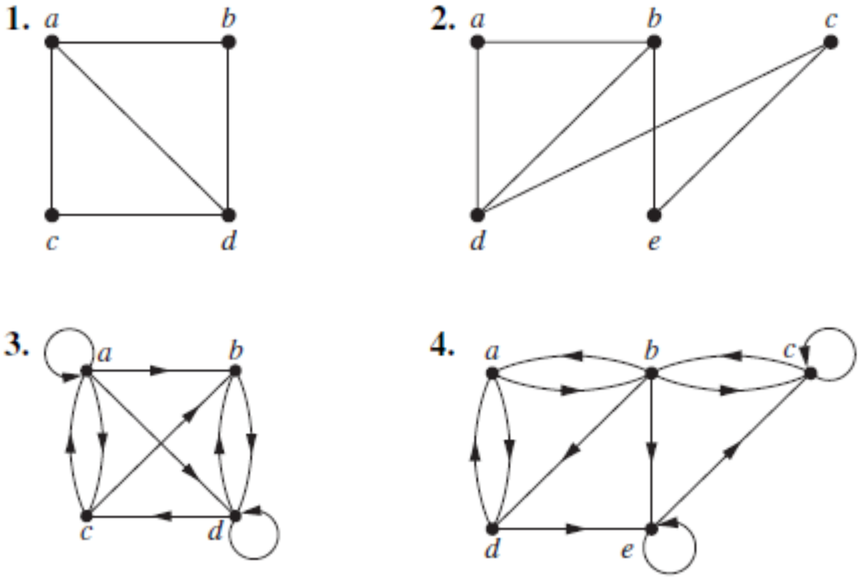
Homework-15

REF : Question Page 675 – 1, 2, 3 & 4

Q. Use an adjacency list to represent the given graph.



Solution: The adjacency list of an undirected graph is simply a list of the vertices of the given graph together with a list of the vertices adjacent to each.

1.

Vertex	Adjacent Vertices
a	b, c, d
b	a, d
c	a, d
d	a, b, c

2.

Vertex	Adjacent Vertices
a	b, d
b	a, d, e
c	e, d
d	a, b, c
e	b, c

Solution: To form the adjacency list of a directed graph, we list, for each vertex in the graph the terminal vertex of each edge that has the given vertex at its initial vertex.

3.

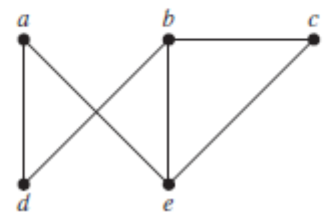
Vertex	Adjacent Vertices
a	a, b, c, d
b	D
c	a, b
d	d, b, c

4.

Vertex	Adjacent Vertices
a	b, d
b	a, c, d, e
c	b, c
d	a, e
e	e, c

REF : Question Page 689 – 1 & 2

Q.1 Does each of these lists of vertices from a path in the following graph? Which paths are simple? Which are circuits? What are the lengths of those that are paths?



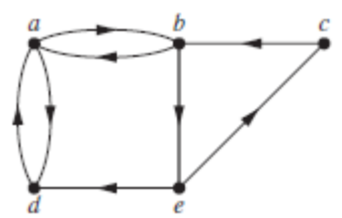
A) a, e, b, c, b : This is a path of length 4(it has 4 edges in it), but it is not simple, since the edge bc is used twice. It is not a circuit, since it ends at a different vertex from the one at which it began.

B) a, e, a, d, b, c, a : This is not a path, since there is no edge from e to a.

C) e, b, a, d, b, e : This is not a path, since there is no edge from b to a.

D) c, b, d, a, e, c: This is a path of length 5, it is simple, since no edge repeated. It is a circuit, since it ends at the same vertex at which it began.

2. Does each of these lists of vertices from a path in the following graph? Which paths are simple? Which are circuits? What are the lengths of those that are paths?



A) a, b, e, c, b: Path is simple, as none of edges are used more than once, length of the path is 4, which is the number of edges in the path, not circuit

B) a, d, a, d, a: Path is not simple as edges (a,d) and (d,a) are used twice, length is 4, not a simple circuit

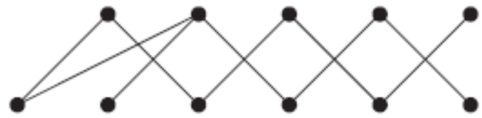
C) a, d, b, e, a : NOT a PATH as in c, there is no edge from d to b and e to a

D) a, b, e, c, b, d, a: NOT a PATH , there is no edge from b to d

REF : Question Page 689 – 3& 4

Determine whether the given graph is connected.

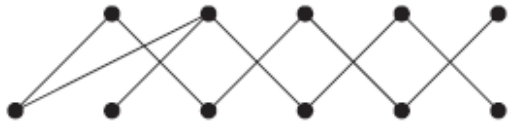
3. 

4. 

Solution:

3. 

One of the vertex in the graph is isolated vertex and there does not exist a path between every pair of distinct vertices of the given graph. Hence the given graph is **not connected** graph.

4. 

There is a path between every pair of distinct
vertices of the given undirected graph.
Therefore the given graph is **connected** graph

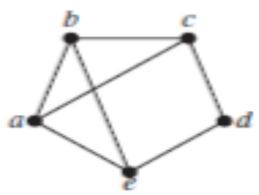
REF : Question Page 703 – 1, 2, 3 & 4

Recall

- **I.** **Euler Circuit:** An Euler circuit in a graph G is a simple circuit containing every edge of G
- **II.** **Euler Path**: An Euler path in G is a simple path containing every edge of G.
- **III.** A connected multigraph with at least two vertices has an Euler circuit if and only if each of its vertices has an even degree.
- **IV.** A connected multigraph has an Euler path but not an Euler circuit if and only if it has exactly two vertices of odd degree.

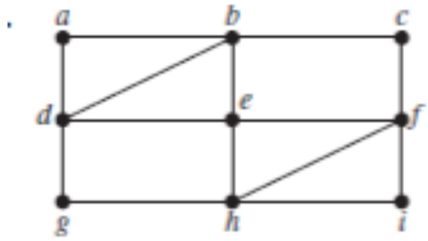
Q. Determine whether the given graph has an Euler circuit. Construct such a circuit when one exists. If no Euler circuit exists, determine whether the graph has an Euler path and construct such a path if one exists.

1)



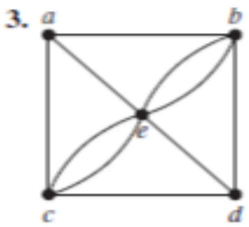
Solution: The graph has more than two vertices of odd degree, the graph has neither an Euler circuit nor Euler Path.

2)



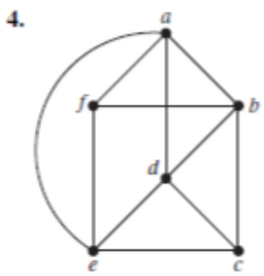
Solution: The graph is Euler circuit by Recall (III) and circuit is as: a-b- c- f- i- h- f- e- h- g- d- b- e- d- a but not Euler Path by Recall (IV)

3)



Solution: This graph does not have an Euler circuit because we have two vertices with odd degrees (a and d). This graph does have an Euler path by Recall (IV). The path is as follows: a-e- c- e- b- e- d- b- a- c- d.

4)



Solution: This graph does not have an Euler circuit because we have two vertices with odd degrees (c and f). This graph does have an Euler path by Recall (IV). The path is as follows: f-a-b-f-e-a-d-e-c-d-b-c

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