UNIT – 2 INFERENTIAL STATISTICS II

PART – A

1. Why do we need hypothesis test?

Ans :

- ❖ The purpose of hypothesis testing is to determine whether there is enough statistical evidence in favor of a certain Belief, or hypothesis, about a parameter.

❖ Hypothesis testing is an essential procedure in statistics. A hypothesis test evaluates two mutually exclusive statements about a population to determine which statement is best supported by the sample data.

❖ Hypothesis tests permit us to draw conclusions that go beyond a limited set of actual observations.

❖ There is a crucial link between hypothesis tests and the need of investigators or researchers, to generalize beyond existing data.

2. What is the importance of standard Error?

Ans : To evaluate the effect of chance, we use the concept of a sampling distribution, that is , the concept of the sample means for all possible random outcomes. A key element in this concept is the standard error of the mean, a measure of the average amount by which sample means differ, just by chance, from the population mean.

3. Define strong decisions?

Ans : H0 is rejected whenever the observed z qualifies as a rare out-Come—one that could have occurred just by chance with a probability of .05 or less—On the assumption that H0 Is true. This suspiciously rare outcome implies that H0 is Probably false (and conversely, that H1 is probably true). Therefore, the rejection Can be viewed as a strong decision.

4. What is weak decision?

Ans : H0 is retained whenever the observed z qualifies as a Common outcome on the assumption that H0 is true. Therefore, H0 could be true. However, the same observed result also would qualify as a common outcome when The original value in H0 is replaced with a Slightly different value. Thus, the Retention of H0 must be viewed as a relatively weak decision.

5. What is one - tailed test?

Ans : A one-tailed test is a statistical test in which the critical area of a distribution is one-sided so that it is either greater than or less than a certain value, but not both. If the sample being tested falls Into the one-sided critical area, the alternative hypothesis will be accepted instead of the null hypothesis.

6. What is a two – tailed test?

Ans : A two-tailed test, in statistics, is a method in which the critical area of a distribution is two-sided And tests whether a sample is greater than or less than a certain range of values. It is used is Null-hypothesis testing and testing for statistical significance.

7. List the consequences of standard Error.

Ans : 1. It shrinks the upper retention region back toward the hypothesized population Mean of 100.

2. It shrinks the entire true sampling distribution toward the true population mean Of 103.

8. What is a power curve?

Ans : basically, a power curve shows how the likelihood of detecting any possible Effect—ranging

from very small to very large—varies for a fixed sample size

9. Define the term estimation.

Ans : Estimation (or estimating) is the process of finding an estimate, or approximation, which is a value that is usable for some purpose even if input data may be incomplete, uncertain, or unstable.

10. Give the formula of confidence intervals.

Ans : A confidence interval for μ uses a range of values that, with a known degree of Certainty,

Includes the unknown population mean.

$$\overline{X} \pm 1.96\sigma_{\overline{X}},$$

11. What is level of confidence in hypothesis

Ans : The level of confidence indicates the percent of time that a series of confidence intervals includes the unknown population characteristic, such as the population mean.

12. What is the effect of sample size in hypothesis?

Ans : The larger the sample size, the smaller the standard error and, hence, the more precise (narrower) the Confidence interval will be. Indeed, as the sample size grows larger, The standard error will approach Zero and the confidence interval will shrink to a point Estimate. Given this perspective, the sample size for a confidence interval, unlike that For a hypothesis test, never can be too large.

13. What are the two types of error.

Ans : Type I Error

Rejecting a true null hypothesis.

Type II Error

Retaining a false null hypothesis.

14. What is the probability of type I error.

Ans : The probability of a type I error, that is, the probability of rejecting a true null hypothesis, the level of significance.

15. What is The probability of detecting a Particular effect.

Ans : Power $(1 - \beta)$

16. Comment on point Estimate.

Ans : A range of values that, with a Known degree of certainty, includes An unknown population characteristics, such as a population mean.

PART – B

1. For each of the following situations, indicate whether H0 should Be retained or rejected

Given a one-tailed test, lower tail critical with $\alpha = .01$, and

(a) Z = – 2.34 (b) z = – 5.13 (c) z = 4.04

Given a one-tailed test, upper tail critical with $\alpha = .05$, and

(D) z = 2.00 (e) z = – 1.80 (f) z = 1.61

Ans : (a) Reject H0 at the .01 level of significance because z = –2.34 is more negative than –2.33.

(b) Reject H0 at the .01 level of significance because z = –5.13 is more negative than –2.33.

(c) Retain H0 at the .01 level of significance because z = 4.04 is less negative than –2.33. (The value of the observed z is in the direction of no concern.)

(d) Reject H0 at the .05 level of significance because z = 2.00 is more positive than 1.65.

(e) Retain H0 at the .05 level of significance because z = –1.80 is less positive than 1.65. (The value of the observed z is in the direction of no concern.)

(f) Retain H0 at the .05 level of significance because z = 1.61 is less positive Than 1.65.

2. Specify the decision rule for each of the following situations (referring to Table 11.1 to find critical z values):
   (a) A two-tailed test with $\alpha = .05$
   (b) A one-tailed test, upper tail critical, with $\alpha = .01$
   (c) a one-tailed test, lower tail critical, with $\alpha = .05$
   (d) A two-tailed test with $\alpha = .01$

Ans : (a) Reject H0 at the .05 level of significance if z equals or is more positive than 1.96 of if z equals or is more negative than –1.96.

(b) Reject H0 at the .01 level of significance if z equals or is more positive than 2.33.

(c) Reject H0 at the .05 level of significance if z equals or is more negative than –1.65.

(d) Reject H0 at the .01 level of significance if z equals or is more positive than 2.58 or if z equals or is more negative than –2.58.

3. (a) List the four possible outcomes for any hypothesis test.

(b) Under the U.S. Criminal Code, a defendant is presumed innocent until proven guilty. Viewing a criminal trial as a hypothesis test (with H0 specifying that the defendant is innocent), Describe each of the four possible outcomes.

Ans: (a) Correct decision (True H0 is retained); Type I error

Correct decision (False H0 is rejected); Type II error

(b)

	STATUS OF H_0	
DECISION	**TRUE H_0 (INNOCENT)**	**FALSE H_0 (GUILTY)**
Retain H_0 (Release)	*Correct Decision:* Innocent defendant is released.	*Type II Error:* Guilty defendant is released (Miss).
Reject H_0 (Sentence)	*Type I Error:* Innocent defendant is sentenced (False Alarm).	*Correct Decision:* Guilty defendant is sentenced.

4. In order to eliminate the type I error, someone decides to use The .00 level of significance. What's wrong with this procedure.

   Ans: A false H0 will never be rejected.

5. Indicate whether the following statements, all referring To Figure, are true or false:
   (a) He value of the true population mean (103) dictates the location of the true sampling Distribution.
   (b) The critical value of z (1.65) is based on the true sampling distribution.
   (c) Since the hypothesized population mean of 100 really is false, it would be impossible to Observe a sample mean value less than or equal to 100.
   (d) A correct decision would be made if the one observed sample mean has a value of 105.

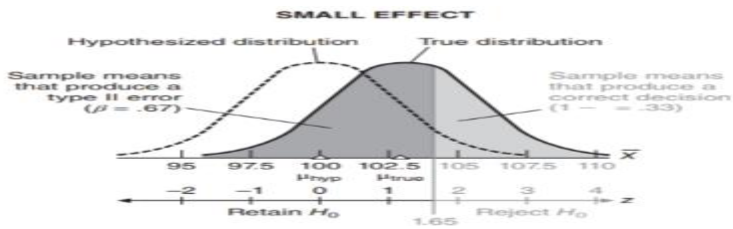
   Ans : (a) True

   (b) True

(c) False. The one observed sample means originates from the true sampling Distribution.

(d) False. If the one observed sample means has a value of 103, an incorrect Decision.

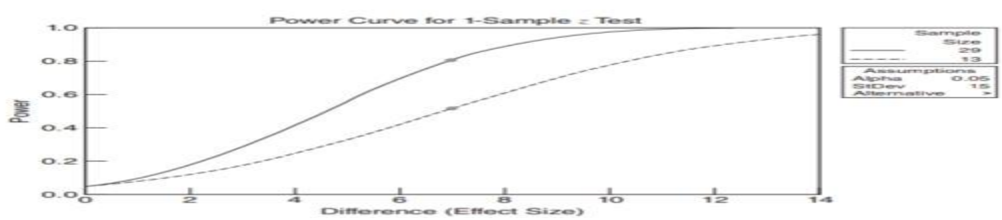
Would be made because the false H0 would be retained.



6. Consult the power curves in Figure to estimate the approximate detection rates, rounded to the nearest tenth, for the following situations:
   (a) A three-point effect, with a sample size of 29
   (b) A six-point effect, with a sample size of 13
   (c) a twelve-point effect, with a sample size of 13

Ans :

(a) .3
(b) .4
(c) .9



7. An investigator consults a chart to determine the sample size Required to detect an eight-point effect with a probability of .80. What happens to this detection rate of .80—will it actually be smaller, the same, or larger—if, unknown to the investigator, the true effect actually equals
(a) Twelve points?
(b) Five points?

Ans : (a) The power for the 12-point effect is larger than .80 because the true sampling distribution is shifted further into the rejection region for the false H0.

(b) The power for the 5-point effect is smaller than .80 because the true sampling distribution is shifted further into the retention region for the false H0.

8. Give two reasons why the research hypothesis is not tested directly.

Ans : Even though H0, the null hypothesis, is the focus of a statistical test, it is usually

of secondary concern to the investigator. Nevertheless, there are several reasons why

although, of primary concern, the research hypothesis is identified with H1 and tested

indirectly. In order to eliminate the type I error, someone decides to use the .00 level of significance. A false H0 will never be rejected

9. How should a projected hypothesis test be modified if you're particularly concerned About
(a) The type I error?
(b) the type II error?

Ans : (a) Type I error is an omission that happens when a null hypothesis is reprobated during hypothesis testing. This is when it is indeed precise or positive and should not have been initially disapproved. So if a null hypothesis is erroneously rejected when it is positive, it is called a Type I error.

(B) A Type Il error means a researcher or producer did not disapprove of the alternate hypothesis when it is in fact negative or false. This does not mean the null hypothesis is accepted as positive as hypothesis testing only indicates if a null hypothesis should be rejected. A Type Il error means a conclusion on the effect of the test wasn't recognized when an effect truly existed. Before a test can be said to have a real effect, it has to have a power level that is 80% or more.

10. A random sample of 200 graduates of U.S. colleges reveals a Mean annual income of $62,600. What is the best estimate of the unknown mean annual Income for all graduates of U.S. colleges?

Ans : 80,100 ± 2.58 6,000

100 = 81,648

78,552

(b) We can claim, with 99 percent confidence, that the interval between $78,552 and $81,648 includes the true population mean salary for all female members of the American Psychological Association. All of these values suggest that, on average, females' salaries are less than males' salaries.

11.Reading achievement scores are obtained for a group of fourth Graders. A score of 4.0 indicates a level of achievement appropriate for fourth grade, a score Below 4.0 indicates underachievement, and a score above 4.0 indicates overachievement. Assume that the population standard deviation equals 0.4. A random sample of 64 fourth graders reveals a mean achievement score of 3.82.

(a) Construct a 95 percent confidence interval for the unknown population mean. (Remember To convert the standard deviation to a standard error.)
(b) Interpret this confidence interval; that is, do you find any consistent evidence either of Overachievement or of underachievement

Ans :

(a) $3.82 \pm 1.96\left(\frac{.4}{\sqrt{64}}\right) = \begin{cases} 3.92 \\ 3.72 \end{cases}$

(b) We can claim, with 95 percent confidence, that the interval between 3.72 and 3.92 includes the *true population mean* reading score for the fourth graders. All of these values suggest that, on average, the fourth graders are underachieving.

12. We can claim, with 99 percent confidence, that the interval between $78,552 and $81,648 includes the true population mean salary for all female members of the American Psychological Association. All of these values suggest that, on average, females' salary s Received special training on how to take the test. After analyzing their scores on the GRE, the investigator reported a dramatic gain, relative to the national average of 500, as indicated by A 95 percent confidence interval of 507 to 527. Are the following interpretations true or false?

(a) About 95 percent of all subjects scored between 507 and 527.
(b) The interval from 507 to 527 refers to possible values of the population mean for all students who undergo special training.
(c) The true population mean definitely is between 507 and 527.
(d) This particular interval describes the population mean about 95 percent of the time.
(e) In practice, we never really know whether the interval from 507 to 527 is true or false.
(f)) We can be reasonably confident that the population mean is between 507 and 527.

Ans :

(a) False. We can be 95 percent confident that the mean for all subjects will be between 507 and 527.
(b) True
(c) False. We can be reasonably confident—but not absolutely confident—that the true population mean lies between 507 and 527.
(d) False. This particular interval either describes the one true population mean or fails to describe the one true population mean.
(e) True
(f) True

13. On the basis of a random sample of 120 adults, a pollster reports, With 95 percent confidence, that between 58 and 72 percent of all Americans believe in life After death.

(a) If this interval is too wide, what, if anything, can be done with the existing data to obtain a Narrower confidence interval?
(b) What can be done to obtain a narrower 95 percent confidence interval if another similar

Investigation is being planned?

Ans : (a) Switch to an interval having a lesser degree of confidence, such as 90 percent or75 percenan

(B) Increase the sample size.

14. In a recent scientific sample of about 900 adult Americans, 70 percent favor stricter gun control of assault weapons, with a margin of error of ±4 percent for A 95 percent confidence interval. Therefore, the 95 percent confidence interval equals 66 to 74 percent. Indicate whether the following interpretations are true or false:

(a) The interval from 66 to 74 percent refers to possible values of the sample percent.
(b) The true population percent is between 66 and 74 percent.
(c) In the long run, a series of intervals similar to this one would fail to include the population percent about 5 percent of the time.
(d) We can be reasonably confident that the population percent is between 66 and 74 percent.

Ans : (a) False. The interval from 66 to 74 percent refers to possible values of the population

Proportion.

(b) False. We can be reasonably confident—but not absolutely confident—that the

true population proportion is between 66 and 74 percent.

(c) True

15.Imagine that one of the following 95 percent confidence intervals estimates the Effect of vitamin C on IQ scores:

95% CONFIDENCE INTERVAL	LOWER LIMIT	UPPER LIMIT
1	100	102
2	95	99
3	102	106
4	90	111
5	91	98

(a) Which one most strongly supports the conclusion that vitamin C increases IQ scores?
(b) Which one implies the largest sample size?
(c) Which one most strongly supports the conclusion that vitamin C decreases IQ Scores?
(d) Which one would most likely stimulate the investigator to conduct an additional Experiment using larger sample sizes?

Ans : (a) 3 (b) 1 (c) 5 (d) 4

15. Case study

16. Explain in detail about confidence interval (CI) by giving relevant example

Ans : Refer topic 12.2 pg 222 to 225

17. Comment on hypothesis test for confidence interval.

Ans : Refer topic 12.6 pg 228

18.

1. A factory has a machine that dispenses 80 mL of fluid in a bottle. An employee believes the average amount of fluid is not 80 mL. Using 40 samples, he measures the average amount dispensed by the machine to be 78 mL with a standard deviation of 2.5. (a) State the null and alternative hypotheses. (b) At a 95% confidence level, is there enough evidence to support the idea that the machine is not working properly?

Ans : https://youtu.be/zJ8e_wAWUzE

19.

In a local teaching district a technology grant is available to teachers in order to install a cluster of four computers in their classrooms. From the 6250 teachers in the district, 250 were randomly selected and asked if they felt that computers were an essential teaching tool for their classroom. Of those selected, 142 teachers felt that computers were an essential teaching tool.

1. Calculate a 99% confidence interval for the proportion of teachers who felt that computers are an essential teaching tool.
2. How could the survey be changed to narrow the confidence interval but to maintain the 99% confidence interval?

Ans : https://youtu.be/SeQeYVJZ2gE

20. In a mythical national survey, 225 students are randomly selected from Those taking calculus, and asked if calculus is their favorite subject. 100 Students reply that calcululs is their favorite subject. Give a 95% confidence Interval for the proportion of all students taking calculus who consider it their Favorite subject.

Ans :

2. In a mythical national survey, 225 students are randomly selected from those taking calculus, and asked if calculus is their favorite subject. 100 students reply that calcululs is their favorite subject. Give a 95% confidence interval for the proportion of all students taking calculus who consider it their favorite subject.

SOLUTION

We will plug into the 95%confidence interval formula for population proportion,

$$\left(\widehat{p} - 1.96\frac{\sqrt{\widehat{p}(1-\widehat{p})}}{\sqrt{n}},\ \widehat{p} + 1.96\frac{\sqrt{\widehat{p}(1-\widehat{p})}}{\sqrt{n}}\right)$$

Here $\widehat{p} = 100/225 = 20/45 = 4/9$ and $n = 225$, so the interval is

$$= \left(4/9 - 1.96\frac{\sqrt{(4/9)(5/9)}}{\sqrt{225}},\ 4/9 + 1.96\frac{\sqrt{(4/9)(5/9)}}{\sqrt{225}}\right)$$

$$= \left(4/9 - 1.96\frac{\sqrt{20}}{(9)(15)},\ 4/9 + 1.96\frac{\sqrt{20}}{(9)(15)}\right)$$

$$\approx (.38,\ .51)$$

21. Suppose X1, . . . , X100 are i.i.d random variables which have uniform distribution 100 [a – 2, + 2], where a is unknown. Suppose the random sample Produces sample mean equal to 3.Compute a 95% confidence interval for a.

Ans :

A random variable with uniform distribution on $[a-2, a+2]$ has mean $\mu = a$. So, a confidence interval for μ is a confidence interval for a. Because $n = 100$ is large, the confidence interval provided by the Central Limit Theorem applies:

$$\left(\overline{X} - 1.96\frac{\sigma}{\sqrt{n}},\ \overline{X} + 1.96\frac{\sigma}{\sqrt{n}}\right)$$

A random variable with uniform distribution on $[a-2, a+2]$ has standard deviation $\sigma = 4/\sqrt{12}$. Our sample mean is 3. Substituting, we get

$$\left(3 - (1.96)\frac{(4/\sqrt{12})}{\sqrt{100}},\ 3 + (1.96)\frac{(4/\sqrt{12})}{\sqrt{100}}\right)$$
$$= (2.73,\ 3.27)\ .$$