Kalkulus Sesi 13

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Deret Taylor

$f(x) = \frac{1}{2x+1}$ di $x = 1$

Ketika $x = 1$, maka:

$$f(1) = \frac{1}{2 \cdot 1 + 1} = \frac{1}{3}$$

$$f'(x) = -\frac{2}{(2x+1)^2}$$

$$f'(1) = -\frac{2}{(2 \cdot 1 + 1)^2} = -\frac{2}{4} = -\frac{1}{2}$$

$$f''(x) = \frac{4}{(2x+1)^3}$$

$$f''(x) = \frac{4}{(2 \cdot 1 + 1)^3} = \frac{4}{8} = \frac{1}{2}$$

Jadi deret Taylor untuk $f(x) = \frac{1}{2x+1}$ di $x = 1$ adalah:

$$f(x) = \frac{1}{3} - \left(\frac{1}{2}\right)(x-1) + \left(\frac{1}{2}\right)(x-1)^2 + \cdots$$

$f(x) = \cos(2x - 1)$ di $x = 2$

Ketika $x = 2$, maka:

$$f(x) = \cos(2x-1) = sum\left(\frac{(-1)^n (2x-1)^{2n}}{2n}!\right), n = 0 \text{ sampai } \infty$$

$$f(2) = \cos(3)$$

$$f'(x) = -2\sin(2x-1)$$

$$f'(2) = -2\sin(3)$$

$$f''(x) = -4\cos(2x-1)$$

$$f''(2) = -4\cos(3)$$

$$f'''(x) = 8\sin(2x-1)$$

$$f'''(2) = 8\sin(3)$$

$$f''''(x) = 16\cos(2x-1)$$

$$f''''(2) = 16\cos(3)$$

Sehingga deret Taylor dari fungsi $f(x) = \cos(2x-1)$ di $x = 2$ adalah:

$$f(x) \approx f(2) + f'(2)(x-2) + \frac{1}{2}!\, f''(2)(x-2)^2 + \frac{1}{3}!\, f'''(2)(x-2)^3 + \frac{1}{4}!\, f''''(2)(x-2)^4 + \cdots$$

Deret Maclaurin

$f(x) = e^{x^2-1}$.

Rumus umum:

$$f(x) = f(0) + f'(0) \cdot \frac{x}{1!} + f''(0) \cdot \frac{x^2}{2!} + f'''(0) \cdot \frac{x^3}{3!} + \cdots$$

Kita dapat menentukan $f'(0)$, $f''(0)$, dst.

$$f'(x) = 2xe^{x^2-1}$$

$$f''(x) = 2e^{x^2-1} + 4x^{2e^{x^2-1}}$$

$$f'''(x) = 8xe^{(x^2-1)} + 8x^{3e^{x^2-1}}$$

Deret Maclaurin dari fungsi $f(x) = e^{x^2-1}$ adalah

$$\frac{1}{e} + \frac{x^2}{e}$$

$f(x) = \sin^2 x$.

Rumus umum:

$$f(x) = f(0) + f'(0)x + \left(\frac{f''(0)}{2!}\right)x^2 + \left(\frac{f'''(0)}{3!}\right)x^3 + \cdots$$

Kita dapat menentukan $f'(0)$, $f''(0)$, dst.

$$f(x) = \sin^2 x$$

$$f'(x) = 2\sin(x)\cos(x)$$

$$f''(x) = 2cos^{2(x)} - 2sin^{2(x)} = 2(cos(2x) - 1)$$

$$f'''(x) = -8sin(x)cos(x)$$

$$f^{(4)(x)} = -8cos(2x) = -8\left(cos^{2(x)} - sin^{2(x)}\right)$$

Deret Maclaurin dari fungsi $f(x) = \sin^2 x$ adalah

$$f(x) = f(0) + f'(0)x + \left(\frac{f''(0)}{2!}\right)x^2 + \left(\frac{f'''(0)}{3!}\right)x^3 + \cdots$$

$$f(x) = 0 + 0x + \left(\frac{2}{2!}\right)x^2 + \left(\frac{0}{3!}\right)x^3 + \cdots$$

$$f(x) = \frac{x^2}{2}$$