Summation Formula

$$\sum_{k=1}^{m} 1 = m, \qquad \sum_{k=1}^{m} k = \frac{m(m+1)}{2}$$

$$\sum_{k=1}^{m} k^2 = \frac{m(m+1)(2m+1)}{6}$$

k	j	Add/Sub flops	Mult Flops	Div Flops
1	$2:n = n-1$ rows	$(n-1)n$	$(n-1)n$	$n-1$
2	$3:n = n-2$	$(n-2)(n-1)$	$(n-2)(n-1)$	$n-2$
3	$4:n = n-3$	$(n-3)(n-2)$	$(n-3)(n-2)$	$n-3$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
k	$k+1:n = n-k-1$	$(n-k)(n-k+1)$	$(n-k)(n-k+1)$	$(n-k)$

$$f(n) = \underbrace{\sum_{k=1}^{n-1} (n-k)}_{\text{division}} + 2\underbrace{\sum_{k=1}^{n-1} (n-k)(n-k+1)}_{\text{Add, Mult}}$$

$$= \sum_{s=1}^{n-1} s + 2\sum_{k=1}^{n-1} (n-k)(n-k+1)$$

$$= \sum_{s=1}^{n-1} s + 2\sum_{s=1}^{n-1} s(s+1)$$

$$= \frac{1}{2}(n-1)n + \frac{2}{3}(n^2-1)n$$

$$\simeq \frac{2}{3}n^3.$$

$$4.03x_1 + 2.16x_2 = -4.61$$
$$6.21x_1 + 3.35x_2 = -7.19$$

3 digit Rounding Arithmetic

WITHOUT PIVOTING

$$\left[\begin{array}{cc|c} 4.03 & 2.16 & -4.61 \\ 6.21 & 3.35 & -7.19 \end{array}\right]$$

$$m = \frac{6.21}{4.03} = 1.54 \text{ (3 digit rounding)}$$

$$R_2 \rightarrow R_2 - R_1 \times m$$

$$\left[\begin{array}{cc|c} 4.03 & 2.16 & -4.61 \\ 0 & 0.02 & -0.09 \end{array}\right]$$

Solving $\boxed{\begin{array}{l} x_2 = -4.5 \\ x_1 = 1.27 \end{array}}$

WITH PIVOTING

$$\left[\begin{array}{cc|c} 6.21 & 3.35 & -7.19 \\ 4.03 & 2.16 & -4.61 \end{array}\right]$$

$$m = \frac{4.03}{6.21} = 0.65$$

$$R_2 \rightarrow R_2 - R_1 \times m$$

$$\left[\begin{array}{cc|c} 6.21 & 3.35 & -7.19 \\ 0 & -0.01 & 0.06 \end{array}\right]$$

$\boxed{\begin{array}{l} x_2 = -6 \\ x_1 = 2.08 \end{array}}$

4 digit Rounding Arithmetic

WITHOUT PIVOTING

$$\left[\begin{array}{cc|c} 4.03 & 2.16 & -4.61 \\ 6.21 & 3.35 & -7.19 \end{array}\right]$$

$$m = \frac{6.21}{4.03} = 1.541 \text{ (4 digit rounding)}$$

$$R_2 \rightarrow R_2 - R_1 \times m.$$

$$\left[\begin{array}{cc|c} 4.03 & 2.16 & -4.61 \\ 0 & 0.021 & -0.086 \end{array}\right]$$

Solving

$\boxed{\begin{array}{l} x_2 = -4.095 \\ x_1 = 1.051 \end{array}}$

WITH PIVOTING

$$\left[\begin{array}{cc|c} 6.21 & 3.35 & -7.19 \\ 4.03 & 2.16 & -4.61 \end{array}\right]$$

$$m = \frac{4.03}{6.21} = 0.649$$

$$R_2 \rightarrow R_2 - R_1 \times m$$

$$\left[\begin{array}{cc|c} 6.21 & 3.35 & -7.19 \\ 0 & -0.014 & 0.056 \end{array}\right]$$

Solving $\boxed{\begin{array}{l} x_2 = -4 \\ x_1 = 1 \end{array}}$

same as EXACT solution.

POSITIVE DEFINITE MATRIX

$x^T A x > 0$ for all non zero x in $\mathbb{R}^n$

Quadratic Form

Eg) Identity Matrix $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is positive definite

$$x^T A x = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = x^2 + y^2$$

Properties of Positive definite matrices

i) They are NON SINGULAR ($\det(A) > 0$) and LU decomposition can be performed on them without having to worry about dividing by zero.

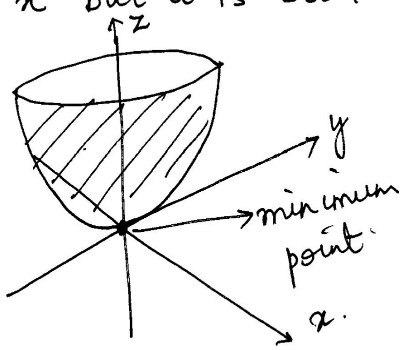
ii) If A is Positive definite matrix, every EIGEN VALUE of A is POSITIVE

iii) Every leading submatrix of A is symmetric and positive definite

iv) All of the diagonal entries of A are POSITIVE

NOTE:- Only symmetric matrix can be called Positive definite

A non symmetric matrix A can still satisfy $x^T A x > 0$ for any nonzero vector x but it is still not considered positive definite



Necessity for Pivoting

If PIVOT is zero
↓
entire process fails

If PIVOT is close to zero
↓
Round off errors may occur.

$a_{11} = 0 \longrightarrow$ Interchange two rows by finding largest available coefficient in the column below pivot equation (PARTIAL PIVOTING)

Complete Pivoting – More Complicated

- Involves Exchange of both rows and columns
- which can change the order of the unknowns.
- Full pivoting is less susceptible to Round off errors but this increase in numerical stability comes at the cost of an increase in the work associated with searching and in the amount of data movement involved.

LU Factorization

Doolittle method – Non zero diagonal elements in the U matrix

Crout – Non zero diagonal elements in the L matrix

Cholesky – Positive definite coefficient matrix symmetric (does not have condition for main diagonal entries)
↓
frequently used in optimisation pbms.