Homework -8

Q1 JOBCO produces two products on two machines. A unit of product 1 requires 2hours on machine 1 and 1 hour on machine 2. For product 2, a unit requires 1 hour on machine 1 and 3hours on machine 2. The revenues per unit of products 1 and 2 are \$30 and \$20 respectively. The total daily processing time available for each machine is 8 hours.

i) Formulate the above as a LPP.

ii) Solve the LPP using the simplex method and observe as to what happens in each of the tables, with the graphical method.

iii) Get the shadow price, minimum capacity and maximum capacity of ma- chines 1 and 2, using TORA and by plotting the lines physically.

Solution:

i) LPP : Maximize $z = 30x_1 + 20x_2$

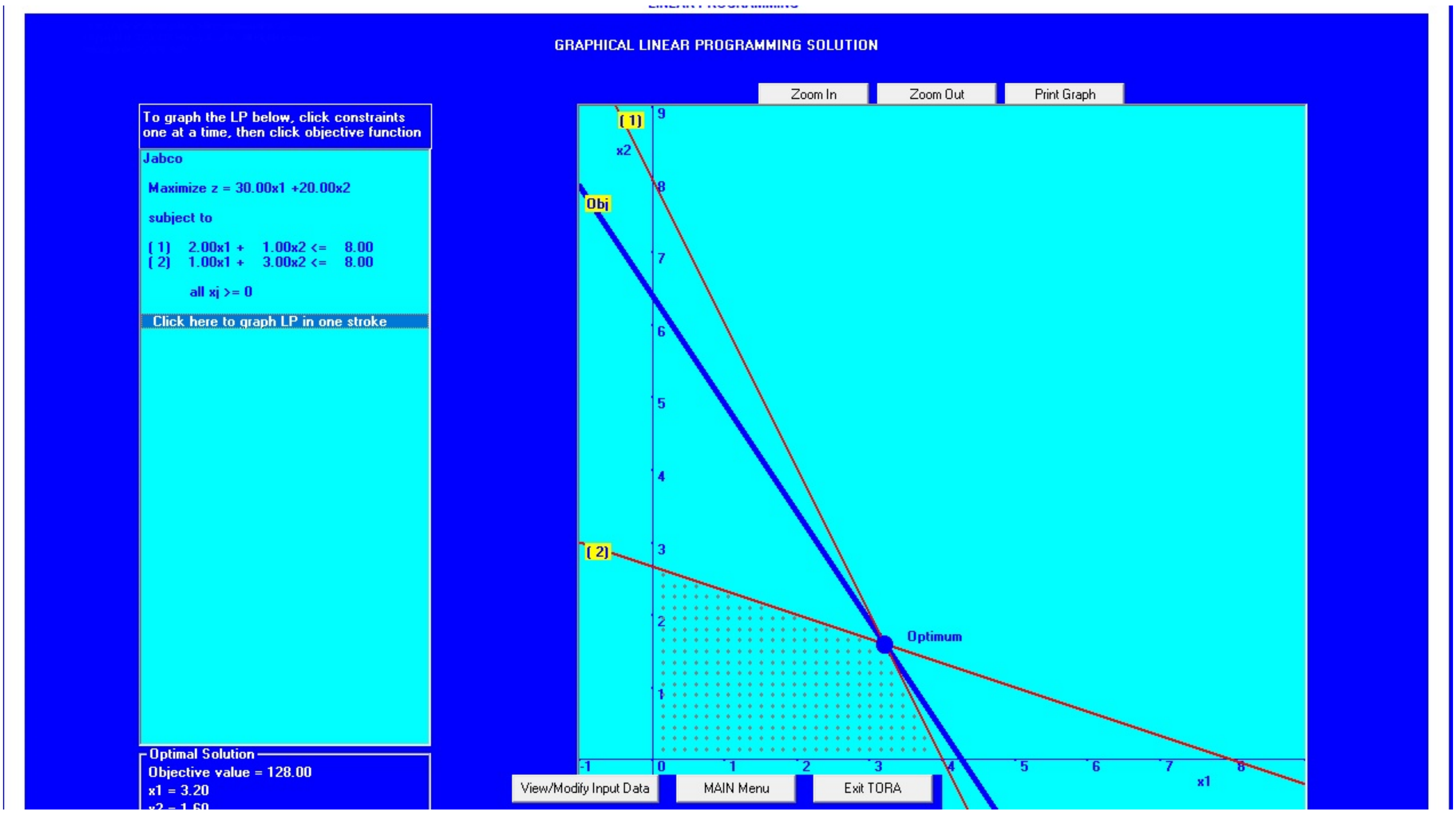
subject to

$2x_1 + x_2 \leq 8$ (machine 1)

$x_1 + 3x_2 \leq 8$ (machine 2)

$x_1 , x_2 \geq 0$ (non-negativity)

ii) Graphical solution by using TORA



Simplex method:

After introducing slack variables

$$\text{Max } Z = 30\ x1 + 20\ x2 + 0\ S1 + 0\ S2$$

subject to

$$2\ x1 + x2 + S1 = 8$$

$$x1 + 3\ x2 + S2 = 8$$

and $x1,x2,S1,S2 \geq 0$

Iteration-1		*Cj*	30	20	0	0	
B	***CB***	***XB***	***x*1**	***x*2**	***S*1**	***S*2**	**Min Ratio *XB/x*1**
***S*1**	0	8	**(2)**	1	1	0	8/2=4→
*S*2	0	8	1	3	0	1	8/1=8
***Z*=0**		***Zj***	**0**	**0**	**0**	**0**	
		Cj - Zj	30↑	20	0	0	

Table 1: This gives a solution at point origin (0,0) in the graphical method.

By $R1$(new)=$R1$(old)÷2 and $R2$(new)=$R2$(old)- $R1$(new)

Iteration-2		*Cj*	30	20	0	0	
B	***CB***	***XB***	***x*1**	***x*2**	***S*1**	***S*2**	**MinRatio *XB/x*2**
*x*1	30	4	1	1/2	1/2	0	4/(1/2)=8
***S*2**	0	4	0	**(5/2)**	-1/2	1	4/(5/2)=8/5=1.6→
***Z*=120**		***Zj***	**30**	**15**	**15**	**0**	
		Cj - Zj	0	5↑	-15	0	

Table 2: This gives a solution at corner point (4,0) in the graphical method.

by $R2$(new)=$R2$(old) ×2/5 and $R1$(new)=$R1$(old) – (1/2)$R2$(new)

Iteration-3		*Cj*	30	20	0	0	
B	***CB***	***XB***	***x*1**	***x*2**	***S*1**	***S*2**	**MinRatio**
*x*1	30	16/5	1	0	3/5	-1/5	
*x*2	20	8/5	0	1	-1/5	2/5	
***Z*=128**		***Zj***	**30**	**20**	**14**	**2**	
		Cj - Zj	0	0	-14	-2	

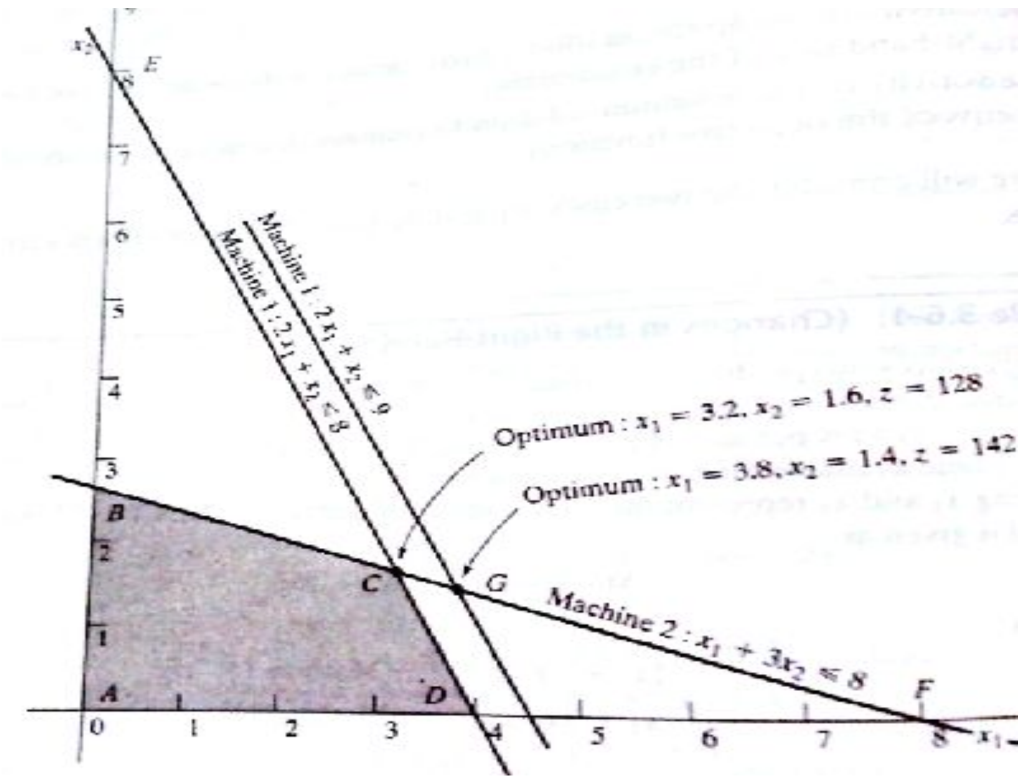
Table 3: This gives a solution at corner point (16/5, 8/5) in the graphical method. Thus in simplex method we are moving from one corner point to next corner point.

Since all $Cj - Zj \leq 0$

Hence, optimal solution is arrived with value of variables as : $x1=16/5$, $x2=8/5$

Max Z = 128 $

iii)



Machine 1 :	Machine 2 :
Change in constraint : $2x_1 + x_2 \leq 9$	Change in constraint : $x_1 + 3x_2 \leq 9$
New optimum solution :	New optimum solution :
$x_1 = 3.8$, $x_2 = 1.4$; $z = 142$	$x_1 = 2.7$, $x_2 = 2.1$; $z = 140$
Dual price = (142-128)/(9-8)= \$14 / hr.	Dual price = (142-140)/(9-8)= \$2 / hr.
Min. capacity at B(0,2.67)=2X0+2.67=2.67hr	Min. capacity at B(4,0)=4+3X0=4hr
Max. capacity at F(8,0)=2X8+0=16hr	Max. capacity at F(0,8)=0+3X8=24hr
.	

Q2 Show that the following objective function can be presented in equation form.

Minimize $Z = \max\{|x_1 - x_2 + 3x_3|, |-x_1 + 3x_2 - x_3|\}; x_1, x_2, x_3 \geq 0$ Hints:

i) **$|a| \leq b$ is equivalent to $a \leq b$ and $a \geq -b$**

ii) **Set $y = \max\{|x_1 - x_2 + 3x_3|, |-x_1 + 3x_2 - x_3|\}$ and observe that in such a case $y \geq |x_1 - x_2 + 3x_3|$ and $y \geq |-x_1 + 3x_2 - x_3|$ and use i).**

Solution: Let $y = \max\{|x_1 - x_2 + 3x_3|, |-x_1 + 3x_2 - x_3|\}$

Therefore Min $z = y$ and either $|x_1 - x_2 + 3x_3| \leq y$ ----(1)

or $|-x_1 + 3x_2 - x_3| \leq y$ ----(2)

by (1) $x_1 - x_2 + 3x_3 \leq y$ and $-x_1 + x_2 - 3x_3 \leq y$

by (2) $-x_1 + 3x_2 - x_3 \leq y$ and $x_1 - 3x_2 + x_3 \leq y$

Thus the problem in equation form is

Min $z = y$

Subject to

$$x_1 - x_2 + 3x_3 \leq y$$
$$-x_1 + x_2 - 3x_3 \leq y$$
$$-x_1 + 3x_2 - x_3 \leq y \text{ and}$$
$$x_1 - 3x_2 + x_3 \leq y$$
$$x_1, x_2, x_3, y \geq 0$$

Q3 Solve the following LPP

Maximize $z = 15x_1 + 6x_2 + 9x_3 + 2x_4$

Subject to

$$2x_1 + x_2 + 5x_3 + 0.6x_4 \leq 10$$
$$3x_1 + x_2 + 3x_3 + 0.25x_4 \leq 12$$
$$7x_1 + x_4 \leq 35$$
$$x_j \geq 0, j = 1, 2, 3, 4.$$

Using TORA and by hand and verify the solution.

Solution : Using TORA:

Iteration 1	1	2	3	4				
Basic	x1	x2	x3	x4	sx5	sx6	sx7	Solution
z (max)	-15.00	-6.00	-9.00	-2.00	0.00	0.00	0.00	0.00
sx5	2.00	1.00	5.00	0.60	1.00	0.00	0.00	10.00
sx6	3.00	1.00	3.00	0.25	0.00	1.00	0.00	12.00
sx7	7.00	0.00	0.00	1.00	0.00	0.00	1.00	35.00
Lower Bound	0.00	0.00	0.00	0.00				
Upper Bound	infinity	infinity	infinity	infinity				
Unrestr'd (y/n)?	n	n	n	n				
Iteration 2	1	2	3	4				
Basic	x1	x2	x3	x4	sx5	sx6	sx7	Solution
z (max)	0.00	-1.00	6.00	-0.75	0.00	5.00	0.00	60.00
sx5	0.00	0.33	3.00	0.43	1.00	-0.67	0.00	2.00
x1	1.00	0.33	1.00	0.08	0.00	0.33	0.00	4.00
sx7	0.00	-2.33	-7.00	0.42	0.00	-2.33	1.00	7.00
Lower Bound	0.00	0.00	0.00	0.00				
Upper Bound	infinity	infinity	infinity	infinity				
Unrestr'd (y/n)?	n	n	n	n				
Iteration 3	1	2	3	4				
Basic	x1	x2	x3	x4	sx5	sx6	sx7	Solution
z (max)	0.00	0.00	15.00	0.55	3.00	3.00	0.00	66.00
x2	0.00	1.00	9.00	1.30	3.00	-2.00	0.00	6.00
x1	1.00	0.00	-2.00	-0.35	-1.00	1.00	0.00	2.00
sx7	0.00	0.00	14.00	3.45	7.00	-7.00	1.00	21.00
Lower Bound	0.00	0.00	0.00	0.00				
Upper Bound	infinity	infinity	infinity	infinity				
Unrestr'd (y/n)?	n	n	n	n				

Thus using TORA we have Max z = 66 at $x_1 = 2$, $x_2 = 6$, $x_3 = 0$, $x_4 = 0$

By Hand solution is same as below

After introducing slack variables

Max $Z = 15x1 + 6x2 + 9x3 + 2x4 + 0S1 + 0S2 + 0S3$

subject to

$$\begin{aligned} 2x1 + x2 + 5x3 + 0.6\ x4 + S1 \quad &= 10 \\ 3x1 + x2 + 3x3 + 0.25x4 \quad + S2 \quad &= 12 \\ 7x1 \quad + \quad x4 \quad + S3 &= 35 \end{aligned}$$

and $x1, x2, x3, x4, S1, S2, S3 \geq 0$

Iteration-1		*Cj*	15	6	9	2	0	0	0	
B	***CB***	***XB***	***x1***	***x2***	***x3***	***x4***	***S1***	***S2***	***S3***	**MinRatio** ***XB/x1***
S1	0	10	2	1	5	0.6	1	0	0	10/2=5
S2	0	12	**(3)**	1	3	0.25	0	1	0	12/3=4→
S3	0	35	7	0	0	1	0	0	1	35/7=5
***Z*=0**		***Zj***	**0**	**0**	**0**	**0**	**0**	**0**	**0**	
		Cj - Zj	15↑	6	9	2	0	0	0	

By

$R2$(new)=$R2$(old)÷3,

$R1$(new)=$R1$(old) - 2$R2$(new)

$R3$(new)=$R3$(old) - 7$R2$(new)

Iteration-2		*Cj*	15	6	9	2	0	0	0	
B	***CB***	***XB***	***x*1**	***x*2**	***x*3**	***x*4**	***S*1**	***S*2**	***S*3**	**MinRatio *XB/x*2**
***S*1**	0	2	0	**(1/3)**	3	0.4333	1	-2/3	0	2/(1/3)=6→
*x*1	15	4	1	1/3	1	0.0833	0	1/3	0	4/(1/3)=12
*S*3	0	7	0	-7/3	-7	0.4167	0	-7/3	1	---
***Z*=60**		***Zj***	**15**	**5**	**15**	**1.25**	**0**	**5**	**0**	
		Cj - Zj	0	1↑	-6	0.75	0	-5	0	

By
$R1$(new)=$R1$(old) ×3

$R2$(new)=$R2$(old) – (1/3) $R1$(new)

$R3$(new)=$R3$(old) +(7/3)$R1$(new)

Iteration-3		*Cj*	15	6	9	2	0	0	0	
B	***CB***	***XB***	***x*1**	***x*2**	***x*3**	***x*4**	***S*1**	***S*2**	***S*3**	**MinRatio**
*x*2	6	6	0	1	9	1.3	3	-2	0	
*x*1	15	2	1	0	-2	-0.35	-1	1	0	
*S*3	0	21	0	0	14	3.45	7	-7	1	
***Z*=66**		***Zj***	**15**	**6**	**24**	**2.55**	**3**	**3**	**0**	
		Cj - Zj	0	0	-15	-0.55	-3	-3	0	

Since all $Cj - Zj \le 0$

Hence, optimal solution is arrived with value of variables as : $x1$=2, $x2$=6, $x3$=0, $x4$=0

Max Z=66