Unit - III

T - test

T - test :

It is a type of inferential statistic used to determine if there is a significant difference between the means of 2 groups, which may be relgated in certain features.

There are 3 Key data values :-

1) difference between the mean values from each data set (called mean difference).
2) SD of each group.
3) No. of data values for each group.

The t-test is one of many tests used for the purpose of hypothesis testing in statistics.

There are several different types of t-test than can be performed depending on the data & type of analysis required.

Example :
Take a sample of students from class A and another sample of students from Class B, we would not expect them to have exactly the same mean and SD.

Mathematically, the t-test takes ~~from~~ a sample from each of two sets and establishes the problem statement by assuming a null hypothesis that the two means are equal.

Based on the applicable formulas, certain values are calculated and compared against the standard values, and the assumed null hypothesis is accepted or rejected accordingly.

If the null hypothesis qualifies to be rejected, it indicates that data readings are strong and are probably not due to chance.

The t-test is just one of many tests used for this purpose.

Three types of t-test :-

1) One Sample t-test
2) Paired Sample t-test
3) Independent Sample t-test.

One Sample t-test :-

It is used to determine whether an unknown population mean is different from a specific value.

Assumptions :-

For a valid test, we need data values that are :

1) Independent (values are not related to one another).

2) Continuous.

3) Obtained via a simple random sample from the population.

Example :-

your Company wants to improve sales. past Sales data indicate that the avgerage sale was \$100 per transaction. After training your sales forces, recent Sales data (taken from a Sample of 25 salesmen) indicates an average sale of \$130, with SD of \$15. Did the training work? Test your hypothesis at a 5% alpha level?

sol:-

null hypothesis

$H_0 : \mu = \$100$

Alternate hypothesis

$H_1 : \mu > \$100$

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

$\bar{x} = \$130$

$\mu = \$100$

$S = \$15$

$n = 25$

$$t = \frac{130 - 100}{15/\sqrt{25}}$$

$$= \frac{30}{3}$$

$$\boxed{t = 10}$$

$\alpha = 0.05$

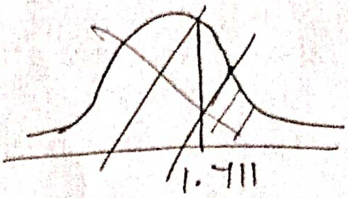
$df = n-1$

$df = 25-1$

$\boxed{df = 24}$

Table value $= 1.711$

t-score $= 10$



$\therefore$ t-value $= 1.711$

$\therefore$ Reject null hypothesis:-

$10 > 1.711$, reject H_0.

② we have the potato yield from 12 different farms. We know that the standard potato yield for the given variety is $\mu = 20$?

$X = [21.5, 24.5, 18.5, 17.2, 14.5, 23.2, 22.1, 20.5, 19.4, 18.1, 24.1, 18.5]$ Test if the potato yield from these farms is significantly better than the standard yield?

$H_0 : \bar{x} = 20$

$H_1 : \bar{x} > 20$

$n = 12$; $\alpha = 0.05$

To calculate the test statistic (T):-

• Calculate the Sample mean:

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

$$\bar{x} = \frac{21.5 + 24.5 + \cdots + 18.5}{12}$$

$$\boxed{\bar{x} = 20.175}$$

- Calculate Sample Standard deviation :-

$$\bar{\sigma} = \frac{(x_1-\bar{x})^2 + (x_2-\bar{x})^2 + (x_3-\bar{x})^3 + \dots + (x_n-\bar{x})^2}{n-1}$$

$$\bar{\sigma} = \frac{(21.5-20.175)^2 + (24.5-20.175)^2 + \dots + (18.5-20.175)^2}{12-1}$$

$$\bar{\sigma} = 3.0211$$

$$T = \frac{\bar{x}-\mu}{se}$$

(or)

$$T = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$$

$$T = \frac{(20.175-20)}{\frac{3.0211}{\sqrt{12}}}$$

$$T = \frac{0.175}{\frac{3.0211}{3.46}}$$

$$T = \frac{0.175}{0.8731}$$

$$\boxed{T = 0.2004}$$

Find the T-Critical :-

$\alpha = 0.05$

$df = n-1$
$= 12-1$; $df = 11$.

T - critical = 1.796

Tstatistic is less than (<) T-critical, it does not fall in the rejection region.

0.26004 < 1.796, we do not reject the null hypothesis.

Sampling distribution of t - test :-

It is the probability distribution of a given random - sample based statistic.

They are important in statistics because they provide a major simplification to statistical inference.

Normally sampling distribution of t represents the distribution that would be obtained if a value of t were calculated for each sample mean for all possible random samples of a given size from some population.

The t distribution is symmetrical but flatter than a normal distribution.

Each t- distribution is associated with the special number referred to as degrees of freedom.

normal distribution

t - distribution

degrees of freedom :-

* refers to the maximum number of logically independent values, which are values that have the freedom to vary in the data sample.

* It is said to be amount of information in the sample.

* changes depending on the design and statistic

Example :-

Consider a data Sample with 5 positive integers.

(4) Four samples are {3, 8, 5 and 4} and the avg. of entire data sample is revealed to be 6.

Sol:-

$$\frac{3+8+5+4+x}{5} = 6$$

$$x + 20 = 30$$

$$\boxed{x = 10}$$

The fifth ~~samgple~~ integer is 10. It does not have the freedom ~~to~~ vary.

∴ The degrees of freedom for this ~~damp~~ta sample is 4.

degrees of freedom formula for :-

- one - sample t - test $\Rightarrow df = n-1$
- Two - Sample t - test (independent) $\Rightarrow df = n_1 + n_2 - 2$
- paired t - test (dependent) $\Rightarrow df = n-1$.

t - test procedure :-

3 Key data values :-

1) difference between the mean values from each data set (called the mean difference).

2) the SD of each group.

3) the number of data values of each group.

The outcome of the t-test procedures the t-value. This calculated t-value is then compared against a value obtained from a critical value table (called the T-distribution table).

This comparison helps to determine the effect of chance alone on the difference, and whether the difference between is outside that chance range.

The t-test produces two values :-

- t - value
- degrees of freedom

- t - value :-

ratio of the difference between the mean of the two sample sets and the variation that exists within (9)

the sample sets.

It is also called as t-score.

A large t-score indicates that the groups are different

A small t-score indicates that the groups are similar.

Estimating Standard error:-

[A measure of the variability in the mean from sample to sample is given by the standard error of the mean — Standard error]

Standard error $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$.

estimated Standard error $S_{\bar{x}} = \frac{S}{\sqrt{n}}$

$S_{\bar{x}}$ → estimated Standard error of the mean.

n → sample Size.

S is defined as,

$$S = \sqrt{\frac{SS}{n-1}} = \sqrt{\frac{SS}{df}}$$

S → Sample Standard deviation.

df → degrees of freedom.

SS has been defined as

$$SS = \Sigma(x-\bar{x}) = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

It is used whenever the unknown population standard deviation must be estimated.

T- test for two independent Samples:-

Independent is also called as Unpaired Sample t-test

The data sets in the two groups don't refer to the same values.

Example:-

They include cases like a group of 100 patients being split into two sets of 50 patients each. One of the groups becomes the control group and is given a placebo, while the other group receives the prescribed treatment.

This Constitutes two independent sample groups which are unpaired with each other.

2 types:-

Equal variance or pooled T-test.

unequal Variance T-test.

Equal variance or pooled T-test :-

* It is used when the number of samples in each group is same or the variance of the two data sets is similar.

* The following formula is used for calculating t-value and degrees of freedom for equal variance t-test.

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Unequal Variance T-test :-

It is used when the number of samples in each group is different, and the variance of the two data sets is also different.

This test is also called as "Welch's t-test".

The following formula is used for calculating t-value and degrees of freedom for an unequal variance t-test.

$$t = \frac{(x_1 - x_2)}{\sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}}}$$

$$df = n_1 + n_2 - 2.$$

Statistical hypothesis :-

- null hypothesis :-
  The means for the two populations are equal.
- alternate hypothesis :-
  The means for the two populations are not equal.

Sampling distribution :

It represents the entire spectrum of difference between sample means based on all possible pairs of random samples from the two underlying population.

Sampling distribution $\overline{x_1} - \overline{x_2}$

Mean of the Sampling distribution ($\mu_{\bar{x}_1 - \bar{x}_2}$):

Mean of the sampling distribution of $\bar{x}$ equals the population mean, $\mu_{\bar{x}} - \mu$.

$\mu_{\bar{x}} \rightarrow$ mean of the sampling distribution.

$\mu \rightarrow$ population mean.

The mean of the new sampling distribution of $\bar{x}_1 - \bar{x}_2$ equals the difference between population means,

$$\mu_{\bar{x}_1 - \bar{x}_2} = \mu_1 - \mu_2 .$$

$\mu_{\bar{x}_1 - \bar{x}_2} \rightarrow$ mean of the new sampling distribution

$\mu_1 - \mu_2 \rightarrow$ difference between the population means.

Standard error of the sampling distribution:

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

where $\sigma_{\bar{x}_1 - \bar{x}_2} \rightarrow$ new standard error.

σ_1^2 and $\sigma_2^2 \rightarrow$ two population variances.

n_1 and $n_2 \rightarrow$ two sample sizes.

test procedures:

1) Define null and alternate/ive hypothesis.
2) State alpha.
3) Calculate degrees of freedom
4) State decision rule
5) Calculate test statistic

6) State results.

7) State Conclusion.

Example :-

A statistic teacher wants to compare his two classes to see if they performed any differently on the tests he gave that semester. Class A had 25 students with an average score of 70, SD 15. Class B has 20 students with an average score of 74, SD 25. Using alpha 0.05 did these two classes perform differently on the test?

$H_0 : \mu_{classA} = \mu_{class B}$

$H_1 : \mu_{class A} \neq \mu_{class B}$

$\alpha = 0.05$

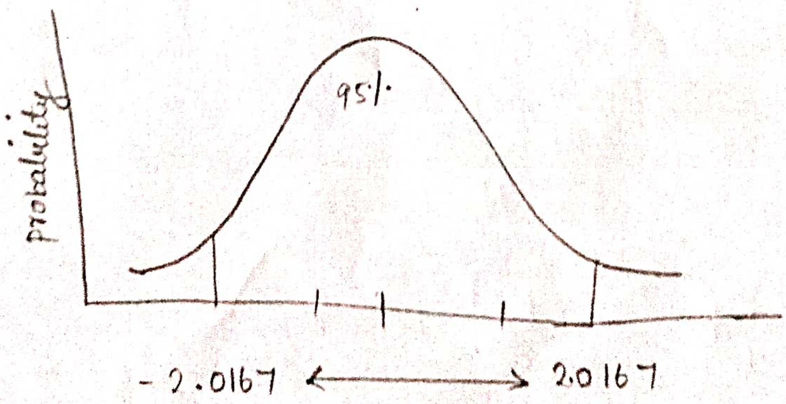
degrees of freedom formula:-

$df = (n_1 - 1) + (n_2 - 1)$

$df = (25-1) + (20-1)$

$= 24 + 19$

$\boxed{df = 43}$



If t is less than −2.0167 (or) greater than 2.0167, reject the null hypothesis.

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}}$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}} \qquad S_p^2 = \frac{SS_1 + SS_2}{df_1 + df_2}$$

$df_1 = n - 1 = 25 - 1$
$\Rightarrow 24$

$$S_p^2 = \frac{SS_1 + SS_2}{df_1 + df_2}$$

$df_2 = n_2 - 1 = 20 - 1$
$\Rightarrow 19$

$$S^2p = \frac{5400 + 11875}{24 + 19}$$

$SS_1 = S_1^2(df_1) = (15^2)(24)$
$\Rightarrow 5400$

$SS_2 = S_2^2(df_2) = (25^2)(19)$
$\Rightarrow 11875$

$$= \frac{17,275}{43}$$

$$S^2p = 401.74$$

$$t = \frac{70 - 74}{\sqrt{\frac{401.74}{25} + \frac{401.74}{20}}}$$

$$= \frac{-4}{\sqrt{36.16}}$$

$$\boxed{t = -0.67}$$

If t is less than -2.0167 or greater than 2.0167, reject the null hypothesis.

t = -0.67

Do not reject Ho.

∴ There was no significant difference between the test performances of class A and class B,

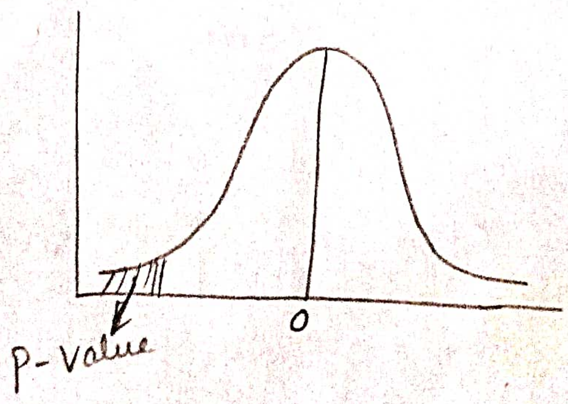
$t = -0.67, P > 0.05$.

P-value :-

* P-value is the probability that you would obtain the effect observed in your sample or larger if the null hypothesis true for the populations.

* P-values are calculated based on your sample data and under the assumption that the null hypothesis is true.

* Lower p-value indicate greater evidence against the null hypothesis.



Statistically Significant results:-

It is a determination, that a relationship between two or more variables is caused by something other than chance.

* It is used to provide evidence concerning the plausibility of the null hypothesis, which hypothesizes that there is nothing more than random chance at work in the data.

* It is used to determine whether the result of a data set is statistically significant.

* Generally, a p-value of 5% or lower is considered statistically significant.

Estimating effect Size :-

The effect size for a t-test for independent samples is usually calculated using Cohen's d.

$$\text{Cohen's } d = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S_p^2}}$$

$d \rightarrow$ Standardized estimate of the effect Size.

$\bar{x}_1$ and $\bar{x}_2 \rightarrow$ two Sample means.

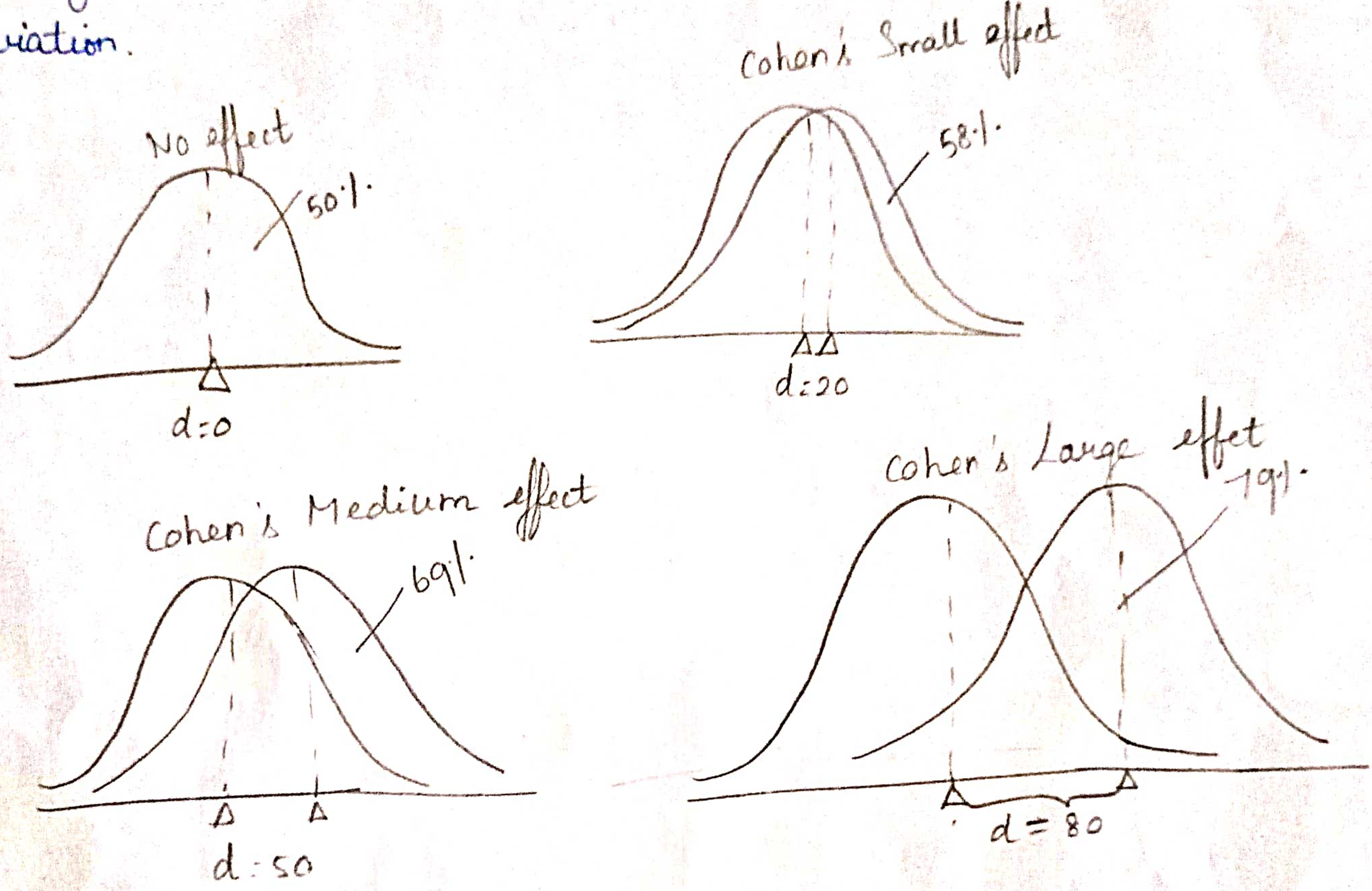
$\sqrt{S_p^2} \rightarrow$ Sample standard deviation obtained from the square root of the pooled Variance estimate.

Cohen's Guidelines for d :-

Effect size is small if d is less than or in the Vicinity of 0.20, that is one-fifth of a standard deviation.

Effect size is medium, if d is in the vicinity of 0.50, that is, one-half of a standard deviation.

effect Size is large if d is more than or in the vicinity of 0.80 that is, four-fifths of standard deviation.



Point estimate :

* It is the most straight forward type of estimate.

* It identifies the observed difference for $\bar{x}_1 - \bar{x}_2$, estimate of the unknown effect.

* $\mu_1 - \mu_2 \rightarrow$ unknown difference between population means.

Confidence interval :-

Confidence intervals for $\mu_1 - \mu_2$ specify ranges of values that, in the long-run, include the unknown effect (difference between population means) a certain percent of the time.

Confidence interval for $\mu_1 - \mu_2$ (two independent samples).

$$\bar{x}_1 - \bar{x}_2 \pm (t\,conf)(S_{\bar{x}_1 - \bar{x}_2})$$

$\bar{x}_1 - \bar{x}_2 \rightarrow$ difference between sample means.

$t\,conf \rightarrow$ number distributed with $n_1 + n_2 - 2$ degrees of freedom from the t-tables.

$S_{\bar{x}_1 - \bar{x}_2} \rightarrow$ estimated Standard error.

Meta analysis :

* Effect size can be conceptualized as a standardized difference.
* In the simplest form, effect size which is denoted by the symbol "d", is the mean difference between groups in standard score form i.e., the ratio of the difference between the means to the standard deviation

This concept is derived from a school of methodology named meta-analysis.

*A single study might lack statistical power due to small sample size. Nevertheless, when many prior studies are combined together statistical power increases.

* An individual study might over-estimate or under-estimate the effect size. Again, when many studies are pooled together, the precision of the estimation can be substantially improved.

T-test for two related Samples:-

*The paired t-test is also known as the dependent sample t-test, the paired difference t-test, the matched pairs t-test and the repeated-samples t-test.

*It is a statistical procedure used to determine whether the mean difference between two sets of observation is zero.

Sampling distribution :-

→ can be denoted as $\bar{D}$.

$$\mu_{\bar{D}} = \mu_D .$$

Esimated Sampling error :

$$\sigma_{\bar{D}} = \frac{\sigma_D}{\sqrt{n}}$$

Standard error of $\bar{D}$ equals the corresponding population standard deviation (for different scores) divided by the square root of the sample Size.

estimating effect size :-

Sample standard deviation

$$S_D = \sqrt{\frac{SS_D}{n-1}}$$

estimated standard error

$$S_{\bar{D}} = \frac{S_D}{\sqrt{n}}$$

confidence interval :-

$$\bar{D} \pm (t_{conf})(S_{\bar{D}})$$

standarized effect Size,

$$\text{Cophen's } d = \frac{\bar{D}}{S_D}$$

Example :-

Researchers want to test a new anti-hunger weight loss pill. They have 10 people rate their hunger both before and after taking the pill. Does the pill do anything? Use alpta =0.05?

Before	after	difference
9	7	(9-7) = 2
10	6	4
7	5	2
5	4	1
7	4	3

5	6	-1
9	7	2
6	5	1
8	5	3
7	7	0

H_0 ; $\mu_{before} = \mu_{after}$

H_1 ; $\mu_{before} \neq \mu_{after}$

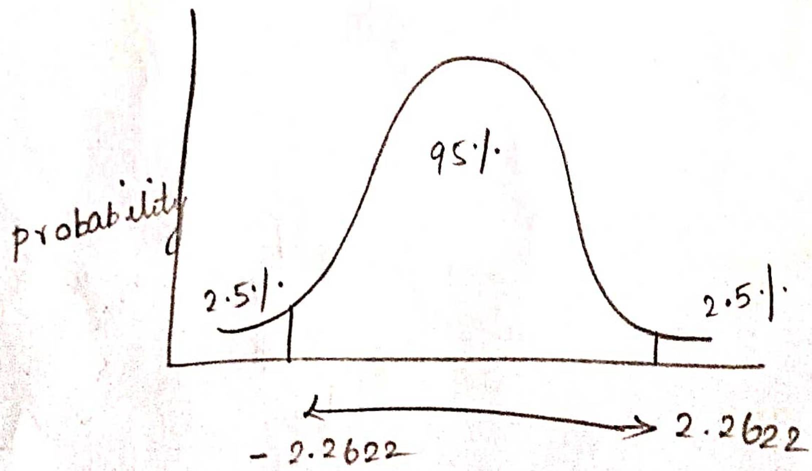
$\alpha = 0.05$

degrees of freedom :-

$df = N - 1$

$df = 10 - 1$

$\boxed{df = 9}$



If t is less than -2.2622 or greater than 2.2622, reject the null hypothesis.

$$t = \frac{\overline{X_D}}{S_D / \sqrt{n}}$$

$$\overline{X}_D = \frac{2+4+2+1+3+(-1)+2+1+3+0}{10}$$

$$\boxed{\bar{X}_D = 1.7}$$

$$S_D = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}}$$

$$= \sqrt{\frac{49 - \frac{(17)^2}{10}}{9}}$$

$$\boxed{S_D = 1.49}$$

$$t = \frac{1.7}{\frac{1.49}{\sqrt{10}}}$$

$$\boxed{t = 3.61}$$

If t is less than -2.2622 or greater than 2.2622, reject the null hypothesis.

$t = 3.61$

∴ The anti-hunger weight loss pill significantly affected hunger, $t = 3.61$, $p < 0.05$.