLUMN to_cost double precision,
    ADD COLUMN rule text,
    ADD COLUMN isolated integer;

SELECT pgr_createTopology('edge_table', 0.000001, 'the_geom', 'id');
```

The function *pgr_createTopology()* will create the `vertices_tmp` table and populate the `source` and `target` columns. The following example populated the remaining columns. In this example, the `fcc` column contains feature class code and the `CASE` statements converts it to an average speed.

```
UPDATE edge_table SET x1 = st_x(st_startpoint(the_geom)),
                      y1 = st_y(st_startpoint(the_geom)),
                      x2 = st_x(st_endpoint(the_geom)),
                      y2 = st_y(st_endpoint(the_geom)),
  cost_len  = st_length_spheroid(the_geom, 'SPHEROID["WGS84",6378137,298.25728]'),
  rcost_len = st_length_spheroid(the_geom, 'SPHEROID["WGS84",6378137,298.25728]'),
  len_km = st_length_spheroid(the_geom, 'SPHEROID["WGS84",6378137,298.25728]')/
↪1000.0,
  len_miles = st_length_spheroid(the_geom, 'SPHEROID["WGS84",6378137,298.25728]')
            / 1000.0 * 0.6213712,
  speed_mph = CASE WHEN fcc='A10' THEN 65
                   WHEN fcc='A15' THEN 65
                   WHEN fcc='A20' THEN 55
                   WHEN fcc='A25' THEN 55
                   WHEN fcc='A30' THEN 45
                   WHEN fcc='A35' THEN 45
                   WHEN fcc='A40' THEN 35
                   WHEN fcc='A45' THEN 35
                   WHEN fcc='A50' THEN 25
                   WHEN fcc='A60' THEN 25
                   WHEN fcc='A61' THEN 25
                   WHEN fcc='A62' THEN 25
                   WHEN fcc='A64' THEN 25
                   WHEN fcc='A70' THEN 15
                   WHEN fcc='A69' THEN 10
                   ELSE null END,
  speed_kmh = CASE WHEN fcc='A10' THEN 104
                   WHEN fcc='A15' THEN 104
                   WHEN fcc='A20' THEN 88
                   WHEN fcc='A25' THEN 88
                   WHEN fcc='A30' THEN 72
                   WHEN fcc='A35' THEN 72
                   WHEN fcc='A40' THEN 56
                   WHEN fcc='A45' THEN 56
                   WHEN fcc='A50' THEN 40
                   WHEN fcc='A60' THEN 50
                   WHEN fcc='A61' THEN 40
                   WHEN fcc='A62' THEN 40
                   WHEN fcc='A64' THEN 40
                   WHEN fcc='A70' THEN 25
                   WHEN fcc='A69' THEN 15
                   ELSE null END;

-- UPDATE the cost information based on oneway streets

UPDATE edge_table SET
    cost_time = CASE
        WHEN one_way='TF' THEN 10000.0
        ELSE cost_len/1000.0/speed_kmh::numeric*3600.0
        END,
    rcost_time = CASE
        WHEN one_way='FT' THEN 10000.0
        ELSE cost_len/1000.0/speed_kmh::numeric*3600.0
        END;

-- clean up the database because we have updated a lot of records

VACUUM ANALYZE VERBOSE edge_table;
```

Now your database should be ready to use any (most?) of the pgRouting algorithms.

Graph Analytics

Overview

It is common to find problems with graphs that have not been constructed fully noded or in graphs with z-levels at intersection that have been entered incorrectly. An other problem is one way streets that have been entered in the wrong direction. We can not detect errors with respect to "ground" truth, but we can look for inconsistencies and some anomalies in a graph and report them for additional inspections.

We do not current have any visualization tools for these problems, but I have used mapserver to render the graph and highlight potential problem areas. Someone familiar with graphviz might contribute tools for generating images with that.

Analyze a Graph

With *pgr_analyzeGraph* the graph can be checked for errors. For example for table "mytab" that has "mytab_vertices_pgr" as the vertices table:

```
SELECT pgr_analyzeGraph('mytab', 0.000002);
NOTICE:  Performing checks, pelase wait...
NOTICE:  Analyzing for dead ends. Please wait...
NOTICE:  Analyzing for gaps. Please wait...
NOTICE:  Analyzing for isolated edges. Please wait...
NOTICE:  Analyzing for ring geometries. Please wait...
NOTICE:  Analyzing for intersections. Please wait...
NOTICE:              ANALYSIS RESULTS FOR SELECTED EDGES:
NOTICE:                    Isolated segments: 158
NOTICE:                            Dead ends: 20028
NOTICE:  Potential gaps found near dead ends: 527
NOTICE:               Intersections detected: 2560
NOTICE:                      Ring geometries: 0
pgr_analyzeGraph
----------
   OK
(1 row)
```

In the vertices table "mytab_vertices_pgr":

- Deadends are identified by `cnt=1`
- Potencial gap problems are identified with `chk=1`.

```
SELECT count(*) as deadends  FROM mytab_vertices_pgr WHERE cnt = 1;
deadends
----------
    20028
 (1 row)

SELECT count(*) as gaps  FROM mytab_vertices_pgr WHERE chk = 1;
 gaps
 -----
   527
 (1 row)
```

For isolated road segments, for example, a segment where both ends are deadends. you can find these with the following query:

```
SELECT *
    FROM mytab a, mytab_vertices_pgr b, mytab_vertices_pgr c
    WHERE a.source=b.id AND b.cnt=1 AND a.target=c.id AND c.cnt=1;
```

If you want to visualize these on a graphic image, then you can use something like mapserver to render the edges and the vertices and style based on `cnt` or if they are isolated, etc. You can also do this with a tool like graphviz, or geoserver or other similar tools.

Analyze One Way Streets

pgr_analyzeOneway analyzes one way streets in a graph and identifies any flipped segments. Basically if you count the edges coming into a node and the edges exiting a node the number has to be greater than one.

This query will add two columns to the vertices_tmp table `ein int` and `eout int` and populate it with the appropriate counts. After running this on a graph you can identify nodes with potential problems with the following query.

The rules are defined as an array of text strings that if match the `col` value would be counted as true for the source or target in or out condition.

Example

Lets assume we have a table "st" of edges and a column "one_way" that might have values like:

- 'FT' - oneway from the source to the target node.
- 'TF' - oneway from the target to the source node.
- 'B' - two way street.
- '' - empty field, assume twoway.
- <NULL> - NULL field, use two_way_if_null flag.

Then we could form the following query to analyze the oneway streets for errors.

```
SELECT pgr_analyzeOneway('mytab',
            ARRAY['', 'B', 'TF'],
            ARRAY['', 'B', 'FT'],
            ARRAY['', 'B', 'FT'],
            ARRAY['', 'B', 'TF'],
            );

-- now we can see the problem nodes
SELECT * FROM mytab_vertices_pgr WHERE ein=0 OR eout=0;

-- and the problem edges connected to those nodes
SELECT gid FROM mytab a, mytab_vertices_pgr b WHERE a.source=b.id AND ein=0 OR
↪eout=0
UNION
SELECT gid FROM mytab a, mytab_vertices_pgr b WHERE a.target=b.id AND ein=0 OR
↪eout=0;
```

Typically these problems are generated by a break in the network, the one way direction set wrong, maybe an error related to z-levels or a network that is not properly noded.

The above tools do not detect all network issues, but they will identify some common problems. There are other problems that are hard to detect because they are more global in nature like multiple disconnected networks. Think of an island with a road network that is not connected to the mainland network because the bridge or ferry routes are missing.

2.1.5 Performance Tips

- *For the Routing functions*
- *For the topology functions:*

For the Routing functions

To get faster results bound your queries to the area of interest of routing to have, for example, no more than one million rows.

Use an inner query SQL that does not include some edges in the routing function

```
SELECT id, source, target from edge_table WHERE
        id < 17 and
        the_geom  && (select st_buffer(the_geom,1) as myarea FROM  edge_table␣
↪where id = 5)
```

Integrating the inner query to the pgRouting function:

```
SELECT * FROM pgr_dijkstra(
        'SELECT id, source, target from edge_table WHERE
            id < 17 and
            the_geom  && (select st_buffer(the_geom,1) as myarea FROM  edge_table␣
↪where id = 5)',
    1, 2)
```

For the topology functions:

When "you know" that you are going to remove a set of edges from the edges table, and without those edges you are going to use a routing function you can do the following:

Analize the new topology based on the actual topology:

```
pgr_analyzegraph('edge_table',rows_where:='id < 17');
```

Or create a new topology if the change is permanent:

```
pgr_createTopology('edge_table',rows_where:='id < 17');
pgr_analyzegraph('edge_table',rows_where:='id < 17');
```

2.1.6 How to contribute

Wiki

- Edit an existing pgRouting Wiki[28] page.
- Or create a new Wiki page
  - Create a page on the pgRouting Wiki[29]
  - Give the title an appropriate name
- Example[30]

[28] https://github.com/pgRouting/pgrouting/wiki
[29] https://github.com/pgRouting/pgrouting/wiki
[30] https://github.com/pgRouting/pgrouting/wiki/How-to:-Handle-parallel-edges-(KSP)

Adding Functionaity to pgRouting

Consult the developer's documentation[31]

Indices and tables

- genindex
- search

Reference

pgr_version - to get pgRouting's version information.

2.2 pgr_version

2.2.1 Name

`pgr_version` — Query for pgRouting version information.

2.2.2 Synopsis

Returns a table with pgRouting version information.

```
table() pgr_version();
```

2.2.3 Description

Returns a table with:

Column	Type	Description
version	`varchar`	pgRouting version
tag	`varchar`	Git tag of pgRouting build
hash	`varchar`	Git hash of pgRouting build
branch	`varchar`	Git branch of pgRouting build
boost	`varchar`	Boost version

History

- New in version 2.0.0

2.2.4 Examples

- Query for full version string

[31] http://docs.pgrouting.org/doxy/2.4/index.html

```
SELECT version FROM pgr_version();
 version
---------
 2.5.1
(1 row)
```

- Query for `version` and `boost` attribute

```
SELECT version, boost FROM pgr_version();

  version   | boost
-----------+--------
 2.2.0-dev | 1.49.0
(1 row)
```

2.2.5 See Also

Indices and tables

- genindex
- search

CHAPTER 3

Data Types

- *pgr_costResult[]* - A set of records to describe a path result with cost attribute.
- *pgr_costResult3[]* - A set of records to describe a path result with cost attribute.
- *pgr_geomResult* - A set of records to describe a path result with geometry attribute.

3.1 pgRouting Data Types

The following are commonly used data types for some of the pgRouting functions.

- *pgr_costResult[]* - A set of records to describe a path result with cost attribute.
- *pgr_costResult3[]* - A set of records to describe a path result with cost attribute.
- *pgr_geomResult* - A set of records to describe a path result with geometry attribute.

3.1.1 pgr_costResult[]

Name

`pgr_costResult[]` — A set of records to describe a path result with cost attribute.

Description

```
CREATE TYPE pgr_costResult AS
(
    seq integer,
    id1 integer,
    id2 integer,
    cost float8
);
```

seq sequential ID indicating the path order

id1 generic name, to be specified by the function, typically the node id

id2 generic name, to be specified by the function, typically the edge id

cost cost attribute

3.1.2 pgr_costResult3[] - Multiple Path Results with Cost

Name

`pgr_costResult3[]` — A set of records to describe a path result with cost attribute.

Description

```
CREATE TYPE pgr_costResult3 AS
(
    seq integer,
    id1 integer,
    id2 integer,
    id3 integer,
    cost float8
);
```

seq sequential ID indicating the path order

id1 generic name, to be specified by the function, typically the path id

id2 generic name, to be specified by the function, typically the node id

id3 generic name, to be specified by the function, typically the edge id

cost cost attribute

History

- New in version 2.0.0
- Replaces `path_result`

See Also

- *Introduction*

Indices and tables

- genindex
- search

3.1.3 pgr_geomResult[]

Name

`pgr_geomResult[]` — A set of records to describe a path result with geometry attribute.

Description

```
CREATE TYPE pgr_geomResult AS
(
    seq integer,
    id1 integer,
    id2 integer,
    geom geometry
);
```

seq sequential ID indicating the path order

id1 generic name, to be specified by the function

id2 generic name, to be specified by the function

geom geometry attribute

History

- New in version 2.0.0
- Replaces `geoms`

See Also

- *Introduction*

Indices and tables

- genindex
- search

3.1.4 See Also

Indices and tables

- genindex
- search

CHAPTER 4

Topology Functions

- *pgr_createTopology* - to create a topology based on the geometry.
- *pgr_createVerticesTable* - to reconstruct the vertices table based on the source and target information.
- *pgr_analyzeGraph* - to analyze the edges and vertices of the edge table.
- *pgr_analyzeOneway* - to analyze directionality of the edges.
- *pgr_nodeNetwork* -to create nodes to a not noded edge table.

4.1 Topology - Family of Functions

The pgRouting's topology of a network, represented with an edge table with source and target attributes and a vertices table associated with it. Depending on the algorithm, you can create a topology or just reconstruct the vertices table, You can analyze the topology, We also provide a function to node an unoded network.

- *pgr_createTopology* - to create a topology based on the geometry.
- *pgr_createVerticesTable* - to reconstruct the vertices table based on the source and target information.
- *pgr_analyzeGraph* - to analyze the edges and vertices of the edge table.
- *pgr_analyzeOneway* - to analyze directionality of the edges.
- *pgr_nodeNetwork* -to create nodes to a not noded edge table.

4.1.1 pgr_createTopology

Name

`pgr_createTopology` — Builds a network topology based on the geometry information.

Synopsis

The function returns:

- `OK` after the network topology has been built and the vertices table created.

- FAIL when the network topology was not built due to an error.

```
varchar pgr_createTopology(text edge_table, double precision tolerance,
                       text the_geom:='the_geom', text id:='id',
                       text source:='source',text target:='target',
                       text rows_where:='true', boolean clean:=false)
```

Description

Parameters

The topology creation function accepts the following parameters:

edge_table `text` Network table name. (may contain the schema name AS well)

tolerance `float8` Snapping tolerance of disconnected edges. (in projection unit)

the_geom `text` Geometry column name of the network table. Default value is `the_geom`.

id `text` Primary key column name of the network table. Default value is `id`.

source `text` Source column name of the network table. Default value is `source`.

target `text` Target column name of the network table. Default value is `target`.

rows_where `text` Condition to SELECT a subset or rows. Default value is `true` to indicate all rows that where `source` or `target` have a null value, otherwise the condition is used.

clean `boolean` Clean any previous topology. Default value is `false`.

Warning: The `edge_table` will be affected

- The `source` column values will change.
- The `target` column values will change.
  - An index will be created, if it doesn't exists, to speed up the process to the following columns:
    * `id`
    * `the_geom`
    * `source`
    * `target`

The function returns:

- `OK` after the network topology has been built.
  - Creates a vertices table: <edge_table>_vertices_pgr.
  - Fills `id` and `the_geom` columns of the vertices table.
  - Fills the source and target columns of the edge table referencing the `id` of the vertices table.
- `FAIL` when the network topology was not built due to an error:
  - A required column of the Network table is not found or is not of the appropriate type.
  - The condition is not well formed.
  - The names of source , target or id are the same.
  - The SRID of the geometry could not be determined.

The Vertices Table

The vertices table is a requirement of the *pgr_analyzeGraph* and the *pgr_analyzeOneway* functions.

The structure of the vertices table is:

id `bigint` Identifier of the vertex.

cnt `integer` Number of vertices in the edge_table that reference this vertex. See *pgr_analyzeGraph*.

chk `integer` Indicator that the vertex might have a problem. See *pgr_analyzeGraph*.

ein `integer` Number of vertices in the edge_table that reference this vertex AS incoming. See *pgr_analyzeOneway*.

eout `integer` Number of vertices in the edge_table that reference this vertex AS outgoing. See *pgr_analyzeOneway*.

the_geom `geometry` Point geometry of the vertex.

History

- Renamed in version 2.0.0

Usage when the edge table's columns MATCH the default values:

The simplest way to use pgr_createTopology is:

```
SELECT  pgr_createTopology('edge_table', 0.001);
NOTICE:  PROCESSING:
NOTICE:  pgr_createTopology('edge_table', 0.001, 'the_geom', 'id', 'source',
↪'target', rows_where := 'true', clean := f)
NOTICE:  Performing checks, please wait .....
NOTICE:  Creating Topology, Please wait...
NOTICE:  -------------> TOPOLOGY CREATED FOR  18 edges
NOTICE:  Rows with NULL geometry or NULL id: 0
NOTICE:  Vertices table for table public.edge_table is: public.edge_table_vertices_
↪pgr
NOTICE:  ----------------------------------------------
 pgr_createtopology
--------------------
 OK
(1 row)
```

When the arguments are given in the order described in the parameters:

We get the same result AS the simplest way to use the function.

```
SELECT  pgr_createTopology('edge_table', 0.001,
    'the_geom', 'id', 'source', 'target');
NOTICE:  PROCESSING:
NOTICE:  pgr_createTopology('edge_table', 0.001, 'the_geom', 'id', 'source',
↪'target', rows_where := 'true', clean := f)
NOTICE:  Performing checks, please wait .....
NOTICE:  Creating Topology, Please wait...
NOTICE:  -------------> TOPOLOGY CREATED FOR  18 edges
NOTICE:  Rows with NULL geometry or NULL id: 0
NOTICE:  Vertices table for table public.edge_table is: public.edge_table_vertices_
↪pgr
```

```
NOTICE:  ----------------------------------------------
 pgr_createtopology
--------------------
 OK
(1 row)
```

Warning:

An error would occur when the arguments are not given in the appropriate order:
In this example, the column `id` of the table `ege_table` is passed to the function as the geometry column, and the geometry column `the_geom` is passed to the function as the id column.

```
SELECT  pgr_createTopology('edge_table', 0.001,
    'id', 'the_geom');
NOTICE:  PROCESSING:
NOTICE:  pgr_createTopology('edge_table', 0.001, 'id', 'the_geom', 'source',
↪'target', rows_where := 'true', clean := f)
NOTICE:  Performing checks, please wait .....
NOTICE:  ----> PGR ERROR in pgr_createTopology: Wrong type of Column id:the_geom
NOTICE:  Unexpected error raise_exception
 pgr_createtopology
--------------------
 FAIL
(1 row)
```

When using the named notation

Parameters defined with a default value can be omitted, as long as the value matches the default And The order of the parameters would not matter.

```
SELECT  pgr_createTopology('edge_table', 0.001,
    the_geom:='the_geom', id:='id', source:='source', target:='target');
 pgr_createtopology
--------------------
 OK
(1 row)
```

```
SELECT  pgr_createTopology('edge_table', 0.001,
    source:='source', id:='id', target:='target', the_geom:='the_geom');
 pgr_createtopology
--------------------
 OK
(1 row)
```

```
SELECT  pgr_createTopology('edge_table', 0.001, source:='source');
 pgr_createtopology
--------------------
 OK
(1 row)
```

Selecting rows using rows_where parameter

Selecting rows based on the id.

```
SELECT  pgr_createTopology('edge_table', 0.001, rows_where:='id < 10');
 pgr_createtopology
--------------------
 OK
(1 row)
```

Selecting the rows where the geometry is near the geometry of row with `id = 5`.

```
SELECT  pgr_createTopology('edge_table', 0.001,
    rows_where:='the_geom && (SELECT st_buffer(the_geom, 0.05) FROM edge_table␣
↪WHERE id=5)');
 pgr_createtopology
--------------------
 OK
(1 row)
```

Selecting the rows where the geometry is near the geometry of the row with `gid` =100 of the table `othertable`.

```
CREATE TABLE otherTable AS  (SELECT 100 AS gid,  st_point(2.5, 2.5) AS other_geom);
SELECT 1
SELECT  pgr_createTopology('edge_table', 0.001,
    rows_where:='the_geom && (SELECT st_buffer(other_geom, 1) FROM otherTable␣
↪WHERE gid=100)');
 pgr_createtopology
--------------------
 OK
(1 row)
```

Usage when the edge table's columns DO NOT MATCH the default values:

For the following table

```
CREATE TABLE mytable AS (SELECT id AS gid,  the_geom AS mygeom, source AS src ,␣
↪target AS tgt FROM edge_table) ;
SELECT 18
```

Using positional notation:

The arguments need to be given in the order described in the parameters.

Note that this example uses clean flag. So it recreates the whole vertices table.

```
SELECT  pgr_createTopology('mytable', 0.001, 'mygeom', 'gid', 'src', 'tgt', clean␣
↪:= TRUE);
 pgr_createtopology
--------------------
 OK
(1 row)
```

Warning:

An error would occur when the arguments are not given in the appropiriate order:
In this example, the column `gid` of the table `mytable` is passed to the function AS the geometry column, and the geometry column `mygeom` is passed to the function AS the id column.

```
SELECT  pgr_createTopology('mytable', 0.001, 'gid', 'mygeom', 'src', 'tgt');
NOTICE:  PROCESSING:
NOTICE:  pgr_createTopology('mytable', 0.001, 'gid', 'mygeom', 'src', 'tgt',␣
↪rows_where := 'true', clean := f)
NOTICE:  Performing checks, please wait .....
NOTICE:  ----> PGR ERROR in pgr_createTopology: Wrong type of Column id:mygeom
NOTICE:  Unexpected error raise_exception
 pgr_createtopology
--------------------
 FAIL
(1 row)
```

When using the named notation

In this scenario omitting a parameter would create an error because the default values for the column names do not match the column names of the table. The order of the parameters do not matter:

```
SELECT  pgr_createTopology('mytable', 0.001, the_geom:='mygeom', id:='gid',␣
↪source:='src', target:='tgt');
 pgr_createtopology
--------------------
 OK
(1 row)
```

```
SELECT  pgr_createTopology('mytable', 0.001, source:='src', id:='gid', target:='tgt
↪', the_geom:='mygeom');
 pgr_createtopology
--------------------
 OK
(1 row)
```

Selecting rows using rows_where parameter

Based on id:

```
SELECT  pgr_createTopology('mytable', 0.001, 'mygeom', 'gid', 'src', 'tgt', rows_
↪where:='gid < 10');
 pgr_createtopology
--------------------
 OK
(1 row)

SELECT  pgr_createTopology('mytable', 0.001, source:='src', id:='gid', target:='tgt
↪', the_geom:='mygeom', rows_where:='gid < 10');
 pgr_createtopology
--------------------
 OK
(1 row)
```

```
SELECT  pgr_createTopology('mytable', 0.001, 'mygeom', 'gid', 'src', 'tgt',
    rows_where:='mygeom && (SELECT st_buffer(mygeom, 1) FROM mytable WHERE gid=5)
↪');
 pgr_createtopology
--------------------
 OK
(1 row)

SELECT  pgr_createTopology('mytable', 0.001, source:='src', id:='gid', target:='tgt
↪', the_geom:='mygeom',
    rows_where:='mygeom && (SELECT st_buffer(mygeom, 1) FROM mytable WHERE gid=5)
↪');
 pgr_createtopology
--------------------
 OK
(1 row)
```

Selecting the rows where the geometry is near the geometry of the row with `gid` =100 of the table `othertable`.

```
SELECT  pgr_createTopology('mytable', 0.001, 'mygeom', 'gid', 'src', 'tgt',
    rows_where:='mygeom && (SELECT st_buffer(other_geom, 1) FROM otherTable WHERE␣
↪gid=100)');
 pgr_createtopology
--------------------
 OK
(1 row)

SELECT  pgr_createTopology('mytable', 0.001, source:='src', id:='gid', target:='tgt
↪', the_geom:='mygeom',
    rows_where:='mygeom && (SELECT st_buffer(other_geom, 1) FROM otherTable WHERE␣
↪gid=100)');
 pgr_createtopology
--------------------
 OK
(1 row)
```

Examples with full output

This example start a clean topology, with 5 edges, and then its incremented to the rest of the edges.

```
SELECT pgr_createTopology('edge_table',  0.001, rows_where:='id < 6', clean :=␣
↪true);
NOTICE:  PROCESSING:
NOTICE:  pgr_createTopology('edge_table', 0.001, 'the_geom', 'id', 'source',
↪'target', rows_where := 'id < 6', clean := t)
NOTICE:  Performing checks, please wait .....
NOTICE:  Creating Topology, Please wait...
NOTICE:  -------------> TOPOLOGY CREATED FOR  5 edges
NOTICE:  Rows with NULL geometry or NULL id: 0
NOTICE:  Vertices table for table public.edge_table is: public.edge_table_vertices_
↪pgr
NOTICE:  ----------------------------------------------
 pgr_createtopology
--------------------
 OK
(1 row)

SELECT pgr_createTopology('edge_table',  0.001);
NOTICE:  PROCESSING:
```

```
NOTICE:  pgr_createTopology('edge_table', 0.001, 'the_geom', 'id', 'source',
↪'target', rows_where := 'true', clean := f)
NOTICE:  Performing checks, please wait .....
NOTICE:  Creating Topology, Please wait...
NOTICE:  -------------> TOPOLOGY CREATED FOR  13 edges
NOTICE:  Rows with NULL geometry or NULL id: 0
NOTICE:  Vertices table for table public.edge_table is: public.edge_table_vertices_
↪pgr
NOTICE:  ----------------------------------------------
 pgr_createtopology
--------------------
 OK
(1 row)
```

The example uses the *Sample Data* network.

See Also

- *Routing Topology* for an overview of a topology for routing algorithms.
- *pgr_createVerticesTable* to reconstruct the vertices table based on the source and target information.
- *pgr_analyzeGraph* to analyze the edges and vertices of the edge table.

Indices and tables

- genindex
- search

4.1.2 pgr_createVerticesTable

Name

`pgr_createVerticesTable` — Reconstructs the vertices table based on the source and target information.

Synopsis

The function returns:

- `OK` after the vertices table has been reconstructed.
- `FAIL` when the vertices table was not reconstructed due to an error.

```
    pgr_createVerticesTable(edge_table, the_geom, source, target, rows_where)
RETURNS VARCHAR
```

Description

Parameters

The reconstruction of the vertices table function accepts the following parameters:

edge_table `text` Network table name. (may contain the schema name as well)

the_geom `text` Geometry column name of the network table. Default value is `the_geom`.

source `text` Source column name of the network table. Default value is `source`.

target `text` Target column name of the network table. Default value is `target`.

rows_where `text` Condition to SELECT a subset or rows. Default value is `true` to indicate all rows.

> **Warning:** The `edge_table` will be affected
>
> - An index will be created, if it doesn't exists, to speed up the process to the following columns:
>   - `the_geom`
>   - `source`
>   - `target`

The function returns:

- `OK` after the vertices table has been reconstructed.
  - Creates a vertices table: <edge_table>_vertices_pgr.
  - Fills `id` and `the_geom` columns of the vertices table based on the source and target columns of the edge table.
- `FAIL` when the vertices table was not reconstructed due to an error.
  - A required column of the Network table is not found or is not of the appropriate type.
  - The condition is not well formed.
  - The names of source, target are the same.
  - The SRID of the geometry could not be determined.

The Vertices Table

The vertices table is a requierment of the *pgr_analyzeGraph* and the *pgr_analyzeOneway* functions.

The structure of the vertices table is:

id `bigint` Identifier of the vertex.

cnt `integer` Number of vertices in the edge_table that reference this vertex. See *pgr_analyzeGraph*.

chk `integer` Indicator that the vertex might have a problem. See *pgr_analyzeGraph*.

ein `integer` Number of vertices in the edge_table that reference this vertex as incoming. See *pgr_analyzeOneway*.

eout `integer` Number of vertices in the edge_table that reference this vertex as outgoing. See *pgr_analyzeOneway*.

the_geom `geometry` Point geometry of the vertex.

History

- Renamed in version 2.0.0

Usage when the edge table's columns MATCH the default values:

The simplest way to use pgr_createVerticesTable is:

```
SELECT  pgr_createVerticesTable('edge_table');
```

When the arguments are given in the order described in the parameters:

```
SELECT  pgr_createVerticesTable('edge_table','the_geom','source','target');
```

We get the same result as the simplest way to use the function.

Warning: An error would occur when the arguments are not given in the appropriate order: In this example, the column source column `source` of the table `mytable` is passed to the function as the geometry column, and the geometry column `the_geom` is passed to the function as the source column.

```
SELECT  pgr_createVerticesTable('edge_table','source','the_geom','target');
NOTICE:  pgr_createVerticesTable('edge_table','source','the_geom','target','true
↪')
NOTICE:  Performing checks, please wait .....
NOTICE:  ----> PGR ERROR in pgr_createVerticesTable: Wrong type of Column␣
↪source: the_geom
HINT:    ----> Expected type of the_geom is integer,smallint or bigint but USER-
↪DEFINED was found
NOTICE:  Unexpected error raise_exception
pgr_createverticestable
-------------------------
  FAIL
(1 row)
```

When using the named notation

The order of the parameters do not matter:

```
SELECT  pgr_createVerticesTable('edge_table',the_geom:='the_geom',source:='source',
↪target:='target');
```

```
SELECT  pgr_createVerticesTable('edge_table',source:='source',target:='target',the_
↪geom:='the_geom');
```

Parameters defined with a default value can be omitted, as long as the value matches the default:

```
SELECT  pgr_createVerticesTable('edge_table',source:='source');
```

Selecting rows using rows_where parameter

Selecting rows based on the id.

```
SELECT  pgr_createVerticesTable('edge_table',rows_where:='id < 10');
```

Selecting the rows where the geometry is near the geometry of row with `id` =5 .

```
SELECT  pgr_createVerticesTable('edge_table',rows_where:='the_geom && (select st_
↪buffer(the_geom,0.5) FROM edge_table WHERE id=5)');
```

Selecting the rows where the geometry is near the geometry of the row with `gid` =100 of the table `othertable`.

```
DROP TABLE IF EXISTS otherTable;
CREATE TABLE otherTable AS  (SELECT 100 AS gid, st_point(2.5,2.5) AS other_geom) ;
SELECT  pgr_createVerticesTable('edge_table',rows_where:='the_geom && (select st_
↪buffer(othergeom,0.5) FROM otherTable WHERE gid=100)');
```

Usage when the edge table's columns DO NOT MATCH the default values:

For the following table

```
DROP TABLE IF EXISTS mytable;
CREATE TABLE mytable AS (SELECT id AS gid, the_geom AS mygeom,source AS src ,
↪target AS tgt FROM edge_table) ;
```

Using positional notation:

The arguments need to be given in the order described in the parameters:

```
SELECT  pgr_createVerticesTable('mytable','mygeom','src','tgt');
```

Warning:

An error would occur when the arguments are not given in the appropriate order: In this example, the column `src` of the table `mytable` is passed to the function as the geometry column, and the geometry column `mygeom` is passed to the function as the source column.

```
SELECT  pgr_createVerticesTable('mytable','src','mygeom','tgt');
NOTICE:  PROCESSING:
NOTICE:  pgr_createVerticesTable('mytable','src','mygeom','tgt','true')
NOTICE:  Performing checks, please wait .....
NOTICE:  ----> PGR ERROR in pgr_createVerticesTable: Table mytable not found
HINT:    ----> Check your table name
NOTICE:  Unexpected error raise_exception
pgr_createverticestable
-------------------------
  FAIL
(1 row)
```

When using the named notation

The order of the parameters do not matter:

```
SELECT  pgr_createVerticesTable('mytable',the_geom:='mygeom',source:='src',target:=
↪'tgt');
```

```
SELECT  pgr_createVerticesTable('mytable',source:='src',target:='tgt',the_geom:=
↪'mygeom');
```

In this scenario omitting a parameter would create an error because the default values for the column names do not match the column names of the table.

Selecting rows using rows_where parameter

Selecting rows based on the gid.

```
SELECT  pgr_createVerticesTable('mytable','mygeom','src','tgt',rows_where:='gid <␣
↪10');
```

```
SELECT  pgr_createVerticesTable('mytable',source:='src',target:='tgt',the_geom:=
↪'mygeom',rows_where:='gid < 10');
```

Selecting the rows where the geometry is near the geometry of row with `gid` =5 .

```
SELECT  pgr_createVerticesTable('mytable','mygeom','src','tgt',
                            rows_where:='the_geom && (SELECT st_buffer(mygeom,0.5)␣
↪FROM mytable WHERE gid=5)');
```

```
SELECT  pgr_createVerticesTable('mytable',source:='src',target:='tgt',the_geom:=
↪'mygeom',
                            rows_where:='mygeom && (SELECT st_buffer(mygeom,0.5)␣
↪FROM mytable WHERE id=5)');
```

Selecting the rows where the geometry is near the geometry of the row with `gid` =100 of the table `othertable`.

```
DROP TABLE IF EXISTS otherTable;
CREATE TABLE otherTable AS  (SELECT 100 AS gid, st_point(2.5,2.5) AS other_geom) ;
SELECT  pgr_createVerticesTable('mytable','mygeom','src','tgt',
                             rows_where:='the_geom && (SELECT st_buffer(othergeom,0.
↪5) FROM otherTable WHERE gid=100)');
```

```
SELECT  pgr_createVerticesTable('mytable',source:='src',target:='tgt',the_geom:=
↪'mygeom',
                             rows_where:='the_geom && (SELECT st_buffer(othergeom,0.
↪5) FROM otherTable WHERE gid=100)');
```

Examples

```
    SELECT pgr_createVerticesTable('edge_table');
    NOTICE:  PROCESSING:
NOTICE:  pgr_createVerticesTable('edge_table','the_geom','source','target','true')
NOTICE:  Performing checks, pelase wait .....
NOTICE:  Populating public.edge_table_vertices_pgr, please wait...
NOTICE:    ----->   VERTICES TABLE CREATED WITH  17 VERTICES
NOTICE:                                         FOR   18  EDGES
NOTICE:    Edges with NULL geometry,source or target: 0
NOTICE:                              Edges processed: 18
NOTICE:  Vertices table for table public.edge_table is: public.edge_table_vertices_
↪pgr
NOTICE:  ----------------------------------------------

     pgr_createVerticesTable
    --------------------
     OK
    (1 row)
```

The example uses the *Sample Data* network.

See Also

- *Routing Topology* for an overview of a topology for routing algorithms.
- *pgr_createTopology* to create a topology based on the geometry.
- *pgr_analyzeGraph* to analyze the edges and vertices of the edge table.

- *pgr_analyzeOneway* to analyze directionality of the edges.

Indices and tables

- genindex
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4.1.3 pgr_analyzeGraph

Name

`pgr_analyzeGraph` — Analyzes the network topology.

Synopsis

The function returns:

- `OK` after the analysis has finished.
- `FAIL` when the analysis was not completed due to an error.

```
varchar pgr_analyzeGraph(text edge_table, double precision tolerance,
                    text the_geom:='the_geom', text id:='id',
                    text source:='source',text target:='target',text rows_where:=
↪'true')
```

Description

Prerequisites

The edge table to be analyzed must contain a source column and a target column filled with id's of the vertices of the segments and the corresponding vertices table <edge_table>_vertices_pgr that stores the vertices information.

- Use *pgr_createVerticesTable* to create the vertices table.
- Use *pgr_createTopology* to create the topology and the vertices table.

Parameters

The analyze graph function accepts the following parameters:

edge_table `text` Network table name. (may contain the schema name as well)

tolerance `float8` Snapping tolerance of disconnected edges. (in projection unit)

the_geom `text` Geometry column name of the network table. Default value is `the_geom`.

id `text` Primary key column name of the network table. Default value is `id`.

source `text` Source column name of the network table. Default value is `source`.

target `text` Target column name of the network table. Default value is `target`.

rows_where `text` Condition to select a subset or rows. Default value is `true` to indicate all rows.

The function returns:

- `OK` after the analysis has finished.
  - Uses the vertices table: <edge_table>_vertices_pgr.

- Fills completely the `cnt` and `chk` columns of the vertices table.
- Returns the analysis of the section of the network defined by `rows_where`

- `FAIL` when the analysis was not completed due to an error.
  - The vertices table is not found.
  - A required column of the Network table is not found or is not of the appropriate type.
  - The condition is not well formed.
  - The names of source , target or id are the same.
  - The SRID of the geometry could not be determined.

The Vertices Table

The vertices table can be created with *pgr_createVerticesTable* or *pgr_createTopology*

The structure of the vertices table is:

id `bigint` Identifier of the vertex.

cnt `integer` Number of vertices in the edge_table that reference this vertex.

chk `integer` Indicator that the vertex might have a problem.

ein `integer` Number of vertices in the edge_table that reference this vertex as incoming. See *pgr_analyzeOneway*.

eout `integer` Number of vertices in the edge_table that reference this vertex as outgoing. See *pgr_analyzeOneway*.

the_geom `geometry` Point geometry of the vertex.

History

- New in version 2.0.0

Usage when the edge table's columns MATCH the default values:

The simplest way to use pgr_analyzeGraph is:

```
SELECT  pgr_createTopology('edge_table',0.001);
SELECT  pgr_analyzeGraph('edge_table',0.001);
```

When the arguments are given in the order described in the parameters:

```
SELECT  pgr_analyzeGraph('edge_table',0.001,'the_geom','id','source','target');
```

We get the same result as the simplest way to use the function.

> **Warning:**
>
> An error would occur when the arguments are not given in the appropriate order: In this example, the column `id` of the table `mytable` is passed to the function as the geometry column, and the geometry column `the_geom` is passed to the function as the id column.

```
SELECT  pgr_analyzeGraph('edge_table',0.001,'id','the_geom','source','target');
NOTICE:  PROCESSING:
NOTICE:  pgr_analyzeGraph('edge_table',0.001,'id','the_geom','source','target',
↪'true')
NOTICE:  Performing checks, please wait ...
NOTICE:  Got function st_srid(bigint) does not exist
NOTICE:  ERROR: something went wrong when checking for SRID of id in table public.
↪edge_table
pgr_analyzegraph
------------------
 FAIL
(1 row)
```

When using the named notation

The order of the parameters do not matter:

```
SELECT  pgr_analyzeGraph('edge_table',0.001,the_geom:='the_geom',id:='id',source:=
↪'source',target:='target');
```

```
SELECT  pgr_analyzeGraph('edge_table',0.001,source:='source',id:='id',target:=
↪'target',the_geom:='the_geom');
```

Parameters defined with a default value can be omitted, as long as the value matches the default:

```
SELECT  pgr_analyzeGraph('edge_table',0.001,source:='source');
```

Selecting rows using rows_where parameter

Selecting rows based on the id. Displays the analysis a the section of the network.

```
SELECT  pgr_analyzeGraph('edge_table',0.001,rows_where:='id < 10');
```

Selecting the rows where the geometry is near the geometry of row with `id` =5 .

```
SELECT  pgr_analyzeGraph('edge_table',0.001,rows_where:='the_geom && (SELECT st_
↪buffer(the_geom,0.05) FROM edge_table WHERE id=5)');
```

Selecting the rows where the geometry is near the geometry of the row with `gid` =100 of the table `othertable`.

```
DROP TABLE IF EXISTS otherTable;
CREATE TABLE otherTable AS  (SELECT 100 AS gid, st_point(2.5,2.5) AS other_geom) ;
SELECT  pgr_analyzeGraph('edge_table',0.001,rows_where:='the_geom && (SELECT st_
↪buffer(other_geom,1) FROM otherTable WHERE gid=100)');
```

Usage when the edge table's columns DO NOT MATCH the default values:

For the following table

```
DROP TABLE IF EXISTS mytable;
CREATE TABLE mytable AS (SELECT id AS gid, source AS src ,target AS tgt , the_geom␣
↪AS mygeom FROM edge_table);
SELECT pgr_createTopology('mytable',0.001,'mygeom','gid','src','tgt');
```

Using positional notation:

The arguments need to be given in the order described in the parameters:

```
SELECT  pgr_analyzeGraph('mytable',0.001,'mygeom','gid','src','tgt');
```

Warning:

An error would occur when the arguments are not given in the appropriate order: In this example, the column `gid` of the table `mytable` is passed to the function as the geometry column, and the geometry column `mygeom` is passed to the function as the id column.

```
SELECT  pgr_analyzeGraph('mytable',0.001,'gid','mygeom','src','tgt');
NOTICE:  PROCESSING:
NOTICE:  pgr_analyzeGraph('mytable',0.001,'gid','mygeom','src','tgt','true')
NOTICE:  Performing checks, please wait ...
NOTICE:  Got function st_srid(bigint) does not exist
NOTICE:  ERROR: something went wrong when checking for SRID of gid in table public.
↪mytable
pgr_analyzegraph
------------------
  FAIL
(1 row)
```

When using the named notation

The order of the parameters do not matter:

```
SELECT  pgr_analyzeGraph('mytable',0.001,the_geom:='mygeom',id:='gid',source:='src
↪',target:='tgt');
```

```
SELECT  pgr_analyzeGraph('mytable',0.001,source:='src',id:='gid',target:='tgt',the_
↪geom:='mygeom');
```

In this scenario omitting a parameter would create an error because the default values for the column names do not match the column names of the table.

Selecting rows using rows_where parameter

Selecting rows based on the id.

```
SELECT  pgr_analyzeGraph('mytable',0.001,'mygeom','gid','src','tgt',rows_where:=
↪'gid < 10');
```

```
SELECT  pgr_analyzeGraph('mytable',0.001,source:='src',id:='gid',target:='tgt',the_
↪geom:='mygeom',rows_where:='gid < 10');
```

Selecting the rows WHERE the geometry is near the geometry of row with `id` =5 .

```
SELECT  pgr_analyzeGraph('mytable',0.001,'mygeom','gid','src','tgt',
                            rows_where:='mygeom && (SELECT st_buffer(mygeom,1) FROM␣
↪mytable WHERE gid=5)');
```

```
SELECT  pgr_analyzeGraph('mytable',0.001,source:='src',id:='gid',target:='tgt',the_
↪geom:='mygeom',
                            rows_where:='mygeom && (SELECT st_buffer(mygeom,1) FROM␣
↪mytable WHERE gid=5)');
```

Selecting the rows WHERE the geometry is near the place='myhouse' of the table `othertable`. (note the use of quote_literal)

```
DROP TABLE IF EXISTS otherTable;
CREATE TABLE otherTable AS  (SELECT 'myhouse'::text AS place, st_point(2.5,2.5) AS␣
↪other_geom) ;
SELECT  pgr_analyzeGraph('mytable',0.001,'mygeom','gid','src','tgt',
         rows_where:='mygeom && (SELECT st_buffer(other_geom,1) FROM otherTable␣
↪WHERE place='||quote_literal('myhouse')||')');
```

```
SELECT  pgr_analyzeGraph('mytable',0.001,source:='src',id:='gid',target:='tgt',the_
↪geom:='mygeom',
        rows_where:='mygeom && (SELECT st_buffer(other_geom,1) FROM otherTable␣
↪WHERE place='||quote_literal('myhouse')||')');
```

Examples

```
    SELECT  pgr_createTopology('edge_table',0.001);
    SELECT pgr_analyzeGraph('edge_table', 0.001);
    NOTICE:  PROCESSING:
    NOTICE:  pgr_analyzeGraph('edge_table',0.001,'the_geom','id','source','target',
↪'true')
    NOTICE:  Performing checks, pelase wait...
    NOTICE:  Analyzing for dead ends. Please wait...
    NOTICE:  Analyzing for gaps. Please wait...
    NOTICE:  Analyzing for isolated edges. Please wait...
    NOTICE:  Analyzing for ring geometries. Please wait...
    NOTICE:  Analyzing for intersections. Please wait...
    NOTICE:              ANALYSIS RESULTS FOR SELECTED EDGES:
    NOTICE:                    Isolated segments: 2
    NOTICE:                            Dead ends: 7
    NOTICE:  Potential gaps found near dead ends: 1
    NOTICE:               Intersections detected: 1
    NOTICE:                      Ring geometries: 0

     pgr_analyzeGraph
    --------------------
     OK
    (1 row)

    SELECT  pgr_analyzeGraph('edge_table',0.001,rows_where:='id < 10');
    NOTICE:  PROCESSING:
    NOTICE:  pgr_analyzeGraph('edge_table',0.001,'the_geom','id','source','target',
↪'id < 10')
    NOTICE:  Performing checks, pelase wait...
    NOTICE:  Analyzing for dead ends. Please wait...
    NOTICE:  Analyzing for gaps. Please wait...
    NOTICE:  Analyzing for isolated edges. Please wait...
    NOTICE:  Analyzing for ring geometries. Please wait...
    NOTICE:  Analyzing for intersections. Please wait...
    NOTICE:              ANALYSIS RESULTS FOR SELECTED EDGES:
    NOTICE:                    Isolated segments: 0
    NOTICE:                            Dead ends: 4
    NOTICE:  Potential gaps found near dead ends: 0
    NOTICE:               Intersections detected: 0
    NOTICE:                      Ring geometries: 0

     pgr_analyzeGraph
    --------------------
```

```
     OK
    (1 row)

    SELECT  pgr_analyzeGraph('edge_table',0.001,rows_where:='id >= 10');
    NOTICE:  PROCESSING:
    NOTICE:  pgr_analyzeGraph('edge_table',0.001,'the_geom','id','source','target',
↪'id >= 10')
    NOTICE:  Performing checks, pelase wait...
    NOTICE:  Analyzing for dead ends. Please wait...
    NOTICE:  Analyzing for gaps. Please wait...
    NOTICE:  Analyzing for isolated edges. Please wait...
    NOTICE:  Analyzing for ring geometries. Please wait...
    NOTICE:  Analyzing for intersections. Please wait...
    NOTICE:              ANALYSIS RESULTS FOR SELECTED EDGES:
    NOTICE:                    Isolated segments: 2
    NOTICE:                            Dead ends: 8
    NOTICE:  Potential gaps found near dead ends: 1
    NOTICE:               Intersections detected: 1
    NOTICE:                      Ring geometries: 0

     pgr_analyzeGraph
    --------------------
     OK
    (1 row)

    -- Simulate removal of edges
    SELECT pgr_createTopology('edge_table', 0.001,rows_where:='id <17');
    SELECT pgr_analyzeGraph('edge_table', 0.001);
    NOTICE:  PROCESSING:
    NOTICE:  pgr_analyzeGraph('edge_table',0.001,'the_geom','id','source','target',
↪'true')
    NOTICE:  Performing checks, pelase wait...
    NOTICE:  Analyzing for dead ends. Please wait...
    NOTICE:  Analyzing for gaps. Please wait...
    NOTICE:  Analyzing for isolated edges. Please wait...
    NOTICE:  Analyzing for ring geometries. Please wait...
    NOTICE:  Analyzing for intersections. Please wait...
    NOTICE:              ANALYSIS RESULTS FOR SELECTED EDGES:
    NOTICE:                    Isolated segments: 0
    NOTICE:                            Dead ends: 3
    NOTICE:  Potential gaps found near dead ends: 0
    NOTICE:               Intersections detected: 0
    NOTICE:                      Ring geometries: 0

     pgr_analyzeGraph
    --------------------
     OK
    (1 row)
SELECT pgr_createTopology('edge_table', 0.001,rows_where:='id <17');
NOTICE:  PROCESSING:
NOTICE:  pgr_createTopology('edge_table',0.001,'the_geom','id','source','target',
↪'id <17')
NOTICE:  Performing checks, pelase wait .....
NOTICE:  Creating Topology, Please wait...
NOTICE:  -------------> TOPOLOGY CREATED FOR  16 edges
NOTICE:  Rows with NULL geometry or NULL id: 0
NOTICE:  Vertices table for table public.edge_table is: public.edge_table_vertices_
↪pgr
NOTICE:  ----------------------------------------------

     pgr_analyzeGraph
    --------------------
     OK
```

```
    (1 row)

SELECT pgr_analyzeGraph('edge_table', 0.001);
NOTICE:  PROCESSING:
NOTICE:  pgr_analyzeGraph('edge_table',0.001,'the_geom','id','source','target',
↪'true')
NOTICE:  Performing checks, pelase wait...
NOTICE:  Analyzing for dead ends. Please wait...
NOTICE:  Analyzing for gaps. Please wait...
NOTICE:  Analyzing for isolated edges. Please wait...
NOTICE:  Analyzing for ring geometries. Please wait...
NOTICE:  Analyzing for intersections. Please wait...
NOTICE:              ANALYSIS RESULTS FOR SELECTED EDGES:
NOTICE:                    Isolated segments: 0
NOTICE:                            Dead ends: 3
NOTICE:  Potential gaps found near dead ends: 0
NOTICE:               Intersections detected: 0
NOTICE:                      Ring geometries: 0

     pgr_analyzeGraph
    --------------------
     OK
    (1 row)
```

The examples use the *Sample Data* network.

See Also

- *Routing Topology* for an overview of a topology for routing algorithms.
- *pgr_analyzeOneway* to analyze directionality of the edges.
- *pgr_createVerticesTable* to reconstruct the vertices table based on the source and target information.
- *pgr_nodeNetwork* to create nodes to a not noded edge table.

Indices and tables

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- search

4.1.4 pgr_analyzeOneway

Name

`pgr_analyzeOneway` — Analyzes oneway Sstreets and identifies flipped segments.

Synopsis

This function analyzes oneway streets in a graph and identifies any flipped segments.

```
text pgr_analyzeOneway(geom_table text,
                       text[] s_in_rules, text[] s_out_rules,
                       text[] t_in_rules, text[] t_out_rules,
                       text oneway='oneway', text source='source', text target=
↪'target',
                       boolean two_way_if_null=true);
```

Description

The analyses of one way segments is pretty simple but can be a powerful tools to identifying some the potential problems created by setting the direction of a segment the wrong way. A node is a *source* if it has edges the exit from that node and no edges enter that node. Conversely, a node is a *sink* if all edges enter the node but none exit that node. For a *source* type node it is logically impossible to exist because no vehicle can exit the node if no vehicle and enter the node. Likewise, if you had a *sink* node you would have an infinite number of vehicle piling up on this node because you can enter it but not leave it.

So why do we care if the are not feasible? Well if the direction of an edge was reversed by mistake we could generate exactly these conditions. Think about a divided highway and on the north bound lane one segment got entered wrong or maybe a sequence of multiple segments got entered wrong or maybe this happened on a round-about. The result would be potentially a *source* and/or a *sink* node.

So by counting the number of edges entering and exiting each node we can identify both *source* and *sink* nodes so that you can look at those areas of your network to make repairs and/or report the problem back to your data vendor.

Prerequisites

The edge table to be analyzed must contain a source column and a target column filled with id's of the vertices of the segments and the corresponding vertices table <edge_table>_vertices_pgr that stores the vertices information.

- Use *pgr_createVerticesTable* to create the vertices table.
- Use *pgr_createTopology* to create the topology and the vertices table.

Parameters

edge_table `text` Network table name. (may contain the schema name as well)

s_in_rules `text[]` source node **in** rules

s_out_rules `text[]` source node **out** rules

t_in_rules `text[]` target node **in** rules

t_out_rules `text[]` target node **out** rules

oneway `text` oneway column name name of the network table. Default value is `oneway`.

source `text` Source column name of the network table. Default value is `source`.

target `text` Target column name of the network table. Default value is `target`.

two_way_if_null `boolean` flag to treat oneway NULL values as bi-directional. Default value is `true`.

Note: It is strongly recommended to use the named notation. See *pgr_createVerticesTable* or *pgr_createTopology* for examples.

The function returns:

- `OK` after the analysis has finished.
  - Uses the vertices table: <edge_table>_vertices_pgr.
  - Fills completely the `ein` and `eout` columns of the vertices table.
- `FAIL` when the analysis was not completed due to an error.
  - The vertices table is not found.
  - A required column of the Network table is not found or is not of the appropriate type.

- The names of source , target or oneway are the same.

The rules are defined as an array of text strings that if match the `oneway` value would be counted as `true` for the source or target **in** or **out** condition.

The Vertices Table

The vertices table can be created with *pgr_createVerticesTable* or *pgr_createTopology*

The structure of the vertices table is:

id `bigint` Identifier of the vertex.

cnt `integer` Number of vertices in the edge_table that reference this vertex. See *pgr_analyzeGgraph*.

chk `integer` Indicator that the vertex might have a problem. See *pgr_analyzeGraph*.

ein `integer` Number of vertices in the edge_table that reference this vertex as incoming.

eout `integer` Number of vertices in the edge_table that reference this vertex as outgoing.

the_geom `geometry` Point geometry of the vertex.

History

- New in version 2.0.0

Examples

```
SELECT pgr_analyzeOneway('edge_table',
ARRAY['', 'B', 'TF'],
ARRAY['', 'B', 'FT'],
ARRAY['', 'B', 'FT'],
ARRAY['', 'B', 'TF'],
oneway:='dir');
NOTICE:  PROCESSING:
NOTICE:  pgr_analyzeGraph('edge_table','{"",B,TF}','{"",B,FT}','{"",B,FT}','{"",B,
↪TF}','dir','source','target',t)
NOTICE:  Analyzing graph for one way street errors.
NOTICE:  Analysis 25% complete ...
NOTICE:  Analysis 50% complete ...
NOTICE:  Analysis 75% complete ...
NOTICE:  Analysis 100% complete ...
NOTICE:  Found 0 potential problems in directionality

pgr_analyzeoneway
-------------------
OK
(1 row)
```

The queries use the *Sample Data* network.

See Also

- *Routing Topology* for an overview of a topology for routing algorithms.
- *Graph Analytics* for an overview of the analysis of a graph.
- *pgr_analyzeGraph* to analyze the edges and vertices of the edge table.
- *pgr_createVerticesTable* to reconstruct the vertices table based on the source and target information.

Indices and tables

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4.1.5 pgr_nodeNetwork

Name

`pgr_nodeNetwork` - Nodes an network edge table.

Author Nicolas Ribot

Copyright Nicolas Ribot, The source code is released under the MIT-X license.

Synopsis

The function reads edges from a not "noded" network table and writes the "noded" edges into a new table.

```
pgr_nodenetwork(edge_table, tolerance, id, text the_geom, table_ending, rows_where,
↪ outall)
RETURNS TEXT
```

Description

A common problem associated with bringing GIS data into pgRouting is the fact that the data is often not "noded" correctly. This will create invalid topologies, which will result in routes that are incorrect.

What we mean by "noded" is that at every intersection in the road network all the edges will be broken into separate road segments. There are cases like an over-pass and under-pass intersection where you can not traverse from the over-pass to the under-pass, but this function does not have the ability to detect and accommodate those situations.

This function reads the `edge_table` table, that has a primary key column `id` and geometry column named `the_geom` and intersect all the segments in it against all the other segments and then creates a table `edge_table_noded`. It uses the `tolerance` for deciding that multiple nodes within the tolerance are considered the same node.

Parameters

edge_table `text` Network table name. (may contain the schema name as well)

tolerance `float8` tolerance for coincident points (in projection unit)dd

id `text` Primary key column name of the network table. Default value is `id`.

the_geom `text` Geometry column name of the network table. Default value is `the_geom`.

table_ending `text` Suffix for the new table's. Default value is `noded`.

The output table will have for `edge_table_noded`

id `bigint` Unique identifier for the table

old_id `bigint` Identifier of the edge in original table

sub_id `integer` Segment number of the original edge

source `integer` Empty source column to be used with *pgr_createTopology* function

target `integer` Empty target column to be used with *pgr_createTopology* function

the geom `geometry` Geometry column of the noded network

History

- New in version 2.0.0

Example

Let's create the topology for the data in *Sample Data*

```
SELECT pgr_createTopology('edge_table', 0.001);
NOTICE:  PROCESSING:
NOTICE:  pgr_createTopology('edge_table',0.001,'the_geom','id','source','target',
↪'true')
NOTICE:  Performing checks, pelase wait .....
NOTICE:  Creating Topology, Please wait...
NOTICE:  -------------> TOPOLOGY CREATED FOR  18 edges
NOTICE:  Rows with NULL geometry or NULL id: 0
NOTICE:  Vertices table for table public.edge_table is: public.edge_table_vertices_
↪pgr
NOTICE:  ----------------------------------------------
pgr_createtopology
--------------------
 OK
(1 row)
```

Now we can analyze the network.

```
SELECT pgr_analyzegraph('edge_table', 0.001);
NOTICE:  PROCESSING:
NOTICE:  pgr_analyzeGraph('edge_table',0.001,'the_geom','id','source','target',
↪'true')
NOTICE:  Performing checks, pelase wait...
NOTICE:  Analyzing for dead ends. Please wait...
NOTICE:  Analyzing for gaps. Please wait...
NOTICE:  Analyzing for isolated edges. Please wait...
NOTICE:  Analyzing for ring geometries. Please wait...
NOTICE:  Analyzing for intersections. Please wait...
NOTICE:              ANALYSIS RESULTS FOR SELECTED EDGES:
NOTICE:                    Isolated segments: 2
NOTICE:                            Dead ends: 7
NOTICE:  Potential gaps found near dead ends: 1
NOTICE:               Intersections detected: 1
NOTICE:                      Ring geometries: 0
pgr_analyzegraph
------------------
 OK
(1 row)
```

The analysis tell us that the network has a gap and an intersection. We try to fix the problem using:

```
SELECT pgr_nodeNetwork('edge_table', 0.001);
NOTICE:  PROCESSING:
NOTICE:  pgr_nodeNetwork('edge_table',0.001,'the_geom','id','noded')
NOTICE:  Performing checks, pelase wait .....
NOTICE:  Processing, pelase wait .....
NOTICE:    Split Edges: 3
NOTICE:   Untouched Edges: 15
NOTICE:       Total original Edges: 18
NOTICE:   Edges generated: 6
NOTICE:   Untouched Edges: 15
NOTICE:         Total New segments: 21
NOTICE:   New Table: public.edge_table_noded
```

```
NOTICE:  ---------------------------------
pgr_nodenetwork
-----------------
 OK
(1 row)
```

Inspecting the generated table, we can see that edges 13,14 and 18 has been segmented

```
SELECT old_id,sub_id FROM edge_table_noded ORDER BY old_id,sub_id;
 old_id | sub_id
--------+--------
 1      |      1
 2      |      1
 3      |      1
 4      |      1
 5      |      1
 6      |      1
 7      |      1
 8      |      1
 9      |      1
 10     |      1
 11     |      1
 12     |      1
 13     |      1
 13     |      2
 14     |      1
 14     |      2
 15     |      1
 16     |      1
 17     |      1
 18     |      1
 18     |      2
(21 rows)
```

We can create the topology of the new network

```
SELECT pgr_createTopology('edge_table_noded', 0.001);
NOTICE:  PROCESSING:
NOTICE:  pgr_createTopology('edge_table_noded',0.001,'the_geom','id','source',
↪'target','true')
NOTICE:  Performing checks, pelase wait .....
NOTICE:  Creating Topology, Please wait...
NOTICE:  -------------> TOPOLOGY CREATED FOR  21 edges
NOTICE:  Rows with NULL geometry or NULL id: 0
NOTICE:  Vertices table for table public.edge_table_noded is: public.edge_table_
↪noded_vertices_pgr
NOTICE:  ----------------------------------------------
pgr_createtopology
--------------------
 OK
(1 row)
```

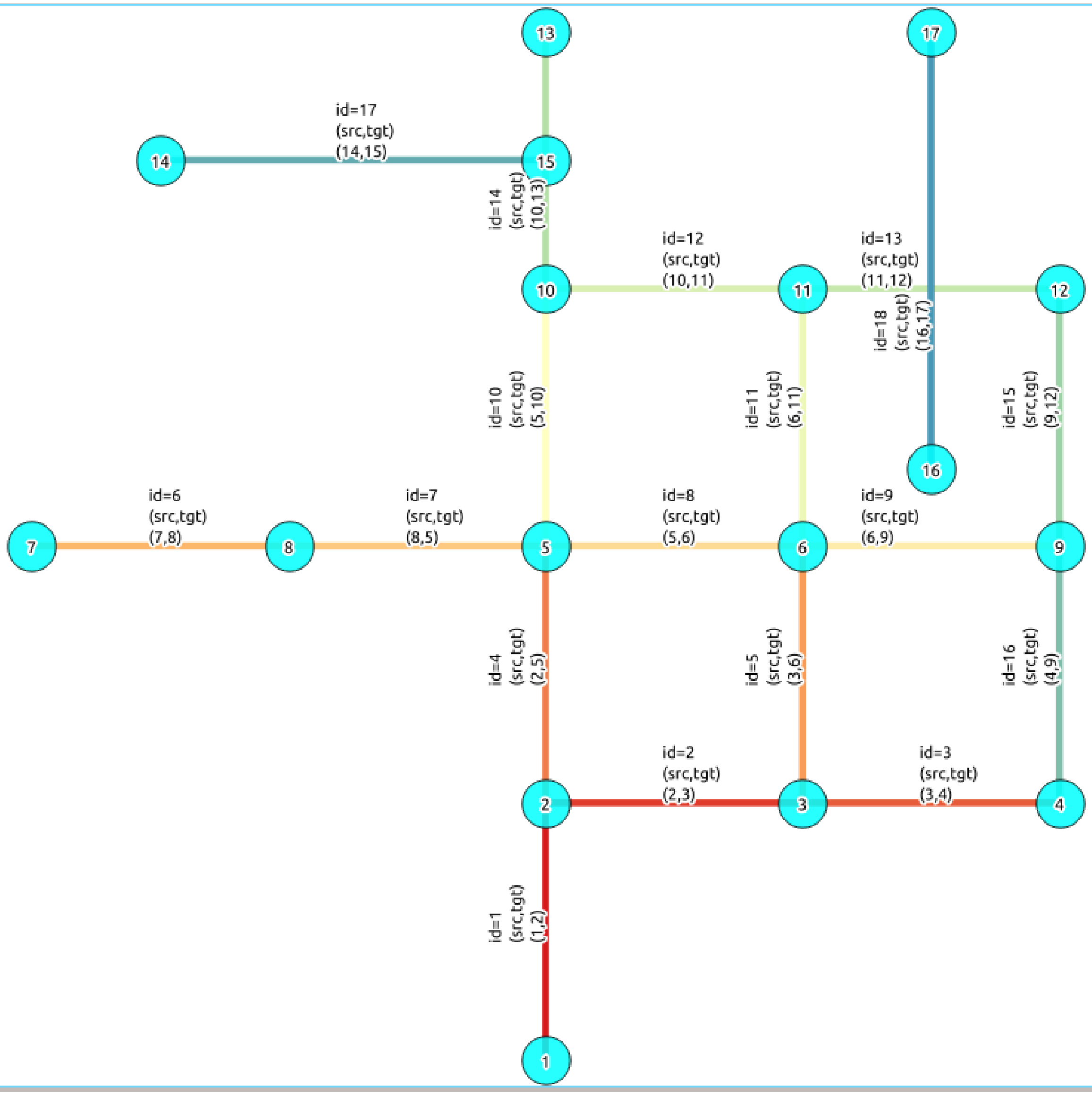
Now let's analyze the new topology

```
SELECT pgr_analyzegraph('edge_table_noded', 0.001);
NOTICE:  PROCESSING:
NOTICE:  pgr_analyzeGraph('edge_table_noded',0.001,'the_geom','id','source','target
↪','true')
NOTICE:  Performing checks, pelase wait...
NOTICE:  Analyzing for dead ends. Please wait...
NOTICE:  Analyzing for gaps. Please wait...
NOTICE:  Analyzing for isolated edges. Please wait...
```

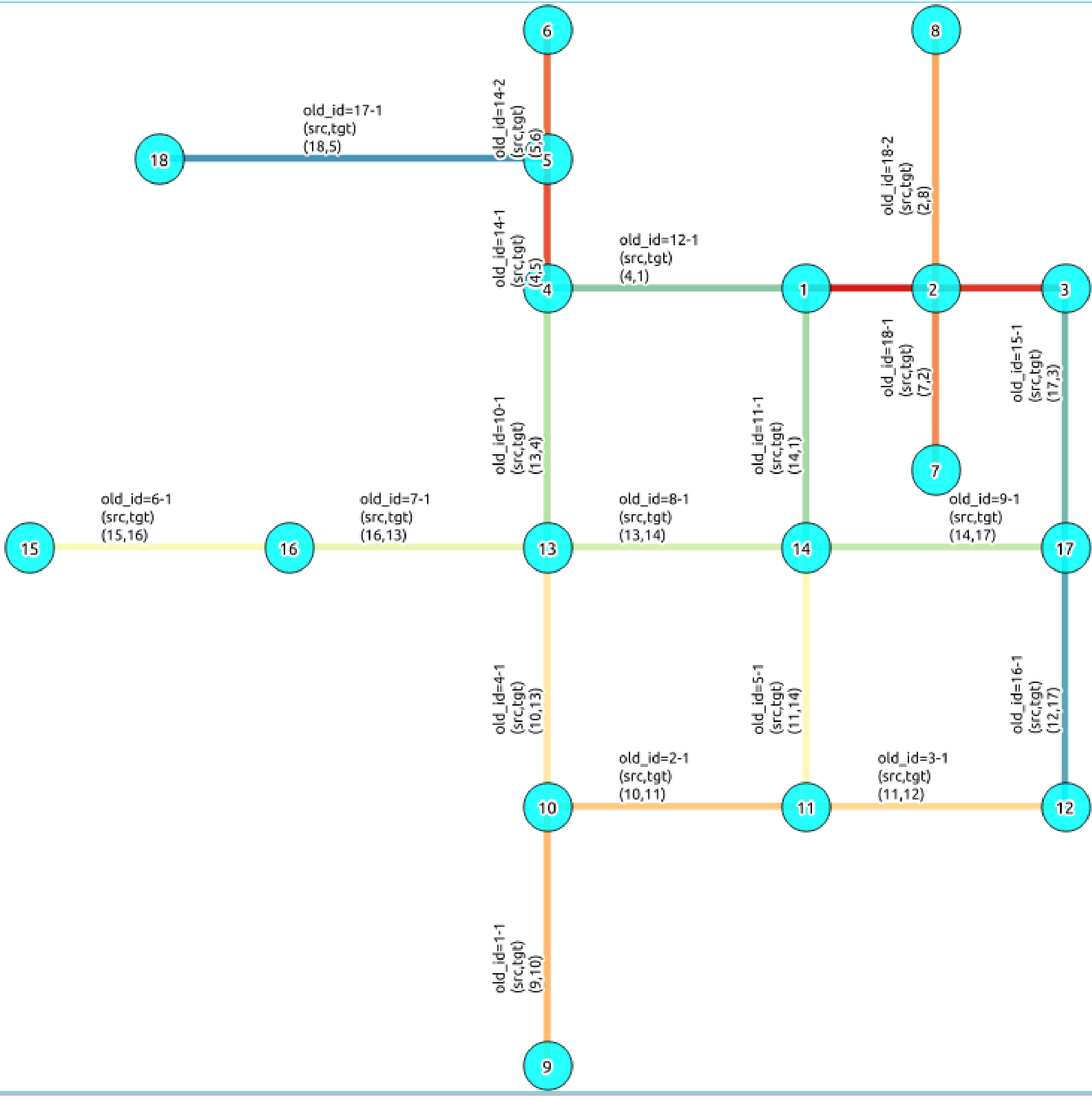
```
NOTICE:  Analyzing for ring geometries. Please wait...
NOTICE:  Analyzing for intersections. Please wait...
NOTICE:              ANALYSIS RESULTS FOR SELECTED EDGES:
NOTICE:                    Isolated segments: 0
NOTICE:                            Dead ends: 6
NOTICE:  Potential gaps found near dead ends: 0
NOTICE:               Intersections detected: 0
NOTICE:                      Ring geometries: 0
pgr_createtopology
--------------------
 OK
(1 row)
```

Images

Before Image



After Image



Comparing the results

Comparing with the Analysis in the original edge_table, we see that.

	Before	After
Table name	edge_table	edge_table_noded
Fields	All original fields	Has only basic fields to do a topology analysis
Dead ends	• Edges with 1 dead end: 1,6,24 • Edges with 2 dead ends 17,18 Edge 17's right node is a dead end because there is no other edge sharing that same node. (cnt=1)	Edges with 1 dead end: 1-1 ,6-1,14-2, 18-1 17-1 18-2
Isolated segments	two isolated segments: 17 and 18 both they have 2 dead ends	**No Isolated segments** • Edge 17 now shares a node with edges 14-1 and 14-2 • Edges 18-1 and 18-2 share a node with edges 13-1 and 13-2
Gaps	There is a gap between edge 17 and 14 because edge 14 is near to the right node of edge 17	Edge 14 was segmented Now edges: 14-1 14-2 17 share the same node The tolerance value was taken in account
Intersections	Edges 13 and 18 were intersecting	Edges were segmented, So, now in the interection's point there is a node and the following edges share it: 13-1 13-2 18-1 18-2

Now, we are going to include the segments 13-1, 13-2 14-1, 14-2 ,18-1 and 18-2 into our edge-table, copying the data for dir,cost,and reverse cost with tho following steps:

- Add a column old_id into edge_table, this column is going to keep track the id of the original edge
- Insert only the segmented edges, that is, the ones whose max(sub_id) >1

```
alter table edge_table drop column if exists old_id;
alter table edge_table add column old_id integer;
insert into edge_table (old_id,dir,cost,reverse_cost,the_geom)
        (with
        segmented as (select old_id,count(*) as i from edge_table_noded group by␣
↪old_id)
        select  segments.old_id,dir,cost,reverse_cost,segments.the_geom
                from edge_table as edges join edge_table_noded as segments on␣
↪(edges.id = segments.old_id)
                where edges.id in (select old_id from segmented where i>1) );
```

We recreate the topology:

```
SELECT pgr_createTopology('edge_table', 0.001);

NOTICE:  PROCESSING:
NOTICE:  pgr_createTopology('edge_table',0.001,'the_geom','id','source','target',
↪'true')
NOTICE:  Performing checks, pelase wait .....
NOTICE:  Creating Topology, Please wait...
NOTICE:  -------------> TOPOLOGY CREATED FOR  24 edges
NOTICE:  Rows with NULL geometry or NULL id: 0
NOTICE:  Vertices table for table public.edge_table is: public.edge_table_vertices_
↪pgr
```

```
NOTICE:  ---------------------------------------------
pgr_createtopology
--------------------
OK
(1 row)
```

To get the same analysis results as the topology of edge_table_noded, we do the following query:

```
SELECT pgr_analyzegraph('edge_table', 0.001,rows_where:='id not in (select old_id␣
↪from edge_table where old_id is not null)');

NOTICE:  PROCESSING:
NOTICE:  pgr_analyzeGraph('edge_table',0.001,'the_geom','id','source','target',
                           'id not in (select old_id from edge_table where old_id␣
↪is not null)')
NOTICE:  Performing checks, pelase wait...
NOTICE:  Analyzing for dead ends. Please wait...
NOTICE:  Analyzing for gaps. Please wait...
NOTICE:  Analyzing for isolated edges. Please wait...
NOTICE:  Analyzing for ring geometries. Please wait...
NOTICE:  Analyzing for intersections. Please wait...
NOTICE:              ANALYSIS RESULTS FOR SELECTED EDGES:
NOTICE:                    Isolated segments: 0
NOTICE:                            Dead ends: 6
NOTICE:  Potential gaps found near dead ends: 0
NOTICE:               Intersections detected: 0
NOTICE:                      Ring geometries: 0
pgr_createtopology
--------------------
OK
(1 row)
```

To get the same analysis results as the original edge_table, we do the following query:

```
SELECT pgr_analyzegraph('edge_table', 0.001,rows_where:='old_id is null')

NOTICE:  PROCESSING:
NOTICE:  pgr_analyzeGraph('edge_table',0.001,'the_geom','id','source','target',
↪'old_id is null')
NOTICE:  Performing checks, pelase wait...
NOTICE:  Analyzing for dead ends. Please wait...
NOTICE:  Analyzing for gaps. Please wait...
NOTICE:  Analyzing for isolated edges. Please wait...
NOTICE:  Analyzing for ring geometries. Please wait...
NOTICE:  Analyzing for intersections. Please wait...
NOTICE:              ANALYSIS RESULTS FOR SELECTED EDGES:
NOTICE:                    Isolated segments: 2
NOTICE:                            Dead ends: 7
NOTICE:  Potential gaps found near dead ends: 1
NOTICE:               Intersections detected: 1
NOTICE:                      Ring geometries: 0
pgr_createtopology
--------------------
OK
(1 row)
```

Or we can analyze everything because, maybe edge 18 is an overpass, edge 14 is an under pass and there is also a street level juction, and the same happens with edges 17 and 13.

```
SELECT pgr_analyzegraph('edge_table', 0.001);

NOTICE:  PROCESSING:
```

```
NOTICE:  pgr_analyzeGraph('edge_table',0.001,'the_geom','id','source','target',
↪'true')
NOTICE:  Performing checks, pelase wait...
NOTICE:  Analyzing for dead ends. Please wait...
NOTICE:  Analyzing for gaps. Please wait...
NOTICE:  Analyzing for isolated edges. Please wait...
NOTICE:  Analyzing for ring geometries. Please wait...
NOTICE:  Analyzing for intersections. Please wait...
NOTICE:              ANALYSIS RESULTS FOR SELECTED EDGES:
NOTICE:                    Isolated segments: 0
NOTICE:                            Dead ends: 3
NOTICE:  Potential gaps found near dead ends: 0
NOTICE:               Intersections detected: 5
NOTICE:                      Ring geometries: 0
pgr_createtopology
--------------------
OK
(1 row)
```

See Also

Routing Topology for an overview of a topology for routing algorithms. *pgr_analyzeOneway* to analyze directionality of the edges. *pgr_createTopology* to create a topology based on the geometry. *pgr_analyzeGraph* to analyze the edges and vertices of the edge table.

Indices and tables

- genindex
- search

4.1.6 See Also

Indices and tables

- genindex
- search

CHAPTER 5

Routing functions

5.1 Routing Functions

All Pairs - Family of Functions

- *pgr_floydWarshall* - Floyd-Warshall's Algorithm
- *pgr_johnson*- Johnson's Algorithm

pgr_aStar - Shortest Path A*

pgr_bdAstar - Bi-directional A* Shortest Path

pgr_bdDijkstra - Bi-directional Dijkstra Shortest Path

Dijkstra - Family of functions

- *pgr_dijkstra* - Dijkstra's algorithm for the shortest paths.
- *pgr_dijkstraCost* - Get the aggregate cost of the shortest paths.
- *pgr_dijkstraCostMatrix - proposed* - Use pgr_dijkstra to create a costs matrix.
- *pgr_drivingDistance* - Use pgr_dijkstra to calculate catchament information.
- *pgr_KSP* - Use Yen algorithm with pgr_dijkstra to get the K shortest paths.
- *pgr_dijkstraVia - Proposed* - Get a route of a seuence of vertices.

pgr_KSP - K-Shortest Path

pgr_trsp - Turn Restriction Shortest Path (TRSP)

Traveling Sales Person - Family of functions

- *pgr_TSP* - When input is given as matrix cell information.
- *pgr_eucledianTSP* - When input are coordinates.

Driving Distance - Category

- *pgr_drivingDistance* - Driving Distance based on pgr_dijkstra
- *pgr_withPointsDD - Proposed* - Driving Distance based on pgr_withPoints
- Post pocessing

  - *pgr_alphaShape* - Alpha shape computation
  - *pgr_pointsAsPolygon* - Polygon around a set of points

5.1.1 All Pairs - Family of Functions

The following functions work an all vertices pair combinations

pgr_floydWarshall

Synopsis

`pgr_floydWarshall` - Returns the sum of the costs of the shortest path for each pair of nodes in the graph using Floyd-Warshall algorithm.

[32]

Fig. 5.1: Boost Graph Inside

Availability: 2.0.0

- Renamed on 2.2.0, previous name pgr_apspWarshall

The Floyd-Warshall algorithm, also known as Floyd's algorithm, is a good choice to calculate the sum of the costs of the shortest path for each pair of nodes in the graph, for *dense graphs*. We make use of the Boost's implementation which runs in $\Theta(V^3)$ time,

Characteristics

The main Characteristics are:

- It does not return a path.
- Returns the sum of the costs of the shortest path for each pair of nodes in the graph.
- Process is done only on edges with positive costs.
- Boost returns a $V \times V$ matrix, where the infinity values. Represent the distance between vertices for which there is no path.
  - We return only the non infinity values in form of a set of *(start_vid, end_vid, agg_cost)*.
- Let be the case the values returned are stored in a table, so the unique index would be the pair: *(start_vid, end_vid)*.
- For the undirected graph, the results are symmetric.
  - The *agg_cost* of *(u, v)* is the same as for *(v, u)*.
- When *start_vid* = *end_vid*, the *agg_cost* = 0.
- **Recommended, use a bounding box of no more than 3500 edges.**

[32] http://www.boost.org/libs/graph/doc/floyd_warshall_shortest.html

Signature Summary

```
pgr_floydWarshall(edges_sql)
pgr floydWarshall(edges_sql, directed)
RETURNS SET OF (start_vid, end_vid,  agg_cost) or EMPTY SET
```

Signatures

Minimal Signature

```
pgr_floydWarshall(edges_sql)
RETURNS SET OF (start_vid, end_vid,  agg_cost) or EMPTY SET
```

Example 1 On a directed graph.

```
SELECT * FROM pgr_floydWarshall(
    'SELECT id, source, target, cost FROM edge_table where id < 5'
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         1 |       2 |        1
         1 |       5 |        2
         2 |       5 |        1
(3 rows)
```

Complete Signature

```
pgr_floydWarshall(edges_sql, directed)
RETURNS SET OF (start_vid, end_vid,  agg_cost) or EMPTY SET
```

Example 2 On an undirected graph.

```
SELECT * FROM pgr_floydWarshall(
    'SELECT id, source, target, cost FROM edge_table where id < 5',
    false
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         1 |       2 |        1
         1 |       5 |        2
         2 |       1 |        1
         2 |       5 |        1
         5 |       1 |        2
         5 |       2 |        1
(6 rows)
```

Description of the Signatures

Description of the edges_sql query (id is not necessary)

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Receives `(edges_sql, directed)`

Parameter	Type	Description
edges_sql	`TEXT`	SQL query as described above.
directed	`BOOLEAN`	(optional) Default is true (is directed). When set to false the graph is considered as Undirected

Description of the return values

Returns set of `(start_vid, end_vid, agg_cost)`

Column	Type	Description
start_vid	`BIGINT`	Identifier of the starting vertex.
end_vid	`BIGINT`	Identifier of the ending vertex.
agg_cost	`FLOAT`	Total cost from `start_vid` to `end_vid`.

History

- Re-design of pgr_apspWarshall in Version 2.2.0

See Also

- *pgr_johnson*
- Boost floyd-Warshall[33] algorithm
- Queries uses the *Sample Data* network.

Indices and tables

- genindex
- search

pgr_johnson

Synopsis

`pgr_johnson` - Returns the sum of the costs of the shortest path for each pair of nodes in the graph using Floyd-Warshall algorithm.

[34]

Fig. 5.2: Boost Graph Inside

Availability: 2.0.0

- Renamed on 2.2.0, previous name pgr_apspJohnson

The Johnson algorithm, is a good choice to calculate the sum of the costs of the shortest path for each pair of nodes in the graph, for *sparse graphs*. It usees the Boost's implementation which runs in $O(V E \log V)$ time,

Characteristics

The main Characteristics are:

- It does not return a path.
- Returns the sum of the costs of the shortest path for each pair of nodes in the graph.
- Process is done only on edges with positive costs.
- Boost returns a $V \times V$ matrix, where the infinity values. Represent the distance between vertices for which there is no path.
  - We return only the non infinity values in form of a set of *(start_vid, end_vid, agg_cost)*.
- Let be the case the values returned are stored in a table, so the unique index would be the pair: *(start_vid, end_vid)*.
- For the undirected graph, the results are symmetric.
  - The *agg_cost* of *(u, v)* is the same as for *(v, u)*.
- When *start_vid* = *end_vid*, the *agg_cost* = 0.

[33] http://www.boost.org/libs/graph/doc/floyd_warshall_shortest.html
[34] http://www.boost.org/libs/graph/doc/johnson_all_pairs_shortest.html

Signature Summary

```
pgr_johnson(edges_sql)
pgr johnson(edges_sql, directed)
RETURNS SET OF (start_vid, end_vid,  agg_cost) or EMPTY SET
```

Signatures

Minimal Signature

```
pgr_johnson(edges_sql)
RETURNS SET OF (start_vid, end_vid,  agg_cost) or EMPTY SET
```

Example 1 On a directed graph.

```
SELECT * FROM pgr_johnson(
    'SELECT source, target, cost FROM edge_table WHERE id < 5
         ORDER BY id'
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         1 |       2 |        1
         1 |       5 |        2
         2 |       5 |        1
(3 rows)
```

Complete Signature

```
pgr_johnson(edges_sql, directed)
RETURNS SET OF (start_vid, end_vid,  agg_cost) or EMPTY SET
```

Example 2 On an undirected graph.

```
SELECT * FROM pgr_johnson(
    'SELECT source, target, cost FROM edge_table WHERE id < 5
         ORDER BY id',
    false
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         1 |       2 |        1
         1 |       5 |        2
         2 |       1 |        1
         2 |       5 |        1
         5 |       1 |        2
         5 |       2 |        1
(6 rows)
```

Description of the Signatures

Description of the edges_sql query (id is not necessary)

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Receives `(edges_sql, directed)`

Parameter	Type	Description
edges_sql	`TEXT`	SQL query as described above.
directed	`BOOLEAN`	(optional) Default is true (is directed). When set to false the graph is considered as Undirected

Description of the return values

Returns set of `(start_vid, end_vid, agg_cost)`

Column	Type	Description
start_vid	`BIGINT`	Identifier of the starting vertex.
end_vid	`BIGINT`	Identifier of the ending vertex.
agg_cost	`FLOAT`	Total cost from `start_vid` to `end_vid`.

History

- Re-design of pgr_apspJohnson in Version 2.2.0

See Also

- *pgr_floydWarshall*
- Boost Johnson[35] algorithm implementation.
- Queries uses the *Sample Data* network.

Indices and tables

- genindex
- search

Performance

The following tests:

- non server computer
- with AMD 64 CPU
- 4G memory
- trusty
- posgreSQL version 9.3

Data

The following data was used

```
BBOX="-122.8,45.4,-122.5,45.6"
wget --progress=dot:mega -O "sampledata.osm" "http://www.overpass-api.de/api/xapi?
↪*[bbox=][@meta]"
```

Data processing was done with osm2pgrouting-alpha

```
createdb portland
psql -c "create extension postgis" portland
psql -c "create extension pgrouting" portland
osm2pgrouting -f sampledata.osm -d portland -s 0
```

Results

Test One

This test is not with a bounding box The density of the passed graph is extremely low. For each <SIZE> 30 tests were executed to get the average The tested query is:

```
SELECT count(*) FROM pgr_floydWarshall(
   'SELECT gid as id, source, target, cost, reverse_cost FROM ways where id <=
↪<SIZE>');

SELECT count(*) FROM pgr_johnson(
   'SELECT gid as id, source, target, cost, reverse_cost FROM ways where id <=
↪<SIZE>');
```

[35] http://www.boost.org/libs/graph/doc/johnson_all_pairs_shortest.html

The results of this tests are presented as:

SIZE is the number of edges given as input.

EDGES is the total number of records in the query.

DENSITY is the density of the data $\dfrac{E}{V \times (V-1)}$.

OUT ROWS is the number of records returned by the queries.

Floyd-Warshall is the average execution time in seconds of pgr_floydWarshall.

Johnson is the average execution time in seconds of pgr_johnson.

SIZE	EDGES	DENSITY	OUT ROWS	Floyd-Warshall	Johnson
500	500	0.18E-7	1346	0.14	0.13
1000	1000	0.36E-7	2655	0.23	0.18
1500	1500	0.55E-7	4110	0.37	0.34
2000	2000	0.73E-7	5676	0.56	0.37
2500	2500	0.89E-7	7177	0.84	0.51
3000	3000	1.07E-7	8778	1.28	0.68
3500	3500	1.24E-7	10526	2.08	0.95
4000	4000	1.41E-7	12484	3.16	1.24
4500	4500	1.58E-7	14354	4.49	1.47
5000	5000	1.76E-7	16503	6.05	1.78
5500	5500	1.93E-7	18623	7.53	2.03
6000	6000	2.11E-7	20710	8.47	2.37
6500	6500	2.28E-7	22752	9.99	2.68
7000	7000	2.46E-7	24687	11.82	3.12
7500	7500	2.64E-7	26861	13.94	3.60
8000	8000	2.83E-7	29050	15.61	4.09
8500	8500	3.01E-7	31693	17.43	4.63
9000	9000	3.17E-7	33879	19.19	5.34
9500	9500	3.35E-7	36287	20.77	6.24
10000	10000	3.52E-7	38491	23.26	6.51

Test Two

This test is with a bounding box The density of the passed graph higher than of the Test One. For each <SIZE> 30 tests were executed to get the average The tested edge query is:

```
WITH
    buffer AS (SELECT ST_Buffer(ST_Centroid(ST_Extent(the_geom)), SIZE) AS geom␣
↪FROM ways),
    bbox AS (SELECT ST_Envelope(ST_Extent(geom)) as box from buffer)
SELECT gid as id, source, target, cost, reverse_cost FROM ways where the_geom &&␣
↪(SELECT box from bbox);
```

The tested queries

```
SELECT count(*) FROM pgr_floydWarshall(<edge query>)
SELECT count(*) FROM pgr_johnson(<edge query>)
```

The results of this tests are presented as:

SIZE is the size of the bounding box.

EDGES is the total number of records in the query.

DENSITY is the density of the data $\dfrac{E}{V \times (V-1)}$.

OUT ROWS is the number of records returned by the queries.

Floyd-Warshall is the average execution time in seconds of pgr_floydWarshall.

Johnson is the average execution time in seconds of pgr_johnson.

SIZE	EDGES	DENSITY	OUT ROWS	Floyd-Warshall	Johnson
0.001	44	0.0608	1197	0.10	0.10
0.002	99	0.0251	4330	0.10	0.10
0.003	223	0.0122	18849	0.12	0.12
0.004	358	0.0085	71834	0.16	0.16
0.005	470	0.0070	116290	0.22	0.19
0.006	639	0.0055	207030	0.37	0.27
0.007	843	0.0043	346930	0.64	0.38
0.008	996	0.0037	469936	0.90	0.49
0.009	1146	0.0032	613135	1.26	0.62
0.010	1360	0.0027	849304	1.87	0.82
0.011	1573	0.0024	1147101	2.65	1.04
0.012	1789	0.0021	1483629	3.72	1.35
0.013	1975	0.0019	1846897	4.86	1.68
0.014	2281	0.0017	2438298	7.08	2.28
0.015	2588	0.0015	3156007	10.28	2.80
0.016	2958	0.0013	4090618	14.67	3.76
0.017	3247	0.0012	4868919	18.12	4.48

See Also

- *pgr_johnson*
- *pgr_floydWarshall*
- Boost floyd-Warshall[36] algorithm

Indices and tables

- genindex
- search

5.1.2 pgr_bdAstar

Name

`pgr_bdAstar` — Returns the shortest path using A* algorithm.

[37]

Fig. 5.3: Boost Graph Inside

[36] http://www.boost.org/libs/graph/doc/floyd_warshall_shortest.html
[37] http://www.boost.org//libs/graph

Availability:

- pgr_bdAstar(one to one) 2.0.0, Signature change on 2.5.0
- pgr_bdAstar(other signatures) 2.5.0

Signature Summary

```
pgr_bdAstar(edges_sql, start_vid, end_vid)
pgr_bdAstar(edges_sql, start_vid, end_vid, directed [, heuristic, factor, epsilon])
RETURNS SET OF (seq, path_seq , node, edge, cost, agg_cost)
  OR EMPTY SET
```

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.
  - pgTap tests might be missing.
  - Might need c/c++ coding.
  - May lack documentation.
  - Documentation if any might need to be rewritten.
  - Documentation examples might need to be automatically generated.
  - Might need a lot of feedback from the comunity.
  - Might depend on a proposed function of pgRouting
  - Might depend on a deprecated function of pgRouting

```
pgr_bdAstar(edges_sql, start_vid, end_vids [, directed, heuristic, factor,␣
↪epsilon])
pgr_bdAstar(edges_sql, start_vids, end_vid [, directed, heuristic, factor,␣
↪epsilon])
pgr_bdAstar(edges_sql, start_vids, end_vids [, directed, heuristic, factor,␣
↪epsilon])

RETURNS SET OF (seq, path_seq [, start_vid] [, end_vid], node, edge, cost, agg_
↪cost)
OR EMPTY SET
```

Using these signatures, will load once the graph and perform several one to one *pgr_bdAstar*

- The result is the union of the results of the one to one *pgr_bdAStar*.
- The extra `start_vid` and/or `end_vid` in the result is used to distinguish to which path it belongs.

Avaliability

- pgr_bdAstar(one to one) 2.0, signature change on 2.5

- pgr_bdAstar(other signatures) 2.5

Signatures

Minimal Signature

```
pgr_bdAstar(edges_sql, start_vid, end_vid)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost)
```

This usage finds the shortest path from the `start_vid` to the `end_vid`

- on a **directed** graph
- with **heuristic**'s value 5
- with **factor**'s value 1
- with **epsilon**'s value 1

Example Using the defaults

```
SELECT * FROM pgr_bdAstar(
    'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
    FROM edge_table',
    2, 3
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    4 |    1 |        0
   2 |        2 |    5 |    8 |    1 |        1
   3 |        3 |    6 |    9 |    1 |        2
   4 |        4 |    9 |   16 |    1 |        3
   5 |        5 |    4 |    3 |    1 |        4
   6 |        6 |    3 |   -1 |    0 |        5
(6 rows)
```

pgr_bdAstar One to One

```
pgr_bdAstar(edges_sql, start_vid, end_vid, directed [, heuristic, factor, epsilon])
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost)
```

This usage finds the shortest path from the `start_vid` to the `end_vid` allowing the user to choose

- **heuristic**,
- and/or **factor**
- and/or **epsilon**.

Note: In the One to One signature, because of the deprecated signature existence, it is compulsory to indicate if the graph is **directed** or **undirected**.

Example Directed using Heuristic 2

```
SELECT * FROM pgr_bdAstar(
    'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
    FROM edge_table',
    2, 3,
    true, heuristic := 2
```

```
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    4 |    1 |        0
   2 |        2 |    5 |    8 |    1 |        1
   3 |        3 |    6 |    9 |    1 |        2
   4 |        4 |    9 |   16 |    1 |        3
   5 |        5 |    4 |    3 |    1 |        4
   6 |        6 |    3 |   -1 |    0 |        5
(6 rows)
```

pgr_bdAstar One to many

```
pgr_bdAstar(edges_sql, start_vid, end_vids [, directed, heuristic, factor,␣
↪epsilon])
RETURNS SET OF (seq, path_seq, end_vid, node, edge, cost, agg_cost) or EMPTY SET
```

This usage finds the shortest path from the `start_vid` to each `end_vid` in `end_vids` allowing the user to choose

- if the graph is **directed** or **undirected**
- and/or **heuristic**,
- and/or **factor**
- and/or **epsilon**.

Example Directed using Heuristic 3 and a factor of 3.5

```
SELECT * FROM pgr_bdAstar(
    'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
    FROM edge_table',
    2, ARRAY[3, 11],
    heuristic := 3, factor := 3.5
);
 seq | path_seq | end_vid | node | edge | cost | agg_cost
-----+----------+---------+------+------+------+----------
   1 |        1 |       3 |    2 |    4 |    1 |        0
   2 |        2 |       3 |    5 |    8 |    1 |        1
   3 |        3 |       3 |    6 |    9 |    1 |        2
   4 |        4 |       3 |    9 |   16 |    1 |        3
   5 |        5 |       3 |    4 |    3 |    1 |        4
   6 |        6 |       3 |    3 |   -1 |    0 |        5
   7 |        1 |      11 |    2 |    4 |    1 |        0
   8 |        2 |      11 |    5 |    8 |    1 |        1
   9 |        3 |      11 |    6 |   11 |    1 |        2
  10 |        4 |      11 |   11 |   -1 |    0 |        3
(10 rows)
```

pgr_bdAstar Many to One

```
pgr_bdAstar(edges_sql, start_vids, end_vid [, directed, heuristic, factor,␣
↪epsilon])
RETURNS SET OF (seq, path_seq, start_vid, node, edge, cost, agg_cost) or EMPTY SET
```

This usage finds the shortest path from each `start_vid` in `start_vids` to the `end_vid` allowing the user to choose

- if the graph is **directed** or **undirected**
- and/or **heuristic**,
- and/or **factor**
- and/or **epsilon**.

Example Undirected graph with Heuristic 4

```
SELECT * FROM pgr_bdAstar(
    'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
    FROM edge_table',
    ARRAY[2, 7], 3,
    false, heuristic := 4
);
 seq | path_seq | start_vid | node | edge | cost | agg_cost
-----+----------+-----------+------+------+------+----------
   1 |        1 |         2 |    2 |    2 |    1 |        0
   2 |        2 |         2 |    3 |   -1 |    0 |        1
   3 |        1 |         7 |    7 |    6 |    1 |        0
   4 |        2 |         7 |    8 |    7 |    1 |        1
   5 |        3 |         7 |    5 |    4 |    1 |        2
   6 |        4 |         7 |    2 |    2 |    1 |        3
   7 |        5 |         7 |    3 |   -1 |    0 |        4
(7 rows)
```

pgr_bdAstar Many to Many

```
pgr_bdAstar(edges_sql, start_vids, end_vids [, directed, heuristic, factor,␣
↪epsilon])
RETURNS SET OF (seq, path_seq, start_vid, end_vid, node, edge, cost, agg_cost) or␣
↪EMPTY SET
```

This usage finds the shortest path from each `start_vid` in `start_vids` to each `end_vid` in `end_vids` allowing the use

- if the graph is **directed** or **undirected**
- and/or **heuristic**,
- and/or **factor**
- and/or **epsilon**.

Example Directed graph with a factor of 0.5

```
SELECT * FROM pgr_bdAstar(
    'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
    FROM edge_table',
    ARRAY[2, 7], ARRAY[3, 11],
    factor := 0.5
);
 seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
-----+----------+-----------+---------+------+------+------+----------
   1 |        1 |         2 |       3 |    2 |    4 |    1 |        0
   2 |        2 |         2 |       3 |    5 |    8 |    1 |        1
   3 |        3 |         2 |       3 |    6 |    9 |    1 |        2
   4 |        4 |         2 |       3 |    9 |   16 |    1 |        3
   5 |        5 |         2 |       3 |    4 |    3 |    1 |        4
   6 |        6 |         2 |       3 |    3 |   -1 |    0 |        5
   7 |        1 |         2 |      11 |    2 |    4 |    1 |        0
   8 |        2 |         2 |      11 |    5 |    8 |    1 |        1
```

```
  9 |         3 |         2 |       11 |      6 |     11 |      1 |           2
 10 |         4 |         2 |       11 |     11 |     -1 |      0 |           3
 11 |         1 |         7 |        3 |      7 |      6 |      1 |           0
 12 |         2 |         7 |        3 |      8 |      7 |      1 |           1
 13 |         3 |         7 |        3 |      5 |      8 |      1 |           2
 14 |         4 |         7 |        3 |      6 |      9 |      1 |           3
 15 |         5 |         7 |        3 |      9 |     16 |      1 |           4
 16 |         6 |         7 |        3 |      4 |      3 |      1 |           5
 17 |         7 |         7 |        3 |      3 |     -1 |      0 |           6
 18 |         1 |         7 |       11 |      7 |      6 |      1 |           0
 19 |         2 |         7 |       11 |      8 |      7 |      1 |           1
 20 |         3 |         7 |       11 |      5 |     10 |      1 |           2
 21 |         4 |         7 |       11 |     10 |     12 |      1 |           3
 22 |         5 |         7 |       11 |     11 |     -1 |      0 |           4
(22 rows)
```

Description of the Signatures

Description of the edges_sql query for astar like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.
x1	`ANY-NUMERICAL`		X coordinate of *source* vertex.
y1	`ANY-NUMERICAL`		Y coordinate of *source* vertex.
x2	`ANY-NUMERICAL`		X coordinate of *target* vertex.
y2	`ANY-NUMERICAL`		Y coordinate of *target* vertex.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Parameter	Type	Description
edges_sql	`TEXT`	Edges SQL query as described above.
start_vid	`ANY-INTEGER`	Starting vertex identifier.
start_vids	`ARRAY[ANY-INTEGER]`	Starting vertices identifierers.
end_vid	`ANY-INTEGER`	Ending vertex identifier.
end_vids	`ARRAY[ANY-INTEGER]`	Ending vertices identifiers.
directed	`BOOLEAN`	• Optional. – When `false` the graph is considered as Undirected. – Default is `true` which considers the graph as Directed.
heuristic	`INTEGER`	(optional). Heuristic number. Current valid values 0~5. Default `5` • 0: h(v) = 0 (Use this value to compare with pgr_dijkstra) • 1: h(v) abs(max(dx, dy)) • 2: h(v) abs(min(dx, dy)) • 3: h(v) = dx * dx + dy * dy • 4: h(v) = sqrt(dx * dx + dy * dy) • 5: h(v) = abs(dx) + abs(dy)
factor	`FLOAT`	(optional). For units manipulation. $factor > 0$. Default `1`. see *Factor*
epsilon	`FLOAT`	(optional). For less restricted results. $epsilon >= 1$. Default `1`.

Description of the return values for a path

Returns set of `(seq, path_seq [, start_vid] [, end_vid], node, edge, cost, agg_cost)`

Column	Type	Description
seq	`INT`	Sequential value starting from **1**.
path_id	`INT`	Path identifier. Has value **1** for the first of a path. Used when there are multiple paths for the same `start_vid` to `end_vid` combination.
path_seq	`INT`	Relative position in the path. Has value **1** for the beginning of a path.
start_vid	`BIGINT`	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	`BIGINT`	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
node	`BIGINT`	Identifier of the node in the path from `start_vid` to `end_vid`.
edge	`BIGINT`	Identifier of the edge used to go from `node` to the next node in the path sequence. `-1` for the last node of the path.
cost	`FLOAT`	Cost to traverse from `node` using `edge` to the next node in the path sequence.
agg_cost	`FLOAT`	Aggregate cost from `start_v` to `node`.

See Also

- *Bidirectional A* - Family of functions*
- *Sample Data* network.
- http://www.boost.org/libs/graph/doc/astar_search.html
- http://en.wikipedia.org/wiki/A*_search_algorithm

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5.1.3 pgr_bdDijkstra

`pgr_bdDijkstra` — Returns the shortest path(s) using Bidirectional Dijkstra algorithm.

[38]

Fig. 5.4: Boost Graph Inside

Availability:

- pgr_bdDijkstra(one to one) 2.0.0, Signature changed 2.4.0
- pgr_bdDijkstra(other signatures) 2.5.0

Signature Summary

```
pgr_bdDijkstra(edges_sql, start_vid,  end_vid)
pgr_bdDijkstra(edges_sql, start_vid, end_vid, directed)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost)
OR EMPTY SET
```

[38] http://www.boost.org/libs/graph/doc

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.
  - pgTap tests might be missing.
  - Might need c/c++ coding.
  - May lack documentation.
  - Documentation if any might need to be rewritten.
  - Documentation examples might need to be automatically generated.
  - Might need a lot of feedback from the comunity.
  - Might depend on a proposed function of pgRouting
  - Might depend on a deprecated function of pgRouting

```
pgr_bdDijkstra(edges_sql, start_vid, end_vids, directed)
pgr_bdDijkstra(edges_sql, start_vids, end_vid, directed)
pgr_bdDijkstra(edges_sql, start_vids, end_vids, directed)

RETURNS SET OF (seq, path_seq [, start_vid] [, end_vid], node, edge, cost, agg_
↪cost)
OR EMPTY SET
```

Signatures

Minimal signature

```
pgr_bdDijkstra(edges_sql, start_vid, end_vid)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost) or EMPTY SET
```

The minimal signature is for a **directed** graph from one `start_vid` to one `end_vid`:

Example

```
SELECT * FROM pgr_bdDijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, 3
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    4 |    1 |        0
   2 |        2 |    5 |    8 |    1 |        1
   3 |        3 |    6 |    9 |    1 |        2
   4 |        4 |    9 |   16 |    1 |        3
   5 |        5 |    4 |    3 |    1 |        4
   6 |        6 |    3 |   -1 |    0 |        5
(6 rows)
```

pgr_bdDijkstra One to One

```
pgr_bdDijkstra(edges_sql, start_vid, end_vid, directed)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from one `start_vid` to one `end_vid`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Example

```
SELECT * FROM pgr_bdDijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, 3,
    false
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    2 |    1 |        0
   2 |        2 |    3 |   -1 |    0 |        1
(2 rows)
```

pgr_bdDijkstra One to many

```
pgr_bdDijkstra(edges_sql, start_vid, end_vids, directed)
RETURNS SET OF (seq, path_seq, end_vid, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from one `start_vid` to each `end_vid` in `end_vids`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Using this signature, will load once the graph and perform a one to one *pgr_dijkstra* where the starting vertex is fixed, and stop when all `end_vids` are reached.

- The result is equivalent to the union of the results of the one to one *pgr_dijkstra*.
- The extra `end_vid` in the result is used to distinguish to which path it belongs.

Example

```
SELECT * FROM pgr_bdDijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, ARRAY[3, 11]);
 seq | path_seq | end_vid | node | edge | cost | agg_cost
-----+----------+---------+------+------+------+----------
   1 |        1 |       3 |    2 |    4 |    1 |        0
   2 |        2 |       3 |    5 |    8 |    1 |        1
   3 |        3 |       3 |    6 |    9 |    1 |        2
   4 |        4 |       3 |    9 |   16 |    1 |        3
   5 |        5 |       3 |    4 |    3 |    1 |        4
   6 |        6 |       3 |    3 |   -1 |    0 |        5
   7 |        1 |      11 |    2 |    4 |    1 |        0
   8 |        2 |      11 |    5 |    8 |    1 |        1
   9 |        3 |      11 |    6 |   11 |    1 |        2
  10 |        4 |      11 |   11 |   -1 |    0 |        3
(10 rows)
```

pgr_bdDijkstra Many to One

```
pgr_bdDijkstra(edges_sql, start_vids, end_vid, directed)
RETURNS SET OF (seq, path_seq, start_vid, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from each `start_vid` in `start_vids` to one `end_vid`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Using this signature, will load once the graph and perform several one to one *pgr_dijkstra* where the ending vertex is fixed.

- The result is the union of the results of the one to one *pgr_dijkstra*.
- The extra `start_vid` in the result is used to distinguish to which path it belongs.

Example

```
SELECT * FROM pgr_bdDijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[2, 7], 3);
 seq | path_seq | start_vid | node | edge | cost | agg_cost
-----+----------+-----------+------+------+------+----------
   1 |        1 |         2 |    2 |    4 |    1 |        0
   2 |        2 |         2 |    5 |    8 |    1 |        1
   3 |        3 |         2 |    6 |    9 |    1 |        2
   4 |        4 |         2 |    9 |   16 |    1 |        3
   5 |        5 |         2 |    4 |    3 |    1 |        4
   6 |        6 |         2 |    3 |   -1 |    0 |        5
   7 |        1 |         7 |    7 |    6 |    1 |        0
   8 |        2 |         7 |    8 |    7 |    1 |        1
   9 |        3 |         7 |    5 |    8 |    1 |        2
  10 |        4 |         7 |    6 |    9 |    1 |        3
  11 |        5 |         7 |    9 |   16 |    1 |        4
  12 |        6 |         7 |    4 |    3 |    1 |        5
  13 |        7 |         7 |    3 |   -1 |    0 |        6
(13 rows)
```

pgr_bdDijkstra Many to Many

```
pgr_bdDijkstra(edges_sql, start_vids, end_vids, directed)
RETURNS SET OF (seq, path_seq, start_vid, end_vid, node, edge, cost, agg_cost) or
↪EMPTY SET
```

This signature finds the shortest path from each `start_vid` in `start_vids` to each `end_vid` in `end_vids`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Using this signature, will load once the graph and perform several one to Many *pgr_dijkstra* for all `start_vids`.

- The result is the union of the results of the one to one *pgr_dijkstra*.
- The extra `start_vid` in the result is used to distinguish to which path it belongs.

The extra `start_vid` and `end_vid` in the result is used to distinguish to which path it belongs.

Example

```
SELECT * FROM pgr_bdDijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[2, 7], ARRAY[3, 11]);
 seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
-----+----------+-----------+---------+------+------+------+----------
   1 |        1 |         2 |       3 |    2 |    4 |    1 |        0
   2 |        2 |         2 |       3 |    5 |    8 |    1 |        1
   3 |        3 |         2 |       3 |    6 |    9 |    1 |        2
   4 |        4 |         2 |       3 |    9 |   16 |    1 |        3
   5 |        5 |         2 |       3 |    4 |    3 |    1 |        4
   6 |        6 |         2 |       3 |    3 |   -1 |    0 |        5
   7 |        1 |         2 |      11 |    2 |    4 |    1 |        0
   8 |        2 |         2 |      11 |    5 |    8 |    1 |        1
   9 |        3 |         2 |      11 |    6 |   11 |    1 |        2
  10 |        4 |         2 |      11 |   11 |   -1 |    0 |        3
  11 |        1 |         7 |       3 |    7 |    6 |    1 |        0
  12 |        2 |         7 |       3 |    8 |    7 |    1 |        1
  13 |        3 |         7 |       3 |    5 |    8 |    1 |        2
  14 |        4 |         7 |       3 |    6 |    9 |    1 |        3
  15 |        5 |         7 |       3 |    9 |   16 |    1 |        4
  16 |        6 |         7 |       3 |    4 |    3 |    1 |        5
  17 |        7 |         7 |       3 |    3 |   -1 |    0 |        6
  18 |        1 |         7 |      11 |    7 |    6 |    1 |        0
  19 |        2 |         7 |      11 |    8 |    7 |    1 |        1
  20 |        3 |         7 |      11 |    5 |   10 |    1 |        2
  21 |        4 |         7 |      11 |   10 |   12 |    1 |        3
  22 |        5 |         7 |      11 |   11 |   -1 |    0 |        4
(22 rows)
```

Description of the Signatures

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Column	Type	Default	Description
sql	`TEXT`		SQL query as described above.
start_vid	`BIGINT`		Identifier of the starting vertex of the path.
start_vids	`ARRAY[BIGINT]`		Array of identifiers of starting vertices.
end_vid	`BIGINT`		Identifier of the ending vertex of the path.
end_vids	`ARRAY[BIGINT]`		Array of identifiers of ending vertices.
directed	`BOOLEAN`	`true`	• When `true` Graph is considered *Directed* • When `false` the graph is considered as *Undirected*.

Description of the return values for a path

Returns set of `(seq, path_seq [, start_vid] [, end_vid], node, edge, cost, agg_cost)`

Column	Type	Description
seq	`INT`	Sequential value starting from **1**.
path_id	`INT`	Path identifier. Has value **1** for the first of a path. Used when there are multiple paths for the same `start_vid` to `end_vid` combination.
path_seq	`INT`	Relative position in the path. Has value **1** for the beginning of a path.
start_vid	`BIGINT`	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	`BIGINT`	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
node	`BIGINT`	Identifier of the node in the path from `start_vid` to `end_vid`.
edge	`BIGINT`	Identifier of the edge used to go from `node` to the next node in the path sequence. `-1` for the last node of the path.
cost	`FLOAT`	Cost to traverse from `node` using `edge` to the next node in the path sequence.
agg_cost	`FLOAT`	Aggregate cost from `start_v` to `node`.

See Also

- The queries use the *Sample Data* network.
- *Bidirectional Dijkstra - Family of functions*
- http://www.cs.princeton.edu/courses/archive/spr06/cos423/Handouts/EPP%20shortest%20path%20algorithms.pdf
- https://en.wikipedia.org/wiki/Bidirectional_search

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5.1.4 Dijkstra - Family of functions

- *pgr_dijkstra* - Dijkstra's algorithm for the shortest paths.
- *pgr_dijkstraCost* - Get the aggregate cost of the shortest paths.
- *pgr_dijkstraCostMatrix - proposed* - Use pgr_dijkstra to create a costs matrix.
- *pgr_drivingDistance* - Use pgr_dijkstra to calculate catchament information.
- *pgr_KSP* - Use Yen algorithm with pgr_dijkstra to get the K shortest paths.
- *pgr_dijkstraVia - Proposed* - Get a route of a seuence of vertices.

pgr_dijkstra

`pgr_dijkstra` — Returns the shortest path(s) using Dijkstra algorithm. In particular, the Dijkstra algorithm implemented by Boost.Graph.

[39] http://www.boost.org/libs/graph/doc/dijkstra_shortest_paths.html

[39]

Fig. 5.5: Boost Graph Inside

Availability

- pgr_dijkstra(one to one) 2.0.0, signature change 2.1.0
- pgr_dijkstra(other signatures) 2.1.0

Synopsis

Dijkstra's algorithm, conceived by Dutch computer scientist Edsger Dijkstra in 1956. It is a graph search algorithm that solves the shortest path problem for a graph with non-negative edge path costs, producing a shortest path from a starting vertex (`start_vid`) to an ending vertex (`end_vid`). This implementation can be used with a directed graph and an undirected graph.

Characteristics

The main Characteristics are:

- Process is done only on edges with positive costs.
- Values are returned when there is a path.
  - When the starting vertex and ending vertex are the same, there is no path.
    - The *agg_cost* the non included values *(v, v)* is *0*
  - When the starting vertex and ending vertex are the different and there is no path:
    - The *agg_cost* the non included values *(u, v)* is ∞
- For optimization purposes, any duplicated value in the *start_vids* or *end_vids* are ignored.
- The returned values are ordered:
  - *start_vid* ascending
  - *end_vid* ascending
- Running time: $O(|start\_vids| * (V \log V + E))$

Signature Summary

```
pgr_dijkstra(edges_sql, start_vid,  end_vid)
pgr_dijkstra(edges_sql, start_vid,  end_vid,  directed:=true)
pgr_dijkstra(edges_sql, start_vid,  end_vids, directed:=true)
pgr_dijkstra(edges_sql, start_vids, end_vid,  directed:=true)
pgr_dijkstra(edges_sql, start_vids, end_vids, directed:=true)

RETURNS SET OF (seq, path_seq [, start_vid] [, end_vid], node, edge, cost, agg_
↪cost)
    OR EMPTY SET
```

Signatures

Minimal signature

```
pgr_dijkstra(TEXT edges_sql, BIGINT start_vid, BIGINT end_vid)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost) or EMPTY SET
```

The minimal signature is for a **directed** graph from one `start_vid` to one `end_vid`.

Example

```
SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, 3
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    4 |    1 |        0
   2 |        2 |    5 |    8 |    1 |        1
   3 |        3 |    6 |    9 |    1 |        2
   4 |        4 |    9 |   16 |    1 |        3
   5 |        5 |    4 |    3 |    1 |        4
   6 |        6 |    3 |   -1 |    0 |        5
(6 rows)
```

pgr_dijkstra One to One

```
pgr_dijkstra(TEXT edges_sql, BIGINT start_vid, BIGINT end_vid,
    BOOLEAN directed:=true);
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from one `start_vid` to one `end_vid`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Example

```
SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, 3,
    FALSE
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    2 |    1 |        0
   2 |        2 |    3 |   -1 |    0 |        1
(2 rows)
```

pgr_dijkstra One to many

```
pgr_dijkstra(TEXT edges_sql, BIGINT start_vid, ARRAY[ANY_INTEGER] end_vids,
    BOOLEAN directed:=true);
RETURNS SET OF (seq, path_seq, end_vid, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from one `start_vid` to each `end_vid` in `end_vids`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Using this signature, will load once the graph and perform a one to one *pgr_dijkstra* where the starting vertex is fixed, and stop when all `end_vids` are reached.

- The result is equivalent to the union of the results of the one to one *pgr_dijkstra*.
- The extra `end_vid` in the result is used to distinguish to which path it belongs.

Example

```
SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    2, ARRAY[3,5],
    FALSE
);
 seq | path_seq | end_vid | node | edge | cost | agg_cost
-----+----------+---------+------+------+------+----------
   1 |        1 |       3 |    2 |    4 |    1 |        0
   2 |        2 |       3 |    5 |    8 |    1 |        1
   3 |        3 |       3 |    6 |    5 |    1 |        2
   4 |        4 |       3 |    3 |   -1 |    0 |        3
   5 |        1 |       5 |    2 |    4 |    1 |        0
   6 |        2 |       5 |    5 |   -1 |    0 |        1
(6 rows)
```

pgr_dijkstra Many to One

```
pgr_dijkstra(TEXT edges_sql, ARRAY[ANY_INTEGER] start_vids, BIGINT end_vid,
    BOOLEAN directed:=true);
RETURNS SET OF (seq, path_seq, start_vid, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from each `start_vid` in `start_vids` to one `end_vid`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Using this signature, will load once the graph and perform several one to one *pgr_dijkstra* where the ending vertex is fixed.

- The result is the union of the results of the one to one *pgr_dijkstra*.
- The extra `start_vid` in the result is used to distinguish to which path it belongs.

Example

```
SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[2,11], 5
);
 seq | path_seq | start_vid | node | edge | cost | agg_cost
-----+----------+-----------+------+------+------+----------
   1 |        1 |         2 |    2 |    4 |    1 |        0
   2 |        2 |         2 |    5 |   -1 |    0 |        1
   3 |        1 |        11 |   11 |   13 |    1 |        0
   4 |        2 |        11 |   12 |   15 |    1 |        1
   5 |        3 |        11 |    9 |    9 |    1 |        2
   6 |        4 |        11 |    6 |    8 |    1 |        3
   7 |        5 |        11 |    5 |   -1 |    0 |        4
(7 rows)
```

pgr_dijkstra Many to Many

```
pgr_dijkstra(TEXT edges_sql, ARRAY[ANY_INTEGER] start_vids, ARRAY[ANY_INTEGER] end_
↪vids,
    BOOLEAN directed:=true);
RETURNS SET OF (seq, path_seq, start_vid, end_vid, node, edge, cost, agg_cost) or␣
↪EMPTY SET
```

This signature finds the shortest path from each `start_vid` in `start_vids` to each `end_vid` in `end_vids`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Using this signature, will load once the graph and perform several one to Many *pgr_dijkstra* for all `start_vids`.

- The result is the union of the results of the one to one *pgr_dijkstra*.
- The extra `start_vid` in the result is used to distinguish to which path it belongs.

The extra `start_vid` and `end_vid` in the result is used to distinguish to which path it belongs.

Example

```
SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[2,11], ARRAY[3,5],
    FALSE
);
 seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
-----+----------+-----------+---------+------+------+------+----------
   1 |        1 |         2 |       3 |    2 |    2 |    1 |        0
   2 |        2 |         2 |       3 |    3 |   -1 |    0 |        1
   3 |        1 |         2 |       5 |    2 |    4 |    1 |        0
   4 |        2 |         2 |       5 |    5 |   -1 |    0 |        1
   5 |        1 |        11 |       3 |   11 |   11 |    1 |        0
   6 |        2 |        11 |       3 |    6 |    5 |    1 |        1
   7 |        3 |        11 |       3 |    3 |   -1 |    0 |        2
   8 |        1 |        11 |       5 |   11 |   11 |    1 |        0
   9 |        2 |        11 |       5 |    6 |    8 |    1 |        1
  10 |        3 |        11 |       5 |    5 |   -1 |    0 |        2
(10 rows)
```

Description of the Signatures

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Column	Type	Default	Description
sql	`TEXT`		SQL query as described above.
start_vid	`BIGINT`		Identifier of the starting vertex of the path.
start_vids	`ARRAY[BIGINT]`		Array of identifiers of starting vertices.
end_vid	`BIGINT`		Identifier of the ending vertex of the path.
end_vids	`ARRAY[BIGINT]`		Array of identifiers of ending vertices.
directed	`BOOLEAN`	`true`	• When `true` Graph is considered *Directed* • When `false` the graph is considered as *Undirected*.

Description of the return values for a path

Returns set of `(seq, path_seq [, start_vid] [, end_vid], node, edge, cost, agg_cost)`

Column	Type	Description
seq	`INT`	Sequential value starting from **1**.
path_id	`INT`	Path identifier. Has value **1** for the first of a path. Used when there are multiple paths for the same `start_vid` to `end_vid` combination.
path_seq	`INT`	Relative position in the path. Has value **1** for the beginning of a path.
start_vid	`BIGINT`	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	`BIGINT`	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
node	`BIGINT`	Identifier of the node in the path from `start_vid` to `end_vid`.
edge	`BIGINT`	Identifier of the edge used to go from `node` to the next node in the path sequence. `-1` for the last node of the path.
cost	`FLOAT`	Cost to traverse from `node` using `edge` to the next node in the path sequence.
agg_cost	`FLOAT`	Aggregate cost from `start_v` to `node`.

Additional Examples

The examples of this section are based on the *Sample Data* network.

The examples include combinations from starting vertices 2 and 11 to ending vertices 3 and 5 in a directed and undirected graph with and with out reverse_cost.

Examples for queries marked as `directed` with `cost` and `reverse_cost` columns

The examples in this section use the following *Network for queries marked as directed and cost and reverse_cost columns are used*

```
SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, 3
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    4 |    1 |        0
   2 |        2 |    5 |    8 |    1 |        1
   3 |        3 |    6 |    9 |    1 |        2
   4 |        4 |    9 |   16 |    1 |        3
   5 |        5 |    4 |    3 |    1 |        4
   6 |        6 |    3 |   -1 |    0 |        5
(6 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, 5
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    4 |    1 |        0
   2 |        2 |    5 |   -1 |    0 |        1
(2 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, ARRAY[3,5]
```

```
);
 seq | path_seq | end_vid | node | edge | cost | agg_cost
-----+----------+---------+------+------+------+----------
   1 |        1 |       3 |    2 |    4 |    1 |        0
   2 |        2 |       3 |    5 |    8 |    1 |        1
   3 |        3 |       3 |    6 |    9 |    1 |        2
   4 |        4 |       3 |    9 |   16 |    1 |        3
   5 |        5 |       3 |    4 |    3 |    1 |        4
   6 |        6 |       3 |    3 |   -1 |    0 |        5
   7 |        1 |       5 |    2 |    4 |    1 |        0
   8 |        2 |       5 |    5 |   -1 |    0 |        1
(8 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    11, 3
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |   11 |   13 |    1 |        0
   2 |        2 |   12 |   15 |    1 |        1
   3 |        3 |    9 |   16 |    1 |        2
   4 |        4 |    4 |    3 |    1 |        3
   5 |        5 |    3 |   -1 |    0 |        4
(5 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    11, 5
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |   11 |   13 |    1 |        0
   2 |        2 |   12 |   15 |    1 |        1
   3 |        3 |    9 |    9 |    1 |        2
   4 |        4 |    6 |    8 |    1 |        3
   5 |        5 |    5 |   -1 |    0 |        4
(5 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[2,11], 5
);
 seq | path_seq | start_vid | node | edge | cost | agg_cost
-----+----------+-----------+------+------+------+----------
   1 |        1 |         2 |    2 |    4 |    1 |        0
   2 |        2 |         2 |    5 |   -1 |    0 |        1
   3 |        1 |        11 |   11 |   13 |    1 |        0
   4 |        2 |        11 |   12 |   15 |    1 |        1
   5 |        3 |        11 |    9 |    9 |    1 |        2
   6 |        4 |        11 |    6 |    8 |    1 |        3
   7 |        5 |        11 |    5 |   -1 |    0 |        4
(7 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[2, 11], ARRAY[3,5]
);
 seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
-----+----------+-----------+---------+------+------+------+----------
   1 |        1 |         2 |       3 |    2 |    4 |    1 |        0
   2 |        2 |         2 |       3 |    5 |    8 |    1 |        1
   3 |        3 |         2 |       3 |    6 |    9 |    1 |        2
```

```
   4  |          4  |           2  |         3  |      9  |    16  |      1  |          3
   5  |          5  |           2  |         3  |      4  |     3  |      1  |          4
   6  |          6  |           2  |         3  |      3  |    -1  |      0  |          5
   7  |          1  |           2  |         5  |      2  |     4  |      1  |          0
   8  |          2  |           2  |         5  |      5  |    -1  |      0  |          1
   9  |          1  |          11  |         3  |     11  |    13  |      1  |          0
  10  |          2  |          11  |         3  |     12  |    15  |      1  |          1
  11  |          3  |          11  |         3  |      9  |    16  |      1  |          2
  12  |          4  |          11  |         3  |      4  |     3  |      1  |          3
  13  |          5  |          11  |         3  |      3  |    -1  |      0  |          4
  14  |          1  |          11  |         5  |     11  |    13  |      1  |          0
  15  |          2  |          11  |         5  |     12  |    15  |      1  |          1
  16  |          3  |          11  |         5  |      9  |     9  |      1  |          2
  17  |          4  |          11  |         5  |      6  |     8  |      1  |          3
  18  |          5  |          11  |         5  |      5  |    -1  |      0  |          4
(18 rows)
```

Examples for queries marked as `undirected` with `cost` and `reverse_cost` columns

The examples in this section use the following *Network for queries marked as undirected and cost and reverse_cost columns are used*

```
SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, 3,
    FALSE
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    2 |    1 |        0
   2 |        2 |    3 |   -1 |    0 |        1
(2 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, 5,
    FALSE
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    4 |    1 |        0
   2 |        2 |    5 |   -1 |    0 |        1
(2 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    11, 3,
    FALSE
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |   11 |   11 |    1 |        0
   2 |        2 |    6 |    5 |    1 |        1
   3 |        3 |    3 |   -1 |    0 |        2
(3 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    11, 5,
    FALSE
```

```
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |   11 |   11 |    1 |        0
   2 |        2 |    6 |    8 |    1 |        1
   3 |        3 |    5 |   -1 |    0 |        2
(3 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[2,11], 5,
    FALSE
);
 seq | path_seq | start_vid | node | edge | cost | agg_cost
-----+----------+-----------+------+------+------+----------
   1 |        1 |         2 |    2 |    4 |    1 |        0
   2 |        2 |         2 |    5 |   -1 |    0 |        1
   3 |        1 |        11 |   11 |   12 |    1 |        0
   4 |        2 |        11 |   10 |   10 |    1 |        1
   5 |        3 |        11 |    5 |   -1 |    0 |        2
(5 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, ARRAY[3,5],
    FALSE
);
 seq | path_seq | end_vid | node | edge | cost | agg_cost
-----+----------+---------+------+------+------+----------
   1 |        1 |       3 |    2 |    2 |    1 |        0
   2 |        2 |       3 |    3 |   -1 |    0 |        1
   3 |        1 |       5 |    2 |    4 |    1 |        0
   4 |        2 |       5 |    5 |   -1 |    0 |        1
(4 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[2, 11], ARRAY[3,5],
    FALSE
);
 seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
-----+----------+-----------+---------+------+------+------+----------
   1 |        1 |         2 |       3 |    2 |    2 |    1 |        0
   2 |        2 |         2 |       3 |    3 |   -1 |    0 |        1
   3 |        1 |         2 |       5 |    2 |    4 |    1 |        0
   4 |        2 |         2 |       5 |    5 |   -1 |    0 |        1
   5 |        1 |        11 |       3 |   11 |   11 |    1 |        0
   6 |        2 |        11 |       3 |    6 |    5 |    1 |        1
   7 |        3 |        11 |       3 |    3 |   -1 |    0 |        2
   8 |        1 |        11 |       5 |   11 |   11 |    1 |        0
   9 |        2 |        11 |       5 |    6 |    8 |    1 |        1
  10 |        3 |        11 |       5 |    5 |   -1 |    0 |        2
(10 rows)
```

Examples for queries marked as `directed` with `cost` column

The examples in this section use the following *Network for queries marked as directed and only cost column is used*

```
SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    2, 3
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
(0 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    2, 5
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    4 |    1 |        0
   2 |        2 |    5 |   -1 |    0 |        1
(2 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    11, 3
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
(0 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    11, 5
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
(0 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    ARRAY[2,11], 5
);
 seq | path_seq | start_vid | node | edge | cost | agg_cost
-----+----------+-----------+------+------+------+----------
   1 |        1 |         2 |    2 |    4 |    1 |        0
   2 |        2 |         2 |    5 |   -1 |    0 |        1
(2 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    2, ARRAY[3,5]
);
 seq | path_seq | end_vid | node | edge | cost | agg_cost
-----+----------+---------+------+------+------+----------
   1 |        1 |       5 |    2 |    4 |    1 |        0
   2 |        2 |       5 |    5 |   -1 |    0 |        1
(2 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    ARRAY[2, 11], ARRAY[3,5]
);
 seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
-----+----------+-----------+---------+------+------+------+----------
   1 |        1 |         2 |       5 |    2 |    4 |    1 |        0
   2 |        2 |         2 |       5 |    5 |   -1 |    0 |        1
(2 rows)
```

Examples for queries marked as `undirected` with `cost` column

The examples in this section use the following *Network for queries marked as undirected and only cost column is used*

```
SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    2, 3,
    FALSE
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    4 |    1 |        0
   2 |        2 |    5 |    8 |    1 |        1
   3 |        3 |    6 |    5 |    1 |        2
   4 |        4 |    3 |   -1 |    0 |        3
(4 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    2, 5,
    FALSE
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    4 |    1 |        0
   2 |        2 |    5 |   -1 |    0 |        1
(2 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    11, 3,
    FALSE
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |   11 |   11 |    1 |        0
   2 |        2 |    6 |    5 |    1 |        1
   3 |        3 |    3 |   -1 |    0 |        2
(3 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    11, 5,
    FALSE
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |   11 |   11 |    1 |        0
   2 |        2 |    6 |    8 |    1 |        1
   3 |        3 |    5 |   -1 |    0 |        2
(3 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    ARRAY[2,11], 5,
    FALSE
);
 seq | path_seq | start_vid | node | edge | cost | agg_cost
```

```
-----+----------+-----------+------+------+------+----------
   1 |        1 |         2 |    2 |    4 |    1 |        0
   2 |        2 |         2 |    5 |   -1 |    0 |        1
   3 |        1 |        11 |   11 |   12 |    1 |        0
   4 |        2 |        11 |   10 |   10 |    1 |        1
   5 |        3 |        11 |    5 |   -1 |    0 |        2
(5 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    2, ARRAY[3,5],
    FALSE
);
 seq | path_seq | end_vid | node | edge | cost | agg_cost
-----+----------+---------+------+------+------+----------
   1 |        1 |       3 |    2 |    4 |    1 |        0
   2 |        2 |       3 |    5 |    8 |    1 |        1
   3 |        3 |       3 |    6 |    5 |    1 |        2
   4 |        4 |       3 |    3 |   -1 |    0 |        3
   5 |        1 |       5 |    2 |    4 |    1 |        0
   6 |        2 |       5 |    5 |   -1 |    0 |        1
(6 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    ARRAY[2, 11], ARRAY[3,5],
    FALSE
);
 seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
-----+----------+-----------+---------+------+------+------+----------
   1 |        1 |         2 |       3 |    2 |    4 |    1 |        0
   2 |        2 |         2 |       3 |    5 |    8 |    1 |        1
   3 |        3 |         2 |       3 |    6 |    5 |    1 |        2
   4 |        4 |         2 |       3 |    3 |   -1 |    0 |        3
   5 |        1 |         2 |       5 |    2 |    4 |    1 |        0
   6 |        2 |         2 |       5 |    5 |   -1 |    0 |        1
   7 |        1 |        11 |       3 |   11 |   11 |    1 |        0
   8 |        2 |        11 |       3 |    6 |    5 |    1 |        1
   9 |        3 |        11 |       3 |    3 |   -1 |    0 |        2
  10 |        1 |        11 |       5 |   11 |   11 |    1 |        0
  11 |        2 |        11 |       5 |    6 |    8 |    1 |        1
  12 |        3 |        11 |       5 |    5 |   -1 |    0 |        2
(12 rows)
```

Equvalences between signatures

Examples For queries marked as `directed` with `cost` and `reverse_cost` columns

The examples in this section use the following:

- *Network for queries marked as directed and cost and reverse_cost columns are used*

```
SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, 3,
    TRUE
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    4 |    1 |        0
```

```
   2 |        2 |    5 |    8 |    1 |        1
   3 |        3 |    6 |    9 |    1 |        2
   4 |        4 |    9 |   16 |    1 |        3
   5 |        5 |    4 |    3 |    1 |        4
   6 |        6 |    3 |   -1 |    0 |        5
(6 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2,3
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    4 |    1 |        0
   2 |        2 |    5 |    8 |    1 |        1
   3 |        3 |    6 |    9 |    1 |        2
   4 |        4 |    9 |   16 |    1 |        3
   5 |        5 |    4 |    3 |    1 |        4
   6 |        6 |    3 |   -1 |    0 |        5
(6 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, ARRAY[3],
    TRUE
);
 seq | path_seq | end_vid | node | edge | cost | agg_cost
-----+----------+---------+------+------+------+----------
   1 |        1 |       3 |    2 |    4 |    1 |        0
   2 |        2 |       3 |    5 |    8 |    1 |        1
   3 |        3 |       3 |    6 |    9 |    1 |        2
   4 |        4 |       3 |    9 |   16 |    1 |        3
   5 |        5 |       3 |    4 |    3 |    1 |        4
   6 |        6 |       3 |    3 |   -1 |    0 |        5
(6 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, ARRAY[3]
);
 seq | path_seq | end_vid | node | edge | cost | agg_cost
-----+----------+---------+------+------+------+----------
   1 |        1 |       3 |    2 |    4 |    1 |        0
   2 |        2 |       3 |    5 |    8 |    1 |        1
   3 |        3 |       3 |    6 |    9 |    1 |        2
   4 |        4 |       3 |    9 |   16 |    1 |        3
   5 |        5 |       3 |    4 |    3 |    1 |        4
   6 |        6 |       3 |    3 |   -1 |    0 |        5
(6 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[2], ARRAY[3],
    TRUE
);
 seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
-----+----------+-----------+---------+------+------+------+----------
   1 |        1 |         2 |       3 |    2 |    4 |    1 |        0
   2 |        2 |         2 |       3 |    5 |    8 |    1 |        1
   3 |        3 |         2 |       3 |    6 |    9 |    1 |        2
   4 |        4 |         2 |       3 |    9 |   16 |    1 |        3
   5 |        5 |         2 |       3 |    4 |    3 |    1 |        4
   6 |        6 |         2 |       3 |    3 |   -1 |    0 |        5
```

```
(6 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[2], ARRAY[3]
);
 seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
-----+----------+-----------+---------+------+------+------+----------
   1 |        1 |         2 |       3 |    2 |    4 |    1 |        0
   2 |        2 |         2 |       3 |    5 |    8 |    1 |        1
   3 |        3 |         2 |       3 |    6 |    9 |    1 |        2
   4 |        4 |         2 |       3 |    9 |   16 |    1 |        3
   5 |        5 |         2 |       3 |    4 |    3 |    1 |        4
   6 |        6 |         2 |       3 |    3 |   -1 |    0 |        5
(6 rows)
```

Examples For queries marked as `undirected` with `cost` and `reverse_cost` columns

The examples in this section use the following:

- *Network for queries marked as undirected and cost and reverse_cost columns are used*

```
SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, 3,
    FALSE
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    2 |    1 |        0
   2 |        2 |    3 |   -1 |    0 |        1
(2 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, ARRAY[3],
    FALSE
);
 seq | path_seq | end_vid | node | edge | cost | agg_cost
-----+----------+---------+------+------+------+----------
   1 |        1 |       3 |    2 |    2 |    1 |        0
   2 |        2 |       3 |    3 |   -1 |    0 |        1
(2 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[2], 3,
    FALSE
);
 seq | path_seq | start_vid | node | edge | cost | agg_cost
-----+----------+-----------+------+------+------+----------
   1 |        1 |         2 |    2 |    2 |    1 |        0
   2 |        2 |         2 |    3 |   -1 |    0 |        1
(2 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[2], ARRAY[3],
    FALSE
);
 seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
-----+----------+-----------+---------+------+------+------+----------
```

```
   1 |         1 |           2 |         3 |      2 |      2 |      1 |           0
   2 |         2 |           2 |         3 |      3 |     -1 |      0 |           1
(2 rows)
```

See Also

- http://en.wikipedia.org/wiki/Dijkstra%27s_algorithm
- The queries use the *Sample Data* network.

Indices and tables

- genindex
- search

pgr_dijkstraCost

Synopsis

`pgr_dijkstraCost`

Using Dijkstra algorithm implemented by Boost.Graph, and extract only the aggregate cost of the shortest path(s) found, for the combination of vertices given.

[40]

Fig. 5.6: Boost Graph Inside

Availability

- pgr_dijkstraCost(all signatures) 2.2.0

The `pgr_dijkstraCost` algorithm, is a good choice to calculate the sum of the costs of the shortest path for a subset of pairs of nodes of the graph. We make use of the Boost's implementation of dijkstra which runs in $O(V \log V + E)$ time.

Characteristics

The main Characteristics are:

- It does not return a path.
- Returns the sum of the costs of the shortest path for pair combination of nodes in the graph.
- Process is done only on edges with positive costs.
- Values are returned when there is a path.
  - The returned values are in the form of a set of *(start_vid, end_vid, agg_cost)*.

[40] http://www.boost.org/libs/graph/doc/dijkstra_shortest_paths.html

    - When the starting vertex and ending vertex are the same, there is no path.
        - The *agg_cost* int the non included values *(v, v)* is *0*
    - When the starting vertex and ending vertex are the different and there is no path.
        - The *agg_cost* in the non included values *(u, v)* is ∞
- Let be the case the values returned are stored in a table, so the unique index would be the pair: *(start_vid, end_vid)*.
- For undirected graphs, the results are symmetric.
    - The *agg_cost* of *(u, v)* is the same as for *(v, u)*.
- Any duplicated value in the *start_vids* or *end_vids* is ignored.
- The returned values are ordered:
    - *start_vid* ascending
    - *end_vid* ascending
- Running time: $O(|start\_vids| * (V \log V + E))$

Signature Summary

```
pgr_dijkstraCost(edges_sql, start_vid, end_vid);
pgr_dijkstraCost(edges_sql, start_vid, end_vid, directed);
pgr_dijkstraCost(edges_sql, start_vids, end_vid, directed);
pgr_dijkstraCost(edges_sql, start_vid, end_vids, directed);
pgr_dijkstraCost(edges_sql, start_vids, end_vids, directed);

    RETURNS SET OF (start_vid, end_vid, agg_cost) or EMPTY SET
```

Signatures

Minimal signature

The minimal signature is for a **directed** graph from one `start_vid` to one `end_vid`:

```
pgr_dijkstraCost(TEXT edges_sql, BIGINT start_vid, BIGINT end_vid)
    RETURNS SET OF (start_vid, end_vid, agg_cost) or EMPTY SET
```

Example

```
SELECT * FROM pgr_dijkstraCost(
    'select id, source, target, cost, reverse_cost from edge_table',
    2, 3);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |       3 |        5
(1 row)
```

pgr_dijkstraCost One to One

This signature performs a Dijkstra from one `start_vid` to one `end_vid`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

```
pgr_dijkstraCost(TEXT edges_sql, BIGINT start_vid, BIGINT end_vid,
                     BOOLEAN directed:=true);
    RETURNS SET OF (start_vid, end_vid, agg_cost) or EMPTY SET
```

Example

```
SELECT * FROM pgr_dijkstraCost(
    'select id, source, target, cost, reverse_cost from edge_table',
    2, 3, false);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |       3 |        1
(1 row)
```

pgr_dijkstraCost One to Many

```
pgr_dijkstraCost(TEXT edges_sql, BIGINT start_vid, array[ANY_INTEGER] end_vids,
        BOOLEAN directed:=true);
    RETURNS SET OF (start_vid, end_vid, agg_cost) or EMPTY SET
```

This signature performs a Dijkstra from one `start_vid` to each `end_vid` in `end_vids`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Example

```
SELECT * FROM pgr_dijkstraCost(
    'select id, source, target, cost, reverse_cost from edge_table',
    2, ARRAY[3, 11]);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |       3 |        5
         2 |      11 |        3
(2 rows)
```

pgr_dijkstraCost Many to One

```
pgr_dijkstraCost(TEXT edges_sql, array[ANY_INTEGER] start_vids, BIGINT end_vid,
                     BOOLEAN directed:=true);
    RETURNS SET OF (start_vid, end_vid, agg_cost) or EMPTY SET
```

This signature performs a Dijkstra from each `start_vid` in `start_vids` to one `end_vid`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Example

```
SELECT * FROM pgr_dijkstraCost(
    'select id, source, target, cost, reverse_cost from edge_table',
    ARRAY[2, 7], 3);
 start_vid | end_vid | agg_cost
```

```
-----------+---------+----------
         2 |       3 |         5
         7 |       3 |         6
(2 rows)
```

pgr_dijkstraCost Many to Many

```
pgr_dijkstraCost(TEXT edges_sql, array[ANY_INTEGER] start_vids, array[ANY_INTEGER]␣
↪end_vids,
        BOOLEAN directed:=true);
    RETURNS SET OF (start_vid, end_vid, agg_cost) or EMPTY SET
```

This signature performs a Dijkstra from each `start_vid` in `start_vids` to each `end_vid` in `end_vids`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Example

```
SELECT * FROM pgr_dijkstraCost(
    'select id, source, target, cost, reverse_cost from edge_table',
    ARRAY[2, 7], ARRAY[3, 11]);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |       3 |         5
         2 |      11 |         3
         7 |       3 |         6
         7 |      11 |         4
(4 rows)
```

Description of the Signatures

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	ANY-INTEGER		Identifier of the edge.
source	ANY-INTEGER		Identifier of the first end point vertex of the edge.
target	ANY-INTEGER		Identifier of the second end point vertex of the edge.
cost	ANY-NUMERICAL		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	ANY-NUMERICAL	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Column	Type	Default	Description
sql	TEXT		SQL query as described above.
start_vid	BIGINT		Identifier of the starting vertex of the path.
start_vids	ARRAY[BIGINT]		Array of identifiers of starting vertices.
end_vid	BIGINT		Identifier of the ending vertex of the path.
end_vids	ARRAY[BIGINT]		Array of identifiers of ending vertices.
directed	BOOLEAN	true	• When true Graph is considered *Directed* • When false the graph is considered as *Undirected*.

Description of the return values for a Cost function

Returns set of `(start_vid, end_vid, agg_cost)`

Column	Type	Description
start_vid	`BIGINT`	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	`BIGINT`	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
agg_cost	`FLOAT`	Aggregate cost from `start_vid` to `end_vid`.

Additional Examples

Example 1 Demonstration of repeated values are ignored, and result is sorted.

```
SELECT * FROM pgr_dijkstraCost(
        'select id, source, target, cost, reverse_cost from edge_table',
            ARRAY[5, 3, 4, 3, 3, 4], ARRAY[3, 5, 3, 4]);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         3 |       4 |        3
         3 |       5 |        2
         4 |       3 |        1
         4 |       5 |        3
         5 |       3 |        4
         5 |       4 |        3
(6 rows)
```

Example 2 Making *start_vids* the same as *end_vids*

```
SELECT * FROM pgr_dijkstraCost(
        'select id, source, target, cost, reverse_cost from edge_table',
            ARRAY[5, 3, 4], ARRAY[5, 3, 4]);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         3 |       4 |        3
         3 |       5 |        2
         4 |       3 |        1
         4 |       5 |        3
         5 |       3 |        4
         5 |       4 |        3
(6 rows)
```

See Also

- http://en.wikipedia.org/wiki/Dijkstra%27s_algorithm
- *Sample Data* network.

Indices and tables

- genindex
- search

pgr_dijkstraCostMatrix - proposed

Name

`pgr_dijkstraCostMatrix` - Calculates the a cost matrix using pgr_dijktras.

Warning: Proposed functions for next mayor release.

- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
  - The functions make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might not change. (But still can)
  - Signature might not change. (But still can)
  - Functionality might not change. (But still can)
  - pgTap tests have being done. But might need more.
  - Documentation might need refinement.

[41]

Fig. 5.7: Boost Graph Inside

Availability: 2.3.0

Synopsis

Using Dijkstra algorithm, calculate and return a cost matrix.

Signature Summary

```
pgr_dijkstraCostMatrix(edges_sql, start_vids)
pgr_dijkstraCostMatrix(edges_sql, start_vids, directed)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Signatures

Minimal Signature

The minimal signature:

- Is for a **directed** graph.

```
pgr_dijkstraCostMatrix(edges_sql, start_vid)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

[41] http://www.boost.org/libs/graph

Example Cost matrix for vertices 1, 2, 3, and 4.

```
SELECT * FROM pgr_dijkstraCostMatrix(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5)
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         1 |       2 |        1
         1 |       3 |        6
         1 |       4 |        5
         2 |       1 |        1
         2 |       3 |        5
         2 |       4 |        4
         3 |       1 |        2
         3 |       2 |        1
         3 |       4 |        3
         4 |       1 |        3
         4 |       2 |        2
         4 |       3 |        1
(12 rows)
```

Complete Signature

```
pgr_dijkstraCostMatrix(edges_sql, start_vids, directed:=true)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Example Cost matrix for an undirected graph for vertices 1, 2, 3, and 4.

This example returns a symmetric cost matrix.

```
SELECT * FROM pgr_dijkstraCostMatrix(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
    false
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         1 |       2 |        1
         1 |       3 |        2
         1 |       4 |        3
         2 |       1 |        1
         2 |       3 |        1
         2 |       4 |        2
         3 |       1 |        2
         3 |       2 |        1
         3 |       4 |        1
         4 |       1 |        3
         4 |       2 |        2
         4 |       3 |        1
(12 rows)
```

Description of the Signatures

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Parameter	Type	Description
edges_sql	`TEXT`	Edges SQL query as described above.
start_vids	`ARRAY[ANY-INTEGER]`	Array of identifiers of the vertices.
directed	`BOOLEAN`	(optional). When `false` the graph is considered as Undirected. Default is `true` which considers the graph as Directed.

Description of the return values for a Cost function

Returns set of `(start_vid, end_vid, agg_cost)`

Column	Type	Description
start_vid	`BIGINT`	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	`BIGINT`	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
agg_cost	`FLOAT`	Aggregate cost from `start_vid` to `end_vid`.

Examples

Example Use with tsp

```
SELECT * FROM pgr_TSP(
    $$
    SELECT * FROM pgr_dijkstraCostMatrix(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table',
        (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
        false
    )
    $$,
    randomize := false
);
 seq | node | cost | agg_cost
-----+------+------+----------
   1 |    1 |    1 |        0
   2 |    2 |    1 |        1
   3 |    3 |    1 |        2
   4 |    4 |    3 |        3
   5 |    1 |    0 |        6
(5 rows)
```

See Also

- *Dijkstra - Family of functions*
- *Cost Matrix - Category*
- *Traveling Sales Person - Family of functions*
- The queries use the *Sample Data* network.

Indices and tables

- genindex
- search

pgr_drivingDistance

Name

`pgr_drivingDistance` - Returns the driving distance from a start node.

[42]

Fig. 5.8: Boost Graph Inside

Availability

- pgr_drivingDistance(single vertex) 2.0.0, signature change 2.1.0
- pgr_drivingDistance(multiple vertices) 2.1.0

[42] http://www.boost.org/libs/graph

Synopsis

Using the Dijkstra algorithm, extracts all the nodes that have costs less than or equal to the value `distance`. The edges extracted will conform to the corresponding spanning tree.

Signature Summary

```
pgr_drivingDistance(edges_sql, start_vid, distance)
pgr_drivingDistance(edges_sql, start_vid, distance, directed)
pgr_drivingDistance(edges_sql, start_vids, distance, directed, equicost)

RETURNS SET OF (seq, [start_vid,] node, edge, cost, agg_cost)
```

Signatures

Minimal Use

```
pgr_drivingDistance(edges_sql, start_vid, distance)
RETURNS SET OF (seq, node, edge, cost, agg_cost)
```

Driving Distance From A Single Starting Vertex

```
pgr_drivingDistance(edges_sql, start_vid, distance, directed)
RETURNS SET OF (seq, node, edge, cost, agg_cost)
```

Driving Distance From Multiple Starting Vertices

```
pgr_drivingDistance(edges_sql, start_vids, distance, directed, equicost)
RETURNS SET OF (seq, start_vid, node, edge, cost, agg_cost)
```

Description of the Signatures

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Column	Type	Description
edges_sql	`TEXT`	SQL query as described above.
start_vid	`BIGINT`	Identifier of the starting vertex.
start_vids	`ARRAY[ANY-INTEGER]`	Array of identifiers of the starting vertices.
distance	`FLOAT`	Upper limit for the inclusion of the node in the result.
directed	`BOOLEAN`	(optional). When `false` the graph is considered as Undirected. Default is `true` which considers the graph as Directed.
equicost	`BOOLEAN`	(optional). When `true` the node will only appear in the closest `start_vid` list. Default is `false` which resembles several calls using the single starting point signatures. Tie brakes are arbitrary.

Description of the return values

Returns set of `(seq [, start_v], node, edge, cost, agg_cost)`

Column	Type	Description
seq	INTEGER	Sequential value starting from **1**.
start_vid	INTEGER	Identifier of the starting vertex.
node	BIGINT	Identifier of the node in the path within the limits from start_vid.
edge	BIGINT	Identifier of the edge used to arrive to node. 0 when the node is the start_vid.
cost	FLOAT	Cost to traverse edge.
agg_cost	FLOAT	Aggregate cost from start_vid to node.

Additional Examples

Examples for queries marked as `directed` with `cost` and `reverse_cost` columns

The examples in this section use the following *Network for queries marked as directed and cost and reverse_cost columns are used*

```
SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table',
        2, 3
      );
 seq | node | edge | cost | agg_cost
-----+------+------+------+----------
   1 |    2 |   -1 |    0 |        0
   2 |    1 |    1 |    1 |        1
   3 |    5 |    4 |    1 |        1
   4 |    6 |    8 |    1 |        2
   5 |    8 |    7 |    1 |        2
   6 |   10 |   10 |    1 |        2
   7 |    7 |    6 |    1 |        3
   8 |    9 |    9 |    1 |        3
   9 |   11 |   12 |    1 |        3
  10 |   13 |   14 |    1 |        3
(10 rows)

SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table',
        13, 3
      );
 seq | node | edge | cost | agg_cost
-----+------+------+------+----------
   1 |   13 |   -1 |    0 |        0
   2 |   10 |   14 |    1 |        1
   3 |    5 |   10 |    1 |        2
   4 |   11 |   12 |    1 |        2
   5 |    2 |    4 |    1 |        3
   6 |    6 |    8 |    1 |        3
   7 |    8 |    7 |    1 |        3
   8 |   12 |   13 |    1 |        3
(8 rows)

SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table',
        array[2,13], 3
      );
 seq | from_v | node | edge | cost | agg_cost
-----+--------+------+------+------+----------
   1 |      2 |    2 |   -1 |    0 |        0
   2 |      2 |    1 |    1 |    1 |        1
   3 |      2 |    5 |    4 |    1 |        1
   4 |      2 |    6 |    8 |    1 |        2
```

```
   5 |       2 |    8 |    7 |    1 |        2
   6 |       2 |   10 |   10 |    1 |        2
   7 |       2 |    7 |    6 |    1 |        3
   8 |       2 |    9 |    9 |    1 |        3
   9 |       2 |   11 |   12 |    1 |        3
  10 |       2 |   13 |   14 |    1 |        3
  11 |      13 |   13 |   -1 |    0 |        0
  12 |      13 |   10 |   14 |    1 |        1
  13 |      13 |    5 |   10 |    1 |        2
  14 |      13 |   11 |   12 |    1 |        2
  15 |      13 |    2 |    4 |    1 |        3
  16 |      13 |    6 |    8 |    1 |        3
  17 |      13 |    8 |    7 |    1 |        3
  18 |      13 |   12 |   13 |    1 |        3
(18 rows)

SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table',
        array[2,13], 3, equicost:=true
      );
 seq | from_v | node | edge | cost | agg_cost
-----+--------+------+------+------+----------
   1 |       2 |    2 |   -1 |    0 |        0
   2 |       2 |    1 |    1 |    1 |        1
   3 |       2 |    5 |    4 |    1 |        1
   4 |       2 |    6 |    8 |    1 |        2
   5 |       2 |    8 |    7 |    1 |        2
   6 |       2 |    7 |    6 |    1 |        3
   7 |       2 |    9 |    9 |    1 |        3
   8 |      13 |   13 |   -1 |    0 |        0
   9 |      13 |   10 |   14 |    1 |        1
  10 |      13 |   11 |   12 |    1 |        2
  11 |      13 |   12 |   13 |    1 |        3
(11 rows)
```

Examples for queries marked as `undirected` with `cost` and `reverse_cost` columns

The examples in this section use the following *Network for queries marked as undirected and cost and reverse_cost columns are used*

```
SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table',
        2, 3, false
      );
 seq | node | edge | cost | agg_cost
-----+------+------+------+----------
   1 |    2 |   -1 |    0 |        0
   2 |    1 |    1 |    1 |        1
   3 |    3 |    2 |    1 |        1
   4 |    5 |    4 |    1 |        1
   5 |    4 |    3 |    1 |        2
   6 |    6 |    8 |    1 |        2
   7 |    8 |    7 |    1 |        2
   8 |   10 |   10 |    1 |        2
   9 |    7 |    6 |    1 |        3
  10 |    9 |   16 |    1 |        3
  11 |   11 |   12 |    1 |        3
  12 |   13 |   14 |    1 |        3
(12 rows)
```

```
SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table',
        13, 3, false
      );
 seq | node | edge | cost | agg_cost
-----+------+------+------+----------
   1 |   13 |   -1 |    0 |        0
   2 |   10 |   14 |    1 |        1
   3 |    5 |   10 |    1 |        2
   4 |   11 |   12 |    1 |        2
   5 |    2 |    4 |    1 |        3
   6 |    6 |    8 |    1 |        3
   7 |    8 |    7 |    1 |        3
   8 |   12 |   13 |    1 |        3
(8 rows)

SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table',
        array[2,13], 3, false
      );
 seq | from_v | node | edge | cost | agg_cost
-----+--------+------+------+------+----------
   1 |      2 |    2 |   -1 |    0 |        0
   2 |      2 |    1 |    1 |    1 |        1
   3 |      2 |    3 |    2 |    1 |        1
   4 |      2 |    5 |    4 |    1 |        1
   5 |      2 |    4 |    3 |    1 |        2
   6 |      2 |    6 |    8 |    1 |        2
   7 |      2 |    8 |    7 |    1 |        2
   8 |      2 |   10 |   10 |    1 |        2
   9 |      2 |    7 |    6 |    1 |        3
  10 |      2 |    9 |   16 |    1 |        3
  11 |      2 |   11 |   12 |    1 |        3
  12 |      2 |   13 |   14 |    1 |        3
  13 |     13 |   13 |   -1 |    0 |        0
  14 |     13 |   10 |   14 |    1 |        1
  15 |     13 |    5 |   10 |    1 |        2
  16 |     13 |   11 |   12 |    1 |        2
  17 |     13 |    2 |    4 |    1 |        3
  18 |     13 |    6 |    8 |    1 |        3
  19 |     13 |    8 |    7 |    1 |        3
  20 |     13 |   12 |   13 |    1 |        3
(20 rows)

SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table',
        array[2,13], 3, false, equicost:=true
      );
 seq | from_v | node | edge | cost | agg_cost
-----+--------+------+------+------+----------
   1 |      2 |    2 |   -1 |    0 |        0
   2 |      2 |    1 |    1 |    1 |        1
   3 |      2 |    3 |    2 |    1 |        1
   4 |      2 |    5 |    4 |    1 |        1
   5 |      2 |    4 |    3 |    1 |        2
   6 |      2 |    6 |    8 |    1 |        2
   7 |      2 |    8 |    7 |    1 |        2
   8 |      2 |    7 |    6 |    1 |        3
   9 |      2 |    9 |   16 |    1 |        3
  10 |     13 |   13 |   -1 |    0 |        0
  11 |     13 |   10 |   14 |    1 |        1
  12 |     13 |   11 |   12 |    1 |        2
  13 |     13 |   12 |   13 |    1 |        3
```

```
(13 rows)
```

Examples for queries marked as `directed` with `cost` column

The examples in this section use the following *Network for queries marked as directed and only cost column is used*

```
SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost FROM edge_table',
        2, 3
      );
 seq | node | edge | cost | agg_cost
-----+------+------+------+----------
   1 |    2 |   -1 |    0 |        0
   2 |    5 |    4 |    1 |        1
   3 |    6 |    8 |    1 |        2
   4 |   10 |   10 |    1 |        2
   5 |    9 |    9 |    1 |        3
   6 |   11 |   11 |    1 |        3
   7 |   13 |   14 |    1 |        3
(7 rows)

SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost FROM edge_table',
        13, 3
      );
 seq | node | edge | cost | agg_cost
-----+------+------+------+----------
   1 |   13 |   -1 |    0 |        0
(1 row)

SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost FROM edge_table',
        array[2,13], 3
      );
 seq | from_v | node | edge | cost | agg_cost
-----+--------+------+------+------+----------
   1 |      2 |    2 |   -1 |    0 |        0
   2 |      2 |    5 |    4 |    1 |        1
   3 |      2 |    6 |    8 |    1 |        2
   4 |      2 |   10 |   10 |    1 |        2
   5 |      2 |    9 |    9 |    1 |        3
   6 |      2 |   11 |   11 |    1 |        3
   7 |      2 |   13 |   14 |    1 |        3
   8 |     13 |   13 |   -1 |    0 |        0
(8 rows)

SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost FROM edge_table',
        array[2,13], 3, equicost:=true
      );
 seq | from_v | node | edge | cost | agg_cost
-----+--------+------+------+------+----------
   1 |      2 |    2 |   -1 |    0 |        0
   2 |      2 |    5 |    4 |    1 |        1
   3 |      2 |    6 |    8 |    1 |        2
   4 |      2 |   10 |   10 |    1 |        2
   5 |      2 |    9 |    9 |    1 |        3
   6 |      2 |   11 |   11 |    1 |        3
   7 |     13 |   13 |   -1 |    0 |        0
```

```
(7 rows)
```

Examples for queries marked as `undirected` with `cost` column

The examples in this section use the following *Network for queries marked as undirected and only cost column is used*

```
SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost FROM edge_table',
        2, 3, false
      );
 seq | node | edge | cost | agg_cost
-----+------+------+------+----------
   1 |    2 |   -1 |    0 |        0
   2 |    1 |    1 |    1 |        1
   3 |    5 |    4 |    1 |        1
   4 |    6 |    8 |    1 |        2
   5 |    8 |    7 |    1 |        2
   6 |   10 |   10 |    1 |        2
   7 |    3 |    5 |    1 |        3
   8 |    7 |    6 |    1 |        3
   9 |    9 |    9 |    1 |        3
  10 |   11 |   12 |    1 |        3
  11 |   13 |   14 |    1 |        3
(11 rows)

SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost FROM edge_table',
        13, 3, false
      );
 seq | node | edge | cost | agg_cost
-----+------+------+------+----------
   1 |   13 |   -1 |    0 |        0
   2 |   10 |   14 |    1 |        1
   3 |    5 |   10 |    1 |        2
   4 |   11 |   12 |    1 |        2
   5 |    2 |    4 |    1 |        3
   6 |    6 |    8 |    1 |        3
   7 |    8 |    7 |    1 |        3
   8 |   12 |   13 |    1 |        3
(8 rows)

SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost FROM edge_table',
        array[2,13], 3, false
      );
 seq | from_v | node | edge | cost | agg_cost
-----+--------+------+------+------+----------
   1 |      2 |    2 |   -1 |    0 |        0
   2 |      2 |    1 |    1 |    1 |        1
   3 |      2 |    5 |    4 |    1 |        1
   4 |      2 |    6 |    8 |    1 |        2
   5 |      2 |    8 |    7 |    1 |        2
   6 |      2 |   10 |   10 |    1 |        2
   7 |      2 |    3 |    5 |    1 |        3
   8 |      2 |    7 |    6 |    1 |        3
   9 |      2 |    9 |    9 |    1 |        3
  10 |      2 |   11 |   12 |    1 |        3
  11 |      2 |   13 |   14 |    1 |        3
  12 |     13 |   13 |   -1 |    0 |        0
```

```
  13 |      13 |    10 |    14 |     1 |         1
  14 |      13 |     5 |    10 |     1 |         2
  15 |      13 |    11 |    12 |     1 |         2
  16 |      13 |     2 |     4 |     1 |         3
  17 |      13 |     6 |     8 |     1 |         3
  18 |      13 |     8 |     7 |     1 |         3
  19 |      13 |    12 |    13 |     1 |         3
(19 rows)

SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost FROM edge_table',
        array[2,13], 3, false, equicost:=true
      );
 seq | from_v | node | edge | cost | agg_cost
-----+--------+------+------+------+----------
   1 |      2 |    2 |   -1 |    0 |        0
   2 |      2 |    1 |    1 |    1 |        1
   3 |      2 |    5 |    4 |    1 |        1
   4 |      2 |    6 |    8 |    1 |        2
   5 |      2 |    8 |    7 |    1 |        2
   6 |      2 |    3 |    5 |    1 |        3
   7 |      2 |    7 |    6 |    1 |        3
   8 |      2 |    9 |    9 |    1 |        3
   9 |     13 |   13 |   -1 |    0 |        0
  10 |     13 |   10 |   14 |    1 |        1
  11 |     13 |   11 |   12 |    1 |        2
  12 |     13 |   12 |   13 |    1 |        3
(12 rows)
```

See Also

- *pgr_alphaShape* - Alpha shape computation
- *pgr_pointsAsPolygon* - Polygon around set of points
- *Sample Data* network.

Indices and tables

- genindex
- search

pgr_KSP

Name

`pgr_KSP` — Returns the “K” shortest paths.

[43]

Fig. 5.9: Boost Graph Inside

[43] http://www.boost.org/libs/graph

Availability: 2.0.0

- Signature change 2.1.0

Synopsis

The K shortest path routing algorithm based on Yen's algorithm. "K" is the number of shortest paths desired.

Signature Summary

```
pgr_KSP(edges_sql, start_vid, end_vid, K);
pgr_KSP(edges_sql, start_vid, end_vid, k, directed, heap_paths)
RETURNS SET OF (seq, path_id, path_seq, node, edge, cost, agg_cost) or EMPTY SET
```

Signatures

Minimal Signature

```
pgr_ksp(edges_sql, start_vid, end_vid, K);
RETURNS SET OF (seq, path_id, path_seq, node, edge, cost, agg_cost) or EMPTY SET
```

Complete Signature

```
pgr_KSP(edges_sql, start_vid, end_vid, k, directed, heap_paths)
RETURNS SET OF (seq, path_id, path_seq, node, edge, cost, agg_cost) or EMPTY SET
```

Description of the Signatures

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Column	Type	Description
edges_sql	`TEXT`	SQL query as described above.
start_vid	`BIGINT`	Identifier of the starting vertex.
end_vid	`BIGINT`	Identifier of the ending vertex.
k	`INTEGER`	The desiered number of paths.
directed	`BOOLEAN`	(optional). When `false` the graph is considered as Undirected. Default is `true` which considers the graph as Directed.
heap_paths	`BOOLEAN`	(optional). When `true` returns all the paths stored in the process heap. Default is `false` which only returns `k` paths.

Roughly, if the shortest path has `N` edges, the heap will contain about than `N * k` paths for small value of `k` and `k > 1`.

Description of the return values

Returns set of `(seq, path_seq, path_id, node, edge, cost, agg_cost)`

Column	Type	Description
seq	INTEGER	Sequential value starting from **1**.
path_seq	INTEGER	Relative position in the path of `node` and `edge`. Has value **1** for the beginning of a path.
path_id	BIGINT	Path identifier. The ordering of the paths For two paths i, j if i < j then agg_cost(i) <= agg_cost(j).
node	BIGINT	Identifier of the node in the path.
edge	BIGINT	Identifier of the edge used to go from `node` to the next node in the path sequence. `-1` for the last node of the route.
cost	FLOAT	Cost to traverse from `node` using `edge` to the next node in the path sequence.
agg_cost	FLOAT	Aggregate cost from `start_vid` to `node`.

Warning: During the transition to 3.0, because pgr_ksp version 2.0 doesn't have defined a directed flag nor a heap_path flag, when pgr_ksp is used with only one flag version 2.0 signature will be used.

Additional Examples

Examples to handle the one flag to choose signatures

The examples in this section use the following *Network for queries marked as directed and cost and reverse_cost columns are used*

```
SELECT * FROM pgr_KSP(
     'SELECT id, source, target, cost, reverse_cost FROM edge_table',
      2, 12, 2,
      directed:=true
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |    2 |    4 |    1 |        0
   2 |       1 |        2 |    5 |    8 |    1 |        1
   3 |       1 |        3 |    6 |    9 |    1 |        2
   4 |       1 |        4 |    9 |   15 |    1 |        3
   5 |       1 |        5 |   12 |   -1 |    0 |        4
   6 |       2 |        1 |    2 |    4 |    1 |        0
   7 |       2 |        2 |    5 |    8 |    1 |        1
   8 |       2 |        3 |    6 |   11 |    1 |        2
   9 |       2 |        4 |   11 |   13 |    1 |        3
  10 |       2 |        5 |   12 |   -1 |    0 |        4
(10 rows)

SELECT * FROM pgr_KSP(
     'SELECT id, source, target, cost, reverse_cost FROM edge_table',
      2, 12, 2
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |    2 |    4 |    1 |        0
   2 |       1 |        2 |    5 |    8 |    1 |        1
   3 |       1 |        3 |    6 |    9 |    1 |        2
   4 |       1 |        4 |    9 |   15 |    1 |        3
   5 |       1 |        5 |   12 |   -1 |    0 |        4
   6 |       2 |        1 |    2 |    4 |    1 |        0
   7 |       2 |        2 |    5 |    8 |    1 |        1
   8 |       2 |        3 |    6 |   11 |    1 |        2
   9 |       2 |        4 |   11 |   13 |    1 |        3
  10 |       2 |        5 |   12 |   -1 |    0 |        4
```

```
(10 rows)
```

Examples for queries marked as `directed` with `cost` and `reverse_cost` columns

The examples in this section use the following *Network for queries marked as directed and cost and reverse_cost columns are used*

```
SELECT * FROM pgr_KSP(
     'SELECT id, source, target, cost, reverse_cost FROM edge_table',
      2, 12, 2
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |    2 |    4 |    1 |        0
   2 |       1 |        2 |    5 |    8 |    1 |        1
   3 |       1 |        3 |    6 |    9 |    1 |        2
   4 |       1 |        4 |    9 |   15 |    1 |        3
   5 |       1 |        5 |   12 |   -1 |    0 |        4
   6 |       2 |        1 |    2 |    4 |    1 |        0
   7 |       2 |        2 |    5 |    8 |    1 |        1
   8 |       2 |        3 |    6 |   11 |    1 |        2
   9 |       2 |        4 |   11 |   13 |    1 |        3
  10 |       2 |        5 |   12 |   -1 |    0 |        4
(10 rows)

SELECT * FROM pgr_KSP(
     'SELECT id, source, target, cost, reverse_cost FROM edge_table',
      2, 12, 2, heap_paths:=true
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |    2 |    4 |    1 |        0
   2 |       1 |        2 |    5 |    8 |    1 |        1
   3 |       1 |        3 |    6 |    9 |    1 |        2
   4 |       1 |        4 |    9 |   15 |    1 |        3
   5 |       1 |        5 |   12 |   -1 |    0 |        4
   6 |       2 |        1 |    2 |    4 |    1 |        0
   7 |       2 |        2 |    5 |    8 |    1 |        1
   8 |       2 |        3 |    6 |   11 |    1 |        2
   9 |       2 |        4 |   11 |   13 |    1 |        3
  10 |       2 |        5 |   12 |   -1 |    0 |        4
  11 |       3 |        1 |    2 |    4 |    1 |        0
  12 |       3 |        2 |    5 |   10 |    1 |        1
  13 |       3 |        3 |   10 |   12 |    1 |        2
  14 |       3 |        4 |   11 |   13 |    1 |        3
  15 |       3 |        5 |   12 |   -1 |    0 |        4
(15 rows)

SELECT * FROM pgr_KSP(
     'SELECT id, source, target, cost, reverse_cost FROM edge_table',
      2, 12, 2, true, true
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |    2 |    4 |    1 |        0
   2 |       1 |        2 |    5 |    8 |    1 |        1
   3 |       1 |        3 |    6 |    9 |    1 |        2
   4 |       1 |        4 |    9 |   15 |    1 |        3
   5 |       1 |        5 |   12 |   -1 |    0 |        4
   6 |       2 |        1 |    2 |    4 |    1 |        0
```

```
   7 |          2 |           2 |     5 |     8 |     1 |          1
   8 |          2 |           3 |     6 |    11 |     1 |          2
   9 |          2 |           4 |    11 |    13 |     1 |          3
  10 |          2 |           5 |    12 |    -1 |     0 |          4
  11 |          3 |           1 |     2 |     4 |     1 |          0
  12 |          3 |           2 |     5 |    10 |     1 |          1
  13 |          3 |           3 |    10 |    12 |     1 |          2
  14 |          3 |           4 |    11 |    13 |     1 |          3
  15 |          3 |           5 |    12 |    -1 |     0 |          4
(15 rows)
```

Examples for queries marked as `undirected` with `cost` and `reverse_cost` columns

The examples in this section use the following *Network for queries marked as undirected and cost and reverse_cost columns are used*

```
SELECT * FROM pgr_KSP(
     'SELECT id, source, target, cost, reverse_cost FROM edge_table',
      2, 12, 2, directed:=false
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |    2 |    2 |    1 |        0
   2 |       1 |        2 |    3 |    3 |    1 |        1
   3 |       1 |        3 |    4 |   16 |    1 |        2
   4 |       1 |        4 |    9 |   15 |    1 |        3
   5 |       1 |        5 |   12 |   -1 |    0 |        4
   6 |       2 |        1 |    2 |    4 |    1 |        0
   7 |       2 |        2 |    5 |    8 |    1 |        1
   8 |       2 |        3 |    6 |   11 |    1 |        2
   9 |       2 |        4 |   11 |   13 |    1 |        3
  10 |       2 |        5 |   12 |   -1 |    0 |        4
(10 rows)

SELECT * FROM pgr_KSP(
     'SELECT id, source, target, cost, reverse_cost FROM edge_table',
      2, 12, 2, false, true
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |    2 |    2 |    1 |        0
   2 |       1 |        2 |    3 |    3 |    1 |        1
   3 |       1 |        3 |    4 |   16 |    1 |        2
   4 |       1 |        4 |    9 |   15 |    1 |        3
   5 |       1 |        5 |   12 |   -1 |    0 |        4
   6 |       2 |        1 |    2 |    4 |    1 |        0
   7 |       2 |        2 |    5 |    8 |    1 |        1
   8 |       2 |        3 |    6 |   11 |    1 |        2
   9 |       2 |        4 |   11 |   13 |    1 |        3
  10 |       2 |        5 |   12 |   -1 |    0 |        4
  11 |       3 |        1 |    2 |    4 |    1 |        0
  12 |       3 |        2 |    5 |   10 |    1 |        1
  13 |       3 |        3 |   10 |   12 |    1 |        2
  14 |       3 |        4 |   11 |   13 |    1 |        3
  15 |       3 |        5 |   12 |   -1 |    0 |        4
  16 |       4 |        1 |    2 |    4 |    1 |        0
  17 |       4 |        2 |    5 |   10 |    1 |        1
  18 |       4 |        3 |   10 |   12 |    1 |        2
  19 |       4 |        4 |   11 |   11 |    1 |        3
  20 |       4 |        5 |    6 |    9 |    1 |        4
```

```
  21 |        4 |         6 |    9 |   15 |    1 |         5
  22 |        4 |         7 |   12 |   -1 |    0 |         6
(22 rows)
```

Examples for queries marked as `directed` with `cost` column

The examples in this section use the following *Network for queries marked as directed and only cost column is used*

```
SELECT  * FROM pgr_KSP(
     'SELECT id, source, target, cost FROM edge_table',
      2, 3, 2
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
(0 rows)

SELECT  * FROM pgr_KSP(
     'SELECT id, source, target, cost FROM edge_table',
      2, 12, 2
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |    2 |    4 |    1 |        0
   2 |       1 |        2 |    5 |    8 |    1 |        1
   3 |       1 |        3 |    6 |    9 |    1 |        2
   4 |       1 |        4 |    9 |   15 |    1 |        3
   5 |       1 |        5 |   12 |   -1 |    0 |        4
   6 |       2 |        1 |    2 |    4 |    1 |        0
   7 |       2 |        2 |    5 |    8 |    1 |        1
   8 |       2 |        3 |    6 |   11 |    1 |        2
   9 |       2 |        4 |   11 |   13 |    1 |        3
  10 |       2 |        5 |   12 |   -1 |    0 |        4
(10 rows)

SELECT   * FROM pgr_KSP(
     'SELECT id, source, target, cost FROM edge_table',
      2, 12, 2, heap_paths:=true
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |    2 |    4 |    1 |        0
   2 |       1 |        2 |    5 |    8 |    1 |        1
   3 |       1 |        3 |    6 |    9 |    1 |        2
   4 |       1 |        4 |    9 |   15 |    1 |        3
   5 |       1 |        5 |   12 |   -1 |    0 |        4
   6 |       2 |        1 |    2 |    4 |    1 |        0
   7 |       2 |        2 |    5 |    8 |    1 |        1
   8 |       2 |        3 |    6 |   11 |    1 |        2
   9 |       2 |        4 |   11 |   13 |    1 |        3
  10 |       2 |        5 |   12 |   -1 |    0 |        4
  11 |       3 |        1 |    2 |    4 |    1 |        0
  12 |       3 |        2 |    5 |   10 |    1 |        1
  13 |       3 |        3 |   10 |   12 |    1 |        2
  14 |       3 |        4 |   11 |   13 |    1 |        3
  15 |       3 |        5 |   12 |   -1 |    0 |        4
(15 rows)

SELECT  * FROM pgr_KSP(
     'SELECT id, source, target, cost FROM edge_table',
```

```
      2, 12, 2, true, true
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |    2 |    4 |    1 |        0
   2 |       1 |        2 |    5 |    8 |    1 |        1
   3 |       1 |        3 |    6 |    9 |    1 |        2
   4 |       1 |        4 |    9 |   15 |    1 |        3
   5 |       1 |        5 |   12 |   -1 |    0 |        4
   6 |       2 |        1 |    2 |    4 |    1 |        0
   7 |       2 |        2 |    5 |    8 |    1 |        1
   8 |       2 |        3 |    6 |   11 |    1 |        2
   9 |       2 |        4 |   11 |   13 |    1 |        3
  10 |       2 |        5 |   12 |   -1 |    0 |        4
  11 |       3 |        1 |    2 |    4 |    1 |        0
  12 |       3 |        2 |    5 |   10 |    1 |        1
  13 |       3 |        3 |   10 |   12 |    1 |        2
  14 |       3 |        4 |   11 |   13 |    1 |        3
  15 |       3 |        5 |   12 |   -1 |    0 |        4
(15 rows)
```

Examples for queries marked as `undirected` with `cost` column

The examples in this section use the following *Network for queries marked as undirected and only cost column is used*

```
SELECT  * FROM pgr_KSP(
     'SELECT id, source, target, cost FROM edge_table',
      2, 12, 2, directed:=false
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |    2 |    4 |    1 |        0
   2 |       1 |        2 |    5 |    8 |    1 |        1
   3 |       1 |        3 |    6 |    9 |    1 |        2
   4 |       1 |        4 |    9 |   15 |    1 |        3
   5 |       1 |        5 |   12 |   -1 |    0 |        4
   6 |       2 |        1 |    2 |    4 |    1 |        0
   7 |       2 |        2 |    5 |    8 |    1 |        1
   8 |       2 |        3 |    6 |   11 |    1 |        2
   9 |       2 |        4 |   11 |   13 |    1 |        3
  10 |       2 |        5 |   12 |   -1 |    0 |        4
(10 rows)

SELECT  * FROM pgr_KSP(
     'SELECT id, source, target, cost FROM edge_table',
      2, 12, 2, directed:=false, heap_paths:=true
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |    2 |    4 |    1 |        0
   2 |       1 |        2 |    5 |    8 |    1 |        1
   3 |       1 |        3 |    6 |    9 |    1 |        2
   4 |       1 |        4 |    9 |   15 |    1 |        3
   5 |       1 |        5 |   12 |   -1 |    0 |        4
   6 |       2 |        1 |    2 |    4 |    1 |        0
   7 |       2 |        2 |    5 |    8 |    1 |        1
   8 |       2 |        3 |    6 |   11 |    1 |        2
   9 |       2 |        4 |   11 |   13 |    1 |        3
  10 |       2 |        5 |   12 |   -1 |    0 |        4
```

```
  11 |         3 |          1 |     2 |     4 |     1 |          0
  12 |         3 |          2 |     5 |    10 |     1 |          1
  13 |         3 |          3 |    10 |    12 |     1 |          2
  14 |         3 |          4 |    11 |    13 |     1 |          3
  15 |         3 |          5 |    12 |    -1 |     0 |          4
(15 rows)
```

See Also

- http://en.wikipedia.org/wiki/K_shortest_path_routing
- *Sample Data* network.

Indices and tables

- genindex
- search

pgr_dijkstraVia - Proposed

Name

`pgr_dijkstraVia` — Using dijkstra algorithm, it finds the route that goes through a list of vertices.

[44]

Fig. 5.10: Boost Graph Inside

Availability: 2.2.0

Synopsis

Given a list of vertices and a graph, this function is equivalent to finding the shortest path between $vertex_i$ and $vertex_{i+1}$ for all $i < size\_of(vertex_via)$.

The paths represents the sections of the route.

Note: This is a proposed function

Signatrue Summary

```
pgr_dijkstraVia(edges_sql, via_vertices)
pgr_dijkstraVia(edges_sql, via_vertices, directed, strict, U_turn_on_edge)
```

[44] http://www.boost.org/libs/graph

```
RETURNS SET OF (seq, path_pid, path_seq, start_vid, end_vid,
    node, edge, cost, agg_cost, route_agg_cost) or EMPTY SET
```

Signatures

Minimal Signature

```
pgr_dijkstraVia(edges_sql, via_vertices)
RETURNS SET OF (seq, path_pid, path_seq, start_vid, end_vid,
    node, edge, cost, agg_cost, route_agg_cost) or EMPTY SET
```

Example Find the route that visits the vertices 1 3 9 in that order

```
SELECT * FROM pgr_dijkstraVia(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table order by id',
    ARRAY[1, 3, 9]
);
 seq | path_id | path_seq | start_vid | end_vid | node | edge | cost | agg_cost |␣
↪route_agg_cost
-----+---------+----------+-----------+---------+------+------+------+----------+--
↪--------------
   1 |       1 |        1 |         1 |       3 |    1 |    1 |    1 |        0 | ␣
↪             0
   2 |       1 |        2 |         1 |       3 |    2 |    4 |    1 |        1 | ␣
↪             1
   3 |       1 |        3 |         1 |       3 |    5 |    8 |    1 |        2 | ␣
↪             2
   4 |       1 |        4 |         1 |       3 |    6 |    9 |    1 |        3 | ␣
↪             3
   5 |       1 |        5 |         1 |       3 |    9 |   16 |    1 |        4 | ␣
↪             4
   6 |       1 |        6 |         1 |       3 |    4 |    3 |    1 |        5 | ␣
↪             5
   7 |       1 |        7 |         1 |       3 |    3 |   -1 |    0 |        6 | ␣
↪             6
   8 |       2 |        1 |         3 |       9 |    3 |    5 |    1 |        0 | ␣
↪             6
   9 |       2 |        2 |         3 |       9 |    6 |    9 |    1 |        1 | ␣
↪             7
  10 |       2 |        3 |         3 |       9 |    9 |   -2 |    0 |        2 | ␣
↪             8
(10 rows)
```

Complete Signature

```
pgr_dijkstraVia(edges_sql, via_vertices, directed, strict, U_turn_on_edge)
RETURNS SET OF (seq, path_pid, path_seq, start_vid, end_vid,
    node, edge, cost, agg_cost, route_agg_cost) or EMPTY SET
```

Example Find the route that visits the vertices 1 3 9 in that order on an undirected graph, avoiding U-turns when possible

```
SELECT * FROM pgr_dijkstraVia(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table order by id',
    ARRAY[1, 3, 9], false, strict:=true, U_turn_on_edge:=false
);
```

```
 seq | path_id | path_seq | start_vid | end_vid | node | edge | cost | agg_cost |␣
↪route_agg_cost
-----+---------+----------+-----------+---------+------+------+------+----------+--
↪--------------
   1 |       1 |        1 |         1 |       3 |    1 |    1 |    1 |        0 |␣
↪             0
   2 |       1 |        2 |         1 |       3 |    2 |    2 |    1 |        1 |␣
↪             1
   3 |       1 |        3 |         1 |       3 |    3 |   -1 |    0 |        2 |␣
↪             2
   4 |       2 |        1 |         3 |       9 |    3 |    5 |    1 |        0 |␣
↪             2
   5 |       2 |        2 |         3 |       9 |    6 |    9 |    1 |        1 |␣
↪             3
   6 |       2 |        3 |         3 |       9 |    9 |   -2 |    0 |        2 |␣
↪             4
(6 rows)
```

Description of the Signature

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Parameter	Type	Default	Description
edges_sql	`TEXT`		SQL query as described above.
via_vertices	`ARRAY[ANY-INTEGER]`		Array of ordered vertices identifiers that are going to be visited.
directed	`BOOLEAN`	`true`	• When `true` Graph is considered *Directed* • When `false` the graph is considered as Undirected.
strict	`BOOLEAN`	`false`	• When `false` ignores missing paths returning all paths found • When `true` if a path is missing stops and returns *EMPTY SET*
U_turn_on_edge	`BOOLEAN`	`true`	• When `true` departing from a visited vertex will not try to avoid using the edge used to reach it. In other words, U turn using the edge with same *id* is allowed. • When `false` when a departing from a visited vertex tries to avoid using the edge used to reach it. In other words, U turn using the edge with same *id* is used when no other path is found.

Description of the parameters of the signatures

Parameter	Type	Description
edges_sql	TEXT	SQL query as described above.
via_vertices	ARRAY[ANY-INTEGER]	Array of vertices identifiers
directed	BOOLEAN	(optional) Default is true (is directed). When set to false the graph is considered as Undirected
strict	BOOLEAN	(optional) ignores if a subsection of the route is missing and returns everything it found Default is true (is directed). When set to false the graph is considered as Undirected
U_turn_on_edge	BOOLEAN	(optional) Default is true (is directed). When set to false the graph is considered as Undirected

Description of the return values

Returns set of `(start_vid, end_vid, agg_cost)`

Column	Type	Description
seq	BIGINT	Sequential value starting from 1.
path_pid	BIGINT	Identifier of the path.
path_seq	BIGINT	Sequential value starting from 1 for the path.
start_vid	BIGINT	Identifier of the starting vertex of the path.
end_vid	BIGINT	Identifier of the ending vertex of the path.
node	BIGINT	Identifier of the node in the path from start_vid to end_vid.
edge	BIGINT	Identifier of the edge used to go from node to the next node in the path sequence. -1 for the last node of the path. -2 for the last node of the route.
cost	FLOAT	Cost to traverse from `node` using `edge` to the next node in the route sequence.
agg_cost	FLOAT	Total cost from `start_vid` to `end_vid` of the path.
route_agg_cost	FLOAT	Total cost from `start_vid` of `path_pid = 1` to `end_vid` of the current `path_pid`.

Examples

Example 1 Find the route that visits the vertices 1 5 3 9 4 in that order

```
SELECT * FROM pgr_dijkstraVia(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table order by id',
    ARRAY[1, 5, 3, 9, 4]
);
 seq | path_id | path_seq | start_vid | end_vid | node | edge | cost | agg_cost |␣
↪route_agg_cost
-----+---------+----------+-----------+---------+------+------+------+----------+--
↪--------------
   1 |       1 |        1 |         1 |       5 |    1 |    1 |    1 |        0 | ␣
↪             0
   2 |       1 |        2 |         1 |       5 |    2 |    4 |    1 |        1 | ␣
↪             1
   3 |       1 |        3 |         1 |       5 |    5 |   -1 |    0 |        2 | ␣
↪             2
   4 |       2 |        1 |         5 |       3 |    5 |    8 |    1 |        0 | ␣
↪             2
   5 |       2 |        2 |         5 |       3 |    6 |    9 |    1 |        1 | ␣
↪             3
   6 |       2 |        3 |         5 |       3 |    9 |   16 |    1 |        2 | ␣
↪             4
```

```
   7 |        2 |         4 |          5 |        3 |      4 |     3 |     1 |         3 |
↪             5
   8 |        2 |         5 |          5 |        3 |      3 |    -1 |     0 |         4 |
↪             6
   9 |        3 |         1 |          3 |        9 |      3 |     5 |     1 |         0 |
↪             6
  10 |        3 |         2 |          3 |        9 |      6 |     9 |     1 |         1 |
↪             7
  11 |        3 |         3 |          3 |        9 |      9 |    -1 |     0 |         2 |
↪             8
  12 |        4 |         1 |          9 |        4 |      9 |    16 |     1 |         0 |
↪             8
  13 |        4 |         2 |          9 |        4 |      4 |    -2 |     0 |         1 |
↪             9
(13 rows)
```

Example 2 What's the aggregate cost of the third path?

```
SELECT agg_cost FROM  pgr_dijkstraVia(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table order by id',
    ARRAY[1, 5, 3, 9, 4]
)
WHERE path_id = 3 AND edge <0;
 agg_cost
----------
        2
(1 row)
```

Example 3 What's the route's aggregate cost of the route at the end of the third path?

```
SELECT route_agg_cost FROM  pgr_dijkstraVia(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table order by id',
    ARRAY[1, 5, 3, 9, 4]
)
WHERE path_id = 3 AND edge < 0;
 route_agg_cost
----------------
              8
(1 row)
```

Example 4 How are the nodes visited in the route?

```
SELECT row_number() over () as node_seq, node
FROM  pgr_dijkstraVia(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table order by id',
    ARRAY[1, 5, 3, 9, 4]
)
WHERE edge <> -1 ORDER BY seq;
 node_seq | node
----------+------
        1 |    1
        2 |    2
        3 |    5
        4 |    6
        5 |    9
        6 |    4
        7 |    3
        8 |    6
        9 |    9
```

```
      10 |     4
(10 rows)
```

Example 5 What are the aggregate costs of the route when the visited vertices are reached?

```
SELECT path_id, route_agg_cost FROM  pgr_dijkstraVia(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table order by id',
    ARRAY[1, 5, 3, 9, 4]
)
WHERE edge < 0;
 path_id | route_agg_cost
---------+----------------
       1 |              2
       2 |              6
       3 |              8
       4 |              9
(4 rows)
```

Example 6 show the route's seq and aggregate cost and a status of "passes in front" or "visits" node 9

```
SELECT seq, route_agg_cost, node, agg_cost ,
CASE WHEN edge = -1 THEN 'visits'
ELSE 'passes in front'
END as status
FROM  pgr_dijkstraVia(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table order by id',
    ARRAY[1, 5, 3, 9, 4])
WHERE node = 9 and (agg_cost  <> 0 or seq = 1);
 seq | route_agg_cost | node | agg_cost |      status
-----+----------------+------+----------+-----------------
   6 |              4 |    9 |        2 | passes in front
  11 |              8 |    9 |        2 | visits
(2 rows)

ROLLBACK;
ROLLBACK
```

See Also

- http://en.wikipedia.org/wiki/Dijkstra%27s_algorithm
- *Sample Data* network.

Indices and tables

- genindex
- search

The problem definition (Advanced documentation)

Given the following query:

pgr_dijkstra($sql, start_{vid}, end_{vid}, directed$)

where $sql = \{(id_i, source_i, target_i, cost_i, reverse\_cost_i)\}$

and

- $source = \bigcup source_i$,
- $target = \bigcup target_i$,

The graphs are defined as follows:

Directed graph

The weighted directed graph, $G_d(V, E)$, is definied by:

- the set of vertices V
  - $V = source \cup target \cup start_{vid} \cup end_{vid}$
- the set of edges E
  - $E = \begin{cases} \{(source_i, target_i, cost_i) \text{ when } cost >= 0\} & \text{if } reverse\_cost = \varnothing \\ & \\ \{(source_i, target_i, cost_i) \text{ when } cost >= 0\} & \\ \cup\{(target_i, source_i, reverse\_cost_i) \text{ when } reverse\_cost_i >= 0\} & \text{if } reverse\_cost \neq \varnothing \end{cases}$

Undirected graph

The weighted undirected graph, $G_u(V, E)$, is definied by:

- the set of vertices V
  - $V = source \cup target \cup start_v vid \cup end_{vid}$
- the set of edges E
  - $E = \begin{cases} \{(source_i, target_i, cost_i) \text{ when } cost >= 0\} & \\ \cup\{(target_i, source_i, cost_i) \text{ when } cost >= 0\} & \text{if } reverse\_cost = \varnothing \\ & \\ \{(source_i, target_i, cost_i) \text{ when } cost >= 0\} & \\ \cup\{(target_i, source_i, cost_i) \text{ when } cost >= 0\} & \\ \cup\{(target_i, source_i, reverse\_cost_i) \text{ when } reverse\_cost_i >= 0)\} & \\ \cup\{(source_i, target_i, reverse\_cost_i) \text{ when } reverse\_cost_i >= 0)\} & \text{if } reverse\_cost \neq \varnothing \end{cases}$

The problem

Given:

- $start_{vid} \in V$ a starting vertex
- $end_{vid} \in V$ an ending vertex
- $G(V, E) = \begin{cases} G_d(V, E) & \text{if6 } directed = true \\ G_u(V, E) & \text{if5 } directed = false \end{cases}$

Then:

- $\pi = \{(path\_seq_i, node_i, edge_i, cost_i, agg\_cost_i)\}$

where:

- $path\_seq_i = i$
- $path\_seq_{|\pi|} = |\pi|$

- $node_i \in V$
- $node_1 = start_{vid}$
- $node_{|\pi|} = end_{vid}$
- $\forall i \neq |\pi|, \quad (node_i, node_{i+1}, cost_i) \in E$
- $edge_i = \begin{cases} id_{(node_i, node_{i+1}, cost_i)} & \text{when } i \neq |\pi| \\ -1 & \text{when } i = |\pi| \end{cases}$
- $cost_i = cost_{(node_i, node_{i+1})}$
- $agg\_cost_i = \begin{cases} 0 & \text{when } i = 1 \\ \displaystyle\sum_{k=1}^{i} cost_{(node_{k-1}, node_k)} & \text{when } i \neq 1 \end{cases}$

In other words: The algorithm returns a the shortest path between $start_{vid}$ and end_{vid} , if it exists, in terms of a sequence o

- $path\_seq$ indicates the relative position in the path of the $node$ or $edge$.
- $cost$ is the cost of the edge to be used to go to the next node.
- agg_cost is the cost from the $start_{vid}$ up to the node.

If there is no path, the resulting set is empty.

See Also

Indices and tables

- genindex
- search

5.1.5 pgr_trsp - Turn Restriction Shortest Path (TRSP)

Name

`pgr_trsp` — Returns the shortest path with support for turn restrictions.

Synopsis

The turn restricted shortest path (TRSP) is a shortest path algorithm that can optionally take into account complicated turn restrictions like those found in real world navigable road networks. Performamnce wise it is nearly as fast as the A* search but has many additional features like it works with edges rather than the nodes of the network. Returns a set of *pgr_costResult* (seq, id1, id2, cost) rows, that make up a path.

```
pgr_costResult[] pgr_trsp(sql text, source integer, target integer,
            directed boolean, has_rcost boolean [,restrict_sql text]);
```

```
pgr_costResult[] pgr_trsp(sql text, source_edge integer, source_pos float8,
                target_edge integer, target_pos float8,
            directed boolean, has_rcost boolean [,restrict_sql text]);
```

```
pgr_costResult3[] pgr_trspViaVertices(sql text, vids integer[],
                directed boolean, has_rcost boolean
                [, turn_restrict_sql text]);
```

```
pgr_costResult3[] pgr_trspViaEdges(sql text, eids integer[], pcts float8[],
                  directed boolean, has_rcost boolean
                  [, turn_restrict_sql text]);
```

Description

The Turn Restricted Shortest Path algorithm (TRSP) is similar to the shooting star in that you can specify turn restrictions.

The TRSP setup is mostly the same as *Dijkstra shortest path* with the addition of an optional turn restriction table. This provides an easy way of adding turn restrictions to a road network by placing them in a separate table.

sql a SQL query, which should return a set of rows with the following columns:

```
SELECT id, source, target, cost, [,reverse_cost] FROM edge_table
```

id `int4` identifier of the edge

source `int4` identifier of the source vertex

target `int4` identifier of the target vertex

cost `float8` value, of the edge traversal cost. A negative cost will prevent the edge from being inserted in the graph.

reverse_cost (optional) the cost for the reverse traversal of the edge. This is only used when the `directed` and `has_rcost` parameters are `true` (see the above remark about negative costs).

source `int4` **NODE id** of the start point

target `int4` **NODE id** of the end point

directed `true` if the graph is directed

has_rcost if `true`, the `reverse_cost` column of the SQL generated set of rows will be used for the cost of the traversal of the edge in the opposite direction.

restrict_sql (optional) a SQL query, which should return a set of rows with the following columns:

```
SELECT to_cost, target_id, via_path FROM restrictions
```

to_cost `float8` turn restriction cost

target_id `int4` target id

via_path `text` comma separated list of edges in the reverse order of `rule`

Another variant of TRSP allows to specify **EDGE id** of source and target together with a fraction to interpolate the position:

source_edge `int4` **EDGE id** of the start edge

source_pos `float8` fraction of 1 defines the position on the start edge

target_edge `int4` **EDGE id** of the end edge

target_pos `float8` fraction of 1 defines the position on the end edge

Returns set of *pgr_costResult[]*:

seq row sequence

id1 node ID

id2 edge ID (`-1` for the last row)

cost cost to traverse from `id1` using `id2`

History

- New in version 2.0.0

Support for Vias

Warning: The Support for Vias functions are prototypes. Not all corner cases are being considered.

We also have support for vias where you can say generate a from A to B to C, etc. We support both methods above only you pass an array of vertices or and array of edges and percentage position along the edge in two arrays.

sql a SQL query, which should return a set of rows with the following columns:

```
SELECT id, source, target, cost, [,reverse_cost] FROM edge_table
```

id `int4` identifier of the edge

source `int4` identifier of the source vertex

target `int4` identifier of the target vertex

cost `float8` value, of the edge traversal cost. A negative cost will prevent the edge from being inserted in the graph.

reverse_cost (optional) the cost for the reverse traversal of the edge. This is only used when the `directed` and `has_rcost` parameters are `true` (see the above remark about negative costs).

vids `int4[]` An ordered array of **NODE id** the path will go through from start to end.

directed `true` if the graph is directed

has_rcost if `true`, the `reverse_cost` column of the SQL generated set of rows will be used for the cost of the traversal of the edge in the opposite direction.

restrict_sql (optional) a SQL query, which should return a set of rows with the following columns:

```
SELECT to_cost, target_id, via_path FROM restrictions
```

to_cost `float8` turn restriction cost

target_id `int4` target id

via_path `text` commar separated list of edges in the reverse order of `rule`

Another variant of TRSP allows to specify **EDGE id** together with a fraction to interpolate the position:

eids `int4` An ordered array of **EDGE id** that the path has to traverse

pcts `float8` An array of fractional positions along the respective edges in `eids`, where 0.0 is the start of the edge and 1.0 is the end of the eadge.

Returns set of *pgr_costResult[]*:

seq row sequence

id1 route ID

id2 node ID

id3 edge ID (`-1` for the last row)

cost cost to traverse from `id2` using `id3`

History

- Via Support prototypes new in version 2.1.0

Examples

Without turn restrictions

```
SELECT * FROM pgr_trsp(
        'SELECT id::INTEGER, source::INTEGER, target::INTEGER, cost FROM edge_table
↪',
        7, 12, false, false
    );
 seq | id1 | id2 | cost
-----+-----+-----+------
   0 |   7 |   6 |    1
   1 |   8 |   7 |    1
   2 |   5 |   8 |    1
   3 |   6 |   9 |    1
   4 |   9 |  15 |    1
   5 |  12 |  -1 |    0
(6 rows)
```

With turn restrictions

Then a query with turn restrictions is created as:

```
SELECT * FROM pgr_trsp(
        'SELECT id::INTEGER, source::INTEGER, target::INTEGER, cost FROM edge_table
↪',
        2, 7, false, false,
        'SELECT to_cost, target_id::int4,
        from_edge || coalesce('','' || via_path, '''') AS via_path
        FROM restrictions'
    );
 seq | id1 | id2 | cost
-----+-----+-----+------
   0 |   2 |   4 |    1
   1 |   5 |  10 |    1
   2 |  10 |  12 |    1
   3 |  11 |  11 |    1
   4 |   6 |   8 |    1
   5 |   5 |   7 |    1
   6 |   8 |   6 |    1
   7 |   7 |  -1 |    0
(8 rows)

SELECT * FROM pgr_trsp(
        'SELECT id::INTEGER, source::INTEGER, target::INTEGER, cost FROM edge_table
↪',
        7, 11, false, false,
        'SELECT to_cost, target_id::int4,
        from_edge || coalesce('','' || via_path, '''') AS via_path
        FROM restrictions'
    );
 seq | id1 | id2 | cost
-----+-----+-----+------
   0 |   7 |   6 |    1
   1 |   8 |   7 |    1
   2 |   5 |   8 |    1
   3 |   6 |   9 |    1
```

```
   4 |    9 |   15 |     1
   5 |   12 |   13 |     1
   6 |   11 |   -1 |     0
(7 rows)
```

An example query using vertex ids and via points:

```
SELECT * FROM pgr_trspViaVertices(
        'SELECT id::INTEGER, source::INTEGER, target::INTEGER, cost FROM edge_table
↪',
        ARRAY[2,7,11]::INTEGER[],
        false,  false,
        'SELECT to_cost, target_id::int4, from_edge ||
        coalesce('',''||via_path,'''') AS via_path FROM restrictions');
 seq | id1 | id2 | id3 | cost
-----+-----+-----+-----+------
   1 |   1 |   2 |   4 |    1
   2 |   1 |   5 |  10 |    1
   3 |   1 |  10 |  12 |    1
   4 |   1 |  11 |  11 |    1
   5 |   1 |   6 |   8 |    1
   6 |   1 |   5 |   7 |    1
   7 |   1 |   8 |   6 |    1
   8 |   2 |   7 |   6 |    1
   9 |   2 |   8 |   7 |    1
  10 |   2 |   5 |   8 |    1
  11 |   2 |   6 |   9 |    1
  12 |   2 |   9 |  15 |    1
  13 |   2 |  12 |  13 |    1
  14 |   2 |  11 |  -1 |    0
(14 rows)
```

An example query using edge ids and vias:

```
SELECT * FROM pgr_trspViaEdges(
        'SELECT id::INTEGER, source::INTEGER, target::INTEGER, cost,
        reverse_cost FROM edge_table',
        ARRAY[2,7,11]::INTEGER[],
        ARRAY[0.5, 0.5, 0.5]::FLOAT[],
        true,
        true,
        'SELECT to_cost, target_id::int4, FROM_edge ||
        coalesce('',''||via_path,'''') AS via_path FROM restrictions');
 seq | id1 | id2 | id3 | cost
-----+-----+-----+-----+------
   1 |   1 |  -1 |   2 |  0.5
   2 |   1 |   2 |   4 |    1
   3 |   1 |   5 |   8 |    1
   4 |   1 |   6 |   9 |    1
   5 |   1 |   9 |  16 |    1
   6 |   1 |   4 |   3 |    1
   7 |   1 |   3 |   5 |    1
   8 |   1 |   6 |   8 |    1
   9 |   1 |   5 |   7 |    1
  10 |   2 |   5 |   8 |    1
  11 |   2 |   6 |   9 |    1
  12 |   2 |   9 |  16 |    1
  13 |   2 |   4 |   3 |    1
  14 |   2 |   3 |   5 |    1
  15 |   2 |   6 |  11 |  0.5
(15 rows)
```

The queries use the *Sample Data* network.

See Also

- *pgr_costResult[]*

Indices and tables

- genindex
- search

5.1.6 Traveling Sales Person - Family of functions

- *pgr_TSP* - When input is given as matrix cell information.
- *pgr_eucledianTSP* - When input are coordinates.

pgr_TSP

Name

- `pgr_TSP` - Returns a route that visits all the nodes exactly once.

Availability: 2.0.0

- Signature changed 2.3.0

Synopsis

The travelling salesman problem (TSP) or travelling salesperson problem asks the following question:

- Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?

This implementation uses simulated annealing to return the approximate solution when the input is given in the form of matrix cell contents. The matrix information must be symmetrical.

Signature Summary

```
pgr_TSP(matrix_cell_sql)
pgr_TSP(matrix_cell_sql,
    start_id, end_id,
    max_processing_time,
    tries_per_temperature, max_changes_per_temperature, max_consecutive_non_
↪changes,
    initial_temperature, final_temperature, cooling_factor,
    randomize,
RETURNS SETOF (seq, node, cost, agg_cost)
```

Signatures

Basic Use

```
pgr_TSP(matrix_cell_sql)
RETURNS SETOF (seq, node, cost, agg_cost)
```

Example

Because the documentation examples are auto generated and tested for non changing results, and the default is to have random execution, the example is wrapping the actual call.

```
WITH
query AS (
    SELECT * FROM pgr_TSP(
        $$
        SELECT * FROM pgr_dijkstraCostMatrix(
            'SELECT id, source, target, cost, reverse_cost FROM edge_table',
            (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 14),
            directed := false
        )
        $$
    )
)
SELECT agg_cost < 20 AS under_20 FROM query WHERE seq = 14;
 under_20
----------
 t
(1 row)
```

Complete Signature

```
pgr_TSP(matrix_cell_sql,
    start_id, end_id,
    max_processing_time,
    tries_per_temperature, max_changes_per_temperature, max_consecutive_non_
↪changes,
    initial_temperature, final_temperature, cooling_factor,
    randomize,
RETURNS SETOF (seq, node, cost, agg_cost)
```

Example:

```
SELECT * FROM pgr_TSP(
    $$
    SELECT * FROM pgr_dijkstraCostMatrix(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table',
        (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 14),
        directed := false
    )
    $$,
    start_id := 7,
    randomize := false
);
 seq | node | cost | agg_cost
-----+------+------+----------
   1 |    7 |    1 |        0
```

```
   2 |     8 |     1 |         1
   3 |     5 |     1 |         2
   4 |     2 |     1 |         3
   5 |     1 |     2 |         4
   6 |     3 |     1 |         6
   7 |     4 |     1 |         7
   8 |     9 |     1 |         8
   9 |    12 |     1 |         9
  10 |    11 |     1 |        10
  11 |    10 |     1 |        11
  12 |    13 |     3 |        12
  13 |     6 |     3 |        15
  14 |     7 |     0 |        18
(14 rows)
```

Description of the Signatures

Description of the Matrix Cell SQL query

Column	Type	Description
start_vid	`BIGINT`	Identifier of the starting vertex.
end_vid	`BIGINT`	Identifier of the ending vertex.
agg_cost	`FLOAT`	Cost for going from start_vid to end_vid

Can be Used with:

- *pgr_dijkstraCostMatrix - proposed*
- *pgr_withPointsCostMatrix - proposed*
- *pgr_floydWarshall*
- *pgr_johnson*

To generate a symmetric matrix

- directed := false.

If using directed := true, the resulting non symmetric matrix must be converted to symmetric by fixing the non symmetric values according to your application needs.

```

Description Of the Control parameters
.....................................................................

The control parameters are optional, and have a default value.

=============================== =========== ============ ␣
↪==================================================
Parameter                       Type        Default      Description
=============================== =========== ============ ␣
↪==================================================
**start_vid**                   ``BIGINT``    `0`          The greedy part of the␣
↪implementation will use this identifier.
**end_vid**                     ``BIGINT``    `0`          Last visiting vertex␣
↪before returning to start_vid.
**max_processing_time**         ``FLOAT``     `+infinity`  Stop the annealing␣
↪processing when the value is reached.
**tries_per_temperature**       ``INTEGER``   `500`        Maximum number of times␣
↪a neighbor(s) is searched in each temperature.
```

```
**max_changes_per_temperature** ``INTEGER``  `60`          Maximum number of times␣
↪the solution is changed in each temperature.
**max_consecutive_non_changes** ``INTEGER``  `100`         Maximum number of␣
↪consecutive times the solution is not changed in each temperature.
**initial_temperature**         ``FLOAT``    `100`         Starting temperature.
**final_temperature**           ``FLOAT``    `0.1`         Ending temperature.
**cooling_factor**              ``FLOAT``    `0.9`         Value between between 0␣
↪and 1 (not including) used to calculate the next temperature.
**randomize**                   ``BOOLEAN``  `true`        Choose the random seed

                                                           - true: Use current␣
↪time as seed
                                                           - false: Use `1` as␣
↪seed. Using this value will get the same results with the same data in each␣
↪execution.

=============================== ===========  ============ ␣
↪=================================================
```

```
Description of the return columns
...............................................................................

Returns set of ``(seq, node, cost, agg_cost)``

============= =========== =================================================
Column           Type              Description
============= =========== =================================================
**seq**       ``INTEGER`` Row sequence.
**node**      ``BIGINT``  Identifier of the node/coordinate/point.
**cost**      ``FLOAT``   Cost to traverse from the current ``node`` ito the next␣
↪``node`` in the path sequence.
                            - ``0`` for the last row in the path sequence.

**agg_cost**  ``FLOAT``   Aggregate cost from the ``node`` at ``seq = 1`` to the␣
↪current node.
                            - ``0`` for the first row in the path sequence.

============= =========== =================================================
```

Examples

Example Using with points of interest.

To generate a symmetric matrix:

- the **side** information of pointsOfInterset is ignored by not including it in the query
- and **directed := false**

```
SELECT * FROM pgr_TSP(
    $$
    SELECT * FROM pgr_withPointsCostMatrix(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id
↪',
        'SELECT pid, edge_id, fraction from pointsOfInterest',
        array[-1, 3, 5, 6, -6], directed := false);
    $$,
```

```
    start_id := 5,
    randomize := false
);
 seq | node | cost | agg_cost
-----+------+------+----------
   1 |    5 |    1 |        0
   2 |    6 |    1 |        1
   3 |    3 |  1.6 |        2
   4 |   -1 |  1.3 |      3.6
   5 |   -6 |  0.3 |      4.9
   6 |    5 |    0 |      5.2
(6 rows)
```

The queries use the *Sample Data* network.

See Also

- *Traveling Sales Person - Family of functions*
- http://en.wikipedia.org/wiki/Traveling_salesman_problem
- http://en.wikipedia.org/wiki/Simulated_annealing

Indices and tables

- genindex
- search

pgr_eucledianTSP

Name

`pgr_eucledianTSP` - Returns a route that visits all the coordinates pairs exactly once.

Availability: 2.3.0

Synopsis

The travelling salesman problem (TSP) or travelling salesperson problem asks the following question:

- Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?

This implementation uses simulated annealing to return the approximate solution when the input is given in the form of coordinates.

Signature Summary

```
pgr_eucledianTSP(coordinates_sql)
pgr_eucledianTSP(coordinates_sql,
    start_id, end_id,
    max_processing_time,
    tries_per_temperature, max_changes_per_temperature, max_consecutive_non_
↪changes,
```

```
    initial_temperature, final_temperature, cooling_factor,
    randomize,
RETURNS SETOF (seq, node, cost, agg_cost)
```

Signatures

Minimal Signature

```
pgr_eucledianTSP(coordinates_sql)
RETURNS SETOF (seq, node, cost, agg_cost)
```

Example

Because the documentation examples are auto generated and tested for non changing results, and the default is to have random execution, the example is wrapping the actual call.

```
WITH
query AS (
    SELECT * FROM pgr_eucledianTSP(
        $$
        SELECT id, st_X(the_geom) AS x, st_Y(the_geom)AS y  FROM edge_table_
↪vertices_pgr
        $$
    )
)
SELECT agg_cost < 20 AS under_20 FROM query WHERE seq = 18;
 under_20
----------
 t
(1 row)
```

Complete Signature

```
pgr_eucledianTSP(coordinates_sql,
    start_id, end_id,
    max_processing_time,
    tries_per_temperature, max_changes_per_temperature, max_consecutive_non_
↪changes,
    initial_temperature, final_temperature, cooling_factor,
    randomize,
RETURNS SETOF (seq, node, cost, agg_cost)
```

Example:

```
SELECT* from pgr_eucledianTSP(
    $$
    SELECT id, st_X(the_geom) AS x, st_Y(the_geom) AS y FROM edge_table_vertices_
↪pgr
    $$,
    tries_per_temperature := 3,
    cooling_factor := 0.5,
    randomize := false
);
 seq | node |       cost        |     agg_cost
-----+------+-------------------+------------------
```

```
   1 |     1 |  1.4142135623731 |                0
   2 |     3 |                1 |  1.4142135623731
   3 |     4 |                1 | 2.41421356237309
   4 |     9 | 0.58309518948453 | 3.41421356237309
   5 |    16 | 0.58309518948453 | 3.99730875185762
   6 |     6 |                1 | 4.58040394134215
   7 |     5 |                1 | 5.58040394134215
   8 |     8 |                1 | 6.58040394134215
   9 |     7 | 1.58113883008419 | 7.58040394134215
  10 |    14 |   1.499999999999 | 9.16154277142634
  11 |    15 |              0.5 | 10.6615427714253
  12 |    13 |              1.5 | 11.1615427714253
  13 |    17 | 1.11803398874989 | 12.6615427714253
  14 |    12 |                1 | 13.7795767601752
  15 |    11 |                1 | 14.7795767601752
  16 |    10 |                2 | 15.7795767601752
  17 |     2 |                1 | 17.7795767601752
  18 |     1 |                0 | 18.7795767601752
(18 rows)
```

Description of the Signatures

Description of the coordinates SQL query

Column	Type	Description
id	`BIGINT`	Identifier of the coordinate. (optional)
x	`FLOAT`	X value of the coordinate.
y	`FLOAT`	Y value of the coordinate.

When the value of **id** is not given then the coordinates will receive an **id** starting from 1, in the order given.

```
Description Of the Control parameters
.....................................................................

The control parameters are optional, and have a default value.

=============================== ===========  ============ ␣
↪==================================================
Parameter                       Type         Default       Description
=============================== ===========  ============ ␣
↪==================================================
**start_vid**                   ``BIGINT``    `0`          The greedy part of the␣
↪implementation will use this identifier.
**end_vid**                     ``BIGINT``    `0`          Last visiting vertex␣
↪before returning to start_vid.
**max_processing_time**         ``FLOAT``    `+infinity`   Stop the annealing␣
↪processing when the value is reached.
**tries_per_temperature**       ``INTEGER``  `500`         Maximum number of times␣
↪a neighbor(s) is searched in each temperature.
**max_changes_per_temperature** ``INTEGER``  `60`          Maximum number of times␣
↪the solution is changed in each temperature.
**max_consecutive_non_changes** ``INTEGER``  `100`         Maximum number of␣
↪consecutive times the solution is not changed in each temperature.
**initial_temperature**         ``FLOAT``    `100`         Starting temperature.
**final_temperature**           ``FLOAT``    `0.1`         Ending temperature.
**cooling_factor**              ``FLOAT``    `0.9`         Value between between 0␣
↪and 1 (not including) used to calculate the next temperature.
```

```
**randomize**                   ``BOOLEAN``  `true`        Choose the random seed

                                                           - true: Use current␣
↪time as seed
                                                           - false: Use `1` as␣
↪seed. Using this value will get the same results with the same data in each␣
↪execution.

=============================== =========== ============ ␣
↪=================================================
```

```
Description of the return columns
...............................................................................

Returns set of ``(seq, node, cost, agg_cost)``

============= =========== =================================================
Column           Type              Description
============= =========== =================================================
**seq**       ``INTEGER`` Row sequence.
**node**      ``BIGINT``  Identifier of the node/coordinate/point.
**cost**      ``FLOAT``   Cost to traverse from the current ``node`` ito the next␣
↪``node`` in the path sequence.
                            - ``0`` for the last row in the path sequence.

**agg_cost**  ``FLOAT``   Aggregate cost from the ``node`` at ``seq = 1`` to the␣
↪current node.
                            - ``0`` for the first row in the path sequence.

============= =========== =================================================
```

Examples

Example Skipping the Simulated Annealing & showing some process information

```
SET client_min_messages TO DEBUG1;
SET
SELECT* from pgr_eucledianTSP(
    $$
    SELECT id, st_X(the_geom) AS x, st_Y(the_geom) AS y FROM edge_table_vertices_
↪pgr
    $$,
    tries_per_temperature := 0,
    randomize := false
);
DEBUG:  pgr_eucledianTSP Processing Information
Initializing tsp class ---> tsp.greedyInitial ---> tsp.annealing ---> OK

Cycle(100)         total changes =0        0 were because  delta energy < 0
Total swaps: 3
Total slides: 0
Total reverses: 0
Times best tour changed: 4
Best cost reached = 18.7796
 seq | node |       cost       |     agg_cost
-----+------+------------------+------------------
```

```
   1 |     1 |  1.4142135623731 |                0
   2 |     3 |                1 |  1.4142135623731
   3 |     4 |                1 | 2.41421356237309
   4 |     9 | 0.58309518948453 | 3.41421356237309
   5 |    16 | 0.58309518948453 | 3.99730875185762
   6 |     6 |                1 | 4.58040394134215
   7 |     5 |                1 | 5.58040394134215
   8 |     8 |                1 | 6.58040394134215
   9 |     7 | 1.58113883008419 | 7.58040394134215
  10 |    14 |   1.499999999999 | 9.16154277142634
  11 |    15 |              0.5 | 10.6615427714253
  12 |    13 |              1.5 | 11.1615427714253
  13 |    17 | 1.11803398874989 | 12.6615427714253
  14 |    12 |                1 | 13.7795767601752
  15 |    11 |                1 | 14.7795767601752
  16 |    10 |                2 | 15.7795767601752
  17 |     2 |                1 | 17.7795767601752
  18 |     1 |                0 | 18.7795767601752
(18 rows)
```

The queries use the *Sample Data* network.

History

- New in version 2.3.0

See Also

- *Traveling Sales Person - Family of functions*
- http://en.wikipedia.org/wiki/Traveling_salesman_problem
- http://en.wikipedia.org/wiki/Simulated_annealing

Indices and tables

- genindex
- search

General Information

Origin

The traveling sales person problem was studied in the 18th century by mathematicians Sir William Rowam Hamilton and **Thomas Penyngton Kirkman**.

A discussion about the work of Hamilton & Kirkman can be found in the book **Graph Theory (Biggs et al. 1976)**.

- ISBN-13: 978-0198539162
- ISBN-10: 0198539169

It is believed that the general form of the TSP have been first studied by Kalr Menger in Vienna and Harvard. The problem was later promoted by Hassler, Whitney & Merrill at Princeton. A detailed description about the connection between Menger & Whitney, and the development of the TSP can be found in On the history of combinatorial optimization (till 1960)[45]

[45] http://www.cwi.nl/~lex/files/histco.ps

Problem Definition

Given a collection of cities and travel cost between each pair, find the cheapest way for visiting all of the cities and returning to the starting point.

Characteristics

- The travel costs are symmetric:
  - traveling costs from city A to city B are just as much as traveling from B to A.
- This problem is an NP-hard optimization problem.
- To calculate the number of different tours through n cities:
  - Given a starting city,
  - There are $n - 1$ choices for the second city,
  - And $n - 2$ choices for the third city, etc.
  - Multiplying these together we get $(n - 1)! = (n - 1)(n - 2)..1$.
  - Now since our travel costs do not depend on the direction we take around the tour:
    * this number by 2
    * $(n - 1)!/2$.

TSP & Simulated Annealing

The simulated annealing algorithm was originally inspired from the process of annealing in metal work.

Annealing involves heating and cooling a material to alter its physical properties due to the changes in its internal structure. As the metal cools its new structure becomes fixed, consequently causing the metal to retain its newly obtained properties. *[C001]*

Pseudocode

Given an initial solution, the simulated annealing process, will start with a high temperature and gradually cool down until the desired temperature is reached.

For each temperature, a neighbouring new solution **snew** is calculated. The higher the temperature the higher the probability of accepting the new solution as a possible bester solution.

Once the desired temperature is reached, the best solution found is returned

```
Solution = initial_solution;

temperature = initial_temperature;
while (temperature > final_temperature) {

    do tries_per_temperature times {
        snew = neighbour(solution);
        If P(E(solution), E(snew), T) >= random(0, 1)
            solution = snew;
    }

    temperature = temperature * cooling factor;
}

Output: the best solution
```

pgRouting Implementation

pgRouting's implementation adds some extra parameters to allow some exit controls within the simulated annealing process.

To cool down faster to the next temperature:

- max_changes_per_temperature: limits the number of changes in the solution per temperature
- max_consecutive_non_changes: limits the number of consecutive non changes per temperature

This is done by doing some book keeping on the times **solution = snew;** is executed.

- max_changes_per_temperature: Increases by one when **solution** changes
- max_consecutive_non_changes: Reset to 0 when **solution** changes, and increased each **try**

Additionally to stop the algorithm at a higher temperature than the desired one:

- max_processing_time: limits the time the simulated annealing is performed.
- book keeping is done to see if there was a change in **solution** on the last temperature

Note that, if no change was found in the first **max_consecutive_non_changes** tries, then the simulated annealing will stop.

```
Solution = initial_solution;

temperature = initial_temperature;
while (temperature > final_temperature) {

    do tries_per_temperature times {
        snew = neighbour(solution);
        If P(E(solution), E(snew), T) >= random(0, 1)
            solution = snew;

        when max_changes_per_temperature is reached
            or max_consecutive_non_changes is reached
            BREAK;
    }

    temperature = temperature * cooling factor;
    when no changes were done in the current temperature
        or max_processing_time has being reached
        BREAK;
}

Output: the best solution
```

Choosing parameters

There is no exact rule on how the parameters have to be chose, it will depend on the special characteristics of the problem.

- Your computational time is crucial, then put your time limit to **max_processing_time**.
- Make the **tries_per_temperture** depending on the number of cities, for example:
  - Useful to estimate the time it takes to do one cycle: use *1*
    - this will help to set a reasonable **max_processing_time**
  - $n * (n - 1)$
  - $500 * n$

- For a faster decreasing the temperature set **cooling_factor** to a smaller number, and set to a higher number for a slower decrease.
- When for the same given data the same results are needed, set **randomize** to *false*.
  - When estimating how long it takes to do one cycle: use *false*

A recommendation is to play with the values and see what fits to the particular data.

Description Of the Control parameters

The control parameters are optional, and have a default value.

Parameter	Type	Default	Description
start_vid	`BIGINT`	*0*	The greedy part of the implementation will use this identifier.
end_vid	`BIGINT`	*0*	Last visiting vertex before returning to start_vid.
max_processing_time	`FLOAT`	*+infinity*	Stop the annealing processing when the value is reached.
tries_per_temperature	`INTEGER`	*500*	Maximum number of times a neighbor(s) is searched in each temperature.
max_changes_per_temperature	`INTEGER`	*60*	Maximum number of times the solution is changed in each temperature.
max_consecutive_non_changes	`INTEGER`	*100*	Maximum number of consecutive times the solution is not changed in each temperature.
initial_temperature	`FLOAT`	*100*	Starting temperature.
final_temperature	`FLOAT`	*0.1*	Ending temperature.
cooling_factor	`FLOAT`	*0.9*	Value between between 0 and 1 (not including) used to calculate the next temperature.
randomize	`BOOLEAN`	*true*	Choose the random seed • true: Use current time as seed • false: Use *1* as seed. Using this value will get the same results with the same data in each execution.

Description of the return columns

Returns set of `(seq, node, cost, agg_cost)`

Column	Type	Description
seq	`INTEGER`	Row sequence.
node	`BIGINT`	Identifier of the node/coordinate/point.
cost	`FLOAT`	**Cost to traverse from the current `node` ito the next** • `0` for the last row in the path sequence.
agg_cost	`FLOAT`	**Aggregate cost from the `node` at `seq = 1` to the c** • `0` for the first row in the path sequence.

See Also

References

- http://en.wikipedia.org/wiki/Traveling_salesman_problem
- http://en.wikipedia.org/wiki/Simulated_annealing

Indices and tables

- genindex
- search

5.1.7 Driving Distance - Category

- *pgr_drivingDistance* - Driving Distance based on pgr_dijkstra
- *pgr_withPointsDD - Proposed* - Driving Distance based on pgr_withPoints
- Post pocessing
  - *pgr_alphaShape* - Alpha shape computation
  - *pgr_pointsAsPolygon* - Polygon around a set of points

pgr_alphaShape

Name

`pgr_alphaShape` — Core function for alpha shape computation.

Synopsis

Returns a table with (x, y) rows that describe the vertices of an alpha shape.

```
table() pgr_alphaShape(text sql [, float8 alpha]);
```

Description

sql `text` a SQL query, which should return a set of rows with the following columns:

```
SELECT id, x, y FROM vertex_table
```

id `int4` identifier of the vertex

x `float8` x-coordinate

y `float8` y-coordinate

alpha (optional) `float8` alpha value. If specified alpha value equals 0 (default), then optimal alpha value is used. For more information, see CGAL - 2D Alpha Shapes[47].

Returns a vertex record for each row:

x x-coordinate

y y-coordinate

If a result includes multiple outer/inner rings, return those with separator row (x=NULL and y=NULL).

History

- Renamed in version 2.0.0
- Added alpha argument with default 0 (use optimal value) in version 2.1.0
- Supported to return multiple outer/inner ring coordinates with separator row (x=NULL and y=NULL) in version 2.1.0

Examples

PgRouting's alpha shape implementation has no way to control the order of the output points, so the actual output might different for the same input data. The first query, has the output ordered, he second query shows an example usage:

Example: the (ordered) results

```
SELECT * FROM pgr_alphaShape(
    'SELECT id::integer, ST_X(the_geom)::float AS x, ST_Y(the_geom)::float AS y
    FROM edge_table_vertices_pgr') ORDER BY x, y;
  x  |  y
-----+-----
   0 |   2
 0.5 | 3.5
   2 |   0
   2 |   4
 3.5 |   4
   4 |   1
   4 |   2
   4 |   3
(8 rows)
```

[47] http://doc.cgal.org/latest/Alpha_shapes_2/group__PkgAlphaShape2.html

Example: calculating the area

Steps:

- **Calculates the alpha shape**
  - the `ORDER BY` clause is not used.
- constructs a polygon
- and computes the area

```
SELECT round(ST_Area(ST_MakePolygon(ST_AddPoint(foo.openline, ST_StartPoint(foo.
↪openline))))::numeric, 2) AS st_area
FROM (
    SELECT ST_MakeLine(points ORDER BY id) AS openline
    FROM (
        SELECT ST_MakePoint(x, y) AS points, row_number() over() AS id
        FROM pgr_alphaShape(
            'SELECT id::integer, ST_X(the_geom)::float AS x, ST_Y(the_geom)::float␣
↪AS y
            FROM edge_table_vertices_pgr')
        ) AS a
   ) AS foo;
 st_area
---------
   11.75
(1 row)
```

The queries use the *Sample Data* network.

See Also

- *pgr_drivingDistance* - Driving Distance
- *pgr_pointsAsPolygon* - Polygon around set of points

Indices and tables

- genindex
- search

pgr_pointsAsPolygon

Name

`pgr_pointsAsPolygon` — Draws an alpha shape around given set of points.

Synopsis

Returns the alpha shape as (multi)polygon geometry.

```
geometry pgr_pointsAsPolygon(text sql [, float8 alpha]);
```

Description

sql `text` a SQL query, which should return a set of rows with the following columns:

```
SELECT id, x, y FROM vertex_result;
```

id `int4` identifier of the vertex

x `float8` x-coordinate

y `float8` y-coordinate

alpha (optional) `float8` alpha value. If specified alpha value equals 0 (default), then optimal alpha value is used. For more information, see CGAL - 2D Alpha Shapes[48].

Returns a (multi)polygon geometry (with holes).

History

- Renamed in version 2.0.0
- Added alpha argument with default 0 (use optimal value) in version 2.1.0
- Supported to return a (multi)polygon geometry (with holes) in version 2.1.0

Examples

In the following query there is no way to control which point in the polygon is the first in the list, so you may get similar but different results than the following which are also correct.

```
SELECT ST_AsText(pgr_pointsAsPolygon('SELECT id::integer, ST_X(the_geom)::float AS
↪x, ST_Y(the_geom)::float AS y
        FROM edge_table_vertices_pgr'));
                      st_astext
------------------------------------------------------
 POLYGON((2 4,3.5 4,4 3,4 2,4 1,2 0,0 2,0.5 3.5,2 4))
(1 row)
```

The query use the *Sample Data* network.

See Also

- *pgr_drivingDistance* - Driving Distance
- *pgr_alphaShape* - Alpha shape computation

Indices and tables

- genindex
- search

[48] http://doc.cgal.org/latest/Alpha_shapes_2/group__PkgAlphaShape2.html

See Also

Indices and tables

- genindex
- search

5.1.8 See Also

Indices and tables

- genindex
- search

All Pairs - Family of Functions

- *pgr_floydWarshall* - Floyd-Warshall's Algorithm
- *pgr_johnson*- Johnson's Algorithm

pgr_aStar - Shortest Path A*

pgr_bdAstar - Bi-directional A* Shortest Path

pgr_bdDijkstra - Bi-directional Dijkstra Shortest Path

Dijkstra - Family of functions

- *pgr_dijkstra* - Dijkstra's algorithm for the shortest paths.
- *pgr_dijkstraCost* - Get the aggregate cost of the shortest paths.
- *pgr_dijkstraCostMatrix - proposed* - Use pgr_dijkstra to create a costs matrix.
- *pgr_drivingDistance* - Use pgr_dijkstra to calculate catchament information.
- *pgr_KSP* - Use Yen algorithm with pgr_dijkstra to get the K shortest paths.
- *pgr_dijkstraVia - Proposed* - Get a route of a seuence of vertices.

pgr_KSP - K-Shortest Path

pgr_trsp - Turn Restriction Shortest Path (TRSP)

Traveling Sales Person - Family of functions

- *pgr_TSP* - When input is given as matrix cell information.
- *pgr_eucledianTSP* - When input are coordinates.

Driving Distance - Category

- *pgr_drivingDistance* - Driving Distance based on pgr_dijkstra
- *pgr_withPointsDD - Proposed* - Driving Distance based on pgr_withPoints
- Post pocessing
  - *pgr_alphaShape* - Alpha shape computation
  - *pgr_pointsAsPolygon* - Polygon around a set of points

CHAPTER 6

Available Functions but not official pgRouting functions

- *Stable Proposed Functions*
- *Experimental Functions*

6.1 Stable Proposed Functions

Warning: Proposed functions for next mayor release.

- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
  - The functions make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might not change. (But still can)
  - Signature might not change. (But still can)
  - Functionality might not change. (But still can)
  - pgTap tests have being done. But might need more.
  - Documentation might need refinement.

As part of the *Dijkstra - Family of functions*

- *pgr_dijkstraCostMatrix - proposed* Use pgr_dijkstra to calculate a cost matrix.
- *pgr_dijkstraVia - Proposed* - Use pgr_dijkstra to make a route via vertices.

Families

aStar - Family of functions

- *pgr_aStar* - A* algorithm for the shortest path.
- *pgr_aStarCost – proposed* - Get the aggregate cost of the shortest paths.
- *pgr_aStarCostMatrix - proposed* - Get the cost matrix of the shortest paths.

Bidirectional A - Family of functions*

- *pgr_bdAstar* - Bidirectional A* algorithm for obtaining paths.
- *pgr_bdAstarCost - Proposed* - Bidirectional A* algorithm to calculate the cost of the paths.
- *pgr_bdAstarCostMatrix - proposed* - Bidirectional A* algorithm to calculate a cost matrix of paths.

Bidirectional Dijkstra - Family of functions

- *pgr_bdDijkstra* - Bidirectional Dijkstra algorithm for the shortest paths.
- *pgr_bdDijkstraCost - Proposed* - Bidirectional Dijkstra to calculate the cost of the shortest paths
- *pgr_bdDijkstraCostMatrix - proposed* - Bidirectional Dijkstra algorithm to create a matrix of costs of the shortest paths.

Flow - Family of functions

- *pgr_maxFlow - Proposed* - Only the Max flow calculation using Push and Relabel algorithm.
- *pgr_boykovKolmogorov - Proposed* - Boykov and Kolmogorov with details of flow on edges.
- *pgr_edmondsKarp - Proposed* - Edmonds and Karp algorithm with details of flow on edges.
- *pgr_pushRelabel - Proposed* - Push and relabel algorithm with details of flow on edges.
- Applications
  - *pgr_edgeDisjointPaths - Proposed* - Calculates edge disjoint paths between two groups of vertices.
  - *pgr_maxCardinalityMatch - Proposed* - Calculates a maximum cardinality matching in a graph.

withPoints - Family of functions

- *pgr_withPoints - Proposed* - Route from/to points anywhere on the graph.
- *pgr_withPointsCost - Proposed* - Costs of the shortest paths.
- *pgr_withPointsCostMatrix - proposed* - Costs of the shortest paths.
- *pgr_withPointsKSP - Proposed* - K shortest paths.
- *pgr_withPointsDD - Proposed* - Driving distance.

categories

Cost - Category

- *pgr_aStarCost – proposed*
- *pgr_bdAstarCost - Proposed*
- *pgr_bdDijkstraCost - Proposed*
- *pgr_dijkstraCost*
- *pgr_withPointsCost - Proposed*

Cost Matrix - Category

- *pgr_aStarCostMatrix - proposed*
- *pgr_bdAstarCostMatrix - proposed*
- *pgr_bdDijkstraCostMatrix - proposed*
- *pgr_dijkstraCostMatrix - proposed*
- *pgr_withPointsCostMatrix - proposed*

KSP Category

- *pgr_KSP* - Driving Distance based on pgr_dijkstra

- *pgr_withPointsKSP - Proposed* - Driving Distance based on pgr_dijkstra

6.1.1 aStar - Family of functions

The A* (pronounced "A Star") algorithm is based on Dijkstra's algorithm with a heuristic that allow it to solve most shortest path problems by evaluation only a sub-set of the overall graph.

- *pgr_aStar* - A* algorithm for the shortest path.
- *pgr_aStarCost – proposed* - Get the aggregate cost of the shortest paths.
- *pgr_aStarCostMatrix - proposed* - Get the cost matrix of the shortest paths.

pgr_aStar

Name

`pgr_aStar` — Returns the shortest path using A* algorithm.

[49]

Fig. 6.1: Boost Graph Inside

Availability:

- pgr_astar(one to one) 2.0.0, Signature changed 2.3.0
- pgr_astar(other signatures) 2.4.0

Characteristics

The main Characteristics are:

- Process is done only on edges with positive costs.
- Vertices of the graph are:
  - **positive** when it belongs to the edges_sql
- Values are returned when there is a path.
  - When the starting vertex and ending vertex are the same, there is no path.
    - The agg_cost the non included values (v, v) is 0
  - When the starting vertex and ending vertex are the different and there is no path:
    - The agg_cost the non included values (u, v) is ∞
- When (x,y) coordinates for the same vertex identifier differ:
  - A random selection of the vertex's (x,y) coordinates is used.
- Running time: $O((E + V) * \log V)$

[49] http://www.boost.org//libs/graph/doc/astar_search.html

Signature Summary

```
pgr_aStar(edges_sql, start_vid, end_vid)
pgr_aStar(edges_sql, start_vid, end_vid, directed, heuristic, factor, epsilon)
```

Warning: Proposed functions for next mayor release.

- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
  - The functions make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might not change. (But still can)
  - Signature might not change. (But still can)
  - Functionality might not change. (But still can)
  - pgTap tests have being done. But might need more.
  - Documentation might need refinement.

```
pgr_aStar(edges_sql, start_vid, end_vids, directed, heuristic, factor, epsilon) --␣
↪proposed
pgr_aStar(edges_sql, starts_vid, end_vid, directed, heuristic, factor, epsilon) --␣
↪proposed
pgr_aStar(edges_sql, starts_vid, end_vids, directed, heuristic, factor, epsilon) --
↪ proposed
RETURNS SET OF (seq, path_seq [, start_vid] [, end_vid], node, edge, cost, agg_
↪cost)
  OR EMPTY SET
```

Signatures

Minimal Signature

```
pgr_aStar(edges_sql, start_vid, end_vid)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost)
```

Example Using the defaults

```
SELECT * FROM pgr_astar(
    'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table
↪',
    2, 12);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    4 |    1 |        0
   2 |        2 |    5 |   10 |    1 |        1
   3 |        3 |   10 |   12 |    1 |        2
   4 |        4 |   11 |   13 |    1 |        3
   5 |        5 |   12 |   -1 |    0 |        4
(5 rows)
```

One to One

```
pgr_aStar(edges_sql, start_vid, end_vid, directed, heuristic, factor, epsilon)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost)
```

Example Undirected using Heuristic 2

```
SELECT * FROM pgr_astar(
    'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table
↪',
    2, 12,
    directed := false, heuristic := 2);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    2 |    1 |        0
   2 |        2 |    3 |    3 |    1 |        1
   3 |        3 |    4 |   16 |    1 |        2
   4 |        4 |    9 |   15 |    1 |        3
   5 |        5 |   12 |   -1 |    0 |        4
(5 rows)
```

One to many

```
pgr_aStar(edges_sql, start_vid, end_vids, directed, heuristic, factor, epsilon) --
↪Proposed
RETURNS SET OF (seq, path_seq, end_vid, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from one `start_vid` to each `end_vid` in `end_vids`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Using this signature, will load once the graph and perform a one to one *pgr_astar* where the starting vertex is fixed, and stop when all `end_vids` are reached.

- The result is equivalent to the union of the results of the one to one *pgr_astar*.
- The extra `end_vid` in the result is used to distinguish to which path it belongs.

Example

```
SELECT * FROM pgr_astar(
    'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table
↪',
    2, ARRAY[3, 12], heuristic := 2);
 seq | path_seq | end_vid | node | edge | cost | agg_cost
-----+----------+---------+------+------+------+----------
   1 |        1 |       3 |    2 |    4 |    1 |        0
   2 |        2 |       3 |    5 |    8 |    1 |        1
   3 |        3 |       3 |    6 |    9 |    1 |        2
   4 |        4 |       3 |    9 |   16 |    1 |        3
   5 |        5 |       3 |    4 |    3 |    1 |        4
   6 |        6 |       3 |    3 |   -1 |    0 |        5
   7 |        1 |      12 |    2 |    4 |    1 |        0
   8 |        2 |      12 |    5 |   10 |    1 |        1
   9 |        3 |      12 |   10 |   12 |    1 |        2
  10 |        4 |      12 |   11 |   13 |    1 |        3
  11 |        5 |      12 |   12 |   -1 |    0 |        4
(11 rows)
```

Many to One

```
pgr_aStar(edges_sql, starts_vid, end_vid, directed, heuristic, factor, epsilon) --␣
↪Proposed
RETURNS SET OF (seq, path_seq, start_vid, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from each `start_vid` in `start_vids` to one `end_vid`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Using this signature, will load once the graph and perform several one to one *pgr_aStar* where the ending vertex is fixed.

- The result is the union of the results of the one to one *pgr_aStar*.
- The extra `start_vid` in the result is used to distinguish to which path it belongs.

Example

```
SELECT * FROM pgr_astar(
    'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table
↪',
    ARRAY[7, 2], 12, heuristic := 0);
 seq | path_seq | start_vid | node | edge | cost | agg_cost
-----+----------+-----------+------+------+------+----------
   1 |        1 |         2 |    2 |    4 |    1 |        0
   2 |        2 |         2 |    5 |   10 |    1 |        1
   3 |        3 |         2 |   10 |   12 |    1 |        2
   4 |        4 |         2 |   11 |   13 |    1 |        3
   5 |        5 |         2 |   12 |   -1 |    0 |        4
   6 |        1 |         7 |    7 |    6 |    1 |        0
   7 |        2 |         7 |    8 |    7 |    1 |        1
   8 |        3 |         7 |    5 |   10 |    1 |        2
   9 |        4 |         7 |   10 |   12 |    1 |        3
  10 |        5 |         7 |   11 |   13 |    1 |        4
  11 |        6 |         7 |   12 |   -1 |    0 |        5
(11 rows)
```

Many to Many

```
pgr_aStar(edges_sql, starts_vid, end_vids, directed, heuristic, factor, epsilon) --
↪ Proposed
RETURNS SET OF (seq, path_seq, start_vid, end_vid, node, edge, cost, agg_cost) or␣
↪EMPTY SET
```

This signature finds the shortest path from each `start_vid` in `start_vids` to each `end_vid` in `end_vids`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Using this signature, will load once the graph and perform several one to Many *pgr_dijkstra* for all `start_vids`.

- The result is the union of the results of the one to one *pgr_dijkstra*.
- The extra `start_vid` in the result is used to distinguish to which path it belongs.

The extra `start_vid` and `end_vid` in the result is used to distinguish to which path it belongs.

Example

```
SELECT * FROM pgr_astar(
    'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table
↪',
    ARRAY[7, 2], ARRAY[3, 12], heuristic := 2);
 seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
-----+----------+-----------+---------+------+------+------+----------
   1 |        1 |         2 |       3 |    2 |    4 |    1 |        0
   2 |        2 |         2 |       3 |    5 |    8 |    1 |        1
   3 |        3 |         2 |       3 |    6 |    9 |    1 |        2
   4 |        4 |         2 |       3 |    9 |   16 |    1 |        3
   5 |        5 |         2 |       3 |    4 |    3 |    1 |        4
   6 |        6 |         2 |       3 |    3 |   -1 |    0 |        5
   7 |        1 |         7 |       3 |    7 |    6 |    1 |        0
   8 |        2 |         7 |       3 |    8 |    7 |    1 |        1
   9 |        3 |         7 |       3 |    5 |    8 |    1 |        2
  10 |        4 |         7 |       3 |    6 |    9 |    1 |        3
  11 |        5 |         7 |       3 |    9 |   16 |    1 |        4
  12 |        6 |         7 |       3 |    4 |    3 |    1 |        5
  13 |        7 |         7 |       3 |    3 |   -1 |    0 |        6
  14 |        1 |         2 |      12 |    2 |    4 |    1 |        0
  15 |        2 |         2 |      12 |    5 |   10 |    1 |        1
  16 |        3 |         2 |      12 |   10 |   12 |    1 |        2
  17 |        4 |         2 |      12 |   11 |   13 |    1 |        3
  18 |        5 |         2 |      12 |   12 |   -1 |    0 |        4
  19 |        1 |         7 |      12 |    7 |    6 |    1 |        0
  20 |        2 |         7 |      12 |    8 |    7 |    1 |        1
  21 |        3 |         7 |      12 |    5 |   10 |    1 |        2
  22 |        4 |         7 |      12 |   10 |   12 |    1 |        3
  23 |        5 |         7 |      12 |   11 |   13 |    1 |        4
  24 |        6 |         7 |      12 |   12 |   -1 |    0 |        5
(24 rows)
```

Description of the Signatures

Description of the edges_sql query for astar like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.
x1	`ANY-NUMERICAL`		X coordinate of *source* vertex.
y1	`ANY-NUMERICAL`		Y coordinate of *source* vertex.
x2	`ANY-NUMERICAL`		X coordinate of *target* vertex.
y2	`ANY-NUMERICAL`		Y coordinate of *target* vertex.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Parameter	Type	Description
edges_sql	`TEXT`	Edges SQL query as described above.
start_vid	`ANY-INTEGER`	Starting vertex identifier.
end_vid	`ANY-INTEGER`	Ending vertex identifier.
directed	`BOOLEAN`	• Optional. – When `false` the graph is considered as Undirected. – Default is `true` which considers the graph as Directed.
heuristic	`INTEGER`	(optional). Heuristic number. Current valid values 0~5. Default `5` • 0: h(v) = 0 (Use this value to compare with pgr_dijkstra) • 1: h(v) abs(max(dx, dy)) • 2: h(v) abs(min(dx, dy)) • 3: h(v) = dx * dx + dy * dy • 4: h(v) = sqrt(dx * dx + dy * dy) • 5: h(v) = abs(dx) + abs(dy)
factor	`FLOAT`	(optional). For units manipulation. $factor > 0$. Default `1`. see *Factor*
epsilon	`FLOAT`	(optional). For less restricted results. $epsilon >= 1$. Default `1`.

Description of the return values for a path

Returns set of `(seq, path_seq [, start_vid] [, end_vid], node, edge, cost, agg_cost)`

Column	Type	Description
seq	`INT`	Sequential value starting from **1**.
path_id	`INT`	Path identifier. Has value **1** for the first of a path. Used when there are multiple paths for the same `start_vid` to `end_vid` combination.
path_seq	`INT`	Relative position in the path. Has value **1** for the beginning of a path.
start_vid	`BIGINT`	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	`BIGINT`	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
node	`BIGINT`	Identifier of the node in the path from `start_vid` to `end_vid`.
edge	`BIGINT`	Identifier of the edge used to go from `node` to the next node in the path sequence. `-1` for the last node of the path.
cost	`FLOAT`	Cost to traverse from `node` using `edge` to the next node in the path sequence.
agg_cost	`FLOAT`	Aggregate cost from `start_v` to `node`.

See Also

- *aStar - Family of functions*

- *Sample Data*
- http://www.boost.org/libs/graph/doc/astar_search.html
- http://en.wikipedia.org/wiki/A*_search_algorithm

Indices and tables

- genindex
- search

pgr_aStarCost – proposed

Name

`pgr_aStarCost` — Returns the aggregate cost shortest path using *aStar - Family of functions* algorithm.

[50]

Fig. 6.2: Boost Graph Inside

Availability: 2.4.0

Signature Summary

> **Warning:** Proposed functions for next mayor release.
>
> - They are not officially in the current release.
> - They will likely officially be part of the next mayor release:
>   - The functions make use of ANY-INTEGER and ANY-NUMERICAL
>   - Name might not change. (But still can)
>   - Signature might not change. (But still can)
>   - Functionality might not change. (But still can)
>   - pgTap tests have being done. But might need more.
>   - Documentation might need refinement.

```
pgr_aStarCost(edges_sql, start_vid, end_vid) -- Proposed
pgr_aStarCost(edges_sql, start_vid, end_vid, directed, heuristic, factor, epsilon)␣
↪-- Proposed
pgr_aStarCost(edges_sql, start_vid, end_vids, directed, heuristic, factor,␣
↪epsilon) -- Proposed
pgr_aStarCost(edges_sql, starts_vid, end_vid, directed, heuristic, factor,␣
↪epsilon) -- Proposed
pgr_aStarCost(edges_sql, starts_vid, end_vids, directed, heuristic, factor,␣
↪epsilon) -- Proposed
```

[50] http://www.boost.org//libs/graph/doc/astar_search.html

```
RETURNS SET OF (start_vid, end_vid, agg_cost) OR EMPTY SET
```

Signatures

Minimal Signature

```
pgr_aStarCost(edges_sql, start_vid, end_vid)
RETURNS SET OF (start_vid, end_vid, agg_cost) OR EMPTY SET
```

Example Using the defaults

```
SELECT * FROM pgr_aStarCost(
    'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table
↪',
    2, 12);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |      12 |        4
(1 row)
```

One to One

```
pgr_aStarCost(edges_sql, start_vid, end_vid, directed, heuristic, factor, epsilon)
RETURNS SET OF (start_vid, end_vid, agg_cost) OR EMPTY SET
```

Example Setting a Heuristic

```
SELECT * FROM pgr_aStarCost(
    'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table
↪',
    2, 12,
    directed := false, heuristic := 2);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |      12 |        4
(1 row)
```

One to many

```
pgr_aStarCost(edges_sql, start_vid, end_vids, directed, heuristic, factor,␣
↪epsilon) -- Proposed
RETURNS SET OF (start_vid, end_vid, agg_cost) OR EMPTY SET
```

This signature finds a path from one `start_vid` to each `end_vid` in `end_vids`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Using this signature, will load once the graph and perform a one to one *pgr_astar* where the starting vertex is fixed, and stop when all `end_vids` are reached.

- The result is equivalent to the union of the results of the one to one *pgr_astar*.

- The extra `end_vid` column in the result is used to distinguish to which path it belongs.

  Example

```
SELECT * FROM pgr_aStarCost(
    'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table
↪',
    2, ARRAY[3, 12], heuristic := 2);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |       3 |        5
         2 |      12 |        4
(2 rows)
```

Many to One

```
pgr_aStarCost(edges_sql, starts_vid, end_vid, directed, heuristic, factor,␣
↪epsilon) -- Proposed
RETURNS SET OF (start_vid, end_vid, agg_cost) OR EMPTY SET
```

This signature finds the shortest path from each `start_vid` in `start_vids` to one `end_vid`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Using this signature, will load once the graph and perform several one to one *pgr_aStar* where the ending vertex is fixed.

- The result is the union of the results of the one to one *pgr_aStar*.
- The extra `start_vid` column in the result is used to distinguish to which path it belongs.

  Example

```
SELECT * FROM pgr_aStarCost(
    'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table
↪',
    ARRAY[7, 2], 12, heuristic := 0);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |      12 |        4
         7 |      12 |        5
(2 rows)
```

Many to Many

```
pgr_aStarCost(edges_sql, starts_vid, end_vids, directed, heuristic, factor,␣
↪epsilon) -- Proposed
RETURNS SET OF (start_vid, end_vid, agg_cost) OR EMPTY SET
```

This signature finds the shortest path from each `start_vid` in `start_vids` to each `end_vid` in `end_vids`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Using this signature, will load once the graph and perform several one to Many *pgr_dijkstra* for all `start_vids`.

- The result is the union of the results of the one to one *pgr_dijkstra*.

- The extra `start_vid` in the result is used to distinguish to which path it belongs.

The extra `start_vid` and `end_vid` in the result is used to distinguish to which path it belongs.

Example

```
SELECT * FROM pgr_aStarCost(
    'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table
↪',
    ARRAY[7, 2], ARRAY[3, 12], heuristic := 2);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |       3 |        5
         2 |      12 |        4
         7 |       3 |        6
         7 |      12 |        5
(4 rows)
```

Description of the Signatures

Description of the edges_sql query for astar like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.
x1	`ANY-NUMERICAL`		X coordinate of *source* vertex.
y1	`ANY-NUMERICAL`		Y coordinate of *source* vertex.
x2	`ANY-NUMERICAL`		X coordinate of *target* vertex.
y2	`ANY-NUMERICAL`		Y coordinate of *target* vertex.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Parameter	Type	Description
edges_sql	`TEXT`	Edges SQL query as described above.
start_vid	`ANY-INTEGER`	Starting vertex identifier.
end_vid	`ANY-INTEGER`	Ending vertex identifier.
directed	`BOOLEAN`	• Optional. – When `false` the graph is considered as Undirected. – Default is `true` which considers the graph as Directed.
heuristic	`INTEGER`	(optional). Heuristic number. Current valid values 0~5. Default `5` • 0: h(v) = 0 (Use this value to compare with pgr_dijkstra) • 1: h(v) abs(max(dx, dy)) • 2: h(v) abs(min(dx, dy)) • 3: h(v) = dx * dx + dy * dy • 4: h(v) = sqrt(dx * dx + dy * dy) • 5: h(v) = abs(dx) + abs(dy)
factor	`FLOAT`	(optional). For units manipulation. $factor > 0$. Default `1`. See *Factor*
epsilon	`FLOAT`	(optional). For less restricted results. $epsilon >= 1$. Default `1`.

Description of the return values for a Cost function

Returns set of `(start_vid, end_vid, agg_cost)`

Column	Type	Description
start_vid	`BIGINT`	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	`BIGINT`	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
agg_cost	`FLOAT`	Aggregate cost from `start_vid` to `end_vid`.

See Also

- *aStar - Family of functions*.
- *Sample Data* network.
- http://www.boost.org/libs/graph/doc/astar_search.html

- http://en.wikipedia.org/wiki/A*_search_algorithm

Indices and tables

- genindex
- search

pgr_aStarCostMatrix - proposed

Name

`pgr_aStarCostMatrix` - Calculates the a cost matrix using *pgr_aStar*.

> **Warning:** Proposed functions for next mayor release.
>
> - They are not officially in the current release.
> - They will likely officially be part of the next mayor release:
>   - The functions make use of ANY-INTEGER and ANY-NUMERICAL
>   - Name might not change. (But still can)
>   - Signature might not change. (But still can)
>   - Functionality might not change. (But still can)
>   - pgTap tests have being done. But might need more.
>   - Documentation might need refinement.

[51]

Fig. 6.3: Boost Graph Inside

Availability: 2.4.0

Synopsis

Using aStar algorithm, calculate and return a cost matrix.

Signature Summary

```
pgr_aStarCostMatrix(edges_sql, vids)
pgr_aStarCostMatrix(edges_sql, vids, directed, heuristic, factor, epsilon)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

[51] http://www.boost.org/libs/graph

Signatures

Minimal Signature

The minimal signature:

- Is for a **directed** graph.

```
pgr_aStarCostMatrix(edges_sql, vids)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Example Cost matrix for vertices 1, 2, 3, and 4.

```
SELECT * FROM pgr_aStarCostMatrix(
    'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table
↪',
    (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5)
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |       1 |        1
         3 |       1 |        2
         4 |       1 |        3
         1 |       2 |        1
         3 |       2 |        1
         4 |       2 |        2
         1 |       3 |        6
         2 |       3 |        5
         4 |       3 |        1
         1 |       4 |        5
         2 |       4 |        4
         3 |       4 |        3
(12 rows)
```

Complete Signature

```
pgr_aStarCostMatrix(edges_sql, vids, directed, heuristic, factor, epsilon)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Example Cost matrix for an undirected graph for vertices 1, 2, 3, and 4.

This example returns a symmetric cost matrix.

```
SELECT * FROM pgr_aStarCostMatrix(
    'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table
↪',
    (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
    directed := false, heuristic := 2
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |       1 |        1
         3 |       1 |        2
         4 |       1 |        3
         1 |       2 |        1
         3 |       2 |        1
         4 |       2 |        2
         1 |       3 |        2
         2 |       3 |        1
         4 |       3 |        1
```

```
         1 |         4 |          3
         2 |         4 |          2
         3 |         4 |          1
(12 rows)
```

Description of the Signatures

Description of the edges_sql query for astar like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.
x1	`ANY-NUMERICAL`		X coordinate of *source* vertex.
y1	`ANY-NUMERICAL`		Y coordinate of *source* vertex.
x2	`ANY-NUMERICAL`		X coordinate of *target* vertex.
y2	`ANY-NUMERICAL`		Y coordinate of *target* vertex.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Parameter	Type	Description
edges_sql	`TEXT`	Edges SQL query as described above.
vids	`ARRAY[ANY-INTEGER]`	Array of vertices_identifiers.
directed	`BOOLEAN`	• Optional. – When `false` the graph is considered as Undirected. – Default is `true` which considers the graph as Directed.
heuristic	`INTEGER`	(optional). Heuristic number. Current valid values 0~5. Default `5` • 0: h(v) = 0 (Use this value to compare with pgr_dijkstra) • 1: h(v) abs(max(dx, dy)) • 2: h(v) abs(min(dx, dy)) • 3: h(v) = dx * dx + dy * dy • 4: h(v) = sqrt(dx * dx + dy * dy) • 5: h(v) = abs(dx) + abs(dy)
factor	`FLOAT`	(optional). For units manipulation. $factor > 0$. Default `1`.
epsilon	`FLOAT`	(optional). For less restricted results. $epsilon >= 1$. Default `1`.

Description of the return values for a Cost function

Returns set of `(start_vid, end_vid, agg_cost)`

Column	Type	Description
start_vid	`BIGINT`	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	`BIGINT`	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
agg_cost	`FLOAT`	Aggregate cost from `start_vid` to `end_vid`.

Examples

Example Use with tsp

```
SELECT * FROM pgr_TSP(
    $$
    SELECT * FROM pgr_aStarCostMatrix(
        'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_
↪table',
        (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
        directed:= false, heuristic := 2
    )
    $$,
    randomize := false
);
```

```
 seq | node | cost | agg_cost
-----+------+------+----------
   1 |    1 |    1 |        0
   2 |    2 |    1 |        1
   3 |    3 |    1 |        2
   4 |    4 |    3 |        3
   5 |    1 |    0 |        6
(5 rows)
```

See Also

- *aStar - Family of functions*
- *Cost Matrix - Category*
- *Traveling Sales Person - Family of functions*
- The queries use the *Sample Data* network.

Indices and tables

- genindex
- search

The problem definition (Advanced documentation)

The A* (pronounced "A Star") algorithm is based on Dijkstra's algorithm with a heuristic, that is an estimation of the remaining cost from the vertex to the goal, that allows to solve most shortest path problems by evaluation only a sub-set of the overall graph. Running time: $O((E + V) * \log V)$

Heuristic

Currently the heuristic functions available are:

- 0: $h(v) = 0$ (Use this value to compare with pgr_dijkstra)
- 1: $h(v) = abs(max(\Delta x, \Delta y))$
- 2: $h(v) = abs(min(\Delta x, \Delta y))$
- 3: $h(v) = \Delta x * \Delta x + \Delta y * \Delta y$
- 4: $h(v) = sqrt(\Delta x * \Delta x + \Delta y * \Delta y)$
- 5: $h(v) = abs(\Delta x) + abs(\Delta y)$

where $\Delta x = x_1 - x_0$ and $\Delta y = y_1 - y_0$

Factor

Analysis 1

Working with cost/reverse_cost as length in degrees, x/y in lat/lon: Factor = 1 (no need to change units)

Analysis 2

Working with cost/reverse_cost as length in meters, x/y in lat/lon: Factor = would depend on the location of the points:

latitude	conversion	Factor
45	1 longitude degree is 78846.81 m	78846
0	1 longitude degree is 111319.46 m	111319

Analysis 3

Working with cost/reverse_cost as time in seconds, x/y in lat/lon: Factor: would depend on the location of the points and on the average speed say 25m/s is the speed.

latitude	conversion	Factor
45	1 longitude degree is (78846.81m)/(25m/s)	3153 s
0	1 longitude degree is (111319.46 m)/(25m/s)	4452 s

See Also

- *pgr_aStar*
- *pgr_aStarCost – proposed*
- *pgr_aStarCostMatrix - proposed*
- http://www.boost.org/libs/graph/doc/astar_search.html
- http://en.wikipedia.org/wiki/A*_search_algorithm

Indices and tables

- genindex
- search

6.1.2 Bidirectional A* - Family of functions

- *pgr_bdAstar* - Bidirectional A* algorithm for obtaining paths.
- *pgr_bdAstarCost - Proposed* - Bidirectional A* algorithm to calculate the cost of the paths.
- *pgr_bdAstarCostMatrix - proposed* - Bidirectional A* algorithm to calculate a cost matrix of paths.

pgr_bdAstarCost - Proposed

Name

`pgr_bdAstarCost` — Returns the shortest path using A* algorithm.

[52] http://www.boost.org//libs/graph

[52]

Fig. 6.4: Boost Graph Inside

Availability: 2.5.0

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.
  - pgTap tests might be missing.
  - Might need c/c++ coding.
  - May lack documentation.
  - Documentation if any might need to be rewritten.
  - Documentation examples might need to be automatically generated.
  - Might need a lot of feedback from the comunity.
  - Might depend on a proposed function of pgRouting
  - Might depend on a deprecated function of pgRouting

Signature Summary

```
pgr_bdAstarCost(edges_sql, start_vid, end_vid)
pgr_bdAstarCost(edges_sql, start_vid, end_vid [, directed , heuristic, factor,␣
↪epsilon])
pgr_bdAstarCost(edges_sql, start_vid, end_vids [, directed, heuristic, factor,␣
↪epsilon])
pgr_bdAstarCost(edges_sql, start_vids, end_vid [, directed, heuristic, factor,␣
↪epsilon])
pgr_bdAstarCost(edges_sql, start_vids, end_vids [, directed, heuristic, factor,␣
↪epsilon])

RETURNS SET OF (start_vid, end_vid, agg_cost)
OR EMPTY SET
```

Using these signatures, will load once the graph and perform several one to one *pgr_bdAstarCost*

- The result is the union of the results of the one to one *pgr_bdAstarCost*.
- The extra `start_vid` and/or `end_vid` in the result is used to distinguish to which path it belongs.

Signatures

Minimal Signature

```
pgr_bdAstarCost(edges_sql, start_vid, end_vid)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

This usage finds the shortest path from the `start_vid` to the `end_vid`

- on a **directed** graph
- with **heuristic**'s value 5
- with **factor**'s value 1
- with **epsilon**'s value 1

Example Using the defaults

```
SELECT * FROM pgr_bdAstarCost(
    'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
    FROM edge_table',
    2, 3
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |       3 |        5
(1 row)
```

pgr_bdAstarCost One to One

```
pgr_bdAstarCost(edges_sql, start_vid, end_vid [, directed, heuristic, factor, ␣
↪epsilon])
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

This usage finds the shortest path from the `start_vid` to each `end_vid` in `end_vids` allowing the user to choose

- if the graph is **directed** or **undirected**
- **heuristic**,
- and/or **factor**
- and/or **epsilon**.

Note: In the One to One signature, because of the deprecated signature existence, it is compulsory to indicate if the graph is **directed** or **undirected**.

Example Directed using Heuristic 2

```
SELECT * FROM pgr_bdAstarCost(
    'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
    FROM edge_table',
    2, 3,
    true, heuristic := 2
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |       3 |        5
```

```
(1 row)
```

pgr_bdAstarCost One to many

```
pgr_bdAstarCost(edges_sql, start_vid, end_vids [, directed, heuristic, factor,
↪epsilon])
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

This usage finds the shortest path from the `start_vid` to each `end_vid` in `end_vids` allowing the user to choose

- if the graph is **directed** or **undirected**
- and/or **heuristic**,
- and/or **factor**
- and/or **epsilon**.

Example Directed using Heuristic 3 and a factor of 3.5

```
SELECT * FROM pgr_bdAstarCost(
    'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
    FROM edge_table',
    2, ARRAY[3, 11],
    heuristic := 3, factor := 3.5
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |       3 |        5
         2 |      11 |        3
(2 rows)
```

pgr_bdAstarCost Many to One

```
pgr_bdAstarCost(edges_sql, start_vids, end_vid [, directed, heuristic, factor,
↪epsilon])
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

This usage finds the shortest path from each `start_vid` in `start_vids` to the `end_vid` allowing the user to choose

- if the graph is **directed** or **undirected**
- and/or **heuristic**,
- and/or **factor**
- and/or **epsilon**.

Example Undirected graph with Heuristic 4

```
SELECT * FROM pgr_bdAstarCost(
    'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
    FROM edge_table',
    ARRAY[2, 7], 3,
    false, heuristic := 4
);
 start_vid | end_vid | agg_cost
```

```
-----------+---------+----------
         2 |       3 |         1
         7 |       3 |         4
(2 rows)
```

pgr_bdAstarCost Many to Many

```
pgr_bdAstarCost(edges_sql, start_vids, end_vids [, directed, heuristic, factor,␣
↪epsilon])
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

This usage finds the shortest path from each `start_vid` in `start_vids` to each `end_vid` in `end_vids` allowing the use

- if the graph is **directed** or **undirected**
- and/or **heuristic**,
- and/or **factor**
- and/or **epsilon**.

Example Directed graph with a factor of 0.5

```
SELECT * FROM pgr_bdAstarCost(
    'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
    FROM edge_table',
    ARRAY[2, 7], ARRAY[3, 11],
    factor := 0.5
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |       3 |         5
         2 |      11 |         3
         7 |       3 |         6
         7 |      11 |         4
(4 rows)
```

Description of the Signatures

Description of the edges_sql query for astar like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.
x1	`ANY-NUMERICAL`		X coordinate of *source* vertex.
y1	`ANY-NUMERICAL`		Y coordinate of *source* vertex.
x2	`ANY-NUMERICAL`		X coordinate of *target* vertex.
y2	`ANY-NUMERICAL`		Y coordinate of *target* vertex.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Parameter	Type	Description
edges_sql	`TEXT`	Edges SQL query as described above.
start_vid	`ANY-INTEGER`	Starting vertex identifier.
start_vids	`ARRAY[ANY-INTEGER]`	Starting vertices identifierers.
end_vid	`ANY-INTEGER`	Ending vertex identifier.
end_vids	`ARRAY[ANY-INTEGER]`	Ending vertices identifiers.
directed	`BOOLEAN`	• Optional. – When `false` the graph is considered as Undirected. – Default is `true` which considers the graph as Directed.
heuristic	`INTEGER`	(optional). Heuristic number. Current valid values 0~5. Default `5` • 0: h(v) = 0 (Use this value to compare with pgr_dijkstra) • 1: h(v) abs(max(dx, dy)) • 2: h(v) abs(min(dx, dy)) • 3: h(v) = dx * dx + dy * dy • 4: h(v) = sqrt(dx * dx + dy * dy) • 5: h(v) = abs(dx) + abs(dy)
factor	`FLOAT`	(optional). For units manipulation. $factor > 0$. Default `1`. see *Factor*
epsilon	`FLOAT`	(optional). For less restricted results. $epsilon >= 1$. Default `1`.

Description of the return values for a Cost function

Returns set of `(start_vid, end_vid, agg_cost)`

Column	Type	Description
start_vid	`BIGINT`	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	`BIGINT`	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
agg_cost	`FLOAT`	Aggregate cost from `start_vid` to `end_vid`.

See Also

- *Bidirectional A* - Family of functions*
- *Sample Data* network.
- Migration Guide[53]
- http://www.boost.org/libs/graph/doc/astar_search.html
- http://en.wikipedia.org/wiki/A*_search_algorithm

[53] https://github.com/cvvergara/pgrouting/wiki/Migration-Guide#pgr_bdastar

Indices and tables

- genindex
- search

pgr_bdAstarCostMatrix - proposed

Name

`pgr_bdAstarCostMatrix` - Calculates the a cost matrix using *pgr_bdAstar*.

[54]

Fig. 6.5: Boost Graph Inside

Availability: 2.5.0

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.
  - pgTap tests might be missing.
  - Might need c/c++ coding.
  - May lack documentation.
  - Documentation if any might need to be rewritten.
  - Documentation examples might need to be automatically generated.
  - Might need a lot of feedback from the comunity.
  - Might depend on a proposed function of pgRouting
  - Might depend on a deprecated function of pgRouting

Synopsis

Using Dijkstra algorithm, calculate and return a cost matrix.

[54] http://www.boost.org/libs/graph

Signature Summary

```
pgr_bdAstarCostMatrix(edges_sql, start_vids)
pgr_bdAstarCostMatrix(edges_sql, start_vids, [, directed , heuristic, factor,␣
↪epsilon])
RETURNS SET OF (start_vid, end_vid, agg_cost)
OR EMPTY SET
```

Signatures

Minimal Signature

```
pgr_bdAstarCostMatrix(edges_sql, start_vids)
RETURNS SET OF (start_vid, end_vid, agg_cost)
OR EMPTY SET
```

This usage calculates the cost from the each `start_vid` in `start_vids` to each `start_vid` in `start_vids`

- on a **directed** graph
- with **heuristic**'s value 5
- with **factor**'s value 1
- with **epsilon**'s value 1

Example Cost matrix for vertices 1, 2, 3, and 4.

```
SELECT * FROM pgr_bdAstarCostMatrix(
    'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table
↪',
    (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5)
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         1 |       2 |        1
         1 |       3 |        6
         1 |       4 |        5
         2 |       1 |        1
         2 |       3 |        5
         2 |       4 |        4
         3 |       1 |        2
         3 |       2 |        1
         3 |       4 |        3
         4 |       1 |        3
         4 |       2 |        2
         4 |       3 |        1
(12 rows)
```

Complete Signature

```
pgr_bdAstarCostMatrix(edges_sql, start_vids, [, directed , heuristic, factor,␣
↪epsilon])
RETURNS SET OF (start_vid, end_vid, agg_cost)
OR EMPTY SET
```

This usage calculates the cost from the each `start_vid` in `start_vids` to each `start_vid` in `start_vids` allowing th

- if the graph is **directed** or **undirected**
- **heuristic**,
- and/or **factor**
- and/or **epsilon**.

Example Cost matrix for an undirected graph for vertices 1, 2, 3, and 4.

This example returns a symmetric cost matrix.

```
SELECT * FROM pgr_bdAstarCostMatrix(
    'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table
↪',
    (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
    false
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         1 |       2 |        1
         1 |       3 |        2
         1 |       4 |        3
         2 |       1 |        1
         2 |       3 |        1
         2 |       4 |        2
         3 |       1 |        2
         3 |       2 |        1
         3 |       4 |        1
         4 |       1 |        3
         4 |       2 |        2
         4 |       3 |        1
(12 rows)
```

Description of the Signatures

Description of the edges_sql query for astar like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	ANY-INTEGER		Identifier of the edge.
source	ANY-INTEGER		Identifier of the first end point vertex of the edge.
target	ANY-INTEGER		Identifier of the second end point vertex of the edge.
cost	ANY-NUMERICAL		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	ANY-NUMERICAL	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.
x1	ANY-NUMERICAL		X coordinate of *source* vertex.
y1	ANY-NUMERICAL		Y coordinate of *source* vertex.
x2	ANY-NUMERICAL		X coordinate of *target* vertex.
y2	ANY-NUMERICAL		Y coordinate of *target* vertex.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Parameter	Type	Description
edges_sql	TEXT	Edges SQL query as described above.
start_vid	ANY-INTEGER	Starting vertex identifier.
start_vids	ARRAY[ANY-INTEGER]	Starting vertices identifierers.
end_vid	ANY-INTEGER	Ending vertex identifier.
end_vids	ARRAY[ANY-INTEGER]	Ending vertices identifiers.
directed	BOOLEAN	• Optional. – When false the graph is considered as Undirected. – Default is true which considers the graph as Directed.
heuristic	INTEGER	(optional). Heuristic number. Current valid values 0~5. Default 5 • 0: h(v) = 0 (Use this value to compare with pgr_dijkstra) • 1: h(v) abs(max(dx, dy)) • 2: h(v) abs(min(dx, dy)) • 3: h(v) = dx * dx + dy * dy • 4: h(v) = sqrt(dx * dx + dy * dy) • 5: h(v) = abs(dx) + abs(dy)
factor	FLOAT	(optional). For units manipulation. $factor > 0$. Default 1. see *Factor*
epsilon	FLOAT	(optional). For less restricted results. $epsilon >= 1$. Default 1.

Description of the return values for a Cost function

Returns set of (start_vid, end_vid, agg_cost)

Column	Type	Description
start_vid	BIGINT	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	BIGINT	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
agg_cost	FLOAT	Aggregate cost from start_vid to end_vid.

Examples

Example Use with tsp

```
SELECT * FROM pgr_TSP(
    $$
    SELECT * FROM pgr_bdAstarCostMatrix(
        'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_
↪table',
        (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
        false
    )
```

```
    $$,
    randomize := false
);
 seq | node | cost | agg_cost
-----+------+------+----------
   1 |    1 |    1 |        0
   2 |    2 |    1 |        1
   3 |    3 |    1 |        2
   4 |    4 |    3 |        3
   5 |    1 |    0 |        6
(5 rows)
```

See Also

- *Bidirectional A* - Family of functions*
- *Cost Matrix - Category*
- *Traveling Sales Person - Family of functions*
- The queries use the *Sample Data* network.

Indices and tables

- genindex
- search

Synopsis

Based on A* algorithm, the bidirectional search finds a shortest path from a starting vertex (`start_vid`) to an ending vertex (`end_vid`). It runs two simultaneous searches: one forward from the `start_vid`, and one backward from the `end_vid`, stopping when the two meet in the middle. This implementation can be used with a directed graph and an undirected graph.

Characteristics

The main Characteristics are:

- Process is done only on edges with positive costs.
- Values are returned when there is a path.
- When the starting vertex and ending vertex are the same, there is no path.
  - The *agg_cost* the non included values *(v, v)* is *0*
- When the starting vertex and ending vertex are the different and there is no path:
  - The *agg_cost* the non included values *(u, v)* is ∞
- Running time (worse case scenario): $O((E + V) * \log V)$
- For large graphs where there is a path bewtween the starting vertex and ending vertex:
  - It is expected to terminate faster than pgr_astar

Description of the Signatures

Description of the edges_sql query for astar like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.
x1	`ANY-NUMERICAL`		X coordinate of *source* vertex.
y1	`ANY-NUMERICAL`		Y coordinate of *source* vertex.
x2	`ANY-NUMERICAL`		X coordinate of *target* vertex.
y2	`ANY-NUMERICAL`		Y coordinate of *target* vertex.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Parameter	Type	Description
edges_sql	`TEXT`	Edges SQL query as described above.
start_vid	`ANY-INTEGER`	Starting vertex identifier.
start_vids	`ARRAY[ANY-INTEGER]`	Starting vertices identifierers.
end_vid	`ANY-INTEGER`	Ending vertex identifier.
end_vids	`ARRAY[ANY-INTEGER]`	Ending vertices identifiers.
directed	`BOOLEAN`	• Optional. – When `false` the graph is considered as Undirected. – Default is `true` which considers the graph as Directed.
heuristic	`INTEGER`	(optional). Heuristic number. Current valid values 0~5. Default `5` • 0: h(v) = 0 (Use this value to compare with pgr_dijkstra) • 1: h(v) abs(max(dx, dy)) • 2: h(v) abs(min(dx, dy)) • 3: h(v) = dx * dx + dy * dy • 4: h(v) = sqrt(dx * dx + dy * dy) • 5: h(v) = abs(dx) + abs(dy)
factor	`FLOAT`	(optional). For units manipulation. $factor > 0$. Default `1`. see *Factor*
epsilon	`FLOAT`	(optional). For less restricted results. $epsilon >= 1$. Default `1`.

See Also

Indices and tables

- genindex
- search

6.1.3 Bidirectional Dijkstra - Family of functions

- *pgr_bdDijkstra* - Bidirectional Dijkstra algorithm for the shortest paths.
- *pgr_bdDijkstraCost - Proposed* - Bidirectional Dijkstra to calculate the cost of the shortest paths
- *pgr_bdDijkstraCostMatrix - proposed* - Bidirectional Dijkstra algorithm to create a matrix of costs of the shortest paths.

pgr_bdDijkstraCost - Proposed

`pgr_bdDijkstraCost` — Returns the shortest path(s)'s cost using Bidirectional Dijkstra algorithm.

[55] http://www.boost.org/libs/graph/doc

[55]

Fig. 6.6: Boost Graph Inside

Availability: 2.5.0

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.
  - pgTap tests might be missing.
  - Might need c/c++ coding.
  - May lack documentation.
  - Documentation if any might need to be rewritten.
  - Documentation examples might need to be automatically generated.
  - Might need a lot of feedback from the comunity.
  - Might depend on a proposed function of pgRouting
  - Might depend on a deprecated function of pgRouting

Signature Summary

```
pgr_dijkstraCost(edges_sql, start_vid,  end_vid)
pgr_bdDijkstraCost(edges_sql, start_vid, end_vid, directed)
pgr_bdDijkstraCost(edges_sql, start_vid, end_vids, directed)
pgr_bdDijkstraCost(edges_sql, start_vids, end_vid, directed)
pgr_bdDijkstraCost(edges_sql, start_vids, end_vids, directed)

RETURNS SET OF (start_vid, end_vid, agg_cost)
OR EMPTY SET
```

Signatures

Minimal signature

```
pgr_bdDijkstraCost(edges_sql, start_vid, end_vid)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost) or EMPTY SET
```

The minimal signature is for a **directed** graph from one `start_vid` to one `end_vid`:

Example

```
SELECT * FROM pgr_bdDijkstraCost(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, 3
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |       3 |        5
(1 row)
```

pgr_bdDijkstraCost One to One

```
pgr_bdDijkstraCost(edges_sql, start_vid, end_vid, directed)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from one `start_vid` to one `end_vid`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Example

```
SELECT * FROM pgr_bdDijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, 3,
    false
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    2 |    1 |        0
   2 |        2 |    3 |   -1 |    0 |        1
(2 rows)
```

pgr_bdDijkstraCost One to many

```
pgr_bdDijkstra(edges_sql, start_vid, end_vids, directed)
RETURNS SET OF (seq, path_seq, end_vid, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from one `start_vid` to each `end_vid` in `end_vids`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Using this signature, will load once the graph and perform a one to one *pgr_dijkstra* where the starting vertex is fixed, and stop when all `end_vids` are reached.

- The result is equivalent to the union of the results of the one to one *pgr_dijkstra*.
- The extra `end_vid` in the result is used to distinguish to which path it belongs.

Example

```
SELECT * FROM pgr_bdDijkstraCost(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, ARRAY[3, 11]);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |       3 |        5
```

```
        2 |       11 |         3
(2 rows)
```

pgr_bdDijkstraCost Many to One

```
pgr_bdDijkstra(edges_sql, start_vids, end_vid, directed)
RETURNS SET OF (seq, path_seq, start_vid, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from each `start_vid` in `start_vids` to one `end_vid`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Using this signature, will load once the graph and perform several one to one *pgr_dijkstra* where the ending vertex is fixed.

- The result is the union of the results of the one to one *pgr_dijkstra*.
- The extra `start_vid` in the result is used to distinguish to which path it belongs.

Example

```
SELECT * FROM pgr_bdDijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[2, 7], 3);
 seq | path_seq | start_vid | node | edge | cost | agg_cost
-----+----------+-----------+------+------+------+----------
   1 |        1 |         2 |    2 |    4 |    1 |        0
   2 |        2 |         2 |    5 |    8 |    1 |        1
   3 |        3 |         2 |    6 |    9 |    1 |        2
   4 |        4 |         2 |    9 |   16 |    1 |        3
   5 |        5 |         2 |    4 |    3 |    1 |        4
   6 |        6 |         2 |    3 |   -1 |    0 |        5
   7 |        1 |         7 |    7 |    6 |    1 |        0
   8 |        2 |         7 |    8 |    7 |    1 |        1
   9 |        3 |         7 |    5 |    8 |    1 |        2
  10 |        4 |         7 |    6 |    9 |    1 |        3
  11 |        5 |         7 |    9 |   16 |    1 |        4
  12 |        6 |         7 |    4 |    3 |    1 |        5
  13 |        7 |         7 |    3 |   -1 |    0 |        6
(13 rows)
```

pgr_bdDijkstraCost Many to Many

```
pgr_bdDijkstra(edges_sql, start_vids, end_vids, directed)
RETURNS SET OF (seq, path_seq, start_vid, end_vid, node, edge, cost, agg_cost) or␣
↪EMPTY SET
```

This signature finds the shortest path from each `start_vid` in `start_vids` to each `end_vid` in `end_vids`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Using this signature, will load once the graph and perform several one to Many *pgr_dijkstra* for all `start_vids`.

- The result is the union of the results of the one to one *pgr_dijkstra*.

- The extra `start_vid` in the result is used to distinguish to which path it belongs.

The extra `start_vid` and `end_vid` in the result is used to distinguish to which path it belongs.

Example

```
SELECT * FROM pgr_bdDijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[2, 7], ARRAY[3, 11]);
 seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
-----+----------+-----------+---------+------+------+------+----------
   1 |        1 |         2 |       3 |    2 |    4 |    1 |        0
   2 |        2 |         2 |       3 |    5 |    8 |    1 |        1
   3 |        3 |         2 |       3 |    6 |    9 |    1 |        2
   4 |        4 |         2 |       3 |    9 |   16 |    1 |        3
   5 |        5 |         2 |       3 |    4 |    3 |    1 |        4
   6 |        6 |         2 |       3 |    3 |   -1 |    0 |        5
   7 |        1 |         2 |      11 |    2 |    4 |    1 |        0
   8 |        2 |         2 |      11 |    5 |    8 |    1 |        1
   9 |        3 |         2 |      11 |    6 |   11 |    1 |        2
  10 |        4 |         2 |      11 |   11 |   -1 |    0 |        3
  11 |        1 |         7 |       3 |    7 |    6 |    1 |        0
  12 |        2 |         7 |       3 |    8 |    7 |    1 |        1
  13 |        3 |         7 |       3 |    5 |    8 |    1 |        2
  14 |        4 |         7 |       3 |    6 |    9 |    1 |        3
  15 |        5 |         7 |       3 |    9 |   16 |    1 |        4
  16 |        6 |         7 |       3 |    4 |    3 |    1 |        5
  17 |        7 |         7 |       3 |    3 |   -1 |    0 |        6
  18 |        1 |         7 |      11 |    7 |    6 |    1 |        0
  19 |        2 |         7 |      11 |    8 |    7 |    1 |        1
  20 |        3 |         7 |      11 |    5 |   10 |    1 |        2
  21 |        4 |         7 |      11 |   10 |   12 |    1 |        3
  22 |        5 |         7 |      11 |   11 |   -1 |    0 |        4
(22 rows)
```

Description of the Signatures

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Column	Type	Default	Description
sql	`TEXT`		SQL query as described above.
start_vid	`BIGINT`		Identifier of the starting vertex of the path.
start_vids	`ARRAY[BIGINT]`		Array of identifiers of starting vertices.
end_vid	`BIGINT`		Identifier of the ending vertex of the path.
end_vids	`ARRAY[BIGINT]`		Array of identifiers of ending vertices.
directed	`BOOLEAN`	`true`	• When `true` Graph is considered *Directed* • When `false` the graph is considered as *Undirected*.

Description of the return values for a Cost function

Returns set of `(start_vid, end_vid, agg_cost)`

Column	Type	Description
start_vid	`BIGINT`	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	`BIGINT`	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
agg_cost	`FLOAT`	Aggregate cost from `start_vid` to `end_vid`.

See Also

- The queries use the *Sample Data* network.
- *pgr_bdDijkstra*
- http://www.cs.princeton.edu/courses/archive/spr06/cos423/Handouts/EPP%20shortest%20path%20algorithms.pdf
- https://en.wikipedia.org/wiki/Bidirectional_search

Indices and tables

- genindex
- search

pgr_bdDijkstraCostMatrix - proposed

Name

`pgr_bdDijkstraCostMatrix` - Calculates the a cost matrix using *pgr_bdDijkstra*.

[56]

Fig. 6.7: Boost Graph Inside

Availability: 2.5.0

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.

[56] http://www.boost.org/libs/graph

- pgTap tests might be missing.
- Might need c/c++ coding.
- May lack documentation.
- Documentation if any might need to be rewritten.
- Documentation examples might need to be automatically generated.
- Might need a lot of feedback from the comunity.
- Might depend on a proposed function of pgRouting
- Might depend on a deprecated function of pgRouting

Synopsis

Using Dijkstra algorithm, calculate and return a cost matrix.

Signature Summary

```
pgr_bdDijkstraCostMatrix(edges_sql, start_vids)
pgr_bdDijkstraCostMatrix(edges_sql, start_vids, directed)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Signatures

Minimal Signature

The minimal signature:

- Is for a **directed** graph.

```
pgr_bdDijkstraCostMatrix(edges_sql, start_vid)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Example Cost matrix for vertices 1, 2, 3, and 4.

```
SELECT * FROM pgr_bdDijkstraCostMatrix(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5)
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         1 |       2 |        1
         1 |       3 |        6
         1 |       4 |        5
         2 |       1 |        1
         2 |       3 |        5
         2 |       4 |        4
         3 |       1 |        2
         3 |       2 |        1
         3 |       4 |        3
         4 |       1 |        3
         4 |       2 |        2
         4 |       3 |        1
(12 rows)
```

Complete Signature

```
pgr_bdDijkstraCostMatrix(edges_sql, start_vids, directed:=true)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Example Cost matrix for an undirected graph for vertices 1, 2, 3, and 4.

This example returns a symmetric cost matrix.

```
SELECT * FROM pgr_bdDijkstraCostMatrix(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
    false
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         1 |       2 |        1
         1 |       3 |        2
         1 |       4 |        3
         2 |       1 |        1
         2 |       3 |        1
         2 |       4 |        2
         3 |       1 |        2
         3 |       2 |        1
         3 |       4 |        1
         4 |       1 |        3
         4 |       2 |        2
         4 |       3 |        1
(12 rows)
```

Description of the Signatures

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Parameter	Type	Description
edges_sql	`TEXT`	Edges SQL query as described above.
start_vids	`ARRAY[ANY-INTEGER]`	Array of identifiers of the vertices.
directed	`BOOLEAN`	(optional). When `false` the graph is considered as Undirected. Default is `true` which considers the graph as Directed.

Description of the return values for a Cost function

Returns set of `(start_vid, end_vid, agg_cost)`

Column	Type	Description
start_vid	`BIGINT`	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	`BIGINT`	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
agg_cost	`FLOAT`	Aggregate cost from `start_vid` to `end_vid`.

Examples

Example Use with tsp

```
SELECT * FROM pgr_TSP(
    $$
    SELECT * FROM pgr_bdDijkstraCostMatrix(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table',
        (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
        false
    )
    $$,
    randomize := false
);
 seq | node | cost | agg_cost
-----+------+------+----------
   1 |    1 |    1 |        0
   2 |    2 |    1 |        1
   3 |    3 |    1 |        2
   4 |    4 |    3 |        3
   5 |    1 |    0 |        6
(5 rows)
```

See Also

- *Bidirectional Dijkstra - Family of functions*
- *Cost Matrix - Category*
- *Traveling Sales Person - Family of functions*
- The queries use the *Sample Data* network.

Indices and tables

- genindex
- search

Synopsis

Based on Dijkstra's algorithm, the bidirectional search finds a shortest path a starting vertex (`start_vid`) to an ending vertex (`end_vid`). It runs two simultaneous searches: one forward from the source, and one backward from the target, stopping when the two meet in the middle. This implementation can be used with a directed graph and an undirected graph.

Characteristics

The main Characteristics are:

- Process is done only on edges with positive costs.
- Values are returned when there is a path.
- When the starting vertex and ending vertex are the same, there is no path.
  - The *agg_cost* the non included values *(v, v)* is *0*
- When the starting vertex and ending vertex are the different and there is no path:
  - The *agg_cost* the non included values *(u, v)* is ∞
- Running time (worse case scenario): $O((V \log V + E))$

- For large graphs where there is a path bewtween the starting vertex and ending vertex:
  - It is expected to terminate faster than pgr_dijkstra

See Also

Indices and tables

- genindex
- search

6.1.4 withPoints - Family of functions

When points are also given as input:

- *pgr_withPoints - Proposed* - Route from/to points anywhere on the graph.
- *pgr_withPointsCost - Proposed* - Costs of the shortest paths.
- *pgr_withPointsCostMatrix - proposed* - Costs of the shortest paths.
- *pgr_withPointsKSP - Proposed* - K shortest paths.
- *pgr_withPointsDD - Proposed* - Driving distance.

pgr_withPoints - Proposed

Name

`pgr_withPoints` - Returns the shortest path in a graph with additional temporary vertices.

> **Warning:** Proposed functions for next mayor release.
>
> - They are not officially in the current release.
> - They will likely officially be part of the next mayor release:
>   - The functions make use of ANY-INTEGER and ANY-NUMERICAL
>   - Name might not change. (But still can)
>   - Signature might not change. (But still can)
>   - Functionality might not change. (But still can)
>   - pgTap tests have being done. But might need more.
>   - Documentation might need refinement.

[57]

Fig. 6.8: Boost Graph Inside

[57] http://www.boost.org/libs/graph

Availability: 2.2.0

Synopsis

Modify the graph to include points defined by points_sql. Using Dijkstra algorithm, find the shortest path(s)

Characteristics:

The main Characteristics are:

- Process is done only on edges with positive costs.
- Vertices of the graph are:
  - **positive** when it belongs to the edges_sql
  - **negative** when it belongs to the points_sql
- Values are returned when there is a path.
  - When the starting vertex and ending vertex are the same, there is no path. - The agg_cost the non included values (v, v) is 0
  - When the starting vertex and ending vertex are the different and there is no path: - The agg_cost the non included values (u, v) is ∞
- For optimization purposes, any duplicated value in the start_vids or end_vids are ignored.
- The returned values are ordered: - start_vid ascending - end_vid ascending
- Running time: $O(|start\_vids| \times (V \log V + E))$

Signature Summary

```
pgr_withPoints(edges_sql, points_sql, start_vid, end_vid)
pgr_withPoints(edges_sql, points_sql, start_vid, end_vid, directed, driving_side,␣
↪details)
pgr_withPoints(edges_sql, points_sql, start_vid, end_vids, directed, driving_side,␣
↪details)
pgr_withPoints(edges_sql, points_sql, start_vids, end_vid, directed, driving_side,␣
↪details)
pgr_withPoints(edges_sql, points_sql, start_vids, end_vids, directed, driving_side,
↪ details)
RETURNS SET OF (seq, path_seq, [start_vid,] [end_vid,] node, edge, cost, agg_cost)
```

Signatures

Minimal Use

The minimal signature:

- Is for a **directed** graph.
- The driving side is set as **b** both. So arriving/departing to/from the point(s) can be in any direction.
- No **details** are given about distance of other points of points_sql query.

```
pgr_withPoints(edges_sql, points_sql, start_vid, end_vid)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost)
```

Example From point 1 to point 3

```
SELECT * FROM pgr_withPoints(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, -3);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |   -1 |    1 |  0.6 |        0
   2 |        2 |    2 |    4 |    1 |      0.6
   3 |        3 |    5 |   10 |    1 |      1.6
   4 |        4 |   10 |   12 |  0.6 |      2.6
   5 |        5 |   -3 |   -1 |    0 |      3.2
(5 rows)
```

One to One

```
pgr_withPoints(edges_sql, points_sql, start_vid, end_vid,
    directed:=true, driving_side:='b', details:=false)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost)
```

Example From point 1 to vertex 3

```
SELECT * FROM pgr_withPoints(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, 3,
    details := true);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |   -1 |    1 |  0.6 |        0
   2 |        2 |    2 |    4 |  0.7 |      0.6
   3 |        3 |   -6 |    4 |  0.3 |      1.3
   4 |        4 |    5 |    8 |    1 |      1.6
   5 |        5 |    6 |    9 |    1 |      2.6
   6 |        6 |    9 |   16 |    1 |      3.6
   7 |        7 |    4 |    3 |    1 |      4.6
   8 |        8 |    3 |   -1 |    0 |      5.6
(8 rows)
```

One to Many

```
pgr_withPoints(edges_sql, points_sql, start_vid, end_vids,
    directed:=true, driving_side:='b', details:=false)
RETURNS SET OF (seq, path_seq, end_vid, node, edge, cost, agg_cost)
```

Example From point 1 to point 3 and vertex 5

```
SELECT * FROM pgr_withPoints(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, ARRAY[-3,5]);
 seq | path_seq | end_pid | node | edge | cost | agg_cost
-----+----------+---------+------+------+------+----------
   1 |        1 |      -3 |   -1 |    1 |  0.6 |        0
   2 |        2 |      -3 |    2 |    4 |    1 |      0.6
   3 |        3 |      -3 |    5 |   10 |    1 |      1.6
   4 |        4 |      -3 |   10 |   12 |  0.6 |      2.6
```

```
   5 |        5 |      -3 |    -3 |    -1 |     0 |       3.2
   6 |        1 |       5 |    -1 |     1 |   0.6 |         0
   7 |        2 |       5 |     2 |     4 |     1 |       0.6
   8 |        3 |       5 |     5 |    -1 |     0 |       1.6
(8 rows)
```

Many to One

```
pgr_withPoints(edges_sql, points_sql, start_vids, end_vid,
    directed:=true, driving_side:='b', details:=false)
RETURNS SET OF (seq, path_seq, start_vid, node, edge, cost, agg_cost)
```

Example From point 1 and vertex 2 to point 3

```
SELECT * FROM pgr_withPoints(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    ARRAY[-1,2], -3);
 seq | path_seq | start_pid | node | edge | cost | agg_cost
-----+----------+-----------+------+------+------+----------
   1 |        1 |        -1 |   -1 |    1 |  0.6 |        0
   2 |        2 |        -1 |    2 |    4 |    1 |      0.6
   3 |        3 |        -1 |    5 |   10 |    1 |      1.6
   4 |        4 |        -1 |   10 |   12 |  0.6 |      2.6
   5 |        5 |        -1 |   -3 |   -1 |    0 |      3.2
   6 |        1 |         2 |    2 |    4 |    1 |        0
   7 |        2 |         2 |    5 |   10 |    1 |        1
   8 |        3 |         2 |   10 |   12 |  0.6 |        2
   9 |        4 |         2 |   -3 |   -1 |    0 |      2.6
(9 rows)
```

Many to Many

```
pgr_withPoints(edges_sql, points_sql, start_vids, end_vids,
    directed:=true, driving_side:='b', details:=false)
RETURNS SET OF (seq, path_seq, start_vid, end_vid, node, edge, cost, agg_cost)
```

Example From point 1 and vertex 2 to point 3 and vertex 7

```
SELECT * FROM pgr_withPoints(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    ARRAY[-1,2], ARRAY[-3,7]);
 seq | path_seq | start_pid | end_pid | node | edge | cost | agg_cost
-----+----------+-----------+---------+------+------+------+----------
   1 |        1 |        -1 |      -3 |   -1 |    1 |  0.6 |        0
   2 |        2 |        -1 |      -3 |    2 |    4 |    1 |      0.6
   3 |        3 |        -1 |      -3 |    5 |   10 |    1 |      1.6
   4 |        4 |        -1 |      -3 |   10 |   12 |  0.6 |      2.6
   5 |        5 |        -1 |      -3 |   -3 |   -1 |    0 |      3.2
   6 |        1 |        -1 |       7 |   -1 |    1 |  0.6 |        0
   7 |        2 |        -1 |       7 |    2 |    4 |    1 |      0.6
   8 |        3 |        -1 |       7 |    5 |    7 |    1 |      1.6
   9 |        4 |        -1 |       7 |    8 |    6 |    1 |      2.6
  10 |        5 |        -1 |       7 |    7 |   -1 |    0 |      3.6
  11 |        1 |         2 |      -3 |    2 |    4 |    1 |        0
```

```
  12 |         2 |         2 |       -3 |     5 |    10 |     1 |         1
  13 |         3 |         2 |       -3 |    10 |    12 |   0.6 |         2
  14 |         4 |         2 |       -3 |    -3 |    -1 |     0 |       2.6
  15 |         1 |         2 |        7 |     2 |     4 |     1 |         0
  16 |         2 |         2 |        7 |     5 |     7 |     1 |         1
  17 |         3 |         2 |        7 |     8 |     6 |     1 |         2
  18 |         4 |         2 |        7 |     7 |    -1 |     0 |         3
(18 rows)
```

Description of the Signatures

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the Points SQL query

points_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Description
pid	`ANY-INTEGER`	(optional) Identifier of the point. • If column present, it can not be NULL. • If column not present, a sequential identifier will be given automatically.
edge_id	`ANY-INTEGER`	Identifier of the "closest" edge to the point.
fraction	`ANY-NUMERICAL`	Value in <0,1> that indicates the relative postition from the first end point of the edge.
side	`CHAR`	(optional) Value in ['b', 'r', 'l', NULL] indicating if the point is: • In the right, left of the edge or • If it doesn't matter with 'b' or NULL. • If column not present 'b' is considered.

Where:

ANY-INTEGER smallint, int, bigint

ANY-NUMERICAL smallint, int, bigint, real, float

Description of the parameters of the signatures

Parameter	Type	Description
edges_sql	TEXT	Edges SQL query as described above.
points_sql	TEXT	Points SQL query as described above.
start_vid	ANY-INTEGER	Starting vertex identifier. When negative: is a point's pid.
end_vid	ANY-INTEGER	Ending vertex identifier. When negative: is a point's pid.
start_vids	ARRAY[ANY-INTEGER]	Array of identifiers of starting vertices. When negative: is a point's pid.
end_vids	ARRAY[ANY-INTEGER]	Array of identifiers of ending vertices. When negative: is a point's pid.
directed	BOOLEAN	(optional). When `false` the graph is considered as Undirected. Default is `true` which considers the graph as Directed.
driving_side	CHAR	**(optional) Value in ['b', 'r', 'l', NULL] indicating if** • In the right or left or • If it doesn't matter with 'b' or NULL. • If column not present 'b' is considered.
details	BOOLEAN	(optional). When `true` the results will include the points in points_sql that are in the path. Default is `false` which ignores other points of the points_sql.

Description of the return values

Returns set of `(seq, [path_seq,] [start_vid,] [end_vid,] node, edge, cost, agg_cost)`

Column	Type	Description
seq	INTEGER	Row sequence.
path_seq	INTEGER	Path sequence that indicates the relative position on the path.
start_vid	BIGINT	Identifier of the starting vertex. When negative: is a point's pid.
end_vid	BIGINT	Identifier of the ending vertex. When negative: is a point's pid.
node	BIGINT	**Identifier of the node:** • A positive value indicates the node is a vertex of edges_sql. • A negative value indicates the node is a point of points_sql.
edge	BIGINT	**Identifier of the edge used to go from `node` to the n** • `-1` for the last row in the path sequence.
cost	FLOAT	**Cost to traverse from `node` using `edge` to the next** • `0` for the last row in the path sequence.
agg_cost	FLOAT	**Aggregate cost from `start_pid` to `node`.** • `0` for the first row in the path sequence.

Examples

Example Which path (if any) passes in front of point 6 or vertex 6 with **right** side driving topology.

```
SELECT ('(' || start_pid || ' => ' || end_pid ||') at ' || path_seq || 'th step:
↪')::TEXT AS path_at,
        CASE WHEN edge = -1 THEN ' visits'
            ELSE ' passes in front of'
        END as status,
        CASE WHEN node < 0 THEN 'Point'
            ELSE 'Vertex'
        END as is_a,
        abs(node) as id
    FROM pgr_withPoints(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id
↪',
        'SELECT pid, edge_id, fraction, side from pointsOfInterest',
        ARRAY[1,-1], ARRAY[-2,-3,-6,3,6],
        driving_side := 'r',
        details := true)
    WHERE node IN (-6,6);
         path_at          |        status         |  is_a  | id
-------------------------+----------------------+---------+----
```

```
 (-1 => -6) at 4th step: |  visits               | Point  |  6
 (-1 => -3) at 4th step: |  passes in front of | Point  |  6
 (-1 => -2) at 4th step: |  passes in front of | Point  |  6
 (-1 => -2) at 6th step: |  passes in front of | Vertex |  6
 (-1 => 3) at 4th step:  |  passes in front of | Point  |  6
 (-1 => 3) at 6th step:  |  passes in front of | Vertex |  6
 (-1 => 6) at 4th step:  |  passes in front of | Point  |  6
 (-1 => 6) at 6th step:  |  visits               | Vertex |  6
 (1 => -6) at 3th step:  |  visits               | Point  |  6
 (1 => -3) at 3th step:  |  passes in front of | Point  |  6
 (1 => -2) at 3th step:  |  passes in front of | Point  |  6
 (1 => -2) at 5th step:  |  passes in front of | Vertex |  6
 (1 => 3) at 3th step:   |  passes in front of | Point  |  6
 (1 => 3) at 5th step:   |  passes in front of | Vertex |  6
 (1 => 6) at 3th step:   |  passes in front of | Point  |  6
 (1 => 6) at 5th step:   |  visits               | Vertex |  6
(16 rows)
```

Example Which path (if any) passes in front of point 6 or vertex 6 with **left** side driving topology.

```
SELECT ('(' || start_pid || ' => ' || end_pid ||') at ' || path_seq || 'th step:
↪')::TEXT AS path_at,
        CASE WHEN edge = -1 THEN ' visits'
            ELSE ' passes in front of'
        END as status,
        CASE WHEN node < 0 THEN 'Point'
            ELSE 'Vertex'
        END as is_a,
        abs(node) as id
    FROM pgr_withPoints(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id
↪',
        'SELECT pid, edge_id, fraction, side from pointsOfInterest',
        ARRAY[1,-1], ARRAY[-2,-3,-6,3,6],
        driving_side := 'l',
        details := true)
    WHERE node IN (-6,6);
         path_at         |        status         |  is_a  | id
-------------------------+----------------------+--------+----
 (-1 => -6) at 3th step: |  visits               | Point  |  6
 (-1 => -3) at 3th step: |  passes in front of | Point  |  6
 (-1 => -2) at 3th step: |  passes in front of | Point  |  6
 (-1 => -2) at 5th step: |  passes in front of | Vertex |  6
 (-1 => 3) at 3th step:  |  passes in front of | Point  |  6
 (-1 => 3) at 5th step:  |  passes in front of | Vertex |  6
 (-1 => 6) at 3th step:  |  passes in front of | Point  |  6
 (-1 => 6) at 5th step:  |  visits               | Vertex |  6
 (1 => -6) at 4th step:  |  visits               | Point  |  6
 (1 => -3) at 4th step:  |  passes in front of | Point  |  6
 (1 => -2) at 4th step:  |  passes in front of | Point  |  6
 (1 => -2) at 6th step:  |  passes in front of | Vertex |  6
 (1 => 3) at 4th step:   |  passes in front of | Point  |  6
 (1 => 3) at 6th step:   |  passes in front of | Vertex |  6
 (1 => 6) at 4th step:   |  passes in front of | Point  |  6
 (1 => 6) at 6th step:   |  visits               | Vertex |  6
(16 rows)
```

Example Many to many example with a twist: on undirected graph and showing details.

```
SELECT * FROM pgr_withPoints(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    ARRAY[-1,2], ARRAY[-3,7],
    directed := false,
    details := true);
 seq | path_seq | start_pid | end_pid | node | edge | cost | agg_cost
-----+----------+-----------+---------+------+------+------+----------
   1 |        1 |        -1 |      -3 |   -1 |    1 |  0.6 |        0
   2 |        2 |        -1 |      -3 |    2 |    4 |  0.7 |      0.6
   3 |        3 |        -1 |      -3 |   -6 |    4 |  0.3 |      1.3
   4 |        4 |        -1 |      -3 |    5 |   10 |    1 |      1.6
   5 |        5 |        -1 |      -3 |   10 |   12 |  0.6 |      2.6
   6 |        6 |        -1 |      -3 |   -3 |   -1 |    0 |      3.2
   7 |        1 |        -1 |       7 |   -1 |    1 |  0.6 |        0
   8 |        2 |        -1 |       7 |    2 |    4 |  0.7 |      0.6
   9 |        3 |        -1 |       7 |   -6 |    4 |  0.3 |      1.3
  10 |        4 |        -1 |       7 |    5 |    7 |    1 |      1.6
  11 |        5 |        -1 |       7 |    8 |    6 |  0.7 |      2.6
  12 |        6 |        -1 |       7 |   -4 |    6 |  0.3 |      3.3
  13 |        7 |        -1 |       7 |    7 |   -1 |    0 |      3.6
  14 |        1 |         2 |      -3 |    2 |    4 |  0.7 |        0
  15 |        2 |         2 |      -3 |   -6 |    4 |  0.3 |      0.7
  16 |        3 |         2 |      -3 |    5 |   10 |    1 |        1
  17 |        4 |         2 |      -3 |   10 |   12 |  0.6 |        2
  18 |        5 |         2 |      -3 |   -3 |   -1 |    0 |      2.6
  19 |        1 |         2 |       7 |    2 |    4 |  0.7 |        0
  20 |        2 |         2 |       7 |   -6 |    4 |  0.3 |      0.7
  21 |        3 |         2 |       7 |    5 |    7 |    1 |        1
  22 |        4 |         2 |       7 |    8 |    6 |  0.7 |        2
  23 |        5 |         2 |       7 |   -4 |    6 |  0.3 |      2.7
  24 |        6 |         2 |       7 |    7 |   -1 |    0 |        3
(24 rows)
```

The queries use the *Sample Data* network.

History

- Proposed in version 2.2

See Also

- *withPoints - Family of functions*

Indices and tables

- genindex
- search

pgr_withPointsCost - Proposed

Name

`pgr_withPointsCost` - Calculates the shortest path and returns only the aggregate cost of the shortest path(s) found, for the combination of points given.

Warning: Proposed functions for next mayor release.

- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
  - The functions make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might not change. (But still can)
  - Signature might not change. (But still can)
  - Functionality might not change. (But still can)
  - pgTap tests have being done. But might need more.
  - Documentation might need refinement.

[58]

Fig. 6.9: Boost Graph Inside

Availability: 2.2.0

Synopsis

Modify the graph to include points defined by points_sql. Using Dijkstra algorithm, return only the aggregate cost of the shortest path(s) found.

Characteristics:

The main Characteristics are:

- It does not return a path.
- Returns the sum of the costs of the shortest path for pair combination of vertices in the modified graph.
- Vertices of the graph are:
  - **positive** when it belongs to the edges_sql
  - **negative** when it belongs to the points_sql
- Process is done only on edges with positive costs.
- Values are returned when there is a path.
  - The returned values are in the form of a set of *(start_vid, end_vid, agg_cost)*.
  - When the starting vertex and ending vertex are the same, there is no path.
    - The *agg_cost* in the non included values *(v, v)* is *0*
  - When the starting vertex and ending vertex are the different and there is no path.
    - The *agg_cost* in the non included values *(u, v)* is ∞
- If the values returned are stored in a table, the unique index would be the pair: *(start_vid, end_vid)*.
- For undirected graphs, the results are symmetric.

[58] http://www.boost.org/libs/graph

  - The *agg_cost* of *(u, v)* is the same as for *(v, u)*.
- For optimization purposes, any duplicated value in the *start_vids* or *end_vids* is ignored.
- The returned values are ordered:
  - *start_vid* ascending
  - *end_vid* ascending
- Running time: $O(|start\_vids| * (V \log V + E))$

Signature Summary

```
pgr_withPointsCost(edges_sql, points_sql, start_vid, end_vid, directed, driving_
↪side)
pgr_withPointsCost(edges_sql, points_sql, start_vid, end_vids, directed, driving_
↪side)
pgr_withPointsCost(edges_sql, points_sql, start_vids, end_vid, directed, driving_
↪side)
pgr_withPointsCost(edges_sql, points_sql, start_vids, end_vids, directed, driving_
↪side)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Note: There is no **details** flag, unlike the other members of the withPoints family of functions.

Signatures

Minimal Use

The minimal signature:

- Is for a **directed** graph.
- The driving side is set as **b** both. So arriving/departing to/from the point(s) can be in any direction.

```
pgr_withPointsCost(edges_sql, points_sql, start_vid, end_vid)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Example

```
SELECT * FROM pgr_withPointsCost(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, -3);
 start_pid | end_pid | agg_cost
-----------+---------+----------
        -1 |      -3 |      3.2
(1 row)
```

One to One

```
pgr_withPointsCost(edges_sql, points_sql, start_vid, end_vid,
    directed:=true, driving_side:='b')
RETURNS SET OF (seq, node, edge, cost, agg_cost)
```

Example

```
SELECT * FROM pgr_withPointsCost(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, 3,
    directed := false);
 start_pid | end_pid | agg_cost
-----------+---------+----------
        -1 |       3 |      1.6
(1 row)
```

One to Many

```
pgr_withPointsCost(edges_sql, points_sql, start_vid, end_vids,
    directed:=true, driving_side:='b')
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Example

```
SELECT * FROM pgr_withPointsCost(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, ARRAY[-3,5]);
 start_pid | end_pid | agg_cost
-----------+---------+----------
        -1 |      -3 |      3.2
        -1 |       5 |      1.6
(2 rows)
```

Many to One

```
pgr_withPointsCost(edges_sql, points_sql, start_vids, end_vid,
    directed:=true, driving_side:='b')
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Example

```
SELECT * FROM pgr_withPointsCost(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    ARRAY[-1,2], -3);
 start_pid | end_pid | agg_cost
-----------+---------+----------
        -1 |      -3 |      3.2
         2 |      -3 |      2.6
(2 rows)
```

Many to Many

```
pgr_withPointsCost(edges_sql, points_sql, start_vids, end_vids,
    directed:=true, driving_side:='b')
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Example

```
SELECT * FROM pgr_withPointsCost(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    ARRAY[-1,2], ARRAY[-3,7]);
 start_pid | end_pid | agg_cost
-----------+---------+----------
        -1 |      -3 |      3.2
        -1 |       7 |      3.6
         2 |      -3 |      2.6
         2 |       7 |        3
(4 rows)
```

Description of the Signatures

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the Points SQL query

points_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Description
pid	`ANY-INTEGER`	(optional) Identifier of the point. • If column present, it can not be NULL. • If column not present, a sequential identifier will be given automatically.
edge_id	`ANY-INTEGER`	Identifier of the “closest” edge to the point.
fraction	`ANY-NUMERICAL`	Value in <0,1> that indicates the relative postition from the first end point of the edge.
side	`CHAR`	(optional) Value in [‘b’, ‘r’, ‘l’, NULL] indicating if the point is: • In the right, left of the edge or • If it doesn’t matter with ‘b’ or NULL. • If column not present ‘b’ is considered.

Where:

ANY-INTEGER smallint, int, bigint

ANY-NUMERICAL smallint, int, bigint, real, float

Description of the parameters of the signatures

Parameter	Type	Description
edges_sql	`TEXT`	Edges SQL query as described above.
points_sql	`TEXT`	Points SQL query as described above.
start_vid	`ANY-INTEGER`	Starting vertex identifier. When negative: is a point's pid.
end_vid	`ANY-INTEGER`	Ending vertex identifier. When negative: is a point's pid.
start_vids	`ARRAY[ANY-INTEGER]`	Array of identifiers of starting vertices. When negative: is a point's pid.
end_vids	`ARRAY[ANY-INTEGER]`	Array of identifiers of ending vertices. When negative: is a point's pid.
directed	`BOOLEAN`	(optional). When `false` the graph is considered as Undirected. Default is `true` which considers the graph as Directed.
driving_side	`CHAR`	**(optional) Value in ['b', 'r', 'l', NULL] indicating if** • In the right or left or • If it doesn't matter with 'b' or NULL. • If column not present 'b' is considered.

Description of the return values

Returns set of `(start_vid, end_vid, agg_cost)`

Column	Type	Description
start_vid	`BIGINT`	Identifier of the starting vertex. When negative: is a point's pid.
end_vid	`BIGINT`	Identifier of the ending point. When negative: is a point's pid.
agg_cost	`FLOAT`	Aggregate cost from `start_vid` to `end_vid`.

Examples

Example With **right** side driving topology.

```
SELECT * FROM pgr_withPointsCost(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    ARRAY[-1,2], ARRAY[-3,7],
    driving_side := 'l');
 start_pid | end_pid | agg_cost
-----------+---------+----------
        -1 |      -3 |      3.2
        -1 |       7 |      3.6
         2 |      -3 |      2.6
         2 |       7 |        3
```

```
(4 rows)
```

Example With **left** side driving topology.

```
SELECT * FROM pgr_withPointsCost(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    ARRAY[-1,2], ARRAY[-3,7],
    driving_side := 'r');
 start_pid | end_pid | agg_cost
-----------+---------+----------
        -1 |      -3 |        4
        -1 |       7 |      4.4
         2 |      -3 |      2.6
         2 |       7 |        3
(4 rows)
```

Example Does not matter driving side.

```
SELECT * FROM pgr_withPointsCost(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    ARRAY[-1,2], ARRAY[-3,7],
    driving_side := 'b');
 start_pid | end_pid | agg_cost
-----------+---------+----------
        -1 |      -3 |      3.2
        -1 |       7 |      3.6
         2 |      -3 |      2.6
         2 |       7 |        3
(4 rows)
```

The queries use the *Sample Data* network.

History

- Proposed in version 2.2

See Also

- *withPoints - Family of functions*

Indices and tables

- genindex
- search

pgr_withPointsCostMatrix - proposed

Name

`pgr_withPointsCostMatrix` - Calculates the shortest path and returns only the aggregate cost of the shortest path(s) found, for the combination of points given.

Warning: Proposed functions for next mayor release.

- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
  - The functions make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might not change. (But still can)
  - Signature might not change. (But still can)
  - Functionality might not change. (But still can)
  - pgTap tests have being done. But might need more.
  - Documentation might need refinement.

[59]

Fig. 6.10: Boost Graph Inside

Availability: 2.2.0

Signature Summary

```
pgr_withPointsCostMatrix(edges_sql, points_sql, start_vids)
pgr_withPointsCostMatrix(edges_sql, points_sql, start_vids, directed, driving_side)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Note: There is no **details** flag, unlike the other members of the withPoints family of functions.

Signatures

Minimal Signature

The minimal signature:

- Is for a **directed** graph.
- The driving side is set as **b** both. So arriving/departing to/from the point(s) can be in any direction.

```
pgr_withPointsCostMatrix(edges_sql, points_sql, start_vid)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Example

```
SELECT * FROM pgr_withPointsCostMatrix(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction from pointsOfInterest',
    array[-1, 3, 6, -6]);
 start_vid | end_vid | agg_cost
```

[59] http://www.boost.org/libs/graph

```
-----------+---------+----------
        -6 |      -1 |      1.3
        -6 |       3 |      4.3
        -6 |       6 |      1.3
        -1 |      -6 |      1.3
        -1 |       3 |      5.6
        -1 |       6 |      2.6
         3 |      -6 |      1.7
         3 |      -1 |      1.6
         3 |       6 |        1
         6 |      -6 |      1.3
         6 |      -1 |      2.6
         6 |       3 |        3
(12 rows)
```

Complete Signature

```
pgr_withPointsCostMatrix(edges_sql, points_sql, start_vids,
    directed:=true, driving_side:='b')
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Example returning a symmetrical cost matrix

- Using the default **side** value on the **points_sql** query
- Using an undirected graph
- Using the default **driving_side** value

```
SELECT * FROM pgr_withPointsCostMatrix(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction from pointsOfInterest',
    array[-1, 3, 6, -6], directed := false);
 start_vid | end_vid | agg_cost
-----------+---------+----------
        -6 |      -1 |      1.3
        -6 |       3 |      1.7
        -6 |       6 |      1.3
        -1 |      -6 |      1.3
        -1 |       3 |      1.6
        -1 |       6 |      2.6
         3 |      -6 |      1.7
         3 |      -1 |      1.6
         3 |       6 |        1
         6 |      -6 |      1.3
         6 |      -1 |      2.6
         6 |       3 |        1
(12 rows)
```

Description of the Signatures

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the Points SQL query

points_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Description
pid	`ANY-INTEGER`	(optional) Identifier of the point. • If column present, it can not be NULL. • If column not present, a sequential identifier will be given automatically.
edge_id	`ANY-INTEGER`	Identifier of the "closest" edge to the point.
fraction	`ANY-NUMERICAL`	Value in <0,1> that indicates the relative postition from the first end point of the edge.
side	`CHAR`	(optional) Value in ['b', 'r', 'l', NULL] indicating if the point is: • In the right, left of the edge or • If it doesn't matter with 'b' or NULL. • If column not present 'b' is considered.

Where:

ANY-INTEGER smallint, int, bigint

ANY-NUMERICAL smallint, int, bigint, real, float

Description of the parameters of the signatures

Parameter	Type	Description
edges_sql	`TEXT`	Edges SQL query as described above.
points_sql	`TEXT`	Points SQL query as described above.
start_vids	`ARRAY[ANY-INTEGER]`	Array of identifiers of starting vertices. When negative: is a point's pid.
directed	`BOOLEAN`	(optional). When `false` the graph is considered as Undirected. Default is `true` which considers the graph as Directed.
driving_side	`CHAR`	**(optional) Value in ['b', 'r', 'l', NULL] indicating if** • In the right or left or • If it doesn't matter with 'b' or NULL. • If column not present 'b' is considered.

Description of the return values for a Cost function

Returns set of `(start_vid, end_vid, agg_cost)`

Column	Type	Description
start_vid	`BIGINT`	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	`BIGINT`	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
agg_cost	`FLOAT`	Aggregate cost from `start_vid` to `end_vid`.

Examples

Example Use with tsp

```
SELECT * FROM pgr_TSP(
    $$
    SELECT * FROM pgr_withPointsCostMatrix(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id
↪',
        'SELECT pid, edge_id, fraction from pointsOfInterest',
        array[-1, 3, 6, -6], directed := false);
    $$,
    randomize := false
);
 seq | node | cost | agg_cost
-----+------+------+----------
   1 |   -6 |  1.3 |        0
```

```
   2 |    -1 |  1.6 |        1.3
   3 |     3 |    1 |        2.9
   4 |     6 |  1.3 |        3.9
   5 |    -6 |    0 |        5.2
(5 rows)
```

See Also

- *withPoints - Family of functions*
- *Cost Matrix - Category*
- *Traveling Sales Person - Family of functions*
- *sampledata* network.

Indices and tables

- genindex
- search

pgr_withPointsKSP - Proposed

Name

`pgr_withPointsKSP` - Find the K shortest paths using Yen's algorithm.

> **Warning:** Proposed functions for next mayor release.
>
> - They are not officially in the current release.
> - They will likely officially be part of the next mayor release:
>   - The functions make use of ANY-INTEGER and ANY-NUMERICAL
>   - Name might not change. (But still can)
>   - Signature might not change. (But still can)
>   - Functionality might not change. (But still can)
>   - pgTap tests have being done. But might need more.
>   - Documentation might need refinement.

[60]

Fig. 6.11: Boost Graph Inside

[60] http://www.boost.org/libs/graph

Availability: 2.2.0

Synopsis

Modifies the graph to include the points defined in the `points_sql` and using Yen algorithm, finds the K shortest paths.

Signature Summary

```
pgr_withPointsKSP(edges_sql, points_sql, start_pid, end_pid, K)
pgr_withPointsKSP(edges_sql, points_sql, start_pid, end_pid, K, directed, heap_
↪paths, driving_side, details)
RETURNS SET OF (seq, path_id, path_seq, node, edge, cost, agg_cost)
```

Signatures

Minimal Usage

The minimal usage:

- Is for a **directed** graph.
- The driving side is set as **b** both. So arriving/departing to/from the point(s) can be in any direction.
- No **details** are given about distance of other points of the query.
- No **heap paths** are returned.

```
pgr_withPointsKSP(edges_sql, points_sql, start_pid, end_pid, K)
RETURNS SET OF (seq, path_id, path_seq, node, edge, cost, agg_cost)
```

Example

```
SELECT * FROM pgr_withPointsKSP(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, -2, 2);
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |   -1 |    1 |  0.6 |        0
   2 |       1 |        2 |    2 |    4 |    1 |      0.6
   3 |       1 |        3 |    5 |    8 |    1 |      1.6
   4 |       1 |        4 |    6 |    9 |    1 |      2.6
   5 |       1 |        5 |    9 |   15 |  0.4 |      3.6
   6 |       1 |        6 |   -2 |   -1 |    0 |        4
   7 |       2 |        1 |   -1 |    1 |  0.6 |        0
   8 |       2 |        2 |    2 |    4 |    1 |      0.6
   9 |       2 |        3 |    5 |    8 |    1 |      1.6
  10 |       2 |        4 |    6 |   11 |    1 |      2.6
  11 |       2 |        5 |   11 |   13 |    1 |      3.6
  12 |       2 |        6 |   12 |   15 |  0.6 |      4.6
  13 |       2 |        7 |   -2 |   -1 |    0 |      5.2
(13 rows)
```

Complete Signature

Finds the K shortest paths depending on the optional parameters setup.

```
pgr_withPointsKSP(edges_sql, points_sql, start_pid, end_pid, K,
    directed:=true, heap_paths:=false, driving_side:='b', details:=false)
RETURNS SET OF (seq, path_id, path_seq, node, edge, cost, agg_cost)
```

Example With details.

```
SELECT * FROM pgr_withPointsKSP(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, 6, 2, details := true);
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |   -1 |    1 |  0.6 |        0
   2 |       1 |        2 |    2 |    4 |  0.7 |      0.6
   3 |       1 |        3 |   -6 |    4 |  0.3 |      1.3
   4 |       1 |        4 |    5 |    8 |    1 |      1.6
   5 |       1 |        5 |    6 |   -1 |    0 |      2.6
   6 |       2 |        1 |   -1 |    1 |  0.6 |        0
   7 |       2 |        2 |    2 |    4 |  0.7 |      0.6
   8 |       2 |        3 |   -6 |    4 |  0.3 |      1.3
   9 |       2 |        4 |    5 |   10 |    1 |      1.6
  10 |       2 |        5 |   10 |   12 |  0.6 |      2.6
  11 |       2 |        6 |   -3 |   12 |  0.4 |      3.2
  12 |       2 |        7 |   11 |   13 |    1 |      3.6
  13 |       2 |        8 |   12 |   15 |  0.6 |      4.6
  14 |       2 |        9 |   -2 |   15 |  0.4 |      5.2
  15 |       2 |       10 |    9 |    9 |    1 |      5.6
  16 |       2 |       11 |    6 |   -1 |    0 |      6.6
(16 rows)
```

Description of the Signatures

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the Points SQL query

points_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Description
pid	`ANY-INTEGER`	(optional) Identifier of the point. • If column present, it can not be NULL. • If column not present, a sequential identifier will be given automatically.
edge_id	`ANY-INTEGER`	Identifier of the "closest" edge to the point.
fraction	`ANY-NUMERICAL`	Value in <0,1> that indicates the relative postition from the first end point of the edge.
side	`CHAR`	(optional) Value in ['b', 'r', 'l', NULL] indicating if the point is: • In the right, left of the edge or • If it doesn't matter with 'b' or NULL. • If column not present 'b' is considered.

Where:

ANY-INTEGER smallint, int, bigint

ANY-NUMERICAL smallint, int, bigint, real, float

Description of the parameters of the signatures

Parameter	Type	Description
edges_sql	`TEXT`	Edges SQL query as described above.
points_sql	`TEXT`	Points SQL query as described above.
start_pid	`ANY-INTEGER`	Starting point id.
end_pid	`ANY-INTEGER`	Ending point id.
K	`INTEGER`	Number of shortest paths.
directed	`BOOLEAN`	(optional). When `false` the graph is considered as Undirected. Default is `true` which considers the graph as Directed.
heap_paths	`BOOLEAN`	(optional). When `true` the paths calculated to get the shortests paths will be returned also. Default is `false` only the K shortest paths are returned.
driving_side	`CHAR`	**(optional) Value in [‘b’, ‘r’, ‘l’, NULL] indicating if** • In the right or left or • If it doesn’t matter with ‘b’ or NULL. • If column not present ‘b’ is considered.
details	`BOOLEAN`	(optional). When `true` the results will include the driving distance to the points with in the `distance`. Default is `false` which ignores other points of the points_sql.

Description of the return values

Returns set of `(seq, path_id, path_seq, node, edge, cost, agg_cost)`

Column	Type	Description
seq	`INTEGER`	Row sequence.
path_seq	`INTEGER`	Relative position in the path of node and edge. Has value 1 for the beginning of a path.
path_id	`INTEGER`	Path identifier. The ordering of the paths: For two paths i, j if i < j then agg_cost(i) <= agg_cost(j).
node	`BIGINT`	Identifier of the node in the path. Negative values are the identifiers of a point.
edge	`BIGINT`	**Identifier of the edge used to go from `node` to the n** • `-1` for the last row in the path sequence.
cost	`FLOAT`	**Cost to traverse from `node` using `edge` to the next** • `0` for the last row in the path sequence.
agg_cost	`FLOAT`	**Aggregate cost from `start_pid` to `node`.** • `0` for the first row in the path sequence.

Examples

Example Left side driving topology with details.

```
SELECT * FROM pgr_withPointsKSP(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, -2, 2,
    driving_side := 'l', details := true);
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |   -1 |    1 |  0.6 |        0
   2 |       1 |        2 |    2 |    4 |  0.7 |      0.6
   3 |       1 |        3 |   -6 |    4 |  0.3 |      1.3
   4 |       1 |        4 |    5 |    8 |    1 |      1.6
   5 |       1 |        5 |    6 |    9 |    1 |      2.6
   6 |       1 |        6 |    9 |   15 |    1 |      3.6
   7 |       1 |        7 |   12 |   15 |  0.6 |      4.6
   8 |       1 |        8 |   -2 |   -1 |    0 |      5.2
   9 |       2 |        1 |   -1 |    1 |  0.6 |        0
  10 |       2 |        2 |    2 |    4 |  0.7 |      0.6
  11 |       2 |        3 |   -6 |    4 |  0.3 |      1.3
  12 |       2 |        4 |    5 |    8 |    1 |      1.6
  13 |       2 |        5 |    6 |   11 |    1 |      2.6
  14 |       2 |        6 |   11 |   13 |    1 |      3.6
  15 |       2 |        7 |   12 |   15 |  0.6 |      4.6
  16 |       2 |        8 |   -2 |   -1 |    0 |      5.2
(16 rows)
```

Example Right side driving topology with heap paths and details.

```
SELECT * FROM pgr_withPointsKSP(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, -2, 2,
    heap_paths := true, driving_side := 'r', details := true);
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |   -1 |    1 |  0.4 |        0
   2 |       1 |        2 |    1 |    1 |    1 |      0.4
   3 |       1 |        3 |    2 |    4 |  0.7 |      1.4
   4 |       1 |        4 |   -6 |    4 |  0.3 |      2.1
   5 |       1 |        5 |    5 |    8 |    1 |      2.4
   6 |       1 |        6 |    6 |    9 |    1 |      3.4
   7 |       1 |        7 |    9 |   15 |  0.4 |      4.4
   8 |       1 |        8 |   -2 |   -1 |    0 |      4.8
   9 |       2 |        1 |   -1 |    1 |  0.4 |        0
  10 |       2 |        2 |    1 |    1 |    1 |      0.4
  11 |       2 |        3 |    2 |    4 |  0.7 |      1.4
  12 |       2 |        4 |   -6 |    4 |  0.3 |      2.1
  13 |       2 |        5 |    5 |    8 |    1 |      2.4
  14 |       2 |        6 |    6 |   11 |    1 |      3.4
  15 |       2 |        7 |   11 |   13 |    1 |      4.4
  16 |       2 |        8 |   12 |   15 |    1 |      5.4
  17 |       2 |        9 |    9 |   15 |  0.4 |      6.4
  18 |       2 |       10 |   -2 |   -1 |    0 |      6.8
  19 |       3 |        1 |   -1 |    1 |  0.4 |        0
  20 |       3 |        2 |    1 |    1 |    1 |      0.4
  21 |       3 |        3 |    2 |    4 |  0.7 |      1.4
  22 |       3 |        4 |   -6 |    4 |  0.3 |      2.1
  23 |       3 |        5 |    5 |   10 |    1 |      2.4
  24 |       3 |        6 |   10 |   12 |  0.6 |      3.4
  25 |       3 |        7 |   -3 |   12 |  0.4 |        4
  26 |       3 |        8 |   11 |   13 |    1 |      4.4
  27 |       3 |        9 |   12 |   15 |    1 |      5.4
  28 |       3 |       10 |    9 |   15 |  0.4 |      6.4
  29 |       3 |       11 |   -2 |   -1 |    0 |      6.8
(29 rows)
```

The queries use the *Sample Data* network.

History

- Proposed in version 2.2

See Also

- *withPoints - Family of functions*

Indices and tables

- genindex
- search

pgr_withPointsDD - Proposed

Name

`pgr_withPointsDD` - Returns the driving distance from a starting point.

> **Warning:** Proposed functions for next mayor release.
>
> - They are not officially in the current release.
> - They will likely officially be part of the next mayor release:
>   - The functions make use of ANY-INTEGER and ANY-NUMERICAL
>   - Name might not change. (But still can)
>   - Signature might not change. (But still can)
>   - Functionality might not change. (But still can)
>   - pgTap tests have being done. But might need more.
>   - Documentation might need refinement.

[61]

Fig. 6.12: Boost Graph Inside

Availability: 2.2.0

Synopsis

Modify the graph to include points and using Dijkstra algorithm, extracts all the nodes and points that have costs less than or equal to the value `distance` from the starting point. The edges extracted will conform the corresponding spanning tree.

Signature Summary

```
pgr_withPointsDD(edges_sql, points_sql, start_vid, distance)
pgr_withPointsDD(edges_sql, points_sql, start_vid, distance, directed, driving_
↪side, details)
pgr_withPointsDD(edges_sql, points_sql, start_vids, distance, directed, driving_
↪side, details, equicost)
RETURNS SET OF (seq, node, edge, cost, agg_cost)
```

Signatures

Minimal Use

The minimal signature:

[61] http://www.boost.org/libs/graph

- Is for a **directed** graph.
- The driving side is set as **b** both. So arriving/departing to/from the point(s) can be in any direction.
- No **details** are given about distance of other points of the query.

```
pgr_withPointsDD(edges_sql, points_sql, start_vid, distance)
    directed:=true, driving_side:='b', details:=false)
RETURNS SET OF (seq, node, edge, cost, agg_cost)
```

Example

```
SELECT * FROM pgr_withPointsDD(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, 3.8);
 seq | node | edge | cost | agg_cost
-----+------+------+------+----------
   1 |   -1 |   -1 |    0 |        0
   2 |    1 |    1 |  0.4 |      0.4
   3 |    2 |    1 |  0.6 |      0.6
   4 |    5 |    4 |  0.3 |      1.6
   5 |    6 |    8 |    1 |      2.6
   6 |    8 |    7 |    1 |      2.6
   7 |   10 |   10 |    1 |      2.6
   8 |    7 |    6 |  0.3 |      3.6
   9 |    9 |    9 |    1 |      3.6
  10 |   11 |   11 |    1 |      3.6
  11 |   13 |   14 |    1 |      3.6
(11 rows)
```

Driving distance from a single point

Finds the driving distance depending on the optional parameters setup.

```
pgr_withPointsDD(edges_sql, points_sql, start_vids, distance,
    directed:=true, driving_side:='b', details:=false)
RETURNS SET OF (seq, node, edge, cost, agg_cost)
```

Example Right side driving topology

```
SELECT * FROM pgr_withPointsDD(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, 3.8,
    driving_side := 'r',
    details := true);
 seq | node | edge | cost | agg_cost
-----+------+------+------+----------
   1 |   -1 |   -1 |    0 |        0
   2 |    1 |    1 |  0.4 |      0.4
   3 |    2 |    1 |    1 |      1.4
   4 |   -6 |    4 |  0.7 |      2.1
   5 |    5 |    4 |  0.3 |      2.4
   6 |    6 |    8 |    1 |      3.4
   7 |    8 |    7 |    1 |      3.4
   8 |   10 |   10 |    1 |      3.4
(8 rows)
```

Driving distance from many starting points

Finds the driving distance depending on the optional parameters setup.

```
pgr_withPointsDD(edges_sql, points_sql, start_vids, distance,
    directed:=true, driving_side:='b', details:=false, equicost:=false)
RETURNS SET OF (seq, node, edge, cost, agg_cost)
```

Description of the Signatures

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the Points SQL query

points_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Description
pid	`ANY-INTEGER`	(optional) Identifier of the point. • If column present, it can not be NULL. • If column not present, a sequential identifier will be given automatically.
edge_id	`ANY-INTEGER`	Identifier of the "closest" edge to the point.
fraction	`ANY-NUMERICAL`	Value in <0,1> that indicates the relative postition from the first end point of the edge.
side	`CHAR`	(optional) Value in ['b', 'r', 'l', NULL] indicating if the point is: • In the right, left of the edge or • If it doesn't matter with 'b' or NULL. • If column not present 'b' is considered.

Where:

ANY-INTEGER smallint, int, bigint

ANY-NUMERICAL smallint, int, bigint, real, float

Description of the parameters of the signatures

Parameter	Type	Description
edges_sql	TEXT	Edges SQL query as described above.
points_sql	TEXT	Points SQL query as described above.
start_vid	ANY-INTEGER	Starting point id
distance	ANY-NUMERICAL	Distance from the start_pid
directed	BOOLEAN	(optional). When `false` the graph is considered as Undirected. Default is `true` which considers the graph as Directed.
driving_side	CHAR	**(optional). Value in ['b', 'r', 'l', NULL] indicating i** • In the right or left or • If it doesn't matter with 'b' or NULL. • If column not present 'b' is considered.
details	BOOLEAN	(optional). When `true` the results will include the driving distance to the points with in the `distance`. Default is `false` which ignores other points of the points_sql.
equicost	BOOLEAN	(optional). When `true` the nodes will only appear in the closest start_v list. Default is `false` which resembles several calls using the single starting point signatures. Tie brakes are arbitrary.

Description of the return values

Returns set of `(seq, node, edge, cost, agg_cost)`

Column	Type	Description
seq	`INT`	row sequence.
node	`BIGINT`	Identifier of the node within the Distance from `start_pid`. If `details =: true` a negative value is the identifier of a point.
edge	`BIGINT`	**Identifier of the edge used to go from `node` to the n** • `-1` when `start_vid` = `node`.
cost	`FLOAT`	**Cost to traverse `edge`.** • `0` when `start_vid` = `node`.
agg_cost	`FLOAT`	**Aggregate cost from `start_vid` to `node`.** • `0` when `start_vid` = `node`.

Examples for queries marked as `directed` with `cost` and `reverse_cost` columns

The examples in this section use the following *Network for queries marked as directed and cost and reverse_cost columns are used*

Example Left side driving topology

```
SELECT * FROM pgr_withPointsDD(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, 3.8,
    driving_side := 'l',
    details := true);
 seq | node | edge | cost | agg_cost
-----+------+------+------+----------
   1 |   -1 |   -1 |    0 |        0
   2 |    2 |    1 |  0.6 |      0.6
   3 |   -6 |    4 |  0.7 |      1.3
   4 |    5 |    4 |  0.3 |      1.6
   5 |    1 |    1 |    1 |      1.6
   6 |    6 |    8 |    1 |      2.6
   7 |    8 |    7 |    1 |      2.6
   8 |   10 |   10 |    1 |      2.6
   9 |   -3 |   12 |  0.6 |      3.2
  10 |   -4 |    6 |  0.7 |      3.3
  11 |    7 |    6 |  0.3 |      3.6
  12 |    9 |    9 |    1 |      3.6
  13 |   11 |   11 |    1 |      3.6
  14 |   13 |   14 |    1 |      3.6
(14 rows)
```

Example Does not matter driving side.

```
SELECT * FROM pgr_withPointsDD(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, 3.8,
    driving_side := 'b',
    details := true);
 seq | node | edge | cost | agg_cost
-----+------+------+------+----------
   1 |   -1 |   -1 |    0 |        0
   2 |    1 |    1 |  0.4 |      0.4
   3 |    2 |    1 |  0.6 |      0.6
   4 |   -6 |    4 |  0.7 |      1.3
   5 |    5 |    4 |  0.3 |      1.6
   6 |    6 |    8 |    1 |      2.6
   7 |    8 |    7 |    1 |      2.6
   8 |   10 |   10 |    1 |      2.6
   9 |   -3 |   12 |  0.6 |      3.2
  10 |   -4 |    6 |  0.7 |      3.3
  11 |    7 |    6 |  0.3 |      3.6
  12 |    9 |    9 |    1 |      3.6
  13 |   11 |   11 |    1 |      3.6
  14 |   13 |   14 |    1 |      3.6
(14 rows)
```

The queries use the *Sample Data* network.

History

- Proposed in version 2.2

See Also

- *pgr_drivingDistance* - Driving distance using dijkstra.
- *pgr_alphaShape* - Alpha shape computation.
- *pgr_pointsAsPolygon* - Polygon around set of points.

Indices and tables

- genindex
- search

Warning: Proposed functions for next mayor release.

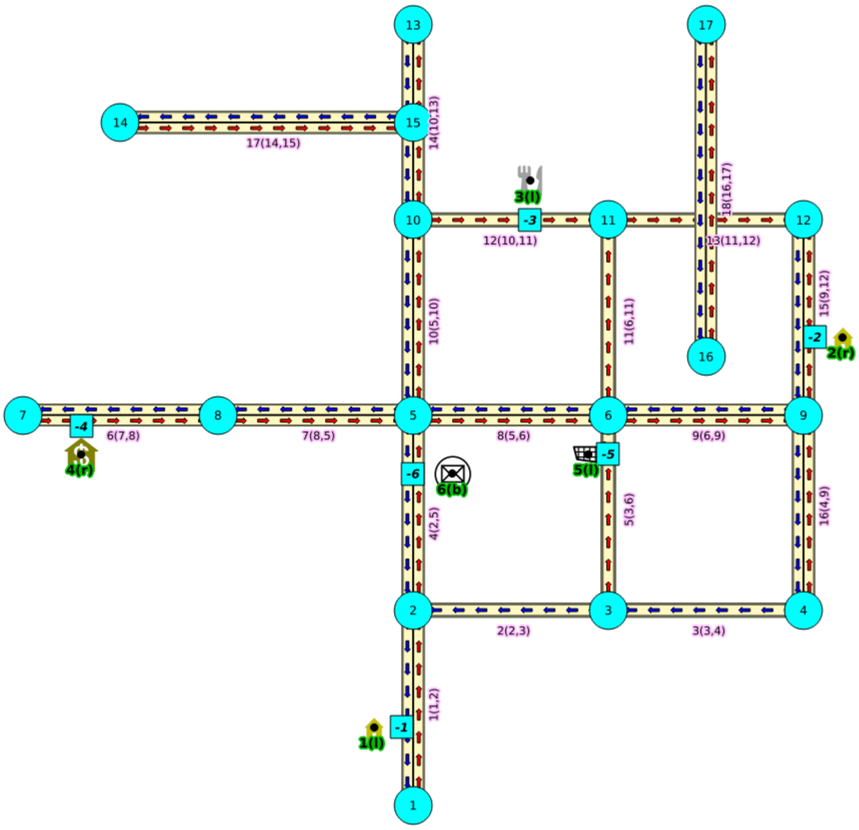
- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
  - The functions make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might not change. (But still can)
  - Signature might not change. (But still can)
  - Functionality might not change. (But still can)
  - pgTap tests have being done. But might need more.

– Documentation might need refinement.

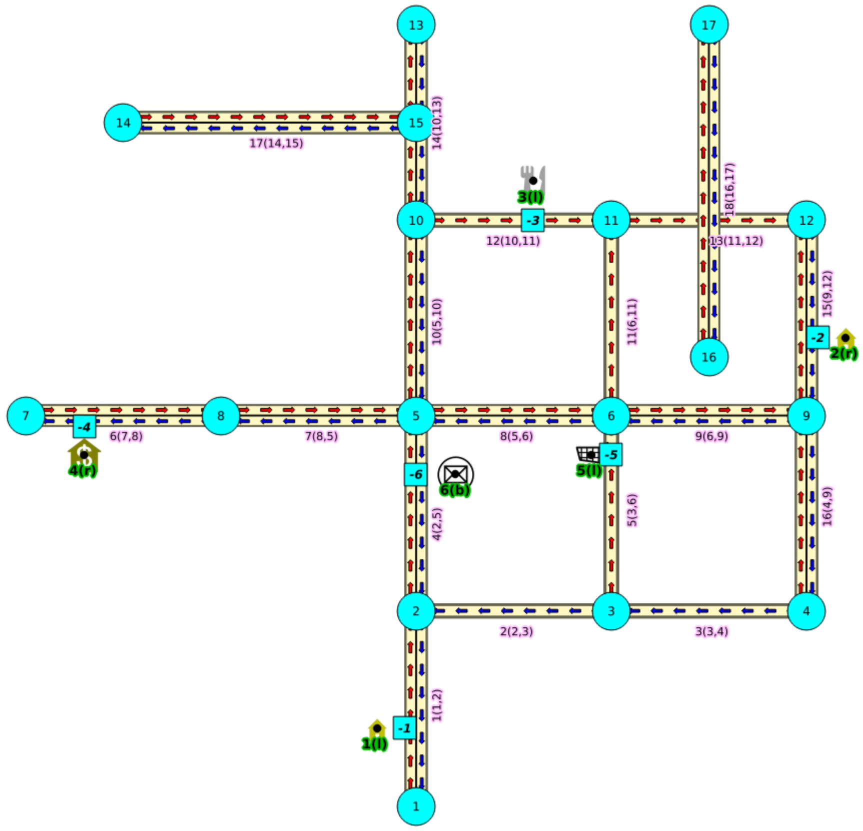
Images

The squared vertices are the temporary vertices, The temporary vertices are added according to the driving side, The following images visually show the differences on how depending on the driving side the data is interpreted.

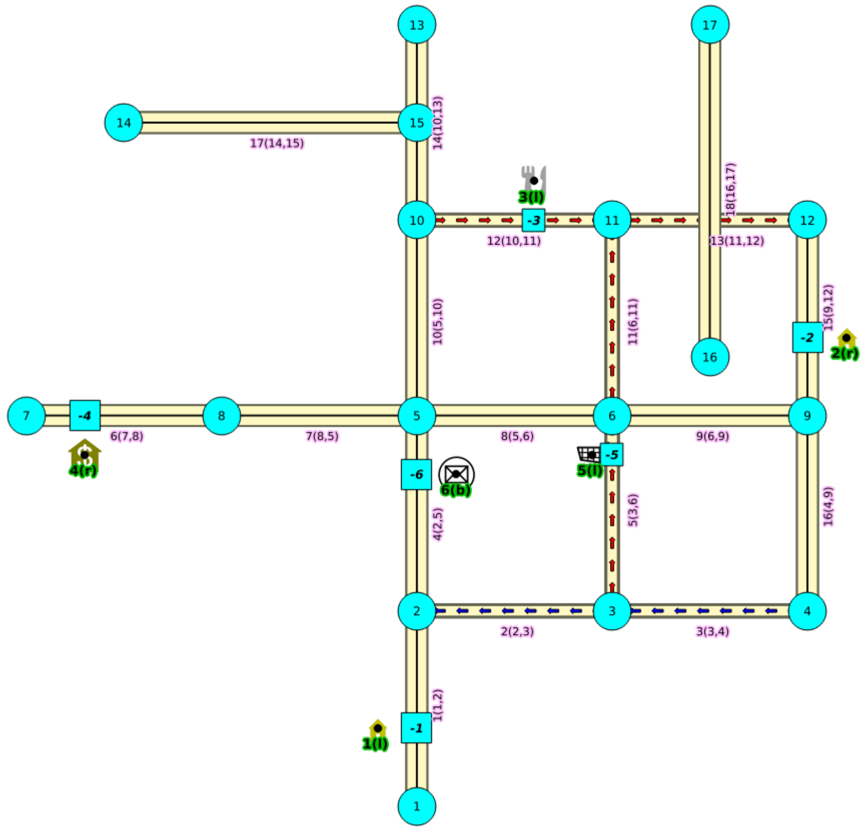
Right driving side



Left driving side



doesn't matter the driving side



Introduction

This family of functions was thought for routing vehicles, but might as well work for some other application that we can not think of.

The with points family of function give you the ability to route between arbitrary points located outside the original graph.

When given a point identified with a *pid* that its being mapped to and edge with an identifier *edge_id*, with a *fraction* along that edge (from the source to the target of the edge) and some additional information about which *side* of the edge the point is on, then routing from arbitrary points more accurately reflect routing vehicles in road networks,

I talk about a family of functions because it includes different functionalities.

- pgr_withPoints is pgr_dijkstra based
- pgr_withPointsCost is pgr_dijkstraCost based
- pgr_withPointsKSP is pgr_ksp based
- pgr_withPointsDD is pgr_drivingDistance based

In all this functions we have to take care of as many aspects as possible:

- Must work for routing:
  - Cars (directed graph)
  - Pedestrians (undirected graph)
- Arriving at the point:

  - In either side of the street.
  - Compulsory arrival on the side of the street where the point is located.
- Countries with:
  - Right side driving
  - Left side driving
- Some points are:
  - Permanent, for example the set of points of clients stored in a table in the data base
  - Temporal, for example points given through a web application
- The numbering of the points are handled with negative sign.
  - Original point identifiers are to be positive.
  - Transformation to negative is done internally.
  - For results for involving vertices identifiers
    - positive sign is a vertex of the original graph
    - negative sign is a point of the temporary points

The reason for doing this is to avoid confusion when there is a vertex with the same number as identifier as the points identifier.

Graph & edges

- Let $G_d(V, E)$ where V is the set of vertices and E is the set of edges be the original directed graph.
  - An edge of the original *edges_sql* is $(id, source, target, cost, reverse\_cost)$ will generate internally
    - $(id, source, target, cost)$
    - $(id, target, source, reverse\_cost)$

Point Definition

- A point is defined by the quadruplet: $(pid, eid, fraction, side)$
  - **pid** is the point identifier
  - **eid** is an edge id of the *edges_sql*
  - **fraction** represents where the edge *eid* will be cut.
  - **side** Indicates the side of the edge where the point is located.

Creating Temporary Vertices in the Graph

For edge (15, 9,12 10, 20), & lets insert point (2, 12, 0.3, r)

On a right hand side driving network

From first image above:

- We can arrive to the point only via vertex 9.
- It only affects the edge (15, 9,12, 10) so that edge is removed.
- Edge (15, 12,9, 20) is kept.
- Create new edges:

  - (15, 9,-1, 3) edge from vertex 9 to point 1 has cost 3
  - (15, -1,12, 7) edge from point 1 to vertex 12 has cost 7

On a left hand side driving network

From second image above:

- We can arrive to the point only via vertex 12.
- It only affects the edge (15, 12,9 20) so that edge is removed.
- Edge (15, 9,12, 10) is kept.
- Create new edges:
  - (15, 12,-1, 14) edge from vertex 12 to point 1 has cost 14
  - (15, -1,9, 6) edge from point 1 to vertex 9 has cost 6

  Remember that fraction is from vertex 9 to vertex 12

When driving side does not matter

From third image above:

- We can arrive to the point either via vertex 12 or via vertex 9
- Edge (15, 12,9 20) is removed.
- Edge (15, 9,12, 10) is removed.
- Create new edges:
  - (15, 12,-1, 14) edge from vertex 12 to point 1 has cost 14
  - (15, -1,9, 6) edge from point 1 to vertex 9 has cost 6
  - (15, 9,-1, 3) edge from vertex 9 to point 1 has cost 3
  - (15, -1,12, 7) edge from point 1 to vertex 12 has cost 7

See Also

Indices and tables

- genindex
- search

6.1.5 Cost - Category

- *pgr_aStarCost – proposed*
- *pgr_bdAstarCost - Proposed*
- *pgr_bdDijkstraCost - Proposed*
- *pgr_dijkstraCost*
- *pgr_withPointsCost - Proposed*

Warning: Proposed functions for next mayor release.

- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
  - The functions make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might not change. (But still can)
  - Signature might not change. (But still can)
  - Functionality might not change. (But still can)
  - pgTap tests have being done. But might need more.
  - Documentation might need refinement.

General Information

Characteristics

The main Characteristics are:

- Each function works as part of the family it belongs to.
- It does not return a path.
- Returns the sum of the costs of the resulting path(s) for pair combination of nodes in the graph.
- Process is done only on edges with positive costs.
- Values are returned when there is a path.
  - The returned values are in the form of a set of *(start_vid, end_vid, agg_cost)*.
  - When the starting vertex and ending vertex are the same, there is no path.
    - The *agg_cost* int the non included values *(v, v)* is *0*.
  - When the starting vertex and ending vertex are the different and there is no path.
    - The *agg_cost* in the non included values *(u, v)* is ∞.
- Let be the case the values returned are stored in a table, so the unique index would be the pair: *(start_vid, end_vid)*.
- Depending on the function and its parameters, the results can be symmetric.
  - The *agg_cost* of *(u, v)* is the same as for *(v, u)*.
- Any duplicated value in the *start_vids* or in *end_vids* are ignored.
- The returned values are ordered:
  - *start_vid* ascending
  - *end_vid* ascending

See Also

Indices and tables

- genindex
- search

6.1.6 Cost Matrix - Category

- *pgr_aStarCostMatrix - proposed*
- *pgr_bdAstarCostMatrix - proposed*
- *pgr_bdDijkstraCostMatrix - proposed*
- *pgr_dijkstraCostMatrix - proposed*
- *pgr_withPointsCostMatrix - proposed*

Warning: Proposed functions for next mayor release.

- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
  - The functions make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might not change. (But still can)
  - Signature might not change. (But still can)
  - Functionality might not change. (But still can)
  - pgTap tests have being done. But might need more.
  - Documentation might need refinement.

General Information

Synopsis

Traveling Sales Person - Family of functions needs as input a symmetric cost matrix and no edge *(u, v)* must value ∞.

This collection of functions will return a cost matrix in form of a table.

Characteristics

The main Characteristics are:

- Can be used as input to *pgr_TSP*.
  - **directly** when the resulting matrix is symmetric and there is no ∞ value.
  - It will be the users responsibility to make the matrix symmetric.
    - By using geometric or harmonic average of the non symmetric values.
    - By using max or min the non symmetric values.
    - By setting the upper triangle to be the mirror image of the lower triangle.
    - By setting the lower triangle to be the mirror image of the upper triangle.
  - It is also the users responsibility to fix an ∞ value.
- Each function works as part of the family it belongs to.
- It does not return a path.
- Returns the sum of the costs of the shortest path for pair combination of nodes in the graph.
- Process is done only on edges with positive costs.
- Values are returned when there is a path.

  - The returned values are in the form of a set of *(start_vid, end_vid, agg_cost)*.
  - When the starting vertex and ending vertex are the same, there is no path.
    * The *agg_cost* int the non included values *(v, v)* is *0*.
  - When the starting vertex and ending vertex are the different and there is no path.
    * The *agg_cost* in the non included values *(u, v)* is ∞.
- Let be the case the values returned are stored in a table, so the unique index would be the pair: *(start_vid, end_vid)*.
- Depending on the function and its parameters, the results can be symmetric.
  - The *agg_cost* of *(u, v)* is the same as for *(v, u)*.
- Any duplicated value in the *start_vids* are ignored.
- The returned values are ordered:
  - *start_vid* ascending
  - *end_vid* ascending
- Running time: approximately $O(|start\_vids| * (V \log V + E))$

See Also

- *pgr_TSP*

Indices and tables

- genindex
- search

6.1.7 KSP Category

- *pgr_KSP* - Driving Distance based on pgr_dijkstra
- *pgr_withPointsKSP - Proposed* - Driving Distance based on pgr_dijkstra

Indices and tables

- genindex
- search

6.2 Experimental Functions

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.

- Signature might change.
- Functionality might change.
- pgTap tests might be missing.
- Might need c/c++ coding.
- May lack documentation.
- Documentation if any might need to be rewritten.
- Documentation examples might need to be automatically generated.
- Might need a lot of feedback from the comunity.
- Might depend on a proposed function of pgRouting
- Might depend on a deprecated function of pgRouting

Contraction - Family of functions - Reduce network size using contraction techniques

- *pgr_contractGraph - Experimental* - Reduce network size using contraction techniques

Graph Analysis

- *pgr_labelGraph - Experimental* - Analyze / label subgraphs within a network

Components - Family of functions - Analyze components within a graph

- *pgr_connectedComponents - Experimental* - Return the connected components of an undirected graph
- *pgr_strongComponents - Experimental* - Return the strongly connected components of a directed graph
- *pgr_biconnectedComponents - Experimental* - Return the biconnected components of an undirected graph
- *pgr_articulationPoints - Experimental* - Return the articulation points of an undirected graph
- *pgr_bridges - Experimental* - Return the bridges of an undirected graph

VRP

- *pgr_gsoc_vrppdtw - Experimental*
- *pgr_vrpOneDepot - Experimental*

6.2.1 Contraction - Family of functions

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.
  - pgTap tests might be missing.
  - Might need c/c++ coding.

- May lack documentation.
- Documentation if any might need to be rewritten.
- Documentation examples might need to be automatically generated.
- Might need a lot of feedback from the comunity.
- Might depend on a proposed function of pgRouting
- Might depend on a deprecated function of pgRouting

pgr_contractGraph - Experimental

pgr_contractGraph - Experimental

`pgr_contractGraph` — Performs graph contraction and returns the contracted vertices and edges.

[62]

Fig. 6.13: Boost Graph Inside

Availability: 2.3.0

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.
  - pgTap tests might be missing.
  - Might need c/c++ coding.
  - May lack documentation.
  - Documentation if any might need to be rewritten.
  - Documentation examples might need to be automatically generated.
  - Might need a lot of feedback from the comunity.
  - Might depend on a proposed function of pgRouting
  - Might depend on a deprecated function of pgRouting

[62] http://www.boost.org/libs/graph

Synopsis

Contraction reduces the size of the graph by removing some of the vertices and edges and, for example, might add edges that represent a sequence of original edges decreasing the total time and space used in graph algorithms.

Characteristics

The main Characteristics are:

- Process is done only on edges with positive costs.
- There are two types of contraction methods used namely,
  - Dead End Contraction
  - Linear Contraction
- The values returned include the added edges and contracted vertices.
- The returned values are ordered as follows:
  - column *id* ascending when type = *v*
  - column *id* descending when type = *e*

Signature Summary:

The pgr_contractGraph function has the following signatures:

```
pgr_contractGraph(edges_sql, contraction_order)
pgr_contractGraph(edges_sql, contraction_order, max_cycles, forbidden_vertices,␣
↪directed)

RETURNS SETOF (seq, type, id, contracted_vertices, source, target, cost)
```

Signatures

Minimal signature

```
pgr_contractGraph(edges_sql, contraction_order)
```

Example Making a dead end contraction and a linear contraction.

```
SELECT * FROM pgr_contractGraph(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[1, 2]);
 seq | type | id | contracted_vertices | source | target | cost
-----+------+----+---------------------+--------+--------+------
   1 | v    |  5 | {7,8}               |     -1 |     -1 |   -1
   2 | v    | 15 | {14}                |     -1 |     -1 |   -1
   3 | v    | 17 | {16}                |     -1 |     -1 |   -1
   4 | e    | -1 | {1,2}               |      3 |      5 |    2
   5 | e    | -2 | {4}                 |      9 |      3 |    2
   6 | e    | -3 | {10,13}             |      5 |     11 |    2
   7 | e    | -4 | {12}                |     11 |      9 |    2
(7 rows)
```

Complete signature

```
pgr_contractGraph(edges_sql, contraction_order, max_cycles, forbidden_vertices,␣
↪directed)
```

Example Making a dead end contraction and a linear contraction and vertex 2 is forbidden from contraction

```
SELECT * FROM pgr_contractGraph(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
ARRAY[1, 2], forbidden_vertices:=ARRAY[2]);
 seq | type | id | contracted_vertices | source | target | cost
-----+------+----+---------------------+--------+--------+------
   1 | v    |  2 | {1}                 |     -1 |     -1 |   -1
   2 | v    |  5 | {7,8}               |     -1 |     -1 |   -1
   3 | v    | 15 | {14}                |     -1 |     -1 |   -1
   4 | v    | 17 | {16}                |     -1 |     -1 |   -1
   5 | e    | -1 | {4}                 |      9 |      3 |    2
   6 | e    | -2 | {10,13}             |      5 |     11 |    2
   7 | e    | -3 | {12}                |     11 |      9 |    2
(7 rows)
```

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Column	Type	Description
edges_sql	`TEXT`	SQL query as described above.
contraction_order	`ARRAY[ANY-INTEGER]`	**Ordered contraction operations.** • 1 = Dead end contraction • 2 = Linear contraction
forbidden_vertices	`ARRAY[ANY-INTEGER]`	(optional). Identifiers of vertices forbidden from contraction. Default is an empty array.
max_cycles	`INTEGER`	(optional). Number of times the contraction operations on *contraction_order* will be performed. Default is 1.
directed	`BOOLEAN`	• When `true` the graph is considered as *Directed*. • When `false` the graph is considered as *Undirected*.

Description of the return values

RETURNS SETOF (seq, type, id, contracted_vertices, source, target, cost)

The function returns a single row. The columns of the row are:

Column	Type	Description
seq	INTEGER	Sequential value starting from **1**.
type	TEXT	**Type of the *id*.** • 'v' when *id* is an identifier of a vertex. • 'e' when *id* is an identifier of an edge.
id	BIGINT	**Identifier of:** • the vertex when *type* = *'v'*. – The vertex belongs to the edge_table passed as a parameter. • the edge when *type* = *'e'*. – The *id* is a decreasing sequence starting from **-1**. – Representing a pseudo *id* as is not incorporated into the edge_table.
contracted_vertices	ARRAY[BIGINT]	Array of contracted vertex identifiers.
source	BIGINT	Identifier of the source vertex of the current edge *id*. Valid values when *type* = *'e'*.
target	BIGINT	Identifier of the target vertex of the current edge *id*. Valid values when *type* = *'e'*.
cost	FLOAT	Weight of the edge (*source*, *target*). Valid values when *type* = *'e'*.

Examples

Example Only dead end contraction

```
SELECT * FROM pgr_contractGraph(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
ARRAY[1]);
 seq | type | id | contracted_vertices | source | target | cost
-----+------+----+---------------------+--------+--------+------
   1 | v    |  2 | {1}                 |     -1 |     -1 |   -1
   2 | v    |  5 | {7,8}               |     -1 |     -1 |   -1
   3 | v    | 10 | {13}                |     -1 |     -1 |   -1
   4 | v    | 15 | {14}                |     -1 |     -1 |   -1
   5 | v    | 17 | {16}                |     -1 |     -1 |   -1
(5 rows)
```

Example Only linear contraction

```
SELECT * FROM pgr_contractGraph(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
ARRAY[2]);
 seq | type | id | contracted_vertices | source | target | cost
-----+------+----+---------------------+--------+--------+------
   1 | e    | -1 | {4}                 |      9 |      3 |    2
   2 | e    | -2 | {8}                 |      5 |      7 |    2
   3 | e    | -3 | {8}                 |      7 |      5 |    2
   4 | e    | -4 | {12}                |     11 |      9 |    2
(4 rows)
```

Indices and tables

- genindex
- search

Introduction

In big graphs, like the road graphs, or electric networks, graph contraction can be used to speed up some graph algorithms. Contraction reduces the size of the graph by removing some of the vertices and edges and, for example, might add edges that represent a sequence of original edges decreasing the total time and space used in graph algorithms.

This implementation gives a flexible framework for adding contraction algorithms in the future, currently, it supports two algorithms:

1. Dead end contraction
2. Linear contraction

Allowing the user to:

- Forbid contraction on a set of nodes.
- Decide the order of the contraction algorithms and set the maximum number of times they are to be executed.

Note: UNDER DISCUSSION: Forbid contraction on a set of edges

Dead end contraction

In the algorithm, dead end contraction is represented by 1.

Dead end nodes

The definition of a dead end node is different for a directed and an undirected graph.

In case of a undirected graph, a node is considered a dead end node if

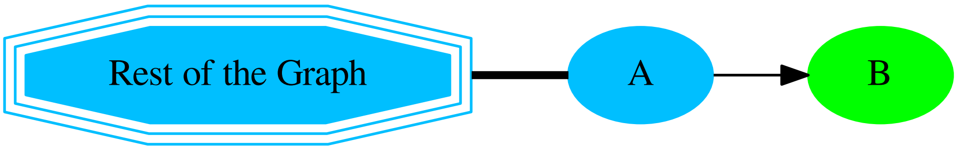
- The number of adjacent vertices is 1.

In case of an directed graph, a node is considered a dead end node if

- There are no outgoing edges and has at least one incoming edge.
- There is one incoming and one outgoing edge with the same identifier.

Examples

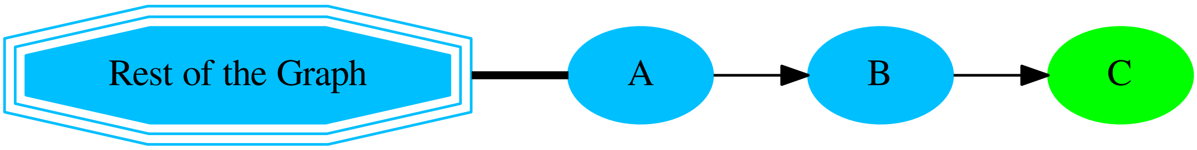
- The green node `B` represents a dead end node
- The node `A` is the only node connecting to `B`.
- Node `A` is part of the rest of the graph and has an unlimited number of incoming and outgoing edges.
- Directed graph



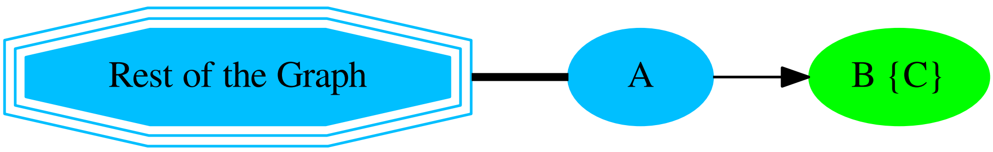
Operation: Dead End Contraction

The dead end contraction will stop until there are no more dead end nodes. For example from the following graph:

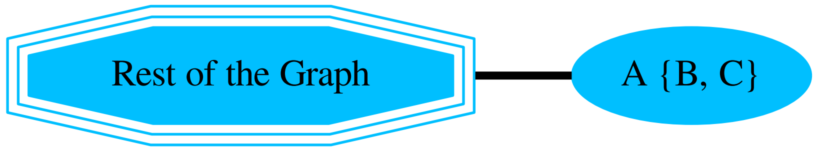
- Node `A` is connected to the rest of the graph by an unlimited number of edges.
- Node `B` is connected to the rest of the graph with one incoming edge.
- Node `B` is the only node connecting to `C`.
- The green node `C` represents a *Dead End* node



After contracting `C`, node `B` is now a *Dead End* node and is contracted:



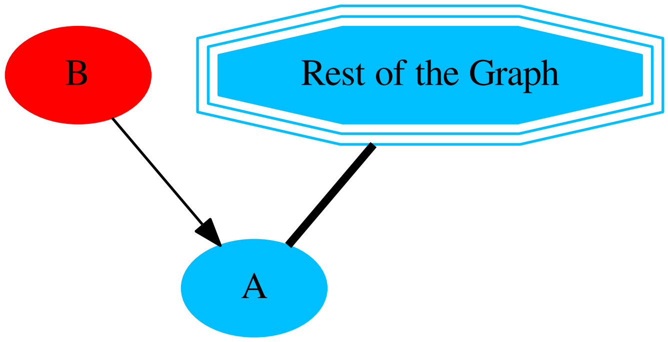
Node `B` gets contracted



Nodes B and C belong to node A.

Not Dead End nodes

In this graph B is not a *dead end* node.



Linear contraction

In the algorithm, linear contraction is represented by 2.

Linear nodes

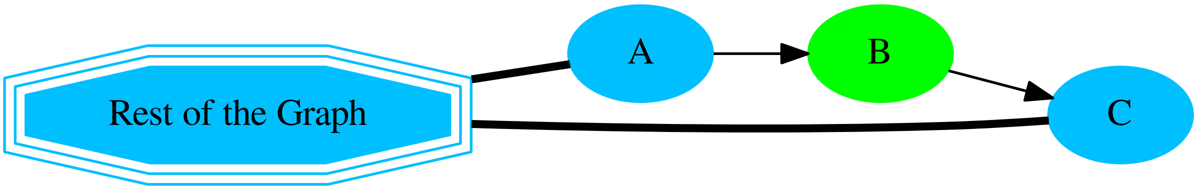
A node is considered a linear node if satisfies the following:

- The number of adjacent vertices are 2.
- Should have at least one incoming edge and one outgoing edge.

Examples

- The green node B represents a linear node
- The nodes A and C are the only nodes connecting to B.
- Node A is part of the rest of the graph and has an unlimited number of incoming and outgoing edges.
- Node C is part of the rest of the graph and has an unlimited number of incoming and outgoing edges.

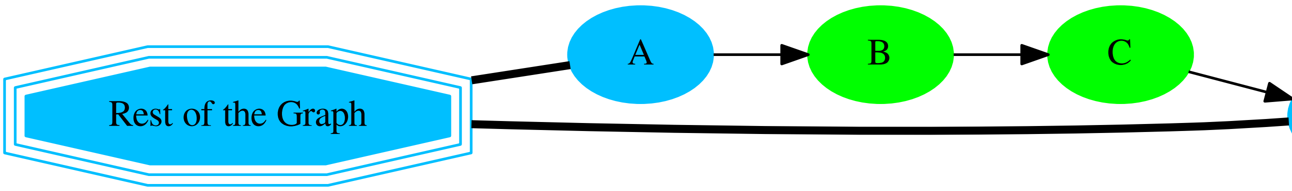
- Directed graph



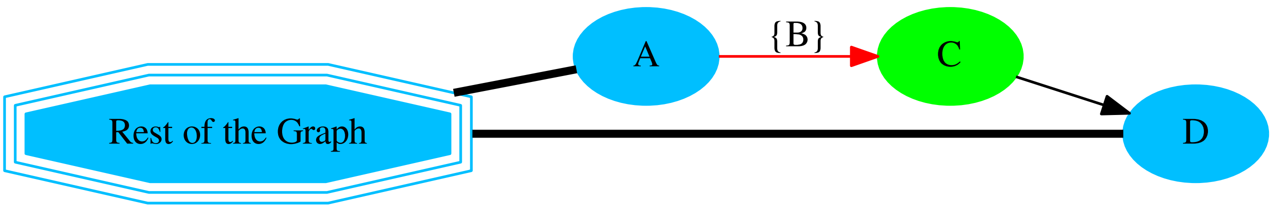
Operation: Linear Contraction

The linear contraction will stop until there are no more linear nodes. For example from the following graph:

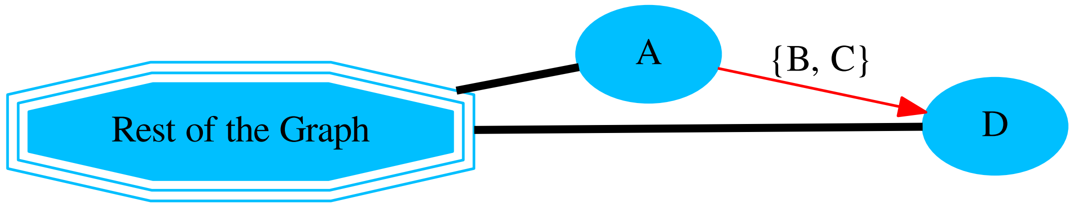
- Node A is connected to the rest of the graph by an unlimited number of edges.
- Node B is connected to the rest of the graph with one incoming edge and one outgoing edge.
- Node C is connected to the rest of the graph with one incoming edge and one outgoing edge.
- Node D is connected to the rest of the graph by an unlimited number of edges.
- The green nodes B and C represents *Linear* nodes.



After contracting B, a new edge gets inserted between A and C which is represented by red color.



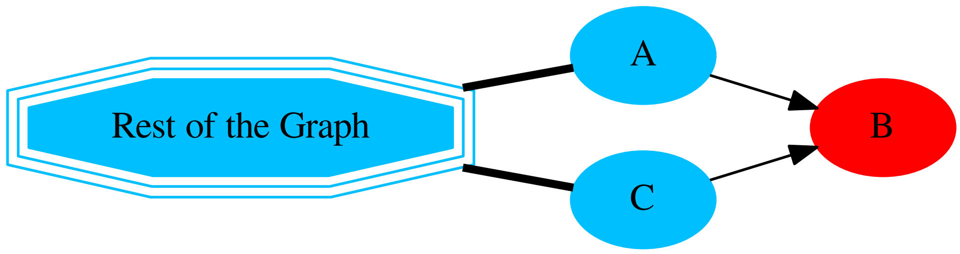
Node C is *linear node* and gets contracted.



Nodes B and C belong to edge connecting A and D which is represented by red color.

Not Linear nodes

In this graph B is not a *linear* node.



The cycle

Contracting a graph, can be done with more than one operation. The order of the operations affect the resulting contracted graph, after applying one operation, the set of vertices that can be contracted by another operation changes.

This implementation, cycles `max_cycles` times through `operations_order` .

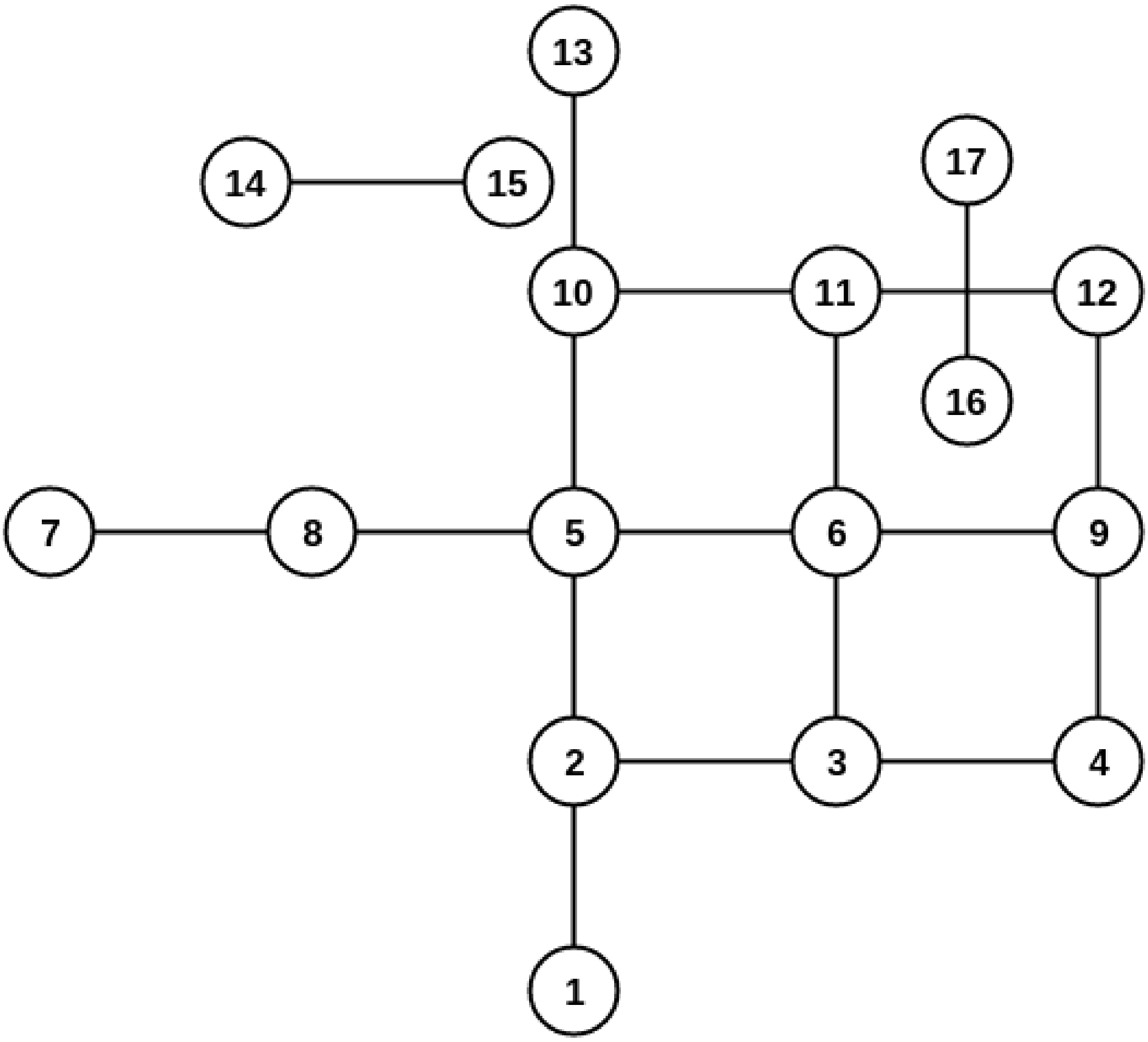
```
<input>
do max_cycles times {
    for (operation in operations_order)
     { do operation }
}
<output>
```

Contracting Sample Data

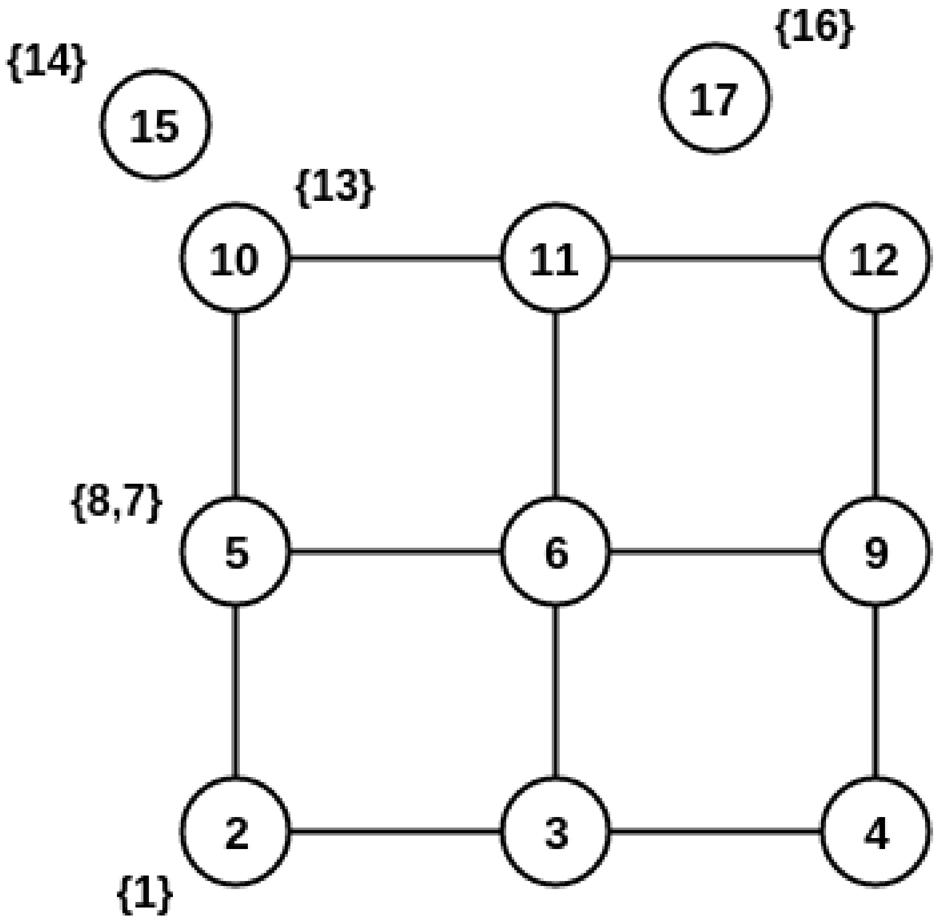
In this section, building and using a contracted graph will be shown by example.

- The *Sample Data* for an undirected graph is used
- a dead end operation first followed by a linear operation.

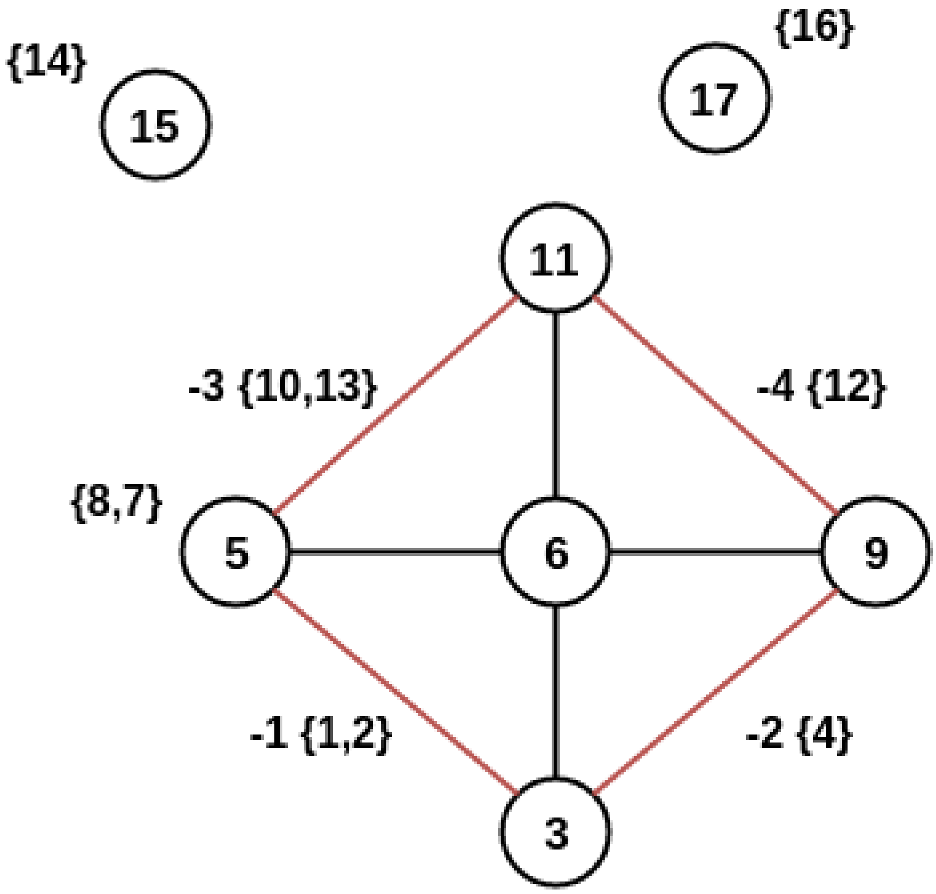
The original graph:



After doing a dead end contraction operation:



Doing a linear contraction operation to the graph above



There are five cases, in this documentation, which arise when calculating the shortest path between a given source and target. In this examples, `pgr_dijkstra` is used.

- **Case 1**: Both source and target belong to the contracted graph.
- **Case 2**: Source belongs to a contracted graph, while target belongs to a edge subgraph.
- **Case 3**: Source belongs to a vertex subgraph, while target belongs to an edge subgraph.
- **Case 4**: Source belongs to a contracted graph, while target belongs to an vertex subgraph.
- **Case 5**: The path contains a new edge added by the contraction algorithm.

Construction of the graph in the database

Original Data

The following query shows the original data involved in the contraction operation.

```
```

Contraction Results

```
SELECT * FROM pgr_contractGraph(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    array[1,2], directed:=true);
 seq | type | id | contracted_vertices | source | target | cost
-----+------+----+---------------------+--------+--------+------
   1 | v    |  5 | {7,8}               |     -1 |     -1 |   -1
```

```
   2 | v     | 15 | {14}                 |      -1 |      -1 |    -1
   3 | v     | 17 | {16}                 |      -1 |      -1 |    -1
   4 | e     | -1 | {1,2}                |       3 |       5 |     2
   5 | e     | -2 | {4}                  |       9 |       3 |     2
   6 | e     | -3 | {10,13}              |       5 |      11 |     2
   7 | e     | -4 | {12}                 |      11 |       9 |     2
(7 rows)
```

The above results do not represent the contracted graph. They represent the changes done to the graph after applying the contraction algorithm. We can see that vertices like 6 and 11 do not appear in the contraction results because they were not affected by the contraction algorithm.

step 1

Adding extra columns to the `edge_table` and `edge_table_vertices_pgr` tables:

Column	Description
contracted_vertices	The vertices set belonging to the vertex/edge
is_contracted	On a *vertex* table: when `true` the vertex is contracted, so is not part of the contracted graph.
is_contracted	On an *edge* table: when `true` the edge was generated by the contraction algorithm.

Using the following queries:

```
ALTER TABLE edge_table ADD contracted_vertices BIGINT[];
ALTER TABLE
ALTER TABLE edge_table_vertices_pgr ADD contracted_vertices BIGINT[];
ALTER TABLE
ALTER TABLE edge_table ADD is_contracted BOOLEAN DEFAULT false;
ALTER TABLE
ALTER TABLE edge_table_vertices_pgr ADD is_contracted BOOLEAN DEFAULT false;
ALTER TABLE
SET client_min_messages TO NOTICE;
SET
```

step 2

For simplicity, in this documentation, store the results of the call to pgr_contractGraph in a temporary table

```
SELECT * INTO contraction_results
FROM pgr_contractGraph(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    array[1,2], directed:=true);
SELECT 7
```

step 3

Update the *vertex* and *edge* tables using the results of the call to pgr_contraction

- In *edge_table_vertices_pgr.is_contracted* indicate the vertices that are contracted.

```
UPDATE edge_table_vertices_pgr
SET is_contracted = true
```

```
WHERE id IN (SELECT  unnest(contracted_vertices) FROM  contraction_results);
UPDATE 10
```

- Add to *edge_table_vertices_pgr.contracted_vertices* the contracted vertices belonging to the vertices.

```
UPDATE edge_table_vertices_pgr
SET contracted_vertices = contraction_results.contracted_vertices
FROM contraction_results
WHERE type = 'v' AND edge_table_vertices_pgr.id = contraction_results.id;
UPDATE 3
```

- Insert the new edges generated by pgr_contractGraph.

```
INSERT INTO edge_table(source, target, cost, reverse_cost, contracted_vertices, is_
↪contracted)
SELECT source, target, cost, -1, contracted_vertices, true
FROM contraction_results
WHERE type = 'e';
INSERT 0 4
```

step 3.1

Verify visually the updates.

- On the *edge_table_vertices_pgr*

```
SELECT id, contracted_vertices, is_contracted
FROM edge_table_vertices_pgr
ORDER BY id;
 id | contracted_vertices | is_contracted
----+---------------------+---------------
  1 |                     | t
  2 |                     | t
  3 |                     | f
  4 |                     | t
  5 | {7,8}               | f
  6 |                     | f
  7 |                     | t
  8 |                     | t
  9 |                     | f
 10 |                     | t
 11 |                     | f
 12 |                     | t
 13 |                     | t
 14 |                     | t
 15 | {14}                | f
 16 |                     | t
 17 | {16}                | f
(17 rows)
```

- On the *edge_table*

```
SELECT id, source, target, cost, reverse_cost, contracted_vertices, is_contracted
FROM edge_table
ORDER BY id;
 id | source | target | cost | reverse_cost | contracted_vertices | is_contracted
----+--------+--------+------+--------------+---------------------+---------------
  1 |      1 |      2 |    1 |            1 |                     | f
  2 |      2 |      3 |   -1 |            1 |                     | f
  3 |      3 |      4 |   -1 |            1 |                     | f
```

```
  4 |       2 |       5 |     1 |              1 |                      | f
  5 |       3 |       6 |     1 |             -1 |                      | f
  6 |       7 |       8 |     1 |              1 |                      | f
  7 |       8 |       5 |     1 |              1 |                      | f
  8 |       5 |       6 |     1 |              1 |                      | f
  9 |       6 |       9 |     1 |              1 |                      | f
 10 |       5 |      10 |     1 |              1 |                      | f
 11 |       6 |      11 |     1 |             -1 |                      | f
 12 |      10 |      11 |     1 |             -1 |                      | f
 13 |      11 |      12 |     1 |             -1 |                      | f
 14 |      10 |      13 |     1 |              1 |                      | f
 15 |       9 |      12 |     1 |              1 |                      | f
 16 |       4 |       9 |     1 |              1 |                      | f
 17 |      14 |      15 |     1 |              1 |                      | f
 18 |      16 |      17 |     1 |              1 |                      | f
 19 |       3 |       5 |     2 |             -1 | {1,2}                | t
 20 |       9 |       3 |     2 |             -1 | {4}                  | t
 21 |       5 |      11 |     2 |             -1 | {10,13}              | t
 22 |      11 |       9 |     2 |             -1 | {12}                 | t
(22 rows)
```

- vertices that belong to the contracted graph are the non contracted vertices

```
SELECT id  FROM edge_table_vertices_pgr
WHERE is_contracted = false
ORDER BY id;
 id
----
  3
  5
  6
  9
 11
 15
 17
(7 rows)
```

case 1: Both source and target belong to the contracted graph.

Inspecting the contracted graph above, vertex 3 and vertex 11 are part of the contracted graph. In the following query:

- vertices_in_graph hold the vertices that belong to the contracted graph.
- when selecting the edges, only edges that have the source and the target in that set are the edges belonging to the contracted graph, that is done in the WHERE clause.

Visually, looking at the original graph, going from 3 to 11: 3 -> 6 -> 11, and in the contracted graph, it is also 3 -> 6 -> 11. The results, on the contracted graph match the results as if it was done on the original graph.

```
SELECT * FROM pgr_dijkstra(
    $$
    WITH
    vertices_in_graph AS (
        SELECT id  FROM edge_table_vertices_pgr WHERE is_contracted = false)
    SELECT id, source, target, cost, reverse_cost
    FROM edge_table
    WHERE source IN (SELECT * FROM vertices_in_graph)
    AND target IN (SELECT * FROM vertices_in_graph)
```

```
    $$,
    3, 11, false);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    3 |    5 |    1 |        0
   2 |        2 |    6 |   11 |    1 |        1
   3 |        3 |   11 |   -1 |    0 |        2
(3 rows)
```

case 2: Source belongs to the contracted graph, while target belongs to a edge subgraph.

Inspecting the contracted graph above, vertex 3 is part of the contracted graph and vertex 1 belongs to the contracted subgr

- expand1 holds the contracted vertices of the edge where vertex 1 belongs. (belongs to edge 19).
- vertices_in_graph hold the vertices that belong to the contracted graph and also the contracted vertices of edge 19.
- when selecting the edges, only edges that have the source and the target in that set are the edges belonging to the contracted graph, that is done in the WHERE clause.

Visually, looking at the original graph, going from 3 to 1: 3 -> 2 -> 1, and in the contracted graph, it is also 3 -> 2 -> 1. The results, on the contracted graph match the results as if it was done on the original graph.

```
SELECT * FROM pgr_dijkstra(
    $$
    WITH
    expand_edges AS (SELECT id, unnest(contracted_vertices) AS vertex FROM edge_
↪table),
    expand1 AS (SELECT contracted_vertices FROM edge_table
        WHERE id IN (SELECT id FROM expand_edges WHERE vertex = 1)),
    vertices_in_graph AS (
        SELECT id  FROM edge_table_vertices_pgr WHERE is_contracted = false
        UNION
        SELECT unnest(contracted_vertices) FROM expand1)
    SELECT id, source, target, cost, reverse_cost
    FROM edge_table
    WHERE source IN (SELECT * FROM vertices_in_graph)
    AND target IN (SELECT * FROM vertices_in_graph)
    $$,
    3, 1, false);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    3 |    2 |    1 |        0
   2 |        2 |    2 |    1 |    1 |        1
   3 |        3 |    1 |   -1 |    0 |        2
(3 rows)
```

case 3: Source belongs to a vertex subgraph, while target belongs to an edge subgraph.

Inspecting the contracted graph above, vertex 7 belongs to the contracted subgraph of vertex 5 and vertex 13 belongs to the contracted subgraph of edge 21. In the following query:

- expand7 holds the contracted vertices of vertex where vertex 7 belongs. (belongs to vertex 5)
- expand13 holds the contracted vertices of edge where vertex 13 belongs. (belongs to edge 21)

- vertices_in_graph hold the vertices that belong to the contracted graph, contracted vertices of vertex 5 and contracted vertices of edge 21.
- when selecting the edges, only edges that have the source and the target in that set are the edges belonging to the contracted graph, that is done in the WHERE clause.

Visually, looking at the original graph, going from 7 to 13: 7 -> 8 -> 5 -> 10 -> 13, and in the contracted graph, it is also 7 -> 8 -> 5 -> 10 -> 13. The results, on the contracted graph match the results as if it was done on the original graph.

```
SELECT * FROM pgr_dijkstra(
    $$
    WITH

    expand_vertices AS (SELECT id, unnest(contracted_vertices) AS vertex FROM edge_
↪table_vertices_pgr),
    expand7 AS (SELECT contracted_vertices FROM edge_table_vertices_pgr
        WHERE id IN (SELECT id FROM expand_vertices WHERE vertex = 7)),

    expand_edges AS (SELECT id, unnest(contracted_vertices) AS vertex FROM edge_
↪table),
    expand13 AS (SELECT contracted_vertices FROM edge_table
        WHERE id IN (SELECT id FROM expand_edges WHERE vertex = 13)),

    vertices_in_graph AS (
        SELECT id  FROM edge_table_vertices_pgr WHERE is_contracted = false
        UNION
        SELECT unnest(contracted_vertices) FROM expand13
        UNION
        SELECT unnest(contracted_vertices) FROM expand7)

    SELECT id, source, target, cost, reverse_cost
    FROM edge_table
    WHERE source IN (SELECT * FROM vertices_in_graph)
    AND target IN (SELECT * FROM vertices_in_graph)
    $$,
    7, 13, false);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    7 |    6 |    1 |        0
   2 |        2 |    8 |    7 |    1 |        1
   3 |        3 |    5 |   10 |    1 |        2
   4 |        4 |   10 |   14 |    1 |        3
   5 |        5 |   13 |   -1 |    0 |        4
(5 rows)
```

case 4: Source belongs to the contracted graph, while target belongs to an vertex subgraph.

Inspecting the contracted graph above, vertex 3 is part of the contracted graph and vertex 7 belongs to the contracted subgraph of vertex 5. In the following query:

- expand7 holds the contracted vertices of vertex where vertex 7 belongs. (belongs to vertex 5)
- vertices_in_graph hold the vertices that belong to the contracted graph and the contracted vertices of vertex 5.
- when selecting the edges, only edges that have the source and the target in that set are the edges belonging to the contracted graph, that is done in the WHERE clause.

Visually, looking at the original graph, going from 3 to 7: 3 -> 2 -> 5 -> 8 -> 7, but in the contracted graph, it is 3 -> 5 -> 8 -> 7. The results, on the contracted graph do not match the results as if it was done on the original graph. This is because the path contains edge 19 which is added by the contraction algorithm.

```
SELECT * FROM  pgr_dijkstra(
    $$
    WITH
    expand_vertices AS (SELECT id, unnest(contracted_vertices) AS vertex FROM edge_
↪table_vertices_pgr),
    expand7 AS (SELECT contracted_vertices FROM edge_table_vertices_pgr
        WHERE id IN (SELECT id FROM expand_vertices WHERE vertex = 7)),
    vertices_in_graph AS (
        SELECT id  FROM edge_table_vertices_pgr WHERE is_contracted = false
        UNION
        SELECT unnest(contracted_vertices) FROM expand7)
    SELECT id, source, target, cost, reverse_cost
    FROM edge_table
    WHERE source IN (SELECT * FROM vertices_in_graph)
    AND target IN (SELECT * FROM vertices_in_graph)
    $$,
    3, 7, false);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    3 |   19 |    2 |        0
   2 |        2 |    5 |    7 |    1 |        2
   3 |        3 |    8 |    6 |    1 |        3
   4 |        4 |    7 |   -1 |    0 |        4
(4 rows)
```

case 5: The path contains an edge added by the contraction algorithm.

In the previous example we can see that the path from vertex 3 to vertex 7 contains an edge which is added by the contraction algorithm.

```
WITH
first_dijkstra AS (
    SELECT * FROM  pgr_dijkstra(
        $$
        WITH
        expand_vertices AS (SELECT id, unnest(contracted_vertices) AS vertex FROM␣
↪edge_table_vertices_pgr),
        expand7 AS (SELECT contracted_vertices FROM edge_table_vertices_pgr
            WHERE id IN (SELECT id FROM expand_vertices WHERE vertex = 7)),
        vertices_in_graph AS (
            SELECT id  FROM edge_table_vertices_pgr WHERE is_contracted = false
            UNION
            SELECT unnest(contracted_vertices) FROM expand7)
        SELECT id, source, target, cost, reverse_cost
        FROM edge_table
        WHERE source IN (SELECT * FROM vertices_in_graph)
        AND target IN (SELECT * FROM vertices_in_graph)
        $$,
        3, 7, false))
SELECT edge, contracted_vertices
    FROM first_dijkstra JOIN edge_table
    ON (edge = id)
    WHERE is_contracted = true;
 edge | contracted_vertices
------+---------------------
   19 | {1,2}
(1 row)
```

Inspecting the contracted graph above, edge 19 should be expanded. In the following query:

- first_dijkstra holds the results of the dijkstra query.
- edges_to_expand holds the edges added by the contraction algorithm and included in the path.
- vertices_in_graph hold the vertices that belong to the contracted graph, vertices of the contracted solution and the contracted vertices of the edges added by the contraction algorithm and included in the contracted solution.
- when selecting the edges, only edges that have the source and the target in that set are the edges belonging to the contracted graph, that is done in the WHERE clause.

Visually, looking at the original graph, going from 3 to 7: 3 -> 2 -> 5 -> 8 -> 7, and in the contracted graph, it is also 3 -> 2 -> 5 -> 8 -> 7. The results, on the contracted graph match the results as if it was done on the original graph.

```
SELECT * FROM pgr_dijkstra($$
    WITH
    -- This returns the results from case 2
    first_dijkstra AS (
        SELECT * FROM  pgr_dijkstra(
            '
            WITH
            expand_vertices AS (SELECT id, unnest(contracted_vertices) AS vertex␣
↪FROM edge_table_vertices_pgr),
            expand7 AS (SELECT contracted_vertices FROM edge_table_vertices_pgr
                WHERE id IN (SELECT id FROM expand_vertices WHERE vertex = 7)),
            vertices_in_graph AS (
                SELECT id  FROM edge_table_vertices_pgr WHERE is_contracted = false
                UNION
                SELECT unnest(contracted_vertices) FROM expand7)
            SELECT id, source, target, cost, reverse_cost
            FROM edge_table
            WHERE source IN (SELECT * FROM vertices_in_graph)
            AND target IN (SELECT * FROM vertices_in_graph)
            ',
            3, 7, false)),

    -- edges that need expansion and the vertices to be expanded.
    edges_to_expand AS (
        SELECT edge, contracted_vertices
        FROM first_dijkstra JOIN edge_table
        ON (edge = id)
        WHERE is_contracted = true),

    vertices_in_graph AS (
        -- the nodes of the contracted solution
        SELECT node FROM first_dijkstra
        UNION
        -- the nodes of the expanding sections
        SELECT unnest(contracted_vertices) FROM edges_to_expand)

    SELECT id, source, target, cost, reverse_cost
    FROM edge_table
    WHERE source IN (SELECT * FROM vertices_in_graph)
    AND target IN (SELECT * FROM vertices_in_graph)
    -- not including the expanded edges
    AND id NOT IN (SELECT edge FROM edges_to_expand)
    $$,
    3, 7, false);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    3 |    2 |    1 |        0
   2 |        2 |    2 |    4 |    1 |        1
   3 |        3 |    5 |    7 |    1 |        2
```

```
   4 |          4 |     8 |      6 |     1 |           3
   5 |          5 |     7 |     -1 |     0 |           4
(5 rows)
```

See Also

- http://www.cs.cmu.edu/afs/cs/academic/class/15210-f12/www/lectures/lecture16.pdf
- http://algo2.iti.kit.edu/documents/routeplanning/geisberger_dipl.pdf
- The queries use *pgr_contractGraph - Experimental* function and the *Sample Data* network.

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6.2.2 Flow - Family of functions

- *pgr_maxFlow - Proposed* - Only the Max flow calculation using Push and Relabel algorithm.
- *pgr_boykovKolmogorov - Proposed* - Boykov and Kolmogorov with details of flow on edges.
- *pgr_edmondsKarp - Proposed* - Edmonds and Karp algorithm with details of flow on edges.
- *pgr_pushRelabel - Proposed* - Push and relabel algorithm with details of flow on edges.
- Applications
  - *pgr_edgeDisjointPaths - Proposed* - Calculates edge disjoint paths between two groups of vertices.
  - *pgr_maxCardinalityMatch - Proposed* - Calculates a maximum cardinality matching in a graph.

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.
  - pgTap tests might be missing.
  - Might need c/c++ coding.
  - May lack documentation.
  - Documentation if any might need to be rewritten.
  - Documentation examples might need to be automatically generated.
  - Might need a lot of feedback from the comunity.
  - Might depend on a proposed function of pgRouting
  - Might depend on a deprecated function of pgRouting

pgr_maxFlow - Proposed

Synopsis

`pgr_maxFlow` — Calculates the maximum flow in a directed graph from the source(s) to the targets(s) using the Push Relabel algorithm.

[63]

Fig. 6.14: Boost Graph Inside

Availability: 2.4.0

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.
  - pgTap tests might be missing.
  - Might need c/c++ coding.
  - May lack documentation.
  - Documentation if any might need to be rewritten.
  - Documentation examples might need to be automatically generated.
  - Might need a lot of feedback from the comunity.
  - Might depend on a proposed function of pgRouting
  - Might depend on a deprecated function of pgRouting

Characteristics

- The graph is **directed**.
- When the maximum flow is 0 then there is no flow and **0** is returned.
  - There is no flow when a **source** is the same as a **target**.
- Any duplicated value in the source(s) or target(s) are ignored.
- Uses the *pgr_pushRelabel* algorithm.
- Running time: $O(V^3)$

[63] http://www.boost.org/libs/graph/doc/push_relabel_max_flow.html

Signature Summary

```
pgr_maxFlow(edges_sql, source,  target)
pgr_maxFlow(edges_sql, sources,  target)
pgr_maxFlow(edges_sql, source,  targets)
pgr_maxFlow(edges_sql, sources,  targets)
RETURNS BIGINT
```

One to One

Calculates the maximum flow from the *source* to the *target*.

```
pgr_maxFlow(edges_sql, source,  target)
RETURNS BIGINT
```

Example

```
SELECT * FROM pgr_maxFlow(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
    , 6, 11
);
 pgr_maxflow
-------------
         230
(1 row)
```

One to Many

Calculates the maximum flow from the *source* to all of the *targets*.

```
pgr_maxFlow(edges_sql, source,  targets)
RETURNS BIGINT
```

Example

```
SELECT * FROM pgr_maxFlow(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
    , 6, ARRAY[11, 1, 13]
);
 pgr_maxflow
-------------
         340
(1 row)
```

Many to One

Calculates the maximum flow from all the *sources* to the *target*.

```
pgr_maxFlow(edges_sql, sources,  target)
RETURNS BIGINT
```

Example

```
SELECT * FROM pgr_maxFlow(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
    , ARRAY[6, 8, 12], 11
);
 pgr_maxflow
-------------
         230
(1 row)
```

Many to Many

Calculates the maximum flow from all of the *sources* to all of the *targets*.

```
pgr_maxFlow(edges_sql, sources,  targets)
RETURNS BIGINT
```

Example

```
SELECT * FROM pgr_maxFlow(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
    , ARRAY[6, 8, 12], ARRAY[1, 3, 11]
);
 pgr_maxflow
-------------
         360
(1 row)
```

Description of the Signatures

Description of the edges_sql query for Max-flow like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
capacity	`ANY-INTEGER`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_capacity	`ANY-INTEGER`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

Description of the Parameters of the Flow Signatures

Column	Type	Default	Description
edges_sql	`TEXT`		The edges SQL query as described above.
source	`BIGINT`		Identifier of the starting vertex of the flow.
sources	`ARRAY[BIGINT]`		Array of identifiers of the starting vertices of the flow.
target	`BIGINT`		Identifier of the ending vertex of the flow.
targets	`ARRAY[BIGINT]`		Array of identifiers of the ending vertices of the flow.

Description of the return value

Type	Description
`BIGINT`	Maximum flow possible from the source(s) to the target(s)

See Also

- *Flow - Family of functions*
- http://www.boost.org/libs/graph/doc/push_relabel_max_flow.html
- https://en.wikipedia.org/wiki/Push%E2%80%93relabel_maximum_flow_algorithm

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pgr_pushRelabel - Proposed

Synopsis

`pgr_pushRelabel` — Calculates the flow on the graph edges that maximizes the flow from the sources to the targets using Push Relabel Algorithm.

[64]

Fig. 6.15: Boost Graph Inside

Availability:

- Renamed 2.5.0, Previous name pgr_maxFlowPushRelabel
- New in 2.3.0

> **Warning:** Experimental functions
>
> - They are not officially of the current release.
> - They likely will not be officially be part of the next release:
>   - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
>   - Name might change.
>   - Signature might change.
>   - Functionality might change.
>   - pgTap tests might be missing.
>   - Might need c/c++ coding.
>   - May lack documentation.
>   - Documentation if any might need to be rewritten.
>   - Documentation examples might need to be automatically generated.
>   - Might need a lot of feedback from the comunity.
>   - Might depend on a proposed function of pgRouting
>   - Might depend on a deprecated function of pgRouting

Characteristics

- The graph is **directed**.

[64] http://www.boost.org/libs/graph/doc/push_relabel_max_flow.html

- Process is done only on edges with positive capacities.
- When the maximum flow is 0 then there is no flow and **EMPTY SET** is returned.
  - There is no flow when a **source** is the same as a **target**.
- Any duplicated value in the source(s) or target(s) are ignored.
- Calculates the flow/residual capacity for each edge. In the output
  - Edges with zero flow are omitted.
- Creates a **super source** and edges to all the source(s), and a **super target** and the edges from all the targets(s).
- The maximum flow through the graph is guaranteed to be the value returned by *pgr_maxFlow* when executed with the same parameters and can be calculated:
  - By aggregation of the outgoing flow from the sources
  - By aggregation of the incoming flow to the targets
- Running time: $O(V^3)$

Signature Summary

```
pgr_pushRelabel(edges_sql, source,  target) - Proposed
pgr_pushRelabel(edges_sql, sources, target) - Proposed
pgr_pushRelabel(edges_sql, source,  targets) - Proposed
pgr_pushRelabel(edges_sql, sources, targets) - Proposed
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

One to One

Calculates the flow on the graph edges that maximizes the flow from the *source* to the *target*.

```
pgr_pushRelabel(edges_sql, source,  target)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_pushRelabel(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
    , 6, 11
);
 seq | edge | start_vid | end_vid | flow | residual_capacity
-----+------+-----------+---------+------+-------------------
   1 |   10 |         5 |      10 |  100 |                30
   2 |    8 |         6 |       5 |  100 |                30
   3 |   11 |         6 |      11 |  130 |                 0
   4 |   12 |        10 |      11 |  100 |                 0
(4 rows)
```

One to Many

Calculates the flow on the graph edges that maximizes the flow from the *source* to all of the *targets*.

```
pgr_pushRelabel(edges_sql, source,  targets)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_pushRelabel(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
    , 6, ARRAY[11, 1, 13]
);
 seq | edge | start_vid | end_vid | flow | residual_capacity
-----+------+-----------+---------+------+-------------------
   1 |    1 |         2 |       1 |  130 |                 0
   2 |    2 |         3 |       2 |   80 |                20
   3 |    3 |         4 |       3 |   80 |                50
   4 |    4 |         5 |       2 |   50 |                 0
   5 |    7 |         5 |       8 |   50 |                80
   6 |   10 |         5 |      10 |   80 |                50
   7 |    8 |         6 |       5 |  130 |                 0
   8 |    9 |         6 |       9 |   80 |                50
   9 |   11 |         6 |      11 |  130 |                 0
  10 |    6 |         7 |       8 |   50 |                 0
  11 |    6 |         8 |       7 |   50 |                50
  12 |    7 |         8 |       5 |   50 |                 0
  13 |   16 |         9 |       4 |   80 |                 0
  14 |   12 |        10 |      11 |   80 |                20
(14 rows)
```

Many to One

Calculates the flow on the graph edges that maximizes the flow from all of the *sources* to the *target*.

```
pgr_pushRelabel(edges_sql, sources,  target)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_pushRelabel(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
    , ARRAY[6, 8, 12], 11
);
 seq | edge | start_vid | end_vid | flow | residual_capacity
-----+------+-----------+---------+------+-------------------
   1 |   10 |         5 |      10 |  100 |                30
   2 |    8 |         6 |       5 |  100 |                30
```

```
   3 |   11 |         6 |       11 |  130 |                  0
   4 |   12 |        10 |       11 |  100 |                  0
(4 rows)
```

Many to Many

Calculates the flow on the graph edges that maximizes the flow from all of the *sources* to all of the *targets*.

```
pgr_pushRelabel(edges_sql, sources,  targets)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_pushRelabel(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
    , ARRAY[6, 8, 12], ARRAY[1, 3, 11]
);
 seq | edge | start_vid | end_vid | flow | residual_capacity
-----+------+-----------+---------+------+-------------------
   1 |    1 |         2 |       1 |   50 |                 80
   2 |    3 |         4 |       3 |   80 |                 50
   3 |    4 |         5 |       2 |   50 |                  0
   4 |   10 |         5 |      10 |  100 |                 30
   5 |    8 |         6 |       5 |  130 |                  0
   6 |    9 |         6 |       9 |   30 |                100
   7 |   11 |         6 |      11 |  130 |                  0
   8 |    7 |         8 |       5 |   20 |                 30
   9 |   16 |         9 |       4 |   80 |                  0
  10 |   12 |        10 |      11 |  100 |                  0
  11 |   15 |        12 |       9 |   50 |                  0
(11 rows)
```

Description of the Signatures

Description of the edges_sql query for Max-flow like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
capacity	`ANY-INTEGER`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_capacity	`ANY-INTEGER`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

Description of the Parameters of the Flow Signatures

Column	Type	Default	Description
edges_sql	`TEXT`		The edges SQL query as described above.
source	`BIGINT`		Identifier of the starting vertex of the flow.
sources	`ARRAY[BIGINT]`		Array of identifiers of the starting vertices of the flow.
target	`BIGINT`		Identifier of the ending vertex of the flow.
targets	`ARRAY[BIGINT]`		Array of identifiers of the ending vertices of the flow.

Description of the Return Values

Column	Type	Description
seq	`INT`	Sequential value starting from **1**.
edge_id	`BIGINT`	Identifier of the edge in the original query(edges_sql).
source	`BIGINT`	Identifier of the first end point vertex of the edge.
target	`BIGINT`	Identifier of the second end point vertex of the edge.
flow	`BIGINT`	Flow through the edge in the direction (source, target).
residual_capacity	`BIGINT`	Residual capacity of the edge in the direction (source, target).

See Also

- *Flow - Family of functions*, *pgr_boykovKolmogorov*, *pgr_edmondsKarp*
- http://www.boost.org/libs/graph/doc/push_relabel_max_flow.html

- https://en.wikipedia.org/wiki/Push%E2%80%93relabel_maximum_flow_algorithm

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pgr_edmondsKarp - Proposed

Synopsis

`pgr_edmondsKarp` — Calculates the flow on the graph edges that maximizes the flow from the sources to the targets using Push Relabel Algorithm.

[65]

Fig. 6.16: Boost Graph Inside

Availability:

- Renamed 2.5.0, Previous name pgr_maxFlowEdmondsKarp
- New in 2.3.0

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.
  - pgTap tests might be missing.
  - Might need c/c++ coding.
  - May lack documentation.
  - Documentation if any might need to be rewritten.
  - Documentation examples might need to be automatically generated.
  - Might need a lot of feedback from the comunity.
  - Might depend on a proposed function of pgRouting
  - Might depend on a deprecated function of pgRouting

[65] http://www.boost.org/libs/graph/doc/push_relabel_max_flow.html

Characteristics

- The graph is **directed**.
- Process is done only on edges with positive capacities.
- When the maximum flow is 0 then there is no flow and **EMPTY SET** is returned.
  - There is no flow when a **source** is the same as a **target**.
- Any duplicated value in the source(s) or target(s) are ignored.
- Calculates the flow/residual capacity for each edge. In the output
  - Edges with zero flow are omitted.
- Creates a **super source** and edges to all the source(s), and a **super target** and the edges from all the targets(s).
- The maximum flow through the graph is guaranteed to be the value returned by *pgr_maxFlow* when executed with the same parameters and can be calculated:
  - By aggregation of the outgoing flow from the sources
  - By aggregation of the incoming flow to the targets
- Running time: $O(V * E^2)$

Signature Summary

```
pgr_edmondsKarp(edges_sql, source,  target) - Proposed
pgr_edmondsKarp(edges_sql, sources, target) - Proposed
pgr_edmondsKarp(edges_sql, source,  targets) - Proposed
pgr_edmondsKarp(edges_sql, sources, targets) - Proposed
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

One to One

Calculates the flow on the graph edges that maximizes the flow from the *source* to the *target*.

```
pgr_edmondsKarp(edges_sql, source,  target)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_edmondsKarp(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
    , 6, 11
);
 seq | edge | start_vid | end_vid | flow | residual_capacity
-----+------+-----------+---------+------+-------------------
   1 |   10 |         5 |      10 |  100 |                30
   2 |    8 |         6 |       5 |  100 |                30
   3 |   11 |         6 |      11 |  130 |                 0
   4 |   12 |        10 |      11 |  100 |                 0
```

```
(4 rows)
```

One to Many

Calculates the flow on the graph edges that maximizes the flow from the *source* to all of the *targets*.

```
pgr_edmondsKarp(edges_sql, source,  targets)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_edmondsKarp(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
   , 6, ARRAY[1, 3, 11]
);
 seq | edge | start_vid | end_vid | flow | residual_capacity
-----+------+-----------+---------+------+-------------------
   1 |    1 |         2 |       1 |   50 |                80
   2 |    3 |         4 |       3 |   80 |                50
   3 |    4 |         5 |       2 |   50 |                 0
   4 |   10 |         5 |      10 |   80 |                50
   5 |    8 |         6 |       5 |  130 |                 0
   6 |    9 |         6 |       9 |   80 |                50
   7 |   11 |         6 |      11 |  130 |                 0
   8 |   16 |         9 |       4 |   80 |                 0
   9 |   12 |        10 |      11 |   80 |                20
(9 rows)
```

Many to One

Calculates the flow on the graph edges that maximizes the flow from all of the *sources* to the *target*.

```
pgr_edmondsKarp(edges_sql, sources,  target)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_edmondsKarp(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
   , ARRAY[6, 8, 12], 11
);
 seq | edge | start_vid | end_vid | flow | residual_capacity
-----+------+-----------+---------+------+-------------------
   1 |   10 |         5 |      10 |  100 |                30
   2 |    8 |         6 |       5 |  100 |                30
```

```
   3 |   11 |          6 |       11 |  130 |                 0
   4 |   12 |         10 |       11 |  100 |                 0
(4 rows)
```

Many to Many

Calculates the flow on the graph edges that maximizes the flow from all of the *sources* to all of the *targets*.

```
pgr_edmondsKarp(edges_sql, sources,  targets)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_edmondsKarp(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
   , ARRAY[6, 8, 12], ARRAY[1, 3, 11]
);
 seq | edge | start_vid | end_vid | flow | residual_capacity
-----+------+-----------+---------+------+-------------------
   1 |    1 |         2 |       1 |   50 |                80
   2 |    3 |         4 |       3 |   80 |                50
   3 |    4 |         5 |       2 |   50 |                 0
   4 |   10 |         5 |      10 |  100 |                30
   5 |    8 |         6 |       5 |  130 |                 0
   6 |    9 |         6 |       9 |   80 |                50
   7 |   11 |         6 |      11 |  130 |                 0
   8 |    7 |         8 |       5 |   20 |                30
   9 |   16 |         9 |       4 |   80 |                 0
  10 |   12 |        10 |      11 |  100 |                 0
(10 rows)
```

Description of the Signatures

Description of the edges_sql query for Max-flow like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
capacity	`ANY-INTEGER`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_capacity	`ANY-INTEGER`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

Description of the Parameters of the Flow Signatures

Column	Type	Default	Description
edges_sql	`TEXT`		The edges SQL query as described above.
source	`BIGINT`		Identifier of the starting vertex of the flow.
sources	`ARRAY[BIGINT]`		Array of identifiers of the starting vertices of the flow.
target	`BIGINT`		Identifier of the ending vertex of the flow.
targets	`ARRAY[BIGINT]`		Array of identifiers of the ending vertices of the flow.

Description of the Return Values

Column	Type	Description
seq	`INT`	Sequential value starting from **1**.
edge_id	`BIGINT`	Identifier of the edge in the original query(edges_sql).
source	`BIGINT`	Identifier of the first end point vertex of the edge.
target	`BIGINT`	Identifier of the second end point vertex of the edge.
flow	`BIGINT`	Flow through the edge in the direction (source, target).
residual_capacity	`BIGINT`	Residual capacity of the edge in the direction (source, target).

See Also

- *Flow - Family of functions*, *pgr_boykovKolmogorov*, *pgr_PushRelabel*
- http://www.boost.org/libs/graph/doc/edmonds_karp_max_flow.html

- https://en.wikipedia.org/wiki/Edmonds%E2%80%93Karp_algorithm

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pgr_boykovKolmogorov - Proposed

Synopsis

`pgr_boykovKolmogorov` — Calculates the flow on the graph edges that maximizes the flow from the sources to the targets using Boykov Kolmogorov algorithm.

[66]

Fig. 6.17: Boost Graph Inside

Availability:

- Renamed 2.5.0, Previous name pgr_maxFlowBoykovKolmogorov
- New in 2.3.0

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.
  - pgTap tests might be missing.
  - Might need c/c++ coding.
  - May lack documentation.
  - Documentation if any might need to be rewritten.
  - Documentation examples might need to be automatically generated.
  - Might need a lot of feedback from the comunity.
  - Might depend on a proposed function of pgRouting
  - Might depend on a deprecated function of pgRouting

[66] http://www.boost.org/libs/graph/doc/boykov_kolmogorov_max_flow.html

Characteristics

- The graph is **directed**.
- Process is done only on edges with positive capacities.
- When the maximum flow is 0 then there is no flow and **EMPTY SET** is returned.
  - There is no flow when a **source** is the same as a **target**.
- Any duplicated value in the source(s) or target(s) are ignored.
- Calculates the flow/residual capacity for each edge. In the output
  - Edges with zero flow are omitted.
- Creates a **super source** and edges to all the source(s), and a **super target** and the edges from all the targets(s).
- The maximum flow through the graph is guaranteed to be the value returned by *pgr_maxFlow* when executed with the same parameters and can be calculated:
  - By aggregation of the outgoing flow from the sources
  - By aggregation of the incoming flow to the targets
- Running time: Polynomial

Signature Summary

```
pgr_boykovKolmogorov(edges_sql, source,  target) - Proposed
pgr_boykovKolmogorov(edges_sql, sources, target) - Proposed
pgr_boykovKolmogorov(edges_sql, source,  targets) - Proposed
pgr_boykovKolmogorov(edges_sql, sources, targets) - Proposed
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

One to One

Calculates the flow on the graph edges that maximizes the flow from the *source* to the *target*.

```
pgr_boykovKolmogorov(edges_sql, source,  target)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_boykovKolmogorov(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
    , 6, 11
);
 seq | edge | start_vid | end_vid | flow | residual_capacity
-----+------+-----------+---------+------+-------------------
   1 |   10 |         5 |      10 |  100 |                30
   2 |    8 |         6 |       5 |  100 |                30
   3 |   11 |         6 |      11 |  130 |                 0
   4 |   12 |        10 |      11 |  100 |                 0
```

```
(4 rows)
```

One to Many

Calculates the flow on the graph edges that maximizes the flow from the *source* to all of the *targets*.

```
pgr_boykovKolmogorov(edges_sql, source,  targets)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_boykovKolmogorov(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
    , 6, ARRAY[1, 3, 11]
);
 seq | edge | start_vid | end_vid | flow | residual_capacity
-----+------+-----------+---------+------+-------------------
   1 |    1 |         2 |       1 |   50 |                80
   2 |    3 |         4 |       3 |   80 |                50
   3 |    4 |         5 |       2 |   50 |                 0
   4 |   10 |         5 |      10 |   80 |                50
   5 |    8 |         6 |       5 |  130 |                 0
   6 |    9 |         6 |       9 |   80 |                50
   7 |   11 |         6 |      11 |  130 |                 0
   8 |   16 |         9 |       4 |   80 |                 0
   9 |   12 |        10 |      11 |   80 |                20
(9 rows)
```

Many to One

Calculates the flow on the graph edges that maximizes the flow from all of the *sources* to the *target*.

```
pgr_boykovKolmogorov(edges_sql, sources,  target)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_boykovKolmogorov(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
    , ARRAY[6, 8, 12], 11
);
 seq | edge | start_vid | end_vid | flow | residual_capacity
-----+------+-----------+---------+------+-------------------
   1 |   10 |         5 |      10 |  100 |                30
   2 |    8 |         6 |       5 |  100 |                30
```

```
   3 |   11 |          6 |        11 |  130 |                    0
   4 |   12 |         10 |        11 |  100 |                    0
(4 rows)
```

Many to Many

Calculates the flow on the graph edges that maximizes the flow from all of the *sources* to all of the *targets*.

```
pgr_boykovKolmogorov(edges_sql, sources,  targets)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_boykovKolmogorov(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
    , ARRAY[6, 8, 12], ARRAY[1, 3, 11]
);
 seq | edge | start_vid | end_vid | flow | residual_capacity
-----+------+-----------+---------+------+-------------------
   1 |    1 |         2 |       1 |   50 |                80
   2 |    3 |         4 |       3 |   80 |                50
   3 |    4 |         5 |       2 |   50 |                 0
   4 |   10 |         5 |      10 |  100 |                30
   5 |    8 |         6 |       5 |  130 |                 0
   6 |    9 |         6 |       9 |   80 |                50
   7 |   11 |         6 |      11 |  130 |                 0
   8 |    7 |         8 |       5 |   20 |                30
   9 |   16 |         9 |       4 |   80 |                 0
  10 |   12 |        10 |      11 |  100 |                 0
(10 rows)
```

Description of the Signatures

Description of the edges_sql query for Max-flow like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
capacity	`ANY-INTEGER`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_capacity	`ANY-INTEGER`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

Description of the Parameters of the Flow Signatures

Column	Type	Default	Description
edges_sql	`TEXT`		The edges SQL query as described above.
source	`BIGINT`		Identifier of the starting vertex of the flow.
sources	`ARRAY[BIGINT]`		Array of identifiers of the starting vertices of the flow.
target	`BIGINT`		Identifier of the ending vertex of the flow.
targets	`ARRAY[BIGINT]`		Array of identifiers of the ending vertices of the flow.

Description of the Return Values

Column	Type	Description
seq	`INT`	Sequential value starting from **1**.
edge_id	`BIGINT`	Identifier of the edge in the original query(edges_sql).
source	`BIGINT`	Identifier of the first end point vertex of the edge.
target	`BIGINT`	Identifier of the second end point vertex of the edge.
flow	`BIGINT`	Flow through the edge in the direction (source, target).
residual_capacity	`BIGINT`	Residual capacity of the edge in the direction (source, target).

See Also

- *Flow - Family of functions*, *pgr_pushRelabel*, *pgr_EdmondsKarp*
- http://www.boost.org/libs/graph/doc/boykov_kolmogorov_max_flow.html

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pgr_maxCardinalityMatch - Proposed

Synopsis

`pgr_maxCardinalityMatch` — Calculates a maximum cardinality matching in a graph.

> **Warning:** Experimental functions
>
> - They are not officially of the current release.
> - They likely will not be officially be part of the next release:
>   - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
>   - Name might change.
>   - Signature might change.
>   - Functionality might change.
>   - pgTap tests might be missing.
>   - Might need c/c++ coding.
>   - May lack documentation.
>   - Documentation if any might need to be rewritten.
>   - Documentation examples might need to be automatically generated.
>   - Might need a lot of feedback from the comunity.
>   - Might depend on a proposed function of pgRouting
>   - Might depend on a deprecated function of pgRouting

[67]

Fig. 6.18: Boost Graph Inside

Availability:

- Renamed 2.5.0, Previous name pgr_maximumCardinalityMatching
- New in 2.3.0

Characteristics

- A matching or independent edge set in a graph is a set of edges without common vertices.

[67] http://www.boost.org/libs/graph/doc/maximum_matching.html

- A maximum matching is a matching that contains the largest possible number of edges.
  - There may be many maximum matchings.
  - Calculates **one** possible maximum cardinality matching in a graph.
- The graph can be **directed** or **undirected**.
- Running time: $O(E * V * \alpha(E, V))$
  - $\alpha(E, V)$ is the inverse of the Ackermann function[68].

Signature Summary

```
pgr_MaximumCardinalityMatching(edges_sql) - Proposed
pgr_MaximumCardinalityMatching(edges_sql, directed) - Proposed

RETURNS SET OF (seq, edge_id, source, target)
    OR EMPTY SET
```

Minimal Use

```
pgr_MaximumCardinalityMatching(edges_sql)
RETURNS SET OF (seq, edge_id, source, target) OR EMPTY SET
```

The minimal use calculates one possible maximum cardinality matching on a **directed** graph.

Example

```
SELECT * FROM pgr_maxCardinalityMatch(
    'SELECT id, source, target, cost AS going, reverse_cost AS coming FROM edge_
↪table'
);
 seq | edge | source | target
-----+------+--------+--------
   1 |    1 |      1 |      2
   2 |    3 |      4 |      3
   3 |    9 |      6 |      9
   4 |    6 |      7 |      8
   5 |   14 |     10 |     13
   6 |   13 |     11 |     12
   7 |   17 |     14 |     15
   8 |   18 |     16 |     17
(8 rows)
```

Complete signature

```
pgr_MaximumCardinalityMatching(edges_sql, directed)
RETURNS SET OF (seq, edge_id, source, target) OR EMPTY SET
```

The complete signature calculates one possible maximum cardinality matching.

Example

68 https://en.wikipedia.org/wiki/Ackermann_function

```
SELECT * FROM pgr_maxCardinalityMatch(
    'SELECT id, source, target, cost AS going, reverse_cost AS coming FROM edge_
↪table',
    directed := false
);
 seq | edge | source | target
-----+------+--------+--------
   1 |    1 |      1 |      2
   2 |    3 |      3 |      4
   3 |    9 |      6 |      9
   4 |    6 |      7 |      8
   5 |   14 |     10 |     13
   6 |   13 |     11 |     12
   7 |   17 |     14 |     15
   8 |   18 |     16 |     17
(8 rows)
```

Description of the Signatures

Description of the SQL query

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Description
id	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex of the edge.
target	ANY-INTEGER	Identifier of the second end point vertex of the edge.
going	ANY-NUMERIC	A positive value represents the existence of the edge (source, target).
coming	ANY-NUMERIC	A positive value represents the existence of the edge (target, source).

Where:

- **ANY-INTEGER** SMALLINT, INTEGER, BIGINT
- **ANY-NUMERIC** SMALLINT, INTEGER, BIGINT, REAL, DOUBLE PRECISION

Description of the parameters of the signatures

Column	Type	Description
edges_sql	TEXT	SQL query as described above.
directed	BOOLEAN	(optional) Determines the type of the graph. Default TRUE.

Description of the Result

Column	Type	Description
seq	INT	Sequential value starting from **1**.
edge	BIGINT	Identifier of the edge in the original query(edges_sql).
source	BIGINT	Identifier of the first end point of the edge.
target	BIGINT	Identifier of the second end point of the edge.

See Also

- *Flow - Family of functions*
- http://www.boost.org/libs/graph/doc/maximum_matching.html
- https://en.wikipedia.org/wiki/Matching_%28graph_theory%29
- https://en.wikipedia.org/wiki/Ackermann_function

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pgr_edgeDisjointPaths - Proposed

Name

`pgr_edgeDisjointPaths` — Calculates edge disjoint paths between two groups of vertices.

[69]

Fig. 6.19: Boost Graph Inside

Availability: 2.3.0

> **Warning:** Experimental functions
>
> - They are not officially of the current release.
> - They likely will not be officially be part of the next release:
>   - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
>   - Name might change.
>   - Signature might change.
>   - Functionality might change.
>   - pgTap tests might be missing.
>   - Might need c/c++ coding.
>   - May lack documentation.
>   - Documentation if any might need to be rewritten.
>   - Documentation examples might need to be automatically generated.
>   - Might need a lot of feedback from the comunity.
>   - Might depend on a proposed function of pgRouting
>   - Might depend on a deprecated function of pgRouting

[69] http://www.boost.org/libs/graph/doc/boykov_kolmogorov_max_flow.html

Synopsis

Calculates the edge disjoint paths between two groups of vertices. Utilizes underlying maximum flow algorithms to calculate the paths.

Characteristics:

The main characterics are:

- Calculates the edge disjoint paths between any two groups of vertices.
- Returns EMPTY SET when source and destination are the same, or cannot be reached.
- The graph can be directed or undirected.
- One to many, many to one, many to many versions are also supported.
- Uses *pgr_boykovKolmogorov - Proposed* to calculate the paths.

Signature Summary

```
pgr_edgeDisjointPaths(edges_sql, start_vid, end_vid)
pgr_edgeDisjointPaths(edges_sql, start_vid, end_vid, directed)
pgr_edgeDisjointPaths(edges_sql, start_vid, end_vids, directed)
pgr_edgeDisjointPaths(edges_sql, start_vids, end_vid, directed)
pgr_edgeDisjointPaths(edges_sql, start_vids, end_vids, directed)

RETURNS SET OF (seq, path_id, path_seq, [start_vid,] [end_vid,] node, edge, cost,
↪agg_cost)
OR EMPTY SET
```

Signatures

Minimal use

```
pgr_edgeDisjointPaths(edges_sql, start_vid, end_vid)
RETURNS SET OF (seq, path_id, path_seq, node, edge, cost, agg_cost)
OR EMPTY SET
```

The minimal use is for a **directed** graph from one `start_vid` to one `end_vid`.

Example

```
SELECT * FROM pgr_edgeDisjointPaths(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    3, 5
);
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |    3 |    2 |    1 |        0
   2 |       1 |        2 |    2 |    4 |    1 |        1
   3 |       1 |        3 |    5 |   -1 |    0 |        2
   4 |       2 |        1 |    3 |    5 |    1 |        0
   5 |       2 |        2 |    6 |    8 |    1 |        1
   6 |       2 |        3 |    5 |   -1 |    0 |        2
(6 rows)
```

One to One

This signature finds the set of dijoint paths from one `start_vid` to one `end_vid`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

```
pgr_edgeDisjointPaths(edges_sql, start_vid, end_vid, directed)
RETURNS SET OF (seq, path_id, path_seq, node, edge, cost, agg_cost)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_edgeDisjointPaths(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    3, 5,
    directed := false
);
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |    3 |    2 |    1 |        0
   2 |       1 |        2 |    2 |    4 |    1 |        1
   3 |       1 |        3 |    5 |   -1 |    0 |        2
   4 |       2 |        1 |    3 |    3 |   -1 |        0
   5 |       2 |        2 |    4 |   16 |    1 |       -1
   6 |       2 |        3 |    9 |    9 |    1 |        0
   7 |       2 |        4 |    6 |    8 |    1 |        1
   8 |       2 |        5 |    5 |   -1 |    0 |        2
   9 |       3 |        1 |    3 |    5 |    1 |        0
  10 |       3 |        2 |    6 |   11 |    1 |        1
  11 |       3 |        3 |   11 |   12 |   -1 |        2
  12 |       3 |        4 |   10 |   10 |    1 |        1
  13 |       3 |        5 |    5 |   -1 |    0 |        2
(13 rows)
```

One to Many

This signature finds the sset of disjoint paths from the `start_vid` to each one of the `end_vid` in `end_vids`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.
- The result is equivalent to the union of the results of the one to one *pgr_edgeDisjointPaths*.
- The extra `end_vid` in the result is used to distinguish to which path it belongs.

```
pgr_edgeDisjointPaths(edges_sql, start_vid, end_vids, directed)
RETURNS SET OF (seq, path_id, path_seq, end_vid, node, edge, cost, agg_cost)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_edgeDisjointPaths(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    3, ARRAY[4, 5, 10]
);
 seq | path_id | path_seq | end_vid | node | edge | cost | agg_cost
-----+---------+----------+---------+------+------+------+----------
   1 |       1 |        1 |       4 |    3 |    5 |    1 |        0
```

```
   2 |        1 |         2 |        4 |      6 |     9 |     1 |          1
   3 |        1 |         3 |        4 |      9 |    16 |     1 |          2
   4 |        1 |         4 |        4 |      4 |    -1 |     0 |          3
   5 |        2 |         1 |        5 |      3 |     2 |     1 |          0
   6 |        2 |         2 |        5 |      2 |     4 |     1 |          1
   7 |        2 |         3 |        5 |      5 |    -1 |     0 |          2
   8 |        3 |         1 |        5 |      3 |     5 |     1 |          0
   9 |        3 |         2 |        5 |      6 |     8 |     1 |          1
  10 |        3 |         3 |        5 |      5 |    -1 |     0 |          2
  11 |        4 |         1 |       10 |      3 |     2 |     1 |          0
  12 |        4 |         2 |       10 |      2 |     4 |     1 |          1
  13 |        4 |         3 |       10 |      5 |    10 |     1 |          2
  14 |        4 |         4 |       10 |     10 |    -1 |     0 |          3
(14 rows)
```

Many to One

This signature finds the set of disjoint paths from each one of the `start_vid` in `start_vids` to the `end_vid`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.
- The result is equivalent to the union of the results of the one to one *pgr_edgeDisjointPaths*.
- The extra `start_vid` in the result is used to distinguish to which path it belongs.

```
pgr_edgeDisjointPaths(edges_sql, start_vids, end_vid, directed)
RETURNS SET OF (seq, path_id, path_seq, start_vid, node, edge, cost, agg_cost)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_edgeDisjointPaths(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[3, 6], 5
);
 seq | path_id | path_seq | start_vid | node | edge | cost | agg_cost
-----+---------+----------+-----------+------+------+------+----------
   1 |       1 |        1 |         0 |    3 |    2 |    1 |        0
   2 |       1 |        2 |         0 |    2 |    4 |    1 |        1
   3 |       1 |        3 |         0 |    5 |   -1 |    0 |        2
   4 |       2 |        1 |         1 |    3 |    5 |    1 |        0
   5 |       2 |        2 |         1 |    6 |    8 |    1 |        1
   6 |       2 |        3 |         1 |    5 |   -1 |    0 |        2
   7 |       3 |        1 |         2 |    6 |    8 |    1 |        0
   8 |       3 |        2 |         2 |    5 |   -1 |    0 |        1
   9 |       4 |        1 |         3 |    6 |    9 |    1 |        0
  10 |       4 |        2 |         3 |    9 |   16 |    1 |        1
  11 |       4 |        3 |         3 |    4 |    3 |    1 |        2
  12 |       4 |        4 |         3 |    3 |    2 |    1 |        3
  13 |       4 |        5 |         3 |    2 |    4 |    1 |        4
  14 |       4 |        6 |         3 |    5 |   -1 |    0 |        5
(14 rows)
```

Many to Many

This signature finds the set of disjoint paths from each one of the `start_vid` in `start_vids` to each one of the `end_vid`

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.
- The result is equivalent to the union of the results of the one to one *pgr_edgeDisjointPaths*.
- The extra `start_vid` and `end_vid` in the result is used to distinguish to which path it belongs.

```
pgr_edgeDisjointPaths(edges_sql, start_vids, end_vids, directed)
RETURNS SET OF (seq, path_seq, start_vid, end_vid, node, edge, cost, agg_cost)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_edgeDisjointPaths(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[3, 6], ARRAY[4, 5, 10]
);
 seq | path_id | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
-----+---------+----------+-----------+---------+------+------+------+----------
   1 |       1 |        1 |         0 |       4 |    3 |    5 |    1 |        0
   2 |       1 |        2 |         0 |       4 |    6 |    9 |    1 |        1
   3 |       1 |        3 |         0 |       4 |    9 |   16 |    1 |        2
   4 |       1 |        4 |         0 |       4 |    4 |   -1 |    0 |        3
   5 |       2 |        1 |         1 |       5 |    3 |    2 |    1 |        0
   6 |       2 |        2 |         1 |       5 |    2 |    4 |    1 |        1
   7 |       2 |        3 |         1 |       5 |    5 |   -1 |    0 |        2
   8 |       3 |        1 |         2 |       5 |    3 |    5 |    1 |        0
   9 |       3 |        2 |         2 |       5 |    6 |    8 |    1 |        1
  10 |       3 |        3 |         2 |       5 |    5 |   -1 |    0 |        2
  11 |       4 |        1 |         3 |      10 |    3 |    2 |    1 |        0
  12 |       4 |        2 |         3 |      10 |    2 |    4 |    1 |        1
  13 |       4 |        3 |         3 |      10 |    5 |   10 |    1 |        2
  14 |       4 |        4 |         3 |      10 |   10 |   -1 |    0 |        3
  15 |       5 |        1 |         4 |       4 |    6 |    9 |    1 |        0
  16 |       5 |        2 |         4 |       4 |    9 |   16 |    1 |        1
  17 |       5 |        3 |         4 |       4 |    4 |   -1 |    0 |        2
  18 |       6 |        1 |         5 |       5 |    6 |    8 |    1 |        0
  19 |       6 |        2 |         5 |       5 |    5 |   -1 |    0 |        1
  20 |       7 |        1 |         6 |       5 |    6 |    9 |    1 |        0
  21 |       7 |        2 |         6 |       5 |    9 |   16 |    1 |        1
  22 |       7 |        3 |         6 |       5 |    4 |    3 |    1 |        2
  23 |       7 |        4 |         6 |       5 |    3 |    2 |    1 |        3
  24 |       7 |        5 |         6 |       5 |    2 |    4 |    1 |        4
  25 |       7 |        6 |         6 |       5 |    5 |   -1 |    0 |        5
  26 |       8 |        1 |         7 |      10 |    6 |    8 |    1 |        0
  27 |       8 |        2 |         7 |      10 |    5 |   10 |    1 |        1
  28 |       8 |        3 |         7 |      10 |   10 |   -1 |    0 |        2
(28 rows)
```

Description of the Signatures

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Column	Type	Default	Description
sql	`TEXT`		SQL query as described above.
start_vid	`BIGINT`		Identifier of the starting vertex of the path.
start_vids	`ARRAY[BIGINT]`		Array of identifiers of starting vertices.
end_vid	`BIGINT`		Identifier of the ending vertex of the path.
end_vids	`ARRAY[BIGINT]`		Array of identifiers of ending vertices.
directed	`BOOLEAN`	`true`	• When `true` Graph is considered *Directed* • When `false` the graph is considered as *Undirected*.

Description of the return values for a path

Returns set of `(seq, path_seq [, start_vid] [, end_vid], node, edge, cost, agg_cost)`

Column	Type	Description
seq	`INT`	Sequential value starting from **1**.
path_id	`INT`	Path identifier. Has value **1** for the first of a path. Used when there are multiple paths for the same `start_vid` to `end_vid` combination.
path_seq	`INT`	Relative position in the path. Has value **1** for the beginning of a path.
start_vid	`BIGINT`	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	`BIGINT`	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
node	`BIGINT`	Identifier of the node in the path from `start_vid` to `end_vid`.
edge	`BIGINT`	Identifier of the edge used to go from `node` to the next node in the path sequence. `-1` for the last node of the path.
cost	`FLOAT`	Cost to traverse from `node` using `edge` to the next node in the path sequence.
agg_cost	`FLOAT`	Aggregate cost from `start_v` to `node`.

See Also

- *Flow - Family of functions*

Indices and tables

- genindex
- search

Flow Functions General Information

Characteristics

- The graph is **directed**.
- Process is done only on edges with positive capacities.
- When the maximum flow is 0 then there is no flow and **EMPTY SET** is returned.
  - There is no flow when a **source** is the same as a **target**.
- Any duplicated value in the source(s) or target(s) are ignored.
- Calculates the flow/residual capacity for each edge. In the output
  - Edges with zero flow are omitted.
- Creates a **super source** and edges to all the source(s), and a **super target** and the edges from all the targets(s).
- The maximum flow through the graph is guaranteed to be the value returned by *pgr_maxFlow* when executed with the same parameters and can be calculated:
  - By aggregation of the outgoing flow from the sources
  - By aggregation of the incoming flow to the targets

pgr_maxFlow is the maximum Flow and that maximum is guaranteed to be the same on the functions *pgr_pushRelabel*, *pgr_edmondsKarp*, *pgr_boykovKolmogorov*, but the actual flow through each edge may vary.

Problem definition

A flow network is a directed graph where each edge has a capacity and a flow. The flow through an edge must not exceed the capacity of the edge. Additionally, the incoming and outgoing flow of a node must be equal except the for source which only has outgoing flow, and the destination(sink) which only has incoming flow.

Maximum flow algorithms calculate the maximum flow through the graph and the flow of each edge.

The maximum flow through the graph is guaranteed to be the same with all implementations, but the actual flow through each edge may vary. Given the following query:

pgr_maxFlow $(edges\_sql, source\_vertex, sink\_vertex)$

where $edges\_sql = \{(id_i, source_i, target_i, capacity_i, reverse\_capacity_i)\}$

Graph definition

The weighted directed graph, $G(V, E)$, is defined as:

- the set of vertices V
  - $source\_vertex \cup sink\_vertex \bigcup source_i \bigcup target_i$
- the set of edges E
  - $E = \begin{cases} \{(source_i, target_i, capacity_i) \text{ when } capacity > 0\} & \text{if } reverse\_capacity = \varnothing \\ \\ \{(source_i, target_i, capacity_i) \text{ when } capacity > 0\} & \\ \cup\{(target_i, source_i, reverse\_capacity_i) \text{ when } reverse\_capacity_i > 0)\} & \text{if } reverse\_capacity \neq \varnothing \end{cases}$

Maximum flow problem

Given:

- $G(V, E)$
- $source\_vertex \in V$ the source vertex
- $sink\_vertex \in V$ the sink vertex

Then:

$$pgr\_maxFlow(edges\_sql, source, sink) = \boldsymbol{\Phi}$$

$$\boldsymbol{\Phi} = (id_i, edge\_id_i, source_i, target_i, flow_i, residual\_capacity_i)$$

Where:

$\boldsymbol{\Phi}$ is a subset of the original edges with their residual capacity and flow. The maximum flow through the graph can be obtained by aggregating on the source or sink and summing the flow from/to it. In particular:

- $id_i = i$
- $edge\_id = id_i$ in edges_sql
- $residual\_capacity_i = capacity_i - flow_i$

See Also

- https://en.wikipedia.org/wiki/Maximum_flow_problem

Indices and tables

- genindex
- search

6.2.3 pgr_labelGraph - Experimental

Name

`pgr_labelGraph` — Locates and labels sub-networks within a network which are not topologically connected.

> **Warning:** Experimental functions
>
> - They are not officially of the current release.
> - They likely will not be officially be part of the next release:
>   - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
>   - Name might change.
>   - Signature might change.
>   - Functionality might change.
>   - pgTap tests might be missing.
>   - Might need c/c++ coding.
>   - May lack documentation.
>   - Documentation if any might need to be rewritten.
>   - Documentation examples might need to be automatically generated.
>   - Might need a lot of feedback from the comunity.
>   - Might depend on a proposed function of pgRouting
>   - Might depend on a deprecated function of pgRouting

Synopsis

Must be run after `pgr_createTopology()`. No use of `geometry` column. Only `id`, `source` and `target` columns are required.

The function returns:

- `OK` when a column with provided name has been generated and populated successfully. All connected edges will have unique similar integer values. In case of `rows_where` condition, non participating rows will have -1 integer values.
- `FAIL` when the processing cannot be finished due to some error. Notice will be thrown accordingly.
- `rows_where condition generated 0 rows` when passed SQL condition has not been fulfilled by any row.

```
varchar pgr_labelGraph(text, text, text, text, text, text)
```

Description

A network behind any routing query may consist of sub-networks completely isolated from each other. Possible reasons could be:

- An island with no bridge connecting to the mainland.
- An edge or mesh of edges failed to connect to other networks because of human negligence during data generation.
- The data is not properly noded.
- Topology creation failed to succeed.

pgr_labelGraph() will create an integer column (with the name provided by the user) and will assign same integer values to all those edges in the network which are connected topologically. Thus better analysis regarding network structure is possible. In case of `rows_where` condition, non participating rows will have -1 integer values.

Prerequisites: Must run `pgr_createTopology()` in order to generate `source` and `target` columns. Primary key column `id` should also be there in the network table.

Function accepts the following parameters:

edge_table `text` Network table name, with optional schema name.

id `text` Primary key column name of the network table. Default is `id`.

source `text` Source column name generated after `pgr_createTopology()`. Default is `source`.

target `text` Target column name generated after `pgr_createTopology()`. Default is `target`.

subgraph `text` Column name which will hold the integer labels for each sub-graph. Default is `subgraph`.

rows_where `text` The SQL where condition. Default is `true`, means the processing will be done on the whole table.

Example Usage

The sample data, has 3 subgraphs.

```
SET client_min_messages TO WARNING;
SET
SELECT pgr_labelGraph('edge_table', 'id', 'source', 'target', 'subgraph');
 pgr_labelgraph
----------------
 OK
(1 row)

SELECT DISTINCT subgraph FROM edge_table ORDER BY subgraph;
 subgraph
----------
        1
        2
        3
(3 rows)
```

See Also

- pgr_createTopology[70] to create the topology of a table based on its geometry and tolerance value.

[70] https://github.com/Zia-/pgrouting/blob/develop/src/common/sql/pgrouting_topology.sql

Indices and tables

- genindex
- search

6.2.4 Components - Family of functions

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.
  - pgTap tests might be missing.
  - Might need c/c++ coding.
  - May lack documentation.
  - Documentation if any might need to be rewritten.
  - Documentation examples might need to be automatically generated.
  - Might need a lot of feedback from the comunity.
  - Might depend on a proposed function of pgRouting
  - Might depend on a deprecated function of pgRouting

- *pgr_connectedComponents - Experimental* - Return the connected components of an undirected graph.
- *pgr_strongComponents - Experimental* - Return the strongly connected components of a directed graph.
- *pgr_biconnectedComponents - Experimental* - Return the biconnected components of an undirected graph.
- *pgr_articulationPoints - Experimental* - Return the articulation points of an undirected graph.
- *pgr_bridges - Experimental* - Return the bridges of an undirected graph.

pgr_connectedComponents - Experimental

`pgr_connectedComponents` — Return the connected components of an undirected graph using a DFS-based approach. In particular, the algorithm implemented by Boost.Graph.

[71]

Fig. 6.20: Boost Graph Inside

[71] http://www.boost.org/libs/graph/doc/connected_components.html

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.
  - pgTap tests might be missing.
  - Might need c/c++ coding.
  - May lack documentation.
  - Documentation if any might need to be rewritten.
  - Documentation examples might need to be automatically generated.
  - Might need a lot of feedback from the comunity.
  - Might depend on a proposed function of pgRouting
  - Might depend on a deprecated function of pgRouting

Synopsis

A connected component of an undirected graph is a set of vertices that are all reachable from each other. This implementation can only be used with an undirected graph.

Characteristics

The main Characteristics are:

- Components are described by vertices
- The returned values are ordered:
  - *component* ascending
  - *node* ascending
- Running time: $O(V + E)$

Signatures

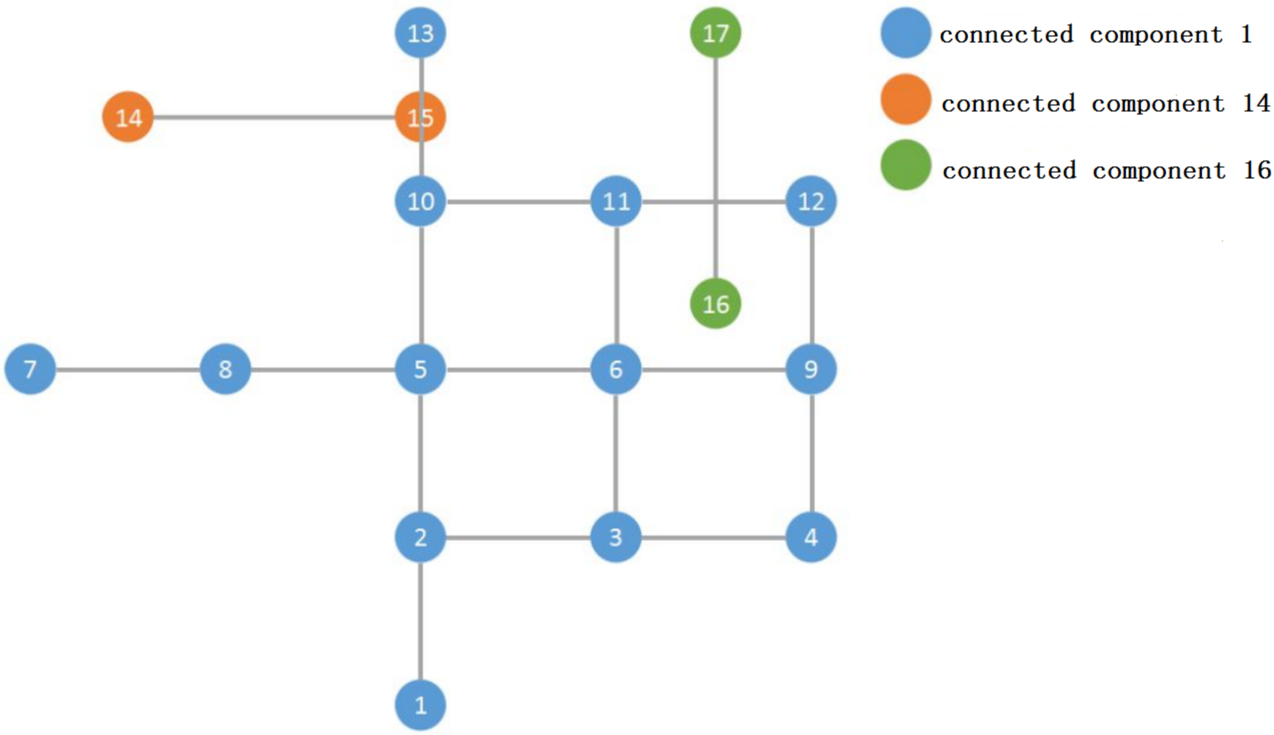
```
pgr_connectedComponents(edges_sql)

RETURNS SET OF (seq, component, n_seq, node)
    OR EMPTY SET
```

The signature is for a **undirected** graph.

Example

```
SELECT * FROM pgr_connectedComponents(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table'
);
 seq | component | n_seq | node
```

```
-----+-----------+-------+------
   1 |         1 |     1 |     1
   2 |         1 |     2 |     2
   3 |         1 |     3 |     3
   4 |         1 |     4 |     4
   5 |         1 |     5 |     5
   6 |         1 |     6 |     6
   7 |         1 |     7 |     7
   8 |         1 |     8 |     8
   9 |         1 |     9 |     9
  10 |         1 |    10 |    10
  11 |         1 |    11 |    11
  12 |         1 |    12 |    12
  13 |         1 |    13 |    13
  14 |        14 |     1 |    14
  15 |        14 |     2 |    15
  16 |        16 |     1 |    16
  17 |        16 |     2 |    17
(17 rows)
```



Description of the Signatures

Description of the edges_sql query for components functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Parameter	Type	Default	Description
edges_sql	`TEXT`		SQL query as described above.

Description of the return values for connected components and strongly connected components

Returns set of `(seq, component, n_seq, node)`

Column	Type	Description
seq	`INT`	Sequential value starting from **1**.
component	`BIGINT`	Component identifier. It is equal to the minimum node identifier in the component.
n_seq	`INT`	It is a sequential value starting from **1** in a component.
node	`BIGINT`	Identifier of the vertex.

See Also

- http://en.wikipedia.org/wiki/Connected_component_%28graph_theory%29
- The queries use the *Sample Data* network.

Indices and tables

- genindex
- search

pgr_strongComponents - Experimental

`pgr_strongComponents` — Return the strongly connected components of a directed graph using Tarjan's algorithm based on DFS. In particular, the algorithm implemented by Boost.Graph.

[72]

Fig. 6.21: Boost Graph Inside

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.
  - pgTap tests might be missing.
  - Might need c/c++ coding.
  - May lack documentation.
  - Documentation if any might need to be rewritten.
  - Documentation examples might need to be automatically generated.
  - Might need a lot of feedback from the comunity.
  - Might depend on a proposed function of pgRouting
  - Might depend on a deprecated function of pgRouting

Synopsis

A strongly connected component of a directed graph is a set of vertices that are all reachable from each other. This implementation can only be used with a directed graph.

Characteristics

The main Characteristics are:

- Components are described by vertices
- The returned values are ordered:

[72] http://www.boost.org/libs/graph/doc/strong_components.html

  - *component* ascending
  - *node* ascending
- Running time: $O(V + E)$

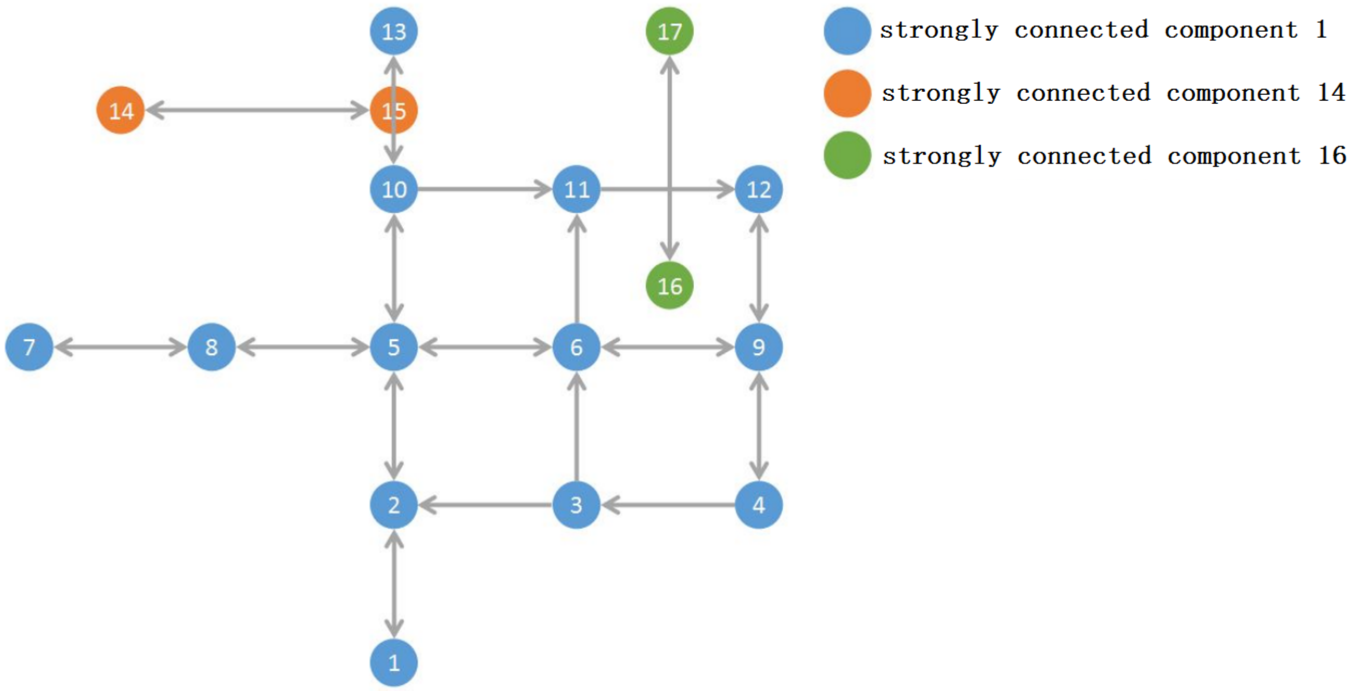
Signatures

```
pgr_strongComponents(edges_sql)

RETURNS SET OF (seq, component, n_seq, node)
    OR EMPTY SET
```

The signature is for a **directed** graph.

Example

```
SELECT * FROM pgr_strongComponents(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table'
);
 seq | component | n_seq | node
-----+-----------+-------+------
   1 |         1 |     1 |    1
   2 |         1 |     2 |    2
   3 |         1 |     3 |    3
   4 |         1 |     4 |    4
   5 |         1 |     5 |    5
   6 |         1 |     6 |    6
   7 |         1 |     7 |    7
   8 |         1 |     8 |    8
   9 |         1 |     9 |    9
  10 |         1 |    10 |   10
  11 |         1 |    11 |   11
  12 |         1 |    12 |   12
  13 |         1 |    13 |   13
  14 |        14 |     1 |   14
  15 |        14 |     2 |   15
  16 |        16 |     1 |   16
  17 |        16 |     2 |   17
(17 rows)
```



Description of the Signatures

Description of the edges_sql query for components functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Parameter	Type	Default	Description
edges_sql	`TEXT`		SQL query as described above.

Description of the return values for connected components and strongly connected components

Returns set of `(seq, component, n_seq, node)`

Column	Type	Description
seq	`INT`	Sequential value starting from **1**.
component	`BIGINT`	Component identifier. It is equal to the minimum node identifier in the component.
n_seq	`INT`	It is a sequential value starting from **1** in a component.
node	`BIGINT`	Identifier of the vertex.

See Also

- http://en.wikipedia.org/wiki/Strongly_connected_component
- The queries use the *Sample Data* network.

Indices and tables

- genindex
- search

pgr_biconnectedComponents - Experimental

`pgr_biconnectedComponents` — Return the biconnected components of an undirected graph. In particular, the algorithm implemented by Boost.Graph.

[73]

Fig. 6.22: Boost Graph Inside

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.

[73] http://www.boost.org/libs/graph/doc/biconnected_components.html

- Signature might change.
- Functionality might change.
- pgTap tests might be missing.
- Might need c/c++ coding.
- May lack documentation.
- Documentation if any might need to be rewritten.
- Documentation examples might need to be automatically generated.
- Might need a lot of feedback from the comunity.
- Might depend on a proposed function of pgRouting
- Might depend on a deprecated function of pgRouting

Synopsis

The biconnected components of an undirected graph are the maximal subsets of vertices such that the removal of a vertex from particular component will not disconnect the component. Unlike connected components, vertices may belong to multiple biconnected components. Vertices can be present in multiple biconnected components, but each edge can only be contained in a single biconnected component. So, the output only has edge version.

This implementation can only be used with an undirected graph.

Characteristics

The main Characteristics are:

- Components are described by edges
- The returned values are ordered:
  - *component* ascending
  - *edge* ascending
- Running time: $O(V + E)$

Signatures

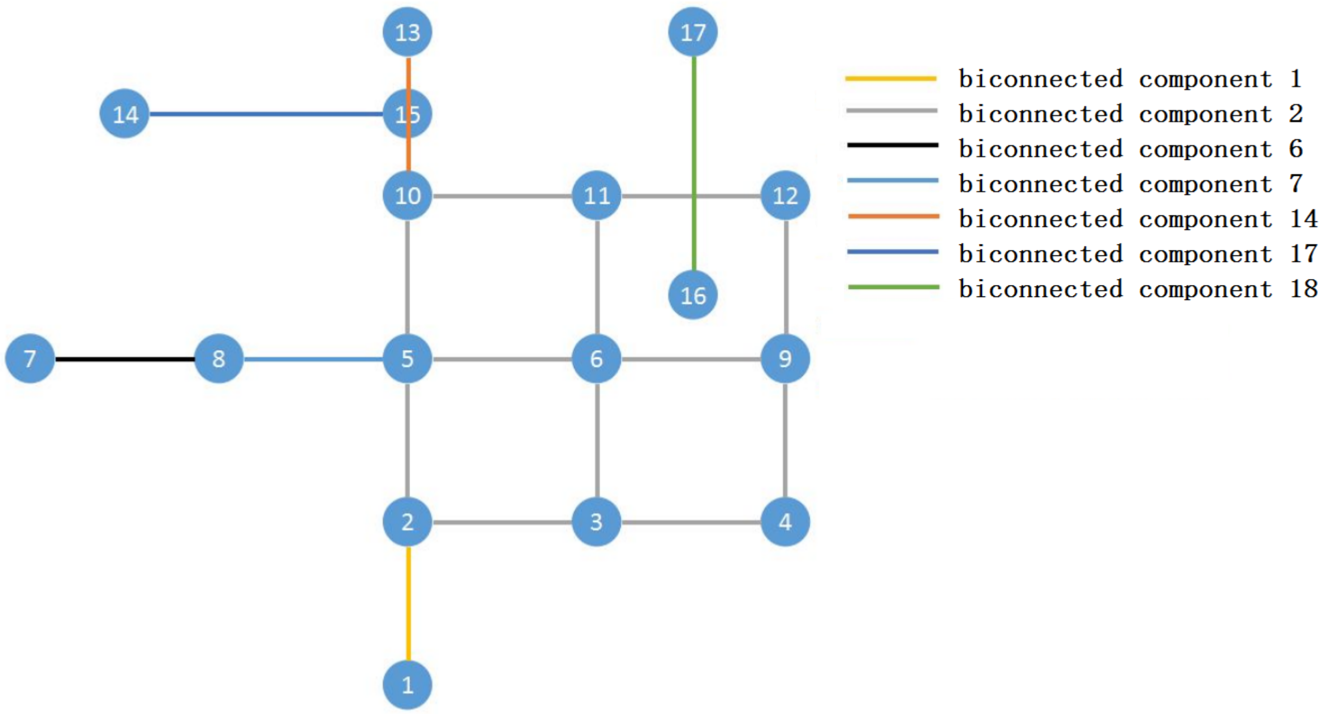
```
pgr_biconnectedComponents(edges_sql)

RETURNS SET OF (seq, component, n_seq, edge)
    OR EMPTY SET
```

The signature is for a **undirected** graph.

Example

```
SELECT * FROM pgr_biconnectedComponents(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table'
);
 seq | component | n_seq | edge
-----+-----------+-------+------
   1 |         1 |     1 |    1
   2 |         2 |     1 |    2
   3 |         2 |     2 |    3
```

```
  4 |          2 |      3 |     4
  5 |          2 |      4 |     5
  6 |          2 |      5 |     8
  7 |          2 |      6 |     9
  8 |          2 |      7 |    10
  9 |          2 |      8 |    11
 10 |          2 |      9 |    12
 11 |          2 |     10 |    13
 12 |          2 |     11 |    15
 13 |          2 |     12 |    16
 14 |          6 |      1 |     6
 15 |          7 |      1 |     7
 16 |         14 |      1 |    14
 17 |         17 |      1 |    17
 18 |         18 |      1 |    18
(18 rows)
```



Description of the Signatures

Description of the edges_sql query for components functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	ANY-INTEGER		Identifier of the edge.
source	ANY-INTEGER		Identifier of the first end point vertex of the edge.
target	ANY-INTEGER		Identifier of the second end point vertex of the edge.
cost	ANY-NUMERICAL		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	ANY-NUMERICAL	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Parameter	Type	Default	Description
edges_sql	TEXT		SQL query as described above.

Description of the return values for biconnected components, connected components (edge version) and strongly connected components

Returns set of `(seq, component, n_seq, edge)`

Column	Type	Description
seq	INT	Sequential value starting from **1**.
component	BIGINT	Component identifier. It is equal to the minimum edge identifier in the component.
n_seq	INT	It is a sequential value starting from **1** in a component.
edge	BIGINT	Identifier of the edge.

See Also

- http://en.wikipedia.org/wiki/Biconnected_component
- The queries use the *Sample Data* network.

Indices and tables

- genindex
- search

pgr_articulationPoints - Experimental

`pgr_articulationPoints` - Return the articulation points of an undirected graph. In particular, the algorithm implemented by Boost.Graph.

[74]

Fig. 6.23: Boost Graph Inside

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.
  - pgTap tests might be missing.
  - Might need c/c++ coding.
  - May lack documentation.
  - Documentation if any might need to be rewritten.
  - Documentation examples might need to be automatically generated.
  - Might need a lot of feedback from the comunity.
  - Might depend on a proposed function of pgRouting
  - Might depend on a deprecated function of pgRouting

Synopsis

Those vertices that belong to more than one biconnected component are called articulation points or, equivalently, cut vertices. Articulation points are vertices whose removal would increase the number of connected components in the graph. This implementation can only be used with an undirected graph.

Characteristics

The main Characteristics are:

- The returned values are ordered:

[74] http://www.boost.org/libs/graph/doc/connected_components.html

  - *node* ascending
- Running time: $O(V + E)$

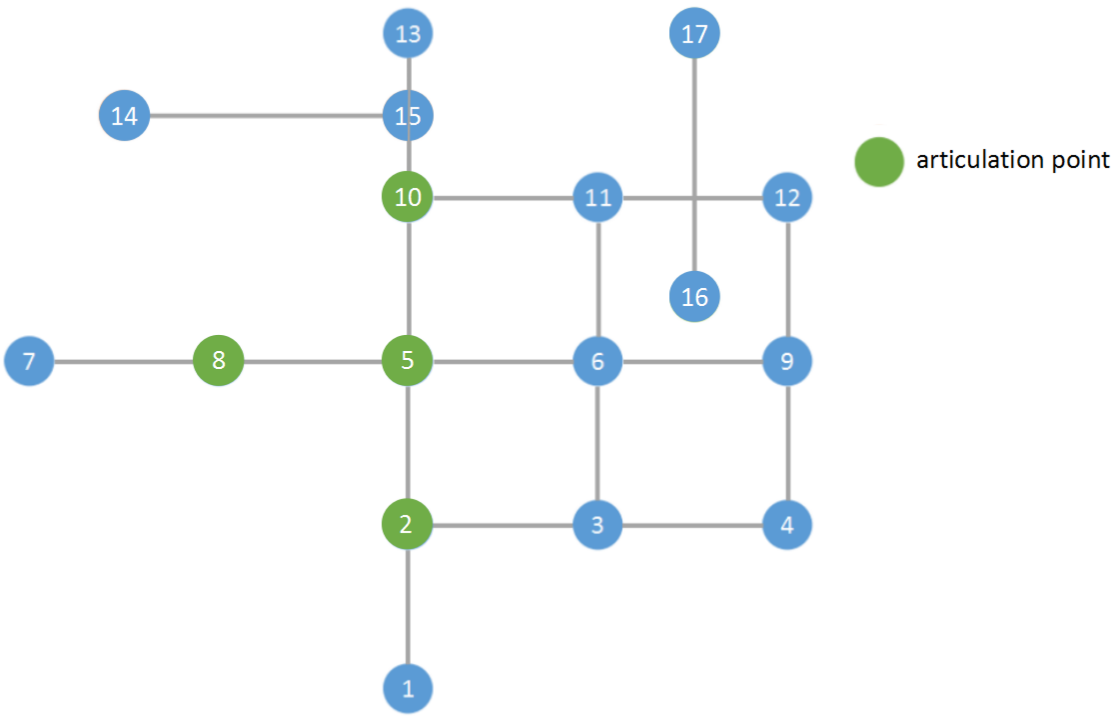
Signatures

```
pgr_articulationPoints(edges_sql)

RETURNS SET OF (seq, node)
    OR EMPTY SET
```

The signature is for a **undirected** graph.

Example

```
SELECT * FROM pgr_articulationPoints(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table'
);
 seq | node
-----+------
   1 |    2
   2 |    5
   3 |    8
   4 |   10
(4 rows)
```



Description of the Signatures

Description of the edges_sql query for components functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Parameter	Type	Default	Description
edges_sql	`TEXT`		SQL query as described above.

Description of the return values for articulation points

Returns set of `(seq, node)`

Column	Type	Description
seq	`INT`	Sequential value starting from **1**.
node	`BIGINT`	Identifier of the vertex.

See Also

- http://en.wikipedia.org/wiki/Biconnected_component
- The queries use the *Sample Data* network.

Indices and tables

- genindex

- search

pgr_bridges - Experimental

`pgr_bridges` - Return the bridges of an undirected graph.

[75]

Fig. 6.24: Boost Graph Inside

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.
  - pgTap tests might be missing.
  - Might need c/c++ coding.
  - May lack documentation.
  - Documentation if any might need to be rewritten.
  - Documentation examples might need to be automatically generated.
  - Might need a lot of feedback from the comunity.
  - Might depend on a proposed function of pgRouting
  - Might depend on a deprecated function of pgRouting

Synopsis

A bridge is an edge of an undirected graph whose deletion increases its number of connected components. This implementation can only be used with an undirected graph.

Characteristics

The main Characteristics are:

- The returned values are ordered:
  - *edge* ascending
- Running time: $O(E * (V + E))$

[75] http://www.boost.org/libs/graph/doc/connected_components.html

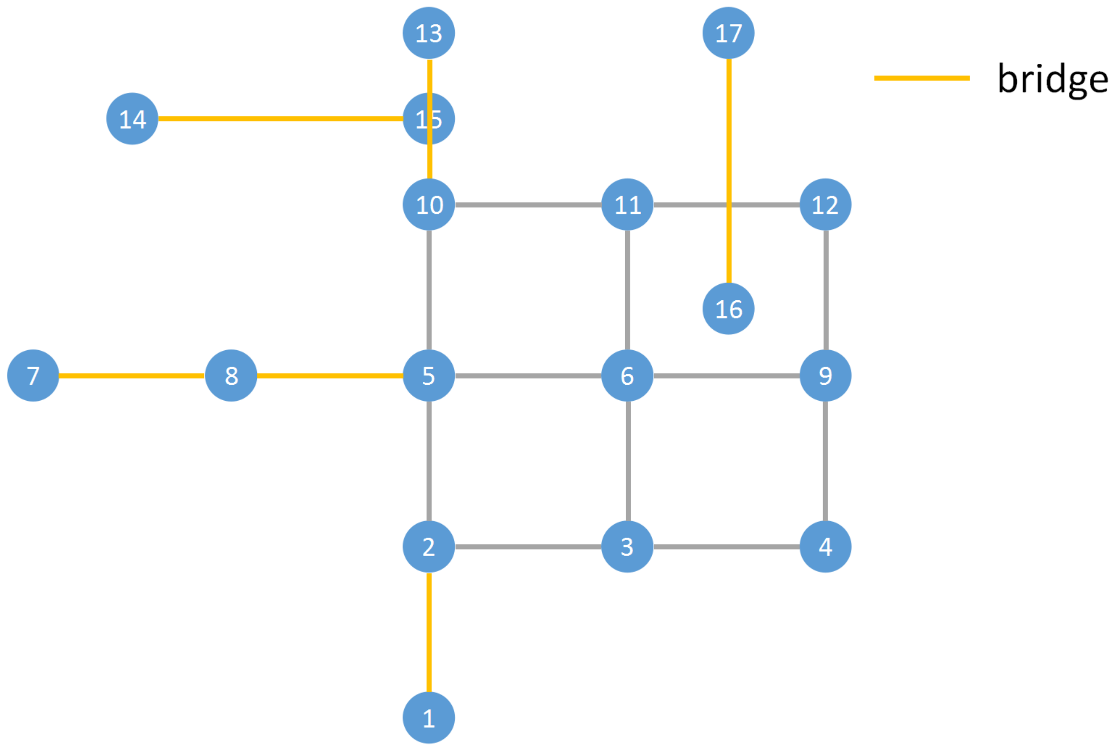
Signatures

```
pgr_bridges(edges_sql)

RETURNS SET OF (seq, node)
    OR EMPTY SET
```

The signature is for a **undirected** graph.

Example

```
SELECT * FROM pgr_bridges(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table'
);
 seq | edge
-----+------
   1 |    1
   2 |    6
   3 |    7
   4 |   14
   5 |   17
   6 |   18
(6 rows)
```



Description of the Signatures

Description of the edges_sql query for components functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Parameter	Type	Default	Description
edges_sql	`TEXT`		SQL query as described above.

Description of the return values for bridges

Returns set of `(seq, node)`

Column	Type	Description
seq	`INT`	Sequential value starting from **1**.
edge	`BIGINT`	Identifier of the edge.

See Also

- http://en.wikipedia.org/wiki/Bridge_%28graph_theory%29
- The queries use the *Sample Data* network.

Indices and tables

- genindex

- search

The problem definition

Connected components

A connected component of an undirected graph is a set of vertices that are all reachable from each other.

Notice: This problem defines on an undirected graph.

Given the following query:

pgr_connectedComponentsV(sql)

where $sql = \{(id_i, source_i, target_i, cost_i, reverse\_cost_i)\}$

and

- $source = \bigcup source_i$,
- $target = \bigcup target_i$,

The graphs are defined as follows:

The weighted undirected graph, $G(V, E)$, is definied by:

- the set of vertices V
  - $V = source \cup target$
- the set of edges E
  - $E = \begin{cases} \{(source_i, target_i, cost_i) \text{ when } cost >= 0\} \\ \cup\{(target_i, source_i, cost_i) \text{ when } cost >= 0\} & \text{if } reverse\_cost = \varnothing \\ \\ \{(source_i, target_i, cost_i) \text{ when } cost >= 0\} \\ \cup\{(target_i, source_i, cost_i) \text{ when } cost >= 0\} \\ \cup\{(target_i, source_i, reverse\_cost_i) \text{ when } reverse\_cost_i >= 0)\} \\ \cup\{(source_i, target_i, reverse\_cost_i) \text{ when } reverse\_cost_i >= 0)\} & \text{if } reverse\_cost \neq \varnothing \end{cases}$

Given:

- $G(V, E)$

Then:

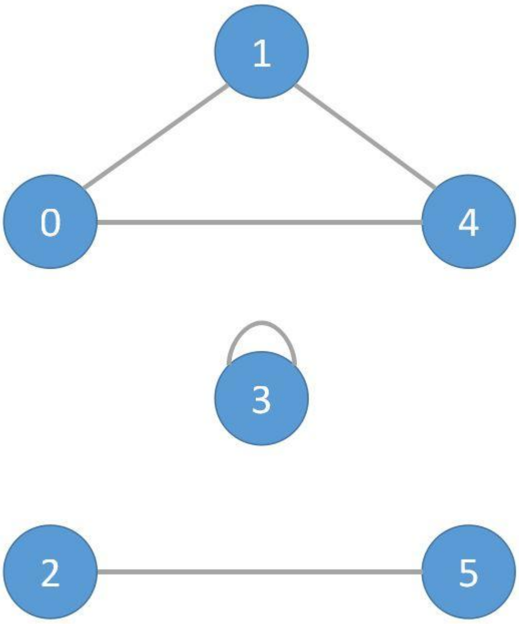
$\pi = \{(component_i, n\_seq_i, node_i)\}$

where:

- $component_i = \min\{node_j | node_j \in component_i\}$
- n_seq_i is a sequential value starting from **1** in a component.
- $node_i \in component_i$
- The returned values are ordered:
  - *component* ascending
  - *node* ascending

Example:

- The first component is composed of nodes `0`, `1` and `4`.
- The second component is composed of node `3`.
- The third component is composed of nodes `2` and `5`.



Strongly connected components

A strongly connected component of a directed graph is a set of vertices that are all reachable from each other.

Notice: This problem defines on a directed graph.

Given the following query:

pgr_strongComponentsV(sql)

where $sql = \{(id_i, source_i, target_i, cost_i, reverse\_cost_i)\}$

and

- $source = \bigcup source_i$,
- $target = \bigcup target_i$,

The graphs are defined as follows:

The weighted directed graph, $G_d(V, E)$, is definied by:

- the set of vertices V
  - $V = source \cup target \cup start_{vid} \cup end_{vid}$
- the set of edges E
  - $E = \begin{cases} \{(source_i, target_i, cost_i) \text{ when } cost >= 0\} & \text{if } reverse\_cost = \varnothing \\ \\ \{(source_i, target_i, cost_i) \text{ when } cost >= 0\} & \\ \cup\{(target_i, source_i, reverse\_cost_i) \text{ when } reverse\_cost_i >= 0)\} & \text{if } reverse\_cost \neq \varnothing \end{cases}$

Given:

- $G(V, E)$

Then:

$\pi = \{(component_i, n\_seq_i, node_i)\}$

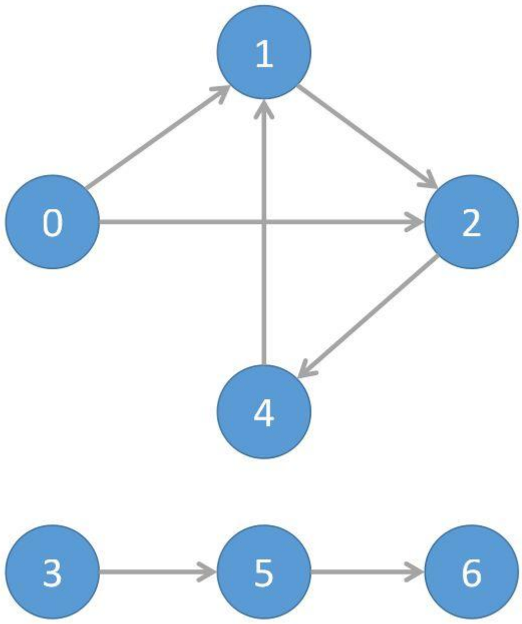
where:

- $component_i = \min node_j | node_j \in component_i$

- n_seq_i is a sequential value starting from **1** in a component.
- $node_i \in component_i$
- The returned values are ordered:
  - *component* ascending
  - *node* ascending

Example:

- The first component is composed of nodes `1`, `2` and `4`.
- The second component is composed of node `0`.
- The third component is composed of node `3`.
- The fourth component is composed of node `5`.
- The fifth component is composed of node `6`.



Biconnected components

The biconnected components of an undirected graph are the maximal subsets of vertices such that the removal of a vertex from particular component will not disconnect the component. Unlike connected components, vertices may belong to multiple biconnected components. Vertices can be present in multiple biconnected components, but each edge can only be contained in a single biconnected component. So, the output only has edge version.

Notice: This problem defines on an undirected graph.

Given the following query:

pgr_biconnectedComponents(sql)

where $sql = \{(id_i, source_i, target_i, cost_i, reverse\_cost_i)\}$

and

- $source = \bigcup source_i$,
- $target = \bigcup target_i$,

The graphs are defined as follows:

The weighted undirected graph, $G(V, E)$, is definied by:

- the set of vertices V
  - $V = source \cup target$
- the set of edges E
  - $E = \begin{cases} \{(source_i, target_i, cost_i) \text{ when } cost >= 0\} & \\ \cup\{(target_i, source_i, cost_i) \text{ when } cost >= 0\} & \quad \text{if } reverse\_cost = \varnothing \\ & \\ \{(source_i, target_i, cost_i) \text{ when } cost >= 0\} & \\ \cup\{(target_i, source_i, cost_i) \text{ when } cost >= 0\} & \\ \cup\{(target_i, source_i, reverse\_cost_i) \text{ when } reverse\_cost_i >= 0)\} & \\ \cup\{(source_i, target_i, reverse\_cost_i) \text{ when } reverse\_cost_i >= 0)\} & \quad \text{if } reverse\_cost \neq \varnothing \end{cases}$

Given:

- $G(V, E)$

Then:

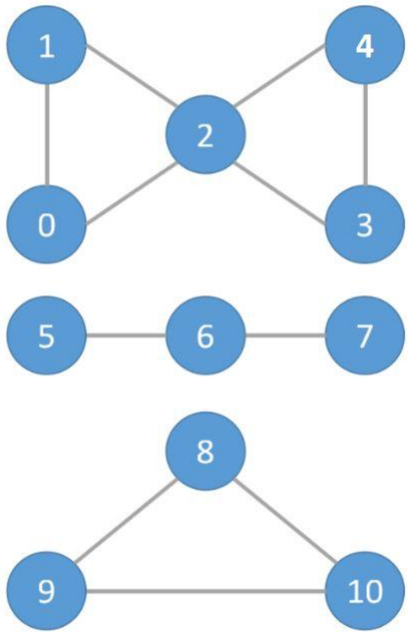
$\boldsymbol{\pi} = \{(component_i, n\_seq_i, node_i)\}$

where:

- $component_i = \min node_j | node_j \in component_i$
- n_seq_i is a sequential value starting from **1** in a component.
- $edge_i \in component_i$
- The returned values are ordered:
  - *component* ascending
  - *edge* ascending

Example:

- The first component is composed of edges `1 - 2`, `0 - 1` and `0 - 2`.
- The second component is composed of edges `2 - 4`, `2 - 3` and `3 - 4`.
- The third component is composed of edge `5 - 6`.
- The fourth component is composed of edge `6 - 7`.
- The fifth component is composed of edges `8 - 9`, `9 - 10` and `8 - 10`.



Articulation Points

Those vertices that belong to more than one biconnected component are called articulation points or, equivalently, cut vertices. Articulation points are vertices whose removal would increase the number of connected components in the graph.

Notice: This problem defines on an undirected graph.

Given the following query:

pgr_articulationPoints(sql)

where $sql = \{(id_i, source_i, target_i, cost_i, reverse\_cost_i)\}$

and

- $source = \bigcup source_i$,
- $target = \bigcup target_i$,

The graphs are defined as follows:

The weighted undirected graph, $G(V, E)$, is definied by:

- the set of vertices V
  - $V = source \cup target$
- the set of edges E
  - $$E = \begin{cases} \{(source_i, target_i, cost_i) \text{ when } cost >= 0\} \\ \cup\{(target_i, source_i, cost_i) \text{ when } cost >= 0\} & \text{if } reverse\_cost = \varnothing \\ \\ \{(source_i, target_i, cost_i) \text{ when } cost >= 0\} \\ \cup\{(target_i, source_i, cost_i) \text{ when } cost >= 0\} \\ \cup\{(target_i, source_i, reverse\_cost_i) \text{ when } reverse\_cost_i >= 0)\} \\ \cup\{(source_i, target_i, reverse\_cost_i) \text{ when } reverse\_cost_i >= 0)\} & \text{if } reverse\_cost \neq \varnothing \end{cases}$$

Given:

- $G(V, E)$

Then:

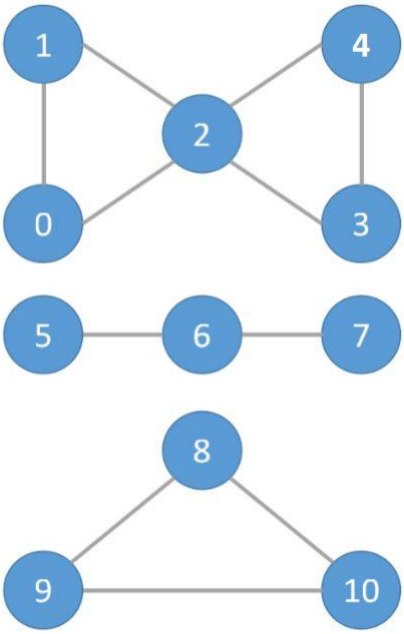
$\pi = \{node_i\}$

where:

- $node_i$ is an articulation point.
- The returned values are ordered:
  - *node* ascending

Example:

- Articulation points are nodes `2` and `6`.



Bridges

A bridge is an edge of an undirected graph whose deletion increases its number of connected components.

Notice: This problem defines on an undirected graph.

Given the following query:

pgr_bridges(sql)

where $sql = \{(id_i, source_i, target_i, cost_i, reverse\_cost_i)\}$

and

- $source = \bigcup source_i$,
- $target = \bigcup target_i$,

The graphs are defined as follows:

The weighted undirected graph, $G(V, E)$, is definied by:

- the set of vertices V
  - $V = source \cup target$
- the set of edges E

– $E = \begin{cases} \{(source_i, target_i, cost_i) \text{ when } cost >= 0\} \\ \cup\{(target_i, source_i, cost_i) \text{ when } cost >= 0\} & \text{if } reverse\_cost = \varnothing \\ \\ \{(source_i, target_i, cost_i) \text{ when } cost >= 0\} \\ \cup\{(target_i, source_i, cost_i) \text{ when } cost >= 0\} \\ \cup\{(target_i, source_i, reverse\_cost_i) \text{ when } reverse\_cost_i >= 0)\} \\ \cup\{(source_i, target_i, reverse\_cost_i) \text{ when } reverse\_cost_i >= 0)\} & \text{if } reverse\_cost \neq \varnothing \end{cases}$

Given:

- $G(V, E)$

Then:

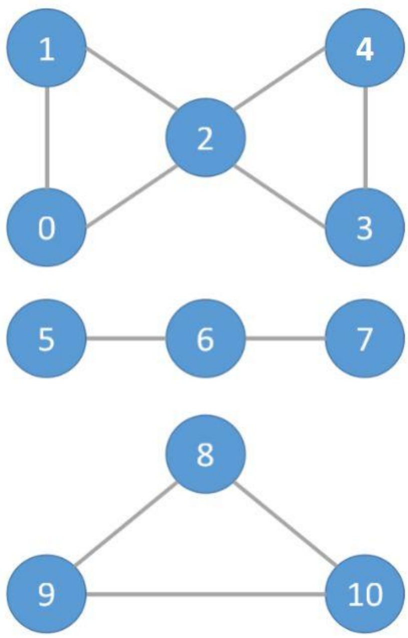
$\pi = \{edge_i\}$

where:

- $edge_i$ is an edge.
- The returned values are ordered:
  - *edge* ascending

Example:

- Bridges are edges `5 <--> 6` and `6 <--> 7`.



Description of the edges_sql query for components functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Parameter	Type	Default	Description
edges_sql	`TEXT`		SQL query as described above.

Description of the return values for connected components and strongly connected components

Returns set of `(seq, component, n_seq, node)`

Column	Type	Description
seq	`INT`	Sequential value starting from **1**.
component	`BIGINT`	Component identifier. It is equal to the minimum node identifier in the component.
n_seq	`INT`	It is a sequential value starting from **1** in a component.
node	`BIGINT`	Identifier of the vertex.

Description of the return values for biconnected components, connected components (edge version) and strongly connected components

Returns set of `(seq, component, n_seq, edge)`

Column	Type	Description
seq	`INT`	Sequential value starting from **1**.
component	`BIGINT`	Component identifier. It is equal to the minimum edge identifier in the component.
n_seq	`INT`	It is a sequential value starting from **1** in a component.
edge	`BIGINT`	Identifier of the edge.

Description of the return values for articulation points

Returns set of `(seq, node)`

Column	Type	Description
seq	`INT`	Sequential value starting from **1**.
node	`BIGINT`	Identifier of the vertex.

Description of the return values for bridges

Returns set of `(seq, node)`

Column	Type	Description
seq	`INT`	Sequential value starting from **1**.
edge	`BIGINT`	Identifier of the edge.

See Also

Indices and tables

- genindex
- search

6.2.5 pgr_gsoc_vrppdtw - Experimental

Name

`pgr_gsoc_vrppdtw` — Returns a solution for *Pick and Delivery* with *time windows* Vehicle Routing Problem

> **Warning:** Experimental functions
>
> - They are not officially of the current release.
> - They likely will not be officially be part of the next release:
>   - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
>   - Name might change.
>   - Signature might change.
>   - Functionality might change.
>   - pgTap tests might be missing.
>   - Might need c/c++ coding.
>   - May lack documentation.

- Documentation if any might need to be rewritten.
- Documentation examples might need to be automatically generated.
- Might need a lot of feedback from the comunity.
- Might depend on a proposed function of pgRouting
- Might depend on a deprecated function of pgRouting

Signature Summary

```
pgr_gsoc_vrppdtw(sql, vehicle_num, capacity)
RETURNS SET OF pgr_costResult[]:
```

Signatures

Complete signature

```
pgr_gsoc_vrppdtw(sql, vehicle_num, capacity)
Returns set of pgr_costResult[]:
```

Example: Show the id1

```
SELECT DISTINCT(id1) FROM pgr_gsoc_vrppdtw(
    'SELECT * FROM customer ORDER BY id', 25, 200)
ORDER BY id1;
 id1
-----
   1
   2
   3
   4
   5
   6
   7
   8
   9
  10
(10 rows)
```

Description of the Signatures

Description of the sql query

Column	Type	Description
id	`ANY-INTEGER`	Identifier of the customer. • A value of `0` identifies the starting location
x	`ANY-NUMERICAL`	`X` coordinate of the location.
y	`ANY-NUMERICAL`	`Y` coordinate of the location.
demand	`ANY-NUMERICAL`	How much is added / removed from the vehicle. • Negative value is a delivery, • Positive value is a pickup,
openTime	`ANY-NUMERICAL`	The time relative to 0, when the customer opens.
closeTime	`ANY-NUMERICAL`	The time relative to 0, when the customer closes.
serviceTime	`ANY-NUMERICAL`	The duration of the loading / unloading.
pIndex	`ANY-INTEGER`	Value used when the current customer is a Delivery to find the corresponding Pickup
dIndex	`ANY-INTEGER`	Value used when the current customer is a Pickup to find the corresponding Delivery

Description of the parameters of the signatures

Column	Type	Description
sql	`TEXT`	SQL query as described above.
vehicle_num	`INTEGER`	Maximum number of vehicles in the result. (currently is ignored)
capacity	`INTEGER`	Capacity of the vehicle.

Description of the result

RETURNS SET OF pgr_costResult[]:

Column	Type	Description
seq	`INTEGER`	Sequential value starting from **1**.
id1	`INTEGER`	Current vehicle identifier.
id2	`INTEGER`	Customer identifier.
cost	`FLOAT`	**Previous `cost` plus *travel time* plus *wait time* plus s** • when `id2 = 0` for the second time for the same `id1`, then has the total time for the current `id1`

Examples

Example: Total number of rows returned

```
SELECT count(*) FROM pgr_gsoc_vrppdtw(
    'SELECT * FROM customer ORDER BY id', 25, 200);
 count
-------
   126
(1 row)
```

Example: Results for only id1 values: 1, 5, and 9

```
SELECT * FROM pgr_gsoc_vrppdtw(
    'SELECT * FROM customer ORDER BY id', 25, 200)
    WHERE id1 in (1, 5, 9);
 seq | id1 | id2 |       cost
-----+-----+-----+------------------
   1 |   1 |   0 |                0
   2 |   1 |  13 | 120.805843601499
   3 |   1 |  17 | 214.805843601499
   4 |   1 |  18 | 307.805843601499
   5 |   1 |  19 | 402.805843601499
   6 |   1 |  15 | 497.805843601499
   7 |   1 |  16 | 592.805843601499
   8 |   1 |  14 | 684.805843601499
   9 |   1 |  12 | 777.805843601499
  10 |   1 |  50 | 920.815276724293
  11 |   1 |  52 | 1013.97755438446
  12 |   1 |  49 | 1106.97755438446
  13 |   1 |  47 | 1198.97755438446
  14 |   1 |   0 | 1217.00531076178
  57 |   5 |   0 |                0
  58 |   5 |  90 | 110.615528128088
  59 |   5 |  87 | 205.615528128088
  60 |   5 |  86 | 296.615528128088
  61 |   5 |  83 | 392.615528128088
  62 |   5 |  82 | 485.615528128088
  63 |   5 |  84 | 581.446480022934
  64 |   5 |  85 |  674.27490714768
  65 |   5 |  88 |  767.27490714768
  66 |   5 |  89 | 860.103334272426
  67 |   5 |  91 |  953.70888554789
  68 |   5 |   0 | 976.069565322888
 105 |   9 |   0 |                0
 106 |   9 |  67 | 102.206555615734
 107 |   9 |  65 | 193.206555615734
 108 |   9 |  63 | 285.206555615734
 109 |   9 |  62 | 380.206555615734
 110 |   9 |  74 | 473.206555615734
 111 |   9 |  72 | 568.206555615734
 112 |   9 |  61 | 661.206555615734
 113 |   9 |  64 | 663.206555615734
 114 |   9 | 102 | 753.206555615734
 115 |   9 |  68 | 846.206555615734
 116 |   9 |   0 | 866.822083743822
(38 rows)
```

See Also

- The examples use *Pick & Deliver Data*
- http://en.wikipedia.org/wiki/Vehicle_routing_problem

Indices and tables

- genindex
- search

6.2.6 pgr_vrpOneDepot - Experimental

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.
  - pgTap tests might be missing.
  - Might need c/c++ coding.
  - May lack documentation.
  - Documentation if any might need to be rewritten.
  - Documentation examples might need to be automatically generated.
  - Might need a lot of feedback from the comunity.
  - Might depend on a proposed function of pgRouting
  - Might depend on a deprecated function of pgRouting

No documentation available

Example:

Current Result

```
BEGIN;
BEGIN
SET client_min_messages TO NOTICE;
SET
SELECT * FROM pgr_vrpOneDepot(
    'SELECT * FROM vrp_orders',
    'SELECT * FROM vrp_vehicles',
    'SELECT * FROM vrp_distance',
    1);
 oid | opos | vid | tarrival | tdepart
-----+------+-----+----------+---------
  -1 |    1 |   5 |        0 |       0
```

```
  66 |    2 |    5 |          0 |         0
  25 |    3 |    5 |          0 |         0
  21 |    4 |    5 |          0 |         0
  84 |    5 |    5 |          0 |         0
  50 |    6 |    5 |          0 |         0
  49 |    7 |    5 |          0 |         0
  24 |    8 |    5 |          0 |         0
  22 |    9 |    5 |          0 |         0
  20 |   10 |    5 |          0 |         0
  19 |   11 |    5 |          0 |         0
  66 |   12 |    5 |         11 |        21
  84 |   13 |    5 |         30 |        45
  24 |   14 |    5 |         71 |        81
  22 |   15 |    5 |         83 |        93
  20 |   16 |    5 |         98 |       108
  19 |   17 |    5 |        114 |       124
  50 |   18 |    5 |        131 |       141
  21 |   19 |    5 |        144 |       154
  25 |   20 |    5 |        158 |       168
  49 |   21 |    5 |        179 |       189
  -1 |   22 |    5 |        234 |       234
  -1 |    1 |    6 |          0 |         0
  31 |    2 |    6 |          0 |         0
  32 |    3 |    6 |          0 |         0
  81 |    4 |    6 |          0 |         0
  94 |    5 |    6 |          0 |         0
  93 |    6 |    6 |          0 |         0
  35 |    7 |    6 |          0 |         0
  33 |    8 |    6 |          0 |         0
  28 |    9 |    6 |          0 |         0
  27 |   10 |    6 |          0 |         0
  93 |   11 |    6 |         15 |        25
  32 |   12 |    6 |         61 |        71
  28 |   13 |    6 |         78 |        88
  31 |   14 |    6 |         97 |       107
  35 |   15 |    6 |        112 |       122
  27 |   16 |    6 |        134 |       144
  33 |   17 |    6 |        152 |       162
  94 |   18 |    6 |        196 |       206
  81 |   19 |    6 |        221 |       231
  -1 |   20 |    6 |        238 |       238
  -1 |    1 |    3 |          0 |         0
  16 |    2 |    3 |          0 |         0
  14 |    3 |    3 |          0 |         0
  48 |    4 |    3 |          0 |         0
  18 |    5 |    3 |          0 |         0
  17 |    6 |    3 |          0 |         0
  15 |    7 |    3 |          0 |         0
  13 |    8 |    3 |          0 |         0
  11 |    9 |    3 |          0 |         0
  10 |   10 |    3 |          0 |         0
  15 |   11 |    3 |         35 |        45
  48 |   12 |    3 |         48 |        58
  13 |   13 |    3 |         64 |        74
  16 |   14 |    3 |         82 |        92
  17 |   15 |    3 |         94 |       104
  10 |   16 |    3 |        115 |       125
  11 |   17 |    3 |        130 |       140
  14 |   18 |    3 |        147 |       157
  18 |   19 |    3 |        169 |       179
  -1 |   20 |    3 |        219 |       219
  -1 |    1 |    8 |          0 |         0
  71 |    2 |    8 |          0 |         0
```

```
   55 |     3 |    8 |          0 |          0
   44 |     4 |    8 |          0 |          0
   43 |     5 |    8 |          0 |          0
   42 |     6 |    8 |          0 |          0
   41 |     7 |    8 |          0 |          0
   40 |     8 |    8 |          0 |          0
   39 |     9 |    8 |          0 |          0
   43 |    10 |    8 |         34 |         44
   40 |    11 |    8 |         49 |         59
   39 |    12 |    8 |         61 |         85
   41 |    13 |    8 |         90 |        100
   42 |    14 |    8 |        111 |        121
   44 |    15 |    8 |        131 |        141
   55 |    16 |    8 |        166 |        176
   71 |    17 |    8 |        198 |        208
   -1 |    18 |    8 |        228 |        228
   -1 |     1 |    1 |          0 |          0
    4 |     2 |    1 |          0 |          0
  101 |     3 |    1 |          0 |          0
   46 |     4 |    1 |          0 |          0
    5 |     5 |    1 |          0 |          0
    3 |     6 |    1 |          0 |          0
   46 |     7 |    1 |         38 |         48
    3 |     8 |    1 |         55 |         65
    2 |     9 |    1 |         96 |         96
    4 |    10 |    1 |        135 |        145
    2 |    11 |    1 |        148 |        158
    5 |    12 |    1 |        165 |        175
  101 |    13 |    1 |        192 |        202
   -1 |    14 |    1 |        222 |        222
   -1 |     1 |   13 |          0 |          0
   92 |     2 |   13 |          0 |          0
   52 |     3 |   13 |          0 |          0
   57 |     4 |   13 |          0 |          0
   85 |     5 |   13 |          0 |          0
   68 |     6 |   13 |          0 |          0
   63 |     7 |   13 |          0 |          0
   63 |     8 |   13 |         29 |         62
   68 |     9 |   13 |         69 |         80
   52 |    10 |   13 |        104 |        114
   85 |    11 |   13 |        123 |        133
   57 |    12 |   13 |        142 |        152
   92 |    13 |   13 |        159 |        177
   -1 |    14 |   13 |        189 |        189
   -1 |     1 |    7 |          0 |          0
   30 |     2 |    7 |          0 |          0
   29 |     3 |    7 |          0 |          0
   38 |     4 |    7 |          0 |          0
   36 |     5 |    7 |          0 |          0
   34 |     6 |    7 |          0 |          0
   34 |     7 |    7 |         51 |         61
   29 |     8 |    7 |         70 |         80
   30 |     9 |    7 |         85 |         95
   38 |    10 |    7 |        149 |        159
   36 |    11 |    7 |        162 |        172
   -1 |    12 |    7 |        217 |        217
   -1 |     1 |    2 |          0 |          0
   89 |     2 |    2 |          0 |          0
   47 |     3 |    2 |          0 |          0
   61 |     4 |    2 |          0 |          0
    9 |     5 |    2 |          0 |          0
    8 |     6 |    2 |          0 |          0
   89 |     7 |    2 |         18 |         77
```

```
   8 |    8 |   2 |       96 |      106
   9 |    9 |   2 |      111 |      121
  47 |   10 |   2 |      124 |      134
  61 |   11 |   2 |      154 |      165
  -1 |   12 |   2 |      192 |      192
  -1 |    1 |  14 |        0 |        0
  97 |    2 |  14 |        0 |        0
  64 |    3 |  14 |        0 |        0
  51 |    4 |  14 |        0 |        0
  96 |    5 |  14 |        0 |        0
  77 |    6 |  14 |        0 |        0
  96 |    7 |  14 |       21 |       44
  64 |    8 |  14 |       63 |       73
  77 |    9 |  14 |       83 |       93
  51 |   10 |  14 |      119 |      129
  97 |   11 |  14 |      154 |      164
  -1 |   12 |  14 |      180 |      180
  -1 |    1 |  15 |        0 |        0
  67 |    2 |  15 |        0 |        0
  73 |    3 |  15 |        0 |        0
  95 |    4 |  15 |        0 |        0
  82 |    5 |  15 |        0 |        0
  72 |    6 |  15 |        0 |        0
  73 |    7 |  15 |       27 |       40
  72 |    8 |  15 |       50 |       75
  82 |    9 |  15 |       91 |      101
  95 |   10 |  15 |      114 |      124
  67 |   11 |  15 |      144 |      154
  -1 |   12 |  15 |      167 |      167
  -1 |    1 |  11 |        0 |        0
  78 |    2 |  11 |        0 |        0
  26 |    3 |  11 |        0 |        0
  87 |    4 |  11 |        0 |        0
  23 |    5 |  11 |        0 |        0
  87 |    6 |  11 |       32 |       97
  23 |    7 |  11 |      118 |      128
  78 |    8 |  11 |      149 |      160
  26 |    9 |  11 |      172 |      182
  -1 |   10 |  11 |      227 |      227
  -1 |    1 |   4 |        0 |        0
  60 |    2 |   4 |        0 |        0
  59 |    3 |   4 |        0 |        0
 100 |    4 |   4 |        0 |        0
  54 |    5 |   4 |        0 |        0
  60 |    6 |   4 |       42 |       52
 100 |    7 |   4 |       74 |       87
  54 |    8 |   4 |      103 |      113
  59 |    9 |   4 |      153 |      163
  -1 |   10 |   4 |      211 |      211
  -1 |    1 |  10 |        0 |        0
  86 |    2 |  10 |        0 |        0
  90 |    3 |  10 |        0 |        0
  65 |    4 |  10 |        0 |        0
  53 |    5 |  10 |        0 |        0
  53 |    6 |  10 |       25 |       62
  65 |    7 |  10 |       82 |       92
  86 |    8 |  10 |      111 |      121
  90 |    9 |  10 |      140 |      154
  -1 |   10 |  10 |      206 |      206
  -1 |    1 |  12 |        0 |        0
   6 |    2 |  12 |        0 |        0
  80 |    3 |  12 |        0 |        0
   7 |    4 |  12 |        0 |        0
```

```
  56 |     5 |  12 |        0 |        0
   6 |     6 |  12 |       40 |       51
  80 |     7 |  12 |       73 |       99
   7 |     8 |  12 |      113 |      123
  56 |     9 |  12 |      142 |      152
  -1 |    10 |  12 |      166 |      166
  -1 |     1 |  19 |        0 |        0
  88 |     2 |  19 |        0 |        0
  70 |     3 |  19 |        0 |        0
  58 |     4 |  19 |        0 |        0
  99 |     5 |  19 |        0 |        0
  70 |     6 |  19 |        9 |       51
  99 |     7 |  19 |       56 |       66
  88 |     8 |  19 |       97 |      107
  58 |     9 |  19 |      125 |      135
  -1 |    10 |  19 |      162 |      162
  -1 |     1 |  17 |        0 |        0
  75 |     2 |  17 |        0 |        0
  98 |     3 |  17 |        0 |        0
  76 |     4 |  17 |        0 |        0
  76 |     5 |  17 |       57 |       84
  98 |     6 |  17 |       97 |      130
  75 |     7 |  17 |      146 |      156
  -1 |     8 |  17 |      192 |      192
  -1 |     1 |  16 |        0 |        0
  69 |     2 |  16 |        0 |        0
  79 |     3 |  16 |        0 |        0
  74 |     4 |  16 |        0 |        0
  74 |     5 |  16 |       39 |       87
  79 |     6 |  16 |       94 |      104
  69 |     7 |  16 |      136 |      154
  -1 |     8 |  16 |      164 |      164
  -1 |     1 |   9 |        0 |        0
  62 |     2 |   9 |        0 |        0
  37 |     3 |   9 |        0 |        0
  45 |     4 |   9 |        0 |        0
  37 |     5 |   9 |       43 |       53
  45 |     6 |   9 |       63 |       74
  62 |     7 |   9 |       94 |      104
  -1 |     8 |   9 |      120 |      120
  -1 |     1 |  18 |        0 |        0
  91 |     2 |  18 |        0 |        0
  12 |     3 |  18 |        0 |        0
  12 |     4 |  18 |       34 |       69
  91 |     5 |  18 |       99 |      109
  -1 |     6 |  18 |      113 |      113
  -1 |     1 |  20 |        0 |        0
  83 |     2 |  20 |        0 |        0
  83 |     3 |  20 |       15 |       52
  -1 |     4 |  20 |       67 |       67
  -1 |     0 |   0 |       -1 |     3712
(241 rows)

ROLLBACK;
ROLLBACK
```

Data

```
drop table if exists vrp_orders cascade;
create table vrp_orders (
```

```
    id integer not null primary key,
    order_unit integer,
    open_time integer,
    close_time integer,
    service_time integer,
    x float8,
    y float8

);

copy vrp_orders (id, x, y, order_unit, open_time, close_time, service_time) from␣
↪stdin;
1        40.000000        50.000000        0        0        240        0
2        25.000000        85.000000        20        145        175        10
3        22.000000        75.000000        30        50        80        10
4        22.000000        85.000000        10        109        139        10
5        20.000000        80.000000        40        141        171        10
6        20.000000        85.000000        20        41        71        10
7        18.000000        75.000000        20        95        125        10
8        15.000000        75.000000        20        79        109        10
9        15.000000        80.000000        10        91        121        10
10        10.000000        35.000000        20        91        121        10
11        10.000000        40.000000        30        119        149        10
12        8.000000        40.000000        40        59        89        10
13        8.000000        45.000000        20        64        94        10
14        5.000000        35.000000        10        142        172        10
15        5.000000        45.000000        10        35        65        10
16        2.000000        40.000000        20        58        88        10
17        0.000000        40.000000        20        72        102        10
18        0.000000        45.000000        20        149        179        10
19        44.000000        5.000000        20        87        117        10
20        42.000000        10.000000        40        72        102        10
21        42.000000        15.000000        10        122        152        10
22        40.000000        5.000000        10        67        97        10
23        40.000000        15.000000        40        92        122        10
24        38.000000        5.000000        30        65        95        10
25        38.000000        15.000000        10        148        178        10
26        35.000000        5.000000        20        154        184        10
27        95.000000        30.000000        30        115        145        10
28        95.000000        35.000000        20        62        92        10
29        92.000000        30.000000        10        62        92        10
30        90.000000        35.000000        10        67        97        10
31        88.000000        30.000000        10        74        104        10
32        88.000000        35.000000        20        61        91        10
33        87.000000        30.000000        10        131        161        10
34        85.000000        25.000000        10        51        81        10
35        85.000000        35.000000        30        111        141        10
36        67.000000        85.000000        20        139        169        10
37        65.000000        85.000000        40        43        73        10
38        65.000000        82.000000        10        124        154        10
39        62.000000        80.000000        30        75        105        10
40        60.000000        80.000000        10        37        67        10
41        60.000000        85.000000        30        85        115        10
42        58.000000        75.000000        20        92        122        10
43        55.000000        80.000000        10        33        63        10
44        55.000000        85.000000        20        128        158        10
45        55.000000        82.000000        10        64        94        10
46        20.000000        82.000000        10        37        67        10
47        18.000000        80.000000        10        113        143        10
48        2.000000        45.000000        10        45        75        10
49        42.000000        5.000000        10        151        181        10
50        42.000000        12.000000        10        104        134        10
51        72.000000        35.000000        30        116        146        10
```

```
52         55.000000        20.000000        19         83         113        10
53         25.000000        30.000000        3          52         82         10
54         20.000000        50.000000        5          91         121        10
55         55.000000        60.000000        16         139        169        10
56         30.000000        60.000000        16         140        170        10
57         50.000000        35.000000        19         130        160        10
58         30.000000        25.000000        23         96         126        10
59         15.000000        10.000000        20         152        182        10
60         10.000000        20.000000        19         42         72         10
61         15.000000        60.000000        17         155        185        10
62         45.000000        65.000000        9          66         96         10
63         65.000000        35.000000        3          52         82         10
64         65.000000        20.000000        6          39         69         10
65         45.000000        30.000000        17         53         83         10
66         35.000000        40.000000        16         11         41         10
67         41.000000        37.000000        16         133        163        10
68         64.000000        42.000000        9          70         100        10
69         40.000000        60.000000        21         144        174        10
70         31.000000        52.000000        27         41         71         10
71         35.000000        69.000000        23         180        210        10
72         65.000000        55.000000        14         65         95         10
73         63.000000        65.000000        8          30         60         10
74         2.000000         60.000000        5          77         107        10
75         20.000000        20.000000        8          141        171        10
76         5.000000         5.000000         16         74         104        10
77         60.000000        12.000000        31         75         105        10
78         23.000000        3.000000         7          150        180        10
79         8.000000         56.000000        27         90         120        10
80         6.000000         68.000000        30         89         119        10
81         47.000000        47.000000        13         192        222        10
82         49.000000        58.000000        10         86         116        10
83         27.000000        43.000000        9          42         72         10
84         37.000000        31.000000        14         35         65         10
85         57.000000        29.000000        18         96         126        10
86         63.000000        23.000000        2          87         117        10
87         21.000000        24.000000        28         87         117        10
88         12.000000        24.000000        13         90         120        10
89         24.000000        58.000000        19         67         97         10
90         67.000000        5.000000         25         144        174        10
91         37.000000        47.000000        6          86         116        10
92         49.000000        42.000000        13         167        197        10
93         53.000000        43.000000        14         14         44         10
94         61.000000        52.000000        3          178        208        10
95         57.000000        48.000000        23         95         125        10
96         56.000000        37.000000        6          34         64         10
97         55.000000        54.000000        26         132        162        10
98         4.000000         18.000000        35         120        150        10
99         26.000000        52.000000        9          46         76         10
100        26.000000        35.000000        15         77         107        10
101        31.000000        67.000000        3          180        210        10
\.

drop table if exists vrp_vehicles cascade;
create table vrp_vehicles (
    vehicle_id integer not null primary key,
    capacity integer,
    case_no integer
);

copy vrp_vehicles (vehicle_id, capacity, case_no) from stdin;
1         200         5
2         200         5
3         200         5
```

```
4        200        5
5        200        5
6        200        5
7        200        5
8        200        5
9        200        5
10        200        5
11        200        5
12        200        5
13        200        5
14        200        5
15        200        5
16        200        5
17        200        5
18        200        5
19        200        5
20        200        5
\.

drop table if exists vrp_distance cascade;
create table vrp_distance (
    src_id integer,
    dest_id integer,
    cost Float8,
    distance Float8,
    traveltime Float8
);

copy vrp_distance (src_id, dest_id, cost, distance, traveltime) from stdin;
1        2        38.078866        38.078866        38.078866
1        3        30.805844        30.805844        30.805844
1        4        39.357337        39.357337        39.357337
1        5        36.055513        36.055513        36.055513
1        6        40.311289        40.311289        40.311289
1        7        33.301652        33.301652        33.301652
1        8        35.355339        35.355339        35.355339
1        9        39.051248        39.051248        39.051248
1        10        33.541020        33.541020        33.541020
1        11        31.622777        31.622777        31.622777
1        12        33.526109        33.526109        33.526109
1        13        32.388269        32.388269        32.388269
1        14        38.078866        38.078866        38.078866
1        15        35.355339        35.355339        35.355339
1        16        39.293765        39.293765        39.293765
1        17        41.231056        41.231056        41.231056
1        18        40.311289        40.311289        40.311289
1        19        45.177428        45.177428        45.177428
1        20        40.049969        40.049969        40.049969
1        21        35.057096        35.057096        35.057096
1        22        45.000000        45.000000        45.000000
1        23        35.000000        35.000000        35.000000
1        24        45.044423        45.044423        45.044423
1        25        35.057096        35.057096        35.057096
1        26        45.276926        45.276926        45.276926
1        27        58.523500        58.523500        58.523500
1        28        57.008771        57.008771        57.008771
1        29        55.713553        55.713553        55.713553
1        30        52.201533        52.201533        52.201533
1        31        52.000000        52.000000        52.000000
1        32        50.289164        50.289164        50.289164
1        33        51.078371        51.078371        51.078371
1        34        51.478151        51.478151        51.478151
1        35        47.434165        47.434165        47.434165
```

```
1        36        44.204072        44.204072        44.204072
1        37        43.011626        43.011626        43.011626
1        38        40.607881        40.607881        40.607881
1        39        37.202150        37.202150        37.202150
1        40        36.055513        36.055513        36.055513
1        41        40.311289        40.311289        40.311289
1        42        30.805844        30.805844        30.805844
1        43        33.541020        33.541020        33.541020
1        44        38.078866        38.078866        38.078866
1        45        35.341194        35.341194        35.341194
1        46        37.735925        37.735925        37.735925
1        47        37.202150        37.202150        37.202150
1        48        38.327536        38.327536        38.327536
1        49        45.044423        45.044423        45.044423
1        50        38.052595        38.052595        38.052595
1        51        35.341194        35.341194        35.341194
1        52        33.541020        33.541020        33.541020
1        53        25.000000        25.000000        25.000000
1        54        20.000000        20.000000        20.000000
1        55        18.027756        18.027756        18.027756
1        56        14.142136        14.142136        14.142136
1        57        18.027756        18.027756        18.027756
1        58        26.925824        26.925824        26.925824
1        59        47.169906        47.169906        47.169906
1        60        42.426407        42.426407        42.426407
1        61        26.925824        26.925824        26.925824
1        62        15.811388        15.811388        15.811388
1        63        29.154759        29.154759        29.154759
1        64        39.051248        39.051248        39.051248
1        65        20.615528        20.615528        20.615528
1        66        11.180340        11.180340        11.180340
1        67        13.038405        13.038405        13.038405
1        68        25.298221        25.298221        25.298221
1        69        10.000000        10.000000        10.000000
1        70        9.219544        9.219544        9.219544
1        71        19.646883        19.646883        19.646883
1        72        25.495098        25.495098        25.495098
1        73        27.459060        27.459060        27.459060
1        74        39.293765        39.293765        39.293765
1        75        36.055513        36.055513        36.055513
1        76        57.008771        57.008771        57.008771
1        77        42.941821        42.941821        42.941821
1        78        49.979996        49.979996        49.979996
1        79        32.557641        32.557641        32.557641
1        80        38.470768        38.470768        38.470768
1        81        7.615773        7.615773        7.615773
1        82        12.041595        12.041595        12.041595
1        83        14.764823        14.764823        14.764823
1        84        19.235384        19.235384        19.235384
1        85        27.018512        27.018512        27.018512
1        86        35.468296        35.468296        35.468296
1        87        32.202484        32.202484        32.202484
1        88        38.209946        38.209946        38.209946
1        89        17.888544        17.888544        17.888544
1        90        52.478567        52.478567        52.478567
1        91        4.242641        4.242641        4.242641
1        92        12.041595        12.041595        12.041595
1        93        14.764823        14.764823        14.764823
1        94        21.095023        21.095023        21.095023
1        95        17.117243        17.117243        17.117243
1        96        20.615528        20.615528        20.615528
1        97        15.524175        15.524175        15.524175
1        98        48.166378        48.166378        48.166378
```

```
1         99       14.142136       14.142136       14.142136
1         100       20.518285       20.518285       20.518285
1         101       19.235384       19.235384       19.235384
2         1       38.078866       38.078866       38.078866
2         3       10.440307       10.440307       10.440307
2         4       3.000000        3.000000        3.000000
2         5       7.071068        7.071068        7.071068
2         6       5.000000        5.000000        5.000000
2         7       12.206556       12.206556       12.206556
2         8       14.142136       14.142136       14.142136
2         9       11.180340       11.180340       11.180340
2         10       52.201533       52.201533       52.201533
2         11       47.434165       47.434165       47.434165
2         12       48.104054       48.104054       48.104054
2         13       43.462628       43.462628       43.462628
2         14       53.851648       53.851648       53.851648
2         15       44.721360       44.721360       44.721360
2         16       50.537115       50.537115       50.537115
2         17       51.478151       51.478151       51.478151
2         18       47.169906       47.169906       47.169906
2         19       82.225300       82.225300       82.225300
2         20       76.902536       76.902536       76.902536
2         21       72.034714       72.034714       72.034714
2         22       81.394103       81.394103       81.394103
2         23       71.589105       71.589105       71.589105
2         24       81.049368       81.049368       81.049368
2         25       71.196910       71.196910       71.196910
2         26       80.622577       80.622577       80.622577
2         27       89.022469       89.022469       89.022469
2         28       86.023253       86.023253       86.023253
2         29       86.683332       86.683332       86.683332
2         30       82.006097       82.006097       82.006097
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2         32       80.430094       80.430094       80.430094
2         33       82.879430       82.879430       82.879430
2         34       84.852814       84.852814       84.852814
2         35       78.102497       78.102497       78.102497
2         36       42.000000       42.000000       42.000000
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2         39       37.336309       37.336309       37.336309
2         40       35.355339       35.355339       35.355339
2         41       35.000000       35.000000       35.000000
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2         44       30.000000       30.000000       30.000000
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2         47       8.602325        8.602325        8.602325
2         48       46.141088       46.141088       46.141088
2         49       81.786307       81.786307       81.786307
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2         51       68.622154       68.622154       68.622154
2         52       71.589105       71.589105       71.589105
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2         55       39.051248       39.051248       39.051248
2         56       25.495098       25.495098       25.495098
2         57       55.901699       55.901699       55.901699
2         58       60.207973       60.207973       60.207973
2         59       75.663730       75.663730       75.663730
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2         69        29.154759        29.154759        29.154759
2         70        33.541020        33.541020        33.541020
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2         76        82.462113        82.462113        82.462113
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2         83        42.047592        42.047592        42.047592
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2         86        72.718636        72.718636        72.718636
2         87        61.131007        61.131007        61.131007
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2         90        90.354856        90.354856        90.354856
2         91        39.849718        39.849718        39.849718
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2         94        48.836462        48.836462        48.836462
2         95        48.918299        48.918299        48.918299
2         96        57.140179        57.140179        57.140179
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5       95      48.918299       48.918299       48.918299
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9        83       38.897301        38.897301        38.897301
9        84       53.712196        53.712196        53.712196
9        85       66.068147        66.068147        66.068147
9        86       74.518454        74.518454        74.518454
9        87       56.320511        56.320511        56.320511
9        88       56.080300        56.080300        56.080300
9        89       23.769729        23.769729        23.769729
9        90       91.263355        91.263355        91.263355
9        91       39.661064        39.661064        39.661064
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9        94       53.851648        53.851648        53.851648
9        95       52.801515        52.801515        52.801515
9        96       59.413803        59.413803        59.413803
9        97       47.707442        47.707442        47.707442
9        98       62.968246        62.968246        62.968246
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10        13       10.198039        10.198039        10.198039
10        14       5.000000        5.000000        5.000000
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10          97          48.846699          48.846699          48.846699
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11          36          72.622311          72.622311          72.622311
11          37          71.063352          71.063352          71.063352
11          38          69.202601          69.202601          69.202601
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11          73          58.600341          58.600341          58.600341
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13          63          57.870545          57.870545          57.870545
13          64          62.241465          62.241465          62.241465
13          65          39.924930          39.924930          39.924930
13          66          27.459060          27.459060          27.459060
13          67          33.955854          33.955854          33.955854
13          68          56.080300          56.080300          56.080300
13          69          35.341194          35.341194          35.341194
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13          71          36.124784          36.124784          36.124784
13          72          57.870545          57.870545          57.870545
13          73          58.523500          58.523500          58.523500
13          74          16.155494          16.155494          16.155494
13          75          27.730849          27.730849          27.730849
13          76          40.112342          40.112342          40.112342
13          77          61.587336          61.587336          61.587336
13          78          44.598206          44.598206          44.598206
13          79          11.000000          11.000000          11.000000
13          80          23.086793          23.086793          23.086793
13          81          39.051248          39.051248          39.051248
13          82          43.011626          43.011626          43.011626
13          83          19.104973          19.104973          19.104973
13          84          32.202484          32.202484          32.202484
13          85          51.546096          51.546096          51.546096
13          86          59.236813          59.236813          59.236813
13          87          24.698178          24.698178          24.698178
13          88          21.377558          21.377558          21.377558
13          89          20.615528          20.615528          20.615528
13          90          71.281134          71.281134          71.281134
13          91          29.068884          29.068884          29.068884
13          92          41.109610          41.109610          41.109610
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14         3         43.462628         43.462628         43.462628
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14         7         42.059482         42.059482         42.059482
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14         21         42.059482         42.059482         42.059482
14         22         46.097722         46.097722         46.097722
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14         24         44.598206         44.598206         44.598206
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14         27         90.138782         90.138782         90.138782
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14         42         66.400301         66.400301         66.400301
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14          88          13.038405          13.038405          13.038405
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14          90          68.876701          68.876701          68.876701
14          91          34.176015          34.176015          34.176015
14          92          44.553339          44.553339          44.553339
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15        93        48.041649        48.041649        48.041649
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16        9        42.059482        42.059482        42.059482
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16        15        5.830952        5.830952        5.830952
16        17        2.000000        2.000000        2.000000
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16        20        50.000000        50.000000        50.000000
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16        26        48.104054        48.104054        48.104054
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17          39          73.783467         73.783467         73.783467
17          40          72.111026         72.111026         72.111026
17          41          75.000000         75.000000         75.000000
17          42          67.742158         67.742158         67.742158
17          43          68.007353         68.007353         68.007353
17          44          71.063352         71.063352         71.063352
17          45          69.202601         69.202601         69.202601
17          46          46.518813         46.518813         46.518813
17          47          43.863424         43.863424         43.863424
17          48          5.385165          5.385165          5.385165
17          49          54.671748         54.671748         54.671748
17          50          50.477718         50.477718         50.477718
17          51          72.173402         72.173402         72.173402
17          52          58.523500         58.523500         58.523500
17          53          26.925824         26.925824         26.925824
17          54          22.360680         22.360680         22.360680
17          55          58.523500         58.523500         58.523500
17          56          36.055513         36.055513         36.055513
17          57          50.249378         50.249378         50.249378
17          58          33.541020         33.541020         33.541020
17          59          33.541020         33.541020         33.541020
17          60          22.360680         22.360680         22.360680
17          61          25.000000         25.000000         25.000000
17          62          51.478151         51.478151         51.478151
17          63          65.192024         65.192024         65.192024
17          64          68.007353         68.007353         68.007353
17          65          46.097722         46.097722         46.097722
17          66          35.000000         35.000000         35.000000
17          67          41.109610         41.109610         41.109610
17          68          64.031242         64.031242         64.031242
17          69          44.721360         44.721360         44.721360
17          70          33.241540         33.241540         33.241540
17          71          45.453273         45.453273         45.453273
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17          73          67.779053         67.779053         67.779053
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17        80        28.635642         28.635642         28.635642
17        81        47.518417         47.518417         47.518417
17        82        52.201533         52.201533         52.201533
17        83        27.166155         27.166155         27.166155
17        84        38.078866         38.078866         38.078866
17        85        58.051701         58.051701         58.051701
17        86        65.253352         65.253352         65.253352
17        87        26.400758         26.400758         26.400758
17        88        20.000000         20.000000         20.000000
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17        91        37.656341         37.656341         37.656341
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17        94        62.169124         62.169124         62.169124
17        95        57.558666         57.558666         57.558666
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18        3        37.202150         37.202150         37.202150
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18        5        40.311289         40.311289         40.311289
18        6        44.721360         44.721360         44.721360
18        7        34.985711         34.985711         34.985711
18        8        33.541020         33.541020         33.541020
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18        10        14.142136         14.142136         14.142136
18        11        11.180340         11.180340         11.180340
18        12        9.433981         9.433981         9.433981
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18        15        5.000000         5.000000         5.000000
18        16        5.385165         5.385165         5.385165
18        17        5.000000         5.000000         5.000000
18        19        59.464275         59.464275         59.464275
18        20        54.671748         54.671748         54.671748
18        21        51.613952         51.613952         51.613952
18        22        56.568542         56.568542         56.568542
18        23        50.000000         50.000000         50.000000
18        24        55.172457         55.172457         55.172457
18        25        48.414874         48.414874         48.414874
18        26        53.150729         53.150729         53.150729
18        27        96.176920         96.176920         96.176920
18        28        95.524866         95.524866         95.524866
18        29        93.214806         93.214806         93.214806
18        30        90.553851         90.553851         90.553851
18        31        89.269256         89.269256         89.269256
18        32        88.566359         88.566359         88.566359
18        33        88.283634         88.283634         88.283634
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18        35        85.586214         85.586214         85.586214
18        36        78.032045         78.032045         78.032045
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18          42          65.299311          65.299311          65.299311
18          43          65.192024          65.192024          65.192024
18          44          68.007353          68.007353          68.007353
18          45          66.287254          66.287254          66.287254
18          46          42.059482          42.059482          42.059482
18          47          39.357337          39.357337          39.357337
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18          49          58.000000          58.000000          58.000000
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18          55          57.008771          57.008771          57.008771
18          56          33.541020          33.541020          33.541020
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18          59          38.078866          38.078866          38.078866
18          60          26.925824          26.925824          26.925824
18          61          21.213203          21.213203          21.213203
18          62          49.244289          49.244289          49.244289
18          63          65.764732          65.764732          65.764732
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18          65          47.434165          47.434165          47.434165
18          66          35.355339          35.355339          35.355339
18          67          41.773197          41.773197          41.773197
18          68          64.070274          64.070274          64.070274
18          69          42.720019          42.720019          42.720019
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18          72          65.764732          65.764732          65.764732
18          73          66.098411          66.098411          66.098411
18          74          15.132746          15.132746          15.132746
18          75          32.015621          32.015621          32.015621
18          76          40.311289          40.311289          40.311289
18          77          68.476273          68.476273          68.476273
18          78          47.885280          47.885280          47.885280
18          79          13.601471          13.601471          13.601471
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18          82          50.695167          50.695167          50.695167
18          83          27.073973          27.073973          27.073973
18          84          39.560081          39.560081          39.560081
18          85          59.203040          59.203040          59.203040
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18          87          29.698485          29.698485          29.698485
18          88          24.186773          24.186773          24.186773
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18          90          78.032045          78.032045          78.032045
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18          92          49.091751          49.091751          49.091751
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18          94          61.400326          61.400326          61.400326
18          95          57.078893          57.078893          57.078893
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18          97          55.731499          55.731499          55.731499
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19         7         74.672619         74.672619         74.672619
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19         15         55.865911         55.865911         55.865911
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19         17         56.222771         56.222771         56.222771
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19         20         5.385165         5.385165         5.385165
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19         28         59.169249         59.169249         59.169249
19         29         54.120237         54.120237         54.120237
19         30         54.918121         54.918121         54.918121
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19         38         79.812280         79.812280         79.812280
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19         41         81.584312         81.584312         81.584312
19         42         71.386273         71.386273         71.386273
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19         47         79.378838         79.378838         79.378838
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19        68        42.059482        42.059482        42.059482
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19        70        48.764741        48.764741        48.764741
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19        73        62.936476        62.936476        62.936476
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19        77        17.464249        17.464249        17.464249
19        78        21.095023        21.095023        21.095023
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19        80        73.573093        73.573093        73.573093
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19        82        53.235327        53.235327        53.235327
19        83        41.629317        41.629317        41.629317
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19        88        37.215588        37.215588        37.215588
19        89        56.648036        56.648036        56.648036
19        90        23.000000        23.000000        23.000000
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19        96        34.176015        34.176015        34.176015
19        97        50.219518        50.219518        50.219518
19        98        42.059482        42.059482        42.059482
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20        5        73.375745        73.375745        73.375745
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20        9        75.026662        75.026662        75.026662
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20        18        54.671748        54.671748        54.671748
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20        38        75.584390        75.584390        75.584390
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20        40        72.277244        72.277244        72.277244
20        41        77.129761        77.129761        77.129761
20        42        66.940272        66.940272        66.940272
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20        45        73.164199        73.164199        73.164199
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20        71        59.413803        59.413803        59.413803
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20        81        37.336309        37.336309        37.336309
20        82        48.507731        48.507731        48.507731
20        83        36.249138        36.249138        36.249138
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20        85        24.207437        24.207437        24.207437
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21          2          72.034714         72.034714         72.034714
21          3          63.245553         63.245553         63.245553
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21          5          68.622154         68.622154         68.622154
21          6          73.375745         73.375745         73.375745
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21          9          70.384657         70.384657         70.384657
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21          13          45.343136         45.343136         45.343136
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21          29          52.201533         52.201533         52.201533
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21          31          48.383882         48.383882         48.383882
21          32          50.159745         50.159745         50.159745
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21          34          44.147480         44.147480         44.147480
21          35          47.423623         47.423623         47.423623
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21          37          73.681748         73.681748         73.681748
21          38          70.837843         70.837843         70.837843
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21          40          67.446275         67.446275         67.446275
21          41          72.277244         72.277244         72.277244
21          42          62.096699         62.096699         62.096699
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21          78         22.472205         22.472205         22.472205
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21          95         36.249138         36.249138         36.249138
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22          7         73.375745         73.375745         73.375745
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22          10         42.426407         42.426407         42.426407
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22         30         58.309519        58.309519        58.309519
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22         33         53.235327        53.235327        53.235327
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22          90          27.000000          27.000000          27.000000
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22          96          35.777088          35.777088          35.777088
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22          98          38.275318          38.275318          38.275318
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23          7          63.906181          63.906181          63.906181
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23          9          69.641941          69.641941          69.641941
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23          11          39.051248          39.051248          39.051248
23          12          40.607881          40.607881          40.607881
23          13          43.863424          43.863424          43.863424
23          14          40.311289          40.311289          40.311289
23          15          46.097722          46.097722          46.097722
23          16          45.486262          45.486262          45.486262
23          17          47.169906          47.169906          47.169906
23          18          50.000000          50.000000          50.000000
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23          21          2.000000          2.000000          2.000000
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23        68        36.124784        36.124784        36.124784
23        69        45.000000        45.000000        45.000000
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23        71        54.230987        54.230987        54.230987
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23        73        55.036352        55.036352        55.036352
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23        76        36.400549        36.400549        36.400549
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24         95         47.010637         47.010637         47.010637
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24         99         48.507731         48.507731         48.507731
24         100         32.310989         32.310989         32.310989
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25         2         71.196910         71.196910         71.196910
25         3         62.096699         62.096699         62.096699
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25         5         67.446275         67.446275         67.446275
25         6         72.277244         72.277244         72.277244
25         7         63.245553         63.245553         63.245553
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25         9         68.949257         68.949257         68.949257
25         10         34.409301         34.409301         34.409301
25         11         37.536649         37.536649         37.536649
25         12         39.051248         39.051248         39.051248
25         13         42.426407         42.426407         42.426407
25         14         38.587563         38.587563         38.587563
25         15         44.598206         44.598206         44.598206
25         16         43.829214         43.829214         43.829214
25         17         45.486262         45.486262         45.486262
25         18         48.414874         48.414874         48.414874
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25         29         56.044625         56.044625         56.044625
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25        69        45.044423        45.044423        45.044423
25        70        37.656341        37.656341        37.656341
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25        73        55.901699        55.901699        55.901699
25        74        57.628118        57.628118        57.628118
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25        78        19.209373        19.209373        19.209373
25        79        50.803543        50.803543        50.803543
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25        87        19.235384        19.235384        19.235384
25        88        27.513633        27.513633        27.513633
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25        90        30.675723        30.675723        30.675723
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25        92        29.154759        29.154759        29.154759
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26        5        76.485293        76.485293        76.485293
26        6        81.394103        81.394103        81.394103
26        7        72.034714        72.034714        72.034714
26        8        72.801099        72.801099        72.801099
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28          72          36.055513          36.055513          36.055513
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28          76          94.868330          94.868330          94.868330
28          77          41.880783          41.880783          41.880783
28          78          78.790862          78.790862          78.790862
28          79          89.498603          89.498603          89.498603
28          80          94.921020          94.921020          94.921020
28          81          49.477268          49.477268          49.477268
28          82          51.429563          51.429563          51.429563
28          83          68.468971          68.468971          68.468971
28          84          58.137767          58.137767          58.137767
28          85          38.470768          38.470768          38.470768
28          86          34.176015          34.176015          34.176015
28          87          74.813100          74.813100          74.813100
28          88          83.725743          83.725743          83.725743
28          89          74.632433          74.632433          74.632433
28          90          41.036569          41.036569          41.036569
28          91          59.228372          59.228372          59.228372
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28          93          42.755117          42.755117          42.755117
28          94          38.013156          38.013156          38.013156
28          95          40.162171          40.162171          40.162171
28          96          39.051248          39.051248          39.051248
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29          4          89.022469          89.022469          89.022469
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29         77         36.715120         36.715120         36.715120
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32        59        77.162167        77.162167        77.162167
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32        61        77.162167        77.162167        77.162167
32        62        52.430907        52.430907        52.430907
32        63        23.000000        23.000000        23.000000
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32        65        43.289722        43.289722        43.289722
32        66        53.235327        53.235327        53.235327
32        67        47.042534        47.042534        47.042534
32        68        25.000000        25.000000        25.000000
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32        70        59.481089        59.481089        59.481089
32        71        62.968246        62.968246        62.968246
32        72        30.479501        30.479501        30.479501
32        73        39.051248        39.051248        39.051248
32        74        89.560036        89.560036        89.560036
32        75        69.634761        69.634761        69.634761
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32        77        36.235342        36.235342        36.235342
32        78        72.449983        72.449983        72.449983
32        79        82.710338        82.710338        82.710338
32        80        88.391176        88.391176        88.391176
32        81        42.720019        42.720019        42.720019
32        82        45.276926        45.276926        45.276926
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32        85        31.575307        31.575307        31.575307
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32        94        31.906112        31.906112        31.906112
32        95        33.615473        33.615473        33.615473
32        96        32.062439        32.062439        32.062439
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32        98        85.702975        85.702975        85.702975
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33        5        83.600239        83.600239        83.600239
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33        13        80.411442        80.411442        80.411442
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33        16        85.586214        85.586214        85.586214
33        17        87.572827        87.572827        87.572827
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33        20        49.244289        49.244289        49.244289
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33        22        53.235327        53.235327        53.235327
33        23        49.335586        49.335586        49.335586
33        24        55.009090        55.009090        55.009090
33        25        51.244512        51.244512        51.244512
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33        30        5.830952        5.830952        5.830952
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33        36        58.523500        58.523500        58.523500
33        37        59.236813        59.236813        59.236813
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33        43        59.363288        59.363288        59.363288
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33        48        86.313383        86.313383        86.313383
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33        59        74.726167        74.726167        74.726167
33        60        77.646635        77.646635        77.646635
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33        77        32.449961        32.449961        32.449961
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34        6        88.459030        88.459030        88.459030
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34        10        75.663730        75.663730        75.663730
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34        47        86.683332        86.683332        86.683332
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34         82         48.836462         48.836462         48.836462
34         83         60.728906         60.728906         60.728906
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34         85         28.284271         28.284271         28.284271
34         86         22.090722         22.090722         22.090722
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34         88         73.006849         73.006849         73.006849
34         89         69.354164         69.354164         69.354164
34         90         26.907248         26.907248         26.907248
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35         2         78.102497         78.102497         78.102497
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35         5         79.056942         79.056942         79.056942
35         6         82.006097         82.006097         82.006097
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35         8         80.622577         80.622577         80.622577
35         9         83.216585         83.216585         83.216585
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35         12         77.162167         77.162167         77.162167
35         13         77.646635         77.646635         77.646635
35         14         80.000000         80.000000         80.000000
35         15         80.622577         80.622577         80.622577
35         16         83.150466         83.150466         83.150466
35         17         85.146932         85.146932         85.146932
35         18         85.586214         85.586214         85.586214
35         19         50.803543         50.803543         50.803543
35         20         49.739320         49.739320         49.739320
35         21         47.423623         47.423623         47.423623
35         22         54.083269         54.083269         54.083269
35         23         49.244289         49.244289         49.244289
35         24         55.758407         55.758407         55.758407
35         25         51.078371         51.078371         51.078371
35         26         58.309519         58.309519         58.309519
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35         31         5.830952         5.830952         5.830952
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35        69        51.478151        51.478151        51.478151
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35        71        60.464866        60.464866        60.464866
35        72        28.284271        28.284271        28.284271
35        73        37.202150        37.202150        37.202150
35        74        86.683332        86.683332        86.683332
35        75        66.708320        66.708320        66.708320
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35        85        28.635642        28.635642        28.635642
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35        89        65.192024        65.192024        65.192024
35        90        34.985711        34.985711        34.985711
35        91        49.477268        49.477268        49.477268
35        92        36.674242        36.674242        36.674242
35        93        32.984845        32.984845        32.984845
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36          13         71.281134        71.281134        71.281134
36          14         79.649231        79.649231        79.649231
36          15         73.783467        73.783467        73.783467
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36          18         78.032045        78.032045        78.032045
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36          21         74.330344        74.330344        74.330344
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36          26         86.162637        86.162637        86.162637
36          27         61.717096        61.717096        61.717096
36          28         57.306195        57.306195        57.306195
36          29         60.415230        60.415230        60.415230
36          30         55.036352        55.036352        55.036352
36          31         58.872744        58.872744        58.872744
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36          49         83.815273        83.815273        83.815273
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36          51         50.249378        50.249378        50.249378
36          52         66.098411        66.098411        66.098411
36          53         69.202601        69.202601        69.202601
36          54         58.600341        58.600341        58.600341
36          55         27.730849        27.730849        27.730849
36          56         44.654227        44.654227        44.654227
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36          58         70.491134        70.491134        70.491134
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36          60         86.452299        86.452299        86.452299
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36        93        44.271887        44.271887        44.271887
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38        23        71.512237        71.512237        71.512237
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40        11       64.031242        64.031242        64.031242
40        12       65.604878        65.604878        65.604878
40        13       62.681736        62.681736        62.681736
40        14       71.063352        71.063352        71.063352
40        15       65.192024        65.192024        65.192024
40        16       70.455660        70.455660        70.455660
40        17       72.111026        72.111026        72.111026
40        18       69.462220        69.462220        69.462220
40        19       76.687678        76.687678        76.687678
40        20       72.277244        72.277244        72.277244
40        21       67.446275        67.446275        67.446275
40        22       77.620873        77.620873        77.620873
40        23       68.007353        68.007353        68.007353
40        24       78.160092        78.160092        78.160092
40        25       68.622154        68.622154        68.622154
40        26       79.056942        79.056942        79.056942
40        27       61.032778        61.032778        61.032778
40        28       57.008771        57.008771        57.008771
40        29       59.363288        59.363288        59.363288
40        30       54.083269        54.083269        54.083269
40        31       57.306195        57.306195        57.306195
40        32       53.000000        53.000000        53.000000
40        33       56.824291        56.824291        56.824291
40        34       60.415230        60.415230        60.415230
40        35       51.478151        51.478151        51.478151
40        36       8.602325        8.602325        8.602325
40        37       7.071068        7.071068        7.071068
40        38       5.385165        5.385165        5.385165
40        39       2.000000        2.000000        2.000000
40        41       5.000000        5.000000        5.000000
```

```
40        42        5.385165        5.385165        5.385165
40        43        5.000000        5.000000        5.000000
40        44        7.071068        7.071068        7.071068
40        45        5.385165        5.385165        5.385165
40        46        40.049969        40.049969        40.049969
40        47        42.000000        42.000000        42.000000
40        48        67.742158        67.742158        67.742158
40        49        77.129761        77.129761        77.129761
40        50        70.342022        70.342022        70.342022
40        51        46.572524        46.572524        46.572524
40        52        60.207973        60.207973        60.207973
40        53        61.032778        61.032778        61.032778
40        54        50.000000        50.000000        50.000000
40        55        20.615528        20.615528        20.615528
40        56        36.055513        36.055513        36.055513
40        57        46.097722        46.097722        46.097722
40        58        62.649820        62.649820        62.649820
40        59        83.216585        83.216585        83.216585
40        60        78.102497        78.102497        78.102497
40        61        49.244289        49.244289        49.244289
40        62        21.213203        21.213203        21.213203
40        63        45.276926        45.276926        45.276926
40        64        60.207973        60.207973        60.207973
40        65        52.201533        52.201533        52.201533
40        66        47.169906        47.169906        47.169906
40        67        47.010637        47.010637        47.010637
40        68        38.209946        38.209946        38.209946
40        69        28.284271        28.284271        28.284271
40        70        40.311289        40.311289        40.311289
40        71        27.313001        27.313001        27.313001
40        72        25.495098        25.495098        25.495098
40        73        15.297059        15.297059        15.297059
40        74        61.351447        61.351447        61.351447
40        75        72.111026        72.111026        72.111026
40        76        93.005376        93.005376        93.005376
40        77        68.000000        68.000000        68.000000
40        78        85.428333        85.428333        85.428333
40        79        57.271284        57.271284        57.271284
40        80        55.317267        55.317267        55.317267
40        81        35.468296        35.468296        35.468296
40        82        24.596748        24.596748        24.596748
40        83        49.578221        49.578221        49.578221
40        84        54.129474        54.129474        54.129474
40        85        51.088159        51.088159        51.088159
40        86        57.078893        57.078893        57.078893
40        87        68.242216        68.242216        68.242216
40        88        73.756356        73.756356        73.756356
40        89        42.190046        42.190046        42.190046
40        90        75.325958        75.325958        75.325958
40        91        40.224371        40.224371        40.224371
40        92        39.560081        39.560081        39.560081
40        93        37.656341        37.656341        37.656341
40        94        28.017851        28.017851        28.017851
40        95        32.140317        32.140317        32.140317
40        96        43.185646        43.185646        43.185646
40        97        26.476405        26.476405        26.476405
40        98        83.546394        83.546394        83.546394
40        99        44.045431        44.045431        44.045431
40        100        56.400355        56.400355        56.400355
40        101        31.780497        31.780497        31.780497
41        1        40.311289        40.311289        40.311289
41        2        35.000000        35.000000        35.000000
41        3        39.293765        39.293765        39.293765
```

```
41         4         38.000000         38.000000         38.000000
41         5         40.311289         40.311289         40.311289
41         6         40.000000         40.000000         40.000000
41         7         43.174066         43.174066         43.174066
41         8         46.097722         46.097722         46.097722
41         9         45.276926         45.276926         45.276926
41         10         70.710678         70.710678         70.710678
41         11         67.268120         67.268120         67.268120
41         12         68.767725         68.767725         68.767725
41         13         65.604878         65.604878         65.604878
41         14         74.330344         74.330344         74.330344
41         15         68.007353         68.007353         68.007353
41         16         73.409809         73.409809         73.409809
41         17         75.000000         75.000000         75.000000
41         18         72.111026         72.111026         72.111026
41         19         81.584312         81.584312         81.584312
41         20         77.129761         77.129761         77.129761
41         21         72.277244         72.277244         72.277244
41         22         82.462113         82.462113         82.462113
41         23         72.801099         72.801099         72.801099
41         24         82.969874         82.969874         82.969874
41         25         73.375745         73.375745         73.375745
41         26         83.815273         83.815273         83.815273
41         27         65.192024         65.192024         65.192024
41         28         61.032778         61.032778         61.032778
41         29         63.631753         63.631753         63.631753
41         30         58.309519         58.309519         58.309519
41         31         61.717096         61.717096         61.717096
41         32         57.306195         57.306195         57.306195
41         33         61.269895         61.269895         61.269895
41         34         65.000000         65.000000         65.000000
41         35         55.901699         55.901699         55.901699
41         36         7.000000         7.000000         7.000000
41         37         5.000000         5.000000         5.000000
41         38         5.830952         5.830952         5.830952
41         39         5.385165         5.385165         5.385165
41         40         5.000000         5.000000         5.000000
41         42         10.198039         10.198039         10.198039
41         43         7.071068         7.071068         7.071068
41         44         5.000000         5.000000         5.000000
41         45         5.830952         5.830952         5.830952
41         46         40.112342         40.112342         40.112342
41         47         42.296572         42.296572         42.296572
41         48         70.455660         70.455660         70.455660
41         49         82.000000         82.000000         82.000000
41         50         75.186435         75.186435         75.186435
41         51         51.419841         51.419841         51.419841
41         52         65.192024         65.192024         65.192024
41         53         65.192024         65.192024         65.192024
41         54         53.150729         53.150729         53.150729
41         55         25.495098         25.495098         25.495098
41         56         39.051248         39.051248         39.051248
41         57         50.990195         50.990195         50.990195
41         58         67.082039         67.082039         67.082039
41         59         87.464278         87.464278         87.464278
41         60         82.006097         82.006097         82.006097
41         61         51.478151         51.478151         51.478151
41         62         25.000000         25.000000         25.000000
41         63         50.249378         50.249378         50.249378
41         64         65.192024         65.192024         65.192024
41         65         57.008771         57.008771         57.008771
41         66         51.478151         51.478151         51.478151
41         67         51.623638         51.623638         51.623638
```

```
41         68         43.185646         43.185646         43.185646
41         69         32.015621         32.015621         32.015621
41         70         43.931765         43.931765         43.931765
41         71         29.681644         29.681644         29.681644
41         72         30.413813         30.413813         30.413813
41         73         20.223748         20.223748         20.223748
41         74         63.158531         63.158531         63.158531
41         75         76.321688         76.321688         76.321688
41         76         97.082439         97.082439         97.082439
41         77         73.000000         73.000000         73.000000
41         78         89.961103         89.961103         89.961103
41         79         59.539903         59.539903         59.539903
41         80         56.612719         56.612719         56.612719
41         81         40.162171         40.162171         40.162171
41         82         29.154759         29.154759         29.154759
41         83         53.413481         53.413481         53.413481
41         84         58.694122         58.694122         58.694122
41         85         56.080300         56.080300         56.080300
41         86         62.072538         62.072538         62.072538
41         87         72.401657         72.401657         72.401657
41         88         77.620873         77.620873         77.620873
41         89         45.000000         45.000000         45.000000
41         90         80.305666         80.305666         80.305666
41         91         44.418465         44.418465         44.418465
41         92         44.384682         44.384682         44.384682
41         93         42.579338         42.579338         42.579338
41         94         33.015148         33.015148         33.015148
41         95         37.121422         37.121422         37.121422
41         96         48.166378         48.166378         48.166378
41         97         31.400637         31.400637         31.400637
41         98         87.321246         87.321246         87.321246
41         99         47.381431         47.381431         47.381431
41         100         60.464866         60.464866         60.464866
41         101         34.132096         34.132096         34.132096
42         1         30.805844         30.805844         30.805844
42         2         34.481879         34.481879         34.481879
42         3         36.000000         36.000000         36.000000
42         4         37.363083         37.363083         37.363083
42         5         38.327536         38.327536         38.327536
42         6         39.293765         39.293765         39.293765
42         7         40.000000         40.000000         40.000000
42         8         43.000000         43.000000         43.000000
42         9         43.289722         43.289722         43.289722
42         10         62.481997         62.481997         62.481997
42         11         59.405387         59.405387         59.405387
42         12         61.032778         61.032778         61.032778
42         13         58.309519         58.309519         58.309519
42         14         66.400301         66.400301         66.400301
42         15         60.901560         60.901560         60.901560
42         16         66.037868         66.037868         66.037868
42         17         67.742158         67.742158         67.742158
42         18         65.299311         65.299311         65.299311
42         19         71.386273         71.386273         71.386273
42         20         66.940272         66.940272         66.940272
42         21         62.096699         62.096699         62.096699
42         22         72.277244         72.277244         72.277244
42         23         62.641839         62.641839         62.641839
42         24         72.801099         72.801099         72.801099
42         25         63.245553         63.245553         63.245553
42         26         73.681748         73.681748         73.681748
42         27         58.258047         58.258047         58.258047
42         28         54.488531         54.488531         54.488531
42         29         56.400355         56.400355         56.400355
```

```
42        30        51.224994        51.224994        51.224994
42        31        54.083269        54.083269        54.083269
42        32        50.000000        50.000000        50.000000
42        33        53.535035        53.535035        53.535035
42        34        56.824291        56.824291        56.824291
42        35        48.259714        48.259714        48.259714
42        36        13.453624        13.453624        13.453624
42        37        12.206556        12.206556        12.206556
42        38        9.899495        9.899495        9.899495
42        39        6.403124        6.403124        6.403124
42        40        5.385165        5.385165        5.385165
42        41        10.198039        10.198039        10.198039
42        43        5.830952        5.830952        5.830952
42        44        10.440307        10.440307        10.440307
42        45        7.615773        7.615773        7.615773
42        46        38.639358        38.639358        38.639358
42        47        40.311289        40.311289        40.311289
42        48        63.529521        63.529521        63.529521
42        49        71.805292        71.805292        71.805292
42        50        65.000000        65.000000        65.000000
42        51        42.379240        42.379240        42.379240
42        52        55.081757        55.081757        55.081757
42        53        55.803226        55.803226        55.803226
42        54        45.486262        45.486262        45.486262
42        55        15.297059        15.297059        15.297059
42        56        31.764760        31.764760        31.764760
42        57        40.792156        40.792156        40.792156
42        58        57.306195        57.306195        57.306195
42        59        77.935871        77.935871        77.935871
42        60        73.000000        73.000000        73.000000
42        61        45.541190        45.541190        45.541190
42        62        16.401219        16.401219        16.401219
42        63        40.607881        40.607881        40.607881
42        64        55.443665        55.443665        55.443665
42        65        46.840154        46.840154        46.840154
42        66        41.880783        41.880783        41.880783
42        67        41.629317        41.629317        41.629317
42        68        33.541020        33.541020        33.541020
42        69        23.430749        23.430749        23.430749
42        70        35.468296        35.468296        35.468296
42        71        23.769[illegible]        [illegible]        [illegible].769729
42        72        21.189[illegible]        [illegible]        [illegible].189620
42        73        11.180[illegible]        [illegible]        [illegible].180340
42        74        57.974[illegible]        [illegible]        [illegible].974132
42        75        66.850[illegible]        [illegible]        [illegible].850580
42        76        87.800911        87.800911        87.800911
42        77        63.031738        63.031738        63.031738
42        78        80.056230        80.056230        80.056230
42        79        53.488316        53.488316        53.488316
42        92        34.205263        34.205263        34.205263
42        93        32.388269        32.388269        32.388269
42        94        23.194827        23.194827        23.194827
42        95        27.018512        27.018512        27.018512
42        96        38.05[illegible]        [illegible]        [illegible]
42        97        21.213203        21.213203        21.213203
42        98        78.517514        78.517514        78.517514
42        99        39.408121        39.408121        39.408121
42        100        51.224994        51.224994        51.224994
```

See Also

- http://en.wikipedia.org/wiki/Vehicle_routing_problem

Indices and tables

- genindex
- search

Graph Transformation

pgr_lineGraph - Experimental

6.2.7 pgr_lineGraph - Experimental

`pgr_lineGraph` — Transforms a given graph into its corresponding edge-based graph.

[76]

Fig. 6.25: Boost Graph Inside

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.

[76] http://www.boost.org/libs/graph/doc/dijkstra_shortest_paths.html

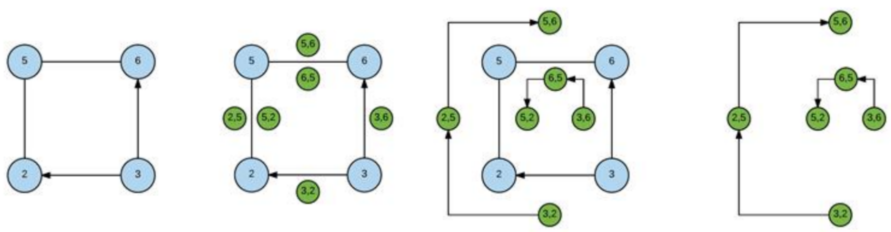
- pgTap tests might be missing.
- Might need c/c++ coding.
- May lack documentation.
- Documentation if any might need to be rewritten.
- Documentation examples might need to be automatically generated.
- Might need a lot of feedback from the comunity.
- Might depend on a proposed function of pgRouting
- Might depend on a deprecated function of pgRouting

Synopsis

Given a graph G, its line graph L(G) is a graph such that:-

- each vertex of L(G) represents an edge of G
- two vertices of L(G) are adjacent if and only if their corresponding edges share a common endpoint in G.

The following figures show a graph (left, with blue vertices) and its Line Graph (right, with green vertices).



Signature Summary

```
pgr_lineGraph(edges_sql, directed)
RETURNS SET OF (seq, source, target, cost, reverse_cost)
    OR EMPTY SET
```

Signatures

Minimal signature

```
pgr_lineGraph(edges_sql)
RETURNS SET OF (seq, source, target, cost, reverse_cost) or EMPTY SET
```

The minimal signature is for a **directed** graph:

Example

```
SELECT * FROM pgr_lineGraph(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table'
);
```

```
 seq | source | target | cost | reverse_cost
-----+--------+--------+------+--------------
   1 |    -16 |     -3 |    1 |           -1
   2 |    -15 |     -9 |    1 |            1
   3 |    -14 |    -10 |    1 |            1
   4 |    -14 |     12 |    1 |           -1
   5 |    -10 |     -7 |    1 |            1
   6 |    -10 |     -4 |    1 |            1
   7 |    -10 |      8 |    1 |            1
   8 |     -9 |     -8 |    1 |            1
   9 |     -9 |     11 |    1 |           -1
  10 |     -8 |     -7 |    1 |            1
  11 |     -8 |     -4 |    1 |            1
  12 |     -7 |     -6 |    1 |            1
  13 |     -4 |     -1 |    1 |            1
  14 |     -3 |     -2 |    1 |           -1
  15 |     -3 |      5 |    1 |           -1
  16 |     -2 |     -1 |    1 |           -1
  17 |     -2 |      4 |    1 |           -1
  18 |      5 |     -8 |    1 |           -1
  19 |      5 |      9 |    1 |           -1
  20 |      5 |     11 |    1 |           -1
  21 |      7 |     -4 |    1 |            1
  22 |      8 |     11 |    1 |           -1
  23 |     10 |     12 |    1 |           -1
  24 |     11 |     13 |    1 |           -1
  25 |     12 |     13 |    1 |           -1
  26 |     13 |    -15 |    1 |           -1
  27 |     16 |     -9 |    1 |            1
  28 |     16 |     15 |    1 |            1
(28 rows)
```

Complete Signature

```
pgr_lineGraph(edges_sql, directed);
RETURNS SET OF (seq, source, target, cost, reverse_cost) or EMPTY SET
```

This signature returns the Line Graph of the current graph:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Example

```
SELECT * FROM pgr_lineGraph(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    FALSE
);
 seq | source | target | cost | reverse_cost
-----+--------+--------+------+--------------
   1 |     -2 |     -1 |    1 |           -1
   2 |     -4 |     -1 |    1 |           -1
   3 |      4 |     -1 |    1 |           -1
   4 |      1 |      4 |    1 |           -1
   5 |     -2 |      4 |    1 |           -1
   6 |     -1 |      4 |    1 |           -1
   7 |     -2 |      1 |    1 |           -1
   8 |     -4 |      1 |    1 |           -1
   9 |      4 |      1 |    1 |           -1
```

```
 10 |      1 |     -2 |     1 |             -1
 11 |     -4 |     -2 |     1 |             -1
 12 |     -1 |     -2 |     1 |             -1
 13 |      4 |     -2 |     1 |             -1
 14 |      1 |     -4 |     1 |             -1
 15 |     -2 |     -4 |     1 |             -1
 16 |     -1 |     -4 |     1 |             -1
 17 |     -3 |     -2 |     1 |             -1
 18 |      5 |     -2 |     1 |             -1
 19 |     -3 |      5 |     1 |             -1
 20 |     -2 |      5 |     1 |             -1
 21 |     -2 |     -3 |     1 |             -1
 22 |      5 |     -3 |     1 |             -1
 23 |    -16 |     -3 |     1 |             -1
 24 |     16 |     -3 |     1 |             -1
 25 |     -3 |     16 |     1 |             -1
 26 |     -3 |    -16 |     1 |             -1
 27 |      7 |     -4 |     1 |             -1
 28 |     -8 |     -4 |     1 |             -1
 29 |    -10 |     -4 |     1 |             -1
 30 |     -7 |     -4 |     1 |             -1
 31 |      8 |     -4 |     1 |             -1
 32 |     10 |     -4 |     1 |             -1
 33 |      4 |     -7 |     1 |             -1
 34 |     -8 |     -7 |     1 |             -1
 35 |    -10 |     -7 |     1 |             -1
 36 |     -4 |     -7 |     1 |             -1
 37 |      8 |     -7 |     1 |             -1
 38 |     10 |     -7 |     1 |             -1
 39 |      4 |      8 |     1 |             -1
 40 |      7 |      8 |     1 |             -1
 41 |    -10 |      8 |     1 |             -1
 42 |     -4 |      8 |     1 |             -1
 43 |     -7 |      8 |     1 |             -1
 44 |     10 |      8 |     1 |             -1
 45 |      4 |     10 |     1 |             -1
 46 |      7 |     10 |     1 |             -1
 47 |     -8 |     10 |     1 |             -1
 48 |     -4 |     10 |     1 |             -1
 49 |     -7 |     10 |     1 |             -1
 50 |      8 |     10 |     1 |             -1
 51 |      7 |      4 |     1 |             -1
 52 |     -8 |      4 |     1 |             -1
 53 |    -10 |      4 |     1 |             -1
 54 |     -7 |      4 |     1 |             -1
 55 |      8 |      4 |     1 |             -1
 56 |     10 |      4 |     1 |             -1
 57 |      4 |      7 |     1 |             -1
 58 |     -8 |      7 |     1 |             -1
 59 |    -10 |      7 |     1 |             -1
 60 |     -4 |      7 |     1 |             -1
 61 |      8 |      7 |     1 |             -1
 62 |     10 |      7 |     1 |             -1
 63 |      4 |     -8 |     1 |             -1
 64 |      7 |     -8 |     1 |             -1
 65 |    -10 |     -8 |     1 |             -1
 66 |     -4 |     -8 |     1 |             -1
 67 |     -7 |     -8 |     1 |             -1
 68 |     10 |     -8 |     1 |             -1
 69 |      4 |    -10 |     1 |             -1
 70 |      7 |    -10 |     1 |             -1
 71 |     -8 |    -10 |     1 |             -1
 72 |     -4 |    -10 |     1 |             -1
```

```
 73 |      -7 |     -10 |     1 |            -1
 74 |       8 |     -10 |     1 |            -1
 75 |       5 |      -8 |     1 |            -1
 76 |      -9 |      -8 |     1 |            -1
 77 |       9 |      -8 |     1 |            -1
 78 |      11 |      -8 |     1 |            -1
 79 |       5 |       9 |     1 |            -1
 80 |       8 |       9 |     1 |            -1
 81 |      -8 |       9 |     1 |            -1
 82 |      11 |       9 |     1 |            -1
 83 |       5 |      11 |     1 |            -1
 84 |       8 |      11 |     1 |            -1
 85 |      -9 |      11 |     1 |            -1
 86 |      -8 |      11 |     1 |            -1
 87 |       9 |      11 |     1 |            -1
 88 |       8 |       5 |     1 |            -1
 89 |      -9 |       5 |     1 |            -1
 90 |      -8 |       5 |     1 |            -1
 91 |       9 |       5 |     1 |            -1
 92 |      11 |       5 |     1 |            -1
 93 |       5 |       8 |     1 |            -1
 94 |      -9 |       8 |     1 |            -1
 95 |       9 |       8 |     1 |            -1
 96 |      11 |       8 |     1 |            -1
 97 |       5 |      -9 |     1 |            -1
 98 |       8 |      -9 |     1 |            -1
 99 |      -8 |      -9 |     1 |            -1
100 |      11 |      -9 |     1 |            -1
101 |      -7 |      -6 |     1 |            -1
102 |       7 |      -6 |     1 |            -1
103 |       6 |       7 |     1 |            -1
104 |      -6 |       7 |     1 |            -1
105 |      -7 |       6 |     1 |            -1
106 |       7 |       6 |     1 |            -1
107 |       6 |      -7 |     1 |            -1
108 |      -6 |      -7 |     1 |            -1
109 |     -15 |      -9 |     1 |            -1
110 |      16 |      -9 |     1 |            -1
111 |      15 |      -9 |     1 |            -1
112 |     -16 |      -9 |     1 |            -1
113 |       9 |      15 |     1 |            -1
114 |      16 |      15 |     1 |            -1
115 |      -9 |      15 |     1 |            -1
116 |     -16 |      15 |     1 |            -1
117 |       9 |     -16 |     1 |            -1
118 |     -15 |     -16 |     1 |            -1
119 |      -9 |     -16 |     1 |            -1
120 |      15 |     -16 |     1 |            -1
121 |     -15 |       9 |     1 |            -1
122 |      16 |       9 |     1 |            -1
123 |      15 |       9 |     1 |            -1
124 |     -16 |       9 |     1 |            -1
125 |       9 |     -15 |     1 |            -1
126 |      16 |     -15 |     1 |            -1
127 |      -9 |     -15 |     1 |            -1
128 |     -16 |     -15 |     1 |            -1
129 |       9 |      16 |     1 |            -1
130 |     -15 |      16 |     1 |            -1
131 |      -9 |      16 |     1 |            -1
132 |      15 |      16 |     1 |            -1
133 |     -14 |     -10 |     1 |            -1
134 |      12 |     -10 |     1 |            -1
135 |      14 |     -10 |     1 |            -1
```

```
136 |       10 |       12 |      1 |              -1
137 |      -14 |       12 |      1 |              -1
138 |      -10 |       12 |      1 |              -1
139 |       14 |       12 |      1 |              -1
140 |       10 |       14 |      1 |              -1
141 |      -10 |       14 |      1 |              -1
142 |       12 |       14 |      1 |              -1
143 |      -14 |       10 |      1 |              -1
144 |       12 |       10 |      1 |              -1
145 |       14 |       10 |      1 |              -1
146 |       10 |      -14 |      1 |              -1
147 |      -10 |      -14 |      1 |              -1
148 |       12 |      -14 |      1 |              -1
149 |       11 |       13 |      1 |              -1
150 |       12 |       13 |      1 |              -1
151 |       12 |       11 |      1 |              -1
152 |       13 |       11 |      1 |              -1
153 |       11 |       12 |      1 |              -1
154 |       13 |       12 |      1 |              -1
155 |       13 |      -15 |      1 |              -1
156 |       15 |       13 |      1 |              -1
157 |      -15 |       13 |      1 |              -1
158 |       13 |       15 |      1 |              -1
(158 rows)
```

Description of the Signatures

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Column	Type	Description
edges_sql	`TEXT`	SQL query as described above.
directed	`BOOLEAN`	• When `true` the graph is considered as *Directed*. • When `false` the graph is considered as *Undirected*.

Description of the return values

RETURNS SETOF (seq, source, target, cost, reverse_cost)

Column	Type	Description
seq	`INTEGER`	Sequential value starting from **1**.
source	`BIGINT`	Identifier of the source vertex of the current edge *id*. • When *negative*: the source is the reverse edge in the original graph.
target	`BIGINT`	Identifier of the target vertex of the current edge *id*. • When *negative*: the target is the reverse edge in the original graph.
cost	`FLOAT`	Weight of the edge (*source*, *target*). • When *negative*: edge (*source*, *target*) does not exist, therefore it's not part of the graph.
reverse_cost	`FLOAT`	Weight of the edge (*target*, *source*). • When *negative*: edge (*target*, *source*) does not exist, therefore it's not part of the graph.

See Also

- https://en.wikipedia.org/wiki/Line_graph
- The queries use the *Sample Data* network.

Indices and tables

- genindex
- search

6.2.8 See Also

Indices and tables

- genindex
- search

CHAPTER 7

Change Log

- *pgRouting 2.5.1 Release Notes*
- *pgRouting 2.5.0 Release Notes*
- *pgRouting 2.4.2 Release Notes*
- *pgRouting 2.4.1 Release Notes*
- *pgRouting 2.4.0 Release Notes*
- *pgRouting 2.3.2 Release Notes*
- *pgRouting 2.3.1 Release Notes*
- *pgRouting 2.3.0 Release Notes*
- *pgRouting 2.2.4 Release Notes*
- *pgRouting 2.2.3 Release Notes*
- *pgRouting 2.2.2 Release Notes*
- *pgRouting 2.2.1 Release Notes*
- *pgRouting 2.2.0 Release Notes*
- *pgRouting 2.1.0 Release Notes*
- *pgRouting 2.0.1 Release Notes*
- *pgRouting 2.0.0 Release Notes*
- *pgRouting 1.x Release Notes*

7.1 Release Notes

To see the full list of changes check the list of Git commits[77] on Github.

[77] https://github.com/pgRouting/pgrouting/commits

Table of contents

7.1.1 pgRouting 2.5.1 Release Notes

To see the issues closed by this release see the Git closed milestone for 2.5.1[78] on Github.

Bug fixes

- Fixed prerequisite minimum version of: cmake

7.1.2 pgRouting 2.5.0 Release Notes

To see the issues closed by this release see the Git closed issues for 2.5.0[79] on Github.

enhancement:

- pgr_version is now on SQL language

Breaking change on:

- pgr_edgeDisjointPaths:
  - Added path_id, cost and agg_cost columns on the result
  - Parameter names changed

[78] https://github.com/pgRouting/pgrouting/issues?utf8=%E2%9C%93&q=milestone%3A%22Release%202.5.1%22%20
[79] https://github.com/pgRouting/pgrouting/issues?q=milestone%3A%22Release+2.5.0%22+is%3Aclosed

  - The many version results are the union of the one to one version

New Signatures:

- pgr_bdAstar(one to one)

New Proposed functions

- pgr_bdAstar(one to many)
- pgr_bdAstar(many to one)
- pgr_bdAstar(many to many)
- pgr_bdAstarCost(one to one)
- pgr_bdAstarCost(one to many)
- pgr_bdAstarCost(many to one)
- pgr_bdAstarCost(many to many)
- pgr_bdAstarCostMatrix
- pgr_bdDijkstra(one to many)
- pgr_bdDijkstra(many to one)
- pgr_bdDijkstra(many to many)
- pgr_bdDijkstraCost(one to one)
- pgr_bdDijkstraCost(one to many)
- pgr_bdDijkstraCost(many to one)
- pgr_bdDijkstraCost(many to many)
- pgr_bdDijkstraCostMatrix
- pgr_lineGraph
- pgr_connectedComponents
- pgr_strongComponents
- pgr_biconnectedComponents
- pgr_articulationPoints
- pgr_bridges

Deprecated Signatures

- pgr_bdastar - use pgr_bdAstar instead

Renamed Functions

- pgr_maxFlowPushRelabel - use pgr_pushRelabel instead
- pgr_maxFlowEdmondsKarp -use pgr_edmondsKarp instead
- pgr_maxFlowBoykovKolmogorov - use pgr_boykovKolmogorov instead
- pgr_maximumCardinalityMatching - use pgr_maxCardinalityMatch instead

Deprecated function

- pgr_pointToEdgeNode

7.1.3 pgRouting 2.4.2 Release Notes

To see the issues closed by this release see the Git closed milestone for 2.4.2[80] on Github.

Improvement

- Works for postgreSQL 10

Bug fixes

- Fixed: Unexpected error column "cname"
- Replace __linux__ with __GLIBC__ for glibc-specific headers and functions

7.1.4 pgRouting 2.4.1 Release Notes

To see the issues closed by this release see the Git closed milestone for 2.4.1[81] on Github.

Bug fixes

- Fixed compiling error on macOS
- Condition error on pgr_withPoints

7.1.5 pgRouting 2.4.0 Release Notes

To see the issues closed by this release see the Git closed issues for 2.4.0[82] on Github.

New Signatures

- pgr_bdDijkstra

New Proposed Signatures

- pgr_maxFlow
- pgr_astar(one to many)
- pgr_astar(many to one)
- pgr_astar(many to many)
- pgr_astarCost(one to one)
- pgr_astarCost(one to many)
- pgr_astarCost(many to one)

[80] https://github.com/pgRouting/pgrouting/issues?utf8=%E2%9C%93&q=milestone%3A%22Release%202.4.2%22%20
[81] https://github.com/pgRouting/pgrouting/issues?utf8=%E2%9C%93&q=milestone%3A%22Release%202.4.1%22%20
[82] https://github.com/pgRouting/pgrouting/issues?q=milestone%3A%22Release+2.4.0%22+is%3Aclosed

- pgr_astarCost(many to many)
- pgr_astarCostMatrix

Deprecated Signatures

- pgr_bddijkstra - use pgr_bdDijkstra instead

Deprecated Functions

- pgr_pointsToVids

Bug fixes

- Bug fixes on proposed functions
  - pgr_withPointsKSP: fixed ordering
- TRSP original code is used with no changes on the compilation warnings

7.1.6 pgRouting 2.3.2 Release Notes

To see the issues closed by this release see the Git closed issues for 2.3.2[83] on Github.

Bug Fixes

- Fixed pgr_gsoc_vrppdtw crash when all orders fit on one truck.
- Fixed pgr_trsp:
  - Alternate code is not executed when the point is in reality a vertex
  - Fixed ambiguity on seq

7.1.7 pgRouting 2.3.1 Release Notes

To see the issues closed by this release see the Git closed issues for 2.3.1[84] on Github.

Bug Fixes

- Leaks on proposed max_flow functions
- Regression error on pgr_trsp
- Types discrepancy on pgr_createVerticesTable

7.1.8 pgRouting 2.3.0 Release Notes

To see the issues closed by this release see the Git closed issues for 2.3.0[85] on Github.

[83] https://github.com/pgRouting/pgrouting/issues?q=milestone%3A%22Release+2.3.2%22+is%3Aclosed
[84] https://github.com/pgRouting/pgrouting/issues?q=milestone%3A%22Release+2.3.1%22+is%3Aclosed
[85] https://github.com/pgRouting/pgrouting/issues?q=milestone%3A%22Release+2.3.0%22+is%3Aclosed

New Signatures

- pgr_TSP
- pgr_aStar

New Functions

- pgr_eucledianTSP

New Proposed functions

- pgr_dijkstraCostMatrix
- pgr_withPointsCostMatrix
- pgr_maxFlowPushRelabel(one to one)
- pgr_maxFlowPushRelabel(one to many)
- pgr_maxFlowPushRelabel(many to one)
- pgr_maxFlowPushRelabel(many to many)
- pgr_maxFlowEdmondsKarp(one to one)
- pgr_maxFlowEdmondsKarp(one to many)
- pgr_maxFlowEdmondsKarp(many to one)
- pgr_maxFlowEdmondsKarp(many to many)
- pgr_maxFlowBoykovKolmogorov (one to one)
- pgr_maxFlowBoykovKolmogorov (one to many)
- pgr_maxFlowBoykovKolmogorov (many to one)
- pgr_maxFlowBoykovKolmogorov (many to many)
- pgr_maximumCardinalityMatching
- pgr_edgeDisjointPaths(one to one)
- pgr_edgeDisjointPaths(one to many)
- pgr_edgeDisjointPaths(many to one)
- pgr_edgeDisjointPaths(many to many)
- pgr_contractGraph

Deprecated Signatures

- pgr_tsp - use pgr_TSP or pgr_eucledianTSP instead
- pgr_astar - use pgr_aStar instead

Deprecated Functions

- pgr_flip_edges
- pgr_vidsToDmatrix
- pgr_pointsToDMatrix

- pgr_textToPoints

7.1.9 pgRouting 2.2.4 Release Notes

To see the issues closed by this release see the Git closed issues for 2.2.4[86] on Github.

Bug Fixes

- Bogus uses of extern "C"
- Build error on Fedora 24 + GCC 6.0
- Regression error pgr_nodeNetwork

7.1.10 pgRouting 2.2.3 Release Notes

To see the issues closed by this release see the Git closed issues for 2.2.3[87] on Github.

Bug Fixes

- Fixed compatibility issues with PostgreSQL 9.6.

7.1.11 pgRouting 2.2.2 Release Notes

To see the issues closed by this release see the Git closed issues for 2.2.2[88] on Github.

Bug Fixes

- Fixed regression error on pgr_drivingDistance

7.1.12 pgRouting 2.2.1 Release Notes

To see the issues closed by this release see the Git closed issues for 2.2.1[89] on Github.

Bug Fixes

- Server crash fix on pgr_alphaShape
- Bug fix on With Points family of functions

7.1.13 pgRouting 2.2.0 Release Notes

To see the issues closed by this release see the Git closed issues for 2.2.0[90] on Github.

[86] https://github.com/pgRouting/pgrouting/issues?q=milestone%3A%22Release+2.2.4%22+is%3Aclosed
[87] https://github.com/pgRouting/pgrouting/issues?q=milestone%3A%22Release+2.2.3%22+is%3Aclosed
[88] https://github.com/pgRouting/pgrouting/issues?q=milestone%3A%22Release+2.2.2%22+is%3Aclosed
[89] https://github.com/pgRouting/pgrouting/issues?q=milestone%3A2.2.1+is%3Aclosed
[90] https://github.com/pgRouting/pgrouting/issues?q=milestone%3A%22Release+2.2.0%22+is%3Aclosed

Improvements

- pgr_nodeNetwork
  - Adding a row_where and outall optional parameters
- Signature fix
  - pgr_dijkstra – to match what is documented

New Functions

- pgr_floydWarshall
- pgr_Johnson
- pgr_dijkstraCost(one to one)
- pgr_dijkstraCost(one to many)
- pgr_dijkstraCost(many to one)
- pgr_dijkstraCost(many to many)

Proposed functionality

- pgr_withPoints(one to one)
- pgr_withPoints(one to many)
- pgr_withPoints(many to one)
- pgr_withPoints(many to many)
- pgr_withPointsCost(one to one)
- pgr_withPointsCost(one to many)
- pgr_withPointsCost(many to one)
- pgr_withPointsCost(many to many)
- pgr_withPointsDD(single vertex)
- pgr_withPointsDD(multiple vertices)
- pgr_withPointsKSP
- pgr_dijkstraVia

Deprecated functions:

- pgr_apspWarshall use pgr_floydWarshall instead
- pgr_apspJohnson use pgr_Johnson instead
- pgr_kDijkstraCost use pgr_dijkstraCost instead
- pgr_kDijkstraPath use pgr_dijkstra instead

Renamed and deprecated function

- pgr_makeDistanceMatrix renamed to _pgr_makeDistanceMatrix

7.1.14 pgRouting 2.1.0 Release Notes

To see the issues closed by this release see the Git closed issues for 2.1.0[91] on Github.

New Signatures

- pgr_dijkstra(one to many)
- pgr_dijkstra(many to one)
- pgr_dijkstra(many to many)
- pgr_drivingDistance(multiple vertices)

Refactored

- pgr_dijkstra(one to one)
- pgr_ksp
- pgr_drivingDistance(single vertex)

Improvements

- pgr_alphaShape function now can generate better (multi)polygon with holes and alpha parameter.

Proposed functionality

- Proposed functions from Steve Woodbridge, (Classified as Convenience by the author.)
  - pgr_pointToEdgeNode - convert a point geometry to a vertex_id based on closest edge.
  - pgr_flipEdges - flip the edges in an array of geometries so the connect end to end.
  - pgr_textToPoints - convert a string of x,y;x,y;… locations into point geometries.
  - pgr_pointsToVids - convert an array of point geometries into vertex ids.
  - pgr_pointsToDMatrix - Create a distance matrix from an array of points.
  - pgr_vidsToDMatrix - Create a distance matrix from an array of vertix_id.
  - pgr_vidsToDMatrix - Create a distance matrix from an array of vertix_id.
- Added proposed functions from GSoc Projects:
  - pgr_vrppdtw
  - pgr_vrponedepot

Deprecated functions

- pgr_getColumnName
- pgr_getTableName
- pgr_isColumnCndexed
- pgr_isColumnInTable
- pgr_quote_ident

[91] https://github.com/pgRouting/pgrouting/issues?q=is%3Aissue+milestone%3A%22Release+2.1.0%22+is%3Aclosed

- pgr_versionless
- pgr_startPoint
- pgr_endPoint
- pgr_pointToId

No longer supported

- Removed the 1.x legacy functions

Bug Fixes

- Some bug fixes in other functions

Refactoring Internal Code

- A C and C++ library for developer was created
  - encapsulates postgreSQL related functions
  - encapsulates Boost.Graph graphs
    - Directed Boost.Graph
    - Undirected Boost.graph.
  - allow any-integer in the id's
  - allow any-numerical on the cost/reverse_cost columns
- Instead of generating many libraries: - All functions are encapsulated in one library - The library has the prefix 2-1-0

7.1.15 pgRouting 2.0.1 Release Notes

Minor bug fixes.

Bug Fixes

- No track of the bug fixes were kept.

7.1.16 pgRouting 2.0.0 Release Notes

To see the issues closed by this release see the Git closed issues for 2.0.0[92] on Github.

With the release of pgRouting 2.0.0 the library has abandoned backwards compatibility to *pgRouting 1.x* releases. The main Goals for this release are:

- Major restructuring of pgRouting.
- Standardization of the function naming
- Preparation of the project for future development.

As a result of this effort:

- pgRouting has a simplified structure

[92] https://github.com/pgRouting/pgrouting/issues?q=milestone%3A%22Release+2.0.0%22+is%3Aclosed

- Significant new functionality has being added
- Documentation has being integrated
- Testing has being integrated
- And made it easier for multiple developers to make contributions.

Important Changes

- Graph Analytics - tools for detecting and fixing connection some problems in a graph
- A collection of useful utility functions
- Two new All Pairs Short Path algorithms (pgr_apspJohnson, pgr_apspWarshall)
- Bi-directional Dijkstra and A-star search algorithms (pgr_bdAstar, pgr_bdDijkstra)
- One to many nodes search (pgr_kDijkstra)
- K alternate paths shortest path (pgr_ksp)
- New TSP solver that simplifies the code and the build process (pgr_tsp), dropped "Gaul Library" dependency
- Turn Restricted shortest path (pgr_trsp) that replaces Shooting Star
- Dropped support for Shooting Star
- Built a test infrastructure that is run before major code changes are checked in
- Tested and fixed most all of the outstanding bugs reported against 1.x that existing in the 2.0-dev code base.
- Improved build process for Windows
- Automated testing on Linux and Windows platforms trigger by every commit
- Modular library design
- Compatibility with PostgreSQL 9.1 or newer
- Compatibility with PostGIS 2.0 or newer
- Installs as PostgreSQL EXTENSION
- Return types re factored and unified
- Support for table SCHEMA in function parameters
- Support for `st_` PostGIS function prefix
- Added `pgr_` prefix to functions and types
- Better documentation: http://docs.pgrouting.org
- shooting_star is discontinued

7.1.17 pgRouting 1.x Release Notes

To see the issues closed by this release see the Git closed issues for 1.x[93] on Github. The following release notes have been copied from the previous `RELEASE_NOTES` file and are kept as a reference.

Changes for release 1.05

- Bug fixes

[93] https://github.com/pgRouting/pgrouting/issues?q=milestone%3A%22Release+1.x%22+is%3Aclosed

Changes for release 1.03

- Much faster topology creation
- Bug fixes

Changes for release 1.02

- Shooting* bug fixes
- Compilation problems solved

Changes for release 1.01

- Shooting* bug fixes

Changes for release 1.0

- Core and extra functions are separated
- Cmake build process
- Bug fixes

Changes for release 1.0.0b

- Additional SQL file with more simple names for wrapper functions
- Bug fixes

Changes for release 1.0.0a

- Shooting* shortest path algorithm for real road networks
- Several SQL bugs were fixed

Changes for release 0.9.9

- PostgreSQL 8.2 support
- Shortest path functions return empty result if they could not find any path

Changes for release 0.9.8

- Renumbering scheme was added to shortest path functions
- Directed shortest path functions were added
- routing_postgis.sql was modified to use dijkstra in TSP search

Indices and tables

- genindex
- search

Bibliography

[C001] Simulated annealing algorithm for beginners[46]

[46] http://www.theprojectspot.com/tutorial-post/simulated-annealing-algorithm-for-beginners/6

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