6.4 Homework problems

Q1 Consider a matrix A of size n×n and do the following

i) Write a code (in a language of your choice) to derive the matrices L and U such that A=I+L+U, as given in the textbook.

ii) Check the code with the examples given in the textbook.

iii) Add a subroutine to the code in i) to calculate the l_1, l_∞ and Frobenius norms.

iv) Compute $\|(I+L)^{-1}U\|$ using the three norms mentioned in iii) to check the criteria for convergence.

v) Calculate the spectral value (the largest eigen value in magnitude) and check if that value is < 1.

MATLAB / OCTAVE would be convenient as most of the routines are available.

Q2 Solve problems 19 and 20 in Problem Set 20.4 (page no. 868) in the textbook T1. Write a code to compute the condition number of a matrix, using part iii) of in problem 1 and use that to evaluate the required quantities for the two problems.

Q3 Write a code to determine the largest eigenvalue and the error associated in determining the same using the power method.

Solutions: -

Q.1. i) Here we assume that A is n×n and the diagonal entries of A are one in the system Ax = B. For if there exist zero on the diagonal then rearranging rows we get non-zero entries on diagonal and if the diagonal entries are not equal to 1, we can divide respective row with the diagonal entry and get 1.

The Scilab function for decomposing A = I + L + U is

```
function []=ILU(A)
   [n,m]=size(A)
   L=zeros(n,n)
   U=zeros(n,n)
   I=eye(n,n)
    for i=1:n-1
    for   j=i+1:n
      L(j,i)=A(j,i)
   end
end
for i=1:n
   for j=i+1:n
   U(i,j)=A(i,j)
   end
end
disp(I)
disp(L)
disp(U)
endfunction
```

ii) This can be verified.

iii) The function norm() can be added to the above code to find l_1, l_∞ and Frobenius norms as follows

One- Norm

```
function []=one_norm(I, L, U)
   C= -(inv(I+L))*U
   disp(C)
[n,m]=size(C)
for i=1:n
```

```
    sum=0
    for j=1:n
    sum=sum+abs(C(i,j))
end
d(i)=sum
end
disp(d)
mprintf("The one norm = %f",max(d))
endfunction
```

Infinity norm

```
function []=two_norm(I, L, U)
    C= -(inv(I+L))*U
    [n,m]=size(C)
    for i=1:n
    sum=0
    for j=1:n
    sum=sum+abs(C(j,i))
end
d(i)=sum
end
disp(d)
mprintf("The Infinity norm = %f",max(d))

endfunction
```

Frobenius Norm

```
function []=frobenius(I, L, U)
    C= -(inv(I+L))*U
    [n,m]=size(C)
    sum=0
    for i=1:n
        for j=1:n
            sum=sum+(C(i,j))^2
    end
    end
    nm=sqrt(sum)
    mprintf("The frobenius norm = %f",nm)
    endfunction
```

Q.2 Let- i) $A = \begin{bmatrix} 2 & 1.4 \\ 1.4 & 1 \end{bmatrix}, b_1 = \begin{bmatrix} 1.4 \\ 1 \end{bmatrix}, b_2 = \begin{bmatrix} 1.44 \\ 1 \end{bmatrix}$. Solve Ax = b_1 and Ax = b_2. Compare solutions and compute condition number of A.

Solution: - Solving the system Ax = b_1, we get solution $[0, 1]^T$, and solving Ax = b_2 we get solution $[1, -0.4]^T$. The condition number for matrix A is given by $\kappa(A) = ||A|| \cdot ||A^{-1}||$
Thus, $\kappa(A) = 2.986637 \cdot 74.665923 = 233.0001$.
Since the condition number is far greater than 1, therefore the system is ill-conditioned.

ii) $A = \begin{bmatrix} 5 & -7 \\ -7 & 10 \end{bmatrix}, b_1 = \begin{bmatrix} -2 \\ 3 \end{bmatrix}, b_2 = \begin{bmatrix} -2 \\ 3.1 \end{bmatrix}$. Solve Ax = b_1 and Ax = b_2. Compare solutions and compute condition number of A.

Solution: - Solving the system Ax = b_1, we get solution $[1, 1]^T$, and solving Ax = b_2 we get solution $[1.7, 1.5]^T$. The condition number for matrix A is given by $\kappa(A) = ||A|| \cdot ||A^{-1}||$
Thus, $\kappa(A) = 14.933185 \cdot 14.933185 = 223.00001$.
Since the condition number is far greater than 1, therefore the system is ill-conditioned.

Q.3 The scilab code to find the largest eigen value and corresponding eigen vector by using power method is

```
function []=powermethod(A)
 u0 =[1 1 1]'
 v= A *u0
a= max (abs(u0))
while abs( max ( abs (v))-a) >0.005
a= max (abs(v))
u0=v/ max ( abs (v))
v= A *u0
end

mprintf("The largest eigenvalue of matrix=%f",max(v))
mprintf("\nThe corresponding eigenvector is")

disp (u0)
endfunction
```

Consider the matrix

```
A=[5,1,0,0;1,3,1,0;0,1,3,1;0,0,1,5]
 A  =

   5.   1.   0.   0.
   1.   3.   1.   0.
   0.   1.   3.   1.
   0.   0.   1.   5.

--> powermethod(A)
The largest eigenvalue of matrix=5.623674
The corresponding eigenvector is
   1.
   0.6236744
   0.6236744
   1.
```