Homework -11 Solution

Q.14 Determine whether $f : Z \times Z \to Z$ is onto if

(a) $f(m,n) = 2m - n$

Solution: For every 'x' belongs to codomain the pair (0, -x) in the domain has image x.

i.e. $f(0,-x) = 0 - (-x) = x$

Hence the function is onto.

(b) $f(m,n) = m^2 - n^2$

Solution: As perfect squares $x^2 : 0,1,4,9,...$

The function is not onto as we cannot able to write 2 as the difference of perfect squares of integers.

(Note that the difference between two perfect squares in the above list is either 1 or greater than 2.)

(c) $f(m,n) = m + n + 1$

Solution: For every 'x' belongs to codomain the pair (0, x-1) in the domain has image x.

i.e. $f(0, x-1) = 0 + (x-1) + 1 = x$

Hence the function is onto.

(d) $f(m,n) = |m| - |n|$

Solution: For every 'x>0' belongs to codomain the pair (x, 0) in the domain has image x.

For every 'x<0' belongs to codomain the pair (0, x) in the domain has image x.

Also for '0' belongs to codomain the pair (0, 0) in the domain has image 0.

Hence the function is onto.

(e) $f(m,n) = m^2 - 4$

Solution: The square of the integer is always positive $m^2 \geq 0$

Thus $m^2 \geq 0 \Rightarrow m^2 - 4 \geq -4$

Note that no integer less than -4 are the image of the element of the domain and Hence **the function is not onto.**

Q.15 Determine whether the function $f : Z \times Z \to Z$ is onto if

(a) $f(m,n) = m + n$

Solution: For every 'x' belongs to codomain the pair (0, x) in the domain has image x.

i.e. $f(0,x) = 0 + x = x$

Hence the function is onto.

(b) $f(m,n) = m^2 + n^2$

Solution: As perfect squares $x^2 : 0,1,4,9,...$

The function is not onto as we cannot able to write 2 as the sum of perfect squares of integers.

(Note that the sum of two perfect squares in the above list is either 1 or greater than 4.)

(c) $f(m,n) = m$

Solution: For every 'x' belongs to codomain the pair (x, 0) in the domain has image x.

i.e. $f(x,0) = x$

Hence the function is onto.

(d) $f(m,n) = |n|$

Solution: As $|x| \geq 0$ for all x, no negative integer are the image of the element of the domain and **Hence the function is not onto.**

(e) $f(m,n) = m - n$

Solution: For every 'x' belongs to codomain the pair (x, 0) in the domain has image x.

i.e. $f(x,0) = x - 0 = x$

Hence the function is onto.

Q.20 Give an example of a function from **N** to **N** that is

a) one-to-one but not onto.

b) onto but not one-to-one.

c) both onto and one-to-one (but different from the identity function).

d) neither one-to-one nor onto.

Solution: Note that N = set of all natural numbers = {0, 1, 2….} = Domain f = Co-domain f

a) one-to-one but not onto.

A function f from N to N such that f(x) = x^2

b) onto but not one-to-one.

A function f from N to N such that $f(x) = \left\lfloor \frac{x}{2} \right\rfloor$ **or** $f(x) = \left\lceil \frac{x}{2} \right\rceil$

c) both onto and one-to-one (but different from the identity function).

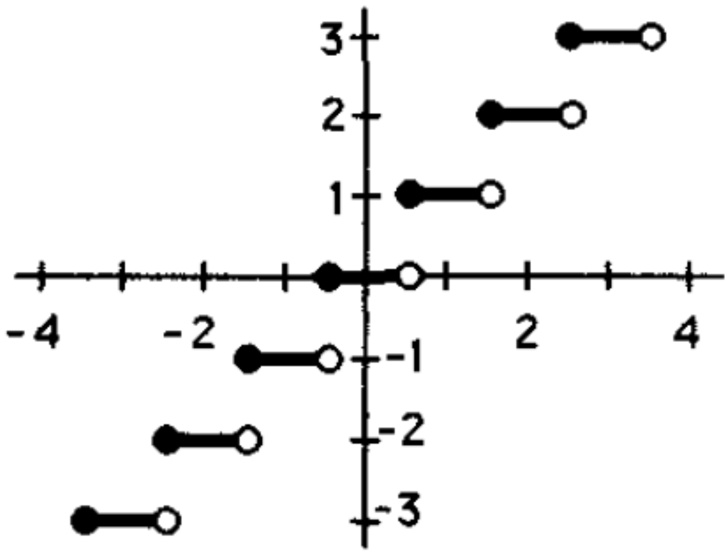
A function f from N to N such that $f(x) = \begin{cases} 1 & \text{for } x=0 \\ 0 & \text{for } x=1 \\ x & \text{otherwise} \end{cases}$

d) neither one-to-one nor onto.

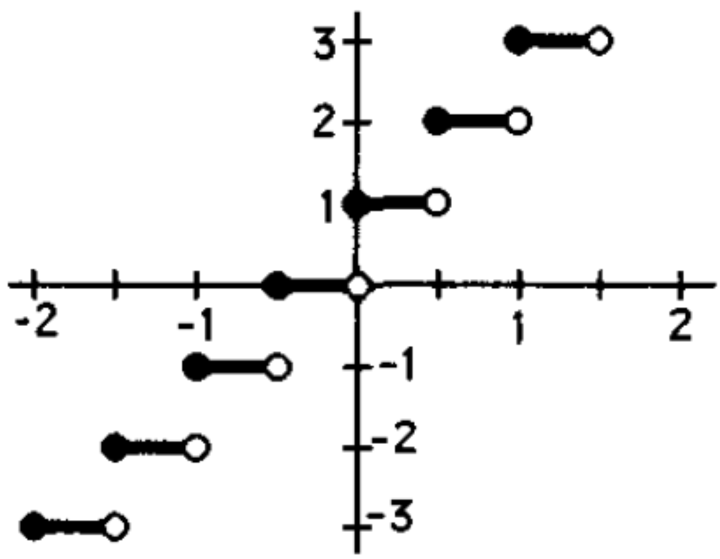
A function f from N to N such that f(x) = 0

Q.67 Draw graphs of each of these functions

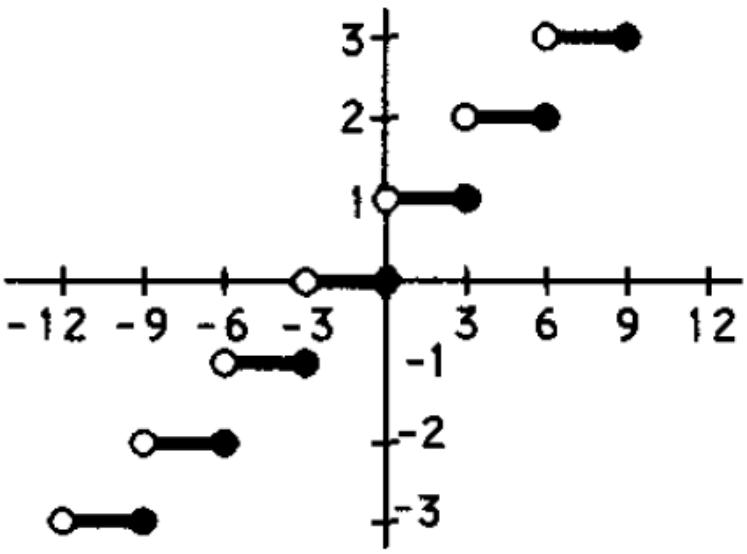
a) $f(x) = \left\lfloor x + \frac{1}{2} \right\rfloor$



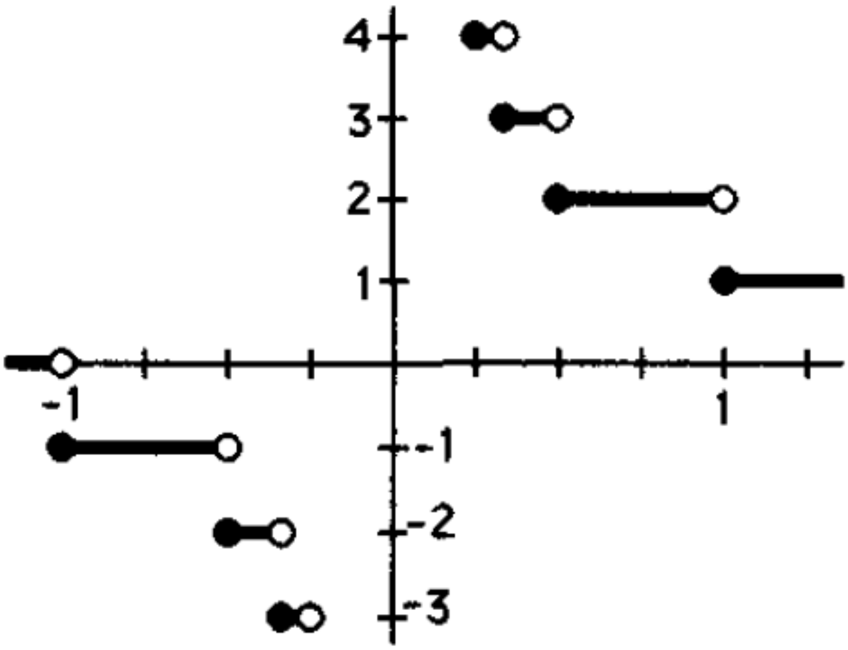
b) $f(x) = \lfloor 2x + 1 \rfloor$



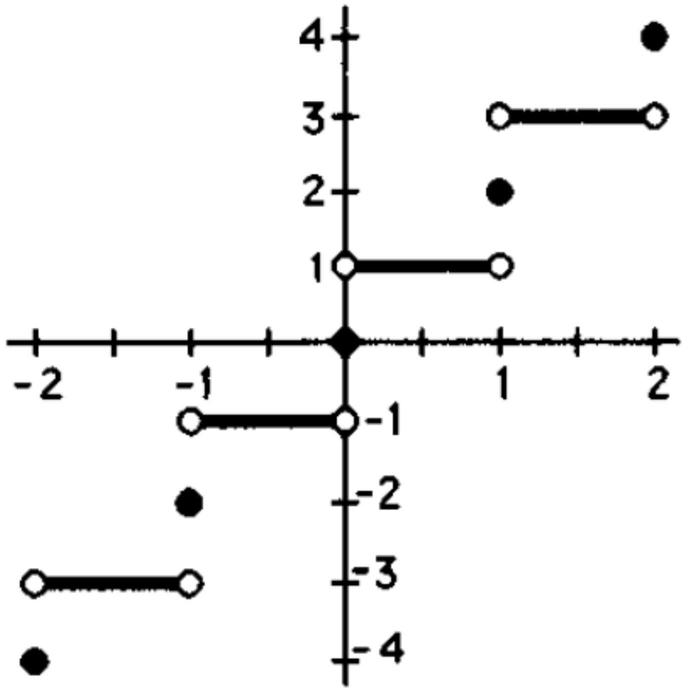
c) $f(x) = \left\lceil \frac{x}{3} \right\rceil$



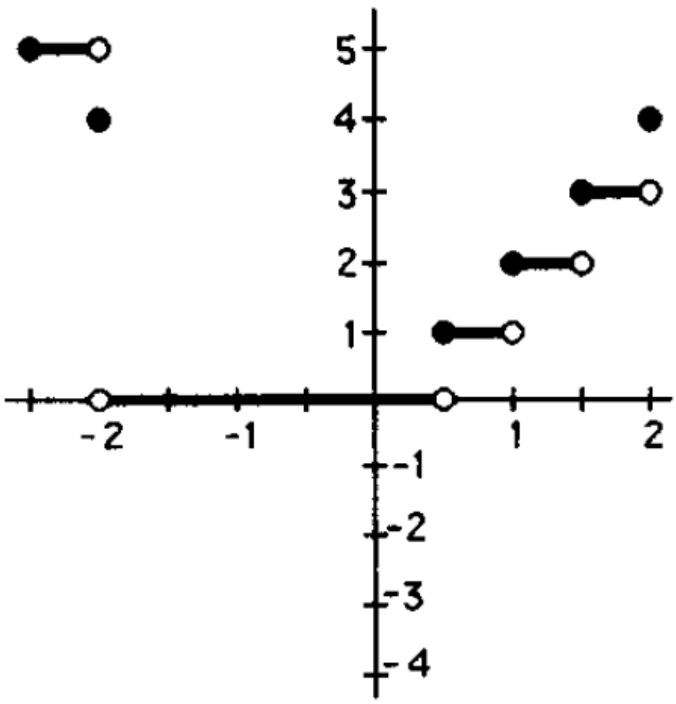
d) $f(x) = \left\lceil \frac{1}{x} \right\rceil$



e) $f(x) = \lceil x - 2 \rceil + \lfloor x + 2 \rfloor$



f) $f(x) = \lceil x/2 \rceil \lfloor 2x \rfloor$



g) $f(x)=\left\lceil \left\lfloor x-\frac{1}{2} \right\rfloor + \frac{1}{2} \right\rceil$