Math178 SU19
Homework 1
Due: Wed, May 29, 2019

Feel free to work with other students, but make sure you write up the homework and code on your own (no copying homework *or* code; no pair programming). Feel free to ask students or instructors for help debugging code or whatever else, though.

Note: You need to create a Github account for submission of the coding part of the homework. Please create a repository on Github to hold all your code and include your Github account username as part of the answer to the coding problems.

1 (**Covariance. (Lecture 1 page 17)**) The covariance between two random variables X and Y is defined as:

$$\text{cov}[X, Y] = \mathbb{E}[(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])].$$

Prove that

$$\text{cov}[X, Y] = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y].$$

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2 (**Correlation. (Lecture 1 page 18)**) The correlation between two random variables X and Y is defined as:

$$\text{corr}[X, Y] = \frac{\text{cov}[X, Y]}{\sqrt{\text{var}[X]\text{var}[Y]}}.$$

Prove that

(a) $-1 \leq \text{corr}[X, Y] \leq 1$;

(b) $\text{corr}[X, Y] = 1$ if and only if $Y = aX + b$ for some parameters a and b.

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3 (**Parametrization. (Lecture 1 page 50)**) Let $\alpha(t)$ be a parametrized curve which does not pass through the origin. If $\alpha(t_0)$ is the point of the trace of α closest to the origin and $\alpha'(t_0) \neq 0$, show that the position vector $\alpha(t_0)$ is orthogonal to $\alpha'(t_0)$.

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4 (**Extra credit. (Lecture 1 page 52)**) How to create a transformation from the data on some helix to the data of the instructors trajectory?

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5 (**Coding. (Lecture 1 page 54-70)**) Please download the H-MOG dataset from: http://www.cs.wm.edu/ qyang/hmog.html (see also Lecture 1 page 54). Please read through the data description and do some visualizations if you have time. If you have any visualization result, please email or print out to submit.

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