Homework-14: Solution Set

Q.1 What kind of graph (from given table) can be used to model a highway system between major cities where

a) there is an edge between the vertices representing cities if there is an interstate highway between them?

b) there is an edge between the vertices representing cities for each interstate highway between them.

c) there is an edge between the vertices representing cities for each interstate highway between them, and there is a loop at the vertex representing a city if there is an interstate highway that circles this city?

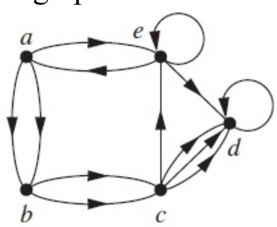
TABLE 1 Graph Terminology.

Type	*Edges*	*Multiple Edges Allowed?*	*Loops Allowed?*
Simple graph	Undirected	No	No
Multigraph	Undirected	Yes	No
Pseudograph	Undirected	Yes	Yes
Simple directed graph	Directed	No	No
Directed multigraph	Directed	Yes	Yes
Mixed graph	Directed and undirected	Yes	Yes

Solution: -

a) A simple as well as multi graph without no direction can be used.
b) A simple undirected graph is used.
c) A graph with self-loops can be used.

Q.2 Determine whether the graph shown has directed or undirected edges, whether it has multiple edges, and whether it has one or more loops. Use your answers to determine the type of graph



Solution: This is a directed graph with multiple directed edges and directed self-loops, there are parallel edges for example edge from a to e and e- a. This is a directed multigraph.

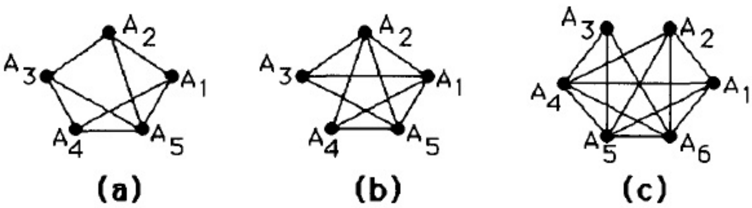
Q.3 The **intersection graph** of a collection of sets $A1, A2, \dots, An$ is the graph that has a vertex for each of these sets and has an edge connecting the vertices representing two sets if these sets have a nonempty intersection. Construct the intersection graph of these collections of sets.

a) $A1 = \{0, 2, 4, 6, 8\}$, $A2 = \{0, 1, 2, 3, 4\}$,
$A3 = \{1, 3, 5, 7, 9\}$, $A4 = \{5, 6, 7, 8, 9\}$,
$A5 = \{0, 1, 8, 9\}$

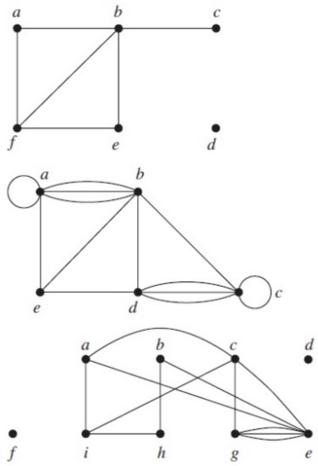
b) $A1 = \{\dots, -4, -3, -2, -1, 0\}$,
$A2 = \{\dots, -2, -1, 0, 1, 2, \dots\}$,
$A3 = \{\dots, -6, -4, -2, 0, 2, 4, 6, \dots\}$,
$A4 = \{\dots, -5, -3, -1, 1, 3, 5, \dots\}$,
$A5 = \{\dots, -6, -3, 0, 3, 6, \dots\}$

c) $A1 = \{x \mid x < 0\}$,
$A2 = \{x \mid -1 < x < 0\}$,
$A3 = \{x \mid 0 < x < 1\}$,
$A4 = \{x \mid -1 < x < 1\}$,
$A5 = \{x \mid x > -1\}$,
$A6 = \mathbf{R}$

Solution: In each case we draw a picture of the graph in question. All are simple graphs. An edge is drawn between two vertices if the sets for the two vertices have at least one element in common. For example, in part (a) there is an edge between vertices A1 and A2 because there is at least one element common to A1 and A2 (in fact there are three such elements). There is no edge between A1 and *A3* since $A1 \cap A3 = \emptyset$



Q.4 find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph. Identify all isolated and pendant vertices. Also find the sum of degrees of all vertices and verify that it is twice the number of edges.



Solution:

	Graph-1	Graph-2	Graph-3
Number of Vertices	6	5	9
Number of Edges	6	13	12
Degree of Vertices	a(2), b(4), c(1), d(0) e(2), f(3)	a(6), b(6), c(6), d(5), e(3)	a(3), b(2), c(4), d(0), e(6), g(4), h(2), i(3), f(0)
Isolated vertices	d	-	d, f
Pendant vertices	c	-	-
Sum of the Degrees	12 = 2(no. of edges)	26 = 2(no. of edges)	24 = 2(no. of edges)