Artificial & Computational Intelligence

DSE CLZG557

M5 : Probabilistic Representation & Reasoning

Raja vadhana P

Assistant Professor,

BITS - CSIS

BITS Pilani
Pilani Campus

Course Plan

M1 Introduction to AI

M2 Problem Solving Agent using Search

M3 Game Playing, Constraint Satisfaction Problem

M4 Knowledge Representation using Logics

M5 Probabilistic Representation and Reasoning

M6 Reasoning over time, Reinforcement Learning

Module 5: Probabilistic Representation and Reasoning

innovate achieve lead

A. Inference using full joint distribution

B. Bayesian Networks

  I. Knowledge Representation

  II. Conditional Independence

  III. Exact Inference (One problem was completed)

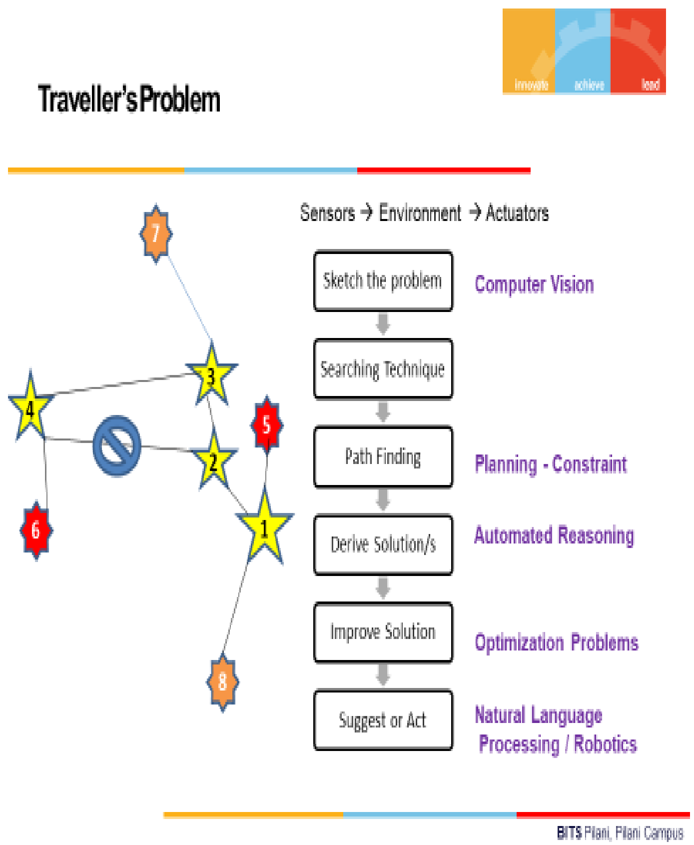
  IV. Introduction to Approximate Inference

Knowledge Representation & Reasoning

Monotonic Reasoning	Non-Monotonic Reasoning		Other Interesting Representations
Propositional Logic	Probabilistic Models		Fuzzy Logic
Predicate Logic			Semantic Nets
			Conceptual Dependency

Reasoning

Reasoning



Traveller's Problem
Sensors → Environment → Actuators
Sketch the problem
Searching Technique
Path Finding
Derive Solution/s
Improve Solution
Suggest or Act
Computer Vision
Planning - Constraint
Automated Reasoning
Optimization Problems
Natural Language Processing / Robotics
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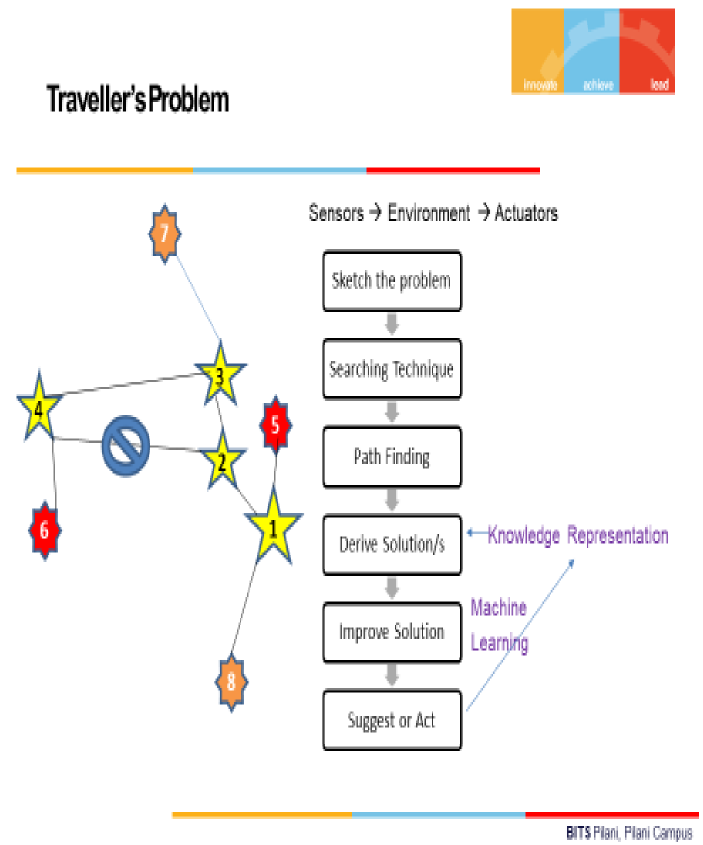

Traveller's Problem
Sensors → Environment → Actuators
Sketch the problem
Searching Technique
Path Finding
Derive Solution/s
Improve Solution
Suggest or Act
Knowledge Representation
Machine Learning
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Reasoning

- Monotonic Reasoning

- Non- Monotonic Reasoning

  Dependency Directed Backtracking: when a statement is deleted as “ no more valid”, other related statements have to be backtracked and they should be either deleted or new proofs have to be found for them. This is called dependency directed backtracking (DDB)

Reasoning

- Monotonic Reasoning
- Non- Monotonic Reasoning

Monotonic	Non-Monotonic
Consistent	Relaxed Consistency
Complete Knowledge	Incomplete Knowledge
Static	Dynamic
Discrete	Continuous & Learning Agent
Predicate Logic	Probabilistic Model

Uncertainty

You can reach Bangalore Airport from MG Road within 90 mins if you go by route A.

- There is uncertainty in this information due to partial observability and non determinism
- Agents should handle such uncertainty

Previous approaches like Logic represent all possible world states

Such approaches can’t be used as multiple possible states need to be enumerated to handle the uncertainty in our information

Uncertainty

You can reach Bangalore Airport from MG Road within 90 mins if you go by route A.

Road Block	Festival Season	Weekend	Observation (20)	Prob
F	F	F	12	0.6
F	F	T	3	0.15
F	T	F	2	0.1
F	T	T	2	0.1
T	F	F	0	0
T	F	T	0	0
T	T	F	1	0.05
T	T	T	0	0
				=1

Belief

You can reach Bangalore Airport from MG Road within 90 mins if you go by route A.

“we are 80% confident that it would be true on any given day”

Augmentation: If we know that it is evening, the Probability of the statement can be 0.4

Probability Theory

Basics

Conditional Probability

Chain Rule

Independence

Conditional Independence

Belief Nets

Joint Probability distribution

Conditional Probability

Towards Chain Rule:

P(a | b) = P(a,b) / P(b)

P(a, b) = P(a | b) P(b)

P(a, b , c) = P(a , x)   where x= b,c

P(a,x)   = P(a | x) . P(x)

= P(a | bc) . P(b, c)

= P(a | bc) . P(b | c). P(c)

Hence : P(a,b,c) = P(a | bc) . P(b | c). P(c)

Chain Rule : Generalization

$$\mathbf{P(X_1, X_2, \ldots X_K) = \prod P(X_i \mid X_{i-1}, \ldots X_1)}$$

Where i = k to 1 (reverse)

Probability Theory

If 'a' and 'b' are conditionally independent in the presence of evidence 'c' then following holds:

<u>Conditional Independence</u>

P(a | b c) = P(a | c)

Extension:

P(a b | c) = P(a | c) . P(b | c)

Independence

If we have two random variables, TimeToBnlrAirport and HyderabadWeather

P(TimeToBnlrAirport, HyderabadWeather)

To determine their relation, use the product rule

= P(TimeToBnlrAirport | HyderabadWeather) / P(HyderabadWeather)

However, we would argue that HyderabadWeather and TimeToBnlrAirport doesn't have any relation and hence

P(TimeToBnlrAirport | HyderabadWeather) = P(TimeToBnlrAirport)

This is called Independence or Marginal Independence

Independence between propositions a and b can be written as

$$P(a \mid b) = P(a) \quad \text{or} \quad P(b \mid a) = P(b) \quad \text{or} \quad P(a \wedge b) = P(a)P(b)$$

Conditional Independence

2 random variables A and B are conditionally independent given C iff

$$P(a, b \mid c) = P(a \mid c)\, P(b \mid c) \text{ for all values } a, b, c$$

More intuitive (equivalent) conditional formulation

- A and B are conditionally independent given C iff

  $$P(a \mid b, c) = P(a \mid c) \text{ OR } P(b \mid a, c) = P(b \mid c), \text{ for all values } a, b, c$$

- Intuitive interpretation:

  $P(a \mid b, c) = P(a \mid c)$ tells us that learning about b, given that we already know c, provides no change in our probability for a, i.e., b contains no information about a beyond what c provides

  $P(R \mid F, P) = P(R \mid P)$

Probabilistic Inference

Computation of posterior probabilities given observed evidence, i.e., full joint probability distribution

	$toothache$		$\neg toothache$	
	$catch$	$\neg catch$	$catch$	$\neg catch$
$cavity$	0.108	0.012	0.072	0.008
$\neg cavity$	0.016	0.064	0.144	0.576

Query: P(cavity ∨ **toothache)**

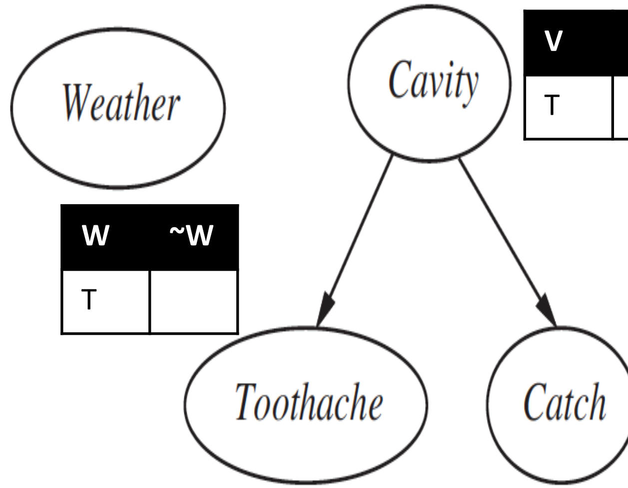
0.108 + 0.012 + 0.072 + 0.008 + 0.016 + 0.064 = 0.28

Building a Bayesian Network

Example Bayesian Net #1

A simple world with four random variables

- Weather, Toothache, Cavity, Catch
- Weather is independent of other variables
- Toothache and Catch are conditionally independent given Cavity
- P(Toothache, Catch | Cavity) = P(Toothache | Cavity) . P(Catch | Cavity)
- Cavity is a direct cause of Toothache and Catch
- No direct relation between Toothache and Catch exists

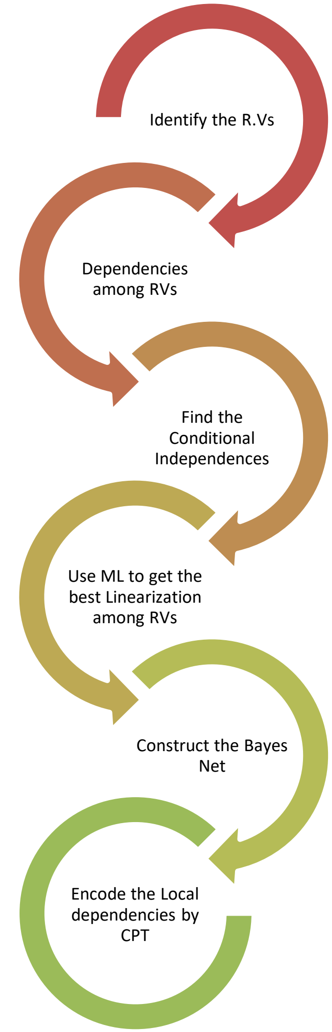


W	~W
T	

V	~V
T	

V	T	~T
T		
F		

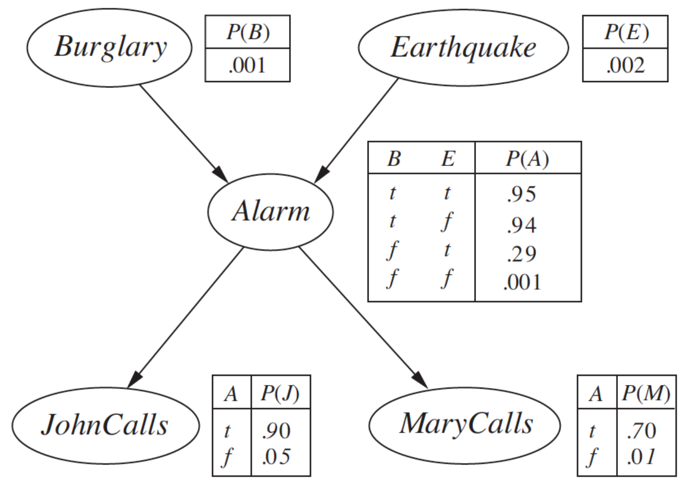
V	C	~C
T		
F		



Example Bayesian Net #2

A Burglary Alarm System

- Fairly reliable on detecting a burglary
- Also responds to earthquakes
- Two neighbors John and Mary a asked to call you at work when Burglary happens and they hear the Alarm
- John nearly always calls when he hears the alarm, however sometimes confuses the telephone ring with alarm and calls then too
- Mary like loud music and often misses the alarm altogether
- **Problem**: Given the information that who has / has not called we need to estimate the probability of a burglary



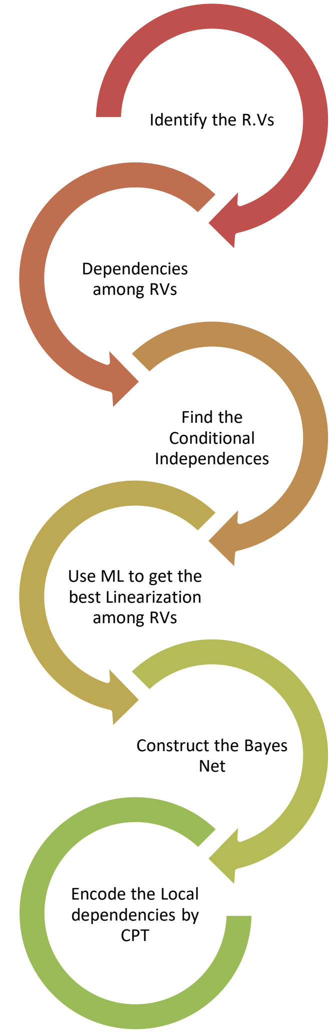
P(B)
.001

P(E)
.002

B	E	P(A)
t	t	.95
t	f	.94
f	t	.29
f	f	.001

A	P(J)
t	.90
f	.05

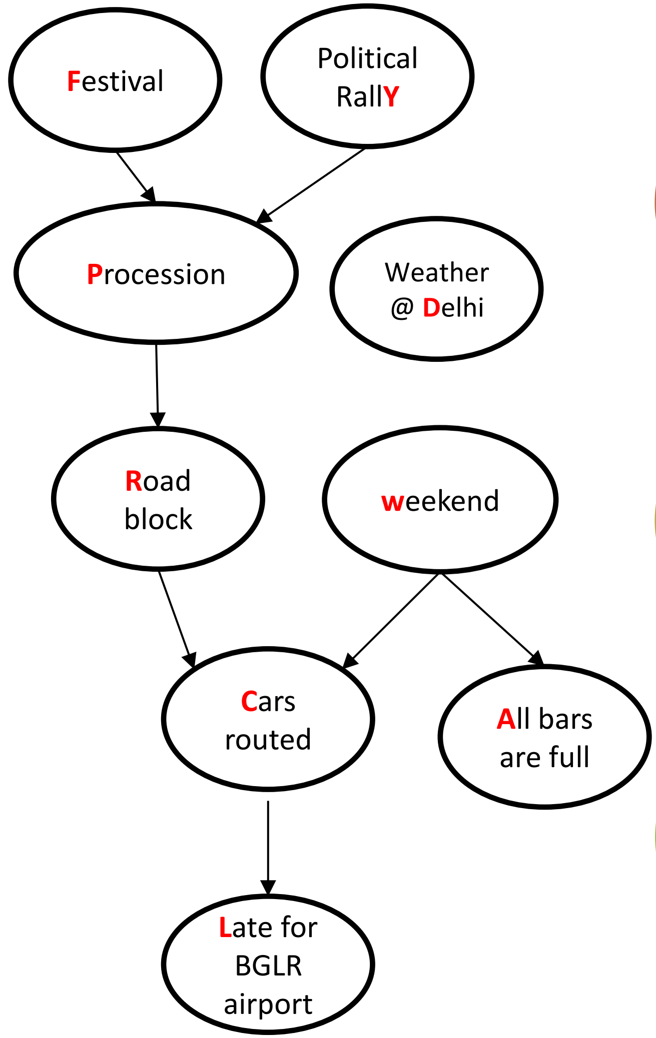
A	P(M)
t	.70
f	.01

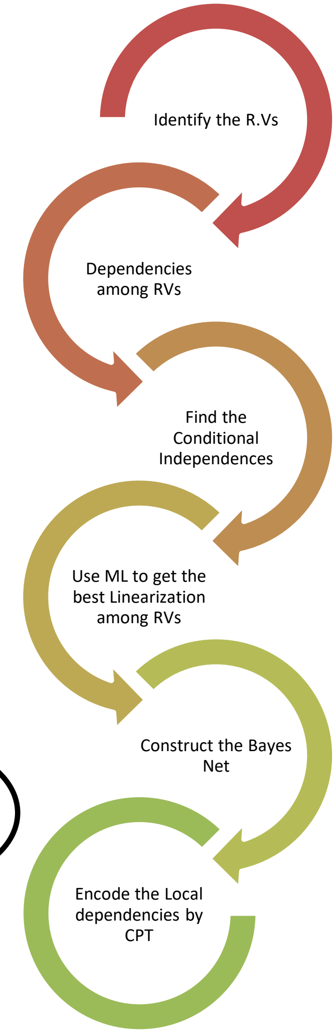


Example Bayesian Net #3

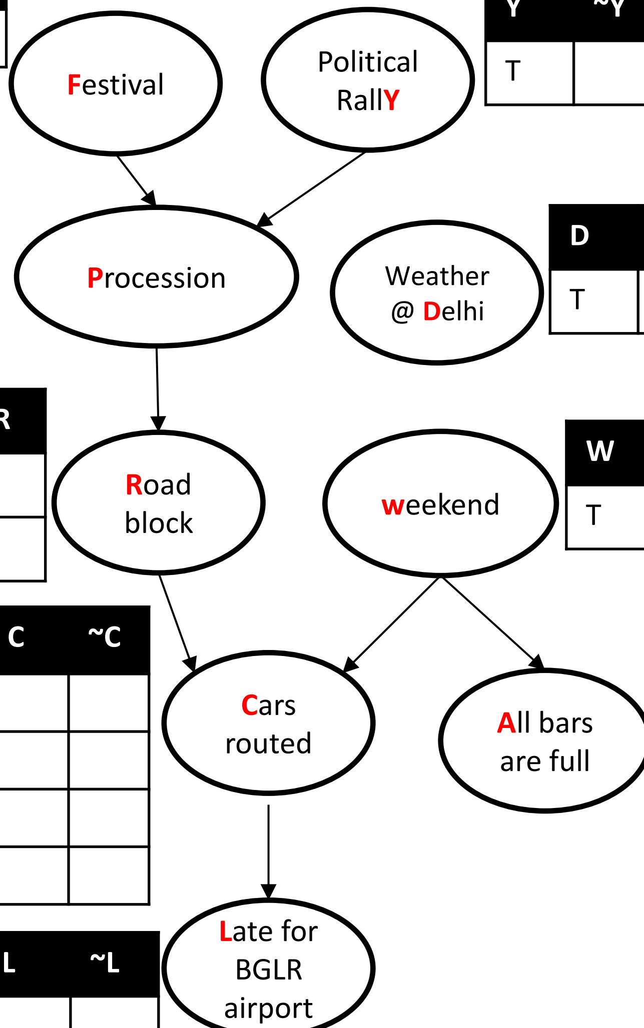
Traffic Prediction -Travel Estimation

- AI system reminds traveler regarding start time
- Travel plan is to reach Delhi and the weather of Delhi may influence the accommodation plans
- Traveler always take car to reach airport
- Car may be rerouted either due to road block or weekday traffic during working hours which delays the arrival to airport
- Bars are always observed to be full on weekends
- Authorities block roads to safe the processions
- Processions observed during festive season or due to the political rally.
- **Problem**: Given the information that there is a political rally expected estimate the probability of late arrival





Example Bayesian Net #3



F	~F
T	

Y	~Y
T	

F	Y	P	~P
T	T		
T	F		
F	T		
F	F		

D	~D
T	

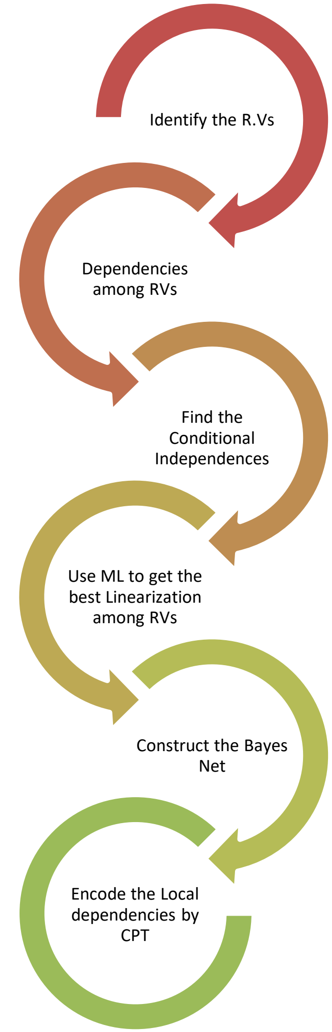
P	R	~R
T		
F		

W	~W
T	

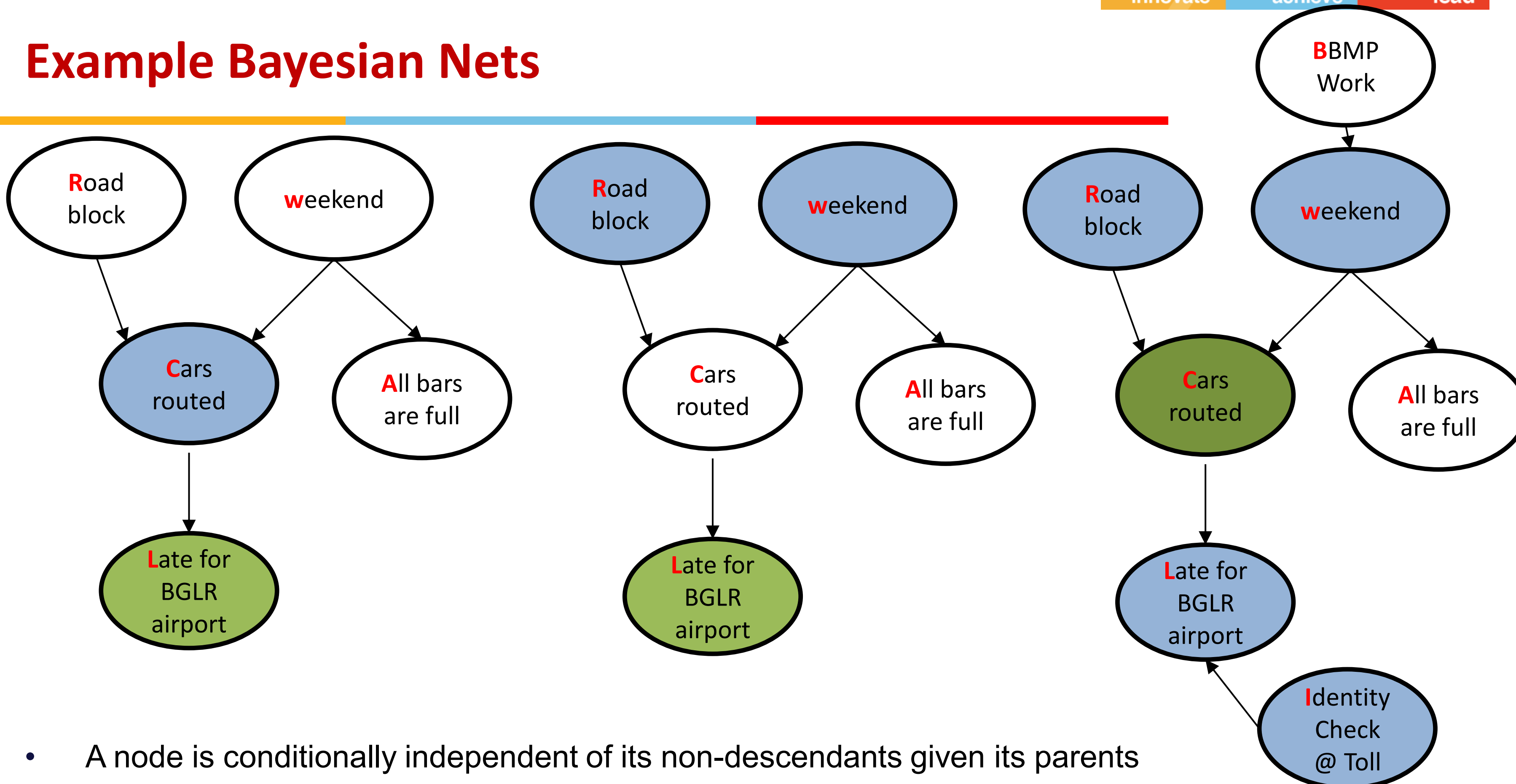
R	W	C	~C
T	T		
T	F		
F	T		
F	F		

W	A	~A
T		
F		

C	L	~L
T		
F		



Example Bayesian Nets

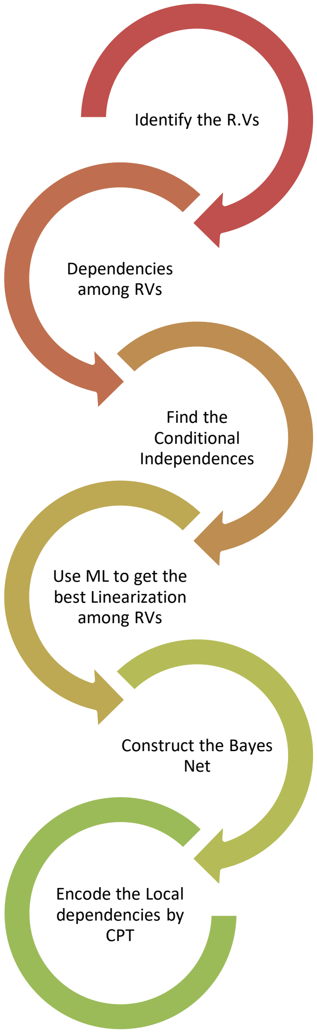


- A node is conditionally independent of its non-descendants given its parents
- A node is conditionally independent of all other nodes in the net , given its parents, children and children's parents.

Bayesian Net???

Wumpus World Problem

- Wumpus Ghost traces of scent in the visited cell
- Earlier visited cell may become unsafe!!!
- **Problem**: Given the information that there is a possibility of apparition of Wumpus anywhere in the cave, AI agent needs to be safely travel with more caution!!



Bayesian Network

Cyber Security

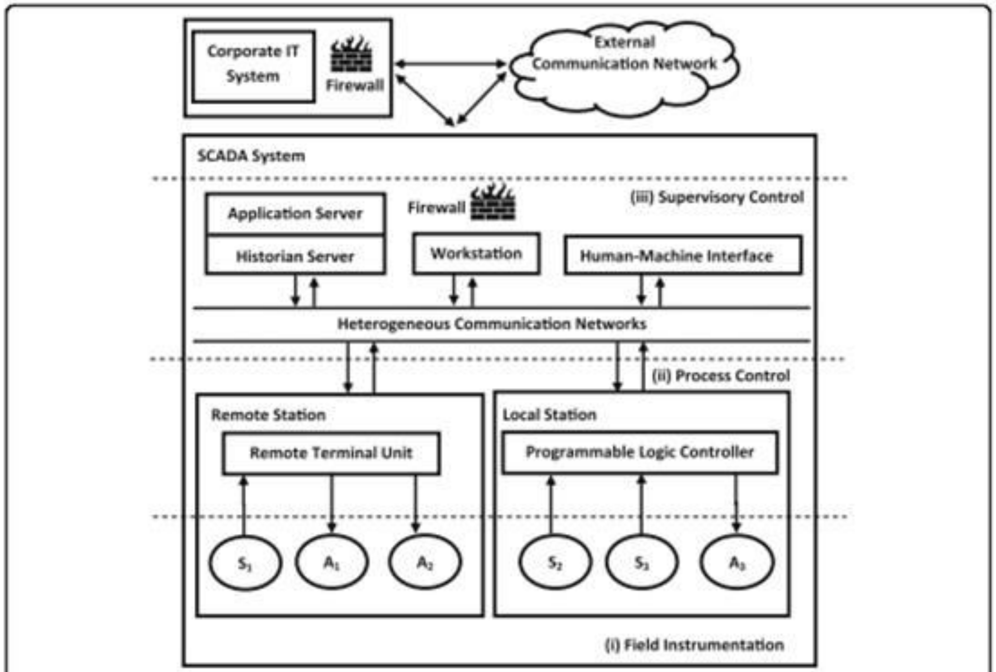


Fig. 1 Typical ICS Architecture and Layers

[Source Credit : 2021 : Chockalingam, S., Pieters, W., Teixeira, A. *et al.* Bayesian network model to distinguish between intentional attacks and accidental technical failures: a case study of floodgates.]

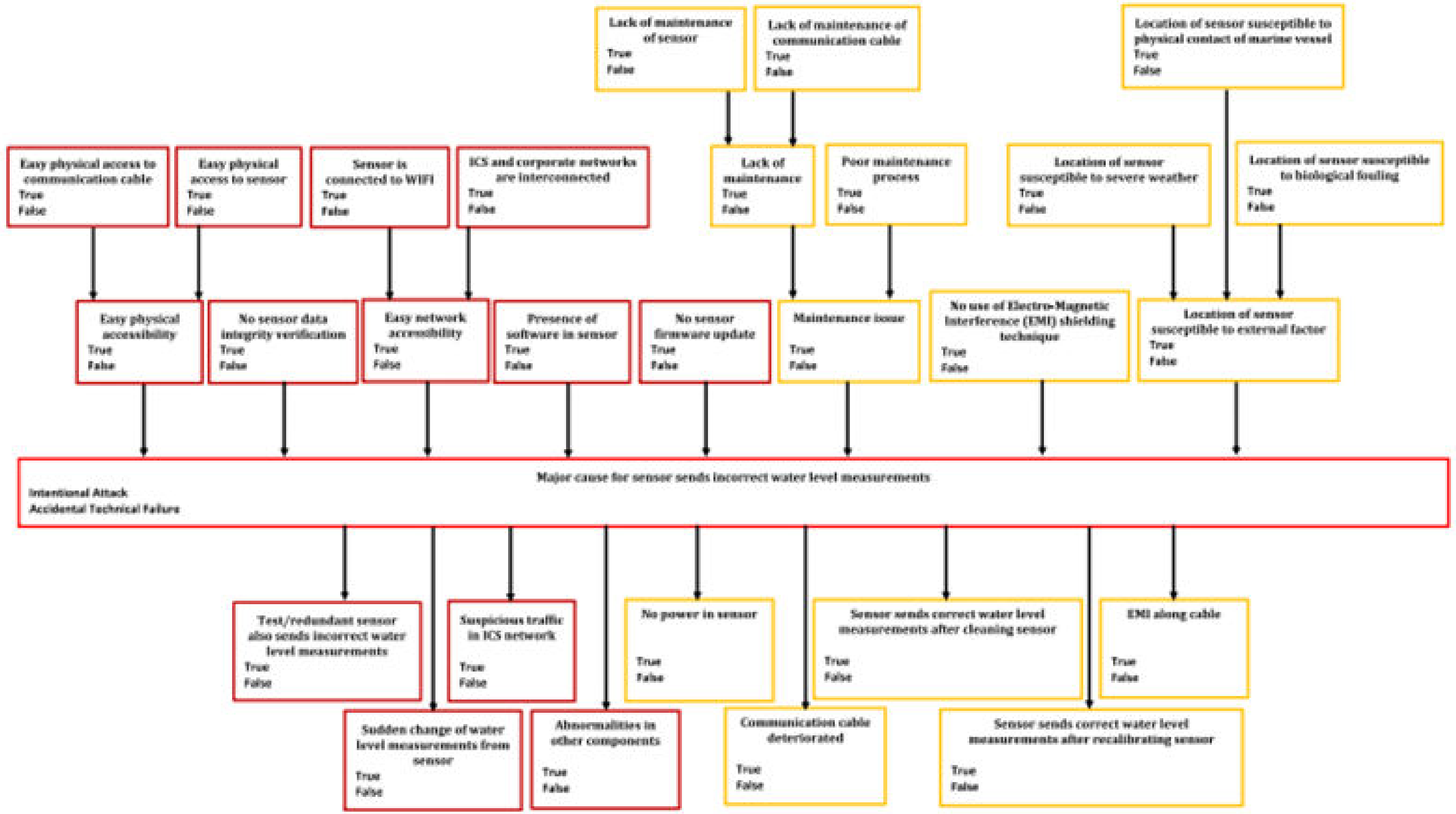


Fig. 4 Constructed Qualitative BN Model. (In this Figure, the presence of contributory factors and observations (or test results) colored in dark red would increase the likelihood of the problem (colored in red) due to an attack on the sensor. Furthermore, the presence of contributory factors and observations (or test results) colored in orange would increase the likelihood of the problem due to sensor failure)

Source Credit : 2021 : Chockalingam, S., Pieters, W., Teixeira, A. *et al.* Bayesian network model to distinguish between intentional attacks and accidental technical failures: a case study of floodgates.

Bayesian Network

Cyber Security

Table 2 CPT Excerpt – Problem Variable

C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	Y: Attack	Y: Failure
True	True	True	True	True	True	True	True	0.02	0.98
True	True	True	True	True	True	True	False	0.09	0.91
True	True	True	True	True	True	False	True	0.06	0.94
True	True	True	True	True	True	False	False	0.24	0.76
True	True	True	True	True	False	True	True	0.09	0.91
True	True	True	True	True	False	True	False	0.38	0.62
True	True	True	True	True	False	False	True	0.24	0.76
True	True	True	True	True	False	False	False	0.97	0.03
True	True	True	True	False	True	True	True	0.02	0.98
True	True	True	True	False	True	True	False	0.09	0.91

In this table, C_1: Easy physical accessibility, C_2: No sensor data integrity verification, C_3: Easy network accessibility, C_4: Presence of software in sensor, C_5: No sensor firmware update, C_6: Maintenance issue, C_7: No use of EMI shielding technique, C_8: Location of sensor susceptible to external factor and Y: Major cause for sensor sends incorrect water level measurements

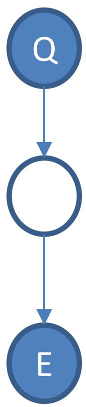
Source Credit : 2021 : Chockalingam, S., Pieters, W., Teixeira, A. *et al.* Bayesian network model to distinguish between intentional attacks and accidental technical failures: a case study of floodgates.

Inferences in Bayesian Nets

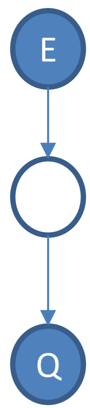
Enumeration

Belief Nets

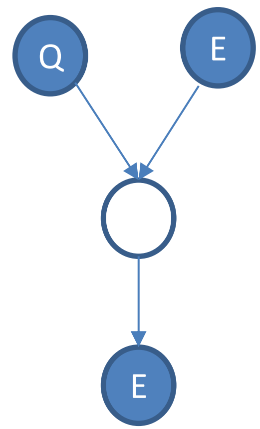
Diagnostic



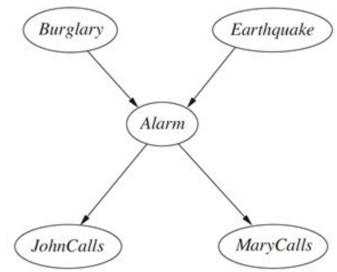
Causal

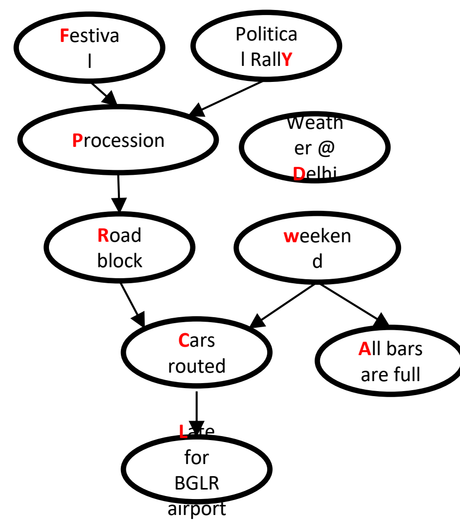


Inter-Casual

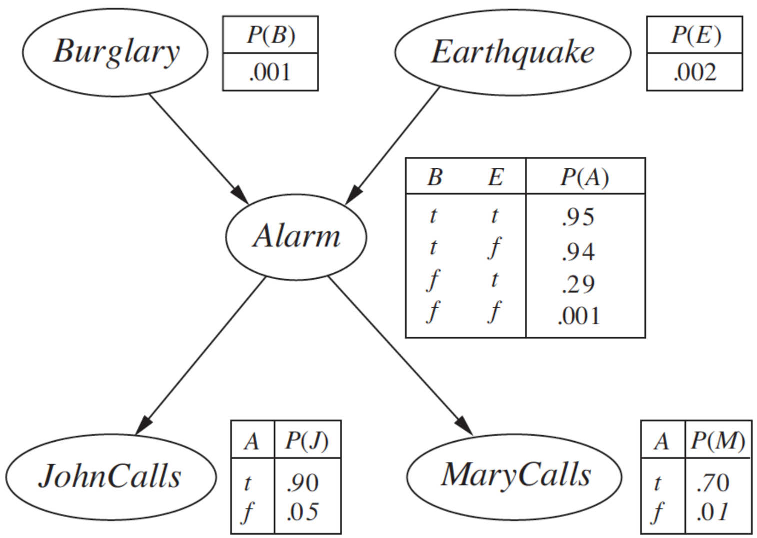


Mixed Inferences





Examples



1. Calculate the probability that alarm has sounded, but neither burglary nor earthquake happened, and both John and Mary called

2. What is the probability that Burglary happened given John & Mary called the police

3. What is the probability that John calls given earthquake occurred?

Next Session Plan:

- Bayesian network : Exact Inference (One more problem)
- Introduction to Approximate Inference
- Reasoning over time (start)

Required Reading: AIMA - Chapter #13, #14.1, #14.2

Thank You for all your Attention

Note : Some of the slides are adopted from AIMA TB materials