BITS Pilani
Pilani Campus

Mathematical Foundations for Data Science

MFDS Team

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DSECL ZC416, MFDS

Lecture No. 8

Agenda

- LPP in Standard Form
- FS, BS, BFS, OBFS
- Motivation for the Simplex Method
- Simplex tables(using excel)
- Sensitivity

Standard LPP

Max Z = **cx**

subject to

Ax = b, with b ≥ 0 ;

x ≥ 0;

To be observed:

- Constraints have the equality
- Non-negativity of b
- Non-negativity of variables

Transformations

- Minimization to Maximization

To convert minimization to maximization problem, multiply objective function by -1

- Negative components of **b**

Multiply by -1 both sides in the constraint

- Constraints with $\leq$

Add SLACK Variable

- Constraints with $\geq$

Subtracting SURPLUS Variable

- Variables unrestricted in sign

Terminologies

Given m equations in n unknowns ($n \geq m$) in standard form

1. Set (n-m) variables to zero and determine m unique values

2. the (n-m) are called non-basic variables

3. the m variables are called basic variables

4. a solution that satisfies the constraints and non-negativity –feasible

5. a solution that is basic and satisfies (4) is – basic feasible solution (bfs)

6. All basic variables are > 0 – non-degenerate bfs

7. A bfs that maximizes the objective function is optimal solution

Naive LPP Solvers vs Simplex

- The optimal solution, if it exists, is a corner point
  - Fundamental theorem of linear programming
- Have ${}^{n}C_{m}$ ways of finding the corner point ($n \geq m$)
- Simplex takes just a fraction of the above
- Simplex is iterative and simple to interpret

Principles of Simplex Method

- Start with an initial ***basic*** feasible solution
- Improve the initial solution, if possible
- Stop, when the bfs cannot be improved

Example: *Maximize* $Z = 5x_1 + 2x_2 + 3x_3 - x_4 + x_5$

Subject to $x_1 + 2x_2 + 2x_3 + x_4 = 8$

$3x_1 + 4x_2 + x_3 + x_5 = 7$

$x_i \geq 0$ for all i=1,2,3,4,5

Reddy Mikks Problem

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Maximize $Z = 5x_1 + 4x_2 + 0s_1 + 0s_2 + 0s_3 + 0s_4$

Subject to

$$
\begin{aligned}
6x_1 + 4x_2 + s_1 &= 24 \\
x_1 + 2x_2 + s_2 &= 6 \\
-x_1 + x_2 + s_3 &= 1 \\
x_2 + s_4 &= 2
\end{aligned}
$$

$$x_1, x_2, s_1, s_2, s_3, s_4 \geq 0$$

(Refer to excel sheet for computational aspects)

Special cases

- Unique optimal value
  - Relative increase < 0 for all non-basic variables
- Alternative optima
  - At least one non-basic variable has relative increase = 0
- Unboundedness
  - The pivot column has entries which are ≤ 0
- Infeasibility (not in our scope)
  - Artificial variable is there in the final table and is >0

Sensitivity Analysis

- Sensitivity Analysis (restricted to graphical solutions)
    - ➢ Changes in right hand side
        - ➢ One or more changes is possible
    - ➢ Changes in objective coefficients
        - ➢ One or more changes possible
    - ➢ Complicated changes require concepts in Duality
        - ➢ Not in the present scope

Changes in RHS(Example 3.6-1)

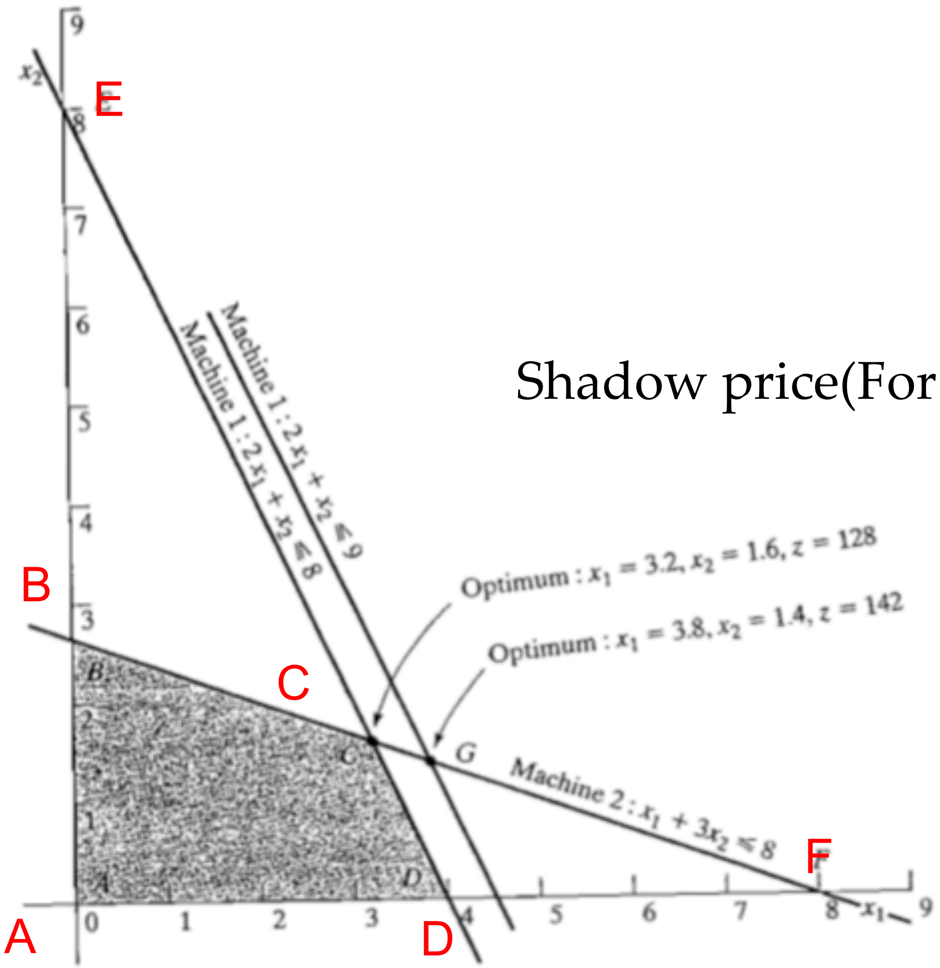
JOBCO produces two products on two machines. A unit of product 1 requires 2 hours on machine 1 and 1 hour on machine 2. For product 2, a unit requires 1 hour on machine 1 and 3 hours on machine 2. The revenues per unit of products 1 and 2 are \$30 and \$20 respectively. The total daily processing time available for each machine is 8 hours.

LPP

Maximize $Z = 30x_1 + 20x_2$

Subject to
$2x_1 + x_2 \leq 8$ (Machine 1)
$x_1 + 3x_2 \leq 8$ (Machine 2)
$x_1, x_2 \geq 0$

Changes in RHS(Example 3.6-1)



$$\text{Maximize } z = 30x_1 + 20x_2$$

$$2x_1 + x_2 \le 8 \quad \text{(Machine 1)}$$
$$x_1 + 3x_2 \le 8 \quad \text{(Machine 2)}$$
$$x_1, x_2 \ge 0$$

Shadow price(For Machine 1) = 142 – 128/9 -8 = $14/hr

Minimum machine 1 capacity at B (0,2.67) = 2.67 hr
Maximum machine 1 capacity at F (8,0) = 16 hr

$2.67 \le$ Machine 1 capacity ≤ 16 hrs

Changes in RHS

	Machine 1	Machine 2
Shadow prce	$ 14/ hour	$ 2 /hour
Minimum capacity	2.67 hours	4 hrs
Maximum capacity	16 hours	24 hours

Questions

1. If JOBCO can increase the capacity of both machines, which machine should receive higher priority?

Ans Machine 1

2. A suggestion is made to increase the capacities of machines 1 and 2 at the additional cost of \$10 / hour. Is this advisable?

Ans Only machine 1 should be increased.

3. If the capacity of machine 1 is increased from the present 8 hours to 13 hours, how will the increase impact the optimum revenue?

Ans Increased to \$198.

4. Suppose the capacity of machine 1 is increased to 20 hours, how will this increase impact the optimum revenue?

Ans We do not have any conclusion

Changes in Objective Coefficients

1. Suppose that the unit revenues for products 1 and 2 are changed to \$35 and \$25 respectively, will the current optimum remain the same?

Ans. Yes, however optimal value changes to \$152.

2. Suppose that the unit revenue of product 2 is fixed at its current value of c_2 = \$20, what is the associated range for c_1, that will keep the optimum unchanged.

Ans. $6.67 \leq c_1 \leq 40$