HOMEWORK-13

Q.2 which of these relations on the set of all people are equivalence relations? Determine the properties of an equivalence relation that the others lack.

a) {(a, b) | a and b are the same age}

b) {(a, b) | a and b have the same parents}

c) {(a, b) | a and b share a common parent}

d) {(a, b) | a and b have met}

e) {(a, b) | a and b speak a common language}

Solutions:

a) a has its same age, so the relation R is Reflexive. If a has the same age of b then b has same age of a thus, R is Symmetric. If a has the same age of b and b has the same age of c then a has the same age of c thus, R is Transitive. Hence Equivalence relation.
b) Obviously, a has its same parents, so R is Reflexive. If a has the same parents of b then b has the same parents of a, thus R is symmetric. If a and b have the same parents, and b has the same of c then a has the same parents of c Thus R is Transitive. Hence Equivalence relation.
c) Is not transitive. Because a can have the same father b, and b have same mother of c but a and c may not have neither the same mother nor the same father.
d) Is not transitive. Because if a have met b and b have met c, it's doesn't mean that a have met c necessarily.
e) Is not transitive. Because if a and b speak a common language say English and b and c speak another common language say Marathi: it doesn't mean that a and c speak a common language.

Q.9 Suppose that A is a nonempty set, and f is a function that has A as its domain. Let R be the relation on A consisting of all ordered pairs (x, y) such that f (x) = f (y).

a) Show that R is an equivalence relation on A.
b) What are the equivalence classes of R.

Solution:

a) This relation is reflexive because it is obvious that f(x) = f(x) for all x ∈ A.
This relation is symmetric because if f(x) = f(y), then it follows that f(y) = f(x).
This relation is transitive because if we have f(x) = f(y) and f(y) = f(z), then it follows that f(x) = f(z). Hence R is an equivalence relation on A.
b) Let x ∈ A. The equivalence relation of x then contains all other values in the set A that have the same image as x.

$$[x]_R = \{y \epsilon A /\ \ y\ has\ the\ same\ image\ as\ x\} \ = \{\ y \epsilon A / f(x) = f(y)\}$$

Q.41 which of these collections of subsets are partitions of {1, 2, 3, 4, 5, 6}?

a) {1, 2}, {2, 3, 4}, {4, 5, 6}

b) {1}, {2, 3, 6}, {4}, {5}

c) {2, 4, 6}, {1, 3, 5}

d) {1, 4, 5}, {2, 6}

Solution:

a) {1, 2}, {2, 3, 4}, {4, 5, 6} Not a partition because these sets are not pairwise disjoint. The element 2 and 4 appear in two of these sets.

b) {1}, {2, 3, 6}, {4}, {5} this is a partition.

c) {2, 4, 6}, {1, 3, 5} This is a partition.

d) {1, 4, 5}, {2, 6}, Not a partition because it is missing the element 3 in any of the sets.

Q.1 which of these relations on {0, 1, 2, 3} are partial orderings? Determine the properties of a partial ordering that the others lack.

a) {(0, 0), (1, 1), (2, 2), (3, 3)}

b) {(0, 0), (1, 1), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)}

c) {(0, 0), (1, 1), (1, 2), (2, 2), (3, 3)}

d) {(0, 0), (1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)}

e) {(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0),(2, 2), (3, 3)}

Solution:

a) R= {(0,0), (1,1), (2,2), (3,3)}
R is Reflexive, because (a, a) ϵ R for every element a ϵ A.
R is antisymmetric, because if (a, b) ϵ R, then a=b (in this case)
R is transitive, because if (a, b) ϵ R and (b, c) ϵ R then a=b=c and (a, c) = (a, a) ϵR

R is **a partial Ordering**, because R is reflexive, antisymmetric and transitive.

b) R= {(0, 0), (1, 1), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)}
R is Reflexive, because (a, a) ϵ R for every element a ϵ A.
R is not antisymmetric, because (2, 3) ϵ R and (3, 2) ϵ R while 2$\neq$3
R is not transitive, because (3, 2) ϵ R and (2, 0) ϵ R while (3, 0)$\notin$ R.

R is **not a partial Ordering**, because R is not antisymmetric and transitive.

c) R={(0, 0), (1, 1), (1, 2), (2, 2), (3, 3)}
R is Reflexive, because (a, a) ϵ R for every element a ϵ A.
R is antisymmetric, because if (a, b) ϵ R and (b, a) ϵ R then a=b
(Since (1, 2) ϵ R and (2, 1) $\notin$ R)
R is **not a partial Ordering**, because R is not antisymmetric and transitive.

d) {(0, 0), (1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)}
R is Reflexive, because (a, a) ϵ R for every element a ϵ A.
R is antisymmetric, because if (a, b) ϵ R and (b, a) ϵ R then a=b
(Since (1, 2) ϵ R and (2, 1) $\notin$ R)
R is **not a partial Ordering**, because R is not antisymmetric and transitive.

e) R= {(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0),(2, 2), (3, 3)}

R is Reflexive, because $(a, a) \in R$ for every element $a \in A$.

R is not antisymmetric, because $(1, 0) \in R$ and $(0, 1) \in R$ while $0 \neq 1$

R is not transitive, because $(2, 0) \in R$ and $(0, 1) \in R$ while $(2, 1) \notin R$.

R is **not a partial Ordering**, because R is not antisymmetric and transitive.

Q.2 which of these relations on {0, 1, 2, 3} are partial orderings? Determine the properties of a partial ordering that the others lack.

a) {(0, 0), (2, 2), (3, 3)}

b) {(0, 0), (1, 1), (2, 0), (2, 2), (2, 3), (3, 3)}

c) {(0, 0), (1, 1), (1, 2), (2, 2), (3, 1), (3, 3)}

d) {(0, 0), (1, 1), (1, 2), (1, 3), (2, 0), (2, 2), (2, 3),(3, 0), (3, 3)}

e) {(0, 0), (0, 1), (0, 2), (0, 3), (1, 0), (1, 1), (1, 2),(1, 3), (2, 0), (2, 2), (3, 3)}

Solution:

A= {0, 1, 2, 3}

a) R= {(0, 0), (2, 2), (3, 3)}

R is not reflexive, because $(1, 1) \notin R$ while $1 \in A$.

R is antisymmetric, because if $(a, b) \in R$ then a=b (in this case)

R is transitive, because $(a, b) \in R$ and $(b, c) \in R$ then a=b=c and $(a, c) = (a, a) \in R$

R is **not a partial Ordering**, because R is not reflexive.

b) R={(0, 0), (1, 1), (2, 0), (2, 2), (2, 3), (3, 3)}

R is Reflexive, because $(a, a) \in R$ for every element $a \in A$.

R is antisymmetric, because if $(a, b) \in R$ and $(b, a) \in R$ then a=b

(Since $(2, 0) \in R$ and $(0, 2) \notin R$; and $(2, 3) \in R$ and $(3, 2) \notin R$

R is transitive, because if $(a, b) \in R$ and $(b, c) \in R$ then a=b or b=c

(Since there are only two elements not of the form (a, a) and that pair does not satisfy $(a, b) \in R$ and $(b, a) \in R$), which implies $(a, c) = (b, c) \in R$ or $(a, c) = (a, b) \in R$

R is a **partial Ordering** because R is reflexive, antisymmetric and transitive.

c) R= {(0, 0), (1, 1), (1, 2), (2, 2), (3, 1), (3, 3)}

R is reflexive, because $(a, a) \in R$ for every element $a \in A$.

R is antisymmetric, because if $(a, b) \in R$ and $(b, a) \in R$, then a=b.

(Since $(1, 2) \in R$ and $(2, 1) \notin R$; $(3, 1) \in R$ and $(1, 3) \notin R$

R is not transitive, because $(3, 1) \in R$ and $(1, 2) \in R$ while $(3, 2) \notin R$

R is **not a partial Ordering**, because R is not transitive.

d) R= {(0, 0), (1, 1), (1, 2), (1, 3), (2, 0), (2, 2), (2, 3),(3, 0), (3, 3)}

R is reflexive, because $(a, a) \in R$ for every element $a \in A$.

R is antisymmetric, because if $(a, b) \in R$ and $(b, a) \in R$, then a=b.

(Since $(1, 2) \in R$ and $(2, 1) \notin R$; similar for all other elements not of the form (a, a).

R is not transitive, because $(1, 2) \in$ R and $(2, 0) \in$ R while $(1, 0) \notin$ R.
R is **not a partial ordering**, because R is not transitive.

e) R= {(0, 0), (0, 1), (0, 2), (0, 3), (1, 0), (1, 1), (1, 2),(1, 3), (2, 0), (2, 2), (3, 3)}
R is Reflexive, because $(a, a) \in$ R for every element $a \in A$.
R is not antisymmetric, because $(1, 0) \in$ R and $(0, 1) \in$ R while $0 \neq 1$
R is not transitive, because $(2, 0) \in$ R and $(0, 3) \in$ R while $(2, 3) \notin$ R.

R is **not a partial Ordering**, because R is not antisymmetric and transitive.

Q.3 Is (S,R) a poset if S is the set of all people in the world and $(a, b) \in R$, where a and b are people, if

a) a is taller than b?

b) a is not taller than b?

c) a = b or a is an ancestor of b?

d) a and b have a common friend?

Solution: S is the set of all people in the world

a) R={(a, b) / a is taller than b}
R is not reflexive, because an individual is not taller than themselves.
R is antisymmetric, because $(a, b) \in$ R and $(b, a) \in$ R cannot both be true at the same time (if a is taller than b, then b cannot be taller than a).
R is transitive, because if $(a, b) \in$ R and $(b, c) \in$ R then a is taller than b and b is taller than c which implies that a is taller than c and thus $(a, c) \in$ R.
(R,S) is **not a poset**, because R is not reflexive.

b) R={(a, b) / a is not taller than b }

R is reflexive, because an individual is not taller than themselves.
R is not antisymmetric, because $(a, b) \in$ R and $(b, a) \in$ R implies a is not taller than b and b is not taller than a. both statements can only be true if a and b have same height. However this does not necessarily imply a=b (that is, a and b are not necessarily the same person)
R is transitive, because if $(a, b) \in$ R and $(b, c) \in$ R then a is not taller than b and b is not taller than c which implies that a is not taller than c and thus $(a, c) \in$ R.
(R, S) is **not a poset**, because R is not symmetric.

c) R={(a, b) / a = b or a is an ancestor of b }
R is reflexive, because all elements with a=b are included in the relation R.

R is antisymmetric, because (a, b) ϵ R and (b, a) ϵ R implies (a is an ancestor of b or a=b) and (b is an ancestor of a or a=b). a cannot be an ancestor of b when b is an ancestor of a thus the statements can only be true if a=b.
R is transitive, because if (a, b) ϵ R and (b, c) ϵ R then (a is an ancestor of b or a=b) and (b is an ancestor of c or b=c) which implies that (a is an ancestor of c or c=a) and thus (a, c) ϵ R.
(R, S) is **a poset,** because R is reflexive, antisymmetric and transitive.

d) R={(a, b) / a and b have a common friend }
R is not reflexive, because an individual has no common friend with himself when the individual has no friends.
R is not antisymmetric, because (a, b) ϵ R and (b, a) ϵ R implies a and b have a common friend, while b and a also have a common friend however a and b are not necessarily the same person.
R is not transitive, because if (a, b) ϵ R and (b, c) ϵ R then a and b have common friend, while b and c have common friend, then a and c do not necessarily have a common friend when the two common friends were different.

(R, S) is **not a poset**, because R is not reflexive, symmetric, and transitive.

Q.4 Is (S,R) a poset if S is the set of all people in the world and (a, b) ∈ R, where a and b are people, if
a) a is no shorter than b?
b) a weighs more than b?
c) a = b or a is a descendant of b?
d) a and b do not have a common friend?

Solution: S is the set of all people in the world
a) R= {(a, b) / a is no shorter than b}
If there are two people a and b with the same height then neither is shorter than the other person, thus (a, b), (b, a) ϵ R even though $a \neq b$.
Thus R is not antisymmetric and
(S, R) is **not a poset.**

b) R= {(a, b) / a weighs more than b}
If a weighs more than b which implies that (a, b) ϵ R then clearly (a, a) $\notin$ R and R is not reflexive,
Hence (S, R) is **not a poset.**

c) R= {(a, b) / a = b or a is a descendant of b }
Clearly, (a, a) ϵ R by definition.
If (a, b),(b, a) ϵ R which $\Longrightarrow$a=b (or both is the descendant of the other, which is not possible).
Again, (a, b), (b, c) ϵ R which $\Longrightarrow$either a=c or a is a descendant of c $\Longrightarrow$ (a, c) ϵ R. Thus R is reflexive, antisymmetric and transitive.
Hence (S, R) is **a poset.**

d) R= {(a, b) / a and b do not have a common friend }
If the person a has a friend then (a, a) $\notin$ R, thus R is not reflexive,
Hence (S, R) is **not a poset.**