Birla Institute of Technology and Science, Pilani

Work Integrated Learning Programmes Division

Cluster Programme - M.Tech. in Data Science and Engg.

I Semester 2019-20

Mathematical Foundation for Data Science

Homework - 3

Q1a) Let P be a real square matrix satisfying $P = P^T$ and $P^2 = P$.

i) Can the matrix P have complex eigenvalues? If so, construct an example, else, justify your answer. No, always real eigenvalues

ii) What are the eigenvalues of P? $\lambda_i = 0$ or 1

b) Given the following matrix $A = \begin{pmatrix} 1 & 2 & r \\ c & 1 & 7 \\ c & 1 & 7 \end{pmatrix}$ where c and r are arbitrary real numbers and $5.5 < r \leq 6.5$, and the fact that $\lambda_1 = 3$ is one of the eigenvalues, is it possible to determine the other two eigenvalues? If so, compute them and give reasons for your answer. 0 and 6

Q2 The Fibonacci sequence is defined by $V_n = V_{n-1} + V_{n-2}$ for $n \geq 2$ with starting values $V_0 = 1$ and $V_1 = 1$. Observe that the calculation of V_k requires the calculation of $V_2, V_3, \ldots, V_{k-1}$. To avoid this, could this problem be written as an eigenvalue problem and solved for V_n directly? If so, find the explicit formula for V_n. $V_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+1}\right]$

Q3 Prove that if A is a square matrix of size $n \times n$, then $A^k \to 0$ as $k \to \infty$ if and only if $|\lambda_i| < 1 \quad \forall i$. The eigenvalues of A^k are λ_i^k and hence $A^k \to 0$ iff $\lambda_i^k = 0$, which is true as long as $|\lambda_i| < 1$

Q4 Construct examples of matrices for which the **defect** is positive, negative and zero wherever possible. Refer to the text book