

# Data Mining

## Study Assignment Set #2

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**Reference Books:**

Introduction to Data Mining by Tan P. N., Steinbach M and Kumar V.  
Pearson Education, 2006

Data Mining: Concepts and Techniques, Second Edition by Jiawei Han and Micheline Kamber  
Morgan Kaufmann Publishers, 2006

### Topic: Classification of Data, Decision Trees, Information Gain

Classification of Data, Decision Trees	Question 1
<p>Learning objectives:</p> <ul style="list-style-type: none"><li>- Basics of statistical learning with Decision Trees.</li><li>- Decision Tree algorithm, and attribute selection methods.</li></ul> <p><b>Basics of statistical learning learning with Decision Trees:</b></p> <pre>graph LR; TD[(Training Data)] -- Induction --&gt; CA[Classification Algorithms]; CA --&gt; CM[(Classifier Model)]; CM -- Deduction / Assigns the class label --&gt; TUD[(Target / Unseen Data)]; CM --&gt; DT[Decision Tree]; DT --&gt; AK[attribute-K]; DT --&gt; AY[attribute-Y]; AK --&gt; CA_A[class A]; AY --&gt; CB[class B];</pre> <p>Below is the generic algorithm to generate a decision tree. It results in a decision tree with nodes split on a certain criterion.</p> <pre>Node Generate_decision_tree (Dj, attribute_list) {   1. Create a Node N   2. if tuple in D are all of the same class, C, then   3.   return N as a leaf node labelled with the class C;   4. if attribute_list is empty then     // majority voting   5.   return N as a leaf node labelled with the majority class in D;   6. apply <b>Attribute_selection_method</b> (D, attribute_list) to find the best     splitting_criterion;   7. label node N with splitting_criterion;   8. if splitting_attribute is discrete-valued and     multiway splits allowed then // not restricted to binary trees     // remove splitting_attribute</pre>	

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9.  attribute_list <- attribute_list -  splitting_attribute;
10. for each outcome j of splitting_criterion
    // partition the tuples and grow subtrees for each partition
11.  let Dj be the set of data tuples in D satisfying outcome j; // a
    partition
12.  if Dj is empty then
13.    attach a leaf labeled with the majority class in D to node N;
14.  else attach the node returned by Generate_decision_tree (Dj,
    attribute_list) to node N;
    endfor
15.  return N;
}
```

Questions:

Q	What are popular attribute selection methods for the above decision tree generation algorithm?
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A	Information Gain	B	Gain Ratio
C	Gini Index	D	All of the above.

Answer	D
Remarks	

Classification of Data

Question 2

**Learning objectives:**

- Basics of statistical learning with Decision Trees.
- Generating Decision Tree (Refer to Question #1)
- Understanding the concept of Entropy.

**Basics of statistical learning learning with Decision Trees:**

Training Data

Induction

Classification Algorithms

Classifier (Model)

Deduction

Assigns the class label

Target / Unseen Data

Decision Tree

attribute-K

attribute-Y

class A

class B

Use of Information Gain for attribute selection.

- Attribute with highest information gain is chosen as the splitting attribute for a node N
- This attribute minimizes the information needed to classify the tuples in the resulting partitions and reflects the least randomness or “impurity” in these partitions.

**Entropy:**

The expected information needed to classify a tuple in D is given by

$$Info(D) = - \sum_{i=1}^m p_i \log_2(p_i)$$

D: Training Data

m: Distinct values of the class label attribute.

p<sub>i</sub>: non-zero probability that an **attribute tuple** in D belongs to a **class Y<sub>i</sub>** and is estimated by |Y<sub>i</sub>, D| / |D|

**\*\*** P(Y<sub>i</sub> | D) = P(Y<sub>i</sub>, D) / P(D) = |Y<sub>i</sub>, D| / |D| **\*\***

[Some use C<sub>i</sub> for class.]

**Example:**

An online computer store uses a Decision Tree classifier with ‘Info Gain’ as a method of attribute selection method.

Let X is a set of attributes of the registered user.

X = {id, age, income, student, credit\_rating}

Let Y is the class variable

Y = buys\_computer = {yes, no}

The **training dataset**, D, is as below.

id	age	income	student	credit_rating	buys_computer
1	youth	high	no	fair	no
2	youth	high	no	excellent	no
3	middle_aged	high	no	fair	yes
4	senior	medium	no	fair	yes
5	senior	low	yes	fair	yes
6	senior	low	yes	excellent	no
7	middle_aged	low	yes	excellent	yes
8	youth	medium	no	fair	no
9	youth	low	yes	fair	yes
10	senior	medium	yes	fair	yes
11	youth	medium	yes	excellent	yes
12	middle_aged	medium	no	excellent	yes
13	middle_aged	high	yes	fair	yes
14	senior	medium	no	excellent	no

**Questions:**

A	Find Info (D) (Entropy of the D) for the class label attribute, Y = buys_computer = {yes, no}.
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Answers:

A	<p>D has a total 14 tuples (training data).</p> <p>m: Distinct values of the class label attribute = 2. buys_computer has two distinct value {yes, no}.</p> <p>p( buys_computer = yes   D) = 9/14</p> <p>p( buys_computer = no   D) = 5/14</p> $Info(D) = - \sum_{i=1}^m p_i \log_2(p_i)$ <p>Info (D) = -9/14 log<sub>2</sub> (9/14) - 5/14 log<sub>2</sub> (5/14)</p> <p>= <b>0.940 bits.</b></p> <p>It is also known as Entropy of D.</p>
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MCQ:

A		B	
C		D	

Answer	
Remarks	

Classification of Data

Question 3

**Learning objectives:**

- Basics of statistical learning with Decision Trees.
- Generating Decision Tree (Refer to Question #1)
- Understanding the concept of Entropy. (Refer to Question #2)
- Understanding attribute selection ('Info Gain').

**Basics of statistical learning learning with Decision Trees:**

Training Data

Induction

Classification Algorithms

Deduction

Classifier (Model)

Assigns the class label

Target / Unseen Data

Decision Tree

attribute-K

attribute-Y

class A

class B

Use of Information Gain for attribute selection.

- Attribute with highest information gain is chosen as the splitting attribute for a node N
- This attribute minimizes the information needed to classify the tuples in the resulting partitions and reflects the least randomness or “impurity” in these partitions.

**Entropy:**

The expected information needed to classify a tuple in D is given by

$$Info(D) = - \sum_{i=1}^m p_i \log_2(p_i)$$

D: Training Data  
m: Distinct values of the class label attribute.  
p<sub>i</sub>: non-zero probability that an **attribute tuple** in D belongs to a **class Y<sub>i</sub>** and is estimated by |Y<sub>i</sub>, D| / |D|

**\*\* P(Y<sub>i</sub> | D) = P(Y<sub>i</sub>, D) / P(D) = |Y<sub>i</sub>, D| / |D| \*\***

[Some use C<sub>i</sub> for class.]

Usually, Info (D) is the entropy of the Root node. Our objective is to find an appropriate ‘attribute’ to perform a split on D.

How much more information would we still need (after partitioning) to arrive at an exact classification? Measure Info<sub>A</sub>(D) for attribute A as below.

$$Info_A(D) = \sum_{j=1}^v \frac{|D_j|}{|D|} \times Info(D_j)$$

Info<sub>A</sub>(D) is the expected information required to classify a tuple from D based on the partition by the attribute A. The smaller the information (still) required, the greater the purity of the partition.

**Gain (A) = Info (D) - Info<sub>A</sub>(D)**  
Gain (A) is an indication of how much would be gained by branching on A (attribute A).

**\*\* Branch on the attribute that gives highest gain \*\***

**Example:**  
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Let Y is the class variable  
Y = buys\_computer = {yes, no}

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6	senior	low	yes	excellent	no
7	middle_aged	low	yes	excellent	yes

8	youth	medium	no	fair	no
9	youth	low	yes	fair	yes
10	senior	medium	yes	fair	yes
11	youth	medium	yes	excellent	yes
12	middle_aged	medium	no	excellent	yes
13	middle_aged	high	yes	fair	yes
14	senior	medium	no	excellent	no

Questions:

A	Find Info (D) (Entropy of the D) for the class label attribute, Y = buys_computer = {yes, no}.
B	Find Info <sub>age</sub> (D), Gain (age)
C	Find Info <sub>income</sub> (D), Gain (income)
D	Find Info <sub>student</sub> (D), Gain (student)
E	Find Info <sub>credit_rating</sub> (D), Gain (credit_rating)

Answers:

A	<p>D has a total 14 tuples (training data).</p> <p>m: Distinct values of the class label attribute = 2. buys_computer has two distinct value {yes, no}.</p> <p>p( buys_computer = yes   D) = 9/14</p> <p>p( buys_computer = no   D) = 5/14</p> $Info(D) = - \sum_{i=1}^m p_i \log_2(p_i)$ <p>Info (D) = -9/14 log<sub>2</sub> (9/14) - 5/14 log<sub>2</sub> (5/14)</p> <p>= <b>0.940 bits.</b></p> <p>It is also known as Entropy of D.</p>
B	<p>Find Info<sub>age</sub>(D), Gain (age)</p> $Info_A(D) = \sum_{j=1}^v \frac{ D_j }{ D } \times Info(D_j)$ <p>Attribute age = {youth, middle_aged, senior}. So, if we split D on <b>age</b>, then it will partition D into 3 partitions. Therefore v = 3.</p>

	$Info_{age} = \frac{ D_{youth} }{ D } \times Info(D_{youth}) + \frac{ D_{middle\_aged} }{ D } \times Info(D_{middle\_aged}) + \frac{ D_{senior} }{ D } \times Info(D_{senior})$ <p><b>Calculating Info(<math>D_{youth}</math>) ----- (1)</b>  <math>D_{youth}</math> has a total 5 tuples (training data).  m: Distinct values of the class label attribute = 2. buys_computer has two distinct value {yes, no}.  <math>p(\text{buys\_computer} = \text{yes} \mid D_{youth}) = 2/5</math>  <math>p(\text{buys\_computer} = \text{no} \mid D_{youth}) = 3/5</math>  Info (<math>D_{youth}</math>) = <math>-2/5 \log_2 (2/5) - 3/5 \log_2 (3/5)</math> bits</p> <p><b>Calculating Info(<math>D_{middle\_aged}</math>) ----- (2)</b>  <math>D_{middle\_aged}</math> has a total 4 tuples (training data).  m: Distinct values of the class label attribute = 2. buys_computer has two distinct value {yes, no}.  <math>p(\text{buys\_computer} = \text{yes} \mid D_{middle\_aged}) = 4/4</math>  <math>p(\text{buys\_computer} = \text{no} \mid D_{middle\_aged}) = 0</math>  Info (<math>D_{middle\_aged}</math>) = <math>-4/4 \log_2 (4/4)</math> bits</p> <p><b>Calculating Info(<math>D_{senior}</math>) ----- (3)</b>  <math>D_{senior}</math> has a total 5 tuples (training data).  m: Distinct values of the class label attribute = 2. buys_computer has two distinct value {yes, no}.  <math>p(\text{buys\_computer} = \text{yes} \mid D_{senior}) = 3/5</math>  <math>p(\text{buys\_computer} = \text{no} \mid D_{senior}) = 2/5</math>  Info (<math>D_{middle\_aged}</math>) = <math>-3/5 \log_2 (3/5) - 2/5 \log_2 (2/5)</math> bits</p> <p><b>Info (<math>D_{age}</math>) = (5/14 * Info(<math>D_{youth}</math>)) + (4/14 * Info(<math>D_{middle\_aged}</math>)) + (5/14 * Info(<math>D_{senior}</math>))</b></p> <p><b>Gain (age) = Info (D) - Info (<math>D_{age}</math>) = 0.246 bits.</b></p>
C	Please do the numerical calculation of Gain (income). $Info_{income} = \frac{ D_{high} }{ D } \times Info(D_{high}) + \frac{ D_{medium} }{ D } \times Info(D_{medium}) + \frac{ D_{low} }{ D } \times Info(D_{low})$
D	Please do the numerical calculation of Gain (student). $Info_{student} = \frac{ D_{yes} }{ D } \times Info(D_{yes}) + \frac{ D_{no} }{ D } \times Info(D_{no})$
E	Please do the numerical calculation of Gain (credit_rating). $Info_{credit\_rating} = \frac{ D_{fair} }{ D } \times Info(D_{fair}) + \frac{ D_{excellent} }{ D } \times Info(D_{excellent})$

MCQ:

A		B	
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C		D	
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Answer	
Remarks	

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