



# Artificial & Computational Intelligence

**DSE CLZG557**

**M2 : Problem Solving Agent using Search**

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# Course Plan



M1 Introduction to AI

M2 Problem Solving Agent using Search

M3 Game Playing, Constraint Satisfaction Problem

M4 Knowledge Representation using Logics

M5 Probabilistic Representation and Reasoning

M6 Reasoning over time, Reinforcement Learning

M7 AI Trends and Applications, Philosophical foundations

## Module 2 : Problem Solving Agent using Search

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A. Uninformed Search

B. Informed Search

C. Heuristic Functions

D. Local Search Algorithms & Optimization Problems

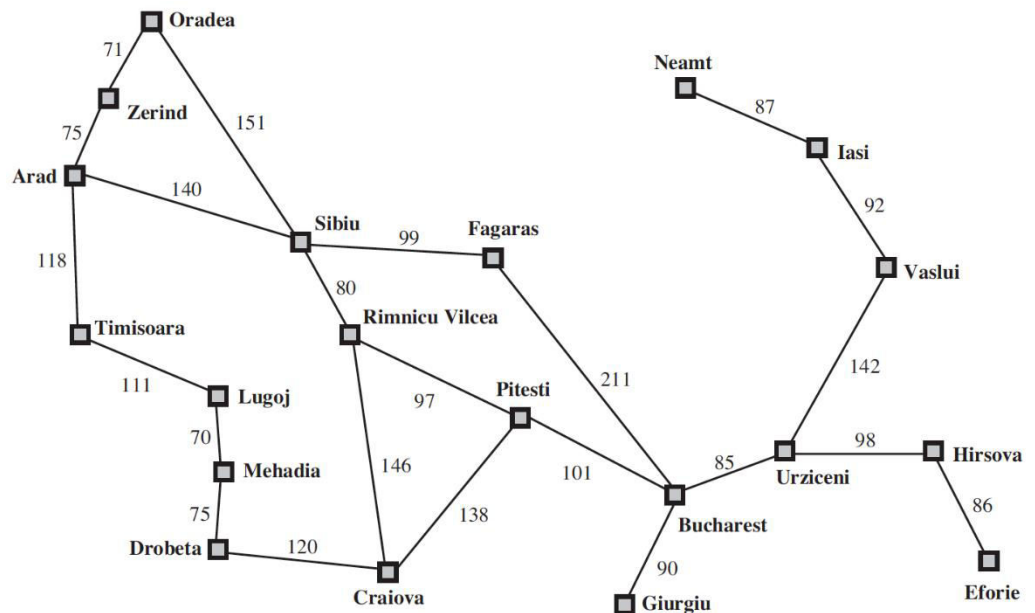
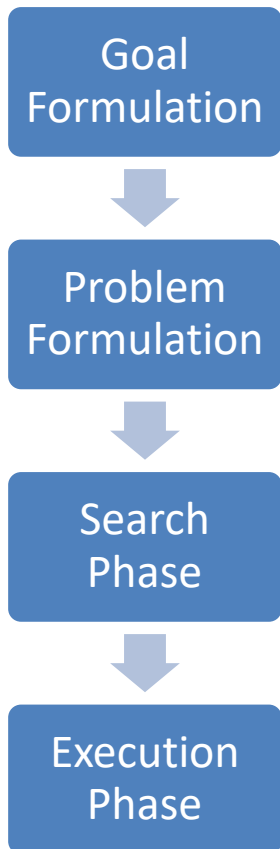
# Problem Formulation

# Problem Solving Agents

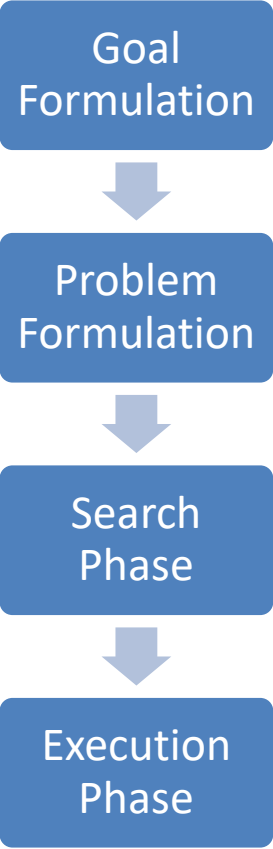
Goal based decision making agents which finds sequence of actions that leads to the desirable stated.

## Phases of Solution Search by PSA

Optimizes the Objective (Local | Global)  
Limits the Actions

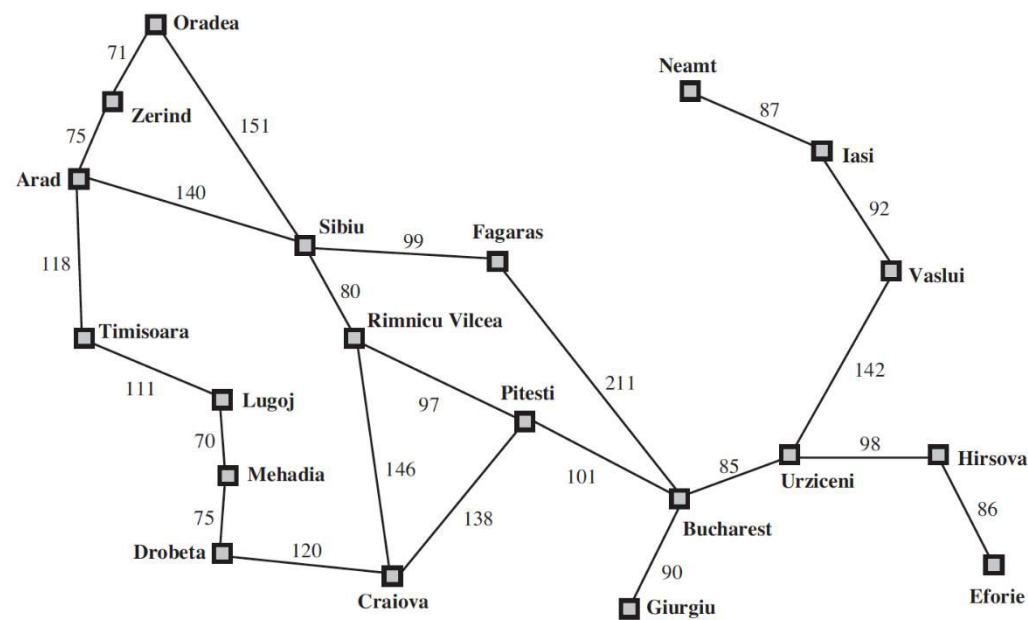


# Problem Solving Agents



## Phases of Solution Search by PSA

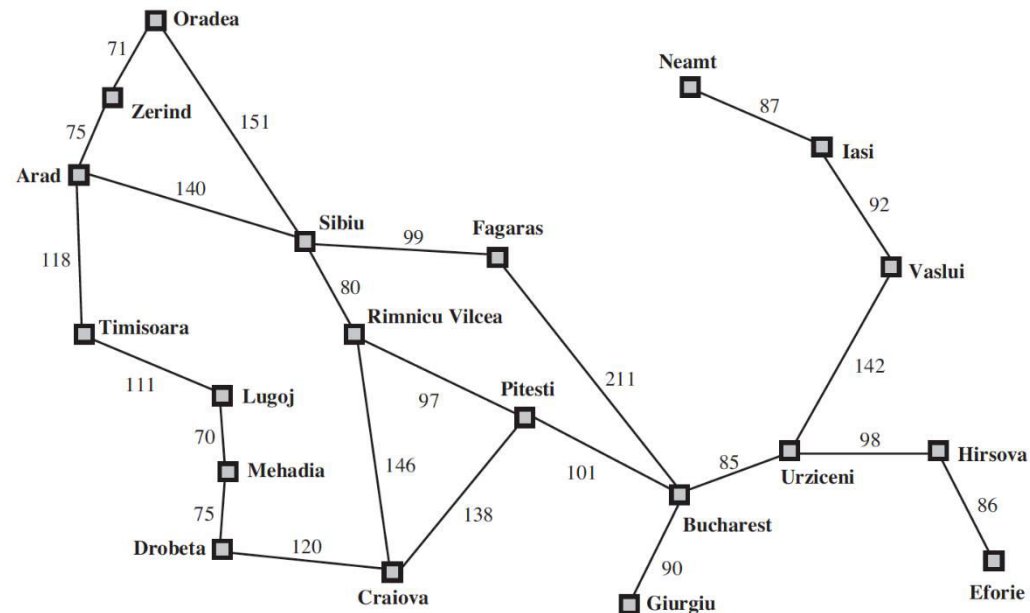
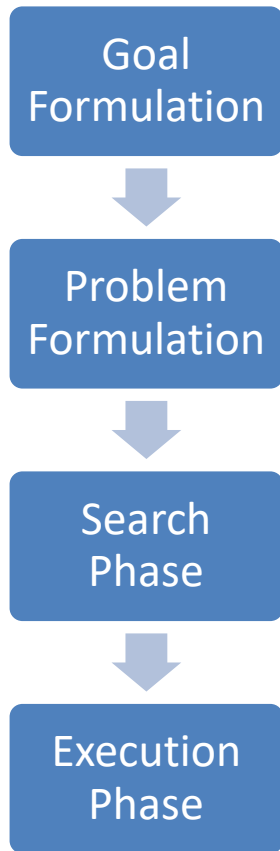
State Space Creations [in the path of Goal]  
Lists the Actions



# Problem Solving Agents

## Phases of Solution Search by PSA

**Assumptions – Environment :**  
**Static**  
**Observable**  
**Discrete**  
**Deterministic**



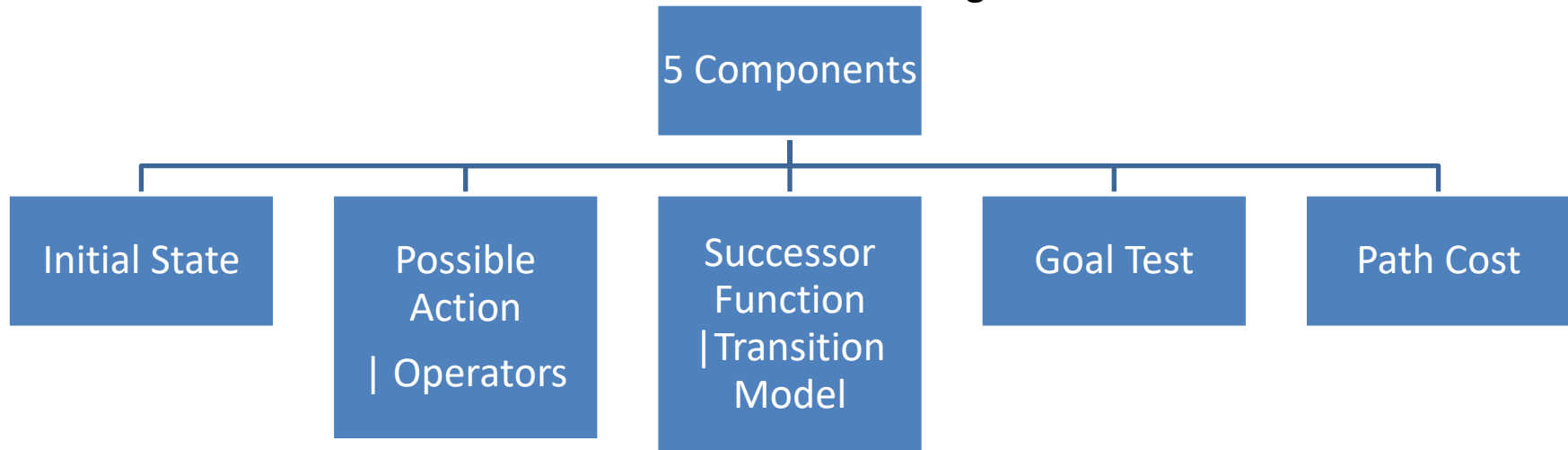




## Problem Solving Agents – Problem Formulation

### Abstraction Representation

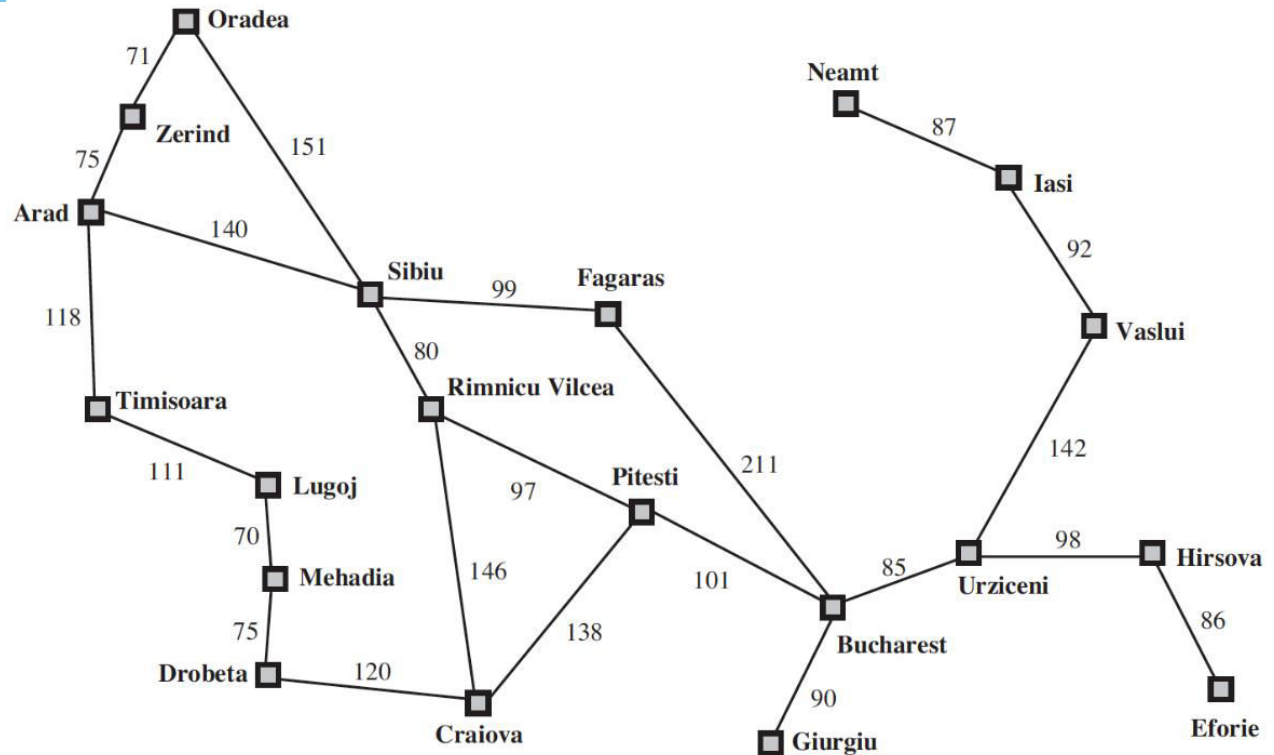
Decide what actions under states to take to achieve a goal



A function that assigns a numeric cost to each path. A path is a series of actions. Each action is given a cost depending on the problem.

**Solution = Path Cost Function + Optimal Solution**

# Problem Solving Agents – Problem Formulation



**Initial State** – E.g.,  $In(Arad)$

**Possible Actions** –  $ACTIONS(s) \rightarrow \{Go(Sibiu), Go(Timisoara), Go(Zerind)\}$

**Transition Model** –  $RESULT(In(Arad), Go(Sibiu)) = In(Sibiu)$

**Goal Test** –  $IsGoal(In(Bucharest)) = Yes$

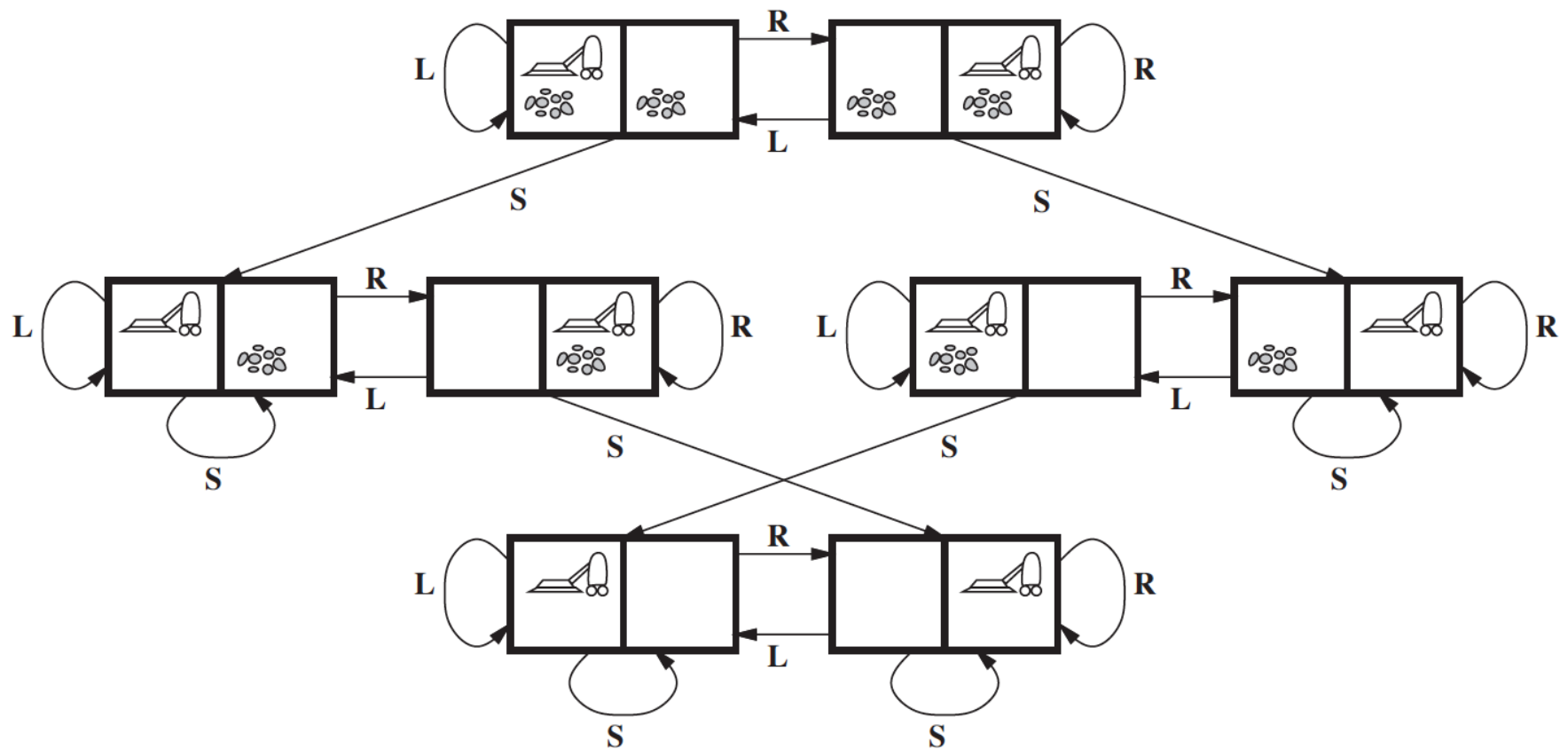
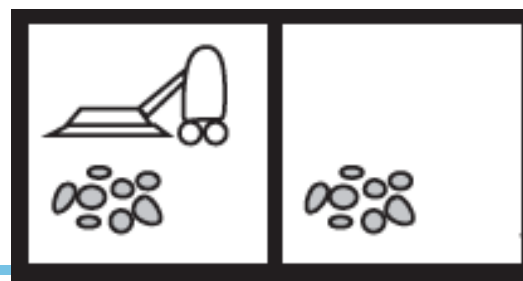
**Path Cost** –  $cost(In(Arad), go(Sibiu)) = 140 \text{ kms}$

# N - Queen

	0	1	2	3	4	5	6	7
0	Q							
1							Q	
2					Q			
3								Q
4		Q						
5				Q				
6						Q		
7			Q					

	0	1	2	3
0			Q	
1	Q			
2				Q
3		Q		

# Vacuum World Problem



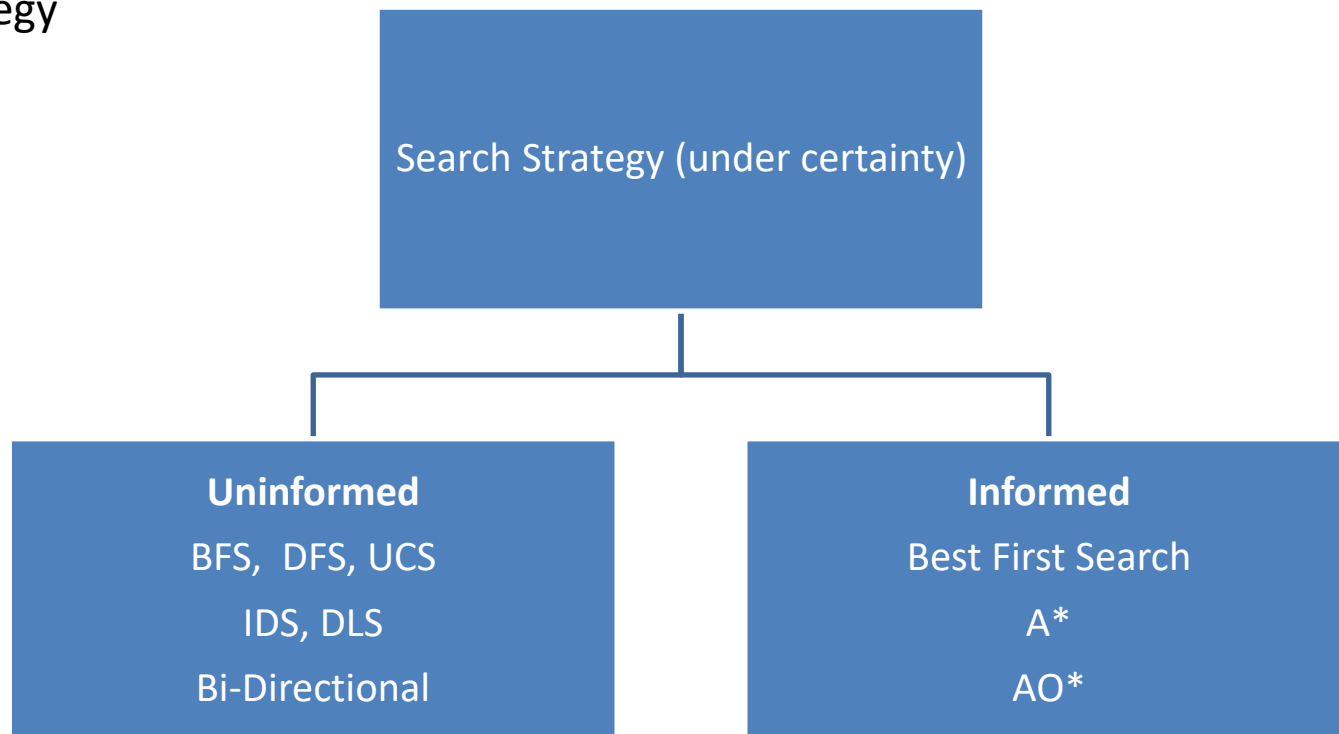
## Example Problem Formulation

	Vacuum World	8 – Queen Problem	Travelling Problem
Initial State	Any	No Queen on board	Based on the problem
Possible Actions	[Move Left, Move Right, Suck]	Add a Queen to any empty square	Take a flight   Train   Shop
Transition Model	[A, ML] = [B , Dirty] [A, ML] = [B, Clean]	[A1, B2] = [FAIL] [A1, B3] = [SAFE]	[A, Go(A->S)] = [S]
Goal Test	Is all room clean? [A, Clean] [B, Clean]	All Queen Safe	Is current = B (destination)
Path Cost	No of steps in path	No of Moves done, backtracking	Cost + Time + Quality

# Searching for Solutions



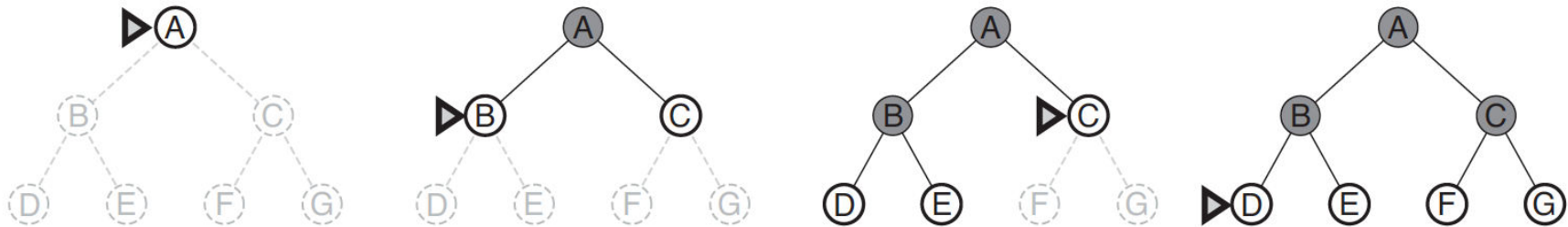
Choosing the current state, testing possible successor function, expanding current state to generate new state is called Traversal. Choice of which state to expand – Search Strategy



# Uninformed Search

## BFS & its Variants

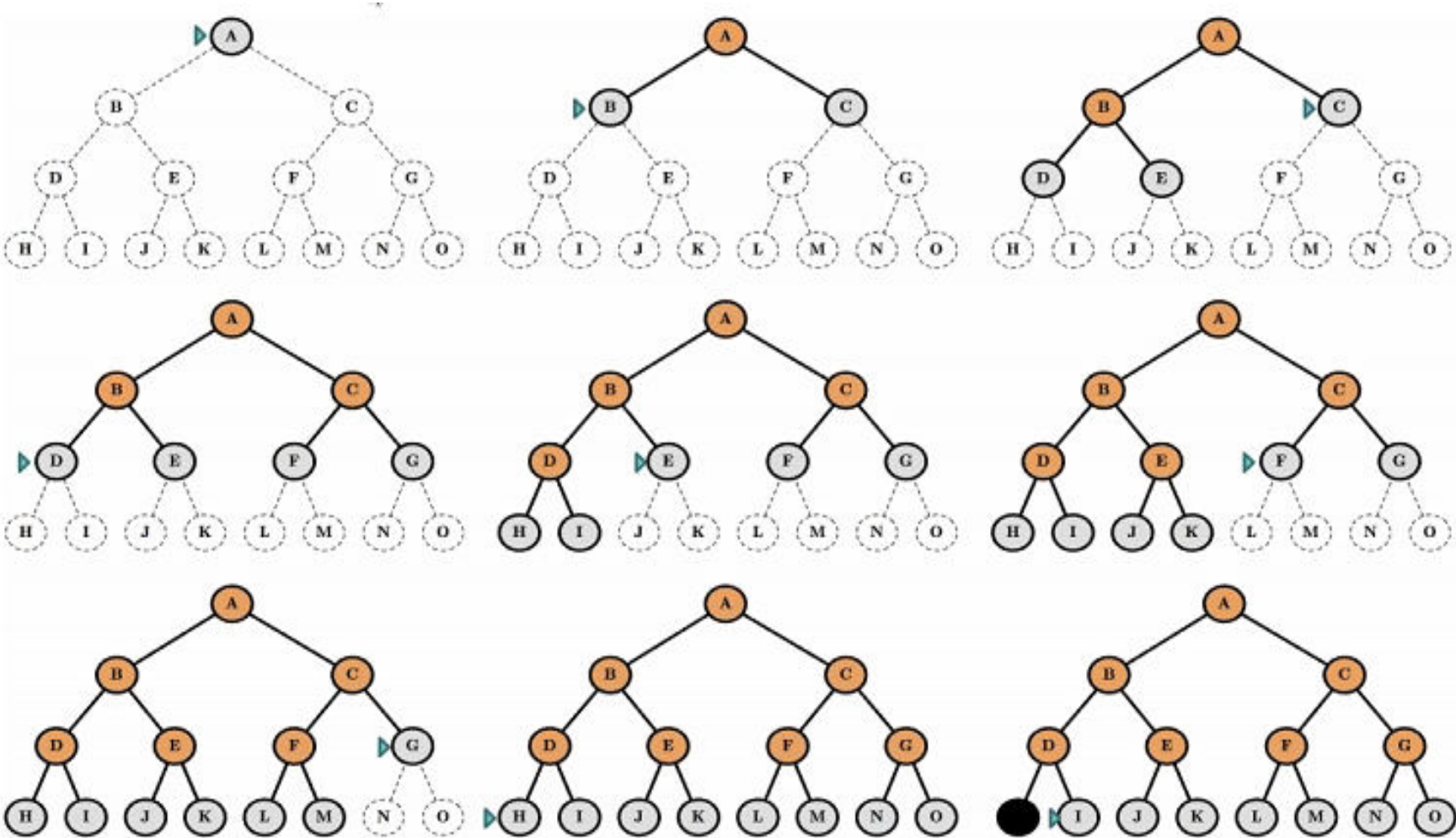
# Breadth First Search (BFS)



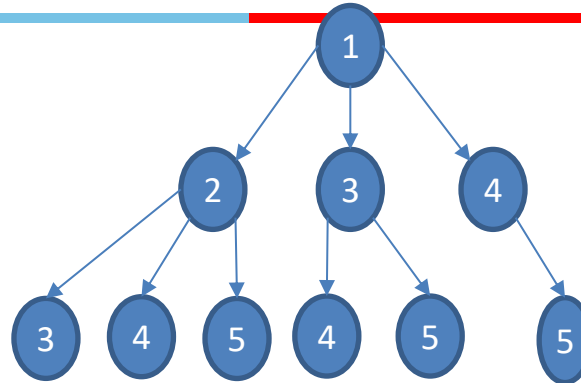
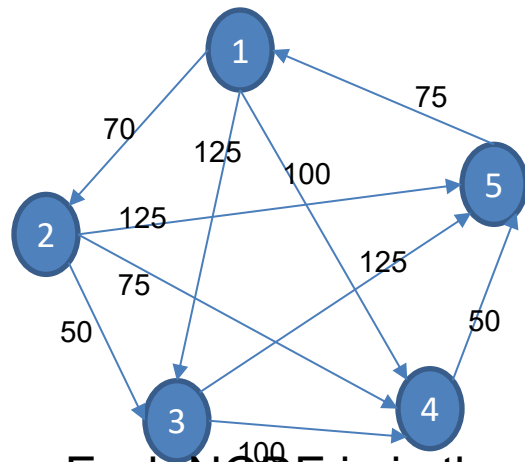
Generate all nodes at a given depth  
before proceeding to deeper nodes



# Breadth First Search (BFS)



# BFS – Uninformed



Each **NODE** in in the search tree denotes an entire **PATH** through the state space graph.

(1)  
 (1 2) (1 3) (1 4)  
 TEST FAILED

(1 3) (1 4) (1 2 3) (1 2 4) (1 2 5)  
 (1 2 3) (1 2 4) (1 2 5) (1 3 4) (1 3 5 ) (1 4 5)  
 TEST PASSED

$C(1-2-5) = 70 + 125 = 195$   
 Expanded : 4  
 Generated : 10  
 Max Queue Length : 6

## Breadth First Search – Evaluation

Depth	Nodes	Time	Memory
2	110	.11 milliseconds	107 kilobytes
4	11,110	11 milliseconds	10.6 megabytes
6	$10^6$	1.1 seconds	1 gigabyte
8	$10^8$	2 minutes	103 gigabytes
10	$10^{10}$	3 hours	10 terabytes
12	$10^{12}$	13 days	1 petabyte
14	$10^{14}$	3.5 years	99 petabytes
16	$10^{16}$	350 years	10 exabytes

Why is Space Complexity a big problem? Imagine a problem with

- branching factor  $b = 10$
- generates 1 million nodes/sec
- Each node requires 1KB

## Breadth First Search – Evaluation

**Complete** – If the shallowest goal node is at a depth  $d$ , BFS will eventually find it by generating all shallower nodes

**Optimal** – Not necessarily. Optimal if path cost is non-decreasing function of depth of node. E.g., all actions have same cost

**Time Complexity** –  $\mathcal{O}(b^d)$   $b$  - branching factor,  $d$  – depth

- Nodes expanded at depth 1 =  $b$
- Nodes expanded at depth 2 =  $b^2$
- Nodes expanded at depth  $d$  =  $b^d$
- Goal test is applied during generation, time complexity would be  $\mathcal{O}(b^{d+1})$

**Space Complexity** –  $\mathcal{O}(b^d)$

- $\mathcal{O}(b^{d-1})$  in explored set
- $\mathcal{O}(b^d)$  in frontier set

# Uniform Cost Search



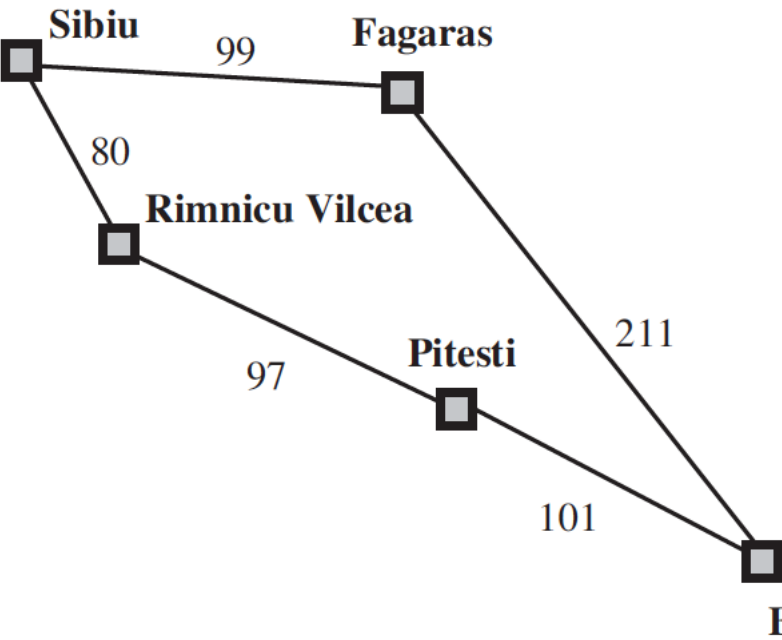
Instead of expanding the shallowest node, Uniform-Cost search expands the node  $n$  with the lowest path cost  $g(n)$

Sorting the Frontier as a priority queue ordered by  $g(n)$

Goal test is applied during expansion

- The goal node if generated may not be on the optimal path
- Find a better path to a node on the Frontier

# Uniform Cost Search



Current State: Sibiu

Frontier: []

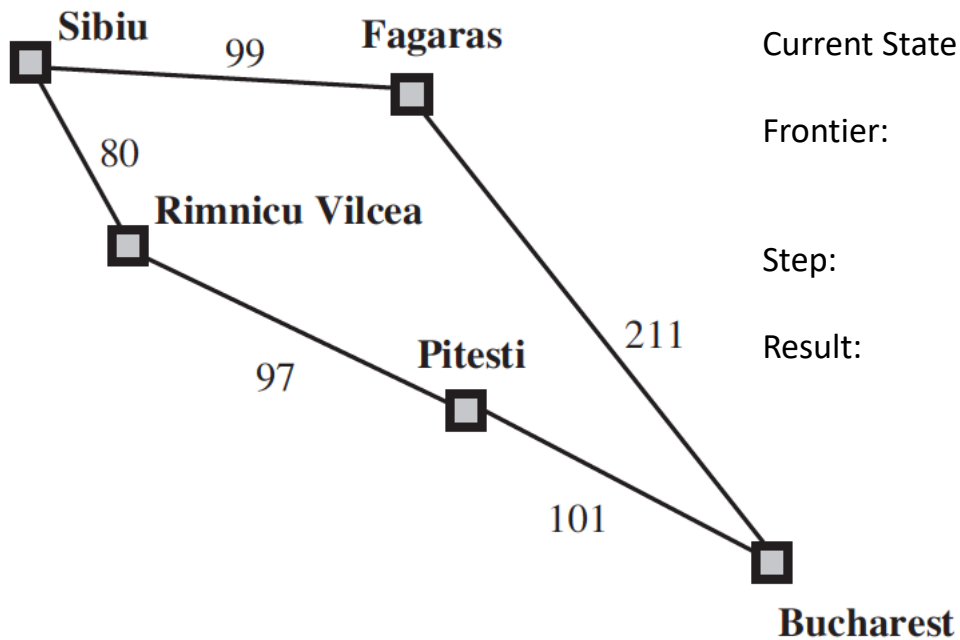
Step: Expand Sibiu

Result: Generates ("Rimnicu Vilcea" 80)  
("Fagaras", 99)  
Add to Frontier

Initial State: Sibiu

Goal State: Bucharest

# Uniform Cost Search



Current State:

Sibiu

Frontier:

[("Rimnicu Vilcea" 80)  
("Fagaras", 99)]

Step:

Expand "Rimnicu Vilcea" (least cost)

Result:

Generates ("Pitesti", 177)  
Add to Frontier

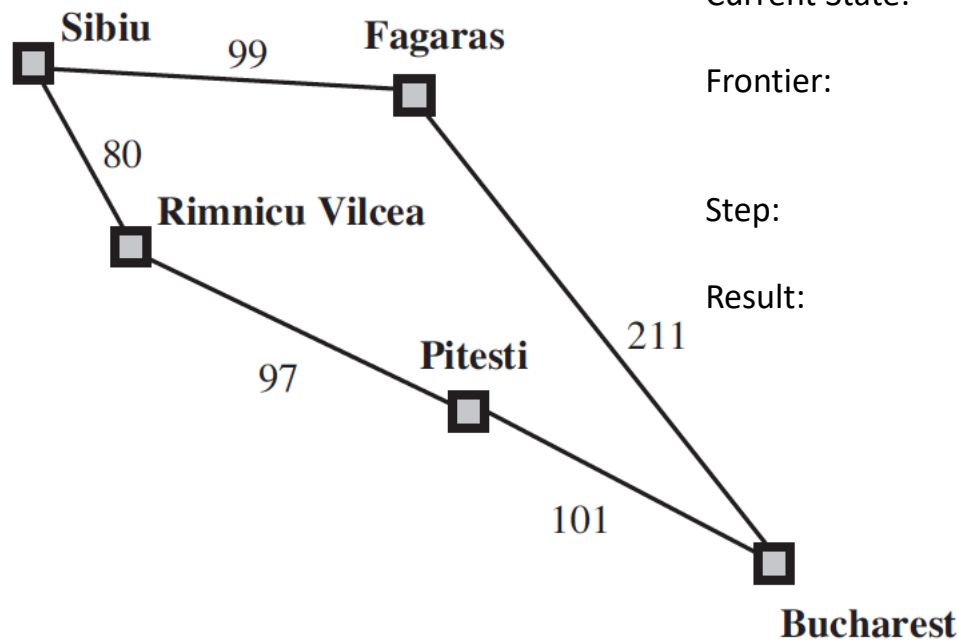
Initial State:

Sibiu

Goal State:

Bucharest

# Uniform Cost Search



Current State:

Rimnicu Vilcea (not a Goal state)

Frontier:

[ ("Fagaras", 99)  
("Pitesti", 177)]

Step:

Expand "Fagaras" (least cost)

Result:

Generates ("Bucharest", 310)  
Add to Frontier  
(It's a Goal State but we won't  
test during generation)

Initial State:

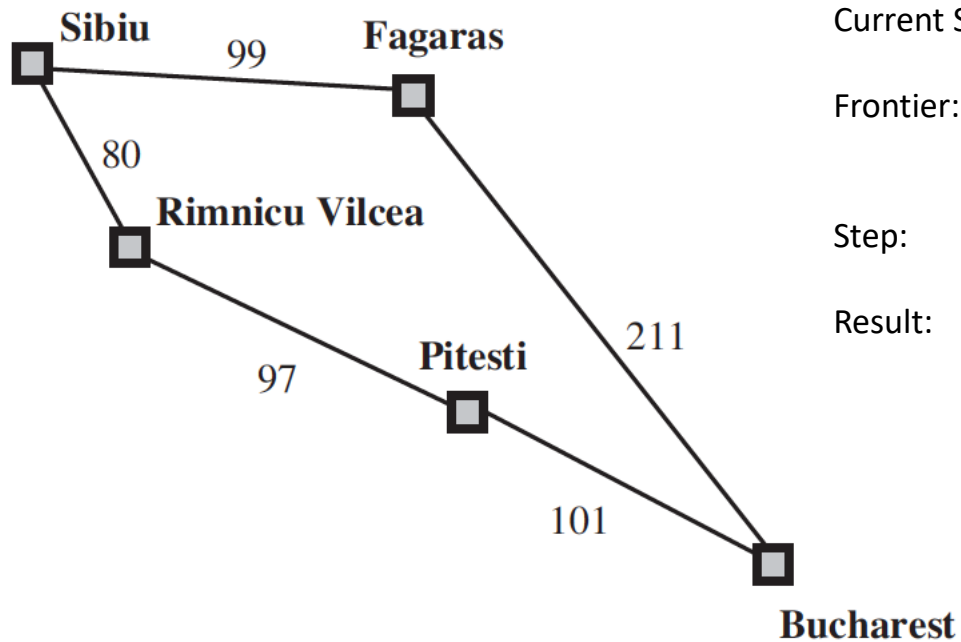
Sibiu

Goal State:

Bucharest



# Uniform Cost Search



Current State:

Fagaras (not a goal state)

Frontier:

[ ("Pitesti", 177)  
("Bucharest", 310)]

Step:

Expand "Pitesti" (least cost)

Result:

Generates ("Bucharest", 278)  
Replace in Frontier  
(It's a Goal State but we won't  
test during generation)

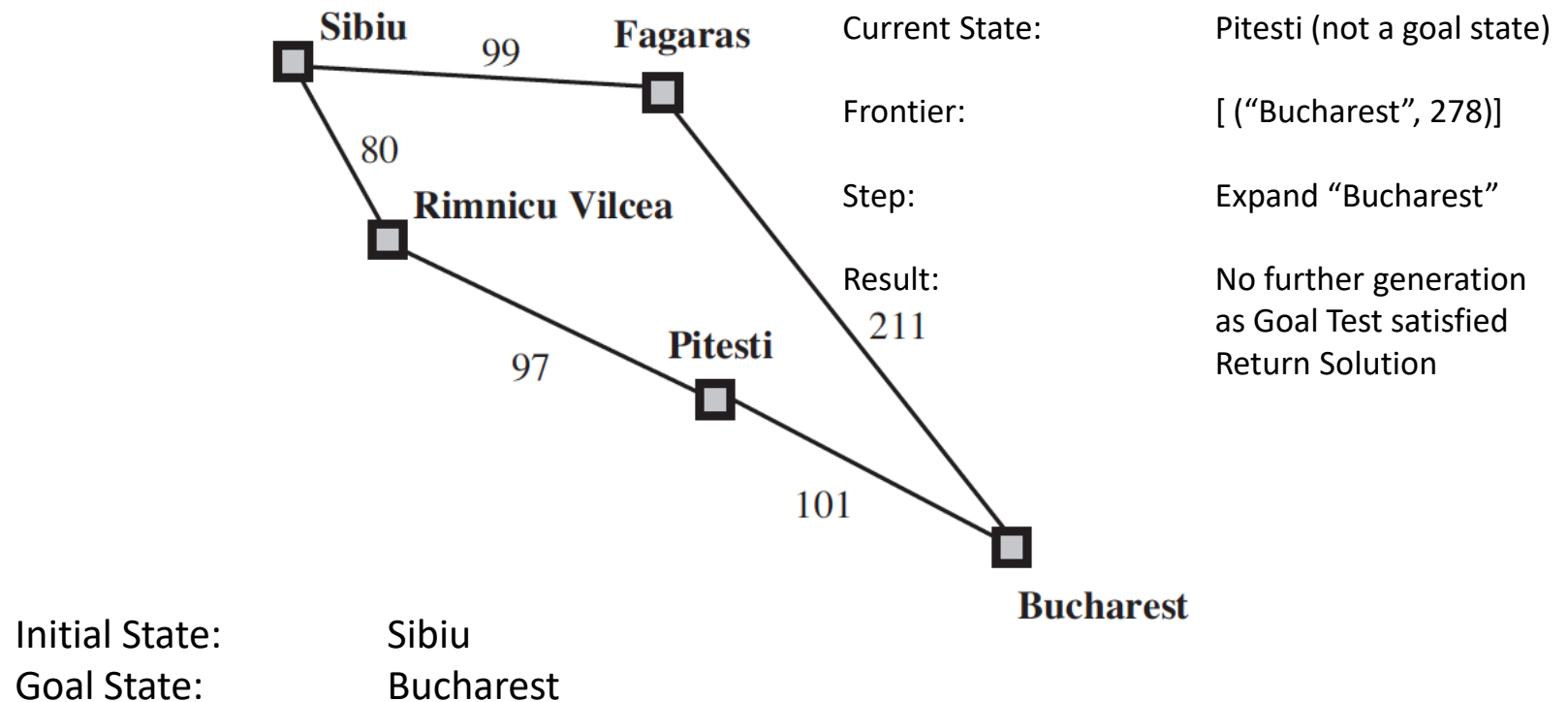
Initial State:

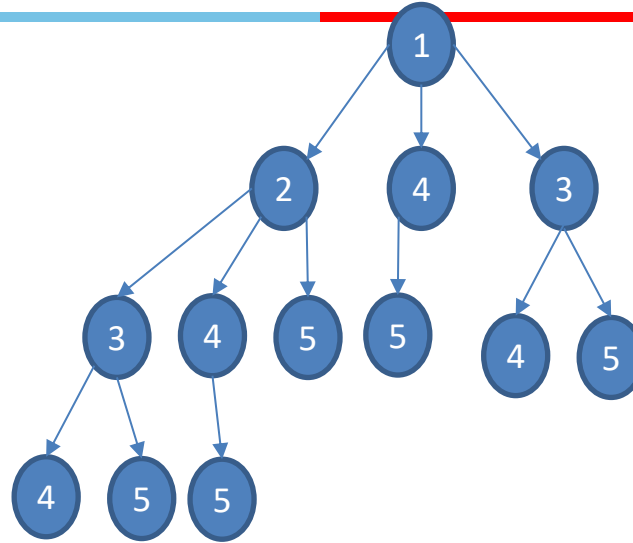
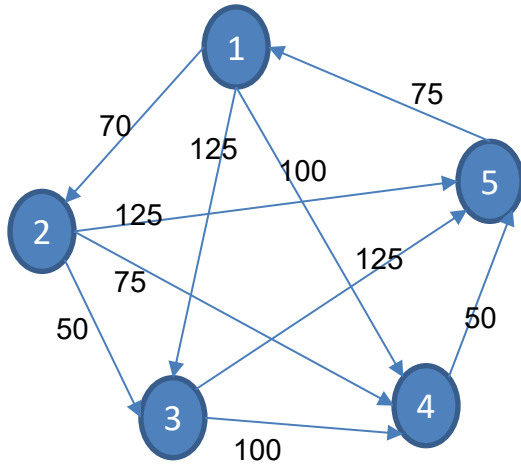
Sibiu

Goal State:

Bucharest

# Uniform Cost Search





(1)

(1 2 : 70) (1 4 : 100) (1 3 : 125)

TEST-F

(1 4 : 100) (1 2 3 : 120) (1 3 : 125) (1 2 4 : 145) (1 2 5 : 195)

TEST-F

(1 2 3 : 120) (1 3 : 125) (1 2 4 : 145) (1 4 5 : 150) (1 2 5 : 195)

TEST-F

(1 3 : 125) (1 2 4 : 145) (1 4 5 : 150) (1 2 3 4 : 170) (1 2 5 : 195) (1 2 3 5 : 245)

TEST-F

(1 2 4 : 145) (1 4 5 : 150) (1 2 3 4 : 170) (1 2 5 : 195) (1 3 4 : 225) (1 2 3 5 : 245) (1 3 5 : 250)

TEST-F

(1 4 5 : 150) (1 2 3 4 : 170) (1 2 4 5 : 195) (1 2 5 : 195) (1 3 4 : 225) (1 2 3 5 : 245) (1 3 5 : 250)

TEST - P

## Uniform Cost Search – Evaluation

- Completeness** – It is complete if the cost of every step  $>$  small +ve constant  $\epsilon$
- It will stuck in infinite loop if there is a path with infinite sequence of zero cost actions
- Optimal** – It is Optimal. Whenever it selects a node, it is an optimal path to that node.
- Time and Space complexity** – Uniform cost search is guided by path costs not depth or branching factor.
- If  $C^*$  is the cost of optimal solution and  $\epsilon$  is the min. action cost
  - Worst case complexity =  $\mathcal{O}(b^{1+\frac{C^*}{\epsilon}})$ ,
  - When all action costs are equal  $\rightarrow \mathcal{O}(b^{d+1})$ , the BFS would perform better
    - As Goal test is applied during expansion, Uniform Cost search would do extra work

# Uninformed Search

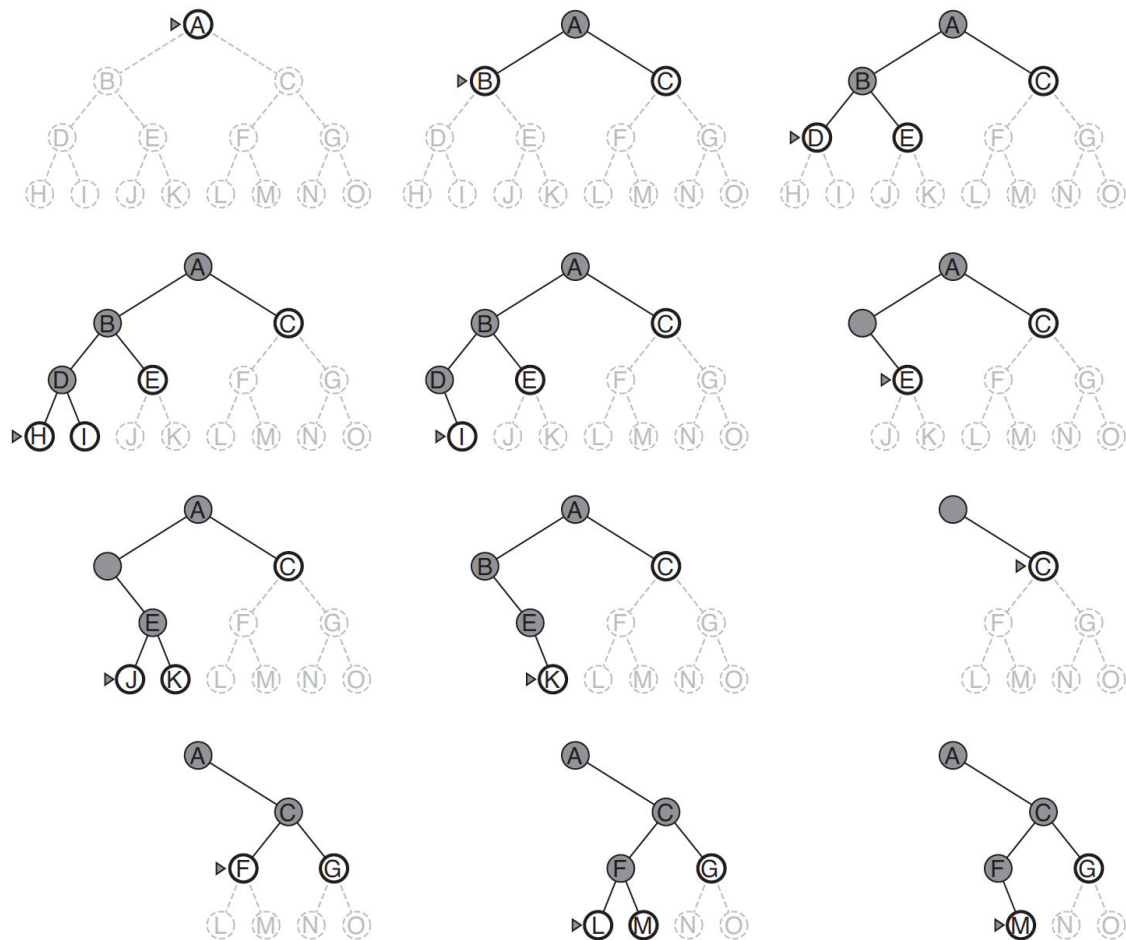
## DFS & its Variants

# Depth First Search (DFS)

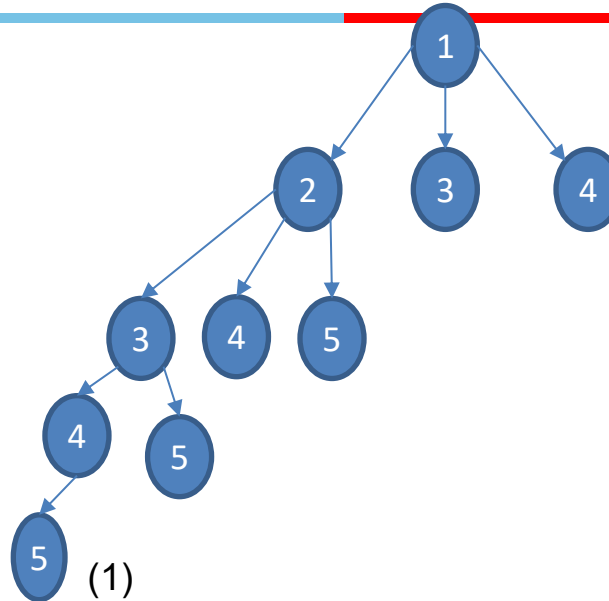
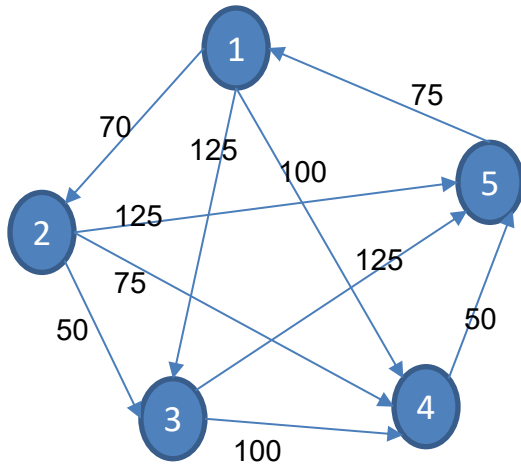
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- Uses LIFO as the Frontier Strategy
- Expands deepest node in current Frontier
- Instance of Graph Search Algorithm

# Depth First Search (DFS)



# DFS – Uninformed



(1)  
 (1 2) (1 3) (1 4)  
 (1 2 3) (1 2 4) (1 2 5) (1 3) (1 4)  
 (1 2 3 4) (1 2 3 5) (1 2 4) (1 2 5) (1 3) (1 4)  
**(1 2 3 4 5)** (1 2 3 5) (1 2 4) (1 2 5) (1 3) (1 4)

$C(1-2-3-4-5) = 70 + 50 + 100 + 50 = 270$

Expanded : 4

Generated : 10

Max Queue Length : 6



# Depth First Search (DFS)

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**Completeness** – Complete in finite state spaces because it will eventually expand every node

**Optimal** – Not Optimal as it would stop when the goal node is reached without evaluating if there is a better path

**Time Complexity** -  $\mathcal{O}(b^m)$  where  $m$  = maximum depth of any node

- Can be much larger than the size of state space
- $m$  can be much larger than  $d$  (shallowest goal)

**Space Complexity** – Needs to store only one path and unexpanded siblings.

- Any node expanded with all its children can be removed from memory
- Requires storage of only  $\mathcal{O}(bm)$ ,  $b$  – branching factor,  $m$  - max depth

# Depth Limited Search (DLS)

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- Running DFS with a predetermined depth limit  $l$
- **Completeness:** No, cannot guarantee a goal if  $l < d$
- **Optimal:** No
- **Time complexity:**  $\mathcal{O}(b^l)$
- **Space Complexity:**  $\mathcal{O}(bm)$

# Application

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## Breadth First Search

- Finding path in a graph (many solutions)
- Finding the Bipartitions in a graph

## Depth First Search

- Find the Connectedness in a graph
- Topological Sorting

# Terminologies Learnt

- Nodes
- States
- Frontier | Fringes
- Search Strategy : LIFO | FIFO | Priority Queue
- Performance Metrics
  - Completeness
  - Optimality
  - Time Complexity
  - Space Complexity
- Algorithm Terminology
  - d Depth of a node
  - b Branching factor
  - n – nodes
  - l – level of a node
  - m – maximum
  - C\* - Optimal Cost
  - E – least Cost
  - N –total node generated

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**Required Reading:** AIMA - Chapter # 3.1, 3.2, 3.3, 3.4(Partial)

Thank You for all your Attention

Note : Some of the slides are adopted from AIMA TB materials