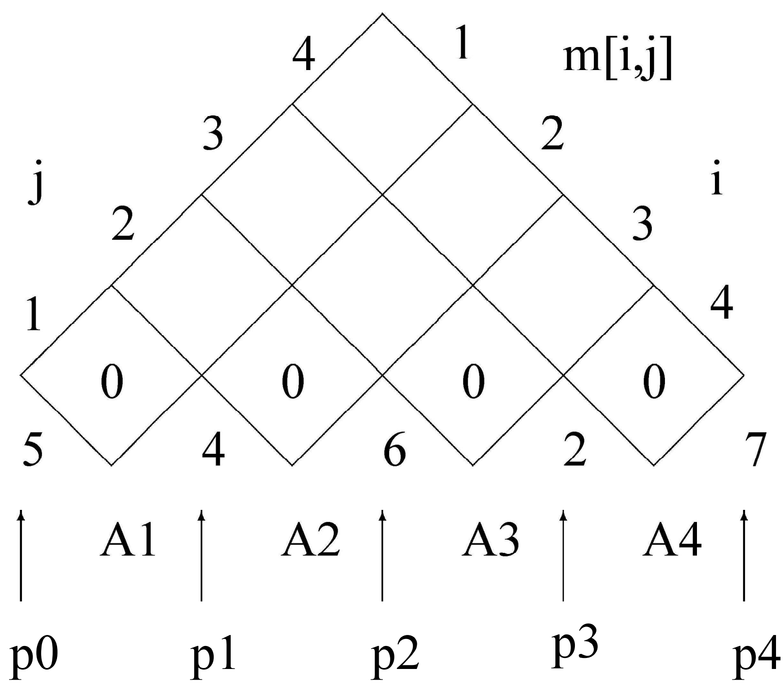
Example for the Bottom-Up Computation

Example: Given a chain of four matrices A_1, A_2, A_3 and A_4, with $p_0 = 5$, $p_1 = 4$, $p_2 = 6$, $p_3 = 2$ and $p_4 = 7$. Find $m[1, 4]$.

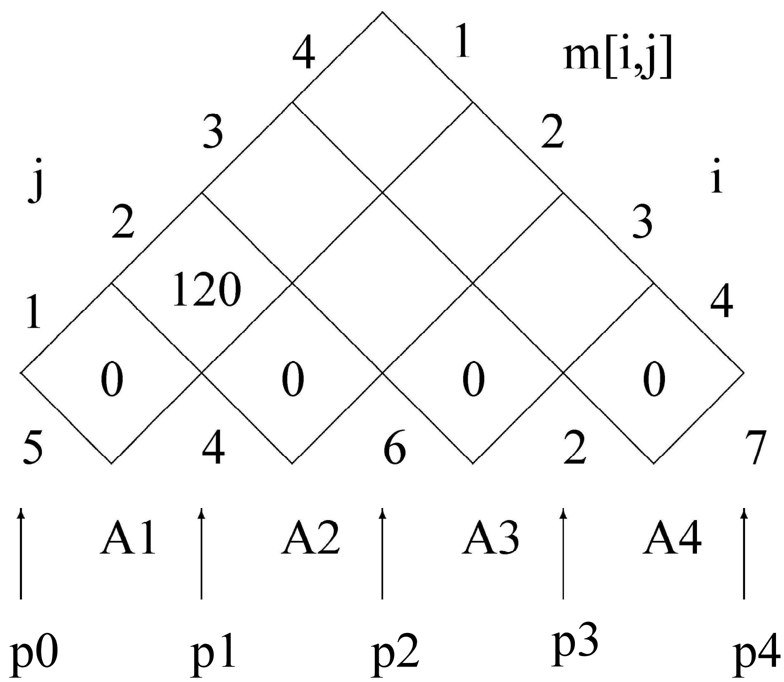
S0: Initialization



Example – Continued

Stp 1: Computing $m[1,2]$ By definition

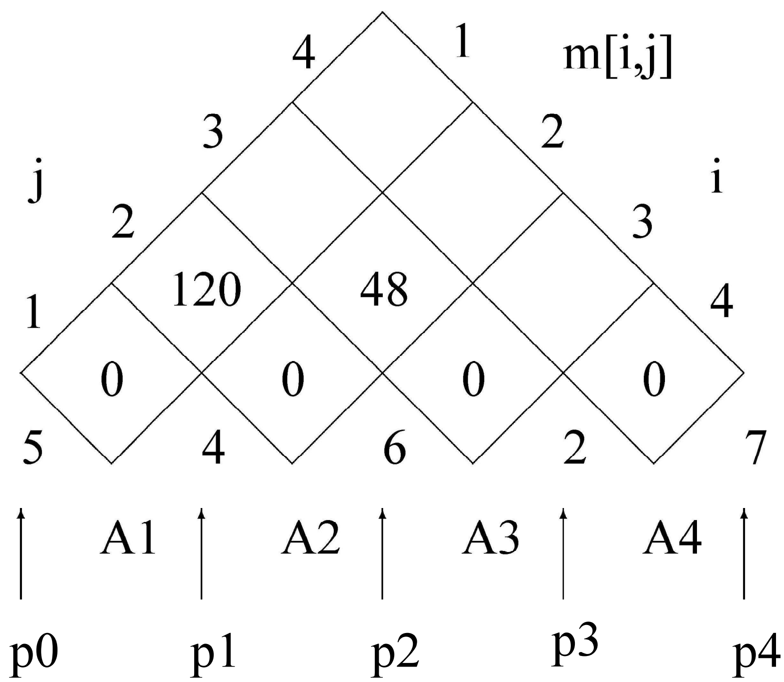
$$\begin{aligned} m[1,2] &= \min_{1 \le k < 2} (m[1,k] + m[k+1,2] + p_0 p_k p_2) \\ &= m[1,1] + m[2,2] + p_0 p_1 p_2 = 120. \end{aligned}$$



Example – Continued

Stp 2: Computing $m[2,3]$ By definition

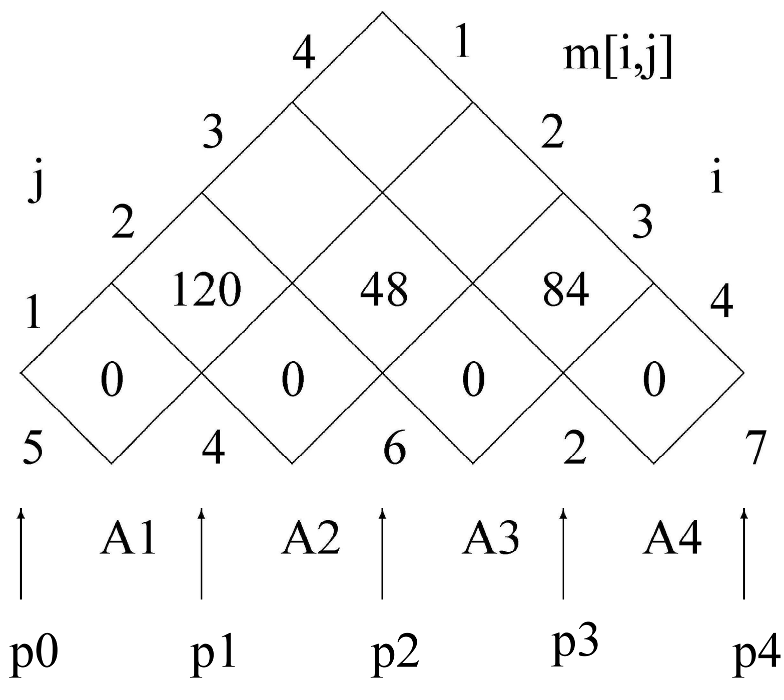
$$
\begin{aligned}
m[2,3] &= \min_{2 \le k < 3} (m[2,k] + m[k+1,3] + p_1 p_k p_3) \\
&= m[2,2] + m[3,3] + p_1 p_2 p_3 = 48.
\end{aligned}
$$



Example – Continued

Stp3: Computing $m[3,4]$ By definition

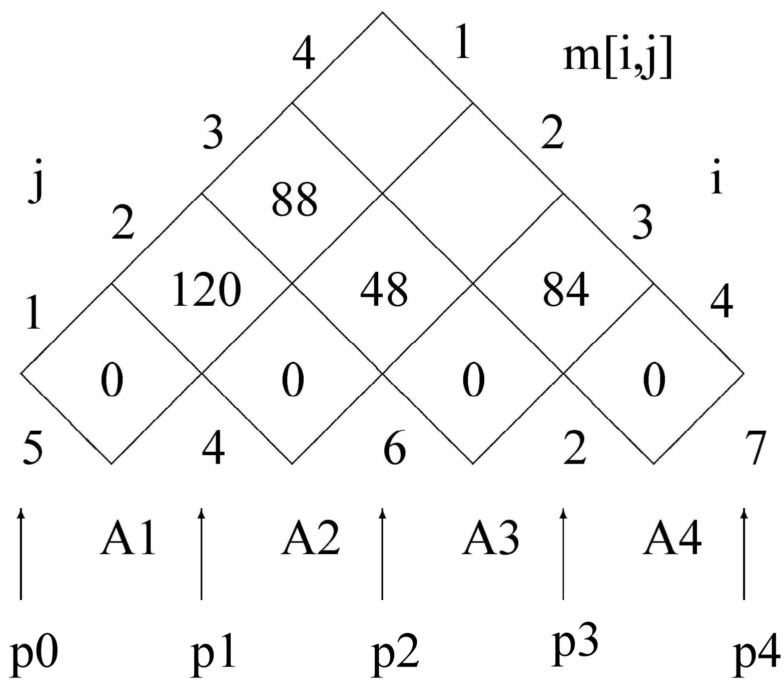
$$m[3,4] = \min_{3 \le k < 4} (m[3,k] + m[k+1,4] + p_2 p_k p_4)$$
$$= m[3,3] + m[4,4] + p_2 p_3 p_4 = 84.$$



Example – Continued

Stp4: Computing $m[1,3]$ By definition

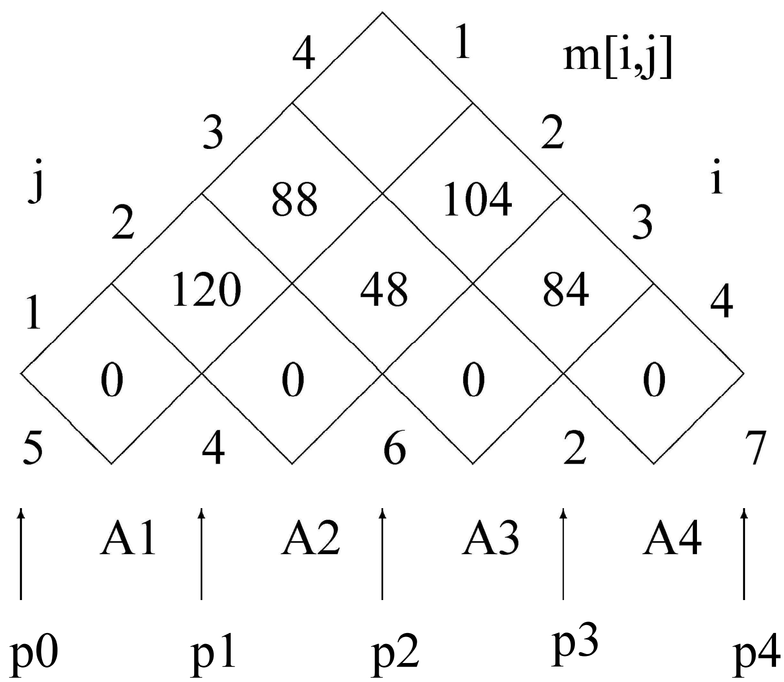
$$\begin{aligned} m[1,3] &= \min_{1 \le k < 3} (m[1,k] + m[k+1,3] + p_0 p_k p_3) \\ &= \min \left\{ \begin{array}{c} m[1,1] + m[2,3] + p_0 p_1 p_3 \\ m[1,2] + m[3,3] + p_0 p_2 p_3 \end{array} \right\} \\ &= 88. \end{aligned}$$



Example – Continued

Stp5: Computing $m[2,4]$ By definition

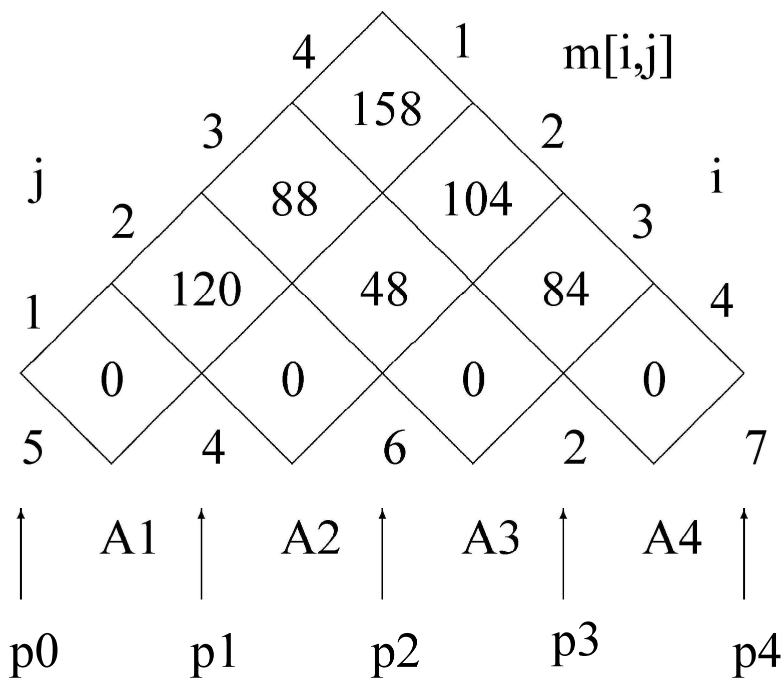
$$
\begin{aligned}
m[2,4] &= \min_{2 \le k < 4} (m[2,k] + m[k+1,4] + p_1 p_k p_4) \\
&= \min \left\{ \begin{array}{l} m[2,2] + m[3,4] + p_1 p_2 p_4 \\ m[2,3] + m[4,4] + p_1 p_3 p_4 \end{array} \right\} \\
&= 104.
\end{aligned}
$$



Example – Continued

St6: Computing $m[1,4]$ By definition

$$m[1,4] = \min_{1 \le k < 4} (m[1,k] + m[k+1,4] + p_0 p_k p_4)$$
$$= \min \left\{ \begin{array}{l} m[1,1] + m[2,4] + p_0 p_1 p_4 \\ m[1,2] + m[3,4] + p_0 p_2 p_4 \\ m[1,3] + m[4,4] + p_0 p_3 p_4 \end{array} \right\}$$
$$= 158.$$



We are done!