



# Artificial & Computational Intelligence

**DSE CLZG557**

## **M2 : Problem Solving Agent using Search**

Raja vadhana P

Assistant Professor,

BITS - CSIS

**BITS Pilani**

Pilani Campus

# Course Plan



M1 Introduction to AI

M2 Problem Solving Agent using Search

M3 Game Playing, Constraint Satisfaction Problem

M4 Knowledge Representation using Logics

M5 Probabilistic Representation and Reasoning

M6 Reasoning over time, Reinforcement Learning

M7 AI Trends and Applications, Philosophical foundations

## Module 2 : Problem Solving Agent using Search

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- A. Uninformed Search
- B. Informed Search
- C. Heuristic Functions
- D. Local Search Algorithms & Optimization Problems

# Problem Formulation

## Learning Objective

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1. Compare given heuristics for a problem and analyze which is the best fit
2. Design relaxed problem with appropriate heuristic design
3. Prove the designed relaxed problem heuristic is admissible
4. Identify the appropriate local search algorithm
5. Ability to design fitness and implement planning problems

# Design of Heuristics

# Heuristic Design

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- **Effective Branching Factor**
- Good Heuristics
- Notion of Relaxed Problems
- Generating Admissible Heuristics

Effective branching factor ( $b^*$ ):

If the algorithm generates  $N$  number of nodes and the solution is found at depth  $d$ , then

$$N + 1 = 1 + (b^*) + (b^*)^2 + (b^*)^3 + \dots + (b^*)^d$$



# Heuristic Design

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- Effective Branching Factor
- Good Heuristics
- **Notion of Relaxed Problems**
- Generating Admissible Heuristics

Simplify the problem

Assume no constraints

Cost of optimal solution to relaxed problem  $\leq$  Cost of optimal solution for real problem



# Heuristic Design

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- Effective Branching Factor
- Good Heuristics
- Notion of Relaxed Problems
- **Generating Admissible Heuristics**

Derive admissible heuristic from exact cost of a solution to a relaxed version of problem

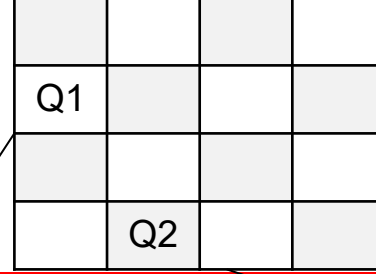
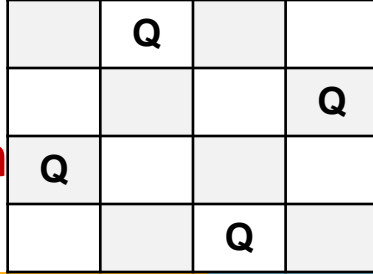
Rule of Dominance : If  $h_2(n) \geq h_1(n)$  for all  $n$ , then choose  $h_2$

Choose  $h(n)$  :  $\max\{ h_1(n), h_2(n), \dots, h_i(n) \}$

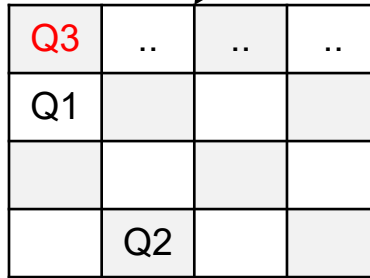
Break the Tie : Combination of  $h_1$  &  $h_2$

# Design of Heuristics

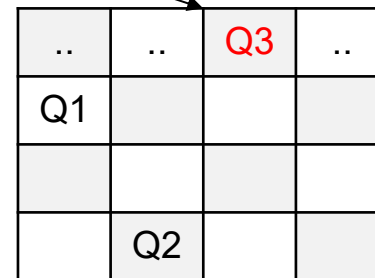
# N-Queen



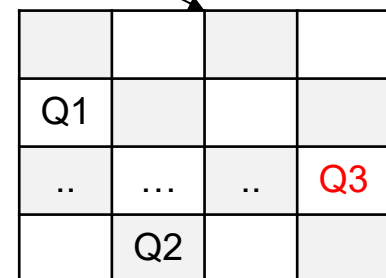
- Construct the search tree by considering one row of the board at a time
- State space graph of relaxed problem is a super graph of original state space because of removal of restrictions



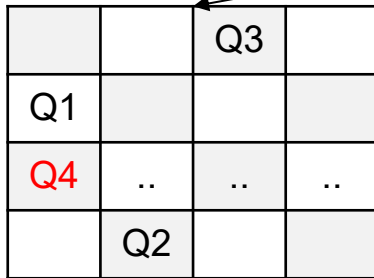
$$1+0+_=1$$



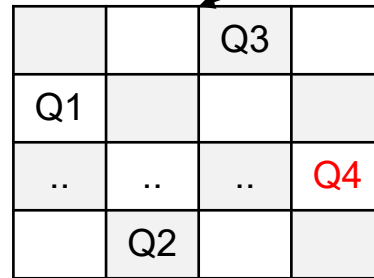
$$0+0+_=0$$



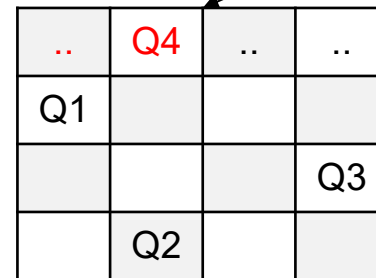
$$0+0+_=0$$



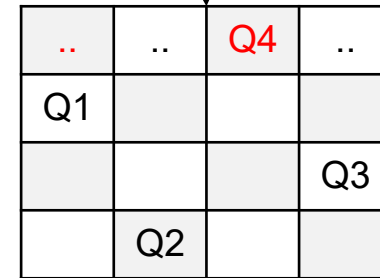
$$1+1+0+_=2$$



$$0+0+0+_=0$$



$$1+1+0+_=2$$



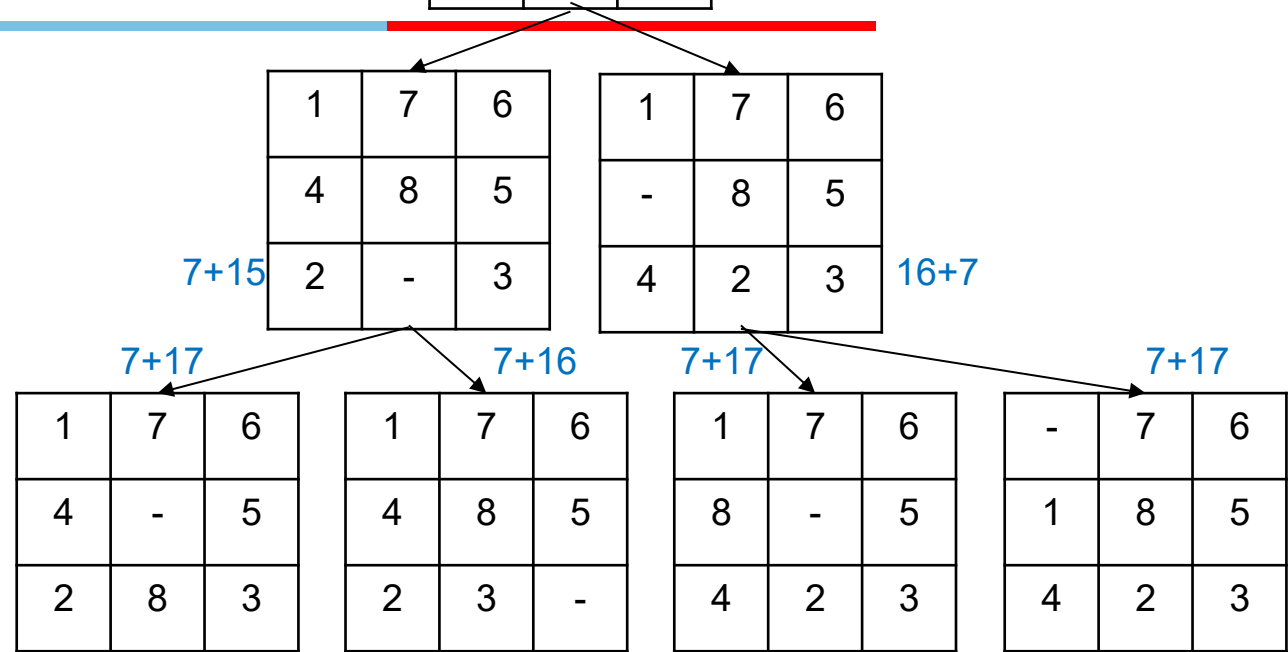
$$0+0+0+_=0$$

Initial State	Possible Actions	Transition Model	Goal Test	Path Cost	No.Of.States
< Xi , Yi >	Place in any non-occupied row in board		isValid Non-Attacking	Transition + Valid Queens	n!

# N-Tile

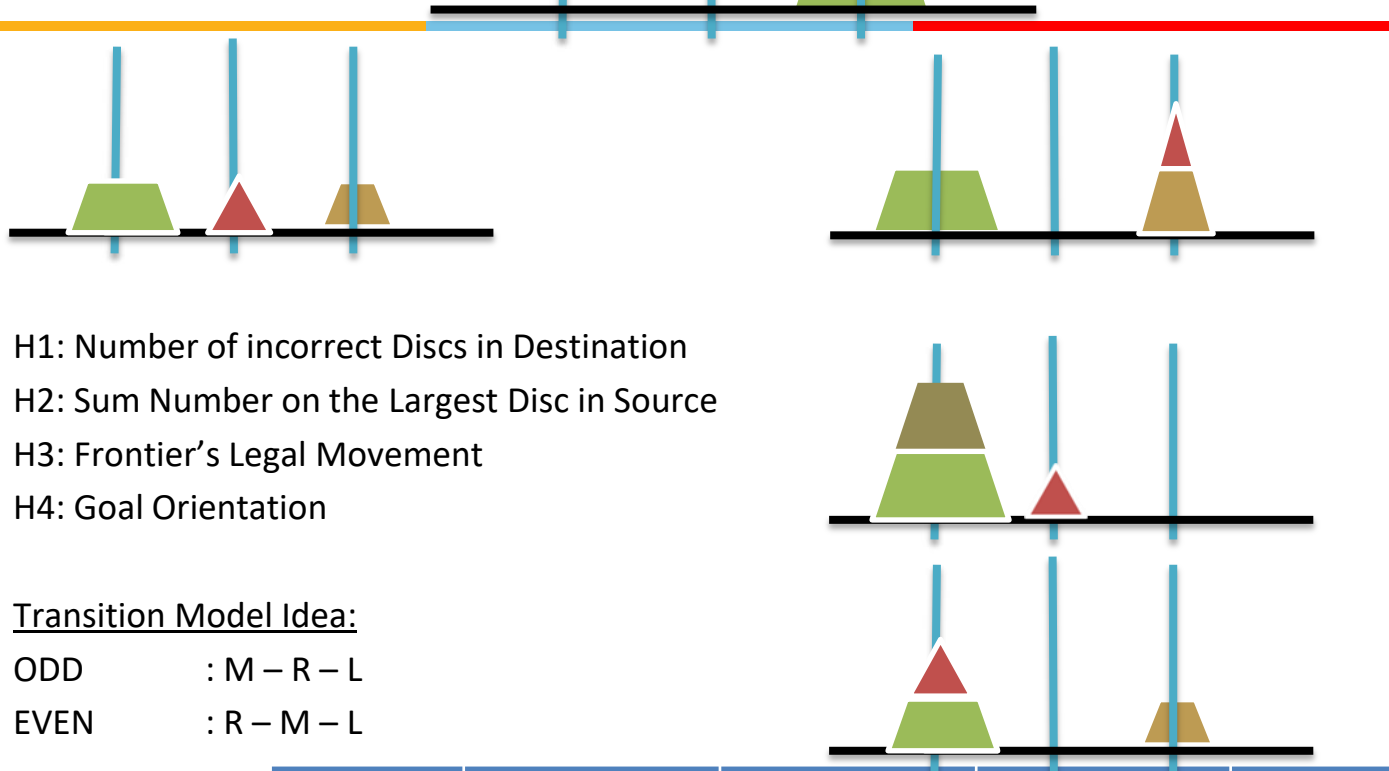
-	1	2
3	4	5
6	7	8

1	7	6
4	8	5
-	2	3



Initial State	Possible Actions	Transition Model	Goal Test	Path Cost	No.Of.States
<LOC, ID>	Move Empty to near by Tile		ID=LOC+1	Transition + Positional + Distance+ Other approaches	9!

# Tower of Hanoi



- H1: Number of incorrect Discs in Destination
- H2: Sum Number on the Largest Disc in Source
- H3: Frontier's Legal Movement
- H4: Goal Orientation

## Transition Model Idea:

ODD : M – R – L  
 EVEN : R – M – L

Initial State	Possible Actions	Transition Model	Goal Test	Path Cost	No.Of.States
( [n], [n-1], [1] )	<ul style="list-style-type: none"> <li>➤ Move X on-top Y : <math>X &lt; Y</math></li> <li>➤ Move X on empty Peg</li> </ul>		( [], [], [1, 2,... n-1, n] )	Optimal : Tested: $2^{n-1}$ <b>No.of.Steps + Direction Cost + Positional Cost</b>	

# Tic Tac Toe



0	X	
0		X
X		0

1 | 2

0	X	X
0		X
X		0

0 | 2

0	X	
0	X	X
X		0

1 | 2

0	X	
0		X
X	X	0

0	X	X
0	0	X
X		0

0 | 1

0	X	X
0		X
X	0	0

1 | 1

0	X	0
0	X	X
X		0

1 | 0

0	X	
0	X	X
X	0	0

1 | 0

0	X	0
0		X
X	X	0

2 | 1

0	X	
0	0	X
X	X	0

0 | 1

Opposite Win | Player Win

Initial State	Possible Actions	Transition Model	Goal Test	Path Cost	No.Of.States
([Xij], [Yij])	Place a coin in unoccupied (i,j)		N : i's N : j's N : i=j	No.of.Steps + Opp.Win + (N-1-Curr.Win)	19,683=3 <sup>9</sup>

## Learn from experience

Trail / Puzzle	X1(n) : No.of.Misplaced Tiles	X2(n): Pair of adjacent tiles that are not in goal	X3(n): Position of the empty tile	.....h'(n)
Example 1	7	10	7	.....
Example 2	5	6	6	.....
.....	..	..	..	.....

-	1	2
3	4	5
6	7	8

1	7	6
4	8	5
-	2	3

Create a suitable model:

$$h(n) = c1*X1(n) + c2*X2(n) + .....$$

# Pattern Database



# Divide and Conquer



Formulate  
Disjoint sub  
problem

Subset of Tiles 1,3,5  
Subset of Tiles 6,7,8  
Compute a DB of tables

Compute  
Optimal Cost of  
this

Eg.,  $f(n) = 10$   
Store Problem : Solution Cost  
**\*\*Do not compute the cost of tiles not in the set**

Repeat for  
multiple  
combination

L1:1, L4:3, L3:5  
L3:1, L5:3, L1:5  
L5:8, L2:7, L3:6

Map the  
solution to the  
original problem

Use BFS from GOAL  
Find best set of states of the same sub problem nearest to G

Lower Limit of  
 $h(n)$

Find Maximum  $h(n)$  among set of states chosen for a particular partition set  
Add the  $h(n)$  for portioned best solution

-	1	2
3	4	5
6	7	8

1	7	6
4	8	5
-	2	3

# Local Search & Optimization

## Optimization Problem

**Goal** : Navigate through a state space for a given problem such that an optimal solution can be found

**Objective** : Minimize or Maximize the objective evaluation function value

**Scope** : Local

**Objective Function** : Fitness Value evaluates the goodness of current solution

**Local Search** : Search in the state-space in the **neighbourhood of current position** until an optimal solution is found

### Single Instance Based

Hill Climbing

Simulated Annealing

Local Beam Search

Tabu Search

### Multiple Instance Based

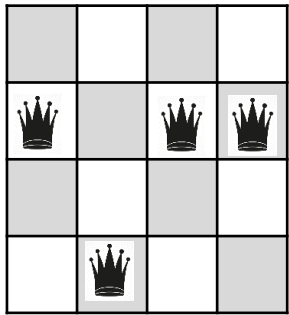
Genetic Algorithm

Particle Swarm Optimization

Ant Colony Optimization

## Terminology

**Local Search** : Search in the state-space in the **neighbourhood** of current position until an optimal solution is found

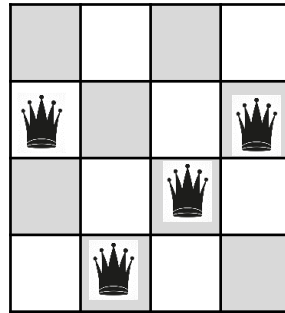


**Feasible State/Solution**

Fitness Value:

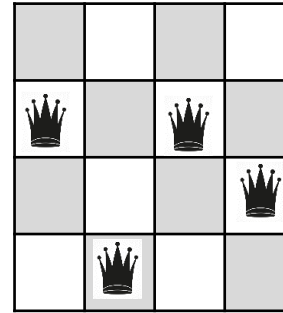
$$h(n) = 4$$

Above is an example of  $h(n) = \text{No.of.Conflicting pairs of queens}$

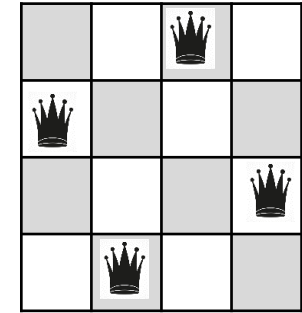


**Neighboring States**

$$h(n) = 4$$



$$h(n) = 2$$



**Optimal Solution**

$$h(n) = 0$$

$$h(n) = 0$$

$$h(n) = 0$$

$$h(n) = 1$$

$$h(n) = 0$$

Above is an example of  $h(n) = \text{No.of.Non-Conflicting Single queens with other queens in the board.}$

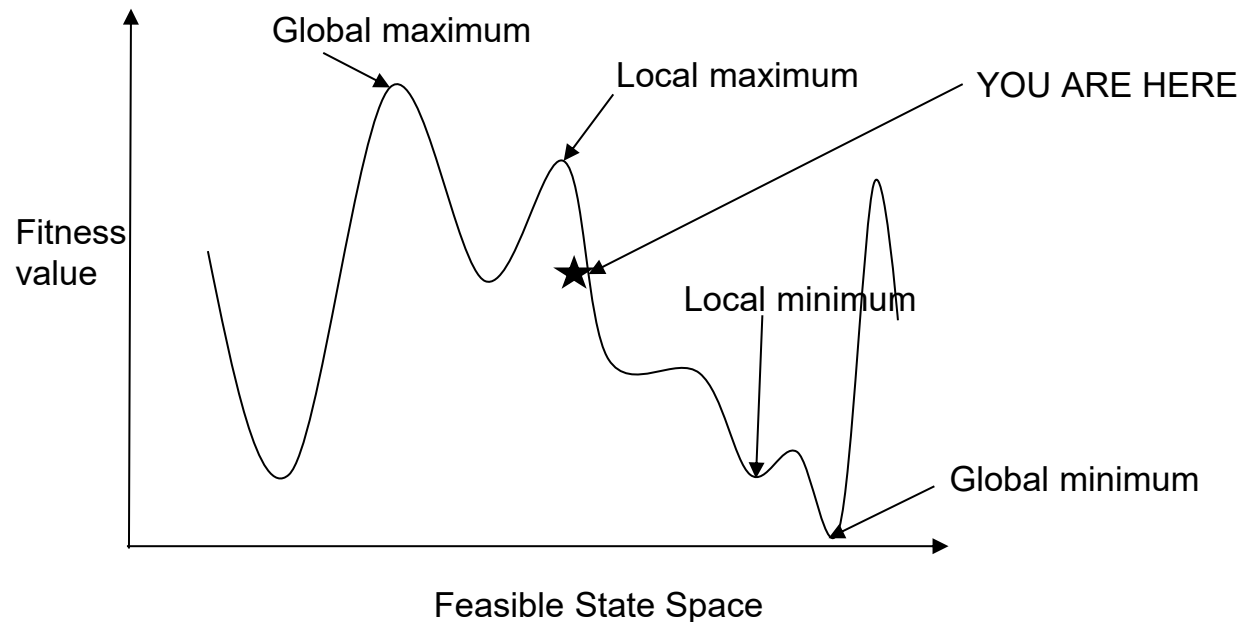
## Terminology

**Local Search** : Search in the state-space in the **neighbourhood** of current position until an optimal solution is found

### Algorithms:

- Choice of Neighbor
- Looping Condition
- Termination Condition

2	5	3	2
♠	6	♠	♠
3	5	4	2
4	♠	4	2

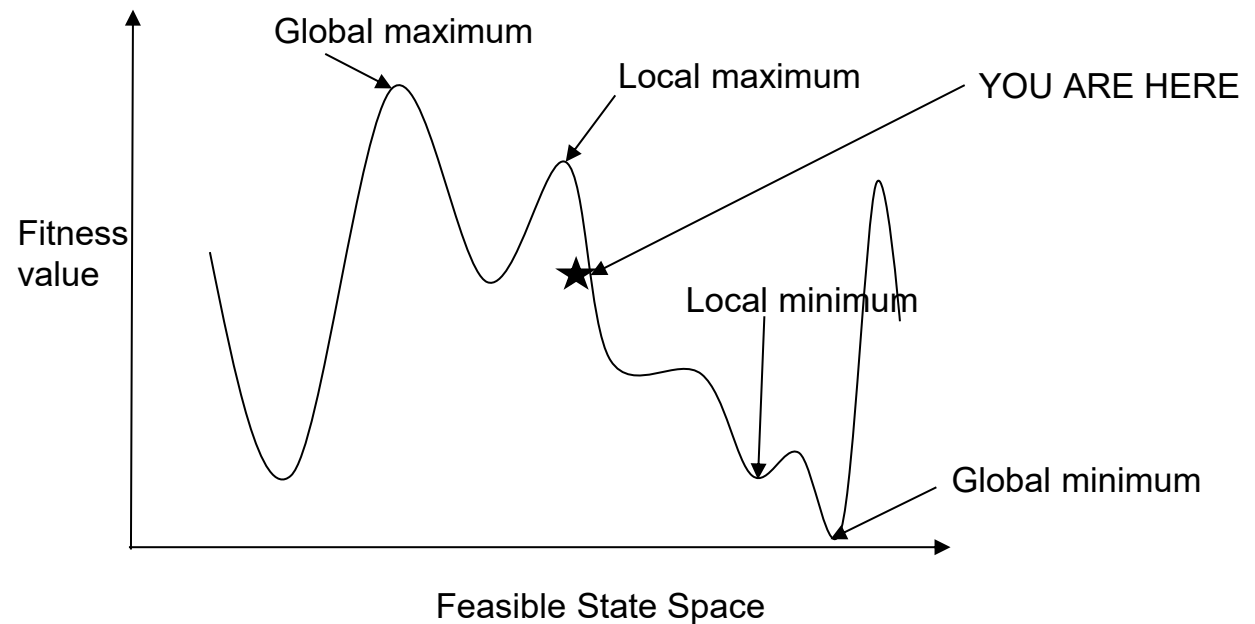


# Hill Climbing

# Hill Climbing



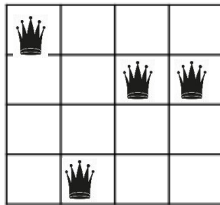
2	5	3	2
♠	6	♠	♠
3	5	4	2
4	♠	4	2



1. Select a random state
2. Evaluate the fitness scores for all the successors of the state
3. Calculate the probability of selecting a successor based on fitness score
4. Select the next state based on the highest probability
5. Repeat from Step 2

**$h(n)$  = No.of non-conflicting pairs of queens in the board.**

Q1-Q2



Q1-Q3

Q1-Q4

Q2-Q3

Q2-Q4

Q3-Q4

1	4	2	2	4
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Note : Steps 3 & 4 in the above algorithm will be a part of variation of Hill climbing



1. Select a random state
2. Evaluate the fitness scores for all the successors of the state
3. Calculate the probability of selecting a successor based on fitness score
4. Select the next state based on the highest probability
5. Repeat from Step 2

**$h(n)$  = No.of non-conflicting pairs of queens in the board.**

Q1-Q2

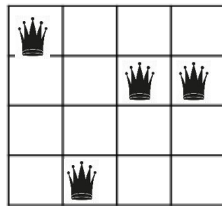
Q1-Q3

Q1-Q4

Q2-Q3

Q2-Q4

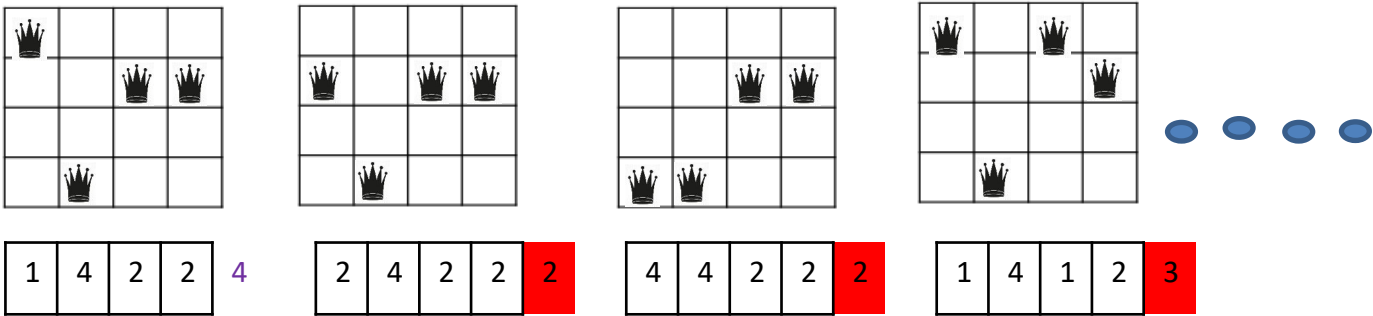
Q3-Q4



1	4	2	2	4
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# Hill Climbing

1. Select a random state
2. Evaluate the fitness scores for all the successors of the state
3. Calculate the probability of selecting a successor based on fitness score
4. Select the best next state
5. Repeat from Step 2



1

4

2

2

4

2

4

2

2

2

4

4

2

2

2

1

4

1

2

3

## Random Restart

1. Select a random state
2. Evaluate the fitness scores for all the successors of the state
3. Calculate the probability of selecting a successor based on fitness score
4. Select the next state based on the highest probability
5. Repeat from Step 2

			👑
👑			
	👑	👑	

3	4	4	2	3
---	---	---	---	---

**function** HILL-CLIMBING(*problem*) **returns** a state that is a local maximum

*current*  $\leftarrow$  MAKE-NODE(*problem*.INITIAL-STATE)

**loop do**

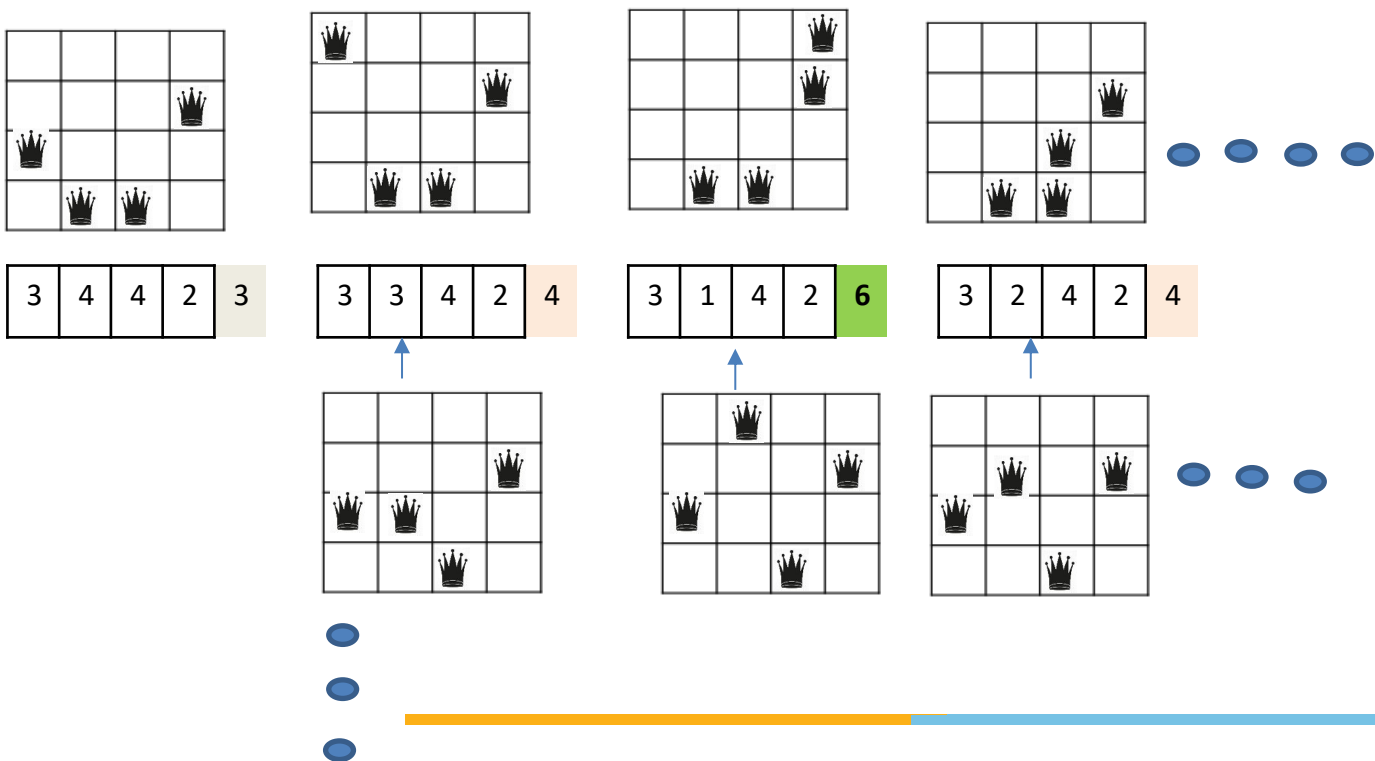
*neighbor*  $\leftarrow$  a highest-valued successor of *current*

**if** *neighbor*.VALUE  $\leq$  *current*.VALUE **then return** *current*.STATE

*current*  $\leftarrow$  *neighbor*

# Hill Climbing –Restart round =2

1. Select a random state
2. Evaluate the fitness scores for all the successors of the state
3. Calculate the probability of selecting a successor based on fitness score
4. Select the next state based on the highest probability
5. Repeat from Step 2





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**Required Reading:** AIMA - Chapter #4.2 , #4.3 (will be continued in next class)

Thank You for all your Attention

Note : Some of the slides are adopted from AIMA TB materials