Artificial & Computational Intelligence

DSE CLZG557

M4 : Knowledge Representation using Logics

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Module 4 :

Knowledge Representation using Logics

A. Logical Representation

B. Propositional Theorem Proving

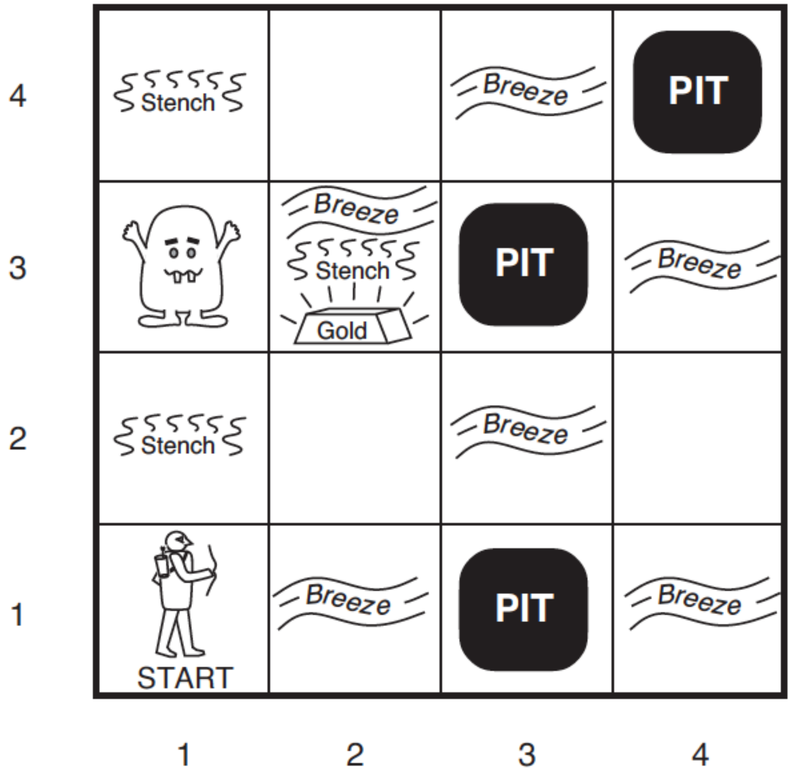
C. DPLL Algorithm

D. First Order Logic

E. FOL Inference

Knowledge based Agent : Model & Represent

Concepts, logic Representation of a sample agent



Wumpus World Problem:

PEAS:

Performance Measure:

+1000 for climbing out with gold,

-1000 for falling into a pit or being eaten by Wumpus,

-1 for each action taken and

-10 for using an arrow

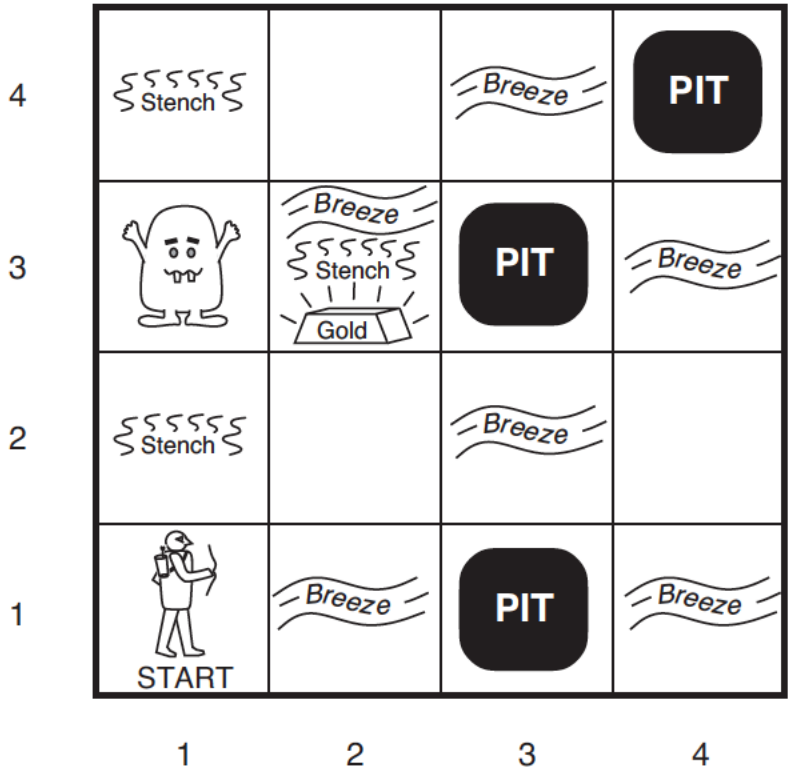
Environment: 4x4 grid of rooms. Always starts at [1, 1] facing right.

The location of Wumpus and Gold are random.

Agent dies if entered a pit or live Wumpus.

Knowledge based Agent : Model & Represent

Concepts, logic Representation of a sample agent



Wumpus World Problem:

PEAS:

Actuators –

Forward,

TurnLeft by 90,

TurnRight by 90,

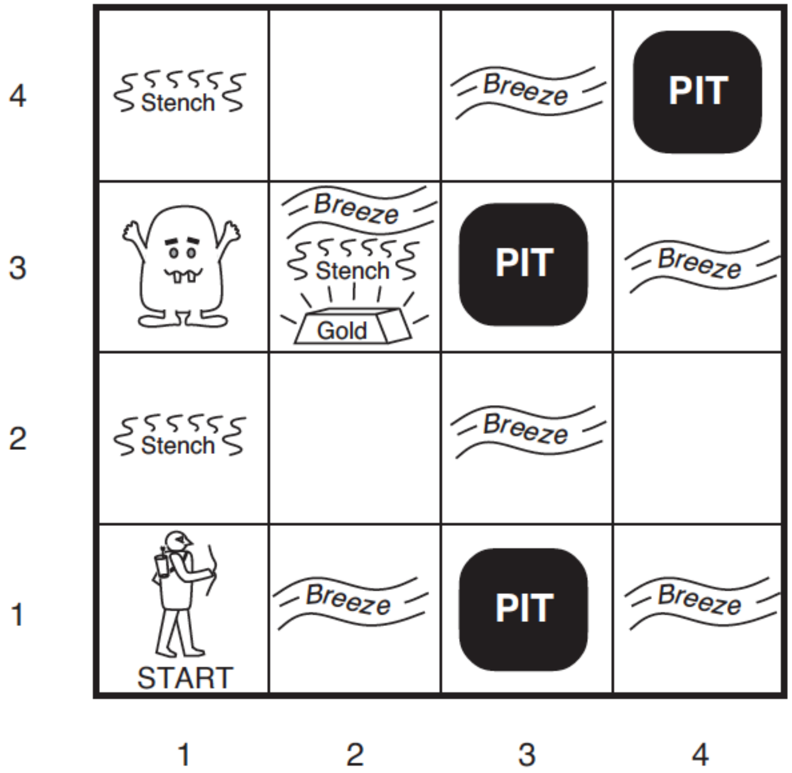
Grab – pick gold if present,

Shoot – fire an arrow, it either hits a wall or kills wumpus. Agent has only one arrow.

Climb – Used to climb out of cave, only from [1, 1]

Knowledge based Agent : Model & Represent

Concepts, logic Representation of a sample agent



Wumpus World Problem:

PEAS:

Sensors. The agent has five sensors

- **S**tench: In all adjacent (but not diagonal) squares of Wumpus
- **B**reeze: In all adjacent (but not diagonal) squares of a pit
- **G**litter: In the square where gold is
- **B**ump: If agent walks into a wall
- **S**cream: When Wumpus is killed, it can be perceived everywhere

Percept Format:

[Stench?, Breeze?, Glitter?, Bump?, Scream?]

E.g., [Stench, Breeze, None, None, None]

Propositional Logic

Agents based on Propositional logic ,TT-Entail for inference from truth table

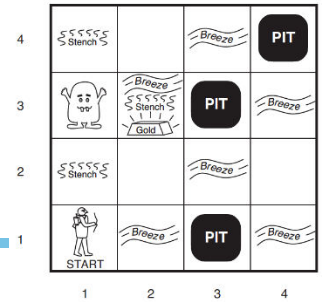
Tie break in search:

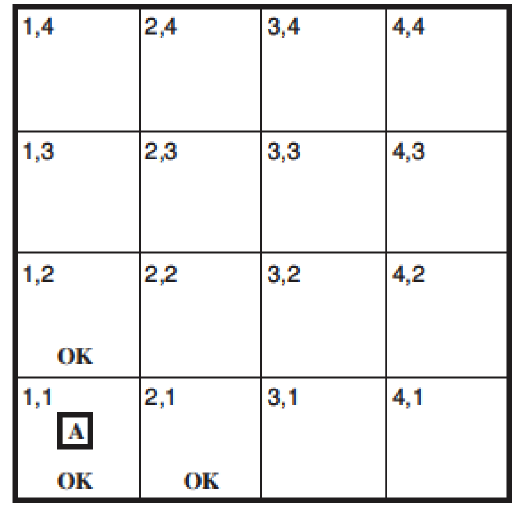
¬ , ∧ , ∨, ⇒ , ⇔

$(\neg A) \wedge B$ has precedence over $\neg(A \wedge B)$

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	*false*	*true*	*false*	*false*	*true*	*true*
false	*true*	*true*	*false*	*true*	*true*	*false*
true	*false*	*false*	*false*	*true*	*false*	*false*
true	*true*	*false*	*true*	*true*	*true*	*true*

Propositional Logic - Modelling





For each [x, y] location

$P_{x,y}$ is true if there is a pit in [x, y]

$W_{x,y}$ is true if there is a wumpus in [x, y]

$B_{x,y}$ is true if agent perceives a breeze in [x, y]

$S_{x,y}$ is true if agent perceives a stench in [x, y]

----------------------- R is the sentence in KB

$R_1 : \neg P_{1,1}$

$R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

$R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$

$R_4 : \neg B_{1,1}$

$R_5 : B_{2,1}$

TT – Entails Inference – Example

$\neg P_{1,2}$ entailed by our KB?

Way – 1 :

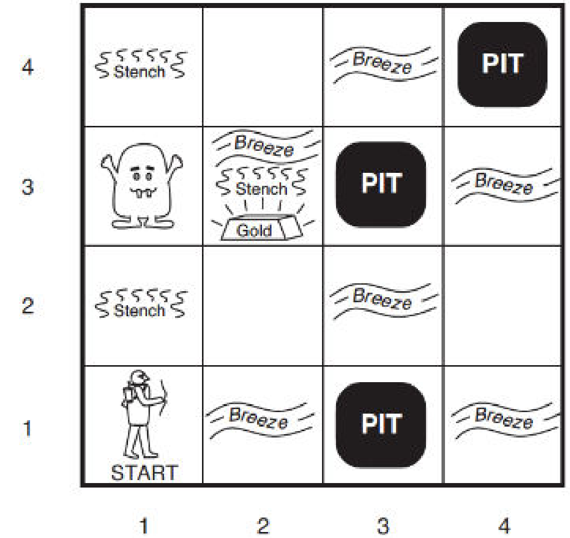
$R_1 : \neg P_{1,1}$

$R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

$R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$

$R_4 : \neg B_{1,1}$

$R_5 : B_{2,1}$



$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	<u>*true*</u>
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	<u>*true*</u>
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	<u>*true*</u>
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

TT – Entails Inference – Example

$\neg P_{1,2}$ entailed by our KB?

Way – 1 :

1. Get sufficient information $B_{1,1}$, $B_{2,1}$, $P_{1,1}$, $P_{1,2}$, $P_{2,1}$, $P_{2,2}$, $P_{3,1}$
2. Enumerate all models with combination of truth values to propositional symbols
3. In all the models, find those models where KB is true, i.e., every sentence R_1, R_2, R_3, R_4, R_5 are true
4. In those models where KB is true, find if query sentence $\neg P_{1,2}$ is true
5. If query sentence $\neg P_{1,2}$ is true in all models where KB is true, then it entails, otherwise it won't

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	<u>*true*</u>
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	<u>*true*</u>
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	<u>*true*</u>
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

Inference : Example – Theorem Proving

Propositional theorem proving - Proof by resolution

Logical Equivalence rules can be used as inference rules

$$
\begin{aligned}
(\alpha \land \beta) &\equiv (\beta \land \alpha) && \text{commutativity of } \land \\
(\alpha \lor \beta) &\equiv (\beta \lor \alpha) && \text{commutativity of } \lor \\
((\alpha \land \beta) \land \gamma) &\equiv (\alpha \land (\beta \land \gamma)) && \text{associativity of } \land \\
((\alpha \lor \beta) \lor \gamma) &\equiv (\alpha \lor (\beta \lor \gamma)) && \text{associativity of } \lor \\
\neg(\neg\alpha) &\equiv \alpha && \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) && \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \lor \beta) && \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha)) && \text{biconditional elimination} \\
\neg(\alpha \land \beta) &\equiv (\neg\alpha \lor \neg\beta) && \text{De Morgan} \\
\neg(\alpha \lor \beta) &\equiv (\neg\alpha \land \neg\beta) && \text{De Morgan} \\
(\alpha \land (\beta \lor \gamma)) &\equiv ((\alpha \land \beta) \lor (\alpha \land \gamma)) && \text{distributivity of } \land \text{ over } \lor \\
(\alpha \lor (\beta \land \gamma)) &\equiv ((\alpha \lor \beta) \land (\alpha \lor \gamma)) && \text{distributivity of } \lor \text{ over } \land
\end{aligned}
$$

Inference : Example – Theorem Proving

1. **Modes Ponens**
2. AND Elimination

α : I walk in rain without the umbrella

β : I get wet

$\alpha \rightarrow \beta$ **α**

β

$$
\begin{aligned}
(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) && \text{commutativity of } \wedge \\
(\alpha \vee \beta) &\equiv (\beta \vee \alpha) && \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) && \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) && \text{associativity of } \vee \\
\neg(\neg \alpha) &\equiv \alpha && \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg \beta \Rightarrow \neg \alpha) && \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg \alpha \vee \beta) && \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) && \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) &\equiv (\neg \alpha \vee \neg \beta) && \text{De Morgan} \\
\neg(\alpha \vee \beta) &\equiv (\neg \alpha \wedge \neg \beta) && \text{De Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) &\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) && \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) && \text{distributivity of } \vee \text{ over } \wedge
\end{aligned}
$$

Inference : Example – Theorem Proving

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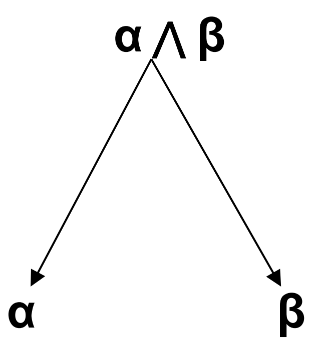

$$
\begin{array}{rcll}
(\alpha \land \beta) & \equiv & (\beta \land \alpha) & \text{commutativity of } \land \\
(\alpha \lor \beta) & \equiv & (\beta \lor \alpha) & \text{commutativity of } \lor \\
((\alpha \land \beta) \land \gamma) & \equiv & (\alpha \land (\beta \land \gamma)) & \text{associativity of } \land \\
((\alpha \lor \beta) \lor \gamma) & \equiv & (\alpha \lor (\beta \lor \gamma)) & \text{associativity of } \lor \\
\neg(\neg\alpha) & \equiv & \alpha & \text{double-negation elimination} \\
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(\alpha \Leftrightarrow \beta) & \equiv & ((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha)) & \text{biconditional elimination} \\
\neg(\alpha \land \beta) & \equiv & (\neg\alpha \lor \neg\beta) & \text{De Morgan} \\
\neg(\alpha \lor \beta) & \equiv & (\neg\alpha \land \neg\beta) & \text{De Morgan} \\
(\alpha \land (\beta \lor \gamma)) & \equiv & ((\alpha \land \beta) \lor (\alpha \land \gamma)) & \text{distributivity of } \land \text{ over } \lor \\
(\alpha \lor (\beta \land \gamma)) & \equiv & ((\alpha \lor \beta) \land (\alpha \lor \gamma)) & \text{distributivity of } \lor \text{ over } \land
\end{array}
$$

Inference : Example – Theorem Proving

$R_1 : \neg P_{1,1}$

$R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$

$R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \lor P_{2,2} \lor P_{3,1})$

$R_4 : \neg B_{1,1}$

$R_5 : B_{2,1}$

$(\alpha \land \beta)$	$\equiv$	$(\beta \land \alpha)$	commutativity of $\land$
$(\alpha \lor \beta)$	$\equiv$	$(\beta \lor \alpha)$	commutativity of $\lor$
$((\alpha \land \beta) \land \gamma)$	$\equiv$	$(\alpha \land (\beta \land \gamma))$	associativity of $\land$
$((\alpha \lor \beta) \lor \gamma)$	$\equiv$	$(\alpha \lor (\beta \lor \gamma))$	associativity of $\lor$
$\neg(\neg\alpha)$	$\equiv$	α	double-negation elimination
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\beta \Rightarrow \neg\alpha)$	contraposition
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\alpha \lor \beta)$	implication elimination
$(\alpha \Leftrightarrow \beta)$	$\equiv$	$((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha))$	biconditional elimination
$\neg(\alpha \land \beta)$	$\equiv$	$(\neg\alpha \lor \neg\beta)$	De Morgan
$\neg(\alpha \lor \beta)$	$\equiv$	$(\neg\alpha \land \neg\beta)$	De Morgan
$(\alpha \land (\beta \lor \gamma))$	$\equiv$	$((\alpha \land \beta) \lor (\alpha \land \gamma))$	distributivity of $\land$ over $\lor$
$(\alpha \lor (\beta \land \gamma))$	$\equiv$	$((\alpha \lor \beta) \land (\alpha \lor \gamma))$	distributivity of $\lor$ over $\land$

Query: $\neg P_{1,2}$. Can we prove if this sentence be entailed from KB using inference rules?---------------------

$R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$	
$R_6 : (B_{1,1} \Rightarrow (P_{1,2} \lor P_{2,1})) \land ((P_{1,2} \lor P_{2,1}) \Rightarrow B_{1,1})$	Biconditional Elimination
$R_7 : ((P_{1,2} \lor P_{2,1}) \Rightarrow B_{1,1})$	And Elimination
$R_8 : (\neg B_{1,1} \Rightarrow \neg (P_{1,2} \lor P_{2,1}))$	Contraposition
$R_9 : \neg (P_{1,2} \lor P_{2,1})$	Modus Ponens
$R_{10} : \neg P_{1,2} \land \neg P_{2,1}$	Demorgans
R11: $\neg \mathbf{P_{1,2}}$	And Elimination

Propositional Logic

Proof by Contradiction

"If there was rain then travelling is difficult. If they had umbrella then travelling is not difficult. They had umbrella. Therefore there was no rain."

S1: Convert the facts into Propositions

Let

p denote the phrase "There was rain"

q denote the phrase "Travelling is difficult"

R denote the phrase "They had umbrella"

S2: Construct the premises from the observations

$(p \rightarrow q)$: If there was rain then travelling is difficult.

$(r \rightarrow \neg q)$: If they had umbrella then travelling is not difficult.

r : They had umbrella

$\neg p$: There was no rain

Proof by Contradiction

"If there was rain then travelling is difficult. If they had umbrella then travelling is not difficult. They had umbrella. Therefore there was no rain."

S2: Construct the premises from the observations

$(p \rightarrow q)$: If there was rain then travelling is difficult.

$(r \rightarrow \neg q)$: If they had umbrella then travelling is not difficult.

r : They had umbrella

$\neg p$: There was no rain

$$(p \rightarrow q) \land (r \rightarrow \neg q) \land r \Rightarrow \neg p$$

S3: Prove the RHS by deriving it from the LHS **OR** Assume RHS's contradiction as one of premise and derive at answer as false to prove that your assumption is wrong. In our example , we are going to follow first method

Propositional Logic

"If there was rain then travelling is difficult. If they had umbrella then travelling is not difficult. They had umbrella. Therefore there was no rain."

R1: $(p \rightarrow q)$

R2: $(r \rightarrow \neg q)$

R3 : r

To Prove : $\neg p$

$$(p \rightarrow q) \wedge (r \rightarrow \neg q) \wedge r \Longrightarrow \neg p$$

Step	Premises	Justification – Rule Applied
1	$(p \rightarrow q)$	Premise Inclusion Rule R1
2	$(r \rightarrow \neg q)$	Premise Inclusion Rule R2
3	r	Premise Inclusion Rule R3
4	$\neg q$	Rule T : Applying Modus Ponens on Step 2 & 3
5	$\neg p$	Rule T : Applying Modus Tollens on Step 1 & 4

Propositional Logic

innovate achieve lead

Proof by Contradiction

R1: $(p \rightarrow q)$

R2: $(r \rightarrow \neg q)$

R3 : r

R4: p

To Prove : $\neg p$

$$(p \rightarrow q) \wedge (r \rightarrow \neg q) \wedge r \Longrightarrow \neg p$$

Step	Premises	Justification – Rule Applied
1	**p**	Negation of RHS . Premise Inclusion R4
2	**$(p \rightarrow q)$**	Premise Inclusion Rule R1
3	**q**	Rule T : Applying Modus Ponens on Step 1 & 2
4	**$(r \rightarrow \neg q)$**	Premise Inclusion Rule R2
5	**$\neg r$**	Rule T : Applying Modus Tollens on Step 3 & 4
6	**r**	Premise Inclusion Rule R3
6	**FALSE**	Contradiction on Step 5 & 6

Propositional Logic

Proof by Contradiction

R1: $(p \rightarrow q)$

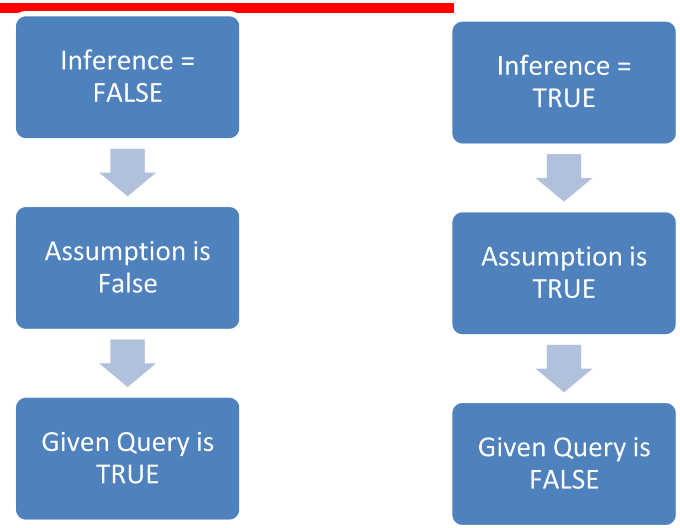
R2: $(r \rightarrow \neg q)$

R3 : r

R4: p

<u>To Prove :</u> $\neg p$

$$(p \rightarrow q) \wedge (r \rightarrow \neg q) \wedge r \Longrightarrow \neg p$$



Step	Premises	Justification – Rule Applied
1	**p**	Negation of RHS . Premise Inclusion R4
2	**$(p \rightarrow q)$**	Premise Inclusion Rule R1
3	**q**	Rule T : Applying Modus Ponens on Step 1 & 2
4	**$(r \rightarrow \neg q)$**	Premise Inclusion Rule R2
5	**$\neg r$**	Rule T : Applying Modus Tollens on Step 3 & 4
6	**r**	Premise Inclusion Rule R3
6	**FALSE**	Contradiction on Step 5 & 6

PL-Resolution : CNF conversion

Wumpus world Book example

$R_1 : \neg P_{1,1}$

$\mathbf{R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})}$

$\mathbf{R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})}$

$R_4 : \neg B_{1,1}$

$R_5 : B_{2,1}$

Query: $\neg P_{1,2}$

Conjunctive Normal Form :

(A V ~B) ^ (A V B V ~C) ^ ~A

Unit Resolution : ~A

Query : Is ‘C’ true?

PL-Resolution : CNF conversion

innovate achieve lead

Wumpus world Book example

$R_1 : \neg P_{1,1}$

$\mathbf{R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})}$

$\mathbf{R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})}$

$R_4 : \neg B_{1,1}$

$R_5 : B_{2,1}$

Query: $\neg P_{1,2}$

Eliminate		$\mathbf{R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})}$	$\mathbf{R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})}$
$\leftrightarrow$	Biconditional Elimination	$(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$	$(B_{2,1} \Rightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})) \wedge ((P_{1,1} \vee P_{2,2} \vee P_{3,1}) \Rightarrow B_{2,1})$
$\rightarrow$	Implication Elimination	$\neg B_{1,1} \vee (P_{1,2} \vee P_{2,1})$ $\neg\mathbf{(P_{1,2}} \vee \mathbf{P_{2,1})} \vee \mathbf{B_{1,1}}$	$\neg B_{2,1} \vee (P_{1,1} \vee P_{2,2} \vee P_{3,1})$ $\neg\mathbf{(P_{1,1}} \vee \mathbf{P_{2,2}} \vee \mathbf{P_{3,1})} \vee \mathbf{B_{2,1}}$
Clause level $\neg$	De Morgan	$(\neg\mathbf{P_{1,2}} \wedge \neg\mathbf{P_{2,1}}) \vee \mathbf{B_{1,1}}$	$(\neg\mathbf{P_{1,1}} \wedge \neg\mathbf{P_{2,2}} \wedge \neg\mathbf{P_{3,1}}) \vee \mathbf{B_{2,1}}$
CNF Form	Distributivity of $\vee$ over $\wedge$	$(\neg\mathbf{P_{1,2}} \vee \mathbf{B_{1,1}}) \wedge (\neg\mathbf{P_{2,1}} \vee \mathbf{B_{1,1}})$	$(\neg\mathbf{P_{1,1}} \vee \mathbf{B_{2,1}}) \wedge (\neg\mathbf{P_{2,2}} \vee \mathbf{B_{2,1}}) \wedge (\neg\mathbf{P_{3,1}} \vee \mathbf{B_{2,1}})$

PL-Resolution

$R_1 : \neg P_{1,1}$

$\cancel{R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})}$

$\cancel{R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})}$

$R_4 : \neg B_{1,1}$

$R_5 : B_{2,1}$

Query: $\neg P_{1,2}$

$R_6 : \neg B_{1,1} \vee P_{1,2} \vee P_{2,1}$

$R_7 : \neg P_{1,2} \vee B_{1,1}$

$R_8 : \neg P_{2,1} \vee B_{1,1}$

$R_9 : \neg B_{2,1} \vee P_{1,1} \vee P_{2,2} \vee P_{3,1}$

$R_{10} : \neg P_{1,1} \vee B_{2,1}$

$R_{11} : \neg P_{2,2} \vee B_{2,1}$

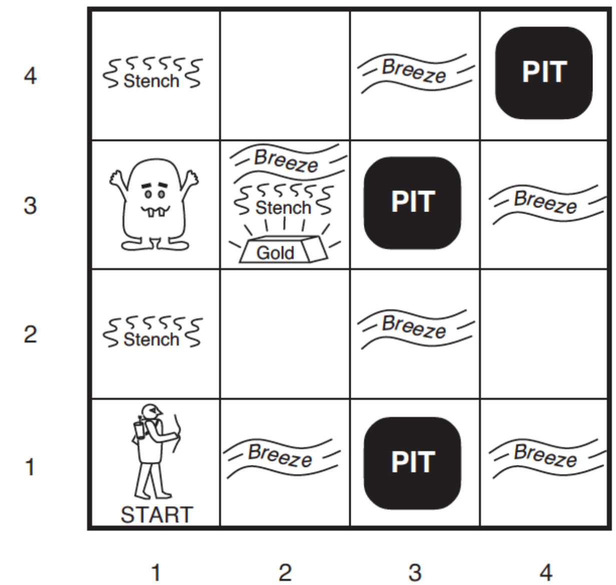
$R_{12} : \neg P_{3,1} \vee B_{2,1}$

PL-Resolution

Unit Resolution: Query: $\neg P_{1,2}$

To find: Is there a pit in location (1,2) using the CNF obtained in previous slide

$$
\begin{tikzcd}
 & \neg B_{1,1} \arrow[dr] & & \neg P_{1,2} \vee B_{1,1} \arrow[dl] \\
P_{1,2} \arrow[dr] & & \neg P_{1,2} \arrow[dl] & \\
 & \{\ \} & &
\end{tikzcd}
$$



PL-Resolution

Unit Resolution: Query: $B_{2,1}$

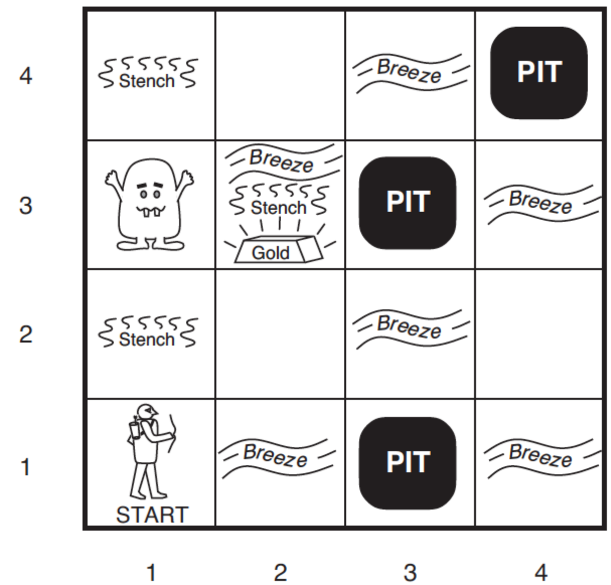
To prove : Is there a breeze in location (2,1) using the CNF obtained in previous slide

$(\neg P_{1,1} \vee B_{2,1}) \wedge (\neg P_{2,2} \vee B_{2,1}) \wedge (\neg P_{3,1} \vee B_{2,1})$

$P_{1,1}$ $\qquad$ $\neg P_{1,1} \vee B_{2,1}$

$\neg B_{2,1}$ $\qquad$ $B_{2,1}$

$\{\ \}$

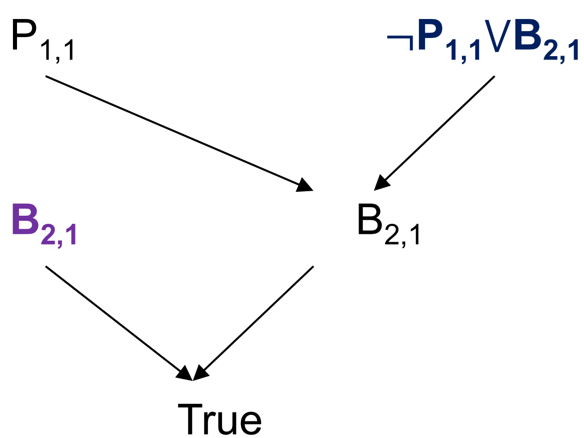


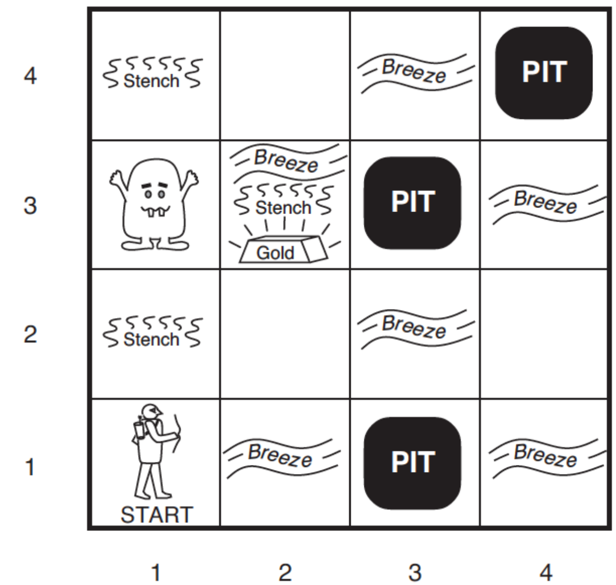
PL-Resolution

Unit Resolution: Query: $\neg B_{2,1}$

To prove : Is there a breeze in location (2,1) using the CNF obtained in previous slide

$(\neg P_{1,1} \vee B_{2,1}) \wedge (\neg P_{2,2} \vee B_{2,1}) \wedge (\neg P_{3,1} \vee B_{2,1})$





PL-Resolution

Unit Resolution: Query: $P_{3,1}$

$R_1 : \neg P_{1,1}$

$\cancel{R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})}$

$\cancel{R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \lor P_{2,2} \lor P_{3,1})}$

$R_4 : \neg B_{1,1}$

$R_5 : B_{2,1}$

R`: $\neg P_{3,1}$

$R_6 : \neg B_{1,1} \lor P_{1,2} \lor P_{2,1}$

$R_7 : \neg P_{1,2} \lor B_{1,1}$

$R_8 : \neg P_{2,1} \lor B_{1,1}$

$R_9 : \neg B_{2,1} \lor P_{1,1} \lor P_{2,2} \lor P_{3,1}$

$R_{10} : \neg P_{1,1} \lor B_{2,1}$

$R_{11} : \neg P_{2,2} \lor B_{2,1}$

$R_{12} : \neg P_{3,1} \lor B_{2,1}$

$\neg P_{1,1}$ $\neg B_{2,1} \lor P_{1,1} \lor P_{2,2} \lor P_{3,1}$

$B_{2,1}$ $\neg B_{2,1} \lor P_{2,2} \lor P_{3,1}$

$\neg P_{3,1}$ $P_{2,2} \lor P_{3,1}$

$\{P_{2,2}\}$

DPLL Algorithm

In logic and computer science, the Davis–Putnam–Logemann–Loveland (**DPLL**) **algorithm** is a complete, backtracking-based search **algorithm** for deciding the satisfiability of propositional logic formulae in conjunctive normal form

Improvements:

1. Early Termination
2. Pure Symbolic Heuristic
3. Unit Clause Heuristic

DPLL Algorithm

$R_7 : \neg P_{1,2} \vee B_{1,1}$

$R_8 : \neg P_{2,1} \vee B_{1,1}$

$\{P_{1,2}, B_{1,1}, P_{2,1}\}$

$R_7 : \neg P_{1,2} \vee B_{1,1}$

$R_8 : \neg P_{2,1} \vee B_{1,1}$

$R_7 : \neg P_{1,2} \vee B_{1,1}$

$R_8 : \neg P_{2,1} \vee B_{1,1}$

$R_7 : \neg P_{1,2} \vee B_{1,1}$

$R_8 : \neg P_{2,1} \vee B_{1,1}$

Example: Propositional Functions (Quantifiers)

All courses are offered and interesting

All offered courses are interesting

Some of the courses are offered and interesting [Atleast one of the offered courses is interesting]

Some of the offered courses are interesting

FOL : Inference

Prove that following statements are valid:

1. Predicate :

"All humans are mortals and Socrates is a human. Therefore Socrates is mortal"

S1: Construct the premises from the predicates:

$\forall x$ [Human(x) → Mortal(x)] denotes the phrase :"All humans are mortals"

Human(socrates) denotes "Socrates is a human"

Mortal(socrates) denotes "Socrates is mortal"

S2: Construct the premises by framing the statement to prove:

$\forall x$ [Human(x) → Mortal(x)] ∧ Human(socrates) ⟹ Mortal(socrates)

FOL : Inference

To Prove :

$\forall x$ [Human(x) → Mortal(x)] ∧ Human(socrates) ⟹ Mortal(socrates)

2 Ways:

1. Prove the RHS by deriving it from the LHS

OR

2. Assume RHS's contradiction as one of premise and derive at answer as false to prove that your assumption is wrong.

In our example , we are going to follow second method for practice

S3 : Assumption to Proof

By assumption, take negation of RHS as one of evidence along with LHS

ie., assume the phrase "Socrates is not mortal" as true.

New Premise to add: ¬ **Mortal(socrates)**

FOL : Inference

To Prove :

$\forall x$ [Human(x) → Mortal(x)] ∧ Human(socrates) ⟹ Mortal(socrates)

$R1$: ¬ Mortal(socrates)

Step	Premises	Justification / Rules Included/ Inference used
1	$\forall x$ [Human(x) → Mortal(x)] ∧ Human(socrates)	Rule P: Taken LHS of to Prove
2	$\forall x$ [Human(x) → Mortal(x)]	Rule T : AND Elimination on Step 1
3	[Human(socrates) → Mortal(socrates)]	Rule T : Universal Specification : Removal of quantifiers
4	¬ Human(socrates) ∨ Mortal(socrates)	Material Implication Law
5	Human(socrates)	Rule T : AND Elimination on Step 1
6	Mortal(socrates)	Disjunctive Syllogism on Step 4 & 5
7	¬ Mortal(socrates)	R1: Premise Inclusion from our assumption
8	FALSE	AND Inclusion on Step 6 & 7
Hence our assumption is wrong. Thus its proved that Mortal(socrates) is true.		

innovate achieve lead

Forward Chaining

- Consider the following problem:

The law says it is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.

- We will prove that West is a criminal

Make the problem Simpler:

1. Convert phrases into definite clause
2. Apply demorgans law (if required) to move all negation to single terms
3. Move all quantifiers to the left – Prenex Normal Form
4. Eliminate the quantifiers

Forward Chaining

- First, we will represent the facts in First Order Definite Clauses

" … it is a crime for an American to sell weapons to hostile nations"

$$American(x) \wedge Weapon(y) \wedge Sells(x,y,z) \wedge Hostile(z) \Rightarrow Criminal(x)$$

"Nono … has some missiles"

$$\exists x\ Owns(Nono,x) \wedge Missile(x)$$

is transformed into two definite clauses by Existential Instantiation

$$Owns(Nono, M_1)$$

$$Missile(M_1)$$

Forward Chaining

- “All of its missiles were sold to it by Colonel West”

$$Missile(x) \wedge Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$$

- Missiles are weapons

$$Missile(x) \Rightarrow Weapon(x)$$

- Hostile means enemy

$$Enemy(x, America) \Rightarrow Hostile(x)$$

- ”West, who is American”

$$American(West)$$

- “The country Nono, an enemy of America”

$$Enemy(Nono, America)$$

Forward Chaining

- Consider the following problem:

The law says it is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.

- We will prove that West is a criminal

Algorithm:

1. Start from the facts
2. Trigger all rules whose premises are satisfied
3. Add the conclusion to known facts
4. Repeat the steps till no new facts are added or the query is answered

Forward Chaining

1. $American(x) \land Weapon(y) \land Sells(x, y, z) \land Hostile(z) \Rightarrow Criminal(x)$
2. $Missile(x) \land Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$
3. $Missile(x) \Rightarrow Weapon(x)$
4. $Enemy(x, America) \Rightarrow Hostile(x)$

Missile(M1)

Owns(Nono, M1)

American (West)

Enemy (Nono, America)

Forward Chaining

1. $American(x) \wedge Weapon(y) \wedge Sells(x,y,z) \wedge Hostile(z) \Rightarrow Criminal(x)$
2. $Missile(x) \wedge Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$
3. $Missile(x) \Rightarrow Weapon(x)$
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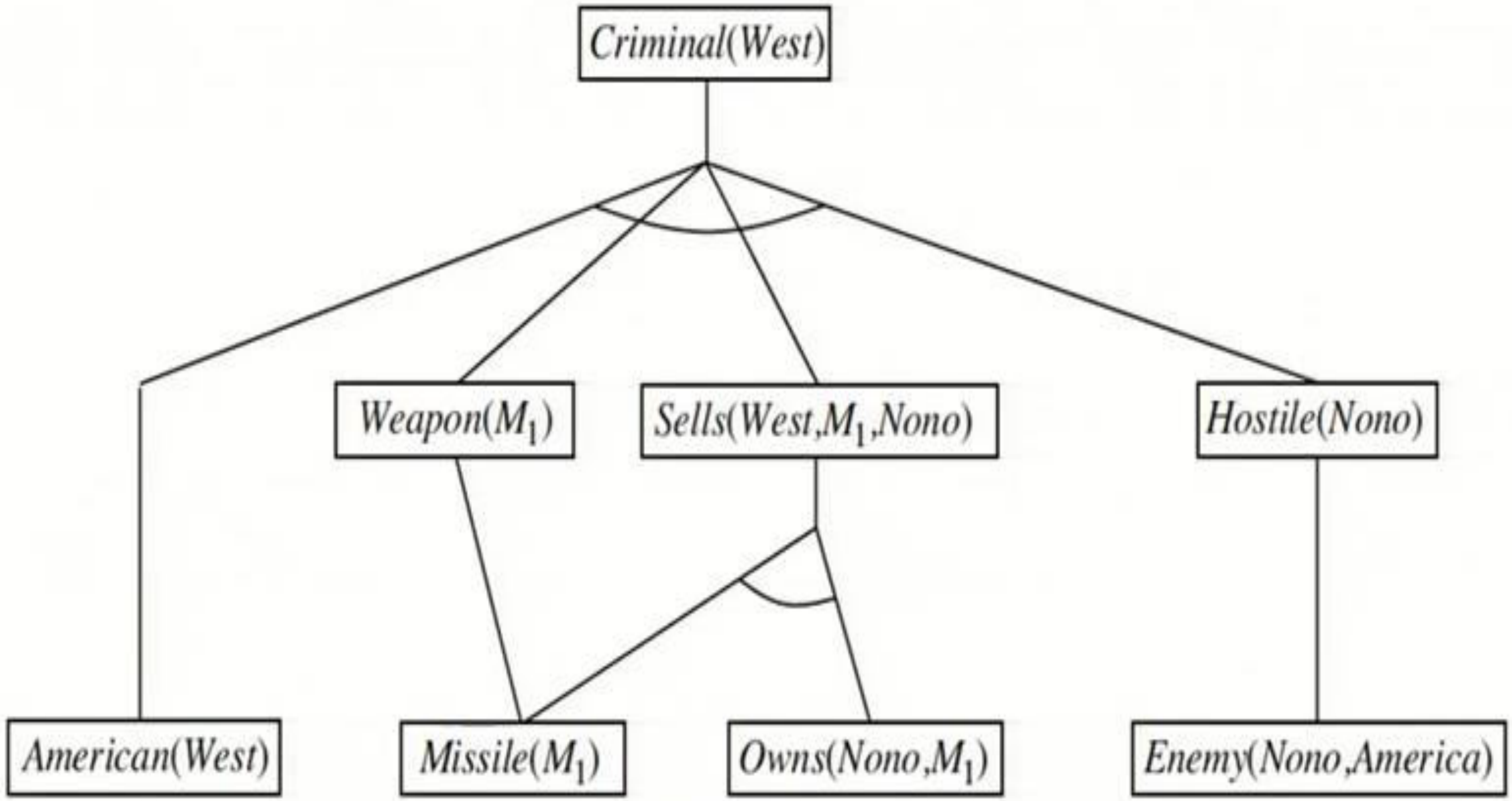


Backward Chaining

Algorithm:

1. Form Definite Clause
2. Start from the Goals
3. Search through rules to find the fact that support the proof
4. If it stops in the fact which is to be proved → Empty Set- LHS

Divide & Conquer Strategy

Substitution by Unification

Backward Chaining

1. $American(x) \land Weapon(y) \land Sells(x,y,z) \land Hostile(z) \Rightarrow Criminal(x)$
2. $Missile(x) \land Owns(Nono,x) \Rightarrow Sells(West,x,Nono)$
3. $Missile(x) \Rightarrow Weapon(x)$
4. $Enemy(x, America) \Rightarrow Hostile(x)$

Missile(M1)
Owns(Nono, M1)
American (West)
Enemy (Nono, America)

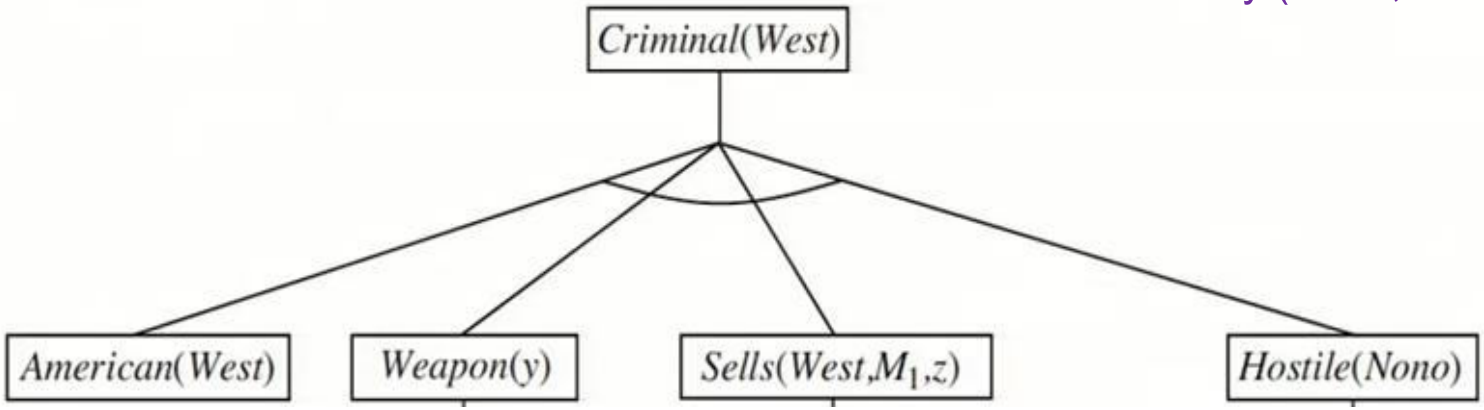
$$American(x) \land Weapon(y) \land Sells(x,y,z) \land Hostile(z) \Rightarrow Criminal(x)$$

$$\boxed{Criminal(West)}$$

Backward Chaining

1. $American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$
2. $Missile(x) \wedge Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$
3. $Missile(x) \Rightarrow Weapon(x)$
4. $Enemy(x, America) \Rightarrow Hostile(x)$

Missile(M1)
Owns(Nono, M1)
American (West)
Enemy (Nono, America)



Backward Chaining

1. $American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$
2. $Missile(x) \wedge Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$
3. $Missile(x) \Rightarrow Weapon(x)$
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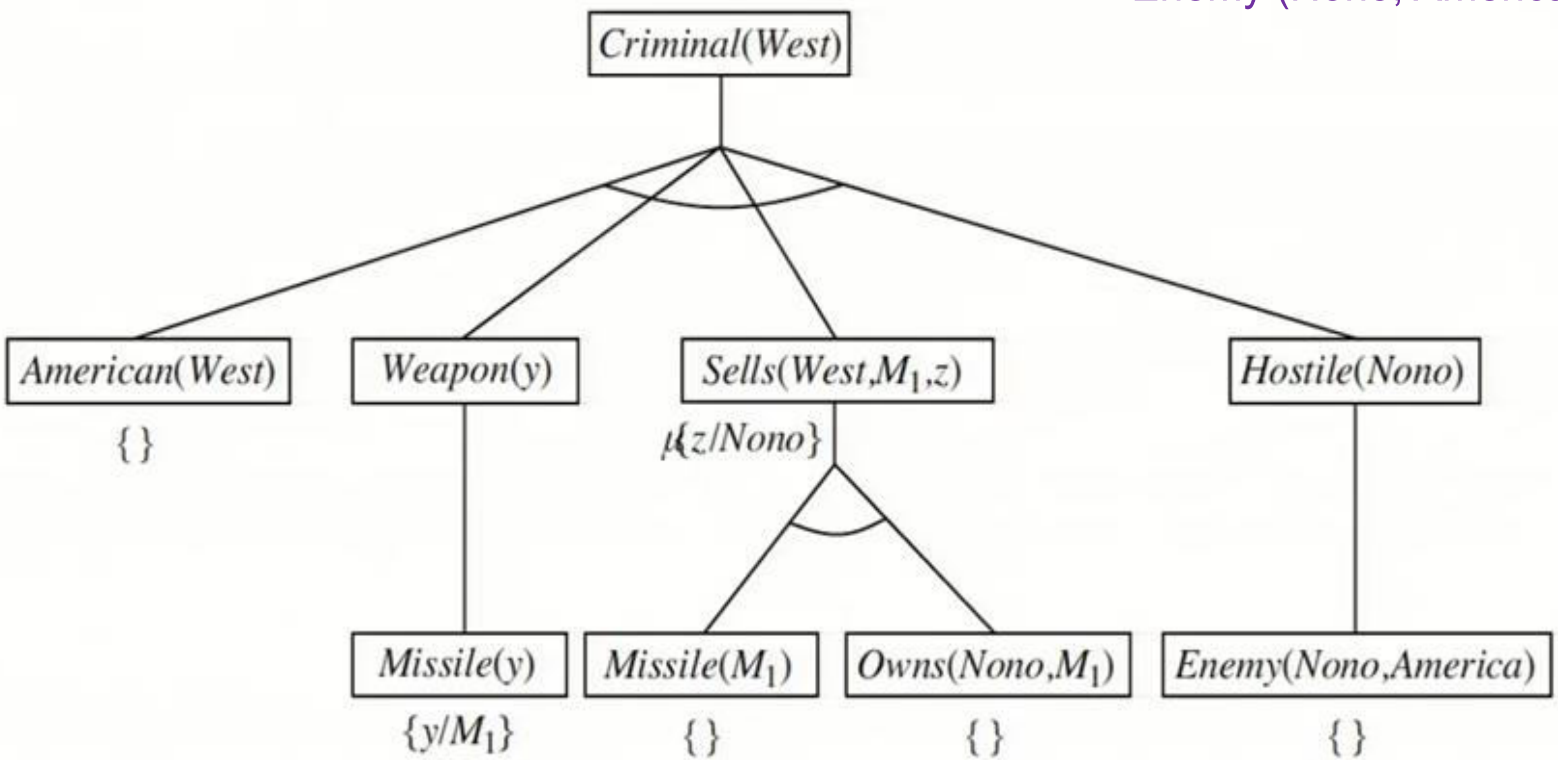
Missile(M1)
Owns(Nono, M1)
American (West)
Enemy (Nono, America)

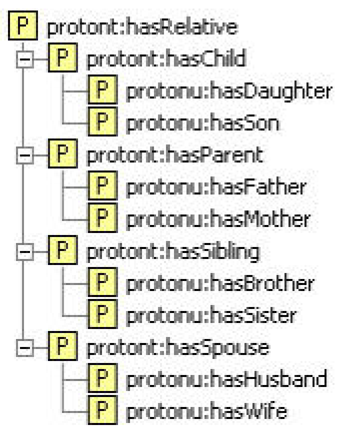


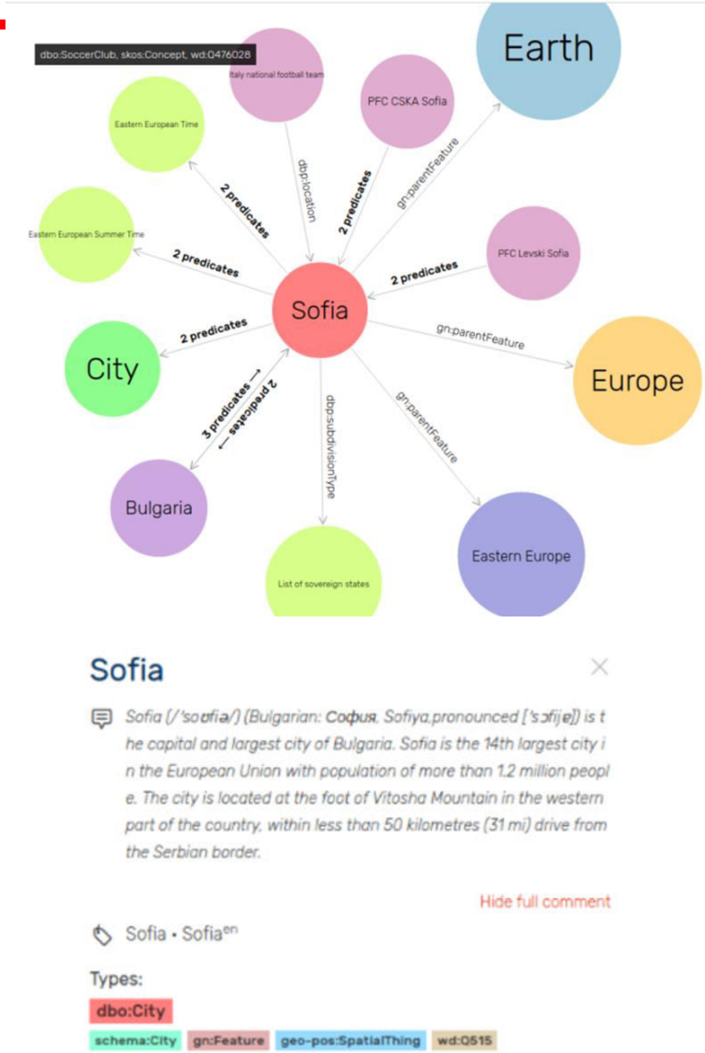
Example #2 : Chaining

All courses offered are interesting	O(Z) → I(Z)
Students like easy to score courses	E(A) → L(s, A)
Data Mining is a Compute Science course	C(dm, cs)
Some of the easy courses are interesting	E(B) → I(B)
All Math are interesting course	C(E,math) → I(E)
Statistics is a math course	C(stat, math)
All computer science course are easy	C(D,cs) → E(D)

Q1: Do students like statistics?
Q2: What course does students like?

In software Product





Source Credit: [GraphDB]

SELECT ?subject ?predicate ?object **WHERE** { ?subject ?predicate ?object . }

Robotic Process Automation

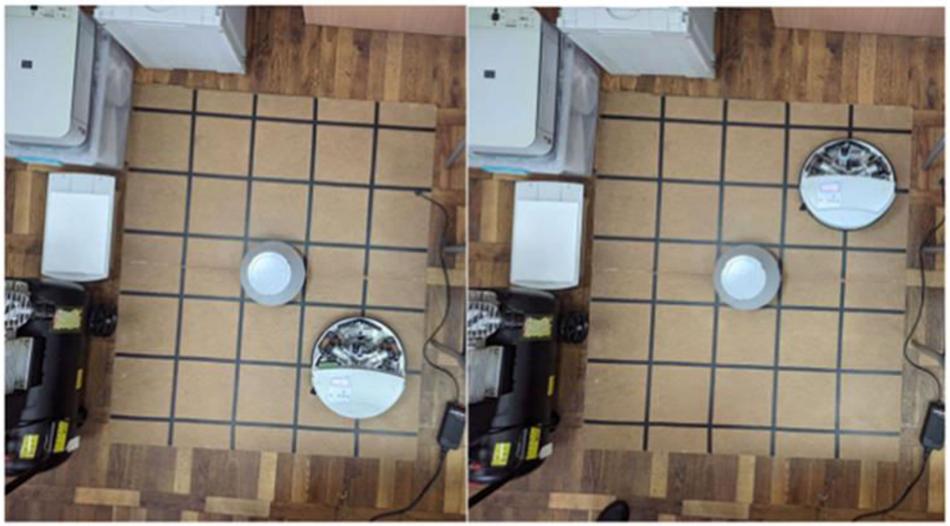


Figure 3. Example of discrete workspace for the Festo Robotino.

States:
is_at(robot, door45, now)
is_with(robot, box1, now))
stands(door12, closed, now)
is_in(box1, room1. now))
Actions:
"go_to";"open_door"; "take_box"; "push_box";

$$pg^0(pr,ps) \leftarrow pg_0^0(pr_0,ps_0,pa_0) \wedge pg_1^0(pr_1,ps_1,pa_1) \wedge \ldots \wedge pg_{n-1}^0(pr_{n-1},ps_{n-1},pa_{n-1}) = \wedge_{i=0}^{n-1} pg_i^0(pr_i,ps_i,pa_i)$$

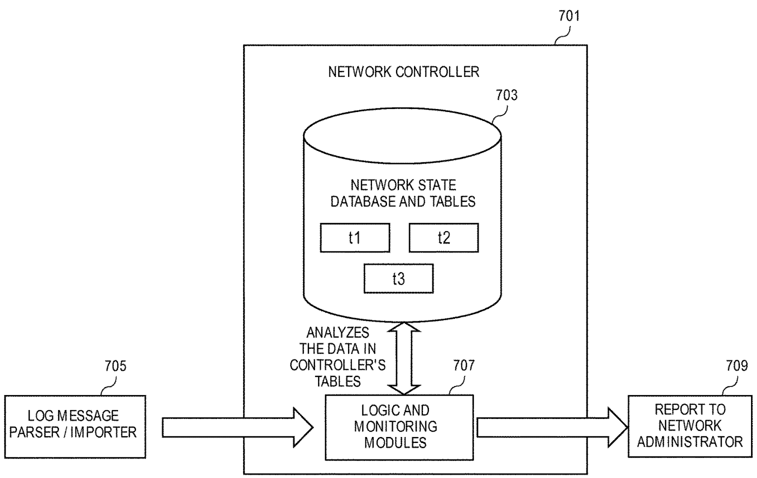
$$pg^m(pr,ps) \leftarrow pg_0^m(pr_0,ps_0,pa_0) \wedge pg_1^m(pr_1,ps_1,pa_1) \wedge \ldots \wedge pg_{n-1}^m(pr_{n-1},ps_{n-1},pa_{n-1}) = \wedge_{i=0}^{n-1} pg_i^m(pr_i,ps_i,pa_i).$$

, the global (final) goal is defined as follows:

$$pg^{total}(pr, ps) \leftarrow \vee_{j=0}^{m-1} \wedge_{i=0}^{n-1} pg_i^m(pr_i, ps_i, pa_i).$$

Source Credit:Tsymbal, O.; Mercorelli, P.; Sergiyenko, O.
[Model of Problem-Solving for Robotic Actions Planning. Mathematics 2021, 9, 3044.]

Software Defined Networking



Path_Untrusted(any_path) :- path(any_path, node a, node b), link(any_path, some link), link untrusted(some link) .

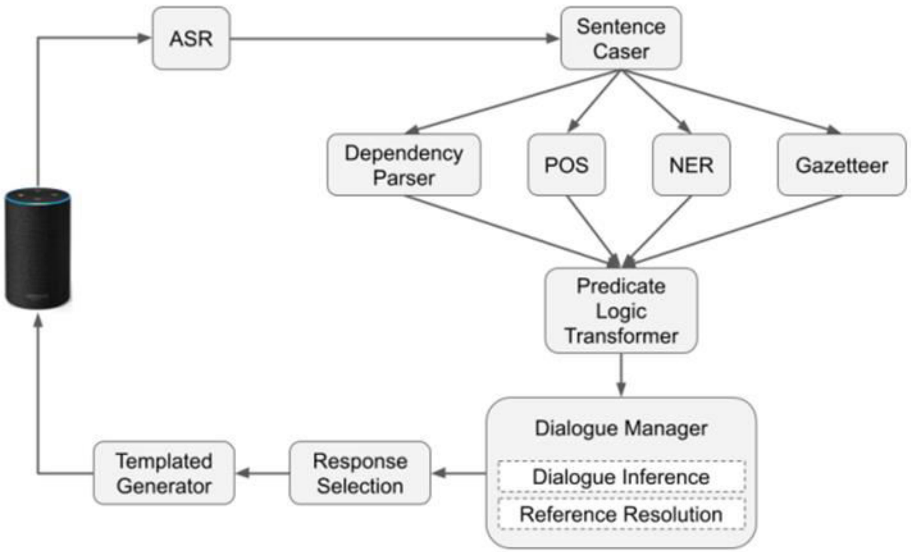
Warning (Path Untrusted).

Expectation El = (| Routes(customer) | <= | Interface(router, intf) , Virtualrouter(customer, router) |).

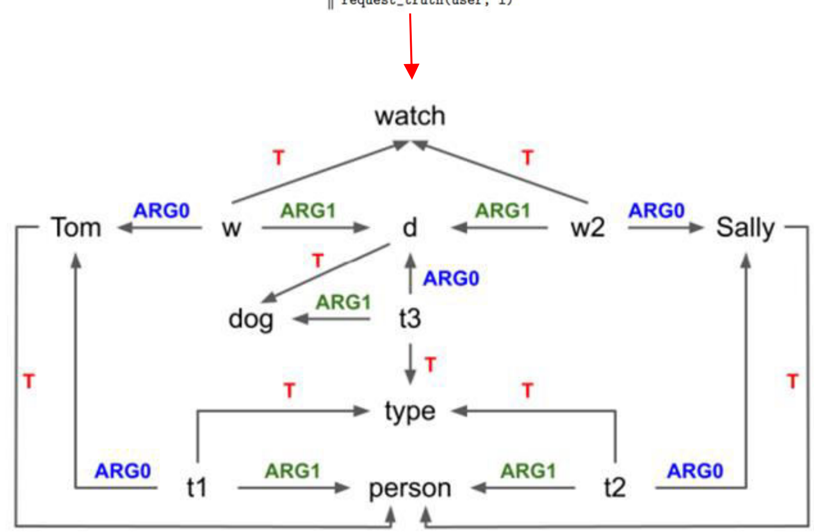
Source Credit: Patent : 2016-2018
Sdn controller logic-inference network troubleshooter (sdn-lint) tool

Logics : Representation & Reasoning

Natural Language – Chat bot



English Sentence	Predicate Logic
I ran on the treadmill.	r/run(user) time(r, past) on(r, t) type(t, treadmill)
I like watching action movies.	l/like(user, watch(user, m)) time(l, now) type(m, movie) type(m, group) action(m)
Your dog is sweet.	s/sweet(d) time(s, now) type(d, dog) possess(bot, d)
I am a math teacher.	b/be(user, t) time(b, now) type(t, teacher) of(t, math)
My grades fell quickly after I stopped studying.	f/fall(g) time(f, past) type(g, grade) type(g, group) possess(user, g) quick(f) after(f, s/stop(user, study(user)) time(s, past)
I didn't eat lunch yet.	e/eat(user, l) type(l, lunch) not(e) time(e, past)
I should eat lunch.	e/eat(user, l) type(l, lunch) should(e) time(e, now)
What musical instrument do you play?	p/play(bot, i) type(i, musical_instrument) time(p, now) request(user, i)
Did you like the book I gave you?	l/like(bot, b) type(b, book) time(l, past) g/give(user, b) recipient(g, bot) time(g, past) request_truth(user, l)



Source Credit: [An Approach to Inference-Driven Dialogue Management within a Social Chatbot](#)
[4th Proceedings of Alexa Prize (Alexa Prize 2020)](#)

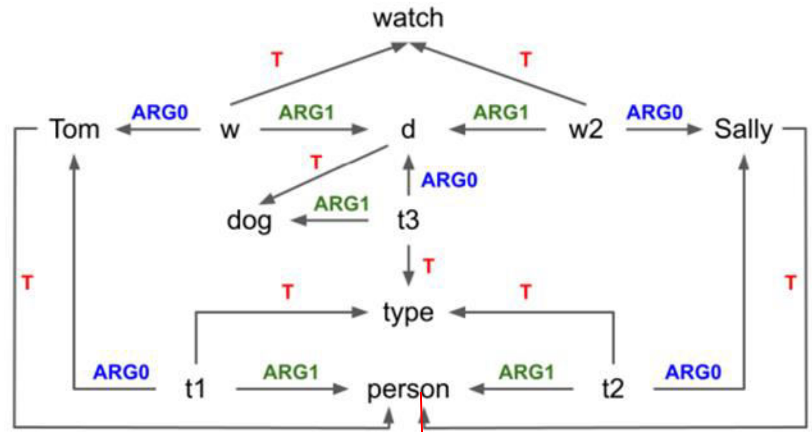
Natural Language – Chat bot

I/like(person(), movie())
→
cause(l, reason())

(a) Inference rule representing the common sense notion that a person likes a movie for a reason

I/like(user, X/item())
cause(l, reason())
→
What do you like about X?

(b) Template rule transforming the logical forms from (a) to an appropriate natural language expression



Rule	Solution	Span Attachments	Concept Attachments
(a)	X: "watched" Y: "Tom"	("watched", ARG0, "Tom")	(w, ARG0, Tom)
(b)	X: "watched" Y: "dog"	("watched", ARG1, "dog")	(w, ARG1, d)
(c)	X: "watched" Y: "stop" Z: "by"	("by", ARG0, "watched") ("by", ARG1, "stop")	(b, ARG0, w) (b, ARG1, bs)
(c)	X: "stop" Y: "park" Z: "near"	("near", ARG0, "stop") ("near", ARG1, "park")	(n, ARG0, bs) (n, ARG1, cp)

Span	Concept
Tom	Tom
watched	$w * /watch(-,-)$
dog	$type(d*, dog)$
by	$b * /by(-,-)$
bus stop	$type(bs*, bus\_stop)$
near	$n * /near(-,-)$

(a) Concepts from Gazetteer

Span	Concept
Tom	$type(t*, per)$
Central Park	$type(cp*, loc)$

(b) Concepts based on NER

Span	Concept
Tom	$type(t*, nnp)$
watched	$type(w*, vbd)$
dog	$type(d*, nn)$
by	$type(b*, in)$
bus	$type(b*, nn)$
stop	$type(s*, nn)$
near	$type(n*, in)$
Central	$type(c*, nnp)$
Park	$type(p*, nnp)$

(c) Concepts based on POS

Source Credit: [An Approach to Inference-Driven Dialogue Management within a Social Chatbot](#)
[4th Proceedings of Alexa Prize (Alexa Prize 2020)](#)

Natural Language – Chat bot

Bot: I'm a big fan of action movies.
User: Yeah, I like the Avengers .
Bot: What do you like about the Avengers?

(a)

User: Let's talk about movies.
Bot: Is there a particular movie that you really like?
User: The Avengers
Bot: What do you like about the Avengers?

(b)

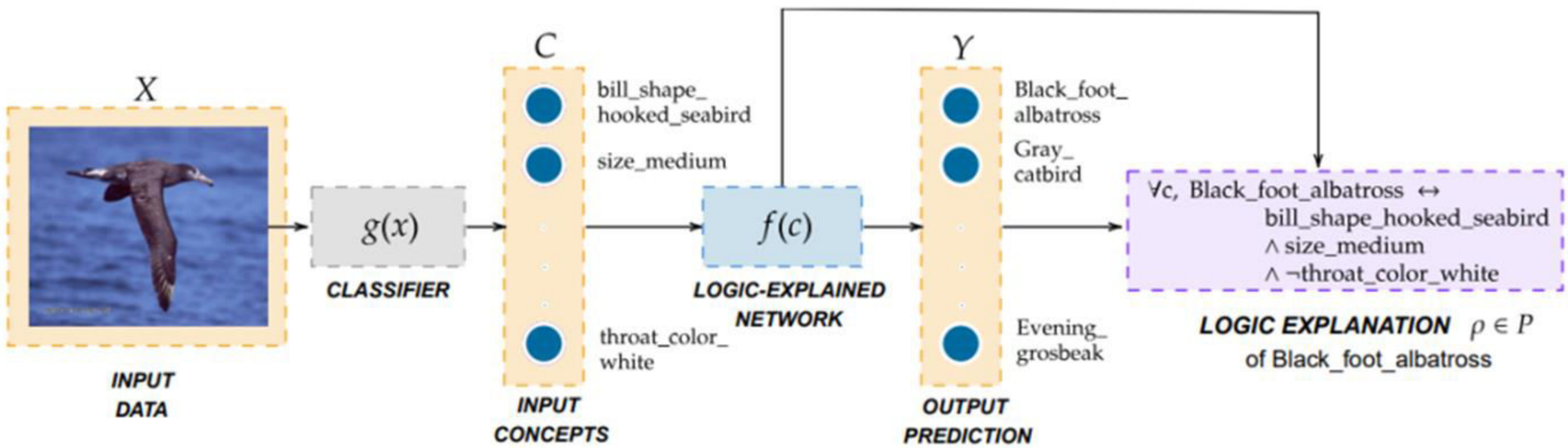
Bot: What did you do this weekend?
User: I watched the Avengers. It's my favorite movie.
Bot: That sounds fun. For my weekend I went hiking.
User: That's cool.
Bot: What do you like about the Avengers?

(c)

Source Credit: [An Approach to Inference-Driven Dialogue Management within a Social Chatbot](#)
[4th Proceedings of Alexa Prize (Alexa Prize 2020)](#)

Logics : Representation & Reasoning

In Deep Learning



Source Credit: [2021: Logic Explained Networks]

Required Reading: AIMA - Chapter #7, #8, #9

Thank You for all your Attention

Note : Some of the slides are adopted from AIMA TB materials

Module 5: Next Session Plan

Probabilistic Representation and Reasoning

A. Inference using full joint distribution

B. Bayesian Networks

I. Knowledge Representation

II. Conditional Independence

III. Exact Inference

IV. Introduction to Approximate Inference

BITS Pilani, Pilani Campus