H.W solving recurrences : iteration method.

(1)
$$T(n) = \begin{cases} 0 & n=0 \\ T(n-1)+1 & \text{otherwise} \end{cases}$$

$T(n) = T(n-1)+1$ — (1)

$T(n-1) = T(n-2)+1$ — (2)

(2) in (1)

$= T(n-2)+1+1$

$= T(n-2)+2$

$= T(n-3)+3$

$\vdots$

$= T(n-i)+i$

when $i=n$,

$T(n) = T(n-n)+n$

$= T(0)+n$

$= n.$

$\Rightarrow T(n) \in O(n)$

Linear.

(2)
$$T(n) = \begin{cases} 0 & n=1 \\ 2T(n/2)+n & n>1. \end{cases}$$

Let $n = 2^k$.

$T(n) = 2T(n/2)+n$

$T(2^k) = 2T(2^k/2)+2^k$ — (1)

$= 2T(2^{k-1})+2^k$

$T(2^{k-1}) = 2T(2^{k-1}/$

$\Rightarrow 2^{k-1} T(1) + 2(\ldots$

$\Rightarrow 2^{k-1}(0) + 2(2^k)$

$\Rightarrow 2'(2^k)$

$T(2^k) \Rightarrow (2)^{k+1}$

$(2)^i T(2^{k-i}) + 2\, 2^{(k-i)} \left[\frac{1}{2} + \frac{1}{(2)^2} + \frac{1}{(2)^3}, \ldots \right] + 2^k$

$T(n) \in O(2^{\log_2 n})$

$$T(2^k) = 2\left[2T(2^{k-2}) + 2^{k-1}\right] + 2^k$$

$$= (2)^2 T(2^{k-2}) + (2)(2)^{k-1} + 2^k$$

$$= (2)^3 T(2^{k-3}) + (2)(2)^{k-2} + (2)(2)^{k-1} + 2^k$$

$$= (2)^4 T(2^{k-4}) + (2)\frac{(2)^k}{(2)^3} + (2)\frac{(2)^k}{2^2} + (2)\frac{2^k}{(2)} + 2^k$$

in terms of i gen eqn. $= (2)^i T(2^{k-i}) + 2(2)^k \underbrace{\left[\frac{1}{(2)^{i-1}} + \frac{1}{(2)^{i-2}} + \cdots + \frac{1}{2^3} + \frac{1}{2^2} + \frac{1}{(2)}\right]}_{G.P.} + 2^k$

$$= (2)^i T(2^{k-i}) + (2)(2^k)\left[\frac{\frac{1}{2}\left(\left(\frac{1}{2}\right)^i - 1\right)}{\left(\frac{1}{2} - 1\right)}\right] + 2^k$$

$a = \frac{1}{2},\ n = i,\ r = \frac{1}{2}$

$a, ar, ar^2, ar^3, \ldots ar^{n-1}$

$$= (2)^i \cdot T(2^{k-i}) + \frac{2^k(2^{-i} - 1)}{\left(-\frac{1}{2}\right)} + 2^k$$

Sum in G.P. =

$r = \cancel{\frac{T_n}{T_{n-1}}}$, $r = T_n / T_{n-1}$

$S_n = \frac{a(r^n - 1)}{(r-1)}$

$$= (2)^i T(2^{k-i}) - 2(2^k)(2^{-i} - 1) + 2^k$$

$r = \frac{\frac{1}{(2)^2}}{\frac{1}{2}} = \frac{2}{2^2} = \frac{1}{2}$

put $i = k$

$$= (2)^k T(2^{k-k}) - 2(2^k)(2^{-k} - 1) + 2^k$$

$$= (2)^k T(1) + 2^k\left[1 - 2(2^{-k} - 1)\right]$$

$$= 0 + 2^k\left[1 - \frac{2}{2^k} + 2\right]$$

$2^k = n$

$\Rightarrow k = \log_2 n$

$$= 2^k\left[3 - 2^{1-k}\right] = (3)2^k - \frac{2}{\cancel{2^k}} \cdot \cancel{2^k}$$

$$T(2^k) = (3)2^k - 2 = (3\, 2^{\log_2 n} - 2)$$

$\boxed{\text{Ans: } O(2^{\log_2 n}) \text{ Exponential order of growth.}}$