Birla Institute of Technology and Science, Pilani

Work Integrated Learning Programmes Division

Cluster Programme - M.Tech. in Data Science and Engg.

I Semester 2019-20

Mathematical Foundation for Data Science

Homework - 2

Q1 Let $\mathbf{B} = (\mathbf{b_1}, \mathbf{b_2}, \ldots, \mathbf{b_{r-1}}, \mathbf{b_r}, \mathbf{b_{r+1}}, \ldots, \mathbf{b_n})$ be a non-singular matrix. If column $\mathbf{b_r}$ is replace by $\mathbf{a}$ and that the resulting matrix is called $\mathbf{B_a}$ along with $\mathbf{a} = \sum_{i=1}^{n} y_i \mathbf{b_i}$, then state the necessary and sufficient condition for $\mathbf{B_a}$ to be non-singular. $y_r \neq 0$

Q2 Let V be a finite dimensional vector space over $\mathbb{R}$. If S is a set of elements in V such that $\text{Span}(S) = V$, what is the relationship between S and the basis of V? Basis of V is a subset of S

Q3 Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by

$$T(x_1, x_2, x_3) = (x_1 - x_2 + 2x_3, 2x_1 + x_2, -x_1 - 2x_2 + 2x_3)$$

then,

(1) show that T is a linear transformation

(2) what are the conditions on a, b, c such that (a, b, c) is in the null space of T. Specifically, find the nullity of T. $a = -2c/3, b = 4c/3$ and $N(T) = 1$

Q4 Construct a linear transformation $T : V \to W$, where V and W are vector spaces over F such that the dimension of the kernel space of T is 666. Is such a transformation unique? Give reasons for your answer. $T(x_1, x_2, \ldots, x_{667}) = (x_1, 0, 0, \ldots, 0)$