Homework 12 (Solutions)

Question 1. ***(Prob 40, Page 583)*** *Let R_1 and R_2 be the "divides" and "is a multiple of" relations on the set of all positive integers, respectively. That is, $R_1 = \{(a, b)|a \;\text{ divides }\; b\}$ and $R_2 = \{(a, b)|a \text{ is a multiple of } b\}$. Find*

a) $R_1 \cup R_2$.

b) $R_1 \cap R_2$.

c) $R_1 - R_2$.

d) $R_2 - R_1$.

e) $R_1 \oplus R_2$.

Ans:

a)

$$\begin{aligned} R_1 \cup R_2 &= \{(a, b)|a \;\text{ divides }\; b \text{ or } a \text{ is a multiple of } b\} \\ &= \{(a, b)|a \;\text{ divides }\; b \text{ or } b \text{ divides } a\}. \end{aligned}$$

b)

$$\begin{aligned} R_1 \cap R_2 &= \{(a, b)|a \;\text{ divides }\; b \text{ and } a \text{ is a multiple of } b\} \\ &= \{(a, b)|a \;\text{ divides }\; b \text{ and } b \text{ divides } a\} \\ &= \{(a, b)|a = \pm b \text{ and } a \neq 0\}. \end{aligned}$$

c)

$$\begin{aligned} R_1 - R_2 &= \{(a, b)|a \;\text{ divides }\; b \text{ and } a \text{ is not a multiple of } b\} \\ &= \{(a, b)|a \;\text{ divides }\; b \text{ and } b \text{ does not divide } a\} \\ &= \{(a, b)|a \;\text{ divides }\; b \text{ and } a \neq \pm b\}. \end{aligned}$$

d)

$$\begin{aligned} R_2 - R_1 &= \{(a, b)|\; a \text{ is a multiple of } b \text{ and} a \;\text{ does not divide }\; b\} \\ &= \{(a, b)|b \;\text{ divides }\; a \text{ and } a \neq \pm b\}. \end{aligned}$$

e)

$$R_1 \oplus R_2 = \{(a, b)|(a \;\text{ divides }\; b \text{ or } b \text{ divides } a) \text{ and } a \neq \pm b\}.$$

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Question 2. ***(Prob 41, Page 583)*** *Let R_1 and R_2 be the "congruent modulo 3" and the "congruent modulo 4" relations, respectively, on the set of integers. That is, $R_1 = \{(a, b)|a \equiv b(\text{mod}3)\}$ and $R_2 = \{(a, b)|a \equiv b(\text{mod}4)\}$. Find*

a) $R_1 \cup R_2$.

b) $R_1 \cap R_2$.

c) $R_1 - R_2$.

d) $R_2 - R_1$.

e) $R_1 \oplus R_2$.

Ans:

a) $R_1 \cup R_2 = \{(a,b)|a - b \equiv 0, 3, 4, 6, 8 \text{ or } 9 (\text{mod } 12)\}$.

b) $R_1 \cap R_2 = \{(a,b)|a \equiv b (\text{mod } 12)\}$.

c) $R_1 - R_2 = \{(a,b)|a - b \equiv 3, 6 \text{ or } 9 (\text{mod } 12)\}$.

d) $R_2 - R_1 = \{(a,b)|a - b \equiv 4 \text{ or } 8 (\text{mod } 12)\}$.

e) $R_1 \oplus R_2 = \{(a,b)|a - b \equiv 3, 4, 6, 8 \text{ or } 9 (\text{mod } 12)\}$.

Question 3. ***(Prob 42, Page 583)*** *List the 16 different relations on the set* $\{0, 1\}$.

Ans: The possible ordered pairs are $\{(0,0), (0,1), (1,0), (1,1)\}$. Here are the 16 different relations on the set $\{0, 1\}$:

- ϕ
- {(0,0)}
- {(0,1)}
- {(1,0)}
- {(1,1)}
- {(0,0),(0,1)}
- {(0,0),(1,0)}
- {(0,0),(1,1)}
- {(0,1),(1,0)}
- {(0,1),(1,1)}
- {(1,0),(1,1)}
- {(0,0),(0,1),(1,0)}
- {(0,0),(0,1),(1,1)}
- {(0,0),(1,0),(1,1)}
- {(0,1),(1,0),(1,1)}
- {(0,0),(1,0),(0,1),(1,1)}.

Question 4. ***(Prob 43, Page 583)*** *How many of the 16 different relations on* $\{0, 1\}$ *contain the pair* $(0, 1)$*?*

Ans: The following 8 relations contain the pair $(0, 1)$:

- {(0,1)}
- {(0,0),(0,1)}
- {(0,1),(1,0)}
- {(0,1),(1,1)}
- {(0,0),(0,1),(1,0)}
- {(0,0),(0,1),(1,1)}
- {(0,1),(1,0),(1,1)}
- {(0,0),(1,0),(0,1),(1,1)}.

Question 5. ***(Prob 44, Page 583)*** *Which of the 16 relations on* $\{0, 1\}$*, which you listed in previos Exercise, are*

a) reflexive?

b) irreflexive?

c) symmetric?

d) antisymmetric?

e) asymmetric?

f) transitive?

Ans:

a) reflexive: $\{(0,0),(0,1)\}, \{(0,0),(0,1),(1,1)\}, \{(0,0),(1,0),(1,1)\}, \{(0,0),(1,0),(0,1),(1,1)\}$.

b) irreflexive: $\phi, \{(0,1)\}, \{(1,0)\}, \{(0,1),(1,0)\}$.

c) symmetric: $\phi, \{(0,0)\}, \{(1,1)\}, \{(0,0),(1,1)\}, \{(0,1),(1,0)\}, \{(0,0),(0,1),(1,0)\}, \{(0,1),(1,0),(1,1)\}, \{(0,0),(1,0),(0,1),(1,1)\}$.

d) antisymmetric: $\phi, \{(0,0)\}, \{(0,1)\}, \{(1,0)\}, \{(1,1)\}, \{(0,0),(0,1)\}, \{(0,0),(1,0)\}, \{(0,0),(1,1)\}, \{(0,1),(1,1)\}, \{(1,0),(1,1)\}, \{(0,0),(0,1),(1,1)\}, \{(0,0),(1,0),(1,1)\}$.

e) asymmetric: $\phi, \{(0,1)\}, \{(1,0)\}$.

f) transitive: $\phi, \{(0,0)\}, \{(0,1)\}, \{(1,0)\}, \{(1,1)\}, \{(0,0),(0,1)\}, \{(0,0),(1,0)\}, \{(0,0),(1,1)\}, \{(0,1),(1,1)\}, \{(1,0),(1,1)\}, \{(0,0),(0,1),(1,1)\}, \{(0,0),(1,0),(1,1)\}, \{(0,0),(1,0),(0,1),(1,1)\}$.

Question 6. ***(Prob 9, Page 590)*** *The* $5-$ *tuples in a* $5-$*ary relation represent these attributes of all people in the United States: name, Social Security number, street address, city, state.*

a) Determine a primary key for this relation.

b) Under what conditions would (name, street address) be a composite key?

c) Under what conditions would (name, street address, city) be a composite key?

Ans:

a) Social security number.

b) No two people with the same name live at the same street address (which is likely to occur).

c) No two people with the same name live at the same street address in the same city.

Question 7. ***(Prob 29, Page 590)***

a) What are the operations that correspond to the query expressed using this SQL statement?
SELECT Supplier
FROM Part_needs
WHERE $1000 \leq Part\_number \leq 5000$

b) What is the output of this query given the database in Table 9 as input?

Ans:

a) $P_1(S_C(R))$ with $C = 1000 \leq \text{Part\_number} \leq 5000$. Here R is Part_needs data set.

b) Suplier- 23,31,33.

Question 8. ***(Prob 13, Page 596)*** *Let R be the relation represented by the matrix*

$$M_R = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Find the matrix representing

a) R^{-1}

b) $\overline{R}$

c) R^2.

Ans:

a) $M_{R^{-1}} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = M_R.$

b) $M_{\overline{R}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$

c) $M_{R^2} = M_R \odot M_R = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$

Question 9. *(**Prob 14, Page 596**) Let R_1 and R_2 be relations on a set A represented by the matrices*

$$M_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \text{ and } M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Find the matrices that represent

a) $R_1 \cup R_2$.

b) $R_1 \cap R_2$.

c) $R_2 \odot R_1$.

d) $R_1 \odot R_1$.

e) $R_1 \oplus R_2$.

Ans:

a) $M_{R_1 \cup R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = M_{R_1} \vee M_{R_2}.$

b) $M_{R_1 \cap R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} = M_{R_1} \wedge M_{R_2}.$

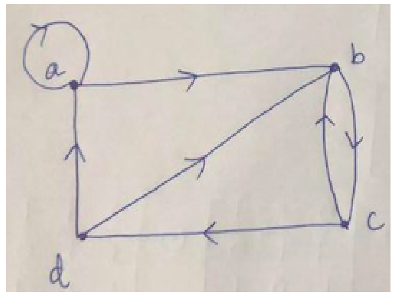
c) $M_{R_2 \odot R_1} = M_{R_1} \odot M_{R_2} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$

d) $M_{R_1 \odot R_1} = M_{R_1} \odot M_{R_1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$

e) $M_{R_1 \oplus R_2} = M_{R_1} \oplus M_{R_2} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$

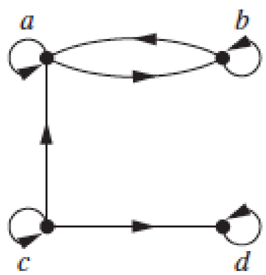
Question 10. ***(Prob 22, Page 597)*** *Draw the directed graph that represents the relation*

$$\{(a,a),(a,b),(b,c),(c,b),(c,d),(d,a),(d,b)\}.$$



Ans:

Question 11. ***(Prob 26, Page 597)*** *List the ordered pairs in the relations represented by the directed graph*



Ans: Ordered pairs in the relations are:

$$\{(a,a),(a,b),(b,a),(c,c),(b,b),(c,a),(c,d),(d,d)\}.$$