MA8391 **PROBABILITY AND STATISTICS** L T P C
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- OBJECTIVES:
- This course aims at providing the required skill to apply the statistical tools in engineering
- problems.
- To introduce the basic concepts of probability and random variables.
- To introduce the basic concepts of two dimensional random variables.
- To acquaint the knowledge of testing of hypothesis for small and large samples which plays
- an important role in real life problems.
- To introduce the basic concepts of classifications of design of experiments which plays very
- important roles in the field of agriculture and statistical quality control.

UNIT I PROBABILITY AND RANDOM VARIABLES 12

Probability – The axioms of probability – Conditional probability – Baye's theorem - Discrete and continuous random variables – Moments – Moment generating functions – Binomial, Poisson, Geometric, Uniform, Exponential and Normal distributions.

UNIT II TWO - DIMENSIONAL RANDOM VARIABLES
12

Joint distributions – Marginal and conditional distributions – Covariance – Correlation and linear regression – Transformation of random variables – Central limit theorem (for independent and identically distributed random variables).

UNIT III TESTING OF HYPOTHESIS 12

Sampling distributions - Estimation of parameters - Statistical hypothesis - Large sample tests based on Normal distribution for single mean and difference of means -Tests based on t, Chi-square and F distributions for mean, variance and proportion - Contingency table (test for independent) - Goodness of fit.

UNIT IV DESIGN OF EXPERIMENTS 12

One way and Two way classifications - Completely randomized design – Randomized block design – Latin square design – 22 factorial design.

UNIT V STATISTICAL QUALITY CONTROL `
12

Control charts for measurements (X and R charts) – Control charts for attributes (p, c and np charts) –
Tolerance limits - Acceptance sampling.

TOTAL : 60 PERIODS

OUTCOMES:

Upon successful completion of the course, students will be able to:

- Understand the fundamental knowledge of the concepts of probability and have knowledge of
- standard distributions which can describe real life phenomenon.
- Understand the basic concepts of one and two dimensional random variables and apply in
- engineering applications.
- Apply the concept of testing of hypothesis for small and large samples in real life problems.
- Apply the basic concepts of classifications of design of experiments in the field of agriculture
- and statistical quality control.
- Have the notion of sampling distributions and statistical techniques used in engineering and
- management problems.

TEXT BOOKS:

1. Johnson, R.A., Miller, I and Freund J., "Miller and Freund's Probability and Statistics for

Engineers", Pearson Education, Asia, 8th Edition, 2015.
2. Milton. J. S. and Arnold. J.C., "Introduction to Probability and Statistics", Tata McGraw Hill, 4th Edition, 2007.

REFERENCES:

1. Devore. J.L., "Probability and Statistics for Engineering and the Sciences‖, Cengage Learning, New Delhi, 8th Edition, 2014.
2. Papoulis, A. and Unnikrishnapillai, S., "Probability, Random Variables and Stochastic Processes", McGraw Hill Education India, 4th Edition, New Delhi, 2010.
3. Ross, S.M., "Introduction to Probability and Statistics for Engineers and Scientists", 3rd Edition, Elsevier, 2004.
4. Spiegel. M.R., Schiller. J. and Srinivasan, R.A., "Schaum‘s Outline of Theory and Problems of Probability and Statistics", Tata McGraw Hill Edition, 2004.
5. Walpole. R.E., Myers. R.H., Myers. S.L. and Ye. K., "Probability and Statistics for Engineers and Scientists", Pearson Education, Asia, 8th Edition, 2007.

COURSE OBJECTIVES

1	To introduce the basic concepts of probability and random variables
2	To acquaint the knowledge of testing of hypothesis for small and large samples which plays an important role in real life problems.
3	To introduce the basic concepts of classifications of design of experiment
4	This course aims at providing the required skill to apply the statistical tools in engineering problems

COURSE OUTCOMES

CO. NO	DESCRIPTION
CO1	Able to understand the fundamental knowledge of the concepts of probability and distributions which can describe the real life phenomenon
CO2	Able to understand the basic concepts of one and two dimensional random variable and apply them in engineering applications
CO3	Able to apply the basic knowledge of testing of hypothesis for small and large samples in engineering field
CO4	Able to understand the basic concepts of classifications of design of experiments which plays very important roles in the field of agriculture and statistical quality control
CO5	Have the notion of sampling distributions and statistical techniques used in engineering and management problems

MA8391 PROBABILITY & STATISTICS

UNIT I

PROBABILITY AND RANDOM VARIABLES 12

Probability – The axioms of probability – Conditional probability – Baye‘s theorem - Discrete and continuous random variables – Moments – Moment generating functions – Binomial, Poisson, Geometric, Uniform, Exponential and Normal distributions.

PART A

1. The probability density function of the rv x is given by $f(x)=k(1-x^2)$ for 0<x<1 find k.**(April/May2019)(U)**

$$\int_{-\infty}^{\infty} f(x)dx = 1 \Rightarrow \int_0^1 k(1-x^2)dx = 1 \Rightarrow k = \frac{3}{2}$$

2. If a random variable X takes the values 1,2,3,4 such that P(X=1)=3P(X=2)=P(X=3)=5P(X=4). Find the probability distribution of X. **(U)** **(Nov. Dec.2012)**

Solution:

Assume P(X=3) = α. By the given equation, $P(X=1)=\frac{\alpha}{2}$ $P(X=2)=\frac{\alpha}{3}$ $P(X=4)=\frac{\alpha}{5}$.

For a probability distribution (and mass function) $\sum P(x) = 1$

P(1)+P(2)+P(3)+P(4) =1

$$\frac{\alpha}{2}+\frac{\alpha}{3}+\alpha+\frac{\alpha}{5}=1 \quad \Rightarrow \quad \frac{61}{30}\alpha = 1 \quad \Rightarrow \alpha = \frac{30}{61}$$

$$P(X=1)=\frac{15}{61};\ P(X=2)=\frac{10}{61};\ P(X=3)=\frac{30}{61};\ P(X=4)=\frac{6}{61}$$

The probability distribution is given by

X	1	2	3	4
$p(x)$	$\frac{15}{61}$	$\frac{10}{61}$	$\frac{30}{61}$	$\frac{6}{61}$

3. Let X be a continuous random variable having the probability density function $f(x)=\begin{cases}\frac{2}{x^3}, & x \ge 1\\ 0, & otherwise\end{cases}$ Find the distribution function of x. (**R)**

Solution:

$$F(x)=\int_1^x f(x)\,dx = \int_1^x \frac{2}{x^3}dx = \left[-\frac{1}{x^2}\right]_1^x = 1-\frac{1}{x^2}$$

4. A random variable X has the probability density function *f(x)* given by $f(x)=\begin{cases} cxe^{-x}, & x \ge 0\\ 0, & otherwise\end{cases}$. Find the value of c and CDF of X.(AP)

Solution:

$$\int_0^\infty f(x)\,dx = 1 \Rightarrow \int_0^\infty cxe^{-x}\,dx = 1 \Rightarrow c\left[-xe^{-x}-e^{-x}\right]_0^\infty = 1 \Rightarrow c(1)=1 \Rightarrow c=1$$

$$F(x)=\int_0^x f(x)\,dx = \int_0^x cxe^{-x}\,dx = \int_0^x xe^{-x}\,dx = \left[-xe^{-x}-e^{-x}\right]_0^x = 1-xe^{-x}-e^{-x}$$

5. A continuous random variable X has the probability density function *f(x)* given by $f(x)=ce^{-|x|}, -\infty<x<\infty$. Find the value of c and CDF of X.(AP)

Solution:

$$f(x,y)=\frac{1}{8}(6-x-y), 0<x<2, 2<y<4$$

$Case\ (i)\ x<0$

$$F(x)=\int_{-\infty}^{x} f(x)\,dx=\int_{-\infty}^{x} c\,e^{-|x|}\,dx=c\int_{-\infty}^{x} e^{x}\,dx=c\left[e^{x}\right]_{-\infty}^{x}=\frac{1}{2}e^{x}$$

$Case(ii)\ x>0$

$$F(x)=\int_{-\infty}^{x} f(x)\,dx=\int_{-\infty}^{x} c\,e^{-|x|}\,dx=c\int_{-\infty}^{0} e^{x}\,dx+c\int_{0}^{x} e^{-x}dx=c\left[e^{x}\right]_{-\infty}^{0}+c\left[-e^{-x}\right]_{0}^{x}$$

$$=c-ce^{-x}+c=c\left(2-e^{-x}\right)=\frac{1}{2}\left(2-e^{-x}\right)$$

$$F(x)=\begin{cases}\frac{1}{2}e^{x}, & x>0\\ \frac{1}{2}\left(2-e^{-x}\right), & x<0\end{cases}$$

6. If a random variable has the probability density $f(x)=\begin{cases}2e^{-2x}, & x\geq 0\\ 0, & otherwise\end{cases}$. Find the probability that it will take on a value between 1 and 3. Also, find the probability that it will take on value greater than 0.5. (AP)

Solution:

$$P(1<X<3)=\int_{1}^{3} f(x)\,dx=\int_{1}^{3} 2e^{-2x}dx=\left[-e^{-2x}\right]_{1}^{3}=e^{-2}-e^{-6}$$

$$P(X>0.5)=\int_{0.5}^{\infty} f(x)\,dx=\int_{0.5}^{\infty} 2e^{-2x}dx=\left[-e^{-2x}\right]_{0.5}^{\infty}=e^{-1}$$

7. Is the function defined as follows a density function? (U)

$$f(x)=\begin{cases}0, & x<2\\ \frac{1}{18}(3+2x), & 2\leq x\leq 4\\ 0, & x>4\end{cases}$$

Solution:

$$\int_{2}^{4} f(x)\,dx=\int_{2}^{4}\frac{1}{18}(3+2x)\,dx=\left[\frac{(3+2x)^2}{72}\right]_{2}^{4}=1$$

Hence it is density function.

8. The cumulative distribution function (CDF) of a random variable X is $F(X)=1-(1+x)e^{-x}$, $x>0$. Find the probability density function of X.(U)

Solution:

$$f(x) = F'(x) = 0 - \left[(1+x)\left(-e^{-x}\right)+(1)\left(e^{-x}\right)\right] = x\ e^{-x}, \quad x>0$$

9. The number of hardware failures of a computer system in a week of operations has the following probability mass function: (AP)

No of failures	0	1	2	3	4	5	6
Probability	0.18	0.28	0.25	0.18	0.06	0.04	0.01

Find the mean of the number of failures in a week.

Solution:

$$E(X) = \sum x\ P(x) = (0)(0.18)+(1)(0.28)+(2)(0.25)+(3)(0.18)+ (4)(0.06)+(5)(0.04)+(6)(0.01)$$
$$= 1.92$$

10. Given the p.d.f of a continuous r.v X as follows $f(x) = \begin{cases} 6x(1-x), & 0 \le x \le 1 \\ 0, & elsewhere \end{cases}$.

Find the CDF of X. (U)

Solution: $F(x) = \int_0^x f(x)\ dx = \int_0^x 6x(1-x)\ dx = \int_0^x 6x - 6x^2\ dx = \left[3x^2 - 2x^3\right]_0^x = 3x^2 - 2x^3$

11. A continuous random variable X has the probability function $f(x) = k(1+x),\ \ 2 \le x \le 5$. Find P(X<4). (AP)

Solution:

$$\int_2^4 f(x)dx = 1 \quad \Rightarrow k\int_2^5 (1+x)dx = 1 \Rightarrow k\left[\frac{(1+x)^2}{2}\right]_2^5 = 1 \Rightarrow k\,\frac{27}{2} = 1 \Rightarrow k = \frac{2}{27}$$

$$P(X<4) = \int_2^4 f(x)\ dx = \frac{2}{27}\int_2^4 (1+x)\ dx = \frac{2}{27}\left[\frac{(1+x)^2}{2}\right]_2^4 = \frac{1}{25}(25-9) = \frac{16}{27}$$

12. Given the p.d.f of a continuous R.V X as follows $f(x) = \begin{cases} 12.5x - 1.25 & 0.1 \le x \le 0.5 \\ 0, & elsewhere \end{cases}$

Find P(0.2 < X < 0.3) (AP)

Solution:

$$P(0.2 < X < 0.3) = \int_{0.2}^{0.3} (12.5x - 1.25)\ dx = \left[12.5\frac{x^2}{2} - 1.25x\right]_{0.2}^{0.3}$$
$$= 1.25\left[5(0.3)^2 - 0.3 - 5(0.2)^2 + 0.2\right]$$
$$= 0.1875$$

13. If the MGF of a continuous R.V X is given by $M_X(t) = \frac{3}{3-t}$. Find the mean and variance of X.(AP)

Solution:

$$M_X(t) = \frac{3}{3-t} = \frac{1}{1-\frac{t}{3}} = \left(1-\frac{t}{3}\right)^{-1} = 1+\frac{t}{3}+\left(\frac{t}{3}\right)^2+\left(\frac{t}{3}\right)^3+\ldots$$

$E(X) = (\text{coefficient of } t)\ 1! = \frac{1}{3}$ is the mean, $E(X^2) = (\text{coefficient of } t^2)2! = \frac{1}{9}\ 2! = \frac{2}{9}$

$$Variance = E(X^2) - E(X)^2 = \frac{2}{9} - \frac{1}{9} = \frac{1}{9}$$

14. If the MGF of a discrete R.V X is given by $M_X(t) = \frac{1}{81}\left(1+2e^t\right)^4$, find the distribution of X.(AP)

Solution:

$$M_X(t) = \frac{1}{81}\left(1+2e^t\right)^4 = \frac{1}{81}\left(1+4C_1\left(2e^t\right)+4C_2\left(2e^t\right)^2+4C_3\left(2e^t\right)^3+4C_4\left(2e^t\right)^4\right)$$

$$= \frac{1}{81} + \frac{8}{81}e^t + \frac{24}{81}e^{2t} + \frac{32}{81}e^{3t} + \frac{16}{81}e^{4t}$$

By the definition of MGF,

$$M_X(t) = \sum e^{tx} p(x) = p(0) + p(1)e^t + p(2)e^{2t} + p(3)e^{3t} + p(4)e^{4t}$$

On comparison with above expansion the probability distribution is

X	0	1	2	3	4
$p(x)$	$\frac{1}{81}$	$\frac{8}{81}$	$\frac{24}{81}$	$\frac{32}{81}$	$\frac{16}{81}$

15. Find the MGF of the R.V X whose p.d.f is $f(x) = \begin{cases} \frac{1}{10}, & 0 < x < 10 \\ 0, & elsewhere \end{cases}$. Hence find its mean.(AP)

Solution:

$$M_X(t) = \int_0^{10} \frac{1}{10}e^{tx}\, dx = \frac{1}{10}\left(\frac{e^{tx}}{t}\right)_0^{10} = \frac{1}{10}\left(\frac{e^{10t}-1}{t}\right) = \frac{1}{10t}\left(1+10t+\frac{100t^2}{2!}+\frac{1000t^3}{3!}+\ldots.-1\right)$$

$$= 1+5t+\frac{1000}{31}t^2+\ldots..$$

$$Mean = \text{coefficient of } t = 5$$

16. Given the probability density function $f(x) = \frac{k}{1+x^2}$, $-\infty < x < \infty$, find k and C.D.F.(U)

Solution:

$$= 5C_0\left(\frac{9}{12}\right)^0\left(\frac{3}{12}\right)^5$$

$$F(x) = \int_{-\infty}^{x} f(x)\, dx = \int_{-\infty}^{\infty} \frac{k}{1+x^2}\, dx = \frac{1}{\pi}\left[\tan^{-1} x\right]_{-\infty}^{x} = \frac{1}{\pi}\left[\left[\tan^{-1}\infty\right]-\left[\tan^{-1}-x\right]\right] = \frac{1}{\pi}\left(\frac{\pi}{2}-\tan^{-1} x\right) = \frac{1}{\pi}\cot^{-1} x$$

17. It has been claimed that in 60 % of all solar heat installation the utility bill is reduced by atleast one-third. Accordingly what are the probabilities that the utility bill will be reduced by atleast one-third in atleast four of five installations?(AP)

Solution:

Given n=5, p=60 % =0.6 and q=1-p=0.4

$$p(x \geq 4) = p[x=4] + p[x=5]$$
$$= 5c_4 (0.6)^4 (0.4)^{5-4} + 5c_5 (0.6)^5 (0.4)^{5-5}$$
$$= 0.337$$

18. The no. of monthly breakdowns of a computer is a r.v. having poisson distribution with mean 1.8. Find the probability that this computer will function for a month with only one breakdown.(AP)

Solution:

$$p(X=x) = \frac{e^{-\lambda}\lambda^x}{x!} \quad , \quad given\ \lambda = 1.8,\ p(x=1) = \frac{e^{-1.8}(1.8)^1}{1!} = 0.2975$$

19. In a company 5 % defective components are produced. What is the probability that atleast 5 components are to be examined in order to get 3 defectives?(AP)

Solution:

To get 3 defectives, 3 or more components must be examined.

p=5 % =0.05, q = 1- p=0.95 and k=success=3

$$p(X=x) = (x-1)c_{k-1}\, p^k q^{x-k} \ ,\ x = k, k+1, k+2, \ldots$$
$$p(x \geq 5) = 1 - p(x<5)$$
$$= 1 - \left[p(x=3) + p(x=4)\right]$$
$$= 1 - \left[2c_2 (0.05)^3 (0.95)^0 + 3c_2 (0.05)^3 (0.95)^1\right]$$
$$= 1 - 0.00048 = 0.9995$$

20. A discrete R.V X has mgf $M_x(t) = e^{2(e^t-1)}$. Find E(x), var(x), and p(x=0)(U).

Solution: Given $M_x(t) = e^{2(e^t-1)}$

We know that mgf of poisson is $M_x(t) = e^{\lambda(e^t-1)}$

Therefore λ = 2. In poisson E(x) = var(x) = λ

$$\therefore\ Mean\ E(x) = \text{var}(x) = 2$$
$$p(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$$
$$\therefore\ p(X=0) = \frac{e^{-\lambda}\lambda^0}{0!} = e^{-\lambda} = e^{-2} = 0.1353$$

21. Find the mean and variance of geometric distribution.(R)

Solution:

The pmf of Geometric distribution is given by $p(X = x) = p\,q^{x-1}\ , x = 1,2,3,.....$

$$Mean\quad E(x)\ = \sum x\ \ p(x) = \sum_{x=1}^{\infty} x\ \ p\ \ q^{x-1} = \ p\sum_{x=1}^{\infty} x\ \ q^{x-1} = \ p\ \left[1\ \ q^{1-1} + 2q^{2-1} + 3q^{3-1} + ...\right]$$

$$= \ p\ \left[1\ \ +2q + 3q^2 + ...\right] = p\left[1-q\right]^2 = \ p\ \ p^{-2} = p^{-1} = \frac{1}{p}$$

$$E(x^2) = \sum_{x=1}^{\infty}\left[x\left(x+1\right) - x\right]\ p\ \ q^{x-1} = \sum_{x=1}^{\infty} x(x+1)\,p\ \ q^{x-1} - \sum_{x=1}^{\infty} x\ \ p\ \ q^{x-1}$$

$$= \ 1(1+1)\,p\ \ q^{1-1} + 2(2+1)\,pq^{2-1} + 3(3+1)\,pq^{3-1} + ... - \frac{1}{p}$$

$$= \ 2p\ \ +2(3)\,pq + 3(4)\,pq^2 + ... - \frac{1}{p} = \ 2p\ \left[1 + 3q + 6q^2 + ...\right] - \frac{1}{p}$$

$$= \ 2p\ \left[1-q\right]^{-3} - \frac{1}{p} = 2\ \ p\ \ p^{-3} - \frac{1}{p} = \frac{2}{p^2} - \frac{1}{p}$$

$$Variance = E(x^2) - \left[E(x)\right]^2 = \frac{2}{p^2} - \frac{1}{p} - \left(\frac{1}{p}\right)^2 = \frac{2}{p^2} - \frac{1}{p} - \frac{1}{p^2}$$

$$= \frac{1}{p^2} - \frac{1}{p} = \frac{1-p}{p^2} = \frac{q}{p^2}$$

22. Find the MGF geometric distribution.(R)

Solution: The PMF of geometric distribution is given by

$$Mgf\ \ M_x(t) = E(e^{tx}) = \sum e^{tx}\ p(x) = \sum_{x=1}^{\infty} e^{tx}\,p\,q^{x-1} = \sum_{x=1}^{\infty} e^{tx}\,p\ q^x\ q^{-1}$$

$$= \frac{p}{q}\sum_{x=1}^{\infty}\left(e^t\right)^x q^x = \frac{p}{q}\sum_{x=1}^{\infty}\left(qe^t\right)^x = \ \frac{p}{q}\left[qe^t + \left(qe^t\right)^2 + \left(qe^t\right)^3 + ...\right]$$

$$= \ \frac{p}{q}\ qe^t\left[1 + qe^t + \left(qe^t\right)^2 + ...\right]$$

$$= \ pe^t\left(1-qe^t\right)^{-1} 1 - qe^t$$

$$= 1 - qe^t$$

23. Show that for the uniform distribution $f(x) = \frac{1}{2a}\ , -a < x < a$, the mgf about origin is $\frac{\sinh\ at}{at}$.(AP)

Solution: Given $f(x) = \frac{1}{2a}\ , -a < x < a$

MGF

$$M_x(t) = E\left[e^{tx}\right] = \int_{-\infty}^{\infty} e^{tx} f(x)dx = \int_{-a}^{a} e^{tx}\frac{1}{2a}dx = \frac{1}{2a}\int_{-a}^{a} e^{tx}dx = \frac{1}{2a}\left[\frac{e^{tx}}{t}\right]_{-a}^{a}$$

$$= \frac{1}{2at}\left[e^{at} - e^{-at}\right] = \frac{1}{2at}2\sinh\, at$$

$$= \frac{\sinh\, at}{at}$$

24. Define exponential density function and find mean and variance of the same.(R)

Solution: The density function of exponential distribution is given by $f(x) = \lambda e^{-\lambda x}, x \geq 0$

$$Mean = E[x] = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{0}^{\infty} x\lambda\, e^{-\lambda x}\,dx = \lambda\int_{0}^{\infty} x\, e^{-\lambda x}\,dx = \lambda\left[\frac{-xe^{-\lambda x}}{\lambda} - \frac{e^{-\lambda x}}{\lambda^2}\right]_0^{\infty}$$

$$= \lambda\left[(0-0) - \left(0 - \frac{1}{\lambda^2}\right)\right] = \lambda\left(\frac{1}{\lambda^2}\right)$$

$$= \frac{1}{\lambda}$$

$$E\left[x^2\right] = \int_{-\infty}^{\infty} x^2 f(x)\,dx = \int_{0}^{\infty} x^2\lambda\, e^{-\lambda x}\,dx = \lambda\int_{0}^{\infty} x^2\, e^{-\lambda x}\,dx = \lambda\left[\frac{-x^2e^{-\lambda x}}{\lambda} - \frac{2xe^{-\lambda x}}{\lambda^2} - \frac{2e^{-\lambda x}}{\lambda^3}\right]_0^{\infty}$$

$$= \lambda\left[(0-0-0) - \left(0 - 0 - \frac{2}{\lambda^3}\right)\right] = \lambda\left(\frac{2}{\lambda^3}\right) = \frac{2}{\lambda^2}$$

$$Variance = E\left(x^2\right) - [E(x)]^2 = \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2 = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$$

25. State memory less property of exponential distribution.(R)

Solution:

If X is exponentially distributed with parameter λ, then for any two positive integers 's' and 't' $P(X > s + t / X > s) = P(X > t)$.

26. A continuous random variable X that can assume any value between $x = 2$ & $x = 5$ has a density function given by $f(x) = k(1 + x)$. Find P(X>4).(U) **(Nov/ Dec. 2012)**

Solution: Since f(x) is a density function,

$$\int_2^5 k(1+x)\ dx = 1 \Rightarrow k\left(x + \frac{x^2}{2}\right)_2^5 = k\left(5 + \frac{25}{2} - 2 - \frac{4}{2}\right) = k\left(\frac{27}{2}\right) = 1 \Rightarrow k = \frac{2}{27}$$

$$P(X > 4) = \int_4^5 \frac{2}{27}(1+x)dx = \frac{2}{27}\left(1 + \frac{25}{2} - \frac{16}{2}\right) = \frac{11}{27}$$

27. Identify the random variable and name the distribution it follows, from following statement: "A realtor claims that only 30% of the houses in a certain neighbourhood appraised at less than Rupees 20 lakhs. A random sample of 10 houses from the neighbourhood is selected and appraised to check the realtor's claims acceptable are not". (U) **(Nov/ Dec. 2012)**

Solution:

X is a random variable that a house is appraised at less than Rs.20 lakhs. And it follows a binomial distribution with n = 10, p=0.30 and q=0.70

28. A coin is tossed 2 times, if 'X' denotes the number of heads, find the probability distribution of X.(**Nov./Dec. 2013**)(AP)

X: No. of heads	0	1	2
P(X=x)	$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$	$2C_1\left(\frac{1}{2}\right)^2$	$2C_2\left(\frac{1}{2}\right)^2$

29. If the probability that a target is destroyed on any one shot is 0.5, find the probability that it would be destroyed on 6^{th} attempt. (AP) (**Nov./Dec.,2013**)

$$P(X = 6) = q^5 p = (0.5)^6$$

30. A continuous random variable X has the probability density function given by $f(x) = a(1+x^2),\ 2 \le x \le 5$, Find 'a' and $P(X < 4)$ (AP) (**May/June 2014**)

$$\int_2^5 a(1+x^2)dx = 1 \Rightarrow a = \frac{1}{42}, \quad P(X < 4) = \int_2^4 \frac{1}{42}(1+x^2)dx = \frac{31}{63}$$

31. For a binomial distribution with mean2 and varience 4/3, Find the first term of the distribution. (AP) **(April/May 2019)**

32. For a binomial distribution with mean 6 and standard deviation $\sqrt{2}$, Find the first two terms of the distribution. (AP) (**May/June 2014**)

From the given data, $n = 9,\ p = \frac{2}{3},\ q = \frac{1}{3}$

I term: $9C_0\left(\frac{1}{3}\right)^9$; II term: $9C_1\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^8$

33. Test whether $f(x) = \begin{cases} |x|; & -1 \le x \le 1 \\ 0; & otherwise \end{cases}$ probability density function of a continuous random variable (U)**(Nov./Dec.2014)**(**April/ May 2015**)

$\int_{-1}^{1} x\,dx = 2\left(\frac{x^2}{2}\right)_0^1 = 2\left(\frac{1}{2}\right) = 1$. Therefore it is a continuous random variable.

34. What do you mean by MGF? Why it is called so? (R) (Nov./Dec.2014)

MGF is a moment generating function which generates all the moments about the origin. It can also be calculated as a coefficient of $\frac{t^r}{r!}$ and also by differentiating the MGF with respect to 't' , r times , i.e.

$$M_X(t) = E(e^{tx})$$

$$\mu_r' = \left[\frac{d^r}{dt^r} M_X(t)\right]_{t=0}$$

35. If the density function of a continuous random variable X is given by $f(x) = \begin{cases} ax; & 0 \le x \le 1 \\ a; & 1 \le x \le 2 \\ 3a - ax; & 2 \le x \le 3 \\ 0; & otherwise \end{cases}$ then find the value of 'a' (U) (**April/ May 2015**)

$$\int_0^1 ax\,dx+\int_1^2 a\,dx+\int_2^3 (3a-ax)dx=1 \qquad \Rightarrow a=\frac{1}{2}$$

36. Suppose that on an average, in every three pages of a book there is one typographical error. IF the number of typographical errors on a single page of the book is a Poisson random variable. What is the probability of at least one error on a specific page of the book?(AP) (**April/ May 2015**)

$$P(X\geq 1)=1-P(X<1)=1-P(X=0)\ =1-\frac{e^{-\lambda}}{0!}=1-e^{-3}$$

37. What are the limitations of Poisson distribution?(U) (**April/ May 2015**)

Poisson distribution is a limiting case of binomial distribution. When $n\to\infty$ *and* $p\to 0$ Instead of Binomial distribution, Poisson distribution will be applied.

38. A continuous random variable X has the probability density function given by $f(x)=a(1+x^2),\ 1\leq x\leq 5$, Find 'a' and $P(X<4)$ (U) (**Nov./Dec. 2015**)

$$\int_1^5 a(1+x^2)dx=1\Rightarrow a=\frac{3}{136}$$

$$P(X<4)=\int_1^4\frac{3}{136}(1+x^2)dx=\frac{18}{34}$$

39. What is meant by memory less property? Which discrete distribution follows this property? **(Nov./Dec. 2015)(R)**

If X is continuously distributed random variable, then for any two positive integers's' and 't' $P(X>s+t/X>s)=P(X>t)$.Geometric distribution follows memory less property.

40. Let X be the random variable which denotes the number of heads in three tosses of a fair coin. Determine the probability mass function of X? (**Nov./ Dec. 2015)(AP)**

X: No. of heads	0	1	2	3
P(X=x)	$\left(\frac{1}{2}\right)^3=\frac{1}{8}$	$3C_1\left(\frac{1}{2}\right)^3$	$3C_2\left(\frac{1}{2}\right)^3$	$3C_3\left(\frac{1}{2}\right)^3$

41. A continuous random variable X has a pdf given by $f(x)=\frac{3}{4}(2x-x^2),\ 0<x<2$. Find P(X>1) (AP)(**Nov./Dec. 2015**)

$$P(X>1)=\int_1^2\frac{3}{4}(2x-x^2)dx=\frac{1}{2}$$

42. Let X be a discrete random variable with pmf $P(X=x)=\frac{x}{10},x=1,2,3,4$, Compute $P(X<3)$ *and* $E\left(\frac{X}{2}\right)$ (AP) (**May/ June 2016**)

Ans:

$$\begin{array}{ccccc} X: & 1 & 2 & 3 & 4 \\ p(x): & \frac{1}{10} & \frac{2}{10} & \frac{3}{10} & \frac{4}{10} \end{array}$$

$$P(X<3)=\frac{1}{10}+\frac{2}{10}=0.3$$

$$E\left(\frac{X}{2}\right)=\sum\frac{x}{2}p(x)=\left(\frac{1}{2}\right)\left(\frac{1}{10}\right)+1\left(\frac{2}{10}\right)+\left(\frac{3}{2}\right)\left(\frac{3}{10}\right)+\left(\frac{4}{2}\right)\left(\frac{4}{10}\right)$$

43. If a random variable X has the moment generating function $M_X(t)=\frac{3}{3-t}$, Compute $E(X^2)$ (AP) (**May/ June 2016**)

$$M_X(t)=\frac{3}{3-t}=\left(1-\frac{t}{3}\right)^{-1}=1+\frac{t}{3}+\left(\frac{t}{3}\right)^2+.....=1+t\left(\frac{1}{3}\right)+\frac{t^2}{2!}\left(\frac{2}{9}\right)+...$$

$$Coefficient\ \ of\ \ \frac{t^2}{2!}=E(X^2)$$

$$E(X^2)=\frac{2}{9}$$

44. If a fair coin is tossed twice. Find $P(X\le 1)$ where X denotes the number of heads in each experiment**.(NOV/Dec.2016)**

X:	**0**	**1**	**2**
P(x):	**1/4**	**2/4**	**¼**

$$P(X\le 1)=P(X=0)+P(X=1)=\frac{3}{4}$$

45. Balls are tossed at random into 50 boxes. Find the expected number of tosses required to get the first ball in the fourth box. **.(NOV/Dec.2016)**
Let Y denote the number of tosses untile the first ball goes into the foruth box. Then Y with p=1/50. The expected value of Y is E(X) =1/p =50

46. If the probability density function of a random variable X is $f(x)=\frac{1}{4}\ in\ -2<x<2$, find $P(|x|>1)$.(**May/ June 2017**)

$$P(|x|>1)=1-P(|X|\le 1)$$

$$=1-\int_{-1}^{1}\frac{1}{4}dx$$

$$=1-\frac{1}{2}=\frac{1}{2}$$

47. If X is a geometric variable, taking values 1,2,3….∞, Find P(X is odd) (**May/ June 2017**)
The pmf of X is $q^{x-1}p$, x=1,2,3…
P(X is odd)=P(X=1)+P(x=3)+P(X=5)+…

$$=p+q^2p+q^4p+....$$

$$=p(1-q^2)^{-1}$$

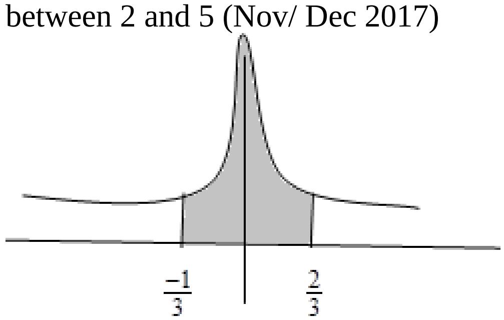
$$=\frac{p}{1-q^2}=\frac{p}{p(1+q)}=\frac{1}{1+q}$$

48. A test engineer discovered that the cumulative distribution function of the lifetime of an equipment (in years) is given by $F(x)=1-e^{-\frac{x}{5}},\ x>0$, what is the expected lifetime of the equipment?(Nov/ Dec. 2017)

$F(x)=1-e^{-\frac{x}{5}},\ x>0$

$f(x)=\frac{1}{5}e^{-\frac{x}{5}}$ is the pdf of continuous random variable which follows exponential distribution with parameter $\frac{1}{5}$, with expected life time of equipment as 5

49. If X is a normal random variable with mean 3 and variance 9, find the probability that X is between 2 and 5 (Nov/ Dec 2017)



$$P(2<X<5)=P\left(\frac{2-3}{3}<Z<\frac{5-3}{3}\right)$$
$$=P\left(\frac{-1}{3}<Z<\frac{2}{3}\right)$$
$$=\phi\left(\frac{2}{3}\right)-\phi\left(\frac{-1}{3}\right)$$
$$=0.11695=11.69\%$$

50. If $M_X(t)=\frac{pe^t}{1-qe^t}$ is a Moment generating function of X then find the mean and variance of X.(Apr/ May 2018)

Mean $\frac{1}{p}$ **,**Variance $\frac{q}{p^2}$

51. The mean of Binomial distribution is 20 and standard deviation is 4. Find the parameters of this distribution(Apr/ May 2018)

$np=20,\ \ npq=16$

$q=\frac{16}{20}=\frac{4}{5}\Rightarrow p=\frac{1}{5}\Rightarrow n=100$

RANDOM VARIABLES AND STANDARD DISTRIBUTIONS
PART B

1. Four boxes A.B,C,D contain fuces. The boxes contain 5000,3000,2000 and 1000 fuses respectively. The percentage of fuses in boxes which are defective are 3%,2%,1% and 0.5% respectively .One fuseis selected at random arbitrarily from one of the boxes. It is found to be defective fuse. Find the probility that it has come from box D .**(April/May2019)(A)**
2. random variable *X* has the following probability distribution:

X=x	*-2*	*-1*	*0*	*1*	*2*	*3*
P(x)	*0.1*	*k*	*0.2*	*2k*	*0.3*	*3k*

(i) Find k, (ii) Evaluate $P(X<2)$ and $P(-2<X<2)$, (iii) Find the PDF of X and (iv) Evaluate the mean of X (U) **(Nov/Dec 2018) (May/ June 2016)**

3. A random variable X has the following probability function (U)

$X=x$	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

(i) Find the value of k, (ii) Evaluate $P(X<6), P(X\geq 6)$,

(iii) If $P(X\leq c)>\frac{1}{2}$, find the minimum value of c.

(iv) $P(1.5<X<4.5/X>2)$ **(April/May 2012)(April/ May 2015)**

4. If X is a Poisson variate such that $P(X=2)=9P(X=4)+90P(X=6)$. Find (i) Mean and $E(X^2)$, (ii) $P(X\geq 2)$. (U) **(April/May 2012)**

5. In a continuous distribution, the probability density is given by $f(x)=kx(2-x), 0<x<2$. Find k, mean, variance and the distribution function.(U) **(May/Jun 2007)**

6. The sales of a convenience store on a randomly selected day are *X* thousand dollars, where *X* is a random variable with a distribution function of the following form☹U)

$$F(x)=\begin{cases}0 & x<0\\ \dfrac{x^2}{2} & 0\leq x<1\\ k(4x-x^2) & 1\leq x<2\\ 1 & otherwise\end{cases}$$

Suppose that this convenience store's total sales on any given day are less than *$2000.*

(a)Find the value of *k*,

(b) Let A and B be the events that 'tomorrow the store's total sales are between 500 and 1500 dollars, and over 1000 dollars,' respectively. Find *P(A)* and *P(B).*

(c) Are *A* and *B* independent events? **(Nov/Dec 2007)**

7. A random variable X has the pdf $f(x)=\begin{cases}2x, & 0<x<1\\ 0, & otherwise\end{cases}$ find (a) $P\left(X<\frac{1}{2}\right)$ (b) $P\left(\frac{1}{4}<X<\frac{1}{2}\right)$ (c) $P\left(X>\frac{3}{4} \middle/ X>\frac{1}{2}\right)$ **(Nov/Dec 2008)(U)**

8. If $p(x)=\begin{cases}xe^{-\frac{x^2}{2}}, x\geq 0\\ 0, x<0\end{cases}$ .Show that *p(x)* is a pdf and also find *F(x)* **(U)(May/Jun 2009)**

9. If the cumulative distribution function of a RV X is given by $F(x)=\begin{cases}1-\dfrac{4}{x^2}, & x>2\\ 0, & x\leq 2\end{cases}$

find **(a)** $P(X<3)$ **(b)** $P(4<X<5)$ **(c)** $P(X\geq 3)$ (U) **(Apr/May 2008)**

10. If the density function of a continuous random variable X is given by $f(x)=\begin{cases} ax, & 0\le x\le 1 \\ a & 1\le x\le 2 \\ 3a-ax & 2\le x\le 3 \\ o & otherwise \end{cases}$

Find a. Find the cdf of X .(U) **(Nov/Dec 2008))(May/ June 2017)**

11. The distribution function of a random variable X is given by $F(x)=1-(1+x)e^{-x};\ x\ge 0$. Find the density function, mean and variance of X **.(Apr/May 2019) (Nov/Dec 2010)(U)**

12. If X is a random variable with a continuous distribution function $F(x)$, prove that $Y=F(x)$ has a uniform distribution in (0, 1). Further if $f(X)=\begin{cases} \frac{1}{2}(x-1), & 1\le x\le 3 \\ 0 & otherwise \end{cases}$, find the range of Y corresponding to the range $1.1\le x\le 2.9$.(AP)

13. The cumulative distribution function (cdf) of a random variable X is given by

$$F(x)=\begin{cases} 0, x<0 \\ x^2, 0\le x<\frac{1}{2} \\ 1-\frac{3}{25}\left(3-x^2\right), \frac{1}{2}\le x<3 \\ 1, x\ge 3 \end{cases}$$ (AP)

Find the pdf of X and evaluate $P(|X|\le 1)$ and $P\left(\frac{1}{3}<X<4\right)$ using both pdf and cdf (PDF).

(May/June2007, (Nov/Dec 2011)

14. The probability function of an infinite discrete distribution is given by $P(X=j)=\frac{1}{2^j}$; $j=1,2,...,\infty$. Verify that the total probability is 1 and find the mean and variance of the distribution. Find also $P(X\ is\ even)$, $P(X\ge 5)$ and $P(X\ is\ divisible\ by\ 3)$. (AP)**(Nov/Dec 2011)**

15. Find the moment –generating function of the binominal random variable with parameters m and p and hence find it mean and variance.(R) **(April/May 2011)(April/ May 2015)**

16. Find the moment generating function of an exponential random variable and hence find its mean and variance. (R) **(April/May 2012)(May/ June 2014)**

17. Find the moment generating function of an exponential random variable and hence find its mean and variance.Also prove that lack of memory property of exponential distribution .

(A) **(April/May 2019)**

18. Find the moment generating function of the geometric random variable with the pdf $f(x)=pq^{x-1}, x=1,2,3,....$ and hence obtain its mean and variance. **(May/Jun 2007)(April/ May 2015)(R)**

19. Derive mean and variance of a Geometric distribution. Also establish the forgetfulness property of the Geometric distribution. **(R)** **(April/May 2011)**

20. Describe the situations in which geometric distribution could be used. Obtain its MGF. **(R)**

(April/May 2010)

21. By calculating the MGF of Poisson distribution with parameter λ, prove that the mean and variance of the Poisson distribution are equal. **(April/May2019,2010)(May/June 2014)(R)**

22. Find the moment generating function of $N(\mu,\sigma^2)$ normal random variable and hence determine the mean and variance. **(Nov/Dec. 2015)(U) .(Nov/ Dec 2017)(Apr/ May 2018)**

23. Find the moment generating function of uniform distribution and hence find its mean and variance. (May/ June 2013)(R)

24. If the density function of X equals $f(x)=\begin{cases} Ce^{-2x} & 0<x<\infty \\ 0 & x<0 \end{cases}$. Find C. What is $P(X>2)$. **(April/May 2010)(U)**

25. A discrete RV has moment generating function $M_X(t)=\left(\frac{1}{4}+\frac{3}{4}e^t\right)^5$. Find $E(X), Var(X)$ and $P(X=2)$. (U) **(Apr/May 2008)**

26. If the moments of a random variable X are defined by $E(X^r)=0.6;\ r=1,2,3,..$ show that $P(X=0)=0.4, P(X=1)=0.6, P(X\geq 2)=0$. (U) **(Nov/Dec 2008)**

27. A coin having probability p of coming up heads is successively flipped until the r^{th} head appears. Argue that X, the number of flips required will be $n, n\geq r$ with probability $P(X=n)=(n-1)C_{r-1}p^r q^{n-r},\ n\geq r$. (U) **(April/May 2010)**

28. A coin is tossed until the first head occurs. Assuming that the tosses are independent and the probability of a head occurring is 'p'. Find the value of 'p' so that the probability that an odd number of tosses required is equal to 0.6. Can you find a value of 'p' so that the probability is 0.5 that an odd number of tosses are required? **(U)** **(Nov/Dec 2010)**

29. Out of 800 families with 4 children each, how many families would be expected to have

(i) 2 boys and 2 girls(AP)
(ii) at least 1 boy
(iii) at most 2 girls
(iv) Children of both sexes.

Assume equal probabilities for boys and girls. **(May/Jun 2009)**

30. 6 dice are thrown 729 times. How many times do you expect atleast three dice to show a five or a Six? (AP)

31. The probability of a man hitting a target is ¼. If he fires 7 times, what is the probability of his hitting the target atleast twice? And how many times must he fire so that the probability of his hitting the target is at least once is greater than 2/3?(**May/ June 2017**)(AP)

32. Ten percent of the tools produced in a certain manufacturing company turn out to be defective. Find the probability that in a sample of 10 tools chosen at random, exactly 2 will be defective by using (1) binomial distribution (2) The Poisson approximation to the binomial distribution. **(Nov/Dec. 2015)(AP)**

33. Suppose that telephone calls arriving at a particular switchboard follow a Poisson process with an average of 5 calls coming per minute. What is the probability that up to a minute will elapse unit 2 calls have come in to the switch board? **(AP)** **(April/May 2011)**

34. A machine manufacturing screws is known to produce 5% defective. In a random sample of 15 screws, what is the probability that there are (AP)

(i) exactly 3 defectives

(ii) not more than 3 defectives **(Nov/Dec 2018)**

35. The number of monthly breakdown of a computer is a random variable having a Poisson distribution with mean equal to 1.8. Find the probability that this computer will function for a month (AP)
(i) without a breakdown
(ii) with only one breakdown
(iii)With at least one break down **(May/Jun 2007), (Nov. /Dec.2012)**

36. In a large consignment of electric bulb, 10 % are defective. A random sample of 20 is taken for inspection. Find the probability that (1) all are good bulbs (2) atmost there are 3 defective bulbs (3) Exactly there are 3 defective bulbs(May/ June 2013)(AP)

37. A manufacturer of pins knows that 2% of his products are defective. If he sells pins in boxes of 100 and guarantees that not more than 4 pins will be defective, what is the probability that a box fail to meet the guaranteed quality?(AP) (**Nov./Dec.2013**)
Hint: Find P(X>4)

38. Messages arrive at a switch board in a poisson manner at an average rate of six per hour. Find the probability for each of the following events: (1) exactly two messages arrive within one hour.(2) no message arrives within one hour (3) Atleast three messages arrive within one hour (AP) (**April/ May 2015**)

39. The number of typing mistakes that a typist makes on a given page has a Poisson distribution with a mean of 3 mistakes. What is the probability that she makes (1) exactly 7 mistakes? (2) fewer than 4 mistakes? (3) no mistakes on a given page? **(Nov/Dec. 2015)(AP)**

40. Messages arrive at a switch board in a poisson manner at an average rate of 6 per hour. Find the probability that exactly two messages arrive within one hour, no message arrives within one hour and at least three messages arrive within one hour. (**Nov./Dec. 2016)(AP)**

41. Messages arrive at a switch board in a poisson manner at an average rate of six per hour. find the probability that atleast three message arrive within one hour. (Nov/ Dec 2017)

42. The mileage which cars owners get with a certain kind of certain kind of radial tire is a random variable having an exponential distribution with mean *40,000* km. Find the probabilities that one of these tires will last (AP)
(i) At least 20,000 km
(ii) At most 30,000 km.

43. The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = \frac{1}{2}$. What is the probability that the repair time exceeds $2h$? What is the conditional probability that a repair takes at least $10h$ given that its duration exceeds $9h$.(U)**(Nov/Dec 2010)** **(April/May 2015)**

44. The lifetime of a TV tube (in years) is an exponential random variable with mean 10. What is the probability that the average lifetime of a random sample of 36 TV tubes is at least 0.5 (AP) **(Nov/Dec 2007)**

45. The lifetime X of particular brand of batteries is exponentially distributed with a mean of 4 weeks. Determine (1) the mean and variance of X (2) what is the probability that the battery life exceeds 2 weeks? (3) Given that the battery has lasted 6 weeks, what is the probability that it will last at least another 5 weeks? **(Nov/Dec. 2015)(AP)**

46. State and prove the forgetfulness property of exponential distribution, using the property, solve the following problem: The length of the shower on a tropical island during rainy seasons has an exponential distribution with parameter λ =2, time being measured in minutes. What is the probability that a shower will last more than 3 minutes? (**Nov./Dec. 2016)(AP)**

47. A component has an exponential time to failure distribution with mean of 10,000 hours.
(1) The component has already been in operation for its mean life. What is the probability that it will fail by 15,000 hours? **(Nov/Dec. 2015)** (AP)
(2) At 15,000 hours the component is still in operation. What is the probability that it will operate for another 5000 hours? **(Nov/Dec. 2015)**

48. Experience has shown that while walking in a certain park, the time X (in minutes) between seeing two people smoking has a density function of the form $f(x) = \begin{cases} \lambda x e^{-x} & x > 0 \\ 0 & otherwise \end{cases}$.

(a) Calculate the value of λ.(AP)
(b) Find the distribution function of X .
(c) What is the probability that Jeff, who has just seen a person smoking, will see another person smoking in 2 to 5 minutes? In atleast 7 minutes? **(Nov/Dec 2007)**

49. The number of personal computers (PC) sold daily at a Computer world is uniformly distributed with a minimum of 2000 PC and a maximum of 5000 PC. Find(AP)
(i) the probability that daily sales will fall between 2,500 and3,000 PC.
(ii) What is the probability that Computer world will sell at least 4000 PCs?
(iii)What is the probability that Computer world will sell exactly 2500 PCs?
(Nov/Dec 2006)

50. Starting at 5.00am every half hour there is a flight from San Francisco airport to Los Angeles International Airport. Suppose that none of these planes is completely sold out and that they always have room for passengers. A person who wants to fly to L.A. arrives at the airport at a random time between 8.45 a.m., and 9.45., am. Find the probability that she waits (AP)
(i)At most 10 minutes (ii) At least 15 minutes **(Nov/Dec 2007)**

51. Trains arrive at a station at 15 minutes intervals starting at 4 a.m. If the passenger arrive at a station at a time that is uniformly distributed between 9.00 and 9.30, find the probability that he has to wait for the train for (1) less than 6 minutes (2) more than 10 minutes (**May/ June 2014**)(AP

52. If a continuous RV X follows uniform distribution in the interval (0,2) and a continuous RV, Y follows exponential distribution with parameter λ such that $P(X<1) = P(Y<1)$ (AP)

53. Let X be a uniformly distributed random variable over [-5, 5], Determine (1) P(X<=2) (2) $P(|X|>2)$, (3) Cumulative distribution function of X, (4) Var(X). (**May/ June 2016)(U)**

54. Let X be uniformly distributed random variable in the interval (a,9) $P(3<x<5) = 2/7$, Find the constant 'a' and compute $P[|x-5|<2]$ (Apr/ May 2018)

55. Define the probability density function of normal distribution and standard normal distribution. Write the important properties of this distribution. (R) **(Nov/Dec 2006)**

56. In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and variance of the distribution.(U) (**Nov./Dec.2014**)

57. The peak temperature T, as measured in degrees Fahrenheit, on a particular day is the Gaussian (85,10) random variable. What is P(T>100), P(T<60) and $P(70 \le T \le 100)$ (**April/ May 2015**)(AP)

58. An electrical firm manufactures light bulbs that have a life, before burn out, that is normally distributed with mean equal to 800 hours and a standard deviation of 40 hours. Find

(i) the probability that a bulb burns more than 834 hours.

(ii) the probability that the bulb burns between 778 and 834 hours.(AP) **(Nov/Dec 2006)**

59. The scores on an achievement test given to 1,00,000 students are normally distributed with mean 500 and standard deviation 100. What should be the score of a student be to place him among the top 10% of all students?(Apr/ May 2018)

60. The average percentage of marks of candidates in an examination is 42 with a standard deviation of 10. If the minimum mark for pass is 50% and 1000 candidates appear for the examination, how many candidates can be expected to get the pass mark if the marks follow normal distribution? If it is required, that the double the number of the candidates should pass, what should be the minimum marks for pass? **(Nov/Dec. 2015)(AP)**

61. The annual rainfall in inches in a certain region has a normal distribution with a mean of 40 and variance of 16. What is the probability that the rainfall in a given year is between 30 and 48 inches? (**Nov./Dec. 2016)(AP)**

62. In the normal population with mean 15 and standard deviation 3.5, it is found that 647 observations exceed 16.25. What is the total number of observations in the population? ?(**May/ June 2017**)(U)

63. A die is cast until 6 appear. What is the probability that it must be cast more than 5 times? **(Nov/Dec 2008)(U)**

64. If the probability that an applicant for a driver's license will pass the road test on any trial is 0.8, what is the probability that he will finally pass the test (1) on the4th trial (2) in fewer than 4 trials? (AP) **(Nov./ Dec.2012)**

65. Suppose that a trainee soldier shoots a target in an independent fashion. If the probability that the target is shot on any one shot is 0.7. (1) What is the probability that the target would be hit on tenth attempt? (2) What is the probability that it takes him less than 4 shots? (3) What is the probability that it takes him an even number of shots? (AP) (**May/ June2014)**

66. Let X be a RV with $E(X)=1$ and $E(X(X-1))=4$. Find $Var\left(\frac{X}{2}\right), Var(2-3X)$ **(Apr/May 2008)(U)**

67. If X is a continuous RV with pdf $f(x)=\begin{cases} x & 0\le x<1 \\ \frac{3}{2}(x-1)^2 & 1\le x<2 \\ 0 & otherwise \end{cases}$ find the cumulative distribution function $F(x)$ of X and use it to find $P\left(\frac{3}{2}<X<\frac{5}{2}\right)$ *(U)* **(Apr/May 2008).**

68. If a RV X has geometric distribution, i.e., $P(X=x)=pq^{x-1}, x=1,2,3$ where $q=1-p, o<p<1$ show that $P(X>x+y/\ X>y)=P(X>x)$ (U) **(Apr/May 2008)**

69. Let the pdf for X be given by $f(x)=\begin{cases} \frac{1}{2}e^{-x^2/2}, x\ge 0 \\ 0 \quad otherwise \end{cases}$.

Find (i) $P\left(X>\frac{1}{2}\right)$, (ii) Moment generating function for X (iii) $E(X)$

(iv) Var(X) (U) **(Apr/May 2008)**

70. If the probability density of X is given by $f(x)=\begin{cases} 2(1-x) & for\ 0<x<1 \\ 0 & otherwise \end{cases}$

(i) Show that $E[X^r]=\frac{2}{(r+1)(r+2)}$

(ii) Use this result to evaluate $E\left[(2x+1)^2\right]$ (U) **(Nov/Dec 2006)**

71. A random variable X has density function given by $f(x)=\begin{cases} \frac{1}{k} & 0<x<k \\ 0 & otherwise \end{cases}$

Find mgf, r^th^ moment, mean, variance (U) **(Nov/Dec 2006)**

72. The pdf of samples of the amplitude of speech wave forms is found to decay exponentially at the rate α so the following pdf is proposed. $f(x)=Ce^{-\alpha|x|}, -\infty<x<\infty$. Find the constant C and also $P(|X|<v)$ and *E(X).(U)* **(Apr/May 2008)**

73. If X is uniformly distributed over $(-\alpha,\alpha), \alpha>0$, find α so that (U)

(i) $P(X>1)=\frac{1}{3}$ (ii) $P(|X|<1)=P(|X|>1)$ **(Nov/Dec 008)**

74. The atoms of radioactive element are randomly disintegrating. If every gram of this element, on average, emits 3.9 alpha particles per second, what is the probability that during the next second the number of alpha particles emitted from 1 gram is (AP)

(i) at most 6 (ii) At least 2 (iii) at least 3 and at most 6 **(Nov/Dec 2007)**

75. A man with *'n'* keys wants to open his door and tries the keys independently and at random. Find the mean and variance of the number of trials required to open the door if unsuccessful keys are not eliminated from further selection. **(AP)** **(Nov/Dec 2007)**

76. A continuous random variable has the pdf $f(x)=kx^4,\ -1<x<1$, Find the value of k and also $P\left(\left(X>\frac{-1}{2}\right)\Big/ X<\frac{-1}{4}\right)$ (May/ June 2013)(u)

77. Find the moment generating function and rth moment for the distribution whose pdf is $f(x) = Ke^{-x}$, $0 \le x \le \infty$. Hence find the mean and variance.(R) **(May/ June 2013)**

78. If a random variable X takes the values 1,2,3,4 such that P(X=1) = 3P(X=2) = P(X=3) = 5P(X=4). Find the probability distribution of X. **(U)** **(Nov. Dec.2012).**

79. Find the MGF of the random variable 'X' having the pdf $f(x) = \begin{cases} x \ , \ for \ 0 < x < 1 \\ 2 - x, \ \ for \ 1 < x < 2 \\ 0, \ otherwise \end{cases}$

(U) (**Nov./Dec.2013**)
(Nov./Dec.2013)
(**Nov./Dec.2013**)

80. If $f(x) = \begin{cases} xe^{-x^2/2}, \ x \ge 0 \\ 0 \ ; \ x < 0 \end{cases}$ then show that $f(x)$ is a pdf and find F(x) (U) (**Nov./Dec.2014)**

81. A random variable X takes the values -2,-1, 0 and 1 with probabilities $\frac{1}{8}, \frac{1}{8}, \frac{1}{4}$ *and* $\frac{1}{2}$ respectively. Find and draw the probability distribution function (AP) (**Nov./Dec.2014**)

82. The CDF of the random variable of x is given by $F(x) = \begin{cases} 0 \ , \ x < 0 \\ x + \frac{1}{2}, \ 0 \le x \le \frac{1}{2} \\ 1, \ x > \frac{1}{2} \end{cases}$ Draw the graph of CDF. Compute $P(X > 1/4)$, $P\left(\frac{1}{3} \le X \le \frac{1}{2}\right)$**(U)** **(April/ May 2015)**

83. A continuous random variable X has the pdf $f(x) = kx^3 e^{-x}, \quad x \ge 0$. Find the rth moment about the origin, moment generating function, mean and variance of X. **(Nov/Dec. 2015)(U)**

84. Let X be a continuous random variable with pdf $f(x) = xe^{-x}, x > 0$, find (i) the cumulative distribution function of X (ii) Moment generating function of X (iii) P(X<2) (iv) E(X) (**May/ June 2016)(u)**

85. Let $P(X = x) = \left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^{x-1}$, $x = 1, 2, 3...$ be the probability mass function of a random variable X, Compute (1) P(X>4) (2) P(X >4/ X>2) (3) E(x) (4) Var (X) (**May/ June 2016**)(u)

86. The probability distribution function of a random variable X is given by $f(x) = \frac{4x(9 - x^2)}{81}, 0 \le x \le 3$ Find the mean, variance and third moment about origin. (**Nov./Dec. 2016)(AP)**

87. Find the MGF of the random variable X having the probability density function $f(x) = \frac{x}{4} e^{-(x/2)}, \ x > 0$, Also find the first four moments about origin?(**May/ June 2017**)(U)

88. Let X be a continuous random variable with the probability density function $f(x) = \frac{1}{4}$, $2 \le x \le 6$, Find the expected value and variance of X (Nov/ Dec 2017)

UNIT – II
TWO DIMENSIONAL RANDOM VARIABLES

Joint distributions – Marginal and conditional distributions – Covariance – Correlation and linear regression – Transformation of random variables – Central limit theorem (for independent and identically distributed random variables).

PART – A

1. Define joint probability density function of two random variables X and Y .(R)

If (X, Y) is a two dimensional continuous random variable such that $P\left[x-\frac{dx}{2} \le X \le x+\frac{dx}{2}\ ,\ y-\frac{dy}{2} \le Y \le y+\frac{dy}{2}\right] = f(x,y)\,dx\,dy$, then $f(x,y)$ is called the joint pdf of (X,Y), provided $f(x,y)$ satisfies the following conditions

$(i)\ f(x,y) \ge 0\ for\ all\ (x,y) \in R$

$(ii) \iint_R f(x,y)\,dx\,dy = 1$

2. State the basic properties of joint distribution of (X,Y) where X and Y are random variables.(R)

Statement:

Properties of joint distribution of (X,Y) are

$(i)\ F[-\infty, y] = 0 = F[x, -\infty]\ and\ F[-\infty, \infty] = 1$

$(ii)\ P[a < X < b, Y \le y] = F(b,y) - F(a,y)$

$(iii)\ P[X \le x, c < Y < d] = F[x,d] - F[x,c]$

$(iv)\ P[a < X < b, c < Y < d] = F[b,d] - F[a,d] - F[b,c] + F[a,c]$

$(v)\ At\ points\ of\ continuity\ of\ f(x,y),\ \frac{\partial^2 F}{\partial x \partial y} = f(x,y)$

3. Can the joint distributions of two random variables X and Y be got if their marginal distributions are random?(R)

Solution:

If the random variables X and Y are independent then the joint distributions of two random variables can be got if their marginal distributions are known.

4. Let X and Y be two discrete random variable with joint pmf $P[X=x, Y=y] = \begin{cases} \frac{x+2y}{18}, & x=1,2;\ y=1,2 \\ 0 & ,otherwise \end{cases}$. Find the marginal pmf of X and $E[X]$?(U)

	1	2

Solution:

The joint PMF of (X,Y) is given by

1	$\frac{3}{18}$	$\frac{4}{18}$
2	$\frac{5}{18}$	$\frac{6}{18}$

Marginal pmf of X **is**

$$P[X=1]=\frac{3}{18}+\frac{5}{18}=\frac{8}{18}=\frac{4}{9}\ , \quad P[X=2]=\frac{4}{18}+\frac{6}{18}=\frac{10}{18}=\frac{5}{9}$$

$$E[X]=\sum x\,p(x)=(1)\left(\frac{4}{9}\right)+(2)\left(\frac{5}{9}\right)=\frac{4}{9}+\frac{10}{9}=\frac{14}{9}.$$

5. Let X and Y be integer valued random variables with $P[X=m,Y=n]=q^2\ p^{m+n-2}\ ,\ n,m=1,2,\ldots\ldots\ldots$ and $p+q=1$. Are $X\ and\ Y$ independent?(U)

Solution:

The marginal PMF of X is

$$p(x)=\sum_{n=1}^{\infty}q^2\ p^{\ m+n-2}=\sum_{n=1}^{\infty}q^2\ p^{m-1}\ p^{n-1}=q^2\ p^{\ m-1}\sum_{n=1}^{\infty}p^{n-1}$$

$$=q^2\ p^{\ m-1}\left[1+p+p^2+p^3+\ldots\ldots\right]=q^2\ p^{\ m-1}(1-p)^{-1}$$

$$=q^2\ p^{m-1}\ q^{-1}=q\ p^{\ m-1}$$

The marginal PMF of Y is

$$p(y)=\sum_{m=1}^{\infty}q^2\ p^{\ m+n-2}=\sum_{m=1}^{\infty}q^2\ p^{m-1}\ p^{n-1}=q^2\ p^{\ n-1}\sum_{m=1}^{\infty}p^{m-1}$$

$$=q^2\ p^{\ n-1}\left[1+p+p^2+p^3+\ldots\ldots\right]=q^2\ p^{\ n-1}(1-p)^{-1}$$

$$=q^2\ p^{n-1}\ q^{-1}=q\ p^{\ n-1}$$

$$p(x)\,p(y)=q\ p^{m-1}.q\ p^{n-1}=q^2\ p^{\ m+n-2}=P[X=m\ \ Y=n]$$

Therefore $X\ and\ Y$ are independent random variables.

6. The joint probability density function of the random variable (X,Y) is given by $f(x,y)=k\,x\,y\,e^{-\left(x^2+y^2\right)},x>0,y>0$. Find the value of k.(U) (**Nov./Dec.2013**)

Solution:

Given $f(x,y)$ is the joint pdf , we have

$$\iint f(x,y)dx\,dy=1 \qquad \text{put } x^2=t$$

$$\int_0^{\infty}\int_0^{\infty}k\,x\,y\,e^{-\left(x^2+y^2\right)}\,dx\,dy=1 \qquad 2x\,dx=dt$$

$$k\int_0^{\infty}\int_0^{\infty}x\,y\,e^{-x^2}\ e^{-y^2}\ dx\,dy=1 \qquad x\,dx=\frac{dt}{2}$$

$$k\int_0^{\infty}y\,e^{-y^2}\left[\int_0^{\infty}x\,e^{-x^2}\ dx\right]dy=1 \qquad when\,x=0,t=0\,and\ when\,x=\infty,t=\infty$$

$$k\int_0^{\infty}y\,e^{-y^2}\left[\int_0^{\infty}e^{-t}\ \frac{dt}{2}\right]dy=1$$

$$\frac{k}{2}\int_0^\infty y e^{-y^2}\left(-e^{-t}\right)_0^\infty dy = 1$$

$$\begin{aligned} &put \quad y^2 = t \\ &2y\,dy = dt \\ &y\,dy = \frac{dt}{2} \\ &when\ y = 0, t = 0\ and\ when\ y = \infty, t = \infty \end{aligned}$$

$$\frac{k}{2}\int_0^\infty y e^{-y^2}(0+1)dy = 1$$

$$\frac{k}{2}\int_0^\infty e^{-t}\frac{dt}{2} = 1 \Rightarrow \frac{k}{4}\left(e^{-t}\right)_0^\infty = 1 \Rightarrow \frac{k}{4}(0+1) = 1 \Rightarrow \frac{k}{4} = 1 \Rightarrow k = 4.$$

7. The joint PDF of the random variable (X,Y) is $f(x,y)=\begin{cases} k(x+y), & 0<x<2;0<y<2 \\ 0 & ,otherwise \end{cases}$.

Find the value of k.(U)

Solution:

Given $f(x,y)$ is the joint pdf , we have

$$\iint f(x,y)\,dx\,dy = 1 \Rightarrow \int_0^2\int_0^2 k(x+y)\,dx\,dy = 1 \Rightarrow k\int_0^2\left[\left(\frac{x^2}{2}\right)_0^2 + y(x)_0^2\right]dy = 1$$

$$\Rightarrow k\int_0^2[(2-0)+y(2-0)]dy = 1 \Rightarrow k\int_0^2(2+2y)dy = 1$$

$$\Rightarrow k\left[2(y)_0^2 + 2\left(\frac{y^2}{2}\right)_0^2\right] = 1 \Rightarrow k = \frac{1}{8}$$

8. The joint pdf of the random variable (X,Y) is $f(x,y)=\begin{cases} cxy & ,0<x<2;0<y<2 \\ 0 & ,otherwise \end{cases}$. Find the value of c.(U)

Solution:

Given $f(x,y)$ is the joint pdf , we have

$$\iint f(x,y)\,dx\,dy = 1 \Rightarrow \int_0^2\int_0^2 c\,x\,y\,dx\,dy = 1 \Rightarrow c\int_0^2 y\left(\frac{x^2}{2}\right)_0^2 dy = 1 \Rightarrow c\int_0^2 y(2-0)dy = 1$$

$$\Rightarrow 2c\left[\frac{y^2}{2}\right]_0^2 = 1 \Rightarrow 4c = 1 \Rightarrow c = \frac{1}{4}$$

9. If two random variables $X\ and\ Y$ have probability density function $f(x,y)=k(2x+y)$ for $0\le x\le 2\ and\ 0\le y\le 3$. Evaluate k.(U)

Solution: Given $f(x,y)$ is the joint PDF, we have

$$\iint f(x,y)\,dx\,dy=1 \Rightarrow \int_0^3\int_0^2 k\left(2x+y\right)dx\,dy=1 \Rightarrow k\int_0^3\left[\;2\left(\frac{x^2}{2}\right)_0^2 + y\left(x\right)_0^2\right]dy=1$$

$$\Rightarrow k\int_0^3\left(4+2y\right)dy=1 \Rightarrow k\left[4\left(y\right)_0^3+2\left(\frac{y^2}{2}\right)_0^3\right]=1$$

$$\Rightarrow k\left[12+9\right]=1 \;\Rightarrow\; 21k=1 \Rightarrow k=\frac{1}{21}$$

10. If the function $f(x,y)=c(1-x)(1-y), 0<x<1\,,\,0<y<1$ is to be a density function, find the value of c.(U)

Solution:

Given $f(x,y)$ is the joint PDF, we have

$$\iint f(x,y)\,dx\,dy=1 \Rightarrow \int_0^1\int_0^1 c(1-x)(1-y)\,dx\,dy=1 \Rightarrow c\int_0^1\int_0^1 (1-x-y+xy)\,dx\,dy=1$$

$$\Rightarrow c\int_0^1\left[(x)_0^1-\left(\frac{x^2}{2}\right)_0^1-y(x)_0^1+y\left(\frac{x^2}{2}\right)_0^1\right]dy=1$$

$$\Rightarrow c\int_0^1\left[1-\frac{1}{2}-y+\frac{y}{2}\right]dy=1 \Rightarrow c\int_0^1\left(\frac{1}{2}-\frac{y}{2}\right)dy=1$$

$$\Rightarrow c\left[\frac{1}{2}(y)_0^1-\frac{1}{2}\left(\frac{y^2}{2}\right)_0^1\right]=1 \Rightarrow c\left[\frac{1}{2}-\frac{1}{4}\right]=1 \;\Rightarrow \frac{c}{4}=1 \Rightarrow c=4$$

Therefore the value of c is $c=4$

11. Find the marginal density functions of $X \text{ and } Y$ if $f(x,y)=\frac{2}{5}(2x+5y), 0\le x\le 1, 0\le y\le 1$.(U)

Solution:

Marginal density of X is

$$f_X(x)=\int f(x,y)\,dy=\frac{2}{5}\int_0^1(2x+5y)\,dy=\frac{2}{5}\left[2x(y)_0^1+5\left(\frac{y^2}{2}\right)_0^1\right]$$

$$=\frac{2}{5}\left[2x+\frac{5}{2}\right]=\frac{4}{5}x+1, \quad 0\le x\le 1$$

Marginal density of Y is

$$f_Y(y)=\int f(x,y)\,dx \Rightarrow =\frac{2}{5}\int_0^1(2x+5y)\,dx=\frac{2}{5}\left[2\left(\frac{x^2}{2}\right)_0^1+5y(x)_0^1\right]$$

$$\Rightarrow =\frac{2}{5}\left[1+5y\right]=\frac{2}{5}+2y\,, \quad 0\le y\le 1$$

12. If $X \text{ and } Y$ have joint pdf $f(x,y)=\begin{cases} x+y & ;0<x<1\,,0<y<1 \\ 0 & ;otherwise\end{cases}$. Check whether $X \text{ and } Y$ are independent.(U)

Solution:

$$f_X(x)=\int f(x,y)dy=\int_0^1 (x+y)dy \;=x(y)_0^1+\left(\frac{y^2}{2}\right)_0^1=x+\frac{1}{2},0<x<1$$

$$f_Y(y)=\int f(x,y)dx=\int_0^1 (x+y)dx \;=\left(\frac{x^2}{2}\right)_0^1+y(x)_0^1=y+\frac{1}{2},0<y<1$$

$$f_X(x).\,f_Y(y)=\left(x+\frac{1}{2}\right)\left(y+\frac{1}{2}\right)=xy+\frac{x}{2}+\frac{y}{2}+\frac{1}{4}\neq x+y\neq f(x,y)$$

Therefore $X\ and\ Y$ are not independent variables.

13. If $X\ and\ Y$ are random variables having the joint density function $f(x,y)=\frac{1}{8}(6-x-y),0<x<2,2<y<4$ **,** find $P[X+Y<3]$ **.(AP)**

Solution:

$$P[X+Y<3]=\iint f(x,y)\,dx\,dy$$

$$=\frac{1}{8}\int_2^3\int_0^{3-y}(6-x-y)\,dx\,dy=\frac{1}{8}\int_2^3\left[(6-y)(x)_0^{3-y}-\left(\frac{x^2}{2}\right)_0^{3-y}\right]dy$$

$$=\frac{1}{8}\int_2^3\left[(6-y)(3-y)-\frac{1}{2}(3-y)^2\right]dy=\frac{1}{8}\int_2^3\left[18-9y+y^2-\frac{1}{2}(3-y)^2\right]dy$$

$$=\frac{1}{8}\left[18(y)_2^3-9\left(\frac{y^2}{2}\right)_2^3+\left(\frac{y^3}{3}\right)_2^3-\frac{1}{2}\left[\frac{(3-y)^3}{-3}\right]_2^3\right]$$

$$=\frac{1}{8}\left[18(3-2)-\frac{9}{2}(9-4)+\frac{1}{3}(27-8)+\frac{1}{6}(0-1)\right]$$

$$=\frac{1}{8}\left[18-\frac{45}{2}+\frac{19}{3}-\frac{1}{6}\right]=\frac{5}{24}$$

14. Let $X\ and\ Y$ be continuous random variable with joint pdf $f_{XY}(x,y)=\frac{3}{2}\left(x^2+y^2\right),0<x<1$, $0<y<1$**.** Find $f_{X/Y}(x/y)$**.(U)**

Solution:

$$f_Y(y)=\int f(x,y)dx=\frac{3}{2}\int_0^1\left(x^2+y^2\right)dx=\frac{3}{2}\left[\left(\frac{x^3}{3}\right)_0^1+y^2(x)_0^1\right]=\frac{3}{2}\left[\frac{1}{3}+y^2\right]=\frac{3}{2}y^2+\frac{1}{2}$$

$$f_{X/Y}(x/y)=\frac{f(x,y)}{f_Y(y)}=\frac{\frac{3}{2}\left(x^2+y^2\right)}{\frac{3}{2}\left(y^2+\frac{1}{3}\right)}=\frac{x^2+y^2}{y^2+\frac{1}{3}}\ .$$

15. If the joint pdf of (X,Y) is given by $f(x,y)=2-x-y;\ 0\le x<y\le 1$, find $E[X]$.(U)

Solution:

$$E[X]=\iint x\, f(x,y)dx\,dy=\int_0^1\int_0^y x[2-x-y]dx\,dy=\int_0^1\int_0^y(2x-x^2-xy)dx\,dy$$

$$=\int_0^1\left[2\left(\frac{x^2}{2}\right)_0^y-\left(\frac{x^3}{3}\right)_0^y-y\left(\frac{x^2}{2}\right)_0^y\right]dy$$

$$=\int_0^1\left(y^2-\frac{y^3}{3}-\frac{y^3}{2}\right)dy=\int_0^1\left(y^2-\frac{5}{6}y^3\right)dy=\left(\frac{y^3}{3}\right)_0^1-\frac{5}{6}\left(\frac{y^4}{4}\right)_0^1$$

$$==\frac{1}{3}-\frac{5}{24}=\frac{3}{24}=\frac{1}{8}$$

16. Let $X\ and\ Y$ be random variable with joint density function

$f_{XY}(x,y)=\begin{cases}4xy\ ;\ 0\le x\le 1\ ,\ 0\le y\le 1\\ 0\quad ,otherwise\end{cases}$. Find $E[XY]$.(U)

Solution:

$$E[XY]=\iint xy\, f(x,y)dx\,dy=\int_0^1\int_0^1 xy(4xy)dx\,dy=4\int_0^1\int_0^1 x^2y^2\,dx\,dy=4\int_0^1 y^2\left(\frac{x^3}{3}\right)_0^1 dy$$

$$=\frac{4}{3}\int_0^1 y^2\,dy=\frac{4}{3}\left(\frac{y^3}{3}\right)_0^1=\frac{4}{3}\left(\frac{1}{3}\right)=\frac{4}{9}.$$

17. Let $X\ and\ Y$ be any two random variables and a,b be constants. Prove that $Cov(aX,bY)=ab\,\text{cov}(X,Y)$ **(R)**

Solution:

$$Cov(X,Y)=E[XY]-E[X]E[Y]$$

$$Cov(aX,bY)=E[aX\ bY]-E[aX]E[bY]$$

$$=ab\,E[XY]-a\,E[X]\,b\,E[Y]=ab[E(XY)-E(X)E(Y)]$$

$$=ab\,Cov(X,Y)$$

18. If $Y=-2X+3$, find $Cov(X,Y)$.(U)

Solution:

$$Cov(X,Y)=E[XY]-E[X]E[Y]$$

$$=E[X(-2X+3)-E[X]E[-2X+3]]$$

$$=E[-2X^2+3X]-E[X](-2E[X]+3)$$

$$=-2E[X^2]+3E[X]+2(E[X])^2-3E[X]$$

$$=-2\left[E[X^2]-(E[X])^2\right]=-2Var\,X$$

19. If X_1 has mean 4 and variance 9 while X_2 has mean -2 and variance 5 and the two are independent, find $Var(2X_1+X_2-5)$.(AP)

Solution:

Given $E[X_1]=4$ $,Var[X_1]=9$

$E[X_2]=-2$ $,Var[X_2]=5$

$Var(2X_1+X_2-5)=4Var\,X_1+Var\,X_2=4(9)+5=36+5=41$

20. Find the acute angle between the two lines of regression.(R)

Solution:

The equations of the regression lines are

$$y-\bar{y} = r\frac{\sigma y}{\sigma x}(x-\bar{x}) \text{ ------------ } (1)$$

$$x-\bar{x} = r\frac{\sigma x}{\sigma y}(y-\bar{y}) \text{ ------------ } (2)$$

Slope of line (1) is $m_1 = r\frac{\sigma y}{\sigma x}$

Slope of line (2) is $m_2 = \frac{\sigma y}{r\sigma x}$

If θ is the acute angle between the two lines, then

$$\tan\theta = \frac{|m_1 - m_2|}{1+m_1 m_2}$$

$$= \frac{\left| r\frac{\sigma y}{\sigma x} - \frac{\sigma y}{r\sigma x}\right|}{1+r\frac{\sigma y}{\sigma x}.\frac{\sigma y}{r\sigma x}} = \frac{\left|\frac{(r^2-1)}{r}\frac{\sigma y}{\sigma x}\right|}{1+\frac{\sigma y^2}{\sigma x^2}} = \frac{\left|-\frac{(1-r^2)}{r}\frac{\sigma y}{\sigma x}\right|}{\frac{\sigma x^2+\sigma y^2}{\sigma x^2}} = \frac{(1-r^2)\sigma x\sigma y}{|r|\left(\sigma x^2+\sigma y^2\right)}$$

21. If $X \text{ and } Y$ are random variables such that $Y = aX + b$ where a and b are real constants, show that the correlation co-efficient $r(X,Y)$ between them has magnitude one.(R)

Solution:

Correlation co-efficient $r(X,Y) = \frac{Cov(X,Y)}{\sigma_X \sigma_Y}$

$$Cov(X,Y) = E[XY] - E[X]E[Y] = E[X(aX+b) - E[X]E[aX+b]]$$

$$= E[aX^2 + bX] - E[X](aE[X]+b)$$

$$= aE[X^2] + bE[X] - a(E[X])^2 - bE[X]$$

$$= a\left[E[X^2] - (E[X])^2\right] = aVar\,X = a\sigma_X{}^2$$

$$\sigma_Y{}^2 = E[Y^2] - (E[Y])^2$$

$$= E\left[(aX+b)^2\right] - (E[aX+b])^2 = E\left[a^2X^2 + 2abX + b^2\right] - (aE[X]+b)^2$$

$$= a^2E[X^2] + 2abE[X] + b^2 - a^2(E[X])^2 - 2abE[X] - b^2$$

$$= a^2\left[E[X^2] - (E[X])^2\right] = a^2\,Var\,X = a^2\sigma_X{}^2$$

Therefore $\sigma_Y = a\sigma_X$ and $r(X,Y) = \frac{a\sigma_X{}^2}{\sigma_X . a\sigma_X} = 1$

Therefore, the correlation co-efficient $r(X,Y)$ between them has magnitude one.

22. If X and Y are two independent random variables with variances 2 and 3, find the variance of 3X+4Y (May/ June 2013)(AP)

Solution: $Var(3X+4Y) = 9Var(X) + 16\ Var(Y) = 9(2) + 16(3) = 66$

23. If the joint pdf of (X,Y) is given by $f(x,y)=2,\ 0\le x\le y\le 1$, Find E(X)(AP) **(May/ June 2013)**

Solution:

The marginal density function of X is $f(x)=\int_x^1 f(x,y)\,dy = 2(1-x)$

$$E(X)=\int_0^1 x(f(x)dx=\int_0^1 x(2)(1-x)dx=\frac{1}{3}$$

24. When will the two regression lines be (A) at right angles (b) coincident?(U) **(Nov./ Dec. 2012)**

Solution:

If $r=\pm 1$, the regression lines will coincide.

If $r=0$, the regression line will be at right angle to each other.

25. A small college has 90 male and 30 female professors. An ad-hoc committee of 5 is selected at random to unite the vision and mission of the college. If X and Y are the number of men and women in the committee, respectively. What is the joint probability mass function of X and Y? (AP) **(Nov./ Dec. 2012)**

X=No. of men Y= No. of female	Probability
X=5, Y=0	$=5C_5\left(\frac{9}{12}\right)^5$
X=4, Y=1	$=5C_4\left(\frac{9}{12}\right)^4\left(\frac{3}{12}\right)^1$
X=3, Y=2	$=5C_3\left(\frac{9}{12}\right)^3\left(\frac{3}{12}\right)^2$
X=2, Y=3	$=5C_2\left(\frac{9}{12}\right)^2\left(\frac{3}{12}\right)^3$
X=1, Y=4	$=5C_1\left(\frac{9}{12}\right)^1\left(\frac{3}{12}\right)^4$
X=0, Y=5	$=5C_0\left(\frac{9}{12}\right)^0\left(\frac{3}{12}\right)^5$

26. Find the value of ‘k’ if the joint density function of (X, Y) is given by $f(x,y)=k(1-x)(1-y),\ 0<x<4, 1<y<5$ (U) **(May/ Nov. 2014)**

$$\int_0^4\int_1^5 k(1-x)(1-y)dxdy \ =1\Rightarrow k=\frac{1}{32}$$

27. Given the joint probability density function of (X,Y) as $f(x,y)=\frac{1}{6},\ 0<x<2,\ 0<y<3$

Determine the marginal density. **(U)** **(May/June 2014)**

$$f(y)=\int_0^2 \frac{1}{6}dx=\frac{1}{3};\quad f(x)=\int_0^3 \frac{1}{6}dy=\frac{1}{2}$$

28. The joint pdf of a two dimensional random variable (X,Y) is given by $f(x,y)=\begin{cases} kxe^{-y}, & 0\le x\le 2,\ y\ge 0 \\ 0, & otherwise \end{cases}$ Find the value of 'k.(U) **(April / May 2015)(Nov./Dec.2014)**

$$\int_0^2\int_0^\infty kxe^{-y}dy\,dx = k\left(\int_0^2 x\,dx\right)\left(\int_0^\infty e^{-y}dy\right)=1\Rightarrow k=\frac{1}{2}$$

29. In a partially destroyed laboratory record of an analysis of correlation data, the following results only are legible: Variance of x = 9; Regression equations are $8X-10Y+66=0$ *and* $40X-18Y-214=0$. What are the mean values of X and Y?(AP) **(April/ May 2015)**

Solving the equations $\begin{aligned}8X-10Y+66=0\\ 40X-18Y-214=0\end{aligned}$

Mean value of X is 13 and the mean value of Y is 17.

30. The joint probability mass function of a two dimensional random variable (X,Y) is given by $p(x,y)=k(2x+3y);\ x=0,1,2;\ y=1,2,3.$ Find the value of k.(U) **(April/ May 2015)**

X / **Y**	**0**	**1**	**2**
1	**3k**	**5k**	**7k**
2	**6k**	**8k**	**10k**
3	**9k**	**11k**	**13k**

$k=\frac{1}{72}$

31. **What do you mean by correlation between two random vairble ?(R) (April/ May 2015)**
When the random variables X and Y are correlated, the correlation coefficient between X and Y lies between -1 and +1
If the correlation coefficient is 0, then the random variable are uncorrelated.
If the correlation coefficient is -1, the random variables are negatively correlated.
If the correlation coefficient is +1, the random variables are positively correlated.

32. Given the two regression lines $3X+12Y=19$ *and* $3Y+9X=46$, Find the coefficient of correlation between X and Y ?(U)(NOV./Dec. 2015)

From the first equations $b_{yx}=-\left(\frac{1}{4}\right)$ and from the second equation $b_{xy}=-\left(\frac{1}{3}\right)$

Then the correlation coefficient between x and y is $\rho_{xy}=\pm\sqrt{b_{xy}\ b_{yx}}=-\sqrt{\frac{1}{12}}=-0.08$

33. The joint probability density function of bivariate random variable (X,Y) is given by $f(x,y)=4xy,\ 0<x<1,\ 0<y<1,$ Find $P(X+Y<1)$ **(U)(Nov./Dec. 2015)**

$$P(X+Y<1)=\int_0^1\int_0^{1-x} 4xy\ dy\ dx=\frac{1}{6}$$

34. Determine the value of the constant 'c' if the joint density function of two discrete random variables X and Y is given by $p(m,n)=cmn,\ m=1,2,3\ \ and\ \ n=1,2,3$ (u)**(Nov./Dec. 2015)**

m / n	1	2	3
1	c	2c	3c
2	2c	4c	6c
3	3c	6c	9c

Sum of the total probabilities =1 implies c= $\frac{1}{36}$

35. The lines of regression in a bivariate distribution are $X+9Y=7,\ and\ Y+4X=\frac{49}{3}$, Find the correlation coefficient? **(U)(Nov./Dec. 2015)**

Let the regression line Y on X be

$$9Y = 7 - X$$

$$Y = \frac{-1}{9}X + \frac{7}{9}$$

And the regression line X on Y be

$$4X = -Y + \frac{49}{3}$$

$$X = \frac{-1}{4}Y + \frac{49}{12}$$

Therefore

$$\rho_{xy} = \pm\sqrt{b_{xy}.\, b_{yx}}$$

$$= -\sqrt{\left(\frac{1}{9}\right)\left(\frac{1}{4}\right)} = \frac{-1}{6}$$

36. Comment on the statement: "If COV(X,Y)=0,then X and Y are uncorrelated".(R) (**Nov./ Dec.2014**)

Since X and Y are uncorrelated, the correlation coefficient between them is zero. Therefore x and Y are independent random variables.

37. The joint pdf of RV (X,Y) is given as $f(x,y)=\frac{1}{x},\ 0<y<x\le 1$, Find the marginal pdf of Y(U). **(May/ June 2016)**

$$f_y(y) = \int_y^1 \frac{1}{x}dx = \log\left(\frac{1}{y}\right)$$

38. Let X and Y be two independent random variables with Var (X)=9, Var(Y)=3, Find the Var(4X-2Y+6) (U) **(May/ June 2016)**

$$Var(4X-2Y+6) = 16\ Var(X) + 4\ Var(Y)\ -16\ \text{cov}(X,Y)$$

$$= 16(9) + 4(3) - 0 = 144 + 12 = 156$$

39. The joint pdf of R.V. (X,Y) is given by $f(x,y)=ke^{-(2x+3y)},\ x>0, y>0$, Find the value of 'k'.**(Nov/Dec. 2016)**

$$\iint f(x,y)\,dx\,dy = 1$$

$$\int_0^\infty\int_0^\infty ke^{-(2x+3y)}\,dx\,dy = 1$$

$$k\left(\frac{e^{-2x}}{-2}\right)_0^\infty\left(\frac{e^{-3y}}{-3}\right)_0^\infty = 1$$

$$k\left(\frac{1}{2}\right)\left(\frac{1}{3}\right) = 1$$

$$k = 6$$

40. Write any two properties of joint cumulative distribution function. **.(Nov/Dec. 2016)**

(1) $F(-\infty) = 0$ and $F(+\infty) = 1$.
(2) F is a non-decreasing function, i.e. if $x_1 \le x_2$, then $F(x_1) \le F(x_2)$.
(3) If $F(x_0) = 0$, then $F(x) = 0$ for every $x \le x_0$.
(4) $P\{X > x\} = 1 - F(x)$.
(5) $P(\{x_1 < X \le x_2\}) = F(x_2) - F(x_1)$.

41. Define conditional distribution of two-dimensional discrete and continuous random variables**.(May/ June 2017)(Apr/ May 2018)**

$P(X = x_i / Y = y_j) = \dfrac{p_{ij}}{P(Y = y_j)} = \dfrac{p_{ij}}{p_{*j}}$ **in case of discrete random variable**

In case continuous random variable,

$f(x/y) = \dfrac{f(x,y)}{f_Y(y)}$ **where** $f_Y(y)$ is the marginal density function of Y

$f(y/x) = \dfrac{f(x,y)}{f_X(x)}$ **where** $f_X(x)$ is the marginal density function of X

are the conditional density functions.

42. If $X = R\cos\phi$ *and* $Y = R\sin\phi$, how are the joint probability density function of (X,Y) and (R,ϕ) are related **(May/ June 2017**)

Let $f(x,y)$ be the joint density functions of X and Y

And $J = \begin{vmatrix} \cos\phi & -R\sin\phi \\ \sin\phi & R\cos\phi \end{vmatrix} = R$ is the jacobian

$g(R,\phi) = Rf(x,y) = R\ f(R\cos\phi, R\sin\phi)$

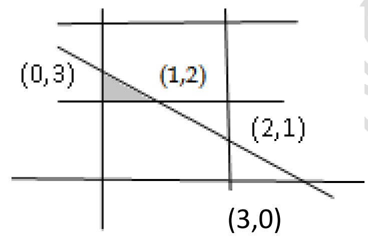
43. Let the joint probability density function of random variable X and Y be given by $f(x,y) = 8xy, 0 < y < x < 1$ Calculate the marginal probability density function of X.(Nov/ Dec 2017)

$$f(x) = \int_0^x 8xy\,dy = 4x^3$$

$$f(y) = \int_y^1 8xy\,dx = 4y(1-y^2)$$

44. If X and Y are random variable having the joint density function $f(x,y) = \frac{1}{8}(6-x-y),\ 0<x<2;\ 2<y<4,$ Find $P(X+Y<3)$ (Nov/ Dec 2017)

$$f(x,y) = \frac{1}{8}(6-x-y),\ 0<x<2;\ 2<y<4,$$



$$P(X+Y<3) = \int_2^3 \int_0^{3-y} \frac{1}{6}(6-x-y)\,dy\,dx = \frac{5}{24}$$

45. Let (X,Y) be a continuous bivariate random variable. If X and Y are independent random variables then show that X and Y are uncorrelated(Apr/ May 2018)

If X and Y are independent random variables then $E(XY) = E(X)E(Y)$

$\Rightarrow \operatorname{cov}(X,Y) = 0$

$\Rightarrow$ X and Y are uncorrelated

UNIT II

PART-B

TWO DIMENSIONAL RANDOM VARIABLES

Marginal &conditional distribution,Covarience

1. Let X and Y have the joint pdf

	X			
Y		0	1	2
	0	0.1	0.4	0.1
	1	0.2	0.2	0

Find

(i) $P(X+Y>1)$

(ii) the probability mass function $P(X = x)$ of the RV X

(iii) $P(Y = 1/ \ X = 1)$

(iv) $E(XY)$ (U) **(Apr/May 2008)**

2. The joint pmf of (X,Y) is given by $p(x,y) = k(3x+5y), x = 1,2,3; y = 0,1,2.$ Find all the marginal and conditional probability distributions $P(X = x_i / Y = 2), P(X \le 2 / Y \le 1)$

(A) **(Apr/May 2019)**

3. The joint distribution of X and Y is given by $f(x,y) = \dfrac{x+y}{21}$, $x = 1,2,3, \ y = 1,2$. Find the marginal distributions and conditional distributions. (**Nov./Dec.2013**) **(Nov/Dec. 2015)(U)**

4. The joint distribution of X and Y is given by $f(x,y) = \dfrac{x+y}{32}$, $x = 1,2, \ y = 1,2,3,4$. Compute the covariance of X and Y . (**May/ June 2016)(U)**

5. The joint pmf of (X,Y) is given by $p(x,y) = k(2x+3y), x = 0,1,2; y = 1,2,3.$ Find all the marginal and conditional probability distributions. Also find the probability distribution of $(X+Y)$. (U) **(Nov/Dec 2007) (Nov/Dec 2011)**

6. Let X and Y be two random variables having the joint probability function $f(x,y) = k(2x+3y)$. where x and y can assume only the integer values 0, 1 and 2. Find all the marginal and conditional distributions. **(U)** **(April/May 2012)**

7. The joint density function of the random variable (X,Y) is given by

$$f(x,y) = \begin{cases} 8xy, 0 < x < 1, 0 < y < x \\ 0, elsewhere \end{cases}$$

(i) Find the marginal density of Y (U)

(ii) Conditional density of $X/Y = y$

(iii) $P\left(X < \dfrac{1}{2}\right)$ **(May/Jun 2007)**

8. The joint probability density function of a two dimensional random variable (X,Y) is given by $f(x,y) = k(xy + y^{2)});0 \le x \le 1, 0 \le y \le 2$ (U)

Compute $P(Y > 1),, P((X+Y) \le 1), P(X > \frac{1}{2}, Y < 1)$. **(April/May 2019)**

9. The joint probability density function of a two dimensional random variable (X,Y) is given by

$f(x,y) = xy^2 + \dfrac{x^2}{8}; 0 \le x \le 2, 0 \le y \le 1$ (U)

Compute $P(X > 1), P\left(Y < \frac{1}{2}\right), P\left(X > 1 \middle/ Y < \frac{1}{2}\right), P\left(Y < \frac{1}{2} \middle/ X > 1\right), P(X < Y), P((X+Y) \le 1)$.

(May/Jun 2009) (Nov/Dec 2007)

10. Given $f(x,y) = cx(x-y)$, $0 < x < 2$, $-x < y < x$ and '0' elsewhere. Evaluate 'c' find $f_X(x)$ and $f_Y(y)$ (U) **(Nov/Dec 2010)**

11. In producing gallium – arsenide microchips, it is known that the ratio between gallium and arsenide is independent of producing a high percentage of workable water, which are main components of microchips. Let X denote the ratio of gallium to arsenide and Y denote the percentage of workable micro wafers retrieved during a 1 hour period. X and Y are independent random variables with the joint density being known as(AP)

$$f(x,y) = \begin{cases} \dfrac{x(1+3y^2)}{4} & 0 < x < 2, 0 < y < 1 \\ 0 & otherwise \end{cases}$$

Show that $E(XY) = E(X)E(Y)$. **(Nov/Dec 2006)**

12. Given the joint density function $f(x,y) = \begin{cases} \dfrac{x(1+3y^2)}{4} & 0 < x < 2,\ 0 < y < 1 \\ 0 & elsewhere \end{cases}$, Find the marginal densities $g(x)$, $h(y)$ and the conditional density $f(x/y)$ and evaluate $P\left[\dfrac{1}{4} < x < \dfrac{1}{2} / Y = 1/3\right]$

(U) **(Apr/May2011)**

13. Two random variables X and Y have the joint density function $f_{XY}(x,y) = x^2 + \dfrac{xy}{3}$ $0 \le x \le 1, 0 \le y \le 2$. Find the conditional density functions. Check whether the conditional density functions are valid. (U) **(Nov/Dec 2006)**

14. Suppose that X and Y are independent non-negative, continuous random variables having densities $f_X(x)$ and $f_Y(y)$ respectively. Compute $P[X < Y]$(U) **(April/May 2010)**

15. The joint density of X and Y is given by $f(x,y) = \begin{cases} \dfrac{1}{2} ye^{-xy} & 0 < x < \infty, 0 < y < 2 \\ 0 & otherwise \end{cases}$.

Calculate the conditional density of X given $Y = 1$. (U) **(April/May 2010)**

16. Determine whether the random variables X and Y are independent , given their joint probability density function as $f(x,y) = \begin{cases} x^2 + \dfrac{xy}{3}, & 0 \le x \le 1,\quad 0 \le y \le 2 \\ 0 & otherwise \end{cases}$

(U)

(April/May 2011)

17. The joint probability density function of a two dimensional random variable (X,Y) is $f(x,y) = \dfrac{6-x-y}{8}$, $0 < x < 2,\ 2 < y < 4$. (U)

Find (1) $P(X < 1 \cap Y < 3)$ (2) $P(X + Y < 3)$ (3) $P(X < 1 / Y < 3)$ **(May/ June 2013)**

18. The joint pdf of random variable X and Y is given by $f(x,y) = \begin{cases} \lambda xy^2, & 0 \le x \le y \le 1 \\ 0\ ; & otherwise \end{cases}$(U)

(1) Determine the value of λ

(2) Find the marginal probability density function of X and Y

(3) Find the conditional pdf f(x/y) **(May/ June 2016)(Nov./ Dec. 2012)**

19. The joint probability density function of two random variables X and Y is given b $f(x,y)=\frac{6}{7}\left(x^2+\frac{xy}{2}\right),\ 0\le x\le 1,\ 0\le y\le 2$, Find the conditional density function of X given Y and the conditional density function of Y given X (U) (**May/ June 2014**)

20. The joint probability density function of a two dimension random variable (X,Y) is given by $f(x,y)=xy^2+\frac{x^2}{8};\ 0\le x\le 2;\ 0\le y\le 1$.Compute (U)

$P(X>1),\ P(Y<\frac{1}{2}),\ P\left(X>1/Y<\frac{1}{2}\right),\ P\left(Y<\frac{1}{2}/X>1\right),\ P(X<Y) and\ P(X+Y\le 1)$

(April/ May 2015)

21. Two random variables X and Y have the following joint probability density function $f(x,y)=xe^{-x(y+1)},\ x\ge 0,\ y\ge 0$. Determine the conditional probability density function of X given Y and the conditional probability density function of Y given X**(Nov/ Dec. 2017)**

22. The joint probability density function of the two dimensional random variable (X,Y) is given by $f(x,y)=\frac{x}{4}(1+3y^2),\ 0<x<2,\ 0<y<1$. Find (i) conditional probability density function of X given Y=y and Y given X =x (ii) $P\left[0.25<X<0.5\ /\ Y=0.33\right]$ **(Apr/ May 2018)**

23. Two dimensional random variable (X,Y) have the joint probability density function $f(x,y)=\begin{cases}8xy, & 0<x<y<1\\ 0 & elsewhere\end{cases}$ (i) Find $P\left[X<\frac{1}{2}\bigcap Y<\frac{1}{4}\right]$, (ii) Find the marginal and conditional distributions. (iii) Are X and Y independent.(**April/May 2012**)(May/ June 2017)(AP)

24. Two random variables X and Y have the following joint probability density functi $f(x,y)=\begin{cases}2-x-y & 0\le x\le 1, 0\le y\le 1\\ 0 & otherwise\end{cases}$

Find

(a) Marginal probability density functions of X and Y

(b) Conditional density functions

(c) $Var(X)$ and $Var(Y)$. (U) **(May/June 2006)**

25. If the joint distribution function of X and Y is given by

$$F(x,y)=\begin{cases}(1-e^{-x})(1-e^{-y}) & for\ x>0, y>0\\ 0 & otherwise\end{cases}$$

(i) the marginal densities of X and Y (U)

(ii) Are X and Y independent

Find $P(1<x<3, 1<Y<2)$

26. Assume that the random variable X and Y have the joint PDF $f(x,y)=\frac{1}{2}x^3y;\ 0\le x\le 2;\ 0\le y\le 1$ Determine if X and Y are independent (**April/ May 2015**)(U)

27. The joint pdf of the random variable X and Y is defined as $f(x,y)=25e^{-5y};\ 0<x<0.2,\ y>0$ (1) find the marginal PDFs of X and Y (2) what is the covariance of X and Y? **(April/ May 2015)(U)**

28. Find the constants k such that $f(x,y)=k(1+x)e^{-y},\ 0<x<1,\ y>0$ is the joint pdf of the continuous random variable (X, Y), Are X and Y independent r.v's Explain. **(May/ June 2016)(U)**

29. Let the joint pdf of (X,Y) be given as $f(x,y)=Cxy^2,\ 0\le x\le y\le 1$, Determine the value of C , Find the marginal pdf of X and Y and find the conditional pdf f(x/ y) (**May/ June 2016**)(U)

30. The probability density function of X and Y is given by $f(x,y)=\frac{6}{7}\left(x^{2+}\frac{xy}{2}\right),0<x<1,\ 0<y<2$ (1) compute the marginal density function of X and Y, (2) Find E(X) and E(Y) (3) Find $P\left(X<\frac{1}{2},Y>\frac{1}{2}\right),0<x<1,\ 0<y<2$. (**Nov/Dec 2015)(U)**

31. The joint CDF of two discrete random variable X and Y is $F(x,y)=\begin{cases}\frac{1}{8}, x=1, y=1\\ \frac{5}{8}, x=1, y=2\\ \frac{1}{4}, x=2, y=1\\ 1, x=2, y=2\end{cases}$. Find the joint probability mass function and the marginal probability mass function of *X* and *Y*.(Nov/ Dec. 2016)

32. The joint probability density function of a two dimensional random variable (X,Y) is given by $f(x,y)=\frac{x(x-y)}{8},0<x<2;-x<y<x$. find the marginal distribution of X and Y and the conditional distribution of Y=y given that X= x(**Nov/Dec. 2016**)

Transformation of random variable

33. If the joint density of X_1, X_2 is given by $f(x_1,x_2)=\begin{cases}6e^{-3x_1-2x_2} & \text{for } x_1>0, x_2>0\\ 0 & \text{otherwise}\end{cases}$ find the probability density of $Y=X_1+X_2$. (U) **(Nov/Dec 2006)**

34. Let X and Y be independent random variables, both uniformly distributed on (0, 1). Calculate the probability density of $X+Y$.(U) **(April/May 2010)**

35. Two random variables X and Y have the following joint probability density function $f(x,y)=\begin{cases}c(4-x-y) & 0\le x\le 2,\ 0\le y\le 2\\ 0 & \text{elsewhere}\end{cases}$ Find $\operatorname{cov}(X,Y)$. Find the equations of two lines of regression. (U) **(Apr/May 2012) (Nov/Dec. 2015)**

36. If X and Y are independent random variables with pdf's e^{-x}, $x \geq 0$ and e^{-y}, $y \geq 0$ respectively, find the density functions of $U = \frac{X}{X+Y}$ and $V = X + Y$. Are U and V independent?(U) **(Nov/Dec 2011)**

37. Let (X,Y) be a two dimensional non negative continuous random variable having the joint density $f(x,y) = \begin{cases} 4xye^{-(x^2+y^2)}, x \geq 0, y \geq 0 \\ 0 \quad elsewhere \end{cases}$ Find the density function of $U = \sqrt{X^2 + Y^2}$ (AP) **(May/Jun 2006)**

38. If the joint probability density of X_1, X_2 is given by $f(x_1,x_2) = \begin{cases} e^{-(x_1+x_2)} & for\, x_1 > 0, x_2 > 0 \\ 0 & elsewhere \end{cases}$. Find the probability of $Y = \frac{X_1}{X_1 + X_2}$ (AP) **(Nov/Dec 2006)**

39. If X and Y are independent random variables having density functions $f(x) = \begin{cases} 2e^{-2x}, & x \geq 0 \\ 0 & x < 0 \end{cases}$ *and* $f(y) = \begin{cases} 3e^{-3y}, & y \geq 0 \\ 0 & y < 0 \end{cases}$ respectively, find the density functions $z = X - Y$. (AP) **(Apr/May 2011)**
(i) **(Nov/Dec 2008)**

40. Let X be a random variable with pdf $f(x) = \left\{ \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, -\infty < x < \infty \right.$, find the pdf of the RV $Y = X^2$. (A) **(Apr/May 2008)**

41. f X is any continuous RV having the pdf $f(x) = \begin{cases} 2x, 0 < x < 1 \\ 0, otherwise \end{cases}$ and $Y = e^{-X}$, find the pdf of RV Y. (U) **(Apr/May 2008)**

42. Find $P(X > 2/Y < 4)$ when the joint pdf of X and Y is given by $g(x,y) = \begin{cases} e^{-(x+y)}, x \geq 0, y \geq 0 \\ o \quad otherwise \end{cases}$. Are X and Y independent RVs? Explain. And find the pdf of the RV $U = \frac{X}{Y}$ (U) **(Apr/May 2008)(May/ June 2016)**

43. If the joint pdf of the RVs X and Y is given by $f(x,y) = \begin{cases} 1, & 0 < x < y < 1 \\ 0 & otherwise \end{cases}$ find the pdf of the RV $U = \frac{X}{Y}$ (U) **(Apr/May 2008)**

44. If X and Y are independent exponential random variables each with parameter 1, find the pdf of $U = X - Y$. (U) **(May/Jun 2007)(May/ June 2013) (Nov/Dec 2015)(Nov/ Dec. 2016)**

45. If the pdf of 'X' is $f(x) = 2x, (0 < x < 1)$, find the pdf of Y= 3X+1 (U) **(Nov./Dec.2013)**

46. If the independent random variables X and Y have the variances 36 and 16 respectively. Find the correlation coefficient r_{uv}, where U =X+Y and V= X-Y (U) **(May/ June, 2014)**

47. If X and Y are independent random variables with pdf's $e^{-x}, x \geq 0$ and $e^{-y}, y \geq 0$ respectively, find the density functions of $U = \frac{X}{X+Y}$. **(Nov/Dec 2011) (Nov/Dec 2015)(U)**

48. If X and Y follows an exponential distribution with parameter 2 and 3 respectively and are independent, find the probability density function of $U = X + Y$ (**May/ June 2017**)

49. If X and Y are two independent random variables each normally distributed with mean 0 and variance σ^2, then find the joint probability density function of $R = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}\left(\frac{y}{x}\right)$ and hence find the probability density function of θ **(May/ Nov 2017)**

50. The joint density function of two random variables X and Y is given by $f(x, y) = \frac{1}{4}e^{-(x+y)/2}, x > 0, y > 0.$ Find the distribution of $\frac{X - Y}{4}$ **(April/May 2019)(A)**

Correlation coeeficient

51. Compute the co-efficient of correlation between X and Y using the following data **(AP) (Nov/Dec 2010)**

X	1	3	5	7	8	10
Y	8	12	15	17	18	20

52. If the correlation coefficient is 0, then can we conclude that they are independent? Justify your answer through an example. What about the converse? **(U)** **(April/May 2010)**

53. Find the coefficient of correlation between industrial production and export using the following data: **(U)** **(Nov/Dec 2008)**

Production(X)	55	56	58	59	60	60	62
Export(Y)	35	38	37	39	44	43	44

54. Calculate the correlation coefficient for the following data: **(May/Jun 2007) (Nov/Dec 2007)(U)**

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

55. Find the coefficient of correlation and obtain the lines of regression from the data given below: **(Nov/Dec 2006)**

X	50	55	50	60	65	65	65	60	60	50
Y	11	14	13	16	16	15	15	14	13	13

56. Find the coefficient of correlation and obtain the lines of regression from the data given below: **(U)(May/Jun 2006)**

X	62	64	65	69	70	71	72	74
Y	126	125	139	145	165	152	180	208

57. Let the random variable X have the marginal density $f_1(x) = 1, -\frac{1}{2} < x < \frac{1}{2}$ and let the conditional density of Y be $f(y/x) = \begin{cases} 1 & x < y < x+1, \quad -\frac{1}{2} < x < 0 \\ 1 & -x < y < 1-x \quad 0 < x < \frac{1}{2} \end{cases}$. Show that the variables are uncorrelated. (AP) **(May/Jun 2006)**

58. Let z be a random variable with probability density $f(z)=\frac{1}{2}$ in the range $-1\le z\le 1$. Let the random variable $X=z$ and the random variable $Y=z^2$. Obviously X and Y are not independent since $X^2=Y$. Show, none the less, that X and Y are uncorrelated. (AP) **(Nov/Dec 2006)**

59. Two random variables X and Y are defined as $Y=4X+9$. Find the correlation coefficient between X and Y. (U) **(Nov/Dec 2006)**

60. The marks obtained by 10 students in Mathematics and Statistics are given below. Find the correlation coefficient between the two subjects (AP) **(May/ June 2013)**

Marks in Mathematics	75	30	60	80	53	35	15	40	38	48
Marks in Statistics	85	45	54	91	58	63	35	43	45	44

61. Let X and Y be random variables having joint density function

$$f(x,y)=\begin{cases}\frac{3}{2}(x^2+y^2), \ 0\le x\le 1, \ 0\le y\le 1\\ 0, \quad elsewhere\end{cases}$$

Find the correlation coefficient γ_{xy} (U) **(Nov./Dec.2013)**

62. The joint probability density function of two random variables X and Y is $f(x,y)=k\left[(x+y)-(x^2+y^2)\right], 0<(x,y)<1,$ Show that X and Y are uncorrelated but not independent?(U) **(May/ June 2014)**

63. Calculate the coefficient of correlation for the following data: (U)

X:	9	8	7	6	5	4	3	2	1
Y:	15	16	14	13	11	12	10	8	9

(**Nov./Dec. 2014**)

64. If X, Y and Z are uncorrelated random variables with zero means and standard deviation 5, 12 and 9 respectively and if U = X + Y and V = Y + Z, find the correlation coefficient between U and V. (**Nov/Dec 2015)(U)**

65. If the joint pdf of (X,Y) is given by $g(x,y)=e^{-(x+y)}, x\ge 0, y\ge 0$ Prove that X and Y are uncorrelated (**May/ June 2017**)

66. Given that X=4Y+5 and Y=kX+4 are regression lines of X on Y and Y on X respectively. Show that $0\le k\le \frac{1}{4}$. If $k=\frac{1}{16}$, find the means of X and Y and the correlation coefficient r_{xy} .**(Apr/ May 2018)**

Regression lines

67. IThe two lines of regression are $\begin{array}{l}8x-10y+66=0\\ 40x-18y+214=0\end{array}$. The variance of X is 9. (U)

Find the mean values of X and Y, Correlation coefficient between X and Y. **(Nov/Dec 2008)**

68. Obtain the equations of the regression lines from the following data, using the method of least squares. Hence find the coefficient of correlation between X and Y. Also estimate the value of Y when $X=38$ and the value of X when $Y=18$. **(May/Jun2009)(April/May 2015)(U)**

X	*22*	*26*	*29*	*30*	*31*	*33*	*34*	*35*
Y	*20*	*20*	*21*	*29*	*27*	*24*	*27*	*31*

69. Can $Y = 5 + 2.8X, X = 3 - 0.5Y$ be the estimated regression equations of Y on X and X on Y respectively? Explain your answer with suitable theoretical arguments. **(Nov/Dec 2007)(U)**

70. For two random variables X and Y with the same mean, the two regression equations are $y = ax + b$ and $x = cy + d$. Find the common mean, ratio of the standard deviations and also show that $\frac{b}{d} = \frac{1-a}{1-c}$. **(AP)** **(Nov/Dec2010)**

71. Obtain the equation of the lines of regression from the following data☹U)

X:	1	2	3	4	5	6	7
Y:	9	8	10	12	11	13	14

(Nov./ Dec. 2012)

72. The regression equations of X and Y is $3y - 5x + 108 = 0$. If the mean value of Y is 44 and the variance of X were $\frac{9}{16}th$ of the variance of Y. Find the mean value of X and the correlation coefficient. (U) **(Nov. / Dec. 2012)**

73. Find the equation of the regression line Y on X from the following data:

X:	3	5	6	8	9	11
Y:	2	3	4	6	5	8

(April/ May 2015)(U)

Central limit theorem

74. Let $X_1, X_2, ..., X_{100}$ be independent identically distributed random variables with $\mu = 2$ And $\sigma^2 = \frac{1}{4}$. Find $P(192 < X_1 + X_2 + ... + X_{100} < 210)$.(U) **(Nov./Dec.2012)**

54. IF $X_1, X_2, ..., X_n$ are Poisson variates with parameter $\lambda = 2$, Use the central limit theorem to estimate $P(120 < S_n < 160)$ where $S_n = X_1 + X_2 + ... + X_n$, $n = 75$ (**Nov./Dec.2014**)

75. If $X_1, X_2, \ldots X_n$ are Poisson variates with parameter $\lambda = 2$, use CLT to estimate $P(120 < S_n < 160)$ where $S_n = X_1 + X_2 + \ldots + X_n$ and n = 75. (**Nov/Dec 2015)**

76. A distribution with unknown mean μ has variance equal to 1.5, Use central limit theorem to find how large a sample should be taken from the distribution in order that the probability will be atleast 0.95 that the sample mean will be within 0.5 of the population mean. (AP) **(May/ June 2013)**

UNIT-III

TESTING OF HYPOTHESIS

Sampling distributions - Estimation of parameters - Statistical hypothesis - Large sample tests based on Normal distribution for single mean and difference of means -Tests based on t, Chi-square and F distributions for mean, variance and proportion - Contingency table (test for independent) - Goodness of fit.

PART-A

1. Write the application of F-test and χ^2 (chi-square test)

F-test(R)

F-distribution is used to test the equality of the variances.

χ^2 (chi-square test)

χ^2 is used to test the goodness of fit.

χ^2 is used to test the independence of attributes.

2. Write two applications of χ^2 test(R)

χ^2 is used to test the goodness of fit.

χ^2 is used to test the independence of attributes.

3. Write the use of 't' distribution(R)

The t-distribution is used to test the significance of the difference between
The mean of a small sample and the mean of the population.
The means of two small samples
The coefficient of correlation in the small sample and that in the population, assumed zero.

4. Define errors in sampling and critical region. (R)

Type I Error

Reject H_0 when it is true.

Type II Error

Accept H_0 when it is false.

5. Define a F- variate. (R)

A random variate F is said to follow Snedecor's F-distribution or simply F-distribution if its probability density function is given by

$$f(F) = \frac{(\gamma_1/\gamma_2)^{\frac{\gamma_1}{2}}}{\beta\left(\frac{\gamma_1}{2}, \frac{\gamma_2}{2}\right)} \cdot \frac{F^{\frac{\gamma_1}{2}-1}}{\left(1+\frac{\gamma_1 F}{\gamma_2}\right)^{\left(\frac{\gamma_1+\gamma_2}{2}\right)}}, F > 0$$

6. A sample of size 13 gave an estimated population variance of 3.0, while another sample of size 15 gave an estimate of 2.5.Could both samples be from populations with the same variance? (A)

Given $n_1 = 13$, $n_2 = 15$, $s_1^2 = 3$, $s_2^2 = 2.5$, $\gamma_1 = 12$, , $\gamma_2 = 14$

H_0: $S_1^2 = S_2^2$ (i.e the two samples have been drawn from population with the same Variance)

$H_1: S_1^2 \neq S_2^2$

$$F = \frac{S_1^{\ 2}}{S_2^{\ 2}} = \frac{3}{2.5} = 1.2$$

$F_{0.05}$ ($\gamma_1 = 12$, $\gamma_2 = 14$) = 2.53 and $F < F_{0.05}$

Therefore H_0 is accepted

(i.e) the two samples could have from two normal population with the same variance

7. **A machine runs on an average of 125 hours/year. A random sample of 49 machines has an annual average use of 126.9 hours with standard deviation of 8.4hours.Does this suggest to believe that machines are used on the average more than 125 hours annually at 0.05 level of significance? (U)**

Given n = 49, $\bar{x} = 126.9, \mu = 165, \sigma = 8.4$

Null Hypothesis: $H_0 : \bar{x} = \mu$

Alternative Hypothesis: $H_1 : \bar{x} > \mu$ (one- tailed test)

Test statistics is

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$= \frac{126.9 - 125}{\frac{8.4}{\sqrt{49}}} = 1.583$$

Tabulated value $z_\alpha = 1.645$

$|z| < z_\alpha$

Therefore, H_0 is accepted.

(i.e) the difference between $\bar{x}$ and μ is not significant.

8. Write down t-test significance single mean formula for given sample and population mean $\bar{x}$ and μ respectively. (R)

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}}$$

9. What are null and alternate hypothesis?. (DEC 2012

Null hypothesis H_0 is based for analyzing the problem Null hypothesis is the hypothesis of no difference. Any hypothesis which is complementary to the null hypothesis is called an alternative hypothesis, denoted by H_1.

10. A random sample of 200 tins of coconut oil gave an average weight of 4.95 kgs. With a standard deviation of 0.21 kg. Do we accept that the net weight is 5 kgs per tin at 5% level? (MAY 2013)

Given $n = 200,\ \bar{x} = 4.95, \mu = 5, \sigma = 0.21$

Null Hypothesis $H_0 : \bar{x} = \mu$

Alternative Hypothesis $H_1 : \bar{x} \neq \mu$ (two - tailed test)

Test statistics is

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$= \frac{4.95 - 5}{\frac{0.21}{\sqrt{200}}} = -3.367$$

Tabulated value $z_\alpha = 1.96$

$|z| \succ z_\alpha$

Therefore, H_0 is rejected

(i.e) the difference between $\bar{x}$ and μ is significant.

11. What are the expected frequencies of 2×2 contingency table,

a	**b**
c	**d**

(MAY 2015)(DEC2012)

The value of χ^2 for the 2×2 contingency table is given by

$$\chi^2 = \frac{N(ad - bc)^2}{(a+b)(c+d)(a+c)(b+d)}$$

12. Write down the formula of test statistic t to test the significant difference between the means of large samples. (MAY 2015) (R)

$$t = \frac{\bar{x} - \bar{y}}{\sqrt{S^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

13. What is random sampling? (DEC.2015,R2013)

A random sampling is one in which each number of population has an equal chance of being included in it. There are NC_n different samples of size n that can be picked up from a population size N.

14.Write about F-test. (DEC.2015,R2013)

F-test is a distribution to test if the estimates S_1^2 and S_2^2 are significantly different or if the samples may be regarded as drawn from the same population or two populations with same variance σ^2.

$$F = \frac{S_1^2}{S_2^2} = \frac{\frac{n_1 s_1^2}{n_1 - 1}}{\frac{n_2 s_2^2}{n_2 - 1}}$$

Where

s_1^2 *and* s_2^2 are the variances of two samples.

15. What are Type – I and Type – II errors? (MAY 2019,2016,R2013)(R)

Type I Error : If H_0 is rejected while it should have been accepted.

Type II Error : If H_0 is accepted while it should have been rejected.

	DECISION	
	Accept H_0	Reject H_1
H_0 True	Correct Decision	Type I error
H_1 False	Type I error	Correct Decision

16. Give the formula for the Chi-Square test of independence for

a	**b**
c	**d**

(MAY2016,R2013)

The value of χ^2 for the 2×2 contingency table is given by

$$\chi^2 = \frac{N(ad-bc)^2}{(a+b)(c+d)(a+c)(b+d)}$$

17. An oil company claims that lessthan 20 percent of all car owners have not tried its gasoline. Test this claims at the .01 level of significance if a random check that 22 of 200 car owners have not tried the oil company's gasoline .

PART-B

Test of significance for large samples

Type I(Test of significance of the difference between sample proportion and population proportion)

1. In a sample of 1000 people in Karnataka 540 are rice eaters and the rest are wheat eaters. Can we assume that both rice and wheat are equally popular in this state at 1% level of significance? **(MAY 2009)(A)**

2. A manufacturer claimed that at least 95% of the equipments which he supplied to a factory conformed to specifications. An examination of a sample of 200 pieces of equipment revealed that 18 were faulty. Test his claim at 5 %level of significance. **(MAY 2011) (A)**

3. In a city 325 men out of 600 men were found to be smokers. Does this information support the conclusion that the majority of men in this city are smokers? **(DEC 2010) (A)**

4. In a sample of 400 parts manufactured by a factory, the number of defective parts was found to be 30.The company, however, claimed that only 5% of their product is defective. Is the claim tenable? **(MAY 2011) (A)**

5. A die was thrown 9000 times and of these 3220 yielded a 3 0r 4. Is this consistent with the hypothesis that the die was unbiased? **(MAY 2009) (A)**

6. Experience has shown that 20% of a manufactured product is of top quality. In one's production of 400 articles, only 50 are of top quality. Show that either the production of the day chosen was not a representative sample or the hypothesis of 20%was wrong. Based on the particular day's production, find also the 95% confidence limits for the percentage of top quality product. **(DEC 2010) (A)**

7. A salesman in a departmental store claims that at most 60 percent of the shoppers entering the store leave without making a purchase. A random sample of 50 shoppers showed that 35 of them left without making a purchase. Are these sample results consistent with the claim of the salesman? **(MAY 2011) (A)**

Type II(Test of significance of the difference between two sample proportions)

8. In a large city A, 20% of a random sample of 900 school boys had a slight physical defect. In another large city b, 18.5% of a random sample of 1600 school boys has the same deft. Is the difference between the proportions significant? **(A)**

9. Before an increase in excise duty on tea, 800 people out of a sample of 1000 were consumers of tea. After the increase in duty, 800 people were consumers of tea in a sample of 1200 persons. Find whether there is a significant decrease in the consumption of tea after the increase in duty. Use 1%level of significance. **(A)**

10. 15.5% of a random sample of 1600 undergraduates were smokers, whereas 20% of a random sample of 900 postgraduates were smokers in a state. Can we conclude that less number of undergraduates are smokers than the postgraduates? **(MAY 2009) (A)**

11. A drug manufacturer claims that the proportion of patients exhibiting side effect to their new arthritis drug is atleast 8% lower than for the standard brand X. In a controlled experiment , 31 out of 100 patients receiving the drug exhibited the side effects, as did 74 out of 150 patients receiving brand X . Teset the manufacturers 's claim at 5% level of significances.**(April/May2019)(A)**

Type III(Test of significance of the difference between sample mean and population mean)

11.A sample of 100 students is taken from large population. The mean height of the students in this sample is 160cm.Can it be reasonably regarded that in the population, the mean height is 165cm and the SD is 10cm?use 1%level of significance. **(A)**

12. The mean breaking strength of the cables supplied by a manufacturer is 1800, with an SD of 100. By a new technique in the manufacturing process, it is claimed that the breaking strength of the cable has increased. To test this claim, a sample of 50 cables is tested and it is found that the mean breaking strength is 1850.Can we support the claim at 1%level of significance? **(A)**

13. A sample of 900 members has a mean of 3.4 cms and SD 2.61cms. Is the sample from a large population of mean 3.25 cm and SD 2.61 cms. If the population is normal and its mean is unknown. Find the 95% fiducial limits of true mean. **(DEC 2010) (A)**

14. An insurance agent has claimed that the average age of policy holders who issue through him is less than the average for all gents which is 0.5 years. A random sample of 100 holders who had

issued through him has the mean 28.8 and SD 6.35.Test his claim at 5% level of significance. **(MAY 2009) (A)**

Type IV (Test of significance of the difference between the means of two samples)

15. In a random sample of size 500,the mean is found to be 20.In another independent sample of size 400,the mean is 150.Could the samples have been drawn from the same population with SD 4?Use 1% LOS. **(MAY 2011) (A)**

16. A simple sample of heights of 6400 Englishmen has mean of 170 cm and a SD of 6.4 cm, While a simple sample of heights of 1600 has mean of 172 cm and an SD of 6.3cm.Do the data indicate that Americans are, on the average, taller than the Englishmen? Use 1%level. **(A)**

17. Test the significance of the difference between the means of the samples drawn from two normal populations with the same SD using the following data: **(A)**

	Size	Mean	SD
Sample 1	100	61	4
Sample 2	200	63	6

18. The average marks scored by 32 boys is 72 with an SD of 8,while that for 36 girls is 70 with an SD of 6.Test 1% LOS whether the boys perform better than girls. **(MAY 2009) (A)**

Test of significance for small samples (Student t- distribution)

Type I (Test of significance of the difference between sample mean and population mean)

19.Tests made on the breaking strength of 10 pieces of a metal gave the following results 578,572,570,568,572,570,570,572,596 and 584 kg. Test if the mean breaking strength of the wire can be assumed as 577kg. **(A)**

20. A machinist is expected to make engine parts with axle diameter 1.75cm. A random sample of 10 parts shows a mean diameter of 1.85cm,with an SD of 0.1cm.On the basis of this sample, would you say that the work of the machinist is inferior? **(A)**

21. A certain injection administered to each of the 12 patients resulted in the following increases of blood pressure:5,2,-1,3,0,6,-2,1,5,0,4.Can it be included that the injection will be ,in general, accompanied by an increase in BP? **(A)**

22. The mean life time of a sample of 25 bulbs is found as 1550 hrs with an SD of 120hrs.The company manufacturing the bulbs claims that the average life of their bulbs is 1600hrs.Is the claim accepted at 5% level of significance? **(MAY 2009) (A)**

23.The heights of 10 males of a given locality are found to be 175,168,155,170,152,170,175,160,160and 165cm.Based on this sample, find the 95% confidence limits for the height of males in that locality. **(A)**

24. Two independent samples of sizes 8 and 7 contained the following values.

Sample1	19	17	15	21	16	18	16	14
Sample2	15	14	15	19	15	18	16	

Is the difference between the sample means significant? **(A)**

25. The following data represent the biological values of protein from cow's milk and buffalo's milk at a certain level. **(A)**

Cow's milk	1.82	2.02	1.88	1.61	1.81	1.54
Buffalo's milk	2.00	1.83	1.86	2.03	2.19	1.88

26. Samples of two types of electric bulbs were tested for length of life and the following data were obtained. **(A)**

	Size	Mean	SD
Sample 1	8	1234HR	36HR
Sample 2	7	1036HR	40HR

Is the difference in the means sufficient to warrant that type 1 bulbs are superior to type 2 bulbs.

27. The mean height and the SD height of 8 randomly chosen soldiers are 166.9 and 8.29 cm respectively. The corresponding values of 6 randomly chosen sailors are 170.3 and 8.50 cm respectively. Based on these data, can we conclude that soldiers are, in general, shorter than sailors?

F-Test

28.A sample of size 13 gave an estimated population variance of 3.0 while another sample of size 15 gave an estimate of 2.5 . Could both samples be from populations with the same variance ? **(A)**

29.Two samples of sizes 9 and 8 gave the sums of squares of deviations from the respective means equal to 160 and 91 respectively .Can they be regarded as drawn from the same normal population. **(MAY 2009) (A)**

30.Two independent samples of 8 and 7 items respectively has the following values of the variable

Sample 1	9	11	13	15	9	12	14	9
Sample 2	10	12	10	14	9	8	10	

Do the two estimates of population variance differ significantly at 5% level of significance? **(MAY 2011)**

31. Two random samples gave the following data

Sample No	Size	Mean	Variance
1	8	9.6	1.2
2	11	16.5	2.5

Can we conclude that the two samples have been drawn from the same normal population?

32. The nicotine contents in two random samples of tobacco are given below:

Sample 1:	21	24	25	26	27	
Sample 2:	22	27	28	30	31	36

Can you say that the two samples came from the same population? **(MAY 2009)**

33.Two random samples are drawn from normal population are given below

Sample 1	17	27	18	25	27	29	13	17
Sample 2	16	16	20	27	26	25	21	

Can we conclude that the two samples have been drawn from the same population?
(A)(April/May2019)

χ^2 -Test(Goodness of fit) (A)

33.The following data show the distribution of digits in the numbers chosen at random from a telephone directory

Digit:	0	1	2	3	4	5	6	7	8	9	Total
Frequency:	1026	1107	997	966	1075	933	1107	972	964	853	10000

Test whether the digits may be taken to occur equally frequently in the directory.

34. The following data give the number of aircraft accidents that occurred during the various days of a week.

Day:	MON	TUE	WED	THUR	FRI	SAT
No .of Accidents:	15	19	13	12	16	15

Test whether the accidents are uniformly distributed over the week. **(A)**

35.The data show defective articles produced by 4 machines

Machine:	A	B	C	D
Production time:	1	1	2	3
No of defectives:	12	30	63	98

Do the figures indicate a significant difference in the performance of the machines?
(MAY 2009)

36. Theory predicts that the proportion of beans in 4 groups A,B,C,D should be 9:3:3:1.In an experiment among 1600 beans, the number in the 4 groups were 882,313,287 and 118.Does the experiment support the theory? **(A)**

37. A survey of 320 families with 5 children revealed the following distribution

No of boys :	0	1	2	3	4	5
No of girls :	5	4	3	2	1	0
No of families:	12	40	88	110	56	14

Is the result consistent with the hypothesis that male and female births are equally probable?

χ^2 -Test(Independence of Attributes)

37. Fit a Poisson distribution to the following data and test the goodness of fit. Test at 5%level of significance (A) **(April/May 2019)**

x	0	1	2	3	4	5
f	142	156	69	27	5	1

38.The following data is collected on two characters . Based on this can you say that there is no relation between smoking and literacy ? **(A)**

	Smokers	Non smokers
Literates	83	57
Illiterates	45	68

39.The following table gives for a sample of married women, the level of education and the marriage adjustment score

Level of education	marriage very low	level low	high	very high	Total
college	24	97	62	58	241
High school	22	28	30	41	121
Middle school	32	10	11	20	73
Total	78	135	103	119	435

Can you conclude from this data that the higher the level of education, the greater is the degree of adjustment in marriage? **(MAY 2009) (A)**

40. In a random sample of 1000 people from city A,400 are found to be a consumers of wheat. In a sample of 800 from city B,400 are found to be consumers of wheat.Does this data give a significant difference between the two cities as far as the proportion of wheat consumers is concerned?

41. 4 coins were tossed 160 times and the following results were obtained:

No of heads:	0	1	2	3	4
Observed frequencies:	17	52	54	31	6

Under the assumption that the coins are unbiased, find the expected frequencies of getting 0,1,2,3,4 heads and test the goodness of fit.

42. The heights of 10 males of a given locality are found to be 70,67,62,68,61,68,70,64,64,66 inches. Is it reasonable to believe that the average height is greater than 64 inches? **(A)**

43. Test of the fidelity and selectivity of 190 radio receivers produced the results shown in the following table:

	FIDELITY		
SELECTIVITY	LOW	AVERAGE	HIGH
LOW	6	12	32
AVERAGE	33	61	18
HIGH	13	15	0

Use the 0.01 level of significance to test whether there is a relationship between fidelity and selectivity.

44. A sample of 10 boys had the IQ's : 70, 120, 110, 101, 88, 83, 95, 98, 100 and 107. Test whether the population mean IQ may be 100. (DEC 2012) **(A)**

45. A mathematics test was given to 50 girls and 75 boys. The girls made an average grade of 76 with a SD of 6, while boys made an average grade of 82 with aSD of 2. Test whether there is any significant difference between the performance of boys and girls. (DEC 2012) **(A)**

46. A machine puts out 16 imperfect articles in a sample of 500. After it was overhauled., it puts out 3 imperfect articles in a sample of 100. Has the machine improved in its performance?.
(DEC 2012)

47. Test whether there is any significant of the difference between the variances of the populations from whichthe following samples are taken:

Sample 1	20	16	26	27	23	22	
Sample 2	27	33	42	35	32	34	38

(DEC 2012)

48. Random samples drawn from two countries gave the following data relating to the heights of adult males. Is the difference between standard deviation significant?

	Country A	Country B
Mean height (in inches)	67.42	67.25
S.D (in inches)	2.58	2.50
Number in samples	1000	1200

(MAY 2013) (A)

49. 1000 students at college level were graded according to their I.Q. and their economic conditions. What conclusion can you draw from the following data:

Economic conditions	I.Q. Level High	Low
Rich	460	140
Poor	240	160

(MAY 2013) (A)

50. The sales manager of a large company conducted a sample survey in states A and B taking 400 samples in each case. The results were in the following table. Test whether the average sales in the same in the 2 states at 1% level.

	State A	State B
Average sales	Rs. 2,500	Rs.2,200
S.D.	Rs. 400	Rs. 550

(MAY 2013) (A)

51. Find if there is any association between extravagance in fathers and extravagance in sons from the following data. Determine the coefficient of association also

	Extravagant father	Miserly father
Extrav. Sons	Under 327	741
Miserly Sons	545	234

(MAY 2013) (A)

52. Test if the variances are significantly different for

X1: 24 27 26 21 25

X2: 27 30 32 36 28 32 **(DEC.2015,R2013)**

53. The number of automobile accidents in a certain locality was 12,8,20,2,14,10,15,6,9,4. Are these frequencies in agreement with the belief that accident conditions were the same during this 10 week period. **(DEC.2015,R2018)**

54. A certain pesticide is packed into bags by a machine. A random sample of 10 bags is chosen and the contents of the bags is found to have the following weights (in kgs) 50,49,52,44,45,48,46,45,49 and 45. Test if the average quantity packed be taken as 50kg.

55. Given

$\overline{X}_1 = 72$, $\overline{X}_2 = 74$

$s_1 = 8$, $s_2 = 6$

$n_1 = 32$, $n_2 = 36$

Test if the means are significant. **(DEC.2015,R2018)**

56. A mathematics test was given to 50 girls and 75 boys. The girls made an average of 76 with an SD of 6 and the boys made an average grade of 82 with an SD of 2. Test whether there is any difference between the performance of boys and girls. **(MAY 2016,R2013)**

57. Theory predits the proportion of beans in the groups A,B,C,D AS 9:3:3:1. In an experiment among beans the numbers in the groups were 882,313,287 and 118. Does the experiment support the theory? **(MAY 2016,R2013)**

58. 400 men and 600women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favour of the proposal. Test whether these two proportions are same. **(MAY 2016,R2013)**

59. The IQ'S of 10 girls are respectively 120,110,70,88,101,100,83,98,95,107. Test whether the population mean IQ is 100. **(MAY 2016,R2013)**

variables.

UNIT IV

DESIGN OF EXPERIMENTS

One way and Two way classifications - Completely randomized design – Randomized block design – Latin square design – 22 factorial design.

PART A

1. What do you understand by Design of Experiments? (R) (MAY 2012) (MAY 2015)

Solution:

By 'experiment' we mean the collection of data (which usually consists of a series of measurement of some feature of an object) for a scientific investigation according to a certain specified sampling procedures.

2. Distinguish between experimental variables and extraneous variables.

Solution:

Consider the example of agricultural experiment which may be performed to verify the claim that a particular manure has got the effect of increasing the yield of paddy. Here the quantity of manure used and amount of yield of paddy are known as experimental variables. And factors such as rainfall, quality of soil and quality of seeds (will also affect the yield of paddy, which are not under study) are called extraneous variables.

3. What is the aim of design of experiments? (R)

Solution:

In any statistical experiment we will have both experimental variables and extraneous variables. The aim of the design of experiments is to control extraneous variables and hence to minimize the error so that the results of experiments could be attributed only to the experimental variables.

4. State the basic principles of design of experiments. (DEC 2012) (R)

(or)

What are the basic principles of design of experiments. (MAY 2015)

Solution:

1. Randomization 2.Replication 3.Local control.

5. What do you mean by experimental group and control group? (R)

Solution:

Consider an agricultural experiment. To analyze the effect of a manure in the yield of paddy, we use the manure in some plots of same size(group of experimental units) is called experimental group and the some other group of plots in which the manure is used and which will provide a basis for comparison is called the control group.

6. Explain the techniques used in the local control of extraneous variables.(U)

Solution:

(i).By grouping, we mean combining sets of homogeneous plots into groups, so that different manures may be used in different groups.

(ii).By blocking, we mean assigning the same number of plots in different blocks

(iii).By balancing, we mean adjusting the procedures of grouping, blocking and assigning the manures in such a manner that a balanced configuration is obtained.

7. Explain replication.(U)

Solution:

In order to study the effects of different manures on the yield are studied, each manure is used in more than one plot. In other words, we resort to replication which means repetition.

8. Name the basic designs of experiments. (R)

Solution:

1.Completely Randomized Design (ANOVA one way classification)
2.Randomized Block Design(ANOVA two way classification)
3.Latin Square Design(ANOVA three way classification)

9. Explain ANOVA. (U) Solution:

ANOVA enables us to divide the total variation (represented by variance) in a group into parts which are accounted to different factors and a residual random variation which could be accounted for by any of these factors. The variation due to any specific factor is compared with the residual variation for significance, and hence the effect of the factors are concluded.

10.Explain CRD. (U)

Solution:

Consider an experiment of agriculture in which "h" treatments (manures) and "n" plots are available. To control the extraneous variables treatment "1" should be replicated on "n_1" plots, treatment 2 should be replicated on "n_2" plots and so on. To reduce the error we have to randomize this process that is which n_1 plots be used treatment 1 and so on. For this we number the plots (from 1 to n) and write the numbers on cards and shuffle well. Now, we select n_1 cards (as cards are selected at random the numbers will not be in order) on which treatments 1 will be used and so on. This process and design is called completely randomized design.

11. Explain RBD. (DEC 2012) (U)

Solution:

Consider an experiment of agriculture in which effects of "k" treatments on the yield of paddy used. For this we select "n" plots. If the quality of soil of these "n" plots is known, then these plots are divided into "h" blocks (each with one quality). Each of these "h" blocks are divided into "k" times (n = hk) and in each one of this "k" plots are applied the "k" treatments in a perfectly randomized manner such that each treatment occurs only once in any of the block. This design is called randomized block design.

12.Explain LSD. (U)

Solution:

Consider an agricultural experiment, in which n^2 plots are taken and arranged in the form of an n X n square,such that plots in each row will be homogeneous as far as possible with respect to one factor of classification, say, quality of soil and plots in each column will be homogeneous with respect to another factor of classification, say, seed quality. Then "n" treatments are given to these plots such that each treatment occurs only once in each tow and only once in each column. The various possible arrangements obtained in this manner are known as Latin squares of order "n" and the design is called Latin Square Design.

13. Is a 2 X 2 Latin Square Design possible? Why? (DEC.2015,R2013)(MAY2016,R2013)

Solution:

In a 2 X 2 LSD, the degree of freedom for the residual variation is (n -1) (n -2) = 0 which is not possible. Therefore a 2 X 2 LSD is not possible.

14. Compare LSD and RBD. (U)

Solution:

RBD	LSD
1.The number of replications of each 2. Can be performed in square field only 3. Number of treatments must be 4.It controls two extraneous variables	1.There is no such restriction treatment is equal to the number of treatments. 2.Can be performed in square or rectangular field. 3. No such restriction 4. Controls only one extraneous variable.

15. Write down the ANOVA table for one way classification. (R) **(MAY 2012)**

Source of Variation	Sum of squares	Degrees of freedom	Mean sum of squares	Variance ratio
Between columns	SSC	C-1	MSC = $\frac{SSC}{C-1}$	MSC = $\frac{MSC}{MSE}$
Within columns	SSE	N-C	MSC = $\frac{SSE}{N-1}$	(or)
Total	TSS	N-1		MSC = $\frac{MSE}{MSC}$

16. Write two advantages of completely randomized experimental design.(DEC.2015,R20113)

1. CRD is most useful in laboratory technique and methodological studies
2. CRD is recommended in situations where an appreciable fraction of units is likely to be destroyed or fail to respond.

17. State the principles of Design of Experiments. (U) **(MAY2016,R2013)**

1. Randomization 2.Replication 3.Local control.

PART B

1. The following table shows the lives in hours of four brands of electric lamps:

Brand								
A:	1610	1610	1650	1680	1700	1720	1800	
B:	1580	1640	1640	1700	1750			
C:	1460	1550	1600	1620	1640	1660	1740	1820
D:	1510	1520	1530	1570	1600	1680		

Perform an analysis of variance and test the homogeneity of the mean lives of the four brands of lamps. **(MAY 2011)**

2. Experiment was performed to judge the effect of four different fuels and three different types of launchers on the range of a certain rocket,.Test on the basis of following ranges in miles,whether there is significant effect due to differences in fuels and whether there is significant effect due to differences in launchers. Use .01 level of significance

	Fuel1	**Fuel1**	**Fuel1**	**Fuel1**
launcherX	**45**	**47**	**48**	**42**
launcherY	**43**	**46**	**50**	**37**
launcherZ	**51**	**52**	**55**	**49**

3. A completely randomized design experiment with 10 plots and 3 treatments gave the following results: (A)

Plot No:	1	2	3	4	5	6	7	8	9	10
Treatment:	A	B	C	A	C	C	A	B	A	B
Yield:	5	4	3	7	5	1	3	4	1	7

Analyze the results for treatment effects. **(DEC 2010)**

4. Three varieties of a crop are tested in a randomized block design with four replications, the layout being as given below: The yields are given in kilograms. Analyze for significance(A)

C48	A51	B52	A49
A47	B49	C52	C51
B49	C53	A49	B50

5. Four experiments determine the moisture content of samples of a powder, each observer taking a sample from each of six consignments. The assessments are given below: (A)

		Consignment					
Observer		1	2	3	4	5	6
	1	9	10	9	10	11	11
	2	12	11	9	11	10	10
	3	11	10	10	12	11	10
	4	12	13	11	14	12	10

Perform an analysis of variance on these data and discuss whether there is any significant difference between consignments or between observers. **(MAY 2009)**

6. The following data resulted from an experiment to compare three burners B_1, B_2,B_3. A Latin square design was used as the tests were made on 3 engines and were spread over 3 days.

	Engine 1	Engine 2	Engine 3
Day 1	B_1 - 16	B_2 – 17	B_3 – 20
Day 2	B_2 – 16	B_3 – 21	B_1 – 15
Day 3	B_3 – 15	B_1 – 12	B_2 – 13

Test the hypotheses that there is no difference between the burners. **(DEC 2010)(Nov/Dec2108)**

7. The following are the number of mistakes made in 5 successive days by 4 technicians working for a photographic laboratory test at a level of significance α= 0.01. Test whether the difference among the four sample means can be attributes to change. (A)

Technician

I	II	III	IV
6	14	10	9
14	9	12	12
10	12	7	8
8	10	15	10
11	14	11	11

8. The following data represent the number of units of production per day turned out by different workers using 4 different types of machines

		Machine type			
		A	B	C	D
workers	1	44	38	47	36
	2	46	40	52	43
	3	34	36	44	32
	4	43	38	46	33
	5	38	42	49	39

(1)Test whether the five men differ with respect to mean productivity
(2)Test whether the mean productivity is the same for the four different machine types
(MAY 2013) (A)

9. what are the basic assumptions involved in ANOVA. **(MAY 2011)** (A)

10. In a Latin square experiment given are the yields in quintals per acre on the paddy crop carried out for testing the effect of five fertilizers A,B,C,D,E. Analyze the data for variations.

B25	A18	E27	D30	C27
A19	D31	C29	E26	B23
C28	B22	D33	A18	E27
E28	C26	A20	B25	D33
D32	E25	B23	C28	A20

(A) **(MAY 2011)**

11. using 2^2 factorial design, draw the graphical presentation of the following table and find the difference in mean without replication term and calculate $(y_1-y_0),(y_{0.1}-y_{0.0})$.
Discuss when PH=2 and 3

	Temp.	PH	Rep1	Rep2	Total
1	300	2	10	14	24
a	350	3	21	19	40
b	300	3	17	15	32
ab	350	3	20	24	44

(A) **(DEC 2010)**

12. Four types of health drinks A,B,C,D were tried on the school children. In order to study the effects of the age groups of the children and localities, four schools from four different localities, were selected and students were divided into four age groups. The latin square designed was arranged, the gain in weights in same units are recorded below,test whether the localities are groups and varieties of food have any significant effect and gain in weight.

		AGE GROUPS			
	5-8	8-11	11-14	14-17	TOTAL
	A2	B1.8	C2.1	D1.5	7.4
	D1.3	A1.4	B1	C1.2	4.9
	B1.7	C1.6	D1.1	A1.9	6.3
TOTAL	5.9	5.8	6.2	6.1	24

13. The following table shows the lives in hours of four pages of electric bulbs.

				BATCHES				
1	1610	1610	1650	1680	1700	1720	1800	
2	1580	1640	1640	1700	1750			
3	1460	1550	1600	1620	1640	1660	1740	1820
4	1510	1520	1530	1570	1600	1680		

Perform and analysis of variance of these data and show that a significance test doesnot reject their homogeneity. **(MAY 2009)** (A)

14. Three varieties of a crop are tested in a randomized block design with four replications. The plot yield in pounds are as follows:

A6	C5	A8	B9
C8	A4	B6	C9
B7	B6	C10	A6

Analyse the experiment yield and state your conclusion. **(MAY 2011)** (A)

15. Setup the analysis of variance table for the following results of a latin square design, use 0.01 level of significance.

A12	C19	B10	D8
C18	B12	D6	A7
B22	D10	A5	C21

D12	A7	C27	B17

16. Analyse the following of latin square experiment

A12	D20	C16	B10
D18	A14	B11	C14
B12	C15	D19	A13
C16	B11	A15	D20

(A) **(MAY 2013)**

17. Compare and contrast the Latin square Design with the Randomised Block Design.

(May 2013).

18. The following is a Latin square of a design when 4 varieties of seeds are being tested. Set up the analysis of variance table and state your conclusion. You may carry out suitable change of origin and scale. (A)

A105	B95	C125	D115
C115	D125	A105	B105
D115	C95	B105	A115
B95	A135	D95	C115

(MAY 2013) (A)

19. A farmer wishes to test the effect of 4 fertilizers A, B, C, D on the yield of wheat. The fertilizers are used in a LSD and the result are tabulated here. Perform an analysis of variance.

A18	C21	D25	B11
D22	B12	A15	C19
B15	A20	C23	D24
C22	D21	B10	A17

(DEC 2012)

20. The sales of 4 salesmaen in 3 seasons are tabulated here. Carry out an analysis of vaiance.

Salesmen

A	B	C	D
36	36	21	35
28	29	31	32
26	28	29	29

(DEC 2012) (A)

21. Given

Detergent	Engine 1	2	3
A	45	43	51
B	47	46	52
C	48	50	55
D	42	37	49

Perform ANOVA and test at .05 level of significance whether these are differences in the detergents or in the engines. (A) **(DEC 2015,R2013)**

22. Find out the main effects and interactions in the following 2^2 - factorial experiment and write down the ANOVA table

	I	a	b	ab
Block	00	10	01	11
I	64	25	30	6
II	75	14	50	33
III	76	12	41	17
IV	75	33	25	10

(DEC 2015,R2013)Nov/Dec2018

23. Three varities of coal were analysed by 4 chemists and the ash content is tabulated here. Perform an analysis of variance. (A)

Chemists

	A	B	C	D
I	8	5	5	7
II	7	6	4	4
III	3	6	5	4

Coal

(MAY 2016, R2013)

24. The result of an RBD experiment on 3 blocks with 4 treatments A,B C,D are tabulated here. Carry out an analysis of variance.

Blocks	Treatment effects
I	A36 D35 C21 B36
II	D32 B29 A28 C31
III	B28 C29 D29 A26

(MAY 2016,R2013)

UNIT V
STATISTICAL QUALITY CONTROL `

Control charts for measurements (X and R charts) – Control charts for attributes (p, c and np charts) –Tolerance limits - Acceptance sampling.

PART-A

1. **What do you mean by Statistical quality control (SQC)?(R)**

SQC is statistical method for finding whether the variation in the quality of the product is due to random causes or assignable causes. It does not involve inspecting each and every item produced for quality standards, but involves inspection of samples of items produced and application of test of significance.

2. What are the advantages of **SQC**

Statistical quality control has several advantages over 100% inspection of system.

(i) Since only a fraction of output a inspected , costs of inspection are greatly reduced.

(ii) It is easy to apply

(iii) Since works of inspection is considerably reduced , efficiency is increased
(iv) SQC ensures an early detection of faults and hence a minimum waste of reject production

3. **What is the difference between Chance variation and assignable variation?(U)**

Chance variation	**assignable variation**
1.It is variation in the quality of the product that occurs due to random causes	1. It is variation in the poor quality of the product that occurs due to non- random causes
2. Examples -Slight changes in temperature , pressure and metal hardness 3. No method is available to control the chance variation	2.Examples – Input raw material , mechanical faults, inexperienced operators 3.Method is available to control the assignable variation

4. **What do you understand by the Process control?(U)**

Process control means control of the quality of the goods while they are in the process of production.

To achieve the process control, repeated random samples are taken from the population of items.

5. **What is the control chart? (Or) shewhart chart? Name the types of control chart**.(R)

Control chart is graphical device mainly used for the study and control of the manufacturing process. The manufacturer can find out , at a glance ,whether or not the process is under control so that proportion of the defective items is not excessive .

There are two types Control chart

(i) Control charts of variable(Mean - $\overline{X}$ chart and Range -R charts)
(ii) Control charts of attributes(p-chart, np-chart and c-chart)

6. **Distinguish between variables and attributes in connection with the quality characteristic of a product** (R)

Variables are the quality characteristic of a product that are measurable (e.g) diameter of the hole board by the drilling machine

Attributes are the quality characteristic of a product that are not amenable for measurement, but are identified by their presence or absence (e.g) Number of defective items in a sample

7. **Find the lower and upper limits for $\overline{X}$ - chart and R –charts, when each sample is of size 4 and $\overline{\overline{X}} = 15.22$ and $\overline{R} = 3.53$**

Solution:

For $\overline{X}$ - chart

Central line y = CL = $\overline{\overline{X}} = 15.22$

Upper control limit y = UCL = $\overline{\overline{X}} + A_2\overline{R}$

$= 15.22 + (0.729)(3.53)$

$= 17.79$

Lower control limit y = LCL = $\overline{\overline{X}} - A_2 \overline{R}$

= 15.22 - (0.729)(3.53)

= 12.65

8. **When do you say that the process is out of control?(R)**
   If one or more plotted points lie outside the control lines (UCL and LCL) then the process is said to be out of control
9. **Under what situation p –chart is drawn instead of np- chart?(R)**
   np –chart is used only when all the samples are of the same size n (constant)
   But p- chart is used when all the samples are of same size or different size
10. **What do you mean by tolerance limit?(U)**
    Tolerance Limits of a quality characteristic are defined as those values between which nearly all the manufactured items will lie.
    If the measurable quality characteristic X is assumed to be normally distributed with mean μ and SD σ, then the Tolerance limit are $\mu \pm 3\sigma$.

    If mean μ and SD σ will not be known then the tolerance limits are $\overline{\overline{X}} \pm 3\dfrac{\overline{R}}{d_2}$
11. **Define Specification limits (R)**
    Specification limits are the targets set for the process / product by the customer or market performance or internet target
12. **Write down the control limit and warning limits (R)**
    Control limit are $\mu \pm 3\sigma$
    Warning limits are $\mu \pm 2\sigma$
13. **How will you decide whether a process is operating at an acceptable level?(R)**
    If the tolerance limits are within the specification limits, then the process is assumed to operate at an acceptable level. If not the process is bound to produce some defective items even though the process may be under control.
14. **When the process is under control and if n=6, $\overline{\overline{X}} = 62.69$ and $\overline{R} = 19.67$ find the tolerance limit .If the Specification for a quality characteristic are (60 $\pm$ 24) in coded values then check whether the process meet the specification or not.(R)**

    The tolerance limit are given by $\overline{\overline{X}} \pm 3\dfrac{\overline{R}}{d_2}$

    = 62.69 $\pm$ 3(19.67)/ 2.534

    = 62.69 $\pm$ 23.29

    The tolerance limit is (39.40, 85.98)
    But specifications limit are (36, 84) Since the tolerance limit does not lie within the specifications limit , the process does not meet the specifications.
15. **What is the difference between c- chart and p-chart?(U)**

	C- chart	p- chart
1	Actual number defects are plotted	Fraction or Proportion defectives are plotted
2	Unit of same size is selected	Sample size may be constant or variable

16. **What is the difference between tolerance limit and control limit?(U)**

An important difference between tolerance limit and control limits is that the former are used to determine whether individual manufactured components are acceptable, whereas the latter are used to manufacturing process.

17. Write the advantages of control chart.(R)

(i) It helps us to rectify the faults and errors during the process or even after the process is over.

(ii) Any change of process in the production line can be tested very easily

(iii) There is a lot of saving time and cost

(iv) Decision can be taken with more reliability and confidence

PART-B

1. The following data give the measurements of 10 samples each of size 5 in the production process taken in an interval of 2 hours. Calculate the sample means and ranges and Draw the appropriate mean and range chart and comment on the state of the control process **(April/May 2018)**

	Sample number									
	1	2	3	4	5	6	7	8	9	10
Observed measurements	49	50	50	48	47	52	49	55	53	54
	55	51	53	53	49	55	49	55	50	54
	54	53	48	51	50	47	49	50	54	52
	49	46	52	50	44	56	53	53	47	54
	53	50	47	53	45	50	45	57	51	56

2. Construct mean and range chart for the following

	Sample number							
	1	2	3	4	5	6	7	8
Observed	32	28	39	50	42	50	44	22
	36	32	52	42	45	29	52	35
	42	40	28	31	34	21	35	44

3. Given below are the values of sample mean and sample range for 10 samples, each of size 5. Draw the appropriate mean and range chart and comment on the state of the control process.(A)

Sample No	1	2	3	4	5	6	7	8	9	10

Mean	43	49	37	44	45	37	51	46	43	47
Range	5	6	5	7	7	4	8	6	4	6

4. A machine fills boxes with dry cereal. 15 samples of 4 boxes are drawn randomly. The weights of the sampled boxes are shown as follows. Draw the appropriate $\overline{X}$ -chart and R -charts and comment on the state of the control process. (A)

Sample No	1	2	3	4	5	6	7	8	9	10
Weight of boxes(X)	10.0	10.3	11.5	11.0	11.3	10.7	11.3	12.3	11.0	11.3
	10.2	10.9	10.7	11.1	11.6	11.4	11.4	12.1	13.1	12.1
	11.3	10.7	11.4	10.7	11.9	10.7	11.1	12.7	13.1	10.7
	12.4	11.7	12.4	11.4	12.1	11.0	10.3	10.7	12.4	11.5

Sample No	11	12	13	14	15
Weight of boxes(X)	12.5	11.9	12.1	11.9	10.6
	11.9	12.1	11.1	12.9	11.9
	11.8	11.6	12.1	13.1	11.7
	11.3	11.4	11.7	12.0	12.1

5. A plastics manufacturer extrudes blanks for use in the manufacture of eye –glass temples. Specifications require that the thickness of these blanks have $\overline{\overline{x}} = 0.150$ inch $\sigma = 0.002$ inch. Calculate 2-sigma and 3- sigma upper and lower control limits for means of samples 5 and prepare control chart
6. A machine is set to deliver packets of a given weight. Ten samples of size 5 each were recorded . Draw the R-chart and comment on its state of control

Sample no	1	2	3	4	5	6	7	8	9	10
Range	7	7	4	9	8	7	12	4	11	5

7. In a factory producing spark plugs, the number of defectives found in the inspection of 15 lots of 100 each is given below: Draw the control chart for the number of defectives and comment on the state of control (A)

Sample no(i)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
No of defectives(np)	5	10	12	8	6	4	6	3	4	5	4	7	9	3	4

8. 15 samples of 200 items each were drawn from the output of a process. The numbers of defective items in the samples are given below. Prepare the control chart for the fraction defective (A)

Sample no(i)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
No of defectives(np)	12	15	10	8	19	15	17	11	13	20	10	8	9	5	8

9. 10 samples of each of size 50 were inspected and the number of defectives in the inspection were 2, 1,1,2,3,5,5,1,2,3 . Draw the np –chart and p-chart for defectives and compare it.
10. The following data refer to visual defects found at inspection of first 10 samples of size 100. Use the data to obtain upper and lower control limit for percentage defective in sample of 100

Sample no	1	2	3	4	5	6	7	8	9	10	total
No of defective	2	1	1	3	2	3	4	2	2	0	20

11. Thirty-five successive samples of 100 castings each, taken from a production line , contained respectively 3,3,5,3,5,0,3,2,3,5,6,5,9,1,2,4,5,2,0,10,3,6,3,2,5,6,3,3,2,5,1,0,7,4 and 3 defectives If the fraction of defectives is to be maintained at 0.02, Construct p- chart for these data
12. Suppose that it is known from the past experience that on the average an aircraft assembly made by a certain company has four missing rivets. Calculate the central line and control limits for the following results of missing rivets in 25 assemblies :
    4,6,5,1,2,3,5,7,1,2,2,4,6,5,3,2,4,1,8,4,5,6,3,4,,2
13. 20 pieces of cloth out of different rolls contained respectively 1,4,3,2,4,5,6,7,2,3,2,5,7,6,4,5,2,1,3 and 8 imperfections ,Ascertain whether the process is in a state of statistical control**(Nov/Dec2018)**
14. In an integrated circuit production line ,sample of 100 units are checked to electrical specifications on alternate days of month and results declared as number of defectives are tabulated below. Draw p-chart and np-chart and comment **(Nov/Dec2018)**
15. The following are the figures for the number of defectives of 10 samples each containing 100 items: 8,10,9,8,10,11,7,9,6,and 12.Draw the control chart for fraction of defectives and comment on state of the control **(Nov/Dec2018)**

Sample no	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
No .of defectives	24	38	62	34	26	36	38	52	33	44	44	52	45	30	34

<u>Control charts for Attributes</u>

There are two basic types of control charts used in attributes

(i) The fraction –defective chart(p chart)

(ii) The number of defects chart (c-chart)

<u>Construction of p-chart</u>

This chart gives best results when the samples size is large . The steps in constructing the chart are;

(i) Calculate The average fraction defective $\bar{p}$

(ii) Central line is drawn with the value $\bar{p}$

(iii) UCL and LCL are determined

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

Note: If LCL= negative then assume assume LCL to be equal to zero

Construction of np-chart

The number of defectives can be plotted directly.

Central line = n $\overline{p}$

$$UCL = \overline{np} + 3\sqrt{\overline{np}(1-\overline{p})}$$

$$LCL = \overline{np} - 3\sqrt{\overline{np}(1-\overline{p})}$$

Construction of C-chart

Let 'C' –represents the number of defects counted in one unit of cloth and $\bar{c}$ represent the mean of the defects counted in several such unit of cloth

Central line = $\bar{c}$

$$UCL = \bar{c} + 3\sqrt{\bar{c}}$$

$$LCL = \bar{c} - 3\sqrt{\bar{c}}$$