Math178 SU19
Homework 1
Due: Wed, May 29, 2019

Feel free to work with other students, but make sure you write up the homework and code on your own (no copying homework *or* code; no pair programming). Feel free to ask students or instructors for help debugging code or whatever else, though. The starter code for problem 2 part c and d can be found under the Resource tab on course website.

*Note:* You need to create a Github account for submission of the coding part of the homework. Please create a repository on Github to hold all your code and include your Github account username as part of the answer to problem 2.

**1** (**Covariance. (Lecture 1 page 17)**) The covariance between two random variables $X$ and $Y$ is defined as:

$$\mathrm{cov}[X, Y] = \mathbb{E}[(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])].$$

Prove that

$$\mathrm{cov}[X, Y] = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y].$$

■

**2** (**Correlation. (Lecture 1 page 18)**) The correlation between two random variables $X$ and $Y$ is defined as:

$$\operatorname{corr}[X, Y] = \frac{\operatorname{cov}[X, Y]}{\sqrt{\operatorname{var}[X]\operatorname{var}[Y]}}.$$

Prove that

(a) $-1 \leq \operatorname{corr}[X, Y] \leq 1$;

(b) $\operatorname{corr}[X, Y] = 1$ if and only if $Y = aX + b$ for some parameters $a$ and $b$.

$\blacksquare$

**3** (**Parametrization. (Lecture 1 page 50)**) Let $\alpha(t)$ be a parametrized curve which does not pass through the origin. If $\alpha(t_0)$ is the point of the trace of $\alpha$ closest to the origin and $\alpha'(t_0) \neq 0$, show that the position vector $\alpha(t_0)$ is orthogonal to $\alpha'(t_0)$.

∎

**4** (**Extra credit. (Lecture 1 page 52)**) How to create a transformation from the data on some helix to the data of the instructors trajectory?

■