Homework - 7

Q1 Reddy Mikks problem:

Example 2.1-1 (The Reddy Mikks Company)

Reddy Mikks produces both interior and exterior paints from two raw materials, M1 and M2. The following table provides the basic data of the problem:

	Tons of raw mat	erial per ton of	Maximum daily
	Exterior paint	Interior paint	availability (tons)
Raw material, M1	6	4	24
Raw material, M2	1	2	6
Profit per ton (\$1000)	5	4	

A market survey indicates that the daily demand for interior paint cannot exceed that for exterior paint by more than 1 ton. Also, the maximum daily demand for interior paint is 2 tons. Reddy Mikks wants to determine the optimum (best) product mix of interior and exterior paints that maximizes the total daily profit.

- i) Use TORA or a simple graph paper to plot the feasible region.
- ii) If the maximum daily availability of M1 is changed from 24 to 25, find the change in profit. This is called the shadow price for M1. In a similar way, find the shadow price of M2.
- iii) If the profits are changed from \$5000 and \$4000 per liter of exterior paint and interior paint to \$c1 and \$c2 respectively, find the range for the ration c1/c2, keeping the same optimal value of x₁ and x₂.
- iv Try to add a constraint that would change / would not change the present optimal solution.

Solution:

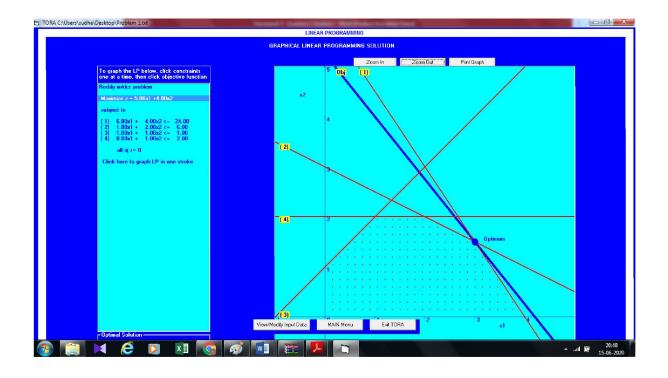
Maximize $z = 5x_1 + 4x_2$ subject to

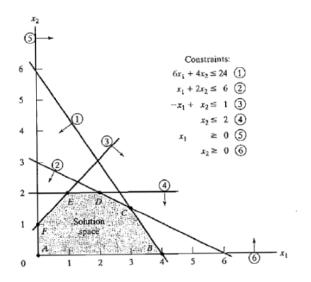
$$6x_1 + 4x_2 \le 24$$
 (1)
 $x_1 + 2x_2 \le 6$ (2)
 $-x_1 + x_2 \le 1$ (3)

$$x_2 \leq 2 \tag{4}$$

$$x_1, x_2 \ge 0 \tag{5}$$

(i) Feasible Region By TORA:





Corner point	(x_1, x_2)	z
A	(0,0)	0
B	(4,0)	20
\boldsymbol{c}	(3, 1.5)	21 (OPTIMUM)
D	(2, 2)	18
$\mathcal E$	(1, 2)	13
F	(0, 1)	4

Maximum Z = \$21,000 occurs at $x_1 = 3$ tons and $x_2 = 1.5$ tons

(ii) If the maximum daily availability of M1 is changed from 24 to 25, find the change in profit. This is called the shadow price for M1. In a similar way, find the shadow price of M2.

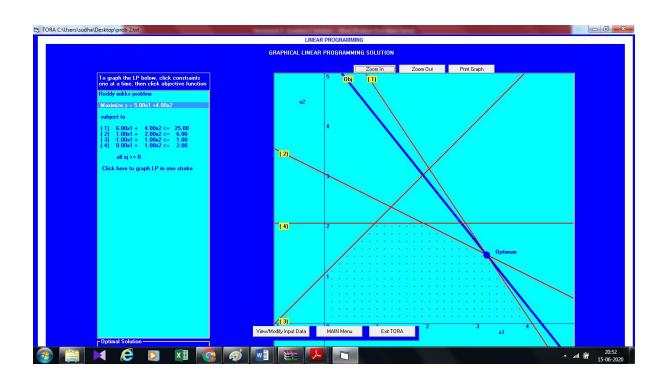
To Find the maximum value of the function $Z = 5 x_1 + 4 x_2$

subject to the constraints:

$$\begin{cases} 6x_1 + 4x_2 \le 25 \\ x_1 + 2x_2 \le 6 \\ -x_1 + x_2 \le 1 \end{cases}$$

$$x_2 \le 2$$

$$x_1 \ge 0 \quad x_2 \ge 0$$



Maximum
$$z = $21,750, x_1 = 3.25, x_2 = 1.375$$

Shadow Price = (21750-21000)/(25-24)=\$750

(iii) If the profits are changed from \$5000 and \$4000 per litre of exterior paint and interior paint to \$c1 and \$c2 respectively, find the range for the ratio c1/c2, keeping the same optimal value of x1 and x2.

The optimal table of the problem by simplex is

		5	4	0	0	0	0	
СВ	Basis	x1	x2	s1	s2	s3	s4	Constants
5	x1	1	0	1/4	- 1/2	0	0	3
4	x2	0	1	- 1/8	3/4	0	0	1 1/2
0	s3	0	0	3/8	-1 1/4	1	0	2 1/2
0	s4	0	0	1/8	- 3/4	0	1	1/2
	cj - zj	0	0	- 3/4	- 1/2	0	0	21

If the profits are changed from \$5000 and \$4000 per litre of exterior paint and interior paint to \$c1 and \$c2 respectively

Optimal table:

		C_1	C_2	0	0	0	0	
СВ	Basis	x1	x2	s1	s2	s3	s4	Constants
C_1	x1	1	0	1/4	- 1/2	0	0	3
C_2	x2	0	1	- 1/8	3/4	0	0	1 1/2
0	s3	0	0	3/8	-1 1/4	1	0	2 1/2
0	s4	0	0	1/8	- 3/4	0	1	1/2
	cj - zj	0	0	c1/4-c2/8	c1/2+3c2/4	0	0	21

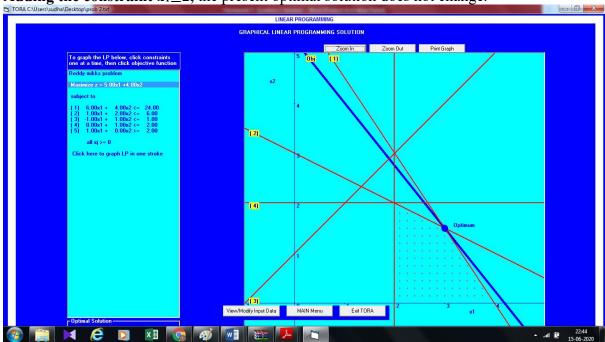
The same optimal retained when $c1/4-c2/8 \le 0$ and $c1/2+3c2/4 \le 0$

we get ratio $C_1/C_2 \le \frac{1}{2}$ and $C_1/C_2 \le -3$

The required ratio is $C_1/C_2 \le -3$

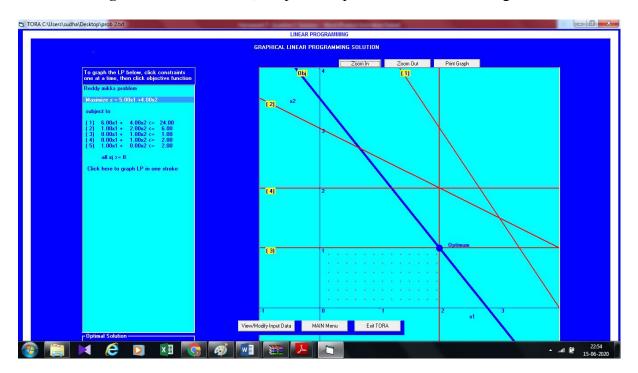
(iv) Try to add a constraint that would change / would not change the present optimal solution.

• Adding the constraint $x_1 \ge 2$, the present optimal solution does not change.



Maximum Z = \$21,000 occurs at $x_1 = 3$ tons and $x_2 = 1.5$ tons

• Adding the constraint $x_1 \le 2$, the present optimal solution will change.



Maximum Z = \$18,000 occurs at $x_1 = 2$ tons and $x_2 = 2$ ton

Q.2 For the Urban planning problem,

- I. Write the complete set of constraints and convert the problem in the standard form.
- II. Would the optimal value change if the maximum number of houses we could demolish is changed to 400?
- III. Observe the decimal nature of the solution (given in the book) and understand the rounding.

Answer:

١.

Objective function Z= $1000x_1 + 1900x_2 + 2700x_3 + 3400x_4$

Subject to,

$$0.18x_1 + 0.28x_2 + 0.4x_3 + 0.5x_4 - 0.2125x_5 \le 0$$

$$x_5 \le 300$$

$$-0.8x_1 + 0.2x_2 + 0.2x_3 + 0.2x_4 \le 0$$

$$0.1x_1 - 0.9x_2 - 0.1x_3 + 0.1x_4 \le 0$$

$$0.25x_1 + 0.25x_2 - 0.75x_3 - 0.75x_4 \le 0$$

$$50x_1 + 70x_2 + 130x_3 + 160x_4 + 2x_5 \le 0$$

$$x_1, x_2, x_3, x_4, x_5 \ge 0$$

Standard form,

Objective function,

$$Z= 1000x_1 + 1900x_2 + 2700x_3 + 3400x_4 + S_1 + S_2 + S_3 + S_4 + S_5 + S_6$$
 Subject to,

$$\begin{aligned} 0.18x_1 + 0.28x_2 + 0.4x_3 + 0.5x_4 - 0.2125x_5 + S_1 &= 0 \\ x_5 + S_2 &= 300 \\ -0.8x_1 + 0.2x_2 + 0.2x_3 + 0.2x_4 + S_3 &= 0 \\ 0.1x_1 - 0.9x_2 - 0.1x_3 + 0.1x_4 + S_4 &= 0 \\ 0.25x_1 + 0.25x_2 - 0.75x_3 - 0.75x_4 + S_5 &\leq 0 \\ 50x_1 + 70x_2 + 130x_3 + 160x_4 + 2x_5 + S_6 &\leq 0 \\ x_1, x_2, x_3, x_4, x_5, S_1, S_2, S_3, S_4, S_5, S_6 &\geq 0 \end{aligned}$$

II.

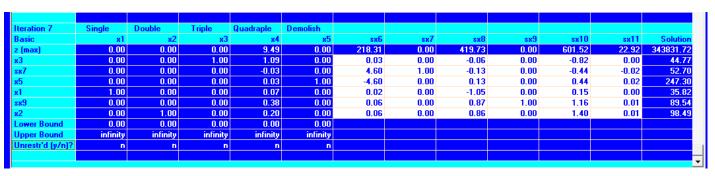
Solution for original problem by TORA

Single	Double	Triple	Quadraple	Demolish							
x1	ж2	кЗ	×4	х5	дже дже	sx7	8кг	exs	sx10	sx11	Solution
-1000.00	-1900.00	-2700.00	-3400.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.18	0.28	0.40	0.50	-0.21	1.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	1.00	0.00	1.00	0.00	0.00	0.00	0.00	300.00
-0.80	0.20	0.20	0.20	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
0.10	-0.90	-0.10			0.00	0.00	0.00	1.00	0.00	0.00	0.00
	0.25	-0.75	-0.75	0.00		0.00	0.00	0.00	1.00	0.00	0.00
	70.00	130.00			0.00	0.00	0.00	0.00	0.00	1.00	15000.00
infinity	infinity	infinity	infinity	infinity							
n	n	n	n	n							
Single			Quadraple								
			×4								Solution
											0.00
											0.00
											300.00
											0.00
											0.00
											0.00
					-320.00	0.00	0.00	0.00	0.00	1.00	15000.00
infinity	infinity	infinity	infinity	infinity							
n	n	n	n	n							
	*1 -1000.00 0.18 0.00 -0.80 0.10 0.25 50.00 0.00 infinity n Single *1 224.00 0.36 0.00 -0.87 0.06 0.52 -7.60 0.00 infinity	x2 x2 -1000.00	X2	X1	x1	X1	X1	X1	X1	No. No.	X1

Iteration 3	Single	Double	Triple	Quadraple	Demolish							
Basic	x1	x2	кЗ	x4	ж5	дже	sx7	8кг	9кг	sx10	sx11	Solution
z (max)	-14600.00	1500.00	700.00	0.00	0.00	0.00	0.00	17000.00	0.00	0.00	0.00	0.00
х4	-4.00	1.00	1.00	1.00	0.00	0.00	0.00	5.00	0.00	0.00	0.00	0.00
sx7	10.38	-1.05	-0.48	0.00	0.00	4.76	1.00	-11.90	0.00	0.00	0.00	300.00
х5	-10.38	1.05	0.48	0.00	1.00	-4.76	0.00	11.90	0.00	0.00	0.00	0.00
ек9	0.50	-1.00	-0.20	0.00	0.00	0.00	0.00	-0.50	1.00	0.00	0.00	0.00
sx10	-2.75	1.00	0.00	0.00	0.00	0.00	0.00	3.75	0.00	1.00	0.00	0.00
sx11	710.76	-92.10	-30.95	0.00	0.00	9.52	0.00	-823.81	0.00	0.00	1.00	15000.00
Lower Bound	0.00	0.00	0.00	0.00	0.00							
Upper Bound	infinity	infinity	infinity	infinity	infinity							
Unrestr'd (y/n)?		n	n	n	n							
Iteration 4	Single	Double	Triple	Quadraple	Demolish							
Basic	x1	x2	кЗ	×4	ж5	дж6	sx7	8x2	9кг	sx10	sx11	Solution
z (max)	0.00	-27700.00	-5140.00	0.00	0.00	0.00	0.00	2400.00	29200.00	0.00	0.00	0.00
x4	0.00	-7.00	-0.60	1.00	0.00	0.00	0.00	1.00	8.00	0.00	0.00	0.00
sx7	0.00	19.71	3.68	0.00	0.00	4.76	1.00	-1.52	-20.76	0.00	0.00	300.00
х5	0.00	-19.71	-3.68		1.00	-4.76	0.00	1.52	20.76	0.00	0.00	0.00
x1	1.00	-2.00			0.00	0.00	0.00	-1.00	2.00	0.00	0.00	0.00
sx10	0.00	-4.50			0.00	0.00	0.00	1.00	5.50	1.00	0.00	0.00
sx11	0.00	1329.43	253.35	0.00	0.00	9.52	0.00	-113.05	-1421.52	0.00	1.00	15000.00
Lower Bound	0.00	0.00	0.00	0.00	0.00							
Upper Bound	infinity	infinity	infinity	infinity	infinity							
11		n	n	n	n							
Unrestr'd (y/n)?	n		<u> </u>									



Optimum solution (iteration 7)



• Would the optimal value change if the maximum number of houses we could demolish is changed to 400?

No, It doesn't change the optimal value.

Iteration 1	Single	Double	Triple	Quadraple	Demolish							
Basic	x1	x2	кЗ	×4	ж5	6xs	sx7	8ж8	9кг	sx10	sx11	Solution
z (max)	-1000.00	-1900.00	-2700.00	-3400.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3ж8	0.18	0.28	0.40	0.50	-0.21	1.00	0.00	0.00	0.00	0.00	0.00	0.00
sx7	0.00	0.00	0.00	0.00	1.00	0.00	1.00	0.00	0.00	0.00	0.00	400.00
8ж2	-0.80	0.20	0.20	0.20	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
9ж2	0.10	-0.90	-0.10	0.10	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00
sx10	0.25	0.25	-0.75	-0.75	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
sx11	50.00	70.00	130.00	160.00	2.00	0.00	0.00	0.00	0.00	0.00	1.00	15000.00
Lower Bound	0.00	0.00	0.00	0.00	0.00							
Upper Bound	infinity	infinity	infinity	infinity	infinity							
Unrestr'd (y/n)?	n	n	n	n	n							
Iteration 2	Single	Double	Triple	Quadraple	Demolish							
Basic	x1	ж2	кЗ	×4	х5	зхб	sx7	8x8	ex9	sx10	sx11	Solution
z (max)	224.00	4.00	20.00	0.00	-1428.00	6800.00	0.00	0.00	0.00	0.00	0.00	0.00
×4	0.36	0.56	0.80	1.00	-0.42	2.00	0.00	0.00	0.00	0.00	0.00	0.00
sx7	0.00	0.00	0.00	0.00	1.00	0.00	1.00	0.00	0.00	0.00	0.00	400.00
8x2	-0.87	0.09	0.04	0.00	0.08	-0.40	0.00	1.00	0.00	0.00	0.00	0.00
sx9	0.06	-0.96	-0.18	0.00	0.04	-0.20	0.00	0.00	1.00	0.00	0.00	0.00
sx10	0.52	0.67	-0.15	0.00	-0.32	1.50	0.00	0.00	0.00	1.00	0.00	0.00
sx11	-7.60	-19.60	2.00	0.00	69.20	-320.00	0.00	0.00	0.00	0.00	1.00	15000.00
Lower Bound	0.00	0.00	0.00	0.00	0.00							
	infinity	infinity	infinity	infinity	infinity							
Upper Bound												
Upper Bound Unrestr'd (y/n)?	ń	n	n	n	n							<u>'</u>

Iteration 3	Single	Double	Triple	Quadraple	Demolish							
Basic	x1	×2	х3	×4	x5	дже дже	sx7	Вкг	еж9	sx10	sx11	Solution
z (max)	-14600.00	1500.00	700.00			0.00	0.00	17000.00	0.00	0.00		0.00
×4	-4.00	1.00	1.00			0.00	0.00	5.00	0.00	0.00	0.00	0.00
sx7	10.38	-1.05	-0.48			4.76	1.00	-11.90	0.00	0.00	0.00	400.00
х5	-10.38	1.05	0.48		1.00	-4.76	0.00	11.90	0.00	0.00	0.00	0.00
еже	0.50	-1.00	-0.20	0.00	0.00	0.00	0.00	-0.50	1.00	0.00	0.00	0.00
sx10	-2.75	1.00	0.00	0.00	0.00	0.00	0.00	3.75	0.00	1.00	0.00	0.00
sx11	710.76	-92.10	-30.95	0.00	0.00	9.52	0.00	-823.81	0.00	0.00	1.00	15000.00
Lower Bound	0.00	0.00	0.00	0.00	0.00							
Upper Bound	infinity	infinity	infinity	infinity	infinity							
Unrestr'd (y/n)?	n	n	n	n	n							
Iteration 4	Single	Double	Triple	Quadraple	Demolish							
Basic	x1	x2	кЗ	×4	х5	дже в ж	sx7	8xs	9ке	sx10	sx11	Solution
z (max)	0.00	-27700.00	-5140.00	0.00	0.00	0.00	0.00	2400.00	29200.00	0.00	0.00	0.00
×4	0.00	-7.00		1.00		0.00	0.00	1.00	8.00	0.00	0.00	0.00
sx7	0.00	19.71		0.00	0.00	4.76	1.00	-1.52	-20.76	0.00	0.00	400.00
х5	0.00	-19.71	-3.68	0.00		-4.76	0.00	1.52	20.76	0.00	0.00	0.00
x1	1.00	-2.00				0.00	0.00	-1.00	2.00	0.00	0.00	0.00
XI			-1.10	0.00	0.00	0.00	0.00	1.00	5.50	1.00	0.00	0.00
sx10	0.00	-4.50								0.00	1.00	15000.00
	0.00	1329.43	253.35	0.00	0.00	9.52	0.00	-113.05	-1421.52	0.00	1.00	10000.00
sx10 ex11 Lower Bound	0.00 0.00	1329.43 0.00	253.35 0.00	0.00 0.00	0.00	9.52	0.00	-113.05	-1421.52	0.00	1.00	13000.00
sx10 sx11 Lower Bound Upper Bound	0.00	1329.43 0.00	253.35	0.00 0.00	0.00	9.52	0.00	-1 13.03	-1421.52	0.00	1.00	13000.00
sx10	0.00 0.00	1329.43 0.00	253.35 0.00 infinity	0.00 0.00 infinity	0.00 infinity	9.52	0.00	-1 (3.03	-1421.52	0.00	1.00	13000.

Iteration 5	Single	Double	Triple	Quadraple	Demolish							
Basic	×1	x2	хЗ	×4	ж5	вж6	sx7	8кг	вж9	sx10	sx11	Solution
z (max)	0.00	0.00	138.86	0.00	0.00	198.44	0.00	44.54	-418.90	0.00	20.84	312540.30
×4	0.00	0.00	0.73	1.00	0.00	0.05	0.00	0.40	0.52	0.00	0.01	78.98
sx7	0.00	0.00	-0.08	0.00	0.00	4.62	1.00	0.15	0.32	0.00	-0.01	177.56
х5	0.00	0.00	0.08	0.00	1.00	-4.62	0.00	-0.15	-0.32	0.00	0.01	222.44
x1	1.00	0.00	-0.02	0.00	0.00	0.01	0.00	-1.17	-0.14	0.00	0.00	22.57
sx10	0.00	0.00	-0.24	0.00	0.00	0.03	0.00	0.62	0.69	1.00	0.00	50.77
x2	0.00	1.00	0.19	0.00	0.00	0.01	0.00	-0.09	-1.07	0.00	0.00	11.28
Lower Bound	0.00	0.00	0.00		0.00							
Upper Bound	infinity	infinity	infinity	infinity	infinity							
Unrestr'd (y/n)?	n	n	n	n	n							
Iteration 6	Single	Double	Triple	Quadraple	Demolish							
Basic	x1	x2	х3	×4	х5	8ж8	sx7	8x2	9кз	sx10	sx11	Solution
z (max)	0.00	0.00	-8.69	0.00	0.00	218.06	0.00	420.27	0.00	608.63	22.90	343442.62
x4	0.00	0.00	0.92	1.00	0.00	0.03	0.00	-0.06	0.00	-0.75	0.00	40.98
sx7	0.00	0.00	0.03		0.00	4.61	1.00	-0.13	0.00	-0.46	-0.02	154.10
х5	0.00	0.00	-0.03		1.00	-4.61	0.00	0.13	0.00	0.46	0.02	245.90
x1	1.00	0.00	-0.07		0.00	0.02	0.00	-1.05	0.00	0.20	0.00	
еж9	0.00	0.00	-0.35		0.00	0.05	0.00	0.90	1.00	1.45	0.00	
х2	0.00	1.00	-0.19		0.00	0.06	0.00	0.87	0.00	1.55	0.01	90.16
Lower Bound	0.00	0.00	0.00		0.00							
Upper Bound	infinity	infinity	infinity	infinity	infinity							
Unrestr'd (y/n)?	n	n	n	n	n							

Iteration 7	Single	Double	Triple	Quadraple	Demolish							
Basic	х1	х2	х3	x4	ж5	вж6	sx7	8ж2	ex9	sx10	sx11	Solution
z (max)	0.00	0.00	0.00	9.49	0.00	218.31	0.00	419.73	0.00	601.52	22.92	343831.72
х3	0.00	0.00	1.00	1.09	0.00	0.03	0.00	-0.06	0.00	-0.82	0.00	44.77
sx7	0.00	0.00	0.00	-0.03	0.00	4.60	1.00	-0.13	0.00	-0.44	-0.02	152.70
х5	0.00	0.00	0.00	0.03	1.00	-4.60	0.00	0.13	0.00	0.44	0.02	247.30
x1	1.00	0.00	0.00	0.07	0.00	0.02	0.00	-1.05	0.00	0.15	0.00	35.82
sx9	0.00	0.00	0.00	0.38	0.00	0.06	0.00	0.87	1.00	1.16	0.01	89.54
х2	0.00	1.00	0.00	0.20	0.00	0.06	0.00	0.86	0.00	1.40	0.01	98.49
Lower Bound	0.00	0.00	0.00	0.00	0.00							
Upper Bound	infinity	infinity	infinity	infinity	infinity							
Unrestr'd (y/n)?	n	n	n	n	n							

 Observe the decimal nature of the solution (given in the book) and understand the rounding. Answer: refer textbook : Introduction to operation research by Hamdy Taha

Page no. 30 remarks .