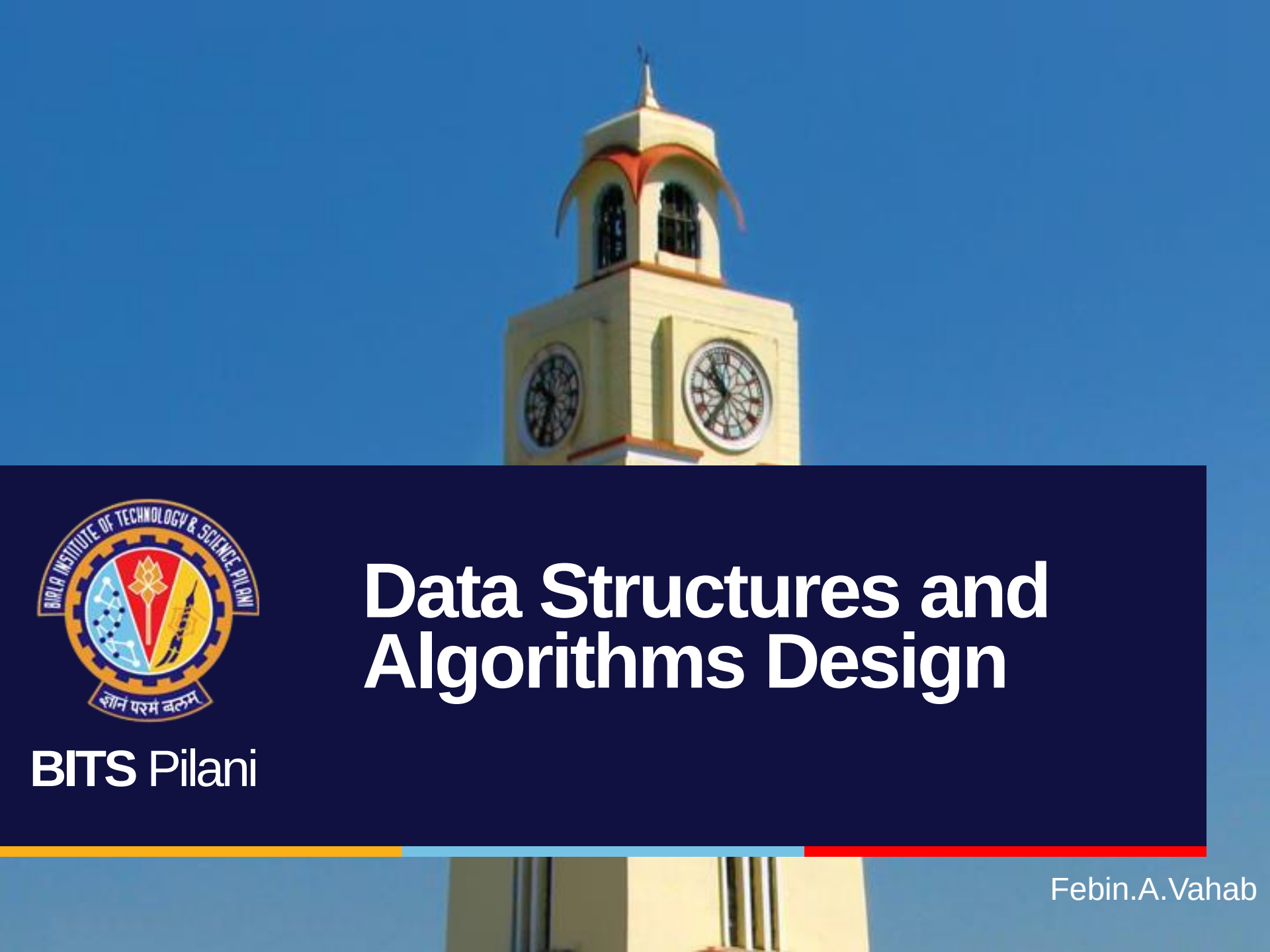


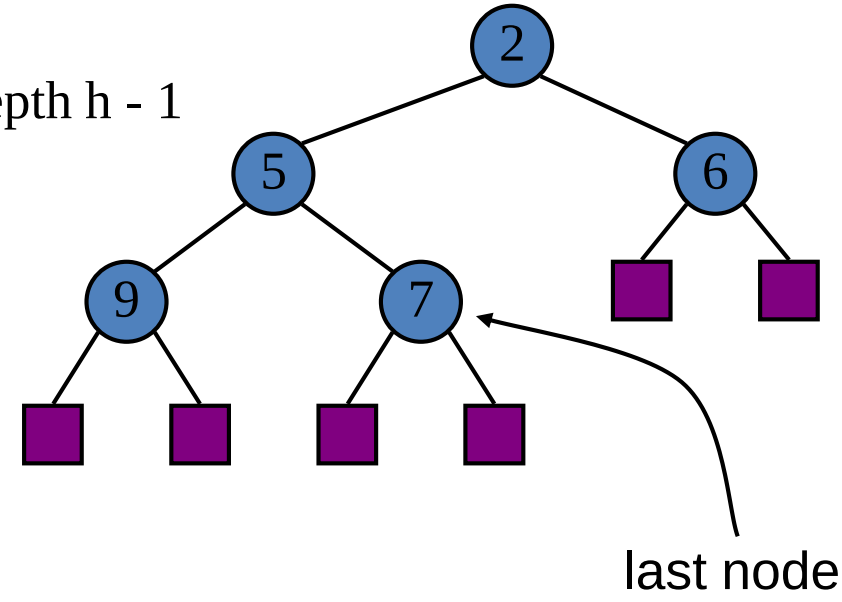
Data Structures and Algorithms Design

BITS Pilani

Febin.A.Vahab

The Heap Data Structure

- Complete Binary Tree: let ***h*** be the height of the heap
  - for levels $i = 0, \ldots, h - 1$, there are 2^i nodes at level ***i***
  - In level ***h*** – 1, the internal nodes are to the left of the external nodes
- All the internal nodes on this level will be visited before any external nodes on this level in an inorder traversal.
- The last node of a heap
  - the rightmost internal node of depth h - 1



Height of a Heap

- Theorem: A heap storing n keys has height $O(\log n)$

  Proof: (we apply the complete binary tree property)
  - Let h be the height of a heap storing n keys
  - Since there are 2^i keys at depth $i = 0, \ldots, h-2$ and at least one key at depth $h-1$, we have $n \geq 1 + 2 + 4 + \ldots + 2^{h-2} + 1$
  - Thus, $n \geq 2^{h-1}$, i.e., $h \leq \log n + 1$