M Tech(Data Science & Engineering)
Introduction to Statistical Methods[ISM]

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Webinar Session No - 1

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Timimgs : 7.30 to 9.00 PM

Learning objectives

- Problems on Basic probability
- Conditional Probability
- Baye's theorem
- Random variables

Problem - 1

If two dice are thrown, What is the probability that the sum is

(i) greater than 8

(ii) neither 7 nor 11.

Solution -1

When two dice are thrown the sample space contains 36 elements

S = { (1,1) , (1,2), (1,3), (1,4), (1,5), (1,6)

.....

.....

(6,1) , (6,2), (6,3), (6,4), (6,5), (6,6) }

Solution -1

(i) Let A be the event that the sum on the two dice

$P(A > 8) = P(A = 9) + P(A = 10) + P(A = 11) + P(A = 12)$

sum is 9 = { (3,6), (6,3), (4,5), (5,4) }

$P(A = 9) = \dfrac{\text{Favourable cases for A}}{\text{Exhaustive number of cases}} = {}^{4}/_{36}$

sum is 10 = { (4,6), (6,4), (5,5) }

$P(A = 10) = \dfrac{\text{Favourable cases for A}}{\text{Exhaustive number of cases}} = {}^{3}/_{36}$

sum is 11 = { (5,6), (6,5) }

$P(A = 11) = {}^{2}/_{36}$

sum is 12 = { (6,6) } $\rightarrow P(A=12) = 1/36$

$P(A) = {}^{1}/_{36}$

Solution -1

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$P(A > 8) = P(A = 9) + P(A = 10) + P(A = 11) + P(A = 12)$

$= {}^{4}/_{36} + {}^{3}/_{36} + {}^{2}/_{36} + {}^{1}/_{36}$

$= {}^{10}/_{36}$

(ii) **Let B denote the event of getting the sum is 7**

sum is 7 = { (1,6), (6,1), (2,5), (5,2), (3,4), (4,3) }

$P(B) = {}^{6}/_{36} = {}^{1}/_{6}$

$\cup$ $\cap$

$P(B' \cap C^c)$

Let C denote the event of getting the sum is 11

sum is 11 = { (5,6), (6,5) }

$P(C) = {}^{2}/_{36}$

$A \cap B = \phi$

Solution -1

Required probability = $P(\overline{B} \cap \overline{C}) = P[(B \cup C)^c]$

$= 1 - P(B \cup C)$

$= 1 - [P(B) + P(C)]$ [A and B are disjoint events] B C

$= 1 - \frac{1}{6} - \frac{1}{18}$

$= \frac{7}{9}$

$P(A \cup B) = P(A) + P(B)$

Problem - 2

Let A denote the event that the next request for assistance from a statistical software consultant relates to the SPSS package, and let B be the event that the next request is for help with SAS. Suppose that $P(A) = 0.30$ *and* $P(B) = 0.50$

a. Calculate $P(A')$.

b. Calculate $P(A \cup B)$.

c. Calculate $P(A' \cap B')$.

1 SPSS

2 SAS

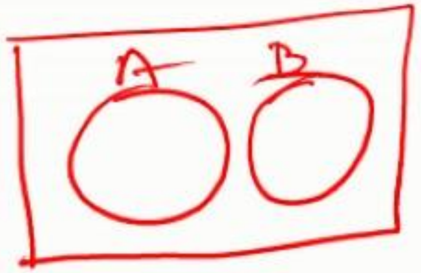
Solution -2

Let A be the event that the next request for assistance from a statistical software consultant relates to the SPSS package.

Let B be the event that the next request is for help with SAS.

$P(A) = 0.30 \; and \; P(B) = 0.50$

$P(A \cap B) = P(A) * P(B)$



$A \cap B = \phi$

i. $\boldsymbol{P(A')} = 1 - P(A)$

$= 1 - 0.30$

$= \mathbf{0.70}$

ii. $\boldsymbol{P(A \cup B)} = P(\mathrm{A}) + P(B) - P(A \cap B)$

$= P(\mathrm{A}) + P(B) - \{P(A) * P(B)\}$ [A and B are independent events]

$= 0.3 + 0.5 - [0.3 * 0.5]$

$= \mathbf{0.65}$

iii. $P(\bar{A} \cap \bar{B}) = P[(A \cup B)^c]$

$= 1 - P(A \cup B)$

$= \mathbf{1 - 0.65 = 0.35}$

$P(A \cap B) = P(A) \times P(B)$

Problem - 3

The route used by a certain motorist in commuting to work contains two intersections with traffic signals. The probability that he must stop at the first signal is .4, the Analogous probability for the second signal is .5, and the probability that he must stop at at least one of the two signals is .6. What is the probability that he must stop

a. At both signals? $\rightarrow P(A \cap B)$

b. At the first signal but not at the second one? $\rightarrow P(A \cap \bar{B})$ 0.6

c. At exactly one signal?

$P[(A \cap \bar{B}) \cup (\bar{A} \cap B)]$

Solution -3

Let A be the event that he must stop at the first signal

Let **B** be the event that he must stop at the second signal

$P(A) = 0.40 \; and \; P(B) = 0.50$

And given that

$P(A \cup B) = 0.6$

i. $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

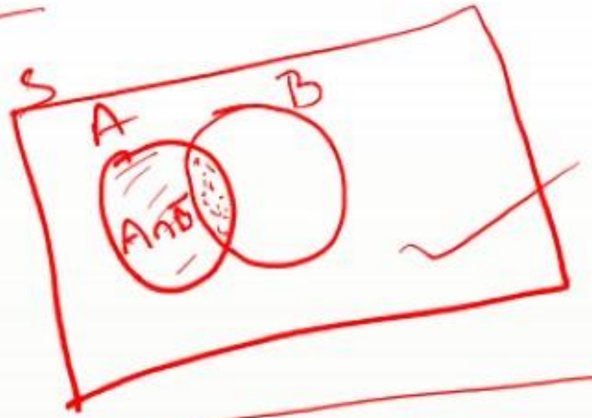
$= 0.4 + 0.5 - 0.6 = 0.3$

ii. $P(A \cap B') = P(A) - P(A \cap B)$

$= 0.4 - 0.3 = 0.1$

iii. P(At exactly one signal) $= P(A \cup B) - P(A \cap B)$

$= 0.6 - 0.3 = 0.3$



$P[(A\cap\bar{B})\cup(\bar{A}\cap B)] = P(A\cap\bar{B}) + P(\bar{A}\cap B)$

$P[(A\cap\bar{B})\cup(\bar{A}\cap B)]$

$= P(A\cap\bar{B}) + P(\bar{A}\cap B)$

$= P(A) - P(A\cap B)$

$+ P(B) - P(A\cap B)$

$= P(A\cup B) - P(A\cap B)$

Problem - 4 ✓

3+2=5

The odds that person X speaks the truth are 3:2 and the odds that person Y Speaks the truth are 5:3. In what percentage of cases are they likely to contradict each on an identical point?

$P(X) = \frac{3}{}$

Solution -4

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Let A = X speaks the truth

$\bar{A}$ = X tell a lie

Let B = Y speaks the truth

$B' = B^c = \bar{B}$ = Y tell a lie

$$P(A) = \frac{3}{3+2}\,;\; P(\bar{A}) = \frac{2}{3+2}$$

$$P(B) = \frac{5}{5+3}\,;\; P(\bar{B}) = \frac{3}{5+3}$$

The event C that X and Y contradict each other on an-identical point.

That can happen in two ways

(i) X speaks the truth and Y tell a lie, i.e, $(A \cap \bar{B})$

(ii) X tell a lie and Y speaks the truth, i.e, $\bar{A} \cap B$

Solution -4

the events $(A \cap \bar{B})$ and $(\bar{A} \cap B)$ mutually exclusive events

$P[(A \cap \bar{B}) \cup (\bar{A} \cap B)]$

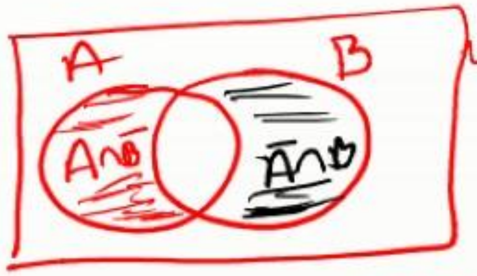
P(C) = P$(A \cap \bar{B})$ + P$(\bar{A} \cap B)$

= $P(A)\,.\,P(\bar{B})$ + $P(\bar{A})\,.\,P(B)$

[A and B are independent events]

$= \frac{3}{5} . \frac{3}{8} + \frac{2}{5} . \frac{5}{8}$

= 0.475



$A \cap \bar{B}$ & $\bar{A} \cap B$

47.5 % **of cases are they likely to contradict each on an identical point.**

$A \ \bar{B}$

A & B

$P(A \cap B) = P(A) . P(B)$

$X \to$ Truth

$Y \to$ Truth / lie

Problem - 5

Consider the following information about travellers on vacation (based partly on a recent Travelocity poll): **40%** check work email, **30%** use a cell phone to stay connected to work, **25%** bring a laptop with them, **23%** both check work email and use a cell phone to stay connected, and **51%** neither check work email nor use a cell phone to stay connected nor bring a laptop. In addition, **88** out of every 100 who bring a laptop also check work email, and **70** out of every 100 who use a cell phone to stay connected also bring a laptop.

$P(A_E' \cap A_C' \cap A_L') = 0.51$

a. What is the probability that a randomly selected traveller who checks work email also uses a cell phone to stay connected?

b. What is the probability that someone who brings a laptop on vacation also uses a cell phone to stay connected?

$P(A_E/A_L) = 0.88$ $P(A_C/A_E) = ?$

Solution - 5

Let $A_E = \{Check\ work\ mail\}$

Let $A_c = \{use\ a\ cell\ phone\ to\ stay\ connected\ to\ work\}$

Let $A_l = \{bring\ a\ laptop\}$

We are given the following probabilities

$P(A_E) = 0.4$

$P(A_c) = 0.3$ and $P(A_l) = 0.25$

$P(A_E \cap A_c) = 0.23$

$P(A'_E \cap A'_c \cap A'_l) = 0.51$

$P\left({}^{A_E}/_{A_l}\right) = 0.88$

$P\left({}^{A_l}/_{A_c}\right) = 0.7$

A_L/A_C

$P(A_C \cap A_L) = \underset{=0.3}{P(A_C)} \cdot \underset{=0.7}{P(A_L/A_C)}$

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a. We need to find the probability that a randomly selected traveller who checks work email also uses a cell phone to stay connected.

Which is a conditional probability of A_c given A_E i.e,

$P(A/B) = \frac{P(A \cap B)}{P(B)}$

$$P\left({}^{A_c}/_{A_E}\right) = \frac{P(A_c \cap A_E)}{A_E}$$

$$= \frac{0.23}{0.4} = 0.575$$

innovate achieve lead

b. The probability that someone who brings a laptop on vacation also uses a cell phone to stay connected.

Which is a conditional probability of A_c given A_l i.e,

$$P\left({}^{A_c}/_{A_l}\right) = \frac{P(A_c \cap A_l)}{A_l} = \frac{P\left({}^{A_l}/_{A_c}\right).P(A_c)}{P(A_l)}$$

$$= \frac{(0.7)*(0.3)}{0.25} = 0.84$$

A_c & A_l

$P(A \cap B) = P(B) \cdot P(A/B)$

or

$= P(A) \cdot P(B/A)$

Problem - 6

A certain shop repairs both audio and video components. Let A denote the event that the next component brought in for repair is an audio component, and let B be the event that the next component is a compact disc player (so the event B is contained in A). Suppose that $P(A) = 0.6$ and $P(B) = 0.05$.

What is $P(B/A)$?

$B \subseteq A$

Solution - 6

Let A denote the event that the next component brought in for repair is an audio component

and

Let B be the event that the next component is a compact disc player

P(A) = 0.6 and P(B) = 0.05

and given that $B \subseteq A$

From sets operations $A \cap B = B$

A∩B

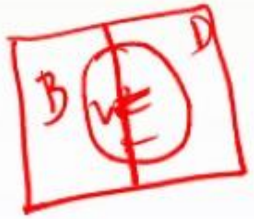
then P($A \cap B$) = P (B) = 0.05

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{0.05}{0.6} = 0.0833$$

Problem - 7

A company that manufactures video camera as produces a basic model and a deluxe model. Over the years 40% of the cameras sold have been the basic model. Of those buying the basic model 30% purchase an extended warranty, whereas 50% of all deluxe purchasers do so. What is the probability that a randomly selected customer has an extended warranty, how likely is it that he or she has a basic model?

$P(B/E) = ?$



B D

$P(B) = 0.4$

$P(D) = 1 - P(B) = 0.6$

Solution - 7

Let B be the event of a basic model.

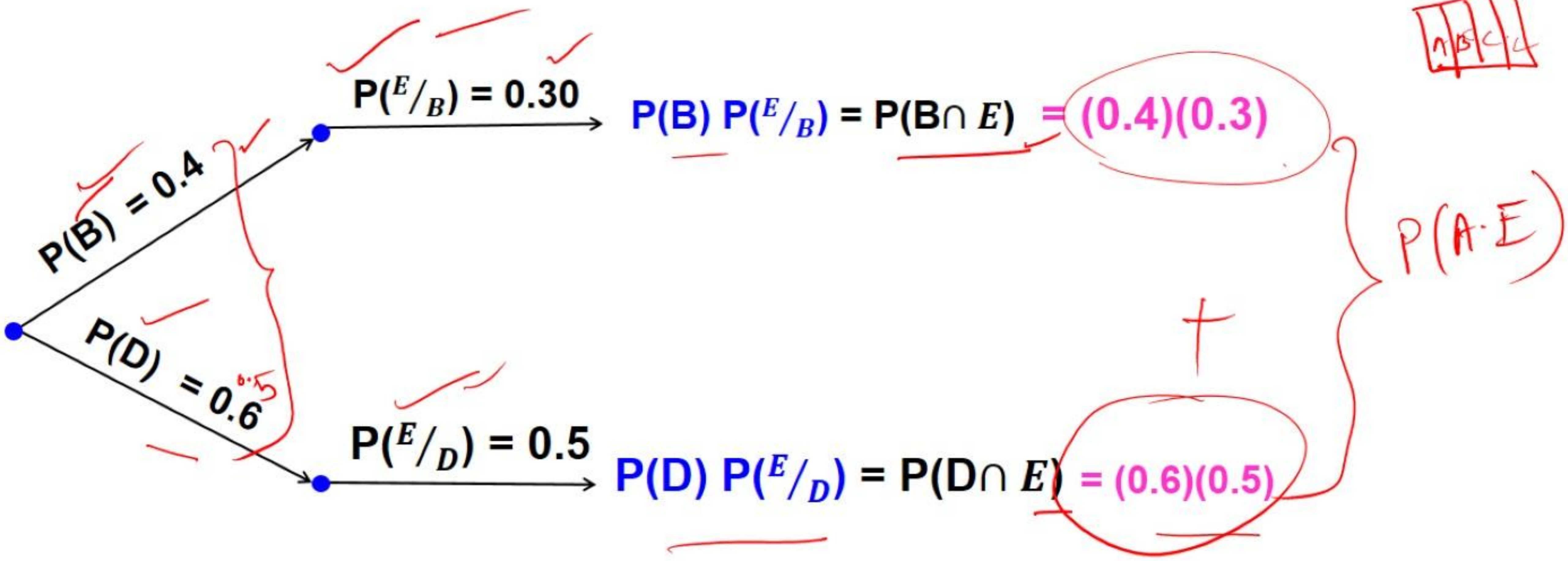
Let D be the event of a deluxe model.

Let E be event of extended warranty.

$P(B) = 0.40$, $P(D) = 0.60$,

$P(E / B) = 0.30$, and $P(E / D) = 0.50$

Probability Tree Diagram



$$P(E) \quad = \quad P(B)\,P(^{E}/_{B}) + P(D)\,P(^{E}/_{D}) \quad = \; 0.42$$

Solution - 7

the probability that a randomly selected customer has an extended warranty, how likely is it that he or she has a basic model is

$$P(B/E) = \frac{P(B \cap E)}{P(E)} = \frac{(0.4)(0.3)}{0.42} = 0.2857$$

$P(B \cap E) = P(E/B) \cdot P(B)$

$P(E) = P[(B \cap E) \cup (D \cap E)]$

$= P(B \cap E) + P(D \cap E)$

E B

Problem - 8

“A company wants to find reasons for the dissatisfaction among employees because of which they are leaving the company. With this view they took the feedback among the employees and identified three reasons i.e. working conditions, pay hike, commuting. The probabilities of dissatisfaction with these three factors are 0.6, 0.3 and 0.1 respectively. And the probabilities that they are leaving the organization with these reasons are 0.3, 0.5and 0.8 respectively.”

As a data scientist, use an appropriate statistical model / method to model this case and suggest the company the probable reasons on priority so that they can focus on it to retain the employees.

$P(W) = 0.6$
$P(P.H) = 0.3$
$P(C) = 0.1$
$\rightarrow 1$

Solution - 8

Let W.C **=** with the working conditions employees are dissatisfaction**.**

$P(W.C) = 0.6$

Let P.H **=** with the pay hike employees are dissatisfaction**.**

$P(P.H) = 0.3$

Let CM **=** with the commuting employees are dissatisfaction**.**

$P(CM) = 0.1$

And

Let L **be event that they are** leaving the company.

Solution - 8

$$P(^{L}/_{W.C}) = 0.3\ ,\quad P(^{L}/_{P.H}) = 0.5,\qquad P(^{L}/_{CM}) = 0.8$$

P[dissatisfaction among employees because of W.C they are leaving the company]

$$P[(W.C) \cap L] = P(W.C)P(^{L}/_{W.C}) = (0.6\,)(0.3) = 0.18$$

P[dissatisfaction among employees because of P.H they are leaving the company]

$$P[(P.H) \cap L] = P(P.H)P(^{L}/_{P.H}) = (0.3\,)(0.5) = 0.15$$

P[dissatisfaction among employees because of CM they are leaving the company]

$$P[(CM) \cap L] = P(CM)P(^{L}/_{CM}) = (0.1\,)(0.8) = 0.08$$

Solution - 8

$P[\text{leaving the company}] = P[L] = P[(W.C) \cap L] + P[(P.H) \cap L] + P[(CM) \cap L]$

$= P(W.C)P(L/_{W.C}) + P(P.H)P(L/_{P.H}) + P(CM)P(L/_{CM})$

$= (0.6)(0.3) + (0.3)(0.5) + (0.1)(0.8)$

$= 0.41$

$P[\text{they are leaving the organization, because of W.C}]$

$= P(W.C/_{L}) = \frac{P[(W.C)\cap L]}{P[L]} = \frac{0.18}{0.41} = 0.4390$

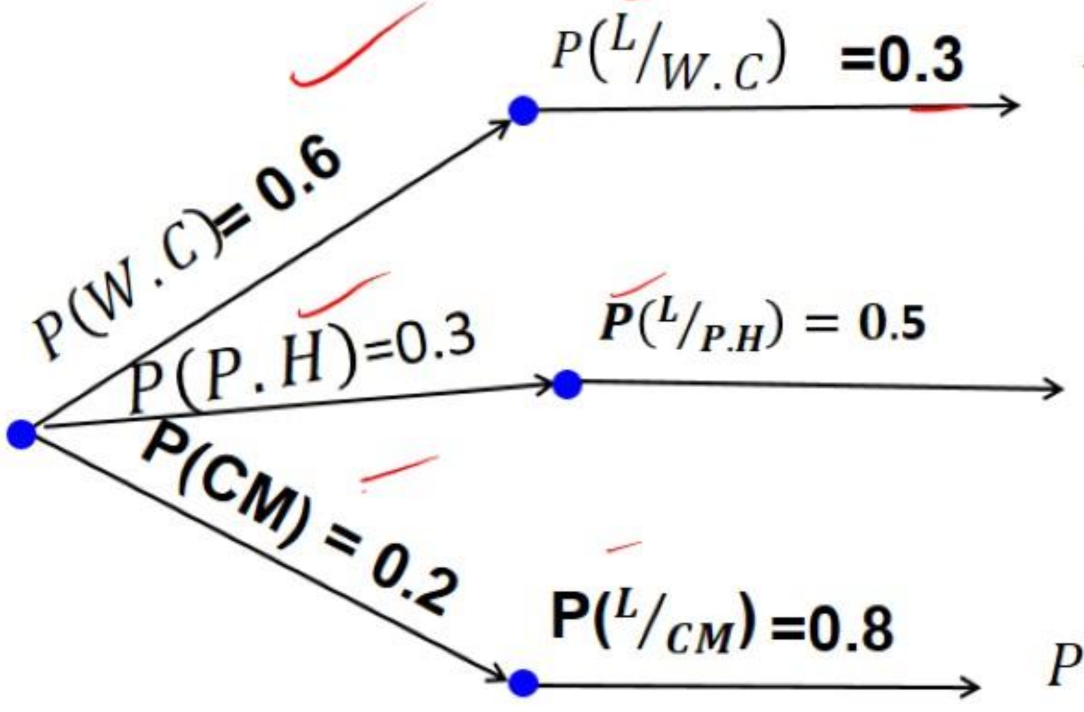
$P(P.H/_{L}) = \frac{P[(P.H)\cap L]}{P[L]} = \frac{0.15}{0.41} = 0.3658$

$P(CM/_{L}) = \frac{P[(CM)\cap L]}{P[L]} = \frac{0.08}{0.41} = 0.1951$

W.C

Most of the employees are leaving the organization with the reason working conditions.

Probability Tree Diagram



P(W.C ∩ L)

$P(W.C)\, P(L/_{W.C})$ = (0.6)(0.3) = 0.18

P(P.H ∩ L]

$P(P.H)\, P(L/_{P.H})$ = (0.3)(0.5) = 0.15

P(CM ∩ L)

$P(CM)\, P(L/_{CM})$ = (0.1)(0.8) = 0.08

P(L) $= P(W.C)\, P(L/_{W.C}) + P(P.H)\, P(L/_{P.H}) + P(CM)\, P(L/_{CM})$ = 0.41

Problem - 9

The chance that doctor A will diagnose a disease X correctly is 60%. The chance that a patient will die by his treatment after correct diagnose is 40% and the chance of death by wrong diagnose is 70%. A patient of doctor A, who had disease X, died. What is the chance that his disease was correctly.

Solution-9

Let E_1= event that disease X is diagnosed correctly by doctor A

Let E_2= event that a patient of doctor A who has disease X died

Then $P(E_1) = \frac{60}{100} = 0.6$ and $P(\bar{E}_1) = 1 - P(E_1) = 0.4$

$P\left({}^{E_2}/_{E_1}\right) = \frac{40}{100} = 0.4$ and $P\left({}^{E_2}/_{\bar{E}_1}\right) = \frac{70}{100} = 0.7$

By Baye's theorem

$$P\left(\frac{E_1}{E_2}\right) = \frac{P(E_1)P\left(\frac{E_2}{E_1}\right)}{P(E_1)P\left(\frac{E_2}{E_1}\right)+P(\bar{E}_1)P\left(\frac{E_2}{\bar{E}_1}\right)}$$

$$= \frac{0.6 * 0.4}{0.6 * 0.4 + 0.4 * 0.7} = \frac{6}{13}$$

Problem - 10

In answering a question on a multiple choice test a student either knows the answer or he guesses.

Let p be the probability that he knows the answer and 1-p be the probability that he guesses. Assume that a student who guesses at the answer will be correct probability is 1/5, where 5 is the number of multiple-choice alternatives.

What is the probability that a student knows the answer to a question given that he answer it correctly?

Solution - 10

Let A be the event the student knows the right answer .

$P(A) = p$

Let B be the event the student guesses the right answer .

$P(B) = 1 - p$

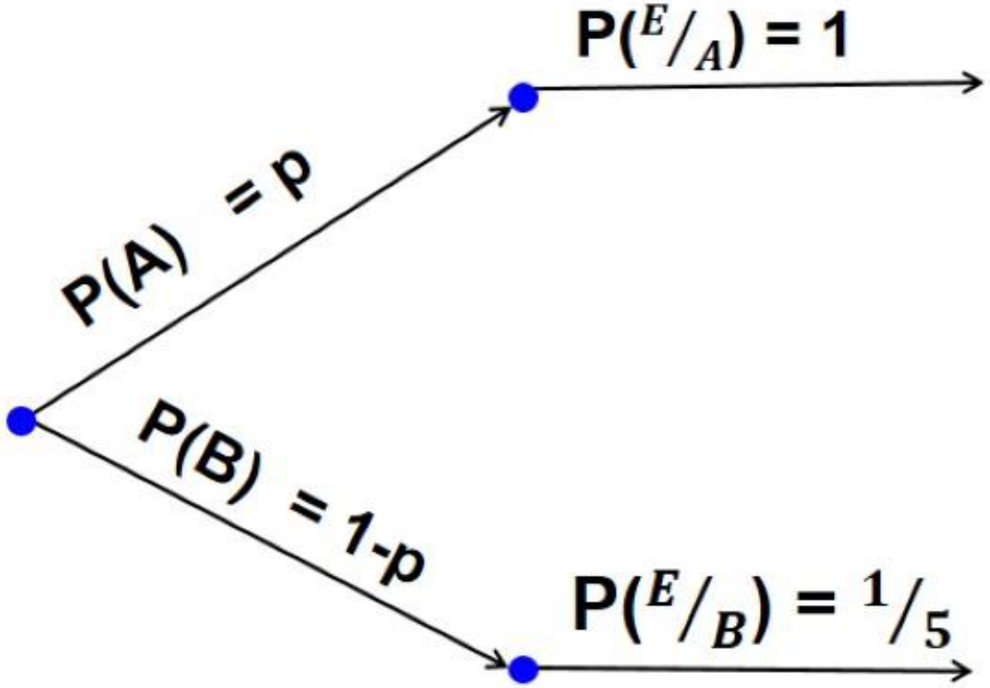
And

Let E be event the student gets the right answer .

$P(E/A) = 1$ [P(student gets the right answer given that he knows the right answer)]

$P(E/B) = 1/5$

Probability Tree Diagram



$P(A)\,P(E/A) = P(A \cap E) = (p)(1)$

$P(B)\,P(E/B) = P(B \cap E) = (1-p)(1/5)$

$$P(E) = P(A)\,P(E/A) + P(B)\,P(E/B) = (4p+1)/5$$

Solution - 10

The probability that a student knows the answer to a question given that he answer it correctly

$$P(A/E) = \frac{P(A \cap E)}{P(E)} = \frac{p}{(4p+1)/5} = \frac{5p}{(4p+1)}$$

Probability Mass function(Pmf)

(Probability distribution function)

Let 'X' be a discrete random variable taking values $x_1, x_2, \dots \dots . x_n$ then the probability mass function $P(X = x)$ is defined under the following conditions

(i) $P(X = x) \geq 0$

(ii) $\sum_{\forall x} P(X = x) = 1$

Continuous Probability Distributions

A random variable is called continuous when it assumes values in a given interval.

The probability that a random variable X assumes different values x in a given interval, say [a, b], is denoted by f(x) = P(a ≤ X ≤ b), called probability density function (pdf).

And

$$\mathrm{f(x)} = P(a \le X \le b) = \int_a^b f(x)\,\mathrm{dx}$$

Properties of continuous probability distribution

A function f(x) to be a probability density function (pdf),
if it satisfies the following conditions

1. $f(X) \geq 0$
2. $\int_{-\infty}^{\infty} f(x)dx = 1$
3. For any $a \leq$ b,

$$P(a \leq X \leq b) = \int_{a}^{b} f(x)dx$$

Let X be a continuous random variable.

Then $P(a \le X \le b) = P(a < X \le b)$

$= P(a \le X < b)$

$= P(a < X < b)$

NOTE: The Cumulative Distribution Function of continuous random variable X is denoted by F(X) and is defined as

$$F(X) = P(X \le x) = \int_{-\infty}^{x} f(x)dx$$

and $\mathbf{f(X)} = \frac{\mathbf{d}}{\mathbf{dx}}[\mathbf{F(X)}]$

Problem - 11

Verify that $P(X) = \frac{x+3}{25}$ $for\ x = 1,2,3,4,5$
Serve as Probability mass function?

Solution:

Given that $P(X) = \frac{x+3}{25} \ for \ x = 1,2,3,4,5$

$P(X = 1) = \frac{1+3}{25} = \frac{4}{25},$

$P(X = 2) = \frac{5}{25}$

$P(X = 3) = \frac{6}{25}$

$P(X = 4) = \frac{7}{25}$

$P(X = 5) = \frac{8}{25}$

$$\sum_{x=1}^{5} P(X = x) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)$$

$$= \frac{4}{25} + \frac{5}{25} + \frac{6}{25} + \frac{7}{25} + \frac{8}{25}$$

$$= \frac{30}{25} > 1$$

Given P(X) is not a Probability mass function

[we know that Total probability is 1]

Problem - 12

For the discrete probability distribution

X=x	0	1	2	3	4	5	6	7
P(X=x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

find (i) k (ii) $P(X < 6)$ (iii) $P(X \geq 6)$

(iv) Mean and Variance

Solution - 12

(i) We know that the Total probability is 1

$$\sum_{i=0}^{7} P(X = x_i) = 1$$

$$0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$$

$$10k^2 + 9k - 1 = 0$$

$$k = -1 \; and \; k = {}^{1}/_{10}$$

then $k = \frac{1}{10}$ [since P(X) ≥ 0, so $k \neq -1$]

Solution - 12

(ii) $P(X < 6) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$

$+P(X = 4) + P(X = 5)$

$= 0 + k + 2k + 2k + 3k + k^2$

$= k^2 + 8k$

$= 0.81$ [$k = \frac{1}{10} = 0.1$]

(iii) $P(X \geq 6) = P(X = 6) + P(X = 7)$

or

$P(X \geq 6) = 1 - P(X < 6) = 1 - 0.81$

$= 0.19$

Solution - 12

Mean = $E[x] = \sum_{x=0}^{7} xP(X = x)$

= 0 + k + 4k + 6k + 12k + $5k^2$ + $12k^2$ + $49k^2 + 7k$

= $66k^2 + 30k = 3.66$

X=x	0	1	2	3	4	5	6	7
P(X=x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

$\boldsymbol{var(x) = E(x^2) - [E(x)]^2}$

$E(x^2) = \sum_{x=0}^{7} x^2 P(X = x)$

= 0 + k + 8k + 18k + 48k + $25k^2$ + $72k^2$ + $343k^2 + 49k$

= $440k^2 + 75k = 16.8$

Then $var(x) = E(x^2) - [E(x)]^2 = 16.8 - (3.66)^2$

= 3.4044

Problem - 13

If a random variable has the probability density f(X) as

$$f(x) = \begin{cases} k(1-x^2), for\ 0 < x < 1 \\ 0\ , \quad\quad otherwise \end{cases}$$

Find (i) k (ii) $P(0.1 \leq X \leq 0.2)$ (iii) $P(X > 0.5)$

(iv) Mean and Variance

Solution-13

Given that f(x) $= \begin{cases} k(1-x^2), for\ 0 < x < 1 \\ 0, \quad otherwise \end{cases}$

(i) We know that $\int_{-\infty}^{\infty} f(x)dx = 1$

$$\int_{-\infty}^{0} f(x)dx + \int_{0}^{1} f(x)dx + \int_{1}^{\infty} f(x)dx = 1$$

$$0 + \int_{0}^{1} k(1-x^2)\, dx + 0 = 1$$

$$k\left(x - \frac{x^3}{3}\right)_0^1 = 1$$

$$\mathrm{k} = \frac{3}{2}$$

Solution- 13

(ii) $P(0.1 \leq X \leq 0.2) = \int_{0.1}^{0.2} f(x)dx$

$$= \int_{0.1}^{0.2} k(1 - x^2)\, dx$$

$$= \frac{3}{2}\left(x - \frac{x^3}{3}\right)_{0.1}^{0.2} \; [\mathrm{k} = \frac{3}{2}\,]$$

$$= \frac{3}{2}(0.1 - \frac{0.007}{3})$$

$$= 0.2965$$

Solution-13

(iii) $P(X > 0.5) = \int_{0.5}^{\infty} f(x)\,dx$

$$= \int_{0.5}^{1} f(x)dx + \int_{1}^{\infty} f(x)dx$$

$$= \int_{0.5}^{\infty} k(1 - x^2)dx$$

$$= \frac{3}{2}\left(x - \frac{x^3}{3}\right)_{0.5}^{1} \; [\mathrm{k} = \frac{3}{2}]$$

$$= \frac{3}{2}(\frac{2}{3} - 0.4583)$$

$$= 0.3125$$

Solution- 13

Mean = $E(x) = \int_{-\infty}^{\infty} x f(x) dx$

$$= \int_{-\infty}^{0} x f(x) dx + \int_{0}^{1} x f(x) dx + \int_{1}^{\infty} x f(x) dx$$

$$= 0 + \frac{3}{2}\int_{0}^{1} x(1 - x^2)\, dx + 0$$

$$= \frac{3}{2}\left(\frac{x^2}{2} - \frac{x^4}{4}\right)_0^1$$

$$= \mathbf{\frac{3}{8}}$$

Solution-13

$var(x) = E(x^2) - [E(x)]^2$

$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)dx$

$$= \int_{-\infty}^{0} x^2 f(x)dx + \int_{0}^{1} x^2 f(x)dx + \int_{1}^{\infty} x^2 f(x)dx$$

$$= 0 + \frac{3}{2}\int_{0}^{1} x^2(1 - x^2)\, dx + 0$$

$$= \frac{3}{2}\left(\frac{x^3}{3} - \frac{x^5}{5}\right)_0^1$$

$$= \frac{2}{10}$$

Then $var(x) = E(x^2) - [E(x)]^2 = \frac{2}{10} - \left(\frac{3}{8}\right)^2$

$= 0.0593$

Thanks