Artificial & Computational Intelligence

DSE CLZG557

M6 : Reasoning over time & Reinforcement Learning

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Course Plan

M1 Introduction to AI

M2 Problem Solving Agent using Search

M3 Game Playing, Constraint Satisfaction Problem

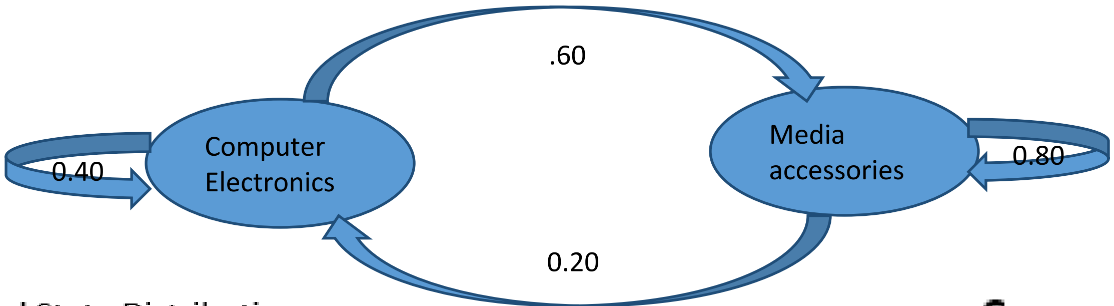
M4 Knowledge Representation using Logics

M5 Probabilistic Representation and Reasoning

M6 Reasoning over time, Reinforcement Learning

Reasoning Over Time

Morkov Model



Current State: Initial State Distribution

$$\begin{array}{c} \left|\begin{array}{c} 1 \\ 0 \end{array}\right| \end{array}\begin{array}{c} C \\ M \end{array}$$

$$\begin{array}{cc|c} C & M & \\ \hline 0.40 & 0.20 & C \\ 0.60 & 0.80 & M \end{array}$$

Next State : Likely to buy Media accessories on next visit

$$\left|\begin{array}{c} 0.40 \\ 0.60 \end{array}\right| \begin{array}{c} C \\ M \end{array}$$

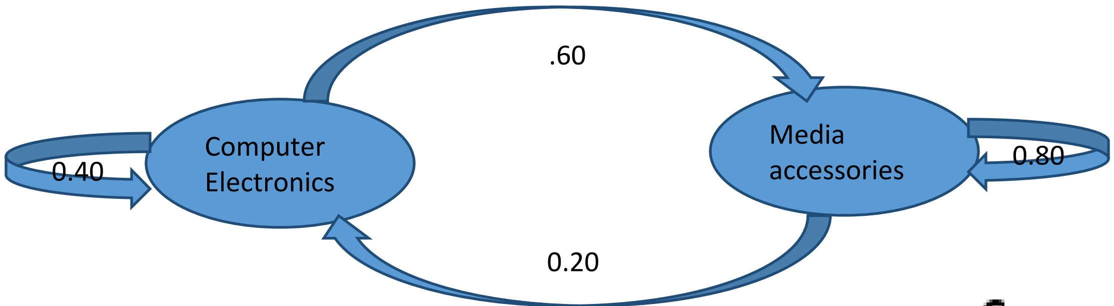
Next State : Likely to buy Media accessories on next visit

$$\left|\begin{array}{c} 0.28 \\ 0.72 \end{array}\right| \begin{array}{c} C \\ M \end{array}$$

Inference in Temporal Models

Morkov Model

Inference Type -1



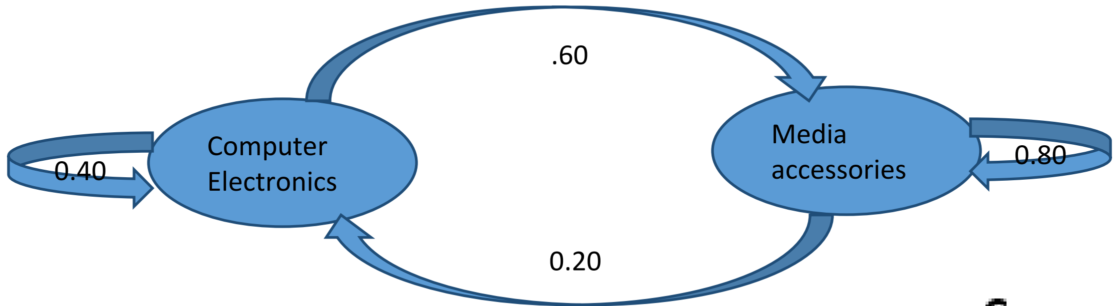
$$\begin{array}{cc|c} C & M & \\ 0.40 & 0.20 & C \\ 0.60 & 0.80 & M \end{array}$$

What is the probability that the purchasing behavior of the customer is in the below order sequentially observed?

(Computer , Media, Media, Computer)

Morkov Model

Inference Type -2

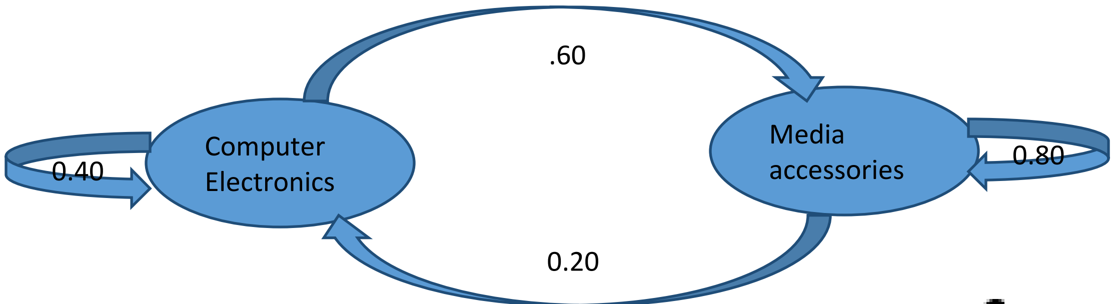


$$
\begin{array}{cc}
\begin{array}{cc} C & M \end{array} & \\
\left|\begin{array}{cc} 0.40 & 0.20 \\ 0.60 & 0.80 \end{array}\right| & \begin{array}{c} C \\ M \end{array}
\end{array}
$$

What is the probability that a customer who purchased Media accessories will return back and keep purchasing Media accessories for only 2 consecutive visits?

Morkov Model

Inference Type -3



$$
\begin{array}{cc|c}
C & M & \\
\hline
0.40 & 0.20 & C \\
0.60 & 0.80 & M
\end{array}
$$

Given that a customer walked into a store and bought a computer electronics, find the expected purchase pattern in his next 3 visits.

[HMM Veterbi]

Markov Process

innovate achieve lead

States | Observations | Assumptions

Modelling sequences of random events and transitions between states over time is known as Morkov chain

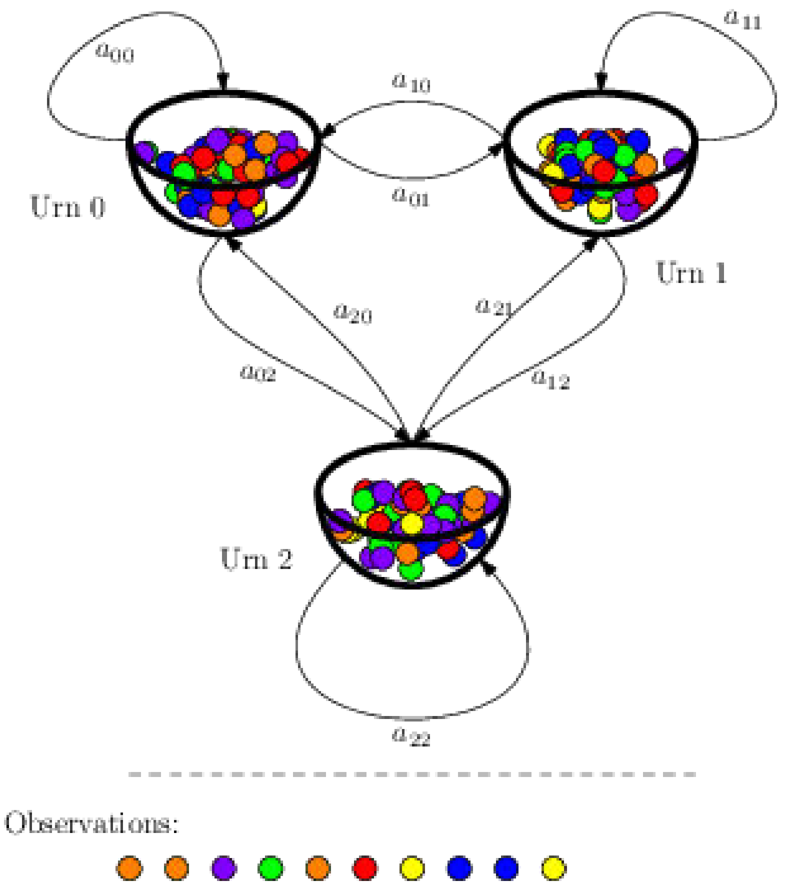
Transition Model / Probability Matrix :
Current state depends only finite number of previous states. :

Overview of HMM

Markov Process

States | Observations | Assumptions

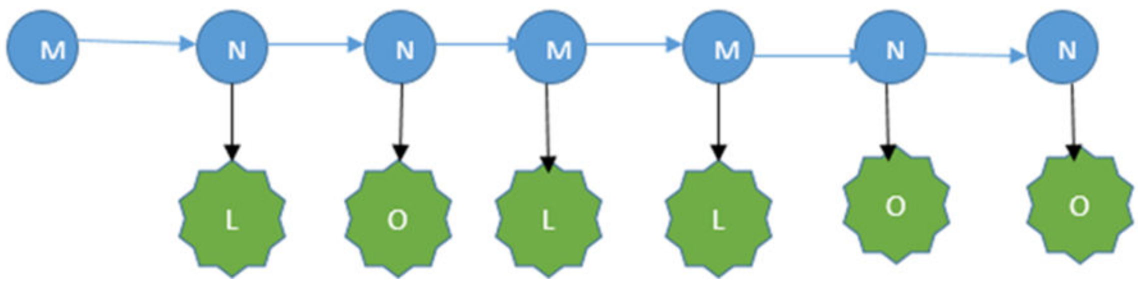
Standard Mathematical Example:
Urn & Ball Model



Hidden Markov Model

States | Observations | Assumptions

Time Slice (**t**)	0	1	2	3	4	5	6	P(**O_t | O_{t-1}**)
Observed Evidence (**O_t/ E_t**)	-	Late	OnTime	Late	Late	Ontime	Ontime	……
Unobserved State (**U_t /X_t/ Q_t**)	Meeting	No Meeting	No Meeting	Meeting	Meeting	No Meeting	No Meeting	……



Transition Model / Probability Matrix

P(**U_{t-1} = No Meeting**)	P(**U_{t-1} = Meeting**)	←Previous
0.5	0.67	P(**U_t = No Meeting**)
0.5	0.33	P(**U_t = Meeting**)

Evidence / Sensor Model/ Emission Probability Matrix

P(**U_t = No Meeting**)	P(**U_t = Meeting**)	←Unobserved Evidence v
0.9	0.3	P(**O_t = OnTime**)
0.1	0.7	P(**O_t = Late**)

Hidden Markov Process

States | Observations | Assumptions

Modelling sequences of random events and transitions between states over time is known as Morkov chain

Hidden Markov Process models events as the state sequences that are not directly observable but only be approximated from the sequence of observations produced by the system

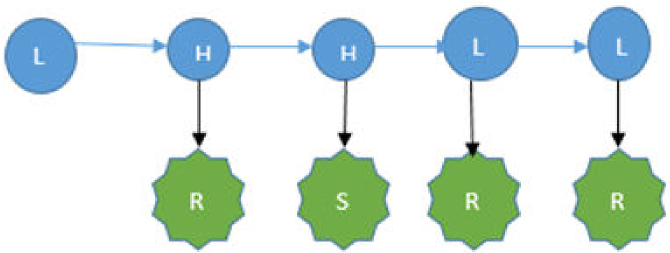
Transition Model / Probability Matrix :
Current state depends only finite number of previous states. :

Evidence / Sensor Model/ Emission Probability Matrix :
Current Evidence or Observation depends Current State of the world. Given the Current State Knowledge of the world, observation doesn't depend on history:

Hidden Morkov Model

States | Observations | Assumptions

Time Slice (t)	0	1	2	3	4	………..	$P(O_t \mid O_{t-1}, O_{t-2})$
Observed Evidence (O_t)	-	Rainy	Sunny	Rainy	Rainy		
Unobserved State(U_t)	Low Pressure	High Pressure	High Pressure	Low Pressure	Low Pressure		



Transition Model / Probability Matrix

$P(U_{t-2} = LP, U_{t-1} = HP)$	$P(U_{t-2} = HP, U_{t-1} = HP)$	$P(U_{t-2} = HP, U_{t-1} = LP)$	$P(U_{t-2} = LP, U_{t-1} = LP)$	←Previous
0.2	0.40	0.85	0.5	$P(U_t = LP)$
0.8	0.60	0.15	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

Hidden Morkov Model

Filtering

$P(L_3 \mid R\text{-}S\text{-}R\text{-}R)$

$P(X_t \mid E_{1...t})$

Prediction

$P(L_3 \mid R\text{-}S)$

$P(X_{t+k} \mid E_{1...t})$

Smoothing

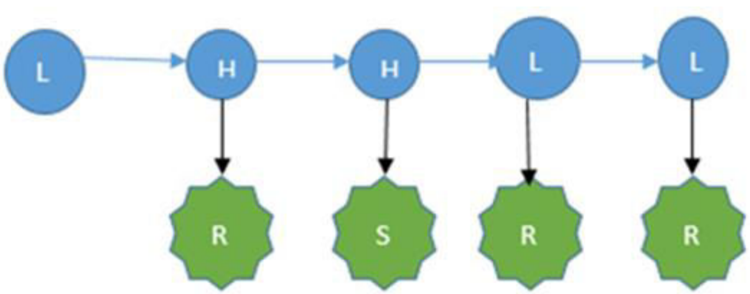
$P(H_2 \mid R\text{-}S\text{-}R\text{-}R)$

$P(X_{k,\ o>k>t} \mid E_{1...t})$

Most Likely Explanation

$P(H\text{-}H\text{-}L\text{-}L \mid R\text{-}S\text{-}R\text{-}R)$

$\text{argmax } X_{1...t} : P(X_{1...t} \mid E_{1...t})$



Hidden Morkov Model

Filtering : P(SecondUrnIsSelected $_3$ | Red-Blue-Blue-Yellow)

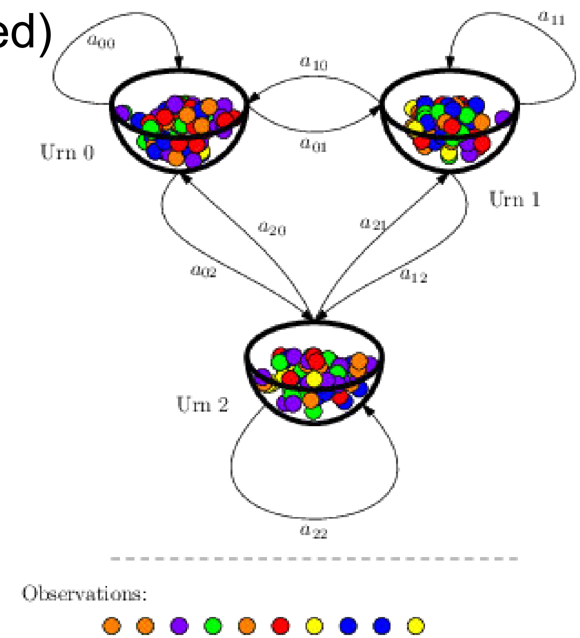
$P(X_t \mid E_{1\ldots t})$

Prediction: P(FirstUrnWillbeSelected $_3$ | Red-Yellow)

$P(X_{t+k} \mid E_{1\ldots t})$

Smoothing: P(ThirdUrnWasSelected $_2$ | Red-Yellow-Red-Red)

$P(X_{k,\ o>k>t} \mid E_{1\ldots t})$



Most Likely Explanation (or) Viterbi Algorithm

P(Urn1-Urn2-Urn1 | Red-Yellow-Yellow)

argmax $X_{1\ldots t}$: $P(X_{1\ldots t} \mid E_{1\ldots t})$

Hidden Morkov Model

Inference: Type -2

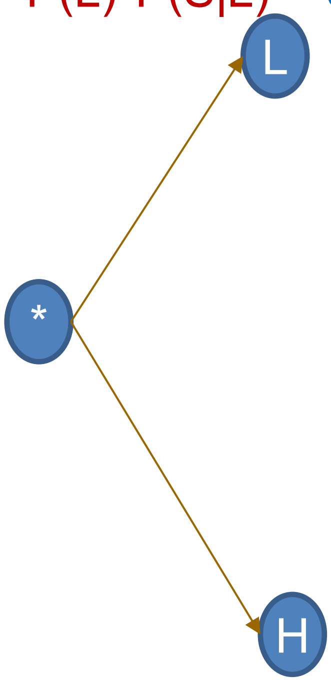
Most Likely Explanation : Veterbi Algorithm

Find the pattern in pressure that might have caused this observation: **S-S-R**

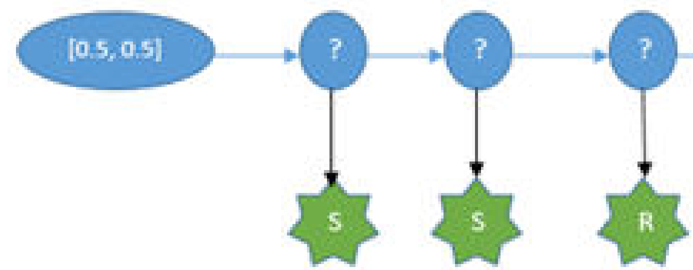
argmax $X_{1...t}$: $P(X_{1...t} \mid E_{1...t})$

MM Inf

$P(L)*P(S|L) = 0.5*0.2 = 0.1 \rightarrow 0.25$



$P(H)*P(S|H) = 0.5*0.6 = 0.3 \rightarrow 0.75$



Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

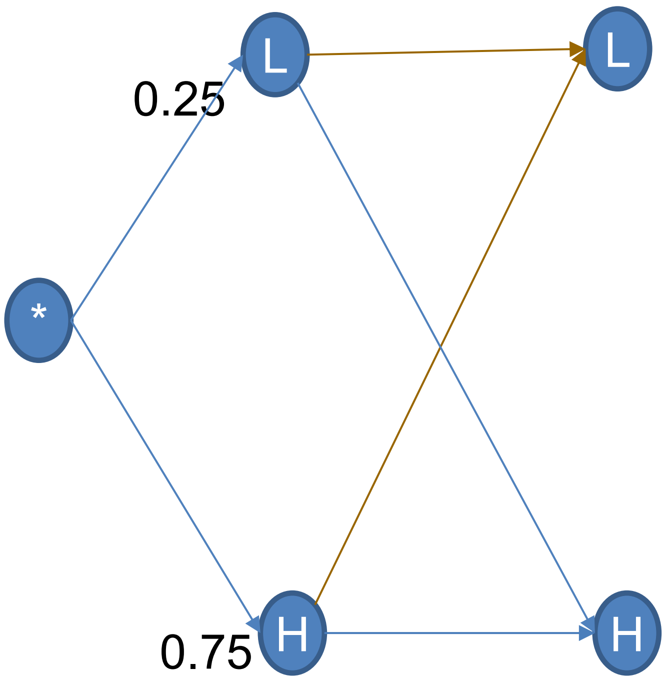
$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

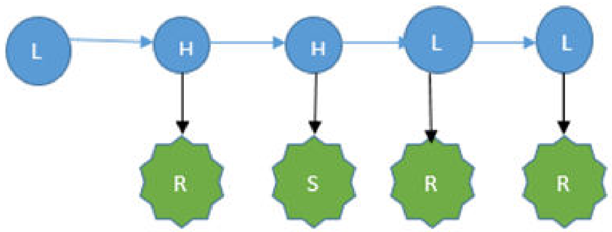
Hidden Morkov Model

Veterbi Algorithm : S-S-R

P(L)*P(L|L)*P(S|L) = 0.25*0.5*0.2 = 0.025

P(H)*P(L|H)*P(S|L) = 0.75*0.2*0.2 = **0.03**





Transition Model / Probability Matrix

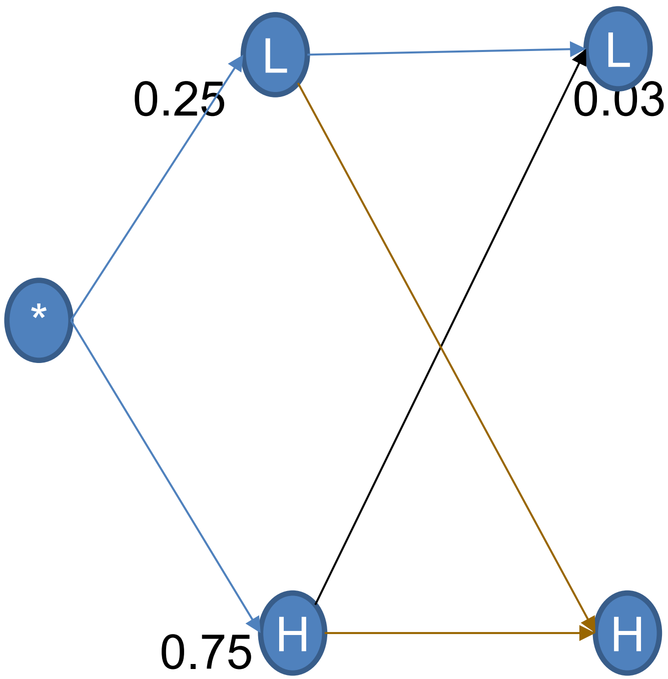
$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

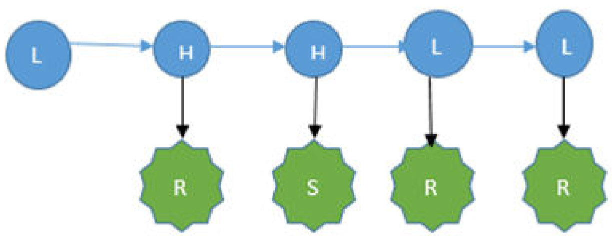
Hidden Morkov Model

Veterbi Algorithm : S-S-R



P(L)*P(H|L)*P(S|H) = 0.25*0.5*0.6 = 0.075

P(H)*P(H|H)*P(S|H) = 0.75*0.8*0.6 = **0.36**



Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

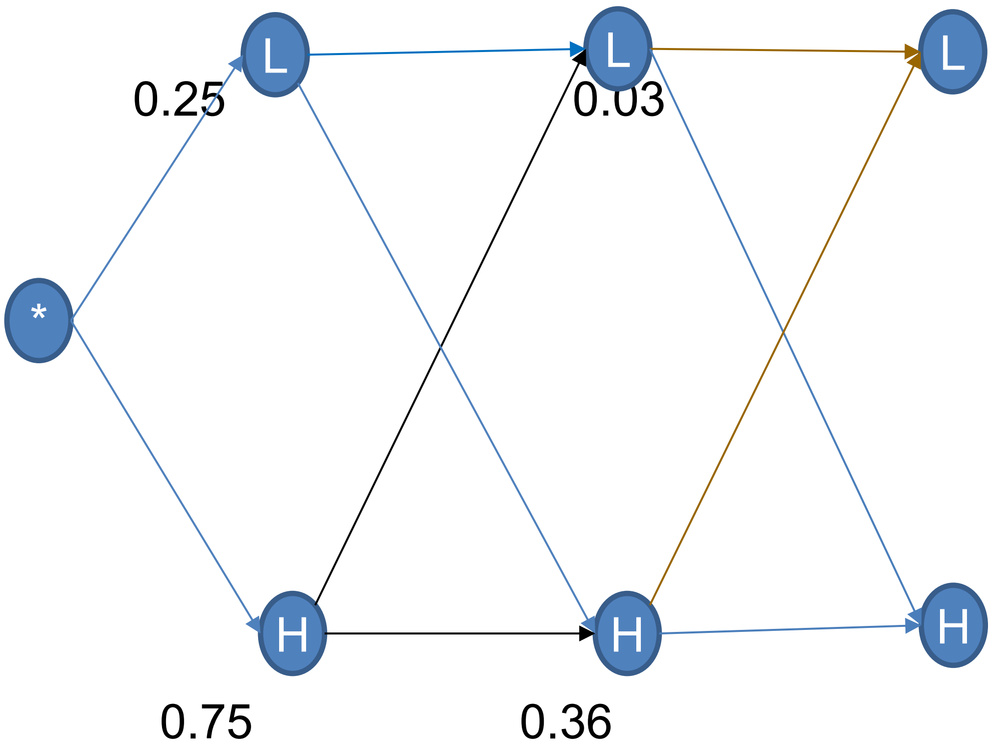
$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

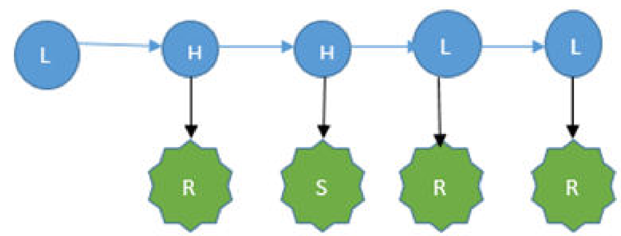
Hidden Morkov Model

Veterbi Algorithm : S-S-R

P(L)*P(L|L)*P(R|L) = 0.03*0.5*0.8 = 0.012

P(H)*P(L|H)*P(R|L) = 0.36*0.2*0.8 = **0.0576**





Transition Model / Probability Matrix

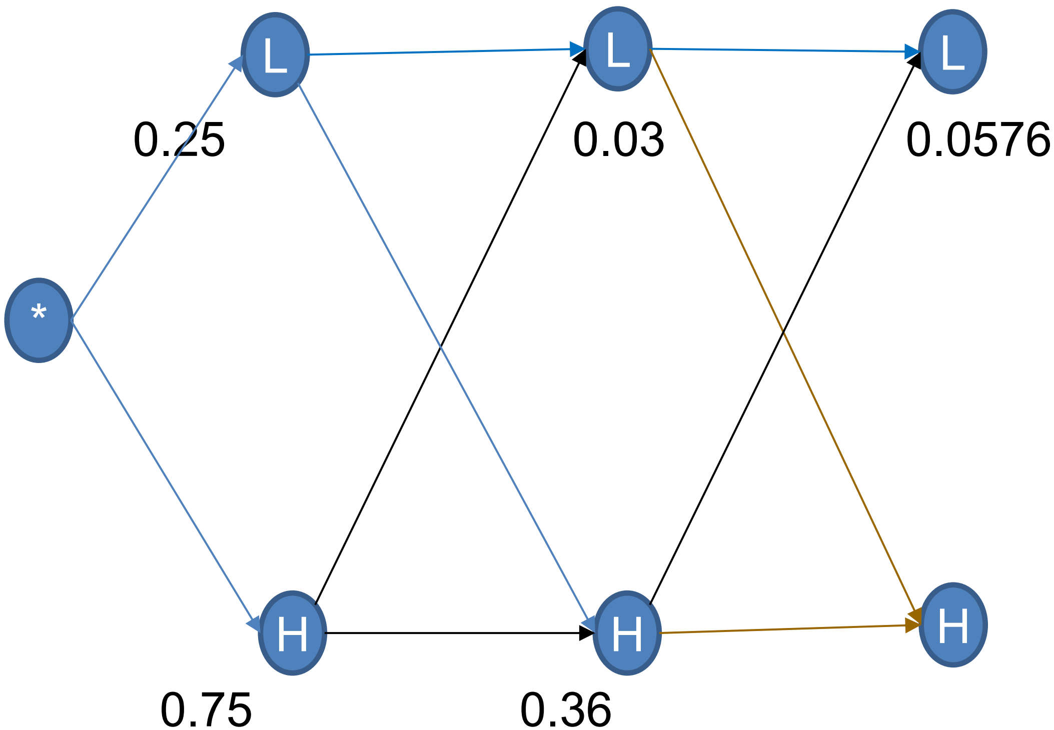
$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

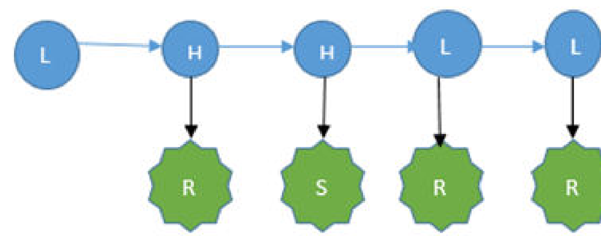
Hidden Morkov Model

Veterbi Algorithm : S-S-R



P(L)*P(H|L)*P(R|H) = 0.03*0.5*0.4 = 0.006

P(H)*P(H|H)*P(R|H) = 0.36*0.8*0.4 = **0.1152**



Transition Model / Probability Matrix

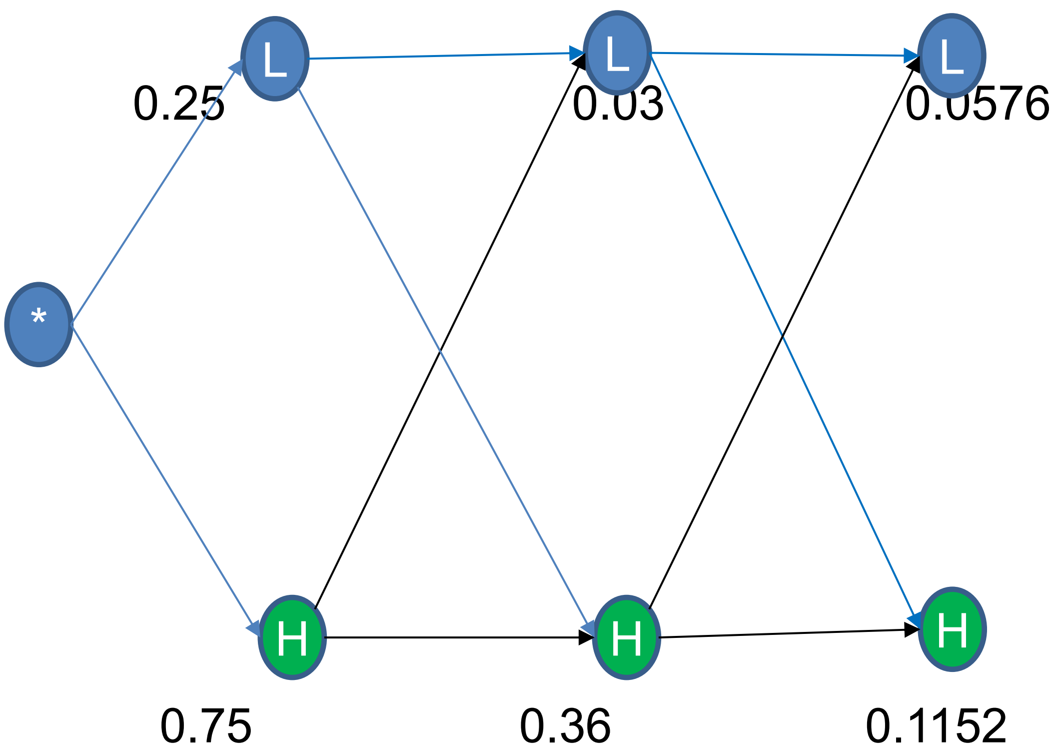
$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

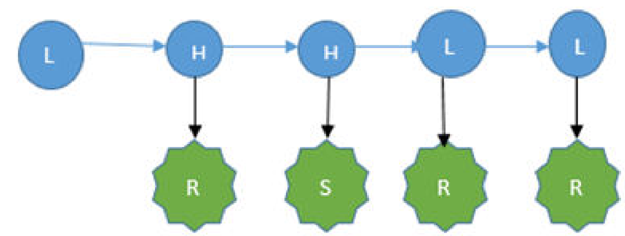
Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

Hidden Morkov Model

Veterbi Algorithm : S-S-R





Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	← Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	← Unobserved Evidence v
0.8	0.4	$P(E_t = \text{Rainy})$
0.2	0.6	$P(E_t = \text{Sunny})$

Initial	Prob
Noun	
aDverb	
Verb	
adJective	
Preposition	
Determiner	

N	D	V	J	P	
					N
					D
					V
					J
					A
					P
					E

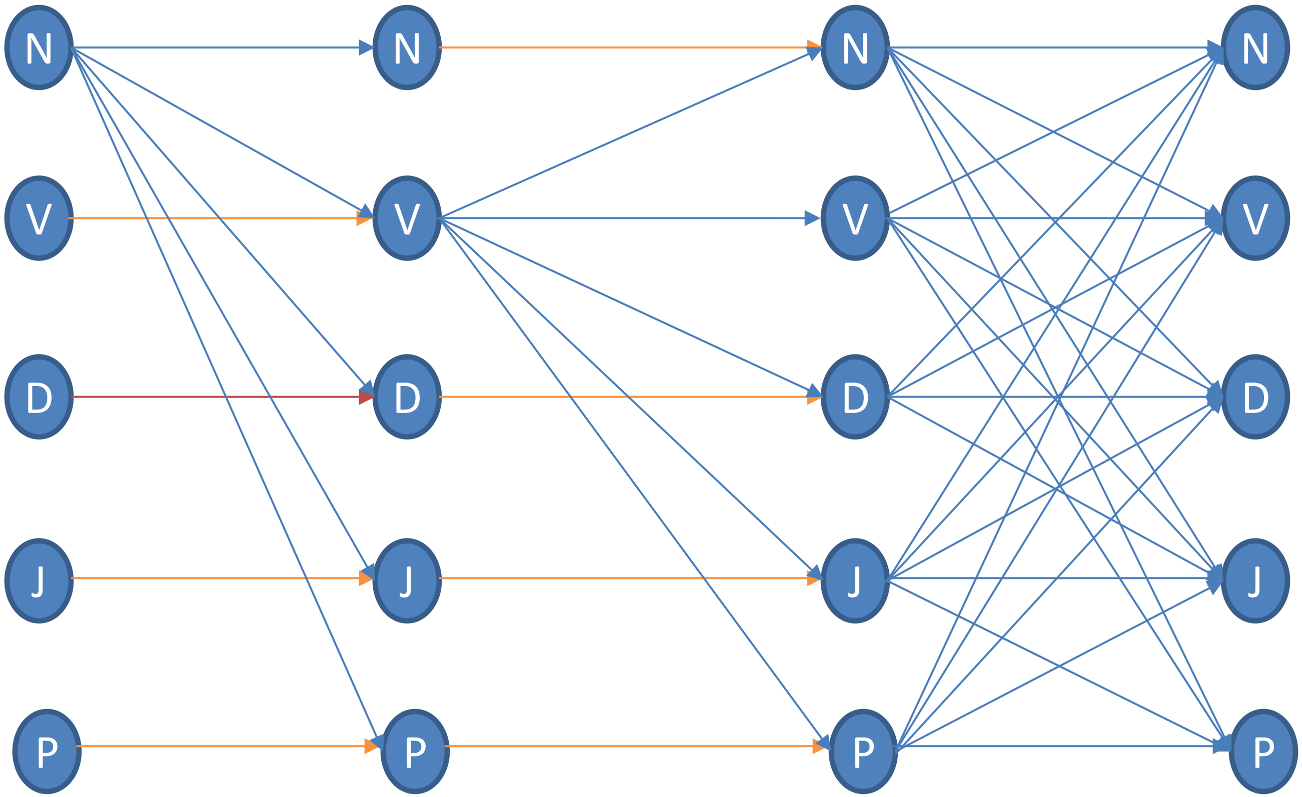
Given the corpus with tags to build training data:

1. Create initial probability matrix.
2. Transition probability matrix
3. Emission probability matrix
4. Use HMM Veterbi algorithm to predict the sequence of PoS Tags for given test data / sentence.

In the HMM model , the PoS tags act as the hidden states and the word in the given test sentence as the observed states.

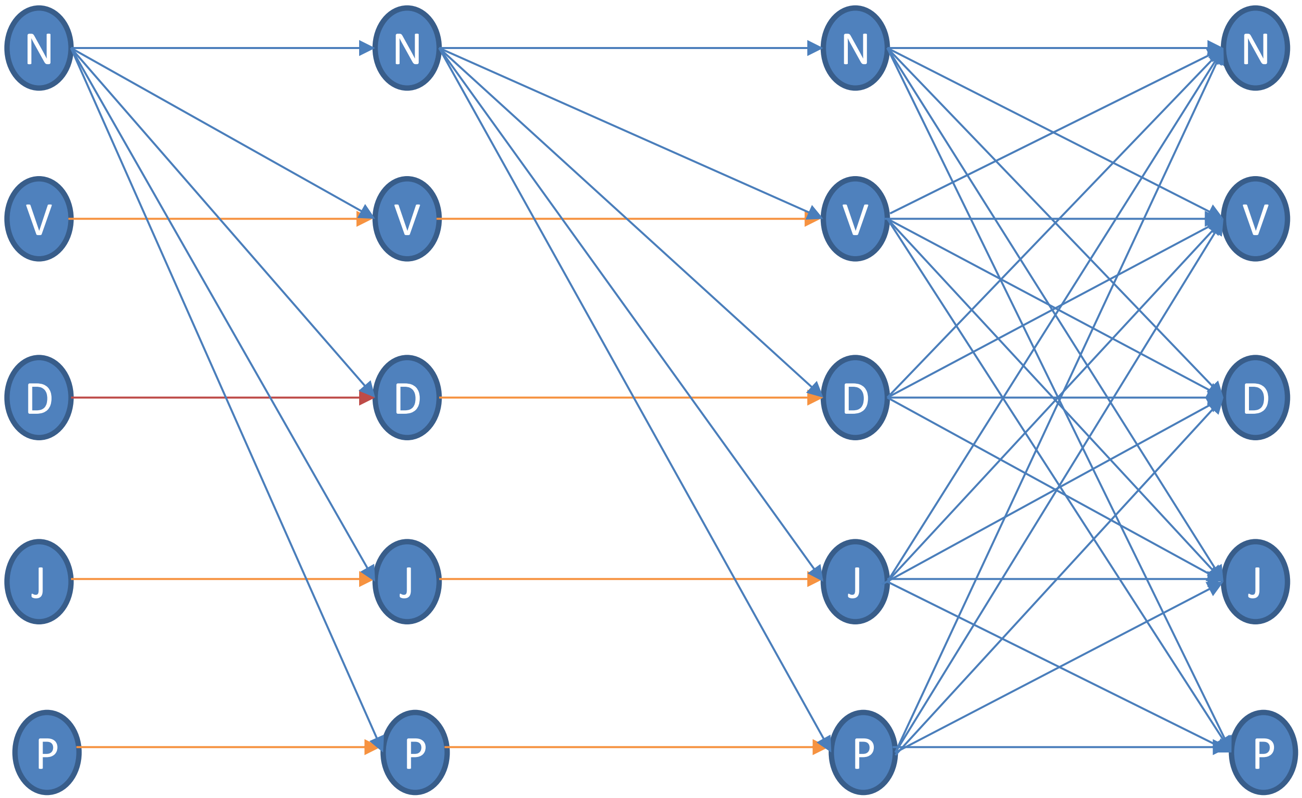
Food N	is V	good J		
Restaurant N	serves V	Food N		
Eating V	good J	food N	is V	health J
The D	food N	is V	ready J	
Eating V	fast A	is V	unhealthy J	

Sample Sequence under Test: Start → Noun → Verb → ………
Assume Noun→ Verb is the maximum Value

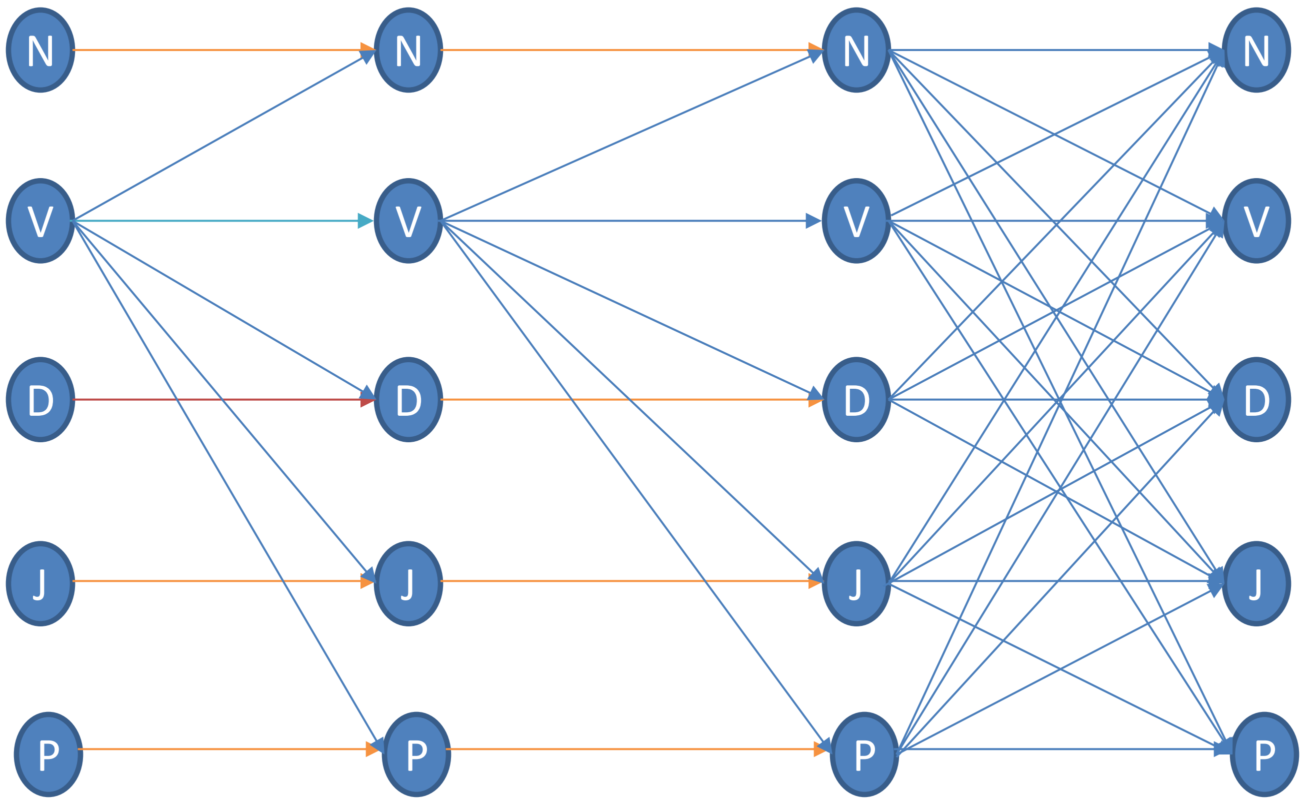


Sample Sequence under Test: Start → Noun → Noun→ ………

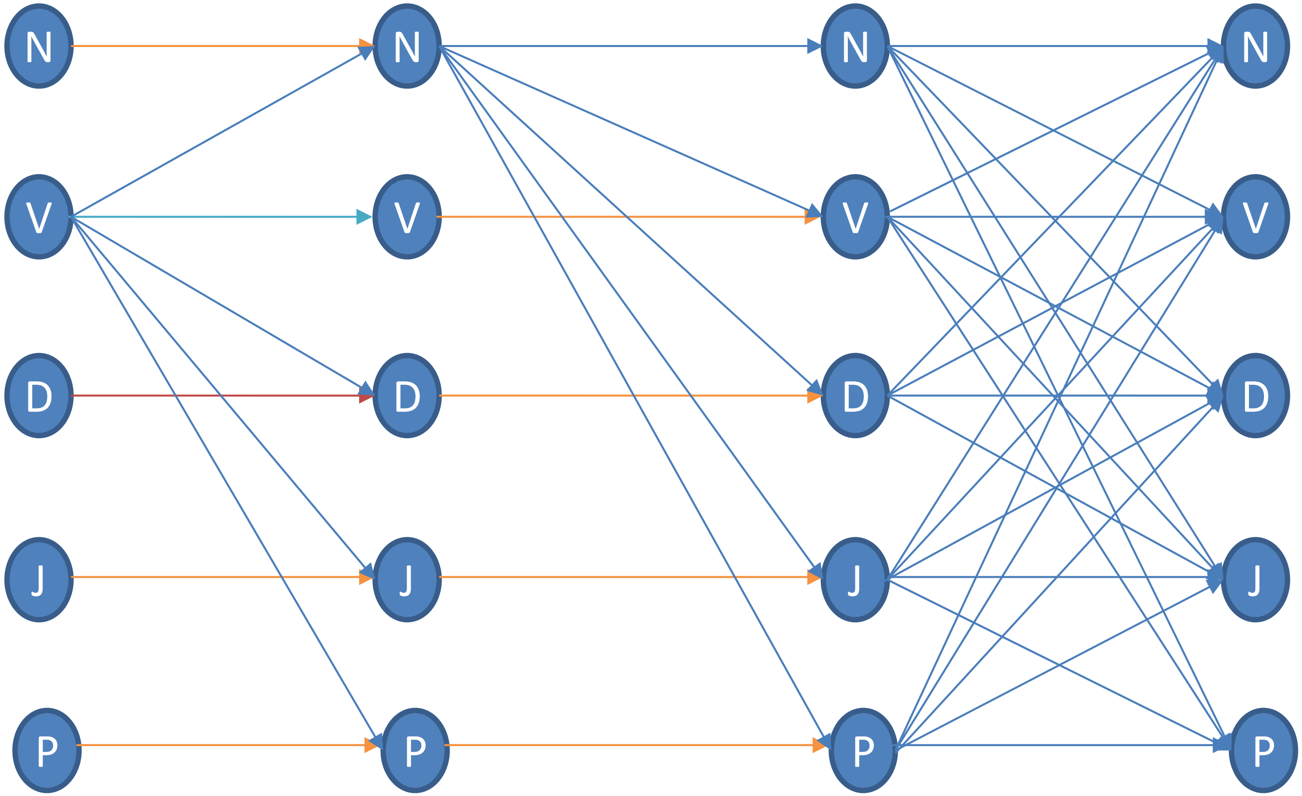
Assume Noun→ Noun is the maximum Value



Sample Sequence under Test: Start → Verb→ Verb → ……….
Assume Verb→ Verb is the maximum Value



Sample Sequence under Test: Start → Verb→ Noun→ ……….
Assume Verb→ Noun is the maximum Value



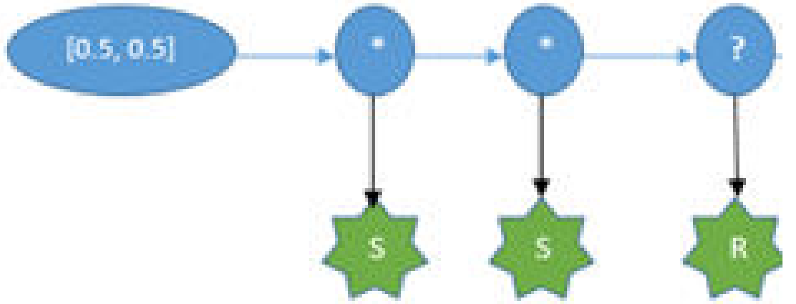
Hidden Morkov Model

Inference: Type -3

Filtering : Forward Propagation Algorithm

Find the Current Pressure if sequence of weather observations recorded are: **S-S-R**

Intuition: $P(E_{1\ldots t}) = \sum_{i=1}^{N} P(E_{1\ldots t} \mid X_{1\ldots t}) * P(X_{1\ldots t}) = \sum_{i=1}^{N} \prod_{j=1}^{t} P(Ej \mid Xj) * P(Xj \mid Xj_{-1})$



Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

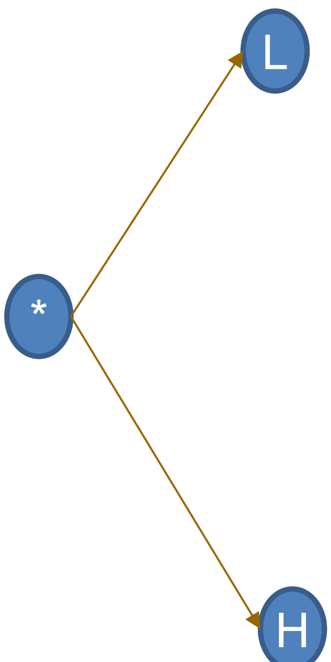
Hidden Morkov Model

Forward Propagation Algorithm

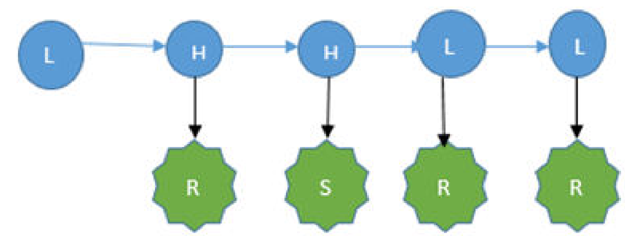
Pressure sequence observation: **S-S-R**

<u>Initialization Phase:</u>

P(L)*P(S|L) = 0.5*0.2 = 0.1 → 0.25



P(H)*P(S|H) = 0.5*0.6 = 0.3 → 0.75



Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

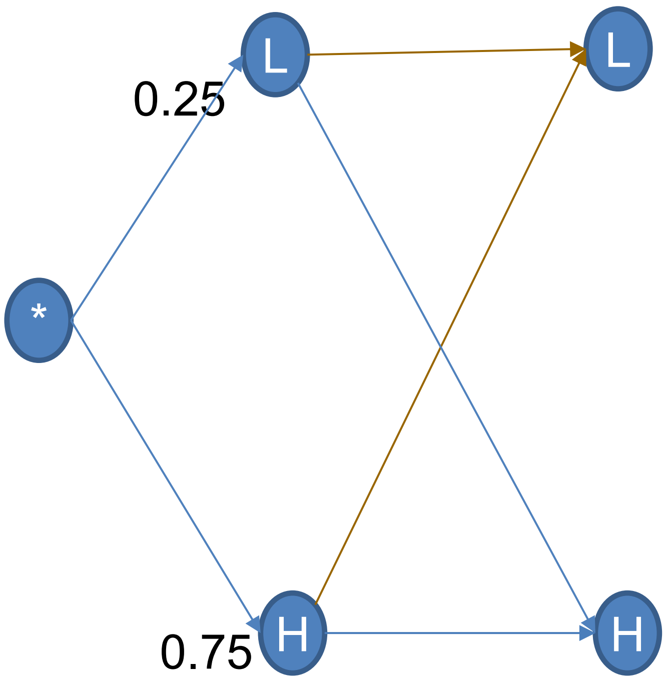
$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

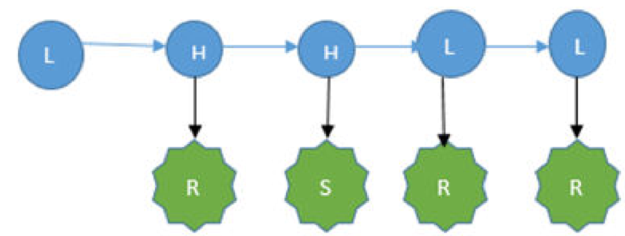
Hidden Morkov Model

Forward Propagation Algorithm : S-S-R

P(L)*P(L|L)*P(S|L) = 0.25*0.5*0.2 = **0.025**

P(H)*P(L|H)*P(S|L) = 0.75*0.2*0.2 = **0.03**





Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

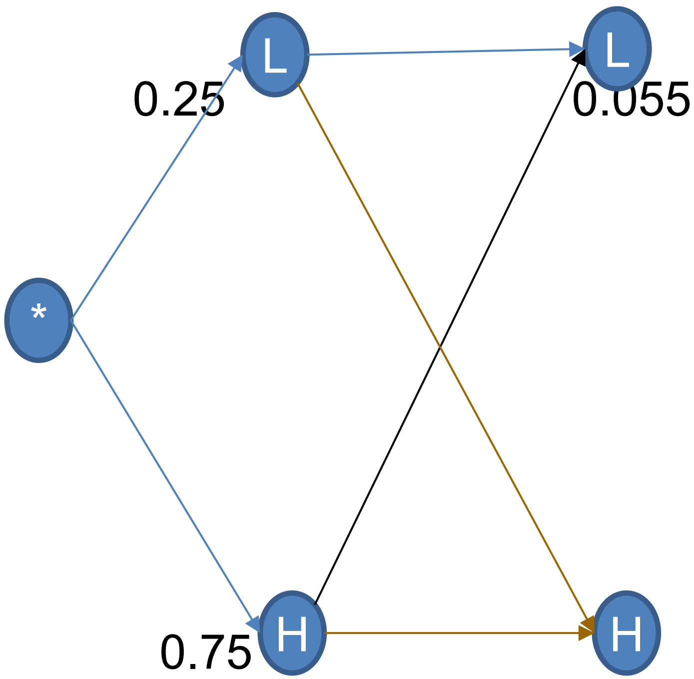
Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

Recursion Phase:

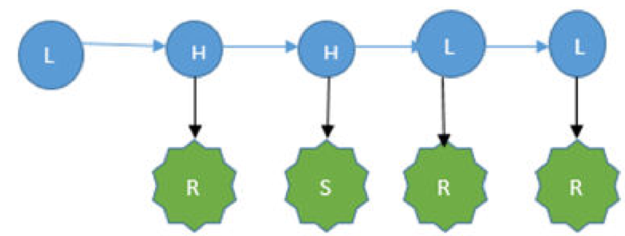
Hidden Morkov Model

Forward Propagation Algorithm : S-S-R



P(L)*P(H|L)*P(S|H) = 0.25*0.5*0.6 = **0.075**

P(H)*P(H|H)*P(S|H) = 0.75*0.8*0.6 = **0.36**



Transition Model / Probability Matrix

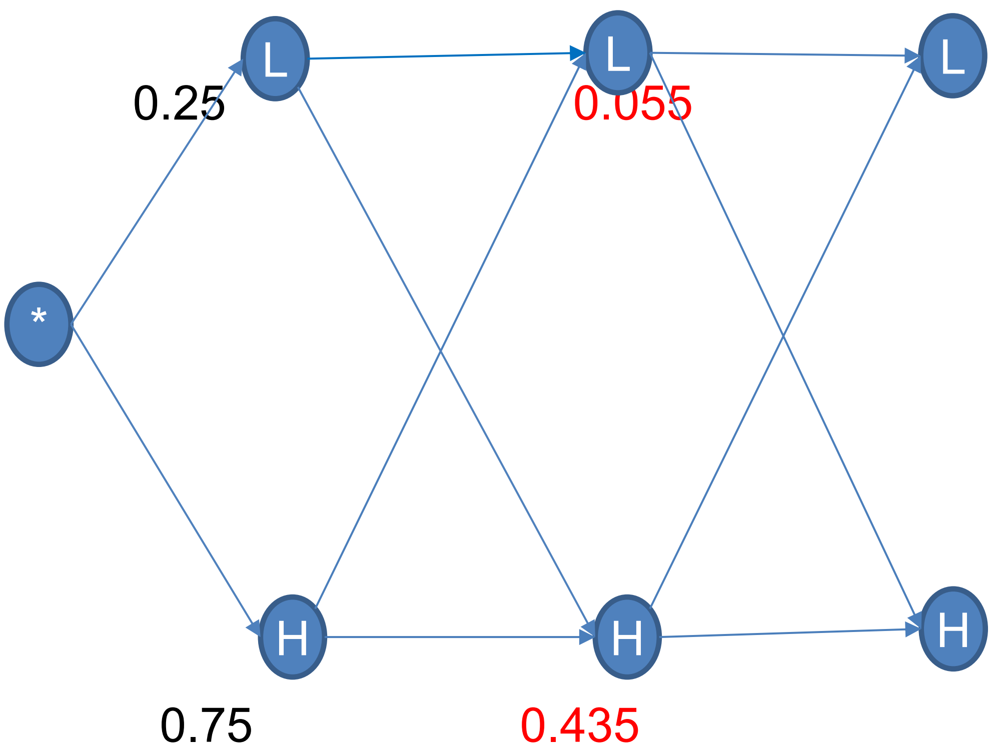
$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

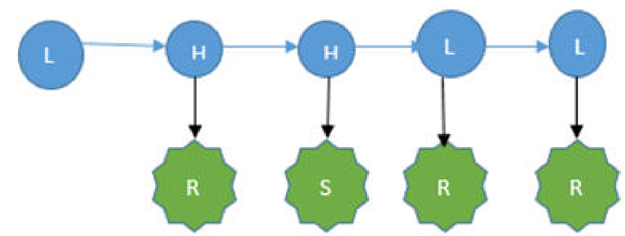
Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = \text{Rainy})$
0.2	0.6	$P(E_t = \text{Sunny})$

Hidden Morkov Model

Forward Propagation Algorithm : S-S-R





Transition Model / Probability Matrix

P(U_{t-1} = HP)	P(U_{t-1} = LP)	←Previous
0.2	0.5	P(U_t = LP)
0.8	0.5	P(U_t = HP)

Evidence / Sensor Model/ Emission Probability Matrix

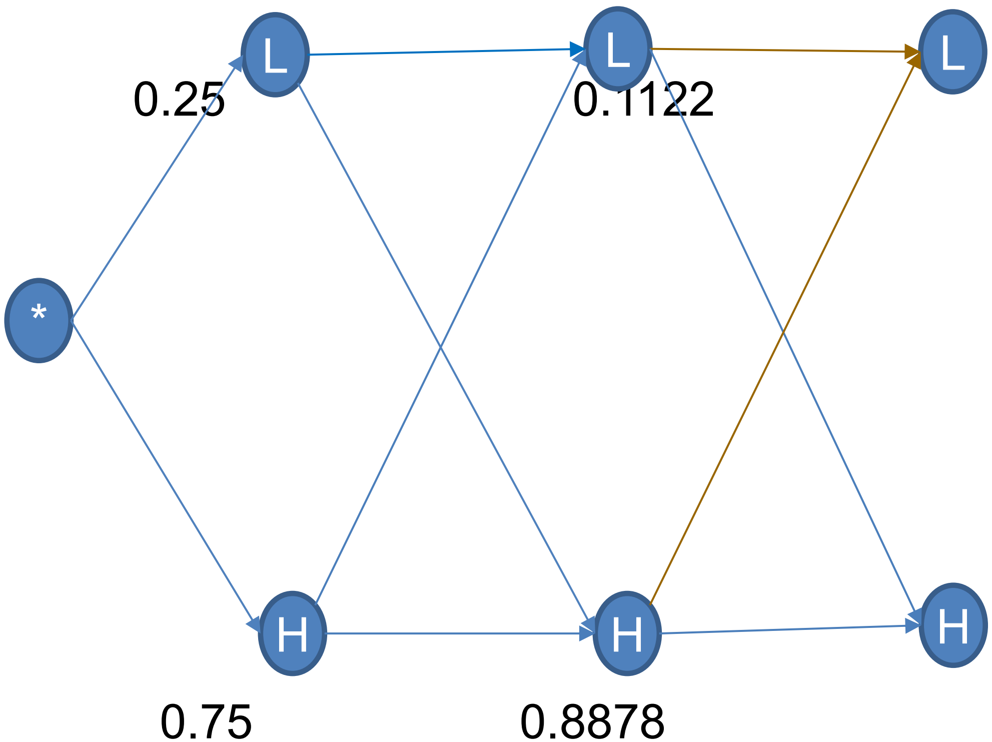
P(X_t = LP)	P(X_t = HP)	←Unobserved Evidence v
0.8	0.4	P(E_t = Rainy)
0.2	0.6	P(E_t = Sunny)

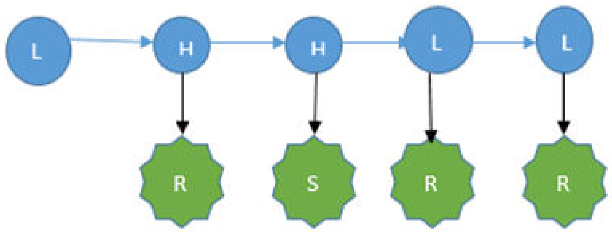
Hidden Morkov Model

Forward Propagation Algorithm : S-S-R

P(L)*P(L|L)*P(R|L) = 0.1122*0.5*0.8 = **0.04488**

P(H)*P(L|H)*P(R|L) = 0.8878*0.2*0.8 = **0.142048**





Transition Model / Probability Matrix

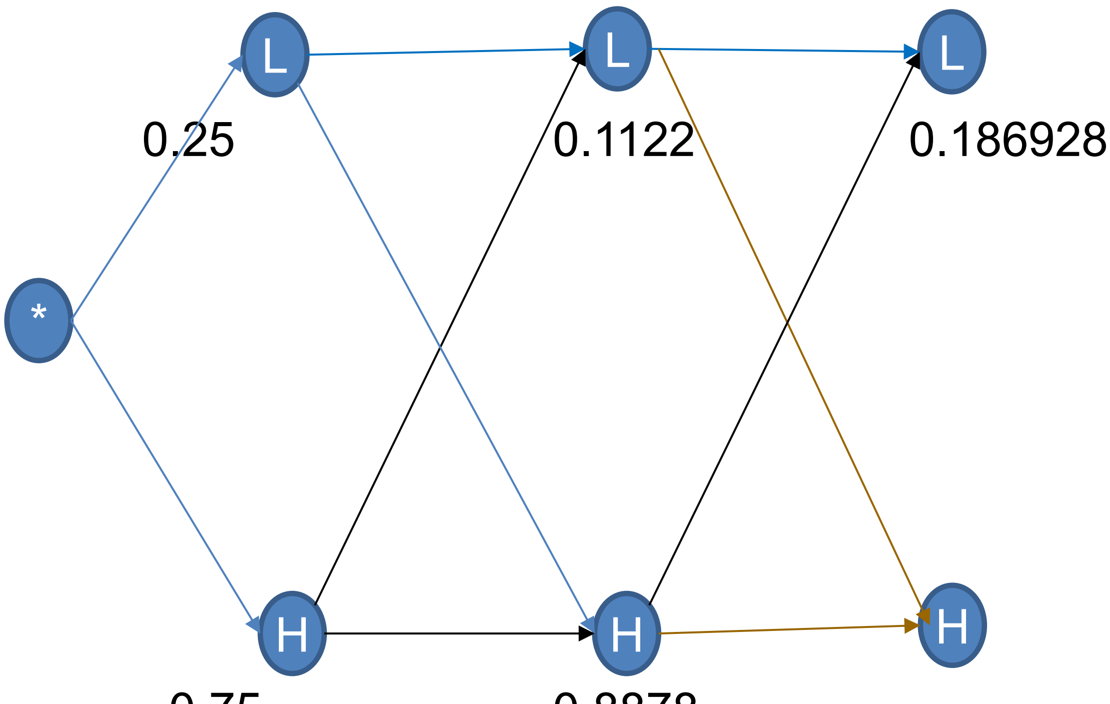
$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

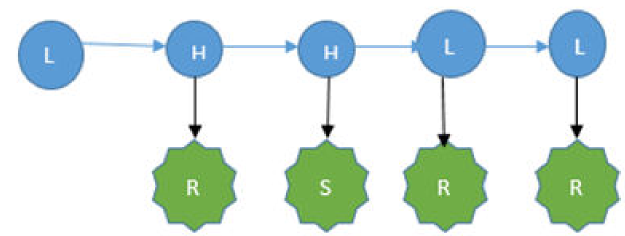
Hidden Morkov Model

Forward Propagation Algorithm : S-S-R



P(L)*P(H|L)*P(R|H) = 0.1122*0.5*0.4 = **0.02244**

P(H)*P(H|H)*P(R|H) = 0.8878*0.8*0.4 = **0.284096**



Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

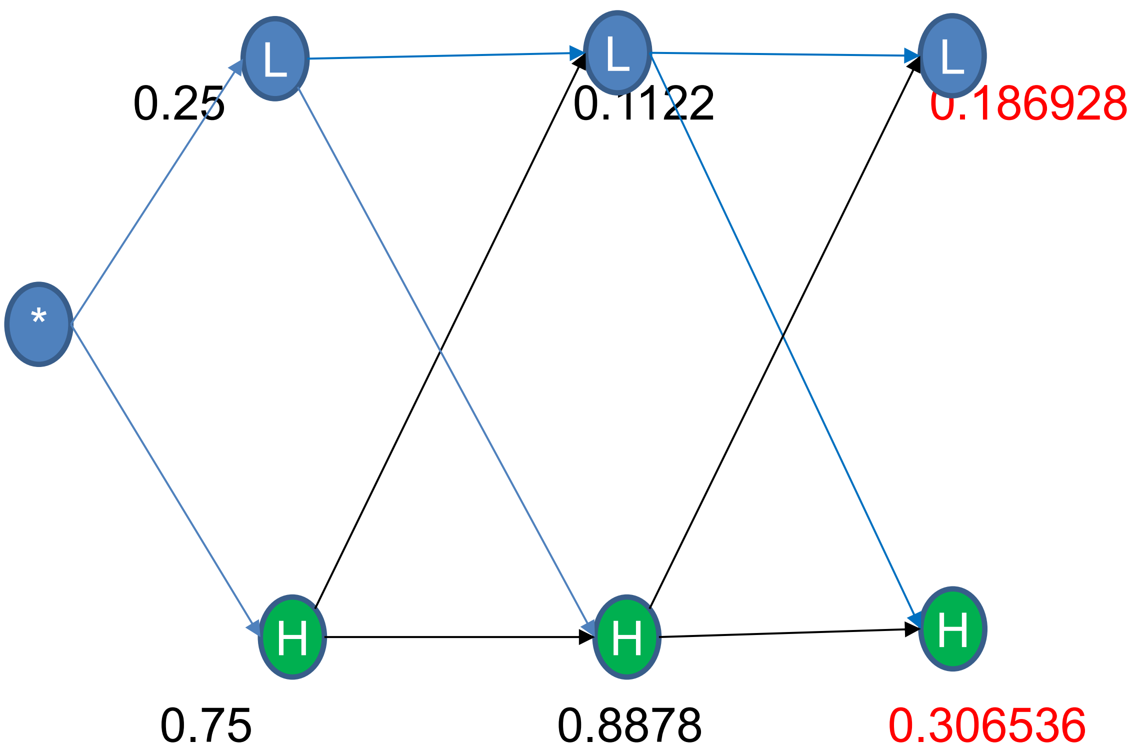
Evidence / Sensor Model/ Emission Probability Matrix

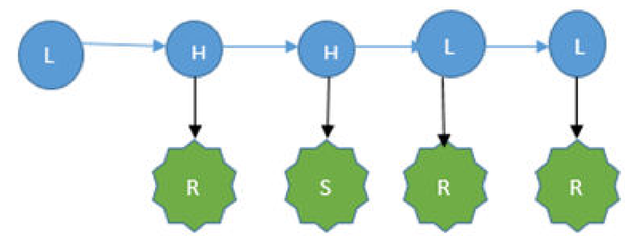
$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

Hidden Morkov Model

Forward Propagation Algorithm : S-S-R

Termination Phase:





Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

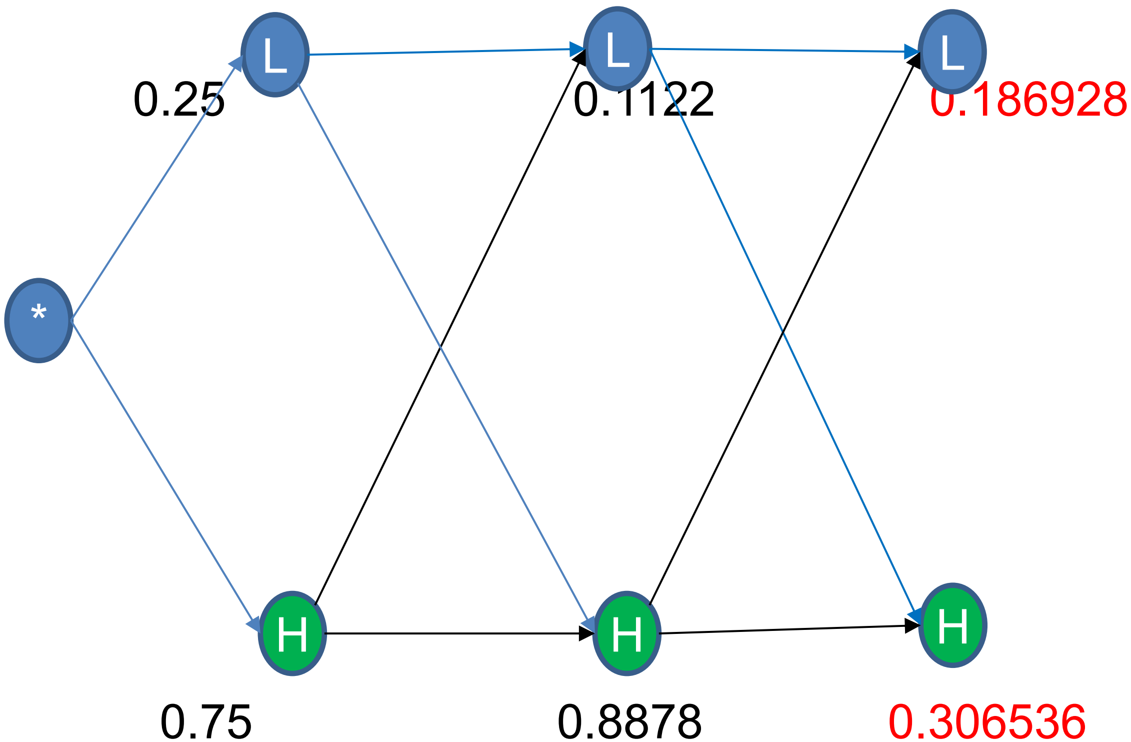
$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

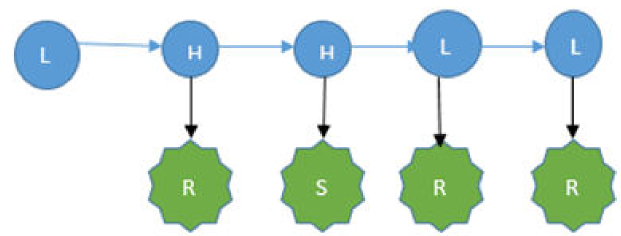
Hidden Morkov Model

Forward Propagation Algorithm : S-S-R

Termination Phase:

(0.37881,**0.62119**)





Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

Hidden Morkov Model

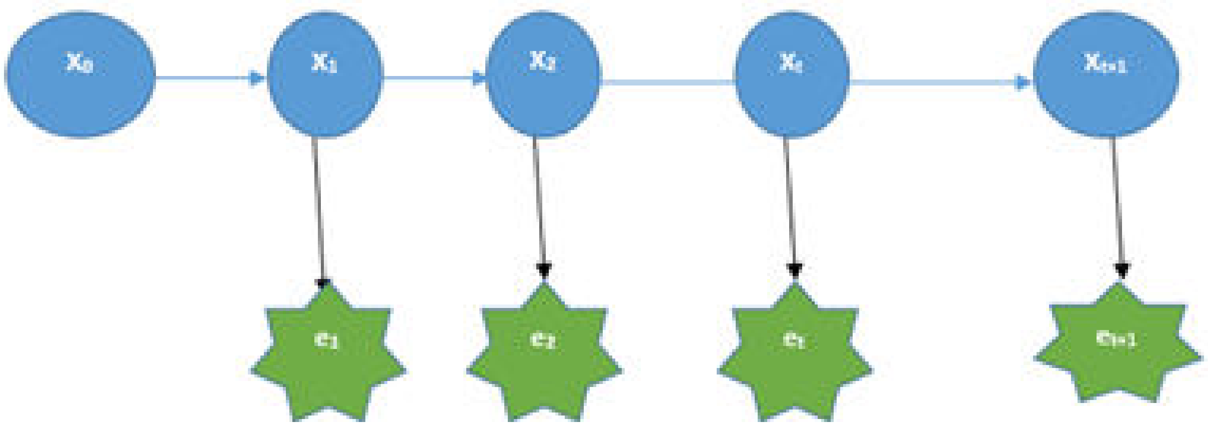
Inference: Type -3

Filtering : Forward Propagation Algorithm

Find the Current Pressure if sequence of weather observations recorded are: **S-S-R**

Intuition: $P(Xt_{+1} | E_{1...t+1}) = \boldsymbol{\alpha}\, P(et_{+1} | Xt_{+1}) * \sum_{X_t} P(X_{t+1}|X_t) * \boldsymbol{P}(\mathbf{X_t}| \mathbf{E_{1..t}})$

$$P(Xt_{+1} | E_{1...t+1}) = \boldsymbol{\alpha}\, P(e_{t+1} | Xt_{+1}) * \sum_{X_t} P(X_{t+1}|X_t) * \boldsymbol{P}(\mathbf{X_t}| \mathbf{E_{1..t}})$$



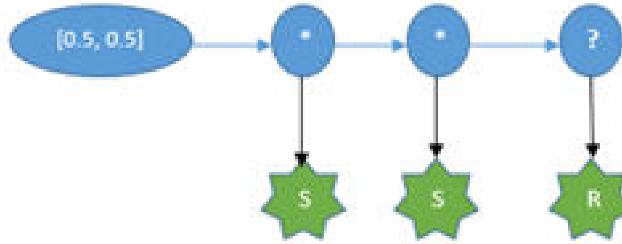
Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

Hidden Morkov Model



-3

Filtering : Forward Propagation Algorithm

Find the Current Pressure if sequence of weather observations recorded are: **S-S-R**

Intuition: $P(Xt_{+1} |E_{1...t+1}) = \boldsymbol{\alpha}\, P(et_{+1} | Xt_{+1}) * \sum_{X_t} P(X_{t+1}|X_t) * \boldsymbol{P(X_t| E_{1..t})}$

$$P(X_3 | SSR) = P(X_3 | S,S,R)$$

$$= \frac{\boldsymbol{P(R\,|X_3\ S,S)} * P(X_3|\,S,S)}{P(R)}$$

$$= \frac{P(R\,|X_3) * \boldsymbol{P(X_3|\,S,S)}}{P(R)}$$

$$= \frac{P(R\,|X_3) * \{\sum_{X_2} P(X_3|X_2) * \boldsymbol{P(X_2|\,S,S)}\}}{P(R)}$$

$$= \frac{P(R\,|X_3) * \{\sum_{X_2} P(X_3|X_2) * P(R\,|X_3) * \{\sum_{X_1} P(X_2|X_1) * \boldsymbol{P(X_1|\,S)}\}\}}{P(R) * P(S)}$$

Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

$$P(Xt_{+1} |E_{1...t+1}) = \boldsymbol{\alpha}\, P(e_{t+1} | Xt_{+1}) * \sum_{X_t} P(X_{t+1}|X_t) * \boldsymbol{P(X_t| E_{1..t})}$$

Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

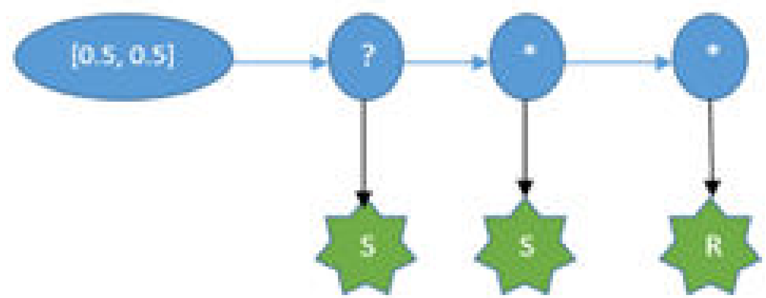
Hidden Morkov Model

Inference: Type -4

Smoothing : Backward Propagation Algorithm (Most Likely State Estimation)

Find the Pressure in past instance of time if sequence of following future weather observations recorded are: **S-S-R**

Intuition: $P(E_{1\ldots t}) = \sum_{i=1}^{N} P(E_{1\ldots t} \mid X_{1\ldots t}) * P(X_{1\ldots t}) = \sum_{i=1}^{N} \prod_{j=1}^{t} P(Ej \mid Xj) * P(Xj \mid Xj_{-1})$



Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

Hidden Morkov Model

Inference: Type -4

Smoothing : Backward Propagation Algorithm

Find the Pressure in past instance of time if sequence of following future weather observations recorded are: **S-S-R**

Intuition: $P(Xt_{+1}\,|E_{1\ldots t+1}) = \boldsymbol{\alpha}\, P(et_{+1}\,|\,Xt_{+1}) * \sum_{X_t} P(X_{t+1}|X_t) * \boldsymbol{P}(\mathbf{X_t}|\,\mathbf{E_{1..t}})$

$P(X_1\,|\,SSR) = P(X_1\,|\,S,S,R)$

$$= \frac{\boldsymbol{P}(\boldsymbol{SR}\,|\mathbf{X_1}\,\boldsymbol{S}) * P(X_1|\,S)}{P(SR)}$$

$$= \frac{P(X_1|\,S) * \{\sum_{X_2} P(X_2|X_1) * \boldsymbol{P}(\mathbf{SR}\,|\,\mathbf{X_2}\,\mathbf{X_1})\,\}}{P(SR)}$$

$$= \frac{P(X_1|\,S) * \{\sum_{X_2} P(X_2|X_1) * \boldsymbol{P}(\mathbf{SR}\,|\,\mathbf{X_2})\,\}}{P(SR)}$$

$$= \frac{P(X_1|\,S) * \{\sum_{X_2} P(X_2|X_1) * P(S\,|\,X_2) * \boldsymbol{P}(\mathbf{R}\,|\,\mathbf{X_2})\}}{P(SR)}$$

$$= \frac{P(X_1|\,S) * \{\sum_{X_2} P(X_2|X_1) * P(S\,|\,X_2) * \{\sum_{X_3} P(X_3|X_2) * P(R\,|\,X_3) * \boldsymbol{P}(.\,|\,\mathbf{X_3})\}\}}{P(SR)}$$

Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probab

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

$$P(Xt\,|E_{t+1,\,t+2\,\ldots\,z}) = \boldsymbol{\alpha} * \text{fwd msg} * \sum_{X_{t+1}} P(X_{t+1}|X_t) * P(e_{t+1}|X_{t+1}) * \boldsymbol{P}(\mathbf{E_{t+2..z}}|\,\mathbf{X_{t+1}})$$

Hidden Morkov Model

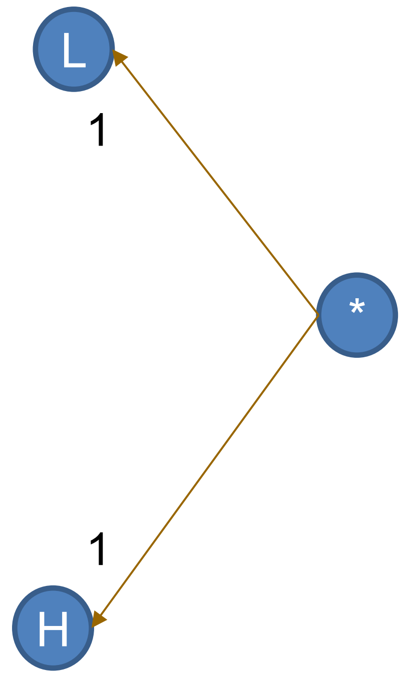
Backward Propagation Algorithm

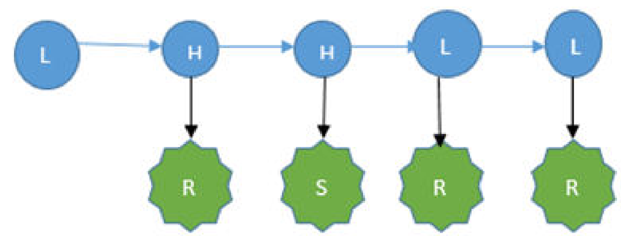
Pressure sequence observation: **S-S-R**

Initialization Phase: Set value 1 for the terminal state

P(L|L)*P(R|L)*P(.|L) = 0.5*0.8 * 1= 0.40

P(H|L)*P(R|H)*P(.|H) = 0.5*0.4 *1= 0.2





Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

P(L|H)*P(R|L)*P(.|L) = 0.2*0.8 * 1= 0.16

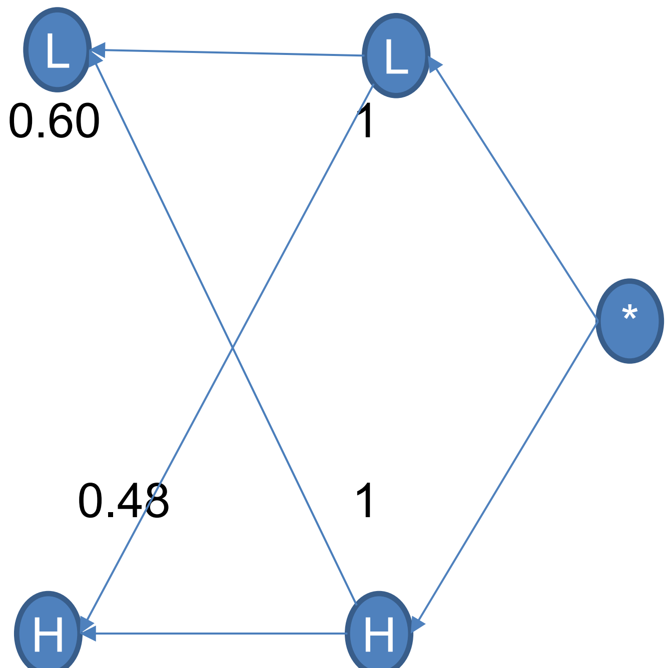
P(H|H)*P(R|H)*P(.|H) = 0.8*0.4 *1= 0.32

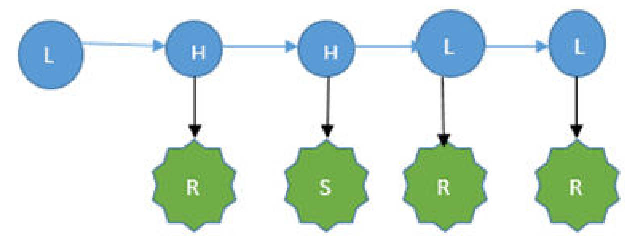
Hidden Morkov Model

Backward Propagation Algorithm : S-S-R

P(L|L)*P(S|L)*MSG(L`) = 0.5*0.2 * 0.60= 0.06

P(H|L)*P(S|H)*MSG(H`) = 0.5*0.6*0.48= 0.144





Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

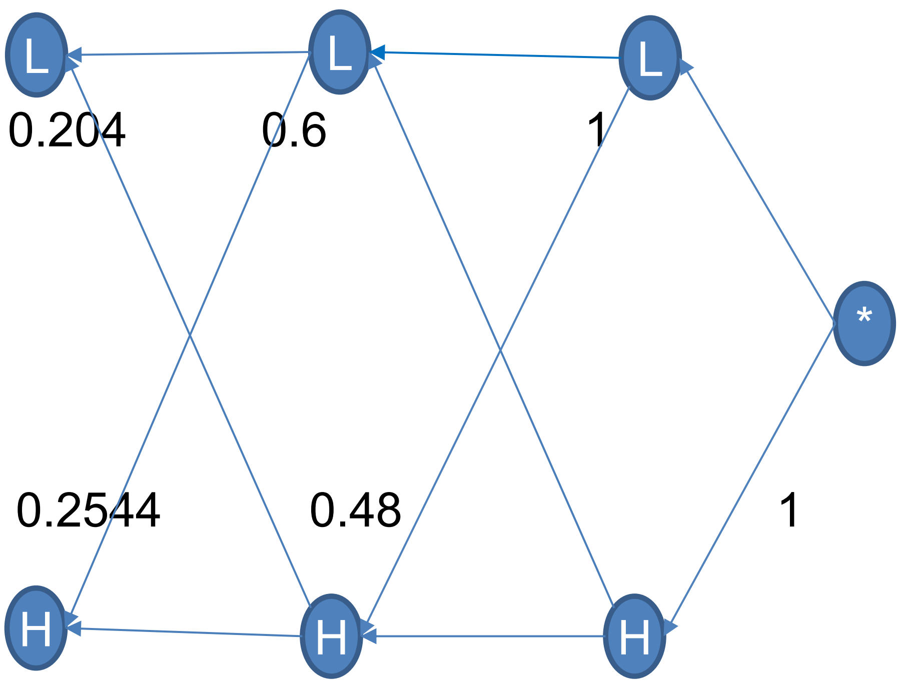
P(L|H)*P(S|L)*MSG(L`) = 0.2*0.2 * 0.6= 0.024

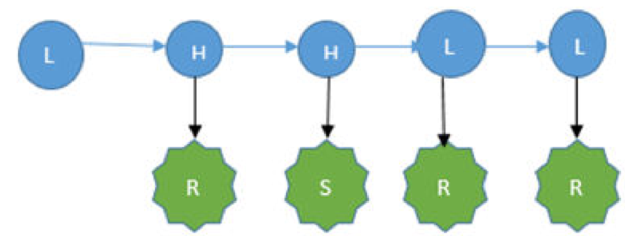
P(H|H)*P(S|H)*MSG(H`) = 0.8*0.6 *0.48= 0.2304

<u>Recursion Phase:</u>

Hidden Morkov Model

Backward Propagation Algorithm : S-S-R





Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

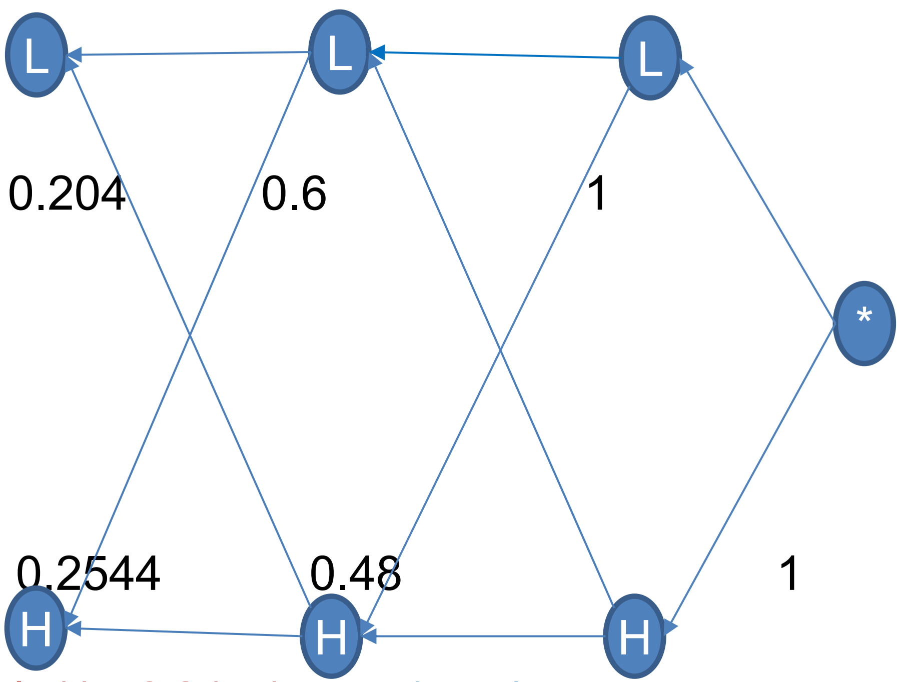
$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

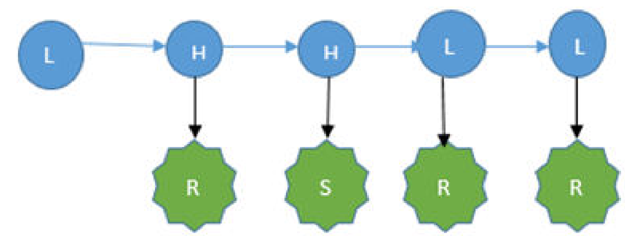
<u>Recursion Phase: If it continues if needed !!!!</u>

Hidden Morkov Model

Backward Propagation Algorithm : S-S-R

P(L)*P(S|L)*MSG(L`) = 0.5*0.2 * 0.204= 0.0204





Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

P(H)*P(S|H)*MSG(H`) = 0.5*0.6 * 0.2544= 0.07632

<u>Termination Phase: </u>(0.2109,0.7891)

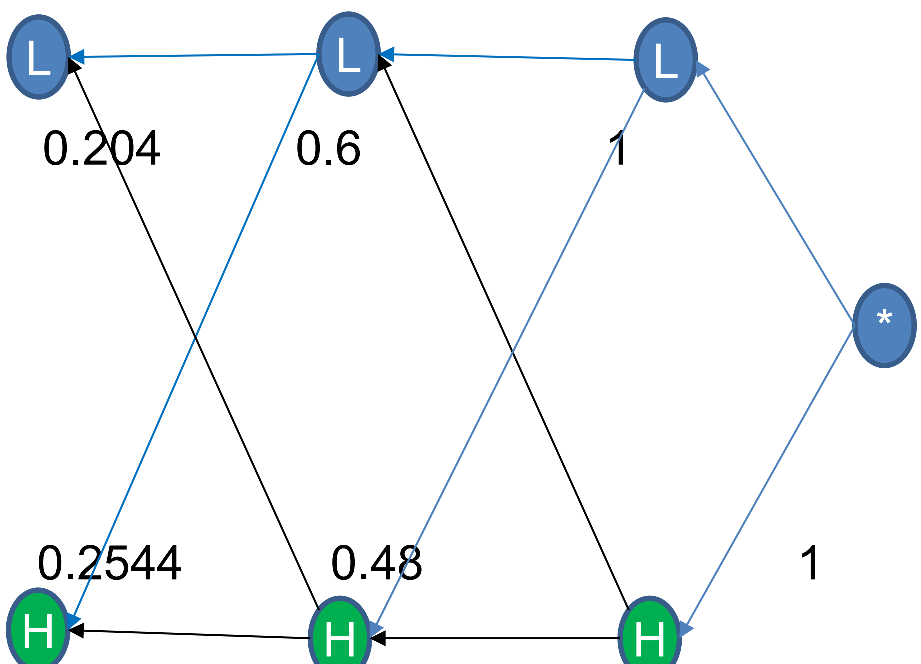
Normalize :Initial value * Emission at start* backMsg

Hidden Morkov Model

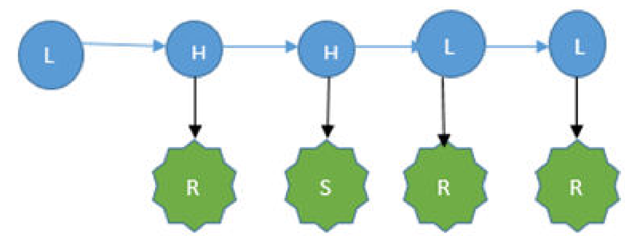
Forward Backward Propagation Algorithm : S-S-R

P(X2 | SSR) = α * P(X2|SS) *P(R|X2)

P(X2 | SSR) = α * (0.1122 , 0.8878)* (0.6, 0.48) = (0.06732, 0.426144) = (0.14,0.86)



Termination Phase: X2 = ??? → **X2 = H**



Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

```
function FORWARD-BACKWARD(ev, prior) returns a vector of probability distributions
   inputs: ev, a vector of evidence values for steps 1, ..., t
           prior, the prior distribution on the initial state, P(X_0)
   local variables: fv, a vector of forward messages for steps 0, ..., t
                    b, a representation of the backward message, initially all 1s
                    sv, a vector of smoothed estimates for steps 1, ..., t

   fv[0] ← prior
   for i = 1 to t do
      fv[i] ← FORWARD(fv[i − 1], ev[i])
   for i = t downto 1 do
      sv[i] ← NORMALIZE(fv[i] × b)
      b ← BACKWARD(b, ev[i])
   return sv
```

Figure 15.4 The forward–backward algorithm for smoothing: computing posterior probabilities of a sequence of states given a sequence of observations. The FORWARD and BACKWARD operators are defined by Equations (15.5) and (15.9), respectively.

Hidden Morkov Model

Cyber Security

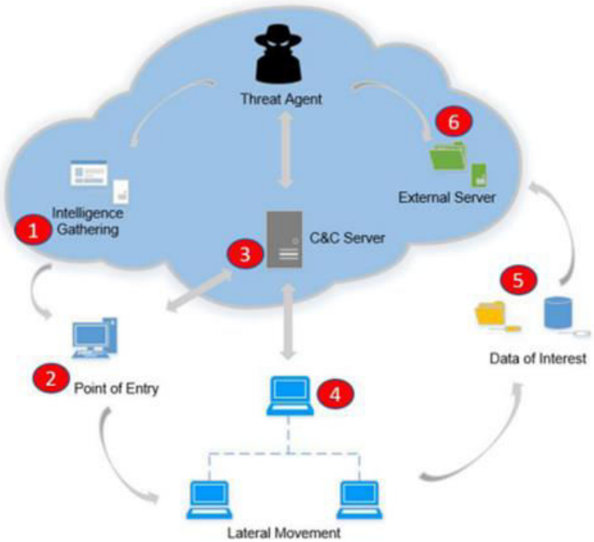
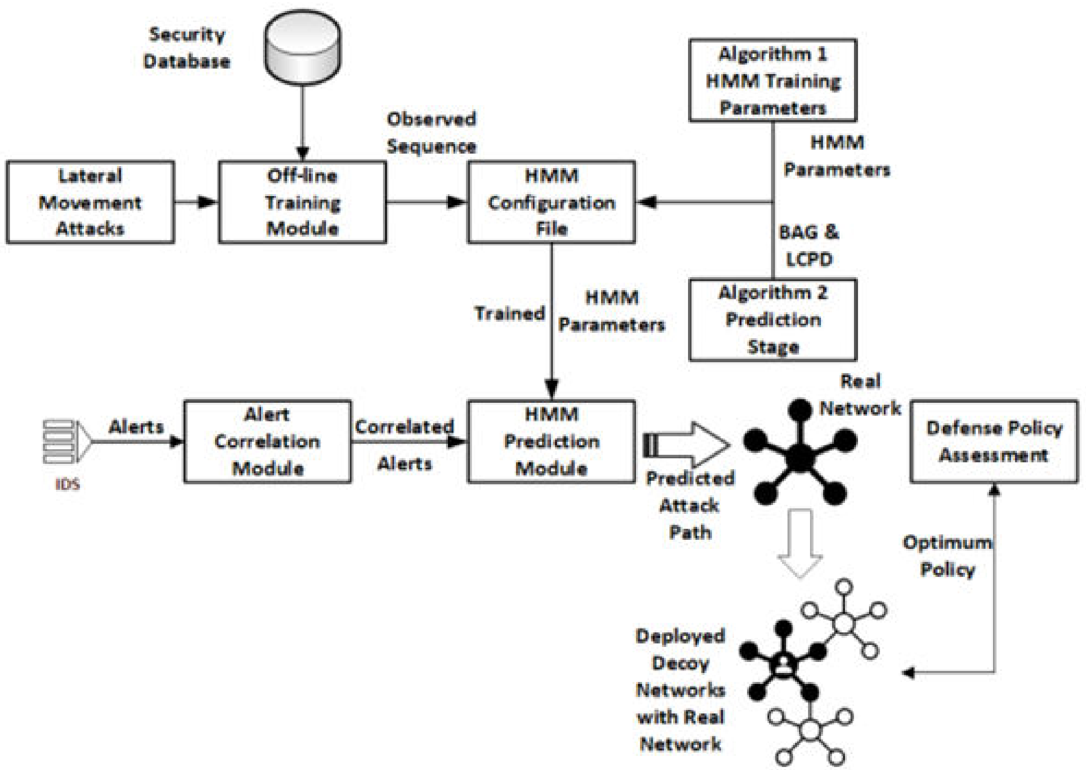


FIGURE 1. Typical stages of APT attack.

Source Credit : [2021 : Hidden Markov Model and Cyber Deception for the Prevention of Adversarial Lateral Movement]

Hidden Morkov Model

Cyber Security



Source Credit : [2021 : Hidden Markov Model and Cyber Deception for the Prevention of Adversarial Lateral Movement]()

Hidden Morkov Model

Cyber Security

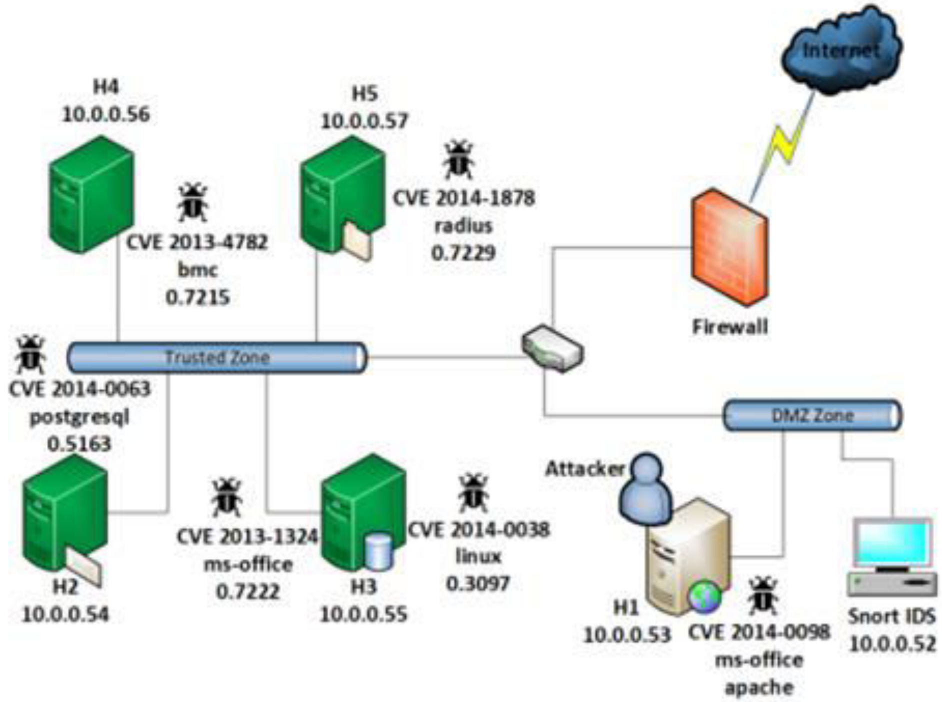


FIGURE 9. Experimental network topology.

Source Credit : [2021 : Hidden Markov Model and Cyber Deception for the Prevention of Adversarial Lateral Movement]()

Hidden Morkov Model

Cyber Security

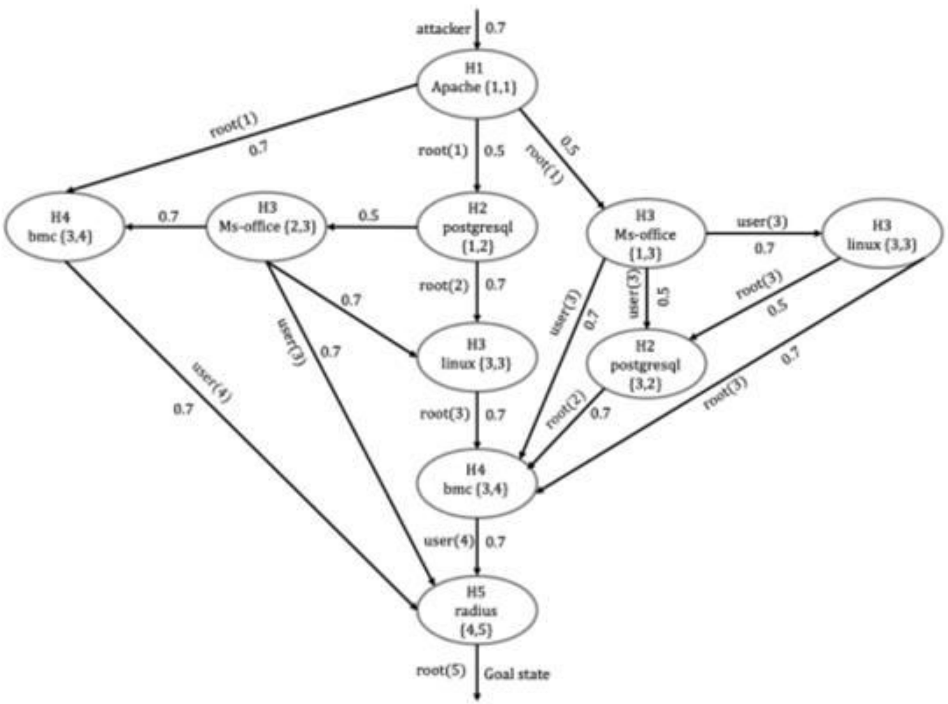


FIGURE 10. Attack graph of the experimental network.

Attack states description.

State	Description
S_1	Initial State
S_2	(H_1,root)
S_3	(H_2,root)
S_4	(H_3,user)
S_5	(H_3,root)
S_6	(H_4,user)
S_7	(H_5,root)

Source Credit : [2021 : Hidden Markov Model and Cyber Deception for the Prevention of Adversarial Lateral Movement]

Hidden Morkov Model

Cyber Security

Attack states description.

State	Description
S_1	Initial State
S_2	(H_1,root)
S_3	(H_2,root)
S_4	(H_3,user)
S_5	(H_3,root)
S_6	(H_4,user)
S_7	(H_5,root)

TABLE 6. Possible attack paths.

Path Number	Attack Path
1	$S_1 \rightarrow S_2 \rightarrow S_6 \rightarrow S_7$
2	$S_1 \rightarrow S_2 \rightarrow S_3 \rightarrow S_4 \rightarrow S_7$
3	$S_1 \rightarrow S_2 \rightarrow S_3 \rightarrow S_6 \rightarrow S_7$
4	$S_1 \rightarrow S_2 \rightarrow S_3 \rightarrow S_4 \rightarrow S_5 \rightarrow S_6 \rightarrow S_7$
5	$S_1 \rightarrow S_2 \rightarrow S_3 \rightarrow S_5 \rightarrow S_6 \rightarrow S_7$
6	$S_1 \rightarrow S_2 \rightarrow S_4 \rightarrow S_6 \rightarrow S_7$
7	$S_1 \rightarrow S_2 \rightarrow S_4 \rightarrow S_3 \rightarrow S_6 \rightarrow S_7$
8	$S_1 \rightarrow S_2 \rightarrow S_4 \rightarrow S_5 \rightarrow S_3 \rightarrow S_6 \rightarrow S_7$
9	$S_1 \rightarrow S_2 \rightarrow S_4 \rightarrow S_5 \rightarrow S_6 \rightarrow S_7$

Source Credit : [2021 : Hidden Markov Model and Cyber Deception for the Prevention of Adversarial Lateral Movement]

Required Reading: AIMA - Chapter #15.1, #15.2, #15.3 (#20.3.3)

Thank You for all your Attention

Note : Some of the slides are adopted from AIMA TB materials