Chapter 5

STATISTICAL QUALITY CONTROL

0 INTRODUCTION :

Quality control is a management tool that enhances quality of product being produ id increases profit. Quality implies fitness for use or conformance to requirem tatistical Quality Control (SQC) **is a method that uses sampling techniques atistical analysis in production process and reduce variability systematically olate sources of difficulties during production.**

In any production process there are two types of variations observed.

(1) Common Cause variation or Random variation.

(2) Special cause variation or assignable variation.

Variation is a hindrance to quality. The variation that result from minor causes s the natural variation that exists in materials, machinery and people behave in a r way and produce slight difference in product characteristic.

Such variations are called **common cause variations or random variations** cannot be eliminated completely.

The **assignable variations** are due to special causes such as excessive to poorly trained operator, poor quality raw material etc.

Nothing can be done about the first variation, whereas the second one detected and the process can be brought **under control.**

A production process which is experiencing only chance variation or random is said to be in **statistical control.**

A production process which is experiencing assignable cause variation is **out of control.** In this case we have to identify and correct the cause of that

A production process is considered successful if it operates in an **in contro** a long period. During this period the product of the process is acceptable.

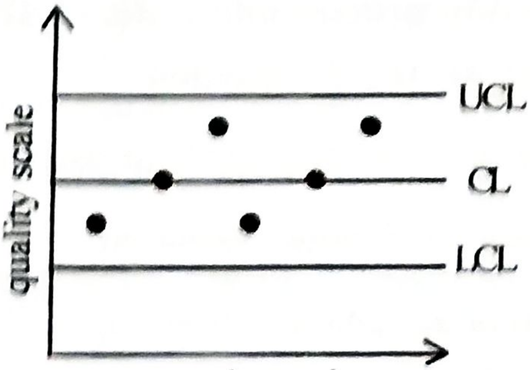
5.1 CONTROL CHART

A control chart is a graphic device invented by Dr. Walter A. Shewart for mo process outputs to identify when they slip out of control. The plot of point: measurable parameter such as $\bar{X}$, R or P is generally called a control chart. The structure of a control chart consists of three horizontal lines

(i) A Central Line (CL) to indicate the **average quality** at which the pr perform or the level of the process

(ii) Upper Control Limit (UCL) line

(iii) Lower Control Limit (LCL) line



From time to time, a sample is taken and the data are plotted on the graph point on the control chart falls outside the control limits, the process is said to statistical control. However, it is desirable for a process to be in control.

5.1.1 Types of control charts

Control charts may be classified into two types.

(i) Control charts of variables

and (ii) Control charts of attributes.

The terms "variables" and "attributes" are associated with the type of dat on the process.

When measurable characteristics such as length, time, weight etc. are cons resulting data is considered continuous and it is called **variable data.**

A qualitative variable that can take on only two values is called an **attr** example a product is defective or non-defective. The data obtained by co number of defective in a sample is called **attribute data.**

Variable data are considered to be of a higher level than attribute data.

2 Control charts for variables (or) measurements

When dealing with variable data, it is usual to exercise control over the average ity of a process as well as its variability. Separate control charts are necessary for ing with these two concepts.

Taking the means of periodic samples we get the control chart for means, the hart. Variability is controlled by the sample ranges and is called the R-Chart.

3 X-bar Chart (or) $\overline{X}$ -Chart

An X-bar chart can be understood by considering a process with mean μ and dard deviation σ. But in practice μ and σ are usually unknown and it is necessary to nate them from samples.

Suppose the process is monitored by taking periodic samples called **subgroups,** of n and computing the sample mean $\overline{X}$ for each sample. Then by central limit theorem mean of the sample means is μ and standard deviation of the sample is $\frac{\sigma}{\sqrt{n}}$.

The central line is μ and the control limits are taken as 3-sigma limits $\mu - 3\frac{\sigma}{\sqrt{n}}$ and $\frac{\sigma}{\sqrt{n}}$.

rking Rule for $\overline{X}$-Chart

Suppose the process is monitored by m periodic samples each of size n.

1. Obtain the means $\overline{X}_1, \overline{X}_2, \ldots, \overline{X}_m$ of the samples where $\overline{X}_i = \frac{\Sigma X}{n}$ is the mean of the i^{th} sample.
2. Obtain the mean of sample means

$$\overline{\overline{X}} = \frac{1}{m}\sum_{i=1}^{m}\overline{X}_i \text{ and S.D of sample means } \sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}$$

3. The control limits are $UCL = \overline{\overline{X}} + 3\frac{\sigma}{\sqrt{n}}$

$$LCL = \overline{\overline{X}} - 3\frac{\sigma}{\sqrt{n}}$$

Since σ is not known, we find $\overline{R} = \frac{\sum_{i=1}^{m} R_i}{m}$,

and **R_i = max. value – min value of the i^{th} sample.**

$\overline{\overline{X}}$ is unbiased estimator of μ.

But $\overline{R}$ does not provide an unbiased estimate of σ.

But $A_2\,\overline{R}$ is an unbiased estimator, where A_2 is a constant from the table given a end of the book for different values of n.

$\therefore$ Control chart $\overline{X}$ is

$$\boxed{\begin{aligned} \text{Central line} &= \overline{\overline{X}} \\ UCL &= \overline{\overline{X}} + A_2\,\overline{R} \\ LCL &= \overline{\overline{X}} - A_2\,\overline{R} \end{aligned}}$$

5.1.4 R-Chart for process variability :

Process variability can be controlled through the use of plots of the sample ra

The R-chart (when σ is not known) is

$$\boxed{\begin{aligned} \text{Central line} &= \overline{R} \\ UCL &= D_4\,\overline{R} \\ LCL &= D_3\,\overline{R} \end{aligned}}$$

where D_3, D_4 values can be found from the table at the end of the book for d values of n.

Note : (1) The R-chart is used for small samples of size less than 15 units.

$$LCL = \overline{\overline{X}} - 3\frac{\sigma}{\sqrt{n}}$$

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(2) If a process is to be under control, then all the sample values in both $\overline{X}$ and R-chart must be within control limits.

(3) If a value jumps out of the control limits, eliminating it we can set up new control limits for testing of quality.

WORKED EXAMPLES

e 1. The following data provides the values of sample mean $\overline{X}$ and the R for the samples of size 5 each. Calculate the values for central line and limits for mean-chart and range chart and determine whether the process ntrol.

No.	1	2	3	4	5	6	7	8	9	10
X)	11.2	11.8	10.8	11.6	11.0	9.6	10.4	9.6	10.6	10
R	7	4	8	5	7	4	8	4	7	9

ersion factors for n = 5 are A_2 = 0.577, D_3 = 0 and D_4 = 2.115)

n : Given 10 samples each of size 5

$$\therefore\ n = 5,\ m = 10$$

Sample number	$\overline{X}$	R
1	11.2	7
2	11.8	4
3	10.8	8
4	11.6	5
5	11.0	7
6	9.6	4
7	10.4	8
8	9.6	4
9	10.6	7
10	10	9
Total	106.6	63

Control limit for chart $\bar{X}$ Chart :

$$\text{Central line} = \bar{\bar{X}}$$

$$UCL = \bar{\bar{X}} + A_2\,\bar{R}$$

$$LCL = \bar{\bar{X}} - A_2\,\bar{R}$$

$$\bar{\bar{X}} = \frac{\Sigma\bar{X}}{10} = \frac{106.6}{10} = 10.66$$

$$\bar{R} = \frac{\Sigma R}{10} = \frac{63}{10} = 6.3$$

For $n = 5$, $A_2 = 0.577$ (given)

$$\text{Central line } \bar{\bar{X}} = 10.66$$

$$UCL = 10.66 + 0.577 \times 6.3$$

$$= 14.2951$$

$$LCL = 10.66 - 0.577 \times 6.3$$

$$= 7.0249$$

Control limits for R-chart :

$$\text{Central line} = \bar{R} = 6.3$$

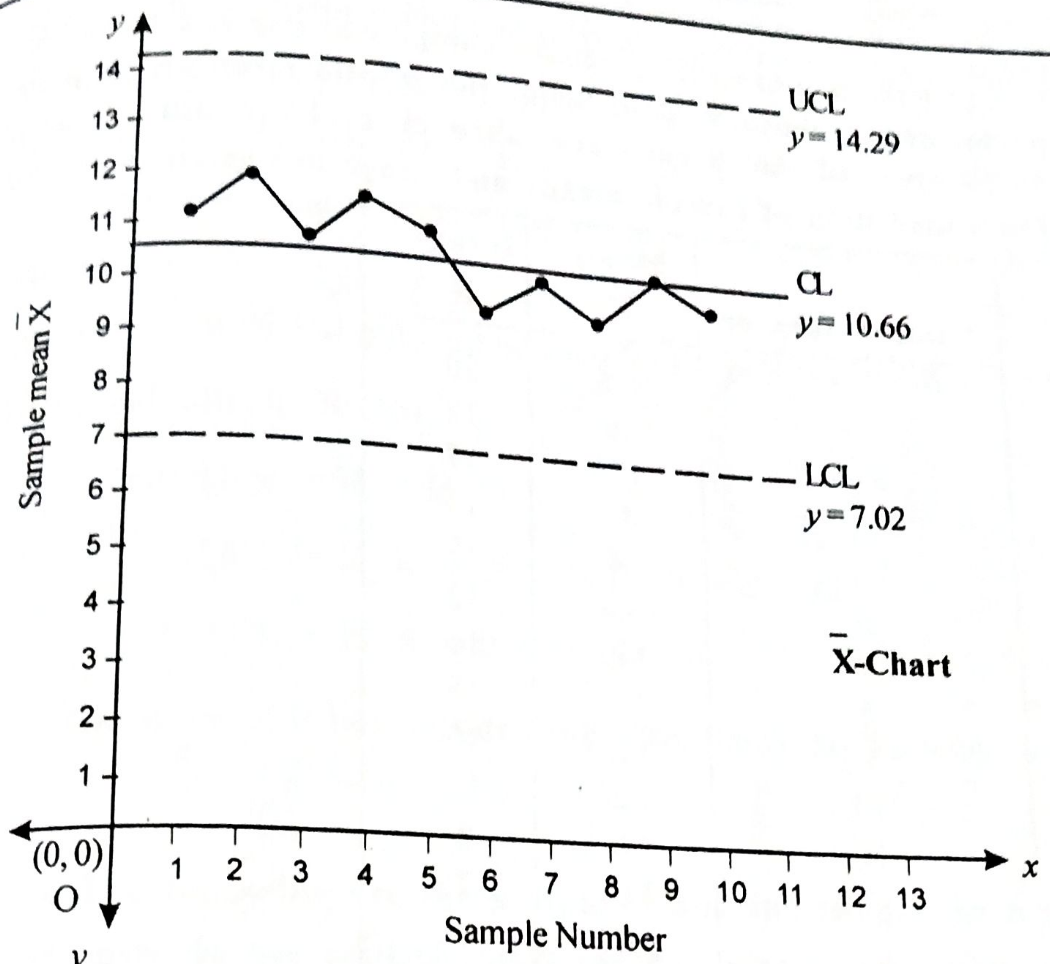
$$UCL = D_4\,\bar{R}$$

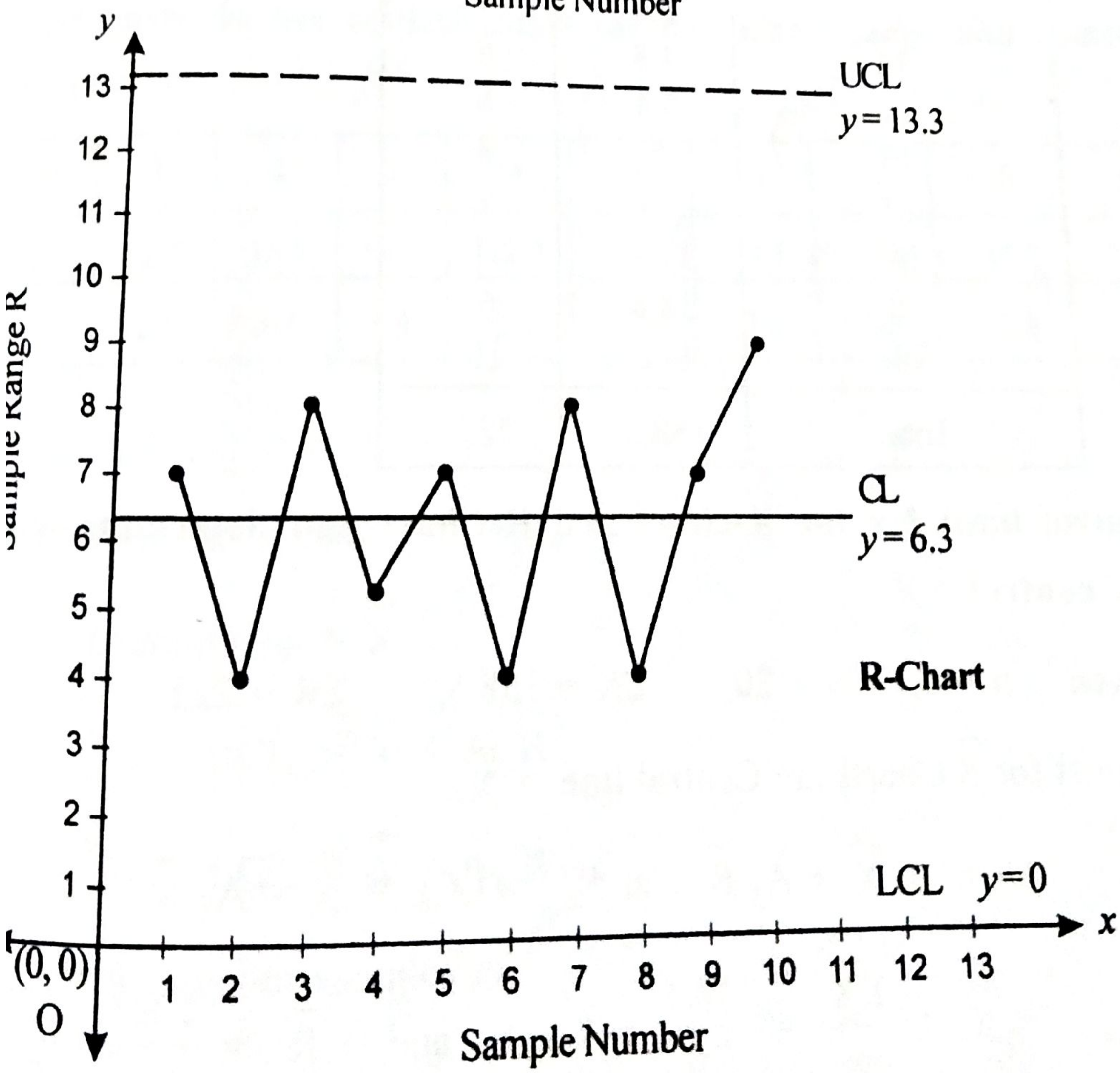
$$= 2.115 \times 6.3 = 13.3245$$

$$LCL = D_3\,\bar{R}$$

$$= 0 \times 6.3 = 0$$

Since all the sample mean values lie between the control limits 7.0249 and
nd the range values lie between the control limits 0 and 13.3245, the process
ate of control.


UCL
y = 14.29
CL
y = 10.66
LCL
y = 7.02
X̄-Chart
Sample mean X̄
Sample Number
(0, 0)
O


UCL
y = 13.3
CL
y = 6.3
R-Chart
LCL y = 0
Sample Range R
Sample Number
(0, 0)
O

Example 2. A process manufacturing missile component parts is being controlled with the performance characteristic being the tensile strength in pounds per square inch. Samples of size 5 each are taken every hour and 20 samples are reported. The coded data of sample means and range are given in the table.

Sample number	Mean $\overline{X}$	Range R
1	10.8	20
2	4.6	12
3	15	17
4	4	13
5	5.4	15
6	18.4	12
7	4.2	14
8	9.6	15
9	4.2	10
10	4.2	11
11	2.6	8
12	2.6	7
13	2.4	9
14	4.8	8
15	5.4	8
16	8	8
17	9.4	7
18	12.6	11
19	18.8	6
20	11.6	11
Total	158.5	223

Find the control limit for the $\overline{X}$-chart and R-Chart and determine whether process is in control.

Solution : Given $n = 5$, $m = 20$ $\Sigma X = 158.5$, $\Sigma R = 223$

Control limits for $\overline{X}$-Charts are Central line $= \overline{\overline{X}}$

$$UCL = \overline{\overline{X}} + A_2 \overline{R} \quad \text{and} \quad LCL = \overline{\overline{X}} - A_2 \overline{R}$$

$$\overline{\overline{X}} = \frac{\Sigma \overline{X}}{m} = \frac{158.5}{20} = 7.925 \quad \text{and} \quad \overline{R} = \frac{\Sigma R}{m} = \frac{223}{20} = 11.$$

From the table for control chart constants, we find for $n = 5$, $A_2 = 0.577$

$\therefore$ UCL $= 7.925 + 0.577 \times 11.15 = 14.3586$

LCL $= 7.925 - 0.577 \times 11.15 = 1.4915$

We find that two sample means (the 6th and 19th sample means) fall outside the control limits.

$\therefore$ the control limits $\bar{X}$ should not be used for the process quality control.

The control limits, for R-chart

$$\text{Central line} = \bar{R} = 11.15$$

$$\text{UCL} = D_4 \bar{R} = 2.115 \times 11.15 = 23.5823$$

$$\text{LCL} = D_3 \bar{R} = 0 \times 11.15 = 0$$

We find no value of R lies outside the control limits. So the range variation is under control.

Example 3. The following are the sample means and ranges for ten samples, each of size 5. Construct the control chart for mean and range and comment on the nature of control.

Sample No.	1	2	3	4	5	6	7	8	9	10
Mean ($\bar{X}$)	12.8	13.1	13.5	12.9	13.2	14.1	12.1	15.5	13.9	14.2
Range R	2.1	3.1	3.9	2.1	1.9	3.0	2.5	2.8	2.5	2

Solution :

Control limits for $\bar{X}$-Charts :

$$\text{Central line} = \bar{\bar{X}}$$

$$\text{UCL} = \bar{\bar{X}} + A_2 \bar{R};$$

$$\text{LCL} = \bar{\bar{X}} - A_2 \bar{R}$$

Here, $n = 5$, number samples $m = 10$

Sample number	$\bar{X}$	R
1	12.8	2.1
2	13.1	3.1
3	13.5	3.9
4	12.9	2.1
5	13.2	1.9
6	14.1	3.0
7	12.1	2.5
8	15.5	2.8
9	13.9	2.5
10	14.2	2
Total	135.3	25.9

$$\bar{\bar{X}} = \frac{\Sigma \bar{X}}{m} = \frac{135.3}{10} = 13.53$$

$$\bar{R} = \frac{\Sigma R}{m} = \frac{25.9}{10} = 2.59$$

The Central line is $\bar{\bar{X}} = 13.53$

The Control limits are $\quad UCL = \bar{\bar{X}} + A_2 \bar{R}$;

$$LCL = \bar{\bar{X}} - A_2 \bar{R}$$

where A_2 is constant for $n = 5$ to be found from the table for control chart con

$A_2 = 0.577$,

$\therefore \quad UCL = 13.53 + 0.577 \times 2.59 = 15.02$

$\quad LCL = 13.53 - 0.577 \times 2.59 = 12.04$

From the given value of $\overline{X}$, we find the value 15.5 for sample 8 lies outside the trol limit UCL = 15.02

So, the process is out of control.

Control limits for R-chart :

$$\text{Central line} = \overline{R} = 2.59$$

$$\text{UCL} = D_4\,\overline{R} \text{ and } \text{LCL} = D_3\,\overline{R}$$

Where D_3, D_4 are constants to be found from the table for control chart constants for 5. We find $D_3 = 0$ and $D_4 = 2.115$

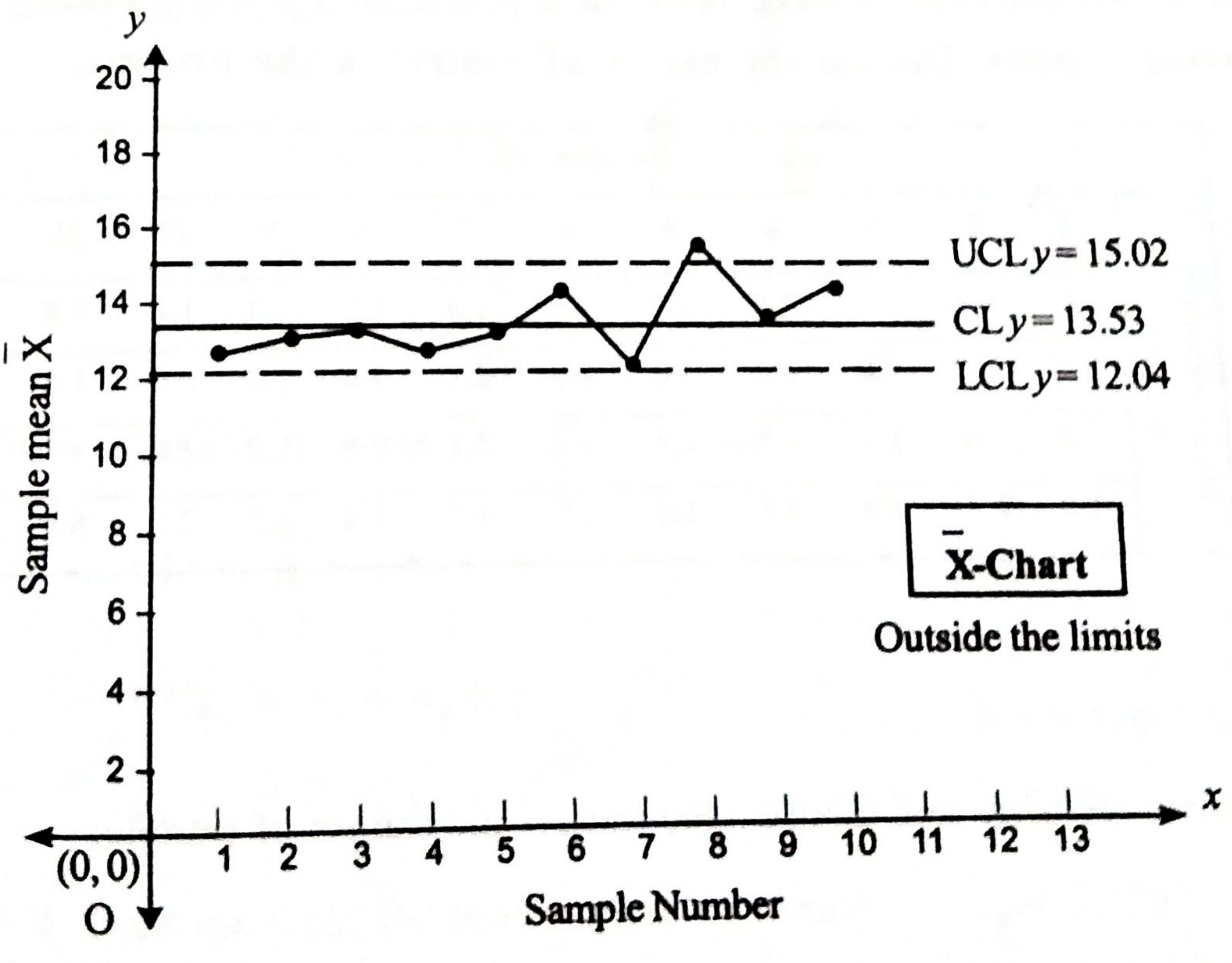
$\therefore$ UCL $= 2.115 \times 2.59 = 5.48$

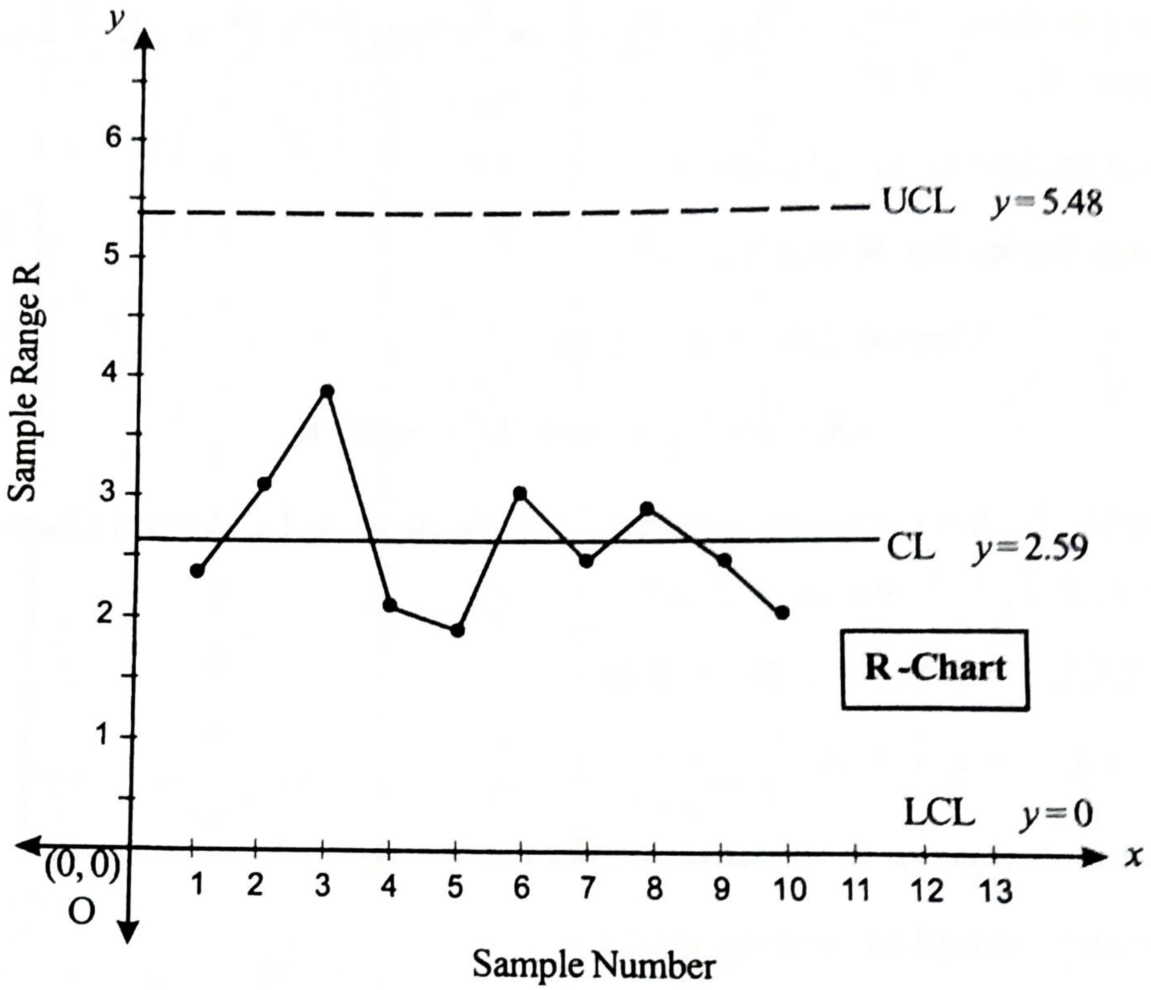
LCL $= 0 \times 2.59 = 0$

We find all the given R values lie within the limits.

So the range variation is under control.

We shall now draw the charts.





Example 4. Measurements were made in a production line by taking 12 sample each having 4 units. Discuss the nature of control in the process.

	Sample No.											
Measurements in (mm)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
	1	**1.4**	**1.6**	**2.9**	**1.7**	**2.6**	**2.3**	**1.9**	**1.7**	**1.8**	**0.8**	**2.0**
	1.4	**2.3**	**1.0**	**2.0**	**3.6**	**2.8**	**2.1**	**1.6**	**2.2**	**2.0**	**1.5**	**2.5**
	1.3	**2.8**	**1.5**	**0.5**	**2.5**	**3.2**	**2.1**	**1.8**	**1.9**	**1.5**	**2.1**	**1.6**
	1.0	**2.7**	**2.0**	**2.2**	**1.8**	**1.5**	**1.7**	**1.4**	**1.2**	**2.0**	**0.9**	**1.8**

lution :

Sample size $n = 4$

We shall calculate and tabulate the means and ranges of samples.

Sample number	ΣX	$\bar{X}$	Range R = Max. value – Min. value
1	4.7	1.18	0.4
2	9.2	2.3	1.4
3	5.1	1.28	0.6
4	7.6	1.9	2.4
5	9.6	2.4	1.9
6	10.1	2.53	1.7
7	8.2	2.05	0.6
8	6.7	1.68	0.5
9	7.0	1.75	1
10	7.3	1.83	0.5
11	5.3	1.33	1.3
12	7.9	1.99	0.9
Total		22.22	13.2

Here, $n = 4$, number samples $m = 12$

$$\bar{\bar{X}} = \frac{\Sigma \bar{X}}{m} = \frac{22.22}{12} = 1.85$$

$$\bar{R} = \frac{\Sigma R}{m} = \frac{13.2}{12} = 1.1$$

The Control limits $\bar{X}$-chart :

$$CL = \bar{\bar{X}} = 1.85$$

$$UCL = \bar{\bar{X}} + A_2 \bar{R} ;$$

$$LCL = \bar{\bar{X}} - A_2 \bar{R}$$

For $n = 4$, from the table for control chart constants $A_2 = 0.729$

$\therefore$ UCL $= 1.85 + 0.729 \times 1.1 = 2.6519$

LCL $= 1.85 - 0.729 \times 1.1 = 1.0481$

Control limits for R-chart :

Central line $= \bar{R} = 1.1$

UCL $= D_4 \bar{R}$ and LCL $= D_3 \bar{R}$

For $n = 4$, from the table for control chart constants $D_3 = 0$, $D_4 = 2.282$

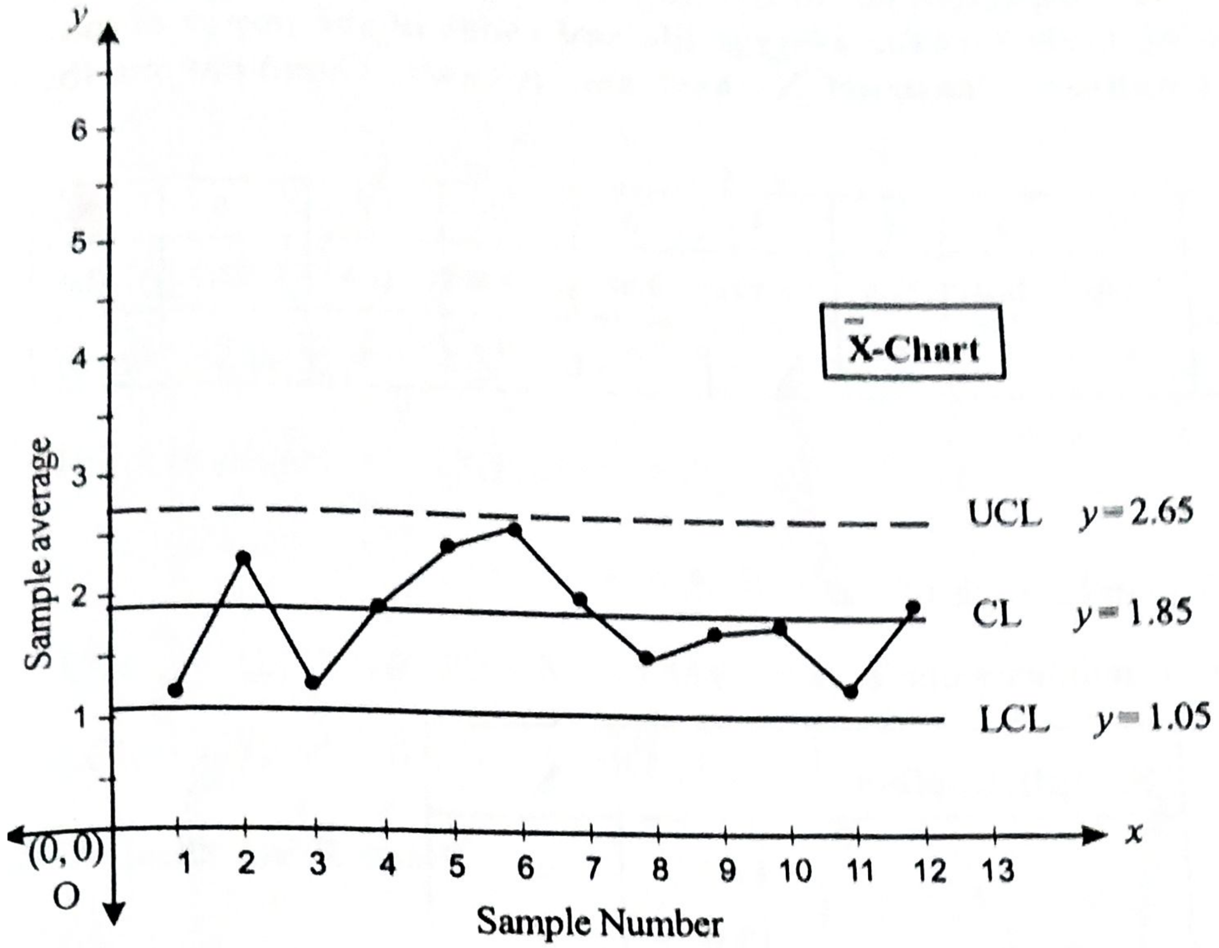
$\therefore$ UCL $= 2.282 \times 1.1 = 2.5102$

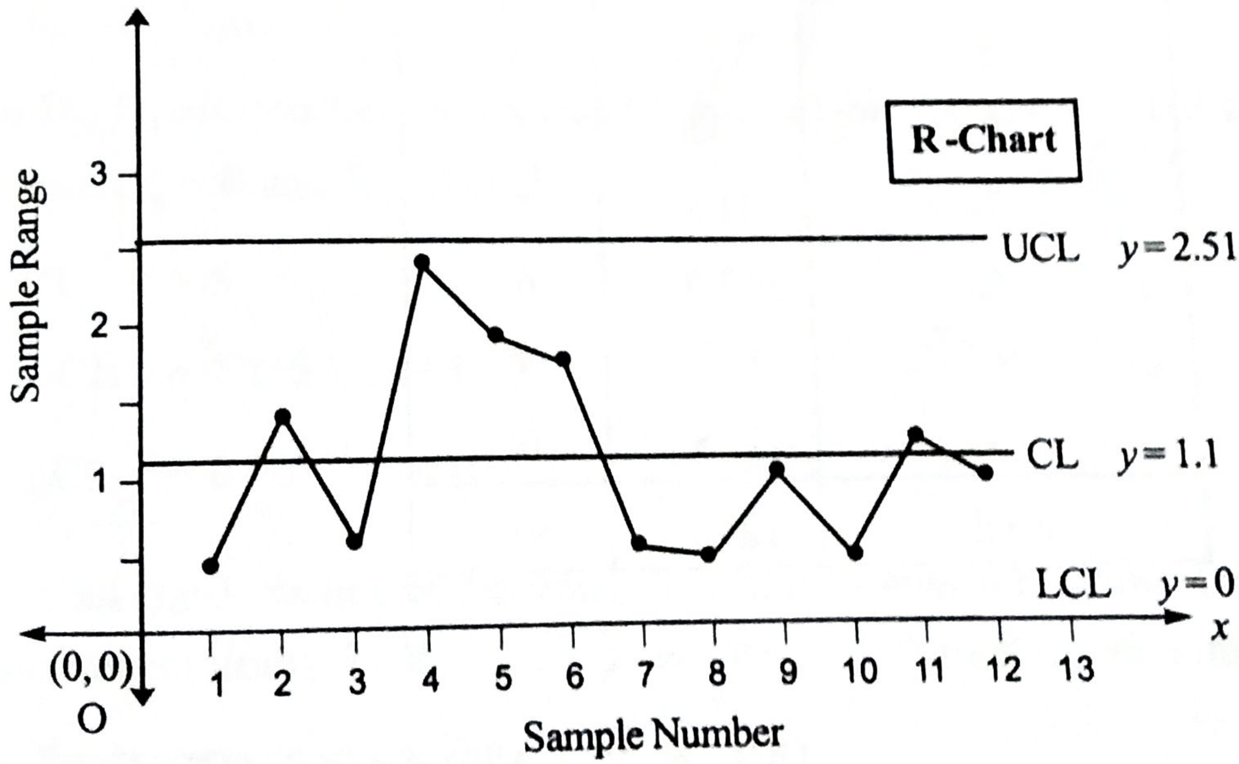
LCL $= 0 \times 1.1 = 0$

We find all the values of $\bar{X}$ lie between the control limits.

Similarly all the values of R lie between the control limits. Hence the production line n control.

We shall next draw the graph to visualize.


X-Chart
y
Sample average
UCL y = 2.65
CL y = 1.85
LCL y = 1.05
(0, 0)
O
x
1 2 3 4 5 6 7 8 9 10 11 12 13
Sample Number


R-Chart
Sample Range
UCL y = 2.51
CL y = 1.1
LCL y = 0
(0, 0)
O
x
1 2 3 4 5 6 7 8 9 10 11 12 13
Sample Number

Example 5. Ten inspection lots of five amplifiers each are drawn from a production. The following table lists the average life and range of the power output obtained for each amplifier. Construct $\bar{X}$-chart and R-chart. Comment on the state of control.

Sample No.	1	2	3	4	5	6	7	8	9	10
Mean ($\bar{X}$)	11.0	12.0	12.8	14.0	13.6	12.8	11.8	12.9	13.0	11.8
Range R	4	4	6	4	6	5	5	6	4	6

[AU 2015]

Solution :

Given 10 samples each of size 5

Here, $n = 5$, number samples $m = 10$

Sample number	$\bar{X}$	R
1	11.0	4
2	12.0	4
3	12.8	6
4	14.0	4
5	13.6	6
6	12.8	5
7	11.8	5
8	12.9	6
9	13.0	4
10	11.8	6
Total	125.7	50

Control limits for $\bar{X}$-Chart : $\quad CL = \bar{\bar{X}}$

$$UCL = \bar{\bar{X}} + A_2 \bar{R};$$

$$LCL = \bar{\bar{X}} - A_2 \bar{R}$$

$$\bar{\bar{X}} = \frac{\Sigma \bar{X}}{m} = \frac{125.7}{10} = 12.57$$

$$\bar{R} = \frac{\Sigma R}{m} = \frac{50}{10} = 5$$

For $n = 5$, $A_2 = 0.577$ [From control chart of constants for $n = 5$]

The Central line is $\bar{\bar{X}} = 12.57$ $\Rightarrow CL = 12.57$

The Control limits are $UCL = \bar{\bar{X}} + A_2 \bar{R}$;

$$LCL = \bar{\bar{X}} - A_2 \bar{R}$$

$\therefore$ $UCL = 12.57 + 0.577 \times 5 = 15.455$

$LCL = 12.57 - 0.577 \times 5 = 9.685$

Control limits for R-chart :

$$\text{Central line} = \bar{R} = 5$$

$$UCL = D_4 \bar{R} \text{ and } LCL = D_3 \bar{R}$$

Where D_3, D_4 are constants to be found from the table for control chart constants for $n = 5$. We find $D_3 = 0$ and $D_4 = 2.115$

$CL = 5$

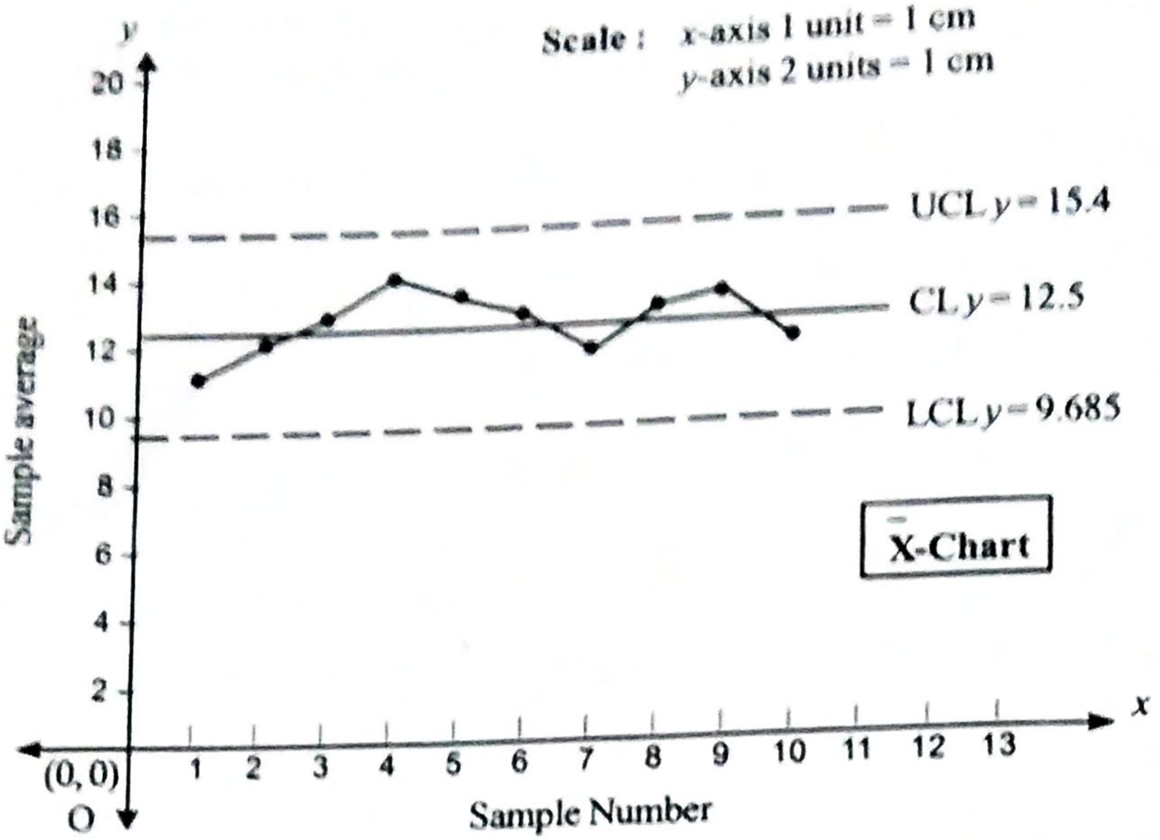
$\therefore$ $UCL = 2.115 \times 5 = 10.575$

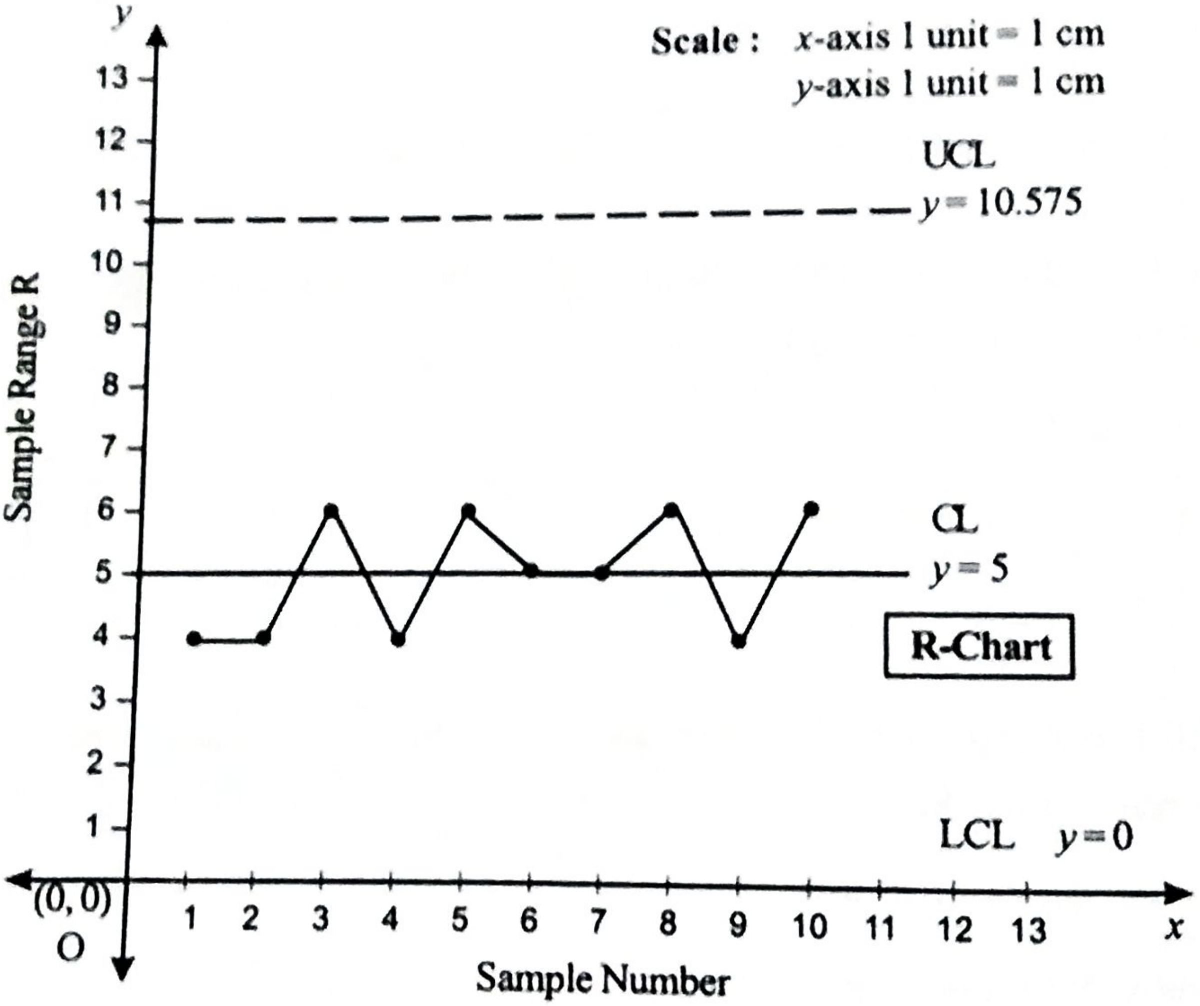
$LCL = 0 \times 5 = 0$

We find all the $\bar{X}$ values be between the control limits for $\bar{X}$ and the R values lie between the control limits for R.

Hence the process is in control.

We shall now draw the charts.


Scale : x-axis 1 unit = 1 cm
y-axis 2 units = 1 cm
UCL y = 15.4
CL y = 12.5
LCL y = 9.685
X̄-Chart
Sample average
Sample Number
(0, 0)
O


Scale : x-axis 1 unit = 1 cm
y-axis 1 unit = 1 cm
UCL
y = 10.575
CL
y = 5
R-Chart
LCL y = 0
Sample Range R
Sample Number
(0, 0)
O

Example 6. The following data give the average life in hours and range in hours of 12 sample each of 5 lamps. Construct the control chats for $\bar{X}$ and R and comment on the state of control.

$\bar{X}$	120	127	152	157	160	134	137	123	140	144	120	127
R	30	44	60	34	38	35	45	62	39	50	35	41

Solution : **[AU 2014, 2019]**

Given sample size $n = 5$

Number of samples $m = 12$

Sample no.	$\bar{X}$	R
1	120	30
2	127	44
3	152	60
4	157	34
5	160	38
6	134	35
7	137	45
8	123	62
9	140	39
10	144	50
11	120	35
12	127	41
Total	1641	513

$$\bar{\bar{X}} = \frac{\Sigma \bar{X}}{m} = \frac{1641}{12} = 136.75$$

$$\bar{R} = \frac{\Sigma R}{m} = \frac{513}{12} = 42.75$$

$\bar{X}$-Chart : $CL = \bar{\bar{X}} = 136.75$

$$UCL = \bar{\bar{X}} + A_2\,\bar{R} \quad \text{and} \quad LCL = \bar{\bar{X}} - A_2\,\bar{R}$$

where value of A_2 from control chart constants for $n = 12$ is $A_2 = 0.266$

$$\therefore\ UCL = 136.75 + 0.266 \times 42.75$$

$$= 136.75 + 11.3715 = 148.1215$$

$$LCL = 136.75 - 11.3715 = 125.3785$$

$\bar{R}$ - Chart : $CL = \bar{R} = 42.75$

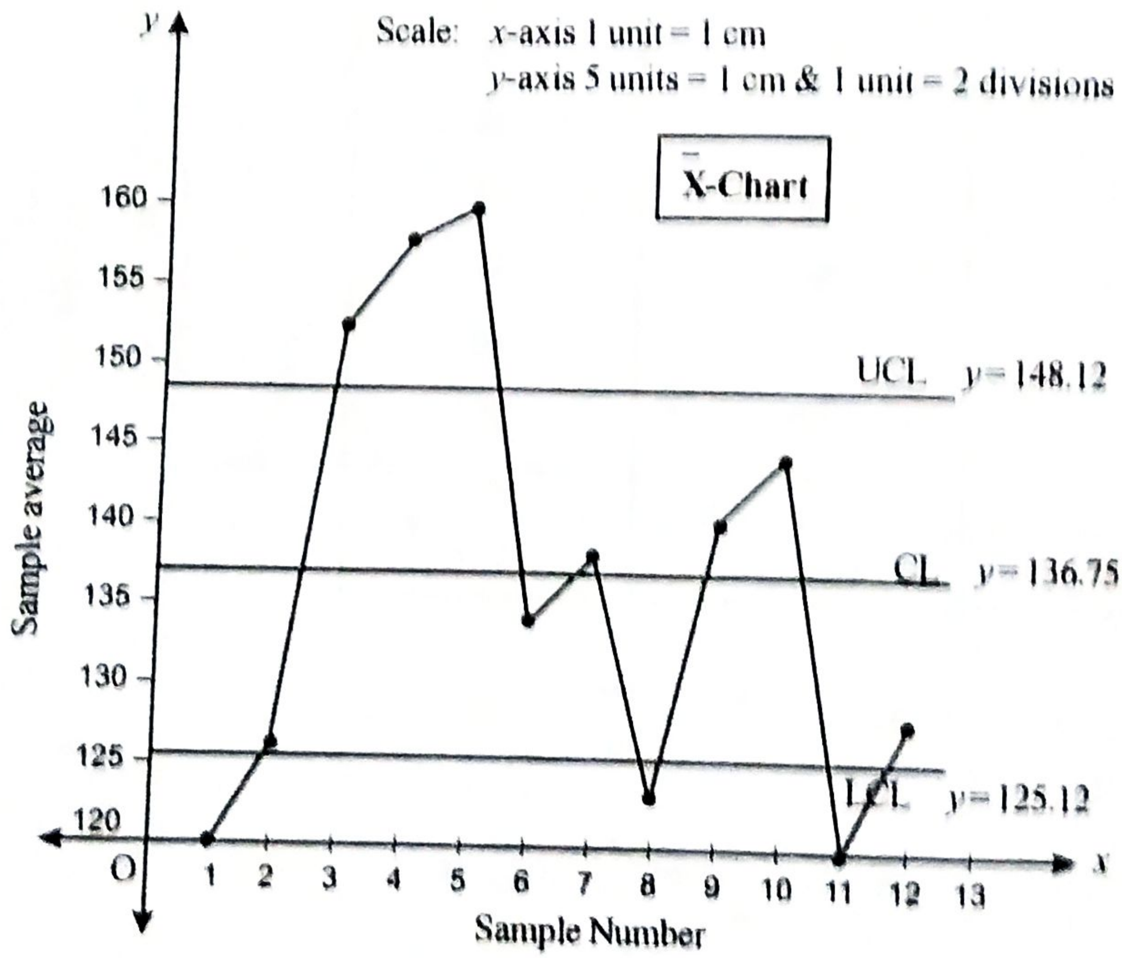
$$UCL = D_4\bar{R} \quad \text{and} \quad LCL = D_3\bar{R}$$

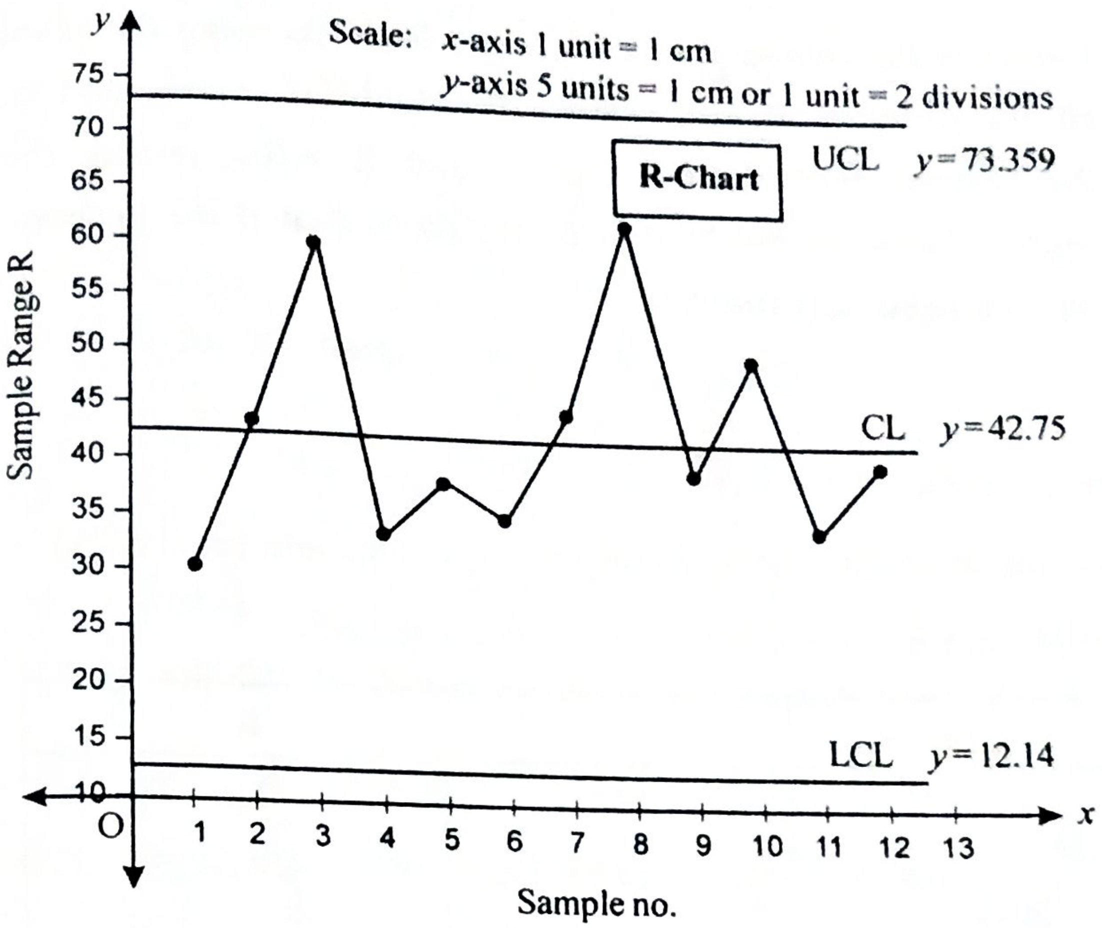
From table for control chart constants for $n = 12$, $D_3 = 0.284$, $D_4 = 1.716$

$$\therefore\ UCL = 1.716 \times 42.75 = 73.359$$

and $$LCL = 0.284 \times 42.75 = 12.141$$

We shall now draw the control charts for $\bar{X}$ and R





From the X-chart, we find of the sample averages are outside the control limits. Whereas the values of R lie between the control limits. Hence the process is out of control.

Example 7. The specification for a certain quality characteristic are 15 ± 6 (in coded values). 15 samples of 14 readings each gave the following values for $\overline{X}$ and R.

Sample no.	1	2	3	4	5	6	7	8	9	10
$\overline{X}$	16.1	15.2	14.2	13.9	15.4	15.7	15.2	15.0	16.9	14.9
R	3	2.1	5.6	2.4	4.1	2.7	2.3	3.8	5.0	2.9

Sample no.	11	12	13	14	15
$\overline{X}$	15.3	17.8	15.9	14.6	15.2
R	13.8	14.2	4.8	5.0	2.2

Compute the control limits for $\bar{X}$ and R charts using the above data for all the samples. If not, remove the doubtful sample and re-compute the revised control limits for $\bar{X}$ and R. After testing the state of control, estimate the tolerance limits and find if the process will meet the required specifications. **[AU 2014]**

Solution :

Given specifications 15 ± 6

That is the quality characteristic should conform to the interval (19, 21)

Given sample size $n = 4$ and number of samples $m = 15$.

Sample no.	$\bar{X}$	R
1	16.1	3.0
2	15.2	2.1
3	1.42	5.6
4	13.9	2.4
5	15.4	4.1
6	15.7	2.7
7	15.2	2.3
8	15.0	3.8
9	16.9	5.0
10	14.9	2.9
11	15.3	13.8
12	17.8	14.2
13	15.9	4.8
14	14.6	5.0
15	15.2	2.2
Total	$\Sigma\bar{X} = 231.3$	$\Sigma R = 73.9$

$$\bar{\bar{X}} = \frac{\Sigma\bar{X}}{m} = \frac{231.3}{15} = 15.42$$

$$\bar{R} = \frac{\Sigma R}{m} = \frac{73.9}{15} = 4.93$$

Control limits for $\bar{X}$-Chart : $\quad CL = \bar{\bar{X}} = 15.42$

$$UCL = \bar{\bar{X}} + A_2 \bar{R};$$

$$LCL = \bar{\bar{X}} - A_2 \bar{R}$$

For $n = 4$, $A_2 = 0.729$ [From control chart of constants for $n = 4$]

$\therefore$ UCL $= 15.42 + 0.729 \times 4.93 = 15.42 + 3.59 = 19.01$

LCL $= 15.42 - 0.729 \times 4.93 = 15.42 - 3.59 = 11.83$

Control limits for R-chart :

$$CL = \bar{R} = 4.3$$

$$UCL = D_4 \bar{R} \text{ and } LCL = D_3 \bar{R}$$

From table for control chart constants, for $n = 4$, $D_3 = 0$, $D_4 = 2.282$

$\therefore$ UCL $= 2.282 \times 4.3 = 9.81$

LCL $= 0 \times 0.43 = 0$

Control limits for $\bar{X}$ are (11.83, 19.01) and all the values of $\bar{X}$ lie in this interval.

But the control limits for R are (0, 9.81)

We find the R values for samples 11 and 12 jumps out of the limits.

Removing these samples, we reconstruct the charts.

Then new $\Sigma \bar{X} = 231.3 - (15.3 + 17.8) = 198.2$

$$\therefore \quad \bar{\bar{X}} = \frac{198.2}{13} = 15.25$$

and $\quad \Sigma R = 73.9 - (13.8 + 14.2) = 45.9$

$$\therefore \quad \bar{R} = \frac{45.9}{13} = 3.53$$

The revised $\bar{X}$-Chart :

$$CL = \bar{\bar{X}} = 15.25, \bar{R} = 3.53$$

$$UCL = \bar{\bar{X}} + A_2 \bar{R}$$

$$= 15.25 + 0.729 \times 3.53$$

$$= 15.25 + 2.57 = 17.82$$

$$LCL = \bar{\bar{X}} - A_2 \bar{R}$$

$$= 15.25 - 2.57 = 12.68$$

So new contol limits fo $\bar{X}$ are (12.68, 17.82)

The revised R-chart :

$$CL = \bar{R} = 3.53$$

$$UCL = D_4 \bar{R} = 2.282 \times 3.53 = 8.06$$

$$LCL = D_3 \bar{R} = 0 \times 3.53 = 0$$

We find all the values of $\bar{X}$ lie between the new control limits (12.68 and 17.8 all values of R lie between the new control limits (0, 8.06)

Hence the process is in control.

The interval of control (12.68, 17.82) is contained in the specification interval

So, the process will meet the required specifications.

Example 8. A machine fills boxes with dry cereal. 15 samples of 4 boxes are drawn randomly. The weights of the samples boxes are shown as follows. Draw the control charts for the sample mean and sample range and determine whether the process is in a state of control.

Sample No.	Weights of boxes			
	Box 1	**Box 2**	**Box 3**	**Box 4**
1	10.0	10.2	11.3	12.4
2	10.3	10.9	10.7	11.7
3	11.5	10.7	11.4	12.4
4	11.0	11.1	10.7	11.4
5	11.3	11.6	11.9	12.1
6	10.7	11.4	10.7	11.0
7	11.3	11.4	11.1	10.3
8	12.3	12.1	12.7	10.7
9	11.0	13.1	13.1	12.4
10	11.3	12.1	10.7	11.5
11	12.5	11.9	11.8	11.3
12	11.9	12.1	11.6	11.4
13	12.1	11.1	12.1	11.7
14	11.9	12.1	13.1	12.0
15	10.6	11.9	11.7	12.1

[AU 2015]

Solution :

Sample size $n = 4$, Number of samples $m = 15$

We have to draw the $\overline{X}$-chart and R-chart.

Sample No.	ΣX	$\bar{X} = \frac{\Sigma X}{4}$	Range R
1	43.9	10.975	2.4
2	43.6	10.9	1.4
3	46.0	11.5	1.7
4	44.2	11.05	0.7
5	46.9	11.725	0.8
6	43.8	10.95	0.7
7	44.1	11.025	1.1
8	47.8	11.95	2.0
9	49.6	12.4	2.1
10	45.6	11.4	1.4
11	47.5	11.875	1.2
12	47.0	11.75	0.7
13	47.0	11.75	1.0
14	49.1	12.275	1.2
15	46.3	11.575	1.5
Total		173.1	19.9

$\therefore$ $$\bar{\bar{X}} = \frac{\Sigma \bar{X}}{m} = \frac{173.1}{15} = 11.54$$

and $$\bar{R} = \frac{\Sigma R}{m} = \frac{19.9}{15} = 1.33$$

$\bar{X}$-Chart : $$CL = \bar{\bar{X}} = 11.54$$

$$UCL = \bar{\bar{X}} + A_2 \bar{R}$$

rom table control constants for $n = 4$, $A_2 = 0.729$

$$\therefore \quad UCL = 11.54 + 0.729 \times 1.33$$

$$= 11.54 + 0.96962 = 12.51$$

and $$LCL = \bar{\bar{X}} - A_2 \bar{R}$$

$$= 11.54 - 0.729 \times 1.33 = 11.54 - 0.9696 = 10.57$$

R - Chart :

$$CL = \bar{R} = 1.33$$

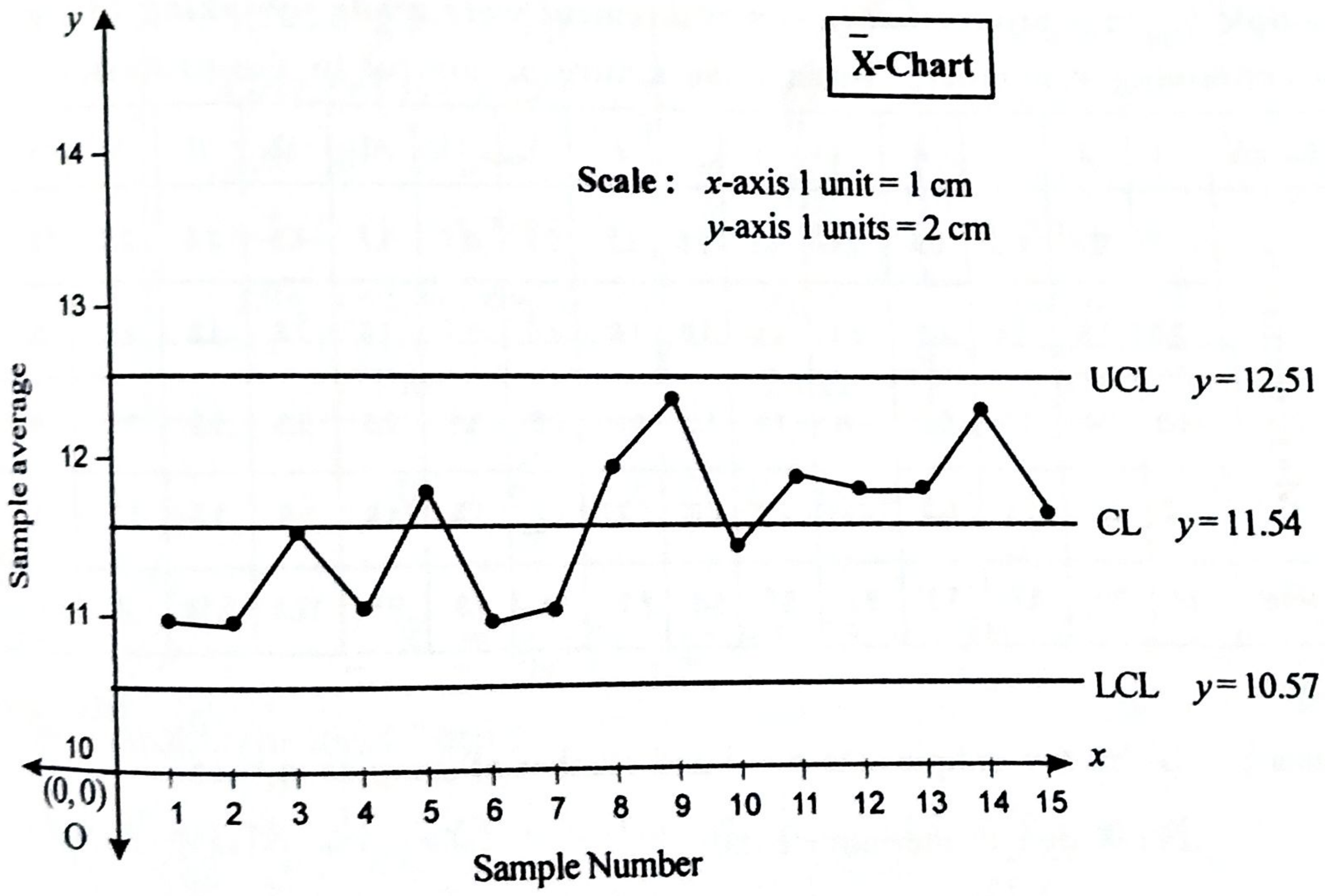
$$UCL = D_4 R$$

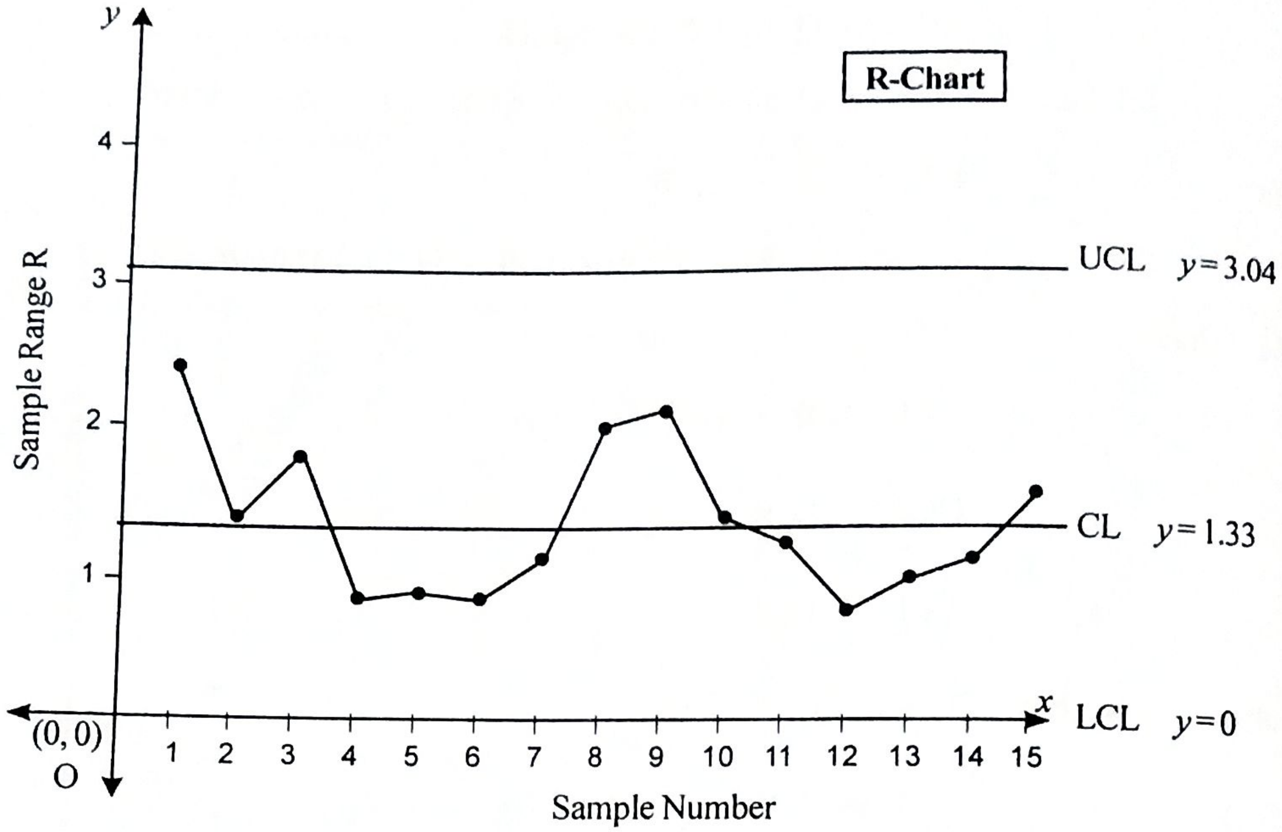
$$LCL = D_3 R$$

For $n = 4$, we have $D_3 = 0$, $D_4 = 2.282$

$$\therefore \quad UCL = 2.282 \times 1.33 = 3.04$$

$$LCL = 0 \times 1.33 = 0$$





In both the $\overline{X}$-chart and R-chart all the values of $\overline{X}$ and all the values R lie within the respective control limits. Hence the process is in control.

Example 9. In a production line, measurement were made by taking 15 samples, eah containing 4 numbers. Discuss the nature of control in the process.

Sample	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Measurements	1.7	0.8	1.0	0.4	1.4	1.8	1.6	2.5	2.9	1.1	1.7	4.6	2.6	2.3	1.9
	2.2	1.5	1.4	0.6	2.3	2.0	1.0	1.6	2.0	1.1	3.6	2.8	1.8	2.1	1.6
	1.9	2.1	1.0	0.7	2.8	1.1	1.5	1.8	0.5	3.1	2.5	3.5	3.2	2.1	1.8
	1.2	0.9	1.3	0.2	2.7	0.1	2.0	1.2	2.2	1.6	1.8	1.9	1.5	1.7	1.4
Total	7.0	5.3	4.7	1.9	9.2	5.0	6.1	7.1	7.6	1.9	9.6	12.8	9.18	2	6.7

[AU 2018]

Solution : Given the sample size $n = 4$ and number of samples $m = 15$

Let X denote measurements.

We shall calculate and tabulate the means and range of samples.

Sample no.	Total measurement for each sample $= \Sigma X$	$\bar{X} = \frac{\Sigma X}{4}$	Range R = max. value– min. value
1	1.7+2.2+1.9+1.2=7	1.75	2.2–1.2=1
2	0.8+1.5+2.1+0.9=5.3	1.33	2.1–0.8=1.3
3	1.0+1.4+1.0+1.3=4.7	1.15	1.4–1=0.4
4	0.4+0.6+0.7+0.2=1.9	0.475	0.7–0.2=0.5
5	1.4+2.3+2.8+2.7=9.2	2.3	2.8–1.4=1.4
6	1.8+2.0+1.1+0.1=5.0	1.25	2.1–0.1=1.9
7	1.6+1.0+1.5+2.0=6.1	1.525	2.0–1=1
8	2.5+1.6+1.8+1.2=7.1	1.775	2.5–1.2=1.3
9	2.9+2.0+0.5+2.2=7.6	1.9	1.9–0.5=2.4
10	1.1+1.1+3.1+1.6=6.9	1.75	3.1–1.1=2
11	1.7+3.6+2.5+1.8=9.6	2.4	3.6–1.7=1.9
12	4.6+2.8+3.5+1.9=12.8	3.2	4.6–1.9=2.7
13	2.6+1.8+3.2+1.5=9.1	2.275	3.2–1.5=1.7
14	2.3+2.1+2.1+1.7=8.2	2.05	2.3–1.7=0.6
15	1.9+1.6+1.8+1.4=6.7	1.675	1.9–1.4=0.5
	Total	26.82	20.6

$$\therefore \qquad \bar{\bar{X}} = \frac{\Sigma \bar{X}}{m} = \frac{26.82}{15} = 1.788 = 1.79$$

and

$$\bar{R} = \frac{\Sigma R}{m} = \frac{20.6}{15} = 1.3733 = 1.37$$

The control limits for $\bar{X}$-Chart :

$$CL = \bar{\bar{X}} = 1.79 \quad ; \quad UCL = \bar{\bar{X}} + A_2 \bar{R} \quad ; \quad LCL = \bar{\bar{X}} - A_2 \bar{R}$$

For $n = 4$, from the table for control chart constants $A_2 = 0.729$

$$\therefore \quad UCL = 1.79 + 0.729 \times 1.37 = 1.79 + 0.998 = 2.788 = 2.79$$

$$LCL = 1.79 - 0.729 \times 1.37 = 1.79 - 0.998 = 0.792 = 0.79$$

Control limits for $\overline{R}$ - Chart :

$$\text{Central line : } CL = \overline{R} = 1.37$$

$$UCL = D_4 \overline{R} \quad \text{and} \quad LCL = D_3 \overline{R}$$

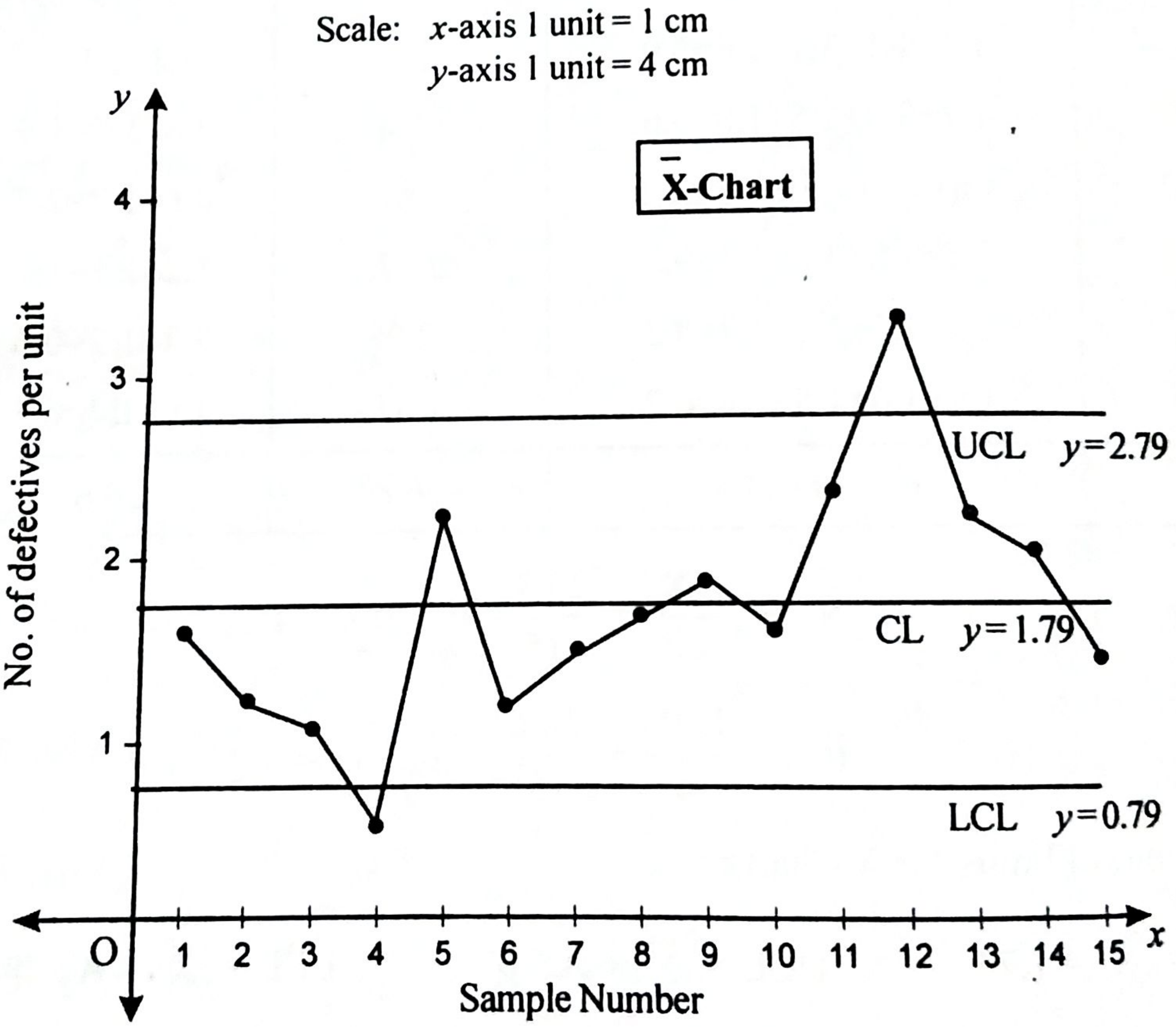
From $n = 4$, from the table for control chart constants

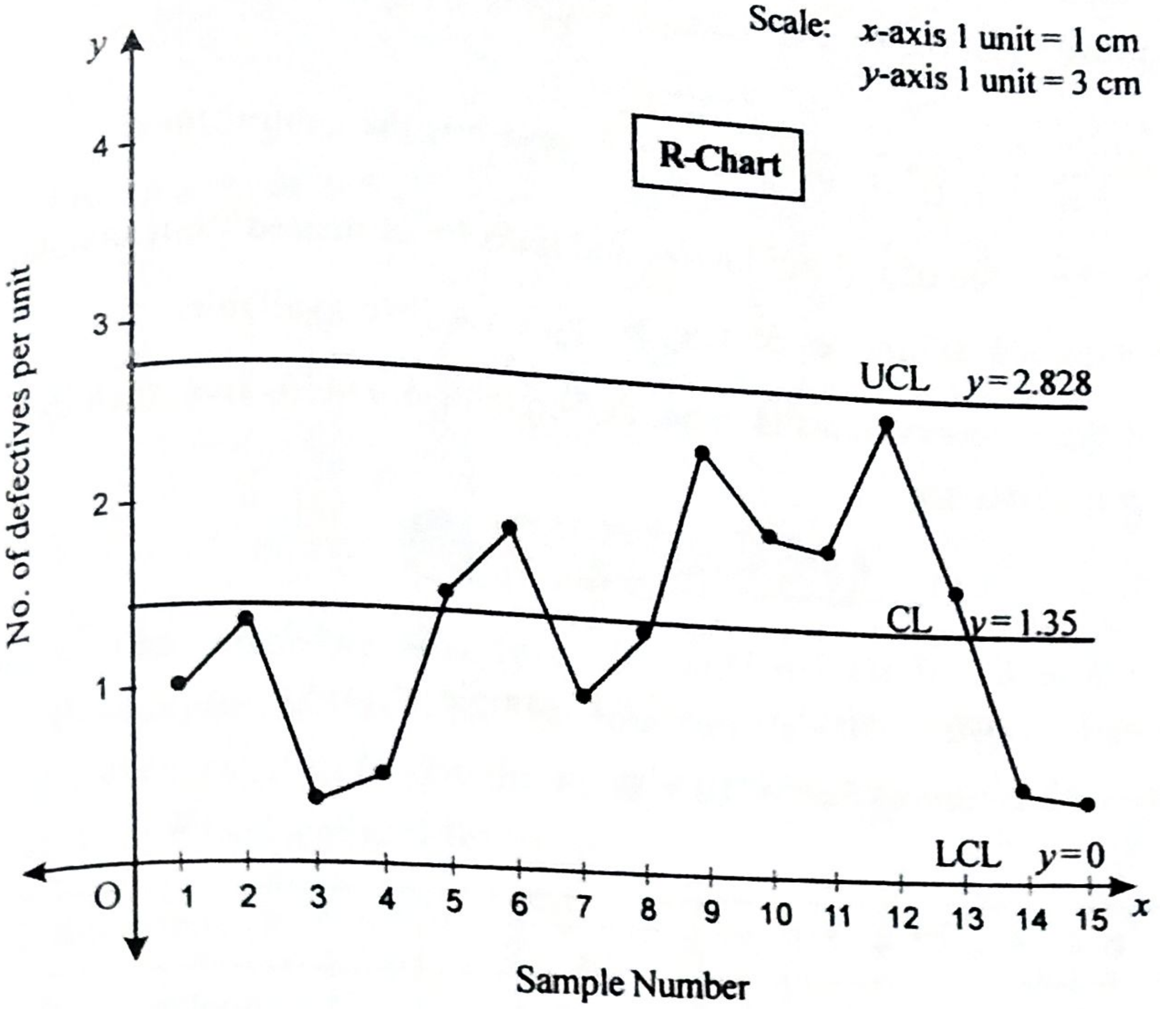
$$D_3 = 0, \quad D_4 = 2.282$$

$$\therefore \quad UCL = 2.282 \times 1.37 = 3.126 = 3.13$$

$$\text{and} \quad LCL = 0 \times 1.37 = 0$$

Since not all the value of $\overline{X}$ lie between the control limits, the process is out of contr even though all the value of $\overline{R}$ lie inside the control line.





5.2 CONTROL CHARTS FOR ATTRIBUTES

There are two fundamental types of control charts that are used in dealing with attribute data.

(1) the p-Chart or the fraction defective chart

(2) the C-Chart or the number of defects chart.

An item is considered defective if it has at least one defect. For example a sample cloth may have several defects like imperfections in thread, colour, material, etc.

(1) **The p-Chart** is designed to control proportion of defective per sample or fraction of defective per sample, when the sample sizes are different.

The p-chart has its theoretical basis in the binomial distribution. Suppose for all items the probability of a defective item is p, and that all items are produced independently. Then in a random sample of n items, if X denotes the number of defectives then $E(X) = np$, $Var(X) = npq = np(1 - p)$, where $q = 1 - p$.

If p is known, then the control limits of 3-sigma are given by

$$p - 3\sqrt{\frac{p(1-p)}{n}} \text{ and } p + 3\sqrt{\frac{p(1-p)}{n}} \text{ and } p \text{ is the central line.}$$

Generally, the value of p is not known and must be estimated from samples.

Suppose there are m samples of sizes $n_1, n_2, \ldots, n_m$ are available.

In each of the n_i observations is either defective or non defective, then the unbiased estimator for p is estimated by

$$\bar{p} = \frac{d_1 + d_2 + \ldots + d_m}{n_1 + n_2 + \ldots + n_m} = \frac{\Sigma d_i}{\Sigma n_i}$$

where d_i is the number defectives in the i^{th} sample of size n_i

Then for p-Chart, central line (CL) $= \bar{p}$

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

Note : When p is small, the LCL may be negative. In this case we take LCL as zero. When p is small we use Poisson distribution as an approximation of binomial.

5.2.1 *np*-Chart or Control Chart for number of defectives

The *np*-chart monitors the number of defectives *np* rather than the proportion of defectives for each sample of size *n* (constant). The *np*-chart is preferable to *p*-chart because the number of defectives is easier for quality technicians and operators to understand rather than the proportion of defective.

The control chart for number of defective chart has central line $n\bar{p}$ and the 3-sigma control limits are

$$\text{UCL} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$\text{LCL} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$$

Note that $\text{UCL} = n\bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = n\left[\bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}\right]$

and $\text{LCL} = n\left[\bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}\right]$

WORKED EXAMPLES

Example 1. The following data refer to visual defects found in the inspection of the first 10 samples of size 100. Use this data to obtain upper and lower control limits for percentage defective in sample of 100. Represent in a suitable chart with central line and control limits.

Sample No.	1	2	3	4	5	6	7	8	9	10
No. of defects	2	1	1	3	2	3	4	2	2	0

Solution :

Sample size is constant and $n = 100$. We shall prepare *np*-chart.

Number of samples $m = 10$

Total no. of defectives $= \Sigma d_i = 2 + 1 + 1 + 3 + 2 + 3 + 4 + 2 + 2 + 0$

$$= 20$$

and $\Sigma n_i = 10 \times 100$, $\bar{p} = \dfrac{\Sigma d_i}{\Sigma n_i}$

$$\therefore \quad \bar{p} = \frac{20}{10 \times 100}$$

$$\therefore \quad \therefore n\bar{p} = 100 \times \frac{20}{10 \times 100} = 2$$

$$\therefore \quad n\bar{p}\,(1-\bar{p}) = 2\left(1-\frac{20}{10\times 100}\right)$$

$$= 2 - \frac{1}{25} = 2 - 0.04 = 1.96$$

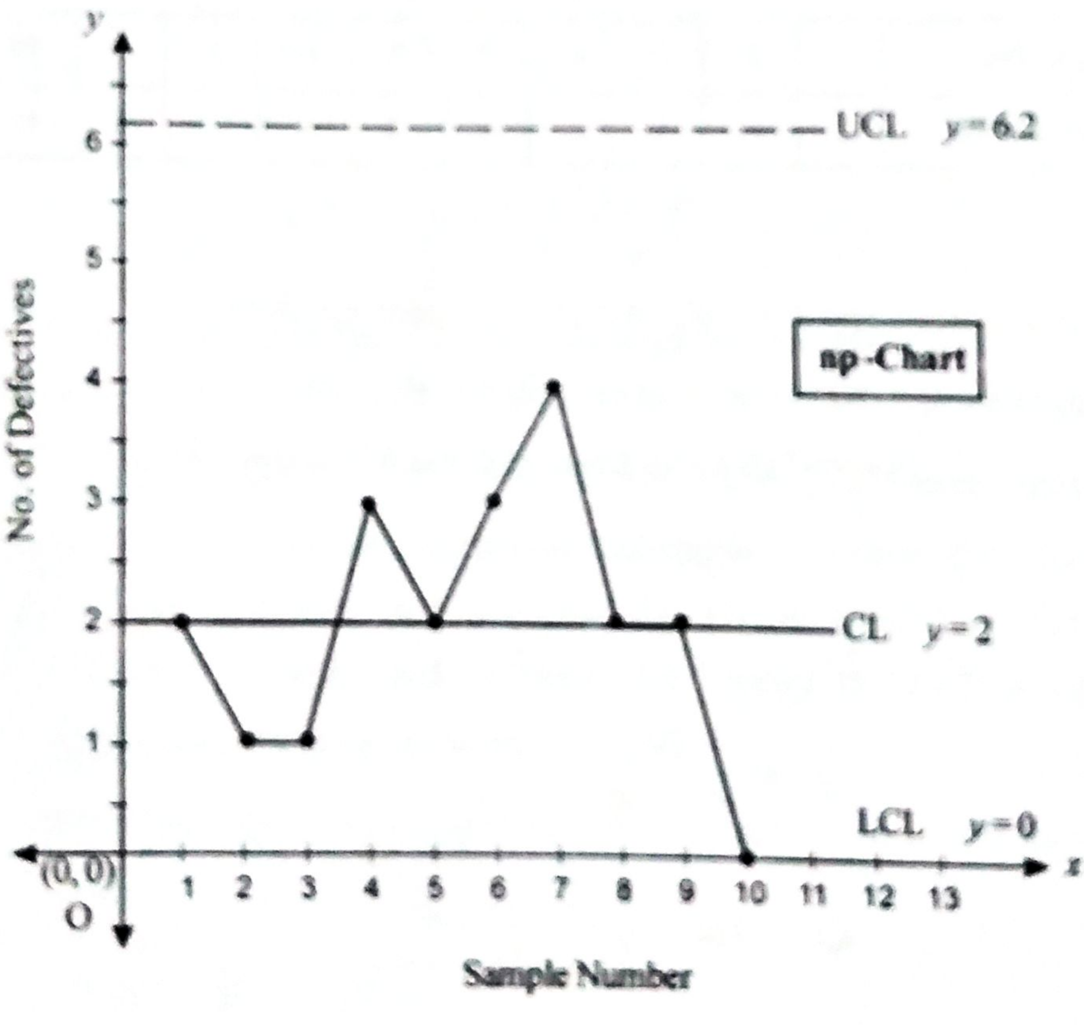
$$\therefore \quad \sqrt{n\bar{p}\,(1-\bar{p})} = \sqrt{1.96} = 1.4$$

$$\therefore \quad np\text{-chart}: \quad CL = n\bar{p} = 2$$

$$UCL = n\bar{p} + 3\sqrt{n\bar{p}\,(1-\bar{p})} = 2 + 3\times 1.4 = 6.2$$

$$LCL = n\bar{p} - 3\sqrt{n\bar{p}\,(1-\bar{p})} = 2 - 3\times 1.4 = -2.2$$

We take LCL = 0, as it cannot be –ve.



Since all the values fall within the control limits, the process is in control.

Note : Since the sample size is constant, we have used *np*-chart. We can also use *p*-chart. But when sample size varies we should use only *p*-chart.

Example 2. From the output of a process that produces several thousand electric tubes daily, samples of 100 tubes are drawn randomly. Sample items are inspected for quality and defective tubes are rejected. The results of 15 samples are shown below. Construct a p-chart and a np-chart and comment on the results.

Sample No.	1	2	3	4	5	6	7	8	9	10
No. of defective tubes	8	10	13	10	14	6	9	8	10	13

Sample No.	11	12	13	14	15
No. of defective tubes	18	9	14	12	15

Solution :

Here the sample size is constant for all samples $\therefore\ n = 100$

Number of samples $m = 15$

Total of all defectives $= \Sigma d_i$

$= 8+10+13+10+14+6+9+8+10+13+18+9+14+12+15$

$= 169$

and $\Sigma n_i = 15 \times 100$

$$\therefore \bar{p} = \frac{\Sigma d_i}{\Sigma n_i} = \frac{169}{15 \times 100} = 0.113$$

***p*-Chart :** Central line $CL = \bar{p} = 0.113$

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$= 0.113 + 3\sqrt{\frac{0.113 \times 0.887}{100}}$$

$$\text{UCL} = 0.113 + 0.095 = 0.208$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$= 0.113 - 0.095 = 0.018$$

Sample No.	1	2	3	4	5	6	7	8	9	10
Fraction defective	0.08	0.10	0.13	0.1	0.14	0.06	0.09	0.08	0.10	0.13
Sample No.	11	12	13	14	15					
Fraction defective	0.18	0.09	0.14	0.12	0.15					

***np*-chart :**

$$\text{CL} = n\bar{p}$$

$$= 100 \times 0.113$$

$$= 11.3$$

$$\text{UCL} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$= n\left[\bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}\right]$$

$$= 100 \times 0.208$$

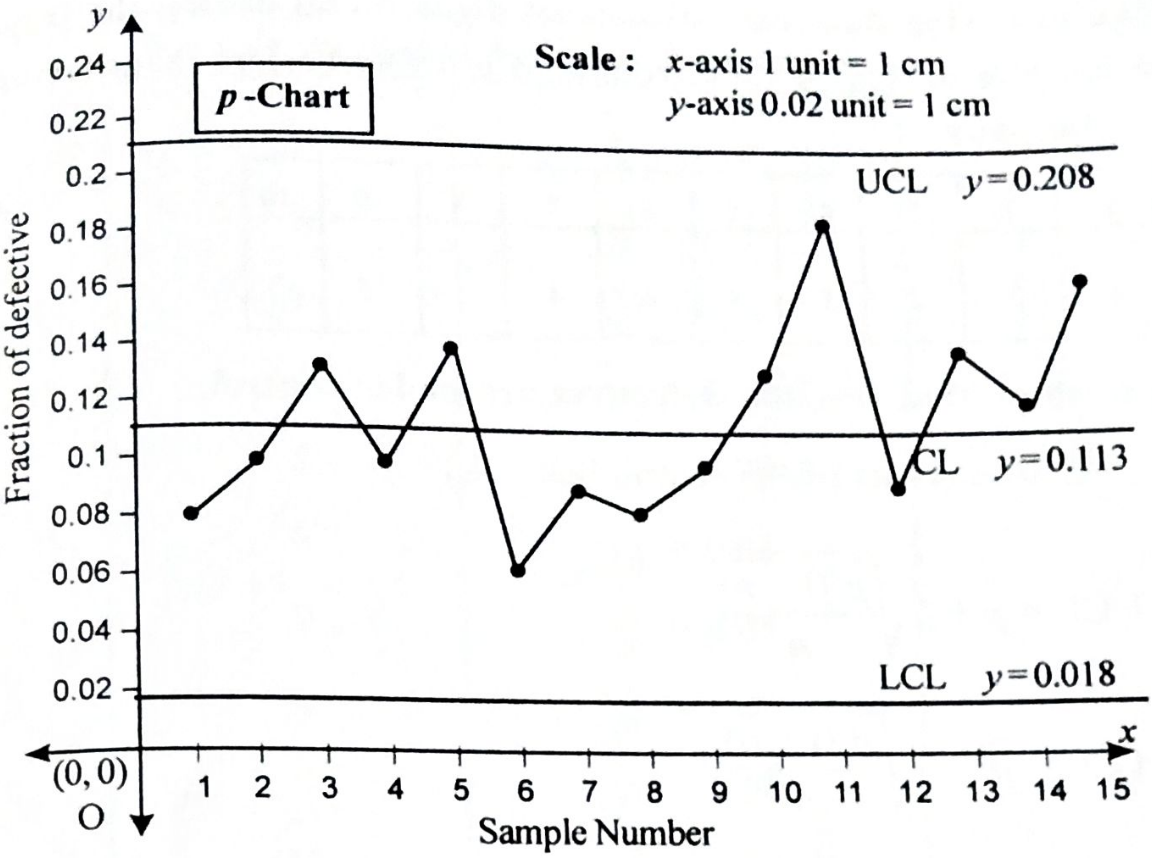
$$= 20.8$$

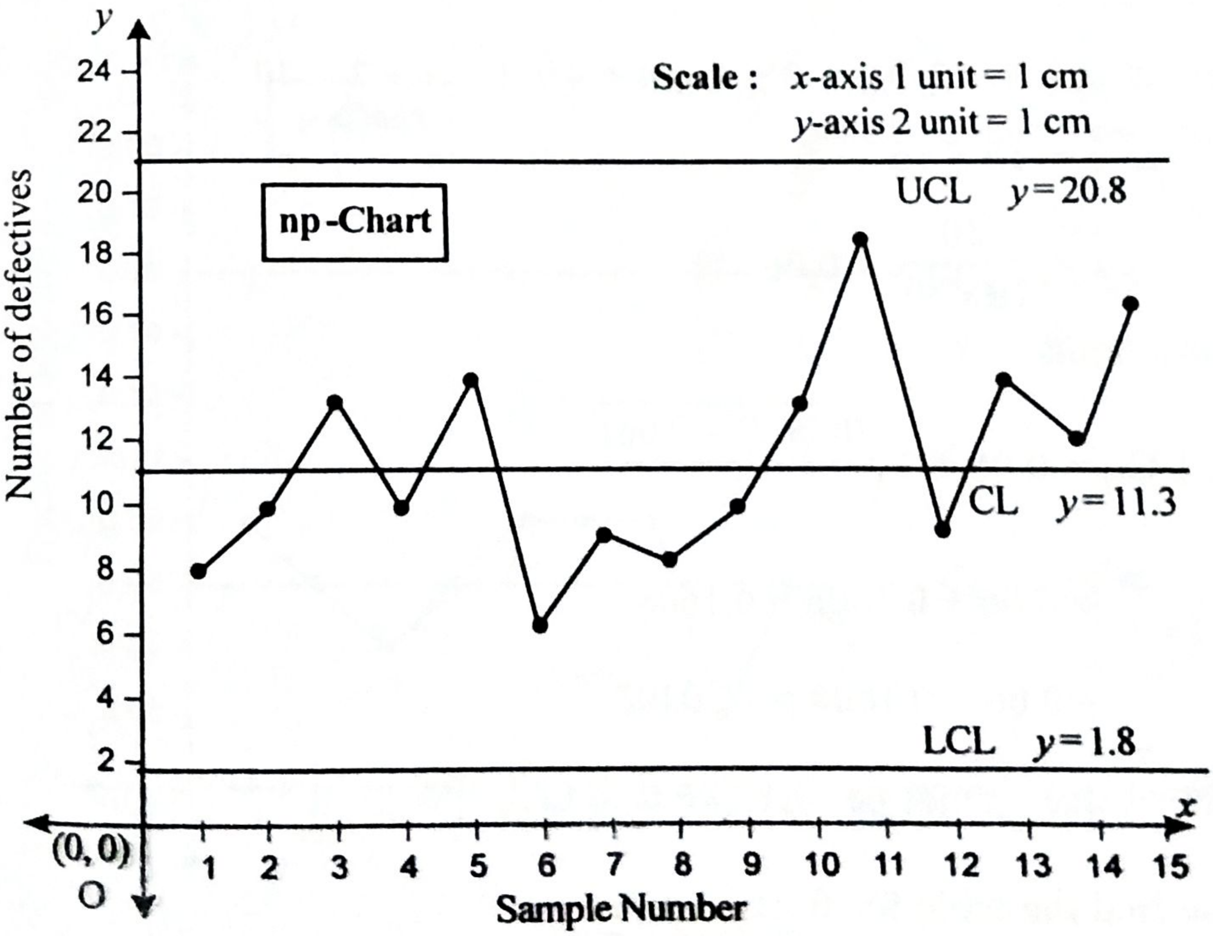
$$\text{LCL} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$= n\left[\bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}\right]$$

$$= 100 \times 0.018 = 1.8$$

We shall now draw the *p*-chart and *np*-chart.


p-Chart
Scale : x-axis 1 unit = 1 cm
y-axis 0.02 unit = 1 cm
UCL y=0.208
CL y=0.113
LCL y=0.018
Fraction of defective
y
0.24
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
(0, 0)
O
x
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
Sample Number


np-Chart
Scale : x-axis 1 unit = 1 cm
y-axis 2 unit = 1 cm
UCL y=20.8
CL y=11.3
LCL y=1.8
Number of defectives
y
24
22
20
18
16
14
12
10
8
6
4
2
(0, 0)
O
x
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
Sample Number

Example 3. The following data refers to visual digits found during the inspection of the first 10 samples of size 50 each from a lot of two-wheelers manufactured by an automobile company.

Sample No.	1	2	3	4	5	6	7	8	9	10
No. of defectives	4	3	2	3	4	4	4	1	3	2

Draw p-chart to show that fraction defectives are under control.

Solution : The p-Chart consists of the central line (CL) $= \bar{p}$

Note that $\quad UCL = \bar{p} + 3\sqrt{\dfrac{\bar{p}\,(1-\bar{p})}{n}}$

and $\quad LCL = \bar{p} - 3\sqrt{\dfrac{\bar{p}\,(1-\bar{p})}{n}}$

where $\quad \bar{p} = \dfrac{\Sigma d_i}{\Sigma n_i}$

Now $\Sigma\, d_i = 4 + 3 + 2 + 3 + 4 + 4 + 4 + 1 + 3 + 2 = 30$

$\Sigma\, n_i = 10 \times 50$

$\therefore \quad \bar{p} = \dfrac{30}{10 \times 50} = 0.06$

$\therefore$ the control limits

$$UCL = 0.06 + 3\sqrt{\frac{0.06\,(1-0.06)}{50}}$$

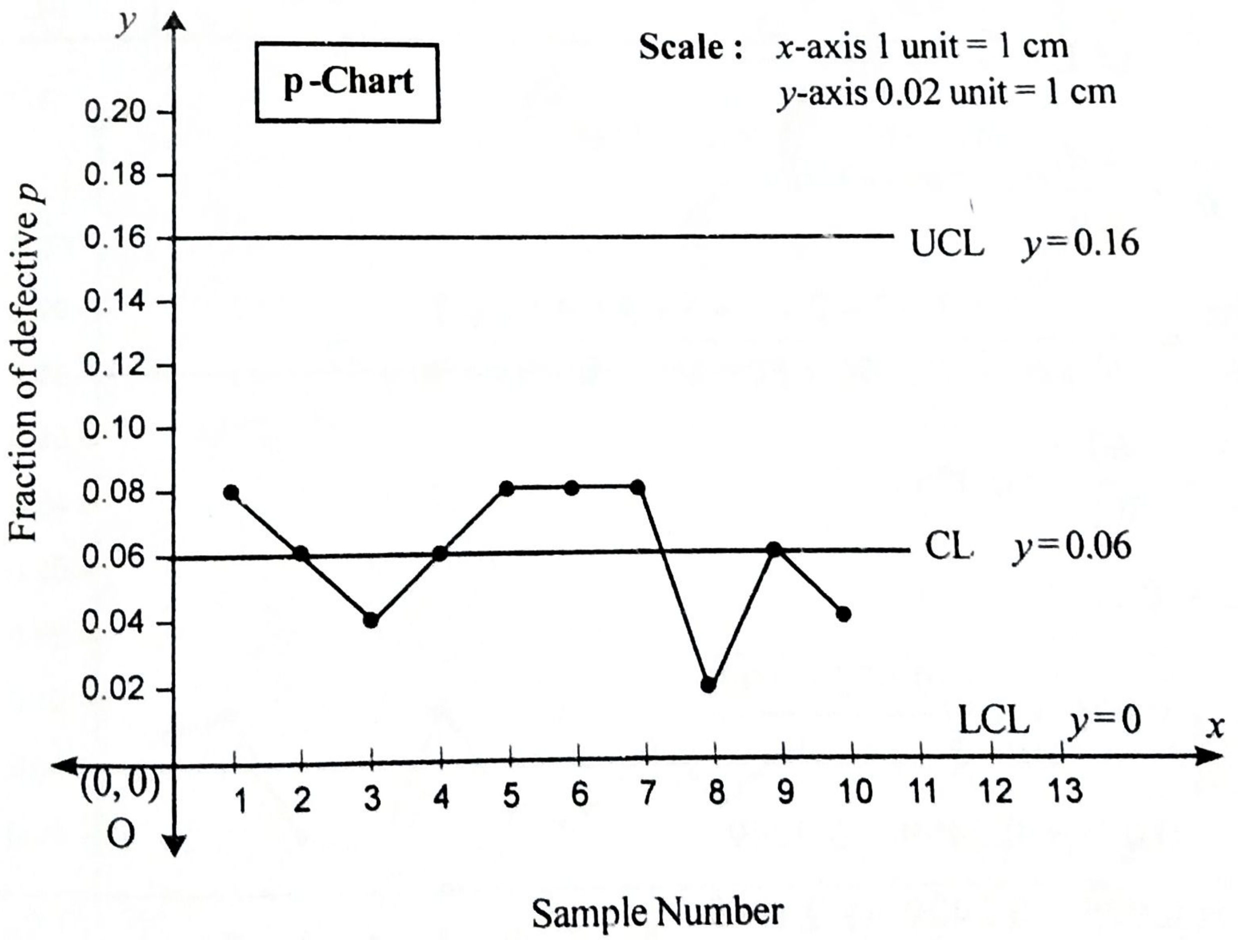
$$= 0.06 + 0.1008 = 0.1608$$

$$= 0.06 - 0.1008 = -0.0408$$

Since the limit value cannot be –ve, we take LCL = 0

We shall now find the table for fraction defectives.

Sample number	Fraction defective p
1	$\frac{4}{50} = 0.08$
2	$\frac{3}{50} = 0.06$
3	$\frac{2}{50} = 0.04$
4	$\frac{3}{50} = 0.06$
5	$\frac{4}{50} = 0.08$
6	$\frac{4}{50} = 0.08$
7	$\frac{4}{50} = 0.08$
8	$\frac{1}{50} = 0.02$
9	$\frac{3}{50} = 0.06$
10	$\frac{2}{50} = 0.04$



From the graph it is clear that all the fraction values lie within the control limits. Hence the process is under control.

Example 4. Construct a control chart for the proportion of defectives for the following data

Sample No.	1	2	3	4	5	6	7	8	9	10
No. of inspected	90	65	85	70	80	80	70	95	90	75
No. of defectives	9	7	3	2	9	5	3	9	6	7

Comment on the nature of the process. **[AU 2015]**

Solution : For fraction defectives we use p-chart. Here $n = 10$.

The central line (CL) $= \bar{p}$

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}\,(1-\bar{p})}{n}}$$

and
$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}\,(1-\bar{p})}{n}}$$

where
$$\bar{p} = \frac{\Sigma d_i}{\Sigma n_i}$$

$$= \frac{9+7+3+2+9+5+3+9+6+7}{90+65+85+70+80+80+70+95+90+75}$$

$$= \frac{60}{800} = 0.075$$

$\therefore$ CL $= 0.075$

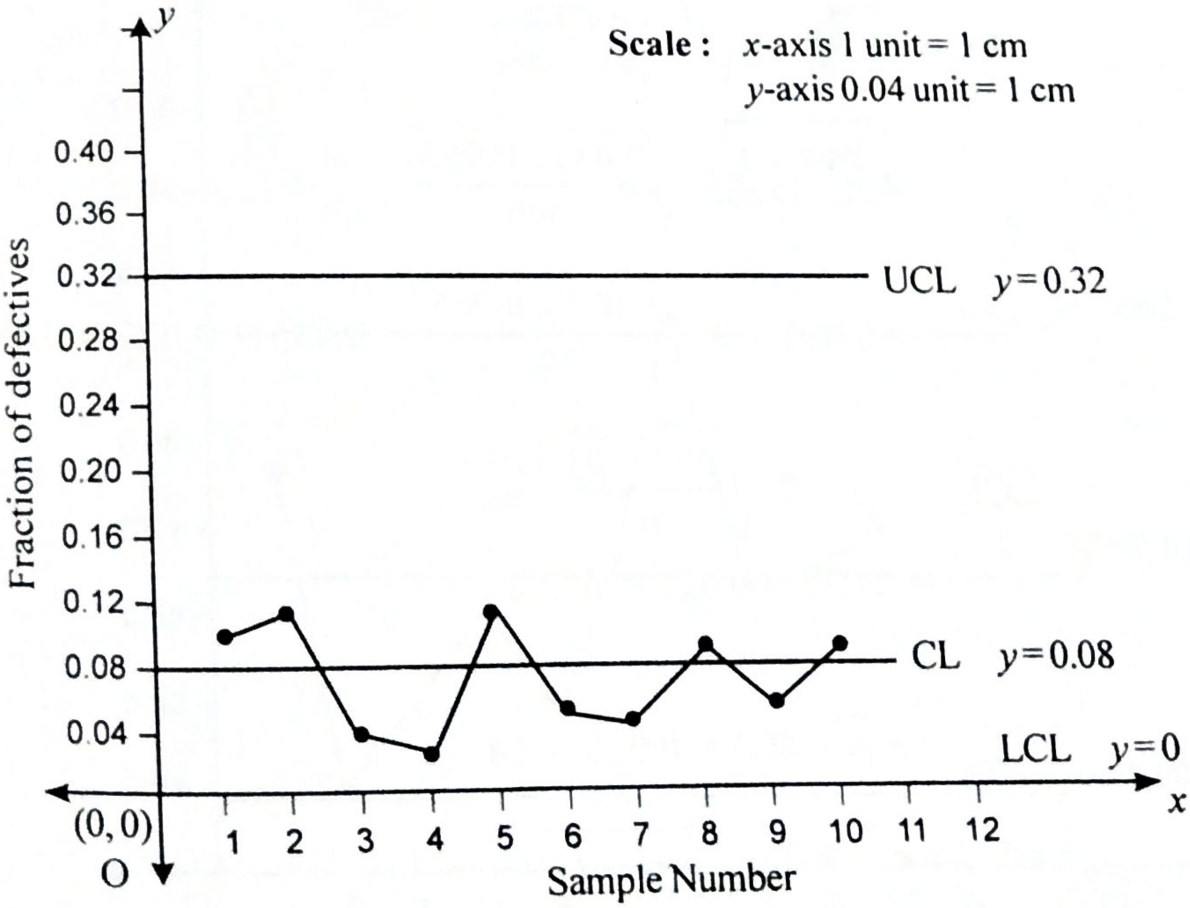
$$\text{UCL} = 0.075 + 3\sqrt{\frac{0.075 \times 0.925}{10}}$$

$$= 0.075 + 0.2499 = 0.3249$$

$$\text{LCL} = 0.075 - 0.2499 = -0.1949$$

We shall now draw the p-chart.

Sample number	Number of inspected n_i	Number of defectives d_i	Fraction defectives
1	90	9	$\frac{9}{90} = 0.1$
2	65	7	$\frac{7}{65} = 0.1077$
3	85	3	$\frac{3}{85} = 0.0353$
4	70	2	$\frac{2}{70} = 0.0286$
5	80	9	$\frac{9}{80} = 0.1125$
6	80	5	$\frac{5}{80} = 0.0625$
7	70	3	$\frac{3}{70} = 0.0429$
8	95	9	$\frac{9}{95} = 0.0947$
9	90	6	$\frac{6}{90} = 0.0667$
10	75	7	$\frac{7}{75} = 0.0933$



p-Chart

From the fraction defective table we find 5 values of p are outside the control limits. So the process is out of control.

Example 5. On inspection of 10 samples, each of size 400, the numbers of defective articles were 19, 4, 9, 12, 9, 15, 26, 14, 15, 17. Draw the *np*-chart and *p*-chart and comment on the state of control. **[AU 2014]**

Solution :

Sample size is constant for all samples and $n = 400$

Total number of defectives $= \Sigma d_i$

$$= 19 + 4 + 9 + 12 + 9 + 15 + 26 + 14 + 15 + 17 = 140$$

$$\Sigma n_i = 10 \times 400 = 4000$$

and $$\bar{p} = \frac{\Sigma d_i}{\Sigma n_i} = \frac{140}{4000} = 0.035$$

P-chart :

$$\text{CL} = \bar{p} = 0.035$$

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$= 0.035 + 3\sqrt{\frac{0.035 \times 0.965}{400}}$$

$$= 0.035 + 3\,\frac{\sqrt{0.035 \times 0.965}}{20} = 0.035 + 0.027 = 0.062$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$= 0.035 - 0.027 = 0.008$$

***np*-chart :**

$$\text{CL} = n\bar{p} = 400 \times 0.035 = 14$$

$$\text{UCL} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$= n\left[\overline{p}+3\sqrt{\frac{\overline{p}\,(1-\overline{p})}{3}}\right] = 400 \times 0.062 = 24.8$$

$$\text{LCL} = n\left[\overline{p}-3\sqrt{\frac{\overline{p}\,(1-\overline{p})}{3}}\right]$$

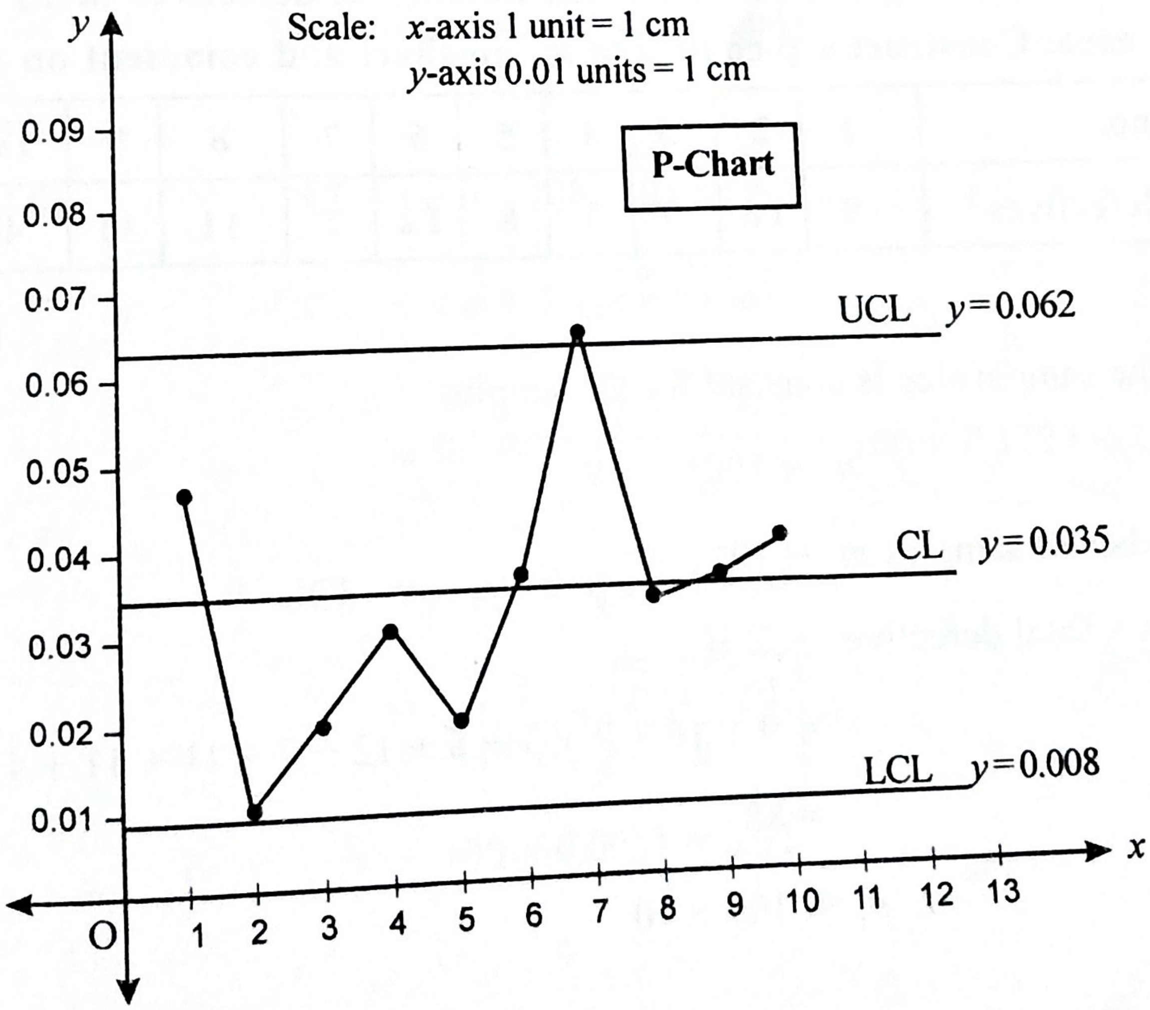
$$= 400 \times 0.008 = 3.2$$

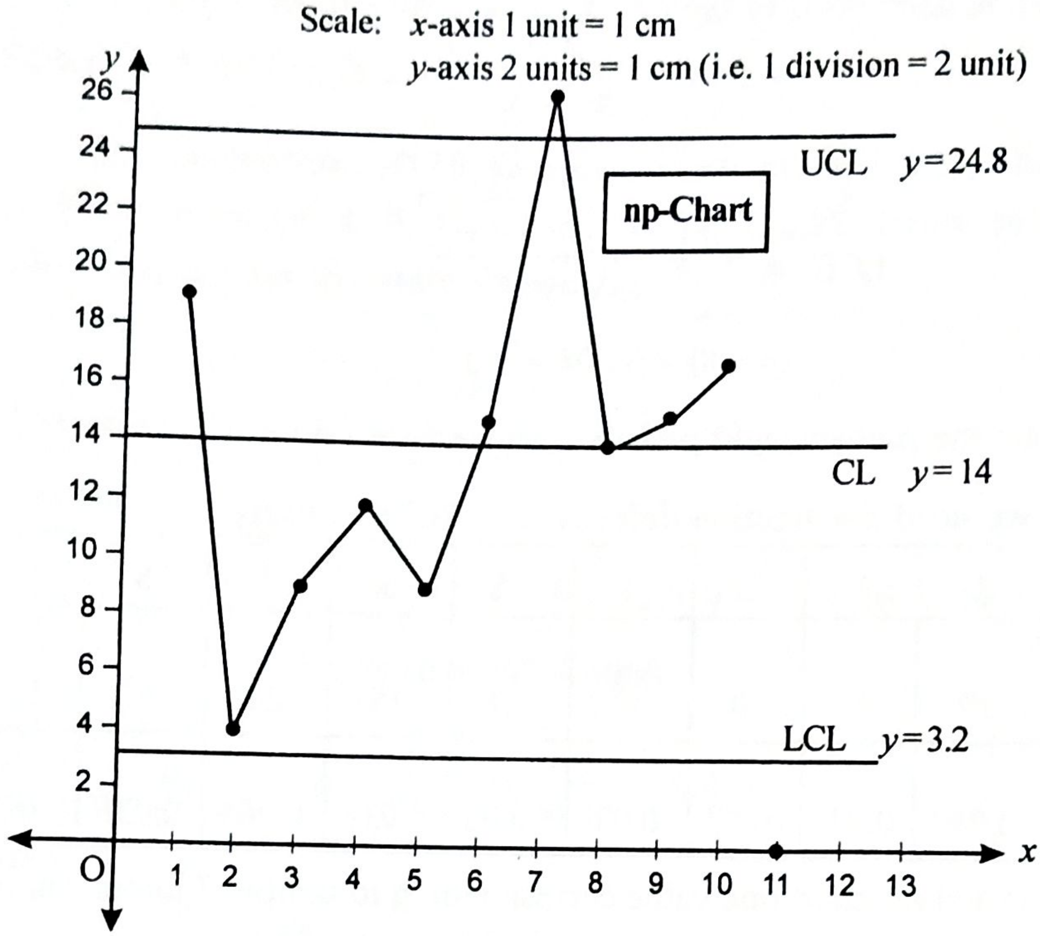
We shall draw the p-chart and np-chart.

For p-chart we need the fraction defective.

Sample no.	1	2	3	4	5	6	7	8	9	10
No. of defectives :	19	4	9	12	9	15	26	14	15	17
Fraction of defectives :	0.048	0.01	0.02	0.03	0.02	0.038	0.065	0.035	0.038	0.04

In the p-chart and np-chart one value corresponding to sample 7 jumps out of control limits and hence the process is out of control.





Example 6. The data given below are the number of defectives in 10 samples of 100 items each. Construct a p-chart and an np-chart and comment on the results.

Sample no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
No. of defectives	**9**	**16**	**7**	**3**	**8**	**12**	**7**	**11**	**11**	**4**

Solution : **[AU 2018]**

Given the sample size is constant for all samples.

$$\therefore \quad n = 100$$

$$\text{Number of samples } m = 10$$

$$\text{Total defective} = \Sigma d_i$$

$$= 9 + 16 + 7 + 3 + 8 + 12 + 7 + 11 + 11 + 4$$

$$= 88$$

$$\Sigma n_i = 100 \times 10$$

and $\quad \bar{p} = \dfrac{\Sigma d_i}{\Sigma n_i} = \dfrac{88}{100 \times 10} = 0.088$

P-chart :

Control Line $\quad CL = \bar{p} = 0.088$

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

Now $\dfrac{\bar{p}(1-\bar{p})}{n} = \dfrac{0.088(1-0.088)}{100} = \dfrac{0.088 \times 0.912}{100} = 0.00080$

$\therefore \quad \sqrt{\dfrac{\bar{p}(1-\bar{p})}{n}} = 0.02828$

$\therefore \quad 3\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n}} = 0.0849$

$\therefore \quad UCL = 0.088 + 0.0849 = 0.1729$

$\quad LCL = 0.088 - 0.0849 = 0.0031$

***np*-chart :**

Central line $\quad CL = n\bar{p} = 100 \times 0.088 = 8.8$

$$UCL = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$= n\left[\bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}\right] = 100 \times 0.1729 = 17.29$$

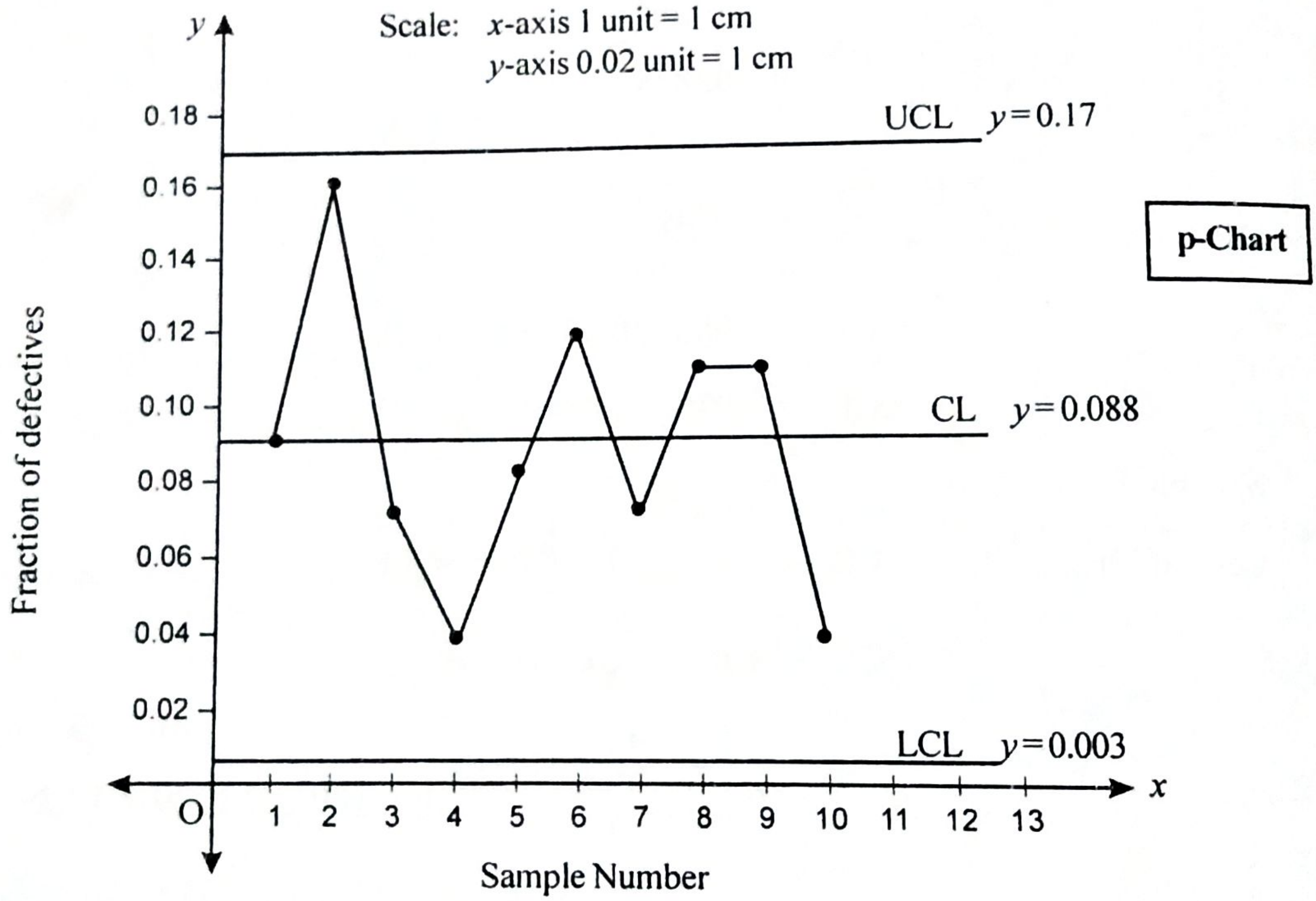
$$UCL = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$$

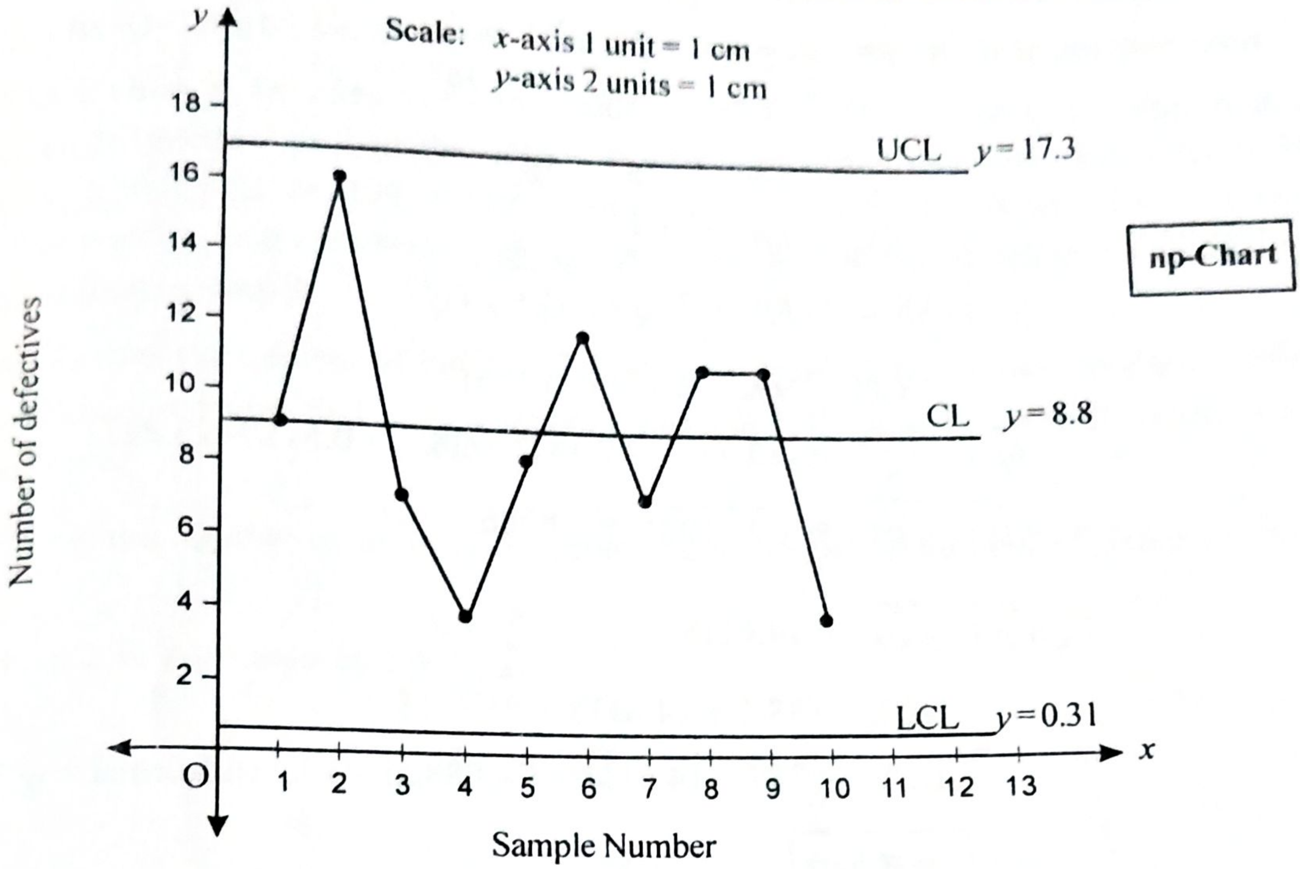
$$= n\left[\bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}\right]$$

$$= 100 \times 0.0031 = 0.31$$

We shall now complete the fraction defective p.

Sample no.	1	2	3	4	5	6	7	8	9	10
No. of defectives	9	16	7	3	8	12	7	11	11	4
Fraction of defectives	$\frac{9}{100}$ $=0.09$	$\frac{16}{100}$ $=0.16$	$\frac{7}{100}$ $=0.07$	$\frac{3}{100}$ $=0.03$	$\frac{8}{100}$ $=0.08$	$\frac{12}{100}$ $=0.12$	$\frac{7}{100}$ $=0.07$	$\frac{11}{100}$ $=0.11$	$\frac{11}{100}$ $=0.11$	$\frac{4}{100}$ $=0.04$





In both the p-chart and np-chart all the values lie within the control limits. Hence the process is under control.

Example 7. The following data gives the number of defectives in 10 samples each of size 100. Construct a *np*-chart for three data and also determine whether the process is in control.

Sample No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
No. of defects	**24**	**38**	**62**	**34**	**26**	**36**	**38**	**52**	**33**	**44**

[AU 2019]

Solution :

Sample size is constant and $n = 100$. We shall prepare *np*-chart.

Number of samples $m = 10$

Total no. of defectives $= \Sigma d_i = 24 + 38 + 62 + 34 + 26 + 36 + 38 + 52 + 33 + 44$

$= 387$

and $\Sigma n_i = 10 \times 100 = 1000$

$$\bar{p} = \frac{\Sigma d_i}{\Sigma n_i} = \frac{387}{1000} = 0.387$$

$\therefore$ $\qquad \therefore\ n\bar{p} = 100 \times 0.387 = 38.7$

np-chart : $\qquad$ CL $= n\bar{p} = 38.7$

$$\text{UCL} = n\bar{p} + 3\sqrt{n\bar{p}\,(1-\bar{p})}$$

$$\text{LCL} = n\bar{p} - 3\sqrt{n\bar{p}\,(1-\bar{p})}$$

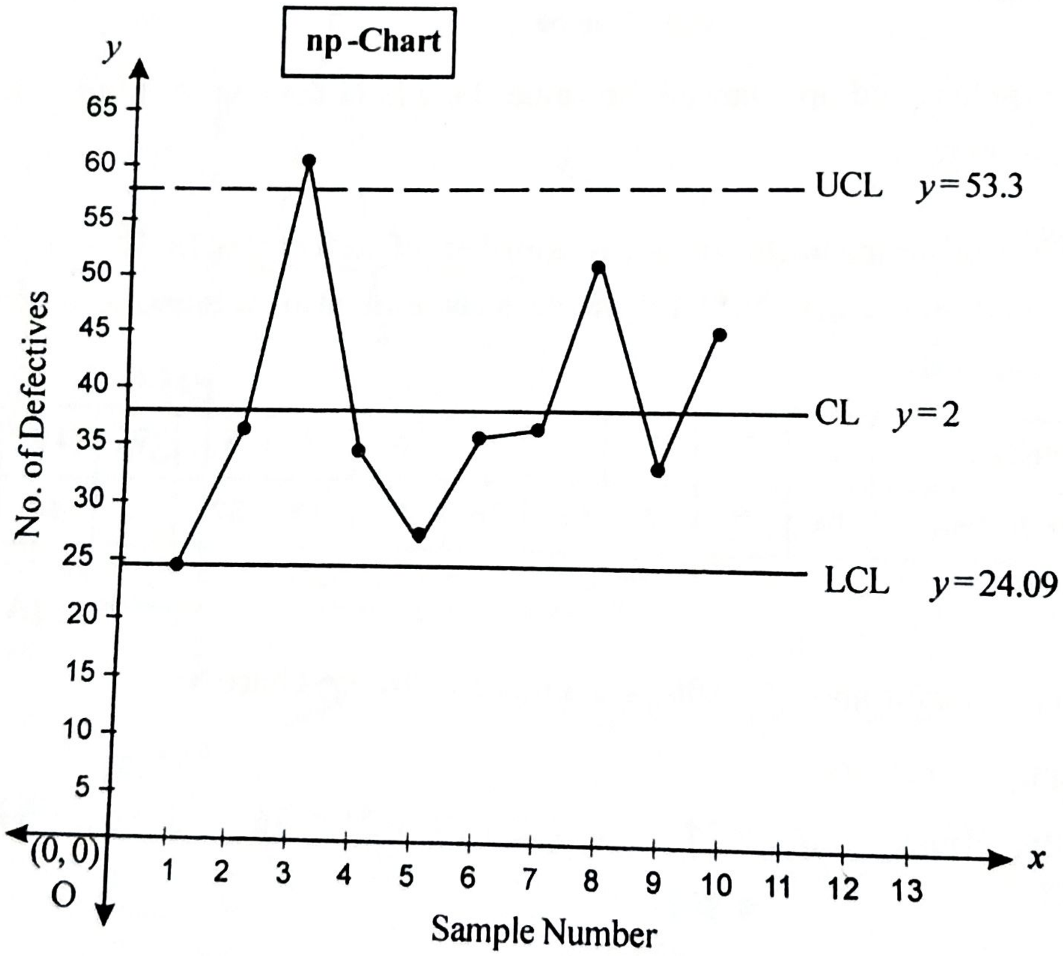
$\therefore$ $\qquad n\bar{p}\,(1-\bar{p}) = (38.7)\,(1-0.387) = 38.7 \times 0.613 = 23.7231$

$\therefore$ $\qquad \sqrt{n\bar{p}\,(1-\bar{p})} = \sqrt{23.7231} = 4.8706$

$\therefore$ $\qquad 3\sqrt{n\bar{p}\,(1-\bar{p})} = 14.6118$

$\therefore$ $\qquad$ UCL $= 38.7 + 14.6118 = 53.3118$

LCL $= 387 - 14.6118 = 24.0882$



Since not all the values lie within control limits, we find the process is out of control.

5.2.2. The C-Chart - or Control chart for the number of defects per unit

The C-chart is designed to control the number of defects per unit. It is important to define an **inspection unit** of the output to be sampled and examined for defects. For example, a unit may be 100 meters of manufactured pipe line, where the number of defective welding is the object of quality control or the number of defects in a 100 meter of manufactured carpet.

In practice the number of defects per unit is very small. If c is the number of defects per manufactured unit then c is a random variable following Poisson distribution with parameter λ.

If m is the number of units of the product and if c_i is the number of defects in the i^{th} unit, then λ is estimated by $\bar{c} = \frac{1}{m}\sum_{i=1}^{m} c_i$

The Central line of C-chart is CL $= \bar{c}$

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} \quad \text{and} \quad \text{LCL} = \bar{c} - 3\sqrt{\bar{c}}$$

Note : To estimate the values of λ (i.e., mean $\bar{c}$) at least 20 values of c must be known. It is desirable to have samples of same size.

WORKED EXAMPLES

Example 1. 20 pieces of cloth out of different rolls contained respectively 1, 4, 3, 2, 4, 5, 6, 7, 2, 3, 2, 5, 7, 6, 4, 5, 2, 1, 3 and 8 imperfections. Determine whether the process is in a state of statistical control.

Solution :

Let c denote the number of imperfections per unit.

$$\bar{c} = \frac{\Sigma c}{n} = \frac{1}{20}[1+4+3+2+4+5+6+7+2+3+2+5+7+6+4+5+2+1+3+8]$$

$$= \frac{80}{20} = 4$$

$\sqrt{\bar{c}} = \sqrt{4} = 2$

For c-chart : CL $= 4$

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 4 + 3 \times 2 = 10$$

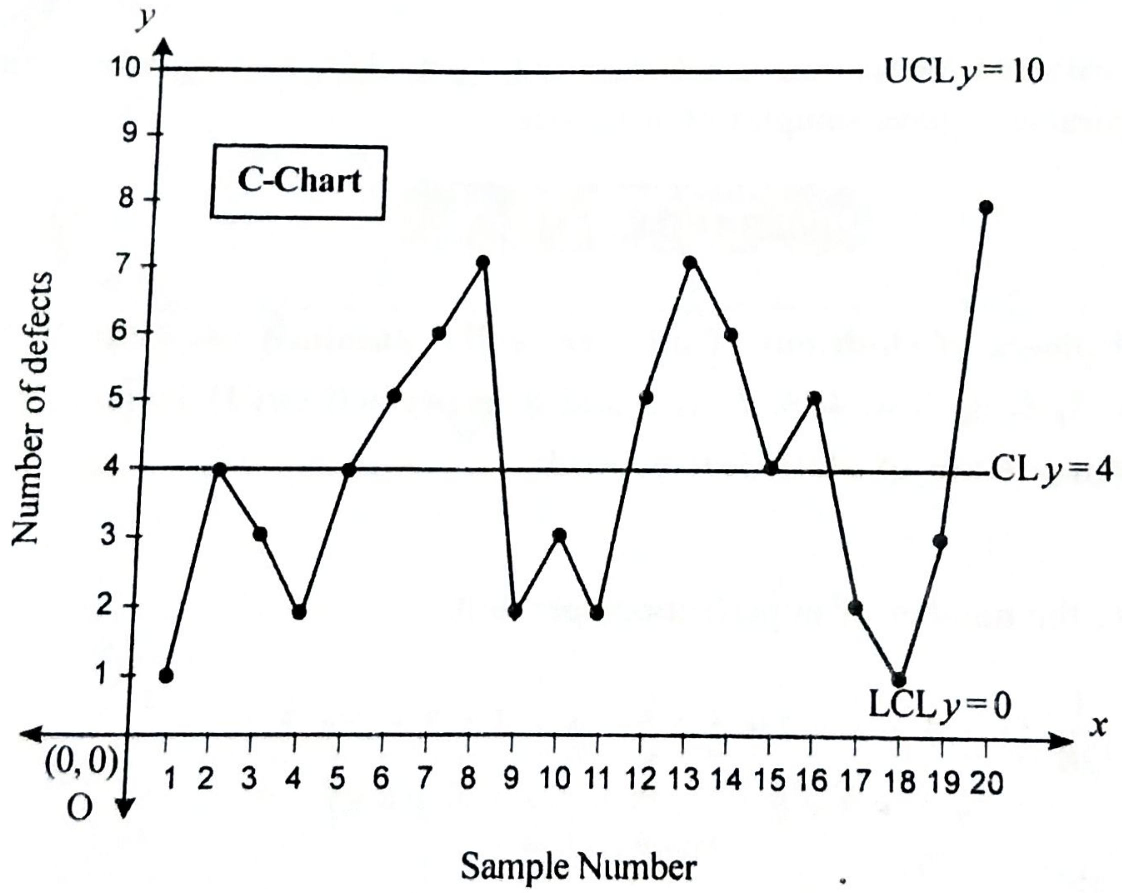
$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 4 - 3 \times 2 = -2$$

But LCL can not be negative. $\therefore$ we take LCL = 0

Since all the values of c lie between the control limits 0 and 10. W find the process is in statistical control.

We shll now draw the c-chart.

Sample No.	1	2	3	4	5	6	7	8	9	10
No. of imperfections	1	4	3	2	4	5	6	7	2	3
Sample No.	11	12	13	14	15	16	17	18	19	20
No. of imperfections	2	5	7	6	4	5	2	1	3	8



Example 2. The following table gives the number of defects in carpets manufactured :

Carpet Serial No.	1	2	3	4	5	6	7	8	9	10
No. of defects	3	4	5	6	3	3	5	3	6	2

Determine the Central line and control limits for c-charts.

Solution :

Let c denote the number of defects per unit (1 carpet).

Here $n = 10$

$$\bar{c} = \frac{\Sigma c}{n}$$

$$= \frac{3+4+5+6+3+3+5+3+6+2}{10}$$

$$= \frac{40}{10} = 4$$

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}}$$

$$= 4 + 3\sqrt{4} = 4 + 6 = 10$$

$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}}$$

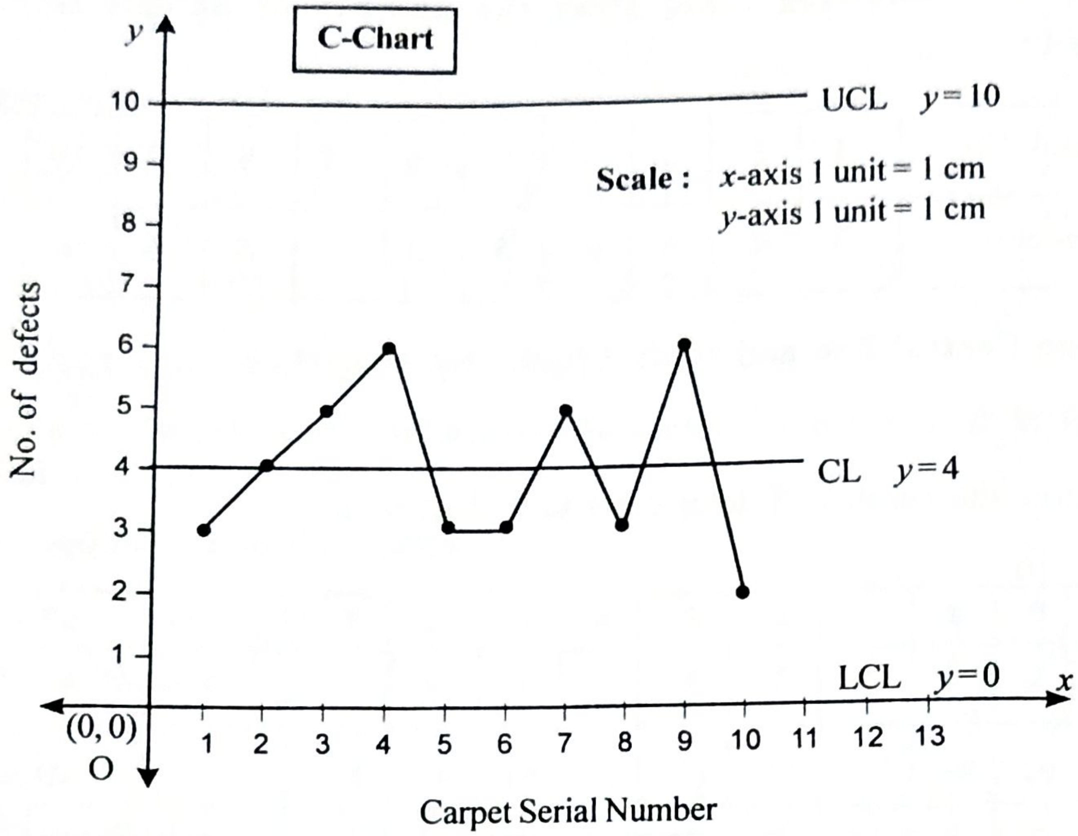
$$= 4 - 3\sqrt{4} = 4 - 6 = -2$$

Since the limit cannot be negative, we take LCL = 0.

From the data we find all the values of c lie between the control limits 0 and 10.

Hence we find the process is in control.

We shall draw the c-chart.



Example 3. From the following data which give the number of defects in 15 pieces of cloth of equal length, construct a control chart for the number of defects.

Piece No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
No. of defects	**3**	**4**	**2**	**7**	**8**	**7**	**5**	**4**
Piece No.	**9**	**10**	**11**	**12**	**13**	**14**	**15**	
No. of defects	**8**	**10**	**5**	**8**	**8**	**6**	**5**	

Solution :

Let c denote the number of defects per piece.

$$\bar{c} = \frac{\Sigma c}{n} = \frac{1}{15}[3 + 4 + 2 + 7 + 8 + 7 + 5 + 4 + 8 + 10 + 5+8]$$

$$= \frac{3+4+2+7+8+7+5+4+8+10+5+8+8+6+5}{15} = \frac{90}{15} = 6$$

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 6 + 3\sqrt{6} = 6 + 7.3485 = 13.3485$$

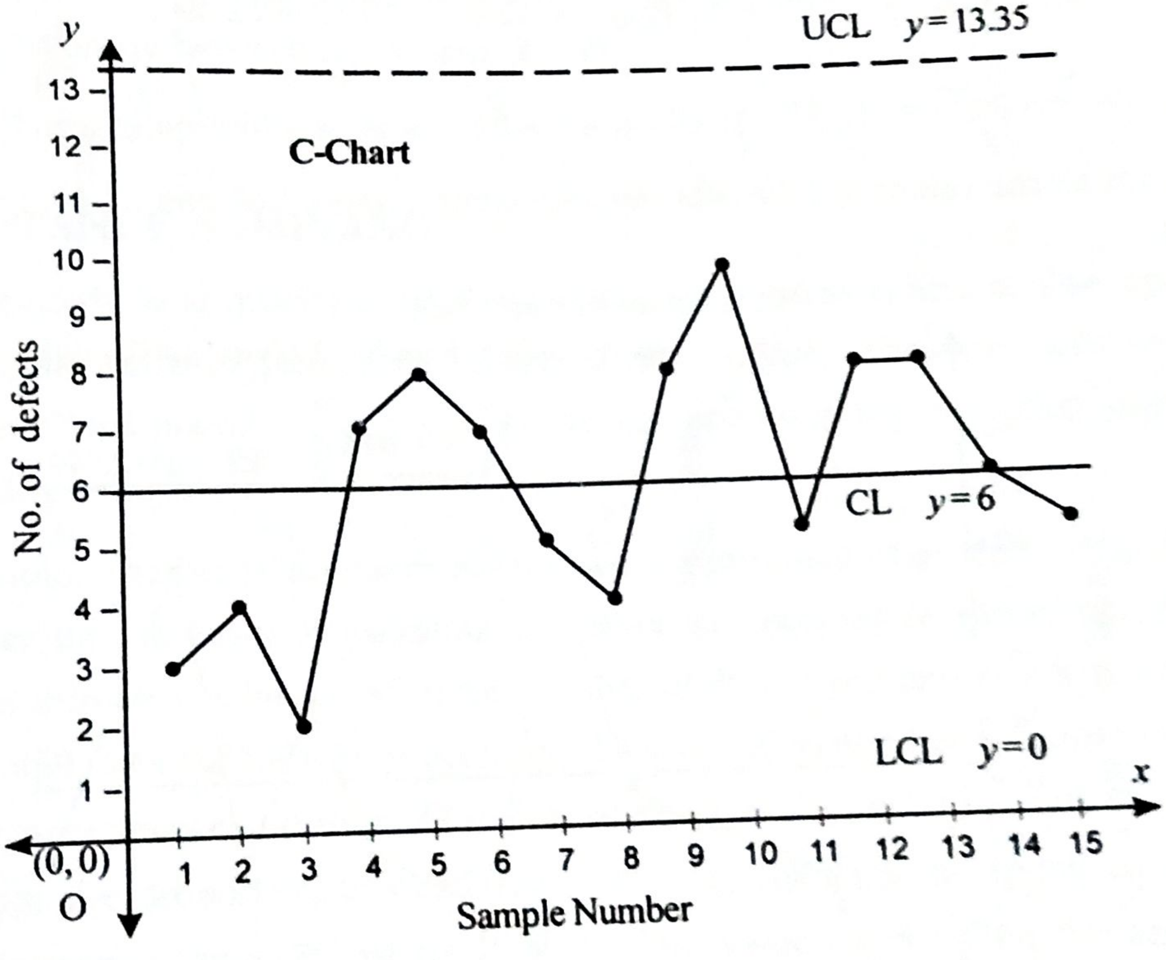
$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 6 - 3\sqrt{6} = 6 - 7.3485 = -1.3485$$

Since the control limit cannot be negative, we take it as 0.

$\therefore$ LCL = 0.

Central line (CL) $= \bar{c} = 6$

Since all the values of c lie between the control limits, the process is in control.



Example 4. The following data relate to the number of defects in each of 15 units drawn randomly from a production process. Draw the control chart or the number of defects and comment on the state of control.

6, 4, 9, 10, 11, 12, 20, 10, 9, 10, 15, 10, 20, 15, 10. **[AU 2014**

Solution :

Given the number of defects per unit and so the suitable control chart is the c-chart.

Here number of samples is $n = 15$.

Sample no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
No. of defects	6	4	9	10	11	12	20	10	9	10	15	10	20	15	10

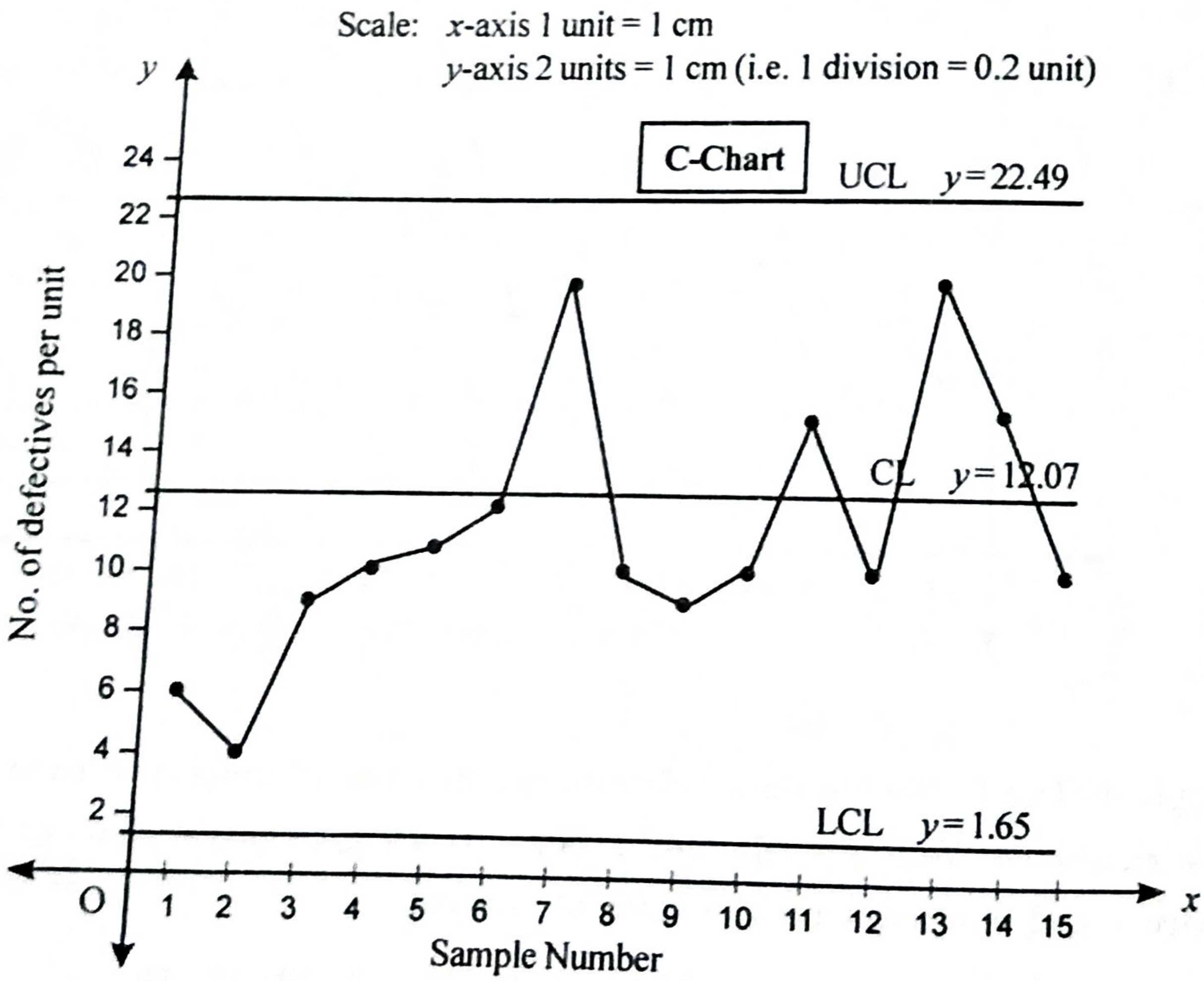
$$\bar{c} = \frac{\Sigma e}{n} = \frac{1}{15}[6+4+9+10+11+12+20+10+9+10+15+10+20+15+10] = \frac{181}{15} = 12.07$$

$\therefore$ CL = 12.07

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 12.07 + 3\sqrt{12.07} = 12.07 + 10.42 = 22.49$$

$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 12.07 - 10.42 = 1.65$$

Since all the values of c lies between the control limits 1.65 and 22.49, the process in control.



5.2.3 Tolerance limits

We have seen confidence limits are used to estimate a parameter of a population. But tolerance limits are used to indicate the limits within which a certain proportion of the population lie.

For example, from a normal population with known mean μ and variance σ^2, the limits which evers the middle 95% of the population observations is $\mu - 1.96\sigma$, $\mu + 1.96\sigma$. This interval $(\mu - 1.96\sigma, \mu + 1.96\sigma)$ is called **tolerance** interval.

But when μ and σ^2 are not known in a normal population, they are estimated by sample mean $\bar{x}$ and variance s^2. Then the **tolerance limits** are given by $\bar{x} \pm ks$, where k is determined such that one can assert with $100\,(1 - \alpha)\%$ confidence that the proportion p of the population lie between $\bar{x} - ks$ and $\bar{x} + ks$.

Thus for large sample of size n, k is 1.96 for $p = 0.95$

5.3 ACCEPTANCE SAMPLING

Two commonly used quality control techniques are **control charts** and **acceptance sampling.** We have already seen control charts of various types. They are useful only for the regulation of the manufacturing process. Acceptance sampling is another important aspect of quality control.

For example, whether to accept or not to accept a shipment of product on inspection by a customer on the basis of sampling is known as **acceptance sampling.** When a sample of the shipment is inspected, if the number of defective items is not more than a specified number, then the lot will be accepted. This specified number is denoted by c and is called the **acceptance number.** This procedure is equivalent to a test of the null hypothesis that the proportion of defective p in a lot equals some specified value P_0 against an alternative value P_1, where $P_1 > P_0$. The value P_0 is called the **acceptable quality level (AQL)** and P_1 is called the **lot tolerance percent defectives (LTPD).**

The statistical techniques used in acceptance sampling will be the familiar hypothesis testing ideas. In acceptance sampling there is always a possibility of committing two type of errors.

(1) Type I error or producer's risk. That is a sampling plan leading to the rejection of a lot which is good.

(2) Type II error or consumer's risk. That is a sampling plan leading to acceptance of a lot which is not good.

5.3.1 Types of acceptance sampling plans.

1. Single sampling plan

When decision is taken to accept or reject a lot on the basis of only one sample, the acceptance plan is known as a single sampling plan. In this plan, the sample size n from a lot containing N items and the acceptance number c to be used and its choice is based on a specified AQL and (or) LTPD in association with given producers risk and (or) consumer's risk.

The sampling plan may be specified as

$$N = 250 ; \quad n = 25 ; \quad c = 1$$

This means "take a random sample of 25 from a lot containing 250 items. If the sample contains more than 1 defective, reject the lot, otherwise accept the lot."

2. Double sampling plan

Double sampling involves the possibility of putting off the decision on a lot until a second sample is taken. A lot is accepted readily if the first sample is good enough. If the first sample is neither good nor bad, the second sample will be taken the decision is based on the first and second samples combined. In double sampling plan five things are specified $n_1, c_1, n_2, n_1 + n_2$ and c_2

n_1 = size of the first sample from a lot.

c_1 = acceptance number of the first sample (i.e., the maximum number of defectives that will permit acceptance of the lot on the basis of first sample)

n_2 = size of the second sample from the lot

$n_1 + n_2$ = size of the combined sample of the first and second samples.

c_2 = acceptance number for the two samples combined.

The double sampling plan may be specified as

$$N = 500$$

$$n_1 = 25 \qquad c_1 = 1$$

$$n_2 = 50 \qquad c_2 = 3$$

This plan involves selection of first a random sample of 25 items from the lot of 500 items. If the sample contains less than or equal to 1 defective, the lot is accepted. If more than 1 defective is found, the lot is rejected.

A second sample of 50 items is taken from the lot and the lot is accepted if the combined sample of 75 items contains less than or equal to 3 defectives and reject the lot otherwise.

3. Multiple or sequential sampling plan

When a sampling plan consists of more than two sample stages, it is called a **multiple sampling plan.**

A sampling procedure is said to be sequential if, after each stage of observation of sample, if the lot is rejected then take another sample and continue this process.

5.3.2 Operating Characteristic Curve (OC - curve)

A sampling plan is best described by its operating characteristic curve or OC curve. The OC curve gives the probability of acceptance for each value that can be assumed by the lot proportion defective P. The OC curve of an acceptance sampling plan shows the ability of the plan to distinguish between good and bad lots.

By an appropriate choice of the sample sizes and the acceptance numbers it is possible to match the OC curve of a double or multiple- sampling plan closely to that of an equivalent single - sampling plan. Thus the degree of protection given by a double or multiple sampling plan can be made essentially the same as that given by single - sampling plan.

It is usual in acceptance sampling the OC curve is constructed by using binomial distribution for the probability of accepting a lot containing the proportion defective P.

If P is very small, say $P < 0.10$ (or np is less than 5) and the lot is atleast 10 times the size of the sample, we use Poisson distribution for constructing the OC curve.

Example 1. For a sampling plan $N = 1000$, $n = 600$, $c = 1$, determine the probability of acceptance of the following lots.

(i) 0.6% defective

(ii) 0.7% defective

(iii) 2% defective

(iv) 3% defective

(v) 5% defective

(vi) 9% defective

Also draw the OC curve.

Solution :

The lot has 0.6% defective, the samples from it will have an average of 0.6% defective. So, in a sample of size 60, the average number of defectives will be $60 \times \frac{0.6}{100} = 0.36$ under the sampling plan with $c = 1$, the sample contains 0 or 1 defectives, we shall find the cumulative probability of drawing a sampling of 6 containing 0 or 1 defective using Poisson approximation.

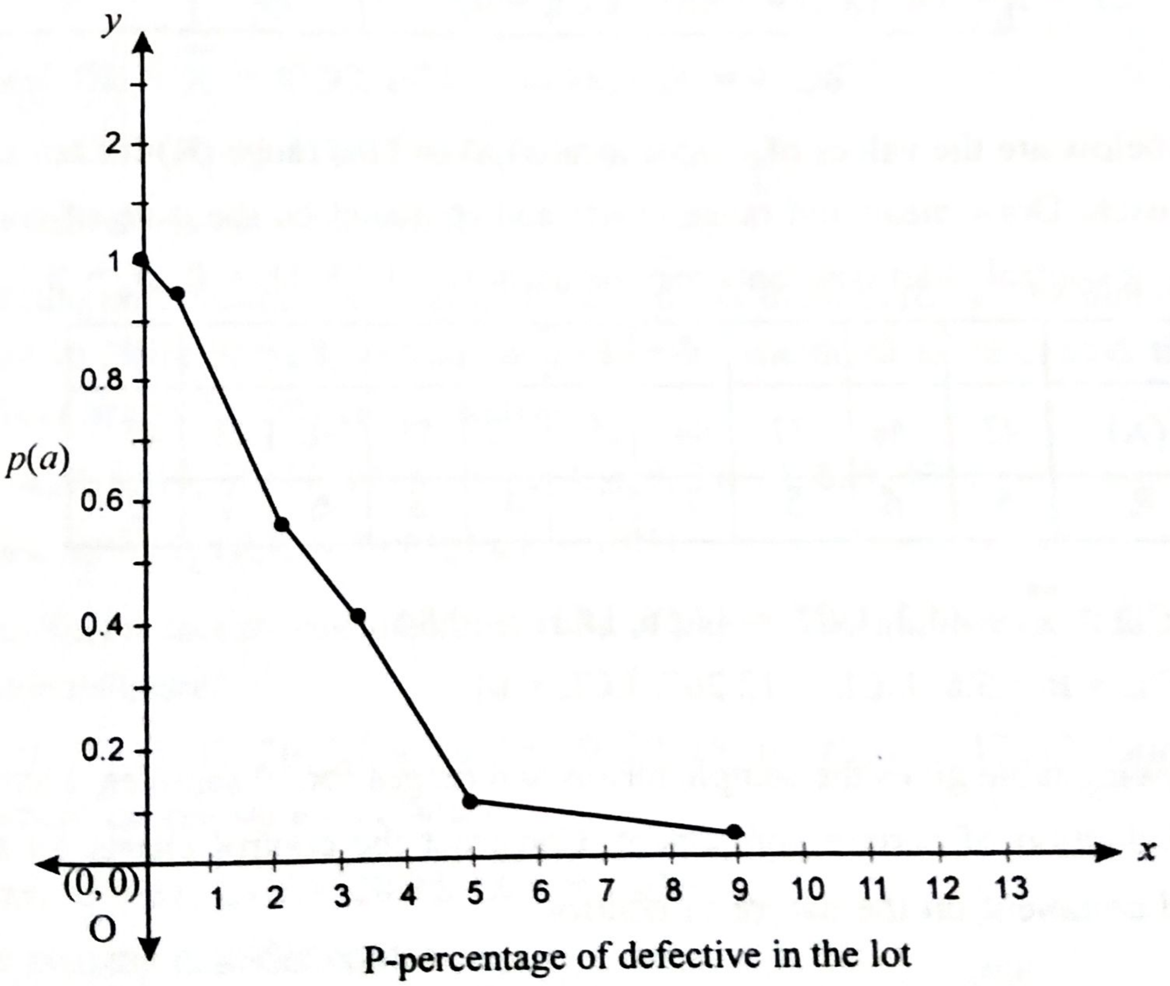
OC curve for single - sampling

$P(a)$: Probability of acceptance

The value $0.9489 \simeq 0.95$ represents the prob. of drawing a sample of size 60 with 0 or 1 defective from a lot known to be 0.6% defective. In words, such a sample will enable acceptance of 95% of lots containing 0.5 percent defective or in a lot of 1000.

960 will be accepted and only 40 will be rejected.

S.No.	% defective in the lot	Average no. of defective in sample	$P(0) = e^{-\lambda}$	$P(1) = e^{-\lambda}.\lambda$	$P(0) + P(1) = P(a)$
1	0.6	$60 \times \frac{0.6}{100} = 0.36$	0.6977	0.2512	0.9489
2	0.7	$60 \times \frac{0.7}{100} = 0.42$	0.6570	0.2759	0.9329
3	2	$60 \times \frac{2}{100} = 1.2$	0.3012	0.3614	0.6626
4	3	$60 \times \frac{3}{100} = 1.8$	0.1653	0.2975	0.4628
5	5	$60 \times \frac{5}{100} = 3.0$	0.0498	0.1494	0.1992
6	9	$60 \times \frac{9}{100} = 5.4$	0.0045	0.0243	0.0288



Taking percentage defective on x-axis and probability of acceptance $p(a)$ on y-axis, the curve obtained is known as operating characteristic curve (OC curve) of the sampling plan.

Probability of rejection is $1 - p(a)$ corresponding to any lot.

EXERCISE 5.1

1. The following data provides the values of sample mean X and the range R for the samples of size 5 each. Calculate the values for Central line and control limit for mean-chart and range chart and determine whether the process is in control.

Sample No.	1	2	3	4	5	6	7	8	9	10
Mean ($\bar{X}$)	15	17	15	18	17	14	18	15	17	16
Range R	7	7	4	9	8	7	12	4	11	5

[**Ans.** CL = $\bar{\bar{X}}$ = 16.2, UCL = 20.47, LCL = 11.93

CL = $\bar{R}$ = 7.4, UCL = 15.651, LCL = 0]

2. Given below are the values of sample means ($\bar{X}$) and the range (R) for ten samples of size 5 each. Draw mean and range charts and comment on the state of control. The following control chart constants may be used $A_2 = 0.58$, $D_3 = 0$, $D_4 = 2.115$.

Sample No.	1	2	3	4	5	6	7	8	9	10
Mean ($\bar{X}$)	43	49	37	44	45	37	51	46	43	47
Range R	5	6	5	7	7	4	8	6	4	6

[**Ans.** CL = $\bar{\bar{X}}$ = 44.2, UCL = 47.56, LCL = 40.84

CL = $\bar{R}$ = 5.8, UCL = 12.267, LCL = 0]

3. The following table gives the sample means and ranges for 10 samples, each of size 6, in the production of certain component. Construct the control charts for mean and range and comment on the nature of control.

Sample No.	1	2	3	4	5	6	7	8	9	10
Mean ($\bar{X}$)	37.3	49.8	51.5	59.2	54.7	34.7	51.4	61.4	70.7	75.3
Range R	9.5	12.8	10	9.1	7.8	5.8	14.5	2.8	3.7	8

[**Ans.** CL = $\bar{\bar{X}}$ = 54.6, UCL = 58.657, LCL = 50.543

CL = $\bar{R}$ = 8.4, UCL = 16.834, LCL = 0

The process is out of control.]

4. Control on measurements of pitch diameter of thread in air-craft fittings is checked with 5 samples each containing 5 items at equal intervals of time.

Sample No.	**Measurements**				
1	46	45	44	43	42
2	41	41	44	42	40
3	42	43	43	42	45
4	40	40	42	40	42
5	43	44	47	47	45

[**Ans.** CL = $\bar{\bar{X}}$ = 42.92, UCL = 44.88, LCL = 40.96

CL = $\bar{R}$ = 3.4, UCL = 7.19, LCL = 0

The process is out of control.]

5. A textile unit produces special cloths and packs them in rolls. The number of defects found in 20 rolls are given below. Determine whether the process is under control. Defects in the 20 rolls are given below :

12, 14, 7, 6, 10, 10, 11, 12, 5 18, 12, 4, 4, 9, 21, 14, 8, 9, 13, 21

[**Ans.** $\bar{C}$ = 11, UCL = 20.95, LCL = 1.05]

6. A newsprint factors manufactures rolls of paper. The number of defects found in 20 sample rolls are

6, 4, 9, 10, 11, 12, 20, 10, 9, 10, 15, 10, 20, 15, 10, 10, 5, 5, 17, 12. Draw C-Chart and comment on the state of control.

[**Ans.** $\bar{C}$ = 11, UCL = 20.95, LCL = 1.05]

The process is under control.

7. The data given below are the number of defectives in 10 samples of size 400 each construct a P-chart and np-chart and comment on the results

Sample No.	1	2	3	4	5	6	7	8	9	10
No. of defects	15	12	4	26	15	9	19	9	14	17

[**Ans.** P-Chart : CL $= \bar{p} = 0.035$, UCL = 0.0626, LCL = 0.0074

np-chart : UCL = 14, LCL = 2.97

The process is out of control]

8. Samples of 100 units are checked to electrical specifications on alternate days of a month in the production line of an integrated circuit and the results of number of defectives are given below draw a P-chart and a np-chart and comment.

Sample No.	1	2	3	4	5	6	7	8
No. of defectors	24	38	62	34	26	36	38	52
Sample No.	9	10	11	12	13	14	15	
No. of defectors	33	44	52	45	30	44	34	

[**Ans.** P-Chart : CL $= \bar{p} = 0.395$, UCL = 0.5416, LCL = 0.2484

np-chart : UCL = 54.16, LCL = 24.84]

9. An inspection of 10 samples of size 400 each from 10 lots revealed the following number of defective units. 17, 15, 14, 26, 9, 4, 19, 12, 9, 15. Calculate the control limits for the number of defective units. Plot the control limits and the observations and state whether the process is under control or not.

[**Ans.** C-Chart : CL = 14, UCL = 25.22, LCL = 2.78]

$c + 3\sqrt{c} = 14 + 11.2249 = 25.2249$]

PART-A QUESTIONS AND ANSWERS

1. **What do you mean by statistical quality control (SQC)?**

Ans. Statistical quality control is a method that uses sampling techniques and statistical analysis in a production process and reduce variability systematically and isolate sources of difficulties during production.

2. **What are the types of variations observed in a production process?**

Ans. There are two types of variations

(1) Common cause variation or random variation

(2) Special cause variation or assignable variation.

3. **What do you mean by process control ?**

Ans. Process control is concerned with the control of the quality of the goods when they are in the process of production.

4. **What is a control chart ? (or) Define a control chart.**

Ans. A control chart is a graphic device for monitoring process outputs to identify when they slip out of control.

5. **What is the general structure of a control chart ?**

Ans. A control chart consists of three horizontal lines.

(1) a central line (CL) to indicate the average quality or the level of the process.

(2) Upper control limit (UCL) line

(3) Lower control limit (LCL) line

6. **What is the basic idea of a control chart ?**

Ans. The basic idea of a control chart is the setting up of lower and upper control limits.

Here $n = 10$

$$\bar{c} = \frac{\Sigma c}{n} = \frac{1}{10}[5 + 1 + 7 + 0 + 2 + 3 + 4 + 0 + 3 + 2]$$

$$= \frac{27}{10} = 2.7$$

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 2.7 + 3\sqrt{2.7} = 2.7 + 3 \times 1.643 = 7.629$$

$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 2.7 - 3\sqrt{2.7} = 2.7 - 3 \times 1.643 = -2.229$$

Since the control limit cannot be negative.

we take LCL = 0

$\therefore$ UCL = 7.629 and UCL = 0

23. Find the lower and upper control limits for the chart when $\bar{c} = 6$ [AU 2014]

Ans. Refer worked example 3, Page No. 5.52

24. What is meant by tolerance limits ? [AU 2014]

Ans. Refer Page No. 5.55

25. What do you understand by control chart for fraction defectives? [AU 2014]

Ans. The control chart for fraction defective is the p-chart where the CL is

$\bar{p} = \dfrac{\Sigma d_i}{\Sigma n_i}$, d_i is the number of defectives in a sample of size n_i.

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}\,(1-\bar{p})}{n}}$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}\,(1-\bar{p})}{n}}$$

26. Distinguish between variable and attributes in connection with the quality characteristic of a product. **[AU 2015]**

Ans. The term variable and attributes are associated with the type of data obtained on the process. When measurable characteristics such as length, weight, time are considered the resulting data is called variable data. A qualitative variable that can take only two values is called an attribute. The data obtained by counting the number of defectives in a sample is called attribute data.

27. Write the formulae for the control chart values (central line), UCL, LCL of a c-chart.

Ans. Central line is

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} \quad \text{and}$$

$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}}$$

28. Control chart for $\bar{X}$ and R to be set up for an imoprtant quality characteristic. The sample size is $n = 5$ and $\bar{X}$ and $\bar{R}$ are computed for each of 35 preminitary samples. The summary data are

$$\sum_{i=1}^{35} \bar{x_i} = 7805, \quad \sum_{i=1}^{35} r_i = 1200.$$

Find the control limits for $\bar{X}$ and R. **[AU 2015]**

Ans. Given : $m = 35$,

$$\sum_{i=1}^{35} \bar{x_i} = 7805$$

$$\text{and} \quad \sum_{i=1}^{35} r_i = 1200$$

$$\therefore \; \bar{\bar{X}} = \frac{\Sigma \bar{x}_i}{m} = \frac{7805}{35} = 223$$

$$\bar{R} = \frac{\Sigma r_i}{m} = \frac{1200}{35} = 34.2857$$

$\bar{X}$-Chart is

$$CL = \bar{\bar{X}} = 223$$

$$UCL = \bar{\bar{X}} + A_2 \bar{R} = 223 + 0.577 \times 34.285 = 242.7828$$

$$LCL = \bar{\bar{X}} - A_2 \bar{R} = 223 - 0.577 \times 34.285 = 203.2172$$

* * *