# Unit - II

## Inferential Statistics - II

why hypothesis test ?

Hypothesis testing :-

Can be used to determine whether a statement about the value of the population parameter should or should not be rejected.

why hypothesis testing is performed ?.

Hypothesis testing is a form of inferential statistics that allows to draw conclusions about an entire population based on a representative sample.

* The gain tremendous benefits by working with a sample.

In most cases, it is simply impossible to observe the entire population to understand its properties.

The only alternative is to collect a random sample and then use statistics to analyze it.

While samples are much more pratical and less expensive to work with, there are trade - offs. exchang one, for the use of another

When you estimate the properties of a population from a sample, the same statistics are unlikely to equal the actual population value exactly.

for instance, the sample mean is unlikely to equal population mean.

The difference between the sample statistic and the population value is the sample error.

population effect:

The effect is the difference between the population value and the null hypothesis value.

The effect is also known as population effect or the difference.

null hypothesis

[refer unit -I notes page 17]

alternate hypothesis:-

[refer Unit -I notes page 18]

P-values:-

Pvalues tells how strongly the sample data contradict the null.

Lower p-values represent stronger evidence against the null.

p-value in conjunction with the significance level to determine whether your data favor the null or alternate hypothesis.

level of Significance ($\alpha$):-

[refer unit -I notes page 18]

# Types of errors in hypothesis testing :-



Statistical hypothesis tests are not 100% accurate because they use a random sample to draw conclusions about entire population.

They are 2 two types of error :-

False positives :-

can reject a null that is true.

Statisticians call this a Type I error.

The type I error rate equals significance level or alpha ($\alpha$).

False negatives :-

~~yo~~ Can fail to reject a null that is false.

Statisticians call this a Type II error.

It is a larger risk when you have a small sample size, noisy data (or) a small effect size.

The type II error rate is known as <u>beta</u> ($\beta$)

Strong or weak decisions :-

$H_0$ – null (retain)

null ah reject pannaradhu Strong decision

Retaining $H_0$ is a weak decisions

rejecting $H_0$ is a strong decision

→ TV consumption influences Sleep (example).

→ people who watch more than three hours of TV daily will ~~kos~~ wake up more frequently during the night than people who watch less than 3 hrs of TV daily.

One tailed and Two tailed Tests:

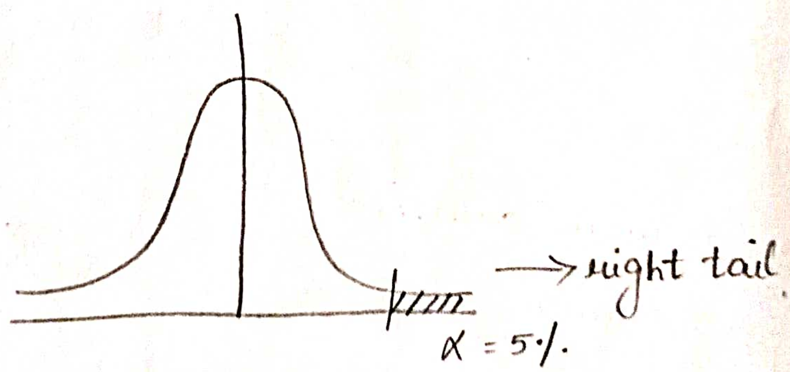
One tailed Test:

One tailed hypothesis tests are also known as directional, because the test for effects is only one direction.

When one tailed test is performed, the entire significance level percentage goes into the extreme end of one tail of the distribution.

$\alpha$ of 5% ; each distribution Must determine whether the critical region is in the right or left tail.

right tail :-



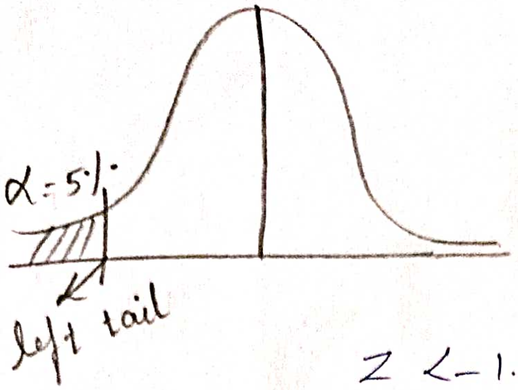

$H_0 : \mu = \mu_0$

$H_1 : \mu > \mu_0$

$Z < 1.645$, $H_0$ is accepted.

$Z > 1.645$, $H_0$ is rejected.

left tail :-




$H_0 : \mu = \mu_0$

$H_1 : \mu < \mu_0$

$Z < -1.645$, $H_0$ is rejected.

$Z > -1.645$, $H_0$ is accepted.

Merits :-

Requires less traffic.

Gains significance faster.

demerits :-

Only accounts for one scenario.

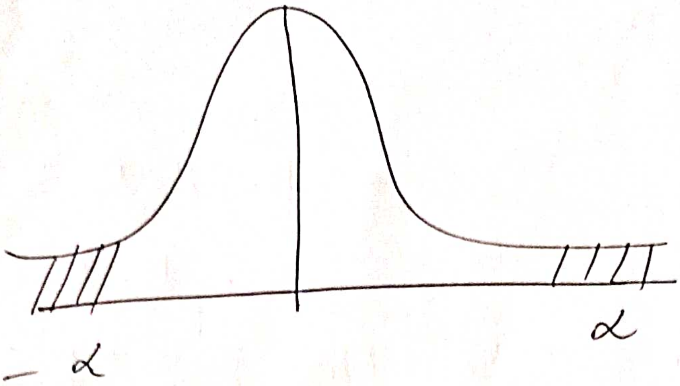
Can lead to inaccurate and biased results.

[example : refer unit -I notes pg 22]

Two-tailed test :-

Two-tailed tests are also known as nondirectional and two-sided tests because you can test for effects in both directions.

When you can perform a two-tailed test, you can split the significance level percentage between both tails of the distribution.




$H_0 : \mu = \mu_0$

$H_1 = \mu \neq \mu_0$

Merits:-

Accounts for all three scenarios.

Leads to accurate and reliable resources.

Demerits :-

requires more traffic.

Takes longer to gain significance.

Conditions :-

| $H_0$ | HA | |
|---|---|---|
| = | $\neq$ | 2 - tailed |
| $\leq$ | > | right tailed |
| $\geq$ | < | left - tailed |

# Influence of Sample Size :-

Sample size refers to the number of participants or observations included in a study.

This number is usually represented by 'n'.

The size of a sample influences two statistical properties :-

* the precision of our estimates.
* the power of the study to draw conclusions.

## Determing the Sample Size in estimation :-

↳ focus on a close value / estimated value close to the mean.

$$Z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}}$$

Suppose a university is performing a survey of the annual earnings of last year's gradates from its business school. It knows from the past experience that the SD of the annual earnings of the entire population 1000 of these graduates is about $1500. How large sample size should the university take in order to estimate the mean annual earnings of last year's class within $500 and 95% confidence level ?

$$Z\sigma_x = \frac{x - \mu}{\sigma_x}$$

$$Z = \frac{(x - \mu)}{\sigma_{\bar{x}}}$$

$$\underset{\text{sample}}{(x} - \underset{\text{pop}}{\mu)} = 500$$

$$\boxed{Z\sigma_{\bar{x}}} = \$500$$

$Z = 1.96$ (95%)

Then $1.96\,\sigma_{\bar{x}} = \$500$ $\qquad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

$$\boxed{\sigma_{\bar{x}} = \frac{\$500}{1.96}}$$

$\sigma_{\bar{x}} = \$255$ (standard error of the mean)

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$\$255 = \frac{\$1500}{\sqrt{n}}$$

$$\sqrt{n} = 5.882$$

$$n = 34.6$$

(A sample of 35 should be taken)

$$\boxed{n \simeq 35}$$

In this, the samples can be large or small.

The larger samples tend to be associated with a smaller margin of error.

However there is a point at which increasing sample size no longer impacts the sampling error.

This phenomenon is known as the "law of diminishing returns".

## Power and Sample Size:-

power refers to the probability of finding a statisti-cally significant result.

The power can be specified by two scerario's.

They are:

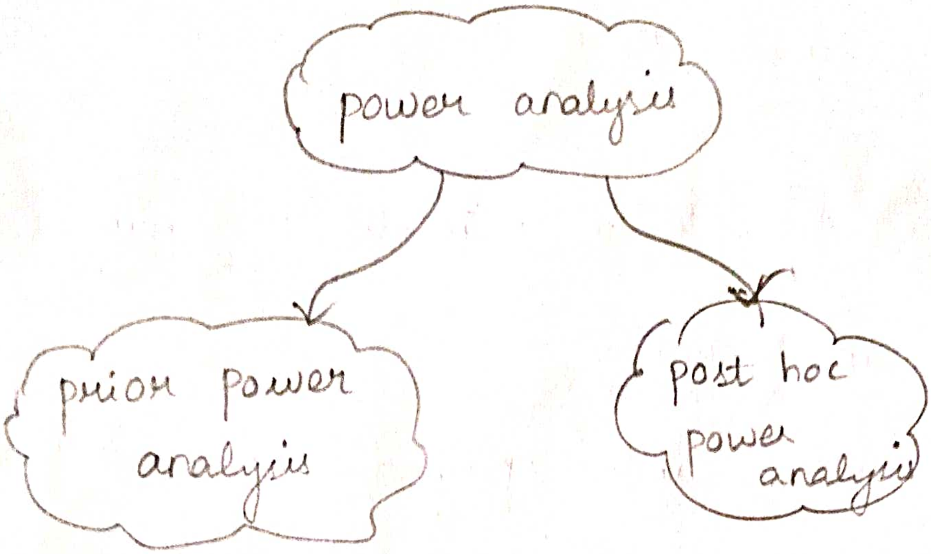
* null hypothesis
* alternate hypothesis

The probability of correctly rejecting the null hypothesis is equal to $(1-\beta)$ which is called power.

The power of of a test refers to its ability to detect what it is looking for.

power $(1-\beta) \rightarrow$ probability that the test will correctly identify a significance difference, effect or association in the sample should one exist in the population.

The larger sample size the study will have the greater power to detect significance difference, effect or association.




**prior power analysis:-**

That examines the relationships among multiple parameters, including the Complexity associated with human participants.

In other words,
Used to calculate the sample Size N, which is necessary to determine the effect Size, desired $\alpha$ level & power level $(1-\beta)$.

**post Hoc power analysis:-**

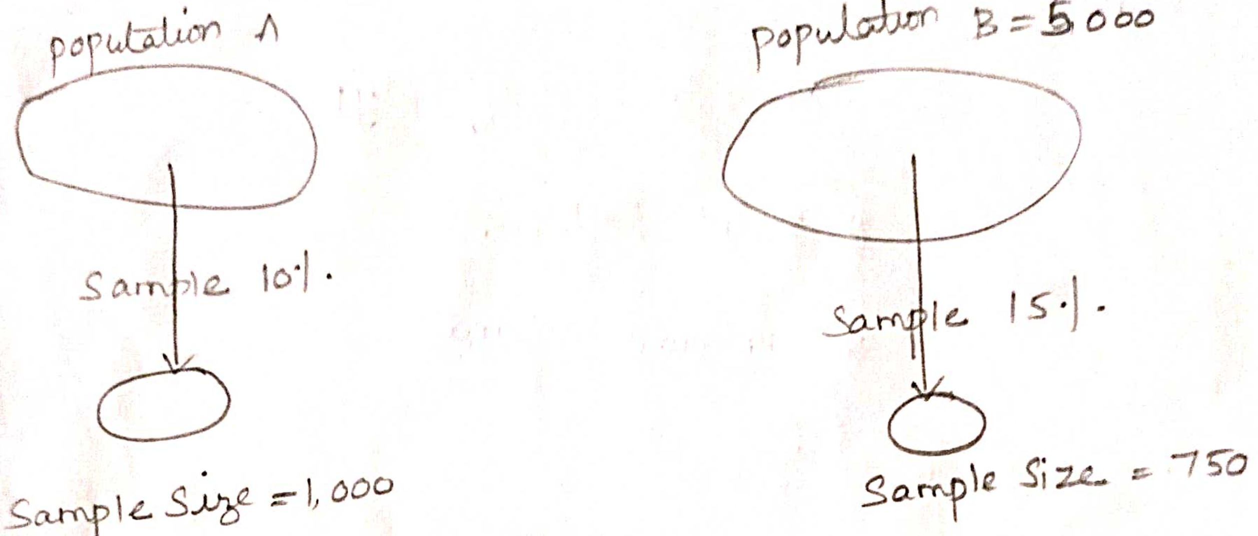
Analyze the results of the experimental data.

**Sample Size:-**

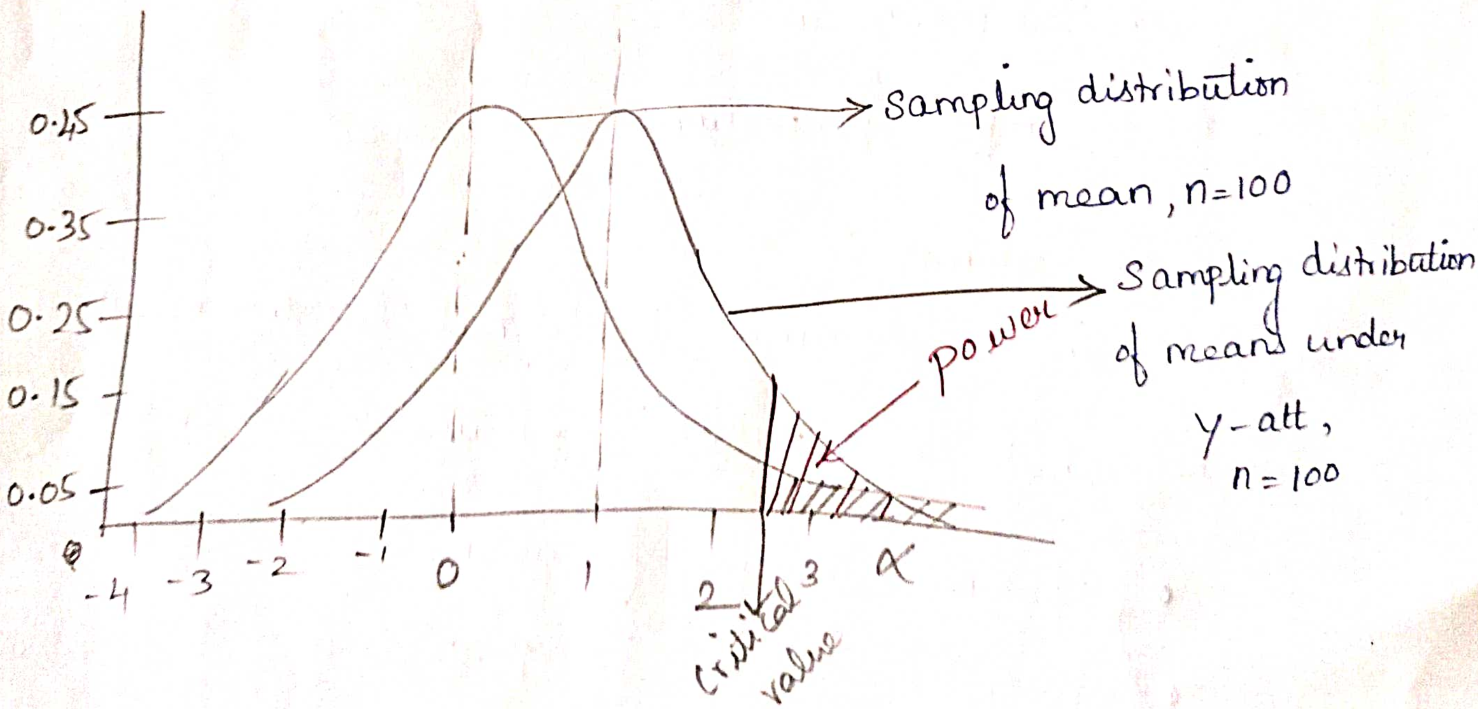
The Goal is to make the inferences about a population from a sample.

In census, the sample size is equal to the popula-tion size.

The larger the sample size will be more accurate from the study.




power of Sample Size :-



## Estimate :-

used to determine the approximate value of a population parameter on the basis of a sample statistic.

Estimate can be of 2 types :-

* point Estimate
* Interval estimate.

## point Estimate :-

A point estimate of a population parameter is a single value of a statistic.

- Sample mean
- Sample proportion.

## Interval estimate :-

A interval estimate is defined by two numbers, between which a population parameter is said to lie.

- Confidence interval for mean.
- Confidence interval for proportion.

It is said to be Confidence interval.

It has lower confidence limit and upper confidence limit.

[point estimate always lies within the interval estimate]

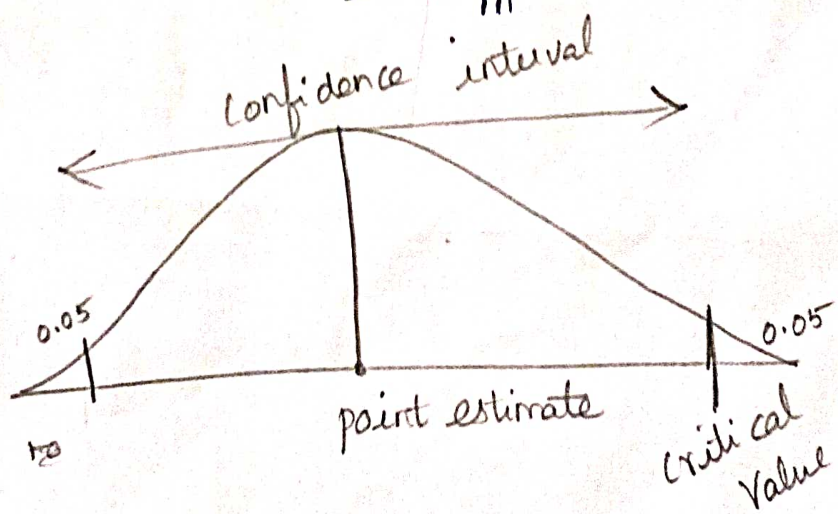
$$\boxed{\text{point estimate} \pm (\text{critical value})\ (\text{standard error})}$$

| $(1-\alpha)*100\%$ | $\alpha$ | $\alpha/2$ | $Z_{\alpha/2}$ |
|---|---|---|---|
| 90% | 0.1 | 0.05 | 1.645 |
| 95% | 0.05 | 0.025 | 1.96 |
| 99% | 0.01 | 0.005 | 2.58 |

The confidence interval can be constructed by 3 factors:-

* The point estimate of the population
* The level of confidence
* The standard deviation of the sample mean.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$



## Margin of error:-

This is the level of precision.

It is the range in which the value that you are trying to measure is estimated to be and is often expressed in percentage points (eg ± 2%)

A narrow margin of error requires a large sample size.

## Effect Size :-

Estimated difference between the groups that we observe in our sample.

To detect the difference with a specified power, a smaller effect size will require a larger sample size.

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